

# Refinance

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## Abstract

I provide a micro-foundation for refinanceable contracts, which are a major class of incomplete contracts. The micro-foundation relates the optimality of refinanceable contracts to the financing relationship being governed by a preference for robustness. As an application, I consider a canonical dynamic financial contracting setting with asymmetric information, where the entity being financed privately observes the state of the world. I show that under preference for robustness, the optimal contract is refinanceable debt. The intuition for the result is a standard “folk intuition” for debt. My preference for robustness based formalization of this folk intuition is robust.

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# 1 Introduction

Most financing contracts contain, either implicitly or explicitly, the option to refinance. That is, the financial contract itself represents only a preliminary financing plan, maturing at some preliminary date  $T$ . However, at any interim date  $t \leq T$ , the entity being financed - from now on, simply called the entity - can *refinance*. This means that the entity can rewrite the remaining terms of the financial contract, including, possibly, extending the maturity date. The only restriction is that the financier's "continuation payoff" must remain the same. In practice, the definition of continuation payoff may change depending on the situation, and additional restrictions may apply to how the entity can refinance. However, roughly speaking, this is what refinance means. Well known examples of refinanceable contracts include mortgages with explicit refinance clauses and short-term corporate debt with implicit rollover options.

This paper seeks to provide some theoretical foundations for refinance by considering a benchmark dynamic financial contracting framework and showing that optimal long-term contracts can be implemented as refinanceable contracts. The key novelty of the paper is that it assumes that the financier is not Bayesian, but, rather, has a preference for robustness. This means that, 1) the financier has an incomplete model of the future repayment capabilities of the entity, 2) the further into the future the financier looks, the more incomplete is the model, and 3) the financier is aware it has an incomplete model and seeks a contract that will do well no matter what the true model is. After deriving the refinance principle, I then apply it to a canonical contracting setting with asymmetric information, where the entity privately observes the contract relevant state of the world. I show that assuming the financier has a preference for robustness implies that the optimal contract is refinanceable debt.

It may come as a surprise to the reader that a micro-foundation for refinance doesn't already exist. After all, refinanceable contracts are ubiquitous and they appear in many applied theory papers. However, there is a good reason for this omission - refinanceable contracts are not very compatible with the standard Bayesian approach to optimal dynamic contracting. With a Bayesian decision maker, it is well known that optimal dynamic contracts generically exhibit a strong form of "completeness." Loosely speaking, there can be a strong disconnect between what is ex-ante optimal and what is ex-post optimal, so that, in general, everything must be determined beforehand. Moreover, distant payments depend in a fine way on what happens before and this dependence is tightly linked to the Bayesian decision maker's ex-ante beliefs. These generic qualities of optimal dynamic contracts in the Bayesian setting go against the spirit of refinance. In refinanceable contracts, which are essentially incomplete contracts, things are not set in stone beforehand - with many details left out entirely; distant payments - if they even occur - are there to provide flexibility; and when refinance occurs, the terms are not tightly governed by ex-ante conditions but, rather, are informed by the current state of the world.

To be fair, one can capture some of the spirit of refinance by focusing on renegotiation-proof optimal contracts. And the literature sometimes does this. Imposing the renegotiation-proof constraint means that the contract must always remain on the Pareto-frontier. However, at the end of the day, even renegotiation-proof optimal contracts exhibit "completeness"

and simply do not resemble anything that might reasonably be interpreted as a refinanceable contract.

All of this suggests that the Bayesian assumption should be replaced. My paper can be seen as essentially arguing that the replacement should be preference for robustness. To get a sense of how preference for robustness is linked to refinance, suppose the financier has some idea about the entity's ability to pay one date into the future but has no idea two or more dates into the future. Then from the perspective of the initial date 0 when the financing contract is signed, contingent repayments by the entity at date 2 and beyond are worthless: The entity can promise the world at date 2, but the financier simply has no confidence the entity can deliver on such promises. Does this then mean that the financing contract should end at date 1? Actually, the answer is no. When today is date 0, date 2 is far away, but when today becomes date 1, date 2 is no longer far away. That is, the financier understands that even though right now it has no idea what the entity can do at date 2, tomorrow things might change. This means that date 2 payments can actually be of value. Again, these distant payments cannot be directly valuable to the financier in the sense that adding such a payment will not increase the financier's ex-ante utility. However, they can provide the entity financial flexibility and it means that in an optimal contract, distant payments must comprise "refinancings" of earlier obligations.

This paper is related to a number of recent papers looking at contracting under various forms of preference for robustness. One difference between this paper and Chung and Ely (2007), Frankel (2014), Garrett (2014), and Carroll (2015) is that I work in a dynamic setting and I'm interested in the implications of preference for robustness for how to structure contracts optimally along the time dimension. The max-min structure of the preference for robustness also relates this paper to the ambiguity aversion literature (e.g. Gilboa and Schmeidler, 1989).

This paper's application of the refinance principle to contracting with asymmetric information and its derivation of refinanceable debt relates it to a long line of work trying to explain the central role of debt as a method of securing financing. See Townsend (1979), Gale and Hellwig (1985), Innes (1990), DeMarzo-Duffie (1999), Yang (2015), and Hebert (2015). One way this paper contributes to the literature is that it not only explains debt but also the implied refinance option.

The intuition for the debt result is simple: The financier demands a flat repayment because - and here is where preference for robustness comes in - it simply does not know enough about the entity and about what the entity knows to be able to pull off a more delicate, clever repayment plan that is both more efficient and incentive-compatible. When the entity can't deliver the flat repayment, then the financier demands everything so that the entity won't be tempted to report that it can't repay when in fact it can.

The above intuition for debt is widely held and is basically a folk intuition. This paper is not the first to attempt to formalize it. The most well known previous example is the costly state verification (CSV) literature initiated by Townsend (1979) and Gale and Hellwig (1985). Indeed, I argue that there is a deep connection between the CSV approach to financial contracting under asymmetric information and this paper's approach despite the fact that this paper does not have costly state verification and those papers do not have preference for robustness.

The starting point is the Bayesian version of the asymmetric information model this paper uses to prove the optimality of refinanceable debt. Due to its canonical structure, and the fact that it has standard Bayesian decision makers, this setting or some enrichment of it would be what most people think of first when contemplating financial contracting under asymmetric information. However, in this setting, a combination of contracts being complete (i.e. contracts completely control both parties' consumption) and the parties being Bayesian leads to optimal contracts that are very state sensitive: Complete consumption control means the contract can be very state sensitive and Bayesian decision makers means that state sensitivity is valuable. Thus, if one wants a simpler optimal contract to emerge under asymmetric information, either complete consumption control or Bayesian decision makers has to go.

The connection between the CSV paper and this paper is that both want a simple optimal contract to emerge - debt - but they choose to discard different parts of the Bayesian asymmetric information model to get there. This paper chooses to get rid of the Bayesian assumption. The CSV papers chose to get rid of complete consumption control and have incomplete consumption control via costly state verification. Both routes lead to formalizations of the aforementioned folk intuition for debt. However, there are well known limitations of the route taken by the CSV literature - there must be bilateral risk-neutrality, the optimal contract is not renegotiation-proof, and random state verification must be disallowed. See Mookherjee and Png (1989) and Attar and Campioni (2003). In contrast, my paper shows that if one, instead, takes the preference for robustness route, then not only can debt be explained, but also the refinance option. Moreover, refinanceable debt is renegotiation proof, the entity can be risk averse, and contracts can be random. Thus, the preference for robustness approach provides a robust formalization of the folk intuition for debt.

## 2 The Refinance Principle

In this section I derive the basic refinance principle in a benchmark perfect information dynamic financial contracting setting. I first assume that the financier and the entity are both Bayesian. Optimal dynamic contracts do not exhibit any salient structures and cannot be interpreted as a refinanceable contracts. I then change the model by assuming the financier has a preference for robustness and show that optimal contracts are refinanceable contracts.

### 2.1 Bayesian Perfect Information Model

The model contains three dates  $t = 0, 1, 2$ . I describe the model starting from the end.

On date 2, the entity realizes a (liquidation) value  $v_2 \geq 0$ . Then the financier and the entity consume  $p_2$  and  $c_2$  respectively. The entity is protected by limited liability, so  $c_2 \geq 0$ . The financier has no such constraint. The budget constraint requires  $p_2 + c_2 \leq v_2$ . After consumption, the model ends.

The entity's date 2 value  $v_2$  is realized according to a distribution that depends on the capital invested at date 1. Specifically, at date 1, the entity first realizes a (liquidation) value  $v_1$ . Then, the financier and the entity consume  $p_1$  and  $c_1$  subject to the same limited liability

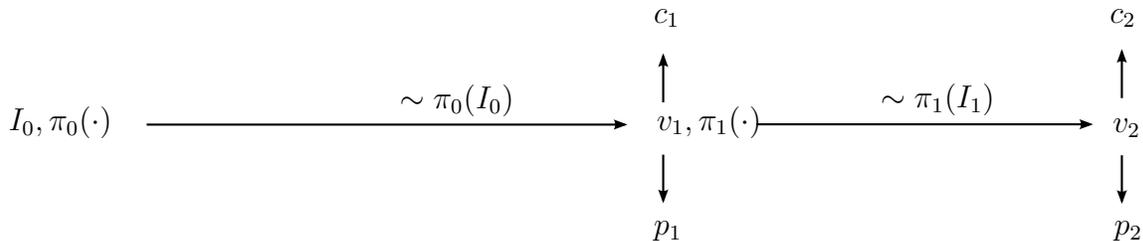


Figure 1

and budget constraints as in date 2. Note, since  $p_1$  can be negative, so can the quantity  $p_1 + c_1$ . A negative  $p_1 + c_1$  means that there is net investment at date 1, with the financier injecting more capital into the entity. Let  $I_1 := -p_1 - c_1$  denote the date 1 investment amount. How much investment  $I_1$  occurs determines what kind of probability distribution over  $v_2$  one can expect. Formally, there is a belief function  $\pi_1(\cdot)$  with domain  $[-v_1, \infty)$  that maps date 1 investment  $I_1$  to full-support probability distributions over the entity's date 2 value  $v_2$  with finite expectation.

The date 1 *state of the world*  $s_1$  is a realization of  $v_1 \geq 0$  and a realization of the belief function  $\pi_1(\cdot)$ .  $\pi_1(\cdot)$  can be any function with the property that higher investment does not lead to a strictly first-order stochastic dominated distribution. The probability distribution of  $s_1$  is  $\pi_0(I_0)$  where  $I_0 \geq 0$  is the investment made by the financier at date 0 and  $\pi_0(\cdot)$  is a belief function mapping date 0 investment to full-support probability distributions over  $s_1$  with finite expected  $v_1$ . See Figure 1 for a graphical representation of the model.

At date 0, the entity and the financier sign a long-term contract spanning the entire model. A contract stipulates a financing plan  $\{I_0, p_1, c_1, p_2, c_2\}$ . Here,  $p_1$  and  $c_1$  can depend on the date 1 state of the world  $s_1$ , and  $p_2$  and  $c_2$  can depend on  $v_2$  as well as a  $s_1$ . All terms of the contract are allowed to be random.

Given  $\pi_0(\cdot)$  and some financier opportunity cost of funds function  $OC(\cdot)$ , the optimal contracting problem is

$$\begin{aligned} \max_{\{I_0, p_1, c_1, p_2, c_2\}} & \mathbf{E}_{\pi_0(I_0)} [u(c_1) + \mathbf{E}_{\pi_1(I_1)} u(c_2)] \\ \text{s.t.} & \mathbf{E}_{\pi_0(I_0)} [p_1 + \mathbf{E}_{\pi_1(I_1)} p_2] \geq OC(I_0) \end{aligned}$$

where  $u(\cdot)$  is a weakly concave function.

The optimal contract is exceedingly simple. Risk-sharing implies that the entity consumes a constant  $c_1 = c_2 = c$  and the financier becomes the full residual claimant. There are three things worth noting here. The contract cannot be interpreted as a refinanceable contract; the contract is not a debt contract; and the simple structure is not robust - if both the financier and the entity had risk-averse preferences, then risk-sharing would imply fully state sensitive consumption for both parties. Overall, there is really not much to say about optimal financial contracting in the Bayesian perfect information model.

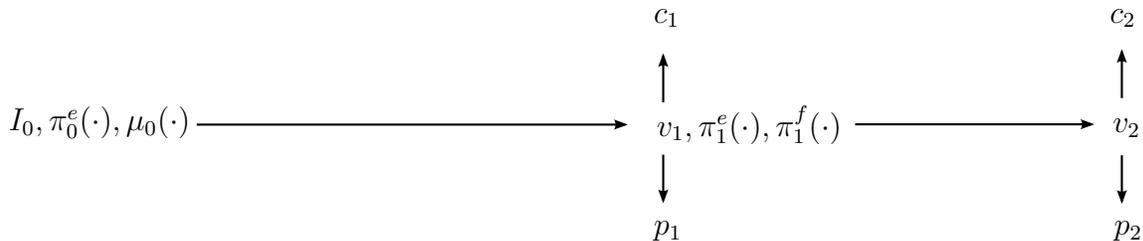


Figure 2

## 2.2 Preference for Robustness, Perfect Information Model

I now replace the Bayesian financier with a financier that has a preference for robustness. The model is changed in the following way:

First, nothing is different at date 2. However, at date 1, the financier and the entity may have differing belief functions, so that given investment  $I_1 = -p_1 - c_1$ , the entity believes that  $v_2$  will be distributed according to some  $\pi_1^e(I_1)$  and the financier believes that  $v_2$  will be distributed according to some  $\pi_1^f(I_1)$ . At date 1, a state of the world  $s_1 := (v_1, \pi_1^e(\cdot), \pi_1^f(\cdot))$  consists of a realization of entity value  $v_1 \geq 0$  along with the realized subjective belief functions for the financier and the entity.

At date 0, the entity has a belief function  $\pi_0^e(\cdot)$ , so that, given date 0 investment  $I_0$ , the entity believes  $s_1$  is distributed according to some full-support  $\pi_0^e(I_0)$ . Unlike the entity, the financier, given  $I_0$ , does not have a single belief about  $s_1$ . Rather, the financier entertains numerous beliefs similar to an ambiguity-averse decision maker. The set of beliefs the financier entertains is defined as follows. The financier has a *confidence belief* function  $\mu_0(\cdot)$ . Given  $I_0$ , the confidence belief  $\mu_0(I_0)$  is a full-support probability distribution over  $v_1$ , not  $s_1$ . The confidence belief  $\mu_0(I_0)$  represents a lower bound distribution for the entity's date 1 value that the financier is confident the entity can achieve. As a result, the financier is willing to entertain any belief  $\pi_0$  about the distribution over the date 1 state of the world so long as the implied distribution over entity's date 1 value weakly first-order-stochastic-dominates its confidence belief  $\mu_0(I_0)$ . See Figure 2.

A contract is exactly the same object as before:  $\{I_0, p_1, c_1, p_2, c_2\}$ . The only difference is how the entity and the financier evaluate contracts. Given a belief function  $\pi_0^e(\cdot)$ , confidence belief function  $\mu_0(\cdot)$  and some financier opportunity cost of funds  $OC(I_0)$ , the optimal contracting problem is,

$$\begin{aligned} & \max_{\{I_0, p_1, c_1, p_2, c_2\}} \mathbf{E}_{\pi_0^e(I_0)} [u(c_1) + \mathbf{E}_{\pi_1^e(I_1)} u(c_2)] \\ \text{s.t.} & \quad \min_{\{\pi_0 \mid \pi_0|_{v_1} \geq \mu_0(I_0)\}} \mathbf{E}_{\pi_0} [p_1 + \mathbf{E}_{\pi_1^f(I_1)} p_2] \geq OC(I_0) \end{aligned}$$

Notice, the financier evaluates contracts based on their worst-case performance where the scenarios the financier is willing to consider are determined by its confidence belief  $\mu_0(I_0)$ .

For each date 1 state of the world  $s_1$ , define  $F_1(s_1) := \mathbf{E}[p_1(s_1) + \mathbf{E}_{\pi_1^f(I_1)} p_2(s_1, \cdot)]$  to be the financier's date 1 continuation payoff. The outer expectation captures the fact that given state  $s_1$ ,  $p_1(s_1)$  may still be random.

**Lemma 1.** *If  $\{I_0, p_1, c_1, p_2, c_2\}$  is an optimal contract, then  $F_1(s_1) = F_1(v_1)$ .*

*Proof.* Pick two date 1 states of the world  $s'_1$  and  $s''_1$  that share the same  $v_1$ . Suppose  $F_1(s'_1) > F_1(s''_1)$ . Then consider the alternate contract that changes the continuation contract -  $\{p_1(s'_1), c_1(s'_1), p_2(s'_1, \cdot), c_2(s'_1, \cdot)\}$  - following  $s'_1$  into the one that maximizes the entity's continuation payoff subject to delivering continuation payoff  $F_1(s''_1)$  to the financier. Given the financier's preference for robustness, it is indifferent between the two contracts. However, the entity is strictly better off.  $\square$

Lemma 1 is the formal statement of the refinance principle under perfect information. It implies that an optimal contract admits a natural implementation as a refinanceable contract.

In the implementation, there is an *initial contract*  $\{I_0, F_1(\cdot)\}$  that specifies only how much the financier invests at date 0 and the entity's financial obligation  $F_1(\cdot)$  to the financier which depends only on the entity's date 1 value  $v_1$ . The initial contract specifies a default option for fulfilling the obligation which involves the entity paying the entire  $F_1(v_1)$  at date 1. However, it also grants the entity the *option to refinance*. This means that at date 1, the entity can renegotiate the default option into another continuation contract  $\{p_1(s_1), p_2(s_1, \cdot)\}$  so long as the financier receives a total expected payoff of at least  $F_1(v_1)$ . It is worth emphasizing that  $p_1(s_1)$  may be negative, so that the renegotiated contract can incorporate situations where the obligation is rolled over or the financier injects even more capital into the entity.

The contrast between an optimal complete contract and its implementation as a refinanceable contract highlights the fact that refinanceable contracts are essentially incomplete contracts. The initial contract only specifies an obligation to the financier but leaves out the details of how the obligation should be fulfilled. The entity is granted control rights over how to specify those details in the future. Moreover, neither the initial contract nor the renegotiated contract specify the entity's consumption schedule, only what the financier is owed. Thus, Lemma 1 can also be interpreted as providing solid theoretical foundations for a major class of incomplete contracts.

### 3 Financing under Asymmetric Information

As an application of the refinance principle and the ideas introduced in the previous section, I now replace the perfect information assumption in the financial contracting model considered in the previous section with a canonical asymmetric information problem. I show that the refinance principle continues to apply and that the optimal contract is debt that can be refinanced. I connect this work with the costly state verification literature and show how my result can be thought of as a robust justification of debt.

#### 3.1 Preference for Robustness, Asymmetric Information Model

The model is based on the preference for robustness model considered in Section 2.2, except I now assume that the entity first privately observes the state of the world at date 1 and can strategically reveal information about the state of the world to the financier. Specifically, after the entity privately observes  $s_1 := (v_1, \pi_1^e(\cdot), \pi_1^f(\cdot))$ , it publicly and credibly reports that

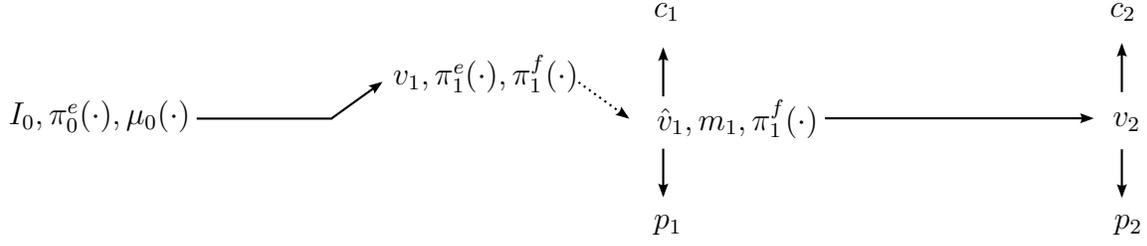


Figure 3

it has value  $\hat{v}_1 \in [0, v_1]$  and reports a message  $m_1$  regarding its own belief function  $\pi_1^e(\cdot)$ . Here the reported entity belief function must have a domain that matches the reported  $\hat{v}_1$ . I assume that a report of  $\hat{v}_1$  leads to a financier belief function which is the true belief function  $\pi_1^f(\cdot)$  except restricted to the domain of  $[-\hat{v}_1, \infty)$ . As an abuse of notation, I will say the financier has belief function  $\pi_1^f(\cdot)$ . Thus, while the entity knows the true state  $s_1$ , the financier observes the reported state of the world  $\hat{s}_1 = (\hat{v}_1, m_1, \pi_1^f(\cdot))$  which is also the state the contract is written over. For simplicity, I continue to assume that  $v_2$  is publicly observable. See Figure 3.

A contract stipulates  $\{I_0, p_1, c_1, p_2, c_2, \hat{v}_1, m_1\}$  where  $p_1$  and  $c_1$  depend on the reported date 1 state of the world  $\hat{s}_1$ ,  $p_2$  and  $c_2$  depend on  $\hat{s}_1$  and  $v_2$ , and the report strategy  $(\hat{v}_1, m_1)$  depends on the privately observed true state of the world  $s_1$ . Given true state  $s_1$  and reported state  $\hat{s}_1$ , the date 1 investment is  $I_1(\hat{s}_1) = -p_1(\hat{s}_1) - c_1(\hat{s}_1)$  and the entity and the financier have beliefs  $\pi_1^e(I_1(\hat{s}_1))$  and  $\pi_1^f(I_1(\hat{s}_1))$  about  $v_2$ . The budget constraint at date 1 is  $p_1 + c_1 \leq \hat{v}_1$ . I focus on truth-telling contracts. In a truth-telling contract, the entity truthfully reports  $\hat{v}_1 = v_1$  and  $m_1 = \pi_1^e(\cdot)$ . Given a true state of the world  $s_1$ , for every possible reported state of the world  $\hat{s}_1$ , the continuation payoff of the entity following  $\hat{s}_1$  is  $\mathbf{E}[u(c_1(\hat{s}_1)) + \mathbf{E}_{\pi_1^e(I_1(\hat{s}_1))} u(c_2(\hat{s}_1, \cdot))]$ . Here, just like in the definition of the financier's continuation payoff in the previous section, the outer expectation captures the fact that after the reported state is realized,  $c_1(\hat{s}_1)$  may still be random.

**Definition 1.** *A truth-telling contract is incentive-compatible if truth-telling maximizes the entity's continuation payoff given any realization of the date 1 state of the world.*

From now on, all contracts are assumed to be incentive-compatible truth-telling contracts.

Fix a contract and any state  $s_1 = (v_1, \pi_1^e(\cdot), \pi_1^f(\cdot))$ . Consider a (possibly off-equilibrium) revealed state  $\hat{s}_1 = (\hat{v}_1, \hat{m}_1, \pi_1^f(\cdot))$ . The continuation contract given  $\hat{s}_1$  is  $\{p_1(\hat{s}_1), c_1(\hat{s}_1), p_2(\hat{s}_1, \cdot), c_2(\hat{s}_1, \cdot)\}$ . A *renegotiation* of this continuation contract is some alternate continuation contract  $\{p'_1, c'_1, p'_2(\cdot), c'_2(\cdot)\}$ . The budget constraint at date 1 is still  $p'_1 + c'_1 \leq \hat{v}_1$ .

Under the renegotiation, the continuation payoffs of the entity and the financier

$$\mathbf{E}[u(c'_1) + \mathbf{E}_{\pi_1^e(-p'_1 - c'_1)} u(c'_2(\cdot))] \quad \mathbf{E}[p'_1 + \mathbf{E}_{\pi_1^f(-p'_1 - c'_1)} p'_2(\cdot)]$$

The entity is strictly better off under the renegotiation if

$$\mathbf{E}[u(c'_1) + \mathbf{E}_{\pi_1^e(-p'_1 - c'_1)} u(c'_2(\cdot))] > \mathbf{E}[u(c_1(s_1)) + \mathbf{E}_{\pi_1^e(I_1(s_1))} u(c_2(s_1, \cdot))]$$

The financier *thinks* it is strictly better off under the renegotiation if

$$\mathbf{E}[p'_1 + \mathbf{E}_{\pi_1^f(-p'_1-c'_1)}p'_2(\cdot)] > \mathbf{E}[p_1(\hat{s}_1) + \mathbf{E}_{\pi_1^f(I_1(\hat{s}_1))}p_2(\hat{s}_1, \cdot)]$$

**Definition 2.** A contract is renegotiation-proof if there does not exist a state  $s_1$  and a renegotiation of the continuation contract  $\{p_1(\hat{s}_1), c_1(\hat{s}_1), p_2(\hat{s}_1, \cdot), c_2(\hat{s}_1, \cdot)\}$  following some reported state  $\hat{s}_1$  that makes the entity strictly better off and makes the financier think it is strictly better off.

There is a natural equivalence between renegotiation-proof contracts and a subset  $\mathcal{C}$  of the set of contracts in the perfect information setting satisfying incentive-constraints implied by Definitions 1 and 2. Thus, the optimal contracting problem can be stated as a perfect information contracting problem where the contract space is constrained to be  $\mathcal{C}$ . Given  $\pi_0^e(\cdot)$ ,  $\mu_0(\cdot)$  and  $OC(\cdot)$ , the optimal contracting problem is

$$\begin{aligned} & \max_{\{I_0, p_1, c_1, p_2, c_2\} \in \mathcal{C}} \mathbf{E}_{\pi_0^e(I_0)} [u(c_1) + \mathbf{E}_{\pi_1^e(I_1)} u(c_2)] \\ \text{s.t.} & \min_{\{\pi_0 \mid \pi_0|_{v_1} \geq \mu_0(I_0)\}} \mathbf{E}_{\pi_0} [p_1 + \mathbf{E}_{\pi_1^f(I_1)} p_2] \geq OC(I_0) \end{aligned}$$

For every state of the world  $s_1 = (v_1, \pi_1^e(\cdot), \pi_1^f(\cdot))$ , define the financier's maximal continuation payoff to be:

$$\bar{F}_1(s_1) = \bar{F}_1(v_1, \pi_1^f(\cdot)) := \max_{\{p'_1, c'_1, p'_2(\cdot), c'_2(\cdot)\}} \mathbf{E}[p'_1 + \mathbf{E}_{\pi_1^f(-p'_1-c'_1)} p'_2(\cdot)]$$

**Definition 3.** A refinable debt contract has the property that  $F_1(s_1) = \bar{F}_1(s_1) \wedge F_1$  for some constant  $F_1$ . Specifically, a contract is called refinable debt if there exists an  $F_1$  such that, for every state of the world  $s_1$ , if  $\bar{F}_1(s_1) = \bar{F}_1(v_1, \pi_1^f(\cdot)) > F_1$ , then

$$\begin{aligned} \{p_1(s_1), c_1(s_1), p_2(s_1, \cdot), c_2(s_1, \cdot)\} &= \arg \max_{\{p'_1, c'_1, p'_2(\cdot), c'_2(\cdot)\}} \mathbf{E}[u(c'_1) + \mathbf{E}_{\pi_1^e(-p'_1-c'_1)} u(c'_2(\cdot))] \\ \text{s.t.} & \mathbf{E}[p'_1 + \mathbf{E}_{\pi_1^f(-p'_1-c'_1)} p'_2(\cdot)] \geq F_1 \end{aligned}$$

and if  $\bar{F}_1(s_1) = \bar{B}_1(v_1, \pi_1^f(\cdot)) \leq F_1$ , then

$$\{p_1(s_1), c_1(s_1), p_2(s_1, \cdot), c_2(s_1, \cdot)\} = \arg \max_{\{p'_1, c'_1, p'_2(\cdot), c'_2(\cdot)\}} \mathbf{E}[p'_1 + \mathbf{E}_{\pi_1^f(-p'_1-c'_1)} p'_2(\cdot)]$$

It is straightforward to verify that refinable debt contracts are members of  $\mathcal{C}$ .

**Theorem 1.** If a truth-telling, renegotiation-proof contract is optimal, then it is refinable debt.

*Proof.* See appendix. □

To get an intuition for the result, consider an alternative option, say, one where  $F_1(s_1) = v_1$ . Depending on the realization of  $(\pi_1^e(\cdot), \pi_1^f(\cdot))$ , the optimal way to deliver continuation

payoff  $v_1$  to the financier may not involve setting  $p_1 = v_1$ . For example, if both parties believe that the entity is going to be sufficiently productive at date 2, the optimal thing to do may be for both parties to consume zero at date 1 and only consume at date 2. Suppose this is the case. Let  $s'_1 = (v'_1, \pi_1^{e'}(\cdot), \pi_1^{f'}(\cdot))$  be such a state and let  $p_2(s'_1, \cdot)$  be the financier's payment at date 2. By assumption then,  $\mathbf{E}_{\pi_1^{f'}(0)} p_2(s'_1, v_2) = v'_1$ . But now consider an alternate state  $s''_1 = (v''_1, \pi_1^{e''}(\cdot), \pi_1^{f''}(\cdot))$  where  $v''_1 < v'_1$ ,  $\pi_1^{e''} = \pi_1^{e'}|_{[-v''_1, \infty)}$ , and  $\pi_1^{f''} = \pi_1^{f'}|_{[-v''_1, \infty)}$ .

If state  $s'_1$  is realized, then, in fact, the entity is strictly better off misreporting that the state is  $s''_1$ . To see why, note that reporting  $s''_1$  means that the entity only needs to deliver continuation payoff  $v''_1$  to the entity. This fact, by itself, does not imply the entity is better off reporting  $s''_1$  because by reporting that the entity has lower value  $v''_1 < v'_1$ , the budget constraint for consumption,  $c_1 + p_1$ , is further tightened. This may be unattractive for the entity if it wants to consume a lot at date 1. But notice, in our situation, this tightening of the budget constraint at date 1 is of no concern: If the entity had reported the truth, they would not have wanted to consume anything at date 1 anyways. Thus, by misreporting the state as  $s''_1$ , the entity can guarantee that it will be better off, by, say renegotiating the continuation contract so that  $c_1 = p_1 = 0$  and choosing a  $p_2(s''_1, \cdot)$  that is strictly less than  $p_2(s'_1, \cdot)$  for every value of  $v_2$ . That the latter is possible is because, by misreporting, the entity is only obligated to choose a  $p_2(s''_1, \cdot)$  that delivers continuation payoff  $v''_1 < v'_1$ . The lesson learned by studying state  $s'$  is:

**Remark.** *Trying to vary the financier's payoff with the entity's value may backfire.*

Of course, this entire analysis was based on examining a state with a particular realization of belief functions that makes the entity tempted to misreport. There may be other realizations of belief functions such that when paired with the same entity value, would make the entity unwilling to misreport to a state with lower entity value. However, recall the financier has preference for robustness. So while the worst case outcome of misreporting may not be always happen, it does matter at the margin.

The proof of Theorem 1 is basically a more fleshed-out version of the above arguments.

### 3.2 Bayesian, Asymmetric Information Model

The lessons learned from the previous subsection evoke the following intuition for (refinance-able) debt: The financier demands a flat obligation from the entity - it doesn't care how the entity fulfills the obligation, just as long as the obligation is fulfilled. The financier acts this way because, frankly, it doesn't know enough about the entity to be able to pull off a more clever, state-dependent repayment scheme that is both more efficient and also incentive-compatible. If, however, the entity is unable to fulfill its flat obligation, then the financier demands everything so as to discourage the entity from claiming it can't pay up when in fact it can.

This intuition for debt is of course not novel. It is, more or less, the folk intuition for debt. The challenge of explaining debt has never been about coming up with the correct intuition, but, rather, coming up with the right formalization of the folk intuition that everyone already knows is correct. The application, in Section 3.1, of the refinance principle to contracting under asymmetric information can be seen as an attempt to take on this challenge.

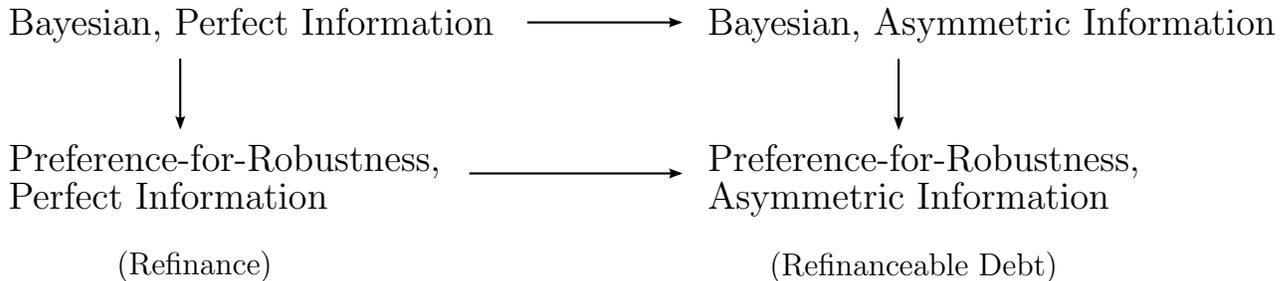


Figure 4

The financial contracting setting I considered in Section 3.1 is standard and general, and the asymmetric information problem I introduce is canonical. Theorem 1 then shows that in this standard setting, all one needs to do is add preference for robustness, and the optimality of (refinanceable) debt results. What I would like to do now is to argue that preference for robustness is not only sufficient but is, in some sense, also necessary for the optimal contract to be refinanceable debt.

Let us start by recalling the sequence of models considered that finally led to refinanceable debt. I began with the benchmark Bayesian, perfect information model. Solving the optimal contracting problem did not lead to any meaningful insights. I then moved to the preference for robustness, perfect information model. This led to the refinance principle. Finally, I considered the preference for robustness, asymmetric information model, which led to refinanceable debt. See Figure 4 for a graphical summary. The diagram in Figure 4 implies there is one more model that I have, thus far, overlooked - the Bayesian, asymmetric information model. Moreover, the diagram suggests that the Bayesian, asymmetric information model might be able to formalize the folk intuition for debt (without refinance).

I will now argue that this suggestion is, to some extent, true, and that moreover, one can view the costly state verification (CSV) approach to financial contracting as a manifestation of this suggestion. However, I will also argue that the link between the Bayesian, asymmetric information model and the optimality of debt is not robust. Ultimately, I will conclude that the right way to explain debt is to actually explain debt and its implied refinance option together, through the preference for robustness, asymmetric information model.

Formally, in the Bayesian, asymmetric information model, at date 1 the entity first privately observes the state of the world at date 1, which is now  $s_1 = (v_1, \pi_1(\cdot))$ . It then reveals  $\hat{v}_1$  and the financier has belief  $\pi_1(\cdot)$  restricted to the domain  $[-\hat{v}_1, \infty)$ . At date 0, the entity and the financier share a belief function  $\pi_0(\cdot)$  taking  $I_0$  to distributions over the date 1 state of the world instead of the financier having a confidence belief function taking  $I_0$  to distributions over the date 1 value of the entity. Once again, the set of incentive-compatible truth-telling, renegotiation-proof contracts can be identified with a certain subset  $\mathcal{C}$  of perfect information contracts. Given  $\pi_0(\cdot)$  and  $OC(I_0)$ , the optimal contracting problem is

$$\begin{aligned} \max_{\{I_0, p_1, c_1, p_2, c_2\} \in \mathcal{C}} & \mathbf{E}_{\pi_0(I_0)} [u(c_1) + \mathbf{E}_{\pi_1(I_1)} u(c_2)] \\ \text{s.t.} & \mathbf{E}_{\pi_0(I_0)} [p_1 + \mathbf{E}_{\pi_1(I_1)} p_2] \geq OC(I_0) \end{aligned}$$

First, consider the risk neutral case where  $u(c_i) = c_i$ . Risk-neutrality means that there is a multiplicity of optimal contracts in the Bayesian, perfect information world. Many of these first-best contracts lie in  $\mathcal{C}$ . Therefore, first-best can be achieved in the Bayesian, asymmetric information model. However, the model lacks predictive power since there is a multitude of very different looking optimal, renegotiation-proof, truth-telling contracts. For example, one optimal contract has the property that  $p_i/(c_i + p_i) = \alpha$  for a constant  $\alpha$  and can be interpreted as an equity contract. Another optimal contract has the property that  $p_1 = 0$ ,  $p_2 = v_2 \wedge F$  for some constant  $F$  and can be interpreted as debt. And yet others have more delicate structures and lack any real-life interpretations.

In order to narrow the set of optimal contracts, one could re-introduce risk-aversion. However, in this case, little can be said about the structure of the optimal contract just like in the Bayesian, perfect information case. In particular, the optimal contract will have a delicate structure, exhibiting full state-dependence and will generally not look like debt or any other recognizable simple contract.

This complexity problem comes from a combination of the contract being able to control everyone's consumption (i.e. complete contracts) and everyone being Bayesian. Complete consumption control means that the contract can be very fine-tuned. Being Bayesian means that fine-tuning contracts is valuable. Thus, if the goal is to get a relatively simple optimal contract, either complete consumption control or the Bayesian assumption must go.

The approach taken by the CSV literature is to essentially get rid of the complete consumption control assumption. That is to say, a unified way to view this paper and the CSV papers is to realize that both encounter the same contract complexity problem and both are trying to arrive at a simple optimal contract - debt. The difference is that the two take different routes to get there. This paper chooses to get rid of the Bayesian assumption. The CSV papers choose to get rid of complete consumption control.

The CSV literature assumes that the contract can only control the financier's consumption but gives the financier the ability to make costly state verifications. Costly state verification represents an incomplete and indirect way for the contract to control the entity's consumption. When the financier verifies the state, it can indirectly control how much the entity consumes by consuming whatever it does not want the entity to consume. However, the financier cannot do this when it does not verify the state because then it is constrained to consume only a portion of what is revealed by the entity. Under this incomplete consumption control plus Bayesian decision makers approach, the CSV literature shows that debt is the optimal contract but only if both players are risk neutral, there is no renegotiation-proof constraint, random state verifications are disallowed, and the model is static. These restrictions have been shown to be quite serious. For example, Mookherjee and Png (1989) shows that if random state verifications are allowed then the optimal contract will look drastically different from debt.

In contrast, this paper shows that by replacing the Bayesian financier with one who has a preference for robustness, not only can debt be explained, but so can the implicit refinance option. Moreover, refinanceable debt is renegotiation-proof, and it is optimal even allowing for random contracts and non risk-neutral preferences.

## 4 Conclusion

This paper provided a robust micro-foundation for refinanceable contracts based on assuming that the financier has a preference for robustness. I considered two versions of a standard dynamic financial contracting setting - a perfect information benchmark and a canonical asymmetric information version where the entity being financed privately observes the contract relevant state of the world. I first showed that optimal contracts in the perfect information model can be implemented as refinanceable contracts. I argued that refinanceable contracts admit a natural interpretation as incomplete contracts with delegation of control rights and details left to be negotiated later. Thus, the optimality of refinance result provides solid theoretical foundations for an important class of incomplete contracts in the financing setting.

I then moved to the asymmetric information setting and showed that the refinance principle continues to hold and that the optimal contract is refinanceable debt. The result represents a robust formalization of a widely accepted folk intuition for debt.

## 5 Appendix

*Proof of Theorem 1.* I begin with a characterization of contracts in  $\mathcal{C}$  before focusing on the optimal ones.

*Claim:* If a contract is in  $\mathcal{C}$ , then  $F_1(s_1) = F_1(v_1, \pi_1^f(\cdot))$ .

Suppose the claim is false. Then consider a let  $s'_1 = (v_1, \pi_1^{e'}(\cdot), \pi_1^f(\cdot))$  and  $s''_1 = (v_1, \pi_1^{e''}(\cdot), \pi_1^f(\cdot))$  be two states with  $F_1(s'_1) > F_1(s''_1)$ . Then, the entity can always report  $v_1$  truthfully but send message  $\hat{m}_1 = \pi_1^{e''}(\cdot)$  and then renegotiate the continuation contract into the optimal one for the firm subject to delivering  $F_1(s''_1) + \varepsilon$  to the financier. If  $\varepsilon > 0$  is sufficiently small, then both the entity and the financier are strictly better off. This violates the assumption that the contract is renegotiation-proof and the claim is proved.

Now, fix a contract in  $\mathcal{C}$ . Consider a pair  $(v'_1, \pi_1^{f'}(\cdot))$  and  $(v''_1, \pi_1^{f''}(\cdot))$  such that  $v'_1 > v''_1$  and  $\pi_1^{f'}(\cdot) \equiv \pi_1^{f''}(\cdot)$  on the domain  $[-v''_1, \infty)$ .

*Claim:* Suppose  $F_1(v''_1, \pi_1^{f''}(\cdot)) < \bar{F}_1(v''_1, \pi_1^{f''}(\cdot))$ . Then  $F_1(v'_1, \pi_1^{f'}(\cdot)) \leq F_1(v''_1, \pi_1^{f''}(\cdot))$ .

Suppose not. There exists a belief function  $\pi_1^{e'}(\cdot)$  such that the continuation contract following state  $s'_1 = (v'_1, \pi_1^{e'}(\cdot), \pi_1^{f'}(\cdot))$  involves date investment  $I_1 \geq -v''_1$ . In this case, I show that the entity is strictly better misreporting the state. Suppose instead the entity reported  $v''_1 < v'_1$ . Then by assumption, the financier would only demand continuation payoff  $F_1(v''_1, \pi_1^{f''}(\cdot)) < F_1(v'_1, \pi_1^{f'}(\cdot))$ . One continuation contract that achieves this is the following: The continuation contract keeps  $p_1$  and  $c_1$  the same. This is possible since by assumption  $I_1 \geq -v''_1$ . This leads to the same date 1 investment which generates the same distribution over  $v_2$ . Since the continuation contract needs to deliver a smaller continuation payoff to the financier, it can design  $p_2$  in a way so that, for every single realization of  $v_2$ , it is strictly smaller than the  $p_2(s'_1, \cdot)$  that would've been enacted had the entity told the truth. In this

new continuation contract, the entity's date 1 consumption is unchanged, but its date 2 consumption strictly increases, so the entity is strictly better off. This contradicts the truth-telling, renegotiation-proof assumption.

Lastly, fix an optimal contract and define the following constant:

$$F_1 := \inf_{\{(v_1, \pi_1^f(\cdot)) \mid F_1(v_1, \pi_1^f(\cdot)) < \bar{F}_1(v_1, \pi_1^f(\cdot))\}} F_1(v_1, \pi_1^f(\cdot))$$

*Claim:* The financier weakly prefers the refinanceable debt contract with face value  $F_1$  to the optimal contract.

Fix an  $\varepsilon > 0$  and pick a  $(v_1'', \pi_1^{f''}(\cdot))$  such that  $F_1(v_1'', \pi_1^{f''}(\cdot)) < \bar{F}_1(v_1'', \pi_1^{f''}(\cdot)) \wedge (F_1 + \varepsilon)$ . The previous claim implies that for every  $v_1' > v_1''$ , there exist states  $s_1'$  with entity value  $v_1'$  such that  $F_1(s_1') \leq F_1 + \varepsilon$ .

Now, let  $\tilde{s}_1 = (\tilde{v}_1, \tilde{\pi}_1^e(\cdot), \tilde{\pi}_1^f(\cdot))$  denote any state such that  $F_1(\tilde{s}_1) > F_1 + \varepsilon$  and let  $\tilde{S}$  denote the set of all such states. Then for any belief  $\pi_0$  satisfying  $\pi_0|_{v_1} \geq \mu_0(I_0)$ , there is another belief  $\pi_0'$  also satisfying  $\pi_0'|_{v_1} \geq \mu_0(I_0)$  that is identical to  $\pi_0$  except that for each state  $\tilde{s}_1 \in \tilde{S}$ , the weight on  $\tilde{s}_1$  is now moved to another state  $s_1'$  with entity value  $v_1' \geq \tilde{v}_1$  and  $F_1(s_1') \leq F_1 + \varepsilon$ . That this is possible comes from the first paragraph of the proof for this claim.

The expected payoff of the debt contract to the financier is the same under both  $\pi_0$  and  $\pi_0'$ . The expected payoff of the optimal contract to the financier weakly decreases going from  $\pi_0$  to  $\pi_0'$ . For the financier, the expected payoff of the optimal contract is weakly smaller than the expected payoff of the debt contract under  $\pi_0'$ . Thus, the financier weakly prefers the refinanceable debt contract with face value  $F_1 + \varepsilon$  to the optimal contract. Finally, since  $\varepsilon$  was arbitrary, the claim is proved.

*Claim:* Unless the optimal contract is refinanceable debt with face value  $F_1$ , the entity strictly prefers the refinanceable debt contract.

Pick a  $(v_1, \pi_1(\cdot))$  such that  $F_1(v_1, \pi_1(\cdot)) < \bar{F}_1(v_1, \pi_1(\cdot))$ . By definition of  $F_1$ ,  $F_1(v_1, \pi_1(\cdot)) \geq F_1$ . If  $F_1(v_1, \pi_1(\cdot)) > F_1$ , then the financier is strictly better off moving from the optimal contract to the refinanceable debt contract with face value  $F_1$ . Contradiction. If for every such  $(v_1, \pi_1(\cdot))$ ,  $F_1(v_1, \pi_1(\cdot)) = F_1$ , then the optimal contract is refinanceable debt with face value  $F_1$ .  $\square$

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