The Economics of Club Bidding in Private Equity

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October, 2009

We are grateful for helpful comments to seminar participants at the Arizona State University and the European University Institute. Contacts: R. Marquez (rmarquez@bu.edu) and R. Singh (rajsingh@umn.edu).

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Abstract

Acquisitions by private equity (PE) firms have gained prominence in recent years. Many of these acquisitions have been conducted by "clubs," where a number of PE firms join together to submit a single bid. We present a novel analysis of the economics of club bidding by private equity firms based on the notion that club formation may create value at the target firm by allowing the different dimensions of value creation of each individual PE firm to be aggregated by the club. The tradeoff is that forming a club decreases competition since it reduces the number of firms that submit a bid for the target. Taking the number of bidders as exogenous, we show that when there is a sufficient number of PE firms, allowing a club to form benefits not only its members but also the target shareholders. However, we also show that when bidding is costly and bidder entry thus endogenous, club formation will generally be bad for the seller. Combining these two sets of results, our findings are useful for understanding the recent evidence analyzing the pro- or anti-competitive effects of club bidding behavior.

Keywords: *Private equity; Club bidding; Joint bidding; Mergers & Acquisitions; Takeover Auctions*

JEL codes: *G24, G34*

1 Introduction

Private equity (PE) firms have become significant players in the market for mergers and acquisitions. Over the last few years leading up to the market turmoil of 2008, these firms raised large amounts of capital from investors: for instance, the top 50 PE firms (as ranked in 2008) had raised about $800 billion in the time period from January 2003 to May 2008.[1] Given that the private equity firms often undertake highly leveraged transactions, this translates into the ability to enter into deals worth somewhere in the range of $3 to $4 trillion dollars. The goal for these private equity firms is to identify underperforming target firms, improve operations, and then profit from the sale (or public offering) of the acquired targets.

At the same time, there has been a growing tendency for private equity firms to form "clubs" and submit bids for target firms in conjunction with other PE firms.[2] In fact, the incidence of club bidding has been sufficiently widespread as to attract the attention of the U.S. Department of Justice, expressing concern that such "bidding rings" may reduce competition in the market for corporate control and therefore harm target shareholders (see Bailey, 2007 for a discussion). The argument is simple: when a club is formed, all things equal there are fewer bidders left who can make offers. Therefore, there is less competition for the target firm, leading to lower bid prices and reduced (target) shareholder value.

This heuristic, however, assumes that nothing changes as a result of the club's formation, so that the only impact is through a reduction in competition. We argue that, particularly for the case of private equity, a club may be formed as a way of exploiting complementary skills and synergies that can be used to create value at the target firm. Joining two (or more) PE firms for the purpose of submitting a bid and acquiring the target may allow for both firms' skill to be used, to the extent that each PE firm may have different expertise that can be used to create value. Indeed, this "value creation" of PE clubs has been recognized by various authors. For example,

> "'Good' club deals will be those formed in order to enhance competition, provide additional buyers, and *take advantage of any number of firm-specific benefits that are compounded when private equity firms bid as a team.* (emphasis added)," (Thane D.

[1] See, The List: Top Private Equity Firms, Financial Week (May 12, 2008), http://www.financialweek.com/apps/pbcs.dll/article?AID=/20080512/CHARTWIDE/771595897

[2] Officer et al. (2008) document about the same number of club deals as the number of deals in which a private equity bidder is acting as a sole bidder (31 club deals and 38 sole PE deals) in the four year period from 2002 to 2006.

Scott, Partner, Edwards Angell Palmer & Dodge LLP in *Competition Law 360°*, publisher Portfolio Media Inc.).

"In addition, club deals enable the private equity firms to spread risks and make the most of complementary skills and management experience that members of the consortia may have." (SJ Berwin, 5/3/2008, AltAssets).

This value creation effect derived from the formation of a PE firm bidding club can offset the possible reduction in competition and lead to increased rather than decreased target shareholder value. The goal of this paper is to study the economic forces underlying such club deals, and to analyze the effect of club formation on the seller's expected revenue, the club's expected profit, and the impact on the remaining non-club bidders.

To study these issues, we present a model that incorporates two main features. First, when a club is formed the value of each bidder in the club is aggregated, at least partly, into the club's value for the target. In other words, we allow for value creation as a result of the club's formation. Second, we assume that there is a limited number of bidders who might benefit from acquiring the target. This would be the case if, for instance, value creation depends on synergies that are in limited supply, or if bidders face financial constraints that limit the number of firms that can participate, or if there are costs to bidding. While this second feature is likely relevant in many takeover contexts, the first is particularly applicable in the context of private equity, where each firm's "private value" from the acquisition is derived from the specific way in which it is able to increase value at the target. As long as the PE firms do not have perfectly overlapping sets of expertise, forming a club should allow for each firm's skill to be used at least to some extent.

This simple setup delivers a number of interesting results. When a club forms, two effects operate. First, there is the "value creation" effect described above. Second, however, there is a "competition effect" since the formation of the club decreases competition by reducing the number of bids that actually get submitted.[3] This reduces the likelihood that a high bid for the target firm will be submitted. Which effect dominates then depends on how many PE firms are willing or are able to submit bids for the target firm.

Taking the number of PE bidders as exogenous, we show that when there is a small number of bidders, allowing a club to form is bad for the target shareholders. This occurs because a further

[3] For this reason club bidding is sometimes referred to as "collusive bidding."

reduction in the number of bidders when there is already not much competition has a large (and negative) impact on the expected value of the winning bid. However, as the number of bidders increases, the value creation effect associated with the club begins to dominate, and the seller is better off since he captures at least some of this value creation. This prediction distinguishes our framework from a pure collusive one where the main effect of forming a club is to reduce competition. Here, the private equity club's value creation can translate into gains for the target shareholders despite the reduction in competition.[4] We also show that the profitability of the club relative to non-club firms is increasing in the number of bidders, suggesting that the incentives to form a club may become even stronger as competition increases.

However, the two effects highlighted above - the value creation and the competition effect - are also reflected in the profits of the PE firms not part of the club. While the club's formation always benefits the members of the club, it has the potential to reduce the profits of the firms not part of the club. This occurs for the same reason as above: even though the reduction in competition that results from the club's formation benefits the remaining independent bidders, the club's value dominates the value of the eliminated bidders. The fact that the club has a higher value reduces the expected profits of the independent bidders. An important implication of this is that, when firms face a cost of bidding, the formation of a club affects competition in an additional way since it leads fewer other firms to enter and submit a bid. The result is that under free-entry the equilibrium number of bidders in the club case will be lower since each independent bidder is negatively affected by the formation of the club, thus introducing a wedge between the number of bidders in the club case versus the non-club case.

The asymmetry in the endogenously determined number of bidders across the two cases has important implications for shareholders at the target firm. Under free entry, we show that a club's formation is always bad for target shareholders. This occurs because of the wedge in the number of bidders across the two cases: a club's formation depresses competition enough that target shareholders always receive a lower expected price for their firm when a club forms. The results for the free entry case therefore represent a stark contrast to those where the number of

[4] This is very different from models of joint bidding in auctions where the formation of a club can enhance competition by allowing for bidders would otherwise not enter to participate. This could be the case, for instance, if bidders are financially constrained and the club allows them to pool resources (see, e.g., Hendricks and Porter (1992) for evidence in the context of auctions by the federal government for offshore leases). Here, competition is unambiguously reduced, yet the value creation by the club partly offsets the competition effect.

bidders is taken exogenously, and qualify those results.

Whether target shareholders in the end benefit from the formation of a club therefore depends on how much value the club creates and on how competitive the private equity market is, both before as well as after the club's formation. If there is a large number of potential bidders ex ante, then allowing a club to form makes the target shareholders worse off. The reason is that, when there is a large number of potential bidders, it is in fact the free-entry condition that determines how competitive the market will be post-formation of the club.

This case can be contrasted to one where the number of potential bidders is not so large and, in particular, is smaller than the number of bidders that would satisfy the industry's free entry condition. Then, as long as the number of potential PE bidders is sufficiently large, club bidding will lead to higher prices and therefore higher surplus for the target shareholders.[5] These findings demonstrate that two primary determinants of whether the formation of a club benefits target shareholders are the number of *potential* bidders and the costs associated with bidding. Interestingly, they also show that increasing the number of potential bidders may in fact make club bidding *less* attractive to the target firm to the extent that firms' willingness to bid starts to become dictated by the need to cover their costs of bidding. At some point, no matter how many PE firms are available to bid, only a subset of them will find it optimal to do so under free entry.

It is worthwhile noting that in our analysis we abstract from other reasons for why club deals may be used. For instance, club deals may arise if they enable several private equity firms, none of whom may have the financial resources to acquire the target alone, to bid by joining their financial strength. Similarly, club deals may enable the PE firms to spread risks and allow them to contemplate acquisitions that would otherwise not be possible, or to share information, thus reducing the overall risk associated with the deal and allowing firms that might not otherwise participate to bid.[6] All of these rationales for club bidding should increase rather than decrease the number of bidders. Information pooling is yet another motivation for bidding clubs to form. Information pooling improves the precision of bidder information but, similar to our model, reduces the potential number of bidders.[7] Our point is not, of course, to suggest that such rationales may

[5]When there are very few bidders, club bidding is unambiguously bad for the seller due to the further reduction of competition, irrespective of the cost of bidding.

[6]Many of these issues have been highlighted in the financial press as possible motivations for club bidding. See, e.g., SJ Berwin, 5/3/2008, AltAssets.

[7]DeBrock and Smith (1983) use simulation analysis to show that the seller might benefit from bidder information sharing as long as the reduction in competition is not too severe. In a more recent paper, Mares and Shor (2008)

not be important, but rather to examine the effect of another factor, namely value creation, that has been identified as particularly important for private equity acquisitions.

A related strand of literature in the economics of auctions has focused on the effect of collusive bidding, or on the opportunities for collusion derived from club bidding or "bidding rings." Much of this literature focuses on whether as a result of collusive bidding the auction is likely to be efficient, in the sense of allocating the good to the bidder with the highest value.[8] Empirical work on this issue has focused on bidding in a common value context (e.g., FCC spectrum auctions, or off shore oil tracts).[9] Recently a few authors have taken bidder collusion to be value reducing for the seller and have developed normative models that design collusion-proof auction rules (see, Che and Kim (2008) and Pavlov (2008)). To the best of our knowledge, none of this literature focuses on the main characteristic we use to distinguish takeovers by private equity clubs from other types of acquisitions, which is the ability to create value by aggregating each firm's synergy into one joint value for the club.

There are some recent empirical studies of the pro- or anti-competitive effects of club bidding in private equity. The evidence on this issue is, however, mixed and inconclusive. Boone and Mulherin (2009) analyze measures of takeover competition such as the number of bidders making offers or the number of bidders receiving confidential information and conclude that there is not much evidence of anticompetitive behavior associated with the presence of a club. They obtain similar evidence when they perform a more traditional "event study" analysis to focus on bidder returns. By contrast, Officer et al. (2008) study bidding behavior primarily in the context of clubs formed by more prominent, larger private equity firms. In this context, they find evidence that club bidding depresses prices and leads to lower gains for target shareholders.

From a theoretical standpoint, reconciling these two sets of findings poses a challenge. One insight offered by our analysis is that some of the differences in target returns across the two

show that if the total level of information in the market is kept constant then joint bidding increases bids due to information pooling. However, the reduction in competition effect always dominates. This reduction is in spite of the effect of the increase in bids in common value auctions due to a reduction in the number of active bidders (see, for example Bulow and Klemperer (2002) and Pinkse and Tan (2005)).

[8]For example, McAfee and McMillan (1992) derive a mechanism for information revelation amongst cartel members and show that efficiency in a sealed-bid first price auction is possible if bidding rings form a 'strong' cartel, i.e., allow for side payments. In a similar vein, Graham and Marshall (1987) show that an outside agent can use a knockout auction to ensure efficiency in a second price auction. Mailath and Zemsky (1991) extend the efficiency result to second price auctions with heterogenous bidders. Also see Athey and Bagwell (2001) for an application to a product market setting.

[9]See Moody Jr and Kruvant (1988) and Hendricks and Porter (1992).

samples may be explained not by focusing on cross-sectional differences in the targets, but rather by differences in the *acquirers*, i.e., the clubs. Our results suggest that club bidding by private equity firms will be beneficial for the seller when the formation of the club is highly efficient, and when there is sufficient competition. We conjecture that more prominent PE firms are likely to have more established (and therefore rigid) corporate cultures that may clash with potential partners, thus leading to less value creation. Moreover, the degree of overlap in expertise between two large PE firms is likely to be larger than for two smaller firms, each of which may be more highly specialized so that the club may be better at aggregating their separate values. Finally, it seems likely that the reduction in competition stemming from the formation of a club comprised of two large, prominent PE firms is likely to be bigger than that stemming from a club composed of smaller, less prominent firms, since size itself may represent a competitive advantage in certain deals. However, a formal test of these implications, and reconciliation with the emerging evidence, is beyond the scope of this paper.

The paper proceeds as follows. In the next section we present the basic model, and in Section 3 we characterize the equilibrium for both the club and the non-club cases. In Section 4 we study the case where the number of bidders is exogenously fixed. In Section 5 we contrast these results to those obtained when the number of bidders is endogenous, being determined by a free entry condition for the PE firms. In Section 6 we combine these results to derive empirical predictions. Section 7 concludes.

2 Model

Suppose there is a fixed number N of independent private equity (PE) firms with the ability to bid for a target and 2 PE firms that have the potential of merging before the start of bidding. Thus, we have a total of $N + 2$ bidders, each of which has a value x_i for a target firm that is drawn from a uniform distribution: $x_i \sim U[0, 1]$ for $i = 1, \ldots, N + 2$. Without loss of generality assume that bidders $1, \ldots, N$ are the independent bidders, and we refer to N as the "base" number of potential bidders. Initially bidders do not know their own value for the target, but learn it before submitting their bids. This captures the fact that a potential acquiring firm may have to do some due diligence in order to identify ways in which value at the target can be improved, with x_i representing how much value added they can provide, or the value of any synergies with the target firm. Therefore, the value x_i may also be specific to the target firm and need not represent an intrinsic property of the bidder. The number of bidders N can be determined by a variety of factors, such as macroeconomic conditions that make it easy or hard for bidders to obtain financing, the number of bidders that are sufficiently "large" that they can bear the exposure to a particular acquisition, the likelihood that any given bidder has expertise that can be used in creating value at a particular target, etc.

We consider two alternative scenarios. In the first scenario, we assume that bidders all learn their private valuations x_i and then all simultaneously submit bids for the target firm. In other words, the firm is auctioned off to the highest bidder. The highest bidder then pays the price bid by the second highest bidder (i.e., it is a second-price auction). Bidders compete for the target by simultaneously submitting a bid price, b_i.

The second scenario is similar to the first in terms of the timing for the bidding, but we first allow two of the PE firms to merge into a "bidding ring" or a "club" and then bid against the remaining N independent bidders. Merging means that the value of the bidding ring is an aggregate of the two individual bidders' values. Specifically, bidders $N+1$ and $N+2$ merge into one "club bidder," CB and this bidder's valuation for the target is $x_{CB} = x_{N+2} + \varphi x_{N+1}$, where $\varphi \in [0, 1]$. This captures the notion that the value each bidder derives for the target represents value increases in the form of operational synergies, cost reductions, streamlining of operations, new management, etc. To the extent that each bidder may have different expertise or may simply have "luck" at identifying different ways of increasing value, allowing two bidders to join together to bid for the target should increase the overall value that is obtained. At the same time, this setting also allows for the possibility that there may be inefficiencies in merging the two bidders, such as if there is a culture clash between the two organizations that prevents full value creation. Likewise, this could reflect overlap in the dimensions along which each bidder can create value in the target. The variable φ represents the scope for possible coordination failures or inefficiencies associated with merging the two bidders, with higher values of φ being indicative of greater efficiency. For $\varphi = 1$, there is no efficiency loss from merging the two bidders. For $\varphi = 0$, there is no value creation associated with the merger of the two private equity firms.

Once bidder CB is formed, all bidders $i = 1, \ldots, N, CB$ simultaneously submit bids. As before, the winner pays the price bid by the second highest bidder.

3 Preliminary analysis

We begin our analysis with a characterization of the revenue to the target firm and expected bidder profits under the two scenarios, no club and club, when all N potential bidders participate. Since the results related to scenario 1 with $N+2$ symmetric bidders are fairly standard (see, e.g., Krishna (2002)), we primarily focus on deriving the results for scenario 2, where bidders are asymmetric. For completeness, it is useful to first summarize the known results for the symmetric bidder case.

Lemma 1 *Suppose there are $N+2$ ex ante symmetric bidders, with private values x_i uniformly distributed in $[0,1]$. In a second price auction:*

1. *The expected revenue to the target firm is $\frac{N+1}{N+3}$, which is increasing in N.*

2. *The expected ex ante profit π_i to any bidder i is given by*

$$\pi_i = \frac{1}{(N+3)(N+2)},$$

which is decreasing in N.

Proof: See appendix. □

Lemma 1 characterizes the selling firm's revenue when all bidders are symmetric and in the absence of "club" bidder representing two merged bidders. Clearly, as the base number of potential bidders N increases, the revenue to the target increases since there are more bidders competing to acquire the target. In the limit the expected revenue to the seller converges to 1, so that all the surplus accrues to the seller. The increase in the number of bidders has the opposite effect on bidder expected profit, which is decreasing in N and in the limit converges to 0.

We now turn to the analysis of scenario two with a club bidder. As described above, when bidders $N+1$ and $N+2$ merge to form a club, their combined synergy for the target is $x_{CB} = x_{N+1} + \varphi x_{N+2} \in [0, 1+\varphi]$. Since x_{CB} is just the sum of two uniformly distributed random variables, it has a triangular distribution $G(.)$ given by

$$G(x_{CB}) = \begin{cases} \frac{1}{2\varphi} x_{CB}^2 & \text{if } x_{CB} \leq \varphi \\ \frac{1}{2}(2x_{CB} - \varphi) & \text{if } \varphi < x_{CB} \leq 1 \\ \frac{1}{2\varphi}\left(2x_{CB} - 1 - (x_{CB} - \varphi)^2\right) & \text{if } 1 < x_{CB} \leq 1 + \varphi \end{cases}.$$

Although defined in a piecewise fashion, notice that $G(x_{CB})$ is continuous and differentiable at φ and 1.

After the club is formed, there will be a total of $N+1$ bidders: N symmetric bidders with values drawn uniformly from $[0,1]$, and 1 bidder with the triangular distribution G. We can now state the following, which summarizes the seller's revenue in the presence of a club bidder.

Lemma 2 *Suppose there are N ex ante symmetric bidders with private values x_i uniformly distributed in $[0,1]$, and one club bidder with value x_{CB} drawn from distribution $G(x_{CB})$ with support in $[0,1+\varphi]$. In a second price auction:*

1. *The expected revenue to the target firm is given by*

$$\Gamma(N,\varphi) = \frac{\varphi}{2}\left(\frac{N}{N+1}\right)+\left(1-\frac{\varphi}{2}\right)\left(\frac{6N+6\varphi-N\varphi+(N+4)(2-\varphi)N^2-2(3+N(1-\varphi))\varphi^{N+1}}{(N+3)(N+2)(N+1)(2-\varphi)}\right),$$

   which is increasing in φ and N.

2. *The expected (ex ante) profit π_{IB} of an independent bidder is given by*

$$\pi_{IB} = \frac{1}{(N+2)(N+1)} - \frac{\varphi}{2N(N+1)} + \frac{\varphi^{N+1}}{N(N+2)(N+1)} - \frac{\varphi^{N+2}}{(N+3)(N+2)(N+1)},$$

   which is decreasing in φ and N, the number of independent bidders.

3. *The expected (ex ante) profit π_{CB} to the club bidder is given by*

$$\pi_{CB} = \frac{1}{(N+2)(N+1)} + \frac{\varphi}{6}\frac{\varphi+N\varphi+3}{N+1} - \frac{\varphi^{N+2}}{(N+3)(N+2)(N+1)}.$$

   Moreover, $\pi_{CB} > \pi_{IB}$, π_{CB} is decreasing in N and increasing in φ.

Proof: See appendix. □

Lemma 2 demonstrates that, as expected, the revenue to the target firm (i.e., to the seller) and the bidder profits are a function of the efficiency or degree of value creation φ of the club. Much like the results from Lemma 1, the revenue to the target increases as the number of potential bidders N increases, while the bidders' profits, whether independent or part of the club, decreases with N.

3.1 The incentives to form a club

Here, we address the question of whether forming a club is beneficial to the private equity firms themselves, or whether they would be better off bidding separately. To address this issue, we focus on the profits to the bidders, comparing the symmetric case to the case with a club bidder.

Proposition 1 *The difference in the expected profits of the club and those of two independent bidders,* $\pi_{CB} - 2\pi_i$*, is*

1. *Increasing in* φ*.*
2. *Positive for all* N *if the club is very efficient* ($\varphi = 1$)*.*
3. *Negative for all* $N > 1$ *if the club is very inefficient* ($\varphi = 0$)*.*
4. *For all* $N > 1$ *there exists a* $\hat{\varphi}(N)$ *such that forming a club is optimal for* $\varphi > \hat{\varphi}(N)$*.*
5. *As* N *increases, the profit to a club bidder relative to that of two independent bidders becomes arbitrarily large: For any* $\Delta > 0$*, there exists* N_Δ *such that* $\frac{\pi_{CB}}{2\pi_i} > \Delta$ *for all* $N > N_\Delta$*.*

Proof: See appendix. □

The incentives of forming a club are best understood by studying the two key effects at play. First, there is a value creation effect that comes from the synergy of the club members and which depends on the degree of efficiency φ of the club. Second, there is the loss of a potential competitor. An increase in efficiency increases the value creation effect and also the dominance of the club. In the limit, when $\varphi = 1$ the club is fully efficient. Moreover, it also benefits from the reduction in competition. On the other hand, if the club is very inefficient, i.e., if $\varphi = 0$, the club loses all benefits of value creation. In this case, individual members of the club would be better off bidding separately, which would significantly increase their chances of having the winning bid and profiting as a result. Putting these two extremes together, Proposition 1 shows that as long as the club is not too inefficient, i.e., $\varphi > \hat{\varphi}(N)$, club formation is beneficial to the PE firms comprising the club.

To show that even for a small number of bidders the threshold efficiency level is relatively small, we plot $\hat{\varphi}(N)$ in the figure below. The figure shows that for a wide range of N, the number of potential bidders, relatively small levels of efficiency are required for the PE firms to find it beneficial to form a club.

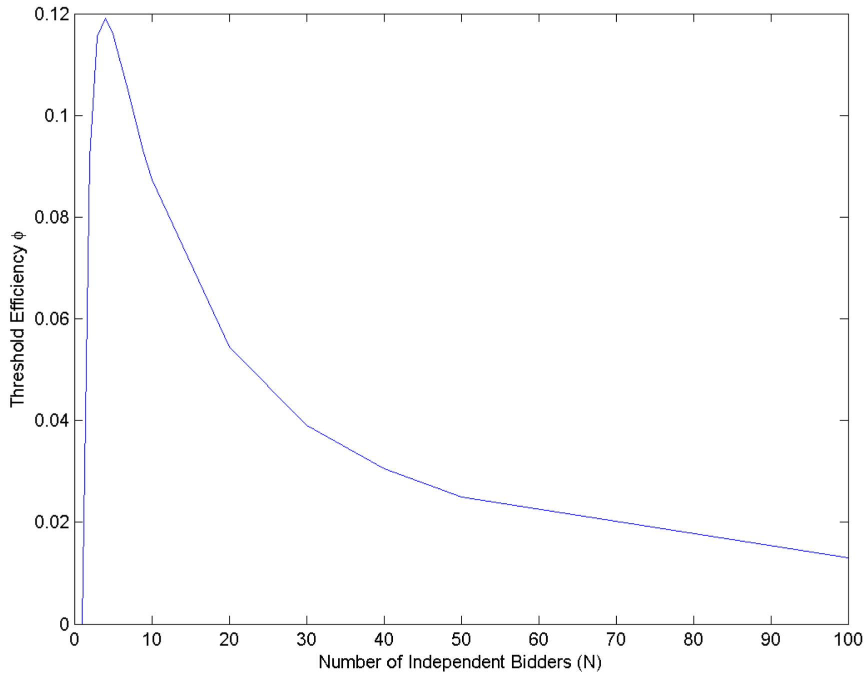


Figure 1: Plot of $\hat{\varphi}$

Proposition 1 also establishes that as the number of bidders becomes large, the incentive for the PE firms to form a club becomes ever more important. This can be seen clearly from focusing on the limiting case as the number of bidders becomes unboundedly large, in which case the profit to a bidder in the symmetric case converges to $\lim_{N\to\infty} \pi_i = 0$, while the profit to the club converges to $\lim_{N\to\infty} \pi_{CB} = \frac{1}{6}\varphi^2 > 0$ for all $\varphi > 0$. This occurs because of the value creation associated with the club. In general, increases in competition raise the expected price the winner must pay to the target firm, and reduce each bidder's probability of winning. With symmetric and independent bidders, the difference between the value of the winning bidder and that of the next highest bidder shrinks quickly, so that the ex ante profit to a bidder becomes vanishingly small.

In contrast, the club benefits greatly through the value it creates since whenever $x_{CB} > 1$ the club is guaranteed of having the highest value, but pays only the value of the second highest bidder. As N increases, the value to the second highest bidder converges to 1, but both $\Pr(x_{CB} > 1)$ and $E[x_{CB}|x_{CB} > 1]$ remain strictly positive (in fact, neither of these terms are functions of N, implying that the club's expected profit is bounded below by $\Pr(x_{CB} > 1)\, E[x_{CB}|x_{CB} > 1] > 0$). Some of the value associated with the club's formation is captured by the target firm, as shown in Lemma 2, but the portion that is obtained by the club remains "large" relative to what the PE firms would obtain if they did not form a club.

4 Do target shareholders benefit from club bidding?

Having characterized the equilibrium profits to the bidders as well as the expected revenue to the seller under the two alternatives concerning whether a club forms or not, we can now compare the two cases. Specifically, we are interested in whether shareholders at the target firm benefit from the formation of a club. For this, we define $\widehat{\Gamma}(N, \varphi)$ as the difference in the expected revenue to the seller when all bidders are symmetric versus when a club forms. More formally, we have

$$\begin{aligned} \widehat{\Gamma}(N, \varphi) &\equiv \frac{N+1}{N+3} - \Gamma(N, \varphi) \\ &= \frac{1}{2} \frac{(4 - 5\varphi - N\varphi) N - 6\varphi + 4 + 2(3 + N(1-\varphi))\varphi^{N+1}}{(N+3)(N+2)(N+1)} \end{aligned} \tag{1}$$

For $\widehat{\Gamma}(N, \varphi) > 0$, target revenue is higher with $N+2$ symmetric bidders. For $\widehat{\Gamma}(N, \varphi) < 0$, the target is better off when a club forms, even if the total number of bidders, $N+1$, is lower.

As a preliminary step, note that an immediate implication of Lemma 2 is that the expected revenue difference, $\widehat{\Gamma}(N, \varphi)$, must be decreasing in φ. In other words, increases in φ increase the rent earned by the target seller stemming from the value creation of the club bidder, and therefore make the club case relatively more attractive for the seller.

We can now establish the following result comparing the two cases.

Proposition 2 *For $\varphi = 0$, the expected revenue with a club bidder is always lower: $\widehat{\Gamma}(N, 0) > 0$ for all $N \geq 1$. However, for $\forall\ \varphi \in (0, 1]$ there exists a unique value $\hat{N}(\varphi)$ such that joint bidding is better for all $N > \hat{N}(\varphi)$. For $\varphi = 1$, $\hat{N} = 1$, and this cutoff value increases as φ gets smaller, becoming arbitrarily large as $\varphi \to 0$.*

Proof: See appendix. □

Proposition 2 establishes that with sufficient competition, any value-creating club generates higher revenue for the sale of the target firm. There are clearly two effects at work. First, the reduction in the number of bidders, from $N + 2$ to $N + 1$, reduces competition and therefore reduces the price the target firm can expect to receive. Second, however, the merger between bidders $N + 1$ and $N + 2$ creates value by allowing them to combine their synergies and further improve efficiencies at the target. This result therefore shows that for N sufficiently large, the second effect dominates and the seller's revenue is increased through the value creation associated with a club bidder. However, it also implies that as the degree of value creation decreases, a significantly greater amount of competition is necessary in order for the target firm to benefit from the value being created by the club. For lower levels of competition, the effect through the reduction in the number of bidders implied by the formation of the club dominates, and the seller would be better off if no club were to form.

5 Free entry and the number of bidders

In the analysis above, we have taken the number of potential independent bidders N as exogenous. More importantly, we have assumed that the base number of bidders is the *same* whether a club forms or not. In this section, we study the case where the number of bidders is determined by a free entry condition and is thus endogenous. We show that, surprisingly, endogenizing the number of bidders dramatically changes many of the results.

Assume now that there is a cost c to each private equity firm of preparing and submitting a bid. Such costs can arise, for instance, if it is costly to put together a bid or if there is a cost associated with due diligence or with an SEC filing fee. Likewise, they can arise if the private equity bidder itself bears a cost associated with identifying a good target for an acquisition since it must identify inefficiencies within the target firm as well as ways in which it can add value. The cost of bidding might also include direct costs like putting together a legal team and arranging financing, as well as indirect opportunity costs. This cost must be paid ex ante, that is, before the bidder knows his own value. This assumption allows us to calculate the free-entry number of bidders by focusing on each bidder's ex ante profits.

Denote by $N_{CB}(c, \varphi)$ the number of independent private equity firms that would enter the bidding contest given that two firms will join forces to become a club bidder with efficiency parameter φ, when bidding costs are c. This leaves a total of $N_{CB} + 1$ bidders. Similarly, let $N_{NC}(c, \varphi) + 2$ denote the number of bidders that would enter if joint bidding was ruled out by fiat. In what follows, we drop the dependence on c and φ for notational convenience. Thus, N_{CB} and N_{NC} are implicitly defined by the following:

$$c = \pi_{IB}\left(N_{CB}, \varphi\right),$$
$$c = \pi_i(N_{NC}),$$

where $\pi_{IB}\left(N_{CB}, \varphi\right)$ is defined in Lemma 2 and $\pi_i(N_{NC})$ in Lemma 1. Equating these two, we can solve for N_{NC} as a function of N_{CB} as follows

$$\pi_i(N_{NC}) = \pi_{IB}\left(N_{CB}, \varphi\right) \Longrightarrow N_{NC} = \sqrt{\frac{1}{4} + \frac{1}{\pi_{IB}\left(N_{CB}, \varphi\right)}} - \frac{5}{2} \tag{2}$$

The relationship defined by (2) is an equilibrium condition. Thus, if the entry costs were such that for a given φ, a number N_{CB} of independent bidders would enter in the joint bidding scenario, then for the same cost (2) specifies the number of independent bidders N_{NC} that would enter in the symmetric bidder scenario.

It is illustrative to examine the relationship between the equilibrium number of bidders N_{NC} and N_{CB} at $\varphi = 0$. In this case, there is no value creation and the only effect is an elimination of a single bidder. Hence, the number of independent bidders that enter in the two cases differs exactly by 1, i.e., $N_{CB} = N_{NC} + 1$, and we are left in fact with the same *total* number of bidders whether a club forms or not. For $\varphi > 0$, however, the results are quite different. The following result represents one of the important implications of endogenizing the number of bidders.

Proposition 3 *For all $c > 0$ and all $\varphi > 0$, $N_{CB} < N_{NC} + 1$.*

Proof: Fix $c > 0$. First, note that for $\varphi = 0$, it is straightforward to see that $N_{CB} = N_{NC} + 1$. Next, note that, since π_{IB} is decreasing in φ (see Lemma 2), we must have that N_{CB} is also decreasing in φ since it satisfies $\pi_{IB}\left(N_{CB}\right) = c$. Since N_{NC} is independent of φ, this tells us that $N_{CB} < N_{NC} + 1$ for $\varphi > 0$. □

For $\varphi > 0$, the reduction in the number of bidders when a club forms is accompanied by the fact that the club-bidder's value now stochastically dominates the eliminated bidder's value. This results in a reduction of the independent bidder's expected profit. In equilibrium, the number of independent bidders (N_{CB}) that enter in the club bidding scenario is strictly less than $N_{NC} + 1$.

This result introduces an important reason for studying the case where firms face a cost of bidding, and which qualifies the result in Propostion 2. As Proposition 3 establishes, once bidding costs are included, the symmetric and the club bidder cases will no longer be characterized by the same base number of independent bidders. An increase in φ improves value creation and also increases the advantage of the club bidder. This results in lowering the number of independent bidders that choose to enter, i.e., $N_{NC} - N_{CB}$ increases. In other words, a *wedge* forms between the number of bidders in the two cases. As we show below, this wedge has important implications for the effect of a club's formation on target shareholders.

To compare the club bidding case against the standard case described in the scenario where all bidders are symmetric, we extend the definition of $\widehat{\Gamma}$ slightly by defining $\widehat{\Gamma}(N_{NC}, N_{CB}, \varphi|c)$ as the difference in the expected revenues to the seller when all bidders are symmetric versus when a club forms. More formally, we have

$$\widehat{\Gamma}(N_{NC}, N_{CB}, \varphi|c) = \frac{N_{NC}+1}{N_{NC}+3} - \Gamma(N_{CB}, \varphi), \tag{3}$$

where $\Gamma(N_{CB}, \varphi)$ is defined in Lemma 2. We can now analyze the effect of endogenous entry on the seller's expected revenue.

Proposition 4 *When equilibrium is determined by a free entry condition, the seller is strictly worse off with club bidding for all* $\varphi \in [0, 1]$.

Proof: See appendix. □

A reduction in the cost of entry increases the number of bidders in both scenarios. We had earlier shown (see Proposition 2) that for a high enough number of bidders the value creation effect dominates the loss of competiton effect. However, in that setting we had assumed that the total number of bidders was exogenously determined. In that case, formation of a club decreased competition because one independent bidder choose not to submit a bid. If, on the other hand, the number of bidders is determined endogenously, then given the dominance of the club the effect on competion is even stronger. Proposition 4 illustrates that the wedge between N_{CB} and N_{NC} becomes wider as the efficiency of the club increases, so much so that the reduction in the number of independent bidders overhelms the value creation effect. The result is surprising because it does not depend on the level of entry costs and, conseqeuntly, the number of independent bidders in equilibrium.

6 Implications for club bidding

The last two sections presented conflicting results concerning the likely impact of having a subset of private equity firms form a club for the purpose of bidding for a target firm. Keeping the base number of possible bidders constant across scenarios, an increase in competition ultimately favors club bidding by allowing some of its value creation to be passed on to the target's shareholders. However, requiring that bidders satisfy a participation constraint for bidding yields the opposite conclusion in that even for very small bidding costs, a club's formation is bad for target shareholders. Given these results, it is straightforward then to see that what determines whether a club's formation benefits target shareholders is whether the number of potential bidders, N, is greater or less than what would obtain in a free entry equilibrium.

Recall now that, from Proposition 2, we know that for all $\varphi > 0$ there exists a value $\hat{N}(\varphi)$ such that joint bidding is better for the seller if the number of independent bidders is $N > \hat{N}(\varphi)$. In this case, there are $N+2$ bidders in the symmetric case, and $N+1$ total bidders in the joint bidding case. If $N < \hat{N}(\varphi)$, then clearly joint bidding will always be bad for the seller. However, if instead $N > \hat{N}(\varphi)$, we then have three possible cases to consider:

1) $N \leq N_{CB}$. In this case, joint bidding is better, in the sense of raising the revenue of the seller.

2) $N_{NC} + 1 \leq N$. In this case, joint bidding is unambiguously bad for the seller.

3) $N_{CB} < N \leq N_{NC} + 1$. In this case, joint bidding may or may not be better, depending on the value of N relative to the other two.

Note that, as c decreases, we always wind up in case (1), where $N \leq N_{CB} < N_{NC}$. On the other hand, for high c we always have case (2). We can now establish the following, under the maintained assumption that $N > \hat{N}(\varphi)$ for a fixed $\varphi > 0$.

Proposition 5 *1) For given φ, assume that $N > \hat{N}(\varphi)$. Then there exists always a value $\bar{c}(\varphi)$ such that, for all $c < \bar{c}(\varphi)$, the seller's profit is higher with joint bidding. 2) The cutoff cost $\bar{c}(\varphi)$ is increasing in φ.*

Proof: Fix φ. As argued above, from Proposition 2, we know that there is a number of bidders, $\hat{N}(\varphi)$, such that joint bidding is better for the seller for all $N > \hat{N}(\varphi)$. From the conditions which define N_{CB} and N_{NC} we know that both are monotonically decreasing in c, and grow unboundedly large as $c \to 0$. Therefore, as c decreases, we can always move N_{CB} sufficiently close to N that joint bidding is better for the seller. □

The first part of the proposition establishes that as long as there is a sufficiently large number of private equity firms that could feasibly put in a bid (i.e., as long as $N > \hat{N}(\varphi)$), allowing for a club to form and submit a joint bid will be optimal for the seller when the cost of bidding, c, is sufficiently low. This is because, as c decreases, the free-entry number of bidders increases, both for the symmetric case as well as for the case with a club bidder. However, since there are only N potential bidders who are actually able to make an offer, we wind up with N bidders for both of the scenarios, and thus can have joint bidding be optimal for the seller. Put differently, the result establishes that when the cost of bidding is sufficiently low, the primary determinant of the impact of a club's formation is the number of firms that can potentially bid, and the free entry condition plays no role. In this case, given that N is lower than N_{CB}, the PE firms obtain positive expected profits in equilibrium.

In Proposition 5 we have analyzed the effect of bidding costs on the seller's profit for a given number of potential bidders. These results allow us to compare the effect of club bidding among scenarios or economies where the cost of bidding may vary but where there is a fixed pool of potential bidders. Alternatively, in many instances it is likely that the cost of bidding is determined by the regulatory or institutional environment, and that the major difference is in the ability PE firms have to bid. The number of PE firms that can potentially bid at any given time may vary for a number of reasons, for instance as a result of macroeconomic factors that determines PE firms' access to credit and financial markets. The next proposition analyzes the economics of club bidding for such scenarios.

Proposition 6 *Fix the cost of bidding c. For a given φ, the seller's profit with joint bidding is lower if either $N < \hat{N}(\varphi)$ or $N \geq N_{NC} + 1$. The seller's profit with joint bidding is higher for intermediate values of N, i.e., for $\hat{N}(\varphi) < N < N_{CB}$.*

Proof: Follows from the definition of $\hat{N}(\varphi)$, N_{CB} and N_{NC}. □

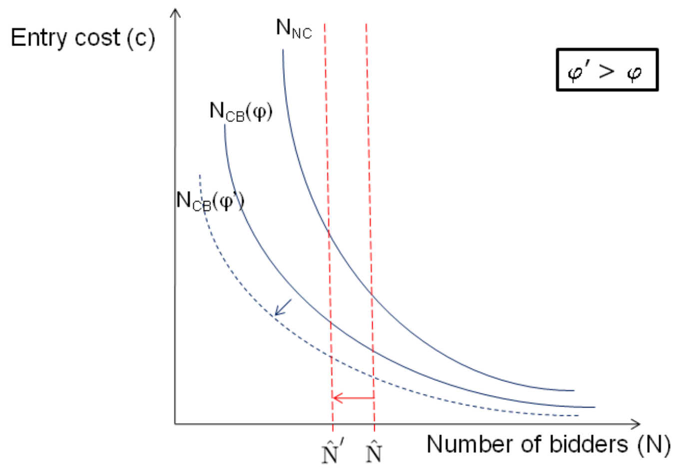


Figure 2: Effect of an increase in φ on the equilibrium number of bidders.

The proposition shows that when the number of potential bidders is either very low or very high, shareholders at the target firm are worse off when a club forms between two of the PE firms. The intuition is fairly straightforward. With a very low number of potential bidders, allowing two of them to form a club significantly reduces competition and harms target shareholders. Conversely, when the number of potential bidders is very large, the primary constraint on bidding is not the number of potential bidders but rather the cost of bidding. In this case, the results on free entry are paramount, and shareholders are again worse off.

The effect of a club's formation, however, is quite different when there is an intermediate number of potential bidders N. When N is greater than the minimum number of bidders that are necessary in order for a club's presence to benefit target shareholders, the expected revenue to the target has the potential to be higher from the presence of the club. The restriction that N be less than N_{CB}, however, is to ensure that there are not so many potential bidders that the free-entry condition becomes the primary determinant of the number of bidders, rather than how many bidders are actually in a position to submit a bid.

The results from Propositions 5 and 6 are illustrated in Figure 2. For any given level of club efficiency φ, the free entry number of bidders in the club case, N_{CB}, is strictly lower than in the case with no club, N_{NC}. As discussed before, this wedge is what makes club bidding worse for the target under free entry. Moreover, as φ increases, the number of bidders that will choose to enter will be even lower in the club case. However, from Proposition 2 we also know that the threshold value $\hat{N}$ decreases as φ increases.

From the figure, it is apparent that, holding the entry cost c constant, club bidding for values of N below $\hat{N}$ will always be bad for the target. Likewise for N greater than N_{NC}, since then the free entry condition will bind. For values in between, club bidding can raise the revenue to the target since there is a sufficient level of competition but the full impact of the wedge across the two scenarios is not felt. A similar argument applies to the case where we treat the number of bidders N as fixed and instead consider changes in the entry cost c. When the cost of bidding is very low, equilibrium will be determined purely by how many PE firms have the potential to bid. Conversely, when the cost of bidding is high, the free entry number of bidders will be low irrespective of how many PE firms could potentially purchase the target.

These results may help in explaining time variations in PE firm bidding behavior and the premiums obtained by target firms. Over the last few years the number of private equity firms have been steadily increasing. Moreover, until the current crisis the funds under management have also shown significant increase. Both of these factors likely contributed to an increase in the number of potential private equity bidders. Over this short time period, it does not seem likely that the fixed cost of bidding has had a remarkable change. The above proposition predicts that, holding all else constant, as the competitiveness of the market increases (an increase in the number of potential bidders due to industry effects), the benefit of joint bidding will initially increase and then decrease, i.e., our model predicts an inverted U-shaped curve for the benefits of joint bidding as a function of the number of bidders. This provides a novel empirical prediction based on the trade-off between value creation and competition identified in this paper.

6.1 Numerical example

Here we present a simple numerical example that illustrates the results above. Suppose that the cost of bidding is $c = \frac{1}{72}$ and that the efficiency parameter for the club is $\varphi = 0.60386$. Using the definitions of $\widehat{\Gamma}$ from (1) and Γ from Lemma 2, we can solve for $\hat{N}(\varphi)$, the threshold number of PE firms beyond which a club's formation is beneficial to target shareholders. This value can be shown to be $\hat{N}(\varphi) = 2.66$.

We can also solve for the free entry number of independent bidders in the club case by equating $\pi_{IB}(N_{CB}, \varphi) = c$ to obtain $N_{CB} = 5$. The number of bidders in the case without a club is just obtained from equating $\pi_i = c$ and solving for N_{NC} to obtain $N_{NC} = 6$.

With this, we can now conclude that, holding costs fixed, if $N \leq 2$ then allowing a club to form destroys value for the seller simply because there are too few bidders already, so that the competition effect dominates. If $N \geq 6$, again club bidding is value destroying for the seller, but this time because of the free entry condition: given the cost of bidding, only 5 independent firms will choose to bid when a club forms. For $N \in \{3, 4, 5\}$, the number of independent bidders in both cases is the same since N is below the free entry equilibrium values of N_{NC} and N_{CB}. In these cases allowing club bidding increase the seller's revenue.

7 Conclusion

This paper has presented a simple analysis of the economics of club bidding in the context of private equity consortia. The key premise in our analysis is that, since private equity firms acquire targets in order to try to improve their value, forming a club may allow some of the individual PE firms' strategies for increasing value at the target to be aggregated. Forming a club thus creates additional value, partly offsetting the effect of the reduction of competition which results from the removal of one potential bidder when a club forms. To the best of our knowledge, our setup is unlike other analyses of private or common value of club bidding that rely on information or risk sharing, and delivers unique predictions.

We show that as long as there is sufficient competition among PE firms, forming a club allows some of the value creation to be appropriated by target shareholders, thus benefitting the target as well as the club's members. However, if bidding is costly then club formation reduces the expected revenue received by target shareholders. This occurs because club formation is generally bad for PE firms not part of the club, which reduces further the number of firms willing to bid.

An interesting extension would be to develop a better understanding of what drives the efficiency of the club, in terms of the club's ability to create value. One would expect that larger PE firms would find less benefit from forming a club with another large firm, since it is less likely that they will have skills that do not overlap significantly. However, such an analysis would call for the addition of size or different dimensions of skill into the model, and issue that we leave for future research.

Appendix

Proof of Lemma 1: It is well-known that in a second price auction, the dominant strategy is for each bidder to bid his private valuation $b_i = x_i$, with the winner paying a price equal to the second highest value among the $N+2$ bidders. Letting z be the second highest of $\{x_i | i \in \{1, \ldots, N+2\}\}$, the distribution function S of the variable z is given by

$$\begin{aligned} S(z) &= (N+2) F^{N+1}(z) - (N+1) F^{N+2}(z) \\ &= (N+2) z^{N+1} - \left(\overline{N}+1\right) z^{N+2} \end{aligned}$$

The expected revenue to the seller can therefore be calculated as

$$\begin{aligned} E[z] &= \int_0^1 zS'(z)dz = 1 - \int_0^1 S(z)dz \\ &= 1 - \int_0^1 \left((N+2) z^{N+1} - (N+1) z^{N+2}\right) dz \\ &= \frac{N+1}{N+3}, \end{aligned}$$

as desired. This expression is clearly increasing in N since $\frac{N+1}{N+3} < 1$.

For the second part, let $y = \max\{x_1, \ldots, x_{N+1}\}$, where y is distributed with CDF $H(y) = F^{N+1}(y)$. The expected profit for a bidder with value x is given by

$$\begin{aligned} \pi_i(x) &= \int_0^x (x-y) H'(y) dy + \int_x^1 0H'(y) dy \\ &= [(x-y) H(y)]_0^x + \int_0^x H(y) dy \\ &= \int_0^x H(y) dy \end{aligned}$$

Ex ante expected profits are

$$\begin{aligned} E[\pi_i(x)] &= \int_0^1 \int_0^x H(y) dy F'(x) dx \\ &= \left[\int_0^x H(y) dy F(x)\right]_0^1 - \int_0^1 H(x) F(x) dx \\ &= \int_0^1 H(y) dy - \int_0^1 H(x) F(x) dx \end{aligned}$$

For $N+2$ symmetric bidders, $F(x) = x$ so that $H(y) = y^{N+1}$. Thus, bidder expected profits are

$$\begin{aligned}
E[\pi_i(x)] &= \int_0^1 y^{N+1} dy - \int_0^1 x^{N+2} dx \\
&= \frac{1}{N+2} - \frac{1}{N+3} \\
&= \frac{1}{(N+3)(N+2)},
\end{aligned}$$

as desired. □

Proof of Lemma 2: To calculate the expected revenue with a club bidder, we note that whenever $x_{CB} > 1$ the value to the club is higher with probability 1 than the value to any of the other N bidders, whose value is at most equal to 1. Therefore, for $x_{CB} \in [1, 1+\varphi]$ the club bidder will always win the auction, and pay a price equal to the highest value among the other N bidders. Defining y to be equal to $\max\{x_i | i \in \{1, \ldots, N\}\}$, the distribution function of y is given by

$$H(y) = [F(y)]^N = y^N$$

On the other hand, whenever $x_{CB} \leq 1$, the club bidder competes with the other N firms, with the winning bidder paying the second highest value among all $N+1$ bidders. Given the distribution function $G(.)$ for x_{CB}, we have that the distribution function for the second highest bid z among N bidders whose values are drawn from the distribution F and one bidder whose value is drawn from G is given by

$$S(z) = F(z)^N + NG(z|z<1)(1-F(z))[F(z)]^{N-1}$$

Substituting for $G(z|z<1)$ and $F(z)$, the distribution function for the second highest value conditional on it being less than 1 is then

$$S(z) = \begin{cases} z^N + N\left(\frac{z^2}{\varphi(2-\varphi)}\right)(1-z) z^{N-1} & \text{if } z \leq \varphi \\ z^N + N\left(\frac{2z-\varphi}{2-\varphi}\right)(1-z) z^{N-1} & \text{if } a \in [\varphi, 1] \end{cases}$$

Denote by $\Gamma(N, \varphi)$ the expected revenue from the bidding ring, which clearly depends on the efficiency parameter φ. This expected revenue is given by

$$\Gamma(N,\varphi) = \Pr(x_{CB} > 1)\,E[y] + \Pr(x_{CB} \leq 1)\,E[z|x_{CB}, x_{1,\ldots,N} \leq 1]$$

After substituting, it can be shown that

$$\Gamma(N,\varphi) = \frac{\varphi}{2}\left(\frac{N}{N+1}\right)+\left(1-\frac{\varphi}{2}\right)\left(\frac{6N+6\varphi-N\varphi+(N+4)(2-\varphi)N^2-2(3+N(1-\varphi))\varphi^{N+1}}{(N+3)(N+2)(N+1)(2-\varphi)}\right),$$

as desired. To show that $\Gamma(N,\varphi)$ is increasing in φ, we start by differentiating Γ with respect to φ

$$\begin{aligned}\frac{\partial\Gamma}{\partial\varphi} &= \frac{1}{2}\left(\frac{N}{N+1}+\frac{6-N-\left(\overline{N}+4\right)N^2+2N\varphi^{N+1}-2((N+1)(3+N(1-\varphi)))\varphi^N}{(N+3)(N+2)(N+1)}\right)\\ &= \frac{\left(6+5N+N^2-2\varphi^N\left(3+(1-\varphi)N^2+2N(2-\varphi)\right)\right)}{2(N+3)(N+2)(N+1)}\end{aligned}$$

The sign of this expression is clearly determined by the term in parentheses in the numerator,

$$\left(6+5N+N^2-2\varphi^N\left(3+(1-\varphi)N^2+2N(2-\varphi)\right)\right) \tag{4}$$

We first show that (4) is decreasing in φ.

$$\begin{aligned}&\frac{\partial}{\partial\varphi}\left(6+5N+N^2-2\varphi^N\left(3+(1-\varphi)N^2+2N(2-\varphi)\right)\right)\\ &= -2N\varphi^{N-1}(N+1)(N(1-\varphi)+3-2\varphi) < 0\end{aligned}$$

Thus, if (4) is positive at $\varphi = 1$ then it is positive for all φ. Evaluating 4 at $\varphi = 1$ we obtain

$$\begin{aligned}&6+5N+N^2-2(3++2N(2-1))\\ &= N(N+1) > 0,\end{aligned}$$

which establishes our result.

For part (2), note that as before the ex ante expected profits are

$$\pi_{IB} = \int_0^1 H(y)\,dy - \int_0^1 H(x)\,F(x)\,dx$$

From the point of an independent bidder

$$H\left(y\right)=F^{N-1}\left(y\right)G\left(y\right)$$

Thus, the ex-ante expected profits of an independent bidder in this case are

$$\begin{aligned}\pi_{IB} &= \int_0^{\varphi} y^{N-1}\left(\frac{y^2}{2\varphi}\right)dy+\int_{\varphi}^{1} y^{N-1}\left(\frac{1}{2}\left(2y-\varphi\right)\right)dy-\int_0^{\varphi} x^{N-1}\left(\frac{x^2}{2\varphi}\right)xdx-\int_{\varphi}^{1} x^{N-1}\left(\frac{1}{2}\left(2x-\varphi\right)\right)xdx \\ &= \frac{1}{\left(N+2\right)\left(N+1\right)}-\frac{\varphi}{2N\left(N+1\right)}+\frac{\varphi^{N+1}}{N\left(N+2\right)\left(N+1\right)}-\frac{\varphi^{N+2}}{\left(N+3\right)\left(N+2\right)\left(N+1\right)},\end{aligned}$$

as above. To show that π_{IB} is decreasing in φ,we take the first derivative of the regular bidder's profit we obtain,

$$\begin{aligned}&\frac{\partial}{\partial\varphi}\left(\frac{1}{\left(N+2\right)\left(N+1\right)}-\frac{\varphi}{2N\left(N+1\right)}+\frac{\varphi^{N+1}}{N\left(N+2\right)\left(N+1\right)}-\frac{\varphi^{N+2}}{\left(N+3\right)\left(N+2\right)\left(N+1\right)}\right) \\ &= \frac{2\left(N^2+4N+3-2N\varphi-N^2\varphi\right)\varphi^N-\left(N+3\right)\left(N+2\right)}{2N\left(N+3\right)\left(N+2\right)\left(N+1\right)}\end{aligned} \tag{5}$$

The sign of the above is clearly determined by its numerator. We show below that the numerator is increasing in φ.

$$\begin{aligned}&\frac{\partial}{\partial\varphi}\left(2\left(N^2+4N+3-2N\varphi-N^2\varphi\right)\varphi^N-\left(N+3\right)\left(N+2\right)\right) \\ &= 2N\varphi^{N-1}\left(N+1\right)\left(N-2\varphi-N\varphi+3\right) \\ &= 2N\varphi^{N-1}\left(N+1\right)\left(\left(1-\varphi\right)\left(N+2\right)+1\right) \\ &> 0\end{aligned}$$

Thus, $\frac{\partial\pi_{IB}}{\partial\varphi}<0\ \forall\varphi$ if it is negative at $\varphi=1$. Evaluating (5) at $\varphi=1$ we obtain

$$\begin{aligned}&2\left(N^2+4N+3-2N\varphi-N^2\varphi\right)\varphi^N-\left(N+3\right)\left(N+2\right)\Big|_{\varphi=1} \\ &= -N\left(N+1\right)<0.\end{aligned}$$

Hence, $\frac{\partial\pi_{IB}}{\partial\varphi}<0\ \forall\varphi$.

Finally, we can show that π_{IB} is decreasing in N as follows:

$$\frac{\partial \pi_{IB}(N,\varphi)}{\partial N} = -\frac{2N+3}{(N+2)^2(N+1)^2} + \varphi\frac{2N+1}{2N^2(N+1)^2} - \varphi^{N+1}\frac{6N-2N\ln\varphi-3N^2\ln\varphi-N^3\ln\varphi+3N^2+2}{N^2(N+2)^2(N+1)^2}$$
$$-\varphi^{N+2}\frac{\left(6\ln\varphi-12\overline{N}+11N\ln\varphi+6N^2\ln\varphi+N^3\ln\varphi-3N^2-11\right)}{(N+3)^2(N+2)^2(N+1)^2}$$

$$\frac{\partial^2 \pi_{IB}(N,\varphi)}{\partial N\partial\varphi} = \frac{2N+1}{2N^2(N+1)^2} - \frac{\varphi^N}{N^2}\frac{2(N+1)-N\ln\varphi(N+2)}{(N+2)^2} - \frac{\varphi^{N+1}\left(\left(N^2+4N+3\right)\ln\varphi-2(N+2)\right)}{(N+3)^2(N+1)^2}$$
$$= \frac{2N+1}{2N^2(N+1)^2} - \frac{\varphi^N}{N^2}\frac{2(N+1)-N(N+2)\ln\varphi}{(N+2)^2} - \frac{\varphi^{N+1}\left(\left(N^2+4N+3\right)\ln\varphi-2(N+2)\right)}{(N+3)^2(N+1)^2}$$

$$\frac{\partial^3 \pi_{IB}(N,\varphi)}{\partial N\partial\varphi^2} = \frac{\varphi^{N-1}}{(N+2)^2}(2\ln\varphi+N\ln\varphi-1) - \frac{\varphi^N}{(N+3)^2}(3\ln\varphi+N\ln\varphi-1)$$
$$= \varphi^{N-1}\left(\left(\frac{N+3-\varphi(N+2)}{(N+3)(N+2)}\right)\ln\varphi - \left(\frac{1}{(N+2)^2}-\frac{\varphi}{(N+3)^2}\right)\right) < 0$$

Evaluating $\frac{\partial^2\pi_{IB}}{\partial N\partial\varphi}$ at $\varphi = 1$ we have $\frac{1}{2}\frac{2N+5}{(N+3)^2(N+2)^2} > 0$. Using $\frac{\partial^3}{\partial N\partial\varphi^2} < 0$ we conclude that $\frac{\partial^2\pi_{IB}}{\partial N\partial\varphi} > 0\ \forall\varphi$. Evaluating $\frac{\partial\pi_{IB}}{\partial N}$ at $\varphi = 1$ we have $-\frac{1}{2}\frac{2N+5}{(N+3)^2(N+2)^2} < 0$. Hence, $\frac{\partial\pi_{IB}}{\partial N} < 0$ for all $\varphi \in (0,1]$, which establishes the result.

Profits for the club bidder with a value of x_{CB} if $x_{CB} \le 1$ are given by,

$$\left(\int_0^{x_{CB}}(x_{CB}-y)H'(y)\,dy\right) = \left[(x_{CB}-y)H(y)\right]_0^{x_{CB}} + \int_0^{x_{CB}}H(y)\,dy$$
$$= \int_0^{x_{CB}}H(y)\,dy.$$

Similarly, if $x_{CB} > 1$ we have,

$$\int_0^1(x_{CB}-y)H'(y)\,dy = \left[(x_{CB}-y)H(y)\right]_0^1 + \int_0^1 H(y)\,dy$$
$$= (x_{CB}-1) + \int_0^1 H(y)\,dy$$

We calculate the ex ante expected profits as

$$\begin{aligned}
\pi_{CB} \equiv & \int_0^{\varphi} \int_0^{x_{CB}} H(y)\, dy\; G'(x_{CB})\, dx_{CB} + \int_{\varphi}^{1} \int_0^{x_{CB}} H(y)\, dy G'(x_{CB})\, dx_{CB} \\
& + \int_1^{1+\varphi} \left((x_{CB} - 1) + \int_0^1 H(y)\, dy \right) G'(x_{CB})\, dx_{CB} \\
& = - \int_0^{\varphi} H(x_{CB})\, G(x_{CB})\, dx_{CB} - \int_{\varphi}^{1} H(x_{CB})\, G(x_{CB})\, dx_{CB} + \varphi \\
& + \int_0^1 H(y)\, dy - \int_1^{1+\varphi} G(x_{CB})\, dx_{CB}
\end{aligned}$$

Substituting the distribution $G(\cdot)$ specified earlier we can write the ex ante expected profits as

$$\begin{aligned}
\pi_{CB} = & - \int_0^{\varphi} (x_{CB})^N \left(\frac{(x_{CB})^2}{2\varphi} \right) dx_{CB} - \int_{\varphi}^{1} (x_{CB})^N \left(\frac{1}{2} (2x_{CB} - \varphi) \right) dx_{CB} + \varphi \\
& + \int_0^1 y^N dy - \int_1^{1+\varphi} \left(\frac{1}{2} \frac{2x_{CB} - 1 - (x_{CB} - \varphi)^2}{\varphi} \right) dx_{CB} \\
& = \frac{1}{(N+2)(N+1)} + \frac{\varphi}{6} \frac{\varphi + N\varphi + 3}{N+1} - \frac{\varphi^{N+2}}{(N+3)(N+2)(N+1)},
\end{aligned}$$

as desired. Taking the derivative of above with respect to φ we have

$$\frac{\partial \pi_{CB}}{\partial \varphi} = \frac{3N + 6\varphi\left(1 - \varphi^N\right) + 8N\varphi + 2N^2\varphi + 9}{6(N+3)(N+1)} > 0$$

We start by calculate $\frac{\partial^3 \pi_{CB}}{\partial N \partial \varphi^2}$ as follows:

$$\begin{aligned}
& \frac{\partial^3}{\partial N \partial \varphi^2} \left(\frac{1}{(N+2)(N+1)} + \frac{\varphi}{6} \frac{\varphi + N\varphi + 3}{N+1} - \frac{\varphi^{N+2}}{(N+3)(N+2)(N+1)} \right) \\
& = \varphi^N \frac{1 - (N+3)\ln \varphi}{(N+3)^2} > 0
\end{aligned}$$

Thus, $\frac{\partial^2 \pi_{CB}}{\partial N \partial \varphi}$ is increasing in φ. Evaluating $\frac{\partial^2 \pi_{CB}}{\partial N \partial \varphi}$ at $\varphi = 1$ we have

$$\begin{aligned}
\left. \frac{\partial^2 \pi_{CB}}{\partial N \partial \varphi} \right|_{\varphi=1} & = \left[\frac{\partial^2}{\partial N \partial \varphi} \left(\frac{1}{(N+2)(N+1)} + \frac{\varphi}{6} \frac{\varphi + N\varphi + 3}{N+1} - \frac{\varphi^{N+2}}{(N+3)(N+2)(N+1)} \right) \right]_{\varphi=1} \\
& = - \frac{1}{2(N+3)^2} < 0
\end{aligned}$$

Thus, $\frac{\partial^2 \pi_{CB}}{\partial N \partial \varphi}$ is negative $\forall \varphi \in [0,1]$, which implies that if $\frac{\partial \pi_{CB}}{\partial N} < 0$ (see below) for $\varphi = 0$ then it negative $\forall \varphi \in [0,1]$.

$$\left.\frac{\partial \pi_{CB}}{\partial N}\right|_{\varphi=0} = \lim_{\varphi \to 0} \frac{\partial}{\partial N} \left(\frac{1}{(N+2)(N+1)} + \frac{\varphi}{6} \frac{\varphi + N\varphi + 3}{N+1} - \frac{\varphi^{N+2}}{(N+3)(N+2)(N+1)} \right)$$
$$= -\frac{2N+3}{(N+2)^2 (N+1)^2} < 0.$$

Note that

$$\pi_{CB} - \pi_{IB} = \frac{\varphi}{6} \frac{\varphi + N\varphi + 3}{N+1} + \frac{\varphi}{2N(N+1)} - \frac{\varphi^{N+1}}{N(N+2)(N+1)}$$
$$= \frac{9N + 2N\varphi + 6\left(1 - \varphi^N\right) + 3N^2\varphi + N^3\varphi + 3N^2}{6N(N+2)(N+1)} \varphi$$
$$> 0$$

□

Proof of Proposition 1: The first part is a straight forward implication of the fact that π_{CB} is increasing in φ and π_i is independent of φ. To see the second part, calculate π_{CB} at $\varphi = 1$ to obtain,

$$\pi_{CB}(\varphi = 1) = \frac{1}{6} \frac{7N + N^2 + 18}{(N+1)(N+3)}.$$

Thus,

$$\pi_{CB} - 2\pi_i = \frac{1}{6} \frac{7N + N^2 + 18}{(N+1)(N+3)} - \frac{2}{(N+3)(N+2)}$$
$$= \frac{1}{6} \frac{\left(20N + 9N^2 + N^3 + 24\right)}{(N+1)(N+2)(N+3)},$$

which is strictly positive for all N.

At $\varphi = 0$

$$\pi_{CB}(\varphi = 0) - 2\pi_i = \frac{1}{(N+2)(N+1)} - \frac{2}{(N+3)(N+2)}$$
$$= -\frac{N-1}{(N+3)(N+2)(N+1)} < 0 \text{ (if } N > 1\text{)}$$

To show the last part all we need is the fact that $\pi_{CB} - 2\pi_i$ is negative at $\varphi = 0$, positive at $\varphi = 1$

and increasing for all φ. $\square$

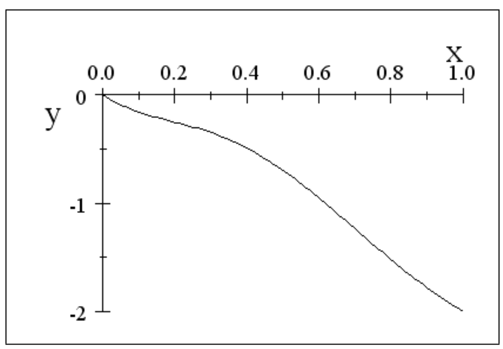
Proof of Proposition 2: Let the numerator of $\widehat{\Gamma}(N,\varphi)$ be

$$\gamma(N,\varphi) = -N^2\varphi + (4-5\varphi)N + 4 - 6\varphi + 2(3 + N(1-\varphi))\varphi^{N+1}$$

We will show first that $\frac{\partial^2 \gamma(N,\varphi)}{\partial N^2}$ is negative for all $N \geq 2$.

$$\begin{aligned}
\frac{\partial}{\partial N}\gamma(N,\varphi) &= -2N\varphi + (4-5\varphi) + 2(3+N(1-\varphi))\varphi^{N+1}\ln\varphi + 2(1-\varphi)\varphi^{N+1} \\
\frac{\partial^2}{\partial N^2}\gamma(N,\varphi) &= -2\varphi + 2(3+N(1-\varphi))\varphi^{N+1}\ln^2\varphi + 2(1-\varphi)\varphi^{N+1}\ln\varphi + 2(1-\varphi)\varphi^{N+1}\ln\varphi \\
&= -2\varphi\left(1 - (3+N(1-\varphi))\varphi^N\ln^2\varphi - (1-\varphi)\varphi^N\ln\varphi - (1-\varphi)\varphi^N\ln\varphi\right) \\
&= -2\varphi\left(1 - [(3+N(1-\varphi))\ln\varphi + 2(1-\varphi)]\varphi^N\ln\varphi\right) \\
&= -2\varphi\left(1 - [(3+2(1-\varphi))\ln\varphi + 2(1-\varphi)]\varphi^2\ln\varphi\right) \text{ (at } N=2),
\end{aligned}$$

which is negative for all $\varphi \in (0,1)$ (see below).



$$\frac{\partial^3}{\partial N^3}\gamma(N,\varphi) = 2\varphi\left(\left[(3+N(1-\varphi))\ln\varphi + 2(1-\varphi)\right]\varphi^N\ln^2\varphi + ((1-\varphi))\varphi^N\ln^2\varphi\right)$$
$$= 2\varphi\left([3+N(1-\varphi)]\ln\varphi + 3(1-\varphi)\right)\varphi^N$$
$$= 2\varphi^{N+1}\ln^2\varphi\left([3+N(1-\varphi)]\ln\varphi + 3(1-\varphi)\right)$$
$$\propto [3+N(1-\varphi)]\ln\varphi + 3(1-\varphi)$$
$$< [3+(1-\varphi)]\ln\varphi + 3(1-\varphi) \text{ (decreasing in } N\text{)}$$
$$= (4-\varphi)\ln\varphi + 3(1-\varphi)$$
$$< 0 \text{ (for all } \varphi < 1\text{)}$$

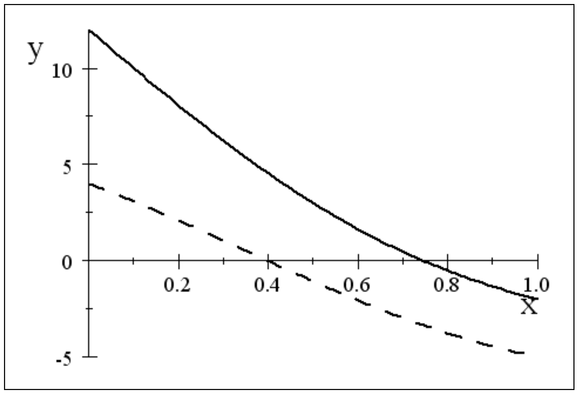
Now

$$(3\ln\varphi - 3\varphi + N\ln\varphi - N\varphi\ln\varphi + 3)$$
$$= ((3+N(1-\varphi))\ln\varphi + 3(1-\varphi))$$
$$< ((4-\varphi)\ln\varphi + 3(1-\varphi))$$
$$< 0 \text{ (Increasing in } \varphi\text{, at } \varphi = 0 \text{ it is 0)}$$

Thus, we have shown that the second derivative is negative at $N = 2$ and is decreasing in N. Thus, $\frac{\partial^2\gamma(N,\varphi)}{\partial N^2}$ is negative for all $N \geq 2$. Now consider the following 4 mutually exclusive and exhaustive cases:

1. $\gamma(2,\varphi) \geq 0$ and $\left[\frac{\partial}{\partial N}\gamma(N,\varphi)\right]_{N=2} \leq 0$
2. $\gamma(2,\varphi) < 0$ and $\left[\frac{\partial}{\partial N}\gamma(N,\varphi)\right]_{N=2} \leq 0$
3. $\gamma(2,\varphi) \geq 0$ and $\left[\frac{\partial}{\partial N}\gamma(N,\varphi)\right]_{N=2} > 0$
4. $\gamma(2,\varphi) < 0$ and $\left[\frac{\partial}{\partial N}\gamma(N,\varphi)\right]_{N=2} > 0$

In case 1 & 3, using the fact that $\frac{\partial^2\gamma}{\partial N^2} < 0$ and $\gamma(N,\varphi)$ in the limit is negative, we know that it there will exist some N_1 such that $\gamma(N_1,\varphi) = 0$ and for all $N > N_1$, $\gamma(N_1,\varphi) < 0$. In case 2, $N_1 \in (1,2)$ everything else follows the same logic as case 1 & 3. In case 4, it is possible that there exists some N_1, $N_2 > 2$ such that $\gamma(N_1,\varphi) = 0$ and $\gamma(N_2,\varphi) = 0$. We thus need to rule out case 4. The figure below plots $\gamma(N,\varphi)$ (solid line) $\frac{\partial}{\partial N}\gamma(N,\varphi)|_{N=2}$ (dashed line). Notice that there does not exist φ such that case 4 is satisfied.



Proof of Proposition 4: For $\varphi = 0$, $N_{NC} = N_{CB} - 1$. The total number of participating bidders in both cases are equal. Moreover, using the fact that $x_{CB} = x_{N+1}$ it is easy to see that the statistical distribution of the second highest value is the same in both scenarios. Hence,the seller obtains the same expected revenue: $\frac{N_{NC}+1}{N_{NC}+3} - \Gamma(0, N_{CB}(0)) = 0$.

Consider now the iso-profit curve of the independent bidders for some fixed entry cost c, i.e., $\pi_{IB}(\varphi, N) = c$. Note that, along the iso-profit curve,

$$\left[\frac{\partial \varphi}{\partial N}\right]_{\pi_{IB}(\varphi,N)=c} = -\frac{\frac{\partial}{\partial N}\pi_{IB}(\varphi, N)}{\frac{\partial}{\partial \varphi}\pi_{IB}(\varphi, N)} < 0$$

since the numerator and the denominator are of the same sign and are both negative.

Now consider the iso-profit curve of the seller at some arbitrary profit K, i.e., $\Gamma(\varphi, N) = K$. Along the seller's iso-profit curve we have

$$\left[\frac{\partial \varphi}{\partial N}\right]_{\Gamma(\varphi,N)=K} = -\frac{\frac{\partial}{\partial N}\Gamma(\varphi, N)}{\frac{\partial}{\partial \varphi}\Gamma(\varphi, N)} < 0$$

since again both the numerator and the denominator have already been shown to be positive (Lemma 2).

We now show that $\left[\frac{\partial \varphi}{\partial N}\right]_{\Gamma(\varphi,N)=K} < \left[\frac{\partial \varphi}{\partial N}\right]_{\pi_{IB}(\varphi,N)=c}$ at any crossing point, i.e., for the same φ

and N. We have

$$\begin{aligned}
&\left[\frac{\partial\varphi}{\partial N}\right]_{\pi_{IB}(\varphi,N)=c}-\left[\frac{\partial\varphi}{\partial N}\right]_{\Gamma(\varphi,N)=K}>0\\
&\Leftrightarrow -\frac{\frac{\partial}{\partial N}\left(\frac{1}{(N+2)(N+1)}-\frac{\varphi}{2N(N+1)}+\frac{\varphi^{N+1}}{N(N+2)(N+1)}-\frac{\varphi^{N+2}}{(N+3)(N+2)(N+1)}\right)}{\frac{\partial}{\partial\varphi}\left(\frac{1}{(N+2)(N+1)}-\frac{\varphi}{2N(N+1)}+\frac{\varphi^{N+1}}{N(N+2)(N+1)}-\frac{\varphi^{N+2}}{(N+3)(N+2)(N+1)}\right)}\\
&\quad-\left(-\frac{\frac{\partial}{\partial N}\left(\frac{\varphi}{2}\left(\frac{N}{N+1}\right)+\left(1-\frac{\varphi}{2}\right)\left(\frac{6N+6\varphi-N\varphi+(N+4)(2-\varphi)N^2-2(3+N(1-\varphi))\varphi^{N+1}}{(N+3)(N+2)(N+1)(2-\varphi)}\right)\right)}{\frac{\partial}{\partial\varphi}\left(\frac{\varphi}{2}\left(\frac{N}{N+1}\right)+\left(1-\frac{\varphi}{2}\right)\left(\frac{6N+6\varphi-N\varphi+(N+4)(2-\varphi)N^2-2(3+N(1-\varphi))\varphi^{N+1}}{(N+3)(N+2)(N+1)(2-\varphi)}\right)\right)}\right)>0\\
&\Leftrightarrow \frac{6N+3N\varphi+4N^2\varphi+N^3\varphi+2N^2+2N\varphi^{N+2}+2N^2\varphi^{N+2}+2\varphi(N+3)(N+1)\left(1-\varphi^N\right)}{N(N+1)\left((N+4N\left(1-\varphi^N(2-\varphi)\right)+N^2\left(1-2\varphi^N(1-\varphi)\right)+6\left(1-\varphi^N\right)\right)\right)}>0.
\end{aligned}$$

The numerator is clearly positive. Therefore, the sign of the expression depends on the signs of the terms $\left(1-2\varphi^N(1-\varphi)\right)$ and $\left(1-\varphi^N(2-\varphi)\right)$ in the denominator. Since $\varphi\leq 1$, both terms are clearly increasing in N. Thus, if we can show that for $N=1$ the terms are both positive, we will be done. Note then that, for the first term, evaluated at $N=1$ gives

$$(1-2\varphi(1-\varphi))>0\Leftrightarrow\varphi^2+(1-\varphi)^2>0,$$

which is clearly true. For the second term, at $N=1$ we have

$$1-\varphi(2-\varphi)>0\Leftrightarrow(1-\varphi)^2>0,$$

which is also clearly positive. This establishes that $\left[\frac{\partial\varphi}{\partial N}\right]_{\Gamma(\varphi,N)=K}<\left[\frac{\partial\varphi}{\partial N}\right]_{\pi_{IB}(\varphi,N)=c}$. Note that, since the inequality is strict, and both the seller's as well as the independent bidders' profits are continuous in N and φ, it also implies that the two iso-profit curves can only cross once.

Let N_{NC} and $N_{CB}(\varphi)$ be the equilibrim number of bidders for a given φ and c. For any $\varphi\geq 0$, consider a slight increase φ to $\varphi+\varepsilon$. From above we know that, in order to stay on the independent bidder's iso-profit curve (with profits equal to c, the entry cost), N_{CB} will go down from $N_{CB}(\varphi)$ to $N_{CB}(\varphi+\varepsilon)$. However, because the iso profit curve for the seller's expected profit curve is steeper, N_{CB} will go down more than the amount that would be required to keep the seller's profit constant.

Thus,

$$\Gamma\left(\varphi+\varepsilon, N_{CB}\left(\varphi+\varepsilon\right)\right) < \Gamma\left(\varphi, N_{CB}\left(\varphi\right)\right) \Rightarrow \frac{N_{NC}+1}{N_{NC}+3} - \Gamma\left(\varphi+\varepsilon, N_{CB}\left(\varphi+\varepsilon\right)\right) > 0.$$

This establishes the proposition since the argument holds for any arbitrary $\varphi \geq 0$ and the fact that the iso-profit curves only cross once. □

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