Investment Horizons and Asset Prices under Asymmetric Information *

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Abstract

I study a generalized OLG economy where asymmetrically informed agents have arbitrary investment horizons. As horizons increase, the *age-adjusted risk aversion* of investors fall, and the *risk transfer* from forced liquidators into voluntary buyers drops. Two equilibria coexist for long enough horizons: a stable, low volatility equilibrium, and an unstable one with higher volatility. Along the stable equilibrium, longer horizons raise prices, lower volatility, and incite aggressive trading by the informed investors, which impound their knowledge into prices and improve market efficiency. For short horizons, cautious trading disaggregates information from prices, and the economy approaches one with no private information.

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1 Introduction

The fact that investors care about returns over a limited horizon is a pervasive feature of financial markets. With trading carried out mostly by intermediaries who care about short-term performance –be it through explicit contracts, or by the threat of fleeing investors– one has good reasons to suspect long-run prospects might often be underweighted in everyday market transactions. From a more cyclical perspective, the outset of financial crises are characterized by widespread investors' withdrawals and fund liquidations, suggesting fund managers' bias towards immediacy might be particularly acute during such episodes. This opens the question of whether the radically different behavior of markets during crises –sharp price drops, heightened volatility, and higher expected returns– could be partly explained by variations in the effective horizons of intermediaries. Moreover, even leaving intermediation out of the story, the fact that households *literally* have finite lifespans suggests concerns about the short-term are relevant.

Despite the apparent importance of understanding the role of finite horizons, our knowledge in this regard seems limited. Mainstream asset pricing often assumes infinitely-lived investors who voluntarily trade at all times.[1] Alternative models with finite horizons, on the other hand, build upon an OLG framework where investors live for two periods.[2] While useful to understand some pricing features and the limitations of arbitrage activity, the lifespan of investors is fixed in these models, making it difficult to compare characteristics across economies with *different* investors' lifespans. The present paper contributes to fill this gap providing a model where investors have arbitrary investment horizons, T. The model then studies how economies with different horizons compare in terms of asset pricing characteristics, with a special focus on the implications for the informational role of prices, or market efficiency.

The model is based on the dynamic rational expectations analysis of Wang (1994). Competitive investors trade an infinitely-lived risky asset to maximize utility of lifetime consumption under CARA preferences. There are two types of investors: those who observe private information about the persistent component of the dividend process (informed investors), and those who infer it from dividends and prices (uninformed investors). Investors also differ in their age. At any point in time, there are T generations of investors coexisting. $T-1$ groups (aged $1, 2, ...T-1$) are still active in the market and can take voluntary positions, while the oldest generation (aged T) is exiting and must unwind its positions at prevailing prices. The net supply of the asset is random and causes prices to fluctuate for reasons orthogonal to fundamentals. This prevents prices from fully revealing the information observed by informed investors.

Generally speaking, the paper finds that investment horizons matter a great deal for asset prices and market efficiency. Along the stable, low volatility equilibrium of the model, longer horizons increase average prices and reduces risk premium and volatility of returns. Moreover, since fundamentals explain a larger fraction of price volatility for long horizons, prices are more informative for investors who learn from them, improving the informational efficiency of the market. The generalized OLG economy developed here highlights two key and novel mechanisms that account for these results. The first relates to the "pricing" of risk, which I label the *age-adjusted risk aversion* effect. Longer lived investors are willing to purchase

[1]See Campbell (2000) for a comprehensive survey.

[2]See De Long et al. (1990), Spiegel (1998), Bacchetta and Van Wincoop (2006), Watanabe (2008), and Banerjee (2011).

the security at lower expected returns since they can smooth consumption over more periods and are less exposed to temporary price fluctuations. The second mechanism relates to the "quantity" of risk that active investors must bear in equilibrium, which I label the *risk transfer* effect. As horizons increase, the relative size of the dying generation shrinks in relation to active investors (voluntary traders), who then bear less aggregate risk. As both mechanisms work in the same direction, longer horizons unambiguously reduce risk premium, mitigate non-fundamental volatility, and improve market efficiency. Interestingly, along the unstable, high volatility equilibrium, increasing investment horizons has the exact opposite implications on volatility, risk premium, and price informativeness.

More specifically, the paper makes three contributions. The first is methodological, and corresponds to the characterization of existence, multiplicity, and stability properties of linear equilibria in generalized OLG models. Nesting arbitrary investment horizons and different information structures, the model studies a variety of economies whose equilibrium properties have not been previously addressed. Regarding existence, economies which fail to exhibit linear equilibria for short horizons will always admit equilibria for large enough T. As a reverse interpretation, market equilibria can break down as horizons shorten.

Regarding multiplicity, a finite horizon economy generically exhibits two equilibria (whenever equilibria exists),[3] a result consistent with the findings of Spiegel (1998), Bacchetta and Van Wincoop (2006), and Watanabe (2008), for the case where $T = 2$ (and a single risky asset). These include a low volatility equilibrium (LVE) in which supply shocks have small price impact, and a high volatility equilibrium (HVE) where they cause large price fluctuations. This paper extends previous results in several dimensions. I show analytically for special cases (symmetric information, and $T = 2$), and numerically for the general economy, that the LVE is stable in the best-response sense, while the HVE is unstable. Moreover, the paper describes pricing moments along these equilibria as a function of the horizons. Along the stable LVE, increases in T lower the price impact of supply, reducing non-fundamental volatility. As $T \to \infty$, the LVE converges smoothly to the infinite-horizon economy of Wang (1994). Along the HVE however, longer horizons leads to unbounded increases in price volatility. In the limit, this equilibrium vanishes as $T \to \infty$. Intuitively, as investors live longer, both the increased willingness to take risks and the smaller proportion of forced liquidations makes the HVE increasingly "difficult" to sustain. While many of these results rely on numerical simulations, I prove analytically novel results for economies with symmetric information. Namely, as $T \to \infty$, a linear equilibrium always exist, and it is unique.[4]

Second, the paper introduces and analyzes the afore-mentioned mechanisms which are the key drivers of the results. These mechanisms are, to the best of my knowledge, new to the literature. To understand the *age-adjusted risk aversion* effect, consider the case of infinitely-lived investors. In this economy the marginal propensity to consume wealth is the ratio between the net and gross rate of interest, r/R. This coefficient is precisely how agents price uncertainty about wealth fluctuations –the "age-adjusted" risk aversion parameter corresponds to $\gamma \cdot r/R$, where γ is the CARA parameter. In the other extreme when agents live two periods, the marginal propensity to consume wealth is one, and the effective risk aversion equals γ. In the present model, the pricing of risk depends on the age of the investor. Importantly, as

[3]This can be proven analytically when $T = 2$ and information is symmetric, and inferred numerically for the general case.
[4]Banerjee (2011) provides an insightful discussion about the properties of each equilibria in the case of $T = 2$.

horizons increase, the average age-adjusted risk aversion declines.

The economics behind the *risk transfer* effect are as follows. Consider once again the infinite-horizon case. Because there is no forced transfer of risk between generations, all agents bear aggregate risk proportionally. In contrast, in a two-period OLG economy, the dying generation (in mass 1/2) must unload all its positions into a single younger generation (also in mass 1/2). In other words, the whole aggregate risk must exchange hands every period! The generalized OLG economy studied here essentially spans the whole intermediate region of horizons left out by these cases, showing how increases in T lower the relative transfer of risk from the dying to all other generations.

The third and most important contribution of the paper is the characterization of asset price informativeness as a function of investors' horizons. I study the behavior of asset prices along the stable LVE for three economies: a full-information benchmark where all investors are informed about the persistent component; a no-information economy in which all investors learn only from dividends; and the asymmetric-information economy where a relatively small mass of investors has access to private information and the rest learn from dividends and prices. A comparison between these economies reveals the following results: a) For long horizons, the asymmetric information economy behaves similarly to the full information benchmark. The low risk environment implied by large T induces active trading by the informed, which impound their knowledge into prices. This increases the share of price volatility explained by fundamentals, improving the precision of prices as endogenous signals. Consequently, the uncertainty of the uninformed is reduced. In this economy, expected returns and return volatility closely mimic the full-information case. b) For short investment horizons, the asymmetric information economy approaches the no-information benchmark. The high risk implied by small T leads informed investors to trade cautiously, disaggregating information from prices and increasing uncertainty about fundamentals for the uninformed. Price movements are largely driven by supply innovations, and expected returns and return volatility line up closely with the no-information case.

After the conceptual discussion on the implications of investment horizons, I include two applications that illustrate how the model can be applied to related questions in macroeconomics and finance. The first studies the effects of a "baby-boomer" generation, and relates more generally to low-frequency shocks to population dynamics. The second discusses how the model can help understand the implications of fund liquidations forced by withdrawals during financial crises. According to this interpretation, increased liquidations during episodes of distress play a similar role as a shortening of effective investment horizons.

The model presented here is related to the finance literature in OLG environments. De Long et al. (1990), as well as Spiegel (1998), Bacchetta and Van Wincoop (2006), and Watanabe (2008), study economies with 2-period lived investors. Bacchetta and Van Wincoop (2006), and Watanabe (2008), are perhaps closest to this paper since they study economies with asymmetrically informed investors. In all these models however, investment horizons are fixed. He and Wang (1995), and Cvitanić et al. (2006), study finite horizon economies with incomplete information. Since agents derive utility only from terminal wealth, the age-adjusted risk aversion coincides with the CARA parameter in both papers. Moreover, in these papers all investors grow old simultaneously, so there is no risk transfer from dying to active generations. The two main forces at work in the present paper are therefore quite different.

Other related papers study short-term investors in the context of 3-period models. In Froot et al. (1992), investors might choose to study information unrelated to fundamentals to the extent it predicts short-term price movements. Kondor (2012) studies an economy with short-term traders, focusing on how public disclosures can simultaneously increase divergence of rational beliefs about returns while lowering conditional uncertainty about fundamentals. Cespa and Vives (2012) find that persistent noise trading can generate multiple equilibria with horizon economy. Albagli (2009) studies the impact of increased fund liquidations during downturns in effectively lowering investors' horizon, and its implications for price informativeness. The key difference with these models remains in their rigid lifespan structure, while variation of horizons constitute the main object of analysis in the present paper.

The rest of the paper is organized as follows. Section 2 introduces the model. Section 3 characterizes existence, multiplicity, and stability, for symmetric information economies. Section 4 studies the impact of horizons in the asymmetric information economy, discussing implications for market efficiency and asset pricing. Section 5 discusses two applications that give a more concrete interpretation of varying investment horizons. Section 6 concludes. All proofs are in the appendix.

2 A Generalized OLG economy with Asymmetric Information

2.1 Basic Setup

2.1.1 Securities

Time is discrete and runs to infinity. There is a risk-free asset in perfectly elastic supply yielding a gross return of $R = 1 + r > 1$, and one risky asset paying an infinite stream of dividends $\{D_\tau\}_{\tau=1}^{\infty}$, where

$$D_t = F_t + \varepsilon_t^D, \tag{1}$$

$$F_t = (1-\rho_F)\bar{F} + \rho_F F_{t-1} + \varepsilon_t^F. \tag{2}$$

F_t is the persistent payoff component –in what follows, the *fundamental*– with mean $\bar{F}$ and persistence ρ_F $(0 \leq \rho_F \leq 1)$. The supply of the risky asset is given by θ_t, a mean-reverting process described by

$$\theta_t = (1-\rho_\theta)\bar{\theta} + \rho_\theta \theta_{t-1} + \varepsilon_t^\theta, \tag{3}$$

with $\bar{\theta} \geq 0$ its unconditional mean, ρ_θ its persistence $(0 \leq \rho_\theta \leq 1)$. The error vector $\epsilon_t \equiv [\varepsilon_t^D \;\; \varepsilon_t^F \;\; \varepsilon_t^\theta]'$ is serially uncorrelated and joint normal with mean zero, and covariance matrix $\Sigma = \text{diag}(\sigma_D^2, \sigma_F^2, \sigma_\theta^2)$.

2.1.2 Investors

The mass of investors in the economy is normalized to unity. A fraction μ are uninformed about F_t, which due to the shocks ε_t^D and ε_t^F, is not revealed by dividends. Letting $\underline{h}_t \equiv \{h_{t-s}\}_{s=0}^{+\infty}$ denote the history of variable h up to time t, public information observed by the uninformed is the history of dividends and prices represented by the filtration $\Omega_t^U = \{\underline{D}_t, \underline{P}_t\}$. The complement share of investors (mass $1-\mu$) are informed: in addition to Ω_t^U, they contemporaneously observe the realization of F_t.

The population in the economy follows a generalized overlapping generation structure with uniform age distribution. At time t, a mass $1/T$ of investors aged T dies and is replaced by an equal mass of new-born investors, who live for T periods. Hence, at any point T different generations of investors coexist (aged $j = 1, 2, ...T$). The mix between informed and uninformed is the same in each generation, so that the age/information distribution is constant. Investors maximize utility of lifetime consumption $\sum_{s=1}^{T} \beta^s U(C_{t+s})$, given by negative exponential $U(C) = -e^{-\gamma C}$ with equal CARA coefficient γ across investors. All investors are born with exogenous wealth w_0.

2.1.3 Asset Markets

Investors can take long/short positions in the risky asset during active trading years $j = \{1, 2...T-1\}$, denoted $X_{j,t}^U$ and $X_{j,t}^I$ for uninformed and informed investors, for which they can borrow/save unlimited amounts of the risk-free asset. The dying generation aged T, however, must liquidate accumulated positions and consume terminal wealth $(X_{T,t}^U = X_{T,t}^I = 0)$. Aggregate asset demand is given by:

$$AD : X_t \equiv \frac{1}{T}(\mu \cdot \sum_{j=1}^{T-1} X_{j,t}^U + (1-\mu) \cdot \sum_{j=1}^{T-1} X_{j,t}^I). \tag{4}$$

Investors are price-takers and submit price-contingent orders (generalized limit orders) to a "Walrasian auctioneer", who sets a price P_t for the risky asset to clear all orders. Defining the dollar excess return of the risky asset by $Q_{t+1} \equiv D_{t+1} + P_{t+1} - RP_t$, the wealth of investor (j, i) (j=age, $i = \{U, I\}$) satisfies

$$W_{j+1,t+1}^i = (W_{j,t}^i - C_{j,t}^i)R + X_{j,t}^i Q_{t+1}. \tag{5}$$

2.2 Equilibrium Characterization

2.2.1 Recursive Representation

The solution approach builds on a 3-step "guess and verify" procedure. First, conjecture that prices are linear in the state variables, and update beliefs of future return moments accordingly. Second, solve for investors' demands. Third, impose market clearing and find the price coefficients of the linear conjecture. Because the economy is dynamic, the evolution of the state variables must be expressed in recursive form. Let $\Psi_{t+1} \equiv [1 \quad F_{t+1} \quad \theta_{t+1}]'$, and define $\hat{F}_t^U \equiv \mathbb{E}[F_t|\Omega_t^U]$ as the uninformed investors' forecast of the persistent component F_t. Given equations (1), (2), and (3), the evolution of Ψ_{t+1} can be written as

$$\Psi_{t+1} = A_\psi \Psi_t + B_\psi \epsilon_{t+1}^U, \tag{6}$$

where $\epsilon_{t+1}^U \equiv [\varepsilon_{t+1}^D \quad \varepsilon_{t+1}^F \quad \varepsilon_{t+1}^\theta \quad \tilde{F}_t^U]'$ is the expanded error vector faced by the uninformed investors, including the exogenous shocks and their own forecast error $\tilde{F}_t^U \equiv \hat{F}_t^U - F_t$ (see appendix). The evolution of beliefs, asset demands, and prices, can now be expressed with this recursive structure. In particular, I conjecture the following linear equilibrium price:

$$P_t = p_0 + \hat{p}_F \hat{F}_t^U + p_F F_t + p_\theta \theta_t. \tag{7}$$

I now introduce the equilibrium concept formally. For any filtration Ω, let $H(x|\Omega) : R \rightarrow [0,1]$ denote the conditional posterior cdf of a random variable x. Let (j,i) be the age/information type of each investor in the economy, with $j = \{1, 2, ...T\}$, and $i = \{U, I\}$, and let the filtration $\Omega_t^U = \{\underline{D}_t, \underline{P}_t\}$ and $\Omega_t^I = \{\underline{D}_t, \underline{P}_t, F_t\}$ represent the information available at time t to uninformed and informed investors. *A competitive rational expectations equilibrium is: 1. A price function given by (7), 2. A risky asset demand $X_{j,t}^i = x(P_t, \Omega_t^i, j)$ by investor (j,i), 3. Posterior beliefs $H(\Psi_t|\Omega_t^U)$ and $H(\Psi_t|\Omega_t^I)$ for uninformed and informed investors, such that $\forall$ (j,i): (i) Asset demands are optimal given prices and posterior beliefs; (ii) Asset markets clear at all times; and (iii) Posterior beliefs satisfy Bayes law.*

2.2.2 Investors' Problem

An investor aged j in period t, with information given by filtration Ω_t^i, solves

$$\max_{X_{j,t}^i, C_{j,t}^i} \quad \mathbb{E}[-\sum_{s=0}^{T-j} \beta^s e^{-\gamma C_{j+s,t+s}^i} \mid \Omega_t^i], \text{ s.t. } W_{j+1,t+1}^i = (W_{j,t}^i - C_{j,t}^i)R + X_{j,t}^i Q_{t+1}, \;\; W_{1,t}^i = w_0. \tag{8}$$

This optimization remains tractable if wealth is conditionally normally distributed: the value function takes a linear-quadratic form in the state variables, and portfolio and consumption policies are obtained in closed form. For informed investors, establishing normality of excess returns Q_{t+1} is immediate. From the observation of public information, they back out the forecast $\hat{F}_t^U$ of the uninformed. Since they also observe F_t, the price reveals supply θ_t. Q_{t+1} is then conditionally gaussian for the informed. For the uninformed, beliefs are characterized by a dynamic filter. From (7), uninformed investors back out a noisy signal about F_t, call it the *informational content of price*, given by $p_t \equiv p_F F_t + p_\theta \theta_t$. Price signals and dividends constitute public information about the state vector Ψ_t, and can be written as

$$S_t \equiv [D_t \;\; p_t]' = A_s \Psi_t + B_s \epsilon_t^U. \tag{9}$$

Theorem 1 describes beliefs and establishes normality of uninformed investors' forecast errors. Let $A(l,m)$ be the lth row, mth column element of a matrix A, and let $\mathbb{O} \equiv \mathbb{E}[(\Psi_t - \mathbb{E}[\Psi_t \mid \Omega_t^U])(\Psi_t - \mathbb{E}[\Psi_t \mid \Omega_t^U])' \mid \Omega_t^U]$ denote the conditional variance of the state vector, then

Theorem 1 (Beliefs with public information): *The distribution of the state vector Ψ_t, conditional on the filtration $\Omega_t^U = \{\underline{D}_t, \underline{P}_t\}$, is normal with mean $\mathbb{E}[\Psi_t \mid \Omega_t^U]$ and variance $\mathbb{O}$, where*

$$\mathbb{E}[\Psi_t \mid \Omega_t^U] = A_\psi \mathbb{E}[\Psi_{t-1} \mid \Omega_{t-1}^U] + K(S_t - \mathbb{E}[S_t \mid \Omega_{t-1}^U]), \tag{10}$$

and the conditional variance and projection matrix K jointly solve

$$\mathbb{O} = (I_3 - KA_s)(A_\psi \mathbb{O} A_\psi' + B_\psi \mathcal{V} B_\psi'), \tag{11}$$

$$K = (A_\psi \mathbb{O} A_\psi' + B_\psi \mathcal{V} B_\psi')A_s'(A_s(A_\psi \mathbb{O} A_\psi' + B_\psi \mathcal{V} B_\psi')A_s' + B_s \mathcal{V} B_s')^{-1}, \tag{12}$$

$$\mathcal{V} = \mathrm{diag}(\sigma_D^2, \sigma_F^2, \sigma_\theta^2, \mathbb{O}(2,2)). \tag{13}$$

We can now characterize optimal consumption and portfolio choices:

Theorem 2 (policy rules): *Let $M_t \equiv [1 \;\; F_t \;\; \theta_t \;\; \tilde{F}_t^U]'$ and $M_t^U \equiv [1 \;\; \hat{F}_t^U \;\; \hat{\theta}_t^U]'$ be the contemporaneous projection of informed and uninformed investors about the expanded state vector including the uninformed forecast error $\tilde{F}_t^U$. Then,*

1. The value function and optimal rules of informed investors correspond to

$$J^I(W_{j,t}^I; M_t; j; t) = -\beta^t e^{-\alpha_j W_{j,t}^I - V_j^I(M_t)}, \tag{14}$$

$$X_{j,t}^I = \Big(\frac{A_Q}{\alpha_{j+1}\Gamma_{j+1}^I} - \frac{h_{j+1}^I}{\alpha_{j+1}\Gamma_{j+1}^I}\Big) \cdot M_t, \tag{15}$$

$$C_{j,t}^I = c_j^I + \Big(\frac{\alpha_{j+1}R}{\alpha_{j+1}R+\gamma}\Big)W_{j,t}^I + \frac{M_t' m_{j+1}^I M_t}{2(\alpha_{j+1}R+\gamma)}. \tag{16}$$

2. The value function and optimal rules of uninformed investors correspond to

$$J^U(W_{j,t}^U; M_t^U; j; t) = -\beta^t e^{-\alpha_j W_{j,t}^U - V_j^U(M_t^U)}, \tag{17}$$

$$X_{j,t}^U = \Big(\frac{A_Q^U}{\alpha_{j+1}\Gamma_{j+1}^U} - \frac{h_{j+1}^U}{\alpha_{j+1}\Gamma_{j+1}^U}\Big) \cdot M_t^U, \tag{18}$$

$$C_{j,t}^U = c_j^U + \Big(\frac{\alpha_{j+1}R}{\alpha_{j+1}R+\gamma}\Big)W_{j,t}^U + \frac{M_t^{U\,\prime} m_{j+1}^U M_t^U}{2(\alpha_{j+1}R+\gamma)}, \tag{19}$$

where c_j^I, c_j^U, are age/information-type dependent constants.

Future dollar returns, Q_{t+1}, depend on the contemporaneous state variables F_t and θ_t, and on the projection about these variables of the uninformed. Form the perspective of the informed, this can be restated as a dependence on the expanded vector $M_t \equiv [1 \;\; F_t \;\; \theta_t \;\; \tilde{F}_t^U]'$ ($\tilde{\theta}_t^U$ is perfectly colinear with $\tilde{F}_t^U$). For the uninformed, the state vector includes their current expectations of the state variables; $M_t^U \equiv [1 \;\; \hat{F}_t^U \;\; \hat{\theta}_t^U]'$.

The consumption rules in expressions (16) and (19) have 3 terms. The first is an age-dependent constant that depends on the inter temporal preference parameter β, and the risk free rate (among other parameters, see appendix). The second term is the product between wealth, and the marginal propensity to consume it: $\alpha_{j+1}R/(\alpha_{j+1}R+\gamma)$. I will come back to this term momentarily. The third term adjusts consumption, for a given realization of contemporaneous wealth, to reflect improvements in the realization of the state through its effect on the value function.

More important for its asset pricing consequences are the optimal portfolios in equations (15) and (18), which take the form found in other dynamic CARA-normal models. Consider the informed investors: the term $A_Q/(\alpha_{j+1}\Gamma^I_{j+1})$ is a mean-variance efficient portfolio capturing the tradeoff between expected returns (numerator) and risk (denominator). Here, A_Q is a (1x4) vector loading on the state vector M_t (A^U_Q a 1x3 vector loading on M^U_t, for uninformed investors), where α_{j+1} is the age-dependent risk aversion coefficient, and the positive scalar Γ^I_{j+1} is the renormalized covariance of returns.[5] In simple terms, this ratio is the response of investors' demand to an increase in expected returns. The second term is a hedging component arising from the fact that shocks affect expected returns further into the future, which is reflected in the dependence of the value function at $t+1$ on ϵ_{t+1}, giving rise to an additional source of risk (see Wang (1994) for more details).

An analogous argument holds for uninformed investors' demands, once the conditional state vector M^U_t and the expanded error vector ϵ^U_t are replaced accordingly. Note in particular that the scalar $\Gamma^U_{j+1} > \Gamma^I_{j+1}$, since the uninformed face more uncertainty about future returns, given their less precise estimates of the contemporaneous state variables.

Barring special cases, the solution relies on numerical procedures. Beginning with a known terminal value function for age T, value functions at earlier ages are computed recursively, which give the optimal portfolios for different age-information types. Equilibrium is then solved imposing market clearing:

$$\frac{1}{T}(\mu \cdot \sum_{j=1}^{T-1} X^U_{j,t} + (1-\mu) \cdot \sum_{j=1}^{T-1} X^I_{j,t}) = \theta_t. \tag{20}$$

The conjecture in (7) can be rewritten as $P_t = P \cdot M_t$, where the vector of price coefficients $P \equiv [p_0 \;\; \hat{p}_F + p_F \;\; p_\theta \;\; \hat{p}_F]$ loads on the expanded vector $M_t \equiv [1 \;\; F_t \;\; \theta_t \;\; \tilde{F}^U_t]'$. This vector of price coefficients is the outcome of a fixed-point. Given a conjecture $P^{(0)}$, investor of type (j,i) chooses portfolio $X^i_{j,t}$, which can be written using (15) and (18) as $X^i_{j,t} = \phi_{j,i} \cdot [1 \;\; \mathbb{E}[F_t|\Omega^i_t] \;\; \mathbb{E}[\tilde{F}^U_t|\Omega^i_t] \;\; P_t]'$.[6] The strategy vector $\phi_{j,i}$, which loads on investor's (j,i) projection about F_t and $\tilde{F}^U_t$, and the price, represent the first relation in the fixed point mapping conjectured prices into actions: $\phi_{j,i} = \phi_{j,i}(P^{(0)})$. We then aggregate and impose market clearing, yielding a second relation that maps individual actions to the price coefficients of the linear price equilibrium, $P^{(1)} = h(\{\phi_{j,i}\}^T_{j=1,i=I,U})$. A fixed-point satisfies $P^* = h(\{\phi_{j,i}(P^*)\}^T_{j=1,i=I,U})$.

3 Symmetric Information Economies

This section characterizes equilibria for the following benchmark cases: the no-information economy where the mass of uninformed is $\mu = 1$, and the full-information economy, with $\mu = 0$. These cases are more tractable, allowing the derivation of important analytical results. Moreover, these cases convey the intuition about the mechanisms triggered from comparing different horizons. I begin with a description of the two mechanisms at the center of the argument (section 3.1), the *age-adjusted risk aversion* (AARA) effect, and the *risk transfer* (RT) effect. I then discuss existence and multiplicity (section 3.2), and stability

[5] In discrete time, $\Gamma^I_{j+1} \equiv B_Q(\Xi^I_{j+1})^{-1}B'_Q$, where $\Xi^I_{j+1} \equiv \Sigma^{-1} + v^{I,bb}_{j+1}$ reflects the (inverse) of the variance covariance matrix of returns, Σ^{-1}, and the changes in the value function that follow from innovations in returns (this is the *renormalized* part). As explained in Merton (1990), the renormalization is unnecessary in continuous time (i.e., $\Xi^I_{j+1} \equiv \Sigma^{-1}$).

[6] Writing demands through the strategy vector $\phi_{j,i}$ maps directly into the analysis of stability (section 3.3).

(section 3.3). Section 3.4 presents an exercise to gauge the relative importance of each mechanism. Section 3.5 provides further discussion on the different equilibria generated by the model.

3.1 The two key mechanisms

3.1.1 Age-adjusted risk aversion effect

The first mechanism is related to the changes in the *pricing* of risk induced by changes in T, which I will refer to as the *age-adjusted risk aversion* (AARA) effect. A key determinant of the risk faced by finitely-lived investors are fluctuations in prices. Indeed, a 2-period lived agent who exits the economy when prices are temporary low due to a large supply shock, for example, will fare much worse than a longer-lived investor not forced to unwind her portfolio at adverse prices. As observed in De long et al. (1990), lengthening horizons is akin to receiving "dividend insurance": by living longer, dividend consumption diminishes the impact non-fundamental risk in the utility of investors.

In particular, the AARA coefficient of an agent aged $j-$years is the endogenous parameter α_j, which can be solved recursively through $\alpha_j = \gamma\alpha_{j+1}R/(\alpha_{j+1}R + \gamma)$ (see appendix), giving

$$\alpha_j = \gamma \frac{r}{R - R^{-(T-j)}}, \quad \text{for all } \ j \in [1, T]. \tag{21}$$

Intuitively, fluctuations in wealth affect utility through its effect on consumption, in turn determined by the marginal propensity to consume. From (16), this propensity corresponds to $\frac{\partial C^I_{j,t}}{\partial W^I_{j,t}} = \frac{\alpha_{j+1}R}{\alpha_{j+1}R+\gamma}$. When we apply the risk-aversion parameter γ, the impact of wealth fluctuations on utility is given by $\gamma \cdot \frac{\alpha_{j+1}R}{\alpha_{j+1}R+\gamma}$; precisely the AARA coefficient α_j. In other words, the coefficient of risk aversion γ is tempered by the ability of the investor to smooth consumption, which depends on the number of periods remaining.

Consider two polar cases for intuition. When agents live 2 periods, the marginal propensity to consume when old is 1, so the AARA in the demand of young agents is $\alpha_{j+1} = \gamma$. At the other extreme of infinite horizons (as in Wang (1994)), equation (21) yields $\alpha^* = \gamma\frac{r}{R}$. For the finite horizon economy T more generally, the AARA increases with age according to (21). If we compare economies with different horizons, the average risk tolerance in economies with longer T will be higher. To illustrate the importance of this effect, Table 1 (col. 2) reports the average risk tolerance across investors of different age vintages, $\sum_{j=1}^{T-1}(1/\alpha_j)/(T-1)$, for a given investment horizon T. I present this statistic since individual demands are proportional to risk tolerance, or the inverse of the AARA. To my knowledge, this mechanism has not been studied formally in OLG financial market models. De long et al. (1990) conjecture that increasing horizons should lead to more risk-taking (and a diminished price impact of "noise traders"), but this insight has not been formally studied.

It is pertinent to compare the AARA mechanism to the literature discussing horizon effects. Samuelson (1969) and Merton (1969) show that i.i.d. returns, power utility investors' portfolio is independent of their horizon. The case of return predictability is the subject of a large literature highlighting inter-temporal hedging motives (See Campbell and Viceira (1999), and references therein). Most related to the present paper are Kim and Omberg (1996), and Barberis (2000), who consider finitely-lived investors with power utility and compare risky allocations under different remaining horizons. They find that inter-temporal hedging motives are stronger for investors with more rebalancing periods left, leading them to overweight the risky asset relative to older agents. How do these results relate to the AARA effect? The present paper finds that risky asset demands are increasing in the remaining horizon of the investor. While one might wonder whether this mechanism is an artifact of CARA, the literature on horizon effects with time-varying investment opportunities is consistent with the qualitative result found here, in that longer-lived investors will have a better disposition towards risk.[7]

3.1.2 Risk transfer effect

The second mechanism is related to how changes in T affect the *amount* of risk that must be absorbed in equilibrium by active generations. I label this mechanism the *risk transfer* (RT) effect. We can illustrate this mechanism by rewriting market-clearing in (20) (dropping information-type subscripts):

$$\begin{aligned} \frac{1}{T}(X_{1,t}+X_{2,t}+\cdot\cdot+X_{T-1,t}) &= \underbrace{\frac{1}{T}(X_{1,t-1}+X_{2,t-1}+\cdot\cdot+X_{T-1,t-1})}_{\theta_{t-1}} + \underbrace{(1-\rho_\theta)(\bar{\theta}-\theta_{t-1})+\varepsilon_t^\theta}_{\Delta\theta_t} \qquad (22) \\ \Rightarrow \qquad & \frac{1}{T}(X_{1,t}+\Delta X_{2,t}+\cdots+\Delta X_{T-1,t}) = \frac{1}{T}X_{T-1,t-1}+\Delta\theta_t \end{aligned}$$

The demand in the left of (22) includes all currently active investors. Of these, ages $2, 3\cdot\cdot\cdot, T-1$ were also present in the previous period. Their net demands correspond to $\Delta X_{2,t} = X_{2,t} - X_{1,t-1}$ for the investor aged 2, $\Delta X_{3,t}$ for age 3, and so on. For these investors, the change in net positions is voluntary. But for the investor aged $T-1$ in the previous period, net demand is exogenously set at $-X_{T-1,t-1}$. In equilibrium, the negative of this amount (per capita), plus the supply innovation $(1-\rho_\theta)(\bar{\theta}-\theta_{t-1})+\varepsilon_t^\theta$, must be absorbed by the aggregate change in the net positions of active investors. The fact that only $T-1$ generations are active (with a total mass of $T-1/T$) to absorb the aggregate risk, in effect implies that the retirees are transferring their proportion of the risk (in aggregate mass of $1/T$) to the active investors. Hence, a risk transfer ratio $1/(T-1)$.

Recall however that AARA increases as investors age, leading to a progressive reduction of demand. The vintage of dying agents therefore hold less supply compared to the young, as discussed in more detail below (section 4.4 on trading volume). This gradual reduction highlights that the inter-generation transfer of risk is a smooth process. The fact however remains that, at any point in time, all vintages who hold less than the average share of supply are transferring risk to those who hold more, an effect induced by the anticipation of each generation that they will die in a finite number of periods. Therefore, the risk transfer ratio $1/(T-1)$ remains the relevant statistic to account for the importance of this effect.

[7]A potential concern is that if dividends are i.i.d., CARA preferences still deliver horizon effects (expression in (21) for α_j is independent of return properties). Arguably, this is of secondary importance if the economy under consideration indeed exhibits return predictability, as then the qualitative effects of horizons on risk taking are similar under CARA and CRRA.

3.2 Existence and multiplicity of equilibria

I begin with the limit cases of investment horizons, $T = 2$, and $T \to \infty$, for which existence and multiplicity are proven analytically. I then discuss the general case ($2 < T < \infty$) relying on numerical simulations.

Proposition 1 (2-period OLG): Let $T = 2$,
a) *Let* $\mu = 0$, *and define* $\sigma_\theta^* \equiv \frac{(R-\rho_\theta)}{4\gamma}(\sigma_D^2 + (\frac{R}{R-\rho_F})^2\sigma_F^2)^{-1/2}$. a.1) *If* $\sigma_\theta > \sigma_\theta^*$, *linear equilibria does not exist.* a.2) *If* $\sigma_\theta \leq \sigma_\theta^*$, *there are two linear equilibria, with price coefficients:*

$$
\begin{aligned}
p_0 &= \frac{1}{r}[(1+p_F)(1-\rho_F)\bar{F} + p_\theta(1-\rho_\theta)\bar{\theta}], \quad p_F = \frac{\rho_F}{R-\rho_F}, \\
p_{\theta,1} &= -\frac{(R-\rho_\theta)\sigma_\theta^{-2}}{4\gamma}(1-\sqrt{1-(\frac{\sigma_\theta}{\sigma_\theta^*})^2}), \;\; p_{\theta,2} = -\frac{(R-\rho_\theta)\sigma_\theta^{-2}}{4\gamma}(1+\sqrt{1-(\frac{\sigma_\theta}{\sigma_\theta^*})^2}).
\end{aligned} \tag{23}
$$

b) *Let* $\mu = 1$, *and define* $\sigma_\theta^{**} \equiv \frac{(R-\rho_\theta)}{4\gamma}(\sigma_D^2 + (\frac{R}{R-\rho_F})^2(\sigma_F^2 + \rho_F^2\sigma_u^2))^{-1/2}w^{-1/2}$. b.1) *If* $\sigma_\theta > \sigma_\theta^{**}$, *linear equilibria does not exist.* b.2) *If* $\sigma_\theta \leq \sigma_\theta^{**}$, *there are two linear equilibria, with price coefficients:*

$$
\begin{aligned}
p_0 &= \frac{1}{r}[(1+\hat{p}_F)(1-\rho_F)\bar{F} + p_\theta(1-\rho_\theta)\bar{\theta}], \quad \hat{p}_F = \frac{\rho_F}{R-\rho_F}, \\
p_{\theta,1} &= -\frac{(R-\rho_\theta)\sigma_\theta^{-2}}{4\gamma}(1-\sqrt{1-(\frac{\sigma_\theta}{\sigma_\theta^{**}})^2}), \;\; p_{\theta,2} = -\frac{(R-\rho_\theta)\sigma_\theta^{-2}}{4\gamma}(1+\sqrt{1-(\frac{\sigma_\theta}{\sigma_\theta^{**}})^2}),
\end{aligned} \tag{24}
$$

where $\sigma_u^2 = \frac{\sigma_D^2(1-\rho_F^2)+\sigma_F^2}{2\rho_F^2}(\sqrt{1+\frac{4\sigma_D^2\sigma_F^2\rho_F^2}{(\sigma_D^2(1-\rho_F^2)+\sigma_F^2)^2}}-1)$, and $w \equiv \frac{\sigma_D^2+(\frac{R}{R-\rho_F})^2(\rho_F^2\sigma_u^2+\sigma_F^2)}{\sigma_D^2+\rho_F^2\sigma_u^2+\sigma_F^2} \geq 1$.

Proposition 1 makes two central points. First, existence of (linear) equilibria is not granted unless we impose parameter restrictions. Indeed, previous OLG models (with $T = 2$) generally assume very small value for the volatility of supply; σ_θ. Second, whenever an equilibrium exists, it is generally multiple (two equilibria). These results are in line with the finding of earlier work, highlighting how the current model nests 2-period OLG economies previously studied. I now explain the intuition for each of these results.

The economics behind non-existence are as follows. Imagine we conjecture a small (in absolute magnitude) price coefficient for supply innovations; $p_\theta^{'}$. When agents live for two periods, their fate is determined in a single trading round. Given the rather high stakes, agents might be unwilling to hold the asset (even if they expect supply shocks to have the modest impact associated with $p_\theta^{'}$), leading them to demand larger price concessions to absorb supply. But this is consistent with a more negative coefficient $p_\theta^{''} < p_\theta^{'}$, and larger volatility of future prices. This iteration goes on without bound, unless volatility of supply remains below a threshold (σ_θ^* and σ_θ^{**} for the full- and no-information), in which case linear price equilibria exists. Notice that whenever $\rho_F > 0$ (a necessary condition for F_t to be predictive about future dividends), the full-information economy allows equilibria at a higher critical σ_θ ($\sigma_\theta^* > \sigma_\theta^{**}$ in this case). Intuitively, with better information about the fundamental, agents tolerate more non-fundamental risk.

Regarding multiplicity, the coefficient p_θ takes the values $p_{\theta,1}$ and $p_{\theta,2}$, given by equation (23) for the case where $\mu = 0$, and (24) for $\mu = 1$. In both cases, $p_{\theta,2} < p_{\theta,1} < 0$. Along the more negative root $p_{\theta,2}$, innovations in supply have a large, negative price impact. This is the high-volatility equilibrium (HVE).

Along the less negative root $p_{\theta,1}$ they have a milder effect in the price, which makes this the low-volatility equilibrium (LVE). Which root obtains determines the value of the constant term p_0. The coefficient associated with innovations in F_t has a unique solution, given by the expected present value of dividends.

These equilibria reflect two self-fulfilling outcomes. Imagine investors expect price volatility to be generated form the LVE. Because non-fundamental fluctuations are relatively modest, investors require low compensation for absorbing supply, which then has a small impact on prices and returns. As a result, prices are in fact relatively stable and high on average (reflected by a large coefficient p_0, and by a negative, but small in magnitude, coefficient p_θ applied to $\bar{\theta}$). But the situation might be the converse. If investors expect the volatility from the HVE, the large risks involved make them reluctant to trade. Supply shocks are then accommodated through large price concessions, causing the HVE to be self-fulfilling as well. The high risk faced by investors in the HVE is compensated through a large average premium (a low, or even negative value of p_0, and a large negative value, in absolute magnitude, of p_θ).

Proposition 2 provides results for the other extreme case in which agents are infinitely lived.

Proposition 2 (infinite horizon limit): *Let $\mu = 0$, or $\mu = 1$. As $T \to \infty$,*
a) A linear equilibrium always exists. b) The equilibrium is unique (in the linear class).

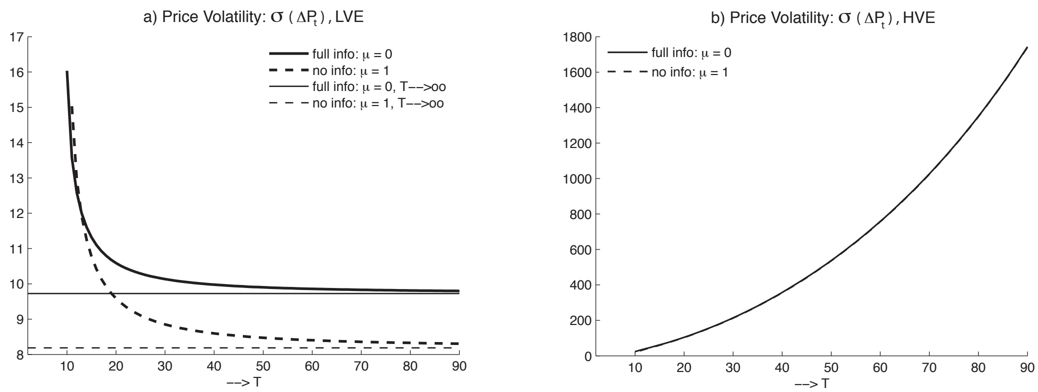
The results of Proposition 2 are new. Wang (1994), for instance, states that an equilibrium price equation similar to expression (7) can be solved for numerically, but is not explicit about whether this is the case for all possible parameters, or whether the solution is unique.

Since a proof of existence and multiplicity for finite investment horizons $T > 2$ is not available, hereon I discuss mostly results from numerical simulations. Panel a) of Table 2 reports baseline parameters. I normalize the variance of F_t shocks, σ_F, to 1, and made them relatively persistent ($\rho_F = 0.95$). In comparison, transitory dividend shocks are relatively volatile ($\sigma_D = 3$). The average supply of the risky asset, $\bar{\theta}$, is normalized to one in accordance to the measure of agents in the economy, with a st. dev. σ_θ set to 10%. Panel b) of Table 2 presents the price coefficients that result from these parameters, as well as the main asset pricing moments.

Figure 1 plots price volatility in the LVE (panel a) and the HVE (panel b), defined as the unconditional st. dev. of price changes. I choose this quantity to make the results comparable to previous work (See Spiegel (1998), and Watanabe (2008)). The solid lines correspond to the full-information economy ($\mu = 0$), while the dashed lines denote the no-information economy ($\mu = 1$). Under the baseline parameters, no equilibria exists for an OLG economy with $T = 2$ (i.e., $\sigma_\theta > \sigma_\theta^*$). The full-information economy exhibits equilibria starting from the critical horizon of $T^* = 10$ onwards, while an horizon of 11 or more is necessary for existence in the no-information economy. The figure also shows that for each investment horizon equal or larger than the minimum required for existence, there are 2 equilibria. These numerical results confirm that multiplicity is a general feature of OLG economies.

Most importantly, the results of Figure 1 (reported in more detail in Table 2) contribute to our understanding of how economies with different investment horizons have distinct equilibrium properties in a generalized OLG framework. Along the LVE, longer horizons increase average prices and reduce price volatility. To understand these result, compare economy A, where investors live for T^A periods,

Figure 1: Multiplicity



with economy B, where they live for $T^B > T^A$. How would the B-economy investors behave if they traded an asset with the volatility characteristics of economy A? Since they live longer, they have a better disposition towards risks (AARA effect), and there is a smaller quantity of risk absorbed per individual (the RT effect). Hence, the volatility of the LVE in economy A cannot be an equilibrium in the B economy, as investors in the latter will have an excess demand for the asset for any given price, compared to economy A. In consequence, supply innovations have a lesser price impact in economy B (as investors are happy to accommodate them), lowering price volatility. Moreover, since expected returns must fall in equilibrium, the average price of the asset increases.

The HVE exhibits the exact opposite features, with average prices falling and volatility increasing with investment horizons. For intuition, recall that the HVE relies on the anticipation of high future price volatility: if agents anticipate high risks in the future, they become reluctant to absorb supply shocks, which then have large price impact. But the longer agents live, the more difficult it is to sustain an equilibrium with high price volatility, due to the mechanisms just explained. Hence, as we increase the horizon, the level of price volatility in the HVE must be even larger than before, in order to induce the cautious behavior on investors (i.e., inelastic demands) that is necessary for the existence of a HVE.

In this context, the fact that multiplicity vanishes as $T \to \infty$ should be intuitive. A HVE is possible only to the extent that investors are finitely lived, since in this case the threat of an adverse price movements late in their lives makes them wary of holding large positions in the asset. But if investors are infinitely lived, they respond to high price volatility by voluntarily buying when prices are low, and selling when high. Hence, the attractiveness of the highly volatile asset cannot remain a feature of this economy. In equilibrium, only moderate levels of volatility (and price discounts) are sustainable.

The numerical results of Figure 1 (and Table 2) have an important connection with the analytical results of Proposition 2. Namely, while the increase in volatility and drop in average prices are unbounded along the HVE, these moments converge smoothly along the LVE to the values of the infinite horizon economy (straight lines in panel a) of Figure 1). This suggests that the unique equilibrium of the infinite horizon economy corresponds to the limit of the LVE for finite horizons.

Figure 2: Existence regions

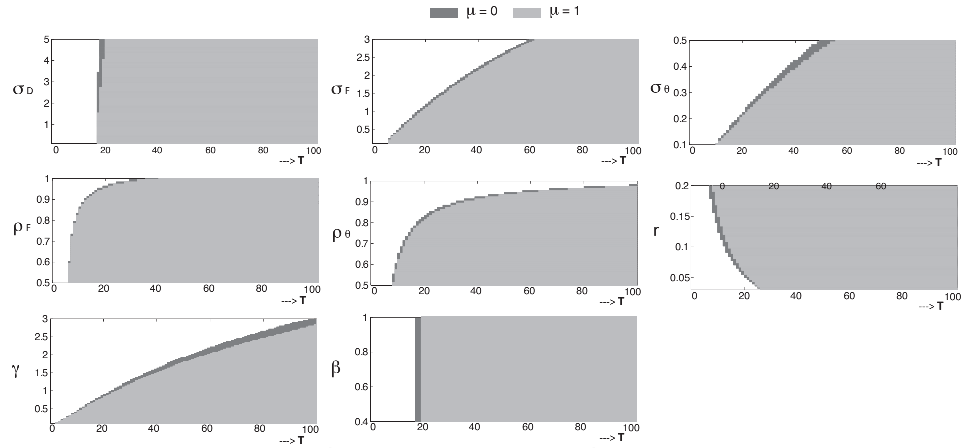


Figure 2 gives a general picture of equilibrium existence regions. Starting from the benchmark parameters, each panel shows the dependence of the critical investment horizon T^* on a particular parameter, for both the full- and no-information economy. Higher volatility of the dividend process (higher σ_D, σ_F, or larger persistence ρ_F) increases fundamental risk and requires longer critical horizons for existence. Similarly, higher non-fundamental risk related to supply (larger σ_θ, or an increase in persistence ρ_θ) also shift up the minimal horizon. Naturally, risk aversion γ increases T^*, while the converse is true for the interest rate, as prices respond less to fundamental innovations when investors discount the future more. Finally, although β matters for the consumption path, it has no effect on portfolio choices (which are wealth-independent), and hence no impact on prices or T^*. Moreover, for all parameter values, the full-information information economy reaches an equilibrium at a (weakly) lower value of T^*.

3.3 Stability

Another feature that differs across equilibria is stability. Following Cespa and Vives (2012), I define an equilibrium to be stable if a small perturbation in the strategy played by other investors, $-i$, triggers an optimal response in investor i's actions which contributes to the restoration of the initial equilibrium.

Formally, let investor's i strategy be given by $X^i_{j,t} = \phi_{0,i} + \phi_{F,i}\mathbb{E}[F_t|\Omega^i_t] + \phi_{P,i}P_t$, and write the price as $P_t = P \cdot \mathbb{E}[\Psi_t|\Omega^i_t]$.[8] The strategy vector $\phi_i \equiv [\phi_{0,i} \;\; \phi_{F,i} \;\; \phi_{P,i}]$ defines a correspondence $\phi_i = g(P)$ between the vector of price coefficients, $P \equiv [p_0 \;\; p_F \;\; p_\theta]$, and the best-response coefficients of investor i derived from individual optimization. In equilibrium, when all investors play the symmetric action $\phi_i = \phi$, these actions imply a price vector $P = h(\phi)$, where $h(\cdot)$ is implied by the aggregation of individual (symmetric) demands, and market-clearing. Put together, these two relations constitute the fixed-point between individual and aggregate strategies $\phi_i = g(h(\phi))$. That is, the strategy of investor i is a function of the equilibrium price vector ($\phi_i = g(P)$), which in turn is a function of the strategies played by

[8]With symmetric information ($\mu = 0$, or $\mu = 1$), the price has only 3 terms: $P_t = p_0 + p_F\mathbb{E}[F_t|\Omega^i_t] + p_\theta\mathbb{E}[\theta_t|\Omega^i_t]$.

all other investors in equilibrium ($P = h(\phi)$). An equilibrium is then defined to be stable, whenever $|\partial\phi_{m,i}/\partial\phi_m| < 1$, for $m = 0, F, P$.

An analytical treatment of stability is possible in the special case where $T = 2$, where we have

Proposition 3 (stability): *Let $\mu = 0$, or $\mu = 1$. For $T = 2$,*

a. The low-volatility equilibrium is stable.

b. The high-volatility equilibrium is unstable.

For the general case (asymmetric information, $2 < T < \infty$), stability can only be inferred from numerical simulations. As discussed in more detail in the appendix, simulations confirm that the LVE is stable, while the HVE is unstable. These stability results shed light into the fragile nature of the HVE. Indeed, if other investors deviate an arbitrarily small ε from the symmetric strategies, investor i's optimal response would tend to reinforce the magnitude of the initial deviation (in absolute terms) driving the economy further away from the initial equilibrium with each iteration of best-response reactions.

As must be noted, this is not the only definition of stability one could entertain. For example, a simpler notion is whether demand functions have negative slope with respect to the price,[9] according to which both equilibria are stable. Yet another notion asks whether the limit equilibrium of a finitely-lived economy coincides with a given equilibrium in the infinite horizon case. For example, Liang (2008) shows that when agents live for two periods and trade a finitely lived asset with terminal date T, there is a unique equilibrium that coincides with the high-volatility solution in the limiting case where $T \to \infty$.[10]

3.4 Comparing equilibria

The low and high volatility equilibria differ in important respects. As discussed by Spiegel (1998), and Watanabe (2008), one appealing property of the HVE is the capacity to generate high excess volatility and large correlation among multiple securities. Another property that has been studied is the effect of higher supply volatility (σ_θ), which tends to *lower* price volatility in the HVE, but to increase it in the LVE. This remains true in the present model (not reported).

The current study unveils several additional properties that differ across equilibria. First, volatility along the LVE falls as investors' horizons increase, while the opposite is true in the HVE. A possible interpretation of this result is that episodes of high volatility in financial markets can be understood as events in which the effective horizons of investors *along the LVE* have become shorter. This contrasts with the previous OLG models (with $T = 2$), which explain high (unconditional) volatility as generated by the HVE. However, one must be careful with this interpretation since, strictly speaking, the model is solved for a stationary economy where changes in horizons are not anticipated by investors (Section 5 provides examples in which such changes are an explicit part of the equilibrium).

Second, along the LVE, longer horizons increase average prices, while the opposite is true for the HVE. To attain positive (average) prices along the LVE, I assume a positive unconditional mean dividend

[9]See Mas-Colel, Whinston, and Green textbook on Microeconomic Theory

[10]This notion of stability is perhaps more related to the uniqueness result of Proposition 2. Liang (2008) compares equilibria between a finitely and an infinitely lived asset, fixing the lifespan of investors (at two periods). In contrast, Proposition 2 compares equilibria between finitely and infinitely lived *investors*, keeping the lifespan of the asset fixed (at infinity).

($\bar{F} = 20$). For the HVE, one can also find positive average prices. As Table 2 (col. 22) shows however, this requires assuming larger $\bar{F}$, the more so the longer the horizon. Third, while the LVE converges smoothly to the infinite horizon case, the HVE vanishes as $T \to \infty$ (Proposition 2). Lastly, the LVE is stable: if investors deviate from the symmetric equilibrium strategies, successive iterations will restore the LVE. In contrast, the economy always moves away from the HVE after an arbitrarily small perturbation.[11]

Which equilibrium should one focus on? Overall, both have desirable properties, depending on the question at hand. The LVE seems more robust due to its stability and convergence properties. On the other hand, the HVE retains its capacity to deliver excess volatility. Moreover, instability might actually be a desirable feature to shed light into anomalous episodes in financial markets, such as trading halts and market breakdowns. In what follows, I will remain agnostic and carry out most of the analysis considering both equilibria explicitly.[12]

3.5 Relative contribution of effects

To gauge the relative importance of the AARA and RT mechanisms I perform the following exercise. Irrespective of the horizon, I fix the total mass of active investors (those aged $T-1$ or younger) to 1. Because this normalization maintains a constant ratio between the mass of active (voluntary) investors, and the moments of the asset supply (mean and variance), it shuts off the RT effect. I then vary the horizon T, letting investors' demands change purely as a result of the AARA effect. The exercise then captures the behavior of traders that expect to live for finite periods, but eliminates the market impact of their liquidations because the aggregate risk absorbed per active investor is fixed, by construction.

Table 1: The two main mechanisms

T	(2) $\sum_{j=1}^{T-1}(1/\alpha_j)/(T-1)$	(3) $1/(T-1)$	(4) $\sigma(\Delta P_t)$	(5) $\sigma(\Delta P,T)$-$\sigma(\Delta P,00)$, both effects	(6) $\sigma(\Delta P,T)$-$\sigma(\Delta P,00)$, AARA only	(7) $\sigma(\Delta P,T)$-$\sigma(\Delta P,00)$, AARA share
2	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
9	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
10	5.20	0.111	257.18	162.59	86.82	0.53
11	5.56	0.100	184.29	89.70	61.59	0.69
13	6.23	0.083	144.01	49.42	38.64	0.78
15	6.86	0.071	128.11	33.53	27.64	0.82
20	8.28	0.053	112.14	17.56	15.35	0.87
25	9.50	0.042	105.92	11.33	10.17	0.90
30	10.56	0.034	102.71	8.12	7.40	0.91
40	12.27	0.026	99.54	4.96	4.59	0.93
50	13.58	0.020	98.04	3.45	3.23	0.93
70	15.40	0.014	96.65	2.06	1.94	0.94
90	18.89	0.011	96.02	1.43	1.36	0.95
Infinity	21.00	--	94.59	0.00	0.00	--

Col. 2) avg. risk tolerance (avg. of inverse AARA). Col 3) risk transfer ratio. Col. 4) variance of price changes,V(ΔP).Col. 5) V(ΔP) with horizon T, minus V(ΔP) with infinite horizon, including both effects. Col. 6) V(ΔP) with horizon T, minus V(ΔP) with infinite horizon, AARA effect only. Col.7) ratio (col.6)/(col5).

[11] Recall this is proven analytically for the special cases in Proposition 3, and inferred numerically for the general case.
[12] For a comprehensive comparison between different equilibria, see Banerjee (2011).

Table 1 presents several statistics to understand the contribution of each mechanism in generating different volatility across economies with different lifespans. For space considerations, I entertain the LVE of the full information economy only. Col. 2 reports the average risk tolerance across investor vintages (the average of the inverse of investors' AARA), which falls with the horizon of investors, while col. 3 reports the risk-transfer ratio. Col. then 4 reports unconditional variance of price changes at different horizons. Since a natural yardstick to asses the importance of horizons is comparing price volatility vis-a-vis an infinite horizon benchmark (where volatility is the lowest along the LVE), col. 5 shows the difference between price variance in an economy with horizon T, and the variance in an economy with $T \to \infty$. Column 6 then shuts off the RT effect as discussed above –normalizing the mass of active investors to 1–, and reports how much price variance increases (with respect to the infinite horizon) as horizons decline, in the hypothetical economy where only the pricing of risk matters. Finally, column 7 calculates the ratio between columns 6 and 5, providing an assessment of relative importance of each mechanism in generating price volatility as a function of investment horizons.

As the table makes clear, both effects are important in delivering the change in price volatility explained by investment horizons, although the AARA effect appears dominant given the baseline parameters. Naturally, the contribution of the RT effect is reduced as horizons increase, since the risk transfer ratio $1/(T-1)$ monotonically diminishes.

4 Asymmetric Information Economies

This section discusses the informational role of prices (market efficiency), and asset pricing moments across economies with different investment horizons, when agents have asymmetric information. In what follows, I assume the mass of uninformed investors is relatively large, at $\mu = 0.8$.

4.1 Existence and multiplicity of equilibria

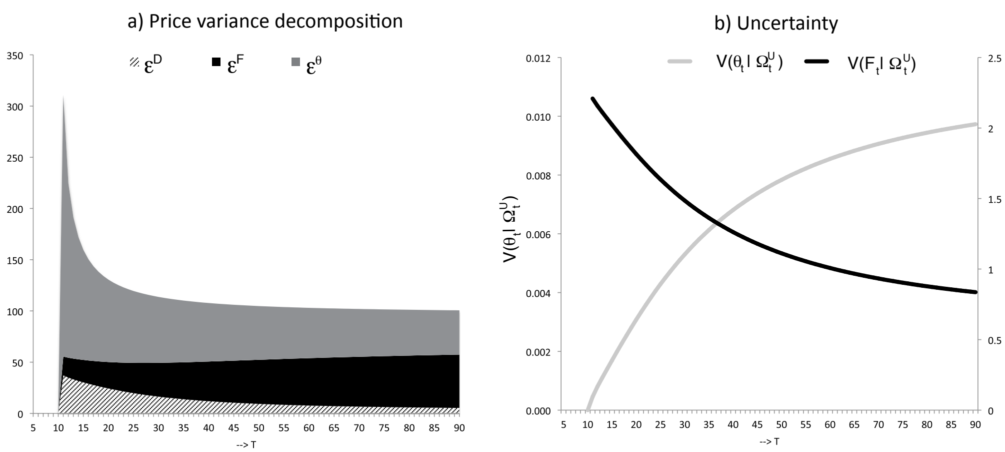
We can learn about the main equilibrium characteristics of the asymmetric information economy from numerical simulations. Qualitatively speaking, the results follow those described in section 3. Regarding existence, an equilibrium typically does not exist (under the baseline parameters) for short horizons, but emerges as T exceeds a minimum threshold. As a function of the underlying parameters, existence regions behave qualitatively similar as those in Figure 2 (not reported). Regarding multiplicity, for any horizon admitting a solution, the numerical exercise always finds two equilibria with similar characteristics as before (I discuss the asset pricing effects of horizons momentarily). Regarding stability, the LVE is stable to small perturbations, but the HVE is unstable.

4.2 Market efficiency

The key aspect of the asymmetric information economy is that the knowledge of a large mass of agents (the uninformed) about the underlying state variables depends on the informativeness of the price. Specifically, the endogenous price signal derived in section 2, $p_t \equiv p_F \cdot F_t + p_\theta \cdot \theta_t$, will be relatively more informative about F_t the when the price impact of fundamentals –private information observed by the informed– is relatively large compared to the price impact of supply shocks. Because information about F_t can only move prices to the extent informed investors react to it, the coefficient p_F is tightly linked to the demand elasticity of the informed to changes in F_t. On the other hand, the impact of supply shocks on prices (p_θ) depends on the willingness of all investors (informed and uninformed) to absorb these.

One of the key contributions of this paper is describing how the informativeness of prices depends on investment horizons. To understand this dependence, cols. 10-14 in panel b) of Table 2 perform a variance decomposition exercise for the LVE. Col. 11-13 report how much price variance is caused by transitory dividend, fundamental, and supply shocks (see appendix for details). Col. 10 reports the st. dev. of price changes, whose square equals the sum of cols. 11-13. Let's focus now on the asymmetric information economy, which is presented in the bottom of the Table. When the horizon is short, supply shocks have a large impact on total variance; 255.4 for $T = 11$. The contribution of fundamentals, on the other hand, is just 18.4, or only about 6% of total price variance. Conversely, for a long horizon such as $T = 90$, supply shocks generate a price variance of only 43.2, compared to 52.1 explained by fundamentals, which is roughly 43% of total variance.

Figure 3: Variance decomposition and uncertainty



Panel a) Price variance attributable to each shock in the asymmetric information economy (μ =0.8), LVE. Panel b) Variance of fundamentals (right axis), and supply (left axis) conditional on public information (complete history of dividends and prices).

What explains the change in variance decomposition as we compare economies with different T? For long investment horizons, both the AARA and RT effects lead to investor demands which are more elastic to expected returns. In consequence, the market is deep and supply innovations have modest price impact. In response to this low risk environment, competitive CARA-investors with superior knowledge – the informed– trade on this information aggressively. Hence, fundamental shocks account for an increasing share of the total variance of price changes, with supply shocks reducing their contribution. Conversely, for short horizons, the aforementioned effects incite cautious trading and demands become more inelastic to expected returns. This enhances the price impact of supply shocks and reduces the contribution of private information, worsening the informational efficiency of the market and increasing the uncertainty of the uninformed.[13] To underscore the link between price variance decomposition and the uncertainty of the uninformed, Figure 3 reproduces these statistics for the LVE. Notice also that with noisier prices, the uninformed rely more on dividends to forecast F_t. Consequently, the transitory shock ϵ_t^D –orthogonal to future payoffs– has a higher contribution for short horizons, and monotonically declines as horizons increase and price information improves.

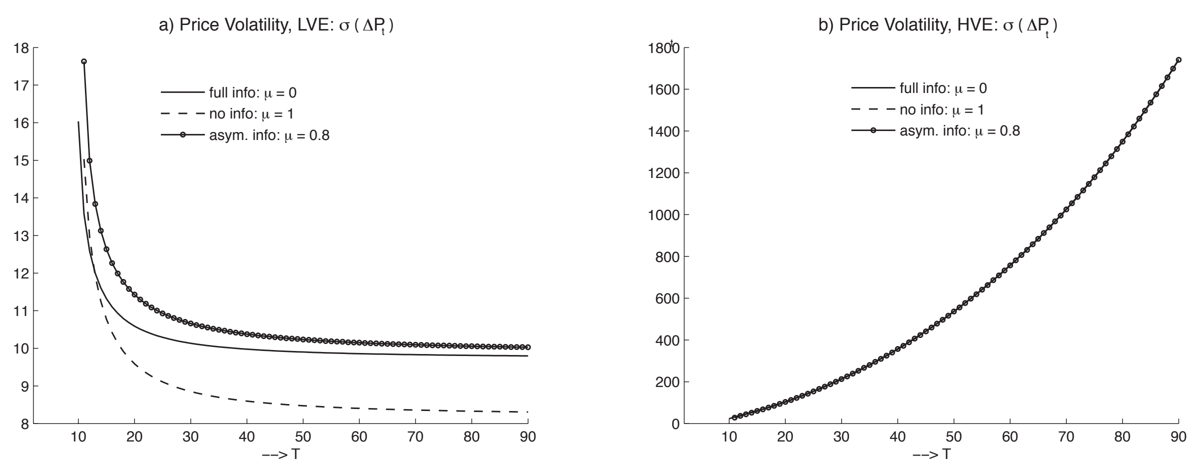
The qualitative impact of horizons on market efficiency along the HVE is reversed. Longer horizons require an increasing amount of non-fundamental volatility for the HVE to be sustained, lowering the signal-to-noise ratio of the price. Because non-fundamental volatility is already high in the HVE (for short horizons), prices are not very informative to begin with, and increasing horizon has an almost negligible effects on the uncertainty of the uninformed under the baseline calibration (see col. 28 of Table 2).

It is important to mention that the relative contribution of supply vs. fundamentals to overall price volatility, as a function of the investment horizons, is robust under alternative parameterizations of the model (for both equilibria).[14]

4.3 Asset pricing moments

I begin discussing price change volatility for the LVE, for the asymmetric information economy as well as the symmetric information cases. Figure 4, panel a) shows the following three patterns. First, volatility declines with horizons in all three economies. Second, the no information economy has less volatility than the full information case for long horizons, but this is reversed for short horizons. Third, volatility is the largest for the asymmetric information economy, for all horizons.

Figure 4: Price volatility: All economies



[13]Interestingly, the more investors know about the fundamental, the more uncertainty they have about supply. See Avdis (2013) for a discussion about the implications of this force on the incentives to acquire information.

[14]Not reported, but available upon request.

Table 2: Baseline parameters and asset pricing moments

a) Parameters

r	β	σ_D	σ_F	ρ_F	$\bar{F}$	σ_θ	ρ_θ	$\bar{\theta}$
0.05	1/(1.05)	3	1	0.95	20	0.1	0.6	1

b) Asset pricing moments

Low volatility equilibrium

T	(2) $\hat{\rho}_F$	(3) p_F	(4) p_θ	(5) $E[Q_t]$	(6) $\sigma(Q_{t+1}\|\Omega_t^U)$	(7) $E[P_t]$	(8) $E[Q_t]$ / $E[P_t]$ (%)	(9) $\sigma(Q_{t+1}\|\Omega_t^U)$/ $E[P]$ (%)	(10) $\sigma(\Delta P_t)$	(11) ε^D	(12) ε^F	(13) ε^θ	(14) ε^F share (%)	(15) $V(F_t\|\Omega_t^U)$	(16) $V(\theta_t\|\Omega_t^U)$
	Full information (μ = 0)														
10*	0.0	9.5	-114.8	45.2	15.8	-504	n.r.	n.r.	16.0	0.0	92.6	164.6	36.0	--	--
11	0.0	9.5	-85.7	34.6	13.9	-293	n.r.	n.r.	13.6	0.0	92.6	91.7	50.2	--	--
13	0.0	9.5	-64.2	26.5	12.7	-131	n.r.	n.r.	12.0	0.0	92.6	51.4	64.3	--	--
15	0.0	9.5	-53.3	22.4	12.2	-47	n.r.	n.r.	11.3	0.0	92.6	35.5	72.3	--	--
20	0.0	9.5	-39.6	16.9	11.6	61	27.7	19.0	10.6	0.0	92.6	19.6	82.5	--	--
25	0.0	9.5	-32.7	14.1	11.4	117	12.1	9.7	10.3	0.0	92.6	13.4	87.4	--	--
30	0.0	9.5	-28.5	12.4	11.3	152	8.2	7.4	10.1	0.0	92.6	10.1	90.1	--	--
40	0.0	9.5	-23.6	10.4	11.2	192	5.4	5.8	10.0	0.0	92.6	7.0	93.0	--	--
50	0.0	9.5	-20.9	9.2	11.1	215	4.3	5.2	9.9	0.0	92.6	5.5	94.4	--	--
70	0.0	9.5	-18.1	8.0	11.1	240	3.3	4.6	9.8	0.0	92.6	4.1	95.8	--	--
90	0.0	9.5	-16.6	7.4	11.0	252	2.9	4.4	9.8	0.0	92.6	3.5	96.4	--	--
Infinity	0.0	9.5	-12.7	5.1	10.9	286	1.8	3.8	9.7	0.0	92.6	2.0	97.9	--	--
T	No information (μ = 1)														
10	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
11*	9.5	0.0	-113.7	44.7	16.4	-494	n.r.	n.r.	15.0	60.9	3.4	161.7	1.5	2.28	0.00
13	9.5	0.0	-78.9	32.1	14.2	-242	n.r.	n.r.	11.9	60.9	3.4	77.8	2.4	2.28	0.00
15	9.5	0.0	-64.4	26.6	13.5	-133	n.r.	n.r.	10.8	60.9	3.4	51.9	2.9	2.28	0.00
20	9.5	0.0	-47.1	20.0	12.7	0	n.r.	n.r.	9.6	60.9	3.4	27.7	3.7	2.28	0.00
25	9.5	0.0	-38.7	16.6	12.4	67	24.6	18.4	9.1	60.9	3.4	18.7	4.1	2.28	0.00
30	9.5	0.0	-33.6	14.6	12.3	108	13.4	11.3	8.9	60.9	3.4	14.1	4.3	2.28	0.00
40	9.5	0.0	-27.8	12.2	12.1	156	7.8	7.8	8.6	60.9	3.4	9.7	4.6	2.28	0.00
50	9.5	0.0	-24.6	10.8	12.1	183	5.9	6.6	8.5	60.9	3.4	7.6	4.7	2.28	0.00
70	9.5	0.0	-21.2	9.4	12.0	212	4.4	5.7	8.4	60.9	3.4	5.6	4.8	2.28	0.00
90	9.5	0.0	-19.5	8.7	12.0	227	3.8	5.3	8.3	60.9	3.4	4.8	4.9	2.28	0.00
Infinity	9.5	0.0	-14.9	6.0	11.8	267	2.2	4.4	8.2	60.9	3.4	2.8	5.0	2.28	0.00
	Asymmetric information (μ = 0.8)														
10	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
11*	7.64	1.86	-129.5	49.4	18.2	-588	n.r.	n.r.	17.6	37.1	18.4	255.4	5.9	2.21	0.00
13	7.56	1.94	-84.7	33.3	15.3	-265	n.r.	n.r.	13.8	33.3	20.3	138.0	10.6	2.11	0.00
15	7.53	1.97	-68.0	27.0	14.4	-141	n.r.	n.r.	12.6	30.3	21.9	107.4	13.7	2.02	0.00
20	7.50	2.00	-48.5	19.6	13.4	8	238.6	163.49	11.4	24.3	25.8	80.4	19.8	1.82	0.00
25	7.50	2.00	-39.1	16.0	13.0	81	19.8	16.09	10.9	19.8	29.6	70.1	24.7	1.64	0.00
30	7.50	2.00	-33.5	13.8	12.7	124	11.1	10.22	10.7	16.4	33.0	64.3	29.0	1.49	0.01
40	7.51	1.99	-27.1	11.3	12.4	174	6.5	7.10	10.4	11.9	38.7	57.1	35.9	1.26	0.01
50	7.52	1.98	-23.6	9.9	12.2	201	4.9	6.03	10.2	9.3	43.0	52.4	41.1	1.11	0.01
70	7.53	1.97	-20.0	8.5	11.9	230	3.7	5.19	10.1	6.6	48.7	46.6	47.8	0.93	0.01
90	7.54	1.96	-18.2	7.8	11.8	244	3.2	4.84	10.0	5.3	52.1	43.2	51.8	0.84	0.01
Infinity	7.56	1.94	-13.5	5.2	10.9	283	1.9	3.85	9.9	2.5	62.8	32.2	64.5	0.56	0.01

High volatility equilibrium

T	(17) $\hat{\rho}_F$	(18) p_F	(19) p_θ	(20) E[Qt]	(21) $\sigma(Q_{t+1}\|\Omega_t^U)$	(22) E[Pt]	(23) $\sigma(\Delta P_t)$	(24) ε^D	(25) ε^F	(26) ε^θ	(27) ε^F share (%)	(28) $V(F_t\|\Omega_t^U)$	(29) $V(\theta_t\|\Omega_t^U)$
	Full information (μ = 0)												
10*	0.0	9.5	-191	71	22	-1,017	23.4	0.0	92.6	454	16.9	--	--
11	0.0	9.5	-270	95	29	-1,504	31.7	0.0	92.6	914	9.2	--	--
13	0.0	9.5	-405	130	42	-2,208	46.3	0.0	92.6	2,055	4.3	--	--
15	0.0	9.5	-543	162	55	-2,834	61.5	0.0	92.6	3,685	2.5	--	--
20	0.0	9.5	-930	233	94	-4,254	104.4	0.0	92.6	10,800	0.8	--	--
25	0.0	9.5	-1,384	300	139	-5,602	155.0	0.0	92.6	23,932	0.4	--	--
30	0.0	9.5	-1,913	365	192	-6,903	214.1	0.0	92.6	45,750	0.2	--	--
40	0.0	9.5	-3,197	496	320	-9,513	357.6	0.0	92.6	127,784	0.1	--	--
50	0.0	9.5	-4,806	631	481	-12,211	537.4	0.0	92.6	288,722	0.0	--	--
70	0.0	9.5	-9,181	937	918	-18,345	1,026.5	0.0	92.6	1,053,534	0.0	--	--
90	0.0	9.5	-15,592	1,330	1,559	-26,200	1,743.3	0.0	92.6	3,039,048	0.0	--	--
Infinity	0.0	9.5	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
T	No information (μ = 1)												
10	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
11*	9.5	0.0	-239	86	27	-1,313	27.9	60.9	3.4	716	0.4	2.28	0.00
13	9.5	0.0	-387	126	40	-2,117	44.0	60.9	3.4	1,875	0.2	2.28	0.00
15	9.5	0.0	-528	158	54	-2,755	59.6	60.9	3.4	3,483	0.1	2.28	0.00
20	9.5	0.0	-917	230	92	-4,208	102.9	60.9	3.4	10,516	0.0	2.28	0.00
25	9.5	0.0	-1,372	298	138	-5,555	153.6	60.9	3.4	23,524	0.0	2.28	0.00
30	9.5	0.0	-1,901	364	191	-6,871	212.7	60.9	3.4	45,196	0.0	2.28	0.00
40	9.5	0.0	-3,185	494	319	-9,480	356.2	60.9	3.4	126,783	0.0	2.28	0.00
50	9.5	0.0	-4,793	629	479	-12,185	535.9	60.9	3.4	287,105	0.0	2.28	0.00
70	9.5	0.0	-9,164	936	916	-18,319	1,024.6	60.9	3.4	1,049,698	0.0	2.28	0.00
90	9.5	0.0	-15,572	1,329	1,557	-26,172	1,741.0	60.9	3.4	3,031,175	0.0	2.28	0.00
Infinity	9.5	0.0	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
	Asymmetric information (μ = 0.8)												
10	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
11*	7.74	1.76	-225	81	25	-1,212	29.0	40.4	125.9	632	0.2	2.28	0.00
13	7.78	1.72	-382	124	40	-2,075	45.1	40.8	125.8	1,829	0.1	2.28	0.00
15	7.79	1.71	-525	157	54	-2,736	60.5	40.9	125.8	3,448	0.0	2.28	0.00
20	7.77	1.73	-916	230	92	-4,196	103.5	40.8	125.8	10,497	0.0	2.28	0.00
25	7.75	1.75	-1,372	298	138	-5,555	154.1	40.5	125.9	23,530	0.0	2.28	0.00
30	7.73	1.77	-1,902	363	191	-6,869	213.2	40.3	125.9	45,225	0.0	2.28	0.00
40	7.70	1.80	-3,186	494	319	-9,483	356.6	40.0	126.0	126,895	0.0	2.28	0.00
50	7.68	1.82	-4,794	629	480	-12,188	536.3	39.8	126.1	287,329	0.0	2.28	0.00
70	7.65	1.85	-9,167	936	917	-18,323	1,025.0	39.4	126.1	1,050,349	0.0	2.28	0.00
90	7.63	1.87	-15,576	1,329	1,558	-26,177	1,741.6	39.3	126.2	3,032,579	0.0	2.28	0.00
Infinity	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.

Panel a. Benchmark parameters. Panel b. Cols. 2-16) LVE moments. Cols. 2-4) price coefficients of eq. (7). Col. 5) uncond. mean dollar return. Col. 6) st.dev. dollar returns, cond. on public info. Col. 7) uncond. mean price. Col. 8) risk premia: uncond. mean dollar return, divided by uncond. mean price. Col 9) return volatility: st.dev.dollar returns cond. on public info., divided by uncond. mean price. Col. 10) uncond. st.dev. of price changes. Col. 11) var. price change due to ε^D shock. Col. 12) var. price change due to ε^F shock (fundamental). Col. 13) var. price change due to ε^θ shock (sum of cols. 11-13 = square of col. 10). Col. 14) share of price change variance due to ε^F shock (fundamentals), %. Col. 15) var. of fundamental F_t, cond. on public info. Col. 16) var. of supply θ_t, cond. on public info. Cols. 17-29) moments for HVE. n.a: not available (equilbrium does not exist). n.r: not reported. *: min. T* (existence).

To understand the first pattern, Table 2 (cols. 10-14) show that as horizons increase, supply driven volatility decreases monotonically in all economies, while fundamental volatility increases. A priori, the relation between overall volatility and horizons could be ambiguous, and not necessarily monotonic. Under the baseline parameters, the increase in fundamental volatility falls short of completely offsetting the fall of supply-driven volatility, leading to a monotonically decreasing relation between horizons and overall volatility. What seems more crucial than volatility per se is its decomposition, which as observed in section 4.2, robustly shift towards more fundamental volatility (and less supply) as horizons increase.

Intuition for the second pattern is as follows. Because investors in the no information economy face larger risks, they are less willing to absorb supply compared to the full information case. This explains why supply driven variance is larger for the no information case relative to full information, at all horizons (col. 13, Table 2). Conversely, in the no information economy fundamentals and transitory dividend shocks together cause less price variability than with full information (cols. 11-12, Table 2). This is because in the former economy there is more uncertainty about F_t, so the variability of posterior expectations about the fundamental is less than the case when it is observed without noise. Which economy will then display larger price volatility? It depends on the horizon. For short horizons, the contribution of supply shocks is relatively large, and overall price volatility in the no information case exceeds the economy with full information. For long horizons, the importance of fundamental volatility grows in both economies, but because it causes more price variance under full information, overall price volatility for this economy is higher than with no information.

In relation to the third pattern in figure 4, where does the asymmetric information economy stand in terms of volatility? Interestingly, price volatility here is larger than for the other two cases. Col. 13 in Table 2 reveals that despite investors having better knowledge about fundamentals here than under no information, supply shocks actually cause *more* price volatility. This is due to the adverse selection effect: with even a small number of informed investors, the uninformed face the risk that a particular purchase of securities is actually a trade against their better informed peers, rather than against a random supply shock. With no informed investors, this is clearly not a risk. In fact, when there are no informed investors, the uninformed observe the supply shock without noise. Hence, uninformed investors in the asymmetric information economy will demand a larger compensation to absorb supply (which they cannot observe) than in the economy with no information (see Wang, 1993, for a detailed discussion). It follows that for short horizons when supply shocks are relatively important, overall price volatility under asymmetric information is larger than under either symmetric information benchmark. For long horizons, on the other hand, fundamental volatility plays a larger role, so in principle the full information economy could display more volatility than the asymmetric information case (as was the case when compared to the no information economy). However, under the benchmark parameters, supply still contributes considerable volatility under asymmetric information, making overall volatility larger than with full information.

Results are reversed for the HVE (Figure 4, panel b). The full information economy has the largest price volatility, mostly due to supply innovations. This is because better informed investors are naturally more prone to absorb risk, and hence need an even larger "threat" in the form of price volatility for the HVE to be sustained (see Table 2, cols. 23-26, to appreciate these differences).

I now turn to average prices, reported along col. 7 of Table 2 for the LVE. For short horizons, investors require a large premium due to non-fundamental price volatility. In the linear model presented in this paper, this produces negative average prices irrespective of the information structure, and the ordering between economies follows closely the relative impact that supply shocks have in each case. Because of adverse selection, the asymmetric information economy has the lowest price, followed closely by the no information case. As horizons increase, the price impact of supply diminished due to the mechanisms highlighted in section 3, and average prices rise. For horizons slightly above $T = 20$ and beyond, average prices are strictly larger in the asymmetric information economy than in the no information benchmark. Moreover, average prices for the asymmetric information economy look closer to the ones exhibited by the full information benchmark, the more so the longer the horizon.[15]

Along the HVE, on the other hand, the increasing levels of volatility implied by longer horizons further reduce prices. Mirroring the results on price volatility, the full information economy has the lowest price. This is because investors here face less fundamental risk, therefore a HVE can be sustained only with higher levels of non-fundamental volatility (and hence lower prices, on average) than in an economy with less information (see col. 22 in Table 2).

Figure 5, panel a) plots expected return (risk premium) and the expected volatility of returns (panel b) across different economies. Because the conditional risk premium is time-varying (depends on the realization of the state variables), I follow the literature in computing the average risk premium as the ratio between the unconditional dollar return, and the unconditional mean price: $\mathbb{E}[Q_t]/\mathbb{E}[P_t]$.[16] This definition circumvents the problem that prices can be negative, or arbitrarily close to zero, in linear pricing models. Naturally, for the definition to be meaningful, one needs to restrict attention to economies with strictly positive average prices, which for the baseline parameters occurs for $T \geq 21$. Return volatility is computed as the ratio between the standard deviation of returns, conditional on public information (which is constant, from Theorem 1), and the mean price.

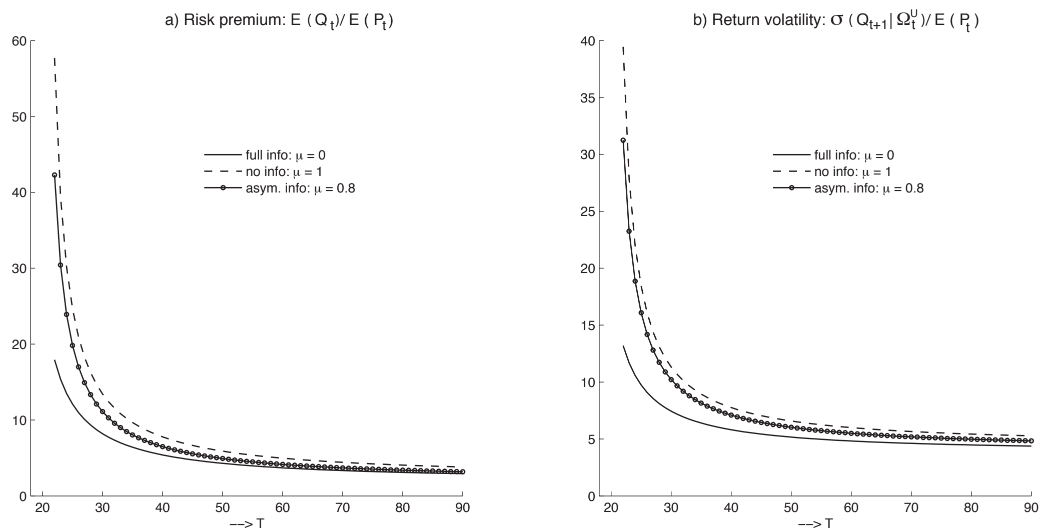
The following patterns emerge in this figure. First, both expected returns, and return volatility, are the lowest for the full information economy. Indeed, although prices are relatively volatile in this economy, they are also the highest on average, so the volatility of returns is diminished. Regarding the risk premium, since investors have the most information about fundamentals in this case their willingness to participate in the market is the highest, which lowers required returns. Second, risk premium and return volatility is highest for the no-information economy, since agents face the largest amount of uncertainty regarding the fundamental value of the asset. While the adverse selection problem provides a force towards increasing the premium in the asymmetric with respect to the no information benchmark, this consideration is of lesser importance at horizons for which average prices are already positive.

Third, and most importantly, how close these asset pricing moments are in the asymmetric informa-

[15]Wang (1993) compares average prices in economies with different proportions of uninformed traders and finds that increasing their proportion strictly decreases prices. Qiu and Wang (2010), and Vayanos and Wang (2012), find that prices can be lower with asymmetric information than in both symmetric information benchmarks if investors trade based on background risks. The results presented here add an additional dimensionality to the problem: for short horizons (along the LVE), asymmetric information has lower prices than with no information, but this is reversed for long horizons.

[16]This definition is also used in Wang (1993), while Banerjee (2011) discusses his results using mean dollar returns, $\mathbb{E}[Q_t]$.

Figure 5: Risk premium and return volatility



tion economy relative to the symmetric information benchmarks *depends on the horizon.* Both expected returns, and return volatility, are closer to no-information for short horizons, but closer to the full-information economy for long horizons. These are the natural implications of horizons on asset pricing when market efficiency is endogenous. For long horizons, prices are accurate signals about fundamentals and the uncertainty of the uninformed traders is small. Hence, although only 20% of the investor population has access to private information, the knowledge of this group actively moves prices, and the average investor is, in effect, pretty well informed in equilibrium. Conversely, when horizons are short supply innovations require large price concessions, increasing their contribution in total price variance and lowering the signal-to-noise ratio of the price. In consequence, information that is private to a group of investors *remains private* to such group. Shorter investment horizons hence set in motion a process of information disaggregation from prices, and the knowledge of the population becomes in effect much closer to the no-information benchmark –with asset pricing moments following suit.

4.4 Trading volume

The present model has rich implications for trading volume patterns. When agents have finite horizons, the gradual unwinding of positions generates trade flows between different age-cohorts. Moreover, with asymmetric information, agents will have an additional motive for trading. I now study the implications of investment horizons on volume along both equilibria, for the asymmetric information economy.

The following volume definitions follow Watanabe (2008) closely. Let the absolute individual trade flow (AIF) generated by individual (j, i) be $AIF^i_{j,t} \equiv |\Delta X^i_{j,t}|$, for $j = 1, 2, ...T$, and $i \in \{U, I\}$. Aggregating the AIF's within investors in the same information category gives the aggregate group trade flow, $AGF^i_t \equiv (1/T) \sum_{j=1}^{T} AIF^i_{j,t}$. We then weight AGF's by the mass of each information category, and include the exogenous trades caused by supply innovations, to get volume:

$$V_t = \frac{1}{2}\left(\mu AGF_t^U + (1-\mu)AGF_t^I + |\Delta\theta_t|\right). \tag{25}$$

The first definition of volume studies here is *expected volume*, $EV \equiv \mathbb{E}[V_t]$. This measure comprises all motives for trade, including trades within groups of the same information type due to inter-temporal rebalancing between older and younger generations; trades between the uninformed and informed agents that arise due to asymmetric information; and exogenous trading arising from supply shocks. To isolate trade flows due to information asymmetries only, the second definition is *net informational volume*,

$$NIV \equiv \frac{1}{2}\left((\mu/T)\mathbb{E}[|\sum_{j=1}^{T}\Delta X_{j,t}^U|] + ((1-\mu)/T)\mathbb{E}[|\sum_{j=1}^{T}\Delta X_{j,t}^I|]\right). \tag{26}$$

Notice that this measure first nets our all trade flows within a given information group i, $\sum_{j=1}^{T}\Delta X_{j,t}^i$, and then takes the expectation of the absolute value. NIV hence captures motives for trading within groups that are exclusively driven by information asymmetries. As a normalization, we have taken flows due to supply shocks out of this definition, so that NIV is zero if agents have common information (The 0.5 factor is applied to avoid double counting of buy and sell orders).

Closed form solutions are obtained from analysis of folded normal distributions (see appendix). Cols. 2-3 of Table 3, panel a), report aggregate AIF for the uninformed and informed investors, for different horizons. As horizons increase, both measures drop considerably. The dominant force here is the intertemporal unwinding of positions between different ages. Panel b) provides some light in this regard, presenting cross-sectional average holdings of selected age-cohorts for an economy with $T = 50$, as well as the average AIF of each cohort (cols. 3 and 5). As investors age, they gradually reduce holdings (cols. 2 and 4), generating inter-cohort trade (cols. 3 and 5). Because the reduction of holdings is more gradual the longer investors live (not reported), AGF's $((1/T)\sum_{j=1}^{T} AIF_{j,t}^i)$ fall with T (cols. 2 and 3 of panel a). Col. 4 then reports *expected volume*, which weights the statistics on cols. 2 and 3 by the masses of uninformed and informed investors, and adds supply volume. Interestingly, expected volume is larger (at all horizons) for the HVE (col. 8). The dominant force here is the increases inter-cohort trade: investors in the HVE reduce positions faster as they age because the risk faced is larger (cols. 6-7, in panel b).

Since volume is also induced by asymmetric information, col. 5, reports NIV. A priori, the relation between volume and asymmetric information is ambiguous. If prices are very noisy, uninformed investors' posteriors respond little to prices. Because the high degree of information asymmetry makes the adverse selection problem acute, information-driven trade between different information types is limited (see Wang, 1994). As price information improves, the uninformed trade more aggressively on price changes, increasing flows between groups. But as information quality rises further, the degree of information asymmetry between groups can be reduced to a point where information-driven flows decrease.

Table 3: Trading volume statistics

a) Aggregate volume statistics

	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Low volatility equilibrium				High volatility equilibrium			
T	$E[AGF_t^U]$	$E[AGF_t^I]$	EV	NIV	$E[AGF_t^U]$	$E[AGF_t^I]$	EV	NIV
10	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
11	0.3731	0.5048	0.2396	0.0460	0.4154	0.5019	0.0850	0.0451
13	0.2969	0.4460	0.2033	0.0475	0.3899	0.4360	0.0847	0.0448
15	0.2534	0.4072	0.1820	0.0486	0.3587	0.3886	0.0846	0.0447
20	0.1922	0.3591	0.1527	0.0506	0.3020	0.3158	0.0846	0.0447
25	0.1591	0.3419	0.1377	0.0520	0.2569	0.2641	0.0845	0.0446
30	0.1380	0.3366	0.1288	0.0529	0.2302	0.2350	0.0845	0.0446
40	0.1123	0.3375	0.1186	0.0540	0.1919	0.1956	0.0843	0.0444
50	0.0971	0.3419	0.1129	0.0545	0.1694	0.1729	0.0842	0.0443
70	0.0801	0.3499	0.1069	0.0549	0.1443	0.1485	0.0839	0.0440
90	0.0710	0.3552	0.1038	0.0550	0.1302	0.1361	0.0834	0.0435
Infinity	0.0440	0.3729	0.0948	0.0549	n.a.	n.a.	n.a.	n.a.

b) Age-cohort volume statistics (T = 50)

age	$E[X_j^U]$	$E[AIF_j^U]$	$E[X_j^I]$	$E[AIF_j^I]$	$E[X_j^U]$	$E[AIF_j^U]$	$E[X_j^I]$	$E[AIF_j^I]$
1	1.3832	1.3832	1.9024	1.9063	3.3843	3.3843	3.4706	3.4706
5	1.3527	0.0874	1.8593	0.4411	2.7217	0.2430	2.7797	0.2492
10	1.3053	0.0846	1.7921	0.4259	2.0265	0.2000	2.0605	0.2040
15	1.2448	0.0811	1.7062	0.4066	1.4564	0.1589	1.4753	0.1614
20	1.1675	0.0767	1.5963	0.3820	0.9977	0.1239	1.0074	0.1254
25	1.0688	0.0715	1.4557	0.3507	0.6435	0.0917	0.6481	0.0925
30	0.9427	0.0654	1.2750	0.3110	0.3803	0.0649	0.3822	0.0653
35	0.7818	0.0589	1.0445	0.2609	0.1984	0.0415	0.1991	0.0417
40	0.5759	0.0545	0.7478	0.1993	0.0838	0.0234	0.0839	0.0235
45	0.3167	0.0561	0.3897	0.1232	0.0232	0.0099	0.0232	0.0099
49	0.0672	0.0640	0.0752	0.0734	0.0026	0.0035	0.0026	0.0035
50	0.0000	0.0672	0.0000	0.0754	0.0000	0.0026	0.0000	0.0026

Panel a. Aggregate volume stats. (2-5: LVE). Col. 2) expected agg. group trade flow, uninf.: $\Sigma_j E[|\Delta X_j^U|]/T$. Col. 3) expected agg. group trade flow, inf.: $\Sigma_j E[|\Delta X_j^I|]/T$. Col. 4) Expected volume: $(1/2)(\mu \cdot \Sigma_j E[|\Delta X_j^U|]/T + (1-\mu) \cdot \Sigma_j E[|\Delta X_j^I|]/T + E[|\Delta\theta|])$. Col. 5) Net information volume: $(1/2)(\mu \cdot E[|\Sigma_j \Delta X_j^U|]/T + (1-\mu) \cdot E[|\Sigma_j \Delta X_j^I|]/T)$. Cols. 6-9) Volume statistics, HVE.
Panel b. Age-cohort volume stats. (2-5: LVE). Col. 2) Average holdings, uninf. aged j. Col. 3) Trading volume uninf. aged j. Cols. 4-5) Stats. for informed. Cols. 6-9) Stats. for HVE.

As the discussion in section 4.3 highlighted, longer horizons along the LVE increase the signal quality of prices, reducing asymmetric information between investors. Col. 5 in Table 3, panel a), is therefore consistent with the theory: NIV is initially low, increases with horizons over some range, and then diminishes. In the benchmark parameterization with $\mu = 0.8$, information asymmetries persist even for very long horizons. In fact, the decreasing range of informational volume can only be seen in the table by comparing NIV at the infinite horizon, with NIV at some long but finite horizon, say $T = 90$. For alternative parameterizations (larger fraction of informed, not reported), the hump in the horizon-NIV relation occurs at shorter lifespans. Along the HVE, on the other hand, NIV falls with horizon (col. 9). This is because prices become more noisy as horizons increase, deepening the adverse selection problem for the uninformed and further reducing trade.

I conclude the discussion of the asymmetric information economy with an observation about the qualitative importance of endogenous information aggregation. As Table 2 shows, although the asymmetric information economy reacts much more to changes in investment horizons than the full-information economy, one could argue that the no-information economy does a reasonable job in delivering comparable changes in risk premium, and return volatility, as the asymmetric information case. What is then the value added of a (more complex) model that highlights endogenous learning?

The answer is that symmetric information benchmarks have no implications for market efficiency. For the no-information economy, prices contain no additional information about the persistent dividend process –over and above from what can be learned from dividends. In the full-information economy, on the other hand, prices are irrelevant as a source of information since agents already know the value of fundamentals. It is in this respect that the analysis of asymmetric information in financial markets becomes crucial. The results in section 4.2 stress that while volatility will be high for short horizons, it is the non-fundamental part of volatility that takes the center stage. To the extent financial markets matter for the allocation of resources, the loss in informational content of the price system could have important implications for the real side of the economy.

5 Applications

I have so far discussed the implications for asset prices and market efficiency of comparing economies with different investment horizons. One might wonder how different T's map into phenomena that affect economies in practice. I now offer two applications of the model to shed light into this question. The first is tailored to understand the asset pricing consequences of demographic change, and discusses the effect of a "baby-boomer" generation such as the one born in the US in the aftermath of WW II. I argue that the effects on several asset pricing statistics are broadly consistent with the trends observed in US stock market in recent decades, and discuss the empirical predictions of its eventual reversal as baby boomers reach retirement. Second, I show how the model can be used to understand financial crisis episodes, in which investment funds suffer increased households' withdrawals and consequent liquidations. In both applications I will focus on the LVE, whose stability properties make the numerical procedure developed in this paper applicable. Unfortunately, this is not the case for the HVE (see appendix for more details).

5.1 Demographic Change and Asset Prices

Several authors have discussed how increased fertility in the US during the decade following WW II is consistent with the marked increased in asset values during the 1990's, as baby-boomers pushed prices up with their increased demand for savings in various financial assets (See Abel, 2003, and references therein). The other side of this coin are the warnings of what will happen to asset prices when this generation reaches retirement and starts unloading assets into the younger generations of smaller size (See for example Siegel, 1998, and Poterba, 2001).

I now study a related demographic change in the context of the model developed above. Specifically, assume that the economy is at a steady-state with a uniform age distribution, and that all agents live to the age of $T = 30$. That is, of the initial mass of investors normalized at one, a mass of $1/T = 0.033$ are 1-year olds, 0.033 are two year olds, and so on. Starting at period t^*, 5 new generations born between t^* until $t^* + 4$ have a size 3 times as large as the others ($3/T = 0.1$). Here, T should be interpreted as the span of time including the active working years and retirement of a typical individual, and t^* as the effective date in which baby boomers enter the workforce and become potential investors in the risky asset. Under this interpretation, it is more natural to assume that other agents already expect the entry of the larger young generations into the stock market by the time they reach working age. I will assume that their birth is at $t^* = 21$, so that the demographic change is anticipated well before it takes place.[17]

Panel a) of Figure 6 plots several asset pricing moments as a function of calendar time. Panel a.1) plots the mass of agents, and the average age of individuals. Notice that the mass of agents increases monotonically between t^* until t^*+4, pushing up aggregate demand. For example, once all baby boomers are alive (t^*+4), market clearing reads:

$$\mu \cdot [(3/30)\sum_{j=1}^{5} X^U_{j,t^*+4} + (1/30)\sum_{j=6}^{29} X^U_{j,t^*+4}] + (1-\mu)\cdot[(3/30)\sum_{j=1}^{5} X^I_{j,t^*+4} + (1/30)\sum_{j=6}^{29} X^I_{j,t^*+4}] = \theta_{t^*+4}.$$

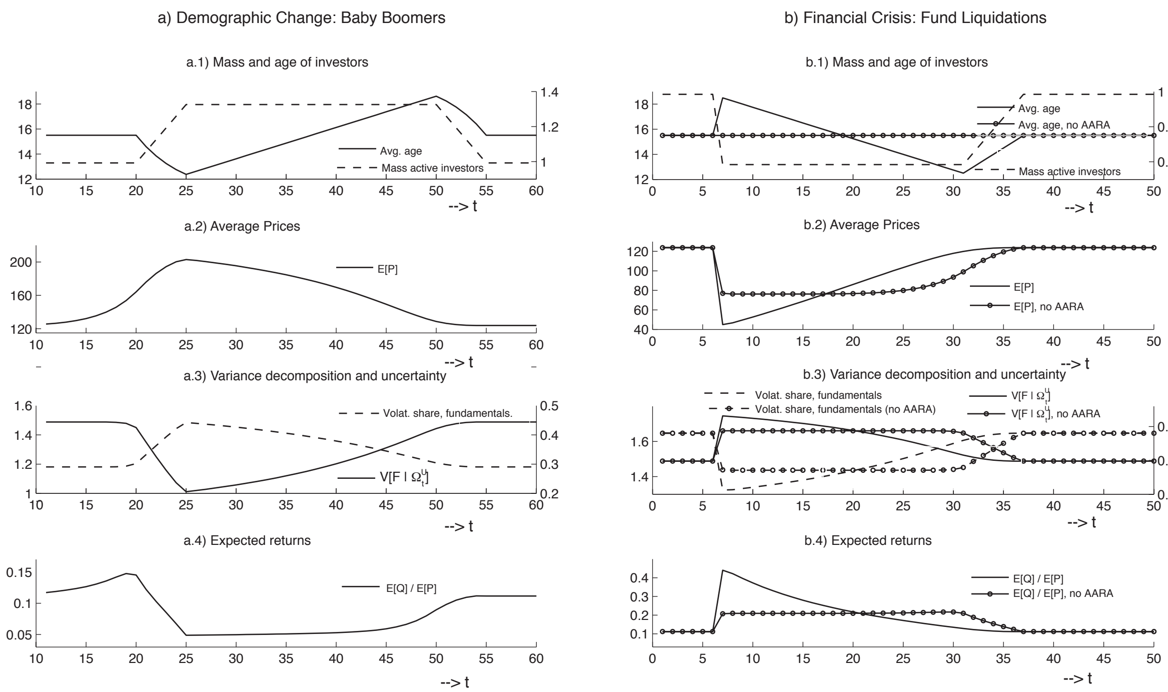
Prices are higher on average during the time interval in which the baby-boomer generation is alive (panel a.2). This is due to two effects. First, there are more active investors during those years so the per capita amount of risk absorbed in equilibrium is lower (specifically, the mass of active investors grows from $T - 1/T = 0.966$ at $t^* - 1$, to 1.3 at $t^* + 4$). This is essentially the *risk-transfer* effect, explicitly shown with the increased mass of agents in panel a.1). Second, because the new borns at t^* are 1 year olds, the age profile of the population is shifted down (panel a.1). This is the *age-adjusted risk aversion* effect: younger individuals have a stronger propensity to invest in the risky asset. Naturally, both effects drive up prices (panel a.2). Note that because we have assumed perfect foresight, prices begin to escalate well before baby boomers are born. Once they have all entered the economy, they age with the passage of time and prices begin a steady process of decline, reaching their old level once the economy returns to its steady state. The demographic change also increases absolute price volatility, but reduce volatility of returns (normalized by average prices, not reported). As plotted in panel a.3), this increase is due to higher fundamental volatility. Consequently, the uncertainty about fundamentals for the uninformed decreases (also panel a.3).

Consistent with large average prices, expected returns in panel a.4) are lower during the life-time of the baby-boomer generation, although the effects are non-monotonic. Before these generations are born, the anticipation of large future prices increases current ones well before before t^*, translating into increasing returns. At t^*, returns drop sharply reflecting the stronger propensity to bear risk by the younger generations (AARA effect), as well as the lower per capita amount of risk absorbed by active investors (RT effect). Moreover, because prices are more informative (panel d), uncertainty falls for the uninformed, bringing down the premium they require to hold the stock. Expected returns bottom by the time all new generations are alive ($t^* + 4$). As these investors start to age, horizon effects reduce their propensity to bear risk and expected returns begin to increase. By the time the last baby-boomer generation is out, returns converge back to their steady state.

The results of this simple exercise are consistent with the rise of stock prices after the 1980's, which as pointed by several observers roughly matches the entry of baby boomers into their prime saving years (see Abel, 2003). It also lends support to the concerns raised by Siegel (1998), that the natural reversal

[17]It is much simpler technically to assume the shock is a one-time, unanticipated event. I assume anticipation as it is closer to full agent rationality, in the spirit of the present paper. Next section considers an unanticipated shock.

Figure 6: Applications: baby boomers; fund liquidations



of this trend should imply a bleak prospect for stock returns, as baby boomers exit and begin to sell their large holdings into the smaller, new young generations.

5.2 Investor withdrawals and fund liquidations

A stylized fact of financial crises is the increased propensity of households' to withdraw funds from intermediaries, triggering forced liquidations of assets. For instance, Ben-David et al. (2012) find that hedge funds reduced their exposure in equity markets in almost 30% during the 2008:Q3-Q4 contraction, which corresponded roughly to 1% of all outstanding equities. Importantly, their results indicate that most of this selling was actually *forced* by investors withdrawing financing. Carhart et al. (2002) study attrition rates in the mutual fund industry, and find that while 3.6% of funds disappear yearly on average over their sample, the standard deviation is quite high, at 2.4%. Chen et al. (2008) measure distress selling of troubled –but still alive– mutual funds, and report average distress-driven sales between 0.6-1% of mutual funds holdings at quarterly frequency (when using the asset-weighted measure of outflows). Moreover, this fraction spikes considerably (nearly doubles) during the main episode of financial turbulence covered in their sample (LTCM).

I now present an exercise in which a financial crisis is captured by a shortening of investors' horizons. While one would expect crisis to be related (and perhaps caused) by a deterioration in fundamentals, the exercise maintains the first and second moments of fundamentals constant, focusing solely on the asset pricing implications of fund liquidations. Regarding anticipation, I will assume here that the shock is unexpected for all investors, which probably captures better the sudden nature of a financial crisis.

One might also want to distinguish which asset pricing effects are brought by the liquidations per se –the larger fraction of forced sellers, relative to voluntary buyers– vs. the effects that arise from the demands of those invests who remain in the market. In practice, both channels are likely to matter: when households redeem and force liquidations, the net supply of the asset for those who remain increases (this is the RT effect). But at the same time, those funds who remain alive are quite aware that their investors are more likely to cash out now than before the crisis, reducing the effective horizon of the investments (this is the AARA effect). To analyze the contribution of these different effects, I perform the following two exercises. Before the shock, the horizon of investors is $T = 30$. In exercise 1, I assume that at t^* the exit date is shortened by 6 periods for each generation. This means that generations aged 24 and older are forced out of the market and must liquidate their assets (and for all the rest, the horizon has become shorter). After that, new investors come into the market replacing the old, so it takes $T = 30$ periods for the age structure to get back to its initial uniform distribution (with aggregate mass of 1). Exercise 1 therefore captures the two effects mentioned: there is a sudden contraction of the investor base (the RT effect), and at the same time the remaining investors live for shorter horizons, which affects their willingness to take risks (the AARA effect).

To isolate the impact of pure liquidations (RT effect), I construct a second example in which the mass of active investors is the same as in the previous case, but the exit of agents is allocated proportionally through different vintages so that the age distribution remains the same as before the shock. For clarification purposes, panel b.1) of Figure 6 plots the average age and mass of the agents. The plain line shows the average age of agents under exercise 1, in which both effects are present (liquidations, and aging of the active generations). The circled lines ("no AARA effect") show the average age of investors under exercise 2. By construction, this statistic is unchanged throughout the exercise. The dashed line shows the mass of active investors, which is also equal (by construction) under both exercises.

Panel b.2) of Figure 6 shows how prices drop immediately. The drop is larger under exercise 1, since the RT and AARA effect work simultaneously. Under exercise 2 (the pure RT effect), the drop is smaller since risk preferences of those who remain in the economy have not changed. Price volatility falls (not reported), driven by a lower contribution of fundamental volatility (panel b.3). Because the signal-to-noise ratio of the price worsens, the uncertainty of the uninformed investors increases (panel b.3). Consistent with the marked contraction in prices at impact, expected returns increase, and then monotonically decline towards the steady state (panel b.4). Notice that while all these effects are more marked at impact when both mechanisms are present (exercise 1), the convergence to the steady state is also faster in this case. This is because after a few periods, the age structure of the population reverses and the average age drops (increasing overall risk taking, compared to the age structure in exercise 2). This of course depends on the particular assumptions made about the age structure of the active investors under each exercise.

The parallel between time-variation in effective investment horizons, and the exercise performed in this last application, is only meant to be suggestive. A rigorous modeling of such episodes should include several factors shaping fund managers' choices, such as pay for performance contracts -either explicit of implicit in fund net flows– as well as career concerns, just to name a few. Moreover, to be consistent with rationality, the ideal model would deal explicitly with time-varying fund liquidations which are likely to differ in some respects from the "once and for all" exercise just presented. Unfortunately, adding such stochastic elements to the model (i.e., time-varying second moments) breaks the normality of returns, which yields the CARA setup intractable.[18] Nevertheless, the model presented above can provide some insights into the main economic channels and the effects for asset prices of a shortening of investment horizons under this high-frequency, cyclical interpretation.

6 Conclusions

This paper analyzed the asset pricing implications of finite investment horizons in financial markets. The main message is twofold. First, horizons matter for the pricing of risk of the average investor (the *age-adjusted risk aversion* effect) and the amount of risk that must be held in equilibrium by the active investors in a generalized OLG economy (the *risk transfer* effect). Both mechanisms have the potential of delivering interesting differences in expected returns and return volatilities when we compare economies with different investment horizons.

Second, horizons matter for market efficiency. While long horizons incite informed investors to actively trade on their information, the high risk environment triggered by short horizons can reduce market efficiency. In particular, even if the fraction of informed investors is relatively low, the degree of information contained in prices can be quite different across economies with different horizons. Asset prices in economies where horizons are long behave similarly to a hypothetical case in which all investors observe economic fundamentals in real time, while they align much closer with a no-information benchmark when horizons are short. This findings suggests information *disaggregation* from the price process can be an important mechanism for understanding variations in economic uncertainty more specifically, and market efficiency more generally.

There are several avenues in which one can extend the current analysis. One natural extension would be to model stochastic variation in effective horizons, due to increased fund liquidations which occur in financial crises. Since these episodes typically display low prices, high expected returns, high return volatility, and pervasive economic uncertainty –all pricing moments consistent with a shortening of horizons in this paper–, it seems natural to explore the role of time-varying forced liquidations for generating these effects in asset pricing moments.

A second question that seems relevant is to study the incentives to acquire information for different investment horizons. While investors have more incentives to become informed when prices are less reliable sources of information, these are also the times when the lifespan over which they plan to use such information is shorter. This leads to non-trivial predictions about the effects of investment horizons on endogenous information acquisition. I leave these questions for future research.

[18]Banerjee and Green (2014) make progress in this regard studying an OLG model where the uninformed infer the quality of other investors' signals from prices. This requires dealing explicitly with learning in non-gaussian environments.

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7 Appendix

Proof of Theorems 1 and 2:

We begin finding the matrices A_ψ and B_ψ in expression (6) from equations (1), (2), and (3):

$$A_\psi = \begin{bmatrix} 1 & 0 & 0 \\ F_0 & \rho_F & 0 \\ \theta_0 & 0 & \rho_\theta \end{bmatrix}, \text{ and } \quad B_\psi = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

where $F_0 \equiv (1-\rho_F)\bar{F}$, and $\theta_0 \equiv (1-\rho_\theta)\bar{\theta}$. I will prove Theorems 1 and 2 through the following 3 steps. 1) For given coefficients in the price equation, solve the Bayesian filtering problem of the uninformed, and find the autoregressive process of their forecast errors. 2) Find the recursive representation of the conditional state vectors M_t and M_t^U, and the conditional distributions of future excess returns, Q_{t+1}. 3) Solve optimal demands, and impose market clearing to find equilibrium prices.

Step 1: The coefficients in the price equation (7) and the dividend process in (1), together with the recursive representation in (6), leads directly to Theorem 1: the Bayesian updating of beliefs described by the Kalman filter, whose derivation can be found in most advanced statistics textbooks. Writing $\mathbb{E}[\Psi_t \mid \Omega_t^U] \equiv \Psi_t^U$ and $\Psi_t^U - \Psi_t \equiv \tilde{\Psi}_t^U$, the evolution of the uninformed investors' forecast error vector can be found from manipulation of (10):

$$\begin{aligned} \tilde{\Psi}_{t+1}^U &= A_U \cdot \tilde{\Psi}_t^U + B_U \cdot \epsilon_{t+1}^U, \text{ with } A_U \equiv (I_3 - KA_s)A_\Psi, \;\; B_U \equiv (K(A_sB_\Psi + B_s) - B_\Psi), \\ A_s &\equiv \begin{bmatrix} 0 & 1 & 0 \\ 0 & \lambda^{-1} & 1 \end{bmatrix}, \text{ and } \quad B_s = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \end{aligned}$$

Here, we define $\lambda \equiv p_\theta/p_F$. But notice that from the observation of the price, forecast errors about F_t and θ_t are perfectly colinear since $F_t + \lambda \cdot \theta_t = F_t^U + \lambda \cdot \theta_t^U$, or $\tilde{\theta}_t^U = -\lambda^{-1} \cdot \tilde{F}_t^U$. This allows to rewrite the vector $\tilde{\Psi}_{t+1}^U$ as $\tilde{\Psi}_{t+1}^U = [0 \;\; \tilde{F}_{t+1}^U \;\; \tilde{\theta}_{t+1}^U]' = A_U \cdot [0 \;\; \tilde{F}_t^U \;\; \tilde{\theta}_t^U]' + B_U \cdot \epsilon_{t+1}^U$, or

$$\tilde{F}_{t+1}^U = \rho_U \cdot \tilde{F}_t^U + b_U \cdot \epsilon_{t+1}^U, \text{with } \rho_U \equiv A_U(2,2) - \lambda^{-1}A_U(2,3), \text{ and } b_U \equiv B_U(2,:). \tag{27}$$

Step 2: Using (27), the evolution of the conditional state vectors M_t and M_t^U can now be found:

$$M_{t+1} = A_M \cdot M_t + B_M \cdot \epsilon_{t+1}, \tag{28}$$

$$M_{t+1}^U = A_M^U \cdot M_t^U + B_M^U \cdot \epsilon_{t+1}^U, \tag{29}$$

$$\text{with} \quad A_M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ F_0 & \rho_F & 0 & 0 \\ \theta_0 & 0 & \rho_\theta & 0 \\ 0 & 0 & 0 & \rho_U \end{bmatrix}, \quad B_M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ B_U(2,1) & B_U(2,2) & B_U(2,3) \end{bmatrix},$$

$$A_M^U = A_\Psi, \;\; B_M^U = K(A_sA_\Psi l_0 v_0 + A_sB_\Psi + B_s), \;\; l_0 \equiv [0 \;\; -1 \;\; \lambda^{-1}]', \text{ and } v_0 \equiv [0 \;\; 0 \;\; 0 \;\; 1].$$

With (28) and (29) we can find the conditional moments of future returns for informed and uninformed investors. Note that the forecast $\hat{F}_t^U$ in the price equation (7) can be replaced to express prices as a function of current state variables and the forecast error of the uninformed,

$$P_t = p_0 + p_1 \cdot F_t + p_2 \cdot \theta_t + p_3 \cdot \tilde{F}_t^U \equiv P \cdot M_t, \tag{30}$$

where $P \equiv [p_0 \quad p_1 \quad p_2 \quad p_3]$ and $p_1 = \hat{p}_F + p_F$, $p_2 = p_\theta$, $p_3 = \hat{p}_F$. Moreover, writing (1) as $D_{t+1} = A_D \cdot \Psi_{t+1} + B_D \cdot \epsilon_{t+1}$, with $A_D \equiv [0 \;\; 1 \;\; 0 \;\; 0]$, $B_D \equiv [1 \;\; 0 \;\; 0]$, future returns Q_{t+1} can now be written as:

$$\begin{aligned} Q_{t+1} \quad &= \quad A_Q \cdot M_t + B_Q \cdot \epsilon_{t+1}, \\ \text{with} \quad & \quad A_Q \equiv A_D A_M + P \cdot (A_M - I_4 R), \text{ and } B_Q \equiv (A_D + P) \cdot B_M + B_D. \end{aligned} \tag{31}$$

For uninformed investors, the corresponding expression is:

$$\begin{aligned} &Q_{t+1} = A_Q^U \cdot M_t^U + B_Q^U \cdot \epsilon_{t+1}^U, \\ &\text{with } A_Q^U \equiv A_Q m_0, \;\; B_Q^U \equiv A_Q m_1 v_0 + B_Q m_0^{'}, \;\; m_0 \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}^{'}, \text{and } m_1 \equiv [0 \quad -1 \quad \lambda^{-1} \quad 1]^{'} \end{aligned} \tag{32}$$

Expressions (31) and (32) confirm future returns are conditional gaussian for all investors.

Step 3: Optimal investment and consumption policies can now be found. We conjecture a linear-quadratic value function at $t+1$ for an informed investor aged j at time t, $J^I(W_{j+1,t+1}^I; M_{t+1}; j+1; t+1) = -\beta^{t+1} \cdot \exp\{-\alpha_{j+1} W_{j+1,t}^I - \frac{1}{2} M_{t+1}^{'} V_{j+1}^I M_{t+1}\}$. The value function at t then takes the form:

$$\begin{aligned} &J^I(W_{j,t}^I; M_t; j; t) = \max_{\{X,C\}} - \exp\{-\gamma C\} - \beta \delta_{j+1}^I \exp\{-\alpha_{j+1} R(W_{j,t}^I - C) - \alpha_{j+1} X A_Q M_t && (33) \\ &-\frac{1}{2} M_t^{'} v_{j+1}^{I,aa} M_t + \frac{1}{2}(\alpha_{j+1} X B_Q^{\prime} + (v_{j+1}^{I,ab})^{\prime} M_t)^{\prime} (\Xi_{j+1}^I)^{-1} (\alpha_{j+1} X B_Q^{\prime} + (v_{j+1}^{I,ab})^{\prime} M_t)\}, \text{ with} \\ &v_{j+1}^{I,aa} \equiv A_M^{'} V_{j+1}^I A_M; \;\; v_{j+1}^{I,bb} \equiv B_M^{'} V_{j+1}^I B_M; \;\; v_{j+1}^{I,ab} \equiv A_M^{'} V_{j+1}^I B_M; && (34) \\ &\Xi_{j+1}^I \equiv \Sigma^{-1} + v_{j+1}^{I,bb}, \;\text{ and }\; \delta_{j+1}^I \equiv |\Sigma \cdot \Xi_{j+1}^I|^{-1/2}. \end{aligned}$$

(see Vives, 2008). Letting the scalar $\Gamma_{j+1}^I \equiv B_Q (\Xi_{j+1}^I)^{-1} B_Q^{'}$, the f.o.c.'s are

$$\begin{aligned} X_{j,t}^I \quad &= \quad \frac{g_{j+1}}{\alpha_{j+1} \Gamma_{j+1}^I} \cdot M_t, \;\text{ with }\; g_{j+1} \equiv A_Q - h_{j+1}^I && (35) \\ C_{j,t}^I \quad &= \quad c_j^I + (\frac{\alpha_{j+1} R}{\alpha_{j+1} R + \gamma}) \cdot W_{j,t}^I + \frac{M_t^{'} m_{j+1}^I M_t}{2(\alpha_{j+1} R + \gamma)}, \text{ where} && (36) \\ c_j^I \quad &\equiv \quad \log(\gamma / (\beta \delta_{j+1}^I \alpha_{j+1} R)) / (\alpha_{j+1} R + \gamma); \;\; h_{j+1}^I \equiv B_Q (\Xi_{j+1}^I)^{-1} v_{j+1}^{I,ab\,\prime}, \text{ and} \\ m_{j+1}^I \quad &\equiv \quad g_{j+1}^{'} g_{j+1} / \Gamma_{j+1}^I + v_{j+1}^{I,aa} - v_{j+1}^{I,ab} (\Xi_{j+1}^I)^{-1} v_{j+1}^{I,ab\,\prime}. \end{aligned}$$

The matrices V_{j+1}^I and scalars α_j, for $j = 1, 2 \ldots T$, can be solved recursively by setting $\alpha_T = \gamma$, $V_T^I = 0_4$, and using the recursive equations found in the f.o.c.'s:

$$\alpha_j = \frac{\gamma \alpha_{j+1} R}{\alpha_{j+1} R + \gamma}, \text{ and } V_j^I = m_{j+1}^I \cdot (\frac{\alpha_j}{\alpha_{j+1} R}) + 2 \cdot i_{1,1} \cdot (\gamma c_{j+1}^I + \log \frac{\alpha_j}{\gamma}). \tag{37}$$

where $i_{1,1}$ is a 4x4 matrix with first element equal to one, and all others equal to zero. Moreover, defining $\omega(h) \equiv \frac{1}{\alpha_{T-h}}$ as the inverse of the age-adjusted risk aversion of an agent who is h years from exiting the economy, and replacing it into the previous recursion, we can write

$$\omega(h) = \frac{1}{\gamma} + \frac{1}{R}\omega(h-1), \text{ or } \quad \omega(h) = \frac{1}{\gamma}\sum_{k=0}^{s}(\frac{1}{R})^k = \frac{R - R^{-s}}{\gamma r}.$$

This implies that for an agent aged j years ($h = T - j$ years from exiting), the age-adjusted risk aversion is given by $\alpha_j = \frac{1}{\omega(T-j)} = \gamma \frac{r}{R - R^{-(T-j)}}$, which corresponds to expression (21) in the main text. For the uninformed investors the procedure is similar, but one must replace M_t, A_Q, B_Q, and Σ, for M_t^U, A_Q^U, B_Q^U, and $\mathcal{V}$, respectively. This allows solving demand and consumption policies using the appropriate superscript U instead of I.

We now impose the market clearing condition (20). Note that although the conditional state vector has different dimensions for the informed and uninformed investors, the uninformed forecasts can be replaced by the actual values of the state variables plus the forecast noise. This leads to the following equation that must be satisfied by the price:

$$\begin{aligned}
&P = Y(P) \cdot Z(P)^{-1}, \quad \text{where} \qquad (38)\\
&Z \equiv [(A_M - I_4 R) \cdot ((m_0 \bar{w}_0^U - m_1 v_0 \bar{w}_1^U) \cdot (m_0^{'} + x_0) + I_4 \bar{w}_0^I) - B_M((m_0^{'} \bar{w}_1^U \cdot (m_0^{'} + x_0) + I_4 \bar{w}_1^I)],\\
&Y \equiv [A_\theta - (A_D A_M m_0 \bar{w}_0^U - (A_D A_M m_1 v_0 + (A_D B_M + B_D) m_0^{'}) \bar{w}_1^U) \cdot (m_0^{'} + x_0) - A_D A_M \bar{w}_0^I + (A_D B_M + B_D) \bar{w}_1^I],\\
&\bar{w}_0^U \equiv \frac{1}{T}\sum_{s=2}^{T}(\alpha_s \Gamma_s^U)^{-1}, \;\; \bar{w}_0^I \equiv \frac{1}{T}\sum_{s=2}^{T}(\alpha_s \Gamma_s^I)^{-1}, \;\; \bar{w}_1^U \equiv \frac{1}{T}\sum_{s=2}^{T}(\Xi_s^U)^{-1} {v_s^{U,ab}}'(\alpha_s \Gamma_s^U)^{-1}, \bar{w}_1^I \equiv \frac{1}{T}\sum_{s=2}^{T}(\Xi_s^I)^{-1} {v_s^{I,ab}}'(\alpha_s \Gamma_s^I)^{-1},\\
&A_\theta \equiv [\bar{\theta} \;\; 0 \;\; 1 \;\; 0], \;\; \text{and } x_0 \equiv [0 \;\; 1 \;\; -\lambda^{-1}]' \cdot v_0.
\end{aligned}$$

Equation (38) is the fixed point problem that determines the equilibrium vector of price coefficients, P. The first part of the fixed-point is the dependence of informed and uninformed investors strategies on the conjectured price vector, $P^{(0)}$. The second part is the relation between strategies and the implied equilibrium price function, which follows from the aggregation of individual strategies and the market clearing requirement. For an initial conjecture $P^{(0)}$, individual choices and their aggregation implies an equilibrium price given by (38), $P^{(1)}$. In the fixed point, the input and output price vectors are equal. Indeed, notice the dependence of matrices Y and Z on A_M, B_M, A_Q, B_Q, A_M^U, B_M^U, A_Q^U, and B_Q^U, all of which are functions of the initial guess $P^{(0)}$.

Proof of Proposition 1:

a) For the full-information economy ($\mu = 0$), the price vector $P^I = [p_0 \quad p_F \quad p_\theta]$ loads on the state vector $M_t \equiv [1 \quad F_t \quad \theta_t]'$. The market clearing condition then reads (dropping age subscripts): $A_Q \Psi_t / 2\gamma\Gamma^I =$

$[0\ 0\ 1]\Psi_t$, where $A_Q = [-p_0 r + p_F F_0 + p_\theta \theta_0 + F_0 \;\; p_F(\rho_F - R) + \rho_F \;\; p_\theta(\rho_\theta - R)]$, and $\Gamma^I = \sigma_D^2 + (1+p_F)^2\sigma_F^2 + p_\theta^2\sigma_\theta^2$. This yields the following three equations for the price coefficients p_0, p_F and p_θ:

$$\begin{aligned} 0 &= -p_0 r + p_F F_0 + p_\theta \theta_0 + F_0, \\ 0 &= p_F(\rho_F - R) + \rho_F, \\ p_\theta(\rho_\theta - R) &= 2\gamma(\sigma_D^2 + (1+p_F)^2\sigma_F^2 + p_\theta^2\sigma_\theta^2), \end{aligned} \tag{39}$$

which yield the quadratic equation for the price coefficient p_θ stated in equation (23). Clearly, when $\sigma_\theta > \sigma_\theta^*$, the term in the square root is negative and there exist no real solution. Hence, there is no equilibria when $\sigma_\theta > \sigma_\theta^*$, and two equilibria whenever $\sigma_\theta \leq \sigma_\theta^*$ (except for the knife-edge case where they coincide).

b) For the no-information economy ($\mu = 1$), the price vector $P^U = [p_0 \;\; \hat{p}_F \;\; p_\theta]$ loads on the state vector $M_t^U \equiv [1 \;\; F_t^U \;\; \theta_t]'$ (as the price reveals the random supply θ_t in the absence of informed traders). The market clearing condition in this case reads (dropping age subscripts): $A_Q^U M_t^U / 2\gamma\Gamma^U = [0\ 0\ 1]M_t^U$, where $A_Q^U = [-p_0 r + \hat{p}_F F_0 + p_\theta\theta_0 + F_0 \;\; \hat{p}_F(\rho_F - R) + \rho_F \;\; p_\theta(\rho_\theta - R)]$.To find $\Gamma^U = B_Q^U(\Xi_{j+1}^U)^{-1}B_Q^{U\,\prime}$, note that we can write $B_Q^U = (A_D + P^U)B_M^U + \tilde{B}_D$ in this case, where $B_M = K(A_s B_\Psi - A_s A_\Psi m_1 v_0 + B_s)$ (using $\lambda^{-1} = 0$ in m_1), and $\tilde{B}_D = B_D - A_D m_1 j_0$, with $j_0 \equiv [B_U(2,1) \;\; B_U(2,2) \;\; B_U(2,3) \;\; \rho_U]$. The matrix K gives the weighting of the uninformed investors' forecast of the state A_Ψ on the vector of signals. When all investors are uninformed, the signal vector includes the dividend –which gives information about the fundamental F_t–, and the price –which is fully informative about the noisy supply θ_t, but contains no information about F_t. K can be solved through the iteration presented in equations (11) and (12), which in this case allows to solve for the conditional variance of F_t (element (2,2) in the $\mathbb{O}$ matrix) and matrix K in closed form. Labeling $\mathbb{O}(2,2) \equiv \sigma_u^2$, we can write:

$$K = \begin{bmatrix} 0 & 0 \\ \frac{\rho_F^2\sigma_u^2 + \sigma_F^2}{\sigma_D^2 + \rho_F^2\sigma_u^2 + \sigma_F^2} & 0 \\ \theta_0 & 0 \end{bmatrix}, \text{ and } \mathbb{O} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{\sigma_D^2(\rho_F^2\sigma_u^2 + \sigma_F^2)}{\sigma_D^2 + \rho_F^2\sigma_u^2 + \sigma_F^2} & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

This gives a quadratic equation for σ_u^2 with positive root $\sigma_u^2 = \frac{\sigma_D^2(1-\rho_F^2)+\sigma_F^2}{2\rho_F^2}\left(\sqrt{1 + \frac{4\sigma_D^2\sigma_F^2\rho_F^2}{(\sigma_D^2(1-\rho_F^2)+\sigma_F^2)^2}} - 1\right)$, as stated in Proposition 1, part b). We can now write $\Gamma^U = \frac{(\sigma_D^2 + (1+\hat{p}_F)^2(\rho_F^2\sigma_u^2 + \sigma_F^2))^2}{\sigma_D^2 + \rho_F^2\sigma_u^2 + \sigma_F^2}$, which yields the following three equations for the price coefficients p_0, $\hat{p}_F$ and p_θ:

$$\begin{aligned} 0 &= -p_0 r + (1+\hat{p}_F)F_0 + p_\theta\theta_0, \\ 0 &= \hat{p}_F(\rho_F - R) + \rho_F, \\ p_\theta(\rho_\theta - R) &= 2\gamma(w(\sigma_D^2 + (1+\hat{p}_F)^2(\rho_F^2\sigma_u^2 + \sigma_F^2)) + p_\theta^2\sigma_\theta^2), \end{aligned}$$

with $w \equiv \frac{\sigma_D^2 + (1+\hat{p}_F)^2(\rho_F^2\sigma_u^2 + \sigma_F^2)}{\sigma_D^2 + \rho_F^2\sigma_u^2 + \sigma_F^2} \geq 1$. This yield the quadratic equation for the price coefficient p_θ stated in equation (24). Clearly, when $\sigma_\theta > \sigma_\theta^{**}$, the term in the square root is negative and there exist no real solution. Hence, there is no equilibria when $\sigma_\theta > \sigma_\theta^{**}$, and two equilibria whenever $\sigma_\theta \leq \sigma_\theta^{**}$. This completes the proof. (Note also that since $w \geq 1$, and $\sigma_u > 0$, we have that $\sigma_\theta^{**} \geq \sigma_\theta^{*}$. This means that equilibria in the no-information economy obtains in a strict subset of parameters than in the full-information economy).

Proof of Proposition 2:

This proof consists of three steps. 1) Derive the price equation that arises from the market clearing condition. This leads to a quadratic equation for the supply coefficient, p_θ. I will show that both roots of the equation depend on a particular coefficient of the value function matrix V (the stationary value function matrix in the infinite horizon case). This coefficient is the ninth element of the matrix, associated with the utility impact of supply innovations, which I label V_θ. 2) Derive a second equation which describes the element V_θ as a function of the price coefficient p_θ. An equilibrium is then a pair $\{p_\theta, V_\theta\}$ satisfying both equations. 3) show that a) an intersection between these functions always exists –part a) of the proposition–, and b) it is unique –part b) of the proposition. Since the problem of investors in the no-information economy can always be represented as an equivalent full-information economy with scaled up noise, and the proof is valid for all possible parameter values, I will prove here the proposition for the full-information economy only.

Step 1: In the infinite horizon case with full information ($\mu = 0$), there exists only one type of investor whose asset demand (equation (35)) can be restated (dropping the age and information subscripts) as $X_t = (A_Q - B_Q \Xi^{-1} v^{ab\,\prime})/(\alpha\Gamma)$, where $A_Q = [-p_0 r + p_F F_0 + p_\theta \theta_0 + F_0 \;\; p_F(\rho_F - R) + \rho_F \;\; p_\theta(\rho_\theta - R)]$ and $B_Q = [1 \;\; (1+p_F) \;\; p_\theta]$ are the row vectors associated with the loadings of future returns on the vector of contemporary state variables, and future disturbances, respectively. It is straightforward to show that for this economy,

$$\Xi = \begin{bmatrix} \sigma_D^{-2} & 0 & 0 \\ 0 & \sigma_F^{-2} & 0 \\ 0 & 0 & \sigma_\theta^{-2} + V_\theta, \end{bmatrix}, \quad \Gamma = \sigma_D^2 + (1+p_F)^2 \sigma_F^2 + p_\theta^2 \frac{1}{\sigma_\theta^{-2} + V_\theta}, \quad \text{and} \quad \alpha = \frac{\gamma r}{R}.$$

Moreover, from equation (34), it can be shown that the second term in the demand's numerator corresponds to $B_Q \Xi^{-1} v^{ab\,\prime} = [p_\theta \frac{V_{1,3} + V_\theta \theta_0}{\sigma_\theta^{-2} + V_\theta} \;\; 0 \;\; p_\theta \frac{\rho_\theta V_\theta}{\sigma_\theta^{-2} + V_\theta}]$, where $V_{1,3}$ is the 3rd term of the (symmetric) matrix V. The market clearing condition then reads

$$\frac{[-p_0 r + p_F F_0 + p_\theta(\theta_0 \frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+V_\theta} - \frac{V_{1,3}}{\sigma_\theta^{-2}+V_\theta}) + F_0 \quad p_F(\rho_F - R) + \rho_F \quad p_\theta(\rho_\theta \frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+V_\theta} - R)]}{\gamma(r/R)(\sigma_D^2 + (1+p_F)^2 \sigma_F^2 + p_\theta^2 \frac{1}{\sigma_\theta^{-2}+V_\theta})} = [0 \quad 0 \quad 1],$$

which gives the three equations determining the price coefficients p_0, p_F and p_θ:

$$0 = -p_0 r + p_F F_0 + p_\theta(\theta_0 \frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2} + V_\theta} - \frac{V_{1,3}}{\sigma_\theta^{-2} + V_\theta}) + F_0 \tag{40}$$

$$0 = p_F(\rho_F - R) + \rho_F \tag{41}$$

$$p_\theta(\rho_\theta \frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2} + V_\theta} - R) = p_\theta^2 \frac{\gamma(r/R)}{\sigma_\theta^{-2} + V_\theta} + \gamma(r/R)(\sigma_D^2 + (\frac{R}{R - \rho_F})^2 \sigma_F^2). \tag{42}$$

(42) is the first equation we will use to find the equilibrium pair $\{p_\theta, V_\theta\}$. Defining $k \equiv (2\gamma/R)(r/R)^2(\sigma_D^2 + (\frac{R}{R-\rho_F})^2\sigma_F^2)^{1/2}$, and $\sigma^{-2} \equiv \sigma_\theta^{-2}\frac{R-\rho_\theta}{R}$, we can rewrite the two (negative) roots of (42) as

$$p_{\theta,1}^{+}(V_\theta) = -\frac{R^2(\sigma^{-2}+V_\theta)}{2\gamma r} + \frac{R^2(\sigma^{-2}+V_\theta)}{2\gamma r}\sqrt{1 - k^2\frac{\sigma_\theta^{-2}+V_\theta}{(\sigma^{-2}+V_\theta)^2}}, \tag{43}$$

$$p_{\theta,1}^{-}(V_\theta) = -\frac{R^2(\sigma^{-2}+V_\theta)}{2\gamma r} - \frac{R^2(\sigma^{-2}+V_\theta)}{2\gamma r}\sqrt{1 - k^2\frac{\sigma_\theta^{-2}+V_\theta}{(\sigma^{-2}+V_\theta)^2}}. \tag{44}$$

Step 2: to find the dependence of the value function term V_θ on the model parameters and the price coefficient p_θ, we adapt equation (37) to the infinite horizon case to write $V = \frac{m}{R} + 2 \cdot i_{1,1} \cdot (\gamma c + \log\frac{\alpha}{\gamma})$, but since $i_{1,1}$ is zero for all terms besides the first, the expression for the ninth term in V reduces to $V_\theta = \frac{m(3,3)}{R}$. But recall that

$$m = \frac{(A_Q - h)' \cdot (A_Q - h)}{\Gamma} + v^{aa} - v^{ab}(\Xi)^{-1}v^{ab\,\prime},$$

which combined with the market clearing condition in (42), allows to write

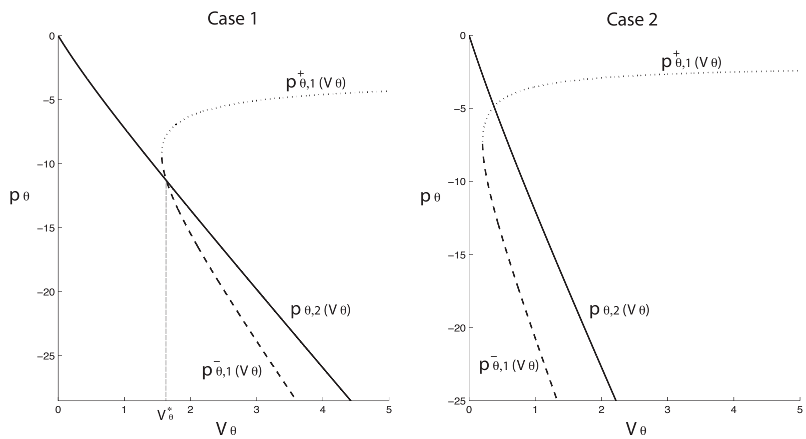
$$\begin{aligned} V_\theta &= p_\theta(\rho_\theta\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+V_\theta} - R)\frac{r\gamma}{R^2} + V_\theta\rho_\theta^2\frac{\sigma_\theta^{-2}}{\sigma_\theta^{-2}+V_\theta}, \quad \text{or} \\ p_{\theta,2}(V_\theta) &= -V_\theta\frac{R}{\gamma r}(1 + \frac{\rho_\theta(1-\rho_\theta)\sigma_\theta^{-2}/R}{\sigma^{-2}+V_\theta}) \end{aligned} \tag{45}$$

Step 3: I now study the two roots of equation (42), and equation (45). Figure 7 provides the loci of these equations in the $\{V_\theta, p_\theta\}$ space. To find existence, it suffices to show that (45) intersects either root of (42) at least once. Uniqueness amounts to showing this intersection is a singleton. I will prove existence and uniqueness by establishing the following:
i) $\partial p_{\theta,1}^{+}(\cdot)/\partial V_\theta > 0$;
ii) $\partial p_{\theta,1}^{-}(\cdot)/\partial V_\theta < 0$, $\partial^2 p_{\theta,1}^{-}(\cdot)/\partial V_\theta^2 > 0$, and $\lim_{V_\theta\to\infty}\partial p_{\theta,1}^{-}(\cdot)/\partial V_\theta = \frac{-R^2}{\gamma r}$,
iii) $\partial p_{\theta,2}(\cdot)/\partial V_\theta < 0$, $\partial^2 p_{\theta,2}(\cdot)/\partial V_\theta^2 > 0$, and $\lim_{V_\theta\to\infty}\partial p_{\theta,2}(\cdot)/\partial V_\theta = \frac{-R}{\gamma r}$, and
iv) At V_θ^*, $|\partial p_{\theta,1}^{-}(V_\theta^*)/\partial V_\theta| > |\partial p_{\theta,2}(V_\theta^*)/\partial V_\theta|$, where V_θ^* satisfies $p_{\theta,1}^{-}(V_\theta^*) = p_{\theta,2}(V_\theta^*)$, whenever an intersection between $p_{\theta,2}(\cdot)$ and the root $p_{\theta,1}^{-}(\cdot)$ exists.

Facts i)-iii) imply that an intersection between equations (42) and (45) always exist. This is because equation $p_{\theta,2}(\cdot)$ in (45), which begins in the origin and has a strictly negative slope that converges to a constant, will either intersect the root $p_{\theta,1}^{-}(\cdot)$ in (44) (case 1), whose strictly negative slope converges to a constant of larger absolute magnitude than the limit of the slope of $p_{\theta,2}(\cdot)$, or it will intersect the root $p_{\theta,1}^{+}(\cdot)$ in (43) (case 2), which has strictly positive slope.

Fact iv) provides uniqueness. To see this, notice that in case 1 (left panel of figure 7), the condition $|\partial p_{\theta,1}^{-}(V_\theta^*)/\partial V_\theta| > |\partial p_{\theta,2}(V_\theta^*)/\partial V_\theta|$ implies the two lines can only intersect once, otherwise at least in one of these intersections condition iv) would be violated. Regarding case 2, whenever equation $p_{\theta,2}(\cdot)$ intersects

Figure 7: Infinite horizon economy with symmetric information



the root $p^{+}_{\theta,1}(\cdot)$, condition iv) implies it cannot also intersect the root $p^{-}_{\theta,1}(\cdot)$, even once. This is because whenever $p^{+}_{\theta,1}(\cdot)$ and $p_{\theta,2}(\cdot)$ intersect, the first intersection with $p^{-}_{\theta,1}(\cdot)$ would be from above, implying $|\partial p^{-}_{\theta,1}(V^*_\theta)/\partial V_\theta| < |\partial p_{\theta,2}(V^*_\theta)/\partial V_\theta|$, or a violation of condition iv). Hence, condition iv) establishes the global uniqueness of the solution. I now prove each of these claims.

To establish fact i), derive equation (43) w.r.t. V_θ, which yields the result immediately. For fact ii), derive the root $p^{-}_{\theta,1}(\cdot)$ in (44) twice, which gives

$$\frac{\partial p^{-}_{\theta,1}(\cdot)}{\partial V_\theta} = -\frac{R^2}{2\gamma r}[1+\frac{2(\sigma^{-2}+V_\theta)-k^2}{2((\sigma^{-2}+V_\theta)^2-k^2(\sigma_\theta^{-2}+V_\theta))^{1/2}}]<0, \quad \frac{\partial^2 p^{-}_{\theta,1}(\cdot)}{\partial V_\theta^2} = \frac{R^2}{2\gamma r}[\frac{k^2\sigma_\theta^{-2}\rho_\theta/R+k^4/4}{((\sigma^{-2}+V_\theta)^2-k^2(\sigma_\theta^{-2}+V_\theta))^{3/2}}]>0,$$

$$\text{and} \qquad \lim_{V_\theta\to\infty}\frac{\partial p^{-}_{\theta,1}(\cdot)}{\partial V_\theta} = \frac{-R^2}{\gamma r}.$$

Similarly, for fact iii) we derive equation $p_{\theta,2}(\cdot)$ in (45) twice to find

$$\frac{\partial p_{\theta,2}(\cdot)}{\partial V_\theta} = -\frac{R}{\gamma r}[1+\frac{\sigma^{-2}\sigma_\theta^{-2}\rho_\theta(1-\rho_\theta)/R}{(\sigma^{-2}+V_\theta)^2}]<0, \quad \frac{\partial^2 p_{\theta,2}(\cdot)}{\partial V_\theta^2} = \frac{R}{\gamma r}[\frac{2\sigma^{-2}\sigma_\theta^{-2}\rho_\theta(1-\rho_\theta)/R}{(\sigma^{-2}+V_\theta)^3}]>0, \text{ and} \lim_{V_\theta\to\infty}\frac{\partial p_{\theta,2}(\cdot)}{\partial V_\theta} = \frac{-R}{\gamma r}.$$

Finally, to establish fact iv), let's define the following objects:

$$a(V_\theta) \equiv \frac{\sigma_\theta^{-2}\rho_\theta(1-\rho_\theta)/R}{\sigma^{-2}+V_\theta}, \quad b(V_\theta) \equiv \frac{V_\theta}{\sigma^{-2}+V_\theta}, \quad c(V_\theta) \equiv 1-k^2\frac{\sigma_\theta^{-2}+V_\theta}{(\sigma^{-2}+V_\theta)^2}, \quad \text{and} \quad d(V_\theta) \equiv \frac{\rho_\theta\sigma_\theta^{-2}}{\sigma_\theta^{-2}+V_\theta}.$$

Intersection of equations $p^{-}_{\theta,1}(\cdot)$ and $p_{\theta,2}(\cdot)$ at $V_\theta = V^*_\theta$ implies we can write

$$-\frac{R^2}{2\gamma r}(\sigma^{-2}+V^*_\theta)(1+c(V^*_\theta)^{1/2}) = -\frac{R}{\gamma r}V^*_\theta(1+a(V^*_\theta)), \text{ or } c(V^*_\theta) = \frac{4}{R}b(V^*_\theta)(1+a(V^*_\theta))(\frac{b(V^*_\theta)(1+a(V^*_\theta))}{R}-1)+1. \tag{46}$$

We now compare the slopes between equations $p^{-}_{\theta,1}(\cdot)$ and $p_{\theta,2}(\cdot)$ at $V_\theta = V^*_\theta$. Manipulation of $\partial p^{-}_{\theta,1}(V^*_\theta)/\partial V_\theta$

using the equality condition in (46) allows to write (omitting the dependence on V_θ)

$$\partial p^{-}_{\theta,1}(\cdot)/\partial V_\theta|_{V_\theta=V^*_\theta} = -\frac{1}{\gamma r}\frac{[b^2(1+a)^2 + b(1+a)(1-\frac{b(1+a)}{R})d]}{\frac{2}{R}b(1+a)-1}, \tag{47}$$

while the slope of equation $p_{\theta,2}(\cdot)$ takes the form

$$\partial p_{\theta,2}(\cdot)/\partial V_\theta|_{V_\theta=V^*_\theta} = -\frac{R}{\gamma r}(1+a(1-b)). \tag{48}$$

Proving condition iv) then amounts to establishing

$$\frac{[b^2(1+a)^2 + b(1+a)(1-\frac{b(1+a)}{R})d]}{\frac{2}{R}b(1+a)-1} > R(1+a(1-b)). \tag{49}$$

but notice that we can rewrite $a(\cdot)$ as $a = \rho_\theta(1-\rho_\theta)(1-b)/(R-\rho_\theta)$. Using this last transformation in (49) gives after some (tedious) manipulation the desired result. This completes the proof.

Proof of Proposition 3

I will prove the proposition for the full-information economy first ($\mu = 0$), and then for the no-information case ($\mu = 1$) (I drop the information superscripts in what follows). The proof consists in three steps. In step 1, I write the demand of agent i as a strategy $X_{t,i} = X(\phi_i; F_t, P_t)$, where $\phi_i \equiv [\phi_{0,i} \;\; \phi_{F,i} \;\; \phi_{P,i}]$ is a vector that loads on a constant, the persistent component F_t, and the price P_t. Starting form a proposed equilibrium price function $P_t = P \cdot [1 \;\; F_t \;\; \theta_t]'$, this implies a correspondence $\phi_i = g(P)$ between the price coefficients vector $P \equiv [p_0 \;\; p_F \;\; p_\theta]$, and the vector of best-response coefficients of investor i, ϕ_i, which is derived from individual optimization. In step 2, we write the equilibrium price vector P as a function of the equilibrium strategies played by investors. Because all active investors are identical (1 year old, informed), we impose symmetry in their strategies; $\phi_i = \phi$, and then aggregate across agents to find $P = h(\phi)$. Here, the function $h(\cdot)$ is implied by the market-clearing condition which maps agents (symmetric) strategies into equilibrium price coefficients. In step 3, we write the fixed-point problem implied by the previous steps as $\phi = g(h(\phi))$: that is, the strategy of players is a function of the equilibrium price ($\phi = g(P)$), which in turn is a function of the equilibrium strategies ($P = h(\phi)$). We then study the shape of the strategy of individual i as a function of the equilibrium strategies played by all other investors: $\phi_i(\phi)$, which by the previous argument is given by $\phi_i = g(h(\phi))$. In particular, we define an equilibrium to be stable in the *best-response sense* whenever whenever $|\partial\phi_{m,i}/\partial\phi_m| < 1$, for $m = 0, F, P$. Here we will analyze the stability of each strategy coefficient, ϕ_0, ϕ_F, and ϕ_P.[19]

Step 1: Let $P \equiv [p_0 \;\; p_F \;\; p_\theta]$ be an equilibrium price vector. We can represent the demand of investor i as the following linear strategy: $X_{t,i} = \phi_{0,i} + \phi_{F,i}F_t + \phi_{P,i}P_t$. Optimization for $T = 2$ gives $X_{t,i} =$

[19]I am indebted to Masahiro Watanabe for the proof that follows. The previous version of this paper included a different proof, which while reaching similar conclusions, was not directly analyzing the impact on best-response functions from perturbations in the conjectured equilibrium.

$\mathbb{E}[Q_{t+1}|\Omega_t^I]/(\gamma\mathbb{V}[Q_{t+1}|\Omega_t^I])$. Since the excess dollar return can be written as $Q_{t+1} = D_{t+1} + P_{t+1} - R \cdot P_t$, we can write the first two moments as a function of the price coefficients through

$$
\begin{aligned}
\mathbb{E}[Q_{t+1}|\Omega_t^I] &= (1+p_F)F_0 + p_0(1-\rho_\theta) + p_\theta\theta_0 + [\rho_F + p_F(\rho_F - \rho_\theta)]F_t + (\rho_\theta - R)P_t \\
\mathbb{V}[Q_{t+1}|\Omega_t^I] &= (p_\theta)^2\sigma_\theta^2 + (1+p_F)^2\sigma_F^2 + \sigma_D^2.
\end{aligned}
$$

The strategies' coefficients must then satisfy

$$
\begin{aligned}
\phi_{0,i} &= \frac{p_0(1-\rho_\theta) + p_\theta\theta_0 + (1+p_F)F_0}{\gamma[(p_\theta)^2\sigma_\theta^2 + (1+p_F)^2\sigma_F^2 + \sigma_D^2]}, && (50)\\
\phi_{F,i} &= \frac{\rho_F + p_F(\rho_F - \rho_\theta)}{\gamma[(p_\theta)^2\sigma_\theta^2 + (1+p_F)^2\sigma_F^2 + \sigma_D^2]}, \text{ and} && (51)\\
\phi_{P,i} &= \frac{\rho_\theta - R}{\gamma[(p_\theta)^2\sigma_\theta^2 + (1+p_F)^2\sigma_F^2 + \sigma_D^2]}, && (52)
\end{aligned}
$$

which defines the mapping between price coefficients, and the best-response functions o individual i; $\phi_i = g(P)$.

Step 2: we now impose symmetry in equilibrium strategies, $\phi_i = \phi$. Aggregating across traders and imposing market clearing gives

$$
\begin{aligned}
\theta_t &= \frac{X_t}{2}, \quad \text{or} \quad P_t = \frac{-\phi_0 - \phi_F F_t + 2\theta_t}{\phi_P}, && (53)\\
\text{implying } p_0 &= -\frac{\phi_0}{\phi_P}, \quad p_F = -\frac{\phi_F}{\phi_P}, \text{ and } p_\theta = \frac{2}{\phi_P}, && (54)
\end{aligned}
$$

which defines the correspondence between price coefficients and symmetric strategies, $P = h(\phi)$.

Step 3: we now analyze the fixed-point defined by the functional $\phi = g(h(\phi))$. Note that from investor's i perspective, her best-response function (strategy) satisfies $\phi_i = g(h(\phi))$. That is, given that all other investors play strategy ϕ, this implies an equilibrium price function $P = h(\phi)$, which in turn defines the optimal response $\phi_i = g(P) = g(h(\phi))$. We now analyze, for the case of individual i, the derivative of each of her strategy coefficients with respect to the corresponding aggregate (symmetric) coefficients, $\partial\phi_{0,i}(\cdot)/\partial\phi_0$, $\partial\phi_{F,i}(\cdot)/\partial\phi_F$, and $\partial\phi_{P,i}(\cdot)/\partial\phi_P$.

We begin with the intercept: $\phi_{0,i}$. From equation (50), (52), and (53) we have

$$
\phi_{0,i}(\phi_0) = \frac{-\phi_0(1-\rho_\theta)/\phi_P + (1+p_F)F_0 + p_\theta\theta_0}{\gamma\mathbb{V}[Q_{t+1}|\Omega_t^I]}; \; \phi'_{0,i}(\cdot) = -\frac{1-\rho_\theta}{R-\rho_\theta}, \text{ hence } |\phi'_{0,i}(\cdot)| < 1.
$$

Turning now to the fundamental coefficient, we have (again from (50), (52), and (53))

$$
\phi_{F,i}(\phi_F) = \frac{\rho_F + p_F(\rho_F - \rho_\theta)}{\gamma\mathbb{V}[Q_{t+1}|\Omega_t^I]} = \frac{\rho_F\phi_P - \phi_F(\rho_F - \rho_\theta)}{\rho_\theta - R}; \; \phi'_{F,i}(\cdot) = \frac{\rho_F - \rho_\theta}{R - \rho_\theta}, \text{ hence } |\phi'_{F,i}(\cdot)| < 1.
$$

We now analyze the stability of the two equilibrium values of ϕ_P, associated with the low and high volatility equilibrium. First, note that from (39), $(1+p_F)\rho_F - Rp_F = 0$. Using this in (54), we can rewrite (52) as:

$$\phi_P = \frac{\rho_\theta - R}{\gamma[\frac{4}{\phi_P^2}\sigma_\theta^2 + (\frac{R}{R-\rho_F})^2\sigma_F^2 + \sigma_D^2]}, \quad \text{or more conviniently} \tag{55}$$

$$1/\phi_P = \frac{\gamma[\frac{4}{\phi_P^2}\sigma_\theta^2 + (\frac{R}{R-\rho_F})^2\sigma_F^2 + \sigma_D^2]}{\rho_\theta - R}. \tag{56}$$

Momentarily, we will use the two roots of equation (56), $1/\phi_{P,1}$ and $1/\phi_{P,2}$, which are given by

$$\frac{\rho_\theta - R}{8\gamma\sigma_\theta^2} \pm \sqrt{\frac{(\rho_\theta - R)^2 - 16\gamma^2\sigma_\theta^2[(\frac{R}{R-\rho_F})^2\sigma_F^2 + \sigma_D^2]}{(\rho_\theta - R)^2}} \cdot \frac{\rho_\theta - R}{8\gamma\sigma_\theta^2} \tag{57}$$

We now calculate the slope $\phi'_{i,P}(\cdot)$ by taking the appropriate derivative in equation (55). This gives

$$\phi'_{i,P}(\cdot) = \frac{1}{\phi_P}\frac{8\gamma\sigma_\theta^2}{\rho_\theta - R}, \tag{58}$$

but $\frac{\rho_\theta - R}{8\gamma\sigma_\theta^2}$ equals the average of the two roots of (56), hence

$$\phi'_{i,P}(\cdot) = \frac{1/\phi_P}{(1/\phi_{P,1} + 1/\phi_{P,2})/2}.$$

Notice that from (55), $\phi_P < 0$. It follows that the root with the more negative value $1/\phi_P$ constitutes the unstable equilibrium, and the one with the less negative value, the stable one. Because we just showed in equation (54) that $p_\theta = 2/\phi_P$, it follows that the high volatility equilibrium (the one with the more negative value of p_θ) is unstable, while the low volatility equilibrium is stable.

I now prove equivalent results for the no-information economy ($\mu = 1$). For brevity, I will only highlight the results that are different between these cases. First, the demand vector is now given by $X_{t,i} = \phi_{0,i} + \phi_{F,i}\hat{F}_t^U + \phi_{P,i}P_t$, and the price is given by $P_t = P \cdot [1 \;\; \hat{F}_t^U \;\; \theta_t]'$, where $P \equiv [p_0 \;\; \hat{p}_F \;\; p_\theta]$, and $\hat{F}_t^U$ is the expectation of the uninformed investors about F_t. Notice in particular that the uninformed beliefs in the no-information case satisfy i) $\hat{\theta}_t^U = \theta_t$ (the supply is known, since the price is observed and there are no other agents with superior information), ii) $\hat{F}_{t+1}^U = \rho_F\hat{F}_t^U + k(D_{t+1} - F_0 - \rho_F\hat{F}_t^U)$. Here, $k = (\rho_F^2\mathbb{O}_F + \sigma_F^2)/(\rho_F^2\mathbb{O}_F + \sigma_F^2 + \sigma_D^2)$, and $\mathbb{O}_F = Var[F_t|\Omega_t^U]$. With these minor changes, its easy to show that the corresponding expressions for the uninformed investors' strategies are given by:

$$\phi_0 = \frac{p_0(1-\rho_\theta) + p_\theta\theta_0 + (1+\hat{p}_F)F_0}{\gamma\mathbb{V}[Q_{t+1}|\Omega_t^U]}, \quad \phi_F = \frac{\rho_F + \hat{p}_F(\rho_F - \rho_\theta)}{\gamma\mathbb{V}[Q_{t+1}|\Omega_t^U]}, \quad \phi_P = \frac{\rho_\theta - R}{\gamma\mathbb{V}[Q_{t+1}|\Omega_t^U]}, \tag{59}$$
$$\text{with} \quad \mathbb{V}[Q_{t+1}|\Omega_t^U] = p_\theta^2\sigma_\theta^2 + \mathbb{O}_F(1+\hat{p}_F k\rho_F)^2 + \sigma_F^2(1+\hat{p}_F k)^2 + \sigma_D^2(1+\hat{p}_F k)^2.$$

From these results, the proofs of the stability of ϕ_0 and ϕ_F are immediate. For the case of ϕ_P, the roots of the inverted equation $1/\phi_P$ are now given by

$$\frac{\rho_\theta - R}{8\gamma\sigma_\theta^2} \pm \sqrt{\frac{(\rho_\theta - R)^2 - 16\gamma^2\sigma_\theta^2[(\frac{(R-\rho_F(1-k))}{R-\rho_F})^2(\sigma_F^2 + \sigma_D^2) + \mathbb{O}_F(\frac{(R-\rho_F(1-k\rho_F))}{R-\rho_F})^2]}{(\rho_\theta - R)^2}} \cdot \frac{\rho_\theta - R}{8\gamma\sigma_\theta^2}.$$

While this expression differs from (57) because $\mathbb{V}[Q_{t+1}|\Omega_t^U]$ is different for the uninformed investors, it is still the case that $\phi'_{i,P}(\cdot) = \frac{1/\phi_P}{(1/\phi_{P,1}+1/\phi_{P,2})/2}$, as can be seen by taking the appropriate derivative of $\phi_{i,P}$ in (59). This completes the proof.

Volatility Decomposition

For the asymmetric information economy, we can write $P_t = P \cdot M_t$, where $P = [p_0 \;\; p_1 \;\; p_2 \;\; p_3]$ is defined in equation (30). The unconditional volatility of price changes is then $\mathbb{V}(\Delta P_t) = P \cdot \mathbb{V}[\Delta M_t] \cdot P'$, where

$$\mathbb{V}[\Delta M_t] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \cdot & \frac{2\sigma_F^2}{1+\rho_F} & 0 & \frac{(2-\rho_U-\rho_F)b_U\mathcal{V}i_F'}{1-\rho_U\rho_F} \\ \cdot & \cdot & \frac{2\sigma_\theta^2}{1+\rho_\theta} & \frac{(2-\rho_U-\rho_\theta)b_U\mathcal{V}i_\theta'}{1-\rho_U\rho_\theta} \\ \cdot & \cdot & \cdot & \frac{2b_U\mathcal{V}b_U'}{1+\rho_U} \end{bmatrix}, \tag{60}$$

is the symmetric covariance matrix of the changes in the state variables defined by M_t, $i_F = [0\ 1\ 0\ 0]$, and $i_\theta = [0\ 0\ 1\ 0]$. Because the vector b_U has zero loadings on the endogenous forecast error $\tilde{F}_t^U$, the variance of price changes can be decomposed into the exogenous shocks in ϵ_t. We can then write the total variance of price changes as $\mathbb{V}[\Delta M_t] = V^D + V^F + V^\theta$, where

$$\begin{aligned} V^D &= \frac{2p_3^2 b_U\mathcal{V}i_{1,1}b_U'}{1+\rho_U}, \;\; V^F = \frac{2p_1^2\sigma_F^2}{1+\rho_F} + \frac{2p_3^2 b_U\mathcal{V}i_{2,2}b_U'}{1+\rho_U} + \frac{2p_1p_3(2-\rho_U-\rho_F)b_U\mathcal{V}i_F'}{1-\rho_U\rho_F}, \\ \text{and} \;\; V^\theta &= \frac{2p_2^2\sigma_\theta^2}{1+\rho_\theta} + \frac{2p_3^2 b_U\mathcal{V}i_{3,3}b_U'}{1+\rho_U} + \frac{2p_2p_3(2-\rho_U-\rho_\theta)b_U\mathcal{V}i_\theta'}{1-\rho_U\rho_\theta}. \end{aligned} \tag{61}$$

For the symmetric information cases, we perform the same analysis with the following minor modifications. When all investors are informed ($\mu = 0$), we simply replace $p_3 = 0$. When all investors are uninformed ($\mu = 1$), the price takes the form $P_t = [p_0 \quad p_1 \quad p_2] \cdot M_t^U$. Writing $F_t^U = F_t + \tilde{F}_t^U$, we then have $P_t = p_0 + p_1 F_t + p_2\theta_t + p_1\tilde{F}_t^U$. We then use p_1 in place of p_3, and the results in equation (61) go through. The main difference is that now investors update beliefs about F_t using dividends only. This is reflected in a different vector b_U, error persistence ρ_U, and steady-state value of $\mathbb{V}[F_t|\Omega_t^U]$ entering matrix $\mathcal{V}$.

Trading Volume Measures

For a normally distributed variable $x_t \sim \mathcal{N}(\bar{x}, \sigma_x^2)$, we apply the standard calculation of moments for folded normal distributions in order to compute the expectation of absolute values.[20] In particular, $\mathbb{E}[|x_t|] = \sigma_x\sqrt{2/\pi} \cdot exp\{-\bar{x}^2/2\sigma_x^2\} + \bar{x}\,(1 - \Phi\,(-\bar{x}/\sigma_x))$. We begin with absolute individual trade flows,

[20] See for instance, Ahsanullah, Kibria, and Shakil, (2014).

$AIF^i_{j,t}$. For informed investors, positions on the asset satisfy (from equation (35)) $X^I_{j,t} = G^I_{j,t} \cdot M_t$, with $G^I_{j,t} \equiv \frac{A_Q - h^I_{j+1}}{\alpha_{j+1}\Gamma^I_{j+1}}$ a (1x4) vector. We can write the change in demand as $\Delta X^I_{j,t} \equiv X^I_{j,t} - X^I_{j-1,t-1} = G^I_{j,t}\Delta M_t + (\Delta G^I_{j,t})M_{t-1}$. It follows that $\Delta X^I_{j,t} \sim \mathcal{N}(\mu^I_{x,j}, (\sigma^I_{x,j})^2)$, where $\mu^I_{x,j} = \Delta G^I_{j,t}[1 \quad \bar{F} \quad \bar{\theta} \quad 0]'$, and $(\sigma^I_{x,j})^2 = G^I_{j,t}\mathbb{V}[\Delta M_t](G^I_{j,t})' + (\Delta G^I_{j,t})\mathbb{V}[M_{t-1}](\Delta G^I_{j,t})' + G^I_{j,t}Cov[\Delta M_t, M_{t-1}](\Delta G^I_{j,t})'$. Matrix $\mathbb{V}[\Delta M_t]$ is given by expression (60). The (symmetric) steady state covariance matrix of the state vector M_{t-1}, $\mathbb{V}[M_{t-1}]$, and $Cov[\Delta M_t, M_{t-1}]$, can be similarly calculated as

$$\mathbb{V}[M_{t-1}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ \cdot & \frac{\sigma_F^2}{1-\rho_F^2} & 0 & \frac{b_U \mathcal{V} i'_F}{1-\rho_U\rho_F} \\ \cdot & \cdot & \frac{\sigma_\theta^2}{1-\rho_\theta^2} & \frac{b_U \mathcal{V} i'_\theta}{1-\rho_U\rho_\theta} \\ \cdot & \cdot & \cdot & \frac{b_U \mathcal{V} b'_U}{1-\rho_U^2} \end{bmatrix}, \quad Cov[\Delta M_t, M_{t-1}] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{-\sigma_F^2}{1+\rho_F} & 0 & \frac{-(1-\rho_F) b_U \mathcal{V} i'_F}{1-\rho_U\rho_F} \\ 0 & 0 & \frac{-\sigma_\theta^2}{1+\rho_\theta} & \frac{-(1-\rho_\theta) b_U \mathcal{V} i'_\theta}{1-\rho_U\rho_\theta} \\ 0 & \frac{-(1-\rho_U) b_U \mathcal{V} i'_F}{1-\rho_U\rho_F} & \frac{-(1-\rho_U) b_U \mathcal{V} i'_\theta}{1-\rho_U\rho_\theta} & -\frac{b_U \mathcal{V} b'_U}{1+\rho_U} \end{bmatrix}.$$

For the uninformed traders, demands are given by $X^U_{j,t} = G^U_{j,t} \cdot M^U_t$, where $G^U_{j,t} \equiv \frac{A^U_Q - h^U_{j+1}}{\alpha_{j+1}\Gamma^U_{j+1}}$ is a (1x3) vector. The first two moments of the change in demands are given by $\mu^U_{x,j} = \Delta G^U_{j,t}[1 \quad \bar{F} \quad \bar{\theta}]'$, and $(\sigma^U_{x,j})^2 = G^U_{j,t}\mathbb{V}[\Delta M^U_t](G^U_{j,t})' + (\Delta G^U_{j,t})\mathbb{V}[M^U_{t-1}](\Delta G^U_{j,t})' + G^U_{j,t}Cov[\Delta M^U_t, M^U_{t-1}](\Delta G^U_{j,t})'$. Writing $\theta^U_t = \theta_t + \tilde{\theta}^U_t = \theta_t - \lambda^{-1}\tilde{F}^U_t$, we can express the state vector of the uninformed as $M^U_t = [1 \quad F_t + \tilde{F}^U_t \quad \theta_t - \lambda^{-1}\tilde{F}^U_t]'$, and $\Delta M^U_t = [0 \quad \Delta F_t + \Delta\tilde{F}^U_t \quad \Delta\theta_t - \lambda^{-1}\Delta\tilde{F}^U_t]'$. The covariance matrices can now be calculated as before,

$$\mathbb{V}[\Delta M^U_t] = \begin{bmatrix} 0 & 0 & 0 \\ \cdot & \frac{2\sigma_F^2}{1+\rho_F} + \frac{2b_U \mathcal{V} b'_U}{1+\rho_U} + \frac{2(2-\rho_U-\rho_F) b_U \mathcal{V} i'_F}{1-\rho_U\rho_F} & -\frac{\lambda^{-1}(2-\rho_U-\rho_F) b_U \mathcal{V} i'_F}{1-\rho_U\rho_F} - \frac{2\lambda^{-1} b_U \mathcal{V} b'_U}{1+\rho_U} + \frac{(2-\rho_U-\rho_\theta) b_U \mathcal{V} i'_\theta}{1-\rho_U\rho_\theta} \\ \cdot & \cdot & \frac{2\sigma_\theta^2}{1+\rho_\theta} + \frac{2\lambda^{-2} b_U \mathcal{V} b'_U}{1+\rho_U} - \frac{2\lambda^{-1}(2-\rho_U-\rho_\theta) b_U \mathcal{V} i'_\theta}{1-\rho_U\rho_\theta} \end{bmatrix},$$

$$\mathbb{V}[M^U_{t-1}] = \begin{bmatrix} 0 & 0 & 0 \\ \cdot & \frac{\sigma_F^2}{1-\rho_F^2} + \frac{b_U \mathcal{V} b'_U}{1-\rho_U^2} + \frac{2b_U \mathcal{V} i'_F}{1-\rho_U\rho_F} & \frac{b_U \mathcal{V} i'_\theta}{1-\rho_U\rho_\theta} - \frac{\lambda^{-1} b_U \mathcal{V} i'_F}{1-\rho_U\rho_F} - \frac{\lambda^{-1} b_U \mathcal{V} b'_U}{1-\rho_U^2} \\ \cdot & \cdot & \frac{\sigma_\theta^2}{1-\rho_\theta^2} + \frac{\lambda^{-2} b_U \mathcal{V} b'_U}{1-\rho_U^2} - \frac{2\lambda^{-1} b_U \mathcal{V} i'_\theta}{1-\rho_U\rho_\theta} \end{bmatrix},$$

$$Cov[\Delta M_t, M_{t-1}] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \frac{-\sigma_F^2}{1+\rho_F} \frac{-(2-\rho_F-\rho_U) b_U \mathcal{V} i'_F}{1-\rho_U\rho_F} - \frac{b_U \mathcal{V} b'_U}{1+\rho_U} & \frac{\lambda^{-1}(1-\rho_F) b_U \mathcal{V} i'_F}{1-\rho_U\rho_F} - \frac{(1-\rho_U) b_U \mathcal{V} i'_\theta}{1-\rho_U\rho_\theta} + \frac{\lambda^{-1} b_U \mathcal{V} b'_U}{1+\rho_U} \\ 0 & \frac{-(1-\rho_\theta) b_U \mathcal{V} i'_\theta}{1-\rho_U\rho_\theta} + \frac{\lambda^{-1}(1-\rho_U) b_U \mathcal{V} i'_F}{1-\rho_U\rho_F} + \frac{\lambda^{-1} b_U \mathcal{V} b'_U}{1+\rho_U} & \frac{-\sigma_\theta^2}{1+\rho_\theta} + \frac{\lambda^{-1}(2-\rho_\theta-\rho_U) b_U \mathcal{V} i'_\theta}{1-\rho_U\rho_\theta} - \frac{\lambda^{-2} b_U \mathcal{V} b'_U}{1+\rho_U} \end{bmatrix}.$$

With these expressions, we can solve for $\mathbb{E}[AIF^i_{j,t}]$, as well as for expected aggregate group trade flows, $\mathbb{E}[AGF^i_{j,t}]$, for $i \in \{U, I\}$. This allows the calculation of expected volume, $EV = 0.5(\mu\mathbb{E}[AGF^U_t] + (1-\mu)\mathbb{E}[AGF^I_t] + \mathbb{E}[|\Delta\theta_t|])$, by applying the formula for the mean absolute value. To compute net informational flows, we simply add demands within groups before applying the absolute value. Let the average demand vector of the uninformed and informed investors be defined as $\bar{G}^U_t \equiv \sum_{j=1}^T G^U_{j,t}/T$ and $\bar{G}^I_t \equiv \sum_{j=1}^T G^I_{j,t}/T$. The first two moments of net uninformed flows are given by $\bar{G}^U_t M^U_t \sim \mathcal{N}(0, (\sigma^U_x)^2)$, with $(\sigma^U_x)^2 = \bar{G}^U_t\mathbb{V}[\Delta M^U_t](\bar{G}^U_t)'$. Similarly, for the informed investors $\bar{G}^I_t M^I_t \sim \mathcal{N}(0, (\sigma^I_x)^2)$, with $(\sigma^I_x)^2 = \bar{G}^I_t\mathbb{V}[\Delta M_t](\bar{G}^I_t)'$. Using σ^I_x and σ^U_x, we can calculate the expected net informational flow for each group (note that they are not equal since there is also the trade flow coming from supply). This allows the calculation of NIV.

Numerical solution method:

I now describe the numerical methodology used to find the equilibrium coefficients in the price function of equation (7). The problem consists in finding the (possibly multiple) price coefficients that satisfy the fixed point representation in expression (38). As initial price coefficients, I use a matrix whose rows correspond to different starting values for the price vector, where the only element changing across rows is the coefficient associated with the random supply, p_θ. Due to its convergence properties, the low volatility equilibrium is straightforward to find as initial guesses using relatively small negative values of p_θ (in absolute magnitude) quickly converge to the the same fixed point price vector. Actually, any guess for p_θ whose absolute value is below the corresponding solution for the HVE, will converge to the LVE solution.

The complications arise when trying to find the price coefficients in the unstable price vector (the HVE equilibrium). To find it I postulate successively decreasing (more negative) supply coefficients as initial guesses. If the iteration brings the price coefficient to the low-volatility equilibrium, this means that the conjectured supply coefficient is still not negative enough. Beyond some threshold, the postulated value of p_θ is too negative and the iteration diverges. This implies that the second equilibrium value of p_θ must lie in between the last initial guess that produced a convergence, and the first guess that produced the divergence. I then zoom into this region, creating a new matrix of price vectors that span this narrower range of price coefficients.

I then repeat the process, every time defining a new range of price coefficients between the last converging and first diverging row vector of the initial matrix. After a few iterations, the range of values where the second equilibrium p_θ lies can be made arbitrarily narrow, providing an arbitrarily close approximation to the true equilibrium value.

This routine works well for solving the steady state HVE, since here there is one coefficient to be found and the "zoom in" numerical approach just discussed remains manageable. When one needs to find a complete path for the equilibrium outside of the steady state (as in the applications discussed in section 5), the numerical problem becomes unmanageable. To understand why, let's imagine we correctly guess with arbitrarily high precision the equilibrium price coefficients for all, but a single period, in the economy's path outside of the steady state. Because the HVE is unstable, this small mistake in the equilibrium of a single period will make the (correctly guessed) price coefficients of the other periods move further away in the next iteration. Hence, successive iterations of the routine make the economy diverge. In practical terms, one would have to guess with arbitrarily high precision the complete path of the economy for the equilibrium to be found.