

Heterogeneous Beliefs and Short-term Credit Booms*

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Abstract

We study the financing of speculative asset-market booms in a standard framework with heterogeneous beliefs and short-sales constraints. Cash-constrained optimists use their asset holdings as collateral to raise debt financing from less optimistic creditors. Through state-contingent refinancing, short-term debt allows the optimists to reduce debt payment in upper states which they assign higher probabilities to, but at the expense of greater rollover risk if the asset fundamental deteriorates at the debt maturity. In contrast, long-term debt allows the optimists to hedge their financing cost in downturns. Our model identifies distinctive effects of initial and future belief dispersion in driving a short-term credit boom, which can in turn fuel an asset-market boom. Our model highlights the important effects of agents' debt-maturity and leverage choices on asset market equilibrium.

Keywords: Asset bubble, Debt maturity, Leverage, Rollover risk

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1 Introduction

Standard economic theories emphasize agents' consumption and portfolio choices as the key drivers of the asset market equilibrium. The recent credit crisis of 2007-2008 painted a different picture, in which the financing of market participants' investment positions, such as leverage and debt maturity, emerge at the center of the boom-and-crisis cycle. In the aftermath of the crisis, many commentators, e.g., Adrian and Shin (2009), Brunnermeier (2009), Gorton and Metrick (2009), and Krishnamurthy (2010), observed that during the housing-market boom before the crisis many financial institutions used large leverages to finance investment positions in mortgage backed securities. Furthermore, as we discuss in Section 4, the large leverages were accompanied by a shortening of debt maturity due to the dramatically increased use of short-term debt, such as overnight repos and asset-backed commercial paper. The difficulties of these financial institutions in rolling over their short-term debt after the housing market started to decline in 2007 had eventually led to a financial market meltdown.¹ The significant roles played by the short-term credit boom in financing the initial housing-market boom and in triggering the later financial crisis highlight the important effects of market participants' financing choices on asset-market dynamics. To the extent that short-term debt exposes the borrowers to rollover risk, their financing decisions appear non-trivial and deserve a systematic analysis.

In this paper, we develop a dynamic model to analyze the interactions between investors' financing choices and asset market dynamics. In particular, we focus on the role of debt maturity, jointly with that of leverage. We build on the framework recently proposed by Geanakoplos (2009), who nicely demonstrates that investors' leverage cycle can directly affect the asset-market cycle. This framework is ideal for our purpose as it involves optimists who hold more optimistic beliefs about the fundamental value of an asset using the asset as collateral to borrow from not so optimistic creditors to finance their asset purchases. This framework also nests a widely used asset-market setting for analyzing asset overvaluation and bubbles as joint effects of heterogeneous beliefs and short-sales constraints, e.g., Miller (1977), Harrison and Kreps (1978), Morris (1996), Chen, Hong, and Stein (2002), and Scheinkman and Xiong (2003). In this setting, short-sales constraints cause the equilibrium asset prices

¹Interestingly, short-term credit booms were also present in several other historical asset-market boom-and-crisis episodes. See White (1990) and Eichengreen and Mitchener (2003) for the stock market boom and crash of 1929, and Rodrik and Velasco (1999) and Reinhart and Rogoff (2009) for the emerging-market debt crises in 1990s.

to bias toward the beliefs of the optimists and, as a result, higher belief dispersion can lead to higher asset prices.

Specifically, our model has two periods and a risky asset whose fundamental value is unobservable. We consider two groups of risk-neutral agents holding heterogeneous and state-contingent beliefs, which originate from their heterogeneous prior beliefs and learning processes, about the asset fundamental. If the optimists have sufficient funds, they would acquire all the asset and bid up the asset price to their optimistic valuation. If the optimists have insufficient funds, then they have to use their asset holdings as collateral to raise debt financing from the pessimists who have excess funds. Like Geanakoplos (2009) and Simsek (2009), we restrict the optimists to standard non-contingent debt contracts, which are widely used in practice.² Different from them, we focus on the optimists' debt maturity choice in raising financing from the pessimistic creditors. The optimists' financing choices, which include both maturity and leverage, determine the credit they can obtain and thus the price they can offer for the asset. In this way, the asset market equilibrium is jointly determined with the credit market equilibrium.

A key advantage of short-term debt is that state-contingent refinancing allows the optimists to structure state-contingent debt payoffs to reduce financing cost and increase leverage. Because the possible asset fundamental decline during a short period is small, the optimists are able to raise a large leverage at the risk-free rate despite the creditors' not so optimistic belief. Thus, short-term debt provides a powerful leverage tool to fuel the optimists' speculative incentives. The downside of short-term borrowing is that if the asset fundamental declines during the initial period, the borrower will have to promise a higher debt payment to obtain refinancing or even to lose the collateralized asset in whole. If he still holds the more optimistic view about the asset fundamental, his greater promise (or the asset if forfeited) is under-valued by the creditor. Such under-valuation represents the so-called rollover risk, which has been recognized as a key trigger of short-term debt crises.³ On the other hand, by locking in the financing for a longer period, long-term debt can act as a hedging device against rollover risk during downturns. Thus, the optimists' initial speculative incentive

²Non-contingent debt contract is shown to be optimal in the costly state verification model of Townsend (1979), the monitoring model of Diamond (1984), and the contingent future financing model of Bolton and Scharfstein (1990). In these models, the unobservability of cash flows is important for the debt contract to be optimal.

³See Acharya, Gale, and Yorulmazer (2009) and He and Xiong (2009a, 2009b). In contrast to these models, the under-valuation (or the so-called firesale discount) in our model is endogenously determined by the heterogeneous beliefs between the borrowers and creditors.

and the subsequent rollover risk jointly determine whether short-term or long-term debt is desirable.

Our model highlights the distinctive roles of belief dispersion at different times. A higher initial belief dispersion about the asset fundamental over the first period creates a greater speculative incentive for the optimists and thus makes short-term debt more desirable, while a higher belief dispersion on the interim date after the fundamental deteriorates increases the optimists' rollover risk and discourages the use of short-term debt. The tradeoff between the initial and future belief dispersion implies that in a dynamic environment with time-varying beliefs, the intuitive argument made by Geanakoplos (2009) that optimists always prefer to max out risk-free short-term borrowing can be sharpened. Our model identifies the conditions for such intuition to hold and not to hold.

Our model suggests that the emergence of a short-term credit boom reflects excessive heterogeneous beliefs between agents. To the extent that short-term debt reduces the borrowers' financial stability, our model suggests that the increasing short-term leverages during a short-term credit boom reflects the borrowers' optimism about the future asset fundamental, as well as the creditors' concerns about the borrowers' ability to repay in the long term. Furthermore, as the rollover risk increases with the belief dispersion between the short-term borrowers and the creditors in the future down state, the heavy use of short-term debt involves not only a large initial belief dispersion, but also the borrowers' expectation that beliefs will soon converge, or, at least, not to diverge further in the future down state.

Our model also shows that if chosen by the optimists, short-term debt can fuel an asset-market boom by allowing them to raise large leverages at low interest rates. It can also act as the bridge from boom to crisis as the rollover risk amplifies the downturn after the asset fundamental deteriorates.

Through the financing-cost channel, our model shows that the common perception that in the presence of short-sales constraints higher belief dispersion leads to higher asset prices may not always hold. This is because the optimists' financing becomes more costly as their beliefs diverge further from the creditors'. In particular, our model highlights the distinctive effects of initial and future belief dispersion. To the extent that the optimists can use short-term debt to mitigate the increased financing cost caused by an increase in the initial belief dispersion, the equilibrium asset price remains increasing with the initial belief dispersion under a reasonable condition in our model. However, an increase of belief dispersion in the

future down state raises the rollover risk and thus discourages the use of short-term debt. This effect exacerbates the financing-cost effect and causes the equilibrium asset price to decrease with the belief dispersion in the future down state. The existing empirical studies of asset-price effects of heterogeneous beliefs, e.g., Chen, Hong, and Stein (2002) and Diether, Malloy, and Scherbina (2002), commonly treat investors' belief dispersion in different time horizons as qualitatively similar and focus on that in short term. In contrast, our model suggests distinctive effects between short-term and long-term belief dispersion.

The emphasis of our model differs from those focusing on the tightening of credit during crises (e.g., Brunnermeier and Pedersen (2009)) and those on the shortening of debt maturity during crises (e.g., He and Xiong (2009a) and Brunnermeier and Oehmke (2009)). Instead, our model identifies the environment that fosters short-term credit booms, which tend to initially fuel asset bubbles and later trigger financial crises.

Our model is related to the literature that studies the pervasive use of short-term debt by banks and financial firms. The existing literature has emphasized several advantages of short-term debt. First, short-term debt is a natural solution to a variety of agency problems inside a firm, e.g., Calomiris and Kahn (1991) and Diamond and Rajan (2001). By choosing short-term financing, creditors keep the option to pull out if they discover that firm managers are pursuing value-destroying projects. Second, the short commitment period also makes short-term debt less information sensitive and thus less exposed to adverse-selection problems, e.g., Gorton and Pennacchi (1990). While these theories imply that firms regularly use certain amounts of short-term debt, they do not explain the increasing use of short-term debt during asset-market booms.

Our model complements Garmaise (2001), who studies the security-design problem of a cash-constrained firm facing investors with heterogeneous beliefs. His model contrasts the optimal security design under heterogeneous beliefs to that under rational expectations, while our model focuses on the role played by debt structure in fueling asset-market speculation driven by heterogeneous beliefs. Landier and Thesmar (2008) derives a model in which optimism leads entrepreneurs to use short-term debt. Their model assumes that the entrepreneurs assign zero probability to the future down state, in which short-term debt will force them give up the control of their firms. As a result, the model does not consider rollover risk and the tradeoff between the borrowers' speculative incentives and rollover risk, which is the focus of our model.

This tradeoff is analogous to that considered by Diamond (1991), who analyzes debt maturity choice for borrowers with private information about their future credit rating. In his model, borrowers face a tradeoff between a preference for short maturity due to expecting their credit rating to improve, against liquidity risk due to the loss of their private rents that cannot be assigned to the creditors. Our model ties both sides of the tradeoff—the borrowers’ speculative incentives and rollover risk—to the heterogeneous beliefs between the borrowers and creditors, which in turn allows us to link the borrowers’ financing choices to the asset market equilibrium.

The paper is organized as follows. Section 2 presents a baseline model with two groups of agents holding exogenously specified beliefs. Section 3 extends the baseline model with learning and three groups of agents. We discuss the implications of the model in Section 4, and conclude in Section 5. All technical proofs are provided in the Appendix.

2 The Model

2.1 Asset and Agents

Consider a model with three dates and two periods. The date is indexed by $t = 0, 1, 2$. There is a long-term risky asset, which we interpret either as a house or a mortgage backed security. The asset pays a final payoff on date 2. The final payoff is determined by the final realization of a publicly observable binomial tree. Figure 1 illustrates the tree. The tree can go either up or down from $t = 0$ to $t = 1$ and from $t = 1$ to $t = 2$. The tree has four possible paths, which we denote by uu , ud , du , and dd (here, u stands for “up” and d stands for “down”), and three possible final nodes (paths ud and du lead to the same final node). We normalize the final payoff of the risky asset at the end of path uu as 1, at the end of paths ud and du as θ , and at the end of paths dd as θ^2 , where $\theta \in (0, 1)$. We denote the asset payoff by $\tilde{\theta} \in \{1, \theta, \theta^2\}$.

The probability of the tree going up in each period is unobservable. Suppose that there are two groups of risk-neutral agents, who differ in their beliefs about these probabilities. In this section, we exogenously specify two sets of beliefs for the agents. Ultimately, the difference in the agents’ beliefs is driven by their prior beliefs and learning processes, and we will extend the model with learning in Section 3.

There are three intermediate nodes on the tree, one on date 0 and two on date 1 (u and d depending on whether the tree goes up or down in the first period). We collect these

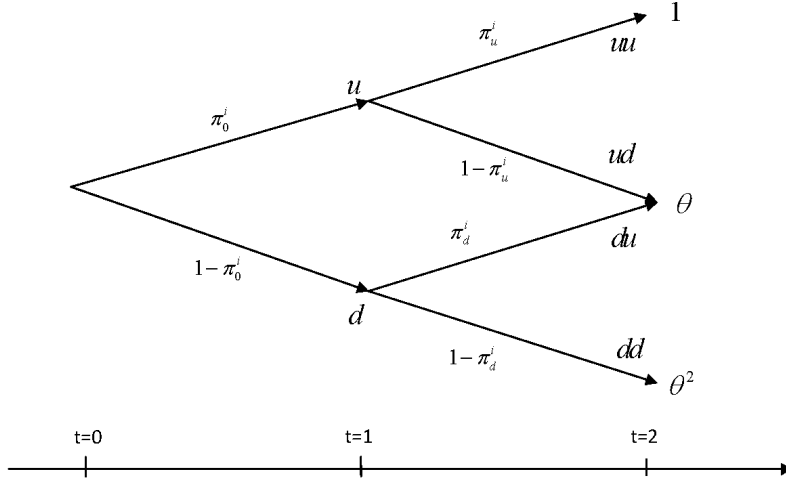


Figure 1: Timeline.

intermediate nodes in the following set: $\{0, u, d\}$. At each of the nodes, each agent has a belief about the probability of the tree going up in the following period. We collect each agent's beliefs in the following set: $\{\pi_0^i, \pi_u^i, \pi_d^i\}$, where $i \in \{h, l\}$ indicates the agent's type. Throughout this section, we assume that the h -type agents are always more optimistic than the l -type agents across all the intermediate nodes (here, the superscript “ h ” and “ l ” stands for high and low.) That is, $\pi_n^h > \pi_n^l$ for any $n \in \{0, u, d\}$. Based on the relative order, we call the h -type agents optimists and the l -type pessimists.

In particular, we emphasize that the belief dispersion between the optimists and pessimists is not constant. Standing at $t = 0$, the difference between π_0^h and π_0^l represents the initial belief dispersion between the two groups about the asset fundamental from date 0 to 1, while the difference between π_d^h and π_d^l represents the future belief dispersion about the asset fundamental from the date-1 state d to date 2. As we will show later, these two types of belief dispersion play distinctive roles in determining the optimal debt maturity choice.

We summarize the final asset payoffs at the end of the four possible tree paths and the optimists' and pessimists' belief about each of the paths in Table 1. Note that the optimists assign a higher probability to path uu and a lower probability to path dd . But his beliefs about the middle paths ud and du can be higher or lower than those of the pessimists, which we will specifically discuss later.

We normalize the total supply of the asset to be one unit. There are $\mu \in (0, 1)$ units of

Table 1: Asset Payoff and Agent's Belief across Different Paths

	Tree Paths			
	uu	ud	du	dd
Asset payoff	1	θ	θ	θ^2
Optimists' belief	$\pi_0^h \pi_u^h$	$\pi_0^h (1 - \pi_u^h)$	$(1 - \pi_0^h) \pi_d^h$	$(1 - \pi_0^h) (1 - \pi_d^h)$
Pessimists' belief	$\pi_0^l \pi_u^l$	$\pi_0^l (1 - \pi_u^l)$	$(1 - \pi_0^l) \pi_d^l$	$(1 - \pi_0^l) (1 - \pi_d^l)$

optimists, who are homogeneous. On date 0, each optimist is initially endowed with 1 unit of the risky asset and c dollars of cash. Given the optimists' optimism, it is natural for them to purchase the rest of the asset ($1 - \mu$ unit) from the pessimists. Following Miller (1977), Harrison and Kreps (1978), Morris (1996), Chen, Hong, and Stein (2002), and Scheinkman and Xiong (2003), we assume that short-sales of the asset are not allowed. As a result, the pessimists cannot speculate on the asset price falling in the future and will sit on the sideline.

The focus of our analysis is on the financing of the optimists' asset purchases. Since they may not have sufficient cash, they may need to borrow from the pessimists who sit on the sideline with cash. As the pessimists' beliefs affect the cost of financing to the optimists, their beliefs can indirectly affect the equilibrium asset price. For simplicity, we assume that both the risk-free interest rate and the agents' discount rate are zero, and that the pessimists on the sideline will always have sufficient cash. Therefore, in equilibrium they always demand zero expected return in financing the optimists.

2.2 Collateralized Debt Financing

Like Geanakoplos (2009), we assume that the optimists use their asset holdings as collateral to obtain debt financing. We focus on non-contingent debt contracts. A non-contingent debt contract specifies a constant debt payment (face value) at maturity unless the borrower defaults. Non-contingent debt contracts are widely used in practice. Townsend (1979) explains its popularity based on the cost of verifying the state of the world. That is, non-contingent debt contracts circumvent the cost of verifying the value of the collateral as long as the borrower makes the promised payment. Diamond (1984) and Bolton and Scharfstein (1990) also derive the optimality of non-contingent debt based on unobservability of cash flows. In

Table 2: Asset Payoff and Debt Payment across Different Paths

	Tree Path			
	uu	ud	du	dd
Asset payoff	1	θ	θ	θ^2
Long-term debt face value $F_L \in [\theta^2, \theta]$	F_L	F_L	F_L	θ^2
Short-term debt face value $F_S \in [\theta^2, K_d]$	F_S	F_S	$F_{S,1} \geq F_S$	θ^2
Short-term debt face value $F_S \in [K_d, \theta]$	F_S	F_S	θ	θ^2

this model, we will restrict the optimists to use only non-contingent debt.

We will first discuss long-term debt contracts, and then short-term ones. We will restrict our attention to contracts with face values in $[\theta^2, \theta]$. We will show in Lemma 2 in Section 2.3.2 that this is without loss of generality in the equilibrium.

2.2.1 Long-term Debt

Consider a long-term debt contract, which is collateralized by one unit of the asset. The contract matures on date 2 and has a face value of $F_L \in [\theta^2, \theta]$. The debt payment is

$$\tilde{D}_L(F_L) = \min(F_L, \tilde{\theta}).$$

Depending on the four possible paths of the tree, the asset payoff and debt payment are listed in Table 2. Given the debt payment, a pessimistic creditor is willing to provide the following credit on date 0:

$$C_L(F_L) = \mathbb{E}_0^l[\tilde{D}_L] = (1 - (1 - \pi_0^l)(1 - \pi_d^l))F_L + (1 - \pi_0^l)(1 - \pi_d^l)\theta^2, \quad (1)$$

where $\mathbb{E}_n^i[\cdot]$ denotes the conditional expectation of a type- i agent on node $n \in \{0, u, d\}$. On the other hand, from the optimistic borrower's perspective, the expected cost of using this debt contract is

$$\mathbb{E}_0^h[\tilde{D}_L] = (1 - (1 - \pi_0^h)(1 - \pi_d^h))F_L + (1 - \pi_0^h)(1 - \pi_d^h)\theta^2. \quad (2)$$

The difference between (1) and (2) highlights a key feature of our model—the borrower and creditor use different probabilities in assessing the cost and value of a debt contract.

In particular, as the borrower is optimistic and assigns a higher probability to path uu , the promised payment F_L at the end of this path is more costly to the borrower than valued by the creditor. Thus, the first-best allocation of asset payoffs between the borrower and creditor would be to assign all of the asset payoff at the end of path uu to the borrower. However, such a non-monotonic allocation is infeasible under standard non-contingent debt contracts, which stipulates monotonic payoffs.

Interestingly, as we will show next, the standard non-contingent short-term debt—through refinancing—can generate non-monotone debt payments, which is the main advantage of short-term debt over long-term debt.

2.2.2 Short-term Debt

We now consider a short-term debt contract collateralized by one unit of asset, and with a promised payment $F_S \in [\theta^2, \theta]$ due on date 1. Different from long-term debt, short-term debt requires refinancing (or rollover) at date 1. As we will prove in Propositions 5 and 6, no optimist saves cash in the equilibrium. This means that it is impossible for any optimist to sell his asset holding to another optimist on date 1. As a result, we only need to consider the optimist's refinancing on date 1 if he uses short-term debt financing. Moreover, if the optimist fails to roll over his debt, he has to forfeit the asset to the creditor who values it at a lower value.

A key insight of our model is that state-contingent refinancing of short-term debt makes it possible for the borrower to reduce debt payment at the end of path uu by trading up payments at the end of some lower paths. Specifically, in the upper interim state u , the borrower can always get a new contract with the same face value F_S , because the asset payoff is always sufficient to pay off the debt regardless of the subsequent final state being 1 or θ . However, in the lower interim state d , the borrower will get a worse term and may even lose the asset if the initially promised payment is too large. Specifically, the face value of the new contract $F_{S,1}$ needs to ensure that the pessimistic lender's valuation of the new debt contract is sufficient for offsetting the initially promised payment F_S :

$$\mathbb{E}_d^l \left[\min \left(F_{S,1}, \tilde{\theta} \right) \right] = F_S.$$

Since the highest possible date-2 payment the borrower can promise is θ , the maximum amount of credit the borrower can raise in this state is:

$$K_d \equiv \mathbb{E}_d^l \left[\min \left(\theta, \tilde{\theta} \right) \right] = \mathbb{E}_d^l \left[\tilde{\theta} \right] = \pi_d^l \theta + (1 - \pi_d^l) \theta^2 < \theta. \quad (3)$$

This implies that the borrower will fail to refinance his short-term debt in the interim state d if the initially promised date-1 debt payment F_S is higher than K_d . Therefore, we have the following two cases to consider:

1. If $F_S \in [\theta^2, K_d]$, the initial short-term debt is riskless and the borrower can obtain a credit of $C_S(F_S) = F_S$ on date 0. In the lower interim state d , the borrower can roll into a new short-term debt contract with face value $F_{S,1}$:

$$F_{S,1} = \frac{F_S}{\pi_d^l} - \frac{1 - \pi_d^l}{\pi_d^l} \theta^2 \geq F_S. \quad (4)$$

The borrower has to promise more as the value of the collateral has deteriorated.

2. If $F_S \in (K_d, \theta]$, the initial debt contract is risky. In the lower interim state d , the payment due exceeds the maximum amount of debt the borrower can refinance from any pessimistic creditor using the asset as the collateral. The borrower thus defaults and loses the asset to the creditor. This outcome is equivalent to the final debt payment at the end of paths du and dd being θ and θ^2 , respectively. By using this debt contract, on date 0 the borrower can get a credit of:

$$C_S(F_S) = \pi_0^l F_S + (1 - \pi_0^l) (\pi_d^l \theta + (1 - \pi_d^l) \theta^2) \text{ if } F_S \in (K_d, \theta].$$

Table 2 summarizes the final debt payments across the four possible tree paths for the two cases discussed above. In both cases, the state-contingent refinancing makes the final debt payment non-monotonic with respect to the final asset payoff, i.e., the debt payment at the end of paths uu and ud is lower than at the end of path du . This rearrangement of debt payment is potentially valuable to the borrower as he assigns a higher probability to path uu than the creditor.

Note that in order to keep the debt risk-free, the highest face value of long-term debt is θ^2 while the highest face value of short-term debt is $K_d > \theta^2$. This means that the borrower can obtain a higher leverage from using short-term debt without increasing the initial-period borrowing cost. Of course, if the asset fundamental deteriorates to state d on date 1, the borrower has to refinance at a higher interest rate for the following period because the debt is now risky.

2.3 The Optimal Debt Contract

To study the optimal debt contract used by an optimistic buyer, we take the asset price p_0 as given. In light of Table 2, we denote a debt contract (either long-term or short-term) by a set of state-contingent debt payment \tilde{D} . Furthermore, we denote $C(\tilde{D}) \equiv \mathbb{E}_0^l[\tilde{D}]$ as the date-0 credit that a borrower can obtain from a pessimist by using the debt contract \tilde{D} .

What is the maximum unit of asset that an optimist can afford on date 0 by using the debt contract \tilde{D} ? He is initially endowed with c dollars of cash and 1 unit of the asset. Suppose that he purchases additional x_i units in the market. His total purchasing power is $c + (1 + x_i)C(\tilde{D})$, the sum of his cash endowment and the credit he can raise by using his asset holding ($1 + x_i$ units in total) as collateral. The budget constraint implies that

$$c + (1 + x_i)C(\tilde{D}) = x_i p_0 \Rightarrow x_i = \frac{c + C(\tilde{D})}{p_0 - C(\tilde{D})}. \quad (5)$$

An implicit assumption in this calculation is that the optimist maxes out his purchasing power, a conjecture that we will verify in Propositions 5 and 6. For each unit of asset, the optimists' date-0 expectation of the date-2 cash flow after netting out the debt payment is $\mathbb{E}_0^h[\tilde{\theta} - \tilde{D}]$. Therefore, the optimist's date-0 value from using the contract \tilde{D} (i.e., the expectation of the final wealth) is

$$V(\tilde{D}) = (1 + x_i)\mathbb{E}_0^h[\tilde{\theta} - \tilde{D}] = \frac{c + p_0}{p_0 - C(\tilde{D})} \left[\mathbb{E}_0^h(\tilde{\theta}) - \mathbb{E}_0^h(\tilde{D}) \right]. \quad (6)$$

This expression illustrates the tradeoff in the optimist's debt choice. On one hand, by promising a collateralized debt payment \tilde{D} on each unit of asset holding, the buyer can raise a credit of $C(\tilde{D})$ and thus establish a larger initial position $\frac{c+p_0}{p_0-C(\tilde{D})}$, which is the first part in $V(\tilde{D})$. This term represents a *leverage* effect. On the other hand, the debt payment reduces the asset payoff to the buyer on date 2. This *debt-cost* effect is reflected in the second part $\mathbb{E}_0^h(\tilde{\theta}) - \mathbb{E}_0^h(\tilde{D})$ in $V(\tilde{D})$.

The debt contract contains two dimensions: *debt maturity* (long-term or short-term) and *promised payment* (i.e., the debt face value). Both are determined by the tradeoff between the leverage effect and debt-cost effect. We will first analyze the agent's maturity choice, and then the face-value choice.

2.3.1 Maturity Choice

To derive the optimal debt maturity, we consider the following question: in order to raise the same amount of credit at $t = 0$, which contract (i.e., long-term or short-term) entails the lower expected cost? Equation (6) implies that the one with the lower cost dominates the other. The following key proposition shows that the optimal maturity choice is determined by the initial and future belief dispersion between the optimists and pessimists.

Proposition 1 *Consider two debt contracts, one short-term and the other long-term. Suppose that both contracts have a face value in $[\theta^2, \theta]$ and give the same date-0 credit to an optimistic borrower. Then, from the borrower's perspective on date 0, the short-term contract requires a (weakly) lower expected cost if and only if*

$$\frac{\pi_0^h}{\pi_0^l} > \frac{(1 - \pi_0^h) \pi_d^h}{(1 - \pi_0^l) \pi_d^l}. \quad (7)$$

Proposition 1 shows that whether the short-term debt contract dominates the long-term debt contract depends on the initial belief ratio between the optimists and pessimists (π_0^h/π_0^l about the first-period fundamental), and the future belief ratio in the lower interim state d (π_d^h/π_d^l about the second-period fundamental.) The short-term contract is dominant if the initial belief ratio is sufficiently large, or if the future belief ratio is sufficiently small.

To understand the intuition, a debt contract not only channels the necessary financing from a creditor to an optimistic borrower to purchase the asset, but also represents an allocation of the asset payoffs between the borrower and creditor. As we have discussed earlier, a long-term contract specifies a monotonic debt payment with respect to the asset payoff, while a short-term contract allows the borrower to reduce the debt payments at the end of paths uu and ud by trading up the payment at the end of path du (the payment at the end of path dd is maxed out.)

Is this tradeoff worthwhile? It first depends on the initial belief ratio π_0^h/π_0^l . If this ratio becomes higher, the reduced future debt payment after the upper interim state u becomes more valuable to the borrower and the increased payment after the lower interim state d becomes less important. Conversely, the creditor finds the reduced payment after u less important, while the increased payment after d more valuable. As a result, the short-term debt contract becomes more desirable to both of the borrower and creditor. This effect reflects the two parties' speculative incentives driven by their initial belief dispersion.

There is also the so-called rollover-risk effect working against short-term debt. In the lower interim state d , the borrower still holds the more optimistic belief going forward. This means that the increased future debt payment is under-valued by the creditor. In other words, the borrower gives up some future asset payoffs at a price lower than his own valuation. In the extreme case, if he cannot obtain sufficient refinancing to repay his maturing debt obligation, he has to give up the asset in whole to the creditor who does not value the asset as much as he does. This under-valuation of the increased debt payment is determined by the belief ratio between the borrower and creditor π_d^h/π_d^l in the interim state d after adjusting for the probability that this state is realized $(1 - \pi_0^h) / (1 - \pi_0^l)$. When the right-hand side of condition 7 becomes large, the rollover-risk effect becomes severe and thus makes it more costly for the borrower to use short-term debt.

The tradeoff between the speculative-incentive effect and rollover-risk effect implies that short-term debt is not always preferable. This result contrasts an intuitive argument made by Geanakoplos (2009) that the optimists always choose to finance their asset purchases using the most aggressive risk-free short-term debt contract (i.e., the contract with a face value of K_d in our model.) This argument ignores the possible rollover risk and does not hold if the future belief dispersion in state d is sufficiently large.

2.3.2 Optimal Debt Face Value

To derive the optimal debt face value, we first characterize its feasible range:

Lemma 2 *If $\mathbb{E}_0^l [\tilde{\theta}] \leq p_0 \leq \mathbb{E}_0^h [\tilde{\theta}]$, then the optimal debt face value is inside $[\theta^2, \theta]$.*

Lemma 2 shows that the optimal debt face value (for either long-term or short-term contract) lies inside the interval $[\theta^2, \theta]$. The condition for this result—the date-0 asset price lies between the pessimists’ and optimists’ asset valuations—is innocuous because it always holds in the equilibrium throughout this section. When we extend the model to incorporate learning in the next section and allow agents’ beliefs to flip on date 1, the equilibrium asset price could be higher than the optimists’ asset valuation because of the asset owner’s resale option. However, the feasible range of the optimal debt face value derived in Lemma 2 still holds after we modify the condition to account for the resale option.

The intuition of Lemma 2 is as follows. If the optimal debt face value is lower than θ^2 , then the debt is risk free. Thus, increasing the face value by a small amount ϵ does not change the risk of the debt, and thus allows the borrower to increase his initial financing by

ϵ at a cost exactly equal to ϵ . Since the asset price is lower than his asset valuation, the increased credit allows him to take a larger asset position and therefore be better off. This shows that the optimal debt face value cannot be smaller than θ^2 . On the other hand, if the optimal debt face value is higher than θ , the borrower always defaults on the debt except at the end of the path uu . This implies that reducing the face value by a small amount ϵ allows the borrower to save debt payment at the end of path uu , which he values more than the creditor. Of course, this also cuts down his initial asset position. In the proof provided in Appendix A.2, we show that as long as the asset price is higher than the pessimistic creditor's asset valuation, the borrower is better off by reducing the debt face value. Thus, the optimal debt face value cannot be higher than θ either.

The following proposition provides the borrower's optimal long-term debt face value conditional on long-term debt being more desirable.

Proposition 3 *Suppose that the condition in (7) does not hold. Thus, it is optimal for the borrower to use long-term debt. Define*

$$P_M \equiv (1 - (1 - \pi_0^l)(1 - \pi_d^l)) \mathbb{E}_0^h [\tilde{\theta} | uu, ud, du] + (1 - \pi_0^l)(1 - \pi_d^l) \mathbb{E}_0^l [\tilde{\theta} | dd].$$

Then, the borrower's optimal debt face value is θ if $p_0 < P_M$; is either θ or θ^2 if $p_0 = P_M$; or is θ^2 if $p_0 > P_M$.

The discrete asset payoff implies that varying the long-term debt face value F_L between θ^2 and θ does not change the risk of the debt. In other words, regardless of the value of F_L in this region, the borrower will always make the promised debt payment F_L at the end of paths uu , ud , and du , and default and thus give up all the asset payoff at the end of dd . Since the borrower is risk-neutral, he will use the highest face value θ to maximize his position if the asset price is below a critical level P_M , which weighs his asset valuation and cost of financing. More precisely, P_M is a weighted average of the borrower's asset valuation in the upper (non-default) states $\{uu, ud, du\}$ and the creditor's valuation in the lower (default) state dd . If we interpret the long-term debt contract as a static contract that spans two periods, then Proposition 3 is analogous to the result of Simsek (2009).

If the borrower uses short-term debt, the default risk of the contract depends on whether the debt face value is higher or lower than K_d , where K_d is the asset's maximum debt capacity in the lower interim state d . If the face value is between θ^2 and K_d , the borrower is always

able to refinance and the initial debt contract is risk free, even though the follow-up contract in the interim state d is risky as the borrower will default on date 2 at the end of path dd . If the face value is between K_d and θ , the borrower cannot get a new debt contract to pay off the initial debt in the state d , and thus default on the debt.

In the same spirit to Proposition 3, the next proposition shows that the borrower will choose to use the highest face value inside the two regions $[\theta^2, K_d]$ and $[K_d, \theta]$ if the asset price is below two thresholds P_H and P_L , respectively. These thresholds reflect the borrower's asset valuation and the cost of using debt in these regions.

Proposition 4 *Suppose that the condition in (7) holds. Thus, it is optimal for the borrower to use short-term debt. Define*

$$P_H \equiv \frac{\pi_d^l (\pi_0^h + (1 - \pi_0^h) \pi_d^h)}{(1 - \pi_0^h) \pi_d^h + \pi_0^h \pi_d^l} \mathbb{E}_0^h [\tilde{\theta} | uu, ud, du] + \frac{(1 - \pi_0^h) \pi_d^h (1 - \pi_d^l)}{(1 - \pi_0^h) \pi_d^h + \pi_0^h \pi_d^l} \mathbb{E}_0^l [\tilde{\theta} | dd]$$

and

$$P_L \equiv \pi_0^l \mathbb{E}_0^h [\tilde{\theta} | uu, ud] + (1 - \pi_0^l) \mathbb{E}_0^l [\tilde{\theta} | du, dd],$$

which satisfy

$$P_L < P_M < P_H.$$

Then, the borrower's optimal short-term debt face value is θ if $p_0 < P_L$; is either θ or K_d if $p_0 = P_L$; is K_d if $P_L < p_0 < P_H$; is either K_d or θ^2 if $p_0 = P_H$; or is θ^2 if $p_0 > P_H$.

The core of Propositions 3 and 4 is that when the asset price becomes cheaper relative to the buyer's own valuation (after adjusting for the financing cost), he will demand a greater position. To finance the greater position, he uses a higher debt face value to obtain more credit. In the next subsection, we will use these two propositions to derive the joint equilibrium of the asset and credit markets.

2.4 The Equilibrium of Asset and Credit Markets

We now derive the equilibrium on dates 0 and 1.

On date 0, the amount of asset purchase by an individual optimistic buyer using a debt contract \tilde{D} is given by Equation (5). Propositions 1, 3, and 4 jointly determine the optimal contract $\tilde{D}(p_0)$ based on the buyer's and creditor's heterogeneous beliefs and the asset price

p_0 . The total measure of buyers in the economy is μ . Their aggregate purchase $\sum_i x_i$ should equal to the total asset endowed by the (pessimistic) sellers $1 - \mu$:

$$\sum_i x_i = 1 - \mu. \quad (8)$$

If all the buyers use the same debt contract $\tilde{D}(p_0)$ to finance their purchases, the market clearing condition

$$\mu \frac{c + C(\tilde{D}(p_0))}{p_0 - C(\tilde{D}(p_0))} = 1 - \mu$$

implies that

$$C(\tilde{D}(p_0)) = (1 - \mu)p_0 - \mu c. \quad (9)$$

This equation illustrates the intricate interaction between the asset price and the endogenously determined amount of credit to the asset buyers. In light of Propositions 3 and 4, the buyers' optimal credit demand—the term $C(\tilde{D}(p_0))$ on the left hand side—decreases with the asset price p_0 . On the other hand, the credit available to the buyers needs to be sufficient to support their asset purchases (market clearing condition), i.e., in equilibrium the buyers' aggregate cash shortfalls should equate their credit demand. The linearly increasing function on the right hand side gives the buyers' cash shortfall—the value of their purchases ($1 - \mu$ shares multiplied by the price p_0) minus their cash endowments μc .

2.4.1 Long-term Debt Equilibrium

Figure 2 plots the two sides of (9) when the condition in (7) fails and borrowers prefer to use long-term debt. As derived in Proposition 3, each buyer's optimal credit demand can take two possible values, $C_L(\theta^2)$ or $C_L(\theta)$. The two-piece horizontal line with a downward jump at P_M represents the credit demand $C(\tilde{D}(p_0))$, i.e., the left hand side of (9). The upward sloping curve represents the necessary credit needed to clear the asset market, the right hand side of (9).

As the intercept of the asset-market clearing condition ($-\mu c$) increases, we encounter three possible cases in equilibrium. First, if each buyer's cash endowment c is high, the optimal credit demand curve and the asset-market clearing condition intersect at a point, where the equilibrium asset price p_0 is higher than P_M and each buyer demands a modest amount of credit $C_L(\theta^2)$. We label this case by case LD1. In this case, the buyers' ample

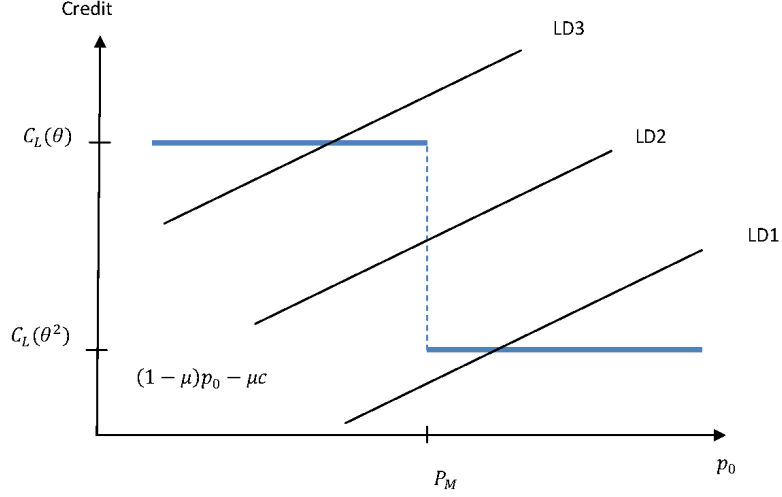


Figure 2: The equilibrium with long-term debt financing.

cash endowments allow them to bid up the asset price to a high level without using much credit. Second, if each buyer's cash endowment c is low, the two curves intersect at a point, where the equilibrium price p_0 is lower than P_M and each buyer demands a large amount of credit $C_L(\theta)$. We label this case by case LD3. In this case, the buyers' limited cash endowments constrain the price from rising high even though each buyer uses an aggressive debt contract.

The case LD2 occurs when the upward sloping asset-market clearing condition passes the middle of the two horizontal levels of the buyers' credit demand curve. In this case, the equilibrium price is exactly P_M and each buyer is indifferent between using long-term debt contracts with face values θ and θ^2 (Proposition 3). Then, the asset market clearing condition (8) is fulfilled by finding a certain mix of buyers using these two contracts. Denote by λ the fraction of buyers using the contract with face value θ . Condition (8) is equivalent to

$$\mu\lambda \frac{c + C_L(\theta)}{P_M - C_L(\theta)} + \mu(1 - \lambda) \frac{c + C_L(\theta^2)}{P_M - C_L(\theta^2)} = 1 - \mu,$$

which implies

$$\lambda = \frac{\frac{1-\mu}{\mu} - \frac{c+C_L(\theta^2)}{P_M-C_L(\theta^2)}}{\frac{c+C_L(\theta)}{P_M-C_L(\theta)} - \frac{c+C_L(\theta^2)}{P_M-C_L(\theta^2)}}. \quad (10)$$

The equilibrium on date 1 depends on the optimistic asset holders' financial conditions in the two possible states: u and d . In the upper state u , each optimist has gained on his

initial position and is now in a strong financial condition. As a result, each optimist is free to buy or sell some share of the asset at a price equal to his own valuation:

$$p_u = \mathbb{E}_u^h \left[\tilde{\theta} \right] = \pi_u^h + (1 - \pi_u^h) \theta. \quad (11)$$

In the lower state d , each optimist suffers a loss from his initial position. In the LD1 case, because of the modest leverage used by each optimist from the long-term debt contract with face value θ^2 , his portfolio is still solvent. At the marginal, he can still afford to buy a small unit of asset at his own valuation

$$p_d = \mathbb{E}_d^h \left[\tilde{\theta} \right] = \pi_d^h \theta + (1 - \pi_d^h) \theta^2. \quad (12)$$

if it is offered in the market. He is also willing to sell the asset at the same price. In the LD2 and LD3 cases, at least some of the optimists have used the aggressive long-term contract with face value θ and their portfolios are effectively under water in state d (i.e., regardless of whether the final state is θ or θ^2 , the asset payoff all goes to the creditor.) Their financial distress leads to a market freeze. On one hand, the asset holders have no incentive to sell their assets at any price below θ because they cannot get any benefit from such a trade. This is exactly the so-called debt overhang problem coined by Myers (1977). This problem is also widely recognized as an important issue in the understand the dry up of asset market liquidity during the recent credit crisis, e.g., Diamond and Rajan (2009). On the other hand, only pessimists have cash and they only value the asset at a value of $\mathbb{E}_d^l \left[\tilde{\theta} \right] = \pi_d^l \theta + (1 - \pi_d^l) \theta^2 < \theta$. This gap between the lower ask and highest bid prices implies that there cannot be any trade in the equilibrium. For the sake of our illustration later, we will use a price

$$p_d = \theta \quad (13)$$

in these scenarios, even though this is not a market price.

Based on the market equilibrium discussed above, it is direct to see that there is no incentive for any optimist to save cash on date 0. Regardless of the debt contracts used by other optimists on date 0 and the state realized on date 1, saving cash on date 0 does not lead to any possible gain on date 1.

In the following proposition, we summarize the discussion on the joint equilibrium of the asset and credit markets in which the buyers only use long-term debt contracts..

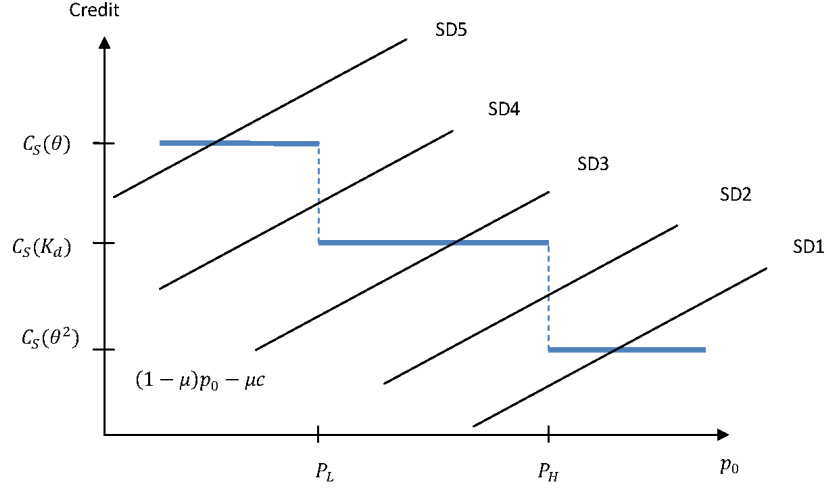


Figure 3: The equilibrium with short-term debt financing.

Proposition 5 *Suppose that the condition in (7) fails and the buyers use long-term debt contracts to finance their asset purchases. Then, there is no incentive for any buyer to save cash and the equilibrium can be broken down into the following three cases:*

- LD1: *If $C_L(\theta^2) > (1 - \mu)P_M - \mu c$, then $p_0 = \frac{\mu c + C_L(\theta^2)}{1 - \mu}$, all the buyers use the same long-term debt contract with face value θ^2 on date 0, and the asset price in the two states on date 1 are given by equations (11) and (12);*
- LD2: *If $C_L(\theta^2) \leq (1 - \mu)P_M - \mu c \leq C_L(\theta)$, then $p_0 = P_M$, each buyer is indifferent between the long-term debt contracts with face values of θ and θ^2 on date 0 with the fraction of buyers using the former contract given in (10), and the asset price in the two states on date 1 are given by equations (11) and (13);*
- LD3: *If $C_L(\theta) < (1 - \mu)P_M - \mu c$, then $p_0 = \frac{\mu c + C_L(\theta)}{1 - \mu}$, all the buyers use the same long-term debt contract with face value θ on date 0, and the asset price in the two states on date 1 are given by equations (11) and (13)*

2.4.2 Short-term Debt Equilibrium

If the condition in (7) holds, the buyers would prefer short-term debt. Figure 3 shows five possible cases for the equilibrium. Proposition 6 lists these cases. The logic for these cases is similar to that for Proposition 5. It is worth mention that on date 1 if any optimist runs into

financial distress in the lower state d from using the aggressive short-term debt contracts with face value of either θ or K_d , he has to forfeit the asset to the creditor at a price equal to the creditor's valuation

$$p_d = \mathbb{E}_d^l [\tilde{\theta}] = \pi_d^l \theta + (1 - \pi_d^l) \theta^2. \quad (14)$$

Furthermore, we can directly verify that the optimists have no incentive to save cash on date 0 by comparing the marginal values of saving cash and of establishing a greater asset position.

Proposition 6 *Suppose that the condition in (7) holds and the buyers use short-term debt contracts to finance their asset purchases. Then, there is no incentive for any buyer to save cash and the equilibrium can be broken down into the following five cases:*

- SD1: *If $C_S(\theta^2) > (1 - \mu) P_H - \mu c$, then $p_0 = \frac{\mu c + C_S(\theta^2)}{1 - \mu}$, all the buyers use the same short-term debt contract with face value θ^2 on date 0, and the asset price in the two states on date 1 are given by equations (11) and (12);*
- SD2: *If $C_S(\theta^2) \leq (1 - \mu) P_H - \mu c \leq C_S(K_d)$, then $p_0 = P_H$, each buyer is indifferent between the short-term debt contracts with face values of K_d and θ^2 on date 0 with the fraction of buyers using the former contract as $\frac{\frac{1 - \mu}{\mu} - \frac{c + C_S(\theta^2)}{P_H - C_S(\theta^2)}}{\frac{c + C_S(K_d)}{P_H - C_S(K_d)} - \frac{c + C_S(\theta^2)}{P_H - C_S(\theta^2)}}$, and the asset price in the two states on date 1 are given by equations (11) and (14);*
- SD3: *If $(1 - \mu) P_L - \mu c < C_S(K_d) < (1 - \mu) P_H - \mu c$, then $p_0 = \frac{\mu c + C_S(K_d)}{1 - \mu}$, all the buyers use the same short-term debt contract with face value K_d on date 0, and the asset price in the two states on date 1 are given by equations (11) and (14);*
- SD4: *If $C_S(K_d) \leq (1 - \mu) P_L - \mu c \leq C_S(\theta)$, then $p_0 = P_L$, each buyer is indifferent between the short-term debt contracts with face values of θ and K_d on date 0 with the fraction of buyers using the former contract as $\frac{\frac{1 - \mu}{\mu} - \frac{c + C_S(K_d)}{P_L - C_S(K_d)}}{\frac{c + C_S(\theta)}{P_L - C_S(\theta)} - \frac{c + C_S(K_d)}{P_L - C_S(K_d)}}$, and the asset price in the two states on date 1 are given by equations (11) and (14);*
- SD5: *If $C_S(\theta) < (1 - \mu) P_L - \mu c$, then $p_0 = \frac{\mu c + C_S(\theta)}{1 - \mu}$, all the buyers use the same short-term debt contract with face value θ on date 0, and the asset price in the two states on date 1 are given by equations (11) and (14).*

2.5 Heterogeneous Beliefs and Asset Price Dynamics

In this subsection, we analyze the effects of agents' heterogeneous beliefs in driving the boom-and-bust cycle of asset prices. In particular, we focus on the different effects of initial and future belief dispersion on the asset price dynamics through their different effects on the optimists' financing choices. The standard result of Miller (1977) suggests that a higher belief dispersion (in either the first or second period) leads to higher asset prices in the presence of short-sales constraints. However, financing cost can always significant deviation from this result.

As we observed during the period around the recent credit crisis, the debt contracts used by financial institutions to finance their investments in mortgage backed securities are mostly safe from the creditors' perspectives. These contracts include repo transactions and asset backed commercial paper. To tie our analysis close to this observation, we focus on the region where if the optimists can only use long-term debt, they will choose the risk-free debt with face value θ^2 (the LD1 case in Proposition 5). To facilitate the analysis, we examine the equilibrium asset price dynamics when 1) only LD (long-term debt) is available, and 2) both LD and SD (long-term and short-term debt) are available. The difference between these two cases highlights the role of debt maturity.

We use the following baseline parameter values:

$$\mu = 0.3, c = 0.6, \theta = 0.4, \pi_0^h = 0.7, \pi_0^l = 0.3, \pi_u^h = 0.6, \pi_u^l = 0.4, \pi_d^h = 0.6, \pi_d^l = 0.4. \quad (15)$$

These numbers imply the following: Optimists consist of 30% of the population and each is endowed with 0.5 dollar in cash. The final asset payoff can be 1, 0.4, or 0.16. We let the objective probability of the tree going up each period be 0.5 and the optimists and pessimists' beliefs be equally spread around the objective probability. As learning is likely to cause belief dispersion to decrease over time, we make the beliefs of the optimists and pessimists on date 0 to be 0.7 and 0.3, and on date 1 in both of the u and d states to be 0.6 and 0.4.

2.5.1 Initial Belief Dispersion on Date 0

We first examine the effect of the initial belief dispersion on date 0. We let the values of π_0^h and π_0^l to deviate from their baseline values and instead take the following ones:

$$\pi_0^h = 0.5 + \delta_0 \text{ and } \pi_0^l = 0.5 - \delta_0$$

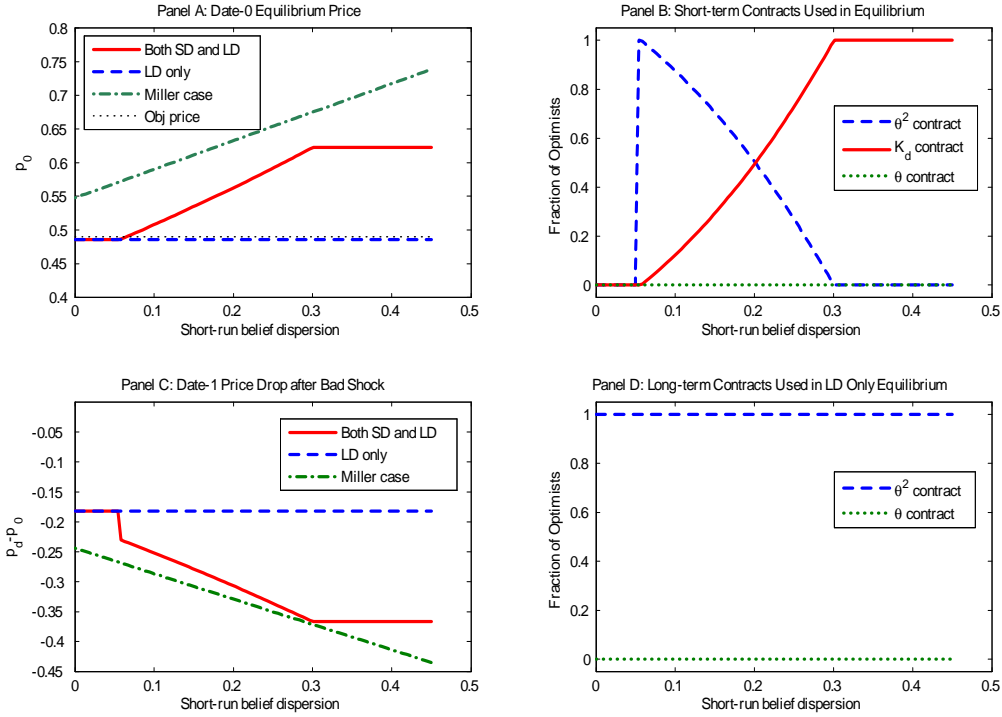


Figure 4: The equilibrium effects of initial belief dispersion on asset and credit markets.

where δ_0 changes from 0 to 0.45 and drives the initial belief dispersion between the optimists and pessimists. Figure 4 illustrates the asset market and credit market equilibrium. Panel A plots the date-0 asset price p_0 with respect to δ_0 . The horizontal dotted line at the 0.49 level represents the asset's fundamental value by the objective probabilities. The dotted upward sloping line represents the asset price in the Miller setting, where optimists always have sufficient funds to execute their purchases. As δ_0 increases from 0 to 0.45, the optimists become more optimistic and p_0 increases from 0.548 to 0.74 (p_0 is higher than 0.49 at $\delta_0 = 0$ because of the belief dispersion on date 1).

LD-only equilibrium The horizontal dashed line at 0.485 plots p_0 when the optimists have access only to long-term debt to finance their asset purchases. Interestingly, despite the wide range of belief dispersion between the optimists and pessimists, the equilibrium asset price is independent of the belief dispersion and is slightly below the asset's fundamental value shown by the horizontal dotted line. Both of these characteristics reflect the important effects of optimists' financing cost. Panel D provides a breakdown of the long-term debt

contracts used by the optimists in the LD only equilibrium. Throughout the wide range of δ_0 value, the optimists always use the same risk-free long-term debt contract with face value θ^2 to finance their asset purchases. This is because the alternative contract with a higher face value θ is risky and more costly under the specified parameters. The use of the risk-free debt contract determines the optimists' purchasing power, which can only support a price level lower than the asset's fundamental value even though the pessimists cannot short sell the asset.

Equilibrium with both SD and LD available The solid line in Panel A plots p_0 when the optimists can choose between long-term and short-term debt. p_0 stays flat at 0.485 as δ_0 increases from 0 to 0.05, then monotonically increases to 0.623 as δ_0 increases further to 0.30, and finally stays flat at 0.623 as δ_0 continues to rise. This pattern is dramatically different from that of the asset price in the LD only equilibrium. This difference highlights the important roles of financing choices in affecting the asset price dynamics.

Panel B provides a breakdown of the short-term debt contracts used by the optimists. Over the region $\delta_0 \in [0, 0.05]$, the optimists do not use any short-term debt. The reason is Proposition 1: Short-term debt is advantageous to long-term debt only when the speculative incentives caused by both parties' initial belief dispersion dominates the rollover-risk effect due to their future belief dispersion. Once δ_0 rises above 0.05, the optimists start to use a mix of short-term debt contracts with face values θ^2 and K_d . The fraction of optimists using the more aggressive K_d contract rises monotonically from 0 to 1 as δ_0 rises from 0.05 to 0.30, and stays at 1 as δ_0 rises further. This panel shows that the increase of the equilibrium price with δ_0 in the region $\delta_0 \in [0.05, 0.30]$ is financed by the optimists' increasing reliance on the short-term debt with face value K_d . Taken together, even though the asset price in the collateral equilibrium is lower than that in the standard Miller setting, short-term debt allows the optimists to manage their financing cost more effectively and thus ensures that the equilibrium price increases with agents' initial belief dispersion δ_0 .

Date-1 crash Panel C of Figure 4 plots the price change on date 1 when the lower state d is realized, i.e., $p_d - p_0$, under different settings. After the realization of the negative shock, the asset price drops and the optimistic asset holders suffer losses on their positions. In the Miller setting, as the date-0 price p_0 monotonically increases with the initial belief dispersion δ_0 , the price drop in state d (i.e., $|p_d - p_0|$) also increases with δ_0 . In the LD

only equilibrium, the optimists always choose the same risk-free long-term debt contract to finance their asset purchases. As a result, the date-0 price is independent of δ_0 and so is the price drop in state d . In contrast, the price drop in the setting with both LD and SD available is generally increasing with δ_0 . In fact, the slope of the price drop with respect to δ_0 in the LD-and-SD equilibrium is even steeper than that in the Miller setting in the middle region. This is because of the rollover risk effect. When the optimists finance their purchases by the aggressive short-term debt with face value K_d , they are forced to turn over their asset to the pessimistic creditor after the negative shock. This debt contract is still risk-free to the creditor as the forfeited asset is just enough to offset the debt payment. However, the shift of the asset's marginal investor from the optimists to the pessimists amplifies the price impact of the negative fundamental shock. This effect shows that not only can short-term debt fuel the asset over-valuation on date 0, but can also exacerbate the downturn after a negative shock.

2.5.2 Future Belief Dispersion on Date 1

Next, we examine the effects of the belief dispersion between the optimists and pessimists on date 1. We will focus on the dispersion in the lower state d . Proposition 1 suggests that the belief dispersion in this state introduces rollover risk, which discourages optimists from using short-term debt. Specifically, we deviate from the baseline parameters in (15) by specifying the following beliefs for the optimists and pessimists:

$$\pi_d^h = 0.5 + \delta_d, \text{ and } \pi_d^l = 0.5 - \delta_d$$

where δ_d changes from 0 to 0.45. Figure 5 illustrates the impact on the asset and credit markets.

Panel A of Figure 5 plots p_0 with respect to δ_d . In the Miller setting, p_0 is again increasing with δ_d for the same reason as before—an increase in δ_d makes the optimists more optimistic about the asset fundamental. In contrast, p_0 decreases with δ_d in both the equilibria with LD and SD and with LD only. In the LD only equilibrium, p_0 decreases with δ_d when δ_d is smaller than 0.05, and is independent of δ_d is larger than 0.05. Panel D further shows that when δ_d is less than 0.05, the optimists use a mix of long-term debt contracts with face values of θ and θ^2 . As δ_d increases in this region, the fraction of optimists who use the aggressive θ contract decreases because the increased future dispersion makes this contract more costly. The optimists all switch to the risk-free θ^2 contract once δ_d rises above 0.05.

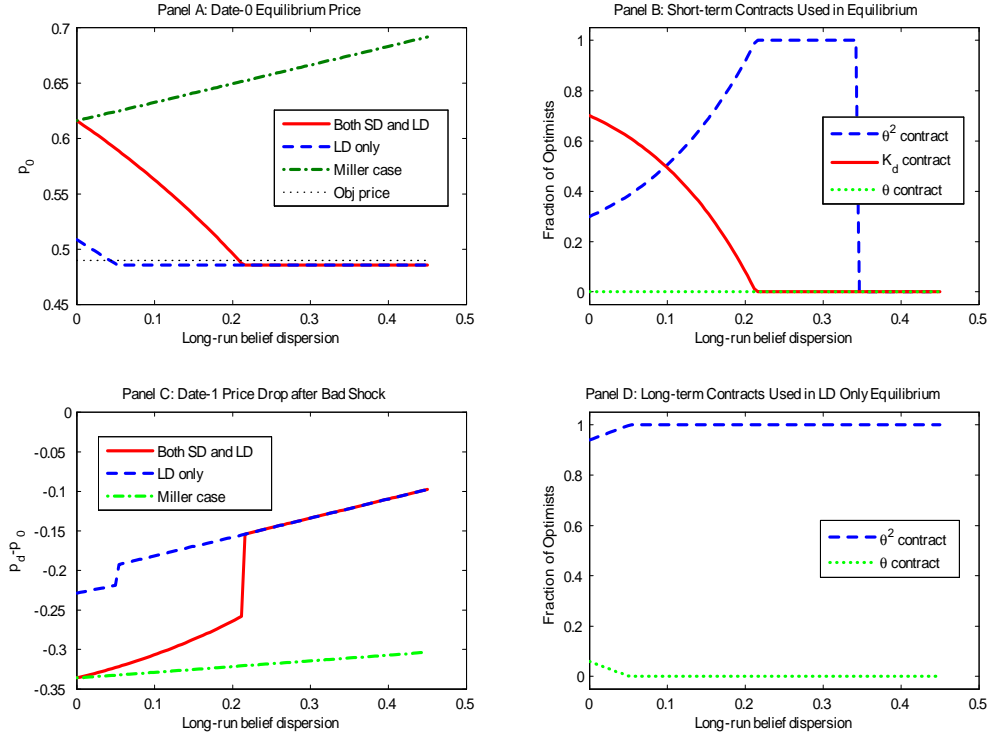


Figure 5: The equilibrium effects of long-run belief dispersion in state d on the asset and credit markets.

In the equilibrium with both LD and SD, p_0 is higher than that in the LD only equilibrium when δ_d is less than 0.22 because in this region the condition (7) in Proposition 7 holds and short-term debt gives the optimists cheaper financing than long-term debt. Indeed, Panel B shows that the optimists use a mix of short-term debt contracts with face values K_d and θ^2 . But, nevertheless, p_0 decreases with δ_d even in this region. This is because an increase of δ_d raises the rollover risk of those optimists who choose to use the K_d contract and causes their fraction to reduce. When δ_d rises above 0.22, all the optimists switch to the θ^2 contract and p_0 becomes insensitive to δ_d and identical to that in the LD only equilibrium.

Taken together, Figures 4 and 5 demonstrate substantial differences in the equilibrium effects of initial and future belief dispersion. While both types of belief dispersion tend to increase the optimists' belief and thus increase the equilibrium asset price, they have different effects on the optimists' financing. In particular, an increase in the future belief dispersion after the asset fundamental deteriorates increases the optimists' rollover risk and thus discourages them from using short-term debt, a useful tool in controlling financing

cost from their perspectives. Through this rollover risk mechanism from the financing cost side, the increase in future belief dispersion can decrease the asset price, in contrast to the standard Miller result. On the other hand, short-term debt makes it possible for the optimists to control their financing cost when the initial belief dispersion increases. As a result, the equilibrium asset price increases with the initial belief dispersion.

In general, we can prove the following proposition regarding the effects of these two types of belief dispersion on the date-0 equilibrium price.

Proposition 7 *Suppose that $(1 - \mu)P_M - \mu c < C_S(\theta^2)$, i.e., the equilibrium falls to the LD1 case of Proposition 3 if the optimists have only access to long-term debt. Then, the date-0 asset price p_0 increases with the belief dispersion between the optimists and pessimists on date 0 and decreases with their belief dispersion in the lower interim state d on date 1.*

3 An Extended Model with Learning

In this section, we extend the baseline model with learning. We allow each agent to update his belief about the asset fundamental on date 1 based on the realized fundamental shock. Such learning justifies the state-contingent beliefs specified in the baseline model. Furthermore, learning motivates two interesting effects. First, learning can lead to flips of beliefs across agents, which in turn intensifies their speculative incentives through asset holders' resale options, a la Harrison and Kreps (1978). Second, learning can also lead to more divergent beliefs between optimists and their creditors when the fundamental deteriorates, and thus more severe rollover risk discussed earlier. We will focus on isolating these two distinctive effects of fluctuating heterogeneous beliefs. The learning technology we adopt is analogous to that used by Morris (1996).

3.1 The Model Setting

Suppose that the fundamental move on the tree is independently and identically distributed in each period. Let π be the unobservable probability of an upward jump. On date 0, each agent has a prior about the distribution of π . There are three groups of risk-neutral agents, who differ in their priors about the distribution of π . We label these groups by A , B , and C . Suppose that the prior of a group- i agent ($i \in \{A, B, C\}$) has a beta distribution with

parameters (α^i, β^i) .⁴ We denote the mean of this distribution as the agent's prior belief:

$$\pi_0^i \equiv \frac{\alpha^i}{\gamma^i}$$

where $\gamma^i \equiv \alpha^i + \beta^i$ represents the agent's confidence about his prior belief. This confidence determines how much the agent reacts to new information on date 1.

On date 1, each agent will update his belief in response to the information $s_1 = 1$ or 0, which corresponds to the up or down move of the tree. This signal has a precision of 1 and improves the confidence of the agent's posterior to $\gamma^i + 1$. The agent's posterior belief is a weighted average of the prior belief π_0^i and the signal s_1 :

$$\pi_1^i = \frac{\gamma^i}{\gamma^i + 1} \pi_0^i + \frac{1}{\gamma^i + 1} s_1.$$

If the agent is more confident about his prior, he puts more weight on the prior but less weight on the information shock. On the other hand, if the agent is less confident about his prior, he reacts more to the information shock. Put explicitly, the agent's posterior belief in the two interim states u and d on date 1 are

$$\pi_u^i = \frac{\gamma^i}{\gamma^i + 1} \pi_0^i + \frac{1}{\gamma^i + 1}$$

and

$$\pi_d^i = \frac{\gamma^i}{\gamma^i + 1} \pi_0^i.$$

To facilitate our analysis, we let the group- A agents be the optimists on date 0 and the group- B and group- C agents share the same pessimistic prior belief:

$$\pi^h \equiv \pi_0^A > \pi^l \equiv \pi_0^B = \pi_0^C.$$

Furthermore, we assume that the two groups of pessimists differ in the confidence of their priors:

$$\gamma^C > \gamma^A > \gamma^B \tag{16}$$

so that on date 1 the group- B agents will react most strongly to the information shock, while the group- C agents will react most weakly.

⁴A beta distribution with parameters (α, β) with $\alpha > 0$ and $\beta > 0$ is defined on the interval $(0, 1)$, and has density function:

$$f(x; \alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}.$$

The mean of the distribution is $\frac{\alpha}{\alpha + \beta}$.

Splitting the date-0 pessimistic agents into two groups with different prior confidence levels allows us to isolate two effects on date 1: one is the optimists' rollover risk in the lower state d and the other is their resale option in the upper state u . The condition in (16) implies that the group- C agents are most insensitive to the information shock on date 1 and in particular their belief does not fall much after the negative shock in state d . In light of our analysis in the previous section, they are the natural creditor to the optimistic group- A agents on date 0 because the financing cost is determined by the creditor's beliefs about the likelihood of the default states. Since the group- C agents are less responsive to the negative shock in the lower interim state d , credit provided by them is cheaper than that by the group- B agents. We also assume that γ^C is not too large so that the belief of the group- C agents is always lower than that of the group- A agents and that they always have sufficient funds to provide the credit. In this setting, the confidence of group- C agents' prior inversely determines the group- A agents' rollover risk.

The condition in (16) also implies that after the positive shock in the interim state u the group- B agents may become more optimistic than the group- A agents and thus become buyers of their assets. More specifically, the group- B agents become more optimistic than group- A agents if γ^B is sufficiently small so that

$$\pi_u^B = \frac{\gamma^B}{\gamma^B + 1} \pi^l + \frac{1}{\gamma^B + 1} > \pi_u^A = \frac{\gamma^A}{\gamma^A + 1} \pi^h + \frac{1}{\gamma^A + 1}.$$

If so, group- A agents will sell their asset to group- B agents at a price equal to the group- B agents' valuation (assuming that they always have sufficient funds). Note that in state u , the group- C agents' belief is always lower than that of group- A agents because of their pessimistic prior belief and higher confidence about the prior. Thus,

$$p_u = \max \{ \pi_u^A + (1 - \pi_u^A) \theta, \pi_u^B + (1 - \pi_u^B) \theta \}.$$

The option to resell the asset to the group- B agents at a higher price is valuable to the group- A agents, and motivates them to pay a price on date 0 that is higher than their buy-and-hold valuation, even though they already hold the most optimistic valuation, e.g., Harrison and Kreps (1978). This speculative component in asset price has been widely used to study asset-price bubbles, e.g., Scheinkman and Xiong (2003). In our model, the confidence parameter γ^B determines the resale option value of the group- A agents.

Table 3: Asset Payoff and Debt Payment in the Extended Model

	Tree Path			
	uu	ud	du	dd
Asset payoff	1	θ	θ	θ^2
Payoff to the initial buyers	p_u	p_u	θ	θ^2
Long-term debt face value $F_L \in [\theta^2, \theta]$	F_L	F_L	F_L	θ^2
Short-term debt face value $F_S \in [\theta^2, K_d]$	F_S	F_S	$F_{S,1} \geq F_S$	θ^2
Short-term debt face value $F_S \in [K_d, \theta]$	F_S	F_S	θ	θ^2

3.2 Learning and Price Bubble

We adopt the same assumptions from the baseline model on the agents' initial cash and asset endowments. We can easily extend our derivation of the baseline model to cover the extended model. The group- A agents correspond to the optimistic buyers in the baseline model, and the group- C agents correspond to the pessimistic creditors. Their state-dependent beliefs are now determined by their priors on date 0 and learning processes on date 1. The presence of group- B agents provides group- A agents the resale option on date 1 in the upper state u . We summarize the asset payoff to the initial buyers and their debt payments from using different contracts in Table 3, which differs from Table 2 in the asset payoff only on paths uu and ud due to the resale option.

The changes in the asset payoff to the initial buyers do not affect the payments of the equilibrium-relevant debt contracts, and thus do not affect the buyers' optimal debt maturity choice given in Proposition 1. Propositions 3, 4, 5, and 6 also remain the same, except that we have to modify the expressions for P_M , P_L , and P_H to account for the changes in the asset payoff on the uu and ud paths:

$$P_M = (\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l) \frac{\pi_0^h p_u + (1 - \pi_0^h) \pi_d^h \theta}{\pi_0^h + \pi_d^h - \pi_0^h \pi_d^h} + (1 - \pi_0^l) (1 - \pi_d^l) \theta^2,$$

$$P_H = \frac{\pi_d^l [\pi_0^h p_u + (1 - \pi_0^h) \pi_d^h \theta] + (1 - \pi_0^h) \pi_d^h (1 - \pi_d^l) \theta^2}{\pi_0^h \pi_d^l + (1 - \pi_0^h) \pi_d^h},$$

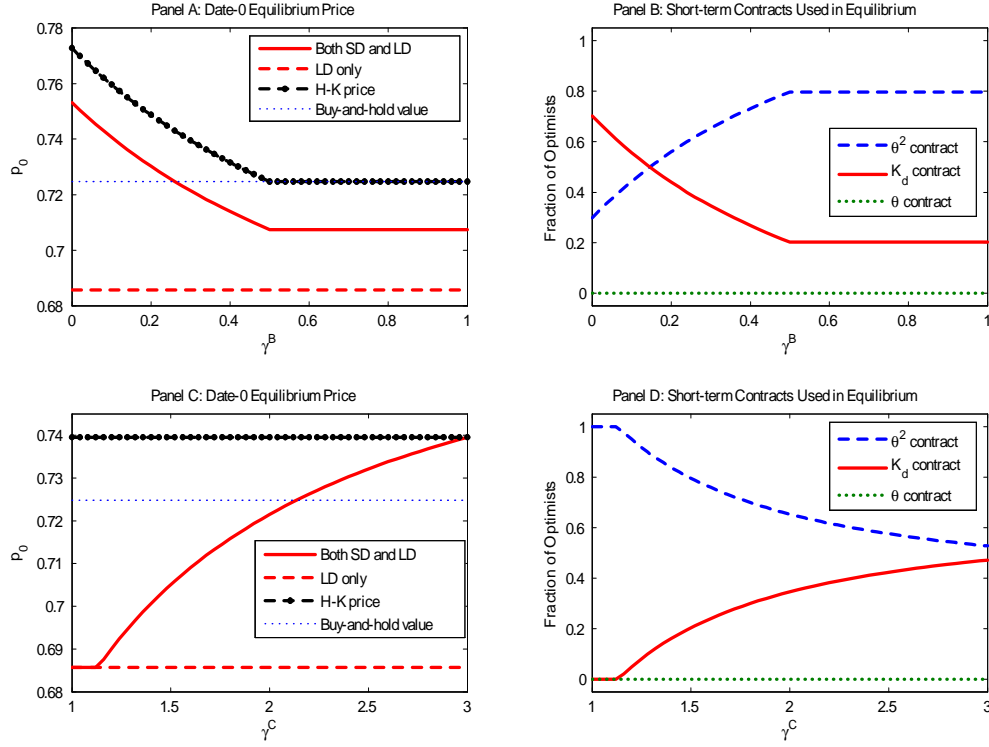


Figure 6: The equilibrium effects of learning.

and

$$P_L = \pi_0^l p_u + (1 - \pi_0^l) [\pi_d^l \theta + (1 - \pi_d^l) \theta^2].$$

To illustrate the effects of financing cost on the asset price bubble, we use a set of numerical examples, based on the following baseline parameters:

$$\mu = 0.3, c = 0.5, \theta = 0.4, \pi^h = 0.6, \pi^l = 0.4, \gamma^A = 1, \gamma^B = 2, \gamma^C = 0.3.$$

We focus on varying the values of γ^B and γ^C , which determine the initial asset holders' resale option value and rollover risk, respectively.

Figure 6 illustrates the equilibrium effects of varying γ^B from 0 to 1 and γ^C from 1 to 3. As γ^B decreases, the belief of the group- B agents (the potential asset buyers) becomes more responsive to the information shock on date 1 and thus increases the initial buyers' resale option value in the upper state u . Panel A plots the equilibrium price p_0 with respect to γ^B . The flat dotted line at 0.725 provides the group- A agents' buy-and-hold value on date 0, while the dashed line with big dots provides their valuation in the Harrison-Kreps setting,

which takes into account the resale option under the assumption that they always have sufficient funds for their asset purchases. The Harrison-Kreps price starts to rise above the buy-and-hold value as γ^B drops below a critical level around 0.48, below which the group- B agents' belief in the u state becomes higher than that of the group- A agents.

The flat dashed line at 0.686 represents the equilibrium price when the buyers have access to only long-term debt. As this line is substantially below the group- A agents' buy-and-hold valuation on date 0, it suggests that the cost of using long-term debt financing severely constrains the optimists from bidding up the asset price. Interestingly, the solid line shows that once the optimists are allowed to use short-term debt, the equilibrium price is always above the price level in the LD only equilibrium, and starts to rise when γ^B drops below 0.48 in parallel with the Harrison-Kreps price. In fact, the price eventually passes the optimists' buy-and-hold valuation when γ^B drops below 0.26. This suggests that the cheap financing provided by short-term debt makes it possible for the optimists to bid up the price to levels closer to their speculative valuations without financing cost. Panel B also plots the types of short-term debt contracts used by the initial buyers. The plot shows that the price increase is financed by their increasing use of the more aggressive debt contract with face value K_d .

As γ^C increases, the belief of the group- C agents (the creditor to the initial buyers) becomes more stable in the lower interim state d and thus reduces the buyers' rollover risk. Panel C of Figure 6 plots the equilibrium price p_0 with respect to γ^C . The plot gives the two benchmark price levels, the group- A agents' buy-and-hold valuation and the Harrison-Kreps price by the two horizontal lines at 0.725 and 0.74, respectively. If group- A agents have access to only long-term debt, the financing cost constrains them to bid up the price only to 0.686, which is substantially below the two benchmark levels. When short-term debt is available, the financing cost becomes lower, especially when the rollover risk is low. Panel C shows that as γ^C increases, the reduced financing cost allows the optimists to bid up the price closer to the Harrison-Kreps price. In fact, the two prices coincide when $\gamma^C = 3$, at which point $\pi_d^A = \pi_d^C$ (i.e., there is no rollover risk.)

4 Discussion

Our model provides useful insights about the inherent instability of credit booms that had accompanied many historical asset market booms, and about the effects of financing cost on asset bubbles. We discuss these implications in this section.

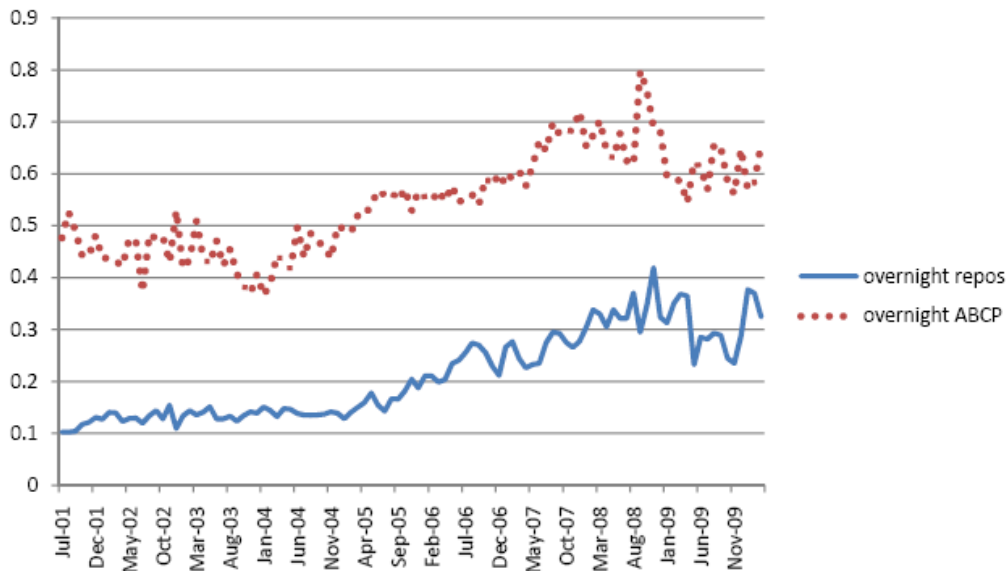


Figure 7: Increasing use of overnight repos and ABCP before the credit crisis of 2007-2008. The solid line is based on the data provided by the Federal Reserve Bank of New York on the aggregate amount of repos used by the primary dealers to finance their positions in mortgage backed securities. It plots the fraction of overnight repos among all repos, which include both overnight and term repos. The dotted line is based on the aggregate amount of issuance of asset-backed commercial paper provided by Federal Reserve Board. It plots the fraction of overnight ABCP (with maturities of 1 to 4 days) among all maturities.

4.1 Short-term Credit Booms

As summarized by Kindleberger (2000), a host of classical economists including Irving Fisher, Henry Simons, and Hyman Minsky emphasized the role of debt contracted to leverage the acquisition of speculative assets for future resale and the role of debt structures in causing financial difficulties. Indeed, there is ample evidence of pronounced cycles of credit expansion and contraction that accompany the boom-and-crisis cycles of asset markets.

The credit boom that preceded the recent credit crisis of 2007-2008 provides a vivid example. As noted by many observers, e.g., Adrian and Shin (2009), Brunnermeier (2009), Gorton and Metrick (2009), and Krishnamurthy (2010), the leverages used by many financial institutions went above 30 in 2007. Interestingly, these institutions had also increasingly relied on overnight repo transactions and asset-backed financial paper to finance their investments in mortgage backed securities. Figure 7 plots the fractions of overnight repos and asset-backed commercial paper from 2001 to 2009. The fraction of overnight repos (which has a maturity of only one day) used by the primary dealers to finance their positions in

mortgage backed securities doubled from about a level around 15% in 2004 to more than 30% in 2007.⁵ There is also a similar pattern in the shortening of maturity in the aggregate issuance of overnight asset-backed commercial paper (the main financing channel of many special investment vehicles for their mortgage backed investments). The fraction of overnight ABCP (with maturities of one to four days) rose from 40% in 2004 to near 70% in 2007.

It is important to differentiate the shortening of debt maturity during the boom from the further shortening of debt maturity during the crisis of 2007-2008. After the crisis disrupted, creditors became more averse to uncertainty and more concerned about the runs by others. As a result, they are less willing to lend and especially less willing to lend for longer terms. See He and Xiong (2009a) and Brunnermeier and Oehmke (2009) for models about maturity shortening during crises based on externalities created by creditors' runs on others. However, these models do not explain the shortening of maturity during the boom. In fact, the runs experienced by the financial institutions represent a severe form of rollover risk resulted from their heavy use of short-term debt. This in turn makes it even more puzzling why the financial institutions had not locked in longer term financing during the boom.

Short-term credit booms were also observed in several other episodes. Before the stock market crash in October 1929, investors had borrowed heavily from their brokers to finance their speculation in the stock market. Rappoport and White (1993) show that the volume of brokers' loans had risen and fallen in sync with the stock market index throughout the boom-and-bust period between 1926 and 1930 and that the maturities of the brokers' loans had also shortened before the crash: "In 1926 and 1927, time loans [loans with maturities of 60 to 90 days] accounted for between 21 and 32 percent of all brokers' loans, but after mid-1928, they declined to under 10 percent." Instead, the investors had shifted to more heavy use of call loans (demand loans).

In the 1990s short-term debt had also been heavily used by many emerging countries to finance their economic booms. According to Rodrik and Velasco (1999), the outstanding stock of debt of emerging-market economy roughly doubled between 1988 and 1997, from \$1 trillion to \$2 trillion. While medium- and long-term debt grew rapidly as well, it was short-term debt that rose particularly rapid during this period. They further show that in a data sample covering 32 emerging-market economies over the period 1988-1998, the ratio of short-term debt to foreign reserve is a robust predictor of financial crises triggered by

⁵Brunnermeier (2009) has also pointed out the increasing use of overnight repos by financial institutions before 2007.

reversals of capital flows.

Reinhart and Rogoff (2009) analyze financial crises over a longer period of time in a larger set of countries. While they confirm the important roles played by short-term debt in some of the debt crises, it was not present in all of the crises. The selective emergence of short-term credit booms makes them special and prompts more careful identification of the environment that had led to their emergence.

Our model provides some useful insights by incorporating both rollover risk and borrowers' debt maturity choices in a unified framework based on heterogeneous beliefs between the borrowers and creditors. Our model characterizes a set of conditions for the emergence of a short-term credit boom. To the extent that short-term debt reduces the borrowers' financial stability, the increasing short-term leverages during a short-term credit boom reflects the borrowers' optimism about the future asset fundamentals, as well as the creditors' concerns about the borrowers' ability to repay in the long term. Furthermore, since the rollover risk faced by short-term borrowers increases with the future belief dispersion between them and the creditors after the asset fundamental deteriorates, the emergence of a short-term credit boom involves not only a large initial belief dispersion, but also the expected belief convergence by the optimists in the future, especially in the future downturns.

Our model also shows that if chosen by the optimists, short-term debt can fuel an asset-market boom by allowing them to raise large leverages at low interest rates. It can also act as the bridge from boom to crisis because the rollover risk amplifies the downturn after the asset fundamental deteriorates. This result highlights the importance of agents' financing choices on asset-market dynamics.

4.2 Heterogeneous Beliefs and Asset Bubbles

There is a large literature modeling asset bubbles generated by agents' heterogeneous beliefs and short-sales constraints, e.g., Miller (1977), Harrison and Kreps (1978), Morris (1996), Chen, Hong, and Stein (2002), and Scheinkman and Xiong (2003). There are two standard results: First, higher belief dispersion tends to lead to higher asset price because optimists become more optimistic and will thus bid up the asset price; second, if learning causes agents' beliefs to fluctuate over time, the possibility of agents' beliefs to cross in the future intensifies their speculative incentives by providing a resale option to the current asset holders. The aforementioned models typically ignore optimists' financing choices. After incorporating

such considerations, our model shows that these standard results require modifications.

Like Simsek (2009), our model shows that financing cost can cause the equilibrium asset price to decrease with belief dispersion. This is because a higher belief dispersion not only makes the optimists more optimistic but also the creditors more pessimistic. As a result, the increased financing cost can limit the ability of the optimists to bid up the asset price. Furthermore, our model also highlights the important difference between initial and future belief dispersion. To the extent that the optimists can use short-term debt to mitigate the increased financing cost caused by an increase in the initial belief dispersion, the equilibrium asset price remains increasing with the initial belief dispersion under a reasonable condition in our model. However, an increase of belief dispersion in the future after the asset fundamental deteriorates raises the optimists' rollover risk and thus discourages the use of short-term debt. This effect exacerbates the financing cost and causes the equilibrium asset price to decrease with the future belief dispersion in downturns.

There are extensive empirical studies of the link between investors' heterogeneous beliefs and asset returns, e.g., Chen, Hong, and Stein (2002) and Diether, Malloy, and Scherbina (2002). It is common to use the dispersion of analyst forecasts as a measure belief dispersion. The existing studies tend to treat the dispersion of analyst forecasts over different time horizons as qualitatively similar and thus mostly focus on analyzing dispersion in short-term forecasts. In contrast to this common practice, our model suggests distinctive economic roles by short-term and long-term belief dispersion. In particular, our model predicts that if the optimists are cash constrained, the asset price tends to be increasing with short-term belief dispersion but decreasing with long-term belief dispersion. This in turn implies that the asset return is decreasing with short-term belief dispersion (which is consistent with the findings of the aforementioned studies) but increasing with long-term belief dispersion (a new prediction to be verified). Furthermore, our model predicts short-term belief dispersion leads to heavier use of short-term debt financing by some investors, while long-term belief dispersion leads to less.

Finally, our model also demonstrates that financing cost can compound the equilibrium effect of agents' belief fluctuations. The seminal work of Harrison and Kreps (1978) suggests that agents' belief fluctuations create a resale option to the asset holders and thus intensify agents' speculative incentives. Our model shows that belief fluctuations can also lead to greater rollover risk for the asset holders who rely on short-term debt to finance their asset

holdings. This concern is especially acute in future down states.

5 Conclusion

Appendix A Proofs for Propositions

A.1 Proof of Proposition 1

There are two cases depending on whether the short-term debt is risky. Suppose that the short-term debt is riskless, i.e., its face value $F_S \in [\theta^2, K_d]$. Then, on date 0 the optimistic borrower can raise $C_S(F_S) = F_S$ from the debt contract. His expected debt payment is (recall (4) and Table II)

$$\mathbb{E}_0^h [\tilde{D}_S] = \pi_0^h F_S + (1 - \pi_0^h) \pi_d^h \left(\frac{F_S}{\pi_d^l} - \frac{1 - \pi_d^l}{\pi_d^l} \theta^2 \right) + (1 - \pi_0^h) (1 - \pi_d^h) \theta^2.$$

On the other hand, the long-term debt contract that delivers the same initial credit as F_S requires a face value of F_L such that

$$C_L(F_L) = (1 - (1 - \pi_0^l) (1 - \pi_d^l)) F_L + (1 - \pi_0^l) (1 - \pi_d^l) \theta^2 = F_S.$$

This implies that

$$F_L = \frac{F_S - (1 - \pi_0^l) (1 - \pi_d^l) \theta^2}{1 - (1 - \pi_0^l) (1 - \pi_d^l)}.$$

Then, the borrower's expected payment by using the long-term contract is

$$\mathbb{E}_0^h [\tilde{D}_L] = (1 - (1 - \pi_0^h) (1 - \pi_d^h)) F_L + (1 - \pi_0^h) (1 - \pi_d^h) \theta^2.$$

Therefore, the difference between the costs of the short-term and long-term debt contracts is

$$\mathbb{E}_0^h [\tilde{D}_L] - \mathbb{E}_0^h [\tilde{D}_S] = \frac{\pi_0^h (1 - \pi_0^l) \pi_d^l - \pi_0^l (1 - \pi_0^h) \pi_d^h}{\pi_d^l (\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l)} (1 - \pi_d^l) (F_S - \theta^2).$$

The short-term debt contract is less costly if and only if (7) is satisfied.

We follow a similar procedure for the case that $F_S \in (K_d, \theta]$. The borrower's expected debt payment by using a short-term debt contract with face value F_S is

$$\mathbb{E}_0^h [\tilde{D}_S] = \pi_0^h F_S + (1 - \pi_0^h) (\pi_d^h \theta + (1 - \pi_d^h) \theta^2),$$

and the date-0 credit that the borrower receives is

$$D_S(F_S) = \pi_0^l F_S + (1 - \pi_0^l) (\pi_d^l \theta + (1 - \pi_d^l) \theta^2).$$

For a long-term debt contract to deliver the same initial credit, its face value F_L has to satisfy

$$(1 - (1 - \pi_0^l) (1 - \pi_d^l)) F_L + (1 - \pi_0^l) (1 - \pi_d^l) \theta^2 = \pi_0^l F_S + (1 - \pi_0^l) (\pi_d^l \theta + (1 - \pi_d^l) \theta^2).$$

This implies that

$$F_L = \frac{\pi_0^l F_S + (1 - \pi_0^l) \pi_d^l \theta}{1 - (1 - \pi_0^l) (1 - \pi_d^l)}.$$

Thus, the borrower's expected debt payment is

$$\mathbb{E}_0^h [\tilde{D}_L] = (1 - (1 - \pi_0^h) (1 - \pi_d^h)) \frac{\pi_0^l F_S + (1 - \pi_0^l) \pi_d^l \theta}{1 - (1 - \pi_0^l) (1 - \pi_d^l)} + (1 - \pi_0^h) (1 - \pi_d^h) \theta^2.$$

Direct algebra gives the difference between the costs of the short-term and long-term contracts:

$$\mathbb{E}_0^h [\tilde{D}_L] - \mathbb{E}_0^h [\tilde{D}_S] = \frac{\pi_0^h (1 - \pi_0^l) \pi_d^l - \pi_0^l (1 - \pi_0^h) \pi_d^h}{\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l} (\theta - F_S).$$

Again, the short-term debt is less costly if and only if (7) holds.

A.2 Proof of Lemma 2

Suppose that the borrower's optimal face value F is lower than θ^2 . The contract could be long-term or short-term. Since the face value is lower than θ^2 , the debt contract is risk free across all the four possible paths, i.e., $\tilde{D} = F$. As a result, the expected debt payment to the borrower is F and the date-0 credit the borrower gets is also F . Then, according to equation (6), the borrower's expected value is

$$\frac{c + p_0}{p_0 - F} \left[\mathbb{E}_0^h (\tilde{\theta}) - F \right].$$

Now, consider increasing the debt face value by a tiny amount ϵ . The debt contract is still risk free, and the borrower's expected value becomes

$$\frac{c + p_0}{p_0 - F - \epsilon} \left[\mathbb{E}_0^h (\tilde{\theta}) - F - \epsilon \right].$$

Since $p_0 \leq \mathbb{E}_0^h (\tilde{\theta})$, this expression is increasing with ϵ . In other words, the borrower is better off by borrowing more. This contradicts with F being the optimal debt face value. Thus, the optimal debt face value cannot be lower than θ^2 .

Next, suppose that the borrower's optimal face value F is higher than θ . The contract could be long-term or short-term. We denote the debt payment on date 2 as \tilde{D}_0 . Since the face value is higher than θ , the borrower always default on the debt contract except at the end of the path uu . That is, $\tilde{\theta} - \tilde{D}_0$ equals $1 - F$ at the end of the path uu , and 0 at the end of the other paths. Then, according to equation (6), the borrower's expected value is

$$\frac{c + p_0}{p_0 - \mathbb{E}_0^l (\tilde{D}_0)} \mathbb{E}_0^h (\tilde{\theta} - \tilde{D}_0).$$

Consider reducing the debt face value by a tiny amount ϵ . We denote the debt payment of the new contract by \tilde{D}_1 . Note that \tilde{D}_1 differs from \tilde{D}_0 only by $-\epsilon$ at the end of the path uu . The borrower's expected value is now

$$\frac{c + p_0}{p_0 - \mathbb{E}_0^l(\tilde{D}_1)} \mathbb{E}_0^h(\tilde{\theta} - \tilde{D}_1) = \frac{c + p_0}{p_0 - \mathbb{E}_0^l(\tilde{D}_0) + \pi_0^l \pi_u^l \epsilon} \left[\mathbb{E}_0^h(\tilde{\theta} - \tilde{D}_0) + \pi_0^h \pi_u^h \epsilon \right].$$

This expression is increasing with ϵ if

$$\frac{\mathbb{E}_0^h(\tilde{\theta} - \tilde{D}_0)}{p_0 - \mathbb{E}_0^l(\tilde{D}_0)} \leq \frac{\pi_0^h \pi_u^h}{\pi_0^l \pi_u^l}.$$

Note that since $p_0 \geq \mathbb{E}_0^l(\tilde{\theta})$,

$$\frac{\mathbb{E}_0^h(\tilde{\theta} - \tilde{D}_0)}{p_0 - \mathbb{E}_0^l(\tilde{D}_0)} \leq \frac{\mathbb{E}_0^h(\tilde{\theta} - \tilde{D}_0)}{\mathbb{E}_0^l(\tilde{\theta} - \tilde{D}_0)} = \frac{\pi_0^h \pi_u^h}{\pi_0^l \pi_u^l}.$$

Thus, the borrower's expected value increases with ϵ , which contradicts with F being the optimal debt face value. This suggests that the optimal debt face value cannot be higher than θ .

A.3 Proof of Proposition 3

On date 0, the borrower's expected value is given in (6). Based on the asset payoff and debt payment listed in Table 2, we have

$$\mathbb{E}_0^h[\tilde{\theta}] - \mathbb{E}_0^h[\tilde{D}_L] = [\pi_0^h \pi_u^h (1 - F_L) + (\pi_0^h (1 - \pi_u^h) + (1 - \pi_0^h) \pi_d^h) (\theta - F_L)].$$

By substituting this and $C_L(F_L)$ in (1) into (6), we derive the borrower's date-0 expected value as

$$V_L(F_L) = (c + p_0) \frac{\pi_0^h \pi_u^h + (\pi_0^h (1 - \pi_u^h) + (1 - \pi_0^h) \pi_d^h) \theta - (\pi_0^h + \pi_d^h - \pi_0^h \pi_d^h) F_L}{p_0 - (1 - \pi_0^l) (1 - \pi_d^l) \theta^2 - (\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l) F_L}.$$

Direct algebra shows that $V_L(F_L)$ is increasing with F_L if and only if $p_0 < P_M$ where

$$P_M = (\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l) \frac{\pi_0^h \pi_u^h + (\pi_0^h (1 - \pi_u^h) + (1 - \pi_0^h) \pi_d^h) \theta}{\pi_0^h + \pi_d^h - \pi_0^h \pi_d^h} + (1 - \pi_0^l) (1 - \pi_d^l) \theta^2. \quad (17)$$

As a result, the borrower's optimal long-term debt leverage is θ if $p_0 < P_M$, is θ^2 if $p_0 > P_M$, and is either θ or θ^2 if $p_0 = P_M$.

A.4 Proof of Proposition 4

We first consider the case in which the face value of the short-term debt $F_S \in [\theta^2, K_d]$. On date 0, the borrower's expected value $V_S(F_S)$ is given in (6). Note that in this case $C_S(F_S) = F_S$. By substituting in the expression of \tilde{D}_S in Table 2 and $F_{S,1}$ in (4), we have

$$V_S(F_S) = (c + p_0) \frac{(1 - \pi_0^h) \pi_d^h + \pi_0^h \pi_d^l}{\pi_d^l} \frac{\pi_d^l [\pi_0^h (\pi_u^h + (1 - \pi_u^h) \theta) + (1 - \pi_0^h) \pi_d^h \theta] + (1 - \pi_0^h) \pi_d^h (1 - \pi_d^l) \theta^2}{(1 - \pi_0^h) \pi_d^h + \pi_0^h \pi_d^l} - F_S. \quad (18)$$

This immediately implies that $V_S(F_S)$ is increasing in F_S if and only if

$$p_0 < \frac{\pi_d^l [\pi_0^h (\pi_u^h + (1 - \pi_u^h) \theta) + (1 - \pi_0^h) \pi_d^h \theta] + (1 - \pi_0^h) \pi_d^h (1 - \pi_d^l) \theta^2}{(1 - \pi_0^h) \pi_d^h + \pi_0^h \pi_d^l} = P_H \quad (19)$$

Next, we consider $F_S \in [K_d, \theta]$. Similarly, by substituting the expression of \tilde{D}_S in Table 2 into (6), we obtain

$$\begin{aligned} V_S(F_S) &= (c + p_0) \frac{\pi_0^h (\pi_u^h + (1 - \pi_u^h) \theta - F_S)}{p_0 - D_S(F_S)} \\ &= (c + p_0) \frac{\pi_0^h}{\pi_0^l} \frac{\pi_0^l \mathbb{E}_0^h [\tilde{\theta} | uu, ud] - \pi_0^l F_S}{p_0 - (1 - \pi_0^l) \mathbb{E}_0^l [\tilde{\theta} | du, dd] - \pi_0^l F_S} \end{aligned} \quad (20)$$

where $\mathbb{E}_0^h [\tilde{\theta} | uu, ud] = \mathbb{E}_u^h [\tilde{\theta}] = \pi_u^h + (1 - \pi_u^h) \theta$ and $\mathbb{E}_0^l [\tilde{\theta} | du, dd] = \mathbb{E}_d^l [\tilde{\theta}] = K_d$. It is easy to show that $V_S(F_S)$ is increasing in F_S if and only if

$$p_0 < \pi_0^l \mathbb{E}_0^h [\tilde{\theta} | uu, ud] + (1 - \pi_0^l) \mathbb{E}_0^l [\tilde{\theta} | du, dd] = P_L. \quad (21)$$

By combining the properties of $V_S(F_S)$ across the intervals of $[\theta^2, K_d]$ and $[K_d, \theta]$, it is direct to verify the borrower's optimal short-term debt face value given in Proposition 4.

Finally we show that $P_L < P_M < P_H$, where three objects are defined in (21), (17), and (19). To show that $P_M < P_H$, we only need to show that

$$(1 - \pi_0^l) (1 - \pi_d^l) > \frac{(1 - \pi_0^h) \pi_d^h (1 - \pi_d^l)}{(1 - \pi_0^h) \pi_d^h + \pi_0^h \pi_d^l}.$$

Simple calculation shows that it is equivalent to (7). Now we show that $P_L < P_M$. The term involving θ^2 has common coefficient $(1 - \pi_0^l) (1 - \pi_d^l) \theta^2$ which cancels out. Because the sum of coefficients of 1, θ , and θ^2 is one for P_L and P_M , it suffices to show that P_L has a smaller coefficient for 1:

$$\pi_0^l \pi_u^h < (\pi_0^l + \pi_d^l - \pi_0^l \pi_d^l) \frac{\pi_0^h \pi_u^h}{\pi_0^h + \pi_d^h - \pi_0^h \pi_d^h}.$$

Simplifying, we find that this is equivalent to (7).

A.5 Proof of Proposition 6

The only non-trivial part of the proposition is that the buyers have no incentive to save cash on date 0. To prove this, we compare the marginal value of establishing an asset position and saving cash on date 0. The marginal value of saving cash is higher than 1 in the equilibrium cases SD2, SD3, SD4, and SD5. In these cases, at least some of the buyers use debt contracts with face values K_d or θ , and thus will run into distress in the lower state d of date 1. What is the marginal value for an optimistic buyer to save a small amount ε ? Going forward, if the state moves into d , he has cash to buy under-valued asset, which is priced at $p_d = \mathbb{E}_d^l[\tilde{\theta}] = \pi_d^l \theta + (1 - \pi_d^l) \theta^2$. He can further lever up by using a debt contract with face value θ^2 (one can show that he has no incentive to use a different face value.) Thus, he can use his cash to a position of size $\frac{\varepsilon}{p_d - \theta^2}$ and thus has an expected value of

$$\frac{\varepsilon}{p_d - \theta^2} \pi_d^h (\theta - \theta^2) = \varepsilon \frac{\pi_d^h}{\pi_d^l}.$$

This in turn implies that his expected value on date 0 is

$$\pi_0^h \varepsilon + (1 - \pi_0^h) \varepsilon \frac{\pi_d^h}{\pi_d^l} = \varepsilon \left[\pi_0^h + (1 - \pi_0^h) \frac{\pi_d^h}{\pi_d^l} \right].$$

Thus, the marginal value of saving cash on date 0 is $\pi_0^h + (1 - \pi_0^h) \frac{\pi_d^h}{\pi_d^l}$.

First, we consider the marginal value of establishing a larger asset position in cases SD2 and SD3. According to equation (18), the marginal value is

$$\frac{(1 - \pi_0^h) \pi_d^h + \pi_0^h \pi_d^l}{\pi_d^l} \frac{P_H - F_S}{p_0 - F_S}$$

where the debt face value F_S could be either K_d or θ^2 . Since $p_0 \leq P_H$ in these cases (Figure 3), the marginal value is higher than

$$\frac{(1 - \pi_0^h) \pi_d^h + \pi_0^h \pi_d^l}{\pi_d^l} = \pi_0^h + (1 - \pi_0^h) \frac{\pi_d^h}{\pi_d^l},$$

which is the marginal value of saving cash on date 0.

Next, we consider the cases SD4 and SD5. According to equation (20), the marginal value of establishing a larger asset position on date 0 is

$$\frac{\pi_0^h \pi_0^l \mathbb{E}_0^h [\tilde{\theta} | uu, ud] - \pi_0^l F_S}{\pi_0^l p_0 - (1 - \pi_0^l) \mathbb{E}_0^l [\tilde{\theta} | du, dd] - \pi_0^l F_S}.$$

Since $p_0 \leq P_L$ in these cases (Figure 3), the marginal value is higher than $\frac{\pi_0^h}{\pi_0^l}$. Note that for the short-term debt to be desirable in equilibrium, the condition in (7) needs to hold. This condition directly implies that

$$\frac{\pi_0^h}{\pi_0^l} > \pi_0^h + (1 - \pi_0^h) \frac{\pi_d^h}{\pi_d^l}.$$

Thus, the marginal value of establishing a larger asset position on date 0 is higher than that of saving cash.

In summary of all the cases considered above, there is no incentive for any buyer to save cash on date 0.

A.6 Proof of Proposition 7

Under the condition given in Proposition 7, the equilibrium falls into the SD2 or SD3 case of Proposition 6 when both LD and SD are available to the optimists. We consider these cases separately.

In the SD3 case, the date-0 asset price is given by

$$p_0 = \frac{\mu c + C_S(K_d)}{1 - \mu} = \frac{\mu c + \pi_d^l \theta + (1 - \pi_d^l) \theta^2}{1 - \mu},$$

which is indifferent to δ_0 and decreases with δ_d .

In the SD2 case, the date-0 price is given by

$$p_0 = P_H = \frac{\pi_d^l [\pi_0^h (\pi_u^h + (1 - \pi_u^h) \theta) + (1 - \pi_0^h) \pi_d^h \theta] + (1 - \pi_0^h) \pi_d^h (1 - \pi_d^l) \theta^2}{(1 - \pi_0^h) \pi_d^h + \pi_0^h \pi_d^l}.$$

First, we consider the comparative static with respect to δ_0 . Note that p_0 depends on only $\pi_0^h = 0.5 + \delta_0$, but not on $\pi_0^l = 0.5 - \delta_0$. Let

$$X = \pi_d^h [\pi_d^l \theta + (1 - \pi_d^l) \theta^2], \text{ and } Y = \pi_d^l [\pi_u^h + (1 - \pi_u^h) \theta].$$

Then,

$$p_0 = \frac{(1 - \pi_0^h) X + \pi_0^h Y}{(1 - \pi_0^h) \pi_d^h + \pi_0^h \pi_d^l} = \frac{X (1 - \pi_0^h) + \pi_0^h \frac{Y}{X}}{\pi_d^h (1 - \pi_0^h) + \pi_0^h \frac{\pi_d^l}{\pi_d^h}}.$$

It is easy to see that p_0 is increasing with δ_0 if and only if $\frac{Y}{X} > \frac{\pi_d^l}{\pi_d^h}$, which is equivalent to $\frac{\pi_u^h + (1 - \pi_u^h) \theta}{\pi_d^l \theta + (1 - \pi_d^l) \theta^2} > 1$. Since $\pi_u^h + (1 - \pi_u^h) \theta > \theta > \pi_d^l \theta + (1 - \pi_d^l) \theta^2$, this inequality holds. Thus, p_0 increases with δ_0 .

We now consider the comparative static with respect to δ_d , which affects p_0 through

$$\pi_d^h = 0.5 + \delta_d \text{ and } \pi_d^l = 0.5 - \delta_d.$$

$$\begin{aligned} \frac{dp_0}{d\delta_d} &\propto \left\{ \begin{array}{l} - [\pi_0^h (\pi_u^h + (1 - \pi_u^h) \theta) + (1 - \pi_0^h) (0.5 + \delta_d) \theta] \\ + (0.5 - \delta_d) (1 - \pi_0^h) \theta + (1 - \pi_0^h) (1 + 2\delta_d) \theta^2 \end{array} \right\} \left[\begin{array}{l} (1 - \pi_0^h) (0.5 + \delta_d) \\ + \pi_0^h (0.5 - \delta_d) \end{array} \right] \\ &- \left\{ \begin{array}{l} (0.5 - \delta_d) [\pi_0^h (\pi_u^h + (1 - \pi_u^h) \theta) + (1 - \pi_0^h) (0.5 + \delta_d) \theta] \\ + (1 - \pi_0^h) (0.5 + \delta_d)^2 \theta^2 \end{array} \right\} (1 - 2\pi_0^h) \\ &\propto \{ (0.5 - \delta_d) \theta + (1 + 2\delta_d) \theta^2 \} [(1 - \pi_0^h) (0.5 + \delta_d) + \pi_0^h (0.5 - \delta_d)] \\ &\quad + (0.5 + \delta_d)^2 \theta^2 (2\pi_0^h - 1) - [\pi_0^h (\pi_u^h + (1 - \pi_u^h) \theta) + (1 - \pi_0^h) (0.5 + \delta_d) \theta] \\ &< \{ (0.5 - \delta_d) \theta + (1 + 2\delta_d) \theta \} [(1 - \pi_0^h) (0.5 + \delta_d) + \pi_0^h (0.5 - \delta_d)] \\ &\quad + (0.5 + \delta_d) (1 - \pi_d^l) \theta (2\pi_0^h - 1) - [\pi_0^h + (1 - \pi_0^h) (0.5 + \delta_d)] \theta \\ &\propto \{ 1.5 + \delta_d \} [(1 - \pi_0^h) (0.5 + \delta_d) + \pi_0^h (0.5 - \delta_d)] + (0.5 + \delta_d)^2 (2\pi_0^h - 1) \\ &\quad - [\pi_0^h + (1 - \pi_0^h) (0.5 + \delta_d)] \\ &= 0 \end{aligned}$$

which proves that p_0 decreases with δ_d .

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