Trading Costs and Informational Efficiency*

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Abstract

This paper studies the effect of trading costs on information diffusion and information acquisition in financial markets. For a given precision of investors' private information, an irrelevance result emerges when investors are ex-ante identical: the level of trading costs does not affect price informativeness or price volatility. This result holds independently of whether investors behave competitively or strategically and applies to both static and dynamic economies. When investors are ex-ante heterogeneous, trading costs reduce (increase) price informativeness if and only if investors who disproportionately trade on information are more (less) elastic than investors who mostly trade due to hedging. Trading costs always reduce information acquisition, even when price informativeness remains unchanged for a given amount of information. Our results matter to understand a) the consequences of cheaper financial trading and b) the effects of transaction taxes.

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1 Introduction

Technological advances have dramatically reduced the cost of trading in financial markets in recent history. However, can we say that this reduction in trading costs have made financial markets better at aggregating information? Has the ability to trade more cheaply encouraged information acquisition in financial markets? More broadly, what are the implications of trading costs for the aggregation and generation of information in financial markets? We seek to provide an answer to these important questions in this paper. More precisely, we carry out a novel systematic analysis of the implications of trading costs for information aggregation and endogenous information acquisition in financial markets.

In our model, investors trade for two reasons. They trade on private information, after they receive a private signal about asset payoffs, and due to a privately known hedging demand, which is stochastic and uncertain in the aggregate. The combination of trading based on private information and the aggregate uncertainty in hedging motives makes prices only partially informative. This forces investors — or any interested external observer — to solve a filtering problem to recover the information about asset payoffs aggregated by asset prices. Using this framework as the core building block, in the spirit of Modigliani and Miller (1958), we structure our paper around several irrelevance results that emerge in different canonical models of financial trading.

Our first main result is an irrelevance theorem that applies to competitive economies with ex-ante identical investors. We show that, for a given precision of investors' private signals, the level of trading costs does not affect price informativeness or price volatility. The logic behind our main result is both simple and powerful. The effect of trading costs on how prices aggregate information is a function of how the relevant signal-to-noise ratio contained in asset prices is affected. For instance, an increase in trading costs necessarily reduces the amount of trading due to information motives, reducing the informational content of prices. However, that increase in trading costs also reduces trading due to hedging needs, reducing the noise component of asset prices. When investors are ex-ante identical, the ratio of these changes — which becomes the relevant signal-to-noise ratio of the economy — remains constant. This allows us to establish our irrelevance result.

We further characterize the conditions under which changes in trading costs affect price volatility and informativeness in economies with ex-ante heterogeneous investors. Importantly, not every form of heterogeneity breaks down our irrelevance result. We show that the crucial dimension of heterogeneity through which trading costs matter is the price-elasticity of demand of the different investors. In our model, demand elasticities are functions of the precisions of investors' private signals and their risk aversion. Whenever investors trading on information are more price sensitive than investors trading for hedging reasons, we expect prices to become less informative when trading costs are higher and viceversa.

Our results highlight the importance of how economic "noise" is modeled when we are interested in understanding the effects of trading costs on information aggregation. For instance, classic noise trading, as in Grossman and Stiglitz (1980), is often modeled as an exogenous stochastic demand or supply shift, but it is often justified as standing for hedging needs of unmodeled traders. Although a classic noise trading formulation may be a useful shortcut in some contexts, it is not satisfactory when we seek to understand the effects of trading costs: it is silent on how noise traders react to changes in the level of trading costs, a form of Lucas (1976) critique. Our formulation, which explicitly models the filtering problem faced by investors from first principles, delivers substantially different results from the classic formulation.

Subsequently, we allow investors to choose how much information to acquire and show, in our benchmark model with ex-ante identical investors, that an increase in trading costs endogenously reduces the amount of information in the economy. Intuitively, high trading costs make it harder for a given investor to profit from acquiring private information. The anticipation of this reduction on the ability to profit from having better private information, induces investors to choose less precise signals. This reduction on the amount of information acquired by investors necessarily reduces price informativeness and can increase or decrease price volatility. Showing that trading costs have different implications for information aggregation and information acquisition is an important takeaway of this paper.

We return to our benchmark model without information acquisition and show that our irrelevance theorem extends to economies with a) random heterogeneous priors as a source of aggregate uncertainty, b) strategic investors, and c) multiple rounds of trade. First, we allow investors to have stochastic privately known heterogeneous priors, which are random in the aggregate. This shows that our result is robust to having other sources of aggregate uncertainty that make investors' filtering problem non-trivial, in addition to hedging. Second, we show that changes in trading costs in economies in which investors' strategic behavior matters (for instance, when there is a finite number of investors) does not affect the level of price informativeness and price volatility. Strategic behavior changes the trading sensitivities of investors but it does so symmetrically. Therefore, the logic underlying the results in the competitive model still applies. Third, we introduce an additional round of trading in the model. The trading sensitivities of forward-looking investors are again affected by the possibility of dynamic trading. In particular, we show that forward-looking investors become more sensitive to trading costs than those in the static benchmark. However, the changes in portfolio sensitivities remain symmetric and the logic underlying the results of the static model still applies.

In addition to improving the understanding of whether the secular trend of reduction in trading costs has affected the role played by financial markets in aggregating information, our results have important practical implications for the broader discussion on the effect of transaction taxes as a policy instrument. It is somewhat surprising that our irrelevance results and our directional result in the model with endogenous information acquisition have not been previously derived and that have been absent from policy discussions. Stiglitz (1989) and Summers and Summers (1989) are good examples of policy-oriented articles which would have benefited from using the results of this paper as a benchmark for policy analysis.

Related Literature This paper lies at the intersection of two major literatures. On the one hand, our results share the emphasis of the work that studies the role played by financial markets in aggregating and originating information, following Grossman (1976), Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981) and many others. From a modeling perspective, our benchmark formulation with a continuum of investors is closest to Admati (1985), and the results on endogenous information acquisition relate to Verrecchia (1982), Kyle (1989), Hellwig and Veldkamp (2009) and Van Nieuwerburgh and Veldkamp (2010), who study endogenous information acquisition without trading costs in models of financial trading. See Brunnermeier (2001), Vives (2008) and Veldkamp (2009) for thorough recent reviews of this line of work.

On the other hand, our results also relate to the body of literature that studies the effects of transaction costs/taxes on financial markets, following Constantinides (1986) and Amihud and Mendelson (1986). More recent contributions are Vayanos (1998), Lo, Mamaysky and Wang (2004), Acharya and Pedersen (2005), Garleanu (2009), Garleanu and Pedersen (2012), Abel, Eberly and Panageas (2013), and Garleanu, Panageas and Yu (2013). We refer the reader to Vayanos and Wang (2012) for a recent survey of this growing literature.

Only a handful of papers lie at the intersection of both literatures, as ours. For instance, Vives (2011) shows in a linear-quadratic market game that introducing a quadratic trading cost can be welfare improving by reducing the degree of private information acquisition. Subrahmanyam (1998) and Dow and Rahi (2000) discuss the effect of trading costs in trading models with strategic agents. The inherent asymmetry among investors embedded in these papers, as well as their use of exogenous supply/demand shocks, explains the differences between their results and ours. Budish, Cramton and Shim (2015) show that a tax on trading is a coarse instrument to reduce high frequency trading in a model with learning. In the context of bilateral trading with information acquisition but no information aggregation, Dang and Morath (2015) compare profit and transaction taxes.

Finally, on the broader set of questions regarding whether structural changes on the financial industry, as those motivated by the reducing on the cost of trading, have affected the role played by financial markets in modern economies, our work relates to Greenwood and Scharfstein (2013), Philippon (2015), Bai, Philippon and Savov (2015) and Turley (2012).

Outline Section 2 describes the environment of our benchmark model and section 3 characterizes the equilibrium of the model for the cases with ex-ante identical and ex-ante heterogeneous investors. It also illustrates the main results with a numerical example. Section 4 allows for endogenous information acquisition and section 5 extends the results to the case of random heterogeneous priors, strategic investors and dynamics. Section 6 concludes. The appendix contains derivations and proofs. The online appendix contains additional results and extensions.

2 Benchmark model: competitive investors with trading costs

As a benchmark, we initially study a competitive model of financial market trading with rational investors who receive private signals about asset payoffs and have stochastic hedging needs. Within this canonical framework, we characterize the conditions under which trading costs affect price volatility and price informativeness. Subsequently, we extend our results in multiple dimensions.

Preferences There are two dates $t = 1, 2$ and a unit measure of investors, indexed by i. Investors choose their portfolio allocation at date 1 and consume at date 2. They maximize constant absolute risk aversion (CARA) expected utility. Therefore, expected utility of investor i is given by

$$\mathbb{E}\left[U_i\left(w_{2i}\right)\right] \quad \text{with} \quad U_i\left(w_{2i}\right) = -e^{-\gamma_i w_{2i}}, \tag{1}$$

where (1) already imposes that investors consume all their terminal wealth w_{2i}. The parameter $\gamma_i > 0$ represents the coefficient of absolute risk aversion $\gamma_i \equiv -\frac{U_i''}{U_i'}$.

Investment opportunities There is a riskless asset in elastic supply with a given gross interest rate R, normalized to 1 for simplicity. There is a single risky asset with an exogenously fixed supply $Q \geq 0$. We denote the price of the risky asset at date 1 by p. This price is quoted in terms of an underlying consumption good (dollar), which acts as numeraire. We denote by q_{0i} the initial holdings of the risky asset at date 1 by investor i. Investors must hold as a whole the total supply Q, so $\int q_{0i} di = Q$. Similarly, market clearing implies that $\int q_{1i} di = Q$, where q_{1i} denotes investor i's final holdings of the risky asset. Investors face no constraints when choosing portfolios: they can borrow and short sell freely.

The per unit asset payoff at date 2 is normally distributed and denoted by θ, where

$$\theta \sim N\left(\overline{\theta}, \sigma_\theta^2\right)$$

This formulation implies that there is aggregate uncertainty about the expected asset payoff. The unconditional expected asset payoff is given by the constant $\overline{\theta} \geq 0$.

Hedging needs Every investor has a stochastic endowment at date 2, denoted by n_{2i}, which is normally distributed and potentially correlated with θ. This endowment captures the fundamental risks associated with each investor's normal economic activity. The covariance $h_i \equiv \mathbb{C}ov\left[n_{2i}, \theta\right]$ determines whether the risky asset is a good hedge for investor i (when $h_i < 0$) or not (when $h_i > 0$). At the beginning of date 1, before trading, every investor i learns the realization of his individual hedging needs h_i, which is given by

$$h_i = \delta + e_{hi},$$

where

$$\delta \sim N\left(0, \sigma_\delta^2\right) \qquad \text{and} \qquad e_{hi} \sim N\left(0, \sigma_{hi}^2\right)$$

This formulation implies that there is aggregate uncertainty about the total magnitude of hedging needs δ. The expected level of total hedging needs is zero. Without loss of generality, we normalize the initial endowment n_{1i} to zero for all investors and assume that $\mathbb{E}\left[n_{2i}\right] - \frac{\gamma_i}{2}\mathbb{V}ar\left[n_{2i}\right] = 0$.

Information structure Investors do not observe the payoff of the risky asset θ. However, every investor observes a private signal s_i about the asset payoff θ, with the following structure:

$$s_i = \theta + \varepsilon_i,$$

where

$$\varepsilon_i \sim N\left(0, \sigma_{\varepsilon i}^2\right)$$

In principle, we allow for the variance of the private signal to be different for every investor. For now, the variances $\sigma_{\varepsilon i}^2$ are a primitive of the economy.

Investors do not observe the aggregate hedging needs in the economy either. Investors only observe their own realization of the hedging need, i.e., h_i is private information of investor i.

Trading costs Investors must pay a quadratic trading cost $\frac{c}{2} \geq 0$ per share traded of the risky asset. In particular, a change in the asset holdings of the risky asset $|q_{1i} - q_{0i}|$ faces a trading cost, in terms of the numeraire, due at the same time the transaction occurs, for both the buyer and the seller of

$$\frac{c}{2} (\Delta q_{1i})^2, \tag{2}$$

where $\Delta q_{1i} \equiv q_{1i} - q_{0i}$. We model trading costs as quadratic to preserve tractability.[1] That said, a number of empirical papers find trading costs to be convex — see Lillo, Farmer and Mantegna (2003) or Engle, Robert and Jeffrey (2008).

Given our CARA utility specification, whether trading costs are rebated or not is irrelevant to determine the investors' trading behavior. To isolate the changes in behavioral responses induced by trading costs from the direct effect on the level of resources, at times we assume that trading costs are rebated lump-sum to investors. We explicitly state in the text when we adopt this approach. Whether c corresponds to the use of economic resources (a trading cost) or whether it is a transfer (a transaction tax) is irrelevant for the positive implications or our model. However, that distinction is important from a welfare perspective.

The consumption/wealth of a given investor i at $t = 2$ is given by the stochastic endowment n_{2i}, the stochastic payoff of his asset holdings $q_{1i}\theta$, and the return on the investment in the riskless asset. This includes the net purchase or sale of the risky asset $(q_{0i} - q_{1i})\, p$, the total tax liability $-\frac{c}{2} (\Delta q_{1i})^2$ and the lump-sum transfer T_{1i}. Formally, the final wealth of investor i is

$$w_{2i} = n_{2i} + q_{1i}\theta + q_{0i}p - q_{1i}p - \frac{c}{2} (\Delta q_{1i})^2 + T_{1i} \tag{3}$$

Equilibrium definition We restrict our attention to rational expectations equilibria in linear strategies. A rational expectations equilibrium in linear strategies consists of portfolio allocations q_{1i} for every investor i and a price p such that: a) investors choose q_{1i} to maximize their expected utility in (1), subject to their budget constraint (3), by submitting a linear demand in s_i, h_i and p, as postulated in equation (6), and b) the price p is such the market for the risky asset clears, that is $\int q_{1i} di = Q$.

Because we adopt a formulation with a continuum of investors as, for instance, Admati (1985), our investors do not suffer from the schizophrenia critique of Hellwig (1980).

Remark. There are four relevant dimensions of ex-ante heterogeneity among investors.

[1] It remains an open question how to solve rich equilibrium rational expectations models with linear or fixed costs. It is challenging to solve the filtering problem faced by investors when asset demands have inaction/no-trade regions. We conjecture that our irrelevance results also apply to models with linear and fixed costs of trading.

Ex-ante, investors can have different risk aversion γ_i, different initial asset holdings q_{0i}, different variance of their hedging needs σ^2_{hi} and different precision of their private signals $\sigma^2_{\varepsilon i}$. Ex-post, they will also differ on the realizations of their hedging needs h_i and their signal s_i, which are stochastic.[2]

Remark. Aggregate uncertainty on the level of stochastic hedging needs make the filtering problem non-trivial, given that there are no noise traders in the model.

The presence of aggregate stochastic hedging needs make the filtering problem non-trivial. If $\sigma^2_\delta = 0$, the equilibrium of the economy becomes fully revealing.

In order to have a meaningful filtering problem, many papers studying learning introduce an unmodeled stochastic demand shock or, equivalently, a shock to the number of shares available: this modeling approach is often referred to as having "noise traders". Allowing for noise traders in its standard form — as in Grossman and Stiglitz (1980) — is not appropriate to study the effects of trading costs. In particular, it is hard to understand in those models how the behavior of noise traders varies with the level of trading costs: this is a form of Lucas (1976) critique. Our theoretical results sharply allow us to elaborate on this remark, which we do at the end of section 3.

3 Equilibrium

First, we study the portfolio problem of an individual investor i. Subsequently, we study the equilibrium of the model with ex-ante identical investors, which allows us to introduce our first irrelevance result. Finally, we characterize the equilibrium of the model in the general case with ex-ante heterogeneous investors and qualify the conditions under which trading costs affect learning.

Investors' portfolio choice

Because of the CARA-normal structure of preferences and returns, the demand for the risky asset of every investor is given by the solution to a mean-variance problem in q_{1i}. Note that investor i knows the actual realization of his hedging needs when trading, although those realizations are not known to other investors.

In particular, investors effectively solve

$$\max_{q_{1i}} \left(\mathbb{E}\left[\theta|s_i, p\right] - \gamma_i h_i - p\right) q_{1i} - \frac{\gamma_i}{2}\mathbb{V}ar\left[\theta|s_i, p\right] q_{1i}^2 - \frac{c}{2}\left(\Delta q_{1i}\right)^2 \tag{4}$$

[2]We postpone the analysis of heterogeneous priors to section 5.1.

The first term in the objective function of investor i represents the expected payoff of holding q_{1i} units of the risky asset. The expected payoff increases with his expected value of the fundamental, $\mathbb{E}[\theta|s_i, p]$, decreases with the level of his realized hedging needs, h_i, and decreases with the price he has to pay for the risky asset, p. The second term captures the utility loss suffered by the risk-averse investor due to the uncertainty about the asset payoff. The last term represents the trading cost the investor must pay to adjust his asset holdings from q_{0i} to q_{1i}.

The first order condition of this problem yields the following demand for the risky asset

$$q_{1i} = \frac{\mathbb{E}[\theta|s_i, p] - \gamma_i h_i - p + c q_{0i}}{\gamma_i \mathbb{V}ar[\theta|s_i, p] + c} \tag{5}$$

Intuitively, investor i demands more shares of the risky asset when the expected asset payoff $\mathbb{E}[\theta|s_i, p]$ is high, when the risky asset is a good hedge ($h_i < 0$), when the price of the risky asset is low, and when the variance of risky asset $\mathbb{V}ar[\theta|s_i, p]$ is low. More risk averse investors demand fewer shares of the risky asset.

To interpret the effect of trading costs on investors demands more easily, we can rewrite equation (5) as follows

$$q_{1i} = w_i q_{0i} + (1 - w_i)\,\hat{q}_{1i},$$

where

$$\hat{q}_{1i} = \frac{\mathbb{E}[\theta|s_i, p] - \gamma_i h_i - p}{\gamma_i \mathbb{V}ar[\theta|s_i, p]} \qquad \text{and} \qquad w_i = \frac{c}{\gamma_i \mathbb{V}ar[\theta|s_i, p] + c}$$

The optimal demand for investor i when trading costs are present is given by a weighted average of their initial asset holdings q_{0i} and the hypothetical optimal portfolio demand in the absence of trading costs, that is, when $c = 0$, which we denote by $\hat{q}_{1i}$. The weight investor i puts on the initial allocation is denoted by $w_i \in [0, 1]$. This weight is an increasing function of the trading cost c and it satisfies $\lim_{c\to\infty} w_i = 1$ and $\lim_{c\to 0} w_i = 0$. Alternatively, we can write equation (5) in the form of a net demand as $q_{1i} - q_{0i} = (1 - w_i)(\hat{q}_{1i} - q_{0i})$. Trading costs proportionally reduce the net demand for the risky asset relative to the case with no trading costs, with an attenuation coefficient $1 - w_i$.

The equilibrium of the model is fully characterized by combining the portfolio decision of investors, characterized in (5), with the market clearing condition for the risky asset, after accounting for investors' filtering problem. For reference, to identify which effects arise from learning, we characterize the equilibrium of our model when investors do not learn from prices in the online appendix.

Equilibrium with ex-ante identical investors

As a benchmark, we consider the case in which all investors are ex-ante identical. That is, we assume that all investors have identical risk aversion, initial asset holdings, variance of their hedging motives and precision of the private signal. Formally, $\gamma_i = \gamma$, $q_{0i} = q_0$, $\sigma^2_{hi} = \sigma^2_h$, and $\sigma^2_{\varepsilon i} = \sigma^2_{\varepsilon}$, $\forall i$.

In the symmetric equilibrium in linear strategies that we study, we guess (and subsequently verify) that investor i's optimal portfolio takes the form

$$q_{1i} = \alpha_s s_i - \alpha_h h_i - \alpha_p p + \psi, \tag{6}$$

where α_s, α_h, α_p and ψ are positive scalars. α_s, α_h, and α_p are the demand sensitivities of investor i to his private signal, his realized hedging needs and the price, respectively. In particular, the price sensitivity α_p accounts for the pecuniary cost of acquiring the asset and for the informational content of prices.

Market clearing implies that the equilibrium price takes the form

$$p = \frac{\alpha_s}{\alpha_p}\theta - \frac{\alpha_h}{\alpha_p}\delta + \frac{\tilde{\psi}}{\alpha_p}, \tag{7}$$

where we define $\tilde{\psi} = \psi - Q$. The equilibrium asset price increases with the fundamental value of the asset θ and decreases with the aggregate hedging need in the economy δ. The last two terms in equation (7) represent the risk premium of the asset.

A relevant equilibrium object of interest is the unbiased signal of the payoff θ contained in the asset price, which we define by $\hat{p}$ as follows

$$\hat{p} = \frac{\alpha_p}{\alpha_s} p - \frac{\tilde{\psi}}{\alpha_s}, \qquad \text{where} \qquad \hat{p}|\theta \sim N\left(\theta, \left(\frac{\alpha_h}{\alpha_s}\right)^2 \sigma^2_\delta\right)$$

Because $\mathbb{E}\left[\hat{p}|\theta\right] = \theta$, $\hat{p}$ is an unbiased signal of θ in equilibrium. Therefore, the relevant measure of price informativeness in this context is given by $\sigma^2_{\hat{p}}$, defined by

$$\sigma^2_{\hat{p}} = \mathbb{V}ar\left[\hat{p}|\theta\right] = \left(\frac{\alpha_h}{\alpha_s}\right)^2 \sigma^2_\delta \tag{8}$$

This measure captures the precision of the information about fundamentals contained in the price. An external observer with no information effectively observes a (public) signal about the risky asset's payoff that corresponds to $\hat{p}$.[3] When $\mathbb{V}ar\left[\hat{p}|\theta\right] \to \infty$, observing asset prices

[3]For clarity, we abstract from production in our model. It is easy to append a production side to this model which exclusively uses asset prices as a source of information to guide production decisions. It is somewhat more involved to introduce feedback effects between real and financial markets, as discussed for instance in Bond, Edmans and Goldstein (2012). However, there is no obvious reason for why our results would not extend to that case.

does not reveal any information about the asset payoff θ. Alternatively, when $\mathbb{V}ar[\hat{p}|\theta] \to 0$, observing the asset price perfectly reveals the realization of θ. It is evident from equation (8) that if there is no aggregate risk on hedging needs, that is, $\sigma_\delta^2 = 0$, the equilibrium price is fully revealing and Grossman (1976) paradox applies.

The amount of information revealed by the price depends on how much information about the fundamental is contained in the total demand function. In turn, this informational content depends on the individual demand sensitivities to private information and to individual hedging needs. A high demand for the asset, and thus a high price, can be due to either a high asset payoff θ or a high aggregate hedging need δ. If investors only trade on their information and pay no attention to their hedging needs, i.e., $\alpha_s > 0$ and $\alpha_h = 0$, a high price reflects a high realization of the asset payoff. In fact, in this case, the price is fully revealing. If, instead, investors only trade based on their hedging needs and disregard their private signals, i.e., $\alpha_h > 0$ and $\alpha_s = 0$, a high price reflects high aggregate hedging needs. In this scenario, the price has no informational content at all. In general, investors react to both their private information and their individual hedging needs. In this context, the amount of information contained in the price depends on the ratio of the aggregate demand sensitivities to hedging needs and to private information. The higher the weight investors put on their hedging needs relative to the weight they assign to their private information, i.e., the higher $\frac{\alpha_h}{\alpha_s}$, the lower the informational content of the price, i.e, the higher $\sigma_{\hat{p}}^2$. The signal-to-noise ratio in asset prices is governed by the ratio $\frac{\alpha_h}{\alpha_s}$.[4]

Theorem 1. (Irrelevance theorem with ex-ante identical investors)

a) When investors are ex-ante identical, the price informativeness is independent of the level of trading costs. Formally, $\mathbb{V}ar[\hat{p}|\theta]$ *does not depend on* c*.*

b) When investors are ex-ante identical, asset price volatility is independent of the level of trading costs. Formally, $\mathbb{V}ar[p]$ *does not depend on* c*.*

Theorem 1 establishes the main results of the paper. Theorem 1a) shows that price informativeness is independent of the level of trading costs. Two identical economies with different levels of trading costs c will have equally informative prices. Intuitively, high trading costs make investors less willing to trade on both their private information and their hedging needs, leaving unchanged the total relative demand sensitivities to hedging and information and, consequently, the signal-to-noise ratio in asset prices. Therefore, price informativeness is not affected by changes in the level of trading costs.

[4]When we use the expression signal-to-noise ratio about θ, the word signal refers to investors' private signals s_i, while the word noise refers to the stochastic hedging needs of investors. See our remark in page 8 about the absence of conventional "noise trading" in our model.

Theorem 1b) additionally establishes that price volatility, in addition to price informativeness, is independent of the level of trading costs. Intuitively, given that the reduction on buying and selling pressures is symmetric across all investors, asset prices remain unaffected by variations in the level of trading costs. Theorem 1b) generalizes the results from the literature on trading costs without learning, which shows that asset price levels and volatilities can increase, decrease or remain constant with changes in the level of trading costs.[5]

Theorem 1 provides a natural benchmark to understand the role of trading costs on the informational efficiency of the economy: only departures from ex-ante homogeneity across investors can generate an effect of trading costs on information aggregation.

The key equilibrium object that characterizes the informativeness of the price is $\frac{\alpha_h}{\alpha_s}$. This ratio is determined by the unique real solution to the following cubic equation:[6]

$$\frac{\alpha_h}{\alpha_s} = \gamma \left(1 + \frac{\sigma_\varepsilon^2}{\sigma_\theta^2} + \left(\frac{1}{\frac{\alpha_h}{\alpha_s}} \right)^2 \frac{\sigma_\varepsilon^2}{\sigma_\delta^2} \right) \tag{9}$$

The equilibrium value of $\frac{\alpha_h}{\alpha_s}$ depends on the following set of primitives as follows:

$$\frac{\partial \left(\frac{\alpha_h}{\alpha_s} \right)}{\partial \gamma} > 0, \quad \frac{\partial \left(\frac{\alpha_h}{\alpha_s} \right)}{\partial \sigma_\varepsilon^2} > 0, \quad \frac{\partial \left(\frac{\alpha_h}{\alpha_s} \right)}{\partial \sigma_\theta^2} < 0, \quad \text{and} \quad \frac{\partial \left(\frac{\alpha_h}{\alpha_s} \right)}{\partial \sigma_\delta^2} < 0$$

As described above, the ratio $\frac{\alpha_h}{\alpha_s}$ measures the demand's relative sensitivity to information versus hedging needs. On the one hand, a high risk aversion and a high variance of the noisy signal make investors relatively more willing to trade on hedging needs as opposed to information. On the other hand, a high uncertainty regarding the asset payoff and a high uncertainty about the level of aggregate hedging needs make investors relatively more willing to trade on information, as opposed to trading for hedging needs.

The ratio $\frac{\alpha_h}{\alpha_s}$ fully characterizes the equilibrium objects $\sigma_{\hat{p}}^2$ and $\mathbb{V}ar\left[\theta | s_i, p\right]$. Similarly, the equilibrium values of the conjectured coefficients in the investors' demands are determined by

$$\alpha_s = \frac{\frac{1}{\sigma_\varepsilon^2} \mathbb{V}ar\left[\theta | s_i, p\right]}{\kappa}, \quad \alpha_h = \frac{\gamma}{\kappa}, \quad \alpha_p = \frac{1 - \frac{\frac{1}{\sigma_{\hat{p}}^2}}{\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_{\hat{p}}^2}}}{\kappa}, \quad \text{and}$$

$$\psi = \frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_{\hat{p}}^2}} \frac{\frac{1}{\sigma_\theta^2} \bar{\theta} \mathbb{V}ar\left[\theta | s_i, p\right]}{\kappa} + \frac{\frac{c}{\kappa} \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_{\hat{p}}^2}}{\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_{\hat{p}}^2}} q_0,$$

[5]It is often assumed that asset prices levels decrease and price volatility increases in the level of trading costs. See Vayanos (1998) for a first counterexample and Vayanos and Wang (2012) or Davila (2014) for more systematic analysis.

[6]The fact that equation (9) uniquely pins down $\frac{\alpha_h}{\alpha_s}$ implies that there exists a unique equilibrium in linear strategies.

where we define $\kappa \equiv \gamma \mathbb{V}ar\left[\theta | s_i, p\right] + c$. The coefficient α_s, which determines the sensitivity of the demand for the risky asset with respect to investors' private signals, is increasing in the precision of investors private signals $\frac{1}{\sigma_\varepsilon^2}$ relative the precision of the total information about the payoff θ, $\mathbb{V}ar\left[\theta | s_i, p\right]^{-1}$.

The coefficient α_h determines the sensitivity of the demand for the risky asset with respect to hedging needs. Naturally, more risk averse investors react more to their hedging needs

The coefficient α_p, which determines the sensitivity of the demand for the risky asset with respect to the asset price, features a substitution effect and an information effect. The first term in the numerator of α_p is the standard substitution effect: when prices are high, investors want to buy fewer shares. The second term in the numerator of α_p arises from the filtering problem solved by investors and tends to reduce the sensitivity of investors' demand to prices. When prices are fully uninformative, so $\sigma_{\hat{p}}^2 \to \infty$, $\frac{\partial q_{1i}}{\partial p}$ equals $-\frac{1}{\kappa}$, as in the model without learning. When prices are partially informative, investors are less sensitive to price changes, since high prices induces investors to infer that the expected asset payoff is high and viceversa. To determine the relative sensitivity of investor's demand to the asset price, what matters for α_p is the value of information contained in asset prices $\sigma_{\hat{p}}^2$ relative to the information in private signals σ_ε^2.

The coefficient ψ determines the demand for the risky asset which does not depend on private signals, prices or hedging needs. When $c \to \infty$, it is the case that $\kappa \to \infty$, but the ratio $\frac{c}{\kappa}$ tends to 1. In that case, $\lim_{c\to\infty} \psi = q_{0i}$, as we expect: investors do not trade at all. When $c \to 0$, we find that $\lim_{c\to\infty} \psi = \frac{1}{1+\frac{\sigma_\varepsilon^2}{\sigma_{\hat{p}}^2}} \frac{\frac{\bar{\theta}}{\sigma_\theta^2}}{\gamma} + \frac{\frac{\sigma_\varepsilon^2}{\sigma_{\hat{p}}^2}}{1+\frac{\sigma_\varepsilon^2}{\sigma_{\hat{p}}^2}} q_{0i}$. Hence, when there is no learning and $c \to 0$, so $\sigma_{\hat{p}}^2 \to \infty$, the unconditional component of the demand is given by the $\frac{\frac{\bar{\theta}}{\sigma_\theta^2}}{\gamma}$, as expected.

Importantly, the magnitude of α_s, α_h and α_p is directly modulated by κ, which is a measure of investors risk tolerance and trading costs. The fact that κ enters multiplicatively in all three variables is what makes the ratios $\frac{\alpha_h}{\alpha_s}$ and $\frac{\alpha_h}{\alpha_p}$ independent of the level of trading costs.

Finally, although price volatility and price informativeness are independent of c, portfolio holdings and trading volume are affected by the level of trading costs c. The equilibrium level of risky asset holdings by investor i can be exclusively written as a function of the realization of ε_i and e_{hi} as follows:

$$q_{1i} - q_{0i} = \alpha_s \varepsilon_i - \alpha_h e_{hi}$$

Because α_s and α_h are decreasing in the level of trading costs c, the level of net trading by an individual investors is decreasing in c.

The effects on total trading volume are similar. If we denote the volume of trading in this economy by $\mathcal{V}$, defined as the number of shares traded. Using a law of large numbers, we can exactly express trading volume in this economy as

$$\mathcal{V} = \frac{1}{2}\int |q_{1i} - q_{0i}|\, di = \sqrt{\frac{1}{2\pi}\left(\alpha_s^2\sigma_\varepsilon^2 + \alpha_h^2\sigma_h^2\right)}$$

Because α_s and α_h are decreasing in the level of trading costs c, the level of aggregate trading volume is decreasing in c. Formally, we show that

$$\frac{d\mathcal{V}}{dc} < 0$$

Therefore, even when price informativeness and price volatility remain unchanged, trading volume is always lower in economies with higher trading costs.

Equilibrium with ex-ante heterogeneous investors

Theorem 1 is a important benchmark to understand how trading costs affect informational efficiency. However, there are many situations in which we expect investors to be ex-ante heterogeneous. In this section, we allow for ex-ante asymmetries among investors to highlight which specific sources of heterogeneity affect price informativeness and volatility and break the irrelevance result stated in the previous section. Formally, we let γ_i, q_{0i}, σ_{hi}^2 and $\sigma_{\varepsilon i}^2$ take arbitrary values across the distribution of investors.

Given a price p, equation (5) continues to determine investor i's demand for the risky asset. In the equilibrium in linear strategies that we study, we guess and subsequently verify that the optimal portfolio of investor i takes the form

$$q_{1i} = \alpha_{si}s_i - \alpha_{hi}h_i - \alpha_{pi}p + \psi_i, \tag{10}$$

where α_{si}, α_{hi} and α_{pi} are positive scalars for every investor i and ψ_i can be positive or negative.

Market clearing implies that the equilibrium price takes the form

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\delta + \frac{\overline{\psi}}{\overline{\alpha_p}}, \tag{11}$$

where we define $\overline{\alpha_s} = \int \alpha_{si}di$, $\overline{\alpha_h} = \int \alpha_{hi}di$, $\overline{\psi} = \int \tilde{\psi}_i di$, and $\tilde{\psi}_i = \psi_i - Q$ as cross sectional averages of the individual parameters. The interpretation of equations (10) and (11) is the same as in the model with ex-ante identical investors. We define by $\hat{p} = \frac{\overline{\alpha_p}}{\overline{\alpha_s}}p - \frac{\overline{\psi}}{\overline{\alpha_s}}$ the unbiased signal of θ, which is distributed as $N\left(\theta, \sigma_{\hat{p}}^2\right)$, where

$$\sigma_{\hat{p}}^2 = \left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)^2 \sigma_\delta^2$$

Once again, $\sigma^2_{\hat{p}}$ is the relevant measure of price informativeness, which corresponds to the information about the asset payoff contained in prices. We relegate the exact characterization of the equilibrium to the appendix, but focus on the implications for the effects of trading costs on price informativeness and volatility. In theorem 2 below, we show that not every form of heterogeneity breaks down the irrelevance result we have shown in the symmetric case. In particular, heterogeneity about the variance of hedging needs or initial positions leaves price informativeness and price volatility unaffected by changes in the level of trading costs.

Theorem 2. (Conditions for irrelevance of trading costs with ex-ante heterogeneous investors)

a) When investors are ex-ante heterogeneous, price informativeness is independent of the level of trading costs if and only if $\gamma_i = \gamma, \forall i$ *and* $\sigma^2_{\varepsilon i} = \sigma^2_{\varepsilon}, \forall i$.

b) When investors are ex-ante heterogeneous, price volatility is independent of the level of trading costs if and only if $\gamma_i = \gamma, \forall i$ *and* $\sigma^2_{\varepsilon i} = \sigma^2_{\varepsilon}, \forall i$.

Theorem 2 proves that trading costs matter for price informativeness and volatility whenever they affect the relative demand's sensitivities to hedging needs versus information for different investors asymmetrically. Intuitively, varying c can only affect price informativeness when the signal-to-noise ratio of the filtering problem that investors are trying to solve is affected. In our economy, demand sensitivities are a direct function of $\kappa_i \equiv \gamma_i \mathbb{V}ar\left[\theta | s_i, p\right] + c$. Therefore, whenever γ_i and $\sigma^2_{\varepsilon i}$ are constant, demand sensitivities are identical across all investors, which leaves the signal-to-noise raise unchanged.

Our next theorem, which determines how price informativeness changes as a function of changes in the demand sensitivities of investors, further allows us to understand which type of heterogeneity matters.

Theorem 3. (Directional effects of trading costs with ex-ante heterogeneous investors)

When the difference in relative-to-the-average sensitivities between information and hedging motives for trading, $\frac{\alpha_{si}}{\overline{\alpha}_s} - \frac{\alpha_{hi}}{\overline{\alpha}_h}$*, is positively (negatively) correlated in the cross-section of investors with the demand sensitivity to asset prices in the model without learning* $\frac{1}{\kappa_i}$*, an increase in trading costs* c *increases (decreases) price informativeness. Formally, the sign of* $\frac{d\mathbb{V}ar[\hat{p}|\theta]}{dc}$ *is determined by*

$$\operatorname{sgn}\left(\frac{d\mathbb{V}ar\left[\hat{p}|\theta\right]}{dc}\right) = \operatorname{sgn}\left(\mathbb{C}ov_i\left[\frac{\alpha_{si}}{\overline{\alpha}_s} - \frac{\alpha_{hi}}{\overline{\alpha}_h}, \frac{1}{\kappa_i}\right]\right) \tag{12}$$

Theorem 3 characterizes the directional change in price informativeness caused by a change in trading costs. Higher costs can increase or decrease informativeness. The sign of $\frac{d\mathbb{V}ar[\hat{p}|\theta]}{dc}$ in equation (12) depends on $\mathbb{C}ov_i\left[\frac{\alpha_{si}}{\overline{\alpha}_s} - \frac{\alpha_{hi}}{\overline{\alpha}_h}, \frac{1}{\kappa_i}\right]$. This is the cross-sectional covariance of two terms.

The first term corresponds to the difference between relative sensitivities to private signals and relative sensitivities to hedging. The second term corresponds to the demand sensitivity of investors to prices without filtering: when $\frac{1}{\kappa_i}$ is high, investors are not too sensitivity to price changes. Intuitively, when the investors who are relatively more sensitive to information than to hedging needs, that is, those with a high $\frac{\alpha_{si}}{\alpha_s} - \frac{\alpha_{hi}}{\alpha_h}$, are also the more responsive to changes in demand, that is, $\frac{1}{\kappa_i}$ is low, we show that high trading costs reduce price informativeness and viceversa. We analyze different combinations of primitives that make equation (12) positive or negative in our numerical example.[7]

Remark. (Broader applicability of irrelevance results) Although for tractability, we have studied a CARA-Normal environment with quadratic trading costs, we expect the logic behind our results to apply to more general environments with different utility and risk specifications, as well as different forms of trading costs. In those more general economies, under appropriate symmetry conditions, a change in trading costs should not affect the informativeness of asset prices.

Comparison with standard noise trading formulations

Our irrelevance results crucially depend on the fact that all investors are symmetrically affected by the change in trading costs. Many times, for tractability, models of learning in financial markets assume that there is an ad-hoc supply/demand shock, often referred to as "noise trading". That assumption, which may be natural in other contexts, would lead us to believe that high trading costs reduce price informativeness. In that case, trading costs reduce the amount of information in asset prices since it only affects the trading of informed investors. That assumption effectively makes noise traders perfectly inelastic, and we have just shown that increasing trading costs in an economy with perfectly inelastic investors has to make prices less informative. However, the amount of noise in asset prices, given by exogenously determined noise trading, remains constant. Therefore, a classic noise trading formulation can deliver misleading results in a framework like the one studied here and is not appropriate to study the effects of trading costs.

[7] For clarity, we have opted for not expressing equation (12) as a function of primitives, although it is easy do so. Independently of the primitives of the economy, the behavior of α_{si}, α_{hi} and κ_i determines the change in price informativeness to trading costs changes.

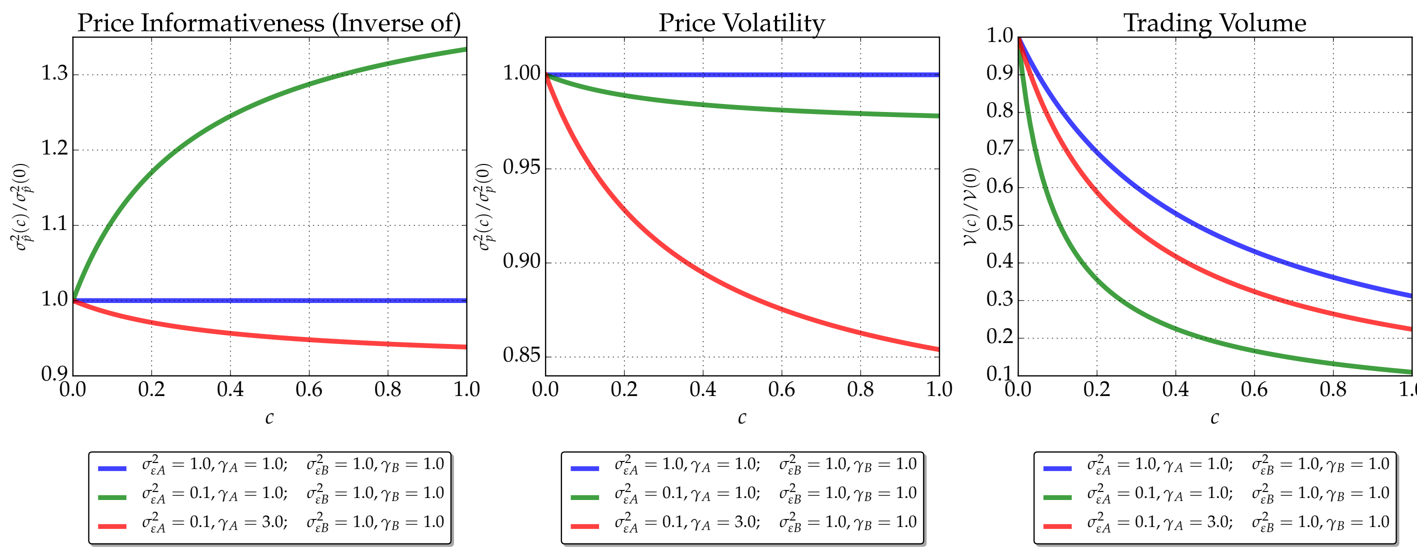

Figure 1: Comparative statics on trading costs c (relative to $c = 0$)

Numerical example

We illustrate our theoretical results with a numerical example. We assume that there are two groups of investors, which we denote by $i = A, B$, each of them accounting for one half of the total population. We assume that all investors have identically distributed hedging needs, i.e., $\sigma^2_{hi} = 1$, $\forall i$, and that they initially own a single share of the the risky asset, so $q_{0i} = Q = 1$, $\forall i$. We further normalize $\sigma^2_{\varepsilon B} = 1$ and $\gamma_B = 1$. The remaining two primitives that determine the equilibrium of the economy are $\sigma^2_{\varepsilon A}$ and γ_A, which can take different values. Table 1 illustrates our parameter choices.

	A-investors	B-investors
Precision Private Information	$\sigma^2_{\varepsilon A}$	$\sigma^2_{\varepsilon B} = 1$
Risk Aversion	γ_A	$\gamma_B = 1$

Table 1: Parameters

We compare three different parameter configurations of $\sigma^2_{\varepsilon A}$ and γ_A. First, we consider the benchmark with ex-ante identical investors and assume that $\sigma^2_{\varepsilon A} = 1$ and $\gamma_A = 1$. In that case, the irrelevance result in theorem 1 applies and $\sigma^2_{\hat{p}}$ is independent of the level of trading costs.

Second, we assume that A-investors are better informed than B-investors by reducing the variance (increasing the precision) of their private signal about the asset payoff. Specifically, we set $\sigma^2_{\varepsilon A} = 0.1$ and $\gamma_A = 1$. In this case, $\sigma^2_{\hat{p}}$ increases with the level of trading costs. With this parametrization, A-investors are more informed and more price sensitive than B-investors. Therefore, as shown in theorem 3, we expect price informativeness to be lower ($\sigma^2_{\hat{p}}$ to be higher) when trading costs increase: the reduction in trading by the more informed and more sensitive A-investors makes prices less informative.

Third, we keep the asymmetry on information precision but now we make A-investors also more risk averse. In particular, we set $\sigma^2_{\varepsilon A} = 0.1$ and $\gamma_A = 3$. In this case, A-investors are more informed and less price sensitive than B-investors at the margin. Again, using our result from theorem 1, we expect an increase in trading costs to increase price informativeness (lower $\sigma^2_{\hat{p}}$). Less informed but more sensitive B-investors disproportionally trade less, while the smaller reduction in trading by the less sensitive and better informed A-investors makes prices more informative.

Figure 1 illustrates how price informativeness, price volatility, and trading volume vary with the level of trading costs c for the different parameter combinations. To ease the interpretation of the results, we express all variables as a ratio relative to the $c = 0$ reference point. For all three parameter configurations, trading volume goes down, as expected. Interestingly, the behavior of price volatility and price informativeness differs. While price informativeness increases or decreases with the level of trading costs, price volatility always decreases with the level of c. An important takeaway of figure 1 is that price volatility and price informativenesss can evolve independently.

4 Endogenous information acquisition

Our analysis so far has treated the precision of the investors' private information as a primitive of the model. This may be a reasonable assumption in some circumstances but, in general, we expect investors to be able to decide how informed they would like to be about the asset payoff. We now allow investors to optimally choose how much information to acquire before trading. In particular, we allow investors to choose the variance/precision of the private signal they receive about the expected asset payoff.

To be able to differentiate the effects of information aggregation and information acquisition, we focus our attention on the case with ex-ante identical investors. Given our irrelevance result for information aggregation in the previous section, it is clear that, in this case, any effect of trading costs on outcomes must be driven by the endogenous choice of how much information to acquire. For now, we'll assume in this section that the risky asset is in zero net supply.

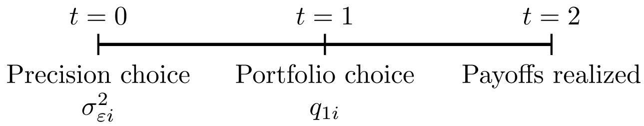

Figure 2: Timeline endogenous information acquisition

Investors' information acquisition problem

The exact timing of the investors' choices is represented in figure 2. As in the benchmark model, investors choose their portfolio allocation q_{1i} at date 1, after observing the realizations of s_i and h_i and filtering the information contained in the asset price. Now, at date 0, before knowing the realization of s_i and h_i, an investor i can choose the variance/precision of his private signal $\sigma^2_{\varepsilon i}$ at a cost $\tau\left(\frac{1}{\sigma^2_{\varepsilon i}}\right)$. Specifically, we assume that investors choose $\sigma^2_{\varepsilon i}$ to solve

$$\max_{\sigma^2_{\varepsilon i}} V\left(\sigma^2_{\varepsilon i}; \{\sigma^2_{\varepsilon j}\}_{j \neq i}\right),$$

where $V\left(\sigma^2_{\varepsilon i}; \{\sigma^2_{\varepsilon j}\}_{j \neq i}\right)$ corresponds to

$$V\left(\sigma^2_{\varepsilon i}; \{\sigma^2_{\varepsilon j}\}_{j \neq i}\right) = \mathbb{E}_i\left[w_{2i}\right] - \frac{\gamma_i}{2}\mathbb{V}ar\left[w_{2i}\right], \tag{13}$$

and where w_{2i} is given by

$$w_{2i} = n_{2i} + q_{1i}\left(\theta - p\right) + q_{0i}p - \frac{c}{2}\left(\Delta q_{1i}\right)^2 + T_{1i} - \frac{1}{2}\tau\left(\frac{1}{\sigma^2_{\varepsilon i}}\right)$$

We assume that the function $\tau(\cdot)$, which captures the cost of acquiring information, is continuous and twice differentiable and it satisfies, $\tau'(\cdot) > 0$, $\tau''(\cdot) \geq 0$ and the Inada condition $\lim_{\frac{1}{\sigma^2} \to \infty} \tau'\left(\frac{1}{\sigma^2}\right) = 0$. When simulating the model, we assume that $\tau(\cdot)$ is quadratic. Our choice of objective function implies a preference for early resolution of uncertainty, which is studied and justified in Veldkamp (2009) and Van Nieuwerburgh and Veldkamp (2010). The expected utility case delivers analogous qualitative insights.[8]

Our new equilibrium definition augments the one used in out previous section by adding optimal precision choices at date 0. As in the rest of the paper, we restrict our attention to equilibria in linear strategies.

[8]Formally, we are assuming that investors' preferences correspond to $\mathbb{E}\left[u_i\left(\mathbb{E}\left[U_i\left(w_{2i}\right)|s_i, p, h_i\right]\right)\right]$, where $U_i\left(w_{2i}\right) = -e^{-\gamma_i w_{2i}}$ and $u_i(x) = -ln(-x)$. The nonlinear transformation $u_i(x)$ introduces a preference for early resolution of uncertainty.

Taking as given the equilibrium of the model starting at date 1, an equilibrium of the model with information acquisition is given by precision choices $\sigma^2_{\varepsilon j}$ for each investor j such that each investor i maximizes $V\left(\sigma^2_{\varepsilon i}; \{\sigma^2_{\varepsilon j}\}_{j\neq i}\right)$, as defined in equation (13), given the other investors' precision choices $\{\sigma^2_{\varepsilon j}\}_{j\neq i}$. The problem solved by investor i to determine his best response given the precision choices of other investors can be written as

$$\max_{\sigma^2_{\varepsilon i}} \underbrace{\frac{\gamma \mathbb{V}ar\left[\theta|s_i,p\right]}{\gamma \mathbb{V}ar\left[\theta|s_i,p\right]+c}}_{\equiv \rho\left(\sigma^2_{\varepsilon i};c\right)} \underbrace{\frac{\sigma^2_m + m^2}{\gamma \mathbb{V}ar\left[\theta|s_i,p\right]}}_{\equiv V_0\left(\sigma^2_{\varepsilon i}\right)} - \tau\left(\frac{1}{\sigma^2_{\varepsilon i}}\right) \tag{14}$$

Investor i's effective objective function of, given by equation (14), has two components. The first component, which captures the value of acquiring information has two terms. The first term, which we define as $\rho\left(\sigma^2_{\varepsilon i};c\right) \in (0,1]$, is an attenuation coefficient, which varies between 0 and 1 depending on the value of the trading costs. The second term, which we define as $V_0\left(\sigma^2_{\varepsilon i}\right)$, is the indirect utility of an investor for a given level of information in an economy without trading costs when the posterior variance is $\mathbb{V}ar\left[\theta|s_i,p\right]$. m and σ^2_m are the first and second moment of the posterior distribution of θ given an information choice $\sigma^2_{\varepsilon i}$ at the beginning of date 1, before the private signals and individual hedging needs are observed. Importantly, the trading cost c only enters the problem of investor i through the attenuation term. For instance, when $c=0$, $\rho\left(\sigma^2_{\varepsilon i};c\right)=1$, and the investors' objective function becomes $V_0\left(\sigma^2_{\varepsilon i}\right) - \tau\left(\frac{1}{\sigma^2_{\varepsilon i}}\right)$. Alternatively, when $c\to\infty$, $\rho\left(\sigma^2_{\varepsilon i};c\right)\to 0$, which eliminates the value of acquiring information. The second component of the investors' objective function in equation (14) is simply the cost of acquiring information.

The presence of the attenuation coefficient implies that the benefit of acquiring information is reduced whenever $c>0$: this is crucial for our results.

Best responses and equilibrium determination

In the appendix, we show that the second order condition for the investors' problem holds. Therefore, the best response of investor i is fully characterized by the first order condition of the investor's problem in equation (14). Formally, the best response $\sigma^2_{\varepsilon i}\left(\{\sigma^2_{\varepsilon j}\}_{j\neq i}\right)$ is determined by

$$\frac{\left(1-2\gamma_i\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)-\gamma_i\left(\sigma^2_m+m^2\right)}{\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^2} = \mathbb{V}ar\left[\theta|s_i,p\right]^{-2}\tau'\left(\frac{1}{\sigma^2_{\varepsilon i}}\right) \tag{15}$$

Given our assumption of ex-ante homogeneity, since each investor is infinitesimal, two investors i and h that anticipate the same $\left\{\sigma^2_{\varepsilon j}\right\}_{j\neq i,h}$ expect the same equilibrium to be

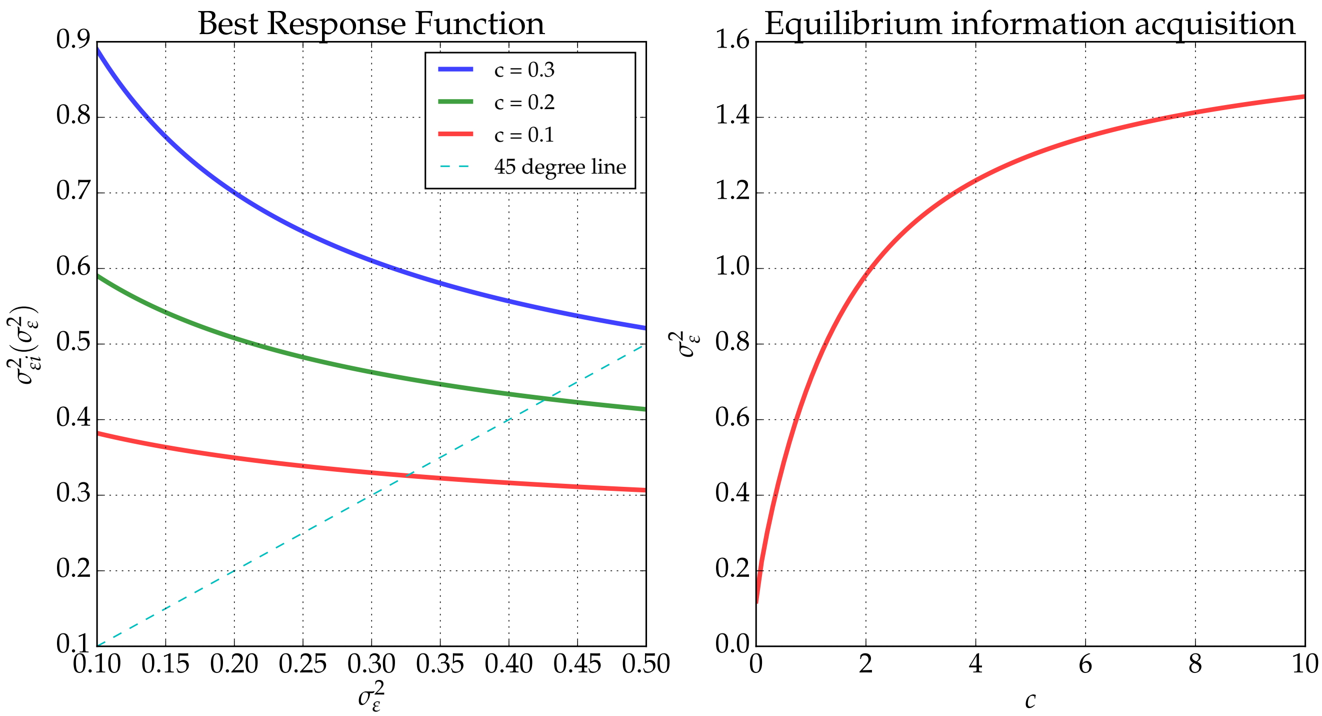

Figure 3: Best response and equilibrium comparative statics

played at date 1 and have the same posterior mean m for the same choice of information precision. In other words, they face the same problem and, thus, they will have the same best response function, i.e., $\sigma^2_{\varepsilon i}\left(\left\{\sigma^2_{\varepsilon j}\right\}_{j\neq i}\right) = \sigma^2_{\varepsilon h}\left(\left\{\sigma^2_{\varepsilon j}\right\}_{j\neq h}\right)$. Therefore, when investors are ex-ante identical, any equilibrium of the information acquisition stage has to be symmetric when investors play linear strategies in the trading stage at date 1. If all investors choose the same precision σ_ε^{-2}, the best response of investor i can be expresses as $\sigma^2_{\varepsilon i}\left(\sigma^2_\varepsilon\right)$.

In the appendix we establish the two following properties of investors' best response. These are illustrated in figure (3) using the same parameters as the benchmark model in our numerical example. First, investors acquire less information when trading costs are higher. This is not surprising given our decomposition of investors' objective function in equation (14). Second, investors acquire less information when other investors decide to be more informed. These comparative statics capture the strategic substitutability inherent to the choice of information in our model. Both results are formally stated as

$$\frac{d\sigma^2_{\varepsilon i}}{dc} > 0 \qquad \text{and} \qquad \frac{d\sigma^2_{\varepsilon i}}{d\sigma^2_\varepsilon} < 0 \tag{16}$$

These results allow to show that there exists a unique symmetric equilibrium in linear strategies. From equation (16), we can further establish that the level of information in the economy is lower when trading costs are higher. This is the main result of this section, formally stated in theorem 4.

Theorem 4. (Effect of trading costs with endogenous information acquisition)

When investors are ex-ante identical, high trading costs reduce information acquisition, which reduces price informativeness but has ambiguous effects on price volatility.

Our irrelevance results derived in the previous sections when the amount of information in the economy was fixed do not extend to situations in which investors acquire information. Intuitively, the presence of tradings costs makes acquiring information less profitable for every individual investor. In equilibrium, even though the reduction of the information acquired by everybody else in the economy due to the trading costs increases the incentives for an individual investor to acquire information, this effect is not large enough to overcome the original reduction of information precision choice caused by the higher trading cost.

Interestingly, even though a lower precision of private signals logically reduces the price informativeness of the economy, the effect on price volatility is ambiguous. More private information increases the average weight investors put on their private signals while it reduces the average weight they assign to hedging shocks. Given that $\mathbb{V}ar\left[p\right]$ takes the following form

$$\mathbb{V}ar\left[p\right] = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \sigma_\theta^2 + \left(\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2 \sigma_\delta^2,$$

the effect on total price volatility will depend on the relative magnitude of the volatilities of σ_θ^2 and σ_δ^2.

5 Extensions

We now extend our analysis to situations with random heterogeneous priors, strategic investors, and multiple rounds of trading. Introducing random heterogeneous priors shows that our main insights do not rely on aggregate hedging noise, but that aggregate uncertainty regarding the level of other motives for trading generates analogous results. Our following two extensions show that strategic behavior and dynamic considerations do not necessarily affect our irrelevance result, which remains a benchmark for more sophisticated models. This is due to the basic nature of our main argument. Needless to say, departures from homogeneity due to strategic behavior or dynamic considerations will also break our irrelevance result in a similar way as we have shown in the analysis with ex-ante heterogeneous investors in section 3.

5.1 Random heterogeneous priors

There are different ways to justify heterogeneity in priors: they may capture intrinsic differences in beliefs (optimistic versus pessimistic investors), they may be the result of having observed different private signals in the past, or, for some situations, they could even reflect different private valuations for the risky asset. We preserve the structure of the symmetric benchmark model, but introduce stochastic heterogeneous priors as follows.[9]

From the point of view of investor i, the payoff of the risky asset is distributed according to

$$\theta \sim N\left(\overline{\theta}_i, \sigma_\theta^2\right),$$

where $\overline{\theta}_i$ denotes his prior expected value for investors, which is also stochastic and distributed according to

$$\overline{\theta}_i = \overline{\theta} + u_i,$$

where

$$u_i \sim N\left(0, \sigma_{ui}^2\right) \qquad \text{and} \qquad \overline{\theta} \sim N\left(\mu_{\overline{\theta}}, \sigma_{\overline{\theta}}^2\right)$$

This formulation implies that the realized average prior mean is unknown, introducing a new source of aggregate uncertainty in addition to the aggregate hedging need and the payoff of the risky asset. Heterogeneity in the variance of stochastic heterogeneous priors σ_{ui}^2 plays a similar role as heterogeneity in the variance of hedging motives σ_{hi}^2: it does not affect the irrelevance results.

The problem solved by investors is similar to the one described for the benchmark model, and it is given by

$$\max_{q_{1i}} \left(\mathbb{E}_i\left[\theta\right] - \gamma_i h_i - p\right) q_{1i} - \frac{c}{2}\left(\Delta q_{1i}\right)^2 - \frac{\gamma}{2} \mathbb{V}ar_i\left[\theta\right] q_{1i}^2,$$

where we denote the asset payoff posterior expected mean and variance for investor i by $\mathbb{E}_i\left[\theta\right] \equiv \mathbb{E}\left[\theta | \overline{\theta}_i, s_i, p\right]$ and $\mathbb{V}ar_i\left[\theta\right] = \mathbb{V}ar_i\left[\theta | \overline{\theta}_i, s_i, p\right]$. The demand for the risky asset is again given by

$$q_{1i} = \frac{\mathbb{E}_i\left[\theta\right] - \gamma h_i - p + c q_{0i}}{\gamma \mathbb{V}ar_i\left[\theta\right] + c}$$

In a symmetric equilibrium in linear strategies, we postulate a demand function given by

$$q_{1i} = \alpha_\theta \overline{\theta_i} + \alpha_s s_i - \alpha_h h_i - \alpha_p p + \psi$$

In contrast to the benchmark model, q_{1i} also depends on the individual realization of the heterogeneous prior $\overline{\theta}_i$. Given this guess, the market clearing condition in the asset market

[9] See Scheinkman and Xiong (2003) or Davila (2014) for models with trading costs and heterogeneous priors.

implies

$$p = \frac{\alpha_\theta}{\alpha_p}\overline{\theta} + \frac{\alpha_s}{\alpha_p}\theta - \frac{\alpha_h}{\alpha_p}\delta - \frac{\tilde{\psi}}{\alpha_p},$$

where we define $\tilde{\psi} = \psi - Q$. In this case, the asset price depends on both the aggregate level of prior heterogeneity $\overline{\theta}$, and the actual payoff realization θ.

Analogously to the benchmark model, the unbiased signal of the payoff θ contained in the asset price is given by $\hat{p}$, which we define as

$$\hat{p} = \frac{\alpha_p}{\alpha_s}p - \frac{\tilde{\psi}}{\alpha_s} - \frac{\alpha_\theta}{\alpha_s}\mu_{\overline{\theta}}$$

The variance of $\hat{p}$, which we denote by $\sigma^2_{\hat{p}}$ is the relevant measure of price informativeness, and is given by

$$\sigma^2_{\hat{p}} = \mathbb{V}ar\left[\hat{p}|\theta\right] = \left(\frac{\alpha_\theta}{\alpha_s}\right)^2 \sigma^2_{\overline{\theta}} + \left(\frac{\alpha_h}{\alpha_s}\right)^2 \sigma^2_\delta$$

Unlike in the benchmark model, even if there are no trading motives due to differences in hedging needs, i.e., $\sigma^2_\delta = 0$, the price of the risky asset is not fully revealing. This occurs because there is a new source of aggregate uncertainty coming from the average level of prior heterogeneity in the economy. Therefore, as long as $\min\left\{\sigma^2_{\overline{\theta}}, \sigma^2_\delta\right\} \neq 0$, the price is not be fully revealing.

Theorem 5. (Irrelevance theorem with random heterogeneous priors)

a) When investors with random heterogeneous priors are ex-ante identical, the informativeness of asset prices is independent of the level of trading costs. Formally, $\mathbb{V}ar\left[\hat{p}|\theta\right]$ does not depend on c.

b) When investors with random heterogeneous priors are ex-ante identical, asset price volatility is independent of the level of trading costs. Formally, $\mathbb{V}ar\left[p\right]$ does not depend on c.

The logic behind theorem 5 is similar to one behind theorem 1. An increase in the level of trading costs equally distorts both the trading due to informational reasons and due to heterogeneity in priors. With ex-ante identical investors, the levels of price informativeness and volatility remain unchanged. Whether some investors heterogeneous priors are more volatile, that is, differences in σ^2_{ui} , does not affect our results.

5.2 Strategic investors

So far, we have derived our results within a perfectly competitive framework. However, we can alternatively assume a market structure in which investors behave strategically.[10] In particular,

[10] Both competitive and strategic models have served as benchmarks for studying trading in financial markets. See Brunnermeier (2001) and Vives (2008) for overviews of the work that studies strategic behavior in financial markets.

we consider the same environment as in our benchmark model, with the exception that now there is a finite number of investors N who internalize the effect of their demand on prices — their price impact. We focus on equilibria in which strategic investors submit demand functions, conditional on the price p. By introducing strategic behavior among investors, we can study the interaction between trading costs and liquidity provision more precisely.

Importantly, our analysis shows that our irrelevance results for price informativeness and price volatility apply without change to the model with strategic ex-ante identical investors. The demand for the risky asset of every investor i is given by the solution to

$$\max_{q_{1i}} \left(\mathbb{E}\left[\theta|s_i, p\right] - \gamma_i h_i - p_{-i}\right) q_{1i} - \frac{\gamma_i}{2}\mathbb{V}ar\left[\theta|s_i, p\right]\left(q_{1i}\right)^2 + p_{-i}q_{0i} - \frac{c}{2}\left(\Delta q_{1i}\right)^2,$$

where p_{-i} corresponds to the residual demand faced by investor i given the portfolio choices of all other investors.

The first order condition of this problem yields the following demand for the risky asset

$$q_{1i} = \frac{\mathbb{E}\left[\theta|s_i, p\right] - \gamma_i h_i - p + \left(c + \frac{\partial p_{-i}}{\partial q_{1i}}\right) q_{0i}}{\gamma_i \mathbb{V}ar\left[\theta|s_i, p\right] + c + \frac{\partial p_{-i}}{\partial q_{1i}}} \tag{17}$$

This expression is identical to the one in the benchmark model, with the exception of the term corresponding to $\frac{\partial p_{-i}}{\partial q_{1i}}$, which we will show is positive in equilibrium, in both the numerator and the denominator. Interestingly, the term corresponding to the price impact of investors is similar to the one corresponding to the trading cost. In fact, the term $c + \frac{\partial p_{-i}}{\partial q_{1i}}$ enters symmetrically into investors' portfolio decisions, with the caveat that $\frac{\partial p_{-i}}{\partial q_{1i}}$ is an equilibrium object while c is a primitive of the model.

Once again, in the symmetric equilibrium in linear strategies that we study, we guess and subsequently verify that the optimal portfolio of investor i takes the form

$$q_{1i} = \alpha_{si}s_i - \alpha_{hi}h_i - \alpha_{pi}p + \psi_i,$$

where α_{si}, α_{hi}, α_{pi} and ψ_i are positive scalars for every investor i. Market clearing, $\sum_{i=1}^{N} q_{1i} = Q$, implies that the equilibrium price takes the form

$$p = \frac{\sum_{i=1}^{N}\left(\alpha_{si}s_i - \alpha_{hi}h_i + \psi_i - \frac{Q}{N}\right)}{\sum_{i=1}^{N}\alpha_{pi}} \tag{18}$$

An important input for the investors' portfolio demands is the residual price elasticity, given by $\frac{\partial p_{-i}}{\partial q_{1i}}$, which takes the value

$$\frac{\partial p_{-i}}{\partial q_{1i}} = \frac{1}{\sum_{j\neq i}\alpha_{pj}} > 0$$

As in the competitive case, a relevant equilibrium object of interest is the unbiased signal of the payoff θ contained in the asset price. In the strategic case, the inference problem is more involved because investor i must discount the non-negligible effect that his signal has on the asset price. We show in the appendix that the variance of the relevant measure of price informativeness is given by

$$\sigma^2_{\hat{p}i} = \frac{\left(\alpha_h^{N-i}\right)^2 \sigma_\delta^2 + \sum_{j\neq i}\left(\frac{\alpha_{sj}}{N-1}\right)^2 \sigma_{\varepsilon j}^2 + \sum_{j\neq i}\left(\frac{\alpha_{hj}}{N-1}\right)^2 \sigma_{hj}^2}{\left(\alpha_s^{N-i}\right)^2}$$

The equilibrium is fully characterized in the appendix. In the remainder of this subsection, we focus exclusively on the effect of trading costs on price informativeness and volatility in the symmetric equilibrium of the model with ex-ante identical investors, in which $\gamma_i = \gamma$, $\sigma^2_{\varepsilon i} = \sigma^2_\varepsilon$, $\sigma^2_{hi} = \sigma^2_h$. We recover identical results to those in the competitive model stated in theorem 1. We use the price informativeness of an external observer as the relevant measure of price informativeness.

Theorem 6. (Irrelevance theorem with strategic investors)

a) When strategic investors are ex-ante identical, the informativeness of asset prices is independent of the level of trading costs. Formally, $\mathbb{V}ar\left[\hat{p}|\theta\right]$ *does not depend on* c*.*

b) When strategic investors are ex-ante identical, asset price volatility is independent of the level of trading costs. Formally, $\mathbb{V}ar\left[p\right]$ *does not depend on* c*.*

The logic behind our results is identical to the one in our benchmark model. Moving from a competitive to a strategic environment changes investor behavior. As we can see from equation (17), strategic investors trade more conservatively than competitive investors, similarly as if they were facing a trading cost. However, this change in behavior affects all investors equally. As long as investors are ex-ante identical, the price sensitivities due to information or hedging needs are symmetrically affected. Hence, any change in the level of trading costs will reduce trading due to information and hedging motives symmetrically, leaving prices unaffected.

5.3 Dynamics

Our benchmark model only considers a single round of trading. However, forward-looking investors are more sensitive to the presence of trading costs, because they anticipate that unwinding a position in the future maybe costly. This makes them more reluctant to trade. To tractably allow for multiple trading rounds within our framework, we assume that investors start with a distribution of asset holdings q_{-1}, and have the opportunity to choose portfolios both at dates 0 and 1.

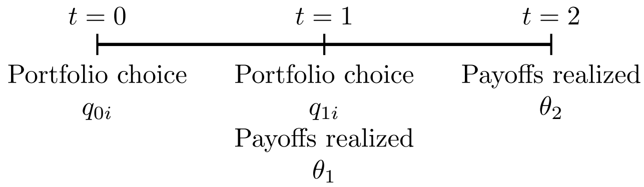

Figure 4: Timeline dynamic model

We further assume a) that investors only maximize expected utility of consumption at the final date 2 and b) that the risky asset pays dividends at dates 1 and 2, respectively denoted by θ_1 and θ_2. The structure of trading at each date is identical to the one in the benchmark model, assuming that all variables have iid realizations. We continue to focus on the model with ex-ante identical investors. The new timing is described in figure 4.

In this environment, the net worth of investors at dates 1 and 2 are respectively are given by

$$w_{2i} = n_{2i} + q_{1i}\theta_2 + w_{1i} - q_{1i}p - \frac{c}{2}\left(\Delta q_{1i}\right)^2 + T_{1i}$$
$$w_{1i} = n_{1i} + q_{0i}\left(\theta_1 + p\right) + w_{0i} - q_{-1i}p - \frac{c}{2}\left(\Delta q_{0i}\right)^2 + T_{0i}$$

The solution to the problem from date 1 onwards is identical to our benchmark model. Hence, we focus our attention on characterizing the equilibrium of the economy at date 0. As we show in the appendix, after calculating the indirect utility of investors at date 1, we can write the objective function of investors at date 0 as

$$\max_{q_{0i}} \underbrace{\mathbb{E}\left[p\right] q_{0i} - \frac{c}{2}\left(q_{0i}\right)^2}_{\text{Forward-Looking Term}} + \left(\mathbb{E}\left[\theta_1 | s_0, p_0\right] - \gamma_i h_{1i}\right) q_{0i} - p_0 \Delta q_{0i} - \frac{c}{2}\left(\Delta q_{0i}\right)^2 - \frac{\gamma_i}{2}\mathbb{V}ar\left[\theta_1 | s_0, p_0\right] q_{0i}^2$$

Investors at date 0 maximize an expression almost identical to the one at date 1, with the exception that now the face a new term incorporate the future benefits and costs of asset holdings. The additional benefit from holding an additional unit of q_{0i} is given by its expected sale price at date 1. The additional cost is determined by the trading cost level c in a quadratic way.

The first order condition to the investors problem yields the following demand for the risky asset at date 0

$$q_{0i} = \frac{\mathbb{E}\left[p\right] + \mathbb{E}\left[\theta_1 | s_0, p_0\right] - \gamma_i h_{1i} - p_0 - c q_{-1i}}{\gamma_i \mathbb{V}ar\left[\theta_1 | s_0, p_0\right] + 2c}$$

This expression is almost identical to the optimal demand at date 1, with the exception that now the level of trading costs in the denominator is effectively doubled: the forward-looking investors trade less in the risky asset, because the internalize the effect of future trading costs when they have to further buy or sell assets.

In the equilibrium in linear strategies that we study, we guess (and subsequently verify) that the optimal portfolio of investor i takes the form

$$q_{0i} = \alpha_{s0} s_i - \alpha_{h0} h_i - \alpha_{p0} p + \psi_0,$$

where α_s, α_h, α_p and ψ are positive scalars. As in our benchmark model, market clearing implies that the equilibrium price takes the form

$$p_0 = \frac{\alpha_{s0}}{\alpha_{p0}} \theta_1 - \frac{\alpha_{h0}}{\alpha_{p0}} \delta_1 + \frac{\tilde{\psi}_0}{\alpha_{p0}},$$

where we define $\tilde{\psi}_0 = \psi_0 - Q$. We again defined by $\hat{p}$ the unbiased signal of θ_1 in equilibrium. Therefore, the relevant measure of price informativeness in this context is given by $\sigma^2_{\hat{p}0}$, defined by

$$\sigma^2_{\hat{p}0} = \mathbb{V}ar\left[\hat{p}_0 | \theta\right] = \left(\frac{\alpha_{h0}}{\alpha_{s0}}\right)^2 \sigma^2_{\delta 1}$$

We can thus prove a new irrelevance theorem in this dynamic context. We provide a more general characterization in the appendix.

Theorem 7. (Irrelevance theorem in dynamic environment)

a) When investors are ex-ante identical, the informativeness of asset prices is independent of the level of trading costs. Formally, $\mathbb{V}ar\left[\hat{p}_0 | \theta_0\right]$ *and* $\mathbb{V}ar\left[\hat{p}_1 | \theta_1\right]$ *do not depend on c.*

b) When investors are ex-ante identical, asset price volatility is independent of the level of trading costs. Formally, $\mathbb{V}ar\left[p_0\right]$ *and* $\mathbb{V}ar\left[p_1\right]$ *do not depend on c.*

The same logic as in our benchmark model applies to the dynamic model. Trading costs will make investors less willing to trade, so trading volume will decrease in equilibrium when trading costs increase. However, as long as the reduction in trading is symmetric across investors trading for informational reasons and those trading for hedging reasons, price informativeness will remain unchanged.

Note that in dynamic environments, small trading costs can generate very large dramatic effects on trading volume. However, our irrelevance results apply regardless. Although, for clarity, we illustrate our results with a two-period model, it is very easy to extend our result to multiperiod dynamic economies.

6 Conclusion

This paper has provided a systematic analysis of the effects of trading costs on information aggregation and information acquisition in financial markets. A simple but powerful set of irrelevance results emerges from our analysis: when investors are ex-ante identical, changes in trading costs equally discourage trading on information and on hedging, leaving price informativeness and price volatility unchanged. This results holds when investors are competitive or strategic, and applies to both static and dynamic economies.

Although we have already explored in this paper several situations in which trading costs affect the informational role of financial markets, looking forward, there is scope to further understand which precise departures from the symmetric benchmarks that we have characterized describe the behavior of modern economies better.

Appendix

Proofs: Section 3

Investors' portfolio problem

Under the assumptions of normality and CARA utility, an investor i maximizes in q_{1i}

$$\mathbb{E}_i\left[w_{2i}\right] - \frac{\gamma_i}{2}\mathbb{V}ar\left[w_{2i}\right],$$

where w_{2i} is given by equation (3) in the text. After getting rid of constants, investor i solves

$$\max_{q_{1i}} \left(\mathbb{E}\left[\theta|s_i, p\right] - \gamma_i h_i - p\right) q_{1i} - \frac{c}{2}\left(\Delta q_{1i}\right)^2 - \frac{\gamma_i}{2}\mathbb{V}ar\left[\theta|s_i, p\right] q_{1i}^2,$$

with an optimality condition given by

$$\begin{aligned} q_{1i} &= \frac{\mathbb{E}\left[\theta|s_i, p\right] - \gamma_i h_i - p + c q_{0i}}{\gamma_i \mathbb{V}ar\left[\theta|s_i, p\right] + c} \\ &= \underbrace{\frac{c}{\gamma_i \mathbb{V}ar\left[\theta|s_i, p\right] + c}}_{w_i} q_{0i} + \underbrace{\frac{\gamma_i \mathbb{V}ar\left[\theta|s_i, p\right]}{\gamma_i \mathbb{V}ar\left[\theta|s_i, p\right] + c}}_{1-w_i} \underbrace{\frac{\mathbb{E}\left[\theta|s_i, p\right] - \gamma_i h_i - p}{\gamma_i \mathbb{V}ar\left[\theta|s_i, p\right]}}_{\hat{q}_{1i}} \\ &= w_i q_{0i} + \left(1 - w_i\right)\hat{q}_{1i} \end{aligned}$$

Note that the demand elasticity of investor i is given by $\frac{\partial q_{1i}}{\partial p} = -\frac{1}{\gamma_i \mathbb{V}ar[\theta|s_i,p]+c}$ and that we can write the net risky asset demand by investor i as

$$\Delta q_{1i} = q_{1i} - q_{0i} = \left(1 - w_i\right)\left(\hat{q}_{1i} - q_{0i}\right)$$

Equilibrium characterization with ex-ante identical investors

In a symmetric equilibrium in linear strategies in which all investors are ex-ante identical, we guess (and verify) that the optimal portfolio of investor i takes the form

$$q_{1i} = \alpha_s s_i - \alpha_h h_i - \alpha_p p + \psi, \tag{19}$$

where α_s, α_h, α_p and ψ are positive scalars. The market clearing condition $\int q_{1i} di = Q$ implies that the equilibrium price takes the form

$$p = \frac{\alpha_s}{\alpha_p}\theta - \frac{\alpha_h}{\alpha_p}\delta + \frac{\tilde{\psi}}{\alpha_p}, \tag{20}$$

where we assume a strong law of large numbers (see, for instance, the appendix of Vives (2008)) to be able to write $\int s_i di = \theta$ and $\int h_i di = \delta$, and we define $\tilde{\psi} = \psi - Q$. As described in the text, θ and δ are distributed as follows

$$\theta \sim N\left(\bar{\theta}, \sigma_\theta^2\right) \qquad \text{and} \qquad \delta \sim N\left(0, \sigma_\delta^2\right)$$

Hence, we can write the distribution of the price p as

$$p \sim N\left(\frac{\alpha_s}{\alpha_p}\bar{\theta} + \frac{\tilde{\psi}}{\alpha_p}, \left(\frac{\alpha_s}{\alpha_p}\right)^2 \sigma_\theta^2 + \left(\frac{\alpha_h}{\alpha_p}\right)^2 \sigma_\delta^2\right)$$

While the conditional distribution of the equilibrium price p given the fundamental θ follows

$$p|\theta \sim N\left(\frac{\alpha_s}{\alpha_p}\theta + \frac{\tilde{\psi}}{\alpha_p}, \left(\frac{\alpha_h}{\alpha_p}\right)^2 \sigma_\delta^2\right)$$

We denote by $\hat{p} = \frac{\alpha_p}{\alpha_s}p - \frac{\tilde{\psi}}{\alpha_s}$ the unbiased signal of θ, which is distributed as follows

$$\hat{p}|\theta \sim N\left(\theta, \sigma_{\hat{p}}^2\right), \quad \text{where} \quad \sigma_{\hat{p}}^2 = \left(\frac{\alpha_h}{\alpha_s}\right)^2 \sigma_\delta^2$$

Solving the optimal filtering problem from the perspective of investor i allows us to write

$$\mathbb{E}\left[\theta|s_i, p\right] = \mathbb{E}\left[\theta|s_i, \hat{p}\right] = \frac{\frac{1}{\sigma_\theta^2}\bar{\theta} + \frac{1}{\sigma_\varepsilon^2}s_i + \frac{1}{\sigma_{\hat{p}}^2}\hat{p}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_{\hat{p}}^2}} \quad \text{and} \quad \mathbb{V}ar\left[\theta|s_i, p\right] = \mathbb{V}ar\left[\theta|s_i, \hat{p}\right] = \frac{1}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_{\hat{p}}^2}}$$

The expected value and the variance of θ, conditional on private signals and equilibrium prices, are the inputs to the portfolio decision of investors as described in equation (5) in the text. If $\sigma_\delta^2 = 0$ the price is fully revealing and $\mathbb{V}ar\left[\theta|s_i, p\right] = 0$. In this case, investors put no weight on their private signals and base their trading behavior exclusively on the information contained in the price. However, the price contains no information at all and the Grossman (1976) paradox arises — see Vives (2008) for a textbook discussion of the implications of this paradox.

Substituting these expressions in investors' demand functions, given by equation (5), we can write q_{1i} as

$$q_{1i} = \frac{\left(\frac{1}{\sigma_\theta^2}\bar{\theta} + \frac{1}{\sigma_\varepsilon^2}s_i + \frac{1}{\sigma_{\hat{p}}^2}\left(\frac{\alpha_p p - \psi + Q}{\alpha_s}\right)\right)\mathbb{V}ar\left[\theta|s_i, p\right] - \gamma h_i - p + cq_0}{\gamma \mathbb{V}ar\left[\theta|s_i, p\right] + c}$$

To ease the notation, we define κ as

$$\kappa \equiv \gamma \mathbb{V}ar\left[\theta|s_i, p\right] + c$$

Matching coefficients with our initial guess in equation (19), we are able to characterize α_s, α_h, α_p and ψ as the solution to the following system of equations

$$\alpha_s = \frac{\frac{1}{\sigma_\varepsilon^2}\mathbb{V}ar\left[\theta|s_i, p\right]}{\kappa} \tag{21}$$

$$\alpha_h = \frac{\gamma}{\kappa} \tag{22}$$

$$\alpha_p = \frac{1 - \frac{\frac{1}{\sigma_{\hat{p}}^2}}{\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_{\hat{p}}^2}}}{\kappa} \tag{23}$$

$$\psi = \frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_{\hat{p}}^2}} \frac{\frac{1}{\sigma_\theta^2}\bar{\theta}\mathbb{V}ar\left[\theta|s_i, p\right]}{\kappa} + \frac{\frac{c}{\kappa}\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_{\hat{p}}^2}}{\frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_{\hat{p}}^2}} q_0 \tag{24}$$

Where we use the fact that $q_0 = Q$. The sensitivity of investors' demand to their own signal α_s depends on the the relative precision of their signal $\frac{1}{\sigma_\varepsilon^2}\mathbb{V}ar\left[\theta|s_i, p\right]$, corrected by the price sensitivity due to the substitution effect of price changes κ. The sensitivity to hedging needs h_i is a function of the curvature of utility γ: more risk averse investors react more to their hedging needs, also account for κ. The demand sensitivity to price changes features a substitution effect and and information effect. When prices are fully uninformative, so $\sigma_{\hat{p}}^2 \to \infty$, $-\frac{\partial q_{1i}}{\partial p}$ is $\frac{1}{\kappa}$, as in the model without learning. When prices are partially informative, investors are less sensitive to price changes, since high prices imply that the expected payoff of the risky asset is high and viceversa. To determine the relative sensitivity of investor's demand to the asset price, what matter for α_p is the value of information contained in asset prices $\sigma_{\hat{p}}^2$ relative to the information in private signals σ_ε^2.

The constant term also has a natural interpretation. On the one hand, when $c \to \infty$, $\kappa \to \infty$, but the ratio $\frac{c}{\kappa} \to 1$. In that case, $\lim_{c\to\infty} \psi = q_{0i}$, as we should expect. Arbitrarily high trading costs moves investors towards no trading at all. On the other hand, when $c \to 0$, $\lim_{c\to\infty} \psi = \frac{1}{1+\frac{\sigma_\varepsilon^2}{\sigma_{\hat{p}}^2}} \frac{\frac{\bar{\theta}}{\sigma_\theta^2}}{\gamma} + \frac{\frac{\sigma_\varepsilon^2}{\sigma_{\hat{p}}^2}}{1+\frac{\sigma_\varepsilon^2}{\sigma_{\hat{p}}^2}} q_{0i}$. When there is no learning and $c \to 0$, so $\sigma_{\hat{p}}^2 \to \infty$, the unconditional component of the demand is given by the $\frac{\frac{\bar{\theta}}{\sigma_\theta^2}}{\gamma}$, as expected.

To fully characterize the equilibrium, we must find the value of $\frac{\alpha_h}{\alpha_s}$, which uniquely determines $\sigma_{\hat{p}}^2$ and consequently $\mathbb{V}ar\left[\theta|s_i, p\right]$. From equations (21) and (22), we can write

$$\frac{\alpha_h}{\alpha_s} = \frac{\gamma}{\frac{1}{\sigma_\varepsilon^2}\mathbb{V}ar\left[\theta|s_i, p\right]}$$

Using the fact that $\mathbb{V}ar\left[\theta|s_i, p\right] = \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_{\hat{p}}^2}\right)^{-1}$ and that $\sigma_{\hat{p}}^2 = \left(\frac{\alpha_h}{\alpha_s}\right)^2 \sigma_\delta^2$ we can characterize $\frac{\alpha_h}{\alpha_s}$ as a function of primitives as the solution to equation (25).

$$\frac{\alpha_h}{\alpha_s} = \gamma \left(1 + \frac{\sigma_\varepsilon^2}{\sigma_\theta^2} + \left(\frac{1}{\frac{\alpha_h}{\alpha_s}}\right)^2 \frac{\sigma_\varepsilon^2}{\sigma_\delta^2}\right) \tag{25}$$

An explicit solution for $\frac{\alpha_h}{\alpha_s}$, which involves solving a cubic equation, is not particularly enlightening. Using standard results on cubic equations — see Abramowitz and Stegun (1964), it is easy to show that there exists a single real solution to equation (25). This guarantees that the symmetric equilibrium we study is unique. It is easy to show that

$$\frac{\partial\left(\frac{\alpha_h}{\alpha_s}\right)}{\partial\gamma} > 0, \quad \frac{\partial\left(\frac{\alpha_h}{\alpha_s}\right)}{\partial\sigma_\varepsilon^2} > 0, \quad \frac{\partial\left(\frac{\alpha_h}{\alpha_s}\right)}{\partial\sigma_\theta^2} < 0 \text{ and } \frac{\partial\left(\frac{\alpha_h}{\alpha_s}\right)}{\partial\sigma_\delta^2} < 0$$

Price informativeness $\sigma_{\hat{p}}^2$ (as well as $\mathbb{V}ar\left[\theta|s_i, p\right]$) is uniquely pinned down once $\frac{\alpha_h}{\alpha_s}$ is known. It inherits some of its properties. Formally

$$\frac{\partial\sigma_{\hat{p}}^2}{\partial\gamma} > 0, \quad \frac{\partial\sigma_{\hat{p}}^2}{\partial\sigma_\varepsilon^2} > 0, \quad \frac{\partial\sigma_{\hat{p}}^2}{\partial\sigma_\theta^2} < 0 \text{ and } \frac{\partial\sigma_{\hat{p}}^2}{\partial\sigma_\delta^2} \gtrless 0 \tag{26}$$

To fully characterize the equilibrium price, from equations (21) and (23), we can write

$$\frac{\alpha_s}{\alpha_p} = \frac{\mathbb{V}ar\left[\theta|s_i, p\right]}{\frac{1}{\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}}} \tag{27}$$

Similarly, from equations (22) and (23), we can write

$$\frac{\alpha_h}{\alpha_p} = \frac{\gamma}{\frac{\frac{1}{\sigma_\varepsilon^2}}{\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}}} \tag{28}$$

Finally, from equations (24) and (23), we can write

$$\frac{\tilde{\psi}}{\alpha_p} = \frac{1}{\sigma_\theta^2}\bar{\theta}\mathbb{V}ar\left[\theta|s_i, p\right] + \gamma\mathbb{V}ar\left[\theta|s_i, p\right] q_0$$

The first term in the expression for $\frac{\tilde{\psi}}{\alpha_p}$ contains an expected payoff and the second term has a risk premium correction. Although we do not emphasize this result in our statement of theorem 1, given that $\frac{\tilde{\psi}}{\alpha_p}$ is independent of c, we can further conclude that asset prices (and trivially asset returns), not only asset price volatility and price informativeness, are invariant to c.

By combining equations (19) and (20), we can write investor i net demand in equilibrium as

$$q_{1i} - q_{0i} = \alpha_s\left(s_i - \theta\right) - \alpha_h\left(h_i - \delta\right) = \alpha_s\varepsilon_i - \alpha_h e_h$$

The equation $\alpha_s\left(s_i - \theta\right) - \alpha_h\left(h_i - \delta\right) = 0$ represents a straight in the space $s_i \times h_i$, with slope $\frac{dh_i}{ds_i} = \frac{\alpha_s}{\alpha_h}$. It denotes the (measure zero) set of investors who decide not to trade. Investors above this line are sellers of the risky asset, while investors below this line are buyers of the risky asset.

Given that the distributions of s_i and h_i are uncorrelated and symmetric, this directly implies that half of the investors will be buyers for any realization of signals and hedging needs, while the other half will be sellers. We can therefore establish that $q_{1i} - q_{0i} \sim N\left(0, \alpha_s^2 \sigma_\varepsilon^2 + \alpha_h^2 \sigma_h^2\right)$. The distribution of $|q_{1i} - q_{0i}|$ is a half normal, with a mean given by $\mathbb{V}ar\left[q_{1i} - q_{0i}\right]\sqrt{\frac{2}{\pi}}$. Therefore, using a strong law of large numbers, we can write volume exactly as

$$\mathcal{V} = \frac{1}{2}\int |q_{1i} - q_{0i}|\, di = \sqrt{\frac{1}{2\pi}}\left(\alpha_s^2 \sigma_\varepsilon^2 + \alpha_h^2 \sigma_h^2\right)$$

Theorem 1. (Irrelevance theorem: symmetric investors)

a) It suffices to show that $\frac{\alpha_h}{\alpha_s}$ is independent of c. The solution to equation (25) does not depend on c, which proves our claim.

b) It suffices to show that $\frac{\alpha_s}{\alpha_p}$ and $\frac{\alpha_h}{\alpha_p}$ are independent of c, which follows directly from equations (27) and (28).

Equilibrium characterization with ex-ante heterogeneous investors

In the equilibrium in linear strategies in which all investors are heterogeneous in the four dimensions discussed in the text, we guess (and verify) that the optimal portfolio of investor i takes the form

$$q_{1i} = \alpha_{si} s_i - \alpha_{hi} h_i - \alpha_{pi} p + \psi_i, \tag{29}$$

where α_{si}, α_{hi}, α_{pi} and ψ_i are positive scalars for every investor. The market clearing condition $\int q_{1i} di = Q$ implies that the equilibrium price takes the form

$$p = \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\delta + \frac{\tilde{\psi}}{\overline{\alpha_p}},$$

where we define

$$\overline{\alpha_s} = \int \alpha_{si} di, \qquad \overline{\alpha_h} = \int \alpha_{hi} di, \qquad \tilde{\psi}_i \equiv \psi_i - Q, \qquad \tilde{\psi} = \int \tilde{\psi}_i di, \quad \text{and} \quad \overline{\alpha_p} = \int \alpha_{pi} di$$

We assume a strong law of large numbers to guarantee that $\int \alpha_{si} \varepsilon_i di \to 0$ and $\int \alpha_{hi} e_{hi} di \to 0$ almost surely, so that we can write $\int \alpha_{si} s_i di = \overline{\alpha_s}\theta$ and $\int \alpha_{hi} h_i di = \overline{\alpha_h}\delta$.

Hence, we can write the distribution of the price p as

$$p \sim N\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\overline{\theta} + \frac{\tilde{\psi}}{\overline{\alpha_p}}, \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \sigma_\theta^2 + \left(\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2 \sigma_\delta^2\right)$$

While the conditional distribution of the equilibrium price p given the fundamental θ follows

$$p|\theta \sim N\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\theta + \frac{\tilde{\psi}}{\overline{\alpha_p}}, \left(\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2 \sigma_\delta^2\right)$$

We denote by $\hat{p} = \frac{\overline{\alpha_p}}{\overline{\alpha_s}} p - \frac{\tilde{\psi}}{\overline{\alpha_s}}$ the unbiased signal of θ, which is distributed as follows

$$\hat{p}|\theta \sim N\left(\theta, \sigma_{\hat{p}}^2\right) \quad \text{where} \quad \sigma_{\hat{p}}^2 = \left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)^2 \sigma_\delta^2$$

Solving the optimal filtering problem from the perspective of investor i allows us to write

$$\mathbb{E}\left[\theta|s_i, p\right] = \mathbb{E}\left[\theta|s_i, \hat{p}\right] = \frac{\frac{1}{\sigma_\theta^2}\bar{\theta} + \frac{1}{\sigma_{\varepsilon i}^2} s_i + \frac{1}{\sigma_{\hat{p}}^2}\hat{p}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_{\varepsilon i}^2} + \frac{1}{\sigma_{\hat{p}}^2}} \quad \text{and} \quad \mathbb{V}ar\left[\theta|s_i, p\right] = \mathbb{V}ar\left[\theta|s_i, \hat{p}\right] = \frac{1}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_{\varepsilon i}^2} + \frac{1}{\sigma_{\hat{p}}^2}}$$

Substituting these expressions in investors' demand functions, given by equation (5), we can write q_{1i} as

$$q_{1i} = \frac{\left(\frac{1}{\sigma_\theta^2}\bar{\theta} + \frac{1}{\sigma_{\varepsilon i}^2} s_i + \frac{1}{\sigma_{\hat{p}}^2}\left(\frac{\overline{\alpha_p}}{\overline{\alpha_s}} p - \frac{\int(\psi_i - Q)di}{\overline{\alpha_s}}\right)\right)\mathbb{V}ar\left[\theta|s_i, p\right] - \gamma_i h_i - p + c q_{0i}}{\gamma_i \mathbb{V}ar\left[\theta|s_i, p\right] + c}$$

To ease the notation, we define

$$\kappa_i \equiv \gamma_i \mathbb{V}ar\left[\theta|s_i, p\right] + c$$

Matching coefficients with our guess in equation (29), we are able to characterize α_{si}, α_{hi}, α_{pi} and ψ_i as the solution to the following system of equations

$$\alpha_{si} = \frac{\frac{1}{\sigma_{\varepsilon i}^2}\mathbb{V}ar\left[\theta|s_i, p\right]}{\kappa_i}$$

$$\alpha_{hi} = \frac{\gamma_i}{\kappa_i}$$

$$\alpha_{pi} = \frac{1 - \frac{1}{\sigma_{\hat{p}}^2}\frac{\overline{\alpha_p}}{\overline{\alpha_s}}\mathbb{V}ar\left[\theta|s_i, p\right]}{\kappa_i} \Rightarrow \alpha_{pi} = \frac{1}{\kappa_i} - \frac{\sigma_{\varepsilon i}^2}{\sigma_{\hat{p}}^2}\frac{\overline{\alpha_p}}{\overline{\alpha_s}}\alpha_{si} \Rightarrow \overline{\alpha_p} = \frac{\sigma_{\hat{p}}^2 \int \frac{1}{\kappa_i} di}{\sigma_{\hat{p}}^2 + \frac{\int \sigma_{\varepsilon i}^2 \alpha_{si} di}{\overline{\alpha_s}}}$$

$$\psi_i = \frac{\left(\frac{1}{\sigma_\theta^2}\bar{\theta} - \frac{1}{\sigma_{\hat{p}}^2}\frac{\int(\psi_i - Q)di}{\overline{\alpha_s}}\right)\mathbb{V}ar\left[\theta|s_i, p\right] + c q_{0i}}{\kappa_i}$$

As in the symmetric case, a full characterization of the equilibrium hinges on finding the equilibrium value of $\frac{\overline{\alpha_h}}{\overline{\alpha_s}}$. In this case, it is given by the solution to the following nonlinear equation:

$$\frac{\overline{\alpha_h}}{\overline{\alpha_s}} = \frac{\int \frac{\gamma_i}{\kappa_i} di}{\int \frac{\frac{1}{\sigma_{\varepsilon i}^2}\mathbb{V}ar(\theta|s_i, p)}{\kappa_i} di} = \frac{\int \frac{1}{\mathbb{V}ar[\theta|s_i,p] + \frac{c}{\gamma_i}} di}{\int \frac{\frac{1}{\sigma_{\varepsilon i}^2}}{\gamma_i + \frac{c}{\mathbb{V}ar(\theta|s_i,p)}} di} \tag{30}$$

Explicitly in terms of $\frac{\overline{\alpha_h}}{\overline{\alpha_s}}$:

$$\frac{\overline{\alpha_h}}{\overline{\alpha_s}} = \frac{\int \frac{1}{\frac{1}{\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_{\varepsilon i}^2}+\frac{1}{\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)^2\sigma_\delta^2}}+\frac{c}{\gamma_i}}di}{\int \frac{\frac{1}{\sigma_{\varepsilon i}^2}}{\gamma_i+\frac{c}{\frac{1}{\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_{\varepsilon i}^2}+\frac{1}{\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)^2\sigma_\delta^2}}}}di} \tag{31}$$

Once the equilibrium value of $\frac{\overline{\alpha_h}}{\overline{\alpha_s}}$ is determined, $\sigma_{\hat{p}}^2$ and $\mathbb{V}ar\left[\theta|s_i,p\right]$ are uniquely pinned down. Implicit differentiation of equation (31) allows us to carry out comparative statics on any of the endogenous variables.

If κ_i is constant, $\frac{\overline{\alpha_h}}{\overline{\alpha_s}}$ is independent of c for any value of c. The reserve result is also true, only when $\kappa_i = \kappa$, $\frac{\overline{\alpha_h}}{\overline{\alpha_s}}$ is independent of c for any value of c. Therefore, $\frac{\overline{\alpha_h}}{\overline{\alpha_s}}$ is independent of c if and only if

$$\kappa_i = \kappa, \quad \forall i$$

$$\kappa_i \equiv \gamma_i \frac{1}{\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_{\varepsilon i}^2}+\frac{1}{\sigma_{\hat{p}}^2}} + c$$

The sign of $\frac{d\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)}{dc}$ can be determined as follows. We can write

$$\frac{\frac{d\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)}{dc}}{\frac{\overline{\alpha_h}}{\overline{\alpha_s}}} = \frac{-\int \frac{\gamma_i}{(\kappa_i)^2}\frac{d\kappa_i}{dc}di}{\overline{\alpha_h}} - \frac{\int \frac{\frac{1}{\sigma_{\varepsilon i}^2}\frac{d\mathbb{V}ar(\theta|s_i,p)}{dc}\kappa_i - \frac{1}{\sigma_{\varepsilon i}^2}\mathbb{V}ar(\theta|s_i,p)\frac{d\kappa_i}{dc}}{(\kappa_i)^2}di}{\overline{\alpha_s}} = \frac{-\int \frac{\alpha_{hi}}{\kappa_i}di}{\overline{\alpha_h}} + \frac{\int \frac{\alpha_{si}}{\kappa_i}di}{\overline{\alpha_s}} - \frac{\int \frac{\frac{1}{\sigma_{\varepsilon i}^2}\frac{d\mathbb{V}ar(\theta|s_i,p)}{dc}}{\kappa_i}di}{\overline{\alpha_s}}$$

$$= \frac{-\int \frac{\alpha_{hi}}{\kappa_i}di}{\overline{\alpha_h}} + \frac{\int \frac{\alpha_{si}}{\kappa_i}di}{\overline{\alpha_s}} - \frac{2}{\sigma_\delta^2}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 \frac{\frac{d\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)}{dc}}{\frac{\overline{\alpha_h}}{\overline{\alpha_s}}} \frac{\int \alpha_{si}\mathbb{V}ar\left(\theta|s_i,p\right)di}{\overline{\alpha_s}}$$

Where we have used the fact that $\frac{d\kappa_i}{dc} = 1$ and that

$$\frac{d\mathbb{V}ar\left(\theta|s_i,p\right)}{dc} = \frac{\frac{1}{\left(\sigma_{\hat{p}}^2\right)^2}\frac{d\sigma_{\hat{p}}^2}{dc}}{\left(\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_{\varepsilon i}^2}+\frac{1}{\sigma_{\hat{p}}^2}\right)^2} = \frac{\frac{2\sigma_\delta^2}{\left(\sigma_{\hat{p}}^2\right)^2}}{\left(\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_{\varepsilon i}^2}+\frac{1}{\sigma_{\hat{p}}^2}\right)^2}\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\frac{d\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)}{dc} = \frac{2\mathbb{V}ar\left(\theta|s_i,p\right)^2}{\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)^3\sigma_\delta^2}\frac{d\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)}{dc}$$

Hence we can write $\frac{\frac{d\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)}{dc}}{\frac{\overline{\alpha_h}}{\overline{\alpha_s}}} = \frac{\frac{-\int \frac{\alpha_{hi}}{\kappa_i}di}{\overline{\alpha_h}}+\frac{\int \frac{\alpha_{si}}{\kappa_i}di}{\overline{\alpha_s}}}{1+\frac{2}{\sigma_\delta^2}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2\frac{\int \alpha_{si}\mathbb{V}ar(\theta|s_i,p)di}{\overline{\alpha_s}}}$, so the sign of $\frac{d\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)}{dc}$ depends on

$$\text{sgn}\left(\frac{d\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)}{dc}\right) = \text{sgn}\left(\frac{\int \frac{\alpha_{si}}{\kappa_i}di}{\overline{\alpha_s}} - \frac{\int \frac{\alpha_{hi}}{\kappa_i}di}{\overline{\alpha_h}}\right)$$

As expected, when $\kappa_i = \kappa$, the sign of this derivative is zero. Note that we can write

$$\frac{\int \frac{\alpha_{si}}{\kappa_i} di}{\overline{\alpha_s}} - \frac{\int \frac{\alpha_{hi}}{\kappa_i} di}{\overline{\alpha_h}} = \frac{\mathbb{E}_i \left[\alpha_{si} \frac{1}{\kappa_i}\right]}{\overline{\alpha_s}} - \frac{\mathbb{E}_i \left[\alpha_{hi} \frac{1}{\kappa_i}\right]}{\overline{\alpha_h}}$$

$$= \frac{\mathbb{E}_i \left[\alpha_{si}\right] \mathbb{E}_i \left[\frac{1}{\kappa_i}\right] + \mathbb{C}ov_i \left[\alpha_{si}, \frac{1}{\kappa_i}\right]}{\overline{\alpha_s}} - \frac{\mathbb{E}_i \left[\alpha_{hi}\right] \mathbb{E}_i \left[\frac{1}{\kappa_i}\right] + \mathbb{C}ov_i \left[\alpha_{hi}, \frac{1}{\kappa_i}\right]}{\overline{\alpha_h}}$$

$$= \mathbb{C}ov_i \left[\frac{\alpha_{si}}{\overline{\alpha_s}}, \frac{1}{\kappa_i}\right] - \mathbb{C}ov_i \left[\frac{\alpha_{hi}}{\overline{\alpha_h}}, \frac{1}{\kappa_i}\right] = \mathbb{C}ov_i \left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}, \frac{1}{\kappa_i}\right]$$

Therefore

$$\text{sgn}\left(\frac{d\left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)}{dc}\right) = \text{sgn}\left(\frac{\int \frac{\alpha_{si}}{\kappa_i} di}{\overline{\alpha_s}} - \frac{\int \frac{\alpha_{hi}}{\kappa_i} di}{\overline{\alpha_h}}\right) = \text{sgn}\left(\mathbb{C}ov_i \left[\frac{\alpha_{si}}{\overline{\alpha_s}} - \frac{\alpha_{hi}}{\overline{\alpha_h}}, \frac{1}{\kappa_i}\right]\right)$$

We can further write

$$\frac{\overline{\alpha_p}}{\overline{\alpha_s}} = \frac{\int \frac{1}{\kappa_i} di - \frac{1}{\sigma_{\hat{p}}^2} \frac{\overline{\alpha_p}}{\overline{\alpha_s}} \int \frac{\mathbb{V}ar(\theta|s_i,p)}{\kappa_i} di}{\int \frac{\frac{1}{\sigma_{\varepsilon i}^2} \mathbb{V}ar(\theta|s_i,p)}{\kappa_i} di} \Rightarrow \frac{\overline{\alpha_p}}{\overline{\alpha_s}} = \frac{\int \frac{1}{\kappa_i} di}{\int \frac{\frac{1}{\sigma_{\varepsilon i}^2} \mathbb{V}ar(\theta|s_i,p)}{\kappa_i} di + \frac{1}{\sigma_{\hat{p}}^2} \int \frac{\mathbb{V}ar(\theta|s_i,p)}{\kappa_i} di}$$

$$\frac{\overline{\alpha_p}}{\overline{\alpha_s}} = \frac{\sigma_{\hat{p}}^2 \int \frac{1}{\kappa_i} di}{\sigma_{\hat{p}}^2 \overline{\alpha_s} + \int \sigma_{\varepsilon i}^2 \alpha_{si} di}$$

Theorem 2. (Conditions for irrelevance of trading costs with ex-ante heterogeneous investors)

a) It suffices to show that $\frac{\overline{\alpha_h}}{\overline{\alpha_s}}$ is independent of c. From equation (12), it is clear that if and only if $\gamma_i = \gamma$ and $\sigma_{\varepsilon i}^2 = \sigma_\varepsilon^2$ price informativeness does not depend on c.

b) It suffices to show that $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ and $\frac{\overline{\alpha_h}}{\overline{\alpha_p}}$ are independent of c. The same logic as point a) applies

Theorem 3. (Directional effects of trading costs with ex-ante heterogeneous investors)

a) It follows directly from the derivations above.

Proofs: Section 4

Investors' information acquisition problem

The expected utility of an investor who participates in the asset market is given by

$$\mathbb{E}\left[u_i\left(\mathbb{E}\left[U_i\left(w_{2i}\right)|s_i, p, h_i\right]\right)\right] \quad \text{with} \quad U_i\left(w_{2i}\right) = -e^{-\gamma_i w_{2i}} \quad \text{and} \quad u_i = -ln\left(-x\right)$$

where

$$w_{2i} = n_{2i} + q_{1i}^{*}\theta + \left(q_{0i}p - q_{1i}^{*}p - \frac{c}{2}\left(\Delta q_{1i}^{*}\right)^{2} + T_{1i}\right) - \frac{1}{2}\tau\left(\frac{1}{\sigma_{\varepsilon i}^{2}}\right)$$

and q_{1i}^{*} is the equilibrium quantity demanded by investor i in a given state.

The objective function of investors can be written, using the law of iterated expectations, as

$$\begin{aligned} V\left(\sigma_{\varepsilon i}^{2};\{\sigma_{\varepsilon j}^{2}\}_{j\neq i}\right) &= \mathbb{E}\left[u_i\left(\mathbb{E}\left[U_i\left(w_{2i}\right)|s_i,p,h_i\right]\right)\right] \\ &= \gamma\left[\mathbb{E}\left[n_{2i}\right] - \frac{\gamma_i}{2}Var\left[n_{2i}\right] + \mathbb{E}\left[v_i\right] - \frac{1}{2}\tau\left(\frac{1}{\sigma_{\varepsilon i}^{2}}\right) + T_{1i}\right] \\ &= \gamma\left[\mathbb{E}\left[v_i\right] - \frac{1}{2}\tau\left(\frac{1}{\sigma_{\varepsilon i}^{2}}\right) + T_{1i}\right] \end{aligned}$$

where we use the assumption that $\mathbb{E}\left[n_{2i}\right] - \frac{\gamma_i}{2}Var\left[n_{2i}\right] = 0$, and where we define

$$v_i = \left(\mathbb{E}\left[\theta|h_i,s_i,p\right] - \gamma_i h_i - p\right)q_{1i} - \frac{c}{2}\left(\Delta q_{1i}\right)^{2} - \frac{\gamma_i}{2}\mathbb{V}ar\left[\theta|s_i,p\right]q_{1i}^{2} + q_{0i}p$$

From now on, we focus on characterizing $\mathbb{E}\left[v_i\right] - \frac{1}{2}\tau\left(\frac{1}{\sigma_{\varepsilon i}^{2}}\right)$, which is the only relevant term for the determination of $\sigma_{\varepsilon i}^{2}$. Using the optimality condition for investors' portfolio demand at date 1, we can write

$$\begin{aligned} v_i &= \left(\kappa_i q_{1i} - cq_{0i}\right)q_{1i} - \frac{c}{2}\left(\Delta q_{1i}\right)^{2} - \frac{\gamma_i}{2}\mathbb{V}ar\left[\theta|s_i,p\right]q_{1i}^{2} + q_{0i}p \\ &= \kappa_i\left(q_{1i}\right)^{2} - cq_{0i}q_{1i} - \frac{c}{2}\left(q_{1i} - q_{0i}\right)^{2} - \frac{\gamma_i}{2}\mathbb{V}ar\left[\theta|s_i,p\right]q_{1i}^{2} + q_{0i}p \\ &= \frac{1}{2}\kappa_i\left(q_{1i}\right)^{2} - \frac{c}{2}\left(q_{0i}\right)^{2} + q_{0i}p, \end{aligned}$$

which can be written as

$$v_i = \frac{1}{2}\frac{\left(\hat{\theta}_i - \gamma_i h_i - p\right)^{2}}{\kappa_i} - \frac{c}{2}\left(q_{0i}\right)^{2} + q_{0i}p,$$

where we define $\hat{\theta}_i \equiv \mathbb{E}\left[\theta|h_i,s_i,p\right]$ and $\kappa_i \equiv \gamma\mathbb{V}ar\left[\theta|s_i,p\right] + c$. The numerator of the first term of v_i represents the expected marginal benefit of increasing the asset holdings of the risky asset. The denominator represents the marginal cost of increasing the risk exposure, because risk averse investors are faced with uncertainty $\gamma_i\mathbb{V}ar\left[\theta|s_i,p\right] \neq 0$ and because investors face trading costs $c \neq 0$.

We can write $\hat{\theta}_i \equiv \mathbb{E}[\theta|h_i, s_i, p]$ as

$$\begin{aligned}\hat{\theta}_i &= \mathbb{V}ar[\theta|s_i, p]\left(\frac{1}{\sigma_\theta^2}\overline{\theta} + \frac{1}{\sigma_{\varepsilon_i}^2}s_i + \frac{1}{\sigma_{\hat{p}}^2}\hat{p}\right) \\ &= \mathbb{V}ar[\theta|s_i, p]\left(\frac{1}{\sigma_\theta^2}\overline{\theta} + \frac{1}{\sigma_{\varepsilon_i}^2}(\theta + \varepsilon_i) + \frac{1}{\sigma_{\hat{p}}^2}\left(\theta - \frac{\overline{\alpha_h}}{\overline{\alpha_s}}\delta\right)\right) \\ &= \mathbb{V}ar[\theta|s_i, p]\left(\frac{1}{\sigma_\theta^2}\overline{\theta} + \left(\frac{1}{\sigma_{\varepsilon_i}^2} + \frac{1}{\sigma_{\hat{p}}^2}\right)\theta + \frac{1}{\sigma_{\varepsilon_i}^2}\varepsilon_i - \frac{\overline{\alpha_s}}{\overline{\alpha_h}}\frac{1}{\sigma_\delta^2}\delta\right)\end{aligned}$$

Therefore, it follows that $\hat{\theta}_i$ is normally distributed as $\hat{\theta}_i \sim N\left(\overline{\theta}, \sigma_{\hat{\theta}_i}^2\right)$, where

$$\begin{aligned}\sigma_{\hat{\theta}_i}^2 &= \mathbb{V}ar[\theta|s_i, p]^2\left(\left(\frac{1}{\sigma_{\varepsilon_i}^2} + \frac{1}{\sigma_{\hat{p}}^2}\right)^2\sigma_\theta^2 + \left(\frac{1}{\sigma_{\varepsilon_i}^2}\right)^2\sigma_{\varepsilon_i}^2 + \left(\frac{1}{\sigma_{\hat{p}}^2}\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)^2\sigma_\delta^2\right) \\ &= \sigma_\theta^2 - \mathbb{V}ar[\theta|s_i, p]\end{aligned}$$

An investor with a more precise private signal has a more precise estimate of θ. A more precise private signal decreases the total amount of uncertainty faced by the investor when he trades and it shifts the updating weight towards his private signal, which is now more informative.

Note that $\sigma_{\hat{\theta}_i}^2$ decreases with $\sigma_{\varepsilon i}^2$. $\sigma_{\hat{\theta}_i}^2$ measures the pre-signal uncertainty and $\mathbb{V}ar[\theta|s_i, p]$ the post-signal one. Overall, the total uncertainty faced by the investor has to be σ_θ^2. A decrease in $\sigma_{\varepsilon i}^2$ increases the precision of the private signal and decreases the uncertainty post-signal while it increases the uncertainty pre-signal.

We can thus write $\mathbb{E}[v_i] - \frac{1}{2}\tau\left(\frac{1}{\sigma_{\varepsilon i}^2}\right)$ as

$$\mathbb{E}[v_i] - \frac{1}{2}\tau\left(\frac{1}{\sigma_{\varepsilon i}^2}\right) = \frac{1}{2}\frac{\mathbb{E}\left[\left(\hat{\theta}_i - \gamma_i h_i - p\right)^2\right]}{\kappa_i} - \frac{c}{2}(q_{0i})^2 + q_{0i}\mathbb{E}[p] + T_{1i} - \frac{1}{2}\tau\left(\frac{1}{\sigma_{\varepsilon i}^2}\right)$$

The marginal net benefit of holding the asset is normally distributed since $\hat{\theta}_i$, h_i and p are normally distributed. We can therefore write

$$\hat{\theta}_i - \gamma_i h_i - p \sim N\left(m, \sigma_{mi}^2\right),$$

where we define

$$m = \left(1 - \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)\overline{\theta} - \frac{\overline{\psi}}{\overline{\alpha_p}}$$

and

$$\sigma_{mi}^2 = \sigma_{\hat{\theta}_i}^2 + \gamma_i^2\left(\sigma_\delta^2 + \sigma_{hi}^2\right) + \sigma_p^2 - 2\gamma_i\mathbb{C}ov\left(\hat{\theta}_i, h_i\right) - 2\mathbb{C}ov\left(\hat{\theta}_i, p\right) + 2\gamma_i\mathbb{C}ov\left(p, h_i\right)$$

Using the fact that $\sigma_{\hat{p}}^2 = \left(\frac{\overline{\alpha_h}}{\overline{\alpha_s}}\right)^2 \sigma_\delta^2$, we can further write $\mathbb{C}ov\left(\hat{\theta}_i, h_i\right)$, $\mathbb{C}ov\left(\hat{\theta}_i, p\right)$ and $\mathbb{C}ov\left(p, h_i\right)$ as

$$\mathbb{C}ov\left(\hat{\theta}_i, h_i\right) = -\mathbb{V}ar\left[\theta|s_i, p\right] \frac{1}{\sigma_{\hat{p}}^2} \frac{\overline{\alpha_h}}{\overline{\alpha_s}} \sigma_\delta^2 = -\frac{\overline{\alpha_s}}{\overline{\alpha_h}} \mathbb{V}ar\left[\theta|s_i, p\right]$$

$$\mathbb{C}ov\left(\hat{\theta}_i, p\right) = \mathbb{V}ar\left[\theta|s_i, p\right] \left(\left(\frac{1}{\sigma_{\varepsilon_i}^2} + \frac{1}{\sigma_{\hat{p}}^2}\right) \frac{\overline{\alpha_s}}{\overline{\alpha_p}} \sigma_\theta^2 + \frac{1}{\sigma_{\hat{p}}^2} \frac{\overline{\alpha_h}}{\overline{\alpha_s}} \frac{\overline{\alpha_h}}{\overline{\alpha_p}} \sigma_\delta^2\right) = \frac{\overline{\alpha_s}}{\overline{\alpha_p}} \sigma_\theta^2$$

$$\mathbb{C}ov\left(p, h_i\right) = -\frac{\overline{\alpha_h}}{\overline{\alpha_p}} \sigma_\delta^2$$

From the expression of σ_m^2, we can show that only $\mathbb{C}ov\left(\hat{\theta}_i, h_i\right)$ and $\sigma_{\hat{\theta}_i}^2$ depend on $\sigma_{\varepsilon i}^2$. Moreover, m^2 is independent of $\sigma_{\varepsilon i}^2$. We can also write $\mathbb{E}\left[\left(\hat{\theta}_i - \gamma_i h_i - p\right)^2\right] = \sigma_m^2 + m^2$. Therefore, leaving aside terms which do not matter for the maximization problem, the choice of $\sigma_{\varepsilon i}^2$ boils down to solving

$$\max_{\sigma_{\varepsilon i}^2} \underbrace{\frac{\gamma \mathbb{V}ar\left[\theta|s_i, p\right]}{\gamma \mathbb{V}ar\left[\theta|s_i, p\right] + c}}_{\rho(c)} \underbrace{\frac{\sigma_m^2 + m^2}{\gamma \mathbb{V}ar\left[\theta|s_i, p\right]}}_{\equiv V_0} - \tau\left(\frac{1}{\sigma_{\varepsilon i}^2}\right)$$

Rewriting σ_m^2

$$\begin{aligned}
\sigma_m^2 &= \sigma_\theta^2 - \mathbb{V}ar\left[\theta|s_i, p\right] + \gamma_i^2\left(\sigma_\delta^2 + \sigma_{hi}^2\right) + \sigma_p^2 + 2\gamma_i \frac{\overline{\alpha_s}}{\overline{\alpha_h}} \mathbb{V}ar\left[\theta|s_i, p\right] - 2\frac{\overline{\alpha_s}}{\overline{\alpha_p}} \sigma_\theta^2 - 2\gamma_i \frac{\overline{\alpha_h}}{\overline{\alpha_p}} \sigma_\delta^2 \\
&= \sigma_\theta^2 + \gamma_i^2\left(\sigma_\delta^2 + \sigma_{hi}^2\right) + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \sigma_\theta^2 + \left(\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2 \sigma_\delta^2 + \left(2\gamma_i \frac{\overline{\alpha_s}}{\overline{\alpha_h}} - 1\right) \mathbb{V}ar\left[\theta|s_i, p\right] - 2\frac{\overline{\alpha_s}}{\overline{\alpha_p}} \sigma_\theta^2 - 2\gamma_i \frac{\overline{\alpha_h}}{\overline{\alpha_p}} \sigma_\delta^2 \\
&= \sigma_\theta^2\left(1 - 2\frac{\overline{\alpha_s}}{\overline{\alpha_p}} + \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\right) + \gamma_i^2 \sigma_{hi}^2 + \left(\gamma_i^2 - 2\gamma_i \frac{\overline{\alpha_h}}{\overline{\alpha_p}} + \left(\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2\right) \sigma_\delta^2 + \left(2\gamma_i \frac{\overline{\alpha_s}}{\overline{\alpha_h}} - 1\right) \mathbb{V}ar\left[\theta|s_i, p\right] \\
&= \sigma_\theta^2\left(1 - \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 + \left(\gamma_i - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2 \sigma_\delta^2 + \gamma_i^2 \sigma_{hi}^2 + \left(2\gamma_i \frac{\overline{\alpha_s}}{\overline{\alpha_h}} - 1\right) \mathbb{V}ar\left[\theta|s_i, p\right] \qquad (32)
\end{aligned}$$

where we used the fact that

$$\sigma_p^2 = \left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 \sigma_\theta^2 + \left(\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2 \sigma_\delta^2$$

Since m is independent of $\sigma_{\varepsilon i}^2$ and σ_m^2 and κ_i depend on $\sigma_{\varepsilon_i}^2$ only through $\mathbb{V}ar\left[\theta|s_i, p\right]$ one can rewrite the investor's problem as if he was choosing $\mathbb{V}ar\left[\theta|s_i, p\right]$.[11] Formally

$$\max_{\mathbb{V}ar[\theta|s_i,p]} \frac{1}{\gamma_i \mathbb{V}ar\left[\theta|s_i, p\right] + c}\left(\sigma_m^2 + m^2\right) - \hat{\tau}\left(\mathbb{V}ar\left[\theta|s_i, p\right]^{-1}\right)$$

where $\hat{\tau}' > 0$ and $\hat{\tau}'' > 0$ The first order condition for this problem is

$$\frac{\left(2\gamma_i \frac{\overline{\alpha_s}}{\overline{\alpha_h}} - 1\right)\left(\gamma_i \mathbb{V}ar\left[\theta|s_i, p\right] + c\right) - \gamma_i\left(\sigma_m^2 + m^2\right)}{\left(\gamma_i \mathbb{V}ar\left[\theta|s_i, p\right] + c\right)^2} + \mathbb{V}ar\left[\theta|s_i, p\right]^{-2} \hat{\tau}'(\cdot) = 0$$

[11]Note that m is independent of $\sigma_{\varepsilon i}^2$ and that σ_m^2 depends on $\sigma_{\varepsilon_i}^2$ only through $\mathbb{V}ar\left[\theta|s_i, p\right]$.

Using the definition of σ_m^2 the first order condition becomes

$$\frac{\left(2\gamma_i\frac{\overline{\alpha_s}}{\overline{\alpha_h}}-1\right)c-\gamma_i\left(\sigma_\theta^2\left(1-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2+\left(\gamma_i-\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2\sigma_\delta^2+\gamma_i^2\sigma_{hi}^2+m^2\right)}{\left(\gamma_i\mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^2}+\mathbb{V}ar\left[\theta|s_i,p\right]^{-2}\hat{\tau}'\left(\cdot\right)=0 \tag{33}$$

The SOC is

$$\begin{aligned}SOC=-\gamma_i&\frac{\left(2\gamma_i\frac{\overline{\alpha_s}}{\overline{\alpha_h}}-1\right)c-\gamma_i\left(\sigma_\theta^2\left(1-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2+\left(\gamma_i-\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2\sigma_\delta^2+\gamma_i^2\sigma_{hi}^2+m^2\right)}{\left(\gamma_i\mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^3}\\&-\mathbb{V}ar\left[\theta|s_i,p\right]^{-4}\hat{\tau}''-2\mathbb{V}ar\left[\theta|s_i,p\right]^{-3}\hat{\tau}'\left(\cdot\right)<0\end{aligned}$$

Since there is a one-to-one mapping between $\mathbb{V}ar\left[\theta|s_i,p\right]$ and $\sigma_{\varepsilon i}^2$, the first order condition (33) defines an implicit best response function $\sigma_{\varepsilon i}^2\left(\left\{\sigma_{\varepsilon j}^2\right\}_{j\neq i}\right)$ where the dependence on $\left\{\sigma_{\varepsilon j}^2\right\}_{j\neq i}$ is solely through $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$, $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, $\sigma_{\hat{p}}^2$ and m which are equilibrium outcomes coming from the trading stage of the game.

If all investors are ex-ante homogeneous, since each investor is infinitesimal, two investors i and h that anticipate the same $\left\{\sigma_{\varepsilon j}^2\right\}_{j\neq i,h}$ face the same $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$, $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$, $\sigma_{\hat{p}}^2$, and m and therefore will have the same best response function $\sigma_{\varepsilon i}^2\left(\left\{\sigma_{\varepsilon j}^2\right\}_{j\neq i}\right)=\sigma_{\varepsilon h}^2\left(\left\{\sigma_{\varepsilon j}^2\right\}_{j\neq h}\right)$. This implies that the equilibrium in the information acquisition game has to be symmetric when investors play linear strategies in the trading stage. If all investors choose the same precision σ_ε^2, the best response of investor i can be expressed as $\sigma_{\varepsilon i}^2\left(\sigma_\varepsilon^2\right)$.

Properties of investors' best responses

Let

$$H\left(\sigma_{\varepsilon i}^2,\sigma_\varepsilon^2\right)\equiv\frac{\left(2\gamma_i\frac{\overline{\alpha_s}}{\overline{\alpha_h}}-1\right)c-\gamma_i\left(\sigma_\theta^2\left(1-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2+\left(\gamma_i-\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2\sigma_\delta^2+\gamma_i^2\sigma_{hi}^2+m^2\right)}{\left(\gamma_i\mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^2}+\mathbb{V}ar\left[\theta|s_i,p\right]^{-2}\hat{\tau}'$$

Using the implicit function theorem, we can write

$$\frac{d\sigma_{\varepsilon i}^2}{dc}=-\frac{\frac{\partial H}{\partial c}|_{\sigma_{\varepsilon i}^{2*}}}{\frac{\partial H}{\partial\sigma_{\varepsilon i}^2}|_{\sigma_{\varepsilon i}^{2*}}}>0$$

$$\frac{d\sigma_{\varepsilon i}^2}{d\sigma_\varepsilon^2}=-\frac{\frac{\partial H}{\partial\sigma_\varepsilon^2}|_{\sigma_{\varepsilon i}^{2*}}}{\frac{\partial H}{\partial\sigma_{\varepsilon i}^2}|_{\sigma_{\varepsilon i}^{2*}}}<0$$

The interpretation of these results is clear. First, $\frac{d\sigma_{\varepsilon i}^2}{d\sigma_\varepsilon^2}<0$ implies there is a unique symmetric equilibrium. Second, $\frac{d\sigma_{\varepsilon i}^2}{dc}>0$ implies that σ_ε^{2*} is decreasing in the trading cost c

Calculations to show the results above

Since

$$\frac{\partial H}{\partial \sigma_{\varepsilon i}^2}|_{\sigma_{\varepsilon i}^{2*}} = \left(\frac{\mathbb{V}ar[\theta|s_i,p]}{\sigma_{\varepsilon i}^2}\right)^2 \quad SOC < 0$$

$$\frac{\partial H}{\partial c}|_{\sigma_{\varepsilon i}^{2*}} = \frac{\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)\left(2\gamma_i\frac{\overline{\alpha_s}}{\overline{\alpha_h}}-1\right)-2\left(\left(2\gamma_i\frac{\overline{\alpha_s}}{\overline{\alpha_h}}-1\right)c-\gamma_i\left(\sigma_\theta^2\left(1-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2+\left(\gamma_i-\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2\sigma_\delta^2+\gamma_i^2\sigma_{hi}^2+m^2\right)\right)}{\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^3}$$

$$= \frac{\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]\left(2\gamma_i\frac{\overline{\alpha_s}}{\overline{\alpha_h}}-1\right)-\left(2\gamma_i\frac{\overline{\alpha_s}}{\overline{\alpha_h}}-1\right)c+2\gamma_i\left(\sigma_\theta^2\left(1-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2+\left(\gamma_i-\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2\sigma_\delta^2+\gamma_i^2\sigma_{hi}^2+m^2\right)}{\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^3}$$

Using the first order condition

$$\frac{\partial H}{\partial c}|_{\sigma_{\varepsilon i}^{2*}} = \frac{\mathbb{V}ar\left[\theta|s_i,p\right]^{-2}\hat{\tau}'\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^2}{\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^3} + \frac{\gamma_i\left(\mathbb{V}ar\left[\theta|s_i,p\right]\left(2\gamma_i\frac{\overline{\alpha_s}}{\overline{\alpha_h}}-1\right)+\sigma_\theta^2\left(1-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2+\left(\gamma_i-\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2\sigma_\delta^2+\gamma_i^2\sigma_{hi}^2+m^2\right)}{\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^3}$$

$$= \frac{\mathbb{V}ar\left[\theta|s_i,p\right]^{-2}\hat{\tau}'\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^2+\gamma_i\left(\sigma_m^2+m^2\right)}{\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^3} > 0$$

where we used the definition of σ_m^2 in (32).

$$\frac{dH}{d\sigma_\varepsilon^2} < 0$$

$$H = \frac{\left(2\gamma_i\frac{\overline{\alpha_s}}{\overline{\alpha_h}}-1\right)c-\gamma_i\left(\sigma_\theta^2\left(1-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2+\left(\gamma_i-\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2\sigma_\delta^2+\gamma_i^2\sigma_{hi}^2+m^2\right)}{\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^2} + \mathbb{V}ar\left[\theta|s_i,p\right]^{-2}\hat{\tau}'$$

$$\frac{dH}{d\sigma_\varepsilon^2} = \frac{2\gamma_i\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{d\sigma_\varepsilon^2}c-\gamma_i\left(2\sigma_\theta^2\left(1-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)}{d\sigma_\varepsilon^2}+2\left(\gamma_i-\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)\frac{d\left(\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)}{d\sigma_\varepsilon^2}\sigma_\delta^2+2m\frac{dm}{d\sigma_\varepsilon^2}\right)\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)}{\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^3}$$

$$-\frac{2\left(\left(2\gamma_i\frac{\overline{\alpha_s}}{\overline{\alpha_h}}-1\right)c-\gamma_i\left(\sigma_\theta^2\left(1-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2+\left(\gamma_i-\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2\sigma_\delta^2+\gamma_i^2\sigma_{hi}^2+m^2\right)\right)\gamma_i\frac{d\mathbb{V}ar[\theta|s_i,p]}{d\sigma_\varepsilon^2}}{\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^3}$$

$$-2\mathbb{V}ar\left[\theta|s_i,p\right]^{-3}\hat{\tau}'\frac{d\mathbb{V}ar\left[\theta|s_i,p\right]}{d\sigma_\varepsilon^2}-\mathbb{V}ar\left[\theta|s_i,p\right]^{-4}\hat{\tau}''\frac{d\mathbb{V}ar\left[\theta|s_i,p\right]}{d\sigma_\varepsilon^2}$$

We can evaluate this expression at $\sigma_{\varepsilon i}^2 = \sigma_{\varepsilon i}^{2*}$

$$\frac{dH}{d\sigma_\varepsilon^2}|_{\sigma_{\varepsilon i}^*} = \frac{2\gamma_i\overbrace{\frac{d\left[\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right]}{d\sigma_\varepsilon^2}}^{<0}c-\gamma_i\left(\overbrace{-2\sigma_\theta^2\left(1-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)\frac{d\left[\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right]}{d\sigma_\varepsilon^2}-2\left(\gamma_i-\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)\frac{d\left[\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right]}{d\sigma_\varepsilon^2}\sigma_\delta^2}^{>0}+2m\overbrace{\frac{dm}{d\sigma_\varepsilon^2}}^{>0}\right)\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)}{\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^3}$$

$$-2\mathbb{V}ar\left[\theta|s_i,p\right]^{-2}\underbrace{\frac{d\mathbb{V}ar\left[\theta|s_i,p\right]}{d\sigma_\varepsilon^2}}_{>0}\underbrace{\left(\frac{\mathbb{V}ar\left[\theta|s_i,p\right]^{-1}c}{\left(\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]+c\right)^3}\hat{\tau}'+\frac{1}{2}\mathbb{V}ar\left[\theta|s_i,p\right]^{-2}\hat{\tau}''\right)}_{>0} < 0$$

In a symmetric equilibrium,

$$\frac{\overline{\alpha_s}}{\overline{\alpha_h}} = \frac{1}{\gamma\left(\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} + \frac{\sigma_\varepsilon^2}{\sigma_\delta^2}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 + 1\right)}.$$

Then, $\frac{\overline{\alpha_s}}{\overline{\alpha_h}}$ is implicitly defined as the unique solution to

$$J(x) \equiv x - \frac{1}{\gamma\left(\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} + \frac{\sigma_\varepsilon^2}{\sigma_\delta^2}x^2 + 1\right)} = 0.$$

Using the implicit function theorem, this implies

$$\begin{aligned}
\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{d\sigma_\varepsilon^2} = -\frac{\frac{\partial J\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{\partial \sigma_\varepsilon^2}}{\frac{\partial J\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{\partial\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}} &= -\frac{\frac{1}{\gamma\left(\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}+\frac{\sigma_\varepsilon^2}{\sigma_\delta^2}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2+1\right)^2}\left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\delta^2}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2\right)}{1 + \frac{1}{\gamma\left(\frac{\sigma_\varepsilon^2}{\sigma_\theta^2}+\frac{\sigma_\varepsilon^2}{\sigma_\delta^2}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2+1\right)^2} 2\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\frac{\sigma_\varepsilon^2}{\sigma_\delta^2}} \\
&= -\frac{\left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\delta^2}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2\right)}{\gamma\left(\frac{\sigma_\varepsilon^2}{\sigma_\theta^2} + \frac{\sigma_\varepsilon^2}{\sigma_\delta^2}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 + 1\right)^2 + 2\frac{\sigma_\varepsilon^2}{\sigma_\delta^2}\frac{\overline{\alpha_s}}{\overline{\alpha_h}}} < 0
\end{aligned}$$

Another way to rewrite this is

$$\frac{d\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}{d\sigma_\varepsilon^2} = -\gamma\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2 \frac{\left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\delta^2}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2\right)}{1 + 2\frac{\sigma_\varepsilon^2}{\sigma_{\hat{p}}^2}\gamma\frac{\overline{\alpha_s}}{\overline{\alpha_h}}}$$

When σ_ε^2 increases, the signal to noise ratio decreases (both the individual signal and the price become less informative) and investors put less weight on their signal relative to their hedging need.

Note that the price informativeness in a symmetric equilibrium is

$$\sigma_{\hat{p}}^2 = \frac{1}{\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2}\sigma_\delta^2$$

and

$$\frac{d\sigma_{\hat{p}}^2}{d\sigma_\varepsilon^2} = = -2\frac{1}{\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^3}\sigma_\delta^2\frac{d\left[\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right]}{d\sigma_\varepsilon^2} = -2\frac{1}{\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}\sigma_{\hat{p}}^2\frac{d\left[\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right]}{d\sigma_\varepsilon^2} > 0$$

A lower precision of the private signals received by investors in the economy decreases the precision of the price. Then,

$$\frac{d\mathbb{V}ar\left[\theta|s_i, p\right]}{d\sigma_\varepsilon^2} = \mathbb{V}ar\left[\theta|s_i, p\right]^2\left(\frac{1}{\sigma_{\hat{p}}^2}\right)^2\frac{d\sigma_{\hat{p}}^2}{d\sigma_\varepsilon^2} > 0$$

Note that

$$\left(-2\sigma_\theta^2\left(1-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2\overbrace{\frac{d\left[\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right]}{d\sigma_\varepsilon^2}}^{<0}-2\left(\gamma_i-\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)\frac{d\left[\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right]}{d\sigma_\varepsilon^2}\sigma_\delta^2+2m\overbrace{\frac{dm}{d\sigma_\varepsilon^2}}^{>0}\right)$$

In a symmetric equilibrium, i.e., if $\sigma_{\varepsilon j}^2=\sigma_\varepsilon^2\forall j\neq i$,

$$\begin{aligned}
m &= \left(1-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)\overline{\theta}-\frac{\int\tilde{\psi}_i dF(i)}{\overline{\alpha_p}} \\
&= \left(1-\frac{\mathbb{V}ar\left[\theta|s_i,p\right]}{\frac{1}{\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}}}\right)\overline{\theta}-\frac{1}{\sigma_\theta^2}\bar{\theta}\mathbb{V}ar\left[\theta|s_i,p\right]+\gamma\mathbb{V}ar\left[\theta|s_i,p\right]Q \\
&= \left(1-\mathbb{V}ar\left[\theta|s_i,p\right]\left(\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}+\frac{1}{\sigma_\theta^2}\right)\right)\overline{\theta}+\gamma\mathbb{V}ar\left[\theta|s_i,p\right]Q \\
&= \gamma\mathbb{V}ar\left[\theta|s_i,p\right]Q
\end{aligned}$$

and, thus,

$$\frac{dm}{d\sigma_\varepsilon^2}=\gamma Q\frac{d\mathbb{V}ar\left[\theta|s_i,p\right]}{d\sigma_\varepsilon^2}>0.$$

$$\begin{aligned}
\frac{d\left[\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right]}{d\sigma_\varepsilon^2} &= \frac{d\left[\frac{\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}}{\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}+\frac{1}{\sigma_\theta^2}}\right]}{d\sigma_\varepsilon^2}=\frac{\partial\left[\frac{\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}}{\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}+\frac{1}{\sigma_\theta^2}}\right]}{\partial\sigma_\varepsilon^2}+\frac{\partial\left[\frac{\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}}{\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}+\frac{1}{\sigma_\theta^2}}\right]}{\partial\sigma_{\hat{p}}^2}\frac{d\sigma_{\hat{p}}^2}{d\sigma_\varepsilon^2} \\
&= -\left(\frac{1}{\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}+\frac{1}{\sigma_\theta^2}}\right)^2\left(\left(\frac{1}{\sigma_\varepsilon^2}\right)^2+\left(\frac{1}{\sigma_{\hat{p}}^2}\right)^2\frac{d\sigma_{\hat{p}}^2}{d\sigma_\varepsilon^2}\right)\frac{1}{\sigma_\theta^2}<0
\end{aligned}$$

$$\begin{aligned}
\frac{d\left[\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right]}{d\sigma_\varepsilon^2} &= \frac{d\left[\gamma\left(1+\frac{\sigma_\varepsilon^2}{\sigma_{\hat{p}}^2}\right)\right]}{d\sigma_\varepsilon^2}=\gamma\frac{-\sigma_\varepsilon^2\frac{d\sigma_{\hat{p}}^2}{d\sigma_\varepsilon^2}+\sigma_{\hat{p}}^2}{\left(\sigma_{\hat{p}}^2\right)^2} \\
&= \frac{\gamma}{\sigma_{\hat{p}}^2}\left(2\frac{\sigma_\varepsilon^2}{\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)}\frac{d\left[\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right]}{d\sigma_\varepsilon^2}+1\right) \\
&= \frac{\gamma}{\sigma_{\hat{p}}^2}\left(-2\gamma\sigma_\varepsilon^2\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\frac{\left(\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_\delta^2}\left(\frac{\overline{\alpha_s}}{\overline{\alpha_h}}\right)^2\right)}{1+2\frac{\sigma_\varepsilon^2}{\sigma_{\hat{p}}^2}\gamma\frac{\overline{\alpha_s}}{\overline{\alpha_h}}}+1\right)
\end{aligned}$$

We must check that

$$X = -2\sigma_\theta^2\left(1-\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)\frac{d\left[\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right]}{d\sigma_\varepsilon^2} - 2\left(\gamma_i - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)\frac{d\left[\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right]}{d\sigma_\varepsilon^2}\sigma_\delta^2 > 0$$

Using the equilibrium expressions for $\frac{\overline{\alpha_s}}{\overline{\alpha_p}}$ and $\frac{\overline{\alpha_h}}{\overline{\alpha_p}}$

$$\begin{aligned} X &= -2\sigma_\theta^2\left(1-\frac{\overline{\mathbb{V}ar}}{\frac{1}{\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}}}\right)\frac{d\left[\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right]}{d\sigma_\varepsilon^2} - 2\gamma\left(1-1-\frac{\sigma_\varepsilon^2}{\sigma_{\hat{p}}^2}\right)\frac{d\left[\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right]}{d\sigma_\varepsilon^2}\sigma_\delta^2 \\ &= -2\sigma_\theta^2\left(1-\frac{\overline{\mathbb{V}ar}}{\frac{1}{\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}}}\right)\frac{d\left[\frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right]}{d\sigma_\varepsilon^2} + 2\gamma\frac{\sigma_\varepsilon^2}{\sigma_{\hat{p}}^2}\frac{d\left[\frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right]}{d\sigma_\varepsilon^2}\sigma_\delta^2 > 0 \end{aligned}$$

where $\overline{\mathbb{V}ar} \equiv \frac{1}{\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}}$.

Theorem 4 (Effect of trading costs with endogenous information acquisition)

a) After having characterize the unique equilibrium in linear strategies, the fact that $\frac{d\sigma_{\varepsilon i}^2}{dc} < 0$ is sufficient to show that investors acquire less information when trading costs are higher. The comparative statics on $\sigma_{\hat{p}}^2$ are is in the benchmark model with ex-ante symmetric investors.

Proofs: Section 5

Random heterogeneous priors

The problem solved by an investor i, given the realization of his prior, is given by

$$\max_{q_{1i}}\left(\mathbb{E}\left[\theta|\overline{\theta}_i, s_i, p\right] - \gamma_i h_i - p\right)q_{1i} - \frac{c}{2}\left(\Delta q_{1i}\right)^2 - \frac{\gamma}{2}\mathbb{V}ar_i\left[\theta|\overline{\theta}_i, s_i, p\right]q_{1i}^2$$

The first order condition for investor i is given by

$$q_{1i} = \frac{\mathbb{E}\left[\theta|\overline{\theta}_i, s_i, p\right] - p - \gamma h_i + cq_{0i}}{c + \gamma\mathbb{V}ar_i\left[\theta|\overline{\theta}_i, s_i, p\right]}$$

In a symmetric equilibrium in linear strategies, investors portfolio demands take the form

$$q_{1i} = \alpha_\theta\overline{\theta_i} + \alpha_s s_i - \alpha_h h_i - \alpha_p p + \psi$$

Market clearing, $\int_i q_{1i}di = Q$, implies that the equilibrium price takes the form

$$p = \frac{\alpha_\theta}{\alpha_p}\overline{\theta} + \frac{\alpha_s}{\alpha_p}\theta - \frac{\alpha_h}{\alpha_p}\delta - \frac{\tilde{\psi}}{\alpha_p},$$

where $\overline{\psi} = \psi - Q$. A relevant equilibrium object of interest is the unbiased signal of the payoff θ contained in the asset price, which we define by $\hat{p}$ as follows

$$\hat{p} = \frac{\alpha_p}{\alpha_s} p - \frac{\tilde{\psi}}{\alpha_s} - \frac{\alpha_\theta}{\alpha_s} \mu_{\overline{\theta}}.$$

Note that $\hat{p}$ is distributed according to $\hat{p}|\theta \sim N\left(\theta, \sigma_{\hat{p}}^2\right)$, where

$$\sigma_{\hat{p}}^2 = \mathbb{V}ar\left[\hat{p}|\theta\right] = \left(\frac{\alpha_\theta}{\alpha_s}\right)^2 \sigma_{\overline{\theta}}^2 + \left(\frac{\alpha_h}{\alpha_s}\right)^2 \sigma_\delta^2.$$

It is the case that that $\mathbb{E}\left[\theta|\overline{\theta}_i, s_i, p\right] = \mathbb{E}\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right]$ and $\overline{\theta}_i, s_i, \hat{p}$ are conditionally independent signals of θ with precisions σ_θ^{-2}, σ_ε^{-2}, and $\sigma_{\hat{p}}^{-2}$ respectively. Then, solving the optimal filtering problem implies that

$$\mathbb{E}\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right] = \frac{\frac{1}{\sigma_\theta^2}\overline{\theta}_i + \frac{1}{\sigma_\varepsilon^2} s_i + \frac{1}{\sigma_{\hat{p}}^2}\hat{p}}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_{\hat{p}}^2}}$$

and

$$\mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right] = \frac{1}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_\varepsilon^2} + \frac{1}{\sigma_{\hat{p}}^2}}$$

This is exactly the same expression we had before with two modifications. First, $\sigma_{\hat{p}}^2$ has a different expression and, second, the prior mean is different for each agent.

Using these expression in investors' optimality condition allows us to write

$$q_{1i} = \frac{\mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right]\left(\frac{1}{\sigma_\theta^2}\overline{\theta}_i + \frac{1}{\sigma_\varepsilon^2} s_i + \frac{1}{\sigma_{\hat{p}}^2}\left(\frac{\alpha_p}{\alpha_s} p - \frac{\tilde{\psi}}{\alpha_s} - \frac{\alpha_\theta}{\alpha_s}\mu_{\overline{\theta}}\right)\right) - p - \gamma h_i + c q_{0i}}{c + \gamma_i \mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, p\right]}$$

Matching coefficients with our initial conjecture, allows us to solve for α_θ, α_s, α_p, α_h and ψ:

$$\alpha_\theta = \frac{\mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right]\frac{1}{\sigma_\theta^2}}{\kappa}$$

$$\alpha_s = \frac{\mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right]\frac{1}{\sigma_\varepsilon^2}}{\kappa}$$

$$\alpha_p = \frac{1 - \mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right]\frac{1}{\sigma_{\hat{p}}^2}\frac{\alpha_p}{\alpha_s}}{\kappa}$$

$$\alpha_h = \frac{\gamma}{\kappa}$$

and

$$\psi = \frac{\mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right]\left(-\frac{\tilde{\psi}}{\alpha_s} - \frac{\alpha_\theta}{\alpha_s}\mu_{\overline{\theta}}\right) + c q_0}{\kappa}$$

where we define

$$\kappa = c + \gamma \mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, p\right]$$

The value of ψ is the same as in the benchmark model with the addition of $\alpha_\theta \overline{\theta}$.

Price informativeness

Theorem 5. (Irrelevance theorem with with random heterogeneous prior)

a) We can write the ratios $\frac{\alpha_\theta}{\alpha_s}$ and $\frac{\alpha_h}{\alpha_s}$ as follows:

$$\frac{\alpha_\theta}{\alpha_s} = \frac{\sigma_\varepsilon^2}{\sigma_\theta^2} \qquad \text{and} \qquad \frac{\alpha_h}{\alpha_s} = \frac{\gamma}{\mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right]\frac{1}{\sigma_\varepsilon^2}},$$

which shows that both ratios are independent of the trading cost c, because$\mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right]$ does not depend on c, since $\sigma_{\hat{p}}^2$ is independent of c.

b) We can write the relevant ratios as follows

$$\frac{\alpha_\theta}{\alpha_p} = \frac{\mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right]\frac{1}{\sigma_\theta^2}}{1 - \mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right]\frac{1}{\sigma_{\hat{p}}^2}\frac{\alpha_p}{\alpha_s}}$$

$$\frac{\alpha_s}{\alpha_p} = \frac{\mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right]\frac{1}{\sigma_\varepsilon^2}}{1 - \mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right]\frac{1}{\sigma_{\hat{p}}^2}\frac{\alpha_p}{\alpha_s}}$$

$$\frac{\alpha_h}{\alpha_p} = \frac{\gamma}{1 - \mathbb{V}ar\left[\theta|\overline{\theta}_i, s_i, \hat{p}\right]\frac{1}{\sigma_{\hat{p}}^2}\frac{\alpha_p}{\alpha_s}}$$

and

$$\frac{\tilde{\psi}}{\alpha_s} = -\frac{\left(\frac{\alpha_\theta}{\alpha_s}\mu_{\overline{\theta}} + \gamma Q\right)}{1 + \frac{1}{\sigma_\varepsilon^2}}$$

Given that all these ratios are independent of c, it follows that the realized distribution of p is independent of c. Then, the irrelevance results from the benchmark model follow when investors have heterogeneous priors.

Strategic investors

The problem solved by a strategic investor i is given by

$$\max_{q_{1i}} \left(\mathbb{E}\left[\theta|s_i, p\right] - \gamma_i h_i - p_{-i}\right) q_{1i} + p_{-i} q_{0i} - \frac{c}{2}\left(\Delta q_{1i}\right)^2 - \frac{\gamma_i}{2}\mathbb{V}ar\left[\theta|s_i, p\right]\left(q_{1i}\right)^2,$$

where p_{-i} is the residual demand faced by investor i given the strategies of all other investors $j \neq i$.

The first order condition for investor i is given by

$$\mathbb{E}\left[\theta|s_i, p\right] - \gamma_i h_i - p_{-i} - c\Delta q_{1i} - \gamma_i \mathbb{V}ar\left[\theta|s_i, p\right] q_{1i} - \frac{\partial p_{-i}}{\partial q_{1i}}\Delta q_{1i} = 0,$$

which can be alternatively written as

$$q_{1i} = \frac{\mathbb{E}\left[\theta|s_i,p\right] - \gamma_i h_i - p + \left(c + \frac{\partial p_{-i}}{\partial q_{1i}}\right) q_{0i}}{\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right] + c + \frac{\partial p_{-i}}{\partial q_{1i}}}$$

As in the competitive case, we can express q_{1i} as a linear combination of initial asset holdings and portfolio demands in the case without trading costs as

$$\begin{aligned} q_{1i} &= \underbrace{\frac{c + \frac{\partial p_{-i}}{\partial q_{1i}}}{\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right] + c + \frac{\partial p_{-i}}{\partial q_{1i}}}}_{w_i} q_{0i} + \underbrace{\frac{\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]}{\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right] + c + \frac{\partial p_{-i}}{\partial q_{1i}}}}_{1-w_i} \underbrace{\frac{\mathbb{E}\left[\theta|s_i,p\right] - \gamma_i h_i - p}{\gamma_i \mathbb{V}ar\left[\theta|s_i,p\right]}}_{\hat{q}_{1i}} \\ &= w_i q_{0i} + (1-w_i)\,\hat{q}_{1i} \end{aligned}$$

Note that the demand elasticity of investor i is given by $\frac{\partial q_{1i}}{\partial p} = -\frac{1}{\gamma_i \mathbb{V}ar[\theta|s_i,p]+c}$ and that we can write in net terms the demand by investor i for the risky asset

$$\Delta q_{1i} = q_{1i} - q_{0i} = (1-w_i)\left(\hat{q}_{1i} - q_{0i}\right)$$

In a symmetric equilibrium in linear strategies, investors portfolio demands take the form

$$q_{1i} = \alpha_{si} s_i - \alpha_{hi} h_i - \alpha_{pi} p + \psi_i$$

Therefore, market clearing implies that

$$\sum_{i=1}^{N} \left(\alpha_{si} s_i - \alpha_{hi} h_i - \alpha_{pi} p + \psi_i\right) = Q$$

or solving for the equilibrium price

$$p = \frac{\sum_{i=1}^{N} \left(\alpha_{si} s_i - \alpha_{hi} h_i + \psi_i - \frac{Q}{N}\right)}{\sum_{i=1}^{N} \alpha_{pi}}. \tag{34}$$

In this case, the residual demand for investor i is given by

$$p_{-1} = \frac{\sum_{j\neq i} \left(\alpha_{sj} s_j - \alpha_{hj} h_j + \psi_j - \frac{Q}{N}\right) + q_i}{\sum_{j\neq i} \alpha_{pj}},$$

which allows us to write the price impact terms for investor i as

$$\frac{\partial p_{-i}}{\partial q_{1i}} = \frac{1}{\sum_{j\neq i} \alpha_{pj}}.$$

For a variable l, we define $\alpha_\ell^N = \frac{\sum_{i=1}^{N} \alpha_{\ell i}}{N}$. Note that $\lim_{N\to\infty} \alpha_\ell^N = \overline{\alpha_\ell}$. Then, given the expression for the equilibrium price in (34) , we can write the distribution of the price p as

$$p \sim N\left(\mu_p, \sigma_p^2\right),$$

where

$$\mu_p = \frac{\alpha_s^N \overline{\theta} + \sum_{i=1}^N \frac{1}{N}\left(\psi_i - \frac{Q}{N}\right)}{\alpha_p^N}$$

and

$$\sigma_p^2 = \frac{\left(\alpha_s^N\right)^2 \sigma_\theta^2 + \left(\alpha_h^N\right)^2 \sigma_\delta^2 + \left(\sum_{i=1}^N \frac{\alpha_{si}}{N}\right)^2 \sigma_{\varepsilon i}^2 + \left(\sum_{i=1}^N \frac{\alpha_{hi}}{N}\right)^2 \sigma_{hi}^2}{\left(\alpha_p^N\right)^2}$$

The conditional distribution of the equilibrium price p given the fundamental θ follows

$$p|\theta \sim N\left(\frac{\alpha_s^N \theta + \sum_{i=1}^N \frac{1}{N}\left(\psi_i - Q\right)}{\alpha_p^N}, \sigma_{p|\theta}^2\right),$$

where

$$\sigma_{p|\theta}^2 = \frac{\left(\alpha_h^N\right)^2 \sigma_\delta^2 + \sum_{i=1}^N \left(\frac{\alpha_{si}}{N}\right)^2 \sigma_{\varepsilon i}^2 + \sum_{i=1}^N \left(\frac{\alpha_{hi}}{N}\right)^2 \sigma_{hi}^2}{\left(\alpha_p^N\right)^2}$$

Importantly, note that with a finite number of investors the price is not fully revealing even if $\sigma_\delta^2 = 0$. There is noise coming from the average signal and the average hedging need.

In this strategic we must make an adjustment to account for investors i signal. The unbiased signal, independent of s_i, contained in p is $\hat{p}_i$, where

$$\begin{aligned}\hat{p}_i &= \frac{\alpha_p^{N-i}}{\alpha_s^{N-i}} p - \frac{1}{\alpha_s^{N-i}} \frac{q_{1i}}{N-1} - \frac{\sum_{j \neq i} \frac{1}{N-1}\left(\psi_j - Q\right)}{\alpha_s^{N-i}} \\ &= \frac{\sum_{j \neq i}\left(\frac{\alpha_{sj}}{N-1} s_j - \frac{\alpha_{hj}}{N-1} h_j\right)}{\alpha_s^{N-i}}\end{aligned}$$

where we define for a variable l, $\alpha_\ell^{N-i} = \frac{\sum_{j \neq i} \alpha_{\ell j}}{N-1}$. We can thus write

$$\hat{p}_i|\theta \sim N\left(\theta, \sigma_{\hat{p}i}^2\right),$$

where

$$\sigma_{\hat{p}i}^2 = \frac{\left(\alpha_h^{N-i}\right)^2 \sigma_\delta^2 + \sum_{j \neq i}\left(\frac{\alpha_{sj}}{N-1}\right)^2 \sigma_{\varepsilon j}^2 + \sum_{j \neq i}\left(\frac{\alpha_{hj}}{N-1}\right)^2 \sigma_{hj}^2}{\left(\alpha_s^{N-i}\right)^2}$$

Solving the optimal filtering problem from the perspective of investor i allows us to write

$$\mathbb{E}\left[\theta|s_i, p\right] = \mathbb{E}\left[\theta|s_i, \hat{p}_i\right] = \frac{\frac{1}{\sigma_\theta^2}\overline{\theta} + \frac{1}{\sigma_{\varepsilon i}^2} s_i + \frac{1}{\sigma_{\hat{p}i}^2}\hat{p}_i}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_{\varepsilon i}^2} + \frac{1}{\sigma_{\hat{p}i}^2}} = \frac{\frac{1}{\sigma_\theta^2}\overline{\theta} + \frac{1}{\sigma_{\varepsilon i}^2} s_i + \frac{1}{\sigma_{\hat{p}i}^2}\left(\frac{\alpha_p^{N-i}}{\alpha_s^{N-i}} p - \frac{1}{\alpha_s^{N-i}}\frac{q_{1i}}{N-1} - \frac{\sum_{j \neq i} \frac{1}{N-1}\left(\psi_j - \frac{Q}{N-1}\right)}{\alpha_s^{N-i}}\right)}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_{\varepsilon i}^2} + \frac{1}{\sigma_{\hat{p}i}^2}}$$

and

$$\mathbb{V}ar\left[\theta|s_i, p\right] = \mathbb{V}ar\left[\theta|s_i, \hat{p}_i\right] = \frac{1}{\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_{\varepsilon i}^2} + \sigma_{\hat{p}i}^2}.$$

Using these expressions in the first order condition for investor i, we can write

$$q_{1i} = \frac{\mathbb{V}ar\left[\theta|s_i, p\right]\left(\frac{1}{\sigma_\theta^2}\overline{\theta} + \frac{1}{\sigma_{\varepsilon i}^2}s_i + \frac{1}{\sigma_{\hat{p}i}^2}\left(\frac{\alpha_p^{N-i}}{\alpha_s^{N-i}}p - \frac{\sum_{j\neq i}\frac{1}{N-1}\left(\psi_j - \frac{Q}{N-1}\right)}{\alpha_s^{N-i}}\right)\right) - \gamma_i h_i - p + cq_{0i}}{\left(\left(\gamma_i + \frac{1}{\sigma_{\hat{p}}^2}\frac{1}{\alpha_s^{N-i}}\frac{1}{N-1}\right)\mathbb{V}ar\left[\theta|s_i, p\right] + c + \frac{1}{\sum_{j\neq i}\alpha_{pj}}\right)}$$

or

$$q_{1i} = \frac{\mathbb{V}ar\left[\theta|s_i, p\right]\left(\frac{1}{\sigma_\theta^2}\overline{\theta} + \frac{1}{\sigma_{\varepsilon i}^2}s_i + \frac{1}{\sigma_{\hat{p}}^2}\left(\frac{\alpha_p^{N}}{\alpha_s^{N}}p - \frac{\sum_{i=1}^{N}\frac{1}{N}\left(\psi_i - \frac{Q}{N}\right)}{\alpha_s^{N}}\right)\right) - \gamma_i h_i - p + cq_{0i}}{\hat{\kappa}_i}$$

where

$$\hat{\kappa}_i = \left(\gamma_i + \frac{1}{\sigma_{\hat{p}i}^2}\frac{1}{\alpha_s^{N-i}}\frac{1}{N-1}\right)\mathbb{V}ar\left[\theta|s_i, p\right] + c + \frac{1}{\sum_{j\neq i}\alpha_{pj}}$$

$$\hat{\kappa}_i = \left(\gamma_i + \frac{\alpha_s^{N-i}}{\left(\alpha_h^{N-i}\right)^2\sigma_\delta^2 + \sum_{j\neq i}\left(\frac{\alpha_{sj}}{N-1}\right)^2\sigma_{\varepsilon j}^2 + \sum_{j\neq i}\left(\frac{\alpha_{hj}}{N-1}\right)^2\sigma_{hj}^2}\right)\mathbb{V}ar\left[\theta|s_i, p\right] + c + \frac{1}{\sum_{j\neq i}\alpha_{pj}}$$

Compared to the competitive case the scale effect is dampened by the price impact $\frac{\partial p_{-i}}{\partial q_{1i}} = \frac{1}{\sum_{j\neq i}\alpha_{pj}} > 0$. Formally, $\hat{\kappa}_i > \kappa_i$. The price impact increases effective risk aversion with respect to the competitive case. As expected, note that

$$\lim_{N\to\infty}\frac{1}{\sigma_{\hat{p}i}^2}\frac{1}{\alpha_s^{N-i}}\frac{1}{N-1} = 0$$

Matching coefficients with the guess we get the following system of equations for the set of parameters we conjectured:

$$\alpha_{si} = \frac{\frac{1}{\sigma_{\varepsilon i}^2}\mathbb{V}ar\left[\theta|s_i, p\right]}{\hat{\kappa}_i},$$

$$\alpha_{hi} = \frac{\gamma_i}{\hat{\kappa}_i},$$

$$\alpha_{pi} = \frac{1 - \mathbb{V}ar\left[\theta|s_i, p\right]\frac{1}{\sigma_{\hat{p}i}^2}\frac{\alpha_p^{N-i}}{\alpha_s^{N-i}}}{\hat{\kappa}_i},$$

and

$$\psi_i = \frac{\mathbb{V}ar\left[\theta|s_i, p\right]\left(\frac{1}{\sigma_\theta^2}\overline{\theta} - \frac{1}{\sigma_{\hat{p}i}^2}\frac{\sum_{j\neq i}\frac{1}{N-1}\left(\psi_i - \frac{Q}{N-1}\right)}{\alpha_s^{N-i}}\right) + cq_{0i}}{\hat{\kappa}_i}$$

The price informativeness from the perspective of investor i is given by

$$\sigma_{\hat{p}i}^2 = \frac{\left(\alpha_h^{N-i}\right)^2\sigma_\delta^2 + \sum_{j\neq i}\left(\frac{\alpha_{sj}}{N-1}\right)^2\sigma_{\varepsilon j}^2 + \sum_{j\neq i}\left(\frac{\alpha_{hj}}{N-1}\right)^2\sigma_{hj}^2}{\left(\alpha_s^{N-i}\right)^2}$$

In the symmetric case in which $\gamma_i = \gamma$, $\sigma_{\varepsilon i}^2 = \sigma_\varepsilon^2$, $\sigma_{hi}^2 = \sigma_h^2$ and $q_{0i} = Q$, $\alpha_\ell^{N-i} = \alpha_{\ell i} = \alpha_\ell$, price informativeness is given by

$$\sigma_{\hat{p}}^2 = \left(\frac{\alpha_h}{\alpha_s}\right)^2 \left(\sigma_\delta^2 + \frac{\sigma_h^2}{N-1}\right) + \frac{\sigma_\varepsilon^2}{N-1},$$

which is independent of c. Note that $\lim_{N\to\infty} \sigma_{\hat{p}}^2 = \left(\frac{\alpha_h}{\alpha_s}\right)^2 \sigma_\delta^2$. As expected, this is the same expression we derived in the competitive case.

Theorem 6. (Irrelevance theorem with strategic investors)

a) Note that we can write

$$\frac{\alpha_h}{\alpha_s} = \frac{\gamma_i}{\frac{1}{\sigma_{\varepsilon i}^2}\mathbb{V}ar\left[\theta | s_i, p\right]} = \frac{\gamma_i \left(\frac{1}{\sigma_\theta^2} + \frac{1}{\sigma_{\varepsilon i}^2} + \frac{1}{\left(\frac{\alpha_h}{\alpha_s}\right)^2 \left(\sigma_\delta^2 + \frac{\sigma_h^2}{N-1}\right) + \frac{\sigma_\varepsilon^2}{N-1}}\right)}{\frac{1}{\sigma_{\varepsilon i}^2}}$$

which is independent of c. Therefore, the price informativeness is independent of the trading cost c.

b) Similarly, it also follows that $\frac{\alpha_p}{\alpha_s}$ is independent of c, which proves our result.

Dynamics

The wealth accumulation equations for investors at dates 1 and 2 are

$$\begin{aligned} w_{2i} &= n_{2i} + q_{1i}\theta_2 + w_{1i} - q_{1i}p - \frac{c}{2}\left(\Delta q_{1i}\right)^2 + T_{1i} \\ w_{1i} &= n_{1i} + q_{0i}\left(\theta_1 + p\right) + w_{0i} - q_{-1i}p - \frac{c}{2}\left(\Delta q_{0i}\right)^2 + T_{0i} \end{aligned}$$

The indirect utility of investor i at date 1 is a function of his initial asset holdings q_{0i}. Formally, we can write $V\left(q_{0i}\right)$ as

$$V\left(q_{0i}\right) = \mathbb{E}\left[U_i\left(w_{2i}\right)\right] = \mathbb{E}\left[\mathbb{E}\left[U_i\left(w_{2i}\right)|h_i, s_i, p\right]\right],$$

where

$$\mathbb{E}\left[U_i\left(w_{2i}\right)|h_i, s_{ii}, p\right] = -e^{-\gamma_i\left[\mathbb{E}[n_{2i}] - \frac{\gamma_i}{2}\mathbb{V}ar[n_{2i}] + v_i + T_{1i}\right]} = -e^{-\gamma_i v_i},$$

after assuming that $\mathbb{E}[n_{2i}] - \frac{\gamma_i}{2} Var[n_{2i}] = 0$ and $T_{1i} = 0$, where

$$\begin{aligned} v_i &= \left(\mathbb{E}[\theta|h_i, s_i, p] - \gamma_i h_i - p\right) q_{1i}^* - \frac{\gamma_i}{2} \mathbb{V}ar[\theta|s_i, p] q_{1i} + w_{1i} - \frac{c}{2}\left(\Delta q_{1i}^*\right)^2 \\ &= \left(\mathbb{E}[\theta|h_i, s_i, p] - \gamma_i h_i - p\right) q_{1i}^* - \frac{\gamma_i}{2} \mathbb{V}ar[\theta|s_i, p] q_{1i} + n_{1i} + q_{0i}(\theta_2 + p) \\ &\qquad + w_{0i} - q_{-1i} p - \frac{c}{2}\left(\Delta q_{0i}\right)^2 - \frac{c}{2}\left(\Delta q_{1i}^*\right)^2 \\ &= \frac{1}{2\kappa_i}\left(\sigma_m^2 + m^2\right) - \frac{c}{2}(q_{0i})^2 + \mathbb{E}[p] q_{0i} + w_{0i} - q_{-1i} p + n_{1i} \end{aligned}$$

Where the last line is derived as in the case with endogenous information acquisition, where we define

$$m = \left(1 - \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)\overline{\theta} - \frac{\int \tilde{\psi}_i dF(i)}{\overline{\alpha_p}}$$

and

$$\sigma_m^2 = \sigma_\theta^2 \left(1 - \frac{\overline{\alpha_s}}{\overline{\alpha_p}}\right)^2 + \left(\gamma_i - \frac{\overline{\alpha_h}}{\overline{\alpha_p}}\right)^2 \sigma_\delta^2 + \gamma_i^2 \sigma_{hi}^2 + \left(2\gamma_i \frac{\overline{\alpha_s}}{\overline{\alpha_h}} - 1\right) \mathbb{V}ar[\theta|s_i, p]$$

Because m and σ_m^2 are independent of q_{0i}, the problem solved by investor i investor's problem can be rewritten as

$$\max_{q_{0i}} \mathbb{E}\left[-e^{-\gamma_i\left[n_{1i} + \mathbb{E}[p] q_{0i} - \frac{c}{2}(q_{0i})^2 + \theta_1 q_{0i} - p_0(q_{0i} - q_{-1i}) - \frac{c}{2}(q_{0i} - q_{-1i})^2\right]} | s_0, p_0\right]$$

We define $h_{1i} = \mathbb{C}ov(n_{1i}, \theta_1)$, where

$$h_{1i} = \delta_1 + \varepsilon_{h1i}$$

and

$$\varepsilon_{h1i} \sim N\left(0, \sigma_{h1}^2\right), \ \forall i$$

Therefore, the problem solved by investor i at date 0 can be written as

$$\max_{q_{0i}} -e^{-\gamma_i\left[\mathbb{E}[n_{1i}] - \frac{\gamma_i}{2}\mathbb{V}ar[n_{1i}] + \mathbb{E}[p] q_{0i} - \frac{c}{2}(q_{0i})^2 + \mathbb{E}[\theta_1|s_0, p_0] q_{0i} - p_0(q_{0i} - q_{-1i}) - \frac{c}{2}(q_{0i} - q_{-1i})^2 - \frac{1}{2}\mathbb{V}ar[\theta_1|s_0, p_0] q_{0i}^2 - \gamma_i h_{1i} q_{0i}\right]}$$

or equivalently

$$\max_{q_{0i}} \gamma_i \left[\begin{array}{c} \mathbb{E}[p] q_{0i} - \frac{c}{2}(q_{0i})^2 + \mathbb{E}[\theta_1|s_0, p_0] q_{0i} - p_0(q_{0i} - q_{-1i}) - \frac{c}{2}(q_{0i} - q_{-1i})^2 \\ -\frac{\gamma_i}{2}\mathbb{V}ar[\theta_1|s_0, p_0] q_{0i}^2 - \gamma_i h_{1i} q_{0i} \end{array}\right]$$

The first order condition for this problem is

$$\mathbb{E}[p] - c q_{0i} + \mathbb{E}[\theta_1|s_0, p_0] - p_0 - \gamma_i h_{1i} - \left(c + \gamma_i \mathbb{V}ar[\theta_1|s_0, p_0]\right) q_{0i} + c q_{-1i} = 0$$

Where $\mathbb{E}[p] - c q_{0i}$ represents the marginal benefit from the additional round of trading. We can rewrite investors; optimal portfolio decision as

$$q_{0i} = \frac{\mathbb{E}[p] + \mathbb{E}[\theta_1|s_0, p_0] - \gamma_i h_{1i} - p_0 - c q_{-1i}}{\kappa_{0i}}$$

where we define $\kappa_{0,i} \equiv 2c + \gamma_i \mathbb{V}ar\left[\theta_1|s_0, p_0\right]$. Intuitively, investors become less reluctant to trade at date 0 because the know it is possible that they have to unwind their position at date 1, paying again a trading cost.

In a symmetric equilibrium in linear strategies, investors have a portfolio demand given by

$$q_{0i} = \alpha_{s0}s_{0i} - \alpha_{p0}p_0 - \alpha_{h0}h_{1i} + \psi_0$$

The market clearing condition $\int q_{1i}di = Q$ implies that the equilibrium price takes the form

$$p_0 = \frac{\alpha_{s0}}{\alpha_{p0}}\theta_1 - \frac{\alpha_{h0}}{\alpha_{p0}} + \frac{\psi_0 - Q}{\alpha_{p0}}.$$

We can therefore write $\mathbb{E}\left[\theta_1|s_0, p_0\right] = \mathbb{E}\left[\theta_1|s_0, \hat{p}_0\right]$, where

$$\hat{p}_0 = \frac{\alpha_{p0}}{\alpha_{s0}}p_0 - \frac{\psi_0 - Q}{\alpha_{s0}}$$

is the unbiased signal of θ_1 contained in the price, which is distributed as

$$\hat{p}_0|\theta_1 \sim N\left(\theta_1, \underbrace{\left(\frac{\alpha_{h0}}{\alpha_{s0}}\right)^2 \sigma_{\delta 1}^2}_{\equiv \sigma_{\hat{p}0}^2}\right).$$

Solving the optimal filtering problem from the perspective of investor i allows us to write

$$\mathbb{V}ar\left[\theta_1|s_0, p_0\right] = \frac{1}{\frac{1}{\sigma_{\theta 1}^2} + \frac{1}{\sigma_{\varepsilon 1}^2} + \frac{1}{\sigma_{\hat{p}0}^2}}$$

and

$$\begin{aligned}
\mathbb{E}\left[\theta_1|s_0, p_0\right] &= \frac{\frac{1}{\sigma_{\theta 1}^2}\overline{\theta}_1 + \frac{1}{\sigma_{\varepsilon 1}^2}s_{1i} + \frac{1}{\sigma_{\hat{p}0}^2}\hat{p}_0}{\frac{1}{\sigma_{\theta 1}^2} + \frac{1}{\sigma_{\varepsilon 1}^2} + \frac{1}{\sigma_{\hat{p}0}^2}} \\
&= \frac{\frac{1}{\sigma_{\theta 1}^2}\overline{\theta}_1 + \frac{1}{\sigma_{\varepsilon 1}^2}s_{1i} + \frac{1}{\sigma_{\hat{p}0}^2}\left(\frac{\alpha_{p0}p - (\psi_0 - Q_0)}{\alpha_{s0}}\right)}{\frac{1}{\sigma_{\theta 1}^2} + \frac{1}{\sigma_{\varepsilon 1}^2} + \frac{1}{\sigma_{\hat{p}0}^2}}
\end{aligned}$$

Combining these expressions with the optimal portfolio demand of investors, we get

$$q_{0i} = \frac{\mathbb{E}\left[p\right] + \mathbb{V}ar\left[\theta_1|s_0, p_0\right]\left(\frac{1}{\sigma_{\theta 1}^2}\overline{\theta}_1 + \frac{1}{\sigma_{\varepsilon 1}^2}s_{1i} + \frac{1}{\sigma_{\hat{p}0}^2}\left(\frac{\alpha_{p0}p - (\psi_0 - Q_0)}{\alpha_{s0}}\right)\right) - \gamma_i h_{1i} - p_0 - cq_{-1i}}{\kappa_{0i}}$$

and matching coefficients gives

$$\alpha_{s0} = \frac{\mathbb{V}ar\left[\theta_1|s_0, p_0\right]\frac{1}{\sigma_{\varepsilon 1}^2}}{\kappa_{0i}}$$

$$\alpha_{h0} = \frac{\gamma_i}{\kappa_{0i}}$$

$$\alpha_{p0} = \frac{1 - \mathbb{V}ar\left[\theta_1|s_0, p_0\right] \frac{1}{\sigma^2_{\hat{p}0}} \frac{\alpha_{p0}}{\alpha_{s0}}}{\kappa_{0i}}$$

$$\psi_0 = \frac{\mathbb{E}\left[p\right] + \mathbb{V}ar\left[\theta_1|s_0, p_0\right] \left(\frac{1}{\sigma^2_{\theta 1}}\overline{\theta}_1 - \frac{1}{\sigma^2_{\hat{p}0}} \frac{\psi_0 - Q}{\alpha_{s0}}\right) - cq_{-1i}}{\kappa_{0i}}$$

The price informativeness in period 0 depends on $\frac{\alpha_h}{\alpha_s}$. If all investors are ex-ante identical, $\kappa_{0i} = \kappa_0$, $\forall i$. Therefore, we can write

$$\frac{\alpha_h}{\alpha_s} = \frac{\gamma\sigma^2_{\varepsilon 1}}{\mathbb{V}ar\left[\theta_1|s_0, p_0\right]} \tag{35}$$

which is independent of c. Similarly, we can write

$$\begin{aligned} \frac{\alpha_{p0}}{\alpha_{s0}} &= \frac{1 - \mathbb{V}ar\left[\theta_1|s_0, p_0\right] \frac{1}{\sigma^2_{\hat{p}0}} \frac{\alpha_{p0}}{\alpha_{s0}}}{\mathbb{V}ar\left[\theta_1|s_0, p_0\right] \frac{1}{\sigma^2_{\varepsilon 1}}} \\ &= \frac{\mathbb{V}ar\left[\theta_1|s_0, p_0\right]}{\left(\frac{1}{\sigma^2_{\varepsilon 1}} + \frac{1}{\sigma^2_{\hat{p}0}}\right)} \end{aligned} \tag{36}$$

And finally, note that

$$\begin{aligned} \frac{\psi_0 - Q}{\alpha_{s0}} &= \frac{\mathbb{E}\left[p\right] + \mathbb{V}ar\left[\theta_1|s_0, p_0\right] \left(\frac{1}{\sigma^2_{\theta 1}}\overline{\theta}_1 - \frac{1}{\sigma^2_{\hat{p}0}} \frac{(\psi_0 - Q_0)}{\alpha_{s0}}\right) - cQ - \kappa_0 Q}{\mathbb{V}ar\left[\theta_1|s_0, p_0\right] \frac{1}{\sigma^2_{\varepsilon 1}}} \\ &= \frac{\mathbb{E}\left[p\right] + \mathbb{V}ar\left[\theta_1|s_0, p_0\right] \left(\frac{1}{\sigma^2_{\theta 1}}\overline{\theta}_1 - \frac{1}{\sigma^2_{\hat{p}0}} \frac{(\psi_0 - Q_0)}{\alpha_{s0}}\right) - \gamma\mathbb{V}ar\left[\theta_1|s_0, p_0\right] Q}{\mathbb{V}ar\left[\theta_1|s_0, p_0\right] \frac{1}{\sigma^2_{\varepsilon 1}}}, \end{aligned}$$

is also independent of c. Hence, the price informativeness and price volatility are independent of the trading costs c.

Theorem 7. (Irrelevance theorem in dynamic environment)

a) At date 1, the results of the benchmark model apply. At date 0, our result follows from equation (35).

b) At date 1, the results of the benchmark model apply. At date 0, our results follows from equations (35) and (36).

Online appendix (not for publication)

Equilibrium with classic noise trading

For the question we study, it is important that we introduce aggregate hedging needs to have a meaningful filtering problem, as oppose to model directly some form of "noise demand". In particular, what matters for our irrelevance result is that the source of noise that makes the filtering problem non trivial affects the primitives of the portfolio problem solved by investors.

Here, we eliminate the aggregate uncertainty arising from hedging needs and solve our model using the more standard stochastic noisy demand for the risky asset. We specifically work with the symmetric competitive benchmark model, and we further assume that $\sigma_{hi}^2 = 0$ and $\delta = 0$. We also introduce noise traders, modeled as a random variable x, such that

$$x \sim N\left(0, \sigma_x^2\right)$$

These assumptions prevent the equilibrium from being fully revealing. We guess and verify that investors' portfolio demands take the form

$$q_{1i} = \alpha_s s_i - \alpha_p p + \psi,$$

where α_s, α_p and ψ are positive scalars. The market clearing condition $\int q_{1i} di = Q$ implies an equilibrium price of the form

$$p = \frac{\alpha_s}{\alpha_p}\theta + \frac{\tilde{\psi}}{\alpha_p} + \frac{x}{\alpha_p},$$

where we define $\tilde{\psi} = \psi - Q$.

We can write the distribution of the price p as

$$p \sim N\left(\frac{\alpha_s}{\alpha_p}\bar{\theta} + \frac{\tilde{\psi}}{\alpha_p}, \left(\frac{\alpha_s}{\alpha_p}\right)^2 \sigma_\theta^2 + \left(\frac{1}{\alpha_p}\right)^2 \sigma_x^2\right)$$

While the conditional distribution of the equilibrium price p given the fundamental θ follows

$$p|\theta \sim N\left(\frac{\alpha_s}{\alpha_p}\theta + \frac{\tilde{\psi}}{\alpha_p}, \left(\frac{1}{\alpha_p}\right)^2 \sigma_x^2\right)$$

We again denote by $\hat{p} = \frac{\alpha_p}{\alpha_s} p - \frac{\tilde{\psi}}{\alpha_s}$ the unbiased signal of θ, which is distributed as follows

$$\hat{p}|\theta \sim N\left(\theta, \sigma_{\hat{p}}^2\right), \quad \text{where} \quad \sigma_{\hat{p}}^2 = \left(\frac{1}{\alpha_p}\right)^2 \sigma_x^2 \tag{37}$$

As in our benchmark model

$$\mathbb{E}\left[\theta|s_i,p\right]=\mathbb{E}\left[\theta|s_i,\hat{p}\right]=\frac{\frac{1}{\sigma_\theta^2}\bar{\theta}+\frac{1}{\sigma_\varepsilon^2}s_i+\frac{1}{\sigma_{\hat{p}}^2}\hat{p}}{\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}} \quad \text{and} \quad \mathbb{V}ar\left[\theta|s_i,p\right]=\mathbb{V}ar\left[\theta|s_i,\hat{p}\right]=\frac{1}{\frac{1}{\sigma_\theta^2}+\frac{1}{\sigma_\varepsilon^2}+\frac{1}{\sigma_{\hat{p}}^2}}$$

Substituting these expressions in investors' demand functions, given by equation (5), we can write q_{1i} as

$$q_{1i}=\frac{\left(\frac{1}{\sigma_\theta^2}\bar{\theta}+\frac{1}{\sigma_\varepsilon^2}s_i+\frac{1}{\sigma_{\hat{p}}^2}\left(\frac{\alpha_p}{\alpha_s}p-\frac{\tilde{\psi}}{\alpha_s}\right)\right)\mathbb{V}ar\left[\theta|s_i,p\right]-p+cq_0}{\gamma\mathbb{V}ar\left[\theta|s_i,p\right]+c}$$

To ease the notation, we define $\kappa\equiv\gamma\mathbb{V}ar\left[\theta|s_i,p\right]+c$. As in our benchmark model, matching coefficients with our initial guess in equation (19), we are able to characterize α_s, α_h, α_p and ψ as the solution to a system of equations. It is clear from equation (37) that $\frac{d\sigma_{\hat{p}}^2}{dc}$ is negative, because α_p is a decreasing function of c, that is

$$\frac{d\sigma_{\hat{p}}^2}{dc}$$

Remark. The model with exogenously given noise trading demand spuriously concludes that high trading costs decrease price informativeness and price volatility. It implicitly models the behavior of a group of investors in the economy as if they were fully inelastic to trading costs.

Equilibrium without learning with competitive investors

For reference, we characterize as a benchmark the equilibrium of the competitive economy when there is no learning. To ease the notation, we use $\mathbb{E}_i\left[\theta\right]$ for $\mathbb{E}\left[\theta|s_i,p\right]$ and $\mathbb{V}ar_i\left[\theta\right]$ for $\mathbb{V}ar_i\left[\theta|s_i,p\right]$. For reference, we derive market clearing in the case without learning as follows:

$$\int\left(1-w_i\right)\left(\hat{q}_{1i}-q_{0i}\right)di=\int\left(1-w_i\right)\left(\frac{\mathbb{E}_i\left[\theta\right]-\gamma_i h_i-p}{\gamma_i\mathbb{V}ar_i\left[\theta\right]}-q_{0i}\right)di=0$$

$$\int\Gamma_i\left(\mathbb{E}_i\left[\theta\right]-\gamma_i h_i-p-\gamma_i\mathbb{V}ar_i\left[\theta\right]q_{0i}\right)di=0,$$

where $\Gamma_i=\frac{1-w_i}{\gamma_i\mathbb{V}ar_i[\theta]}=\frac{1}{\gamma_i\mathbb{V}ar[\theta]+c}$ and $\int\Gamma_i di=\int\frac{1}{\gamma_i\mathbb{V}ar_i[\theta]+c}di$. We can write the equilibrium price as

$$p=\int g_i\left(\mathbb{E}_i\left[\theta\right]-\gamma_i h_i-\gamma_i\mathbb{V}ar_i\left[\theta\right]q_{0i}\right)di,$$

where $g_i=\frac{\Gamma_i}{\int\Gamma_i di}=\frac{\frac{1}{\gamma_i\mathbb{V}ar_i[\theta]+c}}{\int\frac{1}{\gamma_i\mathbb{V}ar_i[\theta]+c}di}$. g_i is the contribution of investor i to the harmonic average of demand sensitivities. When $\gamma_i=\gamma$, we can write $g_i=\frac{\frac{1}{\gamma\mathbb{V}ar_i[\theta]+c}}{\int\frac{1}{\gamma\mathbb{V}ar_i[\theta]+c}di}=1$. In the general case,

$$\frac{dp}{dc}=\int\frac{dg_i}{dc}\left(\mathbb{E}_i\left[\theta\right]-\gamma_i h_i-\gamma_i\mathbb{V}ar_i\left[\theta\right]q_{0i}\right)di,$$

where

$$\frac{dg_i}{dc} = \frac{-\frac{1}{(\gamma_i \mathbb{V}ar_i[\theta]+c)^2}\int \frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c}di + \frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c}\int \frac{1}{(\gamma_i \mathbb{V}ar_i[\theta]+c)^2}di}{\left(\int \frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c}di\right)^2}$$

$$= \frac{1}{\gamma_i \mathbb{V}ar_i [\theta] + c} \frac{-\frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c}\int \frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c}di + \int \frac{1}{(\gamma_i \mathbb{V}ar_i[\theta]+c)^2}di}{\left(\int \frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c}di\right)^2}$$

So $\frac{dg_i}{dc} \gtreqless 0$ if $\int \frac{1}{(\gamma_i \mathbb{V}ar_i[\theta]+c)^2}di \gtreqless \frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c}\int \frac{1}{\gamma_i \mathbb{V}ar_i[\theta]+c}di$. Note that

$$\int \frac{dg_i}{dc}di = 0$$

So we can write

$$\frac{dp}{dc} = \mathbb{C}ov_i \left[\frac{dg_i}{dc}, \mathbb{E}_i [\theta] - \gamma_i h_i - \gamma_i \mathbb{V}ar_i [\theta] q_{0i}\right]$$

Therefore, the price goes up or down when c increases depending on the cross-sectional covariance of $\frac{dg_i}{dc}$, which captures the change induces in demand elasticities, with $\mathbb{E}_i [\theta] - \gamma_i h_i - \gamma_i \mathbb{V}ar_i [\theta] q_{0i}$, which captures the desire for trading unrelated to prices. The main takeaway of this analysis is the following.

Remark. In the model without learning, the equilibrium price is independent of the level of trading costs as long as $\gamma_i \mathbb{V}ar_i [\theta]$ is constant.

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