The Choice Channel of Financial Innovation*

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April 5, 2016

Abstract

Financial innovations in recent decades have vastly expanded investors' portfolio choice. We theoretically analyze the effect of greater choice on investors' savings and asset returns in a model in which investors have possibly heterogeneous beliefs about asset payoffs. Under mild assumptions, we establish a *choice channel* by which greater portfolio choice increases investors' (perceived) return from saving, and induces them to save more. The saving rate increases relatively more for wealthier and less risk averse investors. The general equilibrium effects of the choice channel on asset returns depends on the type of financial innovation. We show that *portfolio customization*, which we capture with improved ability to trade risky assets other than the market portfolio, reduces the expected return on every asset. This result is consistent with the decline in the risk-free interest rate since the early 1980s, and is in contrast with the "precautionary savings" literature that would make the opposite prediction. In contrast, *market participation* (improved ability to trade the market portfolio) reduces the risk premia but typically increases the risk-free rate. We also analyze *securitization*, which we capture with a relaxation of constraints to issue risk-free debt in an environment in which some investors have a high demand for safe assets.

JEL Classification: E21, E43, E44, G11, G12

Keywords: financial innovation, choice channel, customization, participation, securitization, risk premium, interest rate, secular stagnation, belief disagreements, speculation

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[§]We would like to thank Marios Angeletos, Ricardo Caballero, Chen Lian, Andrei Shleifer, and the conference participants at the 2014 Brazilian Society of Econometrics Meetings and the 2015 Lubramacro Workshop for helpful comments. Simsek acknowledges support from the National Science Foundation (NSF) under Grant Number SES-1455319. Any opinions, findings, conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the NSF.

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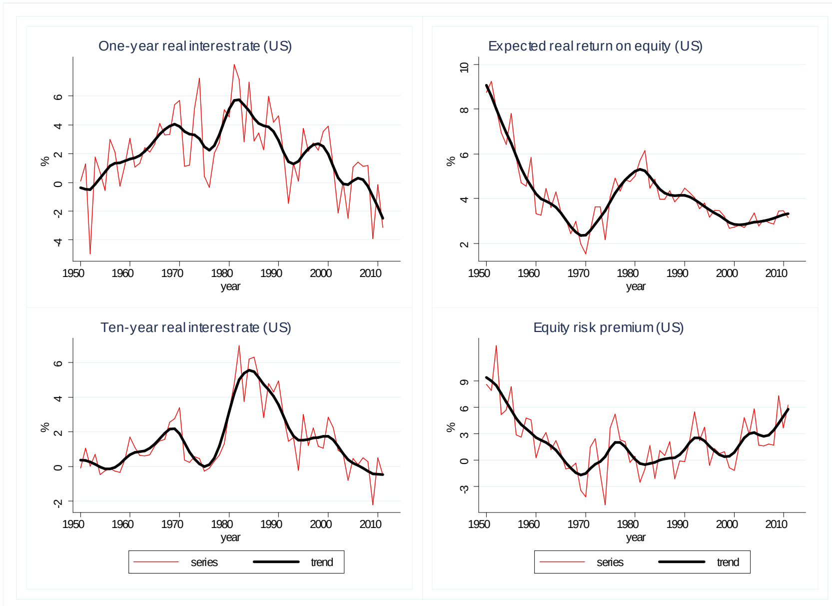

Figure 1: The plots are based on the authors' calculations using the methodology described in Blanchard (1993) and annual returns data from Robert Shiller (available at http://www.econ.yale.edu/ shiller/data.htm). The expected return on equity is calculated by using the dividend yield and the (model based) expected dividend growth.

1 Introduction

A key macroeconomic fact of recent decades is the decline in returns of various asset classes. The panels on the left side of Figure 1 illustrate that the short and long term risk-free real interest rates in the US were on an increasing trend before the 1980s but have been declining since the early 1980s. The same trends also apply to the interest rates in most developed economies. The panels on the right side of Figure 1 illustrate the returns on risky assets and display similar trends with some differences. The (model based) expected return on US stocks has been declining for most of the post-war period, except for an upwards swing in the 1970s. The equity premium—the difference between the expected return on equity and the risk-free rate—also declined in the earlier half of the period, but it appears to be sideways or slightly increasing in recent decades.[1]

Why are asset returns declining? This question is important not only to understand the evolution of wealth inequality, but it is also central for macroeconomic policy. Low interest rates can induce or exacerbate liquidity traps in which monetary policy is constrained by the zero lower bound (Krugman (1998), and Eggertsson and Woodford (2003)). Recent

[1] These trends in the expected risk premium have also been documented in Blanchard (1993), Jagannathan, McGrattan, and Scherbina (2001), Pástor and Stambaugh (2001), and Fama and French (2002). King and Low (2014) also document the declining trend in real interest rates.

research has emphasized various factors that might have contributed to low returns. An aging population or rising income inequality in developed economies might have increased the demand for savings, thereby exerting downward pressure on returns (see, for instance, Summers (2014), Eggertsson and Mehrotra (2014)). High demand for assets—especially safe assets—by fast-growing emerging markets might have also contributed to this pattern (see, for instance, Bernanke (2005), Caballero (2006), and Caballero, Farhi, and Gourinchas (2008)).[2] In this paper, we supplement these explanations for high savings and low returns with a new rationale: financial innovation that expands investors' portfolio choice. Our analysis can help to explain, among other things, why interest rates have been declining since the 1980s but not in earlier decades.

Our starting point is that financial innovation in the post-war years has vastly increased the trading opportunities in financial markets. The changes have been especially dramatic since the early 1980s. In the mid-1970s, the round trip cost of buying and selling a typical stock was about 5% of the stock price (Turley (2012)), whereas it declined to a few cents in recent years. New financial assets, such as futures, options, and other derivatives, enabled trades that were either impossible or too costly in previous years. While these changes were driven by multiple factors, the information technology revolution—which accelerated in the early 1980s—arguably played an important role.

These developments have in turn increased households' portfolio choice. In the 1950s, a typical household in the US arguably did not have much choice in constructing her savings portfolio. She could hold bank deposits, and perhaps save in her own house (or durable assets), but she did not have access to many other financial assets. These days, a comparable household can also hold stocks and other risky assets. Figure 2 shows that stock market participation has indeed increased from about 10% of households in the early 1950s to more than 50% by the end of the 1990s. More importantly, households in recent years can also hold highly customized portfolios. They can choose from a plethora of mutual funds, hedge funds, retirement funds, and ETFs. They can also construct their own portfolios by trading individual stocks, bonds, or more exotic derivatives. Figure 3 shows that mutual funds and exchange traded derivatives, both of which facilitate portfolio customization, have been growing rapidly since the early 1980s (until the recent financial crisis).

Motivated by these observations, we theoretically investigate how financial innovation that increases portfolio choice affects investors' savings and asset prices. In our model, investors with standard Epstein-Zin preferences hold assets to transfer wealth to a future period. Investors optimally choose savings portfolios that consist of the risk-free asset and

[2] Other rationales include increased uncertainty (see Caballero and Farhi (2014)), a slowdown in productivity, or a reduction in the relative price of investment goods (see Summers (2014)).

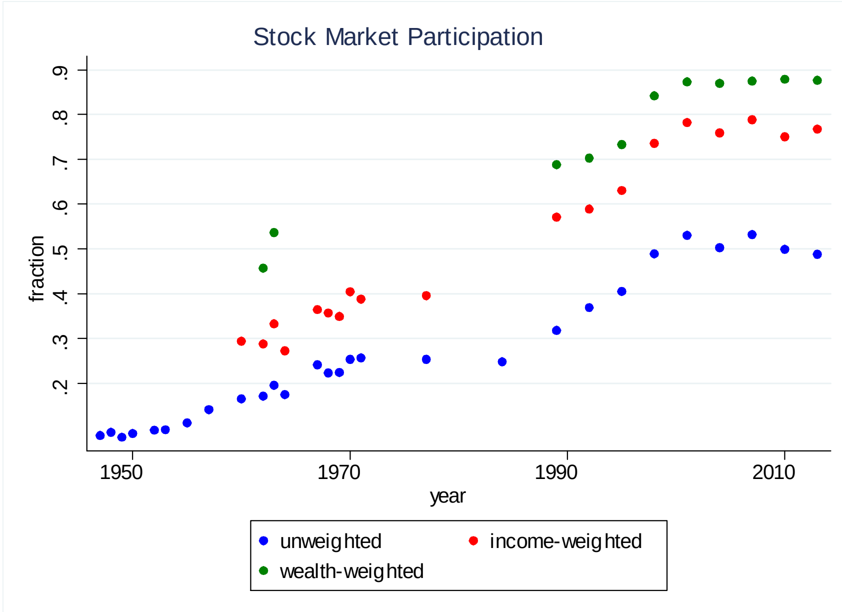

Figure 2: The figure shows the fraction of households in the US that invest in stocks over the period 1947-2013. The plots are based on the authors' calculations using data from the Michigan Survey of Consumer Finances (1947-1977), the PSID (1984), and the Survey of Consumer Finances (1989-2013).

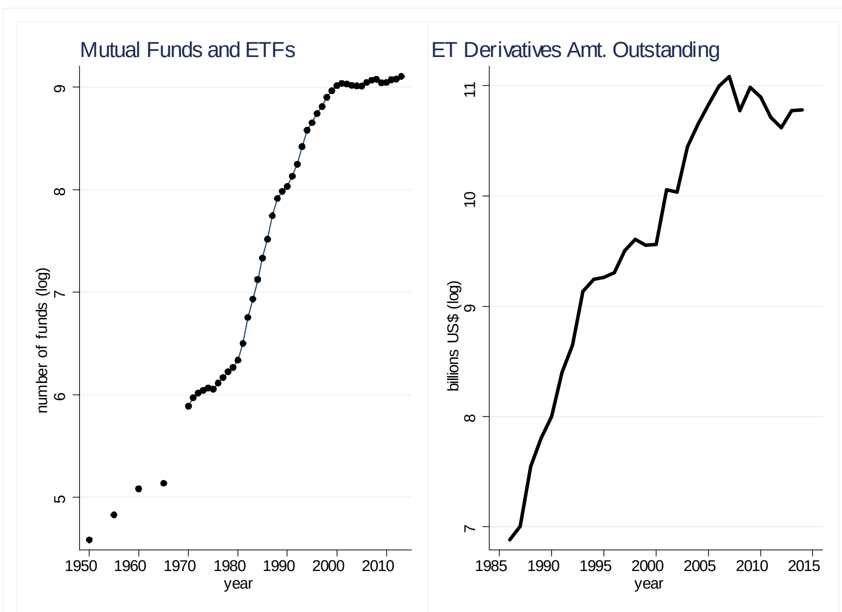

Figure 3: The left panel illustrates the changes in the number of mutual funds and the exchange traded funds in the US (source: Investment Company Institute). The right panel illustrates the changes in outstanding exchange traded derivatives (source: the Bank for International Settlements). Amounts are in constant year 2000 US dollars.

various risky assets. Each investor has access to the risk-free asset, but investors have limited and (possibly) heterogeneous access to risky assets. We capture financial innovation as an improvement in investors' access to risky assets.

Our key assumption is that investors have heterogeneous beliefs about the payoffs of risky financial assets, which provides one rationale for portfolio customization. In fact, in our setup, as well as in many related models, investors with homogeneous beliefs (and without background risks) would only need one risky asset—namely, the market portfolio—to construct their optimal portfolios. Hence, the proliferation of investment funds with heterogeneous strategies is prima facie consistent with our heterogeneous beliefs assumption. The assumption can also be viewed as capturing some other sources of heterogeneity in investors' asset valuations, such as institutional restrictions to hold certain types of assets.

In this setup, we first characterize an investor's savings in partial equilibrium (that is, taking the asset prices as given). Our main result shows that, under relatively weak assumptions, greater choice induces the investor to save more. This result, which we refer to as *the choice channel* of financial innovation, has a simple intuition. Greater portfolio choice increases the investor's (perceived) certainty-equivalent return on her savings portfolio. This creates substitution and income effects that are similar to those created by an increase in the risk-free interest rate. When the substitution effect dominates, which we believe is the empirically relevant case for the majority of investors (see Section 3), then the investor increases her savings. With greater choice in financial markets, saving becomes more attractive, and investors do more of it.

Although the choice channel sounds intuitive, it counters a strand of the "precautionary savings" literature that makes the opposite prediction. This view posits that uninsured background risks induce agents to save for precautionary reasons. The implication is that financial innovation that improves the sharing of background risks should reduce savings, and increase the risk-free interest rate in equilibrium (see Section 1.1 for references and further discussion). Our analysis reveals that the choice channel dominates the precautionary savings channel as long as investors do not face significant background risks, or do not consider these risks when they make portfolio decisions. As we discuss further in Section 1.1, empirical studies often find that investors hold financial portfolios that are quite different from what would be required to hedge background risks. Motivated by these studies, as well as the declining trend in the risk-free interest rate, we shut down the precautionary channel in much of our analysis.

We establish the comparative statics of the choice channel and obtain various additional testable predictions. We find that the choice channel is stronger for wealthier investors. The main reason is that the investors might face a binding borrowing constraint (which we take as exogenous). If the constraint binds, then the saving is determined by the constraint and does not react to choice.[3] The wealthier investors are less likely to face a binding borrowing constraint, so they are more likely to be subject to the choice channel. In addition, wealthier investors start from a greater base level of assets, and thus, their desired asset levels (and savings) react more to an increase in asset returns. We also find that the choice channel is stronger for more risk tolerant investors, because the expansion of choice concerns risky assets.

The result that the choice channel is more relevant for wealthy investors suggests the channel could also affect asset prices in general equilibrium. We investigate the asset pricing implications using a canonical case of our model in which the available financial assets consist of a market portfolio of all cash flows, and several other risky assets in zero net supply. For analytical tractability, we assume a log-normal approximation for portfolio returns (as in Campbell and Viceira (2002)). The choice channel, which increases investors' savings in partial equilibrium, exerts an upwards pressure on asset prices in general equilibrium. However, financial innovation might also generate relative price effects that interfere with the choice channel. The net effect depends on the type of innovation, which we explore in empirically relevant settings.

Our main general equilibrium result concerns portfolio customization, which we capture with improved access to an arbitrary subset of the risky assets other than the market portfolio. Under mild symmetry assumptions on investors' beliefs, we show that greater customization reduces the risk-free rate while leaving the risk premia unchanged. In particular, portfolio customization reduces the expected return on each (risk-free or risky) asset. This result suggests that financial instruments that facilitate portfolio customization, which have become widespread starting in the early 1980s (see Figure 3), can be a contributing factor to the secular decline in the risk-free interest rate as well as the expected return on equity since the early 1980s (see Figure 1).

To understand the intuition, imagine financial assets as a forest that contains several trees. The trees could be a metaphor for individual stocks, industries, or mutual funds with different strategies or styles. Suppose each investor is optimistic for certain trees' yields relative to the average investor. Customization enables investors to trade individual trees as opposed to buying or selling claims on the forest. As a response, investors expand their investments in the trees they like the most, while reducing their positions in the trees they like less. Moreover, for every (relatively) optimistic investor that buys a particular tree, there are (relatively) pessimistic investors that sell that tree. Consequently, investors

[3] In fact, if financial innovation that expands choice happens alongside with other innovations that relax the borrowing constraint, then the saving by constrained investors might actually decline.

collectively like the forest more, in view of the choice channel, but the relative appeal of individual trees remains unchanged. We show that this logic is general, and implies that greater customization increases the valuation (and reduces the expected return) of each tree in tandem.

We also analyze the pricing implications of market participation, which provides a useful contrast with customization. We capture participation as an improvement in investors' access to the market portfolio. This tends to increase asset prices in view of the choice channel, but it also increases the demand for risky assets relative to the safe asset. We find that these relative demand effects are strong, whereas the choice channel is relatively weak in this context. In particular, for empirically relevant parameters, greater participation increases the risk-free rate—in contrast to customization. We also find that participation reduces the risk premium (and the expected return) on the market portfolio. These results suggest that improvements in market participation between the 1950s and the early 1980s (see Figure 2) can be a contributing factor to the trends in the risk-free rate and the equity premium over this period (see Figure 1).

The choice channel from increased participation is relatively weak because of general equilibrium crowd-out effects. Greater participation reduces the risk premium, which in turn reduces the portfolio return of investors that were already participating. This mitigates the increase in aggregate savings and asset prices. These crowd-out effects are absent for increased customization. This is because offsetting positions on a tree do not affect the (relative) price of the tree, and thus, they do not preclude other investors from taking similar positions on other trees (or even the same tree). Thus, while the choice channel can also apply when investors have homogeneous beliefs—as illustrated by our analysis of market participation—its general equilibrium effects are particularly powerful when investors have heterogeneous beliefs, and when they gain access to assets that enable them to express their different beliefs.

We also analyze securitization, which is another important financial innovation in recent decades that converts traditionally illiquid assets (such as mortgages) into marketable securities. Nontraditional forms of securitization (such as CDOs) additionally split the securities into safer and riskier tranches. This type of securitization arguably facilitates portfolio choice by meeting investors' demand for safe assets. In fact, a recent literature has argued that these securities were introduced precisely to meet the growing demand for safety from emerging markets and other sources (see, for instance, Gennaioli, Shleifer, and Vishny (2012)). Figure 4 shows that the growth of nontraditional securitization in the early 2000s has indeed coincided with a dramatic increase in safe asset holdings by emerging market central banks.

We capture the trends in Figure 4 by expanding our model in two dimensions. First, we

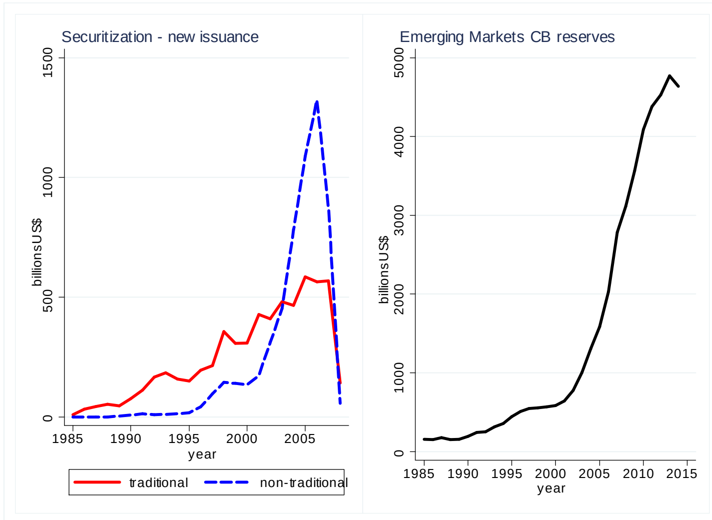

Figure 4: The left panel is from Chernenko, Hanson, and Sunderam (2014), based on data from the Securities Data Company. Traditional securitizations include commercial mortgage backed securities (MBS), prime residential MBS, and asset backed securities. Nontraditional securitizations include nonprime residential MBS and collateral debt obligations. The right panel illustrates the evolution of the reserve assets held by the central banks in emerging markets (source: the World Bank). Amounts are in constant year 2000 US dollars.

introduce "emerging market" investors that have a preference for safe assets, in addition to having a relatively high demand for assets. An increase in the relative mass of these investors decreases the risk-free rate, consistent with the conventional wisdom, while also increasing the risk premium. Second, we also introduce securitization as a relaxation of (other) investors' constraints to issue safe debt so as to make leveraged investments in risky assets. We show that greater securitization mitigates the pricing effects of high savings by emerging markets. In particular, the interest rate declines less compared to the counterfactual without securitization. By keeping the interest rate relatively high, securitization also increases the emerging market investors' (already high) savings. Intuitively, if the US did not provide additional safe assets, the interest rates would decline considerably, and the emerging market investors would be reluctant to save as much as they did. Hence, in this context, the choice channel manifests itself as an increase in savings by emerging markets (or more broadly, investors that demand safe assets). In fact, we find that securitization can reduce other investors' savings in equilibrium, and exacerbate their current account deficits. These results suggest that the collapse of securitization in the aftermath of the recent financial crisis (see Figure 4) can be a contributing factor to the sharper decline in the riskless rate in recent years (see Figure 1), as well as the recent decline in the US current account deficit.

The rest of the paper is organized as follows. Section 1.1 discusses the related literature.

In Section 2 we present an example that illustrates the choice channel and motivates the rest of our analysis. Section 3 introduces the basic environment and establishes our main result, the choice channel, along with its comparative statics. Section 4 extends the basic framework into a general equilibrium model, and its subsections characterize the pricing implications of specific types of financial innovation. Section 4.1 analyzes increased participation. Section 4.2 presents our main general equilibrium result on increased customization. Section 4.3 focuses on increased securitization. We summarize our findings in a concluding section.

1.1 Related literature

Our paper spans various segments of the literature. We contribute to a sizeable literature that investigates financial innovation and security design.[4] We focus on the asset pricing implications of financial innovation in an environment with belief heterogeneity, similar to Fostel and Geanakoplos (2012) and Simsek (2013a) (see Detemple and Selden (1991) for an earlier example). These papers typically take the risk-free rate as given, and characterize how certain types of financial innovations affect the relative price of a single risky asset. In contrast, we focus on the risk-free rate while also characterizing the relative asset prices. We also analyze a broader set of financial innovations, with a focus on portfolio customization, whereas the recent papers in this literature consider securitization and credit derivatives.[5]

Our paper is also related to a large "precautionary savings" literature, which emphasizes that incomplete markets tend to increase agents' idiosyncratic consumption risks and reduce the risk-free interest rate. It is useful to divide this literature into two strands that differ in terms of the sources of risks as well as the implications for aggregate investment.

The first strand focuses on consumption risks driven by idiosyncratic income or background risks (see, for instance, Leland (1968), Dreze and Modigliani (1972), Bewley (1977), Skinner (1988), Kimball (1990), Weil (1992), Huggett (1993), Aiyagari (1994)). As we described earlier, this literature suggests that financial innovation that facilitates the sharing of background risks should reduce savings and increase the interest rate (see Elul (1997) for a formalization and critical evaluation, and Carvajal, Rostek, and Weretka (2012) for a recent application). While we think background risks are clearly important, especially for understanding the financial decisions of economic agents that are net borrowers, we question

[4] A non-exclusive list of contributions includes Allen and Gale (1988, 1991, 1994a), Detemple and Selden (1991), Elul (1995, 1997), Pesendorfer (1995), Calvet, Gonzalez-Eiras, and Sodini (2004), and more recently, Carvajal, Rostek, and Weretka (2012), Fostel and Geanakoplos (2012), and Simsek (2013b). See Duffie and Rahi (1995) and Tufano (2003) for reviews of the older literature.

[5] More broadly, our paper is also related to a large literature on asset pricing with heterogeneous beliefs (e.g., Harrison and Kreps (1978), Scheinkman and Xiong (2003), Geanakoplos (2003), Geanakoplos (2010), He and Xiong (2012), Hong and Sraer (2012)).

their empirical relevance for the types of financial innovations we analyze. At the macro level, the interest rate has been declining since the early 1980s in an environment with rapid financial innovation, which counters the precautionary savings view. At the micro level, most investors (that are net savers) do not seem to be concerned by background risks when constructing their savings portfolios. They tend to overinvest in domestic stocks (French and Poterba (1991)), as well as own company or professionally close stocks (e.g., Benartzi (2001), Poterba (2003), Døskeland and Hvide (2011)). They also seem to trade and adjust their portfolios much more frequently than what could be justified by hedging or liquidity needs (see Hong and Stein (2007)).

A second strand of the precautionary savings literature examines the implications of idiosyncratic investment or rate-of-return risks (e.g. Sandmo (1970), Devereux and Smith (1994), Obstfeld (1994), Krebs (2003), Angeletos and Calvet (2006), Angeletos (2007)). These risks are conceptually different than background risks because they are endogenously taken by economic agents. Building upon this observation, this literature emphasizes that financial innovation that facilitates the sharing of rate-of-return risks can actually increase aggregate investment. The logic of this result is similar to our choice channel, and relies on a relatively large elasticity of intertemporal substitution. That said, our paper is also different from this strand of the literature for two main reasons. First, we analyze the savings decisions of households in financial markets, whereas this literature focuses on firms' (or entrepreneurs') physical investment decisions. In fact, recent contributions in this literature emphasize that financial innovation can raise aggregate investment while still increasing the risk-free interest rate—as in the case with background risks (see Angeletos and Calvet (2006) and Angeletos (2007)). In contrast, our result on customization delineates conditions under which financial innovation reduces the risk-free interest rate. Second, while this literature argues that investment increases in response to better risk sharing, we emphasize that savings increases because households have greater choice—which they might or might not use for risk sharing. In our model, households with heterogeneous beliefs increase their savings because they speculate that their customized portfolios will yield high (risk-adjusted) returns—a phenomenon that could be viewed as the opposite of risk sharing (see Simsek (2013b)).

Our paper is also related to the recent work by Guzman and Stiglitz (GS, 2015), who investigate investors' consumption and savings behavior in an environment with belief disagreements. They emphasize that belief disagreements increase investors' perceived wealth, which they call pseudo-wealth, and contribute to macroeconomic fluctuations. Our paper has several differences. First, we establish comparative statics with respect to financial innovation, as opposed to changes in belief disagreements (and we also focus on the level of asset prices as opposed to consumption volatility). Second, and more importantly, we illustrate that belief disagreements—when unleashed by financial innovation—generate not only a wealth effect as emphasized by GS, but also a substitution effect that tends to induce greater savings. We focus on the cases in which the substitution effect dominates, whereas GS focus on the wealth effect by restricting attention to a class of preferences (quadratic). While our emphasis and results are very different, our model reinforces the broader conceptual point in Guzman and Stiglitz (2015) that belief disagreements affect investors' consumption and savings, with implications for macroeconomic outcomes.[6]

The part of our paper on participation is related to a large literature that documents limited participation in equity markets and examines its implications for asset prices.[7] Our result that greater participation reduces the risk premium has been noted by this literature, which used it as a potential explanation for the historically high levels of the equity premium (see, for instance, Mankiw and Zeldes (1991), Heaton and Lucas (1999), Favilukis (2013)). Our result that greater participation increases the risk-free rate (due to a shift of relative demand towards risky assets) appears to be more novel. Basak and Cuoco (1998) demonstrate a version of this result in a dynamic environment in which participants' and nonparticipants' consumption shares evolve endogenously, and nonparticipants are restricted to have log utility. Relative to Basak and Cuoco (1998), we work with a two period model with exogenous wealth shares and Epstein-Zin preferences for both types of agents. We show that the result is qualitatively robust, and holds for a large range of the elasticity of intertemporal substitution parameter (e.g., away from log utility).

Finally, our analysis contributes to the recent macroeconomics literature on secular stagnation that investigates the sources of low interest rates (see Teulings and Baldwin (2014) for a summary of the recent literature). We identify financial innovation as a novel factor that can lower the risk-free rate. The part of our paper that analyzes the demand for safe assets from emerging markets, together with securitization, is related to a growing literature on the savings glut hypothesis and asset shortages.[8] We argue that securitization counters some of the relative price effects of the demand from emerging markets, while exacerbating

[6]The idea that belief disagreements increase investors' perceived portfolio returns also appears in Simsek (2013b). The idea that this generates income and substitution effects, and affect investors' savings, appears in Brunnermeier, Simsek, and Xiong (2014). They formalize the idea in the context of a model by Sims (2009), and use it as an example of how speculation generates behavioral distortions that can be detected (as inefficient) by their welfare criterion. We focus on the case in which the substitution effect dominates, and analyze the implications for various types of financial innovations and asset returns.

[7]An incomplete list includes Allen and Gale (1994b), Attanasio, Banks, and Tanner (2002), Brav, Constantinides, and Geczy (2002), Vissing-Jorgensen (2002), Vissing-Jørgensen and Attanasio (2003), Calvet, Gonzalez-Eiras, and Sodini (2004), Cao, Wang, and Zhang (2005), Gomes and Michaelides (2008), Guvenen (2009).

[8]In addition to the papers mentioned earlier, see Caballero (2006), Caballero and Krishnamurthy (2009), Bernanke, Bertaut, DeMarco, and Kamin (2011), Blanchard, Furceri, and Pescatori (2014), Justiniano, Primiceri, and Tambalotti (2014).

their (already high) savings. This is consistent with Justiniano, Primiceri, and Tambalotti (2015), who argue that securitization might have relaxed what they refer to as "lending constraints," and analyze the implications of increased savings for mortgage debt and house prices in the US.

2 A Motivating Example

We first present a simple example that illustrates the choice channel, and provides the motivation for our more general model. Consider an economy with two dates, $t \in \{0, 1\}$, and a single consumption good. At date 1, the economy can be in one of two states, denoted by $z \in \{high, low\}$. There is a single fundamental asset in unit supply, which we refer to as the market portfolio and denote by subscript m. The asset yields payoffs only at date 1. The payoff is denoted by $\varphi_m(z)$, and it is greater if the high state is realized, $\varphi_m(high) > \varphi_m(low)$.

There are two types of investors which we refer to as "optimists" and "pessimists," with heterogeneous prior beliefs about the state z. Optimists and pessimists believe the high state is realized with respectively probabilities, $\{\pi^{opt}(high), \pi^{pes}(high)\}$, which satisfy $\pi^{opt}(high) > \pi^{pes}(high)$. Investors are risk-neutral and have a discount factor of one between dates 0 and 1. Thus, they trade financial assets at date 0 to maximize the sum of their expected payoffs at dates 0 and 1. They can take long or short positions in the available financial assets, but they are subject to having nonnegative consumption at each date and state. Investors have large endowments of the asset at date 0, as well as symmetric endowments of the market portfolio, and they have zero endowments of the consumption good at date 1.

First suppose the only available financial asset is the market portfolio. In this case, it can be checked that optimists hold the asset in equilibrium, since they have a higher valuation. In particular, the equilibrium price of the market portfolio is equal to optimists' valuation,[9]

$$P_m = \pi^{opt}(high)\,\varphi_m(high) + \pi^{opt}(low)\,\varphi_m(low)\,. \tag{1}$$

Next suppose that, thanks to financial innovation, there is a second financial asset in zero net supply that has a positive payoff only in the high state. The asset is denoted by h and has payoff $\varphi_h(high) = \varphi_m(high)$ and $\varphi_h(low) = 0$. Together with the market

[9] This price is higher than pessimists' valuation. However, pessimists choose not to short sell since this would induce them to consume a negative amount at date 1. This is because there is no other financial asset in which pessimists could invest the proceeds from their short sale, and investors have no endowment at date 1.

portfolio, this asset completes the financial market and enables investors to take flexible positions on the payoffs in the two states. In equilibrium, it can be checked that optimists hold the payoff in the high state, as they assign a relatively high probability to this state, $\pi^{opt}(high) > \pi^{pes}(high)$. Similarly, pessimists hold the payoff in the low state, since $\pi^{pes}(low) > \pi^{opt}(low)$. The asset prices are then respectively given by,

$$P_h = \pi^{opt}(high) \text{ and } P_m = \pi^{opt}(high)\varphi_m(high) + \pi^{pes}(low)\varphi_m(low). \tag{2}$$

Comparing Eqs. (1) and (2) shows that financial innovation increases the value of the market portfolio. Intuitively, providing investors with greater portfolio choice makes saving more attractive, since investors self-select into holding assets or portfolios that they like relatively more. We refer to this effect as *the choice channel* of financial innovation. In this example, the choice channel raises the equilibrium price of the market portfolio, because the payoff in each state is priced by the investor that values it relatively more.

While this example illustrates the choice channel, it also raises several questions. The example features linear utilities, which implies an infinite elasticity of substitution between date 0 and 1 consumption. One could wonder whether the results are robust to allowing for lower elasticities. The example also does not feature risk aversion or background risks. A natural question, in view of the precautionary savings literature, is whether the presence of background risks can overturn these results. Finally, the example features a single asset (before financial innovation) whose price increases in view of the choice channel. In a more realistic environment with multiple assets, one could wonder how the choice channel affects individual asset prices, e.g., whether it increases the price of the safe asset as well as the market portfolio. In the rest of the paper, we systematically analyze a more general model, which will enable us to address these questions and deliver additional insights.

3 The Choice Channel and Savings

Consider an economy with two dates, denoted by $t \in \{0, 1\}$, and a single consumption good which will be referred to as a dollar. The uncertainty in this economy is described by the random variable, $z \in Z$. There are financial assets denoted by $j \in \{f\} \cup \mathbf{J}$. Each financial asset is a mapping $\varphi_j : Z \to \mathbb{R}_+$ where $\varphi_j(\mathbf{z})$ denotes the payoff at date 1 if state z is realized. The asset f captures the risk-free asset that makes a constant payment in all states, $\varphi_f(\mathbf{z}) = \overline{\varphi}_f > 0$ for each $\mathbf{z}$. The set $\mathbf{J}$ captures risky assets. We assume (until Section 4) that the state space Z is finite, and the vectors, $(\varphi_j(\mathbf{z}))_{z \in Z}$ for $j \in \{f\} \cup \mathbf{J}$, are linearly independent so that each asset is nonredundant. Each asset is traded in a competitive market at some price, $P_j > 0$. In this section, we take these prices as given and analyze how financial innovation that expands an investor's choice affects her savings. We endogenize the prices in the next section.

Specifically, consider an investor denoted by the superscript i. The investor starts with some endowment of the consumption good at date 0, denoted by $Y_0^i > 0$, as well as some positions on financial assets, $\left\{x_{-1,j}^i\right\}_j$. We denote the value of financial assets by $W_0^i = \sum_j x_{-1,j}^i P_j$. Thus, her financial wealth at date 0 is $Y_0^i + W_0^i$. The investor also receives some endowment of the consumption good in state $\mathbf{z}$ of date 1, denoted by $L^i(\mathbf{z})$, which can be thought of as her labor (or other non-financial) income. The investor chooses her consumption and total asset holdings at date 0, denoted by C_0 and A_0, as well as positions in financial assets, denoted by $\left\{x_j^i\right\}_{j\in\{f\}\cup J^i}$, to solve,

$$\begin{aligned}
& \max_{C_0, A_0, \{x_j\}_{j\in\{f\}\cup J^i}} U_0^i\left(C_0, \left(C_1(\mathbf{z})\right)_Z\right) \qquad (3) \\
\text{s.t.} \quad & C_0 + A_0 = Y_0^i + W_0^i \text{ where } A_0 \geq A^{i,\min} \text{ and } A_0 = \sum_{j\in\{f\}\cup J^i} P_j x_j, \\
\text{and} \quad & C_1(\mathbf{z}) = L^i(\mathbf{z}) + \sum_{j\in\{f\}\cup J^i} x_j^i \varphi_j(\mathbf{z}) \text{ for each } \mathbf{z} \in Z.
\end{aligned}$$

Here, $C_1(\mathbf{z})$ denotes the investor's financial wealth in state $\mathbf{z}$ of date 1, which she consumes since there is no subsequent period. The second line captures her budget constraint at date 0 in terms of consumption and asset holdings. We assume $A^{i,\min} \in [-\infty, 0]$ so that the lower bound on assets, $A_0 \geq A^{i,\min}$, allows us to capture an exogenous borrowing constraint in the same spirit as Aiyagari (1994). We allow the limit to depend on the investor's identity so as to capture unmodeled features such as collateralized borrowing (e.g., the limit can also reflect the collateral value of the investor's wealth). The investor allocates her savings portfolio among various assets in J^i. Note that the investor can take unrestricted long or short positions.[10] The last line illustrates that she receives returns from these assets in the next period.

We assume the investor has recursive Epstein-Zin preferences, given by,

$$\begin{aligned}
U_0^i &= C_0^{1-1/\varepsilon^i} + \beta^i \left(V_1^i\right)^{1-1/\varepsilon^i} \qquad (4) \\
\text{where } V_1^i &= \left(E^i\left[C_1(\mathbf{z})^{1-\gamma^i}\right]\right)^{1/\left(1-\gamma^i\right)}.
\end{aligned}$$

Here, the parameter ε^i captures the investor's elasticity of intertemporal substitution (EIS), which will play a central role for our analysis. The parameter γ^i captures the coefficient

[10] We investigate the implications of short-selling constraints in ongoing work.

of relative risk aversion. The variable, V_1^i, captures the certainty equivalent of future consumption. The special case, $\varepsilon^i = 1/\gamma^i$, corresponds to time separable CRRA preferences. The expectations operator, $E^i[\cdot]$, is taken with respect to the investors' belief about the aggregate state. The superscript i on the investor's expectation operator emphasizes that we allow for heterogeneous beliefs. We also assume the beliefs are dogmatic in the sense that investors do not change their beliefs after they observe the prices (formally, investors know each others' beliefs, and thus, they agree to disagree). We will use belief disagreements of this type to capture investors' demand for customized assets.

We also make the following assumption regarding the investor's background risks.

Assumption 1. For each i, there exists scalars, $\left\{l_j^i\right\}_{j\in\{f\}\cup J^i}$, such that $L^i(\mathbf{z}) = \sum_{j\in\{f\}\cup J^i} l_j^i \varphi_j(\mathbf{z})$ for each $z \in Z$.

The assumption holds when the investor's future endowment is constant. It is also satisfied if the investor's future endowment is perfectly correlated with a combination of the risky assets in her access set.[11] Hence, financial innovation that expands the set J^i does not bring any additional benefits in terms of sharing background risks. As we illustrate in the appendix, when this assumption is dropped and several other strong assumptions are added, then one can show that financial innovation that fully completes the market reduces agents' savings. Intuitively, in that alternative scenario, the investor has some precautionary savings motive. Financial innovation enables her to hedge her background risks and mitigates the precautionary savings motive. This precautionary channel of financial innovation is already well understood in the literature. Assumption 1 enables us to abstract away from this channel, and illustrate our novel choice channel.

Under the assumptions we made, there is a unique solution to the investor's problem (3) for a given access set. Our first result describes how financial innovation that expands the investor's access set affects her savings. Specifically, suppose the investor's access set changes from some $J^{i,old}$ to a greater set $J^{i,new} \supset J^{i,old}$. Let $\left(C_0^{i,old}, A_0^{i,old}, \left\{x_j^{i,old}\right\}_{j\in\{f\}\cup J^i}\right)$ and $\left(C_0^{i,new}, A_0^{i,new}, \left\{x_j^{i,new}\right\}_{j\in\{f\}\cup J^i}\right)$ denote the solution corresponding to respectively the old and the new access sets. Note that the investor's savings is given by [cf. Eq. (3)],

$$S_0^i = Y_0^i - C_0^i = A_0^i - W_0^i. \tag{5}$$

In particular, savings equal the change in the value of asset holdings within the period. Since asset prices are constant, it suffices to characterize the effect on asset holdings, A_0^i.

[11] For instance, if an individual's income is perfectly correlated with a broad market index portfolio, and she has access to the index portfolio, then Assumption 1 holds.

Proposition 1 (Choice Channel)**.** *Suppose Assumption 1 holds and $\varepsilon^i > 1$ (so that the investor's savings is increasing in the interest rate). Then, financial innovation increases the investor's asset holdings (and thus, savings), $A_0^{i,new} \geq A_0^{i,old}$, with strict inequality if $A_0^{i,new} > A^{i,\min}$ and $x_j^{i,new} \neq 0$ for some $j \in J^{i,new} \setminus J^{i,old}$.*

The result says that greater portfolio choice induces the investor to save more. Moreover, the inequality is strict as long as the borrowing constraint does not bind for the investor and she takes a nonzero position on some new asset—so that the assets are not completely redundant from her perspective.

We provide a sketch-proof for this result, which is also useful to understand the intuition. In view of Assumption 1, the investor can be hypothetically thought of as selling her labor endowment and repurchasing assets. More specifically, consider a hypothetical investor with zero future endowment, $\tilde{L}^i(\mathbf{z}) = 0$, but instead with financial wealth,

$$\tilde{W}_0^i = Y_0^i + \sum_{j \in \{f\} \cup \mathbf{J}} \left(x_{-1,j}^i + l_j^i\right) P_j.$$

Once we characterize the optimal choice by the hypothetical investor, $\tilde{C}_0^i, \tilde{A}_0^i, \tilde{x}_j^i$, the optimal choice by the original investor can be deduced from,

$$C_0^i = \tilde{C}_0^i, A_0^i = \tilde{A}_0^i - \sum_{j \in \{f\} \cup \mathbf{J}} l_j^i P_j \text{ and } x_j^i = \tilde{x}_j^i - l_j^i \text{ for each } j.$$

Note also that the borrowing constraint, $A_0^i \geq A^{i,\min}$, implies an analogous constraint for the hypothetical investor, $\tilde{A}_0^i \geq \tilde{A}^{i,\min}$, where $\tilde{A}^{i,\min} = A^{i,\min} + \sum_{j \in \{f\} \cup \mathbf{J}} l_j^i P_j$.

Next note that the (hypothetical) investor's problem can be split into two parts. Conditional on asset holdings, $\tilde{A}_0^i$, the investor maximizes her certainty-equivalent payoff at date 1. That is, she solves the portfolio problem,

$$V_1^i\left(\tilde{A}_0^i\right) = \max_{\{\tilde{x}_j\}_{\{f\} \cup J^i}} \left(E^i\left[C_1(\mathbf{z})^{1-\gamma^i}\right]\right)^{1/\left(1-\gamma^i\right)}, \tag{6}$$
$$\text{s.t.} \sum_{\{f\} \cup J^i} P_j \tilde{x}_j = \tilde{A}_0^i \text{ and } C_1(\mathbf{z}) = \sum_{\{f\} \cup J^i} \tilde{x}_j \varphi_j(\mathbf{z})$$

In turn, given the value function, $V_1^i(\cdot)$, she chooses her effective asset holdings, $\tilde{A}_0^i$, by solving the intertemporal problem,

$$\max_{\tilde{A}_0 \geq \tilde{A}^{i,\min}} \left(\tilde{W}_0^i - \tilde{A}_0\right)^{1-1/\varepsilon^i} + \beta^i \left(V_1\left(\tilde{A}_0\right)\right)^{1-1/\varepsilon^i}. \tag{7}$$

The result then follows from three observations. First, the portfolio problem is linearly homogeneous, which implies that the value function is linear in asset holdings,

$$V_1^i\left(\tilde{A}_0\right) = R_{ce}^i \tilde{A}_0. \tag{8}$$

We refer to R_{ce}^i as the investor's certainty-equivalent return. Second, and most importantly, financial innovation increases the certainty-equivalent return, $R_{ce}^{i,new} \geq R_{ce}^{i,old}$, because it expands the choice set of feasible portfolios. Third, in the intertemporal problem, a greater risk-adjusted return implies an increase in asset holdings in view of the assumption $\varepsilon^i > 1$.

Intuitively, with greater portfolio choice, the investor's certainty-equivalent portfolio return can only increase. This creates substitution and income effects. On the one hand, the investor finds savings more attractive, which induces her to save more. On the other hand, the investor also feels richer, which induces her to consume more and save less. The substitution effect dominates, and financial innovation increases savings, whenever the EIS is relatively high.

As this intuition suggests, the result can further be generalized. The particular comparative statics we focus on, the expansion of the access set from $J^{i,old}$ to some $J^{i,new}$, does not play an important role beyond ensuring that the investor has greater choice. Any other financial innovation that expands the investor's choice would induce the investor to save more.[12] In fact, financial innovations that expand the investor's return without affecting her choice would also induce her to save more. For instance, a reduction in trading or intermediation costs works in the same direction as our choice channel.

The result requires two key assumptions: the absence of background risks (Assumption 1) and a relatively high elasticity of intertemporal substitution. As we discuss in Section 1.1, we believe the first assumption is reasonable in our context. Likewise, we also believe a relatively high EIS is appropriate for our context. Using different methodologies, empirical studies find a wide range of estimates for the EIS (see Hall (1988), Blundell, Browning, and Meghir (1994), Attanasio and Browning (1995), Vissing-Jorgensen (2002), Vissing-Jørgensen and Attanasio (2003), Gruber (2013)). Most of the studies assume that investors with separable or Epstein-Zin preferences fully observe the changes in the interest rate and make optimal decisions. Even though we also make the same assumptions, some of these features are not central for our analysis. What is important is that investors have an asset holding (or saving) function that is increasing in their perceived portfolio return. We believe this assumption is plausible, and it can also accommodate some behavioral biases such as limited attention.[13]

[12] In ongoing work, we analyze financial innovations that relax short selling constraints, which increase the investor's savings even though they cannot be mapped into Proposition 1.

[13] To illustrate this, consider an alternative setting in which an investor with Epstein-Zin preferences with

Thus, we view the assumption, $\varepsilon^i > 1$, as a simple way of generating an increasing asset holding function in our setting. In the numerical simulations, we use $\varepsilon^i = 2$, which is the estimate provided by Gruber (2013) based on plausibly exogenous variations in the interest rate that come from tax changes.

For which investors is the choice channel stronger? Eqs. $(6-8)$ suggest that the effect of innovation on savings depends on the increase in the investor's certainty equivalent return, R^i_{ce}, as well as the extent to which the investor reacts to this increase. Our next result formally establishes the comparative statics for the saving rate, $S^i_0/Y^i_0 = \left(A^i_0 - \sum_j x^i_{-1,j} P_j\right)/Y^i_0$ (see (5)).

Observation 1. *The increase in the saving rate,* $\left(S_0^{i,new} - S_0^{i,old}\right)/Y_0^i$, *is*

(i) increasing in the value of financial assets, W_0^i,

(ii) If $\frac{V_1^{i,new}\left(\tilde{A}_0^{i,new}\right)}{C_0^{i,new}} < R_{ce}^{i,new}$, *then increasing in the discount factor (patience),* β^i,

(iii) If $J^{old} = \{f\}$, *then decreasing in risk aversion,* γ^i.

The first part generates the testable implication that the saving rate increases relatively more for wealthier investors. This result follows from two channels that operate in the same direction. First, all else equal, an investor with lower wealth is more likely to face a binding borrowing constraint. When the constraint binds, the choice channel might affect the investor's portfolio but it has a smaller impact (often, no impact) on her savings. These investors' saving rate is primarily determined by the constraint, which we take as exogenous and fixed. In fact, if we were to let the lower bound, $A^{i,\min}$, decline alongside with the expansion of choice (e.g., to capture other financial innovations that might have relaxed borrowing constraints), then the saving rates of the constraint investors would decline. In contrast, a wealthy investor is less likely to face a borrowing constraint. When the constraint does not bind, the investor increases her savings in response to greater choice—even if $A^{i,\min}$ declines.

$\varepsilon^i > 1$ makes consumption and saving decisions over several periods. Suppose the investor has limited attention and observes the asset returns only with some probability. In any period, if she observes the asset returns, then she makes a fully optimal consumption plan as in our model. If she does not observe the returns, then she follows a default rule: say, she consumes and saves according to her earlier plan (many other default rules would also work). This investor's average asset holdings and savings would also increase in response to financial innovation, and our qualitative results would continue to apply in this setting. However, the investor's consumption growth would not increase much after a (surprise) increase in the interest rate. Thus, the empirical strategies that focus on consumption growth can easily (mis-)estimate that $\varepsilon^i < 1$. The key point is that the investor's consumption level would not react much to the changes in the interest rate either. The income effects would still be weaker than the substitution effects, and the investor would actually choose a lower level of consumption and a higher level of savings—albeit not as much as in the full attention case. Moreover, over longer horizons—which is the range over which we apply the comparative statics of our model—the investor's attention would eventually catch up and her savings would arguably increase further.

There is a second reason for why greater wealth is associated with a stronger choice channel. Solving problem (6) reveals that greater choice induces a multiplicative increase in the investor's desired asset holdings, $\tilde{A}_0^i$ (see Eq. (14) below). For instance, greater choice might increase the investor's desired asset holdings by 1%. This type of change generates a greater dollar increase in the assets (and ultimately savings) of wealthier investors, because they start from a greater asset base, $A_0^{i,old}$.

The second part of Observation 1 says that, under appropriate conditions, greater choice increases savings relatively more for more patient investors. While we do not directly observe patience, it is reasonable to think that patience is positively correlated with the value of financial assets, W_0^i, in view of the investors' past savings. Hence, this part reinforces our earlier observation that the choice channel is likely to be stronger for wealthier investors. Intuitively, a more patient investor derives greater utility from future returns, and therefore, she reacts more to an increase in risk-adjusted returns. There is a counterveiling effect which stems from the fact that the investor might be already saving considerably, which leaves little room for further increases in saving. The regularity condition in Observation 1 ensures that the counterveiling effect is not too strong. The condition says that the investor's (expected) certainty-equivalent consumption growth is less than her (expected) certainty-equivalent portfolio return. While this condition concerns endogenous objects, it is likely to hold under reasonable calibrations for the per capita output growth and the risk aversion parameter.[14]

The third part says that greater choice increases savings relatively more for less risk averse investors. While we do not directly observe risk aversion, this is likely to be reflected in the amount of risk in investors' portfolios. Hence, this part generates the additional testable prediction that the choice channel is stronger for investors whose (pre-innovation) portfolios are riskier. The intuition follows from the observation that the choice channel concerns improvements in investors' access to risky assets. Investors that are less risk averse, and therefore more willing to hold risky assets, naturally react more to an expansion of risky choice. The formal result requires the additional condition that the initial choice set consists only of the risk-free asset. However, the result applies more generally—regardless of the initial choice set—once we impose more structure on asset payoffs and convert the portfolio problem into mean-variance optimization (see Eq. (16) in Section 4).

[14]For an example, consider our model in Section 4.1 in which there are no disagreements and financial innovation improves the access to the market portfolio. There, the expected consumption growth is equal to the expected per capita output growth, which is less than 2%. The certainty-equivalent consumption is further below this amount due to risk aversion [see (6)]. In contrast, the certainty equivalent portfolio return is typically above 2% because it incorporates the risk-free rate as well as the gains from the risk premium on the market portfolio. In our calibration, $r_{ce}^{i,new} = \log\left(R_{ce}^{i,new}\right)$ is typically between 2.5% and 3.5% (see the right panel of Figure 5). Hence, the condition comfortably holds.

4 The Choice Channel and Asset Returns

We next investigate how the choice channel affects asset prices and returns in general equilibrium. There are several types of investors denoted by $i \in \{1, .., |I|\}$, each of which has population mass $n^i \geq 0$. We normalize the total population mass to 1, so that, $\sum_i n^i = 1$. To facilitate analytical tractability, we make several simplifying assumptions. We first assume that the following analogue of Assumption 1 holds.

Assumption 1^{G}. $L^i = 0, \varepsilon^i > 1$, and $A^{i,\min} = -\infty$ for each investor i.

As before, the investors' future endowments are correlated with assets in their access sets. Without loss of generality, we also normalize the future endowments to zero and simplify the notation (see Section 3). We also assume each investor has a relatively high EIS so that the choice channel is operational. Finally, we assume the investors do not face borrowing constraints. This assumption does not affect our qualitative results, and it can be dropped at the expense of additional notation.[15]

To endogenize the asset prices, we impose additional structure on asset supplies and payoffs. Each asset $j \in \mathbf{J}$ is in fixed supply denoted by $\eta_j \geq 0$. More importantly, the uncertainty is now described by a $K \times 1$ vector of continuous random variables, $z = (z_1, ..., z_K)'$ (in particular, the state space is now given by, $\mathbf{Z} = \mathbb{R}^K$). The log payoff of a risky asset $j \in \mathbf{J}$ can be written as a linear combination of the underlying uncertainty,

$$\log \varphi_j (\mathbf{z}) = \mathbf{F}_j' \mathbf{z}, \tag{9}$$

where $\mathbf{F}_j$ is a $K \times 1$ vector. We assume investors' beliefs for $\mathbf{z}$ are normally distributed, and thus, their beliefs for asset payoffs are log-normally distributed.

Assumption 2. Investor i's prior belief for $\mathbf{z}$ has a Normal distribution, $N(\boldsymbol{\mu}_{\mathbf{z}}^i, \Lambda_{\mathbf{z}})$, where $\boldsymbol{\mu}_{\mathbf{z}}^i \in \mathbb{R}^K$ is the mean vector and $\Lambda_{\mathbf{z}}$ is the $K \times K$ positive definite covariance matrix. In addition, the $K \times |\mathbf{J}|$ matrix of asset loadings, $\mathbf{F} = [\mathbf{F}_j]_{j \in \mathbf{J}}$, has full rank.

Note that investors can disagree on the mean of the asset payoffs but they agree on the variance of log payoffs (for simplicity). The full rank assumption ensures that risky assets are not redundant. As we noted before, the belief disagreements are dogmatic. Formally, investors know each others' beliefs and they agree to disagree.

In this section, we find it convenient to work with gross and log asset returns, defined respectively by $R_j (\mathbf{z}) = \varphi_j (\mathbf{z}) / P_j$ and $r_j (\mathbf{z}) = \log R_j (\mathbf{z})$, for each $j \in \{f\} \cup \mathbf{J}$. Assumption 2 implies that the asset returns are jointly log-normally distributed with mean and joint

[15] The assumption also arguably has a relatively small impact on our quantitative findings. In practice, the investors who are likely to face binding borrowing constraints command a relatively small amount of aggregate wealth. Thus, they are likely to have a relatively small effect on asset prices.

variance respectively given by,

$$E^i[r_j] = \mu_j^i - \log P_j, \text{ for each } j, \text{ and } var\left(\{r_j\}_{j\in\mathbf{J}}\right) = \Lambda. \tag{10}$$

Here, $\mu_j^i = (\mathbf{F}_j)' \boldsymbol{\mu}_{\mathbf{z}}^i$ (and $\mu_f^i = \log \overline{\varphi}_f$) denote investor i's belief for the log asset payoffs, and $\Lambda = \mathbf{F}'\Lambda_{\mathbf{z}}\mathbf{F}$ denotes the common variance matrix that is common to all investors.

We also define the investor's portfolio return and the log portfolio return, defined respectively by $R_p^i(\mathbf{z}) = C_1^i(\mathbf{z})/A_0^i$ and $r_p^i(\mathbf{z}) = \log R_p^i(\mathbf{z})$. Using the budget constraint in (3), the portfolio return can be written as a linear combination of her asset returns,

$$R_p^i(\mathbf{z}) = \sum_{j\in\{f\}\cup J^i} \omega_j^i R_j(\mathbf{z}), \text{ where } \omega_j^i \equiv x_j^i P_j / A_0^i.$$

Here, ω_j^i denotes the investor's wealth share invested in asset j. Even though each asset has a log-normal distribution, the portfolio return can in general have a complicated distribution. For analytical tractability, we assume the investor optimizes her portfolio after applying a log-normal approximation to portfolio returns as in Campbell and Viceira (2002). This approximation becomes exact in the continuous time limit in which the time horizon between dates 0 and 1 disappears. Over shorter horizons, such as a year, this approach results in reasonable portfolio allocations that are close to optimal.

To describe the approximate problem, we define the investor's (log) risk premium on an asset perceived by an investor i as,

$$\zeta_j^i = E^i[r_j] + \frac{\Lambda_j}{2} - r_f \text{ for each } j \in \mathbf{J}. \tag{11}$$

Intuitively, the risk premium depends on the difference between the expected return on the asset and the risk-free rate. The expected asset return exceeds the expected log return by an amount that depends on the asset's variance, Λ_j, in view of the short-run fluctuations in log returns. Note that different investors can perceive different risk premia in view of the differences in their beliefs for expected returns [cf. (10)]. We also analogously define the investor's risk premium on her portfolio return as,

$$\zeta_p^i = E^i\left[r_p^i\right] + \frac{\Lambda_p}{2} - r_f, \tag{12}$$

where Λ_p denotes the variance of the log portfolio return, r_p^i.

The investor's portfolio problem (13) can then be approximately written as [see Campbell and Viceira (2002) for details],

$$\begin{aligned} r_{ce}^i - r_f &= \max_{\boldsymbol{\omega}_{J^i}, \omega_f} \zeta_p^i - \frac{\gamma^i \Lambda_p}{2}, \\ \text{such that,} \qquad \zeta_p^i &= \sum_{j \in J^i} \omega_j \zeta_j^i \text{ and } \Lambda_p = \boldsymbol{\omega}'_{J^i} \Lambda_{J^i} \boldsymbol{\omega}_{J^i}. \\ \text{where} \qquad \boldsymbol{\omega}_{J^i} &= \left(\omega_j\right)_{j \in J^i} \text{ and } \omega_f = 1 - \sum_{j \in J^i} \omega_j. \end{aligned} \tag{13}$$

Here, the variable, $r_{ce}^i = \log R_{ce}^i$, denotes the log of investor's certainty equivalent return from asset holdings [cf. Eq. (8)]. The investor trades off the portfolio risk premium, ζ_p^i, with the portfolio variance, Λ_p. The third line says that the investor can be thought of as choosing portfolio weights on risky assets, with the residual weight invested in the riskless asset. The second line describes the portfolio risk premium and variance as a function of the risk premia and the variance of the underlying assets, as well as the portfolio weights.[16]

We continue to assume the investor chooses her consumption and savings according to the intertemporal problem (7), given the certainty equivalent return from her portfolio problem (13). The solution can be written as

$$A_0^i = a^i \left(r_{ce}^i\right) \left(Y_0^i + W_0^i\right) \text{ where } a^i \left(r_{ce}^i\right) = \frac{\left(\beta^i\right)^{\varepsilon} \exp\left(r_{ce}^i \left(\varepsilon^i - 1\right)\right)}{1 + \left(\beta^i\right)^{\varepsilon} \exp\left(r_{ce}^i \left(\varepsilon^i - 1\right)\right)}. \tag{14}$$

Here, $a^i \left(r_{ce}^i\right)$ describes the investors' effective asset holding as a fraction of wealth. Importantly, it is an increasing function in view of the assumption $\varepsilon^i > 1$, which captures the choice channel (Proposition 1) in the general equilibrium context.

The asset market clearing conditions can then be written as,

$$\eta_j P_j = \sum_{\{i \quad | \quad j \in \{f\} \cup J^i\}} n^i \omega_j^i a^i \left(r_{ce}^i\right) \left(Y_0^i + W_0^i\right) \text{ for each } j \in \{f\} \cup \mathbf{J}, \tag{15}$$

where $W_0^i = \sum_j P_j x_{-1,j}^i$ for each i. The condition says that the supply of each asset j equals its demand, which is determined by investors' savings as well as their asset allocations. To avoid trivial cases, we assume that each asset that is in positive supply, $\eta_j > 0$, lies in at least one investor's access set. We also assume the economy is closed, so that the assets are initially held by the investors in the economy, that is, $\sum_i n^i x_{-1,j}^i = \eta_j$ for each j.

[16] To obtain these expressions, imagine that log asset returns followed a joint diffusion process over continuous time with instantenaous drifts $\left\{\mu_j^i\right\}_J$ and volatility Λ (starting at $r_j = 0$ for each j). Then, the log portfolio return would also follow a diffusion process. Its mean and variance can be characterized using Ito's Lemma, which leads to the expressions in problem (13).

Definition 1 (Equilibrium)**.** *Under Assumptions 1^G and 2, an (approximate) equilibrium, $\left\{\left(\omega^i_{J^i}, A^i_0\right)_i, P_j\right\}$, is a collection such that portfolio shares solve problem* (13)*, asset holdings satisfy $A^i_0 = a^i\left(r^i_{ce}\right)\left(Y^i_0 + W^i_0\right)$, and markets clear [cf. Eq.* (15)*].*

To characterize the equilibrium, let $\boldsymbol{\zeta}^i_{J^i} = \left(\zeta^i_j\right)_{j\in J^i}$ denote the vector of risk premia for the assets in an investor's access set. Solving problem (13), the investor's portfolio shares and certainty-equivalent return are given by,

$$\boldsymbol{\omega}^i_{J^i} = \frac{1}{\gamma^i}\Lambda^{-1}_{J^i}\boldsymbol{\zeta}^i_{J^i} \text{ and } r^i_{ce} = r_f + \frac{1}{2\gamma^i}\left(\boldsymbol{\zeta}^i_{J^i}\right)'\Lambda^{-1}_{J^i}\boldsymbol{\zeta}^i_{J^i}. \tag{16}$$

Loosely speaking, increasing the risk premia of an asset, while keeping all else constant, tends to shift the investor's portfolio weight towards this asset. Greater risk premium also enables the investor to obtain a greater certainty-equivalent return, which increases her savings. Likewise, keeping risk premia constant, increasing the risk-free rate raises the certainty-equivalent return and savings. Note that the risk free rate as well as the risk premia are endogenous and depend inversely on asset prices according to Eq. (10) and the equation for r_f. The prices (and returns) adjust, so as to satisfy the market clearing conditions in (15).

Proposition 8 in the Appendix establishes the existence of an equilibrium. At this level of generality, we cannot characterize the equilibrium much further. In the rest of this section, we analyze a canonical case that can accommodate the key aspects of various recent financial innovations.

Assumption 3. There exist K risky assets in total, $\mathbf{J} = \{m, 1, ..., K-1\}$. The asset m is in positive supply, $\eta_m > 0$, while the remaining risky assets, as well as the risk-free asset are in zero net supply, $\eta_j = 0$ for $j \neq m$.

The first part ensures that the risky assets collectively complete the market, since the underlying uncertainty is K dimensional (see Assumption 3). The second part says that there exists a single asset, denoted by m, that represents all of the cash flows in positive supply. This asset, which is typically referred to as *the market portfolio*, enables investors to obtain exposure to all assets in proportion to their market valuations. Its practical counterpart could be broad equity or bond indices that proxy for this type of exposure. The remaining risky assets, $j \in \{1, .., K-1\}$, enable investors to customize their exposures according to their specific preferences or beliefs. Their counterparts could be individual stocks, bonds, investment funds, or various types of derivatives. In the rest of the section, we analyze how innovations that improve access to the assets in $\mathbf{J}$ affect the equilibrium returns.

Throughout, we also maintain the following symmetry assumption.

Assumption 4. Investors start with the same endowment of the consumption good, $Y_0^i = Y_0 > 0$ for each i, as well as the risky assets, $x_{-1,j}^i = x_{-1,j}$ for each j and i.

This assumption ensures that investors have the same wealth, $Y_0 + W_0$. Hence, it enables us to abstract away from the effects of financial innovation on wealth redistribution, which is not our focus. In view of this assumption, the wealth shares of different investor groups are captured by the exogenous relative mass parameters, $\{n^i\}_i$.

4.1 Market Participation

We start by investigating increased market participation, which we capture with improved access to asset m. In particular suppose there are two types of investors, $i \in \{1, 0\}$, with market access sets respectively given by $J^1 = \{m\}$ and $J^0 = \emptyset$. Type 1 investors have access to asset m, in addition to the risk-free asset. In contrast, type 0 investors have access to only the risk-free asset. To simplify the exposition, we also assume investors are identical in all other dimensions. In particular, they share the same beliefs, $\boldsymbol{\mu}_{\mathbf{z}}^i = \boldsymbol{\mu}_{\mathbf{z}}$, which implies $\mu_m^i = \mu_m$, for each i.[17] Investors also have the same preference parameters, $\beta^i = \beta, \varepsilon^i = \varepsilon$, and $\gamma^i = \gamma$, which implies, $a^i(\cdot) = a(\cdot)$, for each i.

In this setting, we capture financial innovation as an increase in the relative mass of type 1 investors, n^1 (while keeping the total mass unchanged, $n^1 + n^0 = 1$). In view of the symmetry assumptions, this is equivalent to expanding the access set of some of type 0 investors from $J^{old} = J^0$ to $J^{new} = J^1 \supset J^0$, similar to Section 3. The difference is that financial innovation applies to a positive mass of investors, with potential general equilibrium effects. The following lemma characterizes the equilibrium.

Lemma 1. *Consider the above setup with common beliefs and limited participation in the market portfolio. There exists a unique equilibrium in which ζ_m and r_f jointly solve,*

$$\zeta_m = \gamma\Lambda_m \left(1 + \frac{1-n^1}{n^1} \frac{a(r_f)}{a(r_{ce}^1)}\right), \tag{17}$$

$$\frac{\eta_m P_m}{Y_0 + \eta_m P_m} = \left(1 - n^1\right) a(r_f) + n^1 a\left(r_{ce}^1\right), \tag{18}$$

where $P_m = \exp\left(\mu_m + \frac{\Lambda_m}{2} - r_f - \zeta_m\right)$, *and*

$$r_{ce}^1 = r_f + \frac{\zeta_m^2}{2\gamma\Lambda_m}. \tag{19}$$

[17] This assumption ensures that the market portfolio m is sufficient to construct an efficient portfolio for each investor, providing a justification for the absence of assets $j \in \{1, .., K-1\}$ from investors' access sets. We analyze the introduction of these assets in the next subsection.

To understand Eq. (17), note that the participants' share of the market portfolio satisfies, $\omega_m^1 = \frac{\zeta_m}{\gamma \Lambda_m}$ [cf. Eq. (16)]. With full participation, the equilibrium share of the market portfolio would satisfy, $\omega_m^1 = 1$, which would lead to the risk premium, $\zeta_m = \gamma \Lambda_m$. With limited participation, the equilibrium share of each participant is greater, $\omega_m^1 > 1$, since the aggregate risk is shared among fewer investors. This leads to a greater risk premium relative to the full participation benchmark, $\zeta_m > \gamma \Lambda_m$. Moreover, the degree by which the risk premium exceeds the benchmark depends on the (wealth-averaged) mass of participants, as captured by Eq. (17). Intuitively, the premium must be sufficiently large to compensate the participants for the additional risks they hold in equilibrium.

Eq. (18) is a market clearing condition for all assets. The left side captures the supply of all assets, which is decreasing in the expected return on the market portfolio, $r_f + \zeta_m$. The right side captures the average demand for assets, which is increasing in r_f as well as ζ_m. The equilibrium is found by jointly solving Eqs. (17) and (18). The following result describes the comparative statics with respect to financial innovation.

Proposition 2 (Increased Participation)**.** *Consider the equilibrium characterized in Lemma 1. Financial innovation that increases the relative mass of participants, n_1, decreases the risk premium, ζ_m (increases P_m/P_f), and decreases the expected return on the market portfolio, $r_f + \zeta_m$ (increases P_m).*

The result says that greater participation increases the relative price of the market portfolio, P_m/P_f, as well as its absolute price, P_m. In our numerical simulations, we also find that it also typically decreases the price of the safe asset, P_f, and increases the risk-free rate, r_f, even though this effect is theoretically ambiguous.

The relative price effect follows from Eq. (17). With greater participation, the aggregate risk is shared among a greater set of investors. This reduces the premium for risky assets and raises their relative price. The absolute price effect follows from the choice channel. Investors with access to the market portfolio find saving more attractive, $r_{ce}^1 > r_f$, since they can earn the risk premium. This increases the average demand for assets, as captured by Eq. (18), which in turn increases the asset valuations in equilibrium.

The effect on the price of the safe asset is ambiguous because the relative and the absolute price effects push in opposite directions. For empirically relevant parameters, we find that the relative price effect dominates and increased participation reduces P_f and increases r_f. Intuitively, increased participation creates a large reduction in the relative demand for the safe asset: Some investors who used to invest only in the safe asset move some of their wealth to the risky asset. Investors also increase their asset holdings, which counters the demand shift away from the safe asset to some extent, but cannot fully undo it.

4.1.1 Numerical Illustration

We next illustrate these results using a numerical example. We use the preference parameters, $\gamma = 5$ and $\varepsilon = 2$, along with a yearly calibration. To mitigate the equity premium puzzle, we consider a relatively high level for the standard deviation of the market portfolio, $\sqrt{\Lambda_m} = 5\%$.[18] We also assume 1% growth rate for log output, and calibrate β so that the risk-free rate is equal to its historical average, $r_f = 1\%$, for $n_1 = 0.5$ (conditional on all other parameters). Figure 2 in the introduction suggests that the wealth-weighted participation in the US increased from about 50% in 1960s to about 90% in the 1990s. We therefore investigate the effect of varying n_1 in our model over the range, $[0.5, 0.9]$.

The left panel of Figure 5 illustrates the results of this exercise. The solid lines show that increased participation reduces the risk premium and the expected return on the market portfolio, consistent with Proposition 2, while also increasing the risk-free rate. The dashed lines illustrate the alternative case with $\varepsilon = 1$, which provides a useful benchmark for our results. When $\varepsilon = 1$, the relative price effects are still active but the choice channel is not operational (because the substitution effects of increased choice are exactly countered by strong income effects). Comparing this benchmark with our calibration, $\varepsilon = 2$, shows that the choice channel from increased participation is quantitatively weak. It reduces r_f by a small amount—less than 0.5 percentage points—which does not overturn the increase in r_f due to the relative price effect.

The choice channel is quantitatively weak partly because of crowd-out effects that tend to lower the (marginal) benefit from participation in general equilibrium. Greater n_1 implies a smaller risk premium, and thus a smaller certainty-equivalent return for investors that already participate [cf. Eq. (19)]. These investors react by reducing their asset holdings, which mitigates the effect of increased choice on asset prices. The top right panel of Figure 5 illustrates this crowd-out effect by plotting the gains from participation, $r_{ce}^1 - r_{ce}^0$. The panel also plots the average certainty-equivalent return, $\overline{r}_{ce}$, defined as the solution to $a(\overline{r}_{ce}) = (1 - n^1)\, a(r_{ce}^0) + n^1 a(r_{ce}^1)$. Note that the average return, $\overline{r}_{ce}$, increases by a very small amount: This is because the gain from participation, $r_{ce}^1 - r_{ce}^0$, decreases in the level of participation. For $n_1 = 0.5$, participants earn 2.5pp greater certainty-equivalent return than nonparticipants, but this difference falls to less than 1pp for greater levels of participation. In equilibrium, a relatively small increase in $\overline{r}_{ce}$ translates into a relatively small decrease in the expected return on the market portfolio.

[18] In the data, the volatility of consumption growth is around 1%, which leads to the equity premium puzzle (with relatively standard parameters such as $\gamma = 5$). Our calibration with higher volatility can be thought of as capturing factors omitted from our model, e.g., long-run risk, that could help to explain the equity premium puzzle (Bansal and Yaron (2004)).

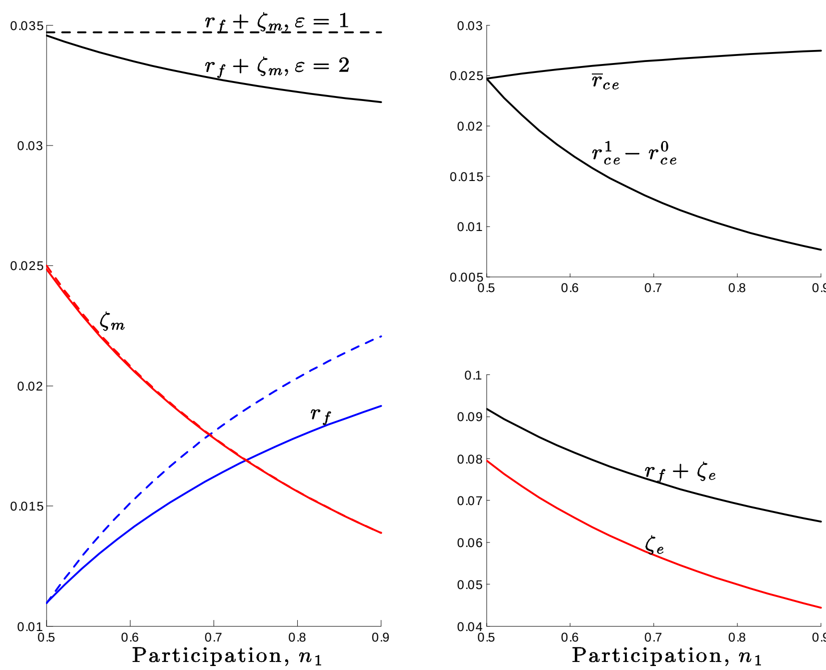

Figure 5: The left panel illustrates the effect of increased participation on asset returns for the main case, $\varepsilon = 2$ (solid lines) and the benchmark case, $\varepsilon = 1$ (dashed lines). The top right panel illustrates the return-gain from participation, as well as the avreage certainty-equivalent return (when $\varepsilon = 2$). The bottom right panel illustrates the effect of participation on the equity premium and the expected return on equity.

We finally investigate the implications of our analysis for the pricing of equity, which we model as a proxy for the market portfolio. Specifically, suppose assets e and m are perfectly correlated (otherwise, the equity premium puzzle becomes even deeper), and calibrate the volatility of e to match its historical average, $\sqrt{\Lambda_e} = 16\%$. By no arbitrage, the risk premium and the expected return on equity are respectively given by $\zeta_e = \zeta_m \frac{\sqrt{\Lambda_e}}{\sqrt{\Lambda_m}}$ and $r_f + \zeta_e$. The bottom right panel of Figure 5 shows that, as we increase participation, the equity premium declines from about 8% to 4%, and its expected return declines from about 9% to 6.5%.

4.2 Portfolio Customization

We next present our main general equilibrium result on customization. We capture increased portfolio customization with improved access to assets $j \in \{1, .., K-1\}$. The practical counterpart of these assets can be thought of as direct trading of individual stocks and bonds; investment funds that specialize in certain industries or styles; or derivatives such as futures, options, and ETFs. These financial instruments, like assets $j \in \{1, .., K-1\}$ in our model, enable investors to construct customized portfolios according to their needs or beliefs.

We model investors' demand for specific cash flows by allowing them to disagree about the underlying uncertainty, that is, $\mu_{\mathbf{z}}^i$ can be different for different i. Investors can also differ in their access to financial assets. Formally, investors' types have two dimensions, $\{i = (i_A, \mathbf{i}_B)\}_i$. The sub-type $i_A \in I_A$ captures the variation in investors' market access, while the sub-type $\mathbf{i}_B \in I_B$ (which itself is a vector) captures the variation in beliefs. Investors' beliefs are drawn independently of their market access. More specifically, the mass of type $i = (i_A, \mathbf{i}_B)$ investors is given by $n^i = n^{i_A} \times n^{\mathbf{i}_B}$, where n^{i_A} denotes the mass of investors with market access type i_A, with $\sum_{i_A} n^{i_A} = 1$, and $n^{\mathbf{i}_B}$ denotes the mass of investors with belief type $\mathbf{i}_B$, with $\sum_{\mathbf{i}_B} n^{\mathbf{i}_B} = 1$. Investors are identical in all dimensions other than possibly their market access and beliefs.

The access types are given by, $I_A = \{0, ..., K-1\}$, such that $J^{i_A} = \{m, 1, .., i_A\}$ for each $i_A \in I_A$. Hence, i_A denotes the number of the nonmarket assets the investor has gained access to (in increasing order). Note that all investors have access to the market portfolio (for simplicity). We model increased customization as a shift of mass from a type with less access to one with more access, that is, $\tilde{n}^{i_A^1} = n^{i_A^1} + \Delta n$ and $\tilde{n}^{i_A^0} = n^{i_A^0} - \Delta n$ where $i_A^1 > i_A^0$ and $\Delta n > 0$. This is equivalent to expanding the access set of a positive mass of investors to include the new financial assets $j \in \{i_A^0 + 1, .., i_A^1\}$.

The belief types are a collection of K-dimensional vectors, $I_B \subset \mathbb{R}^K$, such that type $\mathbf{i}_B$ investors have the belief, $\boldsymbol{\mu}_{\mathbf{z}}^{\mathbf{i}_B} = \boldsymbol{\mu}_{\mathbf{z}} + \mathbf{i}_B$, for the underlying uncertainty. Here, the type describes the deviation of the investor's belief from the average belief, $\boldsymbol{\mu}_{\mathbf{z}} \in \mathbb{R}^K$. We assume that for each type, $\mathbf{i}_B \in I_B$, the opposite type, $-\mathbf{i}_B \in I_B$, also exists and has equal mass,

$$n^{\mathbf{i}_B} = n^{-\mathbf{i}_B} \text{ for each } \mathbf{i}_B \in I_B. \tag{20}$$

This is a mild symmetry assumption that is satisfied for standard distributions. We also make the following assumption that shuts down belief disagreements on the market portfolio,

$$(\mathbf{F}_m)' \, \mathbf{i}_B = 0 \text{ for each } \mathbf{i}_B \in I_B, \text{ which also implies } \mu_m^{\mathbf{i}_B} = \mu_m. \tag{21}$$

This assumption provides analytical tractability but otherwise does not play an important role, as we illustrate below with a numerical example.

The upshot of these assumptions is a closed form characterization of equilibrium, which we present next. To state the result, we define the expected return on a risky asset according to the average belief as $E[r_j] = \mu_j - \log P_j$, where $\mu_j = \mathbf{F}'_j \boldsymbol{\mu}_z$, and the risk premium according to the average belief as $\zeta_j = E[r_j] + \frac{\Lambda_j}{2} - r_f$.

Lemma 2. *Consider the above setting with limited customization of portfolios, full participation in the market portfolio, and belief disagreements that satisfy* (20) *and* (21)*. There exists an equilibrium in which:*

(i) The average risk premium on each risky asset satisfies, $\zeta_j = \frac{\Lambda_{jm}}{\Lambda_m} \zeta_m$*, where* $\zeta_m = \gamma \Lambda_m$*.*

(ii) The risk-free rate, r_f*, is the unique solution to*

$$\frac{\eta_m P_m}{Y_0 + \eta_m P_m} = \sum_{i \in I} n^{i_A} n^{\mathbf{i}_B} a\left(r_{ce}^{(i_A, \mathbf{i}_B)}\right), \tag{22}$$

where the certainty equivalent return for an investor with type $(i_A, \mathbf{i}_B)$ *is,*

$$r_{ce}^{(i_A, \mathbf{i}_B)} = r_f + \frac{1}{2\gamma} \frac{\zeta_m^2}{\Lambda_m} + \frac{1}{2\gamma} \left(\mathbf{F}'_{J^{i_A}}(\mathbf{i}_B)\right)' \Lambda_{J^{i_A}}^{-1} \left(\mathbf{F}'_{J^{i_A}}(\mathbf{i}_B)\right). \tag{23}$$

The first part says that the average risk premium on a risky asset is determined by its "beta" with the market portfolio. It also characterizes the risk-premium on the market portfolio. These are standard asset pricing conditions that would also obtain in a version of our model without any heterogeneity in beliefs or any customization (beyond access to asset m, which we assume). Hence, for the purposes of characterizing the risk premia, or relative asset prices, heterogeneity in beliefs or the degree of customization can be ignored. Loosely speaking, in view of the symmetry assumption (20), for every "optimist" whose portfolio shares deviate from the average portfolio share in a particular direction, there is a "pessimist" whose portfolio deviates in exactly the opposite direction. Since belief heterogeneity does not influence investors' portfolio shares on average, it also does not influence relative asset prices. Customization does not influence relative prices either because, absent belief heterogeneity, the market portfolio m is sufficient to construct efficient portfolios in equilibrium.

The second part shows that, although heterogeneity or customization do not affect relative prices, they can influence absolute asset prices as well as the risk-free interest rate. The market clearing condition (22) characterizes the risk free rate in terms of investors' certainty-equivalent returns. Eq. (23) characterizes the certainty-equivalent returns in terms of investors' beliefs and access sets. The next result describes the effects of greater customization.

Proposition 3 (Customization). *Consider the equilibrium characterized in Lemma 2. Consider financial innovation that increases the scope of customization for some market participants,* $\tilde{n}^{i_A^1} = n^{i_A^1} + \Delta n$ *and* $\tilde{n}^{i_A^0} = n^{i_A^0} - \Delta n$ *where* $i_A^1 > i_A^0$ *and* $\Delta n > 0$*. This change reduces the risk free rate* r_f*, leaves unchanged the average risk premia,* $\left\{\zeta_j\right\}_{j \in \mathbf{J}}$ *(and relative prices,* $\{P_j/P_f\}$*), and decreases the average expected return on risky assets,* $\left\{r_f + \zeta_j\right\}_{j \in \mathbf{J}}$ *(increases* $\{P_j\}$*).*

The intuition follows from Eq. (23), which implies that the investor's certainty-equivalent return, $r_{ce}^{(i_A, \mathbf{i}_B)}$, is increasing in the number of available assets, i_A (see the appendix for a proof, and Eq. (24) below for a special case). Hence, consistent with the choice channel [cf. Proposition 1], greater customization increases investors' savings, $a\left(r_{ce}^{(i_A, \mathbf{i}_B)}\right)$. This in turn increases asset prices, as in Section 4.1. The difference is that increased customization does not generate relative price effects. Thus, unlike increased participation, greater customization increases the absolute price of all assets, while leaving relative asset prices unchanged.

In this context, financial innovation increases investors' certainty equivalent returns by enabling them to construct customized portfolios that feature speculative positions. We illustrate this point further in a special case, which we also use to numerically illustrate the results.

4.2.1 Special Case and Numerical Illustration

Suppose that the market portfolio satisfies, $\log \varphi_m = z_K$, and the sources of risk, $\{z_1, .., z_K\}$, are linearly independent. Hence, the source, z_K, captures a systematic risk factor, and the remaining sources, $\{z_1, .., z_{K-1}\}$, capture nonsystematic (e.g., idiosyncratic) risk factors that are orthogonal to the market portfolio as well as one another. For simplicity, suppose each investor is optimistic or pessimistic about one nonsystematic risk factor. Specifically, for each $k < K$, there are two belief types, $\mathbf{i}_{B,k}$ and $-\mathbf{i}_{B,k}$ that respectively think that the mean of z_k is given by $\mu_k + \Delta_k$ and $\mu_k - \Delta_k$, whereas they agree on the objective mean of the remaining risk factors. Thus, type $\mathbf{i}_{B,k}$ investors are optimistic about z_k (and only z_k), whereas type $-\mathbf{i}_{B,k}$ investors are pessimistic, and the degree of disagreement is captured by the parameter, $\Delta_k \geq 0$. All investors agree on the objective mean of the systematic risk factor, z_K. Note that investors' beliefs satisfy conditions (20) and (21).

Suppose also that there is one (nonmarket) asset per nonsystematic risk factor, that is, $\log \varphi_j = z_j$ for $j \in \{1, .., K-1\}$. These assets can be thought of as the nonsystematic component of stocks, bonds, or other similar financial assets.[19] We also assume investors have symmetric access to these assets, that is, $n^{\bar{i}_A} = 1$ for some $\bar{i}_A \leq K-1$ (and $n^{i_A} = 0$ for $i_A \neq \bar{i}_A$). Hence, the parameter, $\bar{i}_A$, captures the total number of nonmarket assets available to any investor. Using Eq. (23), the investors' certainty-equivalent return can be written as,

$$r_{ce}^{(\bar{i}_A, \mathbf{i}_{B,k})} = r_{ce}^{(\bar{i}_A, -\mathbf{i}_{B,k})} = \begin{cases} r_{ce}^{nonspec} \equiv r_f + \frac{\zeta_m^2}{2\gamma\Lambda_m} & \text{if } \bar{i}_A < k, \\ r_{ce}^{spec} \equiv r_f + \frac{\zeta_m^2}{2\gamma\Lambda_m} + \frac{\Delta_k^2}{2\gamma\Lambda_k} & \text{if } \bar{i}_A \geq k. \end{cases} \tag{24}$$

Here, $r_{ce}^{nonspec}$ is the "nonspeculative" return the investor can obtain by only trading the market portfolio, whereas $r_{ce}^{spec} > r_{ce}^{nonspec}$ is the "speculative" return she can obtain by combining the market portfolio with a position in the risk factor, z_k. The investor is able to obtain the greater speculative return only if the asset k is available for trade. Hence, greater customization (greater $\bar{i}_A$) increases investors' certainty equivalent returns by allowing more of them to construct customized portfolios with speculative positions.

Eq. (24) also implies that increased customization increases the asset holdings, $a\left(r_{ce}^{spec}\right) > a\left(r_{ce}^{nonspec}\right)$, by both the optimists and the pessimists about the risk factor, z_k. It might sound intuitive that optimists increase their savings and cut their consumption, as they need funds to take a long position in the asset. It is perhaps more surprising that the pessimists also increase their savings, since they could in principle consume the funds they generate from the short sale of the asset. As problem (13) illustrates, pessimists use the funds from the short sale to increase their holdings of the safe asset. These safe asset positions, combined with their short positions, is what enables them to obtain a high certainty-equivalent return on their portfolios. They cut consumption and increase asset holdings even further, because they perceive a high return on their overall portfolios.[20]

We next numerically illustrate Proposition 3. Consider the same baseline parameters

[19] When an asset is introduced for trade, investors can combine it with (a proxy of) the market portfolio to make a pure trade on its nonsystematic component, which is similar to trading the assets $j \in \{1, .., K-1\}$ in our example.

[20] Short selling in practice also requires saving in relatively safe assets, because of the margin collateral lenders require for both short and long positions. Collateral in the margin account, just like the investors' safe asset holdings in the model, provides protection for potential losses from the short position.

as in Section 4.1.1. To calibrate disagreements, let $\sqrt{\Lambda_k} = 50\%$ for each $k < K$, which roughly corresponds to the volatility of an average stock return. We assume $\Delta_k = 25\%$ for each $k < K$, so that optimists' perceived Sharpe ratios on asset k is equal to the historical Sharpe ratio on the market portfolio, that is, $\frac{\Delta_k}{\sqrt{\Lambda_k}} = 0.5$. This calibration makes our analysis comparable to the previous subsection on participation, by ensuring the access to assets k and m increase investors' certainty equivalent return by the same amount [cf. Eq. (24)]. In practice, investors are likely to disagree on multiple sources of risks, and arguably to a greater degree, which would lead to even higher speculative Sharpe ratios.

We investigate the effects of varying the degree of customization, $\bar{i}_A/(K-1)$, over the range, $[0, 1]$. Figure 6 illustrates the results of this exercise. The left panel shows that increasing customization reduces the risk-free rate and the expected return on the market portfolio, while leaving the risk premia constant, consistent with Proposition 3. The dashed lines illustrate the solution with $\varepsilon = 1$, which provides a useful benchmark by shutting down the choice channel. Note that, in contrast to Figure 5 on increased participation, the change in returns is entirely driven by the choice channel. Moreover, the choice channel is quantitatively stronger: It reduces the risk-free rate by about 1.5 percentage points, as opposed to less than 0.5pp for increased participation.

The choice channel is relatively strong in this case because, unlike increased participation, increased customization does not feature crowd-out effects. The top right panel of Figure 6 illustrates this by plotting the certainty-equivalent return from speculation for investors who disagree on the return of the newly introduced asset $\bar{i}_A$, which can be thought of as the marginal gain from customization. Note from Eq. (24) that the marginal gain is given by $r_{ce}^{spec} - r_{ce}^{nonspec}$, and does not depend on the level of customization. Consequently, greater customization induces a larger increase on the average certainty-equivalent return and asset holdings compared to greater participation [cf. Figures 6 and 5]. Intuitively, enabling speculation on an asset does not change relative asset prices. Thus, it does not preclude other investors from taking speculative positions on other (or even the same) assets. The absence of crowd-out effects induces a greater reduction in equilibrium asset returns.

We finally illustrate that condition (21) does not play an important role for our analysis beyond facilitating analytical tractability. To this end, suppose investors also disagree about the market portfolio. Specifically, an investor who is optimistic (resp. pessimistic) about a nonsystematic risk $k < K$ is also optimistic (resp. pessimistic) about the systematic risk K, and thus, the market portfolio, $\log \varphi_m = z_K$. We calibrate the level of disagreement on the market portfolio, $\Delta_m = \Delta_k \sqrt{\frac{\Lambda_m}{\Lambda_k}}$, so that investors are "equally" optimistic about systematic and nonsystematic risks (after normalizing by their relative volatilities). The bottom right panel of Figure 6 illustrates the results of increased customization in this case. Compared

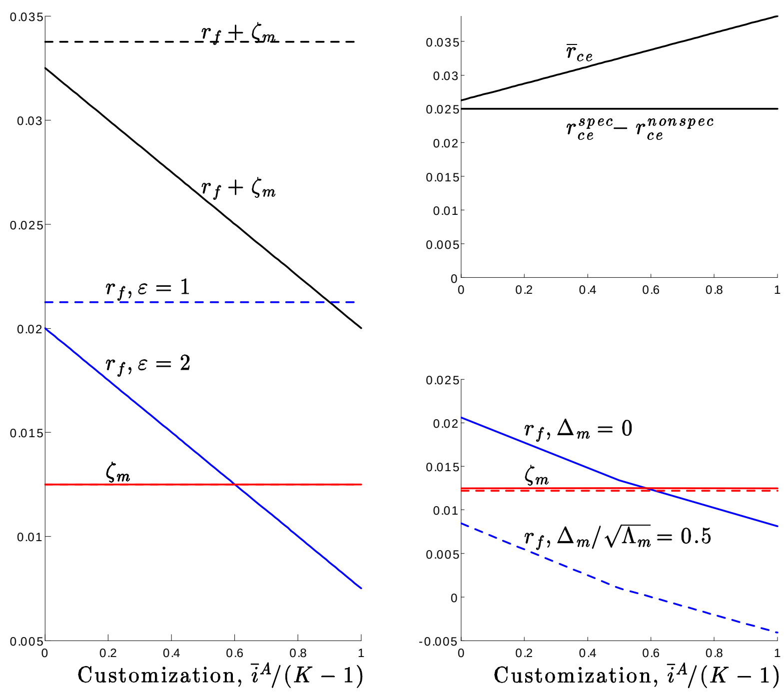

Figure 6: The left panel illustrates the effect of increased customization on asset returns for the main case, $\varepsilon = 2$ (solid lines) and the benchmark case, $\varepsilon = 1$ (dashed lines). The top right panel illustrates the marginal return-gain from customization, as well as the avreage certainty-equivalent return (when $\varepsilon = 2$). The bottom right panel illustrates the asset returns for the main case with no disagreement on the market portfolio (solid lines) and the alternative case with some disagreement on the market portfolio (dashed lines).

to the earlier case with $\Delta_m = 0$, the risk-free rate is uniformly lower. The risk premium is also slightly lower, but the difference is not discernible. More importantly, increased customization reduces the risk-free rate and does not have a discernible effect on the risk premium, as in Proposition 3, even though condition (21) is violated.

Absent condition (21), investors take speculative positions on the market portfolio as well as the nonsystematic risk factors. This generates an additional increase in their certainty-equivalent returns, and reduces the risk-free rate, as illustrated by Figure 6. The difference is that speculation on the market portfolio breaks the symmetry between optimists' and pessimists' returns in Eq. (24). Since the asset m is in positive supply, all investors are its natural buyers. Even if optimists did not adjust their positions (relative to the average investor), their perceived return would be higher simply because they are already holding the market portfolio. Therefore, in equilibrium, optimists obtain a greater certainty-equivalent return—and hold more assets—relative to pessimists. This asymmetry implies that belief disagreements can potentially also affect relative asset prices and risk premia, which makes a theoretical characterization difficult. However, for empirically relevant parameters, these effects are very small, as illustrated by Figure 6, and the results remain qualitatively unchanged.

4.3 Emerging Market Savings and Securitization

Another important innovation in recent years is structured finance, or securitization, which helps to distribute risky cash flows according to investors' risk preferences or beliefs. The growth of securitization in recent years is often linked with another phenomenon: the growing demand for safe assets from fast-growing emerging market economies such as China as well as other sources (see Gennaioli, Shleifer, and Vishny (2012)). In this section, we use a variant of the setup in Section 4.1 to investigate the effect of securitization in an environment with high demand for safe assets.

Suppose there are three types of investors, denoted by $\{(1,+),(1,-),2\}$, that have common beliefs (so it suffices to restrict attention to access to the market portfolio, m). Type 2 investors participate only in the safe asset, $J^2 = \emptyset$, similar to type 0 investors in Section 4.1. We now interpret these investors as corresponding to emerging market investors that have a preference for safe assets (due to unmodeled factors), in addition to having a high demand for assets. To capture the latter feature, we allow these investors to have a different asset holdings function, $a^2(\cdot)$, driven by different parameters, β^2, ε^2. We consider parameters that ensure that type 2 investors hold more assets than the remaining investors in equilibrium (see below for a precise statement).

To model securitization, we also depart from Section 4.1 by assuming that the remaining investors might face an additional constraint that prevents them from short selling the risk-free asset. Securitization relaxes this constraint, and enables a greater fraction of them to borrow and expand their investments in the market portfolio. We view this as capturing the essence of securitization in practice, which enables some investors (or banks) to make leveraged investments in risky assets by issuing relatively safe debt claims.

Specifically, type $(1,+)$ and $(1,-)$ investors are identical in all dimensions except for their ability to securitize. They have access to all (relevant) risky assets, $J^{(1,+)} = J^{(1,-)} = \{m\}$. However, type $(1,-)$ investors face an additional constraint, $\omega_f^{(1,-)} \geq 0$, that prevents them from short-selling the safe asset, whereas $(1,+)$ investors, which we refer to as "the securitizers," do not face this constraint.

Let n^2 and $n^1 = 1 - n^2$ denote the relative mass (or wealth share) of respectively type 2 and type 1 investors, and $n^+ \in [0,1]$ denote the fraction of securitizers within type 1 investors. In this context, our next result characterizes the equilibrium for fixed masses. We then describe the comparative statics of increasing the relative mass of emerging market investors, n^2, as well as the relative mass of securitizers, n^+. Our main analysis endogenizes the level of securitization, n^+, and characterizes how an increase in n^+ that is driven by an increase in n^2 affects the equilibrium. We then illustrate these results with a numerical example, which also shows that securitization has a different effect on type 1 and 2 investors' asset holdings, with implications for their net savings (or current accounts).

Lemma 3. *Consider the above setup with common beliefs, type 1 investors that differ in their ability to securitize, and type 2 investors that invest only in the safe asset. There exists an equilibrium in which ζ_m and r_f jointly solve,*

$$\zeta_m = \gamma \Lambda_m \left(1 + \frac{n^2}{(1-n^2)\, n^+} \frac{a^2(r_f)}{a^1\left(r_{ce}^{(1,+)}\right)} \right), \tag{25}$$

$$\frac{\eta_m P_m}{Y_0 + \eta_m P_m} = n^2 a^2(r_f) + \left(1-n^2\right) \overline{a}^1, \text{ where } \overline{a}^1 = n^+ a^1\left(r_{ce}^{(1,+)}\right) + \left(1-n^+\right) a^1\left(r_{ce}^{(1,-)}\right), \tag{26}$$

and the certainty-equivalent returns of type 1 investors satisfy $r_{ce}^{(1,+)} \geq r_{ce}^{(1,-)}$, where,

$$r_{ce}^{(1,+)} = r_f + \frac{\zeta_m^2}{2\gamma\Lambda_m} \text{ and } r_{ce}^{(1,-)} = r_f + \zeta_m - \frac{\gamma}{2}\Lambda_m. \tag{27}$$

The result is similar to Lemma 1 with limited participation.[21] The main difference is that

[21] Unlike Lemma 1, we are unable to establish the uniqueness of equilibrium, since $a^2(\cdot)$ and $a^1(\cdot)$ are

the aggregate risk that is not held by type 2 investors is now absorbed by a smaller fraction of investors that are able to securitize. Eq. (25) says that the risk premium is determined by the compensation required by the securitizers. Another difference is that type 2 and 1 investors have different asset holding functions, which is captured by the market clearing equation (26). The final difference is that securitizers obtain a greater certainty-equivalent return than non-securitizers. Both of these returns are characterized by Eq. (27). We next describe how an increase in the relative wealth share of type 2 investors affects the equilibrium.

Proposition 4 (Emerging Market Savings)**.** *Consider an equilibrium characterized in Lemma 3 that features greater savings by type* 2 *investors,* $a^2(r_f) \geq \overline{a}^1$*, and satisfies the condition* $\left.\frac{da^2(r_{ce})/dr_{ce}}{a^2(r_{ce})}\right|_{r_{ce}=r_f} \geq \left.\frac{da^1(r_{ce})/dr_{ce}}{a^1(r_{ce})}\right|_{r_{ce}=r_{ce}^{(1,+)}}$. *Then an increase in the relative mass of type* 2 *investors,* n^2*, increases the risk premium,* ζ_m*, and decreases the risk-free rate,* r_f*.*

The result applies for an equilibrium in which type 2 investors hold more assets despite their lower certainty-equivalent returns. This is the case as long as β^2 is sufficiently greater than β^1. The result also requires the technical condition that type 2 investors' asset holdings are more reactive to changes in return relative to the remaining investors. This condition is typically satisfied if ε^2 is sufficiently greater than ε^1, but it does not play an important role beyond facilitating analytical tractability.[22] Under these conditions, the proposition says that an increase in n^2 increases the price of the safe assets, P_f, while reducing the relative price of risky assets, P_m/P_f.

The intuition for the relative price effect is the same as in Proposition 2. The increase in n^2 is similar to a reversal of increased participation, with the implication that it reverses the decline of the risk premium. Unlike in Proposition 2, however, the increase in n^2 has an unambiguous effect on the risk-free rate. This is because type 2 investors not only prefer safe assets but they also demand more assets relative to the remaining investors. The combination of these features ensures that the increase in their relative wealth share decreases the risk-free rate, r_f, consistent with the savings glut hypothesis (see, for instance Bernanke (2005)). We next describe how financial innovation that expands securitization affects the equilibrium in this environment.

Proposition 5 (Securitization)**.** *Consider an equilibrium that satisfies the conditions in Proposition 4. An increase in the relative mass of securitizers,* n^+*, decreases the risk premium,* ζ_m*, and decreases the expected return on the market portfolio,* $r_f + \zeta_m$*.*

potentially different, although the equilibrium is unique in all of our numerical simulations.

[22]In particular, the results in Proposition 4 continue to hold in our numerical simulations even if we assume $\varepsilon^2 = \varepsilon^1$.

Comparing Propositions 2 and 5 illustrates that increased securitization has the same qualitative effects on equilibrium prices as increased participation. It increases the relative price of the risky assets, P_m/P_f, as well as the absolute price of all assets, P_m. In our numerical simulations, it also typically decreases the price of the safe asset, P_f. Intuitively, similar to participation, securitization increases the relative demand for the risky asset, which reduces the risk premium and tends to increase the risk-free rate.

4.3.1 Endogenous Securitization

Securitization in recent years has arguably increased in response to the growing asset demand from emerging markets. We next incorporate this feature into the model by endogenizing the level of securitization, n^+, via free entry. Specifically, suppose all type 1 investors are initially non-securitizers. However, each one of them can become a securitizer by paying a fixed cost $c > 0$ per unit of assets. The optimality condition to become a securitizer can then be written as,

$$r_{ce}^{(1,+)} - r_{ce}^{(1,-)} \geq c \text{ with strict inequality only if } n^+ = 1, \tag{28}$$

where $r_{ce}^{(1,+)}$ and $r_{ce}^{(1,-)}$ are given by Eq. (27). The condition says that the marginal benefit of securitization is equated to its marginal cost, except possibly for a corner solution in which all type 1 investors become securitizers. The equilibrium is defined as before, with the additional requirement that n^+ is endogenous and condition (28) holds. Our next result characterizes this equilibrium.

Lemma 4. *Consider the setup in Lemma 3 with endogenous entry by securitizers. There exists an equilibrium in which ζ_m, r_f, n^+ jointly solve Eqs. (25) – (26) and $(1-n^+)\left(\zeta_m - \overline{\zeta}_m\right) = 0$, along with $\zeta_m \geq \overline{\zeta}_m$, where $\overline{\zeta}_m$ is the unique positive solution to,*

$$\frac{\overline{\zeta}_m^2}{2\gamma\Lambda_m} - \overline{\zeta}_m + \frac{\gamma}{2}\Lambda_m = c. \tag{29}$$

Here, $\overline{\zeta}_m \geq \gamma\Lambda_m$ is the break-even level of the risk premium which ensures that condition (28) holds as an equality. The result says that, as long there is an interior level of securitization, the level of the risk premium is given by the break-even level. We next present the main result in this section, which describes the effect of increasing the relative wealth share of type 2 investors in an environment with endogenous securitization.

Proposition 6 (Emerging Market Savings with Endogenous Securitization)**.** *Consider an equilibrium characterized in Lemma 4 which also satisfies $n^+ < 1$, greater savings by type 2* *investors,* $a^2(r_f) \geq \overline{a}^1$*, and the condition* $\left.\frac{da^2(r_{ce})/dr_{ce}}{a^2(r_{ce})}\right|_{r_{ce}=r_f} \geq \left.\frac{da^1(r_{ce})/dr_{ce}}{a^1(r_{ce})}\right|_{r_{ce}=r_{ce}^{(1,+)}}$*. Then, an increase in the relative mass of type 2 investors,* n^2*, increases the level of securitization,* n^+*. In addition, it leaves unchanged the risk premium,* $\zeta_m = \overline{\zeta}_m$*, decreases the the risk-free rate,* r_f*, and decreases the expected return on all risky assets.*

The result captures the conventional wisdom that the growing demand for safe assets from emerging markets induces greater securitization. It also illustrates that the combined effect of greater asset demand and greater securitization is to reduce the risk-free rate while having a smaller impact (in fact, in our stylized model, no impact) on the risk premium. Put differently, endogenous securitization fully mitigates the impact of emerging market savings on the risk premium, while only partially mitigating its impact on the risk-free rate. Intuitively, for interior levels of securitization, the risk premium is determined by the marginal cost of securitization as illustrated by Eq. (29).[23] Consequently, the increased asset demand—driven by both greater n_2 and greater n^+—translates into a reduction in the risk-free rate.

4.3.2 Numerical Illustration

We next numerically illustrate these results, along with some other features of the equilibrium. We use the baseline parameters as in Section 4.1, e.g., $\gamma = 5, \sqrt{\Lambda_m} = 5\%$, and ε^1, β^1 are the same as before. We let n^2 vary over the range, $[1\%, 10\%]$, which captures the roughly 10-fold growth of the emerging market central bank reserves since late 1990s (cf. Figure 4). We assign a relatively high discount factor to type 2 investors, $\beta^2 = 1.2\beta^1$, along with the elasticity of substitution, $\varepsilon^2 = 1.2\varepsilon^1 = 2.4$. We calibrate the cost of securitization c so that the implied equity premium (with $\sqrt{\Lambda_e} = 0.16$) is equal to 5%, which is close to its level in Section 4.1 with full participation (cf. Figure 5). We also let n^+_{init} denote the endogenous level of securitization that obtains with this cost level and the initial level of type 2 investors, $n^2_{init} = 1\%$.

The panels on the left side of Figure 7 illustrate the effect of increasing n_2 while keeping the securitization constant at its initial level, n^+_{init}. The second panel shows that, consistent with Proposition 4, the risk premium increases and the risk-free rate declines. The third panel illustrates the certainty-equivalent return for type 2 investors as well as the average certainty-equivalent return for type 1 investors.[24] The general decline in asset returns reduces

[23] Our setup with a fixed marginal cost of securitization is admittedly extreme. However, securitization would also greatly—if not fully—mitigate the impact on the risk premium in alternative specifications, as long as the marginal cost is not increasing too fast in the level of securitization.

[24] Here, $\overline{r}^1_{ce}$ is defined as the solution to $a\left(\overline{r}^1_{ce}\right) = (1-n^+)\, a\left(r^{(1,-)}_{ce}\right) + n^+ a\left(r^{(1,+)}_{ce}\right)$ (see Section 4.1).

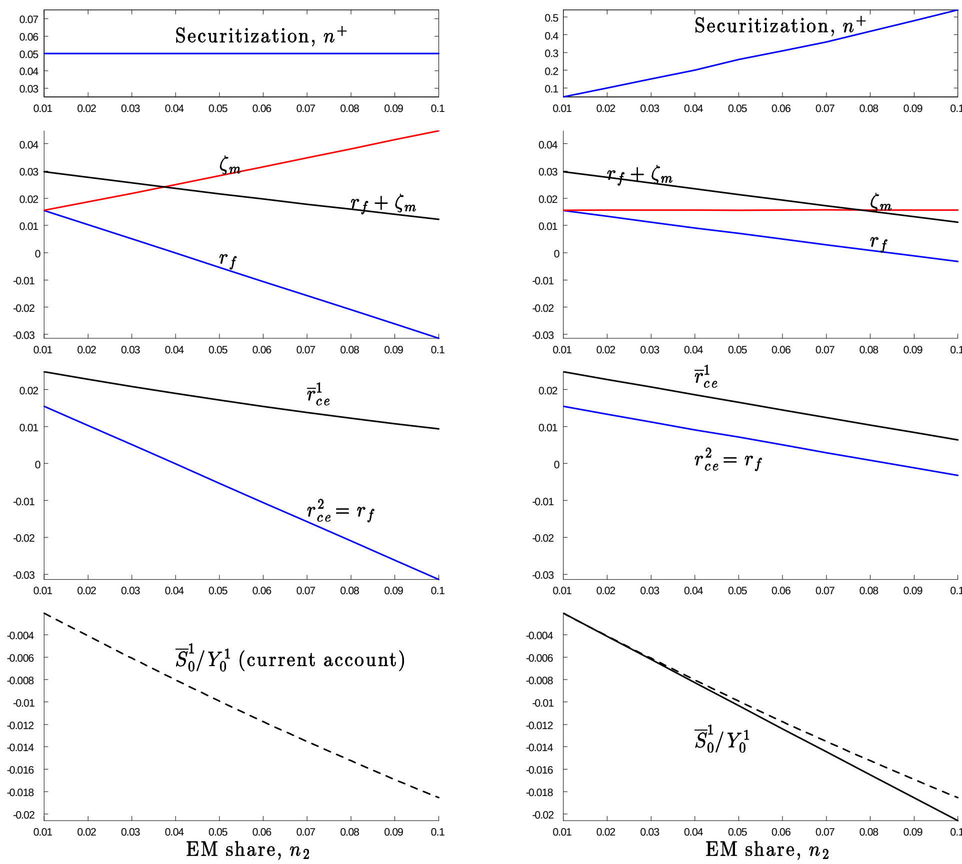

Figure 7: The left panels illustrate the effects of increasing the emerging market share, n_2, on equilibrium variables when the level of securitiation, n^+, remains constant. The right panels illustrate the corresponding effects when the level of securitization endogenously adjusts to the increase in the emerging market share.

the certainty-equivalent return for both types. However, the decline is dampened for type 1 investors since they benefit from the rising risk premium. The bottom panel illustrates that the average (net) savings of type 1 investors as a fraction of their income, $\overline{S}_0^1/Y_0^1 = \left((1-n^+)\,S_0^{(1,-)} + (1-n^+)\,S_0^{(1,+)}\right)/Y_0^1$, which corresponds to their current account in this model. Type 1 investors are running a current account deficit driven by the high asset demand by type 2 investors.

The panels on the right side of Figure 7 illustrate the effect of increasing n_2 in the model with endogenous securitization. Consistent with Proposition 6, greater emerging market share induces greater securitization. The second panel shows that the combined effect leaves the risk-premium unchanged while reducing the risk-free rate. Note, however, that the risk-free rate is greater than what it would be in the absence of the securitization response. The third panel illustrates that this also implies an increase in the certainty-equivalent return for type 2 investors, and therefore, their asset holdings. In contrast, the average certainty-equivalent return of type 1 investors is similar to the case without the securitization response. Intuitively, while greater securitization increases the certainty-equivalent return for type $(1,-)$ investors, it also decreases the certainty-equivalent return for type $(1,+)$ investors due to a crowd-out effect as in Section 4.1. The bottom panel illustrates further that greater securitization exacerbates the current account deficit of type 1 investors, because it increases type 2 investors' asset holdings without affecting much type 1 investors' average asset holdings.

The example also illustrates the additional conceptual point that financial innovation can increase the savings of some investors even when it does not directly expand their portfolio choice. Greater securitization increases the asset holdings of type 2 investors, despite the fact that the safe asset is available to them before and after financial innovation. Intuitively, type 1 and 2 investors would like to split the cash flows from risky assets according to their heterogeneous preferences. Securitization, which expands the portfolio choice of type 1 investors, facilitates the splitting of cash flows and raises the savings of type 2 investors in equilibrium. Hence, the choice channel, which we formalized for an investor in partial equilibrium (cf. Proposition 1), can also have spillover effects on other investors' savings.

5 Conclusion

Rapid financial innovation in recent years has vastly expanded the portfolio choice of investors. We theoretically investigate the implications of financial innovation for investors' savings and asset returns. Our main result establishes a choice channel by which, under mild assumptions, an investor that gains access to greater portfolio choice increases her savings.

The saving rate increases relatively more for wealthier investors, as well as investors that are less risk averse.

In equilibrium, greater savings exert a generally downward pressure on asset returns, but the precise effects also depend on the type of financial innovation. We show that, under mild assumptions, greater portfolio customization reduces the expected return on all assets, including the risk-free rate, without affecting the risk premia. In contrast, for empirically relevant parameters, greater participation increases the risk-free interest rate, and reduces the risk premium (as well as the expected return) on the market portfolio. Greater securitization has similar pricing implications as greater participation, which mitigate—but does not completely undo—the effects of increased savings from emerging markets. We also find that greater securitization facilitates the absorption of emerging market savings, and can exacerbate global current account imbalances.

Our results are broadly consistent with various trends in financial innovation and asset returns in the US over the last half century. Between 1950 and the early 1980s, market participation increased considerably, which might have contributed to the decrease in the risk premium and the increase in the risk-free rate over this period. Starting in the early 1980s, financial instruments that facilitate portfolio customization have become widespread, which might have contributed to the secular decline of the risk-free rate and other asset returns since the 1980s. Securitization started to accelerate in the early 2000s, arguably in response to the increasing demand for safe assets from emerging markets, but collapsed with the recent financial crisis. The collapse of securitization might have contributed to the sharp reduction in the risk-free interest rate, the sharp increase in the risk premium, as well as the reduction in the current account deficit of the US since the financial crisis.

Our results also shed some light on the low interest rates in recent years. These rates are worrisome from a macroeconomic policy point of view, as they increase the likelihood of liquidity trap episodes in which monetary policy is constrained. Our analysis suggests that financial innovation that facilitates portfolio customization might be a contributing factor to low interest rates. We also show that other types of financial innovation that facilitate participation or securitization might help to increase the interest rates.

Even though our analysis has been purely positive, our model also has policy implications. For instance, through the lens of our model, restricting portfolio customization or subsidizing securitization might be beneficial by reducing the incidence of liquidity traps. More generally, our analysis highlights that financial innovation affects investors' consumption and savings decisions, with implications for aggregate demand. Economic agents that introduce or adopt these financial innovations do not internalize their effects on aggregate demand, which might create inefficiencies (see Korinek and Simsek (2014)). We leave a more complete analysis of the interaction between financial innovation and aggregate demand externalities for future work.

A Appendix A: Omitted Extensions

A.1 Precautionary Channel

In the main text, we focused on the cases in which investors effectively do not face any background risks so that they do not have precautionary savings concerns. We next illustrate the effect of financial innovation in an environment with precautionary savings. We obtain an alternative precautionary channel of financial innovation, and contrast it with our choice channel.

Consider the partial equilibrium setup in Section 3. Suppose, in addition, that there is a risk-neutral belief distribution, $\{\pi^n(\mathbf{z})\}_Z$, that prices all assets, that is,

$$P_j = \frac{P_f}{\overline{\varphi}_f} \sum_{\mathbf{z}\in Z} \pi^n(\mathbf{z}) \varphi_j(\mathbf{z}) \text{ for each } j \in \mathbf{J}. \tag{A.1}$$

This assumption holds, for instance, if there is a positive mass of investors that can access all assets (which implies no arbitrage). Suppose also that the investor's belief satisfies the following.

Assumption 1^{P}. $\pi^i(\mathbf{z}) = \pi^n(\mathbf{z})$ for each $\mathbf{z} \in Z$, which also implies $E^i[\cdot] = E^n[\cdot]$.

This assumption is quite strong, as it implies that the investor's (perceived) expected return on every asset is the risk-free interest rate. Under this assumption, a risk-averse investor without background risks would not want to hold any risky financial assets. Hence, the assumption ensures that the investor demands financial assets only to hedge her background risks.

Proposition 7 (Precautionary Channel)**.** *Suppose there is a risk-neutral distribution [cf. (A.1)], Assumption 1^P holds, and the investor has CRRA preferences, $\gamma^i = 1/\varepsilon^i$. Suppose also that financial innovation completes the market in the sense that, for each $\mathbf{z} \in Z$, there exists $j_{\mathbf{z}} \in J^{i,new}$ such that $\varphi_{j_{\mathbf{z}}}(\mathbf{z}) > 0$ and $\varphi_{j_{\mathbf{z}}}(\tilde{\mathbf{z}}) = 0$ for each $\tilde{\mathbf{z}} \neq \mathbf{z}$. Then, financial innovation reduces the investor's asset holdings (and thus, savings), $A_0^{i,new} \leq A_0^{i,old}$, with strict inequality if $C_1^{i,old}(z_1) \neq C_1^{i,old}(z_2)$ for some $z_1, z_2 \in Z$.*

Proof. Let $u^i(c) = c^{1-\gamma^i}$ denote the investor's state utility function. First consider the investor's allocation before financial innovation. The optimality condition for the risk-free asset can then be written as,

$$u'\left(C_0^{i,old}\right) = \left(\beta^i/P_f\right) E\left[u'\left(C_1^{i,old}(\mathbf{z})\right)\right].$$

The key observation is that the CRRA preferences satisfy the prudence condition, $u'''(c) > 0$. In view of this observation (and Jensen's inequality), the optimality condition implies,

$$u'\left(C_0^{i,old}\right) \geq \left(\beta^i/P_f\right) u'\left(E\left[C_1^{i,old}(\mathbf{z})\right]\right). \tag{A.2}$$

This expression illustrates the precautionary savings motive. When $\beta^i/P_f = 1$, the investor would like to have greater average consumption in the future compared to the current period. Next consider the investor's allocation after financial innovation. In view of Assumption 2^{P}, and the assumption that financial assets complete the market, the investor chooses the perfect risk sharing allocation, that is, $C_1^{i,new}(\mathbf{z}) \equiv \overline{C}_1^{i,new}$ for each $\mathbf{z} \in Z$. Consequently, the optimality condition for the safe asset implies,

$$u'\left(C_0^{i,new}\right) = \left(\beta^i / P_f\right) u'\left(\overline{C}_1^{i,new}\right). \tag{A.3}$$

Finally, in view of Assumption 1, the investor's consumption in either case satisfies her lifetime budget constraint,

$$C_0^i + P_f E^n\left[C_1^i(\mathbf{z})\right] = Y_0^i + W_0^i + P_f E^i\left[L_1^i(\mathbf{z})\right].$$

Combining Eqs. $(A.2)$ and $(A.3)$ with the lifetime budget constraint implies that $C_0^{i,new} \geq C_0^{i,old}$, or equivalently, $A_0^{i,new} \leq A_0^{i,old}$. Moreover, the inequality is strict whenever the investor's consumption with old assets features less than perfect risk sharing, completing the proof. □

Hence, consistent with much of the precautionary savings literature (see Section 1.1), financial innovation induces the investor to save less, and strictly so when she faces some income risks before innovation. Intuitively, when markets are incomplete, the investor saves for precautionary reasons. This is because she faces some background risks, and the time-separable CRRA preferences satisfy the prudence condition. Financial innovation enables the investor to hedge her risks. By doing so, it alleviates the precautionary demand for saving, thereby reducing savings.

This intuition also illustrates the fragility of the result. The argument relies on the fact that the investor demands the new financial assets mainly to reduce her portfolio risks. If instead the new financial assets increase the investor's portfolio risks, perhaps because they enable her to participate in sharing the aggregate risk or speculate against other investors, then the argument does not hold. In fact, in the main text, we have argued that the opposite result holds under mild assumptions. Specifically, Proposition 1 in the main text drops assumption 1^{P} and replaces it with the assumption that investors do not face any background risks. This means that the investor holds new risky assets only to share aggregate risks or to speculate against other investors. In this alternative setup, as long as $\varepsilon^i > 1$, providing the investor with greater choice induces her to save more, in sharp contrast to Proposition 7.

B Appendix B: Omitted Proofs

B.1 Proofs for the partial equilibrium analysis in Section 3

Proof of Proposition 1. Included in the main text. □

Proof of Observation 1.

To show parts (i) and (ii), we use the linearity in V_1^i and the Euler equation from the intertemporal problem for an investor, for which the borrowing constraint is not binding, to write

$$\tilde{A}_0^i = \max\left\{\frac{\beta^{\varepsilon^i}\left(R_{ce}^i\right)^{\varepsilon^i-1}}{1+\beta^{\varepsilon^i}\left(R_{ce}^i\right)^{\varepsilon^i-1}}\tilde{W}_0^i, \tilde{A}^{i,\min}\right\}. \tag{B.1}$$

Therefore, for an unconstrained investor,

$$\frac{\partial \tilde{A}_0^i}{\partial R_{ce}^i} = \left(\varepsilon^i - 1\right)\left(R_{ce}^i\right)^{\varepsilon^i}\tilde{W}_0^i\frac{\beta^{\varepsilon^i}}{\left[1+\beta^{\varepsilon^i}\left(R_{ce}^i\right)^{\varepsilon^i-1}\right]^2} > 0, \tag{B.2}$$

and similarly, $\frac{\partial \tilde{A}_0^i}{\partial \tilde{W}_0^i} > 0$. Notice that, since $R_{ce}^{i,new} \geq R_{ce}^{i,old}$, an investor that is unconstrained before financial innovation is also unconstrained after financial innovation. However, if an investor is constrained before financial innovation, she may either remain constrained after financial innovation or become unconstrained. Therefore, one has to examine three cases about whether the borrowing constraint binds for an investor before and after financial innovation.

For part (i), we have $\frac{\partial^2 \tilde{A}_0^i}{\partial R_{ce}^i \partial \tilde{W}_0^i} > 0$. Therefore, since $R_{ce}^{i,new} \geq R_{ce}^{i,old}$, if an investor is initially unconstrained, then,

$$\frac{\partial}{\partial \tilde{W}_0^i}\left(\tilde{A}_0^{i,new} - \tilde{A}_0^{i,old}\right) = \frac{\partial}{\partial \tilde{W}_0^i}\int_{R_{ce}^{i,old}}^{R_{ce}^{i,new}}\frac{\partial \tilde{A}_0^i\left(R, \tilde{W}_0^i\right)}{\partial R_{ce}^i}dR = \int_{R_{ce}^{i,old}}^{R_{ce}^{i,new}}\frac{\partial^2 \tilde{A}_0^i}{\partial R_{ce}^i \partial \tilde{W}_0^i}dR \geq 0.$$

where we can exchange the differentiation and integration since $\frac{\partial \tilde{A}_0^i}{\partial R_{ce}^i}$ is integrable. Since $A_0^{i,new} - A_0^{i,old} = \tilde{A}_0^{i,new} - \tilde{A}_0^{i,old}$, we also have that $A_0^{i,new} - A_0^{i,old}$ is increasing in $\tilde{W}_0^i$, and hence, in W_0^i. Next, if the investor is initially constrained and remains constrained after financial innovation, then

$$A_0^{i,new} - A_0^{i,old} = \tilde{A}_0^{i,new} - \tilde{A}_0^{i,old} = 0,$$

which does not depend on W_0^i. Finally, if the investor becomes unconstrained after financial innovation, then

$$A_0^{i,new} - A_0^{i,old} = A_0^{i,new} - A^{i,\min} = \tilde{A}_0^{i,new} - \tilde{A}^{i,\min},$$

which is increasing in W_0^i by $(B.2)$. It follows that in all three cases $\frac{\partial}{\partial W_0^i}\left(A_0^{i,new} - A_0^{i,old}\right) \geq 0$. Since $\frac{S_0^{i,new} - S_0^{i,old}}{Y_0^i} = \frac{A_0^{i,new} - A_0^{i,old}}{Y_0^i}$, the result follows.

For part (ii), notice that if the investor is unconstrained before financial innovation, then

$$\frac{\partial^2 \tilde{A}_0^i}{\partial R_{ce}^i \partial \beta} = \varepsilon^i \beta^{\varepsilon^i - 1}\left(\varepsilon^i - 1\right)\left(R_{ce}^i\right)^{\varepsilon^i}\tilde{W}_0^i\frac{1-\beta^{\varepsilon^i}\left(R_{ce}^i\right)^{\varepsilon^i-1}}{\left[1+\beta^{\varepsilon^i}\left(R_{ce}^i\right)^{\varepsilon^i-1}\right]^3},$$

which is > 0, iff $\beta^{\varepsilon^i}\left(R_{ce}^i\right)^{\varepsilon^i-1} < 1$. Using the Euler equation of the unconstrained investor, we have

$$\beta^{\varepsilon^i}\left(R_{ce}^{i,new}\right)^{\varepsilon^i-1} = \frac{\tilde{A}_0^{i,new}}{C_0^{i,new}} = \frac{1}{R_{ce}^{i,new}} \frac{V_1^{i,new}\left(\tilde{A}_0^{i,new}\right)}{C_0^{i,new}} < 1.$$

Therefore, $\frac{\partial^2 \tilde{A}_0^i}{\partial R_{ce}^i \partial \beta} > 0$, for each $R_{ce}^i \leq R_{ce}^{i,new}$. This implies,

$$\frac{\partial}{\partial \beta}\left(\tilde{A}_0^{i,new} - \tilde{A}_0^{i,old}\right) = \frac{\partial}{\partial \beta} \int_{R_{ce}^{i,old}}^{R_{ce}^{i,new}} \frac{\partial \tilde{A}_0^i\left(R,\beta\right)}{\partial R_{ce}^i} dR = \int_{R_{ce}^{i,old}}^{R_{ce}^{i,new}} \frac{\partial^2 \tilde{A}_0^i}{\partial R_{ce}^i \partial \beta} dR \geq 0.$$

Since $A_0^{i,new} - A_0^{i,old} = \tilde{A}_0^{i,new} - \tilde{A}_0^{i,old}$, we obtain, $\frac{\partial}{\partial \beta}\left(A_0^{i,new} - A_0^{i,old}\right) \geq 0$. Next, if the investor is constrained before financial innovation and remains constrained, then

$$\frac{\partial}{\partial \beta}\left(\tilde{A}_0^{i,new} - \tilde{A}_0^{i,old}\right) = \frac{\partial}{\partial \beta}\left(A_0^{i,new} - A_0^{i,old}\right) = 0.$$

Finally, if the investor is constrained before financial innovation and is unconstrained after financial innovation, then

$$\frac{\partial}{\partial \beta}\left(A_0^{i,new} - A_0^{i,old}\right) = \frac{\partial}{\partial \beta}\left(\tilde{A}_0^{i,new} - \tilde{A}^{i,\min}\right) = \frac{\partial \tilde{A}_0^{i,new}}{\partial \beta} > 0.$$

It follows that in all three cases $\frac{\partial}{\partial \beta}\left(A_0^{i,new} - A_0^{i,old}\right) \geq 0$. Since $\frac{S_0^{i,new} - S_0^{i,old}}{Y_0^i} = \frac{A_0^{i,new} - A_0^{i,old}}{Y_0^i}$, the result follows.

For part (iii), first of all note that

$$R_{ce}^{i,old} = \overline{\varphi}_f P_f^{-1}.$$

does not depend on the value of γ^i. Next, consider two investors, i_1 and i_2 who are identical except for different risk-aversion coefficients given by γ^{i_1} and γ^{i_2}, with $\gamma^{i_1} > \gamma^{i_2}$. Let $\left\{\tilde{x}_j^{i_1}\left(\tilde{A}_0\right)\right\}_{\{f\}\cup J^{new}}$ denote the utility maximizing portfolio of investor i_1, given asset holdings $\tilde{A}_0$ and let $C_1^{i_1}(\mathbf{z})$ denote the resulting $t = 1$ consumption for state $\mathbf{z} \in \mathbf{Z}$. Therefore,

$$V_1^{i_1}\left(\tilde{A}_0\right) = R_{ce}^{i_1,new}\tilde{A}_0 = \left(E^{i_1}\left[C_1^{i_1}(\mathbf{z})^{1-\gamma^{i_1}}\right]\right)^{1/\left(1-\gamma^{i_1}\right)}.$$

Notice, however, that $\left(E^{i_1}\left[C_1^{i_1}(\mathbf{z})^{1-\gamma^{i_1}}\right]\right)^{1/\left(1-\gamma^{i_1}\right)}$ is the certainty equivalent consumption for investor i_1, and since investor i_2 is less risk averse in the sense of $\gamma^{i_1} > \gamma^{i_2}$, it follows that

$$R_{ce}^{i_1,new}\tilde{A}_0 = \left(E^{i_1}\left[C_1^{i_1}(\mathbf{z})^{1-\gamma^{i_1}}\right]\right)^{1/\left(1-\gamma^{i_1}\right)} \leq \left(E^{i_1}\left[C_1^{i_1}(\mathbf{z})^{1-\gamma^{i_2}}\right]\right)^{1/\left(1-\gamma^{i_2}\right)} \leq V_1^{i_2}\left(\tilde{A}_0\right) = R_{ce}^{i_2,new}\tilde{A}_0.$$

Thus, $R_{ce}^{i_1,new} \leq R_{ce}^{i_2,new}$ Since $\widetilde{A}_0^i$ and A_0^i are increasing in R_{ce}^i by equation $(B.2)$, and since $R_{ce}^{i_1,old} = R_{ce}^{i_2,old}$ implies $A_0^{i_1,old} = A_0^{i_1,old}$, the result follows. □

B.2 Proofs for the general equilibrium analysis in Section 4

B.2.1 Proofs of results in section 4.1

We start with useful lemma about the behavior of the asset holding function.

Lemma 5. *Whenever $\epsilon > 1$, the semi-elasticity $\frac{a'(r_{ce})}{a(r_{ce})}$ is decreasing in r_{ce}.*

Proof. From the Euler Equation in logarithmic form

$$\log a(r_{ce}) - \log(1 - a(r_{ce})) = \epsilon \log \beta + (\epsilon - 1) r_{ce}$$

thus differentiating with respect to r_{ce} and simplifying

$$\frac{a'(r_{ce})}{a(r_{ce})} = (\epsilon - 1)(1 - a(r_{ce})) \tag{B.3}$$

so $\frac{a'(r_{ce})}{a(r_{ce})}$ is decreasing in a and therefore in r_{ce}, whenever $\epsilon > 1$. □

Proof of Lemma 1. To simplify notation, we leave implicit the dependence of $\omega_1(\zeta_m)$,$r_{ce}^1(r_f, \zeta_m)$ and $P_m(r_f + \zeta_m)$ on (r_f, ζ_m). Using Eq. (16), type 1 investors' portfolio share and return are given by,

$$\omega_m^1 = \frac{\zeta_m}{\gamma \Lambda_m} \text{ and } r_{ce}^1 = r_f + \frac{1}{2\gamma\Lambda_m}\zeta_m^2,$$

establishing Eq. (19).

Notice that the market clearing condition for the safe asset can be written as,

$$0 = n^1 \left(1 - \omega_m^1\right) a\left(r_{ce}^1\right) + n^0 a(r_f).$$

Rearranging this expression implies Eq. (17). Finally, Eq. (18) follows by adding all of the market clearing conditions (15).

It remains to show that the system in $(17) - (18)$ has a unique solution. Towards that end let us first define the average level of savings out of wealth as $\overline{a}\left(r_f, \zeta_m, n^1\right) \equiv n^1 a\left(r_{ce}^1\right) + \left(1 - n^1\right) a(r_f)$, and the relative value of the asset endowment as $v(r_f + \zeta_m) \equiv \frac{\eta_m P_m}{Y_0 + \eta_m P_m}$. Combined they characterize

$$\varphi_1(r_f, \zeta_m, n_1) \equiv \overline{a}\left(r_f, \zeta_m, n^1\right) - v(r_f + \zeta_m).$$

Notice that $v'(r_f + \zeta_m) = -Y_0 v(r_f + \zeta_m) < 0$. As a consequence, $\frac{\partial \varphi_1(r_f, \zeta_m, n_1)}{\partial r_f} = \frac{\partial \overline{a}}{\partial r_f} - v' > 0$, and $\frac{\partial \varphi_1(r_f, \zeta_m, n_1)}{\partial \zeta_m} = \frac{\partial \overline{a}}{\partial \zeta_m} - v' > 0$.

Additionally, we define

$$\varphi_2\left(r_f,\zeta_m,n_1\right)\equiv n^1\left(1-\omega^1\right)a\left(r_{ce}^1\right)+\left(1-n^1\right)a\left(r_f\right).$$

An equilibrium then is a solution to $\varphi_1\left(r_f,\zeta_m,n_1\right)=\varphi_2\left(r_f,\zeta_m,n_1\right)=0$.

Notice then that, $\frac{\partial\varphi_2}{\partial r_f}=n^1\left(1-\omega^1\right)a^{'}\left(r_{ce}^1\right)+\left(1-n^1\right)a^{'}\left(r_f\right)$. Additionally, $\varphi_2\left(r_f,\zeta_m,n_1\right)=0\implies\left(1-\omega^1\right)=-\frac{\left(1-n^1\right)}{n^1}\frac{a\left(r_f\right)}{a\left(r_{ce}^1\right)}$ and $\frac{\partial\varphi_2}{\partial r_f}=\frac{\left(1-n^1\right)}{a\left(r_f\right)}\left[\frac{a^{'}\left(r_f\right)}{a\left(r_f\right)}-\frac{a^{'}\left(r_{ce}\right)}{a\left(r_{ce}\right)}\right]$ which is positive whenever $\epsilon>1$, given Lemma 5. Last, $\frac{\partial\varphi_2}{\partial\zeta_m}=-\frac{\partial\omega^1}{\partial\zeta_m}n^1a\left(r_{ce}^1\right)+n^1\left(1-\omega^1\right)a^{'}\left(r_{ce}^1\right)\frac{\partial r_{ce}}{\partial\zeta_m}<0$ since $\left(1-\omega^1\right)<0$ whenever $\varphi_2=0$.

As a consequence, locus $\varphi_1\left(r_f,\zeta_m,n_1\right)=0$ is downward slopping in (r_f,ζ_m)-space while locus $\varphi_2\left(r_f,\zeta_m,n_1\right)=0$ is upward slopping. Both loci are characterized by continuous functions. We can use $\varphi_1\left(r_f,\zeta_m,n_1\right)=0$, with $\frac{\partial\varphi_1}{\partial\zeta_m}\neq0$, and the Implicit Function Theorem to define a decreasing function $\zeta_m^{\varphi_1}\left(\cdot\right)$ of the interest rate r_f over the first locus. We then look for a solution to $\varphi_2\left(r_f,\zeta_m^{\varphi_1}\left(r_f\right),n_1\right)=0$, where the left-hand side is a strictly increasing function of r_f. The existence of a solution is guaranteed by Proposition **8** and uniqueness follows from strict monotonicity. □

Proof of Proposition 2 Let $J\equiv\begin{bmatrix}\frac{\partial\varphi_1}{\partial r_f} & \frac{\partial\varphi_1}{\partial\zeta_m}\\ \frac{\partial\varphi_2}{\partial r_f} & \frac{\partial\varphi_2}{\partial\zeta_m}\end{bmatrix}$ and $\Delta_J<0$ denote its determinant. Then,

$$\begin{bmatrix}\frac{dr_f}{dn^1}\\ \frac{d\zeta_m}{dn^1}\end{bmatrix}=-\frac{1}{\Delta_J}\begin{bmatrix}\frac{\partial\varphi_2}{\partial\zeta_m} & -\frac{\partial\varphi_1}{\partial\zeta_m}\\ -\frac{\partial\varphi_2}{\partial r_f} & \frac{\partial\varphi_1}{\partial r_f}\end{bmatrix}\begin{bmatrix}a\left(r_{ce}^1\right)-a\left(r_f\right)\\ -\frac{a\left(r_f\right)}{n_1}\end{bmatrix}.$$

Therefore, $\frac{d\zeta_m}{dn^1}<0$. Also,

$$\begin{aligned}\frac{d\left[r_f+\zeta_m\right]}{dn^1} &\propto \left(a\left(r_{ce}^1\right)-a\left(r_f\right)\right)\left(\frac{\partial\varphi_2}{\partial\zeta_m}-\frac{\partial\varphi_2}{\partial r_f}\right)+\left(\frac{\partial\varphi_1}{\partial\zeta_m}-\frac{\partial\varphi_1}{\partial r_f}\right)\frac{a\left(r_f\right)}{n_1}\\ &= \left(a\left(r_{ce}^1\right)-a\left(r_f\right)\right)\left(\frac{\partial\varphi_2}{\partial\zeta_m}-\frac{\partial\varphi_2}{\partial r_f}\right)+\left(\frac{a^{'}\left(r_{ce}^1\right)}{a\left(r_{ce}^1\right)}-\frac{a^{'}\left(r_f\right)}{a\left(r_f\right)}\right)\frac{\left(1-n^1\right)}{n_1}\left(a\left(r_f\right)\right)^2<0\end{aligned}$$

again using Lemma 5. □

B.2.2 Proofs of results in section 4.2

Proof of Lemma 2. We define the average portfolio share of an asset j among all investors that have market access $i_A\in I_A$ as,

$$\omega_j^{i_A}=\frac{\sum_{I_B}n^{\mathbf{i}_B}\omega_j^{(i_A,\mathbf{i}_B)}a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right)}{\sum_{I_B}n^{\mathbf{i}_B}a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right)}. \tag{B.4}$$

We will establish the existence of an equilibrium in which prices are uniquely characterized by parts (i)-(ii), investors' certainty-equivalent returns are given by Eq. (23), and their average portfolio shares are given by,

$$\boldsymbol{\omega}^{i_A}_{J^{i_A}} = \frac{1}{\gamma}\Lambda^{-1}_{J^{i_A}}\boldsymbol{\zeta}_{j^{i_A}} = [\omega_m, 0, ..0]' \text{ for each } i_A, \text{ where } \omega_m = \frac{\zeta_m}{\gamma\Lambda_m}. \tag{B.5}$$

Here, $[\omega_m, 0, ..0]$ is a $|J^{i_A}|$-dimensional vector whose first entry is ω_m and the remaining entries are zero. Hence, in addition to the properties in the lemma, we claim that investors' average portfolio shares are independent of the heterogeneity in beliefs or market access.

We first establish Eq. $(B.5)$, given the prices characterized by parts (i)-(ii) and the certainty-equivalent returns in (23). To prove this, consider an investor's perceived risk premium for a risky asset j, which can be written as,

$$\zeta_j^{(i_A,\mathbf{i}_B)} = (\mathbf{F}_j)'\boldsymbol{\mu}^i_{\mathbf{z}} + \frac{\Lambda_j}{2} - \log P_j - r_f = \zeta_j + \mathbf{F}'_j\mathbf{i}_B. \tag{B.6}$$

Using Eq. (16), her demand for the risky assets J^{i_A} (as a proportion of her wealth) is given by the vector,

$$\boldsymbol{\omega}^{(i_A,\mathbf{i}_B)}_{J^{i_A}} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right) = \frac{1}{\gamma}\Lambda^{-1}_{J^{i_A}}\left(\boldsymbol{\zeta}_{j^{i_A}} + \mathbf{F}'_j\mathbf{i}_B\right) a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right).$$

In view of Eq. (23), investors of types $(i_A, \mathbf{i}_B)$ and $(i_A, -\mathbf{i}_B)$ obtain exactly the same certainty equivalent return. Combining these observations, the average demand across belief types $\mathbf{i}_B$ and $-\mathbf{i}_B$ is given by,

$$\frac{\boldsymbol{\omega}^{(i_A,\mathbf{i}_B)}_{J^{i_A}} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right) + \boldsymbol{\omega}^{(i_A,-\mathbf{i}_B)}_{J^{i_A}} a\left(r_{ce}^{(i_A,-\mathbf{i}_B)}\right)}{2} = \frac{1}{\gamma}\Lambda^{-1}_{J^{i_A}}\boldsymbol{\zeta}_{j^{i_A}} \times a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right).$$

Averaging across all belief types, and using Eq. (20), we further obtain,

$$\sum_{\mathbf{i}_B} n^{\mathbf{i}_B}\boldsymbol{\omega}^{(i_A,\mathbf{i}_B)}_{J^{i_A}} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right) = \left(\frac{1}{\gamma}\Lambda^{-1}_{J^{i_A}}\boldsymbol{\zeta}_{j^{i_A}}\right)\left(\sum_{\mathbf{i}_B} n^{\mathbf{i}_B} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right)\right).$$

Using the definition of the average portfolio share in $(B.4)$, we obtain $\boldsymbol{\omega}^{i_A}_{J^{i_A}} = \frac{1}{\gamma}\Lambda^{-1}_{J^{i_A}}\boldsymbol{\zeta}_{j^{i_A}}$. Next note that,

$$\left(\Lambda_{J^{i_A}}[\omega_m, 0, ..0]'\right)_j = \Lambda_{mj}\omega_m = \frac{1}{\gamma}\frac{\Lambda_{mj}\zeta_m}{\Lambda_m} = \frac{1}{\gamma}\zeta_j,$$

where the last equation uses part (i). Applying $\Lambda^{-1}_{J^{i_A}}$ to both sides of the expression implies, $\boldsymbol{\omega}^{i_A}_{J^{i_A}} = \frac{1}{\gamma}\Lambda^{-1}_{J^{i_A}}\boldsymbol{\zeta}_{j^{i_A}} = [\omega_m, 0, ..0]'$, proving Eq. $(B.5)$.

We next check that the investors' certainty-equivalent returns are given by Eq. (23). Using Eqs. (16) and $(B.6)$, we have,

$$
\begin{aligned}
r_{ce}^{i} &= r_f + \frac{1}{2\gamma}\left(\boldsymbol{\zeta}_{J^i} + \mathbf{F}'_{J^i}\mathbf{i}_B\right)' \Lambda_{J^i}^{-1}\left(\boldsymbol{\zeta}_{J^i} + \mathbf{F}'_{J^i}\mathbf{i}_B\right) \\
&= r_f + \frac{1}{2\gamma}\left(\boldsymbol{\zeta}'_{J^i}\Lambda_{J^i}^{-1}\boldsymbol{\zeta}_{J^i} + 2\left(\mathbf{F}'_{J^i}\mathbf{i}_B\right)\left(\Lambda_{J^i}^{-1}\boldsymbol{\zeta}_{J^i}\right) + \left(\mathbf{F}'_{J^i}\mathbf{i}_B\right)' \Lambda_{J^i}^{-1}\left(\mathbf{F}'_{J^i}\mathbf{i}_B\right)\right) \\
&= r_f + \frac{1}{2}\left(\boldsymbol{\zeta}'_{J^i}\left[\omega_m, 0, ..0\right]'\right) + \frac{1}{2\gamma}\left(\mathbf{F}'_{J^i}\mathbf{i}_B\right)' \Lambda_{J^i}^{-1}\left(\mathbf{F}'_{J^i}\mathbf{i}_B\right) + \left(\mathbf{F}'_{J^i}\mathbf{i}_B\right)\left[\omega_m, 0, ..0\right]' \\
&= r_f + \frac{1}{2\gamma}\frac{\zeta_m^2}{\Lambda_m} + \frac{1}{2\gamma}\left(\mathbf{F}'_{J^i}\mathbf{i}_B\right)' \Lambda_{J^i}^{-1}\left(\mathbf{F}'_{J^i}\mathbf{i}_B\right),
\end{aligned}
$$

verifying Eq. (23). Here, the third line uses Eq. (B.5), and the last line uses the assumption (21) that there is no disagreement on the market portfolio, so that $(\mathbf{F}_m)'\,\mathbf{i}_B = 0$.

Next note that parts (i)-(ii) uniquely characterize the equilibrium prices of all assets. We finally check that these prices satisfy the $|\mathbf{J}|+1$ market clearing conditions (15). The conditions for $j \neq m$ hold because $\omega_j^{i_A} = 0$ for each i_A and $j \neq m$. To check the remaining conditions, substitute $\omega_m = 1$ in view of part (i). After this substitution, the market clearing condition for asset f holds since each investor has a zero weight on the risk-free asset, $\omega_f = 1 - \omega_m = 0$. The market clearing condition for asset m also holds, since the condition becomes identical to Eq. (22) in part (ii). This establishes the existence of an equilibrium that satisfies the conditions in the lemma along with Eq. (B.5), completing the proof. □

Proof of Proposition 3. To show this result, notice that a direct extension of the revealed preference argument underlying the Choice Channel (Proposition **1**) to the case of a continuous state space implies that $R_{ce}^{i_A^1} \geq R_{ce}^{i_A^0}$, and hence, $r_{ce}^{(i_A^1,\mathbf{i}_B)} \geq r_{ce}^{(i_A^0,\mathbf{i}_B)}$. Alternatively, a direct inspection of Eq. (23) and the observation that $\left(\mathbf{F}'_{J^{i_A}}\left(\mathbf{i}_B\right)\right)' \Lambda_{J^{i_A}}^{-1}\left(\mathbf{F}'_{J^{i_A}}\left(\mathbf{i}_B\right)\right)$, the square of the speculative Sharpe ratio (Simsek (2013b)), is higher for the i_A^1 investor also gives the same result. Next, re-write (22) as

$$
\sum_{i\in I} n^{i_A} n^{\mathbf{i}_B} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right) - \frac{\eta_m P_m}{Y_0 + \eta_m P_m} = 0 \qquad \text{(B.7)}
$$

and notice that the left-hand side is increasing in r_f, since $r_{ce}^{(i_A,\mathbf{i}_B)}$ is increasing in r_f and $a(.)$ is an increasing function, so the first term is increasing in r_f, and also P_m is decreasing in r_f, so the second term is also increasing in r_f. Finally, since $r_{ce}^{(i_A^1,\mathbf{i}_B)} \geq r_{ce}^{(i_A^0,\mathbf{i}_B)}$, it follows that $\sum_{i\in I} n^{i_A} n^{\mathbf{i}_B} a\left(r_{ce}^{(i_A,\mathbf{i}_B)}\right)$ is increasing in Δn, and so, the left-hand side of (B.7) is increasing in Δn. Hence, r_f is decreasing in Δn.

Showing that $\{\zeta_j\}_{j\in\mathbf{J}}$ remain unchanged follows directly from Lemma **2** (i). Finally, showing that the average expected return on risky assets decreases follows from the behavior of r_f and $\{\zeta_j\}_{j\in\mathbf{J}}$. □

B.3 Proofs of results in section 4.3

Proof of Lemma 3. Type $(1,+)$ investors' portfolio share and return are the same as before,

$$\omega_m^{(1,+)} = \frac{\zeta_m}{\gamma\Lambda_m} > 1 \text{ and } r_{ce}^{(1,+)} = r_f + \frac{1}{2\gamma\Lambda_m}\zeta_m^2.$$

In view of the short selling constraint for the safe asset, type $(1,-)$ investors' portfolio share is given by $\omega_m^{(1,-)} = \max\left(\frac{\zeta_m}{\gamma\Lambda_m}, 1\right)$. In the conjectured equilibrium (and in fact, in any equilibrium), we have $\zeta_m > \gamma\Lambda_m$. Thus, the constraint binds and we have,

$$\omega_m^{(1,-)} = 1 \text{ and } r_{ce}^{(1,-)} = r_f + \zeta_m - \frac{\gamma}{2}\Lambda_m.$$

Note that $\zeta_m > \gamma\Lambda_m$ also implies $r_{ce}^{(1,+)} > r_{ce}^{(1,-)}$, establishing Eq. (27).

Next note that the market clearing condition for the safe asset can be written as,

$$0 = n^{(1,+)}\left(1 - \omega_m^{(1,+)}\right) a\left(r_{ce}^{1s}\right) + n^2 a\left(r_f\right).$$

Rearranging this expression implies Eq. (25). Finally, Eq. (26) follows by adding all of the market clearing conditions (15). Existence is ensured by Proposition 8, the proof of which is not altered by short-selling constraints that only apply to a subset of agents. □

We use the following definitions across the proof of both propositions that follow. First, we define the average levels of savings as $\overline{a}\left(r_f, \zeta_m, n\right) = n^2 a^2\left(r_f\right) + n^1\left(n^+ a^1\left(r_{ce}^{(1,+)}\right) + \left(1 - n^+\right) a^1\left(r_{ce}^{(1,-)}\right)\right)$ and the relative value of asset endowments as $v\left(r_f + \zeta_m\right) \equiv \frac{\eta_m P_m}{e_0 + \eta_m P_m}$.Therefore, $v'\left(r_f + \zeta_m\right) = -e_0 v\left(r_f + \zeta_m\right)$.

The market clearing conditions can be rewritten as

$$\varphi_1\left(r_f, \zeta_m, n\right) \equiv \overline{a}\left(r_f, \zeta_m, n\right) - v\left(r_f + \zeta_m\right) = 0$$

and

$$\varphi_2\left(r_f, \zeta_m, n\right) \equiv n^2 a^2\left(r_f\right) + n^1 n^+\left(1 - \omega^{(1,+)}\right) a^1\left(r_{ce}^{(1,+)}\right) = 0.$$

We have $\frac{\partial\varphi_1}{\partial r_f} = \frac{\partial\overline{a}}{\partial r_f} - v' > 0$ and $\frac{\partial\varphi_1}{\partial\zeta_m} = \frac{\partial\overline{a}}{\partial\zeta_m} - v' > 0$. Notice that $\frac{\partial\varphi_1}{\partial r_f} - \frac{\partial\varphi_1}{\partial\zeta_m} = n^2 a^2\left(r_f\right)\left(\frac{a^{0'}\left(r_f\right)}{a^2\left(r_f\right)} - \frac{a^{1'}\left(r_{ce}^{(1,+)}\right)}{a^1\left(r_{ce}^{(1,+)}\right)}\right) > 0$.

Also, $\frac{\partial\varphi_2}{\partial r_f} = n^2 a^2\left(r_f\right)\left(\frac{a^{0'}\left(r_f\right)}{a^2\left(r_f\right)} - \frac{a^{1'}\left(r_{ce}^{(1,+)}\right)}{a^1\left(r_{ce}^{(1,+)}\right)}\right) > 0$ and $\frac{\partial\varphi_2}{\partial\zeta_m} = -n^1\frac{a^1\left(r_{ce}^{(1,+)}\right)}{\gamma\Lambda_m} - n^2 a^2\left(r_f\right)\frac{a^{1'}\left(r_{ce}^{(1,+)}\right)}{a^1\left(r_{ce}^{(1,+)}\right)}\omega^{(1,+)} < 0$. Let $J \equiv \begin{bmatrix} \frac{\partial\varphi_1}{\partial r_f} & \frac{\partial\varphi_1}{\partial\zeta_m} \\ \frac{\partial\varphi_2}{\partial r_f} & \frac{\partial\varphi_2}{\partial\zeta_m} \end{bmatrix}$ and Δ_J denote its determinant.

Proof of Proposition 4. We have $\frac{\partial\varphi_1}{\partial n^2} = a^2\left(r_f\right) - \left(n^+ a^1\left(r_{ce}^{(1,+)}\right) + \left(1 - n^+\right) a^1\left(r_{ce}^{(1,-)}\right)\right) > 0$, and $\frac{\partial\varphi_2}{\partial n^2} = a^2(r_f) - n^+\left(1-\omega^{(1,+)}\right)a^1\left(r_{ce}^{(1,+)}\right) > 0$.Then,

$$\begin{bmatrix} \frac{dr_f}{dn^2} \\ \frac{d\zeta_m}{dn^2} \end{bmatrix} = -\frac{1}{\Delta_J}\begin{bmatrix} \frac{\partial\varphi_2}{\partial\zeta_m} & -\frac{\partial\varphi_1}{\partial\zeta_m} \\ -\frac{\partial\varphi_2}{\partial r_f} & \frac{\partial\varphi_1}{\partial r_f} \end{bmatrix}\begin{bmatrix} \frac{\partial\varphi_1}{\partial n_0} \\ \frac{\partial\varphi_2}{\partial n_0} \end{bmatrix}.$$

The condition on saving rates ensures that $\Delta_J < 0$. Then $\frac{dr_f}{dn^2} < 0$. Additionally,

$$\begin{aligned} \frac{d\zeta_m}{dn^2} &\propto -\frac{\partial\varphi_2}{\partial r_f}\frac{\partial\varphi_1}{\partial n^2} + \frac{\partial\varphi_1}{\partial r_f}\frac{\partial\varphi_2}{\partial n^2} \\ &= \frac{\partial\varphi_1}{\partial\zeta_m}\frac{\partial\varphi_1}{\partial n^2} + \frac{\partial\varphi_1}{\partial r_f}\left(\frac{\partial\varphi_2}{\partial n^2} - \frac{\partial\varphi_1}{\partial n^2}\right) > 0. \end{aligned}$$

□

Proof of Proposition 5. First notice that $\frac{\partial\varphi_1}{\partial n^+} = n^1\left(a^1\left(r_{ce}^{(1,+)}\right) - a^1\left(r_{ce}^{(1,-)}\right)\right) > 0$ and $\frac{\partial\varphi_2}{\partial n^+} = n^1\left(1-\omega^{1,+}\right)a^1\left(r_{ce}^{(1,+)}\right) = -n^2n^+a^2(r_f) < 0$. So,

$$\begin{bmatrix} \frac{dr_f}{dn^+} \\ \frac{d\zeta_m}{dn^+} \end{bmatrix} = -\frac{1}{\Delta_J}\begin{bmatrix} \frac{\partial\varphi_2}{\partial\zeta_m} & -\frac{\partial\varphi_1}{\partial\zeta_m} \\ -\frac{\partial\varphi_2}{\partial r_f} & \frac{\partial\varphi_1}{\partial r_f} \end{bmatrix}\begin{bmatrix} \frac{\partial\varphi_1}{\partial n^+} \\ \frac{\partial\varphi_2}{\partial n^+} \end{bmatrix}$$

where Δ_J is the determinant defined previously. Under the condition on saving rates, $\Delta_J < 0$ and $\frac{\partial\varphi_2}{\partial r_f} > 0$, so it follows that

$$\frac{d\zeta_m}{dn^+} \propto -\frac{\partial\varphi_2}{\partial r_f}\frac{\partial\varphi_1}{\partial n^+} + \frac{\partial\varphi_1}{\partial r_f}\frac{\partial\varphi_2}{\partial n^+} > 0$$

and

$$\begin{aligned} \frac{d(r_f+\zeta_m)}{dn^+} &\propto \left(\frac{\partial\varphi_2}{\partial\zeta_m} - \frac{\partial\varphi_2}{\partial r_f}\right)\frac{\partial\varphi_1}{\partial n^+} + \left(-\frac{\partial\varphi_1}{\partial\zeta_m} + \frac{\partial\varphi_1}{\partial r_f}\right)\frac{\partial\varphi_2}{\partial n^+} \\ &= \left(\frac{\partial\varphi_2}{\partial\zeta_m} - \frac{\partial\varphi_2}{\partial r_f}\right)\frac{\partial\varphi_1}{\partial n^+} + \left(\frac{\partial\varphi_2}{\partial r_f}\right)\frac{\partial\varphi_2}{\partial n^+} < 0. \end{aligned}$$

□

Proof of Lemma 4. To show existence, first note that Lemma 3 and an application of the implicit function theorem imply that ζ_m is continuous in n^+ for $0 < n^+ \leq 1$. Next, optimality condition for securitization, (28), and the definition of $\overline{\zeta}_m$, (29), imply that $\zeta_m \geq \overline{\zeta}_m$, for any $0 < n^+ \leq 1$. These ensure that equation

$$\left(1-n^+\right)\left(\zeta_m - \overline{\zeta}_m\right) = 0$$

has at least one solution n^+ with $0 < n^+ \leq 1$. Finally, by Lemma 3 there exists an equilibrium for the economy with exogenous n^+ for the value(s) of n^+ that solve the above equation. This ensures existence of an equilibrium with an endogenous value of n^+.

Next, we define the average levels of savings as $\overline{a}(r_f, \zeta_m, n) = n^2 a^2(r_f) + (1-n^2)\left(n^+ a^1\left(r_{ce}^{(1,+)}\right) + (1-n^+)\, a^1\left(r_{ce}^{(1,-)}\right)\right)$ and the relative value of asset endowments as $v(r_f+\zeta_m) \equiv \frac{\eta_m P_m}{e_0+\eta_m P_m}$.Therefore, $v^{'}(r_f+\zeta_m) = -e_0 v(r_f+\zeta_m)$.

We have now a system characterized by

$$\varphi_1(r_f, \zeta_m, n) \equiv \overline{a}(r_f, \zeta_m, n) - v(r_f+\zeta_m) = 0,$$

$$\varphi_2(r_f, \zeta_m, n) \equiv n^2 a^2(r_f) + n^1 n^+ \left(1-\omega^{1,+}\right) a^1\left(r_{ce}^{(1,+)}\right) = 0,$$

and

$$\varphi_3(\zeta_m, n^+) \equiv \left(1-n^+\right)\left(\zeta_m - \overline{\zeta}_m\right) = 0$$

where the first condition follows from market-clearing across both markets, the second is simply market clearing in the riskless asset market and the third reflects endogenous entry, where either $n^+ = 1$ and $\zeta_m \geq \overline{\zeta}_m$ or $n^+ < 1$ and $\zeta_m = \overline{\zeta}_m$. □

Proof of Proposition 6. If a solution features $n^+ < 1$, then by continuity, locally any solution to $\varphi_1(r_f, \zeta_m, n) = \varphi_2(r_f, \zeta_m, n) = \varphi_3(\zeta_m, n^+) = 0$ features $\zeta_m = \overline{\zeta}_m$. That leads to $\varphi_1\left(r_f, \overline{\zeta}_m, n\right) = \varphi_2\left(r_f, \overline{\zeta}_m, n\right) = 0$.

We define the matrix

$$\hat{J} \equiv \begin{bmatrix} \frac{\partial \varphi_1}{\partial r_f} > 0 & \frac{\partial \varphi_1}{\partial n^+} > 0 \\ \frac{\partial \varphi_2}{\partial r_f} > 0 & \frac{\partial \varphi_2}{\partial n^+} < 0 \end{bmatrix}.$$

The condition that $\left(\frac{a^{0'}(r_f)}{a^2(r_f)} - \frac{a^{1'}\left(r_{ce}^{(1,+)}\right)}{a^1\left(r_{ce}^{(1,+)}\right)}\right) \geq 0$ is sufficient for $\Delta_{\hat{J}} < 0$, where $\Delta_{\hat{J}}$ denotes the determinant of $\hat{J}$. Then,

$$\frac{\partial r_f}{\partial n^2} \propto \frac{\partial \varphi_2}{\partial n^+}\left(a^2 - \overline{a}_1\right) + \frac{\partial \varphi_1}{\partial n^+} \frac{n^+\left(1-\omega^{1,+}\left(\overline{\zeta}_m\right)\right) a^1\left(r_{ce}^{(1,+)}\right)}{n^2} < 0.$$

□

C Appendix C: Omitted Results

Proposition 8 (Existence)**.** *Under Assumptions* 2^G *and 3, there exists an approximate equilibrium with* $P_j > 0$ *for each* $j \in \{f\} \cup \mathbf{J}$*. The portfolio weights and the certainty equivalent returns are characterized by Eq.* (16)*, and the prices are characterized as the solution to the demand system* (15)*.*

Proof. Let $P = \{P_j\}_{j\in\{f\}\cup J}$ denote the asset price vector. We work with a truncated economy, where prices satisfy $P_j \leq \alpha$ for each asset $j \in \{f\} \cup J$. We are only interested in sufficiently large

α so that the truncation becomes inconsequential. First, let extended portfolio weights be also defined over assets that agent i cannot trade, so that

$$\hat{\omega}_j^i(P) \equiv \begin{cases} \omega_j^i(P), \text{ whenever } j \in \{f\} \cup J^i \\ 0, \text{ otherwise.} \end{cases}$$

For $P \gg 0$ we have individual excess demand for asset $j \in \{f\} \cup J$ defined as

$$z_j^i(P) \equiv \frac{\hat{\omega}_j^i(P)}{P_j} A_0^i(P) - x_{-1,j}^i \tag{C.1}$$

and we analogously define the excess demand for consumption at date $t = 0$ as $z_0^i(P) \equiv c_0^i(P) - Y_0^i$. Aggregate excess demands are then simply defined as $z_j(P) \equiv \sum_i n^i z_j^i(P)$ and $z_0(P) \equiv \sum_i n^i z_0^i(P)$. Walras' Law, i.e., $z_0(P) + \sum_{j \in J} P_j z_j(P) = 0$ can be trivially verified from individual optimality.

First, we impose a lower bound on prices $\hat{\epsilon} > 0$, which we successively relax later. Define $S_{\hat{\epsilon}} \equiv \left\{P \in \mathbb{R}_{++}^{|J|} | P_j \geq \hat{\epsilon} \text{ and } P_j \leq \alpha, \forall j \in \{f\} \cup J\right\}$ which is compact and convex. We are only interested in $\alpha > \hat{\epsilon}$ as to ensure the non-emptiness of $S_{\hat{\epsilon}}$.

We next define a continuous price updating function. Let each entry, which describes the update to the price of asset $j \in J$, be defined by

$$P_j^{upd}(P, \hat{\epsilon}) \equiv \begin{cases} \hat{\epsilon},\, if\, z_j(P) < \hat{\epsilon} - P_j \\ P_j + z_j(P),\, if\, \hat{\epsilon} - P_j \leq z_j(P) \leq \alpha \\ \alpha,\, if\, z_j(P) > \alpha \end{cases}. \tag{C.2}$$

Then, let the function $P^{upd}(P, \hat{\epsilon}) : S_{\hat{\epsilon}} \to S_{\hat{\epsilon}}$ be defined as $P^{upd}(P, \hat{\epsilon}) = \left\{P_j^{upd}(P, \hat{\epsilon})\right\}_{j \in \{f\} \cup J}$. As excess demand functions are continuous, so is the function $P^{upd}(\cdot, \hat{\epsilon})$, which maps the non-empty, convex, and compact set $S_{\hat{\epsilon}}$ into itself. From Brouwer's Fixed Point Theorem, there exists $P^{\hat{\epsilon}} \in S_{\hat{\epsilon}}$ such that $P_j^{upd}\left(P^{\hat{\epsilon}}, \hat{\epsilon}\right) = P^{\hat{\epsilon}}$.

We now take a sequence $\{\hat{\epsilon}_k\}_{k \in \mathbb{N}}$ such that $\hat{\epsilon}_k \to 0$. Let $\left\{P^{\hat{\epsilon}_k}\right\}_{k \in \mathbb{N}}$ be the associated sequence of fixed points. As each price lies in $[0, \alpha]$ that sequence is bounded and admits a converging subsequence. To save on notation, assume we have selected such subsequence from the start. Define its limit by $P^* = \left(P_1^*, P_2^*, ..., P_{|J|}^*\right)$. Naturally $P^* \in \overline{\cup_k S_{\hat{\epsilon}_k}} = \left\{P \in \mathbb{R}_+^{|J|} | P_j \leq \alpha, \forall \{f\} \cup J\right\}$. We now show that $P^* \in \mathbb{R}_{++}^{|J|}$.

Consider the case with $P_j^* = 0$ for risky assets, which w.l.o.g. we call assets $1, .., m$, while the riskless rate remains bounded away from zero. In this case, the risk premia for assets $1, .., m$ approach $+\infty$, and the risk premia for the remaining assets remain finite. Consider all investors that have access to at least one of the assets $1, .., m$ and call that set $I_{r \to \infty}$. It is easy to check that each of these investors have $r_{ce} \to \infty$, and thus, they save all their wealth.

Now consider the net demand for assets that comes from these investors only, $z_j^{I_{r \to \infty}} \equiv$

$\sum_{i\in I_{r\rightarrow\infty}} n^i z^i_j(P)$. We claim that regardless of how the prices for $1,..,m$ approach 0 (or conversely, regardless of the risk premia approach infinity), there exists at least one asset within $1,..,m$ such that the total demand from these investors for that asset becomes unboundedly positive. Since the demand from the other investors is finite, this will provide a contradiction.

Let us rewrite risk premia along the sequence. Take a given agent $i \in I_{r\rightarrow\infty}$, then the (individually perceived) risk-premium $\zeta^{i,k}_j\left(P^{\hat{\epsilon}_k}\right)$ on any asset $j \in J$ can be appropriately rewritten as $\zeta^{i,k}_j = \left\|\zeta^{i,k}\right\| \hat{\zeta}^{i,k}_j$ where $\left\|\zeta^{i,k}\right\| := \sum_j \left|\zeta^{i,k}_j\right|$ denotes a norm and

$$\hat{\zeta}^{i,k}_j \equiv \frac{\zeta^{i,k}_j}{\left\|\zeta^{i,k}\right\|}$$

denotes the $j-th$ entry of a normalized risk-premium vector.[25] The vector $\hat{\zeta}^{i,k} = \left\{\hat{\zeta}^{i,k}_j\right\}_{j\in J}$ belongs to the surface of the unit ball centered at zero.

As that surface is a compact set, $\left\{\hat{\zeta}^{i,k}\right\}_{k\in\mathbb{N}}$ admits a converging subsequence, which we can index by $k_i \in \mathbb{N}$. That forms another price sequence $\left\{P^{\hat{\epsilon}_{k_i}}\right\}_{k_i\in\mathbb{N}}$, from which we can extract a subsequence to ensure that the analogously defined vector $\hat{\zeta}^{i',k_i}$ converges for any second agent $i' \in I_{r\rightarrow\infty}$. Given that $I_{r\rightarrow\infty}$ is finite, this step can be iteratively repeated until a subsequence, indexed by $\tilde{k} \in \mathbb{N}$, is extracted and ensures that each $\hat{\zeta}^{i,\tilde{k}}$ converges. Additionally, for each $i \in I_{r\rightarrow\infty}$, $\lim_{\tilde{k}\rightarrow\infty} \hat{\zeta}^{i,\tilde{k}} = \hat{\zeta}$, i.e., the limit of the normalized risk-premia are the same and independent of $i \in I_{r\rightarrow\infty}$, since disagreements are bounded, while at least one return goes to infinity.

Take a given agent $i \in I_{r\rightarrow\infty}$. Define $\hat{\zeta}^{i,\tilde{k}}_{J_i}$ and $\hat{\zeta}_{J_i}$ to be respectively the restriction of the normalized risk premia vectors $\hat{\zeta}^{i,\tilde{k}}$ and $\hat{\zeta}$ to the assets that agent i can trade. Notice that along that subsequence portfolio weights of the form $\omega^i_{J^i}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) = \frac{1}{\gamma^i}\Lambda^{-1}_{J^i}\hat{\zeta}^{i,\tilde{k}}_{J_i}\left\|\zeta^{i,\tilde{k}}\right\|$ are optimal from equation (16). Therefore, we take the following limit of an inner product

$$\lim_{\tilde{k}\rightarrow\infty}\left\langle \hat{\zeta}^{i,\tilde{k}}_{J_i}, \frac{\omega^i_{J^i}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)}{\left\|\zeta^{i,\tilde{k}}\right\|}\right\rangle = \frac{1}{\gamma^i}\hat{\zeta}'_{J_i}\Lambda^{-1}_{J^i}\hat{\zeta}_{J_i} > 0$$

from the positive-definiteness of $\Lambda^{-1}_{J^i}$ and the fact that $\hat{\zeta}_{J_i}$ is not null. It follows that it is possible to find $\delta > 0$ and a sufficiently high element $\overline{k}$ such that

$$\left\langle \hat{\zeta}, \frac{\hat{\omega}^i\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)}{\left\|\zeta^{i,\tilde{k}}\right\|}\right\rangle > \delta,$$

whenever $i \in I_{r\rightarrow\infty}$ and $\tilde{k} > \overline{k}$. Given that $A^i_0\left(P^{\hat{\epsilon}_{\tilde{k}}}\right)$ is bounded from below for sufficiently high $\tilde{k}$

[25] As prices are converging to zero, there are finitely many elements with $\sum_j \zeta^{i,k}_j = 0$. We can move to a subsequence that disregards these.

for all $i \in I_{r\to\infty}$, there exists $\delta_1 > 0$

$$\left\langle \hat{\zeta}, \sum_{i\in I_{r\to\infty}} n^i A_0^i \left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \frac{\hat{\omega}^i \left(P^{\hat{\epsilon}_{\tilde{k}}}\right)}{\left\|\zeta^{i,\tilde{k}}\right\|} \right\rangle > \delta_1, \tag{C.3}$$

for all $\tilde{k} > \overline{k}$. This directly implies that there exists one asset $j \in \{1, ..., m\}$ such that $\sum_{i\in I_{r\to\infty}} n^i A_0^i \left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \hat{\omega}^i \left(P^{\hat{\epsilon}_{\tilde{k}}}\right)$ grows without bounds. It follows that excess demand for that asset is unbounded along the subsequence that is indexed by $\tilde{k}$. From (C.2) this means that $P_j^{upd}\left(P^{\hat{\epsilon}_k}, \hat{\epsilon}_k\right) = \alpha$ infinitely many times as $k \to \infty$, reaching a contradiction with $P_j^* = 0$.

Suppose now, towards a different contradiction, that $r_f \to \infty$. Using arguments similar to the previous ones, it is possible to select a subsequence, indexed by $\tilde{k} \in \mathbb{N}$, in which the risk premium, $\zeta_j^i \left(P^{\hat{\epsilon}_{\tilde{k}}}\right)$, perceived by each agent $i \in I$ for each asset $j \in J$ either converges to a finite constant, diverges to $+\infty$ or diverges to $-\infty$. Also, a premium can only diverge for all agents at the same time and in the same direction.

First, we deal with the case in which no premium diverges. In this situation, each asset price converges to zero. Adding equations C.1 over agents and assets, properly multiplied by prices and individual population shares, we get

$$\sum_{i,j} P_j^{\hat{\epsilon}_{\tilde{k}}} n^i z_j^i \left(P^{\hat{\epsilon}_{\tilde{k}}}\right) = \sum_{i,j} n^i \left[\hat{\omega}_j^i \left(P^{\hat{\epsilon}_{\tilde{k}}}\right) A_0^i \left(P^{\hat{\epsilon}_{\tilde{k}}}\right) - P_j^{\hat{\epsilon}_{\tilde{k}}} x_{-1,j}^i\right],$$

which after simplifications leads to

$$\sum_j P_j^{\hat{\epsilon}_{\tilde{k}}} z_j \left(P^{\hat{\epsilon}_{\tilde{k}}}\right) = \sum_i n^i A_0^i \left(P^{\hat{\epsilon}_{\tilde{k}}}\right) - \sum_{i,j} P_j^{\hat{\epsilon}_{\tilde{k}}} n^i x_{-1,j}^i.$$

As $P^{\hat{\epsilon}_{\tilde{k}}} \to 0$, the right hand side converges to $\sum_i n^i Y_0^i > 0$. As a consequence, the excess demand for at least one asset j needs to approach $+\infty$ along a subsequence. Along this subsequence then $P_j^{upd}\left(P, \hat{\epsilon}_{\tilde{k}}\right) = \alpha$ infinitely often, leading to a contradiction of the zero price limit.

For the case in which some premia diverge, we still obtain

$$\lim_{\tilde{k}\to\infty} A_0^i \left(P^{\hat{\epsilon}_{\tilde{k}}}\right) - P_j^{\hat{\epsilon}_{\tilde{k}}} n^i x_{-1,j}^i = Y_0^i > 0$$

and

$$\sum_j P_j^{\hat{\epsilon}_{\tilde{k}}} z_j \left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to \sum n^i Y_0^i > 0$$

If $P^{\hat{\epsilon}_{\tilde{k}}} \to 0$, we find the same contradiction as before. Therefore, for at least one asset $j \in J$, we need to have $P_j^{\hat{\epsilon}_{\tilde{k}}} \to P_j^* \neq 0$ which implies that $\zeta_j^{i,\tilde{k}} \to -\infty$ for each $i \in I$. We can therefore follow all the previous steps leading to C.3, with the exception that $\hat{\zeta}$ can now have negative entries. This means that we can find a subsequence and an asset $j' \in J$, such that either $\zeta_{j'}^{i,\tilde{k}} \to -\infty$ and

$z_{j'}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to -\infty$ or $\zeta_{j'}^{i,\tilde{k}} \to +\infty$ and $z_{j'}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to +\infty$. For the latter case, we would reach the same contradiction as before since $z_{j'}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to +\infty$ implies that $P_j^{upd}\left(P^{\hat{\epsilon}_k}, \hat{\epsilon}_k\right) = \alpha$ infinitely many times which contradicts positive infinity limits for both the riskless rate and the risk premium on j'. Therefore, we need to rule out the former situation. Given that $P_j^{\hat{\epsilon}_{\tilde{k}}} \to P_j^* > 0$, $\zeta_j^{i,\tilde{k}} \to -\infty$ and $\hat{\zeta}_{j'} \neq 0$ together imply that $P_{j'}^* > 0$. But from (C.2), $z_{j'}\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to -\infty$ implies $P_j^{\hat{\epsilon}_{\tilde{k}}} = \hat{\epsilon}_{\tilde{k}}$ infinitely many times with $\hat{\epsilon}_{\tilde{k}} \to 0$, reaching a contradiction with $P_{j'}^* > 0$.

We have, therefore, ruled out any possibility that $P_j^* = 0$ for some asset $j \in J \cup \{f\}$. We still need to show that for sufficiently high α, market clearing is ensured in all markets at prices P^*. Given that $P_j^* \gg 0$, it is possible to find a sufficiently high $\hat{k}$ and $\delta_2 > 0$, such that

$$P_j^{\hat{\epsilon}_k} > \delta_2 > \hat{\epsilon}_k,$$

for all $k > \hat{k}$. As a consequence, from (C.2), for $k > \hat{k}$, $P_j^{\hat{\epsilon}_k} \geq 0$ and $z_j\left(P_j^{\hat{\epsilon}_k}\right) \geq 0$.

Additionally, for each $i \in I$, $c_0^i\left(P_j^{\hat{\epsilon}_k}\right) \in \left[0, Y_0^i + \alpha \sum x_{-1}^i\right]$ implying that

$$-\alpha \sum_i n^i x_{-1}^i \leq \sum_j P_j^{\hat{\epsilon}_{\tilde{k}}} z_j\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \leq \sum_i n^i Y_0^i.$$

For $\alpha^2 > \sum_i n^i Y_0^i$, it follows that $z\left(P^{\hat{\epsilon}_{\tilde{k}}}\right) \to z\left(P^*\right) = 0$ ensuring market-clearing in the limit and existence of a Walrasian Equilibrium.

□

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