

# Contracting on Credit Ratings: Adding Value to Public Information\*

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## Abstract

We show that credit ratings are valuable even if they provide no information beyond what is known publicly. In our model, an investor contracts with a manager who invests in either a risky bond with a state-dependent return or a riskless asset. The state is known to both investor and manager, but unverifiable to a third party and therefore non-contractible. A credit rating on the risky bond provides a verifiable, but noisy signal about the state. If the rating is good, the optimal contract imposes no restriction on actions, and the wage is set to the minimal level that induces investment in the risky bond. If the rating is bad, the optimal contract depends on the precision of the rating. If the rating is precise, the manager is banned from investing in the risky bond. With an imprecise rating, the investor prefers to use a wage contract with no restrictions on action. In an economy with a continuum of investor-manager pairs, the use of the rating in setting contracts in turn determines the equilibrium return of the risky bond. We establish that widespread use of credit ratings may increase asset volatility.

# 1 Introduction

There are many contexts in which market participants use and react to the issuance of reports based on already known, publicly-available information; noteworthy examples are credit ratings on sovereign bonds, or insured municipal bonds. While many models of credit ratings on companies assume that the rating agency possesses information not already reflected in market prices, such a claim is difficult to make for government debt. A credit rating merely provides a summary of information already available, and yet markets react to it.<sup>1</sup>

In this paper, we pursue a novel explanation for the existence of this (seemingly) redundant information aggregation and reporting: When contracts are incomplete, the use of ratings allows market participants to write better contracts. Consider an investor who delegates the management of her portfolio and wants to provide incentives to a manager who is prone to moral hazard. An incentive contract based on portfolio outcomes may not be precise enough to ensure that the manager always acts in her best interests. However, an additional signal (such as a credit rating) can provide her with a useful tool to improve on the contract.

Our framework is (loosely) based on Aghion and Bolton (1992). Briefly, this is a stylized incomplete contracting model between an investor and a manager in which states are observable, but not verifiable. There are two states, high and low and two feasible actions: hold a risky bond or hold a risk-less asset instead. The manager's preferred action depends on the realization of a stochastic private benefit. Thus, in contrast to many contracting frameworks, the size of the private perquisites the manager can extract are unrealized at the time the contract is written and unknown to both parties. The potential inefficiency is that, due to these private benefits, the investor and manager may end up preferring different state-contingent actions. We interpret a credit rating as a verifiable signal about the state; this rating can be either good or bad. The more precise a rating is, the more likely good ratings ensue when the state is high. Contracts may be written on this verifiable signal, potentially improving efficiency in the contracting relationship.

In our model, investors delegate investment to a portfolio manager. An investor offers a contract that has two components. First, the manager is paid a compensation or wage based on the return delivered by him and the credit rating of the risky bond. Second, the investor can restrict the manager's action by requiring him to invest in the risk-less asset (or alternatively, prohibiting investment in the risky asset). Each investor/manager pair is atomistic and takes the possible returns as given.

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<sup>1</sup>For example, Brooks, et al., find that downgrades of sovereign debt adversely affect both the level of the domestic stock market and the dollar value of the country's currency.

We show that the optimal contract has the following features: If the credit rating is good, depending on the precision of the rating, the contract may feature zero or positive wages. In either case, no restriction is imposed on the manager's action. If the rating is bad but imprecise, the contract offers zero wages and does not restrict the manager's action. In an intermediate precision range, the investor uses a wage contract to induce an incentive compatible action from the manager, without restricting the action. However, if the rating is sufficiently precise, the investor prohibits the manager from investing in a risky bond with a bad rating, and offers a zero wage. We note that the zero wage is, of course, a normalization and reflects the manager's outside option.

The key intuition behind this contract is that restricting the manager's actions is costly if the rating is imprecise. Sometimes, the rating will be bad even though the state is high, and prohibiting investment in the risky bond requires the investor to forgo the return she can earn in this scenario. The other option is to use wages to induce an incentive compatible action. In this case, the investor must induce the manager to hold the risky bond when the state is high. Boosting the reward for holding the risk-less bond in the low state requires correspondingly increasing the reward for holding the risky bond in the high state, providing a trade-off for the investor.

We then turn to the market-wide equilibrium implications of credit ratings. The equilibrium return of the risky bond depends on the realization of the portfolio management sector demand. This in turn, is affected by the equilibrium returns, which each infinitesimal investor-manager pair takes as given. The importance of this fixed point is seen most clearly in the ranges of rating precision in which investors are close to indifferent between offering a wage-only contract, and insisting on a prohibitive contract which restricts investment in the risky asset when the rating is bad. For low precisions, all use a wage-only contract, while for high precisions, all use the prohibitive contract. However, for intermediate values, there is a range of ratings precision for which both the prohibitive and wage-only contracts are observed. In the aggregate, investors mix across contracts so that the resulting demand for the risky asset makes each investor indifferent between these two contracts.

Taking the return of the risky asset as given, each investor benefits from using the credit rating. However, when all investors do so, this affects the bond's returns. In particular, even when fundamentals are fixed, the price of the risky bond now depends on its credit rating. Our framework also has implications for the effect of ratings on asset returns and the choice of an optimal rating precision. Asset prices are more volatile than justified by fundamentals when credit ratings are widely used in contracts. Further, ratings may lead to lower returns. The aggregate effects of contracting on credit ratings on the returns of the asset therefore

imply that, even absent any direct cost to producing more precise credit ratings, it may be optimal for a rating to have some noise in it.

Our focus on the use of non-informative credit ratings to mitigate contracting frictions is novel. Other work on non-informative ratings includes Boot, Milbourn, and Schmeits (2006), who present a framework in which a firm's funding costs depend on the market's beliefs about the type of project being chosen. The credit rating agency, by providing a rating, allows infinitesimal investors to coordinate on particular beliefs when multiple equilibria are possible. Further the credit watch procedure provides a mechanism to monitor the firm if it can improve the payoff of its project. Manso (2014), also considers how a credit rating might have real effects, in a model with multiple equilibria self-fulfilling beliefs. In his framework changes in a firm's credit rating affects its ability to raise capital, which then reinforce the original rating.

Much of the work in the literature considers credit ratings that communicate new information about the firm to the market. For example, Opp, Opp and Harris (2013) illustrate how the use of ratings by regulators might have pernicious effects, and Fulghieri, Strobl and Xia (2013) consider rating manipulation by the credit rating agency itself. Mathis, McAndrews and Rochet (2009) demonstrate that when the flow income from new transactions is high, a rating agency's concern for future reputation no longer acts as a disciplining device.

Rating shopping has been examined by Bolton, Freixas and Shapiro (2012) in a world with some boundedly rational consumers who trust the assigned rating. Competition between credit rating firms induces inefficiency, and ratings are more likely to be inflated in booms. Skreta and Veldkamp (2009) examine a similar friction and show that issuers have an incentive to produce complex assets when some consumers are naïve. Subsequent work by Sangiorgi and Spatt (2013) considers rating shopping when all consumers are rational, with the key friction being opacity about how many ratings an issuer has actually obtained. In equilibrium, too many ratings are obtained. While we have a single rating in our model, we consider the effect on different users of the rating, such as firms and portfolio managers.

Donaldson and Piacentino (2013) consider the effect of credit ratings in contracts, and suggest that investment mandates based on ratings lead to inefficiency. We provide a counter-perspective: in our model, the use of ratings leads to better contracts and so increases social surplus. Researchers have also considered the optimal degree of coarseness (see Goel and Thakor (2013) and Kartasheva and Yilmaz (2013)).

We build on the large literature on optimal contracts in a delegated portfolio management problem. Bhattacharyya and Pfleiderer (1985) consider such a problem with asymmetric information and Stoughton (1993) models the moral hazard version in which the manager

chooses the proportion to invest in a risky asset (so the action set is continuous). We focus on the use of an outside signal in the contract, and simplify the action space to be binary.

In other work, Admati and Pfleiderer (1997), Lynch and Musto (1997), and Das and Sundaram (1998) consider the use of benchmark evaluation measures. In our setting, we assume that other investors' performance is not verifiable, ruling out the possibility of relative performance evaluation. Innes (1990) provides optimal contracts in a limited liability setting when there is moral hazard on the part of the investor.

Starting with Dasgupta and Prat (2006), some papers have considered the effects of career concerns on the part of portfolio managers on financial market equilibrium. Dasgupta and Prat (2008) introduce the notion of a reputational premium that a risky bond must earn to compensate for the risk that manager will be fired when a bond defaults. Guerrieri and Kondor (2012) construct a general equilibrium model that endogenizes reputational concerns, and show that the reputational premium amplifies price volatility.

We introduce our model in Section 2. In Section 3, we demonstrate the optimal contract for a single investor-manager pair, holding the price of the risky bond as fixed for each state and credit rating. We then step back to exhibit the equilibrium effects of the contract in Section 4. We provide some implications of our findings and discuss some features of the model in Section 5. All proofs appear in the appendix.

## 2 Model

The delegated portfolio management sector of an economy comprises a continuum of investors and a continuum of portfolio managers, each with mass one. There are two assets, a risky bond and a risk-free one. Investors and managers are randomly matched in pairs, and contract exclusively with each other. The investor–manager relationship continues over four periods,  $t = 1, \dots, 4$ . Contracts are signed at time  $t = 1$ , information is released at time  $t = 2$ , trading occurs at time  $t = 3$  and payoffs are realized at time  $t = 4$ .

At time  $t = 1$ , an investor offers a manager a contract that specifies both a feasible action set at the trading date  $t = 3$ , and compensation or a wage at the final date  $t = 4$ . For simplicity, we assume that each manager may purchase one unit of either the risky or risk-less asset. Restricting the feasible action set captures the notion that credit ratings are often used to restrict the investment set for portfolio managers. The action of investing in the risky asset is denoted  $a_h$ , and purchasing the risk-free bond is denoted  $a_\ell$ . The contract also specifies a wage at time  $t = 4$ , conditional on the portfolio performance and a credit rating for the risky bond.

At time  $t = 2$ , three pieces of information become available to market participants. First, a state, which affects the payoff to holding the risky bond, is realized. The risky security has two possible payoff states,  $h$  and  $\ell$ , which correspond to the risk-return relationship offered by the bond. State  $h$  has probability  $\phi$ , and corresponds to the “solvent” state for the bond, with a relatively high return. State  $\ell$  corresponds to a “default” state, with a relatively low (possible negative) return.<sup>2</sup>

Critically, even though both parties know the state, it is not verifiable, and so not directly contractible. However, a contractible signal  $\sigma$  is available, in the form of a credit rating on the risky bond. We do not model the source of the credit rating. The rating is correlated with the state. Specifically, the signal takes on one of two values,  $g$  or  $b$ , and is potentially informative, with  $\text{Prob}(\sigma = g \mid s = h) = \text{Prob}(\sigma = b \mid s = \ell) = \psi \geq \frac{1}{2}$ . Thus, if  $\psi = \frac{1}{2}$ , the rating is completely uninformative, which is equivalent to the investor and manager being able to contract only on the final value of the investment, and if  $\psi = 1$ , the rating is perfectly informative, which is equivalent to the investor and manager being able to contract directly on the state. We refer to  $\psi$  as the precision of the rating.

The manager obtains a private benefit  $m$  from holding the risky bond in state  $\ell$ , the size of which is realized at time  $t = 2$ . The private benefit corresponds to either synergies with his other funds (“soft money”) or side transfers that he obtains from a sell-side firm if he places the risky bond in an investor’s portfolio. This random private benefit is drawn from a uniform distribution with support  $[0, M]$ . The size of the private benefit is independent across managers; As is customary, the private benefit  $m$  is not verifiable, so cannot be contracted on.

At time  $t = 3$ , portfolio managers each choose an action from their respective feasible sets. Collectively, their actions determine the demand for the risky bond, and hence the return on the bond between times 3 and 4. In state  $s$ , let  $q_\sigma^s$  denote the demand when the credit rating is  $\sigma$ . Let  $r^s(q)$  denote the return on the asset in state  $s$  if the aggregate demand for the asset from the delegated portfolio management (DPM) sector is  $q \in [0, 1]$ . We assume that  $r^s$  is decreasing in  $q$ . That is, a larger demand leads to a higher price and so a lower return. In choosing the contract to offer a hired manager (at  $t = 2$ ), an investor has rational expectations about the returns to the risky bond under different scenarios. That is, she correctly anticipates  $r^s(q_\sigma^s)$  for each  $s = h, \ell$  and  $\sigma = g, b$ . The return to holding the riskless asset is  $r^f$ , regardless of state or signal on the risky bond.

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<sup>2</sup>In the model, for simplicity, we model the return offered by the risky bond in a given state as being deterministic in equilibrium. More broadly, we interpret state  $h$  as offering a high reward-for-risk, and state  $\ell$  as offering a low reward-for-risk.

Let  $\bar{r}^s = r^s(0)$  be the maximal return to the risky asset in state  $s$ . This return is realized if the price of the risky asset is low; that is, the demand for the asset from the DPM sector is zero. Correspondingly, let  $\underline{r}^s = r^s(1)$  be the minimal return to the risky asset in state  $s$ , obtained when its price is high; specifically, when all investors wish to buy the risky asset, so that the demand from the DPM sector is one. The minimal return on a long position is -100%, so  $\underline{r}^s \geq -1$  for each  $s$ . We further assume that  $\underline{r}^h > r^f > \bar{r}^\ell$ ; that is, the return on the risky bond is greater in state  $h$  than in state  $\ell$ , regardless of the credit rating. Under these assumptions, an investor purchasing bonds directly would prefer to buy the risky bond in state  $h$  (when the reward to bearing its risk is high) and the riskless bond in state  $\ell$  (when the reward to bearing the risk on the risky bond is low). Given the agency conflict, managers may sometimes take an inefficient action. Potentially, there are gains to trade from renegotiation between the investor and manager at that time. For now, we assume that renegotiation is costly enough to be infeasible, and return to a discussion of renegotiation in Section 5.

To summarize: There are four dates in the model,  $t = 1$  through 4. Figure 1 shows the sequence of events in the model.

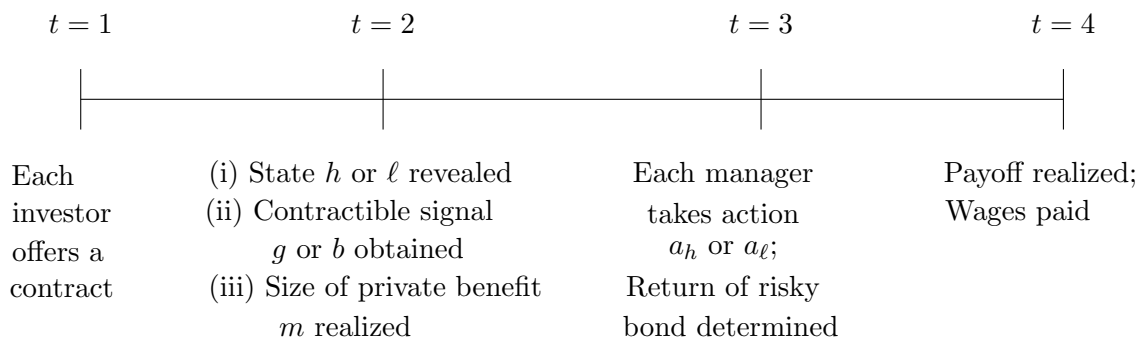


Figure 1: **Sequence of Events**

It is important to note that the contract is written before the state and credit rating are realized. We have in mind a situation in which contracts are written on a periodic basis (say once a year), whereas the state (which could reflect other aspects of the investors' portfolio) can change frequently in between. The credit rating need not be known as soon as the state is revealed, but it must be known before the manager takes an action. The private benefit of the manager reflects the effect of market events on other assets held by the manager or other payments he receives from his relationships, so is known only when the state itself is revealed.

There is no discounting, and we model both parties as risk-neutral. The payoff to the



investor from this relationship is the net return generated by the manager less the total compensation paid to the manager. The payoff to the manager is the sum of the wage and any private benefits he may garner. The manager enjoys limited liability that requires the wage in any state to be non-negative. His reservation utility is zero, so any contract that satisfies limited liability is also individually rational. When the outcome is realized at time 4, the manager is paid the wage specified by the contract signed at time 1 and the investor keeps any extra investment income.

We assume that the investor cannot directly invest in the risky bond on her own. Implicitly, the cost of direct investing is too high for her. This cost may be interpreted as either the opportunity cost of time for the investor or the direct cost of access to certain securities.<sup>3</sup> We also ignore an individual rationality constraint on the investor. That is, for now we assume that the payoff she obtains after contracting with the manager exceeds  $r^f$ , the payoff she could obtain if she invested in the riskless bond by herself. In Section 3, we show that the optimal contract satisfies this feature.

An equilibrium in this model has several components. First, each investor offers an optimal contract to the manager, anticipating the returns on the risky asset. The wage offered to the manager depends on both the rating and the return on the portfolio. In addition, we allow the investor to designate a specified action set for the manager, which depends on the rating. Second, portfolio managers optimally decide whether to buy the risky bond or the riskless asset, given the state, credit rating, returns on the risky bond and (in the case of managers) the contract and the private benefit. Third and finally, the market for the risky bond clears, which determines the return in each state and for each credit rating.

Formally, let  $\mathbf{w} = \{w_\sigma^j\}_{\substack{\sigma=g,b \\ j=h,f,\ell}}$ ,  $\mathbf{r} = \{r_\sigma^j\}_{\substack{\sigma=g,b \\ j=h,f,\ell}}$ , and  $\mathbf{A} = \{A_g, A_b\}$  with  $A_\sigma \subseteq \{a_h, a_\ell\}$  for each  $\sigma$ . A contract offered by investor  $i$  is denoted by  $C_i = \{\mathbf{w}, \mathbf{A}\}_i$ . Then,

**Definition 1** *A market equilibrium in the model consists of:*

- (a) *An optimal contract  $C_i = \{\mathbf{w}, \mathbf{A}\}_i$ , offered by each investor  $i$  to her portfolio manager, where the wages  $\mathbf{w}$  depend on the rating  $\sigma$  and the returns on the risky bond  $\mathbf{r}$ , and the feasible actions  $\mathbf{A}$  depend on the rating  $\sigma$ .*
- (b) *A payoff-maximizing action chosen by each portfolio manager  $i$ , given the returns on the risky bond  $\mathbf{r}$ , the state  $s$ , the credit rating  $\sigma$ , the contract  $C_i$ , and her private benefit  $m$ .*

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<sup>3</sup>For example, under SEC Rule 144A, only qualified institutional buyers may purchase certain private securities.

(c) Returns on the risky bond given state  $s$  and credit rating  $\sigma$  determined by  $r_\sigma^s = r^s(q_\sigma^s)$ , where  $q_\sigma^s$  is the aggregate demand for the risky bond generated by portfolio managers in part (b).

An equilibrium is therefore a Nash equilibrium in contracts. Each investor offers an optimal contract given the returns on the risky bond, where the returns on the risky bond in turn depend on the contracts offered by all other investors. In that sense, each investor is offering an optimal contract given the contracts offered by all other investors.

We make the following additional assumptions on parameters.

**Assumption 1** (i)  $\bar{r}^\ell < r^f$ .

(ii)  $\frac{1-\phi}{\phi}(r^f - \underline{r}^\ell) \leq M < \underline{r}^h - r^f$ .

The first part of the assumption, that the highest possible return in the low state is less than the risk free rate, implies that action  $a_\ell$  (buying the riskless bond) maximizes the investor's payoff in the low state  $\ell$ . Part (ii) ensures that for some realizations of  $m$ , the agency conflict between investor and manager is sufficiently large so that it has bite. However, it is not too large, so that it can be effective to offer a wage contract to induce the manager to take the action preferred by the investor. Notice that  $\underline{r}^h - r^f > M$  implies that  $\underline{r}^h > r^f$  (as  $M > 0$ ).

### 3 Optimal Contract for a Single Investor-Manager Pair

As a first step, consider the optimal contract for a single investor-manager pair. The contract is entered into at time  $t = 1$ , before the state and credit rating are known. In addition, the extent of the moral hazard problem (i.e., the size of the realized private benefit  $m$ ) is unknown to both parties. The demand of each investor and each manager is infinitesimal, so they take as given the return on the risky asset in each scenario (i.e., state-rating pair).

Because all agents know the state, but cannot contract on it, the optimal contract depends on how precise the signal is. If it is perfectly precise ( $\psi = 1$ ), then the investor and manager effectively contract on the state. If it is perfectly imprecise,  $\psi = \frac{1}{2}$ , then it has no benefit. Define a threshold level of precision

$$\hat{\psi}(r_b^\ell) = \frac{1}{1 + \frac{1-\phi}{\phi} \frac{(r^f - r_b^\ell)}{M}}. \quad (1)$$

Because each investor and manager treats  $r_b^\ell$  as fixed, we suppress the dependence of  $\hat{\psi}$  on  $r_b^\ell$  in the notation for the rest of this section.

Our main result in this section is stated in Proposition 1. We state the result first, and build up the intuition in the remainder of the section. We show that if ratings are imprecise (less than  $\hat{\psi}$ ), investors do not use the rating in the contract. That is, neither the wages offered nor the permissible actions rely on the rating. However, as the rating becomes more precise (in particular, above  $\hat{\psi}$ , but below some threshold  $\psi_1$ ), the investor chooses an optimal wage contract that does not restrict the manager's action. In this intermediate precision range, it is too costly to impose a restriction on action: When the state is high but the rating is bad (which can sometimes happen with imprecise ratings), forcing the manager to hold the risk-less asset entails giving up on the high return that can be obtained on the risky bond. Finally, as the rating becomes even more precise (above  $\psi_1$ ), the investor prefers to restrict the manager's action when the rating is bad, rather than relying on wages to induce the right action. In particular, she bans the manager from investing in the risky bond.

**Proposition 1** *The optimal contract for each investor is as follows.*

- (i) *If the rating is  $g$ , no wage is offered and no action restriction is imposed.*
- (ii) *If the rating is  $b$ , zero wages are offered when  $\psi \leq \hat{\psi}$ . Further, there exists a threshold rating precision  $\psi_1 \in (\hat{\psi}, 1)$  such that:*
  - (a) *If  $\psi \in (\hat{\psi}, \psi_1)$ , the contract relies only on wages, with no action restriction.*
  - (b) *If  $\psi > \psi_1$ , the contract prohibits investment in the risky asset, and offers zero wages.*

The rest of this section explains the intuition behind the optimal contract. We establish a number of preliminary results before exhibiting the proof of Proposition 1.

First, as the simplest case, consider a contract in which the investor restricts the manager's actions after some rating. We call this a prohibitive contract. Implicitly, we assume that the investor has a way to enforce a restriction on actions, either through a technological system, or perhaps due to a large reputational or legal penalty suffered by a manager who violates an imposed restriction. As any restriction on actions reduces the feasible action set to a singleton, it is immediate that no wages are offered.

**Lemma 1** *Suppose an optimal contract restricts the manager's actions conditional on some rating  $\sigma$ . Then, it must be that  $w_\sigma^j = 0$  for each  $j = h, f, \ell$ .*

It is important to recognize that, if the rating does not perfectly reflect the state, the moral hazard problem (which arises in the low state,  $\ell$ ) can emerge both when the rating is good and when it is bad. If the investor bans the manager from investing in the risky asset when the rating is bad, she will drive him down to his participation constraint and pay him a wage of zero. However, this does not imply that the wage if the rating is good is also zero (recall that both  $\mathbf{w}$  and  $\mathbf{A}$  can be made contingent on rating) We turn to this in Lemma 2 below.

Further, if the rating is not fully precise, the prohibitive contract may ban an action that is optimal. That is, sometimes the rating will be  $b$  even in state  $h$ , but the prohibitive contract prevents the manager from purchasing the risky asset. As an alternative, consider a contract in which there is no restriction on the manager's actions, so that  $A_g = A_b = \{a_h, a_\ell\}$ . In such a contract, the manager's action depends in part on the wages offered. We term this a wage contract.

In a wage contract, the investor writes a contract for the manager that depends on the possible investment returns and the credit rating. A contract is therefore characterized by a payoff for each rating, state pair or  $\mathbf{w} = \{w_g^h, w_g^\ell, w_b^h, w_b^\ell, w_g^f, w_b^f\}$ , where  $w_\sigma^j$  denotes the compensation to the manager when the credit rating is  $\sigma \in \{g, b\}$  and the portfolio payoff is  $r^j$  for  $j \in \{h, \ell, f\}$ .

At time 1, when the contract is signed, the investor chooses the various wage levels  $\{w_g^h, w_b^h, w_g^f, w_b^f, w_g^\ell, w_b^\ell\}$  to maximize her expected payoff

$$\Pi = \phi\pi^h + (1 - \phi)\pi^\ell. \quad (2)$$

In state  $h$ , the credit rating is  $g$  with probability  $\psi$  and  $b$  with probability  $1 - \psi$ . If the investor induces the action  $a_h$ , her payoff is  $r_\sigma^h - w_\sigma^h$ ; if she induces the action  $a_\ell$ , her payoff is  $r^f - w_\sigma^f$ . In the high-reward state, there is no private benefit, and the manager takes the action that yields him the highest wage. Thus, the expected payoff of the investor in this state  $h$  is:

$$\begin{aligned} \pi^h = & \psi \left( (r_g^h - w_g^h)1_{\{w_g^h \geq w_g^f\}} + (r^f - w_g^f)1_{\{w_g^h < w_g^f\}} \right) \\ & + (1 - \psi) \left( (r_b^h - w_b^h)1_{\{w_b^h \geq w_b^f\}} + (r^f - w_b^f)1_{\{w_b^h < w_b^f\}} \right), \end{aligned} \quad (3)$$

where  $1_{\{x\}}$  is an indicator function that takes on the value of 1 if the event  $x$  occurs, and 0 otherwise. The indicator captures the fact that the manager takes the action that gives him the highest wage in the high return state.

Next, consider the low reward state  $\ell$ . The credit rating is  $g$  with probability  $1 - \psi$  and  $b$  with probability  $\psi$ . Given a signal  $\sigma$ , the manager takes the action  $a_h$  if  $w_\sigma^f \geq w_\sigma^\ell + m$ , or  $m \leq w_\sigma^f - w_\sigma^\ell$ . He takes action  $a_\ell$  if  $w_\sigma^f < w_\sigma^\ell + m$ , or  $m > w_\sigma^f - w_\sigma^\ell$ . Of course, at the time the contract is established, neither party knows  $m$ , the size of the manager's private benefit. The investor therefore has to take expectations over the possible values it may take. Overall, the investor's expected payoff in the low state  $\ell$  is

$$\begin{aligned} \pi^\ell = & (1 - \psi) \left( (r^f - w_g^f) \frac{(w_g^f - w_g^\ell)}{M} + (r_g^\ell - w_g^\ell) \left(1 - \frac{(w_g^f - w_g^\ell)}{M}\right) \right) \\ & + \psi \left( (r^f - w_b^f) \frac{(w_b^f - w_b^\ell)}{M} + (r_b^\ell - w_b^\ell) \left(1 - \frac{(w_b^f - w_b^\ell)}{M}\right) \right). \end{aligned} \quad (4)$$

In the high reward state,  $h$ , suppose that the rating is  $\sigma$ . Notice that the manager takes action  $a_h$  if his payoff from doing so is higher than the payoff from investing in the risk-free bond; that is, if  $w_\sigma^h \geq w_\sigma^f$ . He takes action  $a_\ell$  otherwise. Thus, the payoff to investing in the risk-free asset affects incentive compatibility in both the high and low reward states. Therefore, if incentive compatibility binds in the low state, there has to be a commensurate increase in wage for the manager in the high state. This feature makes the wage contract "expensive" for the investor; in particular, it implies that  $w_\sigma^h = w_\sigma^f$ . That is, for any rating  $\sigma$ , the manager is paid the same compensation when the return is  $r^h$  as he earns by investing in the risk-free asset. In a wage contract, the manager always has the choice of investing in the risk-free asset, so an investor who wants to induce him to hold the risky asset must provide at least as much of a reward for the latter action when the state is high. To minimize the cost of providing this incentive, the investor sets  $w_\sigma^h$  as low as possible, that is, equal to  $w_\sigma^f$ .

In addition, in an optimal wage contract,  $w_\sigma^\ell = 0$ . That is, if the manager invests in the risky asset in the low reward state  $\ell$ , she obtains a zero payoff. The investor does not want to hold the risky bond in this state, so it cannot be worthwhile to reward a manager who holds the risky bond in state  $\ell$ .

**Lemma 2** *The optimal wage contract sets  $w_\sigma^h = w_\sigma^f$  and  $w_\sigma^\ell = 0$  for each credit rating  $\sigma = g, b$ .*

Lemma 2 reduces the investor's problem of finding an optimal wage contract to two choice variables,  $w_g^f$  and  $w_b^f$ . That is, the optimal contract is characterized by the compensation that the manager receives for investing in the risk-free asset, given the rating on the risky bond.

Broadly, the optimal wage contract rewards the manager for avoiding the risky bond in the low return state,  $\ell$ , when its credit rating is bad ( $b$ ). If the signal embodied in the credit rating is sufficiently informative about the state (i.e.,  $\psi$  is sufficiently high), the manager receives a positive wage  $w_b^f$  for buying the riskless asset when the risky bond has a low credit rating. He receives a zero wage for the same action when the risky bond has a good credit rating (i.e.,  $w_g^f = 0$ ). In other words, if the credit rating is sufficiently precise, the investor induces the manager to tilt toward the risky bond when it has a high credit rating and steer clear of the risky bond when it has a bad credit rating. Further, the wage  $w_b^f$  is capped at  $M$ , as it cannot be optimal to pay the manager more than his maximum private benefit.

**Lemma 3** *In the optimal wage contract:*

(i)  $w_g^f = 0$ , regardless of the rating precision  $\psi$ .

(ii)  $w_b^f$  depends on the rating precision  $\psi$ . Specifically,

$$w_b^f = \begin{cases} \min \left\{ \frac{1}{2} \left( r^f - r_b^\ell - \frac{\phi}{1-\phi} \frac{1-\psi}{\psi} M \right), M \right\} & \text{if } \psi \geq \hat{\psi} \\ 0 & \text{if } \psi < \hat{\psi}. \end{cases} \quad (5)$$

The optimal wage, when it is positive, trades off the investor's payoff across states. Suppose the risky bond obtains a bad credit rating  $b$ . A higher wage  $w_b^f$  induces the manager to hold the risk-less bond more often in the low state (i.e., for a larger set of private benefit realizations); this anti-shirking effect increases the investor's payoff. However, in the high state,  $h$ , because of the incentive compatibility constraint ( $w_b^h \geq w_b^f$ ), the investor has to pay the manager a higher amount. This incentive compatibility effect decreases the investor's payoff. The optimal wage  $w_b^f$  balances these two effects. This wage is increasing in signal precision,  $\psi$ . If  $\psi$  is high (say close to 1), the anti-shirking effect dominates, because the bond is likely to get a bad credit rating only in the low state. Conversely, when  $\psi$  is low (close to  $\frac{1}{2}$ ), the incentive compatibility effect becomes more important (the risky bond may get a bad credit rating even in the high state), so the investor sets  $w_b^f$  to zero.

The intuition for setting  $w_g^f = 0$  is similar. On the one hand, in the low return state,  $\ell$ , a positive  $w_g^f$  induces the manager to hold the riskless bond for a higher range of private benefit realizations. On the other, it requires the investor to increase  $w_g^h$  correspondingly, which lowers her payoff in the high return state,  $h$ . Under our assumptions, for a good rating, the incentive compatibility effect always dominates, so the investor sets  $w_g^f$  to zero.

With the wage contract, the stochastic private benefit represents an important friction. If the highest value of the private benefit (i.e.,  $M$ ) is sufficiently high, even with a fully precise rating, the optimal contract does not always elicit the action preferred by the investor. Even if the investor could contract directly on the state, she would prefer to let the manager sometimes deviate to the inefficient action in state  $\ell$  (when the private benefit  $m$  is high enough), because by keeping  $w_b^f$  low, she sometimes obtains the efficient action at lower cost (when the private benefit  $m$  is low). It is therefore natural to consider a contract in which the investor explicitly prohibits the manager from investing in the risky bond when the credit rating is bad.

We focus on a contract that bans investment in the risky bond when the rating is bad. With a good rating the manager will always purchase the risky bond, regardless of state, even when  $w_g^f = 0$ . In state  $h$ , he obtains no benefit from deviating to the riskless bond, and in state  $\ell$ , he obtains his private benefit  $m$  if he purchases the risky bond. Therefore, a prohibitive contract that requires the manager to purchase the risky bond has no further bite when the rating is  $g$ . Even a small monitoring cost will lead to it being dominated.

However, when the rating on the risky bond is  $b$ , the investor may want to prevent the manager from investing in the risky bond; in terms of our notation, by setting  $A_b = \{a_\ell\}$ . The cost of doing so is that sometimes the rating does not reflect the state, and this action is inefficient. To explore this tradeoff, let  $\delta_b = \text{Prob}(s = h \mid \sigma = b) = \frac{\phi(1-\psi)}{\phi(1-\psi) + (1-\phi)\psi}$  be the (objective) probability the state is high given that the rating on the risky bond is  $b$ . From Lemma 2, the optimal wage contract satisfies  $w_b^h = w_b^f$  and  $w_b^\ell = 0$ . The manager buys the risky bond in state  $h$ ; in state  $\ell$  she buys the risky bond if  $m > w_b^f$  and the risk-less bond if  $m \leq w_b^f$ . Therefore, the payoff to the investor from using an optimal wage contract is

$$\Pi_{w,b} = \delta_b(r_b^h - w_b^f) + (1 - \delta_b)\left[\frac{w_b^f}{M}(r^f - w_b^f) + \left(1 - \frac{w_b^f}{M}\right)r_b^\ell\right], \quad (6)$$

where  $w_b^f$  is set as in Lemma 3, and  $\frac{w_b^f}{M}$  represents the mass of managers with  $m \leq w_b^f$  (recall that  $m$  is uniformly distributed over  $[0, M]$ ).

If the investor bans the manager from investing in the risky asset, she offers zero wages (i.e.,  $w_b^h = w_b^f = w_b^\ell = 0$ ), her payoff is

$$\Pi_x = r^f, \quad (7)$$

since the wage rate is optimally set to zero. Equating these payoffs determines the ranges of rating precision defined in Proposition 1. The formal proof of the proposition, showing the

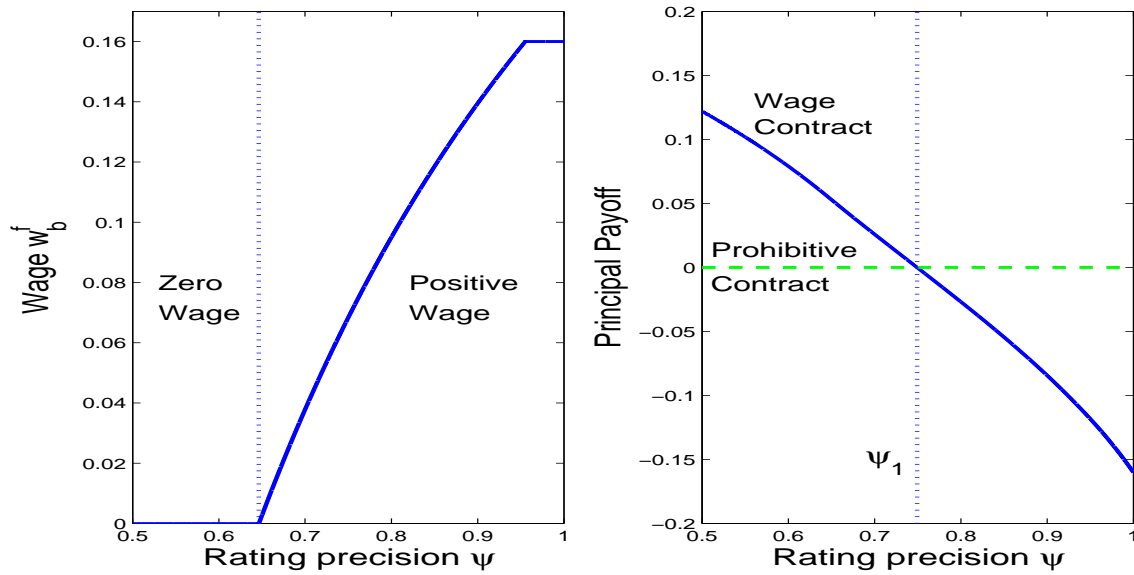
optimality of the wage and prohibitive contracts in the respective ranges, is in the Appendix.

We illustrate the results of this section in a numerical example.

**Example 1**

Set  $\phi = 0.8$ ,  $r^f = 0$ ,  $r^h = r_b^h = 0.24$ ,  $r_b^\ell = -0.35$ , and  $M = 0.16$ .

From Lemma 3, the optimal wage when the rating is  $g$  is zero (i.e.,  $w_g^f = 0$ ), and the manager buys the risky asset. We therefore focus throughout the example on the rating  $b$ . In Figure 2, we illustrate the manager’s wages in the optimal wage contract (Figure (a)) and the investor’s payoff from the optimal wage contract and the prohibitive contract (Figure (b)), when the rating on the risky bond is  $b$ .



(a) Wage  $w_b^f$  in optimal wage contract

(b) Investor’s payoffs from both contracts

Figure 2: Optimal Wage Contract: Manager’s Wage and Investor’s Payoff

Figure 2 (a) illustrates the optimal wage contract when the rating is  $b$ . In the example, the value of  $\hat{\psi}$  is about 0.65. When  $\psi < \hat{\psi}$ , as shown in Lemma 3, it is optimal to set  $w_b^\ell = 0$ . For  $\psi > \hat{\psi}$ , the wage is positive and strictly increasing in  $\psi$  over some range. Of course, the maximal wage the investor will offer in a wage contract is  $M$ . In the example, for  $\psi \geq 0.96$  (approximately), the wage is flat at  $w_b^\ell = M$ . The vertical dotted line denotes  $\hat{\psi}$ .

We plot the investor’s payoffs from the the optimal wage contract and the prohibitive contract when the rating is  $b$  in Figure 2 (b). As expected, the investor’s payoff from the



optimal wage contract decreases in  $\psi$ . There are two reasons for this. First, with an increase in  $\psi$ , a bad rating is more likely to occur in the low state. Second, as  $\psi$  increases above  $\hat{\psi}$ , the wage  $w_b^f$  increases in  $\psi$ , reducing the investor's payoff in the high state as well. The return from the prohibitive contract is constant at  $r^f$ , so there exists a threshold  $\psi_1$  (approximately at 0.75 in the figure) such that for  $\psi \leq \psi_1$ , the investor prefers the wage contract, and for  $\psi > \psi_1$ , the investor prefers the prohibitive contract.

Putting together the two thresholds  $\hat{\psi}$  and  $\psi_1$ , the overall optimal contract in different regions has the features shown in Figure 3. There are three regions to consider when the rating is  $b$ : for  $\psi \leq \hat{\psi}$ , the contract offers zero wages and has no restriction on actions, for  $\psi \in (\hat{\psi}, \psi_1)$ , the contract has positive wages with no restrictions on actions, and for  $\psi \geq \psi_1$ , the contract prohibits investment in the risky asset, and offers zero wages.

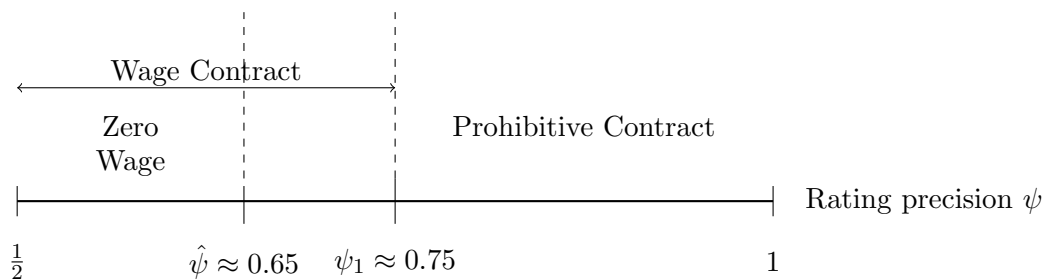


Figure 3: **Optimal Contract for a Single Investor**

Finally, we note that the individual rationality constraint for the investor is satisfied when an optimal contract is offered. A direct investor only has access to the risk-free asset, and earns  $r^f$  for sure. An investor who hires this manager can earn the same payoff by offering a prohibitive contract that prevents the manager from buying the risky asset, and offering a zero wage. When the rating is  $g$ , the optimal contract leaves the investor strictly better off, compared to buying the risk-free asset. Therefore, the overall individual rationality constraint for the investor is satisfied.

## 4 Market Equilibrium

Having determined how each investor-manager pair will behave, we now turn to the overall equilibrium in the market. In a market equilibrium, the return on the risky asset depends on the aggregate actions off all the portfolio managers, and therefore on the contracts offered by all investors. Each investor is infinitesimal in the economy, and takes the returns on the

risky asset as given. In particular, in Proposition 1, we treat  $r_b^\ell$  as fixed. The following complication emerges in a market equilibrium. The threshold values of rating precision in Proposition 1,  $\hat{\psi}$  and  $\psi_1$ , each depend on  $r_b^b$ . It is straightforward to see, for example, that  $\hat{\psi}$  increases in  $r_b^b$ . Further, as  $r_b^\ell$  increases, the wage in the optimal wage contract,  $w_b^f$ , decreases. That further implies that the payoff from the optimal wage contract,  $\Pi_{w,b}$ , increases, so that  $\psi_1$  also increases in  $r_b^\ell$ . There is therefore a fixed point problem in market equilibrium. The contracts offered affect  $r_b^\ell$ , which in turn affects the optimality of the offered contract.

Observe that  $\hat{\psi}$ , as defined in equation (1), is increasing in  $r_b^\ell$ , so is minimized when  $r_b^\ell = \underline{r}^\ell$ . Define  $\underline{\psi} = \hat{\psi}(\underline{r}^\ell)$ . Now, under Assumption 1 (ii), we have  $M \geq \frac{1-\phi}{\phi}(r^f - \underline{r}^\ell) \geq M$ , which implies that  $\underline{\psi} \geq \frac{1}{2}$ .

We begin with the following observation. Suppose the proportion of principals who offer the wage contract is  $\beta$ , so that a proportion  $1 - \beta$  offers the prohibitive contract. Fix  $\beta$  and let  $\psi$ , the rating precision vary. As  $\psi$  varies, the optimal wage in the wage contract will change, which in turn will affect  $r_b^\ell$ . We show in Lemma 4 that after taking into account all effects, the payoff to an investor from using the wage contract,  $\Pi_{w,b}$ , is strictly decreasing in  $\psi$ .

**Lemma 4** *Fix  $\beta$ , the proportion of principals who offer the wage contract. Suppose  $\psi \geq \underline{\psi}$ . Then,  $\Pi_{w,b}$  is strictly decreasing in  $\psi$ .*

We exhibit the overall market equilibrium in Proposition 2. If  $\psi$  remains below  $\underline{\psi}$ , the optimal contract offers zero wages and no restriction on actions. In other words, ratings do not play any role in the contract. As  $\psi$  increases beyond  $\underline{\psi}$ , all principals offer a wage contract over some range of  $\psi$  (so that  $\beta = 1$ ). Over another range of  $\psi$ , the proportion  $\beta$  decreases continuously from 1 to 0, and when ratings become very precise, all principals offer the prohibitive contract (so that  $\beta = 0$ ).

**Proposition 2** *In a market equilibrium, for all values of  $\psi$ , the contracts offered by investors set  $w_g^f = 0$  and have no restriction on actions if the rating is  $g$ . Further, there exist rating thresholds  $\psi_x$  and  $\psi_y$ , with  $\underline{\psi} < \psi_x < \psi_y < 1$  such that, when the rating is  $b$ :*

- (i) *If  $\psi \leq \underline{\psi}$ , the contract offered by all investors has zero wages and no restriction on actions.*
- (ii) *If  $\psi \in (\underline{\psi}, \psi_x)$ , the contract offered by all investors relies only on wages, and does not restrict the manager's action.*

- (iii) If  $\psi \in (\psi_x, \psi_y)$ , a mass of investors,  $\beta(\psi) \in (0, 1)$ , offer a contract that depends only on wages, with the remainder offering a contract that bans investment in a risky asset.
- (iv) If  $\psi > \psi_y$ , the contract offered by all investors sets wages to zero and bans the manager from investing in the risky asset.

The market equilibrium, therefore, recovers some of the features of the single-investor problem. With a good rating, no wages are offered and no action restriction is imposed. With a bad rating, when the rating precision is low (below  $\underline{\psi}$ ), with a bad rating too, all wages are set to zero and there is no restriction on actions. Conversely, when the rating precision is very high (above  $\psi_y$ ), the contract prohibits investment in a risky asset with a bad rating.

However, in contrast to Proposition 1, there are two additional regions of rating precision. In a low intermediate range (rating precision between  $\underline{\psi}$  and  $\psi_x$ ), all investors offer only a wage contract when the rating is  $b$ . At the precision  $\psi_x$ , if all investors offer a wage contract, each investor is indifferent between a wage contract and a prohibitive contract when the rating is  $b$ . However, if all investors were to switch and offer a prohibitive contract, each investor would strictly prefer a wage contract. With a prohibitive contract, the demand for the risky asset is smaller than with a wage contract. Therefore, if all investors were to offer a prohibitive contract,  $r_b^\ell$  increases, which in turn implies that at the threshold  $\psi_x$ , it is optimal for an investor to instead offer a wage contract.

Now, consider a rating precision just above  $\psi_x$ . If all other investors offer a wage contract, investor  $i$  prefers a prohibitive contract. If all other investors offer a prohibitive contract, investor  $i$  prefers a wage contract. In other words, the equilibrium features a mix of contracts, with a fraction  $\beta(\psi)$  of investors offering a wage contract and a fraction  $1 - \beta(\psi)$  offering a prohibitive contract. The fraction  $\beta(\psi)$  decreases as  $\psi$  increases, so that when the rating precision increases to  $\psi_y$ , in equilibrium all investors offer a prohibitive contract.

Note that, as in Proposition 1, in a market equilibrium too, the optimal contract offers zero wages and imposes no restriction on actions if the rating is  $g$ .

The demand for the risky bond determines its return in equilibrium. The demand (and hence the return) depend on both the state and the rating. If the rating is  $g$ , the contract offers zero wages and does not restrict actions. The manager always purchases the risky bond. If the state is  $\ell$ , the manager receives his private benefit  $m$ . If the state is  $h$ , the manager receives a zero wage regardless of action. We consider an equilibrium in which the manager takes the action preferred by the investor in this case (purchasing the risky bond). The

demand for the bond from the delegated portfolio management sector is thus 1, regardless of state.

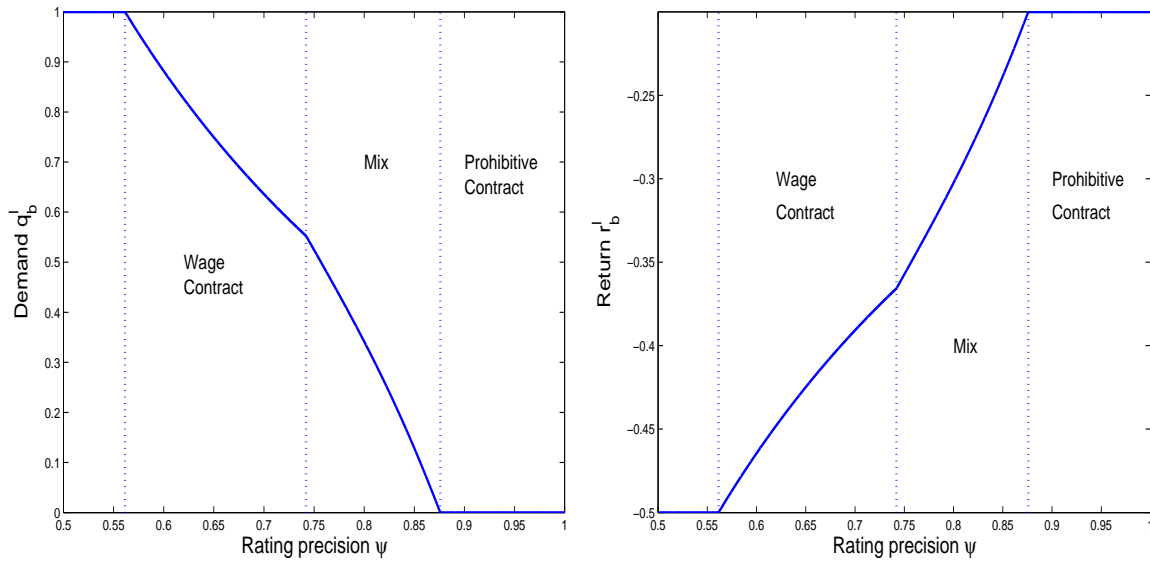
If the rating is  $b$ , the demand for the risky bond depends on  $\psi$ . For  $\psi \in (\hat{\psi}, \psi_x)$ , with wage contracts offered, the demand is 1 in the high state, and  $1 - \frac{w_b^f}{M}$  in the low state. For  $\psi \geq \psi_y$ , the demand for the risky bond is zero, as the manager is prohibited from investing in it. For  $\psi$  between  $\psi_x$  and  $\psi_y$ , the demand is  $\beta(\psi) \left(1 - \frac{w_b^f}{M}\right)$ .

We illustrate these results in the context of Example 1.

### Example 1, continued

Recall that  $\phi = 0.8$ ,  $M = 0.16$ , and  $r^f = 0$ . We set  $\bar{r}^h = 0.32$  and  $\underline{r}_h = 0.24$ , with  $r^h(\cdot)$  being linear in demand over the range  $[\underline{r}^h, \bar{r}^h]$ . Further, we set  $\bar{r}^\ell = -0.2$  and  $\underline{r}^\ell = -0.5$ , with  $r^\ell(\cdot)$  similarly being linear in demand over the range  $[\underline{r}^\ell, \bar{r}^\ell]$ . Recall that the demand for the risky asset from the DPM sector lies between 0 and 1, and is given by the mass of managers who purchase the risky asset in any given scenario.

We continue to focus on the case that the state is  $\ell$  and the rating on the risky asset is  $b$ . Figure 4 shows the equilibrium demand for the risky asset and its return as the rating precision,  $\psi$ , varies.



(a) Demand for risky asset

(b) Return on risky asset

Figure 4: **Equilibrium Demand for and Return on Risky Asset Given  $b$  Rating**

The demand for the risky asset in state  $\ell$  with rating  $b$  is shown in Figure 4 (a). As shown in the figure, in equilibrium there are four relevant regions of  $\psi$ . When  $\psi$  is low

(below about 0.56), the contract is a wage contract with zero wages. The demand for the risky asset from the DPM sector is therefore 1; all managers purchase the risky bond. In an intermediate precision range ( $\psi$  between 0.56 and 0.74), the optimal contract is a wage contract with positive wages. Because wages are increasing in  $\psi$ , the demand decreases in  $\psi$ . A third region emerges for  $\psi$  between 0.74 and 0.82. Here, some investors offer a wage contract and some investors offer the prohibitive contract. We label this region as “Mix” to indicate that a mix of contracts exists in the market. The prohibitive contract prevents investment in the risky asset, so the demand for the risky asset falls steeply in this region. Finally, for high values of  $\psi$  (above 0.82), all investors offer the prohibitive contract, so the demand for the risky asset goes to zero.

The return on the risky asset given state  $\ell$  and rating  $b$  is shown in Figure 4 (b). The return equals  $r^\ell$  for low values of  $\psi$ , and equals  $\bar{r}^\ell$  for high values of  $\psi$ . At intermediate values of  $\psi$ , it is strictly increasing in  $\psi$ .

Next, we consider the effect of an increase in rating precision on the payoffs of the investor and the manager. Noting that over the region  $[\psi_x, \psi_y]$  the investor is indifferent between offering a wage contract and a prohibitive contract, the investor’s expected payoff from an optimal contract is

$$\Pi = \begin{cases} \phi \left[ \psi r_g^h + (1 - \psi)(r_b^h - w_b^f) \right] \\ \quad + (1 - \phi) \left[ (1 - \psi)r_g^\ell + \psi \left( \frac{w_b^f}{M}(r^f - w_b^f) + \left(1 - \frac{w_b^f}{M}\right)r_b^\ell \right) \right] & \text{if } \psi \in [\underline{\psi}, \psi_x] \\ \phi(\psi r_g^h + (1 - \psi)r^f) + (1 - \phi)((1 - \psi)r_g^\ell + \phi r^f) & \text{if } \psi \geq \psi_x. \end{cases} \quad (8)$$

The expected payoff to the manager in a wage contract is

$$A_w = \phi(1 - \psi)w_b^f + (1 - \phi) \left\{ (1 - \psi)\frac{M}{2} + \psi \left( \frac{(w_b^f)^2}{M} + \left(1 - \frac{w_b^f}{M}\right)\frac{w_b^f + M}{2} \right) \right\}, \quad (9)$$

resulting an overall expected payoff to the manager of

$$A = \begin{cases} A_w & \text{if } \psi \in [\underline{\psi}, \psi_x] \\ \beta(\psi)A_w + (1 - \beta(\psi))(1 - \phi)(1 - \psi)\frac{M}{2} & \text{if } \psi \in (\psi_x, \psi_y) \\ (1 - \phi)(1 - \psi)\frac{M}{2} & \text{if } \psi \geq \psi_y. \end{cases} \quad (10)$$

We show that, as the precision of the ratings increases, the payoff to an investor unambiguously increases. At low levels of precision (just above  $\underline{\psi}$ ), the payoff to the manager also increases with  $\psi$ . However, as precision increases further, the manager’s payoff decreases as

$\psi$  goes up. In this range, the rating acts like a device to transfer utility from the manager to the investor.

**Proposition 3** *Suppose that  $\psi \geq \underline{\psi}$ . Then, an increase in the rating precision,  $\psi$ ,*

- (i) *Strictly increases the payoff of the investor.*
- (ii) *Strictly increases the payoff of the manager over some range  $(\underline{\psi}, \psi')$ , and strictly decreases the payoff of the manager over the range  $(\psi_y, 1)$ .*
- (iii) *Strictly increases the surplus in the transaction between investor and manager over the ranges  $(\underline{\psi}, \psi')$  and  $[\psi_y, 1]$ .*

Suppose  $\psi \in [\underline{\psi}, \psi_x]$ , so that all investors offer a wage contract. An increase in the rating precision has the following effects on a manager's payoff. First, it reduces the probability of obtaining the bad rating in the high state, reducing the manager's payoff (recall that  $w_b^f > 0$  and  $w_g^f = 0$  when  $\psi > \underline{\psi}$ ). Second, it increases the manager's payoff when the state the rating is bad—the manager earns  $w_b^f$  in the high state and  $\max\{w_b^f, m\}$  in the low state, both of which increase as  $w_b^f$  increases. When  $\psi = \underline{\psi}$ , the optimal contract sets  $w_b^f = 0$ , so the first effect is not relevant. Thus, the manager's payoff strictly increases as the rating precision increases beyond  $\underline{\psi}$ . As  $\psi$  increases and correspondingly  $w_b^f$  increases, the first effect becomes more important, so that there may be a rating precision beyond which the manager's payoff in decreases as  $\psi$  increases.

Conversely, when the rating precision is greater than  $\psi_y$ , all investors offer a prohibitive contract. The manager only obtains a positive payoff if the state is low and the rating is good, so that he can invest in the risky asset and earn the private benefit  $m$ . The likelihood of attaining this payoff strictly decreases as  $\psi$  increases.

The surplus in the transaction between investor and manager is obtained by adding up their respective payoffs. This surplus of course increases over the range in which both investor's and manager's payoff is increasing. It also increases over the range in which only prohibitive contracts are offered,  $[\psi_y, 1]$ . It is possible there is an intermediate region over which the surplus decreases in  $\psi$ .

We complete our discussion of Example 1 by showing the expected payoff to the investor and the manager, and the surplus, as the precision of the credit rating changes. Figure 5 shows the various payoffs.

Recall that, in the example,  $\psi_x \approx 0.74$  and  $\psi_y \approx 0.87$ . As expected from Proposition 3, the payoff of the investor increases in  $\psi$ . In the example, the payoff of the manager increases

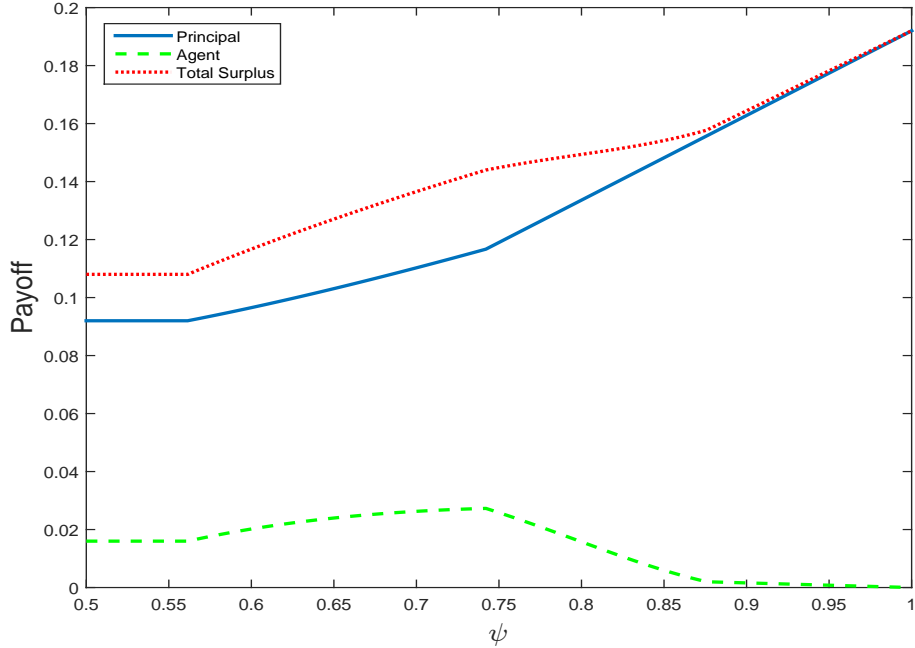


Figure 5: Investor and Manager Payoffs, and Surplus

in  $\psi$  until  $\psi = \psi_x$ , and decreases thereafter. The transaction surplus consistently increases in  $\psi$ .

## 5 Discussion

In this section, we first discuss some implications of our view that a primary role of credit ratings is to aid in writing contracts between investors and portfolio managers. We then discuss the interpretation of the verifiable signal in our model as a credit rating, and finally point out that allowing renegotiation between parties (frequently a feature in incomplete contracts) may affect the quantitative but not the qualitative results of our model.

### 5.1 Implications of the Contracting View

We now turn to the empirical implications of our model. Throughout this section, we describe our approach as the “contracting view,” and compare its implications to those resulting from the “information view,” under which credit ratings communicate new information to investors.

To begin with, we note that market price reactions in response to changes in credit ratings are similar in both approaches.

**Implication 1** *An increase (decrease) in a bond price after a credit rating upgrade is consistent with both the contracting and the information views.*

In our model, whenever  $\psi > \underline{\psi}$ , an increase in the credit rating from  $b$  to  $g$  leads to more managers buying the risky bond in state  $\ell$  regardless of the contract offered, and in state  $h$  as well if the contract is the prohibitive one. Thus, our model is consistent with the results of Tang (2009), who finds that when Moody's refined its rating system to include  $+$  and  $-$  levels, there was a response in bond prices, and Cornaggia, Cornaggia, and Israelsen (2014), who show that when Moody's revised its rating scale for municipal bonds, prices reacted accordingly. In particular, such results cannot establish by themselves that ratings contain new information.

We now turn to possible ways to distinguish between the two views.

**Implication 2** *The release of a credit rating should decrease adverse selection in the market for the stock of a firm under the information view, and leave it unchanged under the contracting view.*

Before a credit rating is released, some informed traders in a firm's stock may already know the information it contains. When that information is released publicly to the entire market, measures of adverse selection (such as, e.g., microstructure or spread-based measures) should decrease. In the contracting view, the release of a credit rating should have no effect on adverse selection in the market for a stock.

**Implication 3** *All else equal, credit ratings lead to a reduced demand for portfolio management services in the information view, and an increased demand for such services in the contracting view.*

If credit ratings contain information, releasing them makes it easier for investors to invest on their own rather than hire a portfolio manager. Conversely, if they facilitate contracting between investors and portfolio managers, widely available lead to an increased demand for asset management services.

**Implication 4** *Under the contracting view, increasing the precision of a credit rating can increase the volatility of asset returns.*

The widespread use of credit ratings in the optimal contract induces a correlation in the actions of portfolio managers. In turn, fixing the reward-for-risk state, this induces a difference



in the returns of a bond with a good rating and one with a low rating. That is, the bond return is volatile even when its fundamentals are held fixed, simply because of a noisy credit rating. In Figure 6, we show the volatility of bond returns in each state in our running Example. This volatility is computed as the standard deviation of returns in each state, across a good and bad rating.

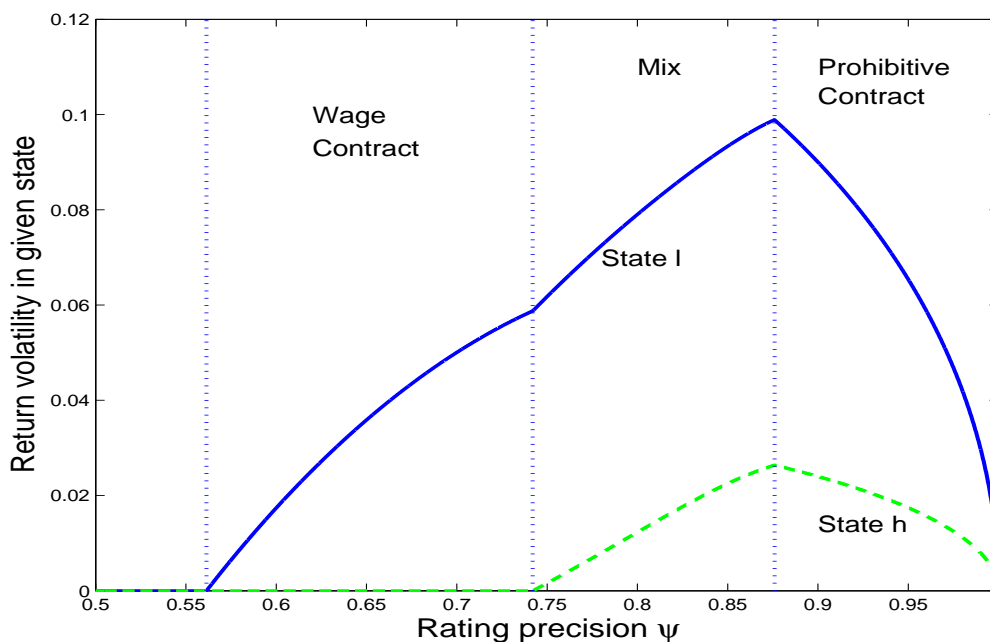


Figure 6: **Equilibrium Return Volatility**

One implication is that a welfare-maximizing regulator who can mandate (or incentivize) a precision level for ratings faces a trade-off. In the information view, all else equal, a higher rating precision should lead to higher welfare. In the contracting view, total surplus in the transaction between the investor and portfolio manager increases with rating precision. However, to the extent that a regulator also cares about market stability, it may not find it desirable to increase precision. For example, in Figure 6, if rating precision is around 0.7 or 0.75, a small further increase in precision leads to greater return volatility.

Our model implies an absence of persistence in the performance of portfolio managers. If all managers are offered the prohibitive contract (i.e., when rating precision  $\psi$  is high), all take the same action, so there is no heterogeneity in performance. If some managers are offered a wage contract, ex post (based on the realization of their private benefits), actions are heterogeneous across these managers in the scenario that the reward-for-risk is low and the rating is bad. The managers that deliver a low return in this scenario will be the ones with

a high realized private benefit. Unless private benefits are correlated through time, there is no persistence of manager performance. If, however, credit ratings communicate information to the market, managers that are skilled at generating that information on their own should exhibit persistent over-performance.

**Implication 5** *In the contracting view, if private benefits are uncorrelated through time, there is no persistence in portfolio manager performance.*

Finally, we note that the contracting approach suggests that competition between rating agencies could have unforeseen and negative consequences. If signals provide information, then having multiple signals from many providers should (through aggregation) provide more precise information about the underlying value of the asset. By contrast, if the signals are used in contracts, then having multiple signals may make the contracting problem more difficult (if different participants in the markets contract on different ratings).

## 5.2 Interpreting the Verifiable Signal as a Credit Rating

If a state cannot be directly contracted on, what properties should a contractible signal have? First, a signal should be forward-looking: For example, a firm that undertakes a new line of business may have payoffs that depend on states that were not obvious when the investor and manager agreed on the contract. Second, the signal cannot be too volatile—contracts have to be enforceable, and if signals change at too high a frequency relative to the actions of the manager, it is difficult to determine if he behaved appropriately given the contract.

Could asset prices be used in investor-manager contracts instead of credit ratings? The relationship between price levels and payoff-relevant states is tenuous, as demonstrated by the failure of asset pricing to establish price levels. Changes in prices or relative prices are potentially useful, but can be excessively volatile. One can imagine ways to control for volatility in contracts. In a CAPM world, price changes are driven by market movements in addition to idiosyncratic risk. Unless the investor strips out the systematic volatility (i.e., by also looking at the price change on an index) a contract that only looks at price changes will conflate the two sources of volatility. Further, relative prices (e.g., perhaps beta-adjusted returns) impose a significant computational burden on the contracting parties. Further, the maintained assumption has to be that the underlying firms' returns are drawn from a stationary distribution. That is, any asset pricing model (which is imperative to distinguish between systematic and idiosyncratic risk) is built on historical data.

Another set of potentially contractible signals are generated by auditors and stock analysts. Auditors verify realized cash flows, and so provide information that is inherently

backward-looking. That is, an auditor is useful to determine if the correct action was taken in the past, as opposed to specifying the correct future action. By contrast, analysts provide an assessment of future prospects. However, analysts' coverage tends to focus on equities and on larger firms. Thus, an investor using analysts recommendations has to restrict her investment universe.

Various government agencies provide information that can be used as a signal of the macro-state (e.g., non-farm payroll). These measures are somewhat less useful for use in investment contracts that concern firms or industries. First, the mapping between the macro-state and performance of a given firm is not clearly defined. Second, the government data aggregates a tremendous amount of information and the figures are frequently revised.

Credit ratings have a few characteristics that make them extremely useful in contracts. First, they are stable, in that they change relatively infrequently, and are cycle-neutral. Second, they are forward-looking, and provide a business analysis of all states relevant to investment in a particular bond. Third, most issues are rated by more than one agency. Fourth, the rating agencies do not sell investment services or actively participate in intermediation markets. Therefore they generally do not present an obvious conflict of interest such as might arise with analysts. It is interesting to note that in comparison to ratings on firms, ratings on assets owned by SPVs are less useful, because they create a conflict of interest when the deal flow is large (see Mathis, McAndrews, and Rochet, 2009).

### 5.3 Renegotiation

Suppose the state is  $\ell$  and the credit rating is  $\sigma$ . Then, a manager with a private benefit in the range  $(w_\sigma^f, r^f - r_\sigma^\ell)$  will take an inefficient action, by buying the risky bond when it would be efficient to hold the risk-less asset. Surplus can be improved if the manager can renegotiate with the investor, a possibility we have so far ignored. It is usual in incomplete contracts models to consider renegotiation, because whenever the manager is taking an inefficient action, renegotiation has the potential to increase the total surplus. Of course, ex ante it also affects how the surplus is split between the investor and the manager. In the delegated portfolio management problem, one suspects that renegotiation is infrequent. After all, an investor delegates her investment decisions because she does not want to monitor her portfolio closely.

Nevertheless, to incorporate the notion of renegotiation, consider the following amendment to the model. At time 3, given his contract and knowledge of the state and signal, the manager may renegotiate the contract. In the low state, if  $m \in (w_\sigma^f, r^f - r_\sigma^\ell)$ , the contract

does not induce the outcome that the investor wants (i.e., holding the risky bond in state  $\ell$ ), and yet the private benefit is low enough that there are gains to renegotiation. Suppose that renegotiation is costly, in the sense the opportunity to renegotiate is stochastic, and occurs with probability  $\lambda$  (so with probability  $1 - \lambda$ , there is no renegotiation). We expect  $\lambda$  to be high, for example, if the manager is a private wealth manager, and negotiates separate contracts with each of his clients. Conversely, if the manager is a bond fund manager with dispersed investors all signing the same contract,  $\lambda$  will be zero.

Suppose further that, when renegotiation is feasible, the manager has all the bargaining power. The manager makes a take-it-or-leave-it offer to the investor that specifies both the action the manager will take and a new wage contract for the manager. If the investor accepts, the old contract is torn up and the new one holds. If the investor rejects, the old contract remains in force. Any gains to trade at the renegotiation stage are therefore captured by the manager.<sup>4</sup> After any possible renegotiation, the manager invests by taking action  $a_h$  or  $a_\ell$ .

In such a set-up, the investor's payoff is not affected by the possibility of renegotiation, because the manager captures all gains from renegotiation. Thus, renegotiation has no effect on the optimal contract, on the investor's payoff, or on the decision to hire the manager. Therefore, most of our results are robust to this amendment.

However, the payoff to the manager changes—the manager now earns not just what was promised in the contract at time 1, but also captures any extra surplus he can garner from renegotiation at time 3. Therefore, an improvement in the precision of the rating may actually hurt the manager, since it reduces the need to renegotiate and therefore reduces the amount he can capture at time 3.

## 6 Conclusion

When contracts are incomplete, credit ratings have value even when they contain no new information about the issuer or the security being rated. They enable contracts to be written on a noisy signal about known but unverifiable states, improving efficiency when asset prices are given, and also increasing the payoff to the investor in the contract. However, when credit ratings are used in contracts economy-wide, there is a feedback effect leading to increased volatility of prices of risky assets.

Our model implies that credit ratings are used only because they improve contracting

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<sup>4</sup>Suppose, instead, we gave all bargaining power at the renegotiation stage to the investor. This would be equivalent to allowing the investor to write a contract *after* the state were known, going against the spirit of the idea that contracts are revised only at periodic intervals, whereas the state may change rapidly in between contract revisions.

efficiency between investors and managers. In the absence of credit ratings, managers sometimes take an inefficient action. In addition, the threat to take an inefficient action results in loss of payoff to the investors. In short, if credit ratings did not exist, investors would have to invent them. Therefore, moves such as those in the European Union in 2012 to ban the use of credit ratings are short-sighted at best.

The incomplete contracting approach suggests that credit ratings are not necessarily the most appropriate tool for investors and managers to use in contracts governing investments in structured finance vehicles. Consider either a company or a government (local or national). Using a forward looking business model, a credit rating might provide a useful summary of states in which a government might change tax or monetary policies. Further, they might provide a useful summary of states in which a company obtains refinancing or sells assets to ensure its financial solvency. However, in the case of structured finance, vehicles comprise pools of assets, for which the servicer does not take analogous actions. It therefore suggests that for these types of assets, whose quality is sunk at the time of origination and no action can be taken to improve quality or viability, an auditor or a business entity that specializes in backward-looking analysis is most appropriate.

## Appendix: Proofs

Note: The sequence of Proofs in the Appendix does not correspond to those presented in the text. The proof of Proposition 1 is presented after the proofs of Lemma 1 through Lemma 3. Expositionally, in the text, we think it is more helpful to state the main proposition in Section 3 first and then develop the supporting lemmata.

### Proof of Lemma 1

The unrestricted action set is  $\{a_h, a_\ell\}$ . Suppose the rating on the risky bond is  $\sigma$ . Any restriction reduces the set to a single feasible action, so that the manager has no choice over actions. It is immediate that it is optimal to set all wages to zero; i.e.,  $w_\sigma^h = w_\sigma^f = w_\sigma^\ell = 0$ . ■

### Proof of Lemma 2

Suppose that the credit rating on the risky bond is  $g$ . In state  $h$  the manager takes action  $a_h$  if  $w_g^h \geq w_g^f$  and action  $a_\ell$  otherwise. In state  $\ell$ , the manager takes action  $a_h$  if  $w_g^f < w_g^\ell + m$ , and action  $a_\ell$  if  $w_g^f \geq w_g^\ell + m$ . It is immediate to see that it cannot be optimal to set  $w_g^h > w_g^f$ : Reducing  $w_g^h$  to  $w_g^f$  does not change the action in either state, and strictly reduces the amount paid to the manager in the  $h$ . So,  $w_g^h \leq w_g^f$ .

Suppose  $w_g^h < w_g^f$ . Then, in state  $h$ , the manager takes action  $a_\ell$ , so that the investor's payoff in this state is  $r^f - w_g^f$ . If the investor increases  $w_g^h$  to set it equal to  $w_g^f$ , the manager switches to action  $a_h$ . The investor's payoff in this state becomes  $r_g^h - w_g^f$ . Under Assumption 1 part (ii),  $r_g^h \geq \underline{r}^h > r^f + M > r^f$ , so this strictly improves the investor's payoff. Therefore, it must be that  $w_g^h = w_g^f$ .

Now, increasing  $w_g^\ell$  above zero has two effects: (i) the probability that the manager takes the inefficient action  $a_h$  in state  $\ell$  is  $\left[1 - \frac{w_g^f - w_g^\ell}{M}\right]$  (recall that  $m$  is uniform over  $[0, M]$ ); this probability increases in  $w_g^\ell$  (ii) conditional on action  $a_h$  being taken in state  $\ell$ , the investor's payoff in that state is  $r_g^\ell - w_g^\ell$ , which decreases in  $w_g^\ell$ . Therefore, it must be optimal to set  $w_g^\ell = 0$  (i.e., for the limited liability constraint to bind when the portfolio return is  $r_g^\ell$ ).

A similar argument applies when the rating is  $b$ . ■

### Proof of Lemma 3

Consider equations (3) and (4) in the text, which show the investor's payoff in states  $h$  and  $\ell$  respectively. Using the fact that  $w_\sigma^h = w_\sigma^f$  and  $w_\sigma^\ell = 0$  for each  $\sigma$ , we can re-write these

equations as

$$\pi_h = \psi(r_g^h - w_g^f) + (1 - \psi)(r_b^h - w_b^f) \quad (11)$$

$$\pi_\ell = (1 - \psi) \left( (r^f - w_g^f) \frac{w_g^f}{M} + r_g^\ell \left(1 - \frac{w_g^f}{M}\right) \right) + \psi \left( (r^f - w_b^f) \frac{w_b^f}{M} + r_b^\ell \left(1 - \frac{w_b^f}{M}\right) \right) \quad (12)$$

The investor's payoff is  $\Pi = \phi\pi_h + (1 - \phi)\pi_\ell$ .

The first-order conditions for interior values of  $w_g^f$  and  $w_b^f$  are  $\frac{\partial \Pi}{\partial w_g^f} = 0$  and  $\frac{\partial \Pi}{\partial w_b^f} = 0$ .

Taking these derivatives, we obtain

$$-\phi\psi + (1 - \phi)(1 - \psi) \left[ (r^f - w_g^f - r_g^\ell) \frac{1}{M} - \frac{w_g^f}{M} \right] = 0 \quad (13)$$

$$-\phi(1 - \psi) + (1 - \phi)\psi \left[ (r^f - w_b^f - r_b^\ell) \frac{1}{M} - \frac{w_b^f}{M} \right] = 0. \quad (14)$$

It is straightforward to see that the second-order conditions  $\frac{\partial^2 \Pi}{\partial (w_g^f)^2} < 0$  and  $\frac{\partial^2 \Pi}{\partial (w_b^f)^2} < 0$  are satisfied.

The solution to the first-order conditions is given by

$$w_g^f = \frac{1}{2} \left( r^f - r_g^\ell - \frac{\phi}{1 - \phi} \frac{\psi}{1 - \psi} M \right) \quad (15)$$

$$w_b^f = \frac{1}{2} \left( r^f - r_b^\ell - \frac{\phi}{1 - \phi} \frac{1 - \psi}{\psi} M \right) .. \quad (16)$$

Notice that the expression for  $w_g^f$  is decreasing in  $\psi$  while  $w_b^f$  is increasing in  $\psi$ . Further, recall that  $\psi \geq \frac{1}{2}$ . We now determine when the optimal wage contract is given by the solution to the first order conditions.

Suppose that  $\psi = \frac{1}{2}$ , the case of a completely uninformative rating. The solution to the first-order condition given rating  $\sigma$  is  $w_\sigma^f = \frac{1}{2} \left( r^f - r_\sigma^\ell - \frac{\phi}{1 - \phi} M \right)$ . From Assumption 1 part (ii),  $M \geq \frac{1 - \phi}{\phi} (r^f - \underline{r}^\ell)$ . By definition,  $r_g^\ell$  and  $r_b^\ell$  are each weakly greater than  $\underline{r}^\ell$ , it follows that the wage that satisfies the first-order condition has the property that  $w_\sigma^f \leq 0$  for  $\psi = \frac{1}{2}$ .

(i) Consider the wage offered when the rating is  $g$ . At  $\psi = \frac{1}{2}$ , the solution to the first order condition is a negative wage. Because the manager enjoys limited liability, the optimal wage at  $\psi = \frac{1}{2}$  is  $w_g^f = 0$ . Further, the solution to the first-order condition, decreases in  $\psi$ . Thus, for all  $\psi > \frac{1}{2}$ , the limited liability constraint binds and we have  $w_g^f = 0$ .

(ii) Now, consider the wage offered when the rating is  $b$ . As argued above, at  $\psi = \frac{1}{2}$ , we have  $w_b^f = 0$ . However, the solution to the first-order condition (16) is increasing in  $\psi$ . Whenever

that solution lies between 0 and  $M$ , it represents the optimal wage. From equation (16),  $w_b^f \geq 0$  is equivalent to the condition

$$\frac{1-\psi}{\psi} \frac{\phi}{1-\phi} M \leq r^f - r_b^\ell,$$

$$\text{Or, } \psi \geq \frac{1}{1 + \frac{1-\phi}{\phi} \frac{r^f - r_b^\ell}{M}},$$

or  $\psi \geq \hat{\psi}(r_b^\ell)$ .

Finally,  $w_b^f \leq M$ ; because when  $w_b^f = M$ , the manager always chooses the riskless asset in state  $\ell$ , so increasing the wage beyond  $M$  has no further effect on the manager's action. Thus, when  $\psi \leq \hat{\psi}$ , we have  $w_b^f = 0$  in the optimal wage contract, and when  $\psi > \hat{\psi}$ , we have  $w_b^f = \min \left\{ \frac{1}{2} \left( r^f - r_b^\ell - \frac{\phi}{1-\phi} \frac{1-\psi}{\psi} M \right), M \right\}$ . ■

### Proof of Proposition 1

(i) Suppose the rating on the risky bond is  $g$ . As shown in Lemma 3, the optimal wage contract sets  $w_g^f = 0$ , and from Lemma 2, we have in turn  $w_g^h = w_g^\ell = 0$ . Therefore, when the rating is  $g$ , it is optimal for the manager to purchase the risky bond. Restriction the action to insist that the manager purchases the risky bond has no further effect, and so the optimal contract has no restriction on action.

(ii) Suppose the rating on the risky bond is  $b$ . The investor can induce the manager to always purchase the risky bond by setting  $w_b^f = 0$ , so the only restriction on action that is meaningful to consider is banning investment in the risky bond; i.e., setting  $A_b = \{a_\ell\}$ . The payoff from the optimal wage contract is shown in equation (6) in the text, and the payoff from banning investment in the risky asset is  $r^f$  (as shown in equation (7)). The wage contract is superior to banning iff

$$\delta_b(r_b^h - w_b^f) + (1 - \delta_b) \left[ \frac{w_b^f}{M} (r^f - w_b^f - r_b^\ell) + r_b^\ell \right] \geq r^f, \quad (17)$$

where  $w_b^f$  is given by the expression in equation (5) and  $\delta_b = \frac{\phi(1-\psi)}{\phi(1-\psi) + (1-\phi)\psi}$ .

Now, consider  $\psi = \hat{\psi}$ . At this value of  $\psi$ , we know from Lemma 3 that  $w_b^f = 0$ . When  $w_b^f = 0$ , the inequality in (17) reduces to

$$\delta_b r_b^h + (1 - \delta_b) r_b^\ell \geq r^f, \quad (18)$$

$$\text{Or, } r_b^h \geq \frac{r^f}{\delta_b} - \frac{1 - \delta_b}{\delta_b} r_b^\ell = r^f + \frac{1 - \delta_b}{\delta_b} (r^f - r_b^\ell). \quad (19)$$



When  $\psi = \hat{\psi}$ , straightforward algebraic calculation shows that  $\frac{1-\delta_b}{\delta_b} = \frac{M}{r^f - r_b^\ell}$ , so equation (19) reduces to

$$r_b^h \geq r^f + M, \quad (20)$$

an inequality that has been assumed to strictly hold in Assumption 1 part (ii). Therefore, when  $\psi = \hat{\psi}$ , the investor strictly prefers an optimal wage contract to a prohibitive contract that bans investment in the risky asset.

Next, consider  $\psi = 1$ . In this case, it is clearly optimal for the investor to ban investment in the risky bond when the rating is  $b$  (because the state is  $\ell$  for sure). Using a contract that relies only on wages has one of two effects:

- (a) the optimal wage is  $M$ , and the manager always buys the riskless bond. Then, the prohibitive contract is strictly superior, because it achieves the same outcome at a lower cost (zero) for the investor, or
- (b) the optimal wage is  $w_b^f < M$ , in which case the manager sometimes buys the risky bond (when  $m > w_b^f$ ). The prohibitive contract achieves the investor's desired action for all  $m$  and at zero cost, so is again strictly superior.

Finally, going back to equation (6), take the derivative of  $\Pi_{w,b}$ , the investor's payoff from a wage contract, with respect to  $\psi$ . We have

$$\frac{\partial \Pi_{w,b}}{\partial \psi} = \frac{\partial \Pi_{w,b}}{\partial \delta_b} \frac{\partial \delta_b}{\partial \psi} + \frac{\partial \Pi_{w,b}}{\partial w_b^f} \frac{\partial w_b^f}{\partial \psi} \quad (21)$$

Whenever  $w_b^f \in (0, M)$ , the wage is chosen to satisfy  $\frac{\partial \Pi_{w,b}}{\partial w_b^f} = 0$ . Conversely, if  $w_b^f = M$ , we have  $\frac{\partial w_b^f}{\partial \psi} = 0$ . In either case,  $\frac{\partial \Pi_{w,b}}{\partial \psi} = \frac{\partial \Pi_{w,b}}{\partial \delta_b} \frac{\partial \delta_b}{\partial \psi} < 0$ , as  $\frac{\partial \Pi_{w,b}}{\partial \delta_b} > 0$  and  $\frac{\partial \delta_b}{\partial \psi} < 0$ .

Therefore,  $\Pi_{w,b}$ , the investor's payoff from a wage-only contract when the rating is  $b$ , is strictly decreasing in the rating precision  $\psi$ , whereas the payoff from banning investment in the risky asset is independent of  $\psi$ , and remains  $r^f$ . Because  $\Pi_{w,b}$  is continuous in  $\psi$ , it then follows that there exists some rating precision  $\psi_1 \in (\hat{\psi}, 1)$  such that when  $\psi \in (\hat{\psi}, \psi_1)$ , the optimal contract conditional on a rating of  $b$  relies on wages with no restriction on actions, whereas when  $\psi > \psi_1$ , the optimal contract relies on banning the manager from investing in the risky asset when the rating is  $b$ , and offering zero wages. ■

#### Proof of Lemma 4

Recall that

$$\Pi_{w,b} = \delta_b(r_b^h - w_b^f) + (1 - \delta_b) \left( \frac{w_b^f}{M}(r^f - w_b^f) + \left(1 - \frac{w_b^f}{M}\right) r_b^\ell \right). \quad (22)$$

Here,  $\delta_b, r_b^h, r_b^\ell$ , and  $w_b^f$  are functions of  $\psi$ , and  $r_b^h, r_b^\ell$  and  $w_b^f$  are functions of  $\beta$ . Fix  $\beta$ ; going forward, in the notation we suppress the dependence on  $\beta$ .

Now,

$$\frac{\partial \Pi_{w,b}}{\partial \psi} = \frac{\partial \Pi_{w,b}}{\partial \delta_b} \frac{\partial \delta_b}{\partial \psi} + \frac{\partial \Pi_{w,b}}{\partial w_b^f} \frac{\partial w_b^f}{\partial \psi} + \frac{\partial \Pi_{w,b}}{\partial r_b^h} \frac{\partial r_b^h}{\partial \psi} + \frac{\partial \Pi_{w,b}}{\partial r_b^\ell} \frac{\partial r_b^\ell}{\partial \psi}. \quad (23)$$

Consider the second term,  $\frac{\partial \Pi_{w,b}}{\partial w_b^f} \frac{\partial w_b^f}{\partial \psi}$ . For  $\psi \geq \psi_x$ , there are two possibilities: (i)  $w_b^f$  satisfies the first-order condition for optimal wages in the principal's problem, in which case  $\frac{\partial \Pi_{w,b}}{\partial w_b^f} = 0$ , or (ii)  $w_b^f = M$ , in which case  $\frac{\partial w_b^f}{\partial \psi} = 0$ . Therefore, the second term is equal to zero. Further, when  $\beta$  is fixed, the demand for the risky bond in the high state is  $q_b^h = \beta$ , which is invariant in  $\psi$ . Hence,  $\frac{\partial r_b^h}{\partial \psi} = 0$ , so that the third term is also zero. We therefore have

$$\frac{\partial \Pi_{w,b}}{\partial \psi} = \frac{\partial \Pi_{w,b}}{\partial \delta_b} \frac{\partial \delta_b}{\partial \psi} + \frac{\partial \Pi_{w,b}}{\partial r_b^\ell} \frac{\partial r_b^\ell}{\partial \psi} \quad (24)$$

Observe that  $r_b^\ell = r^\ell(q_b^\ell)$ , and in turn  $q_b^\ell = \beta \left(1 - \frac{w_b^f}{M}\right)$ . Therefore, we have  $\frac{\partial r_b^\ell}{\partial \psi} = -\frac{\beta}{M} r^{\ell'}(q_b^\ell) \frac{\partial w_b^f}{\partial \psi}$ , where  $r^{\ell'}$  denotes the derivative of  $r^\ell$  with respect to demand.

Further, if  $w_b^f = M$ , then  $\frac{\partial w_b^f}{\partial \psi} = 0$ . Instead, if  $w_b^f$  satisfies the first-order condition in the principal's problem, then  $w_b^f = \frac{1}{2} \left( r^f - r_b^\ell - \frac{\phi}{1-\phi} \frac{1-\psi}{\psi} M \right)$ . Therefore, we have

$$\begin{aligned} \frac{\partial w_b^f}{\partial \psi} &= \frac{1}{2} \left( \frac{\beta}{M} r^{\ell'} \frac{\partial w_b^f}{\partial \psi} + \frac{\phi}{1-\phi} \frac{M}{\psi^2} \right) \\ \frac{\partial w_b^f}{\partial \psi} &= \frac{\phi M}{(1-\phi)\psi^2} \frac{1}{2 - \frac{\beta r^{\ell'}}{M}} = \frac{\phi M^2}{(1-\phi)\psi^2 (2M - \beta r^{\ell'})}. \end{aligned} \quad (25)$$

Substituting the RHS of the last equation into the expression for  $\frac{\partial r_b^\ell}{\partial \psi}$  we have

$$\frac{\partial r_b^\ell}{\partial \psi} = -\frac{\phi M}{(1-\phi)\psi^2} \frac{\beta r^{\ell'}}{2M - \beta r^{\ell'}} \quad (26)$$

Now, consider the various terms on the RHS of equation (24). It is straightforward to compute  $\frac{\partial \Pi_{w,b}}{\partial \delta_b}$  and  $\frac{\partial \Pi_{w,b}}{\partial r_b^\ell}$ . Further, given  $\delta_b = \frac{\phi(1-\psi)}{\phi(1-\psi)+(1-\phi)\psi}$ , we can compute  $\frac{\partial \delta_b}{\partial \psi} = -\frac{\delta_b(1-\delta_b)}{\psi(1-\psi)}$ . Putting all this together, we have

$$\begin{aligned} \frac{\partial \Pi_{w,b}}{\partial \psi} &= -\left(r_b^h - w_b^f - \frac{w_b^f}{M}(r^f - w_b^f) - \left(1 - \frac{w_b^f}{M}\right)r_b^\ell\right) \frac{\delta_b(1-\delta_b)}{\psi(1-\psi)} \\ &\quad - (1-\delta_b) \left(1 - \frac{w_b^f}{M}\right) \frac{\phi M}{(1-\phi)\psi^2} \frac{\beta r^{\ell'}}{2M - \beta r^{\ell'}} \end{aligned} \quad (27)$$

$$\begin{aligned} &= \frac{\phi(1-\delta_b)}{(1-\phi)\psi^2} \left[ - (1-\delta_b) \left(r_b^h - w_b^f - r_b^\ell - \frac{w_b^f}{M}(r^f - w_b^f - r_b^\ell)\right) \right. \\ &\quad \left. - M \left(1 - \frac{w_b^f}{M}\right) \frac{\beta r^{\ell'}}{2M - \beta r^{\ell'}} \right]. \end{aligned} \quad (28)$$

Therefore, a sufficient condition for  $\frac{\partial \Pi_{w,b}}{\partial \psi}$  to be strictly negative is

$$-M \left(1 - \frac{w_b^f}{M}\right) \frac{\beta r^{\ell'}}{2M - \beta r^{\ell'}} < (1-\delta_b) \left(r_b^h - w_b^f - r_b^\ell - \frac{w_b^f}{M}(r^f - w_b^f - r_b^\ell)\right). \quad (29)$$

Observe that when  $w_b^f = 0$ , the LHS of equation (29) is zero, and the RHS is  $(1-\delta_b)(r_b^h - r^f) > 0$ . Thus, the inequality is trivially satisfied when  $w_b^f = 0$ .

Suppose, instead that  $w_b^f < M$ . Then, as  $\psi \geq \psi_x$ , the first-order condition for the optimal wage in the principal's problem is satisfied, so that  $w_b^f = \frac{1}{2} \left(r^f - r_b^\ell - \frac{\delta_b}{1-\delta_b} M\right)$ , or  $r^f - 2w_b^f - r_b^\ell = \frac{\delta_b}{1-\delta_b} M$ . Therefore, the RHS of (29) can be written as

$$\begin{aligned} (1-\delta_b) \left(r_b^h - w_b^f - r_b^\ell - \frac{(w_b^f)^2}{M} - \frac{\delta_b}{1-\delta_b} w_b^f\right) &= (1-\delta_b) \left(r_b^h - \frac{w_b^f}{1-\delta_b} - r_b^\ell - \frac{(w_b^f)^2}{M}\right) \\ &= -w_b^f + (1-\delta_b) \left(r_b^h - r_b^\ell - \frac{(w_b^f)^2}{M}\right) \end{aligned} \quad (30)$$

Recall that  $r^{\ell'}$  lies between  $-\infty$  and 0. The LHS of equation (29) is strictly decreasing in  $r^{\ell'}$ , and so is maximized when  $r^{\ell'} \rightarrow -\infty$ . Its maximum value, in the limit, is  $M - w_b^f$ . Therefore, equation (29) holds if

$$(1-\delta_b) \left(r_b^h - r_b^\ell - \frac{(w_b^f)^2}{M}\right) > M. \quad (31)$$

In Claim 1 below, we show that this inequality holds for all  $\psi \geq \psi_x$ . Equation (31) in turn implies equation (29), which then implies that  $\frac{\partial \Pi_{w,b}}{\partial \psi} < 0$ , so that  $\Pi_{w,b}$  is strictly decreasing in  $\psi$ .

*Claim 1* For  $\psi \geq \psi_x$ ,  $(1 - \delta_b) \left( r_b^h - r_b^\ell - \frac{(w_b^f)^2}{M} \right) > M$ .

*Proof of Claim*

First, consider  $\psi = \psi_x$ . Recall that  $\psi_x = \underline{\psi}(r^\ell)$ . At this value of  $\psi$ , the optimal wage  $w_b^f$  both satisfies the first-order condition for optimality from the principal's problem and is equal to zero. Therefore, the demand for the risky asset is 1 in both states, so that  $r_b^h = \underline{r}^h$  and  $r_b^\ell = \underline{r}^\ell$ . Setting the optimal wage to zero, we have  $\frac{1}{2} \left( r^f - \underline{r}^\ell - \frac{\delta_b}{1-\delta_b} M \right) = 0$ , so that  $\delta_b = \frac{r^f - \underline{r}^\ell}{M + r^f - \underline{r}^\ell}$ , and  $1 - \delta_b = \frac{M}{M + r^f - \underline{r}^\ell}$ .

Now, substituting in the value of  $\delta_b$ ,  $w_b^f = 0$ ,  $r_b^h = \underline{r}^h$  and  $r_b^\ell = \underline{r}^\ell$  into the LHS of equation (31), we obtain

$$\underline{r}^h - \underline{r}^\ell > M + r^f - \underline{r}^\ell, \quad \text{or,} \quad \underline{r}^h - r^f > M, \quad (32)$$

which has been assumed in Assumption 1, part (ii). Therefore, the claim holds for  $\psi = \psi_x$ .

Define the LHS of the claim to be  $\Gamma(\psi) = (1 - \delta_b) \left( r_b^h - r_b^\ell - \frac{(w_b^f)^2}{M} \right)$ . We have shown that  $\Gamma(\psi_x) > M$ . If  $\frac{\partial \Gamma}{\partial \psi} \geq 0$ , then it must be that  $\Gamma(\psi) > M$  for all  $\psi \geq \psi_x$ .

Now,  $\frac{\partial \Gamma}{\partial \psi} = - \left( r_b^h - r_b^\ell - \frac{(w_b^f)^2}{M} \right) \frac{\partial \delta_b}{\partial \psi} - (1 - \delta_b) \left( \frac{\partial r_b^h}{\partial \psi} + \frac{2w_b^f}{M} \frac{\partial w_b^f}{\partial \psi} \right)$ . Substitute in the values of  $\frac{\partial w_b^f}{\partial \psi}$  from equation (25) and  $\frac{\partial r_b^\ell}{\partial \psi}$  from equation (26), and note that  $\frac{\partial \delta_b}{\partial \psi} = -\frac{\delta_b(1-\delta_b)}{\psi(1-\psi)}$ .

Then, the condition  $\frac{\partial \Gamma}{\partial \psi} \geq 0$  is equivalent to

$$(1 - \delta_b) \left( r_b^h - r_b^\ell - \frac{(w_b^f)^2}{M} \right) \geq M \left( \frac{2w_b^f}{2M - \beta r^{\ell'}} \right). \quad (33)$$

Now, at any given value of  $\psi$ , the wage  $w_b^f$  is fixed. Consider the term  $\left( \frac{2w_b^f - \beta r^{\ell'}}{2M - \beta r^{\ell'}} \right)$  on the RHS of equation (33). This term is strictly decreasing in  $r^{\ell'}$  if  $w_b^f < M$ , and invariant in  $r^{\ell'}$  if  $w_b^f = M$ . In either case, a maximum value of  $\frac{w_b^f}{M}$  is attained when  $r^{\ell'} = 0$ . Therefore, the RHS of equation (33) is less than or equal to  $M \times \frac{w_b^f}{M} = w_b^f$ .

That is, equation (33) holds whenever  $(1 - \delta_b) \left( r_b^h - r_b^\ell - \frac{(w_b^f)^2}{M} \right) \geq w_b^f$ ; i.e., whenever  $\Gamma(\psi) \geq w_b^f$ .

Therefore, we have shown that (i)  $\Gamma(\psi_x) \geq M$ , and (ii) whenever  $\Gamma(\psi) \geq w_b^f$ , we have  $\frac{\partial \Gamma}{\partial \psi} \geq 0$ . Consider  $\psi$  increasing just above  $\psi_x$ . As  $\Gamma(\psi_x) > M > w_b^f = 0$ , we have  $\frac{\partial \Gamma}{\partial \psi} > 0$ , so

$\Gamma(\psi)$  remains strictly above  $M$ . The same argument continues to hold as  $\psi$  increases, for any value of  $\psi$ . At some value of  $\psi$ , it is possible that  $w_b^f = M$ , in which case  $\frac{\partial \Gamma}{\partial \psi} = 0$ . However, we never have  $\frac{\partial \Gamma}{\partial \psi} < 0$ , so that  $\Gamma(\psi)$  remains strictly greater than  $M$ . That is,  $\Gamma(\psi) > M$  for all  $\psi \geq \psi_x$ , proving Claim 1. ■

As argued in the paragraph before Claim 1, it now follows that, whenever  $r^{\ell'} > -\infty$ ,  $\Pi_{w,b}$  is strictly decreasing in  $\psi$  for all  $\psi \geq \psi_x$ , proving Lemma 4. ■

### Proof of Proposition 2

First, suppose the rating is  $g$ . From Proposition 1, for all  $\psi \geq \frac{1}{2}$  and for all values of  $r_g^{\ell}$ , it is optimal for each investor to offer a contract that sets  $w_g^h = w_g^f = w_g^{\ell} = 0$ , and imposes no restriction on the manager's action. Therefore, this contract remains the offered contract in a market equilibrium.

Next, consider the rating  $b$ . We consider each part of the Proposition in turn.

(i) Suppose that all investors offer a contract with  $w_b^h = w_b^f = w_b^{\ell} = 0$  and no restriction on the manager's actions. Then, in equilibrium, the demand from the DPM sector is 1, and the return is  $r^{\ell}(1) = \underline{r}^{\ell}$ . Now, from Proposition 1 part (ii), if  $\psi \leq \underline{\psi}$ , it follows that it is a best response for each investor to offer a wage contract with zero wages. Therefore, this contract prevails in a market equilibrium. For completeness, note that under Assumption 1 part (ii), it follows that  $\underline{\psi} \geq \frac{1}{2}$ .

(ii) For a generic  $\psi > \underline{\psi}$ , it is a best response for an investor to offer a wage contract if  $\Pi_{w,b} \geq \Pi_x$ , where  $\Pi_{w,b}$  is shown in equation (6) and  $\Pi_x$  in equation (7). From Lemma 4, for a fixed value of  $\beta$ , the payoff  $\Pi_{w,b}$  overall falls as  $\psi$  increases. Conversely,  $\Pi_x$  remains unchanged at  $r^f$ . Further, for any value of  $\beta$ , when  $\psi = 1$ , we have  $\Pi_{w,b} = \frac{w_b^f}{M}(r^f - w_b^f) + (1 - \frac{w_b^f}{M})r_b^{\ell} < r^f$  (the last inequality holds because  $w_b^f > 0$  and  $r_b^{\ell} < r^f$ ). That is, an investor prefers a prohibitive contract that bans investment in the risky asset.

Fix  $\beta = 1$ . Then, by the mean value theorem, there must exist some  $\psi_x$  at which  $\Pi_{w,b} = \Pi_x$  when  $w_b^f$  and  $r_b^f$  are allowed to adjust as  $\psi$  changes. That is, there exists some  $\psi_x$  such that if all other investors offer a wage contract with no restriction on actions, it is a best response for investor  $i$  to do the same. Then, for  $\psi \in (\underline{\psi}, \psi_x)$ , the contract offered in market equilibrium is a wage contract with no restriction on actions.

(iii) At  $\psi = \psi_x$ , all investors offer a contract with no restriction on actions. The total demand for the risky asset is 1 in state  $h$  (as  $w_b^h = w_b^f$ ) and in state  $\ell$  if the rating is  $g$ , and  $1 - \frac{w_b^f}{M}$  in state  $\ell$  when the rating is  $b$  (as only managers with  $m > w_b^f$  buy the risky asset).

Suppose, instead, all investors except investor  $i$  switched to offering the prohibitive contract that banned investment in the risky asset. Consider the best response of investor  $i$ . As she is infinitesimal in the market, she takes the return on the risky asset as given in each state. The demand for the risky asset from the DPM sector is zero in both states, so  $r_b^h = \bar{r}^h$  and  $r_b^\ell = \bar{r}^\ell$ . The return in both states increases, compared to the case in which all investors offer the wage contract. It is immediate that the payoff to the optimal wage contract,  $\Pi_{w,b}$ , also increases. Therefore, if all other investors switched to the prohibitive contract, investor  $i$  now strictly prefers the wage contract.

Now, consider  $\psi$  just greater than  $\psi_x$ . If all investors offer the wage contract, investor  $i$  strictly prefers the prohibitive contract. If all investors offer the prohibitive contract, investor  $i$  strictly prefers the wage contract. It follows that there exists some fraction  $\beta(\psi)$  such that if  $\beta(\psi)$  investors offer the wage contract and  $1 - \beta(\psi)$  the prohibitive contract, investor  $i$  is indifferent between the two contracts. Therefore, in the market equilibrium, both contracts are offered by some proportion of investors. It follows that  $\beta(\psi)$  must decrease in  $\psi$ ; at a fixed  $\beta(\psi)$ , as  $\psi$  increases, the wage contract becomes strictly inferior to the prohibitive contract.

(iv) Suppose all investors are using a contract that bans investment in the risky asset when the rating is  $b$ , so that  $\beta = 0$ . Then,  $r_b^\ell = \bar{r}^\ell$  and similarly  $r_b^h = \bar{r}^h$ . As argued above in part (ii), when  $\psi = 1$ , we have  $\Pi_{w,b} < \Pi_x$ . As argued in part (iii), if  $\beta = 0$ , at  $\psi = \psi_x$  we have  $\Pi_{w,b} > \Pi_x$ . Because  $\Pi_{w,b}$  decreases in  $\psi$ , there exists some  $\psi_y < 1$  such that when  $\psi \geq \psi_y$ , all investors offer a contract with zero wages which prohibits investment in the risky asset when the rating is  $b$ .

Now, fix  $\beta = 0$ . At  $\psi = \psi_x$ , we have  $\Pi_{w,b} > \Pi_x$ , and at  $\psi = \psi_y$ , we have  $\Pi_{w,b} = \Pi_x$ . As  $\Pi_{w,b}$  is decreasing in  $\psi$ , it follows that  $\psi_y > \psi_x$ . ■

### Proof of Proposition 3

(i) First, suppose  $\psi \in [\underline{\psi}, \psi_x]$ . Over this range of  $\psi$ , the investor's payoff is

$$\begin{aligned} \Pi &= \phi \left[ \psi r_g^h + (1 - \psi)(r_b^h - w_b^f) \right] \\ &\quad + (1 - \phi) \left[ (1 - \psi)r_g^\ell + \psi \left( \frac{w_b^f}{M}(r^f - w_b^f) + \left(1 - \frac{w_b^f}{M}\right)r_b^\ell \right) \right]. \end{aligned} \quad (34)$$

Using the definition of  $\Pi_{w,b}$ , the payoff from a wage contract when the rating is bad, shown in equation (6), we can write this payoff as

$$\Pi = \phi \psi r_g^h + (1 - \phi)(1 - \psi)r_g^\ell + (\phi(1 - \psi) + (1 - \phi)\psi)\Pi_{w,b}. \quad (35)$$

Then,

$$\frac{d\Pi}{d\psi} = \phi r_g^h - (1 - \phi)r_g^\ell - (2\phi - 1)\Pi_{w,b} + [\phi(1 - \psi) + (1 - \phi)\psi] \frac{d\Pi_{w,b}}{d\psi}. \quad (36)$$

This expression is non-negative if

$$\phi r_g^h - (1 - \phi)r_g^\ell \geq (2\phi - 1)\Pi_{w,b} - [\phi(1 - \psi) - (1 - \phi)\psi] \frac{d\Pi_{w,b}}{d\psi}. \quad (37)$$

We show in Lemma 4 that  $\frac{d\Pi_{w,b}}{d\psi} < 0$ . Further, in the proof of Proposition 2, we show that

$$\begin{aligned} \frac{d\Pi_{w,b}}{d\psi} = & -\frac{\phi(1 - \delta_b)}{(1 - \phi)\psi^2} \left[ (1 - \delta_b) \left( r_b^h - w_b^f - r_b^\ell - \frac{w_b^f}{M}(r^f - w_b^f - r_b^\ell) \right) \right. \\ & \left. + M \left( 1 - \frac{w_b^f}{M} \right) \frac{\beta r^{\ell'}}{2M - \beta r^{\ell'}} \right]. \end{aligned} \quad (38)$$

Consider  $\psi \in (\underline{\psi}, \psi_x)$ . Then,  $\beta = 1$ , so that the last term can be written as  $(M - w_b^f) \frac{r^{\ell'}}{M - r^{\ell'}}$ . Further,  $r^{\ell'} \leq 0$ . Observe that  $\frac{r^{\ell'}}{M - r^{\ell'}} = \frac{1}{\frac{M}{r^{\ell'}} - 1}$ . This expression is equal to 0 when  $r^{\ell'} = 0$  and goes to a limit of  $-1$  as  $r^{\ell'} \rightarrow -\infty$ . Therefore, the minimum value of  $\frac{d\Pi_{w,b}}{d\psi}$  (and hence the maximal value of the RHS of equation (37)) is attained when  $r^{\ell'} = 0$ . Making this substitution into equation (37) and simplifying, the inequality in equation (37) holds if

$$\begin{aligned} \phi r_g^h - (1 - \phi)r_g^\ell \geq & (2\phi - 1)\Pi_{w,b} \\ & + \frac{\phi(1 - \phi)}{\phi(1 - \psi) + (1 - \phi)\psi} \left( r_b^h - r^f + \left( 1 - \frac{w_b^f}{M} \right) (r^f - w_b^f - r_b^\ell) \right). \end{aligned} \quad (39)$$

Consider the RHS of equation (39). On the RHS, substitute in  $\frac{\phi(1 - \phi)}{\phi(1 - \psi) + (1 - \phi)\psi} = \delta_b \frac{1 - \phi}{1 - \psi}$  and  $\Pi_{w,b} = \delta_b(r_b^h - w_b^f) + (1 - \delta_b) \left( \frac{w_b^f}{M}(r^f - w_b^f) + (1 - \frac{w_b^f}{M})r_b^\ell \right)$ , and simplify. The RHS can be shown to be equal to:

$$+\delta_b \left( 2\phi - 1 + \frac{1 - \phi}{1 - \psi} \right) \left( 1 - \frac{w_b^f}{M} \right) (r^f - w_b^f - r_b^\ell). \quad (40)$$

Now,  $2\phi - 1 + \frac{1 - \phi}{1 - \psi} = \phi + (1 - \phi) \frac{\psi}{1 - \psi}$ . Therefore,  $\delta_b(2\phi - 1 + \frac{1 - \phi}{1 - \psi}) = \delta_b(\phi + (1 - \phi) \frac{\psi}{1 - \psi}) =$

$\phi\delta_b + \phi(1 - \delta_b) = \phi$ . The expression in (40) simplifies to

$$\phi r_b^h - (1 - \phi)r_b^\ell - \phi w^f - (1 - \phi)\frac{w^f}{M}(r^f - w^f - r_b^\ell). \quad (41)$$

Then, equation (39) may be written as

$$\begin{aligned} \phi r_g^h - (1 - \phi)r_g^\ell &\geq \phi r_b^h - (1 - \phi)r_b^\ell - \phi w^f - (1 - \phi)\frac{w^f}{M}(r^f - w^f - r_b^\ell) \\ \phi w^f + (1 - \phi)\frac{w^f}{M}(r^f - w^f - r_b^\ell) &\geq \phi(r_b^h - r_g^h) - (1 - \phi)(r_b^\ell - r_g^\ell). \end{aligned} \quad (42)$$

Now, for  $\psi \in [\underline{\psi}, \psi_x]$ ,  $r_b^h = r_g^h = \underline{r}^h$  (the manager always invests in the risky bond when the state is  $h$ ) and  $r_b^\ell \geq r_g^\ell = \underline{r}^\ell$ , with strict inequality for all  $\psi > \underline{\psi}$ . Therefore, the RHS of equation (42) is zero at  $\psi = \underline{\psi}$ , and strictly negative for  $\psi > \underline{\psi}$ . The RHS is also zero at  $\psi = \underline{\psi}$ , and strictly negative for  $\psi > \underline{\psi}$ . That is, equation (42) holds as a strict inequality for  $\psi \in (\underline{\psi}, \psi_x]$ .

In turn, this implies that  $\frac{d\Pi}{d\psi} > 0$  for  $\psi \in (\underline{\psi}, \psi_x]$ . Therefore, the investor's payoff  $\Pi$  is increasing in  $\psi$  for  $\psi$  in this range.

Now, consider  $\psi \geq \psi_x$ . For  $\psi \in [\psi_x, \psi_y]$ , each investor is indifferent between an optimal wage contract and a prohibitive contract. For  $\psi > \psi_y$ , the investor strictly prefers the prohibitive contract. Therefore, for all  $\psi \geq \psi_x$ , the investor's payoff is equal to that obtained under the prohibitive contract, and may be written as

$$\Pi = \phi(\psi r_g^h + (1 - \psi)r^f) + (1 - \phi)((1 - \psi)r_g^\ell + \psi r^f). \quad (43)$$

As  $r_g^h > r^f > r_g^\ell$ , it follows that  $\Pi$  is increasing in  $\psi$ .

(ii) Consider the manager's payoff. First, suppose  $\psi \in [\underline{\psi}, \psi_x]$ . Recall that in this region, only a wage contract is offered, and that  $w_g^f = 0$ . Then, the agent's payoff is

$$\begin{aligned} A_w &= \phi(1 - \psi)w_b^f + (1 - \phi) \left\{ (1 - \psi)\frac{M}{2} + \psi \left( \frac{w_b^f}{M}w_b^f + \left(1 - \frac{w_b^f}{M}\right) E(m \mid m \geq w_b^f) \right) \right\} \\ &= \phi(1 - \psi)w_b^f + (1 - \phi)\frac{M}{2} + (1 - \phi)\psi\frac{(w_b^f)^2}{2M}. \end{aligned} \quad (44)$$

Therefore,

$$\frac{dA_w}{d\psi} = -\phi w_b^f + (1 - \phi)\frac{(w_b^f)^2}{2M} + \left( \phi(1 - \psi) + (1 - \phi)\psi\frac{w_b^f}{M} \right) \frac{dw_b^f}{d\psi}. \quad (45)$$



Recalling that  $w_b^f = \frac{1}{2} \left( r^f - r_b^\ell - \frac{\phi}{1-\phi} \frac{1-\psi}{\psi} M \right)$ , we have  $\frac{dw_b^f}{d\psi} = \frac{\phi}{(1-\phi)\psi^2} \frac{M}{2}$ . Therefore,

$$\frac{dA_w}{d\psi} = w_b^f \left( (1-\phi) \frac{w_b^f}{2M} - \phi \left( 1 - \frac{1}{2\psi} \right) \right) + \frac{\phi^2(1-\psi)}{(1-\phi)\psi^2} \frac{M}{2}. \quad (46)$$

Now, at  $\psi = \underline{\psi}$ , we have  $w_b^f = 0$ . It is clear that at this value of  $\psi$ ,  $\frac{dA_w}{d\psi} > 0$ , so that the manager's payoff improves in  $A$ . Because the inequality is strict, it must be that  $\frac{dA_w}{d\psi} > 0$  for  $\psi$  in some range  $(\underline{\psi}, \psi')$ .

Finally, consider  $\psi \in [\psi_y, 1]$ . In this range of  $\psi$ , all investors offer the prohibitive contract. The wage is zero, and the manager earns a payoff only if the state is low and the credit rating is good, so that she can invest in the risky bond. Her payoff is therefore

$$A = (1-\phi)(1-\psi) \frac{M}{2}, \quad (47)$$

which is clearly decreasing in  $\psi$ .

(iii) The surplus in the transaction between the investor and the manager is equal to  $\Lambda = \Pi + A$ . It is immediate from above that the surplus is strictly increasing both for  $\psi \in [\underline{\psi}, \psi')$ , and in the region  $\psi \in [\psi_y, 1]$ . ■

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