Security Design in a Production Economy with Flexible Information Acquisition*

Ming Yang
Duke University

Yao Zeng
Harvard University

This Version: June, 2013
First Draft: June, 2012

Abstract

This paper investigates security design in a production economy through the lens of information acquisition. It highlights investors as information experts, who are capable of acquiring information on the market prospects of entrepreneurs' technologies and then help screen projects through their financing decisions. Thus, real production depends on information production, while these two are separately performed by the entrepreneur and the investor, which constructs a friction. The optimal security reduces this friction. The model predicts two optimal securities and a new pecking order of them. Standard debt is optimal when the dependence of real production on information production is weak and thus the friction is not severe. More interestingly, convertible preferred stock is optimal when the dependence is strong and thus the friction is severe. Both optimal securities and the new pecking order are consistent with empirical evidence. A new approach, flexible information acquisition, enables us to specify securities on continuous states without distributional assumptions and deliver sharper predictions than previous literature.

Keywords: information acquisition, production economy, security design
JEL: D82, D86, G24, G32, L26

*Earlier versions of this paper have been circulated under the title, "Venture Finance under Flexible Information Acquisition." We thank Malcolm Baker, John Campbell, Emmanuel Farhi, Paolo Fulghieri, Diego Garcia (discussant), Simon Gervais, Barney Hartman-Glaser, Ben Hebert, Steven Kaplan, Arvind Krishnamurthy, Josh Lerner, Stephen Morris, Marcus Opp, Jonathan Parker, Raghu Rajan, Adriano Rampini, David Robinson, Hyun Shin, Andrei Shleifer, Alp Simsek, S. Viswanathan, and Michael Woodford; and seminar participants at Berkeley Haas, Duke Fuqua, Harvard, MIT, UNC Kenan-Flagler, Peking University, 2013 Finance Theory Group Workshop, 2013 Western Finance Association Annual Meeting, and 2013 Toulouse TIGER Forum for helpful comments.

1 Introduction

A typical setting of modern corporate finance is that an entrepreneur comes up with a business plan or technology and then gets finance from an investor to start a project. The entrepreneur is often viewed as an expert who is more informed than the investor about the project. Nevertheless, an important real aspect missed in this prevailing approach is that investors are often more capable of accessing projects' uncertain market prospects by their specific and local industry experience, and thus may help screen projects through their financing decisions. For instance, a medical professor who invents a new drug may seek venture finance for potential large scale production, and the venture capitalist acquires information to evaluate the market prospect of the proposed project and then decides whether to finance it. A chef who considers opening a new restaurant may go to a local bank for potential finance, and the bank may better acquire market information about the popularity of the proposed restaurant based on its past experience in financing various local restaurants. In these examples, the entrepreneur may initiate contracting by inviting an investor and proposing securities in exchange for finance, but the investor could be more informed than the entrepreneur by acquiring information and screening the project, which in turn shapes the entrepreneur's security design. As suggested by Tirole (2006), a major disadvantage of conventional corporate finance literature is that such information advantage by investors is largely neglected.[1] Our paper fills the blank by exploring why and how various projects are financed by issuing various securities, through uncovering the role of investors' information acquisition in the financing process.

Highlighting investors' ability of information acquisition, this paper studies security design in a production economy with friction. In our stylized model, an entrepreneur has exclusive access to a technology and could start a risky project, but she has no money for the initial investment. Nevertheless, she may attempt to finance the project through contracting with an investor. Importantly, the investor can acquire costly information about the project's uncertain future cash flow before making a financing decision. Only when the investor believes the project is good enough, can the project be financed and initiated. In other words, the investor does not only finance the entrepreneur's project but also helps to screen the project through information acquisition. Therefore, the real production and the production of information are separated in the economy, but the former depends on the latter, which constructs the fundamental friction in this production economy. The optimal security, which is written on the project's future cash flow, is of interest in this context, because it reduces the friction by reconciling two conflicting forces. On the one hand, it is necessary for the entrepreneur to reward the investor, because it induces

[1] Some exceptions are surveyed by Bond, Edmans and Goldstein (2012). However, none of these papers speaks to the context of security design.

the investor to acquire information more effectively, which leads to a higher social surplus. On the other hand, as the entrepreneur always shares the social surplus with the investor, she also wants to retain as much as possible. These two conflicting forces reflect a tension between the two agents, and the optimal security thus reconciles this tension. Especially, the shapes of the optimal securities in different circumstances also speak to the different roles of information acquisition, accordingly.

Our predictions shed lights on the relationships among optimal security design, information acquisition, and the pecking order theory (Myers and Majluf, 1984) in a production economy. We formulate the optimal securities that are used by various investors to finance various projects differing in nature. The entrepreneur's optimal securities in our model correspond to standard debt and convertible preferred stock in different circumstances, which further constructs new pecking orders along different dimensions. The friction of our production economy and the trade-off between the benefit and cost of information acquisition play crucial roles here. In finding these optimal securities, we do not have any restrictions on the space of feasible securities, and the payoff function is defined on continuous states with arbitrary distributions, as opposed to finite discrete states or continuous states with given distributional assumptions often seen in previous literature. Hence, the predictions are sharp and robust. Our predictions on optimal securities also help to bridge the security design literature and the classic pecking order theory. This is because our new pecking order of optimal securities comes from an optimization over a general security space, as opposed to an exogenously given set of securities like debt and equity.

When the dependence of real production on information production is weak, namely, the friction in the production economy is not severe, the optimal security is standard debt that does not induce the investor to acquire information. This case corresponds to the scenarios in which the project's ex-ante market prospect is clear and good enough relative to the required initial investment. Thus, the benefit of screening by information acquisition does not justify the information cost. Therefore, it is optimal to deter costly information acquisition by issuing a standard debt, which is the least information sensitive security as noted by conventional wisdom (Myers and Majluf, 1984; Gorton and Pennacchi, 1990; DeMarzo and Duffie, 1999). This is also consistent with the empirical evidence that many conventional small businesses and less transformational start-ups rely heavily on standard debt or traditional short-term fixed-income financial instruments for finance (for example, Petersen and Rajan, 1994; Kerr and Nanda, 2009).

In contrast, when the dependence of real production on information production is strong, namely, the friction in the production economy is severe, the optimal security turns out to be a participating convertible preferred stock that induces the investor to acquire information. This prediction is new to the security design literature, but it is closely in line with empirical facts, both qualitatively and quantitatively. As documented in Kaplan and Stromberg (2003), four fifths of contracts between real-world entrepreneurs and venture capitals are convertible preferred stocks, and half of them are participating convertible preferred stocks. The predicted multiple of the convertible preferred stock, defined as the ratio of its face value to the investor's initial investment, is always larger than one, which also matches the empirical documents in Lerner, Leamon and Hardymon (2012). It corresponds to the scenarios when the project's ex-ante market prospect is not good or clear enough relative to the required initial investment. Information here is more valuable as it updates the investor's perceived market prospect of the project, and thus may eventually help to screen in potentially good projects and screen out potentially bad projects. The conventional wisdom of expected net present value (NPV) criterion is also revised, as a project with negative expected NPV may also be financed after the investor acquires information. This is consistent with the screening role of investors that they employ their specific and local industry experience to screen projects. As a result, the entrepreneur is willing to compensate the investor to encourage such beneficial information acquisition. Especially, the entrepreneur wants to design such a security that pays generously in general, but differently across states, which encourages the investor to acquire more information to distinguish between different states. This also suggests that standard equity is not optimal because it pays too little in bad states, which again fits the reality that standard equity is the least used security in financing new projects (Lerner, Leamon and Hardymon, 2012).

In general, combining the predictions that debt is optimal when information acquisition is not worthwhile while convertible preferred stock is optimal when information acquisition is crucial, the optimality of debt and convertible preferred stock in different circumstances constructs new pecking orders along three empirical dimensions: the profitability of the project, the uncertainty of the project, and the difficulty to evaluate the project by the investor. These pecking orders are also consistent with empirical facts. We further perform comparative statics of the shapes of optimal securities along the three dimensions. These comparative statics speak to the fact that different projects are endogenously financed by different securities and potentially different types of investors. Accordingly, the role of information acquisition is different as well. The dependence of real production and information production play important roles in shaping the pecking orders and the comparative statics.

Our framework parsimoniously accommodates a variety of theoretical contexts of corporate finance as well as real-world scenarios of production and entrepreneurship. On the one hand, most importantly, we highlight the investor as an information expert. The acknowledgement of investors' information acquisition in production and entrepreneurship is dated back to Knight (1921) and Schumpeter (1942). Besides numerous modern anecdotal evidence (see Kaplan and

Lerner, 2010; Da Rin, Hellmann and Puri, 2011, for reviews), recent empirical literature (Kerr, Lerner and Schoar, 2011; Chemmanur, Krishnan and Nandy, 2012) have also acknowledged such screening effects of various investors. As the information cost in our model captures the investor's expertise in acquiring information, it allows our analysis to cover various types of investors, including venture capitals, angel investors, specialized or local banks, and other financial institutions. On the other hand, we highlight two aspects of the entrepreneur. First, the entrepreneur has access to the technology but is financially constrained. Second, the entrepreneur's human capital is not transferrable, which means the investor can neither buy the technology nor hire the entrepreneur as a worker, and the entrepreneur initiates contracting by designing and proposing the security. These settings fit with the argument in Rajan (2012) that the differentiation of entrepreneurs is important in the early stage of firms' life cycles, and we actually show that a project transfer is always not optimal even if it is physically feasible. This paper, to the best of our knowledge, is the first to explicitly investigate the interplay between information acquisition and security design in a production economy, and deliver specific predictions that are consistent with empirical evidence regarding the contracts between real-world entrepreneurs and investors (Sahlman, 1990; Gompers, 1999; Kaplan and Stromberg, 2003).

A comparison between production economy and exchange economy helps to highlight the essence of the relationship between security design and information acquisition. Yang (2012a) considers information acquisition in an exchange economy and focuses on the unique optimality of debt. In that model, a seller has an asset in-place and proposes an asset-backed security to a buyer to raise liquidity. The buyer can acquire information about the fundamental of the asset before her purchase decision, which leads to endogenous adverse selection. The buyer collects the most payoff-relevant information determined by the shape of the security, and thus the seller designs the security to induce the buyer to acquire information least harmful to the seller's interest. Debt is shown to be uniquely optimal in this case because it is least information sensitive and mitigates adverse selection to the greatest extent. A key feature driving the unique optimality of debt is that the seller's asset is already in-place. Importantly, in this exchange economy, the social surplus depends negatively on information acquisition. This is because first, information acquisition leads to endogenous adverse selection and thus creates illiquidity, and second, information is costly per se. As a result, to discourage information is desirable. On the contrary, in this paper, the entrepreneur's project can only be initiated if it is financed by the investor. Therefore, it is a production economy in which the net social surplus depends positively on information acquisition. In this case, adverse selection is no longer the focus. Instead, the entrepreneur wants to design such a security that encourages the investor to acquire information in favor of herself. Therefore, standard debt is no longer optimal when information acquisition is desirable.

A new concept, flexible information acquisition, plays an important role in revealing the relationship between security design and information acquisition. Security design interacts with the investor's attention allocation in different aspects of the underlying cash flow when acquiring information. It thus calls for a sufficiently flexible characterization of information acquisition to capture the potential variety of attention allocation. In acquiring information, for instance, a debt holder would allocate her attention to bad states as only the default risk matters, while an equity holder wants to pay more attention to good states as she benefits more from enjoying the upside payments. For an arbitrary security, the investor's incentive of information acquisition, in the sense of attention allocation, would be determined in such a way accordingly, and this in turn affects the entrepreneur's incentive to design the security. In characterizing these effects, the traditional approach of exogenous information asymmetry is inadequate. Moreover, recent models of information acquisition also fail to accommodate such flexibility of incentive, since they implicitly impose some rigid information acquisition technologies with restrictions on signals (see Veldkamp, 2011, for a review). Because of such rigidity, the interplay between information acquisition and security design cannot be fully analyzed. In contrast, our new approach following (Yang, 2012a,b) allows agents to choose not only how much, but also what kind of information to acquire. Concretely, flexible information acquisition employes rational inattention (Sims, 2003) as a foundation. But it switches the gear from macroeconomic decision problems to microeconomic strategic interactions as a new target, in which agents can actively use their information capacities to choose any information structure. It further renders our specific predictions on the shape of security possible, which cannot be achieved in previous literature.

Related Literature. Our model contributes to the literature of venture contract design by highlighting information acquisition as a new channel, which is highly relevant but overlooked in past theoretical research. Although contract and security design is the focus of modern research in innovation and entrepreneurial finance, existing theoretical literature mainly pay attention to the aspects of monitoring and moral hazard (Schmidt, 2003; Casamatta, 2003; Hellmann, Thomas, 2006), refinancing and staging of finance (Bergemann and Hege, 1998; Cornelli and Yosha, 2003; Repullo and Suarez, 2004), as well as allocation of control rights (Berglof, 1994; Hellmann, 1998; Kirilenko, 2001; de Bettignies, 2008), leaving the role of screening and information acquisition largely untouched. Also, existing models often fail to deliver consistent theoretical predictions with real-world securities between entrepreneurs and venture investors. For instance, as documented in Da Rin, Hellmann and Puri (2011), the most commonly used double moral hazard models are able to approximately predict the contracts between entrepreneurs and venture capitals, but these models cannot deliver the precise allocation of cash flow rights between the two parties.

Moreover, our model delivers clear implications on liquidity provision and the associated specific allocation of cash flow rights in various economies and financial markets. In this model, the aggregate risk is variable since it depends on the financing decision of the project in the production economy, so that the circumstance could also be viewed as a primary financial market. In contrast, the circumstance of Yang (2012a), where the aggregate risk is fixed, could be viewed as an exchange economy or a secondary financial market. Such comparison further helps us understand why different optimal securities arise in different circumstances. Earlier mechanism design literature pertaining to information gathering also highlight such comparison between different economies or financial markets and suggest that the value of information would differ accordingly (Cremer and Khalil, 1992; Cremer, Khalil and Rochet, 1998a,b). But these papers do not focus on the security design problem and cannot make specific predictions on the forms of contracts in different economies or financial markets. Together with Yang (2012a), this paper fills this gap by delivering specific forms of securities for liquidity provision in different circumstances.

This paper is also closely related to a series of theoretical work on information, screening and optimal security design. These models predict non-debt-like securities in various circumstances with information asymmetry (see Brennan and Kraus, 1987; Boot and Thakor, 1993; Nachman and Noe, 1994; Fulghieri and Lukin, 2001; Inderst and Mueller, 2006; Hennessy, 2009, 2011; Chakraborty and Yilmaz, 2011; Dow, Goldstein and Guembel, 2011; Fulghieri, Garcia and Hackbarth, 2012). Compared to those papers, most of which focus on ex-ante information asymmetry or rigid information acquisition, our model delivers clearer interaction between security design and the corresponding information asymmetry resulted from information acquisition. This enables us to capture the role of information more explicitly in characterizing the optimal security between entrepreneurs and investors. Also, most models in previous literature can only accommodate either discrete states or continuous states with given distributional assumptions. Thanks to the technique of flexible information acquisition, our framework enables us to model securities on continuous states with arbitrary distributions and thus to characterize the shapes of optimal securities more specifically. Through this line, our model solidifies the standard intuition of information sensitiveness that non-debt-like securities may encourage investors to acquire information and facilitate information aggregation.

The rest of the paper is organized as follows. Section 2 specifies the environment of the production economy and elaborates the concept of flexible information. The optimal securities are characterized and discussed in Section 3. Section 4 gives both theoretical and numerical results on our new pecking orders. Section 5 performs comparative statics on the optimal securities. In the final section, we conclude and discuss possible directions for further research. If otherwise noted, all proofs are attached in the appendix.

2 Model

We present our stylized model of a production economy, focusing on the interplay between financing risky projects and flexible information acquisition. This model does not aim to capture every aspect of production and finance. Instead, we make assumptions to highlight the key friction: dependence and separation of real production and information production.

2.1 Financing Entrepreneurial Production

Consider an economy with two dates, $t = 0, 1$, and a single consumption good. There are two agents: an entrepreneur and a deep-pocket investor, both risk neutral. Their utility function is the sum of consumptions over the two dates:

$$u = c_0 + c_1 \,,$$

where c_t denotes an agent's consumption at date t. We assume that the entrepreneur starts with zero initial wealth, while the deep-pocket investor has large endowment at date 0. In what follows we use subscripts E and I to indicate the entrepreneur and the investor, respectively.

We consider the technology owned by the entrepreneur and the finance of a risky project. To initiate the project at date 0, the technology requires an investment $k > 0$. If initiated, the project generates a non-negative verifiable random cash flow θ at date 1. The project cannot be initiated partially. Hence, the entrepreneur has to raise k, by selling a security to the investor at date 0. The payment of an asset-backed security at date 1 is a mapping $s : \mathbb{R}_+ \to \mathbb{R}_+$ such that $s(\theta) \in [0, \theta]$ for any θ. We only focus on the cash flow aspect of projects and securities.

We specify the processes of security design and information acquisition, both at date 0. The agents have a common prior Π on the future cash flow θ, if initiated. The entrepreneur designs the security, and then proposes a take-it-or-leave-it offer to the investor at price k. Facing the offer, the investor acquires information about the future cash flow θ in the manner of rational inattention (Sims, 2003; Woodford, 2008; Yang, 2012a,b), updates her belief on θ, and then decides whether to accept the offer. This characterizes the process of flexible information acquisition, where the information acquired is measured by reduction of entropy. The information cost per unit reduction of entropy is μ. We will elaborate flexible information acquisition in the next subsection.

The implicit assumptions in the settings above are justified to capture the key features in financing entrepreneurial production, especially from a perspective of information acquisition. First, the entrepreneur can only undertake the risky project if she gets financing from the investor. This is consistent with earlier evidence that entrepreneurs are often financially constrained (Evans and Jovanovic, 1989; Holtz-Eakin, Joulfaian and Rosen, 1994). Even in mature firms, managers seek for outside finance since the internal capital market often does not function well for new risky projects (Stein, 1997; Scharfstein and Stein, 2000). Second, more importantly, the investor is able to acquire information and thus screen the project through her financing decision. This point not only accounts for the empirical evidence that investors screen projects by their information expertise, but also stands our framework out from most previous security design literature that feature the entrepreneur's exogenous information advantage. Taking these two points together leads to the dependence and separation of real production and information production, which is the key friction in our framework.

It is worth highlighting what aspects of the finance in a production economy are abstracted away from the model, and to what extent these aspects affect our work. First, to focus on information acquisition and screening, we put moral hazard aside. To ignore moral hazard is a general practice in the security design literature, especially when information asymmetry is highlighted (see DeMarzo and Duffie, 1999; DeMarzo, 2005). Second, we assume that the entrepreneur's human capital is inalienable, so that she has the bargaining power to design a security and a project transfer is impossible. Together with the differentiation argument in Rajan (2012), this assumption also broadly corresponds to the earlier incomplete contract literature which suggest the ownership go to the entrepreneur when innovative firms are young (Aghion and Tirole, 1994). In a later section we will formally show that even if the project is transferrable, to transfer the project at any fixed price is always not optimal. Moreover, in the Appendix we discuss a general allocation of bargaining powers between the two agents and we show that it does not impact our main results. Third, the bargaining is abstracted as a take-it-or-leave-it offer from its reality. This implies that the allocation of control rights is not our focus. Fourth, we do not model staging of finance, and thus we interpret the cash flow θ in our model as an ex-post one, which already takes the consequence of investors' exiting into account. Last, investors are unlikely to be risk neutral in reality. Nevertheless, the assumption of risk neutrality enables us to first focus on screening as opposed to risk sharing, which is of less interest for our purpose. Actually, our results would be only strengthened if we take investors' risk aversion into account.

2.2 Flexible Information Acquisition

We model the way through which the investor acquires information and screens the project by flexible information acquisition (Yang, 2012a).[2] As we will see, it allows us to deliver sharp predictions on optimal securities over continuous states without any distributional assumptions, which stand our paper out of most existing security design literature. Fundamentally, the entrepreneur in our framework is able to design the security in an arbitrary way, which calls for an equally

[2]For more detailed justification of this environment, see Woodford (2008) and Yang (2012a,b).

flexible account of information acquisition to capture the interaction between the shape of securities and the information to be acquired. This goal is hard to achieve through classic information acquisition technologies.

The key of flexible information acquisition is that it not only captures how much but also what aspects of information that an agent acquires. Consider an agent who chooses a binary action $a \in \{0,1\}$ and receives a payoff $u(a,\theta)$, where $\theta \in \mathbb{R}_+$ is the fundamental, distributed according to a continuous probability measure Π over $\mathbb{R}_+$. Before making the decision, the agent has access to a set of binary-signal information structures, and each signal corresponds to one optimal action.[3] Specifically, she observes binary signals $x \in \{0,1\}$ parameterized by a measurable function $m : \mathbb{R}_+ \to [0,1]$, where $m(\theta)$ is the probability of observing signal 1 if the true state is θ. When observing signal 1 (or 0), the agent's optimal action is 1 (or 0). The conditional probability $m(\theta)$ describes the agent's decision rule of information acquisition. By choosing different functional forms of $m(\theta)$, the agent can make her signal covary with fundamental in any arbitrary way. Intuitively, for instance, if the agent's payoff is sensitive to fluctuations of the state within some range $A \subset \mathbb{R}_+$, she would pay more attention to this range by making $m(\theta)$ covarying more with θ in A. As opposed to classic information acquisition technologies that often involve restrictions on the signals to be acquired, flexible information acquisition allows agents to choose signals drawn from any conditional distribution of the fundamental. As we will see, this flexibility is crucial in characterizing the investor's ability to screen projects and further ensures our sharp predictions on optimal securities.

We then characterize the cost of information. As in Yang (2012a), the amount of information conveyed by an information structure $m(\cdot)$ is defined as the expected reduction of uncertainty through observing the signal generated by $m(\cdot)$, where the uncertainty associated with a distribution is measured by Shannon (1948)'s entropy. Formally, we use the concept of mutual information, which is defined as the difference between agents' prior entropy and expected posterior entropy:

$$\begin{aligned} I(m) &= H(prior) - H(posterior) \\ &= -g(\mathbb{E}[m(\theta)]) - (-\mathbb{E}[g(m(\theta))]) \\ &= \mathbb{E}[g(m(\theta))] - g(\mathbb{E}[m(\theta)]) \ , \end{aligned}$$

where

$$g(x) = x \cdot \ln x + (1-x) \cdot \ln(1-x) \ ,$$

and the expectation operator $\mathbb{E}(\cdot)$ is with respect to θ under the probability measure Π. Denote

[3] In general, an agent can get access to any information structure. But it is shown that the agent always prefers binary-signal information structures in binary decision problems. See Woodford (2008) and Yang (2012b) for more discussions.

by $M = \{m \in L(\mathbb{R}_+, \Pi) : \theta \in \mathbb{R}_+, m(\theta) \in [0,1]\}$ the set of binary-signal information structures, and $c : M \to \mathbb{R}_+$ the cost of information. The cost is assumed to be proportional to the associated mutual information:

$$c(m) = \mu \cdot I(m) \ ,$$

where $\mu > 0$ is the marginal cost of information acquisition.

Built upon flexible information acquisition, the agent's problem is to choose a functional form of $m(\theta)$ to maximize her expected payoff minus the information cost. We characterize the optimal decision rule $m(\theta)$ in the following proposition. We denote $\Delta u(\theta) = u(1,\theta) - u(0,\theta)$, which is the the payoff gain of taking action 1 over action 0. We also assume that $\Pr[\Delta u(\theta) \neq 0] > 0$ to exclude the trivial case where the agent is always indifferent between the two actions. The proof is in Yang (2012a) (see also Woodford, 2008, for an earlier treatment).

Proposition **1.** *Given u, Π, and μ, let $m^*(\theta) \in M$ be an optimal decision rule and*

$$\bar{\pi}^* = \mathbb{E}[m^*(\theta)]$$

be the corresponding unconditional probability of taking action 1. Then,

i) the optimal decision rule of information acquisition is unique;

ii) there are three cases for the optimal decision rule:

a) $\bar{\pi}^* = 1$, i.e., Prob$[m^*(\theta) = 1] = 1$ if and only if

$$\mathbb{E}\left[\exp\left(-\mu^{-1} \cdot \Delta u(\theta)\right)\right] \leqslant 1\,; \tag{2.1}$$

b) $\bar{\pi}^* = 0$, i.e., Prob$[m^*(\theta) = 0] = 1$ if and only if

$$\mathbb{E}\left[\exp\left(\mu^{-1} \cdot \Delta u(\theta)\right)\right] \leqslant 1\,;$$

c) $0 < \bar{\pi}^* < 1$ and Prob$[0 < m^*(\theta) < 1] = 1$ if and only if

$$\mathbb{E}\left[\exp\left(\mu^{-1} \cdot \Delta u(\theta)\right)\right] > 1 \text{ and } \mathbb{E}\left[\exp\left(-\mu^{-1} \cdot \Delta u(\theta)\right)\right] > 1\ ; \tag{2.2}$$

in this case, the optimal decision rule $m^*(\theta)$ is determined by the equation

$$\Delta u(\theta) = \mu \cdot \left(g'(m^*(\theta)) - g'(\bar{\pi}^*)\right) \tag{2.3}$$

for all $\theta \in \mathbb{R}_+$, where

$$g'(x) = \ln\left(\frac{x}{1-x}\right) \ .$$

Proposition 1 fully characterizes the agent's possible optimal decisions of information acquisition. Case a) and Case b) correspond to the scenarios where there exists an ex-ante optimal action 1 or 0. These two cases do not involve information acquisition. They correspond to the scenarios where the prior is extreme or the cost of information acquisition is sufficiently high. In contrast, Case c), the more interesting one, involves information acquisition. Especially, the optimal decision rule $m^*(\theta)$ is not constant in this case, and neither action 1 nor action 0 is ex-ante optimal. Intuitively, this case corresponds to the scenario where the prior is not extreme, or the cost of information acquisition is sufficiently low. In Case c) where information acquisition is involved in the optimal decision rule, the agent equates the marginal benefit of information to the marginal cost of information. By doing so, the agent chooses the shape of $m^*(\theta)$ according to the shape of payoff gain $\Delta u(\theta)$ and her prior Π.[4] In the next section we will see that the shape of $m^*(\theta)$ plays a critical role in characterizing how the investor screen a project.

3 Security Design

We consider the entrepreneur's security design problem. Denote the entrepreneur's optimal security by $s^*(\theta)$. The strategic circumstance between the entrepreneur and the investor is a dynamic Bayesian game with sequential moves. Concretely, the entrepreneur first designs the security, and then the investor acquires information according to the security and decides whether to accept it. Hence, we apply the results in Proposition 1 to the investor's decision problem, given the entrepreneur's security, and then solve for the entrepreneur's optimal security by backward induction. To distinguish from the general decision problem above, we denote the investor's optimal decision rule of information acquisition as $m_s(\theta)$, given the security $s(\theta)$. The investor's optimal decision rule of information acquisition given the entrepreneur's optimal security $s^*(\theta)$ will be denoted by $m_s^*(\theta)$.

We formally define the equilibrium as follows.

DEFINITION **1.** *The sequential equilibrium is defined as a collection of the entrepreneur's optimal security $s^*(\theta)$ and the investor's optimal decision rule of information acquisition $m_s^*(\theta)$ based on which:*

i) Given u, Π, k and μ, $s^*(\theta)$ and $m_s^*(\theta)$ maximize the expected payoffs of the entrepreneur and the investor, respectively.

ii) Both agents use the Bayes' rule to update their beliefs about the fundamental θ, and follow sequential rationality.

According to Proposition 1, there are three cases pertaining to the investor's behavior, given

[4] See Woodford (2008), Yang (2012a,b) for more examples on this decision problem.

the entrepreneur's optimal security. First, the investor may optimally choose not to acquire information and accept the entrepreneur's optimal security directly. This implies that the project would be financed for sure. Second, the investor may optimally acquire some information, induced by the entrepreneur's optimal security, and then accept the entrepreneur's optimal security with positive (but less than one) probability . In this case, the project would be financed with positive (but less than one) probability from an ex-ante perspective. Third, the investor may directly reject the entrepreneur's optimal security without acquiring information, which implies that the project would not be financed. All the three cases are accommodated by the equilibrium definition. This last case, however, represents the outside option of the entrepreneur, who can always propose nothing to the investor and skip the project. Thus, it is less interesting to consider the last case, and we will focus on the first two types of equilibrium.

The following lemma helps to distinguish the first two cases from the last case. If the condition given is satisfied, the optimal security in equilibrium would be a nontrivial one and the project would be financed with positive probability.

LEMMA **1.** *The project would be financed with positive probability in equilibrium if and only if*

$$\mathbb{E}\left[\exp(\mu^{-1} \cdot (\theta - k))\right] > 1 \,. \tag{3.1}$$

Lemma 1 is an intuitive investment criterion. It implies that the security would more likely be accepted by the investor, if the prior of the project's market prospect is better, if the initial investment k is smaller, or if the investor's information cost μ is lower. When condition (3.1) is violated, the investor would reject the security, whatever it is.

It is interesting to note that condition (3.1) is different from the conventional expected NPV criterion, which suggests that a project should be initiated when $\mathbb{E}[\theta] - k > 0$. In particular, according to condition (3.1), some projects with ex-ante negative expected NPV may be initiated with positive probability. This observation is consistent with our main idea that real production depends on information production. Thanks to such dependence, investment and information acquisition are performed simultaneously, so that the conventional expected NPV criterion based on a fixed prior is generalized to accommodate the potential of belief updating. Instead, we have a new information-adjusted investment criterion. This also calls for a new efficiency criterion, which will be elaborated later.

Lemma 1 implies that the entrepreneur will never propose all the cash flow to the investor if the project would be financed. This corollary is straightforward, but we highlight it as it is useful in establishing some important results later. Intuitively, to retain a little bit more would still result in a finance with positive probability and give the entrepreneur a positive expected payoff.

Corollary 1. *When the project would be financed with positive probability, $s^*(\theta) = \theta$ is not an optimal security.*

In what follows, we assume that condition (3.1) is satisfied, and characterize the entrepreneur's optimal security, focusing on the first two types of equilibrium. As we will see, the entrepreneur's optimal securities in these two cases are different, which implies that the investor acquires information in different manners. We also show that to transfer the project at a given price is always not optimal, which justifies the security design approach.

3.1 Optimal Security without Inducing Information Acquisition

In this subsection, we consider the case in which the entrepreneur's optimal security is directly accepted by the investor without information acquisition. In other words, it is optimal for the entrepreneur to design such a security that does not induce the investor to acquire information. Concretely, this means $Pr\left[m_s(\theta) = 1\right] = 1$. We first consider the investor's problem of information acquisition, given the entrepreneur's security, then we characterize the optimal security.

Given a security $s(\theta)$, the investor's payoff gain by accepting the security over rejecting it is

$$\Delta u_I(\theta) = u_I(1, \theta) - u_I(0, \theta) = s\left(\theta\right) - k\,. \tag{3.2}$$

According to Proposition 1 and conditions (2.1) and (3.2), any security $s(\theta)$ that is accepted by the investor without information acquisition must satisfy

$$\mathbb{E}\left[\exp\left(-\mu^{-1} \cdot \left(s\left(\theta\right) - k\right)\right)\right] \leqslant 1\,. \tag{3.3}$$

If the left hand side of the inequality (3.3) is strictly less than one, the entrepreneur could lower $s(\theta)$ to some extent to increase her expected payoff gain, without changing the investor's incentive. Hence, condition (3.3) always holds as an equality in equilibrium.

By backward induction, the entrepreneur's problem is to choose a security $s(\theta)$ to maximize her expected payoff

$$u_E(s(\cdot)) = \mathbb{E}\left[\theta - s(\theta)\right]$$

subject to the investor's information acquisition constraint

$$\mathbb{E}\left[\exp\left(-\mu^{-1} \cdot \left(s\left(\theta\right) - k\right)\right)\right] = 1\,,$$

and the feasibility condition $0 \leqslant s(\theta) \leqslant \theta$.[5]

[5]With this feasibility condition, the entrepreneur's individual rationality constraint $\mathbb{E}\left[\theta - s(\theta)\right] \geqslant 0$ is automatically satisfied, which is also true for the later case with information acquisition. This comes from the fact that

As we would see, the entrepreneur's optimal security in this case follows a standard debt. We analytically characterize this optimal security by the following proposition, along with its graphical illustration.

PROPOSITION **2.** *If the entrepreneur's optimal security $s^*(\theta)$ induces the investor to accept the security without acquiring information in equilibrium, then it takes the form of a standard debt:*

$$s^*(\theta) = \min(\theta, D^*)$$

where the face value D^* is determined by

$$D^* = k - \mu \cdot \ln(\lambda^{-1} \cdot \mu) > k\,,$$

in which λ is a positive constant determined in equilibrium.

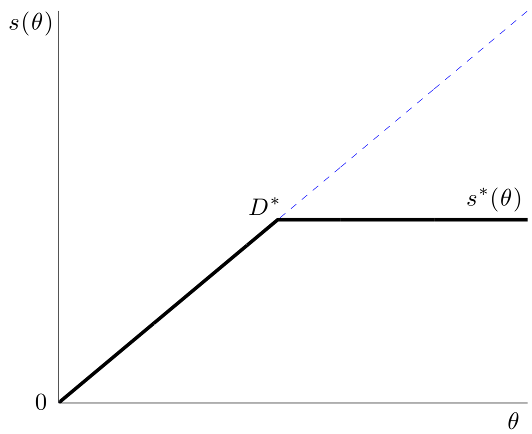

Figure 1: The Unique Optimal Security without Information Acquisition

It is intuitive to have standard debt as the optimal security when the entrepreneur finds it optimal not to induce information acquisition. Since screening is not worthwhile and thus the entrepreneur wants to design a security to deter it, debt is the least information sensitive one to provide the desired expected payoff to the seller. From another perspective, the optimal security renders the investor to break even between acquiring and not acquiring information. Hence, thanks to flexible information acquisition, any mean-preserving spread of the optimal security, which gives the entrepreneur the same expected payoff, would induce the investor to acquire

the entrepreneur is assumed to have no money to start. This fact also implies that the entrepreneur always prefers to start the project, which is consistent with real-world practices. Note that, it is not correct to interpret this in a way that the entrepreneur would like to contract with any individual investor, whatever the security is. This is because we do not model the competition between different investors, and the investor in our model represents a collection of all investors in the market.

unnecessary information. This implies that the optimal security should be as flat as possible when the limited liability constraint is not binding, which leads to standard debt.

Standard debt accounts for the real-world scenarios in which some projects are financed by fixed-income financial instruments. On the one hand, when a project's prospect is clear and thus no information is needed, it is often optimal to deter investor's costly information acquisition by issuing standard debt, which is the least information sensitive (Myers and Majluf, 1984; Gorton and Pennacchi, 1990; DeMarzo and Duffie, 1999). On the other hand, empirical evidence also suggest that many small businesses and less transformational start-ups with conventional projects rely heavily on direct bank finance (for example, Petersen and Rajan, 1994; Kerr and Nanda, 2009), as opposed to more exotic financial instruments with venture capitals or buyout funds.

3.2 Optimal Security Inducing Information Acquisition

In this subsection, we characterize the entrepreneur's optimal security if it induces the investor to acquire information and accept the security with positive probability (but less than one). In other words, it is optimal for the entrepreneur to design such a security that induces the investor to acquire information. According to Proposition 1, this means $Prob\,[0 < m_s(\theta) < 1] = 1$.

Again, according to Proposition 1 and conditions (2.2) and (3.2), any security $s(\theta)$ that induces the investor to acquire information must satisfy

$$\mathbb{E}\left[\exp\left(\mu^{-1}\left(s\left(\theta\right)-k\right)\right)\right] > 1 \tag{3.4}$$

and

$$\mathbb{E}\left[\exp\left(-\mu^{-1}\left(s\left(\theta\right)-k\right)\right)\right] > 1 \ , \tag{3.5}$$

Given such a security $s(\theta)$, Proposition 1 and condition (2.3) also prescribe that the investor's optimal decision rule of information acquisition $m_s(\theta)$ is uniquely characterized by

$$s\left(\theta\right)-k = \mu\cdot\left(g'\left(m_s\left(\theta\right)\right)-g'\left(\overline{\pi}_s\right)\right) \ , \tag{3.6}$$

where

$$\overline{\pi}_s = \mathbb{E}\left[m_s\left(\theta\right)\right]$$

is the investor's unconditional probability of accepting the security and it does not depend on θ. In what follows, we also denote this unconditional probability induced by the entrepreneur's optimal security by $\overline{\pi}_s^*$.

We derive the entrepreneur's optimal security by backward induction. Taking into account of investor's response m_s, the entrepreneur chooses a security $s(\theta)$ to maximize her expected payoff

$$u_E(s(\cdot)) = \mathbb{E}[m_s(\theta) \cdot (\theta - s(\theta))] \tag{3.7}$$

subject to (3.4), (3.5),[6] (3.6), and the feasibility condition $0 \leqslant s(\theta) \leqslant \theta$.[7]

To fix idea, we first offer an intuitive roadmap to look into the optimal security and the associated optimal information acquisition rule, highlighting their key properties. Then we give a formal proposition to characterize the optimal security and discuss its implications. The detailed derivation of the optimal security and related formal proofs are presented in the Appendix.[8]

First, the investor's optimal information acquisition rule $m_s^*(\theta)$, induced by the optimal security $s^*(\theta)$, must increase in θ. When the entrepreneur finds it optimal to induce the investor to acquire information, she benefits from screening by the investor. Importantly, screening makes sense only if the investor screens in a good project and screens out a bad project; otherwise it incurs a lower social surplus. Thanks to flexible information acquisition, this implies that an $m_s^*(\theta)$ should more likely generate a good signal and result in a successful finance for higher fundamental θ, while generate a bad signal and result in a rejection for lower θ. Therefore, $m_s^*(\theta)$ should be increasing in θ. As we will see, the shapes of $m_s^*(\theta)$ and $s^*(\theta)$ are highly related to each other. This relationship is crucial for us to deliver predictions on optimal securities over continuous states without distributional assumptions, and flexible information acquisition plays an important role in characterizing this relationship precisely.

To induce an increasing optimal information acquisition rule $m_s^*(\theta)$, the optimal security $s^*(\theta)$, proposed by the entrepreneur, must be increasing in θ as well, according to the first order condition of information acquisition (3.6). Intuitively, this monotonicity reflects the dependence of real production on information production: the entrepreneur is willing to compensate the investor more in an event of higher cash flow to encourage effective screening. As opposed to the classic security design literature that often restrict the feasible set to non-decreasing securities (for example, Innes, 1990; Nachman and Noe, 1994; DeMarzo and Duffie, 1999; DeMarzo, 2005), our prediction of an increasing optimal security without such constraints is significant.

We also argue that the non-negative constraint $s(\theta) \geqslant 0$ is not binding for the optimal security $s^*(\theta)$ for any $\theta > 0$. Suppose $s^*(\widetilde{\theta}) = 0$ for some $\widetilde{\theta} > 0$. Since $s^*(\theta)$ is increasing in θ, for all $0 \leqslant \theta \leqslant \widetilde{\theta}$ we must have $s^*(\theta) = 0$. This violates the above argument that $s^*(\theta)$ must be increasing in θ. Intuitively, zero payoffs in bad states give the investor too little incentive to

[6] It is worth noting that, according to Proposition 1, both conditions (3.4) and (3.5) should not be binding for the optimal security; otherwise the investor would not acquire information.

[7] Again, the entrepreneur's individual rationality constraint $\mathbb{E}[m_s(\theta) \cdot (\theta - s(\theta))] \geqslant 0$ is automatically satisfied.

[8] To facilitate understanding, the intuitive investigation of the optimal security is not organized in the same order as the proofs in the Appendix, but all the claims in the main text are guaranteed by the formal proofs.

acquire information and screen a project, which is not optimal for the entrepreneur. Actually, the security with zero payoffs in bad states look closest to common stock, which is the least used security between entrepreneurs and investors (Kaplan and Stromberg, 2003; Kaplan and Lerner, 2010; Lerner, Leamon and Hardymon, 2012).

To shot a closer look into the optimal security, it is instructive to follow a perturbation argument on the entrepreneur's security design problem, which gives the entrepreneur's first order condition. Specifically, denote by $r^*(\theta)$ the marginal contribution to the entrepreneur's expected payoff $u_E(s(\cdot))$ by any feasible perturbation to the optimal security $s^*(\theta)$.[9] As $s^*(\theta) > 0$ for any $\theta > 0$, it is intuitive to show that for any $\theta > 0$:

$$r^*(\theta) \begin{cases} = 0 & \text{if} \quad 0 < s^*(\theta) < \theta \\ \geqslant 0 & \text{if} \quad \quad s^*(\theta) = \theta \end{cases},$$

which is further shown to be equivalent to

$$(1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \begin{cases} = \mu & \text{if} \quad 0 < s^*(\theta) < \theta \\ \geqslant \mu & \text{if} \quad \quad s^*(\theta) = \theta \end{cases}, \tag{3.8}$$

where w^* is a constant determined in equilibrium.

We now argue that the optimal security $s^*(\theta)$ follows the 45° degree line in bad states and then increases in θ in good states with a smaller slope. That is, the residual of the optimal security, $\theta - s^*(\theta)$, increases in θ as well in good states. Due to the entrepreneur's first order condition (3.8) and the monotonicity of $m_s^*(\theta)$, if $s^*(\widehat{\theta}) = \widehat{\theta}$ for some $\widehat{\theta} > 0$, it must be $s^*(\theta) = \theta$ for any $0 < \theta < \widehat{\theta}$. Similarly, if $s^*(\widehat{\theta}) < \widehat{\theta}$ for some $\widehat{\theta} > 0$, it must be $s^*(\theta) < \theta$ for any $\theta > \widehat{\theta}$. In addition, Corollary 1 rules out $s^*(\theta) = \theta$ for all $\theta > 0$ as an optimal security. Thus, since $s^*(\theta)$ is increasing in θ, the limited liability constraint can only be binding in bad states.[10] Importantly, according to condition (3.8) and again the monotonicity of $m_s^*(\theta)$, when the limited liability constraint is not binding in good states, not only $s^*(\theta)$ but also $\theta - s^*(\theta)$ is increasing in θ. In other words, $s^*(\theta)$ is dual monotone when it deviates from the the 45° degree line in good states.

The shape of the optimal security $s^*(\theta)$ reflects the economy's friction in a clear way. Recall that the monotonicity of $s^*(\theta)$ reflects the dependence of information and real production. The monotonicity of $\theta - s^*(\theta)$ however reflects their separation: the entrepreneur wants to retain as much as possible even if she needs the investor to screen the project. Specifically, the area between $s^*(\theta)$ and the 45° degree line not only captures the entrepreneur's retained benefit, but

[9]Formally, $r^*(\theta)$ is the Frechet derivative, the functional derivative used in the calculus of variations, of $u_E(s(\cdot))$ at $s^*(\theta)$. It is analogical to the commonly used derivative of a real-valued function of a single real variable but generates it to functions on Banach spaces.

[10]In the proofs we show that the limited liability constraint must be binding for some states $(0, \widehat{\theta})$ with $\widehat{\theta} > 0$.

also reflects the degree to which the allocation of resources is inefficient due to the friction. The coincidence is intuitive: the dependence renders the investor to get all the resources but the separation prevents the entrepreneur from proposing such a deal. The competition of the two is alleviated in a least inefficient way: to reward the investor more but also retain more in better states, and it also becomes the best way for the entrepreneur to encourage the investor to acquire information. In this sense, our prediction of the dual monotonicity comes endogenously from the friction of the economy, rather than by assumption like existing literature (for example, Biais and Mariotti, 2005).

We have the following proposition to characterize the optimal security $s^*(\theta)$ that induces the investor to acquire information. We interpret it as a participating preferred stock, according to its shape.

PROPOSITION **3.** *If the entrepreneur's optimal security $s^*(\theta)$ induces the investor to acquire information in equilibrium, then it takes the following form of a participating preferred stock with a face value $\widehat{\theta} > 0$:*

$$s^*(\theta) = \begin{cases} \theta & \text{if} \quad 0 \leqslant \theta \leqslant \widehat{\theta} \\ \widehat{s}(\theta) & \text{if} \quad \theta > \widehat{\theta} \end{cases},$$

where $\widehat{\theta}$ is determined in equilibrium and the unconstrained part $\widehat{s}(\theta)$ satisfies:

i) $\widehat{\theta} < \widehat{s}(\theta) < \theta$;

ii) $0 < d\widehat{s}(\theta)/d\theta < 1$.[11]

Finally, the corresponding optimal rule of information acquisition satisfies $dm_s^*(\theta)/d\theta > 0$.

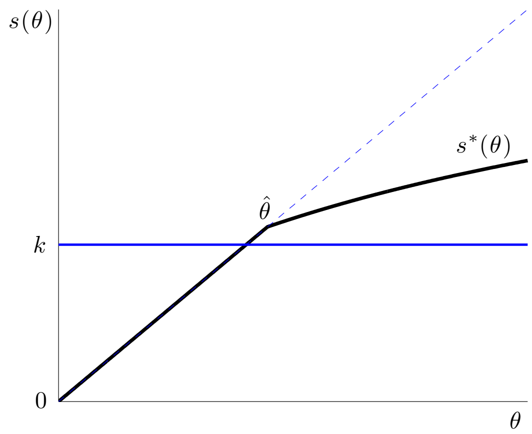

Figure 2: The Unique Optimal Security with Information Acquisition

Proposition 3 offers a clear prediction on the entrepreneur's optimal security in financing risky projects. In particular, it is closest to participating convertible preferred stock, which grants the

[11]In the appendix, we have provided the specific function form of $d\widehat{s}(\theta)/d\theta$.

holder a right to receive both the face value and their equity participation as if the stock were converted, in the event of a sale or liquidation. This is consistent with the empirical evidence of venture capital contracts, in which convertible preferred stocks account for 79.8% of total contracts used, and 48.2% of them are participating (Kaplan and Stromberg, 2003). It also fits in line with earlier evidence (Sahlman, 1990; Bergemann and Hege, 1998; Gompers, 1999) on the popularity of convertible preferred stock in financing new projects. This prediction cannot be achieved by classic models that are silent on the information expertise of investors. In particular, we are able to specify our securities on continuous states with arbitrary distributions, thanks to flexible information acquisition. For simplicity, in what follows we will refer to the optimal security in this case as convertible preferred stock, and we call a security convertible preferred stock if it can be characterized by Proposition 3.

It is interesting to contrast the results here to Yang (2012a), where the optimal security is always debt. As the seller's asset is already in-place in Yang (2012a), information acquisition is socially wasteful in that case. Concretely, liquidity provision leads to conflicting interests of the two parties, so that the information acquired by the buyer makes herself better off at the expense of the seller through endogenous adverse selection. As a result, the seller designs the debt to optimally discourage information acquisition harmful to her own interest. In contrast, in a production economy, the entrepreneur and the investor jointly expose themselves to the risk of the project if the investor accepts the security, and are not so if the security is rejected. In this case, information acquisition could be socially valuable and the conflicting interests of the two parties could be partly reconciled. Therefore, the entrepreneur may design a security to encourage the investor to acquire information that helps to screen projects in favor of herself. More generally, this comparison sheds lights on the different implications of a production economy (or a primary financial market) and an exchange economy (or a secondary financial market). In a production economy (or a primary financial market), real production depends on information production; while in an exchange economy (or a secondary financial market) information acquisition only helps reallocate existing resources. Therefore, when financing involves information acquisition, different economies (or financial markets) may require different forms of optimal securities, because the incentives of information acquisition and the underlying values of information are different accordingly.

For convertible preferred stock as the optimal security, the following corollary further supports its consistency with the empirical evidence. It speaks to the multiple of the optimal convertible preferred stock, which is defined as the ratio of the face value $\widehat{\theta}$ of convertible preferred stock to the investor's initial investment k. The multiple is viewed as a key characteristic of a convertible preferred stock, which is analogous to the role of returns for other securities.

Corollary **2.** *The multiple of convertible preferred stock, as the optimal security $s^*(\theta)$ that induces information acquisition in equilibrium, is greater than one. In other words, $\widehat{\theta} > k$.*

Corollary 2 is consistent with the empirical documents (Kaplan and Stromberg, 2003; Lerner, Leamon and Hardymon, 2012). In practice, the multiple would always be greater than one to ensure the investors' keen interest in converting when they exit. In our context, this property still comes from the logic that the investor should be sufficiently compensated for both the physical investment requirement and the screening service through information acquisition. Although we do not explicitly model the exiting of investors, it is incorporated in the distribution of the future cash flow. Hence, the correct prediction of the multiple gives us another perspective to illustrate the power of our model.

3.3 Project Transfer

This subsection considers the potential for transferring the project at a fixed non-negative price and shows that it is not optimal. Equivalently, as there is no moral hazard, a transfer also represents a scenario where the entrepreneur works for the investor as a worker and gets a fixed wage payment. According to Rajan (2012), entrepreneurs' human capital is often inalienable because of the high differentiation of young firms, which justifies our security design approach. Here, we further argue that, even if the project is transferrable, the entrepreneur still finds such a transfer not optimal.

The key to understand the idea is to interpret project transfer as a feasible security, and show that it is not optimal. When the entrepreneur proposes a project transfer to the investor at a fixed price $p \geqslant 0$, it is equivalent for her to propose a security $s(\theta) = \theta - p$ without the non-negative constraint $s(\theta) \geqslant 0$. To see why, if the investor accept the offer of transfer and undertake the project, she gets the entire cash flow θ and pay the fixed price p as an upfront cost. This interpretation allows us to analyze project transfer in our standard security design framework.

To see why the equivalent security $s(\theta) = \theta - p$ is feasible but not optimal, it is important to observe that the non-negative constraint $s(\theta) \geqslant 0$ is binding in neither case of security design. Hence, it is equivalent to consider a larger set of feasible securities, which is still restricted by the limited liability constraint $s(\theta) \leqslant \theta$ but allows negative payoffs to the investor in some states. As standard debt and convertible preferred stock are still the only two optimal securities in this generalized problem, and the security $s(\theta) = \theta - p$ representing project transfer is feasible, we conclude that project transfer is not optimal to the entrepreneur for any transfer price p. Intuitively, project transfer is not optimal because it does not follow the least costly way to compensate the investor, no matter whether information acquisition is induced.

Proposition 4. *When the project can be financed with positive probability, to transfer the project at a fixed price $p \geqslant 0$ is not optimal for the entrepreneur.*

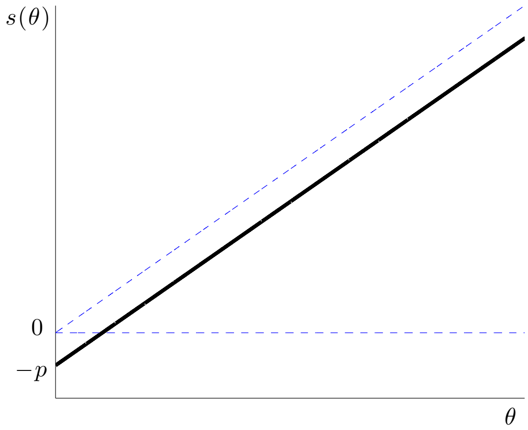

Figure 3: Project Transfer as a Security

The timing and the presence of friction in our production economy are important for Proposition 4. Think of an alternative timing in which the investor can only acquire information after the deal of transfer. The friction of the production economy is no longer present, because real production and information production are both performed by the investor after the transfer. Hence, following this timing, project transfer is optimal to the entrepreneur through setting a price that is equivalent to the expected profit of the investor. In this case, the entrepreneur's bargaining power is too strong in the sense that she can prevent the investor from acquiring information when evaluating the project, which essentially removes the friction of the economy.

4 Pecking Order of Optimal Securities

Having characterized the optimal securities with and without information acquisition, we take them together and determine the optimal security given characteristics of the production economy. This delivers new insights regarding the pecking order theory (Myers and Majluf, 1984) in the presence of production and information acquisition: the entrepreneur chooses different optimal securities and thus different capital structures in different circumstances. This pecking order helps unify debt and convertible preferred stock, which are often viewed as two distinct securities in many aspects. Our approach also bridges the security design literature and the classic pecking order theory, since our new pecking order comes from security design over a general space of feasible securities, rather than a given set of commonly observed securities like debt and equity.

Our new pecking order is presented against two benchmarks: one is the conventional NPV investment criterion and the other is an efficiency criterion, which allow us to reveal the relationship between the friction of the production economy and the optimal security in a clearer manner. The friction that real production depends on information production but they are separately performed by the entrepreneur and the investor, drives the new pecking order. Whether convertible preferred stock or standard debt is optimal depends on whether or not the entrepreneur wants to encourage information acquisition and screening, which further depends on the different extent of dependence of real production on information production. If the dependence is strong, the friction of the economy is more severe. Screening is also more valuable in this case, so that the entrepreneur finds it more worthwhile to induce information acquisition and proposes convertible preferred stock. Otherwise, the friction is less severe and inducing costly screening is not necessary for the entrepreneur, so that standard debt is optimal.

4.1 An NPV Benchmark for Pecking Order

We first benchmark our new pecking order to the conventional expected NPV criterion.

Proposition **5.** *When the project is financed with positive probability:*

i). If $\mathbb{E}[\theta] \leqslant k$, the optimal security $s^*(\theta)$ is convertible preferred stock; or

ii). If $\mathbb{E}[\theta] > k$, $s^*(\theta)$ is either convertible preferred stock or standard debt.

Intuitively, the only security to finance a negative NPV project is convertible preferred stock, which induces information acquisition. The role of screening is crucial: only through information acquisition could a negative NPV project be potentially found good and worth financing. This is also consistent with the traditional wisdom of the NPV criterion that a negative NPV project can never be financed by standard debt since the belief is given and fixed.

Interestingly, convertible preferred stock may be optimal with both negative and positive NPV projects, but the underlying mechanisms are subtly different. In both cases, the dependence of real production on information production is strong.

When the project has a zero or negative NPV, convertible preferred stock is used to encourage the investor to screen in a potentially good project. In this case, the investor will never finance it if she is unable to acquire information, because it only incurs an expected loss even if the entrepreneur proposes all the cash flow to the investor. Thus, if it would be financed, the only way is to use convertible preferred stock to encourage information acquisition. This implies that the dependence of real production on information production is strong due to the relatively poor prior, and thus the friction is severe. When the investor acquires information, we may expect either a good signal that leads to a successful deal or a bad signal that results in a rejection, but the ex-ante probability of finance becomes positive since a potentially good project can be screened in, thanks to information acquisition. Hence, the entrepreneur is better off by proposing convertible preferred stock.

In contrast, when the project has a positive NPV, convertible preferred stock may still be used in some circumstances, but to encourage the investor to screen out a potentially bad project. Here, the dependence of real production on information production is still strong due to a relatively mediocre prior. In a status quo where the investor is unable to screen a project, the entrepreneur can finance the positive NPV project for sure by using standard debt with a high face value. However, when the investor can acquire information, the entrepreneur might find such a sure finance too expensive or even impossible, because by doing so she retains too little for herself. Instead, the entrepreneur could retain more by offering a less generous convertible preferred stock and invite the investor to acquire information and screen the project. Although this results in a probability of finance less than one, the entrepreneur's total expected profit is higher since a potentially bad project may be screened out, which justifies convertible preferred stock as optimal.

Finally, standard debt may be optimal for some circumstances with a positive NPV project. When the prior is sufficiently good, the dependence of real production on information production is weak, and thus the benefit from encouraging information acquisition and screening does not justify the information cost. Hence, it is optimal for the entrepreneur to propose standard debt with a sufficiently high face value to deter costly information acquisition while still retain enough for herself. A project can be financed for sure in this case and the investor does not screen it.

4.2 An Efficiency Benchmark for Pecking Order

We then benchmark our pecking order to a more fundamental efficiency criterion. To understand how the optimal security evolves with the severity of the friction, we consider a frictionless centralized economy in which real production and information production are aligned. We define an efficiency criterion with help of this centralized economy and use it to benchmark the equilibria and optimal securities in the decentralized economy. We show that, if and only if the friction in the decentralized economy is not severe in the sense that an optimal security can achieve efficiency, the optimal security is standard debt and information acquisition is not induced in equilibrium. If and only if the friction is severe in the sense that even an optimal security cannot achieve efficiency, the optimal security is convertible preferred stock and information acquisition is induced. This dichotomy again highlights the close connection among the shape of the optimal security, the role of information acquisition, and the extent of the friction in the production economy.

We start by defining the expected social surplus and an efficiency criterion. The expected surplus of our decentralized economy is the difference between the expected profit of the project and the information cost, both of which are functions of the information acquisition rule. Thus, an optimal security in the decentralized economy achieves efficiency if the associated optimal information acquisition rule maximizes expected social surplus in equilibrium.

Definition **2.** *An optimal security in the decentralized economy achieves efficiency if and only if the associated optimal information acquisition rule $m_s^*(\theta)$ satisfies:*

$$m_s^*(\theta) \in \arg \max_{0 \leqslant m(\theta) \leqslant 1} \mathbb{E}[m(\theta) \cdot (\theta - k)] - \mu \cdot I(m(\theta)) \,. \tag{4.9}$$

To facilitate discussion, we characterize a frictionless centralized economy that helps to benchmark the friction in the corresponding decentralized economy. In the centralized economy, u, Θ, Π, k and μ are given as same. However, we suppose the entrepreneur has enough initial wealth and can also acquire information. Thus, real production and information production are aligned. In this economy, security design is irrelevant. The entrepreneur's problem is to decide whether to acquire information and to undertake the project. The entrepreneur's payoff gain by undertaking the project over skipping it is

$$\Delta u_I(\theta) = u_I(1, \theta) - u_I(0, \theta) = \theta - k \,.$$

We denote the information acquisition rule in the centralized economy by $m_c(\theta)$ and the optimal one by $m_c^*(\theta)$. Thus, the entrepreneur's problem in the centralized economy is

$$\max_{0 \leqslant m_c(\theta) \leqslant 1} \mathbb{E}[m_c(\theta) \cdot (\theta - k)] - \mu \cdot I(m_c(\theta)) \,. \tag{4.10}$$

By construction, the entrepreneur's objective function (4.10) in the centralized economy is just the expected social surplus in the decentralized economy. Hence, it is convenient for us to work with the centralized economy to analyze the efficiency of equilibria in the corresponding decentralized economy.

Since the optimal rules of information acquisition are unique for both the centralized and decentralized economies, efficiency is achieved if and only if information is acquired in the same manner in both the decentralized economy and the centralized economy.

Lemma **2.** *An optimal security in the decentralized economy achieves efficiency if and only if the associated optimal rule of information acquisition $m_s^*(\theta)$ satisfies*

$$Prob[m_s^*(\theta) = m_c^*(\theta)] = 1 \,,$$

where $m_c^*(\theta)$ is the optimal information acquisition rule in the corresponding centralized economy.

The efficiency criterion in Lemma 2 highlights the role of information acquisition in the production economy and provides a natural measure of the severity of friction in the decentralized economy. Fundamentally, we may view the expected social surplus (4.10) or (4.9) in our production economy as a production function, information characterized by the rule of information acquisition $m_c(\theta)$ or $m(\theta)$ as the sole input. This again fits in line with our idea that real production depends on information production. Consequently, efficiency is achieved if and only if the optimal security in the decentralized economy delivers the same equilibrium input allocation as what the centralized economy does. If the optimal security does this job, the friction in the decentralized economy is not severe as it can be effectively removed by the optimal security. Otherwise, the friction is severe since it cannot be completely removed even if an optimal security is used.

With the efficiency criterion in Definition 2 and Lemma 2, we are able to characterize our pecking order of optimal securities as follows.

PROPOSITION **6.** *In the decentralized production economy, when the project is financed with positive probability:*

i) The optimal security $s^*(\theta)$ is standard debt if and only if the friction of the decentralized economy is not severe in the sense that an optimal security achieves efficiency; and

ii) $s^*(\theta)$ is convertible preferred stock if and only if the friction is severe in the sense that even an optimal security cannot achieve efficiency.

This pecking order against the efficiency criterion is important not only because it distinguishes between the two optimal securities given characteristics of the production economy, but also because it wraps up the roles of optimal securities and information acquisition in reducing the friction of our economy. In the decentralized economy, real production is performed by the entrepreneur while information production by the investor. The separation is always present and unchanged in spite of different exogenous characteristics of the economy. Hence, the severity of friction is reflected by the extent to which real production depends on information production. If the friction is severe, the dependence is strong, which makes information acquisition valuable and thus renders convertible preferred stock as optimal. Similarly, if the friction is not severe, the dependence is weak. As a consequence, the benefit of information acquisition does not justify its cost and thus standard debt it optimal.

Our pecking order shed new lights on unifying empirical evidence. Debt financing is popular for conventional and standard projects and for investors who have less expertise in information acquisition, which represent the cases where the friction is not severe. Financing with convertible preferred stock is common for innovative projects that need more screening and for investors who are more capable of doing that, which represents the cases where the friction is severe.

4.3 Numerical Examples of Pecking Order

In this subsection, we use numerical examples to illustrate our new pecking order of optimal securities along three empirical dimensions: the profitability of the project, the uncertainty of the project, and the investor's information cost. These examples are consistent with our theoretical pecking orders against the NPV criterion and the efficiency criterion. Again, the dependence of real production on information production shapes the optimal securities in different circumstances as well as the pecking orders in different dimensions.

The numerical examples are organized as follows. For some given prior distributions of θ, the fundamental of the project, we compare the optimal securities within a set of (k, μ) pairs, namely the investment requirement and the information cost. We pick up four distributions of θ with the same mean but increasing uncertainty. Concretely, we take normal distributions with mean 0.5 and a series of standard deviation 0.125, 0.25, and 2, and then truncate these distributions to interval $[0, 1]$. The family also consists of a fourth distribution which represents an extreme case, in which the project is so uncertain that it has more chances to take either a best-case realization or a worst-case realization. In other words, the fourth extreme distribution is a mean-preserving spread of the first three distributions. All the distributions are normalized after the truncation. We illustrate the family of four distributions in Figure 4. As for the set of (k, μ) pairs, our k varies within the unit interval $[0, 1]$ and μ varies within the unit interval $[0.2, 1.2]$.

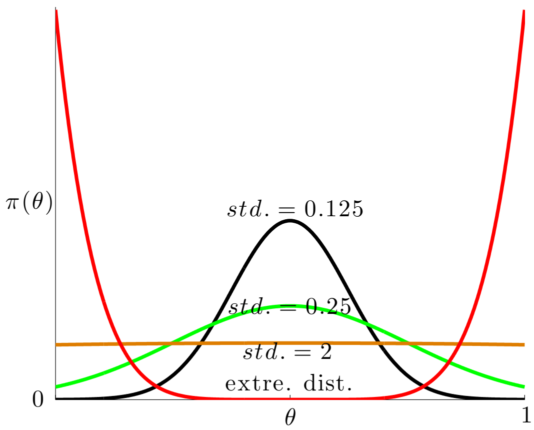

Figure 4: Typical Distributions with Increasing Uncertainty

Together with the characterization of the two nontrivial optimal securities in the previous two cases (without and with inducing information acquisition when the project gets financed), Proposition 1 allows us to map the two optimal securities to the associated probabilities of finance in their corresponding equilibria. Specifically, when the optimal security is standard debt, the probability of finance is 1, while the probability of finance is between 0 and 1 when the optimal security is convertible preferred stock. When the project is directly rejected without inducing information acquisition, the optimal security is a trivial one. Therefore, we can plot the equilibrium probability of finance against the (k, μ) pairs within the unit square $[0, 1] \times [0.2, 1.2]$. Figure 5 illustrates how the type of optimal security evolves according to the changes of investment requirement, information cost, as well as the uncertainty of the project.

The pecking orders of optimal securities is displayed in Figure 5 as the evolution from the red upper platform to the rainbow-colored cliff then to the dark blue lower platform. Concretely, the upper platform consists of equilibria in which the probability of finance is 1, which implies the underlying optimal security is standard debt. The cliff consists of equilibria in which the probabilities of finance are numbers between 0 and 1, which implies the underlying optimal security is convertible preferred stock. The lower platform represents the equilibria in which the project is rejected for sure without inducing information acquisition and the optimal security is trivial.

We interpret the new pecking orders of optimal securities illustrated in Figure 5 within our framework of production economy with information acquisition, and again we focus on the cases with nontrivial optimal securities when the project gets financed.

The first numerical pecking order is on the dimension of project profitability. Given other parameters fixed, when the investment requirement k increases, the optimal security evolves from standard debt to convertible preferred stock. Intuitively, the larger k is, the less profitable the project looks initially to the investor who is able to screen it by information acquisition, and it is thus more costly for the entrepreneur to finance the project for sure. Hence, real production depends more strongly on information production and information acquisition is more worthwhile. Therefore, it is more desirable for the entrepreneur to propose a convertible preferred stock to encourage the investor to acquire information, and thus to screen projects in a favorable way to her total profit. Specifically, when k is low, the project is optimally financed for sure by standard debt. When k is intermediate but the expected NPV is still positive, the project is optimally financed with positive probability by convertible preferred stock, and the investor acquires information to screen out potentially bad projects. When k is high and thus the expected NPV is negative, if the project would be financed, it is optimally financed with positive probability by convertible preferred stock, and the investor acquires information to screen in potentially good projects.

Second, given other parameters, especially, the prior mean of the fundamental $\mathbb{E}[\theta]$, fixed, there is also a larger chance for convertible preferred stock to be optimal when the the project is more uncertain. The mechanism here is however slightly different from the earlier case regarding the profitability of the project. When the project is more uncertain ex-ante, more information is payoff-relevant and thus the investor has a higher incentive to acquire information, so that to induce favorable information acquisition is less costly to the entrepreneur, and thus convertible

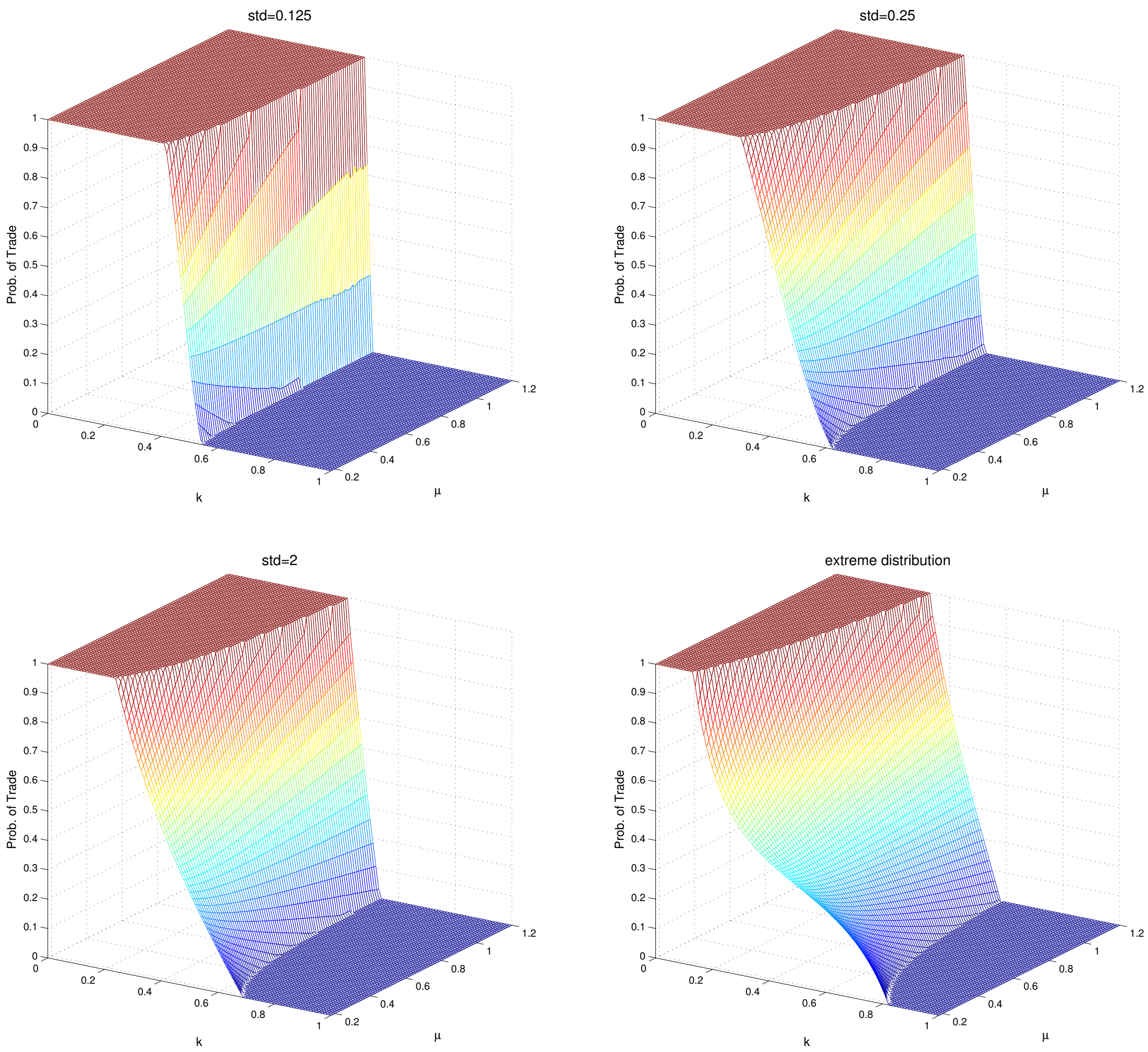

Figure 5: Pecking Orders of Optimal Securities

preferred stock is more likely to be used. This is again consistent with the intuition that the dependence of real production on information production matters for the optimal security.

Last, given other parameters fixed, the optimal security evolves from convertible preferred stock to standard debt when the information cost μ gets larger. The mechanism is as follows. When the cost of information acquisition is higher, it is less likely that the benefit from information acquisition and potential screening would justify the information cost. In particular, the investor's information cost is taken into account and compensated when the entrepreneur design the optimal security. So it turns out convertible preferred stock that encourages information acquisition is less likely to be used. Again, in this case, the dependence of real production on information production gets weaker because of the higher information cost.

5 Comparative Statics of the Optimal Security

In this section, we discuss the evolution of the shape of optimal securities, when the nature of the underlying project and the investors' information cost vary. As we emphasize throughout the paper, security design and information acquisition are closely linked in the context. When the parameters of the environment vary, the way through which the entrepreneur wants to use the investor's information expertise also changes. This eventually leads to different shapes of optimal security in equilibrium.

We unify the intuitions of all the comparative statics. Conflicting interests present between the entrepreneur and the investor, because they share the total surplus of the production economy. Nevertheless, the conflict could be partly alleviated through costly information acquisition that helps screen projects and thus increases social surplus. Consequently, a natural tension, consisting of two competing forces, faces the entrepreneur when she proposes the security. On the one hand, as only the investor is able to acquire information, which is helpful in screening projects, the entrepreneur would like to reward the investor more in order to encourage information and thus elevates the total surplus. On the other hand, however, as the entrepreneur's payoff comes from the opposite of the proposed security, she would like to keep more and compensate less to the investor as long as the investor would not reject the offer. When the first force gets more powerful, the resulting optimal security would move closer to the 45° line, which encourages information acquisition to the most efficient extent. In contrast, when the second force dominates, the resulting optimal security would move closer to standard debt with a low face value, which discourages information acquisition and leaves the most to the entrepreneur.

5.1 Profitability of Project

First, we consider the effect of changing the initial investment k on the optimal security $s^*(\theta)$. In doing this, we fix the market prospect of the project as well as the investor's information cost. Concretely, we fix the distribution of θ and the value of μ. When the nature of the project is fixed, a decrease in the initial investment represents that the project is more profitable.

We show that, when the project is financed with positive probability, the face value $\widehat{\theta}$ of convertible preferred stock is increasing in the required investment k. The cash flow after converting is also increasing in k. Moreover, the optimal security will be standard debt when k is sufficiently small. Intuitively, when the project is more profitable, it is more likely to be financed by standard debt without inducing information acquisition. In contrast, when the project is less profitable, it is more likely to be financed by convertible preferred stock, where information acquisition in induced and the compensation to the investor tends to be higher.

We offer the following numerical simulations to illustrate our result, as shown in Figure 6. The distribution of θ is fixed and generated as follows. We take a normal distribution with mean 0.5 and standard deviation 0.125, and then truncate this distribution to interval $[0, 1]$. The information cost μ is fixed at 0.2. The initial investment k will be taking there different values: 0.4, 0.475, and 0.525. We would see the optimal security is a standard debt when $k = 0.4$, and is a convertible preferred stock as described above when k is taking larger values.

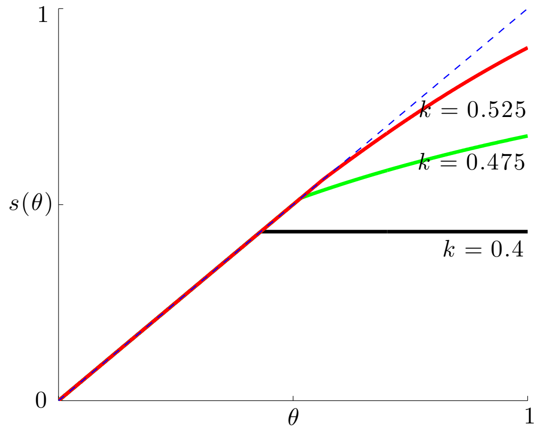

Figure 6: Change of Initial Investment: $\mathbb{E}[\theta] = 0.5; \mu = 0.2$

The intuition for the comparative statics for the profitability of project is clear. When the initial investment k is smaller, the project is more profitable ex-ante, so that it is less desirable to compensate the investor to induce costly information acquisition. In other words, the first force as described, namely, the value of information, decreases. As a result, the resulting security moves towards standard debt. On the contrary, when the initial investment k is greater, the project is less profitable ex-ante and screening becomes more necessary. Hence, the entrepreneur is more willing to propose a more generous convertible preferred stock to the investor to induce information acquisition.

5.2 Uncertainty of Project

We consider the effect of changing the project's uncertainty on the optimal security $s^*(\theta)$. Especially, when considering the change of uncertainty, we fix the mean of the prior of the fundamental θ, and consider a family of distributions which are ranked by second order stochastic dominance. In doing this, we also fix the initial investment k of the project as well as the investor's information cost μ. Interestingly, the comparative statics in this case depends on the relationship between the mean of the fundamental $\mathbb{E}[\theta]$ and the initial investment k, as separated by the two cases in Proposition 5. It is also worth noting that, the effect of changing in uncertainty cannot be accounted by the argument of risk attitude, because both the entrepreneur and the investor in our context is risk neutral. Instead, we still link the change of uncertainty with the tension and the two forces we discussed before.

First, we consider Case i) in Proposition 5, namely, $\mathbb{E}[\theta] > k$. This implies that the project has a positive expected NPV, given the prior. We show that, in this case, the face value $\widehat{\theta}$ of convertible preferred stock is increasing in the uncertainty of the project. The cash flow after converting is also increasing in the uncertainty. Moreover, the optimal security will be standard debt when the uncertainty is sufficiently small. Intuitively, when the project is less uncertain, it is more likely to be financed by standard debt without inducing information acquisition. In contrast, when the project involves more uncertainty, it is more likely to be financed by convertible preferred stock, where information acquisition is induced and the compensation to the investor tends to be higher.

We offer the following numerical simulations to illustrate our result, as in Figure 7. The initial investment is fixed at $k = 0.4 < \mathbb{E}[\theta]$. The information cost μ is fixed at 0.2. The families of distributions of θ are generated as we did in Figure 4. The left panel illustrates the distributions of θ we used, and the right panel illustrates the evolution of the optimal security.

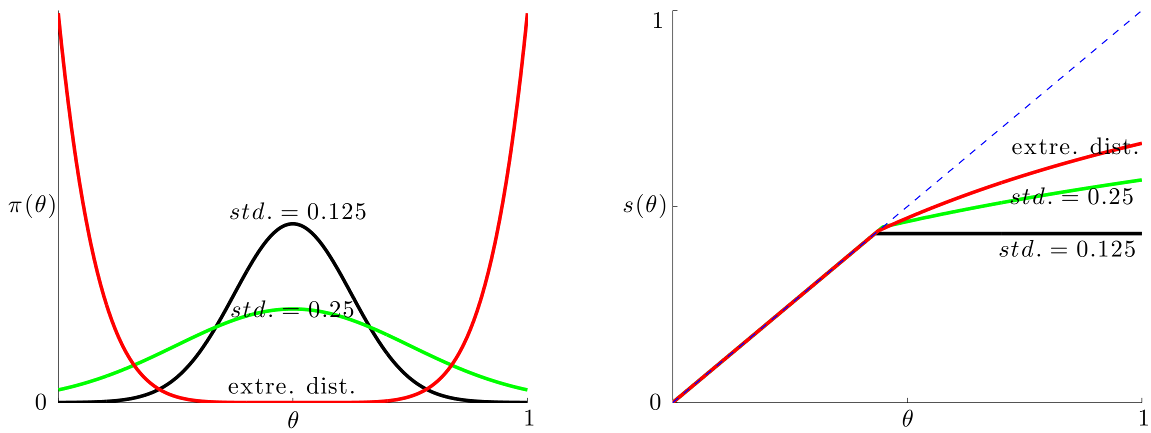

Figure 7: Change of Uncertainty: $k = 0.4 < \mathbb{E}[\theta] = 0.5$, $\mu = 0.2$

The intuition for this case is the following. Also, keep in mind that we have $k < \mathbb{E}(\theta)$ in this case, which means the project has a standard positive expected NPV and investor may acquire information to screen out potentially bad projects. When the uncertainty of the project is smaller, the project is more likely to be good, which implies screening out is of less relevance and it is less desirable to compensate the investor to induce costly information acquisition. Again, the first force as described, namely, the value of information, decreases. Hence, the resulting security moves towards standard debt. On the contrary, when the uncertainty is greater, the project is more likely to be bad and screening out becomes more necessary. Hence, the entrepreneur is more willing to propose a more generous convertible preferred stock to the investor to induce information acquisition.

Then we consider Case ii) in Proposition 5, namely, $\mathbb{E}[\theta] \leqslant k$. This implies that the project has a non-positive expected NPV, given the prior. But it might still be financed with positive probability in equilibrium, thanks to screening in good projects by the investor. In contrast, in this case, the face value $\widehat{\theta}$ of convertible preferred stock, is decreasing in the uncertainty of the project. The cash flow after converting is also decreasing in the uncertainty. This means, when the project is more uncertain, the compensation to the investor tends to be lower.

The numerical simulations are as in Figure 8. The initial investment is fixed at $k = 0.525 \geqslant \mathbb{E}[\theta]$ and the information cost μ is still fixed at 0.2. Similarly, the families of distributions of θ are generated as we did in Figure 4. All the distributions are normalized after the truncation. Again, the left panel illustrates the distributions of θ we used, and the right panel illustrates the evolution of the optimal security.

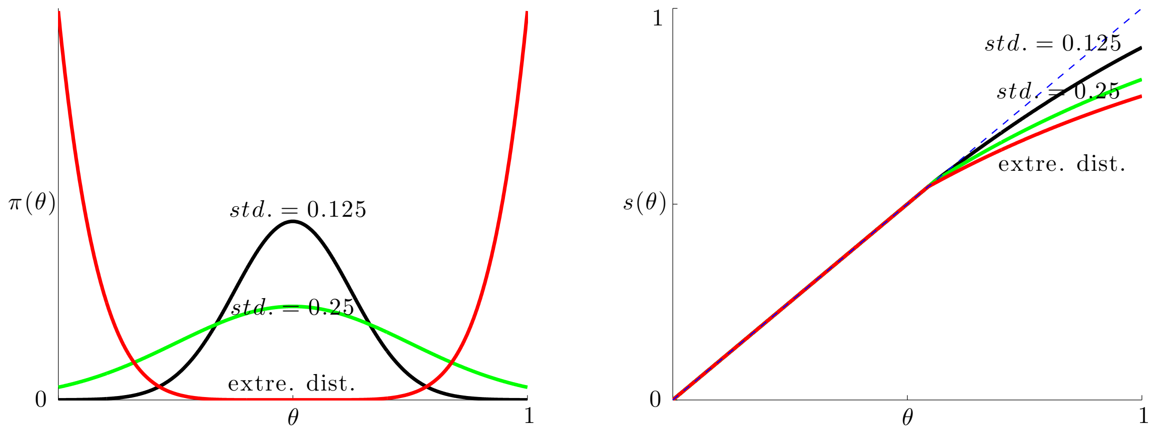

Figure 8: Change of Uncertainty: $k = 0.525 \geqslant \mathbb{E}[\theta] = 0.5$, $\mu = 0.2$

The intuition for this case is similar. Keep in mind that we have $k \geqslant \mathbb{E}(\theta)$ in this case, which means the project has a non-positive expected NPV and the investor acquires information to screen in potentially good projects. In contrast to the previous case, when the project is less uncertain, the project is most like to be bad and screening in potentially good projects becomes more necessary. This means the first force, namely, the value of information increases. Hence, the entrepreneur is more willing to propose a more generous convertible preferred stock to the investor to induce information acquisition. Rather, when the project is more uncertain, the project is more likely to be good and thus screening in becomes less necessary, so that the resulting security moves away from the 45° degree line.

5.3 Investor's Information Advantage

Last, we consider the effect of changing the investor's information cost μ on the optimal security $s^*(\theta)$, with the distribution of θ and the initial investment k fixed. Again, interestingly, the comparative statics in this case depends on the relationship between the mean of the fundamental $\mathbb{E}[\theta]$ and the initial investment k, as the two cases in Proposition 5.

First, in Case i) of $\mathbb{E}[\theta] > k$, we show that, the face value $\widehat{\theta}$ of convertible preferred stock, if it is the optimal security in equilibrium, is increasing in the information cost of the project. The concavity of the cash flow after converting is also decreasing in the information cost. Interestingly, the converting rate is always decreasing in the information cost, which means the investor is less compensated in good states, if she has a higher information cost. When the information cost is sufficiently large, the investor would not acquire information anymore, and the optimal security would be standard debt, with a higher face value.

In the numerical simulations as in Figure 9, the initial investment is fixed at $k = 0.4 < \mathbb{E}(\theta)$. The distribution of θ is also fixed and generated as follows. We take a normal distribution with mean 0.5 and standard deviation 2, and then truncate this distribution to interval $[0, 1]$. The information cost μ will be taking there different values: 0.2, 0.4, and 1.

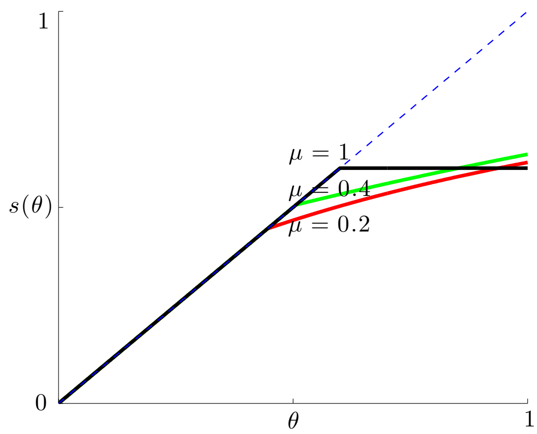

Figure 9: Change of Information Cost: $k = 0.4 < \mathbb{E}[\theta] = 0.5$

The intuition for this case is as follows. We have $k < \mathbb{E}(\theta)$ in this case, which means the project has a positive expected NPV and the investor may acquire information to screen out potentially bad projects. When the investor's information cost becomes larger, it is harder for the entrepreneur to incentivize the investor to screen out. Hence, the entrepreneur has to give up more to the investor by raising the face value of the proposed security, which make the investor comfortable in financing some potentially bad projects. Moreover, as information acquisition is more costly, the entrepreneur would like to encourage less information acquisition by proposing flatter cash flow in good states, which help lower the investor's information cost in equilibrium. As a result, the cash flow after converting would become less in this case.

Then we consider Case ii) of $\mathbb{E}[\theta] \leqslant k$. We show that, the face value $\widehat{\theta}$ of convertible preferred stock is still increasing in the information cost of the project, and the concavity of the cash flow after converting is also decreasing in the information cost. Nevertheless, the converting rate is always increasing in the information cost, which means the investor is more compensated in good states, if she has a higher information cost.

The numerical simulations are in Figure 10. The initial investment is fixed at $k = 0.6 \geqslant \mathbb{E}(\theta)$. The distribution of θ is also fixed and generated as follows. Again, we take a normal distribution with mean 0.5 and standard deviation 2, and then truncate this distribution to interval $[0, 1]$. Similarly, The information cost will be taking there different values: 0.075, 0.125, and 0.225.

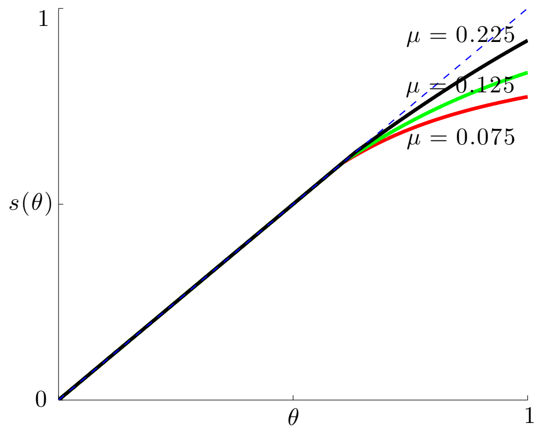

Figure 10: Change of Information Cost: $k = 0.6 \geqslant \mathbb{E}[\theta] = 0.5$

The intuition for this case is straightforward under our unified explanation. Recall we have $k \geqslant \mathbb{E}(\theta)$ in this case, which means the project has a non-positive expected NPV and the investor may acquire information to screen in good projects. When the investor's information cost becomes larger, it is harder for the entrepreneur to incentivize the investor to screen in, so that the entrepreneur has to offer more. Especially, as information becomes more costly in this case, the entrepreneur wants to encourage information acquisition by proposing more cash flow after converting to compensate for the investor's higher information cost.

6 Conclusion

Highlighting information acquisition, this paper have investigated security design in a production economy. The key friction in the economy is that, real production depends on information production while these two are separately performed by an entrepreneur and an investor. A new pecking order of optimal securities has been predicted: standard debt, which does not induce information acquisition, is optimal when the dependence is weak and thus the friction is not severe, and convertible preferred stock, which induces information acquisition, is optimal when the dependence is strong and thus the friction is severe. Both the optimal securities and the pecking order are supported by empirical evidence.

This paper contributes on several fronts of security design and information acquisition. Information insensitive and sensitive securities have been unified to a single origin: in screening and financing different projects, the investor's information expertise is called for in different extents and manners, which further shapes the optimal securities. The comparison between the production economy and an exchange economy further highlights this origin. Thanks to the new concept of flexible information acquisition, we have been able to characterize optimal securities on continuous states without any distributional assumptions.

References

Admati, Anat and Pfleiderer, Paul (1994). "Robust Financial Contracting and the Role of Venture Capitalists." *Journal of Finance*, 49: 371-402.

Aghion, Philippe and Tirole, Jean (1992). "On The Management of Innovation." *Quarterly Journal of Economics*, 109: 1185-1207.

Bergemann, Dirk and Hege, Ulrich (1998). "Venture Capital Financing, Moral Hazard, and Learning." *Journal of Banking and Finance*, 22: 703-35.

Berglof, Erik (1994). "A Control Theory of Venture Capital Finance." *Journal of Law, Economics and Organization*, 10: 247-67.

Biais, Bruno and Mariotti, Thomas (2005). "Strategic Liquidity Supply and Security Design." *Review of Economic Studies*, 72: 617-49.

Bond, Philip, Edmans, Alex and Goldstein, Itay (2012). "The Real Effects of Financial Markets." *Annual Review of Finance and Economics*, 4(2): 1-22.

Boot, Arnoud and Thakor, Anjan (1993). "Security Design." *Journal of Finance*, 48: 1349-1378.

Brennan, Michael and Kraus, Alan (1987). "Efficient Financing Under Asymmetric Information." *Journal of Finance*, 42: 1225-1243.

Casamatta, Catherine (2003). "Financing and Advising: Optimal Financial Contracts with Venture Capitalists." *Journal of Finance*, 58(5): 2059-2085.

Chakraborty, Archishman and Yilmaz, Bilge (2011). "Adverse Selection and Covertible Bonds." *Review of Economic Studies*, 78: 148-175.

Chemmanur, Thomas, Krishnan, Karthik and Nandy, Debarshi (2012). "How Does Venture Capital Financing Improve Efficiency in Private Firms? A Look Beneath the Surface." *Review of Financial Studies*, forthcoming.

Cremer, Jacques and Khalil, Fahad (1992). "Gathering Information Before Signing a Contract." *American Economic Review*, 82: 566-578.

Cremer, Jacques, Khalil, Fahad and Rochet, Jean-Charles (1998a). "Strategic Information Gathering before a Contract Is Offered." *Journal of Economic Theory*, 81: 163-200.

Cremer, Jacques, Khalil, Fahad and Rochet, Jean-Charles (1998b). "Contracts and Productive Information Gathering." *Games and Economic Behavior*, 25: 174-193.

Cornelli, Francesca and Yosha, Oved (2003). "Stage Financing and the Role of Convertible Securities." *Review of Economic Studies*, 70: 1-32.

Da Rin, Marco, Hellmann, Thomas and Puri, Manju (2011). "A Survey of Venture Capital Research." Forthcoming in George Constantinides, Milton Harris, and Rene Stulz (eds), *Handbook of the Economics of Finance*, 2.

Dang, Tri Vi, Gorton, Gary and Holmstrom, Bengt (2011). "Ignorance and the Optimality of Debt for Liquidity Provision." Working paper. http://sloanweb.mit.edu/finance/pdf/Gary%20Gorton.pdf

de Bettignies, Jean-Etienne (2008). "Financing the Entrepreneurial Venture." *Management Science*, 54: 151-166.

DeMarzo, Peter and Duffie, Darrell (1999). "A Liquidity-Based Model of Security Design." *Econometrica*, 67: 65-99.

DeMarzo, Peter (2005). "The Pooling and Tranching of Securities: A Model of Informed Intermediation." *Review of Financial Studies*, 18: 1-35.

Dow, James, Goldstein, Itay and Guembel, Alexander (2011). "Incentives for Information Production in Markets where Prices Affect Real Investment." Working Paper. http://finance.wharton.upenn.edu/~itayg/Files/informationproduction.pdf

Evans, David and Jovanovic, Boyan (1989). "An Estimated Model of Entrepreneurial Choice under Liquidity Constraints." *Journal of Political Economy*, 97: 808-827.

Fulghieri, Paolo and Lukin, Dmitry (2001). "Information Production, Dilution Costs, and Optimal Security Design." *Journal of Financial Economics*, 61: 3-42.

Fulghieri, Paolo, Garcia, Diego and Hackbarth, Dirk (2012). "Asymmetric Information and the Pecking Order." Working Paper. http://public.kenan-flagler.unc.edu/faculty/fulghiep/Fulghieri-Garcia-Hackbarth-Nov%2014-2012.pdf

Gompers, Paul (1999). "Ownership and Control in Entrepreneurial Firms: An Examination of Convertible Securities in Venture Capital Investment." Working Paper.

Gorton, Gary and Pennacchi, George (1990). "Financial Intermediaries and Liquidity Creation." *Journal of Finance*, 45: 49-72.

Hellmann, Thomas (1998). "The Allocation of Control Rights in Venture Capital Contracts." *RAND Journal of Economics*, 29: 57-76.

Hellmann, Thomas (2006). "IPOs, Acquisitions, and the Use of Convertible Securities in Venture Capital." *Journal of Financial Economics*, 81: 649-679.

Hennessy, Christopher (2009). "Security Design, Liquidity, and the Informational Role of Prices." Working Paper. http://faculty.london.edu/chennessy/assets/documents/DRAFT18.pdf

Hennessy, Christopher (2011). "Equilibrium Security Design and Liquidity Creation by Privately Informed Issuers." Working Paper. http://faculty.london.edu/chennessy/assets/documents/Equilibrium_Security_Design_and_Liquidity_Creation_by_Privately_Informed_Issuers.pdf

Holtz-Eakin, Douglas, Joulfaian, David and Rosen, Harvey (1994). "Sticking It Out: Entrepreneurial Survival and Liquidity Constraints." *Journal of Political Economy*, 102: 53-75.

Inderst, Roman and Mueller, Holger (2006). "Informed Lending and Security Design." *Journal of Finance*, 61: 2137-2162.

Innes, Robert (1990). "Limited Liability and Incentive Contracting with Ex-ante Action Choices." *Journal of Economic Theory*, 52: 45-67.

Kaplan, Steven and Lerner, Josh (2010). "It Ain't Broke: The Past, Present, and Future of Venture Capital." *Journal of Applied Corporate Finance*, 22: 36-47.

Kaplan, Steven and Stromberg, Per (2003). "Financial Contracting Theory Meets the Real World: An Empirical Analysis of Venture Capital Contracts." *Review of Economic Studies*, 70: 281-315.

Kerr, William, Lerner, Josh and Schoar, Antoinette (2011). "The Consequences of Entrepreneurial Finance: Evidence from Angel Financings." *Review of Financial Studies*, forthcoming.

Kerr, William and Nanda, Ramana (2009). "Democratizing Entry: Banking Deregulations, Financing constraints, and Entrepreneurship." *Journal of Financial Economics*, 94: 124-149.

Kirilenko, Andrei (2001). "Valuation and Control in Venture Finance." *Journal of Finance* 56: 565-587.

Knight, Frank (1921). *Risk, Uncertainty, and Profit.* Boston: Houghton Mifflin.

Lerner, Josh, Leamon, Ann and Hardymon, Felda (2012). *Venture Capital, Private Equity, and the Financing of Entrepreneurship.* John Wiley & Sons, Inc.

Myers, Stewart and Majluf, Nicholas (1984). "Corporate Financing and Investment Decisions when Firms Have Information that Investors Do Not Have." *Journal of Financial Economics*, 13: 187-221.

Morris, Stephen and Shin, Hyun Song (1998). "Unique Equilibrium in a Model of Self-Filling Currency Attacks." *American Economic Review*, 88: 587-597.

Nachman, David and Noe, Thomas (1994). "Optimal Design of Securities under Asymmetric Information." *Review of Financial Studies*, 7:1-44.

Petersen, Mitchell and Rajan, Raghuram (1994). "The Benefits of Lending Relationships Evidence from Small Business Data." *Journal of Finance*, 49: 3-37.

Rajan, Raghu (2012). "Presidential Address: The Corporation in Finance." *Journal of Finance*, 67: 1173-1217.

Repullo, Rafael and Suarez, Javier (2004). "Venture Capital Finance: A Security Design Approach." *Review of Finance*, 8: 75-108.

Sahlman, William (1990). "The Structure and Governance of Venture-Capital Organizations." *Journal of Financial Economics*, 27: 473-521.

Scharfstein, David and Stein, Jeremy (2000). "The Dark Side of Internal Capital Markets: Divisional Rent-Seeking and Inefficient Investment." *Journal of Finance,* 55: 2537-2564.

Schmidt, Klaus (2003). "Convertible Securities and Venture Capital Finance." *Journal of Finance*, 58:1139-1166.

Schumpeter, Joseph (1942). *Capitalism, Socialism, and Democracy.* New York: Harper Brothers.

Shannon, Claude (1948). "A Mathematical Theory of Communication." *Bell System Technical Journal*, 27: 379-423.

Sims, Christopher (2003). "Implications of Rational Inattention." *Journal of Monetary Economics*, 50: 665-690.

Stein, Jeremy (1997). "Internal Capital Markets and the Competition for Corporate Resources." *Journal of Finance*, 52: 111-133.

Tirole, Jean (2006). *The Theory of Corporate Finance.* Princeton University Press.

Townsend, Robert (1979). "Optimal Contracts and Competitive Markets with Costly State Verification." *Journal of Economic Theory*, 21: 265-293.

Veldkamp, Laura (2011). *Information Choice in Macroeconomics and Finance.* Princeton University Press.

Woodford, Michael (2008). "Inattention as A Source of Randomized Discrete Adjustment." Working paper. http://www.columbia.edu/~mw2230/Random%20Discrete%20Adjustment.pdf

Woodford, Michael (2009). "Information-Constrained State-Dependent Pricing." *Journal of Monetary Economics*, 56: S100-S124.

Yang, Ming (2012a). "Optimality of Debt with Flexible Information Acquisition." Working paper. http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2103971

Yang, Ming (2012b). "Coordination with Flexible Information Acquisition." http://papers.ssrn.com/sol3/papers.cfm?abstract_id=2103970

A Appendix

A.1 Derivation of Convertible Preferred Stock as Optimal Security

This appendix derives the optimal security $s^*(\theta)$ when it induces information acquisition. To make the intuition clearer, we proceed by two steps.

First, we solve for an "unconstrained" optimal security in this case without the feasibility condition $0 \leqslant s(\theta) \leqslant \theta$. We denote the solution by $\widehat{s}(\theta)$. We also denote the corresponding information acquisition rule by $\widehat{m}_s(\theta)$.[12] We try to highlight that the feasibility condition is a mechanical restriction on the potential securities. Importantly, it is not directly relevant to the entrepreneur's motives in designing the securities, which are aiming to induce the investor to acquire information that is most favorable to the entrepreneur. Hence, the unconstrained optimal security $\widehat{s}(\theta)$ helps to reveal the relationship between security design and information acquisition in a clearer manner. After that, we resume the feasibility condition and characterize the exact optimal security $s^*(\theta)$. This two-step approach streamlines our presentation and makes the intuition clearer.

For the unconstrained optimal security $\widehat{s}(\theta)$ in the case with information acquisition, we have the following lemma.

Lemma **3.** *In an equilibrium with information acquisition, the unconstrained optimal security $\widehat{s}(\theta)$ and its corresponding information acquisition rule $\widehat{m}_s(\theta)$ are determined by*

$$\widehat{s}(\theta) - k = \mu \cdot \left(g'\left(\widehat{m}_s(\theta)\right) - g'\left(\overline{\pi}_s^*\right)\right) \ , \tag{A.1}$$

where

$$\overline{\pi}_s^* = \mathbb{E}\left[m_s^*(\theta)\right] \ ,$$

and

$$(1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) = \mu \, , \tag{A.2}$$

where

$$w^* = \mathbb{E}\left[(\theta - s^*(\theta)) \frac{g''(\overline{\pi}_s^*)}{g''(m_s^*(\theta))}\right] \left(1 - \mathbb{E}\left[\frac{g''(\overline{\pi}_s^*)}{g''(m_s^*(\theta))}\right]\right)^{-1} \ ,$$

in which $\overline{p}_s^*$ and w^* are two constants that do not depend on θ .

Lemma 3 exhibits the relationship between the unconstrained optimal security and the corresponding decision rule for information acquisition. Recall Proposition 1, condition (A.1) specifies how the investor responses to the unconstrained optimal security $\widehat{s}(\theta)$ by adjusting her decision

[12]Note that, $\widehat{m}_s(\theta)$ is not the solution to the entrepreneur's problem without the feasibility condition, but is a translation of that solution. This will be seen clearer in the statement of Lemma 3.

rule of information acquisition $\widehat{m}_s(\theta)$. On the other hand, condition (A.2) is derived from the entrepreneur's optimization problem. It describes the entrepreneur's optimal choices of cash flow across states, which is summarized by the unconstrained optimal security, given the investor's decision rule of information acquisition. In equilibrium, $\widehat{s}(\theta)$ and $\widehat{m}_s(\theta)$ are jointly determined. This again highlights the close relationship between security design and information acquisition in our context.

Although it is mathematically difficult to solve the system of equations (A.1) and (A.2), we are able to deliver some important analytical characteristics of the unconstrained optimal security $\widehat{s}(\theta)$ and the corresponding information acquisition rule $\widehat{m}_s(\theta)$. In particular, we are confident enough to speak to the shape of the unconstrained optimal security $\widehat{s}(\theta)$ as well as the actual optimal security $s^*(\theta)$ by these analytical properties. The following lemma gives the key results.

Lemma **4.** *In an equilibrium with information acquisition, the unconstrained optimal security* $\widehat{s}(\theta)$ *and the corresponding information acquisition rule* $\widehat{m}_s(\theta)$ *satisfy*

$$\frac{\partial \widehat{m}_s(\theta)}{\partial \theta} = \mu^{-1} \cdot \widehat{m}_s(\theta) \cdot (1 - \widehat{m}_s(\theta))^2 > 0 \ , \tag{A.3}$$

and

$$\frac{\partial \widehat{s}(\theta)}{\partial \theta} = 1 - \widehat{m}_s(\theta) > 0 \, . \tag{A.4}$$

We have several interesting observations from Lemma 4. First, condition (A.3) implies that the unconstrained optimal information acquisition rule $\widehat{m}_s(\theta)$ is strictly increasing. Second, condition (A.4) implies that the unconstrained optimal security $\widehat{s}(\theta)$ is also strictly increasing. These are because, according to Proposition 1, we have $Prob[0 < \widehat{m}_s(\theta) < 1] = 1$ in this case, and thus the right hand sides of (A.3) and (A.4) is positive. Last, the unconstrained optimal security $\widehat{s}(\theta)$ is strictly concave. This is because conditions (A.3) and (A.4) imply

$$\frac{\partial^2 \widehat{s}(\theta)}{\partial \theta^2} = -\mu^{-1} \cdot \widehat{m}_s(\theta) \cdot (1 - \widehat{m}_s(\theta))^2 < 0 \ .$$

Therefore, the unconstrained optimal security $\widehat{s}(\theta)$ is an increasing concave function of θ.

The monotonicity of the unconstrained information acquisition rule $\widehat{m}_s(\theta)$ as shown in (A.3) is intuitive. In our context, the investor provides two different types of services to the entrepreneur. The first is the investment required to initiate the project, and the second is the information about the project's market prospect, which implies screening in good projects and screening out bad projects. In other words, a better fundamental would more likely generate a good signal and result in a successful finance, while a worse fundamental would more likely generate a bad signal and result in a rejection. As a result, the screening service makes sense only if the unconstrained optimal information acquisition rule $\widehat{m}_s(\theta)$ is increasing.

Moreover, the monotonicity of $\widehat{m}_s(\theta)$ has two important implications on the shape of the unconstrained optimal security $\widehat{s}(\theta)$. On the one hand, according to condition (A.1), $\widehat{s}(\theta)$ is increasing because $\widehat{m}_s(\theta)$ is increasing. This reflects the dependence of real production on information production: the entrepreneur is willing to compensate the investor more in an event of higher cash flow to encourage screening. On the other hand, according to condition (A.2), $\theta - \widehat{s}(\theta)$ is also increasing because $\widehat{m}_s(\theta)$ is increasing. This however reflects the separation of real production and information production: the entrepreneur wants to retain as much as possible. Again as $\widehat{m}_s(\theta)$ is increasing, the competition of the two effects above implies that the least costly way for the entrepreneur to encourage the investor to acquire information is to reward the investor more but also retain more in higher states.

Given the unconstrained optimal security $\widehat{s}(\theta)$, it is instructive to have the following lemma to illustrate the possible relative positions between the unconstrained optimal security and the feasibility constraints.

Lemma **5.** *Three possible relative positions between the unconstrained optimal security $\widehat{s}(\theta)$ and the feasibility constraints $0 \leqslant s(\theta) \leqslant \theta$ may occur in equilibrium, in the $\theta \sim s$ space:*

i) $\widehat{s}(\theta)$ intersects with the $45°$ line $s = \theta$ at $(\widehat{\theta}, \widehat{\theta})$, $\widehat{\theta} > 0$, and does not intersect with the horizontal axis $s = 0$.

ii) $\widehat{s}(\theta)$ goes through the origin $(0, 0)$, and does not intersect with either the $45°$ line $s = \theta$ or the horizontal axis $s = 0$ for any $\theta \neq 0$.

iii) $\widehat{s}(\theta)$ intersects with the horizontal axis $s = 0$ at $(\widetilde{\theta}, 0)$, $\widetilde{\theta} > 0$, and does not intersect with the $45°$ line $s = \theta$.

In the three different cases, it is easy to imagine that the actual optimal security $s^*(\theta)$ will be constrained by the feasibility condition in different ways. For example, the optimal security will be constrained by the $45°$ line $s = \theta$ in Case i) and by the horizontal axis $s = 0$ in Case iii). Our concern is whether the presence of the two feasibility constraints would significantly change the interplay between security design and information acquisition, and thus the resulting optimal security. Fortunately, the answer is no. As expected, the feasibility constraints are only mechanical. By imposing the feasibility conditions, we have the following characterization for the optimal security:

Lemma **6.** *In an equilibrium with information acquisition, the corresponding optimal security $s^*(\theta)$ satisfies*

$$s^*(\theta) = \begin{cases} \theta & \text{if} \quad \widehat{s}(\theta) > \theta \\ \widehat{s}(\theta) & \text{if} \quad 0 \leqslant \widehat{s}(\theta) \leqslant \theta \\ 0 & \text{if} \quad \widehat{s}(\theta) < 0 \end{cases},$$

where $\widehat{s}(\theta)$ is the corresponding unconstrained optimal security.

Lemma 6 is instructive because it tells us how to construct an optimal security with information acquisition from its corresponding unconstrained optimal security. Concretely, the optimal security $s^*(\theta)$ will follow its corresponding unconstrained optimal security $\widehat{s}(\theta)$ when the latter is within the feasible region $0 \leqslant s \leqslant \theta$. When the unconstrained optimal security is out of the feasible region, the resulting optimal security will follow one of the feasibility constraints that is binding. The expression of Lemma 6 is fairly simple but the result is not trivial. Importantly, the presence of the feasibility constraints indeed changes the shapes of the resulting optimal securities from its unconstrained counterparts, which implies that the investor's incentives of information acquisition is also changed. Nevertheless, Lemma 6 ensures us that such change does not affect the entrepreneur's choice of the cash flow allocations in the states where the feasibility constraints are not binding. Also, in the states where the feasibility constraints are binding, Lemma 6 tells us that it is still optimal for the entrepreneur to just hit the binding constraints to exploit the investor's information advantage to the largest extent.

Therefore, we can apply Lemma 6 to the three cases of the unconstrained optimal security $\widehat{s}(\theta)$ described in Lemma 5. This gives the three potential cases of the optimal security $s^*(\theta)$, respectively.

LEMMA **7.** *In an equilibrium with information acquisition, the optimal security $s^*(\theta)$ may take one of the following three forms:*

i) When the corresponding unconstrained optimal security $\widehat{s}(\theta)$ intersects with the $45°$ line $s = \theta$ at $(\widehat{\theta}, \widehat{\theta})$, $\widehat{\theta} > 0$, we have

$$s^*(\theta) = \begin{cases} \theta & \text{if} \quad 0 \leqslant \theta < \widehat{\theta} \\ \widehat{s}(\theta) & \text{if} \quad \theta \geqslant \widehat{\theta} \end{cases}.$$

ii) When the corresponding unconstrained optimal security $\widehat{s}(\theta)$ goes through the origin $(0,0)$, we have $s^*(\theta) = \widehat{s}(\theta)$ for $\theta \in \mathbb{R}_+$.

iii) When the corresponding unconstrained optimal security $\widehat{s}(\theta)$ intersects with the horizontal axis $s = 0$ at $(\widetilde{\theta}, 0)$, $\widetilde{\theta} > 0$, we have

$$s^*(\theta) = \begin{cases} 0 & \text{if} \quad 0 \leqslant \theta < \widetilde{\theta} \\ \widehat{s}(\theta) & \text{if} \quad \theta \geqslant \widetilde{\theta} \end{cases}.$$

The three potential cases of the optimal security $s^*(\theta)$ take different shapes. Specifically, in Case i), the optimal security follows a standard debt in bad states but increases as a concave function in good states. In Case iii), the payment of optimal security in bad states is zero, while is an increasing and concave function in good states. Case ii) lies in between as a cut-off case, in which the payment of optimal security is an increasing and concave function.

Having characterizing the potential cases of the optimal security $s^*(\theta)$ by its differential properties, we proceed by checking whether these three potential cases are indeed the valid solution to the entrepreneur's problem in an equilibrium with information acquisition. Interestingly, not all the three cases can occur in equilibrium. The following proposition tells us that only the shape in Case i) can actually sustain an equilibrium with information acquisition.

LEMMA **8.** *If the entrepreneur's optimal security $s^*(\theta)$ induces the investor to acquire information in equilibrium, then it must follow Case i) in Lemma 7, which corresponds to a participating convertible preferred stock with a face value $\widehat{\theta}$.*

Together with the results already established, this lemma straightforwardly leads to Proposition 3. Recall that the investor can always reject the offer and enjoy her outside option, which is normalized to zero in the context, if her expected payoff is negative. The rejection of the security, however, is always sub-optimal to the entrepreneur, as long as the project has a positive expected future cash flow. As a result, the entrepreneur wants to make sure that the investor, who makes the initial investment and takes cost to acquire information, is sufficiently compensated so that she is willing to accept the security.

A.2 General Allocation of Bargaining Powers

This appendix extends our benchmark model to a more general setting which allows for arbitrary allocation of the bargaining powers between the entrepreneur and the investor in security design. Suppose a third party in the economy knows the relative bargaining powers of the entrepreneur and the investor. The entrepreneur's bargaining power in security design is $1-\alpha$ and the investor's is α. The third party designs the security and proposes it to the investor. The investor acquire information according to the security she gets and decides whether or not to accept this security. The third party's objective function is a weighted average of the entrepreneur's and the investor's utilities. The weights represent the bargaining powers of the two, respectively. When $\alpha = 0$, this reduces to our benchmark decentralized model.

In this setting, the third-party's objective function, namely, the payoff gain is

$$u_T(s(\theta)) = \alpha \cdot (\mathbb{E}[(s(\theta) - k) \cdot m(\theta)] - \mu \cdot I(m)) + (1 - \alpha) \cdot \mathbb{E}[(\theta - s(\theta)) \cdot m(\theta)] .$$

We show that, the differential equation that governs information acquisition is still as same as condition (3.6):

$$s\left(\theta\right) - k = \mu \cdot \left(g'\left(m_s\left(\theta\right)\right) - g'\left(\overline{\pi}_s\right)\right) \ ,$$

while the other differential equation that characterizes the optimality of the unconstrained security is given as

$$r(\theta) = (2\alpha - 1) \cdot m(\theta) + (1 - \alpha) \cdot \mu^{-1} \cdot m(\theta) \cdot (1 - m(\theta)) \cdot (\theta - s(\theta) + w)\,.$$

We have the following two propositions to characterize the optimal security in the general setting. The proofs are similar to their counterparts in the main text so that we omit them.

PROPOSITION **7.** *When $0 \leqslant \alpha < 1/2$ and information acquisition happens in equilibrium, the unconstrained optimal security $\widehat{s}(\theta)$ and the corresponding information acquisition rule $\widehat{m}_s(\theta)$ satisfy*

$$\frac{d\widehat{s}\,(\theta)}{d\theta} = \frac{1 - \widehat{m}_s\,(\theta)}{1 - \frac{\alpha}{1-\alpha}\widehat{m}_s(\theta)} \in (0, 1)$$

and

$$\frac{d\widehat{m}_s\,(\theta)}{d\theta} = \frac{\mu^{-1} \cdot \widehat{m}_s\,(\theta) \cdot (1 - \widehat{m}_s\,(\theta))^2}{1 - \frac{\alpha}{1-\alpha}\widehat{m}_s(\theta)} > 0\ .$$

Also, all the results from Lemma 5 to Lemma 7 and from Proposition 1 to Proposition 5 still hold.

PROPOSITION **8.** *When $1/2 \leqslant \alpha \leqslant 1$, the optimal security features $s^*(\theta) = \theta$.*

Our generalized results show that, when the investor has some bargain power in security design but it is still the entrepreneur who dominates, all the qualitative results keep the same. Nevertheless, if the investor dominates, the optimal security would favor the investor to the greatest extent. The latter case looks counterintuitive at a first glance. But it is indeed consistent with our context, in which the ability of information acquisition should also be accounted as a part of the investor's bargaining power. In the benchmark model, the entrepreneur proposes the security but the investor can acquire information, which exhibits a balance of total bargaining powers. Instead, when even the role of proposing the security goes to the investor, the investor is too powerful in the sense of total bargain powers.

A.3 Proofs

This appendix provides all proofs omitted above.

PROOF OF LEMMA 1. We first prove the "only if" part.

Suppose that $\mathbb{E}\left[\exp(\mu^{-1}(\theta - k))\right] \leqslant 1$. According to Proposition 1, even if the entrepreneur proposes all the future cash flow to the investor, the investor would reject the offer without acquiring information. Since $s(\theta) \leqslant \theta$, the project cannot be initiated in this case.

Then we prove the "if" part.

Let $t \in (0,1)$. Since $\mathbb{E}\left[\exp(\mu^{-1}(t \cdot \theta - k))\right]$ is continuous in t, there exists $t < 1$ such that

$$\mathbb{E}\left[\exp(\mu^{-1}(t \cdot \theta - k))\right] > 1\,.$$

Hence, according to Proposition 1, the security $s_t(\theta) = t \cdot \theta$ would be accepted by the investor with positive probability. Moreover, let $m_t(\theta)$ be the corresponding information acquisition rule. As $s_t(\theta)$ would be accepted with positive probability, $m_t(\theta)$ cannot be always zero. Hence, the entrepreneur's expected payoff is $\mathbb{E}[(1-t) \cdot \theta \cdot m_t(\theta)]$, which is strictly positive.

Note that the security $s_t(\theta)$ is a feasible security. Hence, the optimal security $s^*(\theta)$ would also be accepted with positive probability and delivers positive expected payoff to the entrepreneur. This concludes the proof. □

Proof of Corollary 1. The proof is straightforward following the above proof of Lemma 1. Proposing $s^*(\theta) = \theta$ gives the entrepreneur a zero payoff, while proposing $s_t(\theta) = t \cdot \theta$ constructed in the proof of Lemma 1 gives her a strictly positive expected payoff. This suggests $s^*(\theta) = \theta$ is not optimal. □

Proof of Proposition 2. The Lagrangian of the entrepreneur's problem is

$$\mathscr{L} = \mathbb{E}\left[\theta - s(\theta) + \lambda \cdot \left(1 - \exp\left(\mu^{-1} \cdot (k - s(\theta))\right)\right) + \eta_1(\theta) \cdot s(\theta) + \eta_2(\theta) \cdot (\theta - s(\theta))\right]\,,$$

where λ, $\eta_1(\theta)$ and $\eta_2(\theta)$ are multipliers.

The first order condition is

$$\frac{d\mathscr{L}}{ds(\theta)} = -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (q - s(\theta))\right) + \eta_1(\theta) - \eta_2(\theta) = 0\,. \tag{A.5}$$

We first consider a special case that is helpful for us to solve the optimal security. If $0 < s(\theta) < \theta$, the two feasibility conditions are not binding. Thus $\eta_1(\theta) = \eta_2(\theta) = 0$, and the first order condition is simplified as

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - s(\theta))\right) = 0\,.$$

By rearrangement, we get

$$s(\theta) = k - \mu \cdot \ln(\lambda^{-1} \cdot \mu)\,. \tag{A.6}$$

We denote the right hand side of (A.6), which is irrelevant of θ, as D^*. By definition, we have $D^* > 0$. Also, it is straightforward to have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right) = 0\,. \tag{A.7}$$

In what follows, we characterize the optimal solution $s^*(\theta)$ for different regions of θ.

First, we consider the region of $\theta > D^*$. We show that $0 < s^*(\theta) < \theta$ in this region by contradiction.

If $s^*(\theta) = \theta > D^*$, we have $\eta_1(\theta) = 0$ and $\eta_2(\theta) \geqslant 0$. From the first order condition (A.5) we obtain

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - \theta)\right) = \eta_2(\theta) \geqslant 0 \,. \tag{A.8}$$

On the other hand, as $\theta > D^*$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right) > -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - \theta)\right) \,. \tag{A.9}$$

Conditions (A.7), (A.8), and (A.9) construct a contradiction. So we must have $s^*(\theta) < \theta$ if $\theta > D^*$.

Similarly, if $s^*(\theta) = 0$, we have $\eta_1(\theta) \geqslant 0$ and $\eta_2(\theta) = 0$. Again from the first order condition (A.5) we obtain

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot k\right) = -\eta_1(\theta) \leqslant 0 \,. \tag{A.10}$$

On the other hand, as $D^* > 0$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right) < -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot k\right) \,. \tag{A.11}$$

Conditions (A.7), (A.10), and (A.11) construct another contradiction. So we must have $s^*(\theta) > 0$ if $\theta > D^*$.

Therefore, we have shown that $0 < s^*(\theta) < \theta$ for $\theta > D^*$. From the discussion above for the special case, we conclude that $s^*(\theta) = D^*$ for $\theta > D^*$.

We then consider the region of $\theta < D^*$. We show that $s^*(\theta) = \theta$ in this region.

Since $s^*(\theta) \leqslant \theta < D^*$, we have

$$-1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - s^*(\theta))\right) > -1 + \lambda \cdot \mu^{-1} \cdot \exp\left(\mu^{-1} \cdot (k - D^*)\right) \,. \tag{A.12}$$

From condition (A.7), the right hand side of this inequality (A.12) is zero. Together with the first order condition (A.5), the inequality (A.12) implies that $\eta_1(\theta) = 0$ and $\eta_2(\theta) > 0$. Therefore, we have $s^*(\theta) = \theta$ in this region.

Also, from the first order condition (A.5) and the condition (A.7), it is obvious that $s^*(D^*) = D^*$.

To sum up, the entrepreneur's optimal security without inducing the investor to acquire information features a standard debt with face value D^* determined by condition (A.6).

We need to check that there exists $D^* > 0$ and the corresponding multiplier $\lambda > 0$ such that

$$\mathbb{E}\left[\exp\left(-\mu^{-1} \cdot (\min(\theta, D^*) - k)\right)\right] = 1\,, \tag{A.13}$$

where D^* is determined by condition (A.6).

Consider the left hand side of condition (A.13). Clearly, it is continuous and monotonically decreasing in D^*. When D^* is sufficiently large, the left hand side of (A.13) approaches $\mathbb{E}\left[\exp\left(-\mu^{-1} \cdot (\theta - k)\right)\right]$, a number less than one, which is guaranteed by the condition (3.3) as well as the feasibility condition $s(\theta) \leqslant \theta$. On the other hand, when $D^* = 0$, it approaches $\exp\left(\mu^{-1} \cdot k\right)$, which is strictly greater than one. Hence, there exists $D^* > 0$ such that the condition (A.13) holds.

Moreover, from the condition (A.6), we also know that D^* is continuous and monotonically increasing in λ. When λ approaches zero, D^* approaches negative infinity, while when λ approaches positive infinity, D^* approaches positive infinity as well. Hence, for any $D^* > 0$ there exists a corresponding multiplier $\lambda > 0$.

Finally, suppose $D^* \leqslant k$. It is easy to see that this debt would be rejected by the investor due to Proposition 1, a contradiction. This concludes the proof. □

Proof of Lemma 3. We derive the entrepreneur's optimal security $s^*(\theta)$ and the corresponding unconstrained optimal security $\widehat{s}(\theta)$ through calculus of variations. Specifically, we characterize how the entrepreneur's expected payoff responds to the perturbation of her optimal security.

Let $s(\theta) = s^*(\theta) + \alpha \cdot \varepsilon(\theta)$ be an arbitrary perturbation of the optimal security $s^*(\theta)$. Note that the investor's optimal decision rule $m_s(\theta)$ appears in the entrepreneur's expected payoff $u_E(s(\cdot))$, according to condition (3.7), and it is implicitly determined by the proposed security $s(\theta)$ through the functional equation (3.6). Hence, we need first characterize how $m_s(\theta)$ varies with respect to the perturbation of $s^*(\theta)$. Taking derivative with respect to α at $\alpha = 0$ for both sides of (3.6) leads to

$$\begin{aligned}\mu^{-1}\varepsilon(\theta) &= g''\left(m_s^*(\theta)\right) \cdot \left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0} \\ &\quad - g''\left(\overline{\pi}_s^*\right) \cdot \mathbb{E}\left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0}\,.\end{aligned}$$

Take expectation of both sides and we get

$$\begin{aligned}&\mathbb{E}\left[\left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0}\right] \\ = \;& \mu^{-1} \cdot \left(1 - \mathbb{E}\left[\left(g''\left(m_s^*(\theta)\right)\right)^{-1}\right] \cdot g''\left(\overline{\pi}_s^*\right)\right)^{-1} \cdot \mathbb{E}\left[\left(g''\left(m_s^*(\theta)\right)\right)^{-1} \varepsilon(\theta)\right]\,.\end{aligned}$$

Combining the above two equations, for any perturbation $s(\theta) = s^*(\theta) + \alpha \cdot \varepsilon(\theta)$, the investor's decision rule of information acquisition $m_s(\cdot)$ is characterized by

$$\begin{aligned} \left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0} &= \mu^{-1} \cdot \left(g''\left(m_s^*(\theta)\right)\right)^{-1} \varepsilon(\theta) \\ &+ \frac{\mu^{-1} \cdot \left(g''\left(m_s^*(\theta)\right)\right)^{-1} \cdot \mathbb{E}\left[\left(g''\left(m_s^*(\theta)\right)\right)^{-1} \varepsilon(\theta)\right]}{\left(g''\left(\overline{\pi}_s^*\right)\right)^{-1} - \mathbb{E}\left[\left(g''\left(m_s^*(\theta)\right)\right)^{-1}\right]} . \end{aligned} \tag{A.14}$$

We interpret condition (A.14) briefly. The first term of the right hand side of (A.14) is the investor's local response to $\varepsilon(\theta)$. It is of the same sign as the perturbation $\varepsilon(\theta)$. When the repayment of the security increases at state θ, the investor is more likely to accept the security at this state. The second term measures the investor's average response to perturbation $\varepsilon(\theta)$ over all states. It is straightforward to verify that the denominator of the second term is positive due to Jensen's inequality. As a result, if the perturbation increases the investor's repayment on average over all states, she is more likely to accept the security as well.

Now we can calculate the variation of the entrepreneur's expected payoff $u_E(s(\cdot))$, according to condition (3.7). Taking derivative of $u_E(s(\cdot))$ with respect to α at $\alpha = 0$ leads to

$$\left.\frac{\partial u_E(s(\cdot))}{\partial \alpha}\right|_{\alpha=0} = \mathbb{E}\left[\left.\frac{\partial m_s(\theta)}{\partial \alpha}\right|_{\alpha=0} (\theta - s(\theta))\right] - \mathbb{E}\left[m_s^*(\theta) \cdot \varepsilon(\theta)\right] . \tag{A.15}$$

Substitute (A.14) into (A.15) and we get

$$\left.\frac{\partial u_E(s(\cdot))}{\partial \alpha}\right|_{\alpha=0} = \mathbb{E}\left[r(\theta) \cdot \varepsilon(\theta)\right] , \tag{A.16}$$

where

$$r(\theta) = -m_s^*(\theta) + \mu^{-1} \cdot \left(g''\left(m_s^*(\theta)\right)\right)^{-1} \cdot \left(\theta - s^*(\theta) + w^*\right) \tag{A.17}$$

and

$$w^* = \mathbb{E}\left[(\theta - s^*(\theta)) \frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*(\theta)\right)}\right] \left(1 - \mathbb{E}\left[\frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*(\theta)\right)}\right]\right)^{-1} .$$

Note that w^* is a constant that does not depend on θ and will be endogenously determined in the equilibrium. Besides, $r(\theta)$ is the Frechet derivative of the entrepreneur's expected payoff $u_E(s(\cdot))$ at $s^*(\theta)$, which measures the marginal contribution of any perturbation to the entrepreneur's expected payoff when the security is optimal. Specifically, the first term of (A.17) is the direct contribution of perturbing $s^*(\theta)$ disregarding the variation of $m_s^*(\theta)$, and the second term measures the indirect contribution through the variation of $m_s^*(\theta)$. This Frechet derivative $r(\theta)$ plays an important role in shaping the entrepreneur's optimal security.

To further characterize the optimal security, we discuss the Frechet derivative $r(\theta)$ in detail.

Recall that the optimal security would be restricted by the feasibility condition $0 \leqslant s^*(\theta) \leqslant \theta$. Let

$$A_0 = \{\theta \in \Theta : \theta \neq 0, s^*(\theta) = 0\} ,$$

$$A_1 = \{\theta \in \Theta : \theta \neq 0, 0 < s^*(\theta) < \theta\} ,$$

and

$$A_2 = \{\theta \in \Theta : \theta \neq 0, s^*(\theta) = \theta\} .$$

Clearly, $\{A_0, A_1, A_2\}$ is a partition of $\Theta \backslash \{0\}$. Since $s^*(\theta)$ is the optimal security, we have

$$\left.\frac{\partial u_E(s(\cdot))}{\partial \alpha}\right|_{\alpha=0} \leqslant 0$$

for any feasible perturbation $\varepsilon(\theta)$.[13] Hence, condition (A.16) implies

$$r(\theta) \begin{cases} \leqslant 0 & \text{if } \theta \in A_0 \\ = 0 & \text{if } \theta \in A_1 \\ \geqslant 0 & \text{if } \theta \in A_2 \end{cases} . \tag{A.18}$$

According to Proposition 1, when the optimal security $s^*(\theta)$ induces the investor to acquire information, we have $0 < m_s^*(\theta) < 1$ for all $\theta \in \Theta$. Hence, condition (A.18) can be rearranged as

$$\frac{r(\theta)}{m_s^*(\theta)} = -1 + \mu^{-1} \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \begin{cases} \leqslant 0 & \text{if } \theta \in A_0 \\ = 0 & \text{if } \theta \in A_1 \\ \geqslant 0 & \text{if } \theta \in A_2 \end{cases} . \tag{A.19}$$

Recall condition (3.6), given the optimal security $s^*(\theta)$, the investor's optimal decision rule of information acquisition $m_s^*(\theta)$ is characterized by

$$s^*(\theta) - k = \mu \cdot \left(g'(m_s^*(\theta)) - g'(\overline{\pi}_s^*)\right) , \tag{A.20}$$

where

$$\overline{\pi}_s^* = \mathbb{E}[m_s^*(\theta)]$$

is the investor's unconditional probability of accepting the optimal security $s^*(\theta)$. Conditions (A.19) and (A.20) as a system of functional equations jointly determine the optimal security $s^*(\theta)$ when it induces the investor's information acquisition.

Finally, when we focus on the unconstrained optimal security $\widehat{s}(\theta)$, note that is would not be

[13] A perturbation $\varepsilon(\theta)$ is feasible with respect to $s^*(\theta)$ if there exists $\alpha > 0$ such that for any $\theta \in \Theta$, $s^*(\theta) + \alpha \cdot \varepsilon(\theta) \in [0, \theta]$.

restricted by the feasibility condition. Hence, the corresponding Frechet derivative $r(\theta)$ would be always zero at the optimum. On the other hand, the investor's optimal decision rule would not be affected. As a result, the conditions (A.20) and (A.19) become

$$\widehat{s}(\theta) - k = \mu \cdot \left(g'\left(\widehat{m}_s\left(\theta\right)\right) - g'\left(\overline{\pi}_s^*\right)\right) \ ,$$

where

$$\overline{p}_s^* = \mathbb{E}\left[m_s^*(\theta)\right] \ ,$$

and

$$(1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) = \mu \, ,$$

where

$$w^* = \mathbb{E}\left[(\theta - s^*(\theta)) \frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*(\theta)\right)}\right]\left(1 - \mathbb{E}\left[\frac{g''\left(\overline{\pi}_s^*\right)}{g''\left(m_s^*(\theta)\right)}\right]\right)^{-1} \ ,$$

in which $\overline{p}_s^*$ and w^* are two constants that do not depend on θ. This concludes the proof. □

Proof of Lemma 4. From Lemma 3, $(\widehat{s}(\theta), \widehat{m}_s(\theta))$ satisfies the two differential equations (A.1) and (A.2). By condition (A.2), we get

$$\widehat{m}_s(\theta) = 1 - \frac{\mu}{\theta - \widehat{s}\left(\theta\right) + w^*} \ . \tag{A.21}$$

Substituting (A.21) into (A.1) leads to

$$\mu^{-1}\left(\widehat{s}(\theta) - k\right) = g'\left(\frac{\mu}{\theta - \widehat{s}\left(\theta\right) + w^*}\right) - g'\left(\overline{\pi}_s^*\right) \ .$$

Taking derivatives of both sides of the above equation with respect to θ leads to

$$\begin{aligned}
\mu^{-1} \cdot \frac{d\widehat{s}\left(\theta\right)}{d\theta} &= g''\left(\widehat{m}_s\left(\theta\right)\right) \cdot \frac{d\widehat{m}_s\left(\theta\right)}{d\theta} \\
&= g''\left(\widehat{m}_s\left(\theta\right)\right) \cdot \frac{\mu \cdot \left(1 - \frac{d\widehat{s}(\theta)}{d\theta}\right)}{\left(\theta - \widehat{s}\left(\theta\right) + w^*\right)^2} \\
&= \frac{1 - \frac{d\widehat{s}(\theta)}{d\theta}}{\theta - \widehat{s}\left(\theta\right) + w^* - \mu} \ ,
\end{aligned}$$

where we use

$$g''\left(x\right) = \frac{1}{x\left(1 - x\right)}$$

while deriving the third equality. Manipulating the above equation we get

$$\begin{aligned}\frac{d\widehat{s}\,(\theta)}{d\theta} &= \frac{\mu}{\theta-\widehat{s}\,(\theta)+w^*}\\ &= 1-\widehat{m}_s\,(\theta)\ \ ,\end{aligned}$$

where the last equality follows (A.21).

Again, taking derivatives of both sides of the above equation with respect to θ leads to

$$\begin{aligned}\mu^{-1}\cdot\frac{d\widehat{s}\,(\theta)}{d\theta} &= g''\left(\widehat{m}_s\,(\theta)\right)\cdot\frac{d\widehat{m}_s\,(\theta)}{d\theta}\\ &= \frac{1}{\widehat{m}_s\,(\theta)\,(1-\widehat{m}_s\,(\theta))}\cdot\frac{d\widehat{m}_s\,(\theta)}{d\theta}\ .\end{aligned}$$

Hence

$$\begin{aligned}\frac{d\widehat{m}_s\,(\theta)}{d\theta} &= \mu^{-1}\cdot\widehat{m}_s\,(\theta)\cdot(1-\widehat{m}_s\,(\theta))\cdot\frac{d\widehat{s}\,(\theta)}{d\theta}\\ &= \mu^{-1}\cdot\widehat{m}_s\,(\theta)\cdot(1-\widehat{m}_s\,(\theta))^2\,.\end{aligned}$$

This completes the proof. □

Proof of Lemma 5. From Lemma 4, it is easy to see that the slope of $\widehat{s}(\theta)$ is always less than one. Hence, Lemma 5 is straightforward. □

Proof of Lemma 6. We proceed by discussing three cases.

Case 1: We show that $\widehat{s}(\theta)>\theta$ would imply $s^*(\theta)=\theta$.

Suppose $s^*(\theta)<\theta$. Then we have $s^*(\theta)<\widehat{s}(\theta)$. Since both $(s^*(\theta),m_s^*(\theta))$ and $(\widehat{s}(\theta),\widehat{m}_s(\theta))$ satisfy condition (3.6), we must have $m_s^*(\theta)<\widehat{m}_s(\theta)$. Therefore,

$$\begin{aligned}\frac{r(\theta)}{m_s^*(\theta)} &= -1+\mu^{-1}\cdot(1-m_s^*(\theta))\cdot(\theta-s^*(\theta)+w^*)\\ &> -1+\mu^{-1}\cdot(1-\widehat{m}_s(\theta))\cdot(\theta-\widehat{s}(\theta)+w^*)\\ &= 0\ ,\end{aligned}$$

which implies $s^*(\theta)=\theta$, a contradiction.

Note that, the logic for the inequality above is as follows. Since $(\widehat{s}(\theta),\widehat{m}_s(\theta))$ satisfies condition (A.2), we must have $\theta-\widehat{\theta}+w^*>0$. Hence, $\widehat{s}(\theta)>s^*(\theta)$ implies that

$$\theta-s^*(\theta)+w^*>\theta-\widehat{s}(\theta)+w^*>0\,.$$

Also note that $1-m_s^*(\theta)>1-\widehat{m}_s(\theta)>0$, we get the inequality above.

Hence, we have $s^*(\theta) = \theta$ in this case.

Case 2: We show that $\widehat{s}(\theta) < 0$ would imply $s^*(\theta) = 0$.

Suppose $s^*(\theta) > 0$. Then we have $s^*(\theta) > \widehat{(}\theta)$. By similar argument we know that $m_s^*(\theta) > \widehat{m}_s(\theta)$. Therefore,

$$\begin{aligned}
\frac{r(\theta)}{m_s^*(\theta)} &= -1 + \mu^{-1} \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \\
&< -1 + \mu^{-1} \cdot (1 - \widehat{m}_s(\theta)) \cdot (\theta - \widehat{s}(\theta) + w^*) \\
&= 0 \ ,
\end{aligned}$$

which implies $s^*(\theta) = 0$. This is a contradiction. Hence, we have $s^*(\theta) = 0$ in this case.

Case 3: We show that $0 \leqslant \widehat{s}(\theta) \leqslant \theta$ would imply $s^*(\theta) = \widehat{s}(\theta)$.

Suppose $\widehat{s}(\theta) < s^*(\theta)$. Then similar argument suggests $r(\theta)/m_s^*(\theta) < 0$, which implies $s^*(\theta) = 0 < \widehat{s}(\theta)$. This is a contradiction.

Similarly, suppose $s^*(\theta) < \widehat{s}(\theta)$. Similar argument suggests that $r(\theta)/m_s^*(\theta) > 0$, which implies $s^*(\theta) = \theta > \widehat{s}(\theta)$. This is, again, a contradiction. Hence, we have $s^*(\theta) = \widehat{s}(\theta)$ in this case.

This concludes the proof. □

Proof of Lemma 7. Apply Lemma 5 to Lemma 6, then Lemma 7 is straightforward. □

Proof of Lemma 8. We prove by contradiction. Suppose that the last two cases in Lemma 7 can occur in equilibrium. Hence, there exists a $\widetilde{\theta} \geqslant 0$, such that $s^*(\theta) = 0$ when $0 \leqslant \theta \leqslant \widetilde{\theta}$ and $s^*(\theta) = \widehat{s}(\theta)$ when $\theta > \widetilde{\theta}$.

Note that, $s^*(\theta)$ is strictly increasing when $\theta > \widetilde{\theta}$. Also, since we focus on the equilibrium with information acquisition, there must exist a θ'' such that $s^*(\theta'') > k$; otherwise the optimal security would be rejected without information acquisition. Therefore, there exists a $\theta' > \widetilde{\theta}$ such that $s^*(\theta') = \widehat{s}(\theta') = k$. Recall condition (A.1), we have

$$m_s^*(\theta') = \bar{\pi}_s^* \, .$$

Moreover, notice that we have $s^*(\theta') \in (0, \theta')$, we have

$$\begin{aligned}
0 = r(\theta') &= -m_s^*(\theta') + \mu^{-1} \cdot m_s^*(\theta') \cdot (1 - m_s^*(\theta')) \cdot (\theta' - s^*(\theta') + w^*) \\
&= -\bar{\pi}_s^* + \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k + w^*) \\
&= \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k) + \mathbb{E}[r(\theta)] \ ,
\end{aligned}$$

where

$$
\begin{aligned}
\mathbb{E}[r(\theta)] &= -\bar{\pi}_s^* + \mu^{-1}\left(\mathbb{E}\left[\frac{(\theta - s(\theta)) \cdot g''(\bar{\pi}_s^*))}{g''(m(\theta))}\right] / g''(\bar{\pi}_s^*) + w^* \mathbb{E}\left[\frac{1}{g''(m(\theta))}\right]\right) \\
&= -\bar{\pi}_s^* + \mu^{-1}\left(w^* \cdot \left(1 - \mathbb{E}\left[\frac{g''(\bar{\pi}_s^*)}{g''(m(\theta))}\right]\right) / g''(\bar{\pi}_s^*) + w^* \mathbb{E}\left[\frac{1}{g''(m(\theta))}\right]\right) \\
&= -\bar{\pi}_s^* + \frac{\mu^{-1} w^*}{g''(\bar{\pi}_s^*)} \\
&= -\bar{\pi}_s^* + \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot w^* \ .
\end{aligned}
$$

We can express the expectation term $\mathbb{E}[r(\theta)]$ in another way. Note that, for any $\theta \in [0, \widetilde{\theta}]$, by definition we have

$$
\begin{aligned}
r(\theta) &= -m_s^*(\theta) + \mu^{-1} \cdot m_s^*(\theta) \cdot (1 - m_s^*(\theta)) \cdot (\theta - s^*(\theta) + w^*) \\
&= -\widehat{m}_s(\widetilde{\theta}) + \mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \cdot (\theta - 0 - \theta^* + \theta^* + w^*) \\
&= r(\widetilde{\theta}) - \mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \cdot (\widetilde{\theta} - \theta) \\
&= -\mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \cdot (\widetilde{\theta} - \theta) \ .
\end{aligned}
$$

Also, as $s^*(\theta) = \widehat{s}(\theta)$ for any $\theta > \widetilde{\theta}$, we have $r(\theta) = 0$ for all $\theta > \widetilde{\theta}$. Hence,

$$
\mathbb{E}[r(\theta)] = -\mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (\widetilde{\theta} - \theta) d\Pi(\theta) \ .
$$

Therefore, we have

$$
\begin{aligned}
\mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - k) &= -\mathbb{E}[r(\theta)] && \text{(A.22)} \\
&= \mu^{-1} \cdot \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (\widetilde{\theta} - \theta) d\Pi(\theta) \ . && \text{(A.23)}
\end{aligned}
$$

Now we take the tangent line of $s^*(\theta)$ at $\theta = \widetilde{\theta}$. The tangent line intersects $s = k$ at $\widetilde{\theta}'$, which is given by

$$
\frac{k}{\widetilde{\theta}' - \theta'} = \left.\frac{ds^*(\theta)}{d\theta}\right|_{\theta'} = 1 - \widehat{m}_s(\widetilde{\theta}) \, .
$$

Hence, we have

$$
\widetilde{\theta}' = \widetilde{\theta} + \frac{k}{1 - \widehat{m}_s(\widetilde{\theta})} \, .
$$

Also, note that we have shown that for any $\theta \geqslant \widetilde{\theta}$, we have

$$
\frac{ds^*(\theta)}{d\theta} = \frac{d\widehat{s}(\theta)}{d\theta} = 1 - \widehat{m}_s(\theta) = 1 - m_s^*(\theta) \, .
$$

Hence,

$$\frac{d^2 s^*(\theta)}{d\theta^2} = -\mu^{-1} \cdot m_s^*(\theta) \cdot (1 - m_s^*(\theta))^2 < 0\,.$$

Therefore, $s^*(\theta)$ is strictly concave for $\theta \geqslant \widetilde{\theta}$, and consequently, we also have $\widetilde{\theta}' < \theta'$. As a result, from conditions (A.22) and (A.23), we have

$$\begin{aligned}
\bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) \cdot (\widetilde{\theta}' - k) \quad &< \quad \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (\widetilde{\theta} - \theta) d\Pi(\theta) \\
&= \quad \bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1 - \widehat{m}_s(\widetilde{\theta})} \cdot k \right) \,.
\end{aligned}$$

By Jensen's inequality, we get

$$\bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) > \mathbb{E}\left[m_s^*(\theta) \cdot (1 - m^*(\theta)) \right]\,.$$

Therefore, we have

$$\begin{aligned}
\widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (\widetilde{\theta} - \theta) d\Pi(\theta) \quad &> \quad \bar{\pi}_s^* \cdot (1-\bar{\pi}_s^*) \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1 - \widehat{m}_s(\widetilde{\theta})} \cdot k \right) \\
&> \quad \mathbb{E}\left[m_s^*(\theta) \cdot (1 - m^*(\theta)) \right] \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1 - \widehat{m}_s(\widetilde{\theta})} \cdot k \right) \,.
\end{aligned}$$

Expand the expectation term above and rearrange, we get

$$\begin{aligned}
& \widehat{m}_s(\widetilde{\theta})^2 \cdot k \cdot Prob[\theta \leqslant \widetilde{\theta}] + \int_{\widetilde{\theta}}^{+\infty} m_s^*(\theta) \cdot (1 - m_s^*(\theta)) d\Pi(\theta) \cdot \left(\widetilde{\theta} + \frac{\widehat{m}_s(\widetilde{\theta})}{1 - \widehat{m}_s(\widetilde{\theta})} \cdot k \right) \\
< \quad & \widehat{m}_s(\widetilde{\theta}) \cdot (1 - \widehat{m}_s(\widetilde{\theta})) \int_0^{\widetilde{\theta}} (-\theta) d\Pi(\theta) \\
\leqslant \quad & 0\,.
\end{aligned}$$

Nevertheless, the left hand side of the above inequality should be positive, which is a contradiction. This concludes the proof. □

Proof of Proposition 4. We first consider the case with a positive transfer price $p > 0$. Suppose the corresponding security $s(\theta) = \theta - p$ is optimal in a generalized security design problem without the non-negative constraint. However, this security can be accommodated by neither Proposition 2 or Proposition 3, which two exclusively characterize the optimal security in the generalized security design problem, a contradiction.

By Corollary 1, we know that the security $s(\theta) = \theta$ that represents transfer with a zero price is not optimal. This concludes the proof. □

Proof of Corollary 2. First, note that $s^*(\theta)$ is strictly increasing and continuous. Also, note that there exists a θ'' such that $s^*(\theta'') > k$; otherwise, the offer will be rejected without information acquisition.

Therefore, there exists an unique θ' such that $s^*(\theta') = k$, which ensures that $m_s^*(\theta') = \bar{\pi}_s^*$, and

$$\begin{aligned} r(\theta') &= -\bar{\pi}_s^* + \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - s^*(\theta') + w^*) \\ &= \mu^{-1} \cdot \bar{\pi}_s^* \cdot (1 - \bar{\pi}_s^*) \cdot (\theta' - s^*(\theta')) + \mathbb{E}[r(\theta')] \ . \end{aligned}$$

Note that $\mathbb{E}[r(\theta')] > 0$ and $\theta' - s^*(\theta') \geqslant 0$, we have $\theta' < \widehat{\theta}$. As $\theta' = s^*(\theta') = k$, it follows that $\widehat{\theta} > \theta' = k$. This concludes the proof. □

Proof of Proposition 5. When we have $\mathbb{E}[\theta] \leqslant k$ and $\mathbb{E}\left[\exp(\mu^{-1}(t \cdot \theta - k))\right] > 1$, according to Proposition 1, even if the entrepreneur proposes all the future cash flow to the investor, the security would induce the investor to acquire information and accept it with positive (but less than one) probability. This concludes the proof. □

Lemma **9.** *A project is initiated with positive probability in the decentralized economy if and only if it is initiated with positive probability in the corresponding centralized economy.*

Proof of Lemma 9. With the objective function (4.10) in the centralized economy, the entrepreneur's optimal decision $m_c^*(\theta)$ is characterized by Proposition 1. Specifically, the investor will initiate the project without information acquisition, i.e., $Prob[m_c^*(\theta) = 1] = 1$ if and only if

$$\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))] \leqslant 1 \, ,$$

will skip the project without information acquisition, i.e., $Prob[m_c^*(\theta) = 0] = 1$ if and only if

$$\mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] \leqslant 1 \, ,$$

and will initiate the project with probability $0 < \bar{\pi}_c^* < 1$, $\bar{\pi}_c^* = \mathbb{E}[m_c^*(\theta)]$, if and only if

$$\mathbb{E}[\exp(-\mu^{-1} \cdot (\theta - k))] > 1 \text{ and } \mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] > 1 \, ,$$

in which $m_c^*(\theta)$ is determined by

$$\theta - k = \mu \cdot (g'(m_c^*(\theta)) - g'(\bar{\pi}_c^*)) \, .$$

It is straightforward to observe that, the project is initiated with positive probability in the frictionless centralized economy if and only if

$$\mathbb{E}[\exp(\mu^{-1} \cdot (\theta - k))] > 1\,. \tag{A.24}$$

Clearly, condition (A.24) is just the same as condition (3.1) in Lemma 1 that gives the investment criterion in a corresponding decentralized production economy. This concludes the proof. $\square$