Bank Capital, Liquid Reserves, and Insolvency Risk*

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Abstract

We develop a dynamic model of banking to assess the effects of liquidity and leverage requirements on banks' insolvency risk. The model features endogenous capital structure, liquid asset holdings, payout, and default decisions. In the model, banks face taxation, flotation costs of securities, and default costs and are financed with equity, insured deposits, and risky debt. Using the model, we demonstrate that (*i*) regulatory requirements affect bank behavior even when they are not binding; (*ii*) liquidity requirements constraining banks to hold a minimum amount of liquid reserves have no long-run effects on default risk but may increase default risk in the short run; (*iii*) leverage requirements increase banks' franchise value and reduce default risk.

Keywords: banks; liquidity buffers; capital structure; insolvency risk; regulation

JEL Classification Numbers: G21, G28, G32, G33.

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Banks can impose major risks on the economy. Avoidance of these risks and the associated social costs is the overwhelming concern of prudential regulation. Given the experience in the financial crisis of 2007-2008, in which insufficient liquidity buffers and excessive leverage made the financial system unable to withstand large negative shocks, the debate on banking regulation has recently evolved around two main ideas, that have been reflected in proposals for regulatory reform. First, equity (or leverage) requirements should be significantly increased, so that if the value of the banks' assets were to decline this would not automatically lead to distress and the resulting losses would be borne by the bank owners (see e.g. Admati and Hellwig (2013)).[1] Second, because most banks assets are illiquid and raising fresh equity at short notice is costly, banks should hold a buffer of liquid reserves in order to be able to cope with short-term losses (see e.g. Gorton (2012)).

While many qualitative discussions of leverage and liquidity requirements are available in the literature, financial theory has made little headway in developing models that provide quantitative guidance for the use of different instruments of prudential regulation. Moreover, as useful as they are, many of the recent discussions on banking regulation ignore important incentive effects that regulatory requirements may have on bank behavior. Our objective in this paper is therefore twofold. First, we seek to develop a dynamic tractable model of banks' choices of liquid asset holdings, financing, payout, and default policies in the presence of realistic market frictions. Second, we want to use this model to characterize the endogenous response of banks to the imposition of liquidity and leverage requirements and to measure the effects of such requirements on banks' insolvency risk.

We begin our analysis by formulating a dynamic structural model in which banks face taxation, issuance costs of securities, and default costs and may be constrained by a regulator to hold a minimum amount of liquid reserves and of equity capital. In the model, banks are

[1] For example, U.S. banking regulators have proposed in July 2013 a new restriction on borrowing for U.S. banks. This new restriction would require a ratio of regulatory (Tier 1) capital to total assets of 6 percent for 8 Systemically Important Financial Institutions and 5 percent for their bank holding companies, independently of the riskiness of these assets. Similarly, the Vickers Commission in the U.K. has recommended a 4 percent ratio rather than the 3 percent global standard set by the Basel III regime.

financed with equity, insured deposits, and risky debt. They hold risky, illiquid assets (e.g. loan portfolios) that yield high returns but are subject to both small and frequent shocks (i.e. Brownian shocks) as well as large negative shocks capturing tail risk (i.e. negative jumps). Banks also hold risk-free, liquid assets (such as cash reserves or safe government bonds) that can be used to absorb these shocks and save on recapitalization costs. In the model, banks maximize the present value of future dividends to incumbent shareholders by choosing their buffers of liquid assets as well as their payout, financing, and default policies.

We start our analysis by solving the policy choices that maximize shareholder value in an hypothetical environment in which banks are unregulated. We show that when outside financing is costly, inside and outside equity are not perfect substitutes and banks find it optimal to hold buffers of liquid assets. Indeed, while cash is an asset with low returns, it is also the most liquid store of value, the one that can be used to absorb potential losses without issuing costly securities. To reduce the risk of default and save on recapitalization costs, banks manage their liquidity buffers dynamically by adjusting their dividend payments to shareholders. Banks facing higher recapitalization costs are less levered, pay less dividends, hold more liquid reserves, and default earlier.

We also show that banks absorb small and intermediate losses using their buffers of liquid assets or by raising outside funds at a cost when these buffers are insufficient. That is, in our model, small and intermediate losses do not lead to insolvency. We show, however, that large losses due to a realization of tail risk may lead banks to default. Notably, we find that banks' insolvency risk increases with tail risk, financial leverage, and financing costs and decreases with franchise value, i.e. the expected cash flow from running the banks' assets. As will become clear, one key feature of our framework is that it highlights a trade-off between managing insolvency risk ex ante via capital structure policy versus ex post via liquidity buffers. That is, in our model, liquidity management, capital structure, and default decisions are jointly and endogenously determined.

After solving for the policy choices of unregulated banks, we examine the effects of prudential regulation on these policy choices and insolvency risk. We first consider liquidity requirements that mandate banks to hold a minimum amount of liquid reserves. We show that when facing such a requirement, banks voluntary choose to hold liquid buffers in excess of the required minimum in order to reduce the recapitalization costs associated with breaches of the requirement. Notably, we show that as liquidity requirements rise, banks raise their *target level* of liquid reserves by a like amount, thereby making their cushion of liquid reserves independent of the level chosen by the regulator. We then investigate the implication of this behavior for default risk. We show that because raising outside funds is costly, an increase in liquidity requirements will not generally lead to an immediate adjustment to the new target buffer. As a result, in the *short-run*, liquidity requirements will lead to a drop in franchise value and to an increase in insolvency risk. We also demonstrate that once there has been an opportunity to build up liquid reserves towards the desired level, changes in liquidity requirements have no impact on insolvency risk.

In addition to liquidity requirements, banks may be subject to leverage (or equity) requirements. Such requirements do not impose any restriction on what assets banks should hold but impose constraints on the way they fund their operations. As argued by Admati and Hellwig (2013), a tightening of leverage requirements transfers a large fraction of the bank's risks to the bank's owners, which otherwise might be passed on to creditors or taxpayers. Leverage requirements generally indicate how much equity capital banks should have relative to their total assets. For example, banks are expected to maintain a Tier 1 capital to asset ratio of 3% under Basel III. In July 2013, the US Federal Reserve Bank announced that the minimum leverage ratio would be 6% for 8 Systemically Important Financial Institutions. Consistent with these recommendations, we consider requirements that fix *ex-ante* a minimum amount of equity capital for banks. Using a calibrated version of the model, we show that such requirements have significant effects on both insolvency risk and bank value. For example, increasing equity capital from 6% (as recommended by U.S.

regulators) to 20% (as recommended by Admati and Hellwig) decreases the probability of default over a one year horizon from 0.1518% to less than 0.0001% and reduces total bank value by 6.25%.

The literature examining default risk in banks has started with the early contributions of Merton (1977, 1978), in which the objective is to determine the cost of deposit insurance and loan guarantees. Although an important milestone, the analysis of Merton has four limitations. First, it assumes that banks default whenever the assets-to-deposits ratio falls below some *exogenous* barrier. This provision approximates the case in which banks face a positive net worth covenant, but it is by no means the only or typical situation. Second, the capital structure of banks is set exogenously and does not depend on the frictions that banks face such as taxes, bankruptcy costs, or issuance costs of securities. Third, the dynamics of the banks assets are governed by an exogenous process, implying that there is no connection between the banks' asset and capital structures. Fourth, raising equity is costless, so that liquid reserves are irrelevant.

Most of the recent quantitative banking models examine variants to the first of these assumptions. By doing so, these studies are able to determine the effects of a given, exogenous capital structure on insolvency risk, while maintaining the Modigliani and Miller (1958) assumption that investment policy is independent of financing policy. In these contributions, insolvency is endogenous and triggered by shareholders' decision to cease injecting funds in the bank (see e.g. Fries, Mella-Barral, and Perraudin (1997), Bhattacharya, Planck, Strobl, and Zechner (2002), or Décamps, Rochet, and Roger (2004)). While identifying some prime determinants of insolvency risk, these theories assume that asset and liability structures are exogenously given. As a result, they leave open the question of how financing structure and asset structure interact and jointly affect insolvency risk. In addition, these models also maintain the assumption that banks can raise outside funds at no cost, thereby leaving no role for liquidity. In that respect, they ignore a number of key determinants of insolvency risk, which are at the centre of most regulatory frameworks, for example those of the Basel Committee on Banking Supervision.

Our analysis inherits some of the assumptions of this literature. For example, bank shareholders are protected by limited liability and the bank's objective is shareholder value maximization. However, it differs from these contributions in three important respects. First, we consider that at least part of a bank's assets are illiquid and that it is costly to issue securities, thereby providing a role for liquidity buffers. Second, we incorporate some of the key market imperfections and regulatory requirements that banks face in practice and relate banks' payout, financing, and default policies to these frictions. Third, in our model, financing structure and asset structure interact and jointly affect insolvency risk. We show that these unique features have important implications. Notably, while in standard models of shareholder value is always increased by making dividend payments to shareholders, this is not the case in our model with frictions, in which shareholders have incentives to protect their franchise value by maintaining adequate liquid reserves.

There is a large literature on bank capital that analyzes its role in regulation (see e.g. Hellman, Murdoch, and Stiglitz (2001), Décamps, Rochet and Roger (2004), Morrison and White (2005), or Repullo and Suarez (2013)). But it is only recently that the question of bank optimal capital structure has begun to be addressed. The papers that are most closely related to ours in this literature are Froot and Stein (1988) and Subramanian and Yang (2013). Subramanian and Yang use a dynamic model to examine banks' insolvency risk. In their paper, there is no role for liquid reserves as outside financing is costless. In addition, the authors mostly focus on regulators' incentives to step in as banks become distressed. In that respect, the two papers are complementary.

Froot and Stein (1998) build a two-period model in which capital is initially costless but may become costly in the future. In their model, decisions are made by bank shareholders in the first period, in which the future increase in financing costs leads the bank to increase equity capital and to engage in risk management activities to hedge future potential losses. Our model differs from Froot and Stein by considering that banks always face the same set of frictions and choose their payout, financing, and default decisions in response to these frictions as well as regulatory requirements. Another difference is that we consider a dynamic model. Indeed, as suggested by Hellwig (1998), a static framework fails to capture important intertemporal effects. For example, in a static model, a regulatory requirement can only affect a bank's behavior if it is binding. In practice however, regulatory requirements are binding for a small minority of banks and yet seem to influence the behavior of other banks.

From a methodological perspective, our paper relates to the inventory models of Milne and Whalley (2001), Milne (2004), Peura and Keppo (2006), or Décamps, Mariotti, Rochet, and Villeneuve (2011), in which financing costs lead banks or corporations to have liquidity buffers. In these models, uncertainty is solely driven by Brownian shocks and there is no risk of default if firms can raise outside equity. Our paper advances this literature in two important dimensions. First, we incorporate jumps in our analysis to account for tail risk and show that banks may find it optimal to default, even if they have access to outside funds. Second, we endogenize not only banks' payout decisions but also their financing and default policies. This allows us to examine the effects of regulation on banks' insolvency risk. As shown in the paper, this problem is difficult to solve because we now have two free boundaries (the default and the payout thresholds) instead of one (the payout threshold). Another contribution of this paper is therefore to develop a new method based on fixed-point arguments to derive the solution to shareholders' optimization problem in this context.

The remainder of the paper is organized as follows. Section I presents the model. Section II derives the equity value-maximizing payout, financing, and default policies for unregulated banks. Section III examines the effects of liquidity and leverage requirements on these policy choices and insolvency risk. Section IV discusses the model's implications. Section 5 concludes. The proofs are gathered in a separate Appendix.

I. Model

We develop a dynamic banking model in which financing structure and asset structure interact and jointly affect insolvency risk. Our focus is on the liquidity management, payout, financing, and default policies that maximize the bank's equity value. We also analyze the effects of leverage and liquidity requirements on these policy choices and banks' insolvency risk. Throughout the paper, agents are risk neutral and discount cash flows at a constant rate ρ. Time is continuous and uncertainty is modeled by a probability space $(\Omega, \mathcal{F}, P; \mathbb{F})$, with the filtration $\mathbb{F} = \{\mathcal{F}_t : t \geq 0\}$ satisfying the usual conditions.

The subject of study is a bank held by shareholders that have limited liability. This bank is subject to taxation at the rate $\theta \in (0, 1)$. It owns a portfolio of risky assets (e.g. a portfolio of risky loans) as well as liquid reserves (i.e. cash reserves or safe government bonds) and is financed by equity, insured deposits, and risky debt. The bank's risky assets generate after-tax cumulative cash flows A_t that evolve according to:

$$dA_t = (1 - \theta)\mu dt + \sigma dB_t - Y_{N_t} dN_t.$$

In this equation, B_t is a Brownian motion, N_t is a Poisson process, (μ, σ) are constants, and $(Y_n)_{n=1}^{\infty}$ is a sequence of independently and identically distributed random variables that are drawn from $\mathbb{R}_+$ according to an exponential distribution with cumulative density function $F(y) = 1 - e^{-\beta y}$ with $\beta > 0$ (a similar modeling assumption is made for example in DeMarzo, Duffie, and Varas (2013)). The increments of the Brownian motion represent small and frequent shocks to the bank cash flows. The jumps of the Poisson process represent large losses that may be due, for example, to defaults across the loan portfolio of the bank. We denote the intensity of the Poisson process by λ so that over an infinitesimal time interval there is a probability λdt that the bank makes a large loss. Because the distribution of the jump magnitudes has unbounded support, the bank will be unable to withstand this loss with some positive probability, leading to insolvency. That is, the bank is subject to tail risk (see for example Acharya, Cooley, Richardson, and Walter (2009) for an analysis of the role of tail risk in the 2007-2009 financial crisis).

The relative illiquidity of bank assets generally constitutes a main source of banking fragility.[2] Indeed, as discussed in Froot and Stein (1998), "one of the fundamental roles of banks and financial intermediaries is to invest in assets which, because of their information-sensitive nature, cannot be frictionlessly traded in the capital markets." The standard example of such an illiquid asset is a loan to a small or medium-sized company. To capture this important feature, we consider that *risky* assets are illiquid and assume for simplicity that they have zero liquidation value, as in Biais, Mariotti, Rochet, and Villeneuve (2010), DeMarzo, Duffie, and Varas (2013), or Rochet and Zargari (2013).

In addition to its risky assets, the bank can (or may be constrained by the regulator to) hold liquid, risk-free reserves. We denote by S_t the liquid reserves of the bank at any time $t \geq 0$. Holding liquid reserves generally involves deadweight costs. We capture these costs by assuming that the rate of return on liquid reserves is zero.[3] When optimizing its liquid asset holdings, the bank trades-off the lower returns of these assets with the benefits of liquidity.

The bank capital structure comprises equity, a fixed volume of insured deposits D, and risky debt. Deposits are insured against bank failure by a deposit insurance fund and require the bank to make a payment $c_D \geq 0$ per unit of time, which includes the interest paid to depositors and a deposit insurance premium.[4] Risky debt requires the bank to make a

[2]This illiquidity problem can be due for example to the special expertise of the bank in evaluating its long-term investments. If the bank tries to sell them before they pay out, other investors might infer that the bank got bad news about the future returns from these investments leading to a decrease of the price at which the bank could sell these assets. That is, liquidity problems may be fundamentally informational. But informational problems are also why we need a banking system in the first place.

[3]A necessary condition for a well-defined payout policy is that there exists a cost of holding liquidity in that the rate of return on liquid reserves is strictly less than the risk-free rate of return ρ.

[4]As in Hanson, Shleifer, Stein and Vishny (2014), "we depart from much of the rest of the literature by downplaying the vulnerability of bank deposits to runs. Indeed, we emphasize precisely the opposite aspect of deposit finance: Relative to other forms of private-money creation that occur in the shadow-banking sector–notably, short-term collateralized claims such as broker-dealer repurchase agreements (repos)–bank

payment $c_L \geq 0$ per unit of time and has a face value L that is endogenized in section III.B. The bank's debt payments are thus $c = c_D + c_L$ and its cumulative earnings satisfy

$$C_t = A_t - (1-\theta)ct = (1-\theta)(\mu - c)t + \sigma B_t - \sum_{n=1}^{N_t} Y_n. \tag{1}$$

Given that sudden losses can eliminate an arbitrarily large fraction of the bank's assets, the bank is subject to default risk. In the following, we use a stock based definition of default whereby the bank services debt as long as equity value is positive (as in e.g. Leland (1994)). That is, default is the result of the optimizing behavior of shareholders. We will show that in our model limited liability only has value if the bank cash flows are subject to tail risk, i.e. only if $\lambda > 0$, for otherwise the bank would not be subject to default risk.

In the model, equity capital and liquid reserves serve as buffers against default risk. The bank can increase its liquid reserves either by retaining earnings or by issuing new equity. We consider that when raising outside funds, the bank has to pay a lump-sum cost ϕ. Because of this fixed cost, the bank will raise new funds through lumpy and infrequent issues. In addition, in order to reduce the costs associated with having to issue fresh equity, the bank will choose to retain earnings and hold liquidity buffers.

A payout and financing strategy for the bank is a pair $\pi = (P^\pi, R^\pi)$ of adapted and non-decreasing processes with initial value zero, where P_t^π and R_t^π respectively represent the cumulative payouts to shareholders and the cumulative net financing raised from investors up until time $t \geq 0$. The liquid reserves process associated with a strategy π is defined by:

$$S_t^\pi = s + C_t - P_t^\pi + R_t^\pi,$$

where C_t is defined in equation (1) and s is the initial level of liquid reserves. This equation is a general accounting identity, in which P_t^π and R_t^π are endogenously determined by the

deposits are noteworthy because, in the modern institutional environment, they are highly sticky and not prone to run at the first sign of trouble."

bank. It shows that liquid reserves increase with the bank's earnings and with the funds raised from outside investors and decrease with payouts to shareholders. The liquidation time associated with the payout and financing strategy π is then defined by:

$$\tau_\pi = \inf\{t \geq 0 : S^\pi_{t+} = \lim_{u \downarrow t} S^\pi_u \leq 0\}.$$

In the model, bank shareholders make their financing, payout, and default decisions after observing the increment of the cash flow process. As a result, we use left-continuous processes in the definition of strategies and right-hand limits in the definition of liquidation times. Notably, the occurrence of a cash flow jump

$$\Delta C_t = C_t - \lim_{s \uparrow t} C_s \leq -S^\pi_{t-}$$

that depletes the liquid reserves of the bank results in default if shareholders do not provide sufficient funds for the right hand limit

$$S^\pi_{t+} = S^\pi_{t-} + \Delta C_t + R^\pi_{t+} - R^\pi_t - P^\pi_{t+} + P^\pi_t$$

to be strictly positive. In the following, we will consider strategies such that $P^\pi_{t+} - P^\pi_t \leq S^\pi_t + R^\pi_{t+} - R^\pi_t$, implying that the bank cannot pay out amounts that it does not hold.

Management chooses the payout, financing, and default policies of the bank to maximize the present value of future dividends to incumbent shareholders, net of the claim of new investors on future cash flows. That is, management solves:

$$v(s) = \sup_{\pi \in \Pi(s)} \mathbb{E}_s \left[\int_0^{\tau_\pi} e^{-\rho t} (dP^\pi_t - d\Phi_t(R^\pi)) \right], \tag{2}$$

where $\Pi(s)$ denotes the set of admissible strategies and dP^π_t and $d\Phi_t(R^\pi)$ are respectively the bank's incremental payout and the contribution of shareholders to the bank (including the cumulative cost of external financing) over the time interval $(t, t + dt)$.[5] Because risky assets have no liquidation value, there is no cash flow to shareholders in default. That is, we have $(S^{\pi}_{\tau_\pi} - D - L)^+ = 0$, where $x^+ \equiv \max(0, x)$. We show in section III.A that this is no longer the case in the presence of a regulatory liquidity requirement.

To gain some intuition on the solution to problem (2), note that because of the fixed costs of financing, the bank raises external funds only when cash flow shocks deplete its liquid reserves, i.e. when $s \leq 0$. In such instances, the bank has to either immediately raise new funds to finance the shortfall and continue operating, or default. The bank may choose to default when the shortfall or the cost of refinancing are large. When this is not the case, the bank pays the financing cost ϕ and raises an amount that maximizes equity value. Taking into account these two possibilities, we thus have that the equity value function satisfies for all $s \leq 0$:

$$v(s) = \sup_{b \geq 0} (v(b) - b + s - \phi)^+ = (\alpha_0^* + s)^+ , \tag{3}$$

where b is the level of the bank's liquid reserves after raising external funds and

$$\alpha_0^* = v(0) = \sup_{b \geq 0} (v(b) - (b + \phi))^+$$

is the maximal shortfall that bank shareholders can accept to refinance. Anticipating, equation (3) shows that two key determinants of default risk in our model are financing costs ϕ and tail risk, i.e. the possibility for s to become (very) negative.

[5]Formally, the cumulative net financing raised from outside investors R^{π}_t and the total contribution of shareholders to the bank $\Phi_t(R^{\pi})$ up until time t are defined by

$$R^{\pi}_t = \sum_{n=1}^{\infty} 1_{\{t > \xi_n\}} r_n \quad \text{and} \quad \Phi_t(R^{\pi}) = R^{\pi}_t + \sum_{n=1}^{\infty} 1_{\{t > \xi_n\}} \phi,$$

for some increasing sequence of stopping times $(\xi_n)_{n=1}^{\infty}$ that represent the dates at which the bank raises funds from outside investors and some sequence of nonnegative random variables $(r_n)_{n=1}^{\infty}$ that represent the net financing amounts. See the Technical Appendix to the paper for more details.

Because the likelihood of costly refinancing decreases with s, we expect the marginal value of liquid reserves to be decreasing and, therefore, the equity value function to be concave. If this conjecture is verified, then there should exist some barrier level b_0^* such that $v'(s) \geq 1$ if and only if $s \leq b_0^*$ and the optimal payout policy should consist in distributing dividends to maintain liquid reserves at or below the target level b_0^*.

The main difficulty in solving problem (2) is that one needs to simultaneously determine the values of α_0^* and b_0^*. To circumvent this difficulty, we proceed in two steps. In the first step, we fix the value of the constant α and solve for the payout policy in an auxiliary problem where the bank cannot raise funds but produces the payoff $(\alpha + s)^+$ to shareholders when it runs out of liquid reserves and defaults. In the second step, we show that the payoff to shareholders in default can be chosen in such a way that the value of this auxiliary problem coincides with the solution to problem (2) and derive the equity value-maximizing payout, financing, and default policies.

Before moving to these two steps, we first determine the equity value and the bank's policy choices when there are no refinancing costs. This allows us to derive a first best *franchise value* of equity that will be of repeated use when solving shareholders' optimization problem.

II. Value of an unregulated bank

A. *First best franchise value*

When there are no costs of raising funds, in that $\phi = 0$, there is no need for the bank to hold liquid reserves because any loss can be covered by issuing equity at no cost. As a result, it is optimal to distribute all earnings and the optimization problem reduces to choosing the default policy that maximizes equity value. That is, management solves

$$v^* = \sup_{\tau \in \mathcal{S}} \mathbb{E}\left[e^{-\rho\tau} |\Delta C_\tau| + \int_0^\tau e^{-\rho t} dC_t \right],$$

where cumulative earnings evolve according to equation (1), $\mathcal{S}$ denotes the set of all stopping times, and $|\Delta C_\tau|$ represents the size of the loss leading to default. Solving this problem leads to the following result.

PROPOSITION 1 (First best franchise value): *When issuing equity is costless, it is optimal to distribute all earnings and the optimal default time is given by*

$$\tau^* = \inf\{t \geq 0 : v^* + \Delta C_t \leq 0\}$$

where the first best franchise value of equity $v^* > 0$ is the unique solution to

$$\rho v^* = (1-\theta)(\mu - c) - \lambda \mathbb{E}\left[\min(v^*, Y_1)\right]. \tag{4}$$

when $c < \mu$ and $v^* = 0$ otherwise. For $c < \mu$, the first best franchise value of equity is increasing in the cash flow rate μ, and decreasing in the coupon rate c, the jump intensity λ, and the mean jump size $1/\beta$.

Proposition 1 shows that for any $c < \mu$, it is optimal for bank shareholders to default the first time that the absolute value of a jump of the cash flow process exceeds equity value. In effect, the quantity $|\Delta C_t| = Y_{N_t}$ plays the role of a cost of investment that shareholders have to pay to keep the bank alive following the occurrence of a large loss. To better understand this feature, we can rewrite the Bellman equation (4) as

$$v^* = p(1-\theta)\left(\frac{\mu - c}{\rho}\right) + (1-p)\mathbb{E}\left[(v^* - Y_1)^+\right].$$

with $p = \frac{\rho}{\rho+\lambda} \in (0, 1]$. Thus, when issuing equity is costless, the banks' problem can be interpreted as a discrete-time, infinite horizon problem in which shareholders earn $(1-\theta)\,\frac{\mu-c}{\rho}$ each period with probability p and otherwise face a random liquidity shock that they can decide to pay, in which case the bank continues, or not, in which case the bank is liquidated.

To determine the effect of limited liability on bank policy choices, let

$$v_0 = \mathbb{E}\left[\int_0^\infty e^{-\rho t} dC_t\right] = \frac{1}{\rho}\left((1-\theta)(\mu - c) - \frac{\lambda}{\beta}\right)$$

stand for the equity value under the assumption that shareholders never default, and denote by $\Omega = v^* - v_0$ the excess value generated by the option of declaring default. Using the Bellman equation (4), it is easily seen that this quantity satisfies:[6]

$$\Omega = \left(\frac{\lambda}{\rho}\right) \mathbb{E}\left[(Y_1 - v_0 - \Omega)^+\right].$$

This equation shows that limited liability only has value if the bank cash flows are subject to jumps, i.e. only if $\lambda > 0$, for otherwise the bank would not be subject to default risk. Importantly, the fact that the first best franchise value of equity v^* is strictly positive for all $c < \mu$, irrespective of the frequency and magnitude of the jumps, implies that it is optimal for the bank to operate even if the present value of cash flows v_0 is negative. That is, in the presence of tail risk, limited liability leads shareholders to overinvest in risky assets.

This behavior is reminiscent of the collapse of AIG who was providing insurance by issuing credit default swaps (CDSs). AIG was collecting premia on these CDSs and was expected to make payments only if certain bonds defaulted, which unexpectedly happened in the wake of the subprime crisis. Once the bonds started defaulting, the CDSs had to pay out and AIG was on the hook for billions, which eventually led them to default.

B. *Equity value and bank policies with no refinancing*

Having determined the first best franchise value of equity, we now turn to the solution of the auxiliary problem in which the bank has no access to outside funds. In this case,

[6] When the jumps Y_n of the cumulative cash flow process are exponentially distributed, the solution to this equation is given in closed form by $\Omega = (1/\beta)\mathcal{W}((\lambda/\rho)e^{-v_0})$ where $\mathcal{W}(x)$ is the Lambert$W$ function (see for example Corless, Gonnet, Hare, Jeffrey, and Knuth (1996)). While this closed form solution is useful in numerical computation, it is not necessary for our analysis of the Bellman equation.

shareholders cannot refinance, have to default if the bank runs out of liquid reserves, and can only optimize equity value over the bank's payout policy.

Let $\alpha \geq 0$ be a constant and consider a bank whose assets produce after-tax net cash flows to shareholders given by dC_t as long as it is in operation, and a cash flow $(\alpha + s)^+$ to shareholders if the bank is liquidated at a point where $S_t = s$. The optimization problem of shareholders in such a bank can be written as

$$w(s; \alpha) = \sup_{\pi \in \Pi_0(s)} \mathbb{E}_s \left[\int_0^{\tau_\pi} e^{-\rho t} dP_t^\pi + e^{-\rho \tau_\pi} (\alpha + S_{\tau_\pi}^\pi)^+ \right], \tag{5}$$

where $\Pi_0(s)$ denotes the subset of admissible strategies such that $R^\pi = 0$ (i.e. no refinancing) and τ_π denotes the first time that liquid reserves are negative. The first term on the right-hand side of (5) captures the present value of all dividend payments until the time of default. The second term gives the present value of the cash flow to shareholders in default.

If the value of α is sufficiently large, it should be optimal for shareholders to liquidate the bank assets and distribute all the available cash. The following result confirms this intuition and shows that the threshold above which immediate liquidation is optimal coincides with the first best franchise value of equity determined in the previous section.

LEMMA 2: *If $\alpha \geq v^*$, it is optimal for shareholders to immediately liquidate.*

Given the above result, we now assume that $\alpha < v^*$ and solve problem (5) by deriving the payout policy that maximizes shareholder value. Following the literature on optimal dividend policies (see for example Albrecher and Thonhauser (2009) for a recent survey), it is natural to conjecture that the optimal payout strategy for shareholders should be of barrier type. Specifically, we conjecture that for any $\alpha < v^*$ there exists a constant barrier $b^*(\alpha)$ such that the optimal policy for problem (5) consists in paying dividends to maintain liquid reserves at or below the level $b^*(\alpha)$. To verify this conjecture, we start by calculating the value of the bank's equity under an arbitrary barrier strategy.

Fix a constant $b > 0$ and consider the strategy $\pi_b = (P^b, 0)$ that consists in paying dividends to maintain the liquid reserves of the bank at or below the level b. The cumulative payout associated with this barrier strategy is

$$P_t^b = \max_{0 \leq u < t} 1_{\{t>0\}} (X_u - b)^+$$

where $X_t = s + C_t$ denotes the uncontrolled liquid reserves process (i.e. assuming that there are no dividend payments) and the corresponding value is defined by

$$w(s; \alpha, b) = \mathbb{E}_s \left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho \tau_{\pi_b}} (\alpha + S_{\tau_{\pi_b}}^{\pi_b})^+ \right].$$

Letting ζ_0 denote the first time that the uncontrolled liquid reserves of the bank X_t become negative and using the dividend–penalty identity (see Gerber, Lin, and Yang (2006) and Gerber and Yang (2010)), we have that this value is given by:

$$w(s; \alpha, b) = \begin{cases} (\alpha + s)^+, & \text{for } s \leq 0, \\ \psi(s; \alpha) + \frac{W(s)}{W'(b)} (1 - \psi'(b; \alpha)), & \text{for } 0 < s \leq b, \\ s - b + w(b; \alpha, b), & \text{for } s > b, \end{cases} \tag{6}$$

where the function

$$\psi(s; \alpha) = \mathbb{E}_s \left[e^{-\rho \zeta_0} (\alpha + X_{\zeta_0})^+ \right]$$

gives the present value of the payment that shareholders receive in liquidation if no dividends are distributed prior to bankruptcy, and $W(s)$ is the $\rho-$scale function of the uncontrolled liquid reserves process. Closed-form expressions for both of these functions as linear combinations of exponentials are provided in the Appendix.

In the payout region (i.e. for $s > b$), the bank pays dividends to maintain its liquid reserves at or below b, so that equity value grows linearly with s. In the earnings retention region (i.e. for $s \leq b$), the term

$$\frac{W(s)}{W'(b)} = w(s, 0, b) = \mathbb{E}_s \left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b \right]$$

gives the present value of the dividend payments that shareholders receive until default. The term

$$\psi(s;\alpha) - \psi'(b;\alpha)\frac{W(s)}{W'(b)} = \mathbb{E}_s \left[e^{-\rho \tau_{\pi_b}} (\alpha + S^{\pi_b}_{\tau_{\pi_b}})^+ \right]$$

gives the present value of the payoff that the bank shareholders receive in liquidation (the second term on the left hand side reflecting the effects of payouts on this present value).

Equation (6) shows that in the earnings retention region, the value of a barrier strategy depends on the barrier level only through the function

$$H(b;\alpha) = \frac{1 - \psi'(b;\alpha)}{W'(b)}.$$

In the Appendix, we show that there exists a unique payout barrier $b^*(\alpha)$ that maximizes this function over $\mathbb{R}_+$ and, relying on a verification theorem for the Bellman equation associated with our optimization problem, we prove that the corresponding strategy is optimal among all strategies. This leads to the following result.

Proposition 3 (Auxiliary value function): *Consider a bank with no access to outside funds that produces a cash flow $(\alpha + s)^+$ to shareholders in default. The equity value of such a bank is concave and twice continuously differentiable over $(0, \infty)$ and given by*

$$w(s;\alpha) = w(s;\alpha, b^*(\alpha)),$$

where $b^*(\alpha)$ is the unique solution to $H'(b^*(\alpha);\alpha) = 0$. Furthermore, the optimal policy for bank shareholders is to distribute dividends to maintain liquid reserves at or below $b^*(\alpha)$.

As can be seen from equation (6), the value function of problem (5) is linear with slope equal to one in the payout region where $s > b^*(\alpha)$. Combining this property with the smoothness established in Proposition 3 shows that this value function satisfies:

$$0 = w'(b^*(\alpha);\alpha) - 1 = w''(b^*(\alpha);\alpha).$$

The first of these two conditions is known as the smooth-pasting condition and indicates that liquid reserves are reflected down through dividend payments at the constant barrier level $b^*(\alpha)$. The second condition is usually referred to as the high-contact condition and guarantees the optimality of the dividend barrier; see for example Dumas (1991).

C. *Equity value and optimal bank policies*

Having solved for the payout policy of a bank that defaults the first time it runs out of liquid reserves, we now show how to obtain the optimal policies for problem (2) by endogenizing the value of the constant α that determines the equity value of the bank at the point where it runs out of liquid reserves.

Let $b > 0$ be a constant and consider the strategy $\hat{\pi}_b$ that consists in paying dividends to maintain liquid reserves at or below b and in raising funds back to b whenever liquid reserves become negative if that is profitable. Denote by

$$v(s;b) = \mathbb{E}_s\left[\int_0^{\tau_{\hat{\pi}_b}} e^{-\rho t}(dP_t^{\hat{\pi}_b} - d\Phi_t(R^{\hat{\pi}_b}))\right]$$

the corresponding equity value. By definition, this function satisfies

$$v(s;b) = (v(b;b) - b + s - \phi)^+, \qquad \forall s \leq 0.$$

Since the bank does not raise funds before the first time τ_{π_b} that liquid reserves are negative, we have that $R_t^{\hat{\pi}_b} = P_t^{\hat{\pi}_b} - P_t^b = 0$ for all $0 \leq t \leq \tau_{\pi_b}$. Therefore, using the above equations together with the law of iterated expectations, we get that

$$\begin{aligned} v(s;b) &= \mathbb{E}_s\left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho\tau_{\pi_b}} v(S^{\pi_b}_{\tau_{\pi_b}}; b)\right] \\ &= \mathbb{E}_s\left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho\tau_{\pi_b}} (v(b;b) - b + S^{\pi_b}_{\tau_{\pi_b}} - \phi)^+\right] \\ &= w(s; v(0;b), b), \end{aligned} \tag{7}$$

where $w(s;\alpha,b)$ is defined as in equation (6). Evaluating the bank's equity at the payout barrier b, subtracting $b+\phi$ from both sides of equation (7), and taking the positive part shows that the equity value associated with zero liquid reserves solves

$$\hat{\alpha}(b) = (v(b;b) - b - \phi)^+ = v(0;b) = (w(b;\hat{\alpha}(b), b) - b - \phi)^+ .$$

Given equation (7), the results of the previous section suggest that to obtain the optimal strategy, we should look for a barrier level b such that $b = b^*(\hat{\alpha}(b))$ or, equivalently, for a fixed point of the function defined by

$$g(\alpha) = (w(b^*(\alpha), \alpha, b^*(\alpha)) - b^*(\alpha) - \phi)^+.$$

We show in the Appendix that this function admits a unique fixed point α_0^* in the interval $[0, v^*)$. If the cost of financing is such that

$$\phi \geq \phi^* \equiv w(b^*(0); 0, b^*(0)) - b^*(0),$$

then this fixed point is given by $\alpha_0^* = 0$ and it is not profitable for shareholders to raise funds when the bank runs out of liquid reserves. If the cost of refinancing is such that $\phi < \phi^*$, then we have that $\alpha_0^* > 0$ and it is profitable for shareholders to refinance when the bank runs out of liquid reserves. In this case, we have $\alpha_0^* = w(b_0^*; \alpha_0^*, b_0^*) - b_0^* - \phi$ with $b_0^* = b^*(\alpha_0^*)$ and it follows that α_0^* gives both the bank's equity value at the point where it runs out of liquid reserves and the size of the maximum loss that shareholders are willing to absorb.

The identity (7) and the definition of the constants α_0^* and b_0^* imply that the equity value associated with the strategy $\hat{\pi}_{b_0^*}$ is then given by

$$v(s; b_0^*) = w(s; \alpha_0^*, b_0^*) = w(s; \alpha_0^*).$$

Using the properties of the auxiliary value function derived in the previous section, we show in the Appendix that this function satisfies

$$v(s; b_0^*) = (\alpha_0^* + s)^+ = \max_{b \geq 0} \left(w(b; \alpha_0^*) - b + s - \phi \right)^+, \qquad s \leq 0,$$

and, relying on a verification theorem for the Bellman equation associated with the bank's optimization problem, we prove that the barrier strategy $\hat{\pi}_{b_0^*}$ is optimal, not only in the class of barrier strategies, but among all strategies. This leads to the following result.

Proposition 4 (Equity value function): *The equity value function is concave and twice continuously differentiable in liquid reserves over* $(0, \infty)$ *and given by*

$$v(s) = v(s; b_0^*) = w(s; \alpha_0^*, b_0^*),$$

where α_0^* *denotes the unique fixed point of* $g(\alpha)$ *in the interval* $[0, v^*)$. *The optimal payout strategy consists in paying dividends to maintain liquid reserves at or below* $b_0^* = b^*(\alpha_0^*)$. *When* $\phi < \phi^*$, *the bank raises funds to move to* b_0^* *whenever liquid reserves become negative with a shortfall smaller than* $\alpha_0^* > 0$, *and liquidate otherwise. When* $\phi \geq \phi^*$, *the bank never raises equity and defaults the first time that liquid reserves reach* $\alpha_0^* = 0$.

Proposition 4 shows that when outside funds are costly, the value of equity is a concave function of the bank's liquid reserves over $(0, \infty)$, so that there are *no* incentives for shareholders to increase cash flow volatility even if the bank is levered. In our model, the bank's risk aversion is endogenous and depends on the level of its liquid reserves. In particular, for liquid reserves above the target level b_0^*, the equity value function satisfies

$$v''(s) = w''(s; \alpha_0^*) = 0,$$

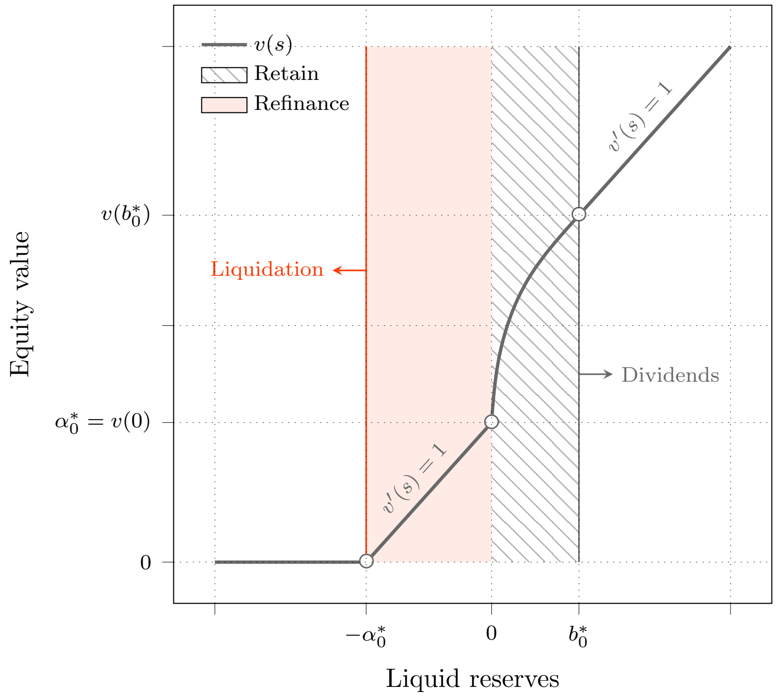
so that the bank becomes risk-neutral. For negative values of liquid reserves, the equity value function is linear as long as shareholders find it optimal to refinance and equal to zero in the default region where $s + \alpha_0^* \leq 0$. The value of equity is therefore convex for negative values of liquid reserves. However, since these values can only be reached following the occurrence of a large loss, the bank does not spend any time in this region. The equity value function is represented in Figure 1.

Under the optimal payout policy, the bank pays dividends in a minimal way to maintain its liquid reserves at or below b_0^*. This is illustrated by the smooth-pasting and high-contact conditions:

$$0 = v'(b_0^*) - 1 = v''(b_0^*),$$

which show that liquid reserves are optimally reflected down at b_0^*. When liquid reserves exceed b_0^*, the bank is fully capitalized and places no premium on internal funds so that it is optimal to make a lump sum payment $s - b_0^*$ to shareholders. The desired level of reserves, b_0^*, results from the trade-off between the cost of raising funds and the cost of holding liquid reserves. The next section investigates how this tradeoff is affected by the imposition of regulatory constraints.

Figure 1: Equity value for an unregulated bank



Notes. This figure illustrates the shape of the equity value function and the optimal strategy for an unregulated bank. The regions ▭ on each side of the graph correspond respectively to liquidation (left) and dividend payments (right) while the two intermediate regions ▨ and ▭ correspond respectively to earnings retention and refinancing.

Lastly, note that in our model, banks are subject to insolvency risk even though they can raise outside funds. Notably, Proposition 4 shows that it is optimal for shareholders to default following a shock that brings liquid reserves below $-\alpha_0^* = -v(0)$. Our results are thus in sharp contrast with those of Brownian-driven inventory models (see e.g. Milne and Whalley (2001), Peura and Keppo (2006), or Décamps, Mariotti, Rochet, and Villeneuve (2011)) in which there is no default if banks can raise outside equity. Also, while in those models firms always raise the same amount of capital, there exists some time series variation in the amount of funds raised from investors in our model.

III. The effects of liquidity and leverage ratio requirements

This section examines the effects of prudential regulation on the policy choices that maximize equity value and on the risk of insolvency. We introduce prudential regulation in our setup by considering that the regulator can set an lower bound on equity capital when making leverage choices and can force the bank to maintain its liquid reserves in excess of a given regulatory level. That is, our focus is on the instruments of regulation that are being considered in the Third Basel Accord, namely leverage and liquidity requirements.

A. Liquidity requirements

We start by examining the effects of liquidity requirements on the bank's policy choices and insolvency risk. To do so, we consider a regulatory constraint that mandates the bank to have a minimum level of liquid reserves $T > 0$ at all times. When $s \leq T$, the bank has the choice of recapitalizing by issuing new equity at a fixed cost ϕ or liquidating. By introducing this liquidity requirement in the model, we are interested in answering the following questions. Can a liquidity requirement affect bank behavior even if it is not binding? If liquidity requirements were to rise, would banks raise their target level of liquid reserves? What are the implications of these behaviors on default risk?

The value of equity in a bank subject to a minimum regulatory level of liquid reserves is defined by the optimization problem

$$v(s;T) = \sup_{\pi \in \Pi(s,T)} \mathbb{E}_s \left[\int_0^{\tau_{\pi,T}} e^{-\rho t} (dP_t^\pi - d\Phi_t(R^\pi)) + e^{-\rho \tau_{\pi,T}} (S^\pi_{\tau_{\pi,T}} - D - L)^+ \right], \qquad (8)$$

where $\tau_{\pi,T} = \inf\{t \geq 0 : S^\pi_{t+} \leq T\}$ denotes the default time associated with the strategy π in the presence of a liquidity requirement at the level T, the constant L is the face value of risky debt, and $\Pi(s,T)$ denotes the set of admissible payout and financing strategies such that $P^\pi_{t+} - P^\pi_t \leq S^\pi_t - T + R^\pi_{t+} - R^\pi_t$. This constraint implies that the regulatory amount of liquid reserves T cannot be distributed and therefore guarantees that creditors will receive at least this amount in liquidation. The first term on the right-hand side of equation (8) captures the present value of all dividend payments to shareholders until the time of default. The second term is the present value of the cash flow to shareholders in default.

Let $\ell(s) = (s - D - L)^+$ denote the payment that shareholders receive if liquidation occurs at point where the bank's liquid reserves are equal to s and let $a \wedge b = \min(a, b)$. Solving shareholders' optimization problem leads to the following result.

PROPOSITION 5 (Liquidity requirements): *Equity value in a bank subject to a minimum regulatory level of liquid reserves $T \geq 0$ is given by:*

$$v(s;T) = \begin{cases} v(s-T;0), & \text{if } \ell(T) \leq \alpha_0^*, \\ w(s-T;\ell(T)), & \text{otherwise.} \end{cases} \tag{9}$$

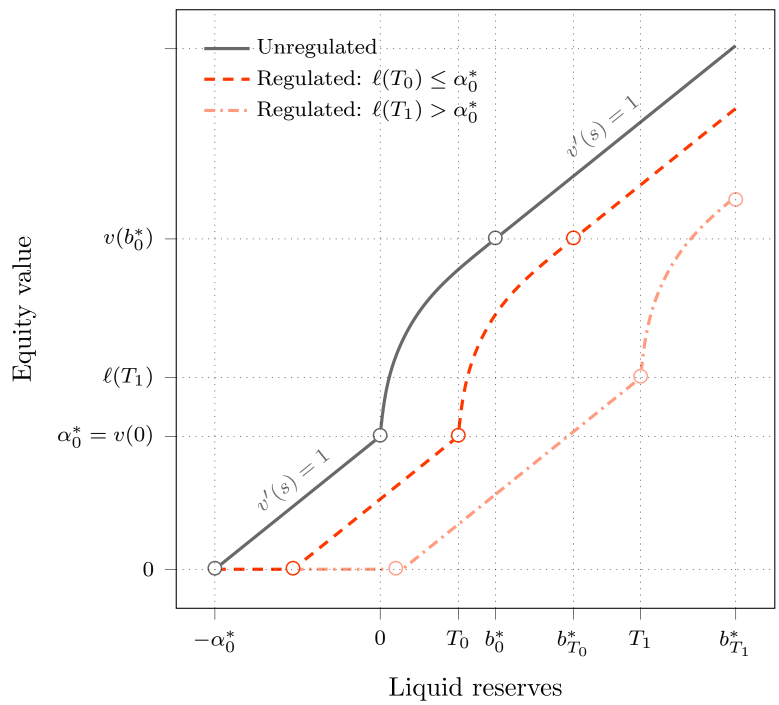
Liquidity requirements increase the target level of liquid reserves b_T^* in that

$$b_T^* = \begin{cases} T + b_0^*, & \text{if } \ell(T) \leq \alpha_0^*, \\ T + b^*(\ell(T) \wedge v^*), & \text{otherwise.} \end{cases} \tag{10}$$

When $\ell(T) \leq \alpha_0^*$, the bank pays dividends to maintain liquid reserves at or below $T + b_0^*$, raises funds to move to $T + b_0^*$ whenever liquid reserves fall below T with a shortfall smaller than α_0^*, and liquidates otherwise. When $\ell(T) > \alpha_0^*$, the bank never raises equity, defaults as soon as $s \leq T$, and pays dividends to maintain liquid reserves at or below $T + b^*(\ell(T) \wedge v^*)$.

Proposition 5 shows that the effect of minimum liquidity requirements on the bank's policy choices depends on their stringency. When the liquidity requirement is such that $\ell(T) > \alpha_0^* = v(0)$, it is never optimal for shareholders to refinance at the minimum regulatory level of liquid reserves T since the cash flow to shareholders in liquidation exceeds the continuation value of equity. In this case, the bank liquidates as soon as liquid reserves fall short of T. By contrast, when $\ell(T) \leq \alpha_0^*$, it is optimal for shareholders to refinance at

Figure 2: Equity value for a regulated bank



Notes. This figure illustrates the change in the equity value when going from an unregulated bank (solid line) to either a regulated bank with T_0 such that $\ell(T_0) \leq \alpha_0^*$ (dashed line) or to a regulated bank with T_1 such that $\ell(T_1) > \alpha_0^*$ (dotted line).

T. In this case, liquidity requirements lead to a drop in equity value as

$$v(s;T) = v(s-T;0) = v(s-T) \leq v(s), \qquad \forall s \geq T,$$

and make it optimal for shareholders to default when liquid reserves fall below $T - \alpha_0^*$. Figure 2 illustrates these effects by plotting the value equity for different liquidity requirements.

Given that liquidity requirements reduce equity value, it is immediate to see that in the short-run, or when the bank has not had the opportunity to move to its new target buffer of liquid reserves, such requirements increase insolvency risk (see also Figure 5 below). That is, liquidity requirements have a negative transitory effect on insolvency risk. We therefore focus below on the long-run effects of liquidity requirements, i.e. on their effects once the bank has had the opportunity to build up liquid reserves. In addition, given that setting $T > \alpha_0^* + D + L$ leads to liquidation whenever liquid reserves fall below the regulatory trigger, we consider that the regulator sets a minimum liquidity requirement satisfying $\ell(T) \leq \alpha_0^*$. In this case, it is optimal for shareholders to refinance when liquid reserves reach the regulatory level T, at which point the bank raises $b_T^\star - T = b_0^\star$ to go to the target level of liquid reserves. As a result, equity value at the regulatory trigger T is given by:

$$v(T; T) = \left(v(b_T^*; T) - (b_T^* - T) - \phi\right)^+ .$$

This value corresponds to the maximum amount that shareholders are willing to contribute to keep the bank alive following a large loss. Using equations (9) and (10), we then have

$$v(T; T) = \left(v(b_T^* - T) - (b_T^* - T) - \phi\right)^+ = \left(v(b_0^*) - b_0^* - \phi\right)^+ = \alpha_0^*,$$

so that shareholders default when liquid reserves fall below T with a shortfall greater than α_0^*. This shows that the presence of liquidity requirement increases the default threshold from the unregulated value $-\alpha_0^*$ to the regulated value $T - \alpha_0^*$. Since the target level of liquid reserves satisfies $b_T^* = b_0^* + T$, we immediately get the following result:

Corollary 6 (Liquidity requirements and insolvency risk): *Liquidity requirements have no effect on insolvency risk when banks are optimally capitalized.*

Given that $b_T^* = b_0^* + T$, the cushion of liquid reserves that the bank holds above the required level does not depend on the level chosen by the regulator. In effect, liquidity requirements simply shifts the support of the distribution of liquid reserves from $(-\alpha_0^*, b_0^*]$ to $(-\alpha_0^* + T, b_0^* + T]$. This is due to the fact that raising the level of liquid reserves does not change the trade-off between the cost of carrying liquidity and the cost of outside capital. Lastly, since the bank's attitude towards risks depends on the amount of liquid reserves held in excess of the refinancing threshold, this result in turn implies that, once there has been an opportunity to build up liquid reserves towards the desired level, liquidity requirements do not affect the attitude of the bank towards risks. Therefore, our analysis implies that liquidity requirements have no long-run impact on either risk taking or insolvency risk.

B. *Endogenous leverage and leverage requirements*

So far, our analysis has considered that the debt level of the bank was exogenously given. In this section, we examine the privately optimal mix between equity and debt on the balance sheet of the bank and the effects of regulatory constraints on this mix.

To do so, consider a bank that raises an amount D of deposits and issues at par a perpetual debt contract, with face value L and coupon rate c_L. Assume that the bank is subject to a constant setup cost $\Psi \geq \phi$ and let us start by determining the set of face values that lead to nonnegative payoffs for both shareholders and debtholders. If creditors agree to the proposed face value L, then the present value to shareholders of setting up the bank is

$$\eta_s(c_L, L, T) = \sup_{e \geq (T+\Psi-D-L)^+} \left(-e + v(D + L + e - \Psi, T|c_L, L)\right),$$

where e represents the amount of equity to be injected by shareholders and $v(s, T|c_L, L)$, defined as in (8), gives the equity value of a bank that holds s in liquid reserves, is subject to a minimum liquidity requirement $T \geq 0$, and has issued a perpetual debt contract with coupon rate c_L and face value L. In order to derive conditions under which this net present value is nonnegative, consider the function

$$N(c_L, T) = T + b_0^*(c_L) + \Psi - D - v(b_0^*(c_L)|c_L),$$

where $b_0^*(c_L)$ and $v(s|c_L)$ respectively give the optimal payout barrier and the equity value of an unregulated bank with coupon rate c_L. Note that, since shareholders in such a bank do not receive any payment in liquidation, these quantities depend neither on the face value of the debt contract L nor on the amount of deposits D that the bank initially raises. In the Appendix, we establish the following result:

Proposition 7 (Net present value): *The net present value of setting up the bank to shareholders is positive if and only if $L \geq N(c_L, T)$, in which case this NPV satisfies:*

$$\eta_s(c_L, L, T)^+ = (L - N(c_L, T))^+,$$

and shareholders' strategy $\Theta^*(c_L) = (T, \alpha_0^*(c_L), b_0^*(c_L))$ consists in paying dividends to maintain liquid reserves at or below $T + b_0^*(c_L)$, raising funds to move to $T + b_0^*(c_L)$ whenever liquid reserves fall below T with a shortfall smaller than $\alpha_0^*(c_L)$, and liquidating otherwise.

The first part of Proposition 7 shows that shareholders break even only if they can obtain at least $N(c_L, T)^+$ from creditors. If creditors agree to such a price, then the second part of the proposition shows that shareholders will first adjust liquid reserves to the optimal level

$$b_T^*(c_L) = T + b_0^*(c_L)$$

and will then run the bank according to the barrier strategy $\Theta^*(c_L)$. Accordingly, the present value to creditors will be

$$\eta_c(c_L, L, T) = d\left(T + b_0^*(c_L), \Theta^*(c_L) \middle| \, c_L, L\right) - L, \tag{11}$$

where the function

$$d(s, \Theta | c_L, L) = \mathbb{E}\left[\int_0^{\tau(s,\Theta)} e^{-\rho t} c_L dt + e^{-\rho\tau(s,\Theta)} \min\left(S_{\tau(s,\Theta)}(s, \Theta) - D, L\right)^+\right] \tag{12}$$

gives the market value of the bank's debt for all s under the assumption that it follows a barrier strategy $\Theta = (T, a, b)$. The first term on the right-hand side of equation (12) captures the present value of all coupon payments to debtholders until the occurrence of the default time $\tau(s, \Theta)$ associated with the use of the strategy Θ starting from the initial level s. The second term is the present value of the cash flow to debtholders in default, where we have assumed that deposits were senior to risky debt.[7]

In the Appendix, we derive a closed-form expression for the value of the bank's debt and show that, for any coupon rate $c_L > 0$, the creditors' present value is strictly decreasing in L and admits a unique root $L^*(c_L, T)$ that lies in $(0, c_L/\rho]$. This result implies that the present value of creditors in equation (11) is positive if and only if the proposed face value lies below $L^*(c_L, T)$. Combining this with Proposition 7 then shows that the set of individually rational face values associated to a coupon rate level is given by the interval

$$\left\{L > 0 : N(c_L, T)^+ \leq L \leq L^*(c_L, T)\right\}.$$

If the chosen coupon rate is such that this set is empty, then there is no face value of debt that generates nonnegative payoffs for both parties. If the coupon rate is such that $0 < N(c_L, T) \leq L^*(c_L, T)$, then any face value between these points generates a nonnegative present value for both parties, and it remains to determine how the surplus is shared between shareholders and creditors. To do so, we assume that creditors are competitive and do not have any bargaining power against the bank. In this case, the usual undercutting argument implies that shareholders will be able to sell debt at the highest possible price $L^*(c_L, T)$.

In our model, issuing debt is beneficial for the bank as it reduces its taxes. At the same time, for any given level of liquid reserves, debt financing increases expected bankruptcy costs and the frequency at which the bank raises costly outside capital. When choosing the coupon payment c_L on risky debt, the objective of shareholders is to maximize the value of equity after debt has been issued plus the proceeds from the debt issue net of the cost to

[7]Helberg and Lindset (2013) examine the effects of alternative priority rules on bank leverage and credit spreads in a Leland (1994) style model.

shareholders of acquiring the bank assets through the provision of required capital. That is, shareholders solve:

$$\sup_{c_L \in \mathcal{C}(T)} \eta_s\left(c_L, L^*(c_L, T), T\right) = \sup_{c_L \in \mathcal{C}(T)} \left(v(b_0^*(c_L)) + D + L^*(c_L, T) - T - b_0^*(c_L) - \Psi\right), \quad (13)$$

where the feasible set is given by

$$\mathcal{C}(T) = \{c_L \geq 0 : N(c_L, T) \leq L^*(c_L, T)\}.$$

One key difference between the optimization problem (13) and leverage choices in dynamic corporate finance models such as Leland (1994) is that the initial contribution of shareholders to the bank's capital is endogenous and depends on the liquidity constraints faced by the bank. Another important difference is that, in our model, the bank determines its optimal capital structure by balancing tax benefits against both issuance costs and bankruptcy costs.

While problem (13) is not amenable to an explicit solution, we can show that as long as the minimum liquidity requirement does not exceed the amount of deposits—which is the relevant case in practice—the presence of a liquidity requirement will either deter shareholders from setting up the bank or will have no effect on the optimal coupon rate. In particular, we establish the following result in the Appendix.

PROPOSITION 8 (Optimal leverage and liquidity requirements): *Assume that the optimal coupon rate* $c_L^*(0) \in \mathcal{C}(0)$ *is well-defined. If the liquidity requirement is such that* $T \leq D$ *then either* $\mathcal{C}(T)$ *is empty or* $c_L^*(T) = c_L^*(0)$.

In addition to liquidity requirements, the bank may also face leverage ratio requirements. That is, in an attempt to reduce insolvency risk, the regulator may constrain the bank to choose a debt level $c_L \in [0, \hat{c})$, where $\hat{c} \geq 0$ is a cap on the coupon payment on risky debt. We investigate in the next section the effects of such requirements on the bank's policy choices and the risk of insolvency.

IV. Model analysis

A. *Parameter values and implied variables*

This section provides additional results and illustrates the effects of frictions and regulatory constraints on bank's policy choices using numerical examples. The values of the model parameters are set as follows: the risk-free rate of return $\rho = 5\%$, the after-tax mean return on risky assets $(1-\theta)\mu - \frac{\lambda}{\beta} = 13.5\%$, the diffusion coefficient $\sigma = 8\%$, the frequency of large losses $\frac{1}{\lambda} = 2$, the mean size of large losses $\frac{1}{\beta} = 9\%$, the corporate tax rate $\theta = 35\%$, the fixed volume of demand deposits $D = 4.5$ and the corresponding cash outflow $c_D = 4.5\%$, and the cost of outside funds $\phi = 0.75\%$. Table 1 summarizes these parameter values.

These parameter values imply that large losses represent on average 50% of yearly income and occur every other year in expectation and that the cost of refinancing represents between $\frac{\phi}{b_T^* + \alpha_T^*} = 1.27\%$ (at the default threshold) and $\frac{\phi}{b_T^*} = 6.36\%$ (at the regulatory threshold) of the capital raised. For these parameter values, the optimal coupon on risky debt is $c_L = c_L^*(0) = 11.49\%$ and the face value of risky debt is $L = 2.25$.

Because for any coupon level, liquid reserves fluctuate between their minimum regulatory level and the target buffer set by bank shareholders, the leverage ratio of the bank effectively remains between two bands as in the dynamic capital structure models of Fischer, Heinkel, and Zechner (1989), Hackbarth, Miao, and Morellec (2006), Strebulaev (2007), and Morellec, Nikolov, and Schürhoff (2012). Notably, the leverage ratio of the bank – defined as debt value plus deposits over total bank value – fluctuates between 91.89% and 93.49% at the optimal coupon level and deposits represent between 61.25% and 62.31% of total bank value, consistent with the figures reported in Gropp and Heider (2010).[8] Lastly, the 1-year default probability for the bank at optimal leverage is 0.1057%, consistent with the values reported

[8]Interestingly, changing the pricing of deposit insurance has little effect on the leverage ratio selected by shareholders. For example, the overall leverage of the bank fluctuates between 91.45% and 93.26% if we increase c_D from 4.5% to 9% and between 91.46% and 93.56% if we increase c_D to 13.5%. This is due to the fact that an increase in c_D leads not only to a decrease in $c_L^*(T)$ but also to a decrease in equity value.

Table I: Parameter values and implied variables

A. Parameter values		
Parameter	Symbol	Value
Drift rate net of taxes	$(1-\theta)\mu$	0.18
Volatility	σ	0.08
Arrival intensity	λ	0.50
Mean jump size	$1/\beta$	0.09
Tax rate	θ	0.35
Face value of debt	L	2.25
Face value of deposits	D	4.50
Coupon rate	c_L	0.1149
Cost of deposits	c_D	0.045
Discount rate	ρ	0.05
Financing cost	ϕ	0.0075
Liquidity requirement	T	0.00
B. Implied variables		
Variable	Symbol	Value
First best value	v^*	0.6220
Equity value	α_0^*	0.4699
Target level of liquid reserves	b_0^*	0.1179
Share of financing cost		0.0636
Leverage		$[0.9189, 0.9349]$
Leverage from deposits		$[0.6125, 0.6231]$

in Crossen and Zhang (2012) or Hamilton, Munves, and Smith (2010) for 1-year default probabilities for financial firms.

B. Default and payout decisions

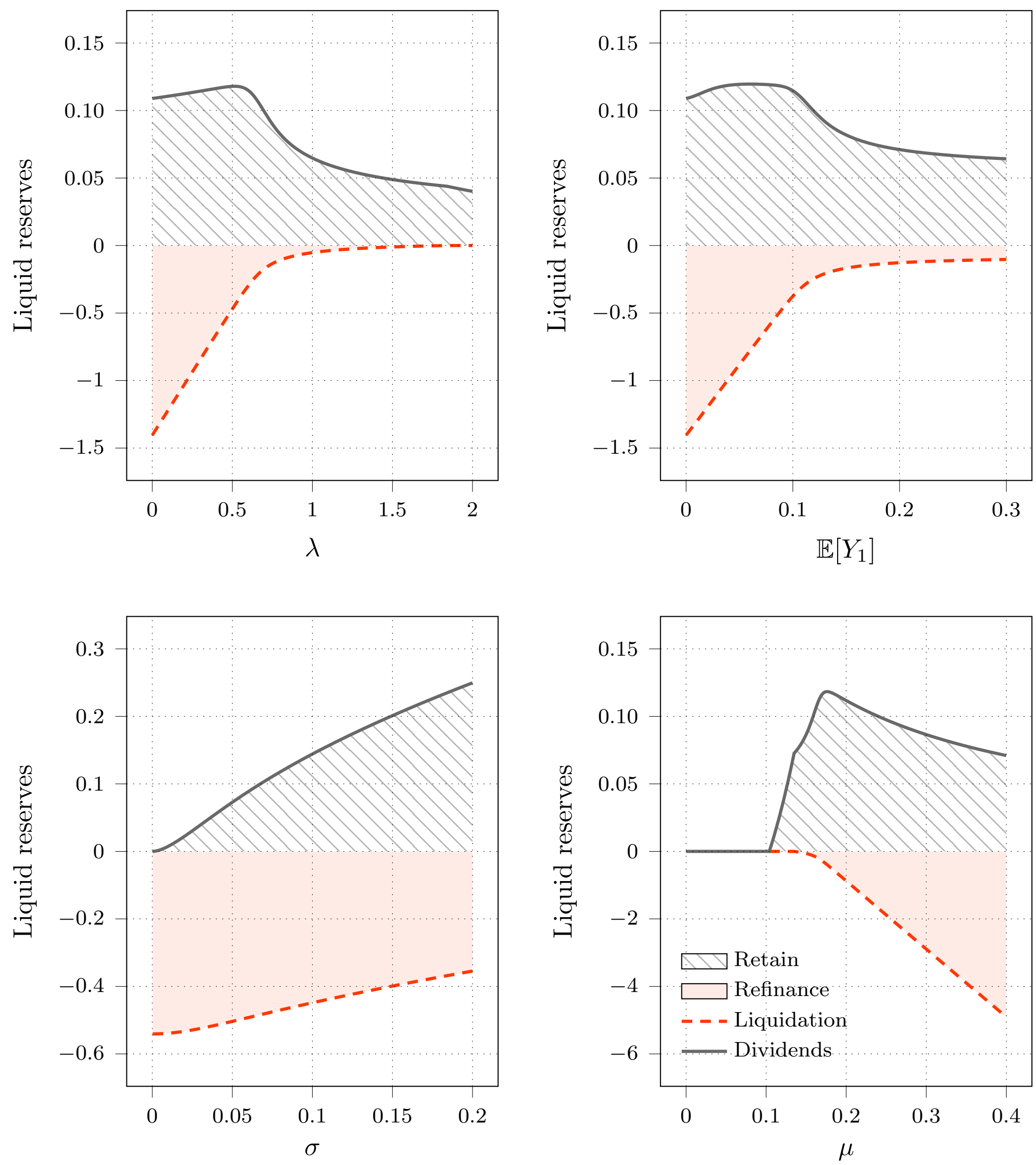
We start our analysis by examining the determinants of the target level of liquid reserves. To do so, we plot in Figure 3 the unconstrained target buffer b_0^* as a function of the intensity of large losses λ, the size of large losses $\frac{1}{\beta}$, the volatility coefficient σ, and the rate of return μ. In the model, the target buffer of liquid reserves maximizes the value of equity and results from a trade-off between the carry cost of liquidity ρ and the cost of raising new capital. Because an increase in volatility increases the likelihood of a costly equity issuance, the target buffer

b_0^* increases with σ. An increase in the frequency of large losses λ or in their size $\frac{1}{\beta}$ has two opposite effects on b_0^*. First, it increases the likelihood of a costly security issuance and, hence, shareholders' incentives to build up liquid reserves. Second, it reduces the expected cash flow from operating the bank's assets (and thus the bank's franchise value) and therefore shareholders' incentives to contribute capital. The first effect dominates for low values of λ and $\frac{1}{\beta}$; the second effect dominates for larger expected losses. The effect of the rate of return μ on the target level of liquid reserves also results from two opposite effects. On the one hand, a rate of return increases the bank's franchise value and shareholders' incentives to contribute capital. On the other hand a higher rate of return increases revenues and reduces the role of liquid reserves as a buffer to absorb losses.

Consider next the determinants of the bank's default decision. As shown in Proposition 4, when there is no liquidity requirement in that $T = 0$, default occurs when the bank is hit with a negative shock that takes its liquid reserves below $-\alpha_0^*$. This threshold is given by the value of equity at the target, i.e. $v(b_0^*)$, net of the new provision of capital b_0^* and of the refinancing cost ϕ. By decreasing $v(b_0^*)$, an increase in the intensity of large losses λ, in their size $\frac{1}{\beta}$, in the cost of financing ϕ, or in the diffusion coefficient σ lead to an increase in the default threshold. By contrast, an increase in the rate of return μ (or a decrease in the coupon rate c) leads to an increase in the franchise value of the bank and, therefore, to a decrease in the default threshold. These effects are illustrated in Figure 3.

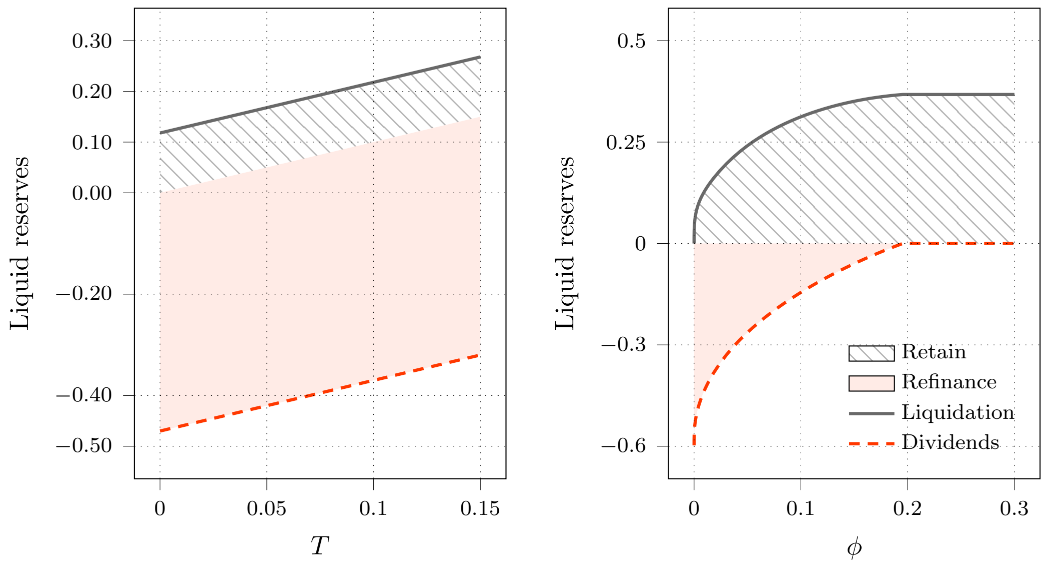
To illustrate the effects of liquidity requirements on bank's policy choices, Figure 4 plots the target level of liquid reserves as well as the default threshold as functions of the minimum requirement T. Consistent with Proposition 5, the figure shows that both thresholds grow linearly with the minimum liquidity requirement T. That is, regulatory requirements affect bank behavior even when they are not binding. The figure also shows that banks will generally hold a significant buffer of liquid reserves in excess of the liquidity requirements in order to reduce refinancing costs. Therefore, our theory predicts that liquidity requirements should be non-binding for most banks. Lastly, Figure 4 shows that both thresholds increase

Figure 3: Effect of the cash-flow parameters



Notes. This figure plots the target level of liquid reserves (solid line) and the liquidation threshold (dashed line) as functions of the jump arrival intensity, the mean jump size, the cash flow volatility and the cash flow drift. In each panel the upper and lower regions ▭ correspond respectively to dividend payments and liquidation while the two intermediate regions ▨ and ▭ correspond respectively to earnings retention and refinancing.

Figure 4: Effect of liquidity requirements and financing costs



Notes. This figure plots the target level of liquid reserves (solid line) and the liquidation threshold (dashed line) as functions of the required level of liquid reserves and the fixed cost of equity financing. In each panel the upper and lower regions ▭ correspond respectively to dividend payments and liquidation while the two intermediate regions ▧ and ▭ correspond respectively to earnings retention and refinancing.

with the cost of external funds ϕ. Indeed, an increase in the cost of raising funds decreases the benefits of refinancing, leading to an increase in the selected default threshold. In addition, an increase in the cost of external funds raises the value of inside equity and therefore shareholders' incentives to build up liquid reserves.

C. Regulation and insolvency risk

An important question that our model allows to answer is whether regulatory requirements affect insolvency risk. Proposition 5 and Corollary 6 show that altering the liquid reserves of banks has no long-run impact for the risk of insolvency (this is also illustrated by Figure 4 in which the spread between the target level of liquid reserves b^*_T and the default threshold

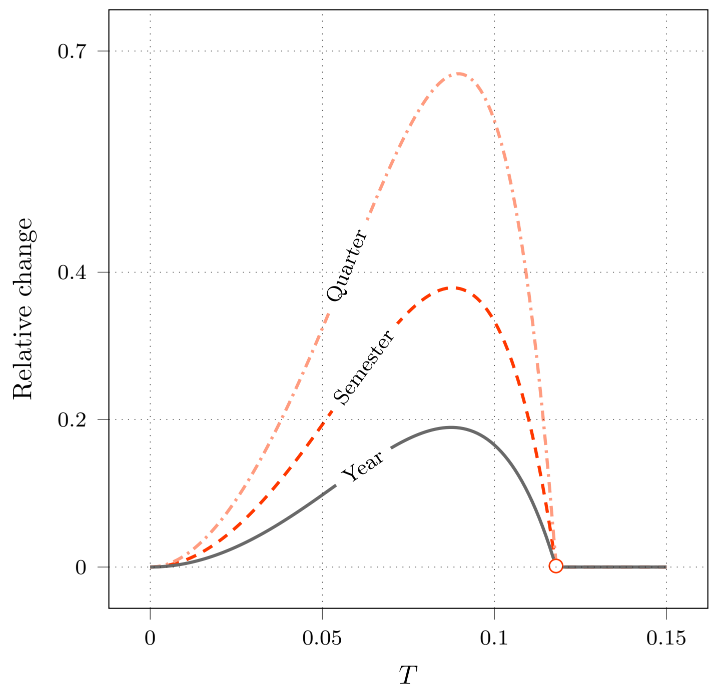
$T - \alpha_0^*$ is independent of T). To examine the short-run effects of liquidity requirements on insolvency risk, Figure 5 plots the relative change in the probability of default of the bank that results from imposing a minimum liquidity requirement. When the bank follows equity value-maximizing policies and is optimally financed, this relative change is given by:

$$\frac{P\left(\tau(b_0^*; T, \alpha_0^*, b_0^*) < t\right)}{P\left(\tau(b_0^*; 0, \alpha_0^*, b_0^*) < t\right)} - 1,$$

where $\tau(b_0^*; T, \alpha_0^*, b_0^*)$ is the time of default for a bank with liquid reserves b_0^*, default trigger at $-\alpha_0^*$, payout trigger at b_0^*, and facing a liquidity requirement T. Figure 5 considers three different horizons to compute the change in the probability of default: A three-month horizon ($t = 0.25$), a six-month horizon ($t = 0.5$), and a one-year horizon ($t = 1$). Consistent with the discussion in section III.A, Figure 5 shows that imposing a minimum liquidity requirement always increases insolvency risk in the short-run. In addition, the figure shows that the increase in the probability of default is larger for shorter horizons, since it is less likely that the bank has had the opportunity to build up liquid reserves. The figure also shows that for large liquidity requirements (i.e. when T is large), it is optimal for the bank to recapitalize immediately. In such cases, liquidity requirements do not change the probability of default.

One way to constrain the bank's leverage is to constrain its liquid asset holdings by imposing liquidity requirements. However, we have just shown that imposing such constraints leads to a short-run increase in insolvency risk for any given coupon c_L. In addition, as shown by Proposition 8, liquidity requirements have no effects on banks' *ex ante* choice of debt level, i.e. on the liability side of the balance sheet, in that $c_L^*(T) = c_L^*(0)$ for all $T \leq D$. As a result, a more direct mechanism, constraining directly banks' leverage ratios, may be necessary. One measure advocated by practitioners and academics to limit default risk in the banking sector is to impose a minimum level of equity capital (or a maximum coupon level) for banks (see e.g. Admati and Hellwig (2013)).

Figure 5: Insolvency risk and liquidity requirements

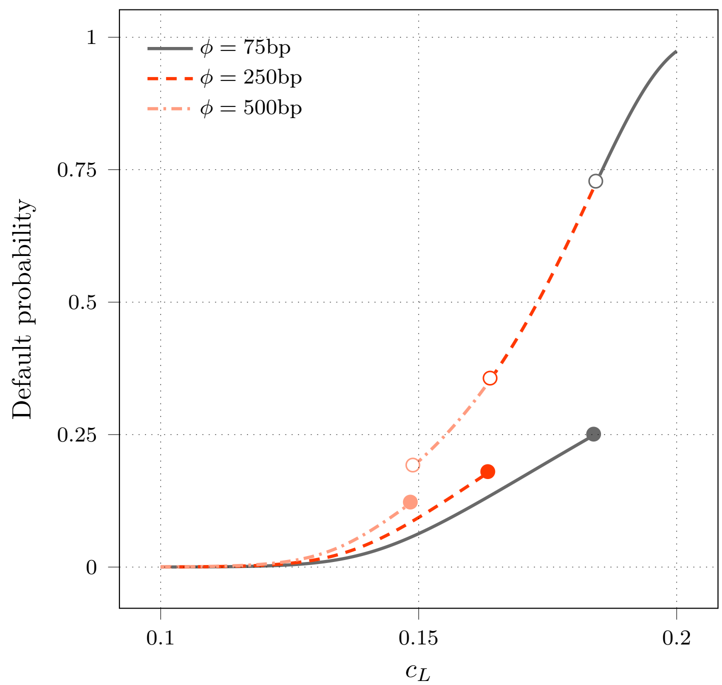


Notes. The figure plots the relative change in default probability at a 3 month (dotted line), 6 month (dashed line) and 1 year horizon (solid line) induced by a tightening of the liquidity requirement for a bank with liquid reserves equal to the unregulated target b_0^*. The dot indicates the point where the liquidity requirement level reaches the unregulated target.

To illustrate the effects of such leverage ratio requirements on insolvency risk, Figure 6 plots the probability of default of the bank over a one-year horizon as a function of its coupon payment c_L, assuming that the bank holds the target amount of liquid reserves $b_T^*(c_L)$. (The Appendix shows how to compute this default probability.) The three curves represent this default probability for different values of the cost of financing ϕ. On each curve the discontinuity occurs at the point where the bank no longer uses outside equity financing, i.e. at the point where $\ell(T)$ becomes larger than α_0^*.

As shown by the figure, leverage ratio requirements have a very strong quantitative effect on insolvency risk. In our base case environment for example, reducing the coupon rate of

Figure 6: Insolvency risk and coupon level



Notes. This figure plots the default probability at an horizon of one year for an optimally capitalized, unregulated bank as a function of the coupon rate c_L on long term debt for different level of the fixed cost of financing. On each curve the discontinuity occurs at the point where the bank no longer uses outside equity financing.

the bank from $c_L = 12.8\%$ – corresponding to a equity capital ratio of 6% advocated by U.S. regulators – to $c_L = 5.34\%$ – corresponding to the equity capital ratio of 20% recommended by Admati and Hellwig (2013) – reduces the probability of default over a one-year horizon from 0.1518% to less than 0.0001%. In effect, leverage ratio requirements leads to an increase in the *ex post* franchise value of the bank (i.e. in the cash flow rate of the bank net of the payments to debtholders) and, hence, in shareholders' willingness to absorb losses.[9]

[9] In our model, the bank only has access to one class of risky assets. As a result, the ex-ante constraint on the coupon level that we consider in the paper is equivalent to a risk-weighted capital ratio. When banks have access to multiple risky assets, risk-based capital charges constrain the bank's asset choices to reflect the trade-off between risk and return. See for example Rochet (1992) for an analysis of the benefits and costs of risk-weights.

D. Regulation and value

Our analysis so far has shown that while liquidity requirements may increase banks' insolvency risk in the short-run, leverage requirements reduce this risk. Another important aspect of regulation that has been widely discussed in the financial press relates to its effects on valuations. To measure the value effects of liquidity and leverage requirements, Figure 7 plots the net present value to shareholders (top left panel) as well as the bank's leverage (top right panel) as functions of the selected coupon payment.

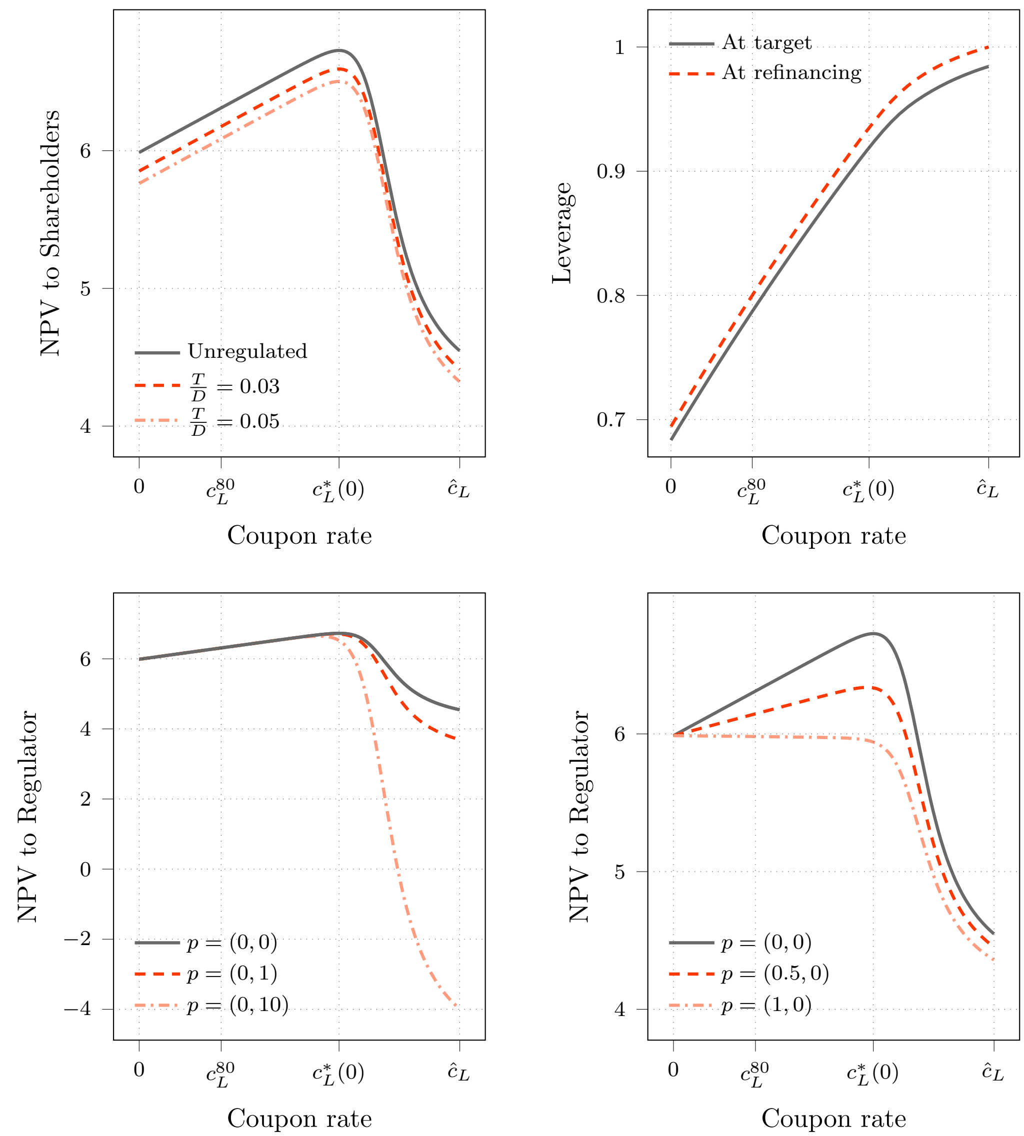
As shown by Figure 7, both liquidity and leverage ratio requirements have significant value effects. For example, the total value of the bank, given by $v(b_0^*(c_L)) + D + L^*(c_L, T)$, is reduced by 6.25% when moving from the equity value-maximizing leverage ratio to a leverage ratio with 20% equity capital. In our model, this decrease in value is due to an increase in the bank's cost of capital. Similarly, the figure shows that liquidity requirements lead not only to a short-run increase in default risk but also to a large reduction in the value of the bank. In our base case environment for example, requiring that liquid reserves represent at least 3% of the value of deposits reduces the value of the bank by 1.97%.

Next, to measure the social benefits of regulation, we compute the social net present value of the bank defined as the net present value to shareholders net of the cost of debt to the regulator. We consider that this cost has two components. First, it includes a flow cost that (partially) balances the tax benefits of debt to the bank shareholders. Second, it includes an additional cost of default, related for example to systemic effects. Specifically, we define the net present value to the regulator $\eta_r(p, c_L)$ or social net present value as:

$$\eta_r(p, c_L) \equiv \eta_s(c_L) - \mathbb{E}_{b_T^*(c_L)} \left[p_0 e^{-\rho \tau_T^*(c_L)} + p_1 \int_0^{\tau_T^*(c_L)} e^{-\rho t} \theta c_L dt \right],$$

where $\eta_s(c_L)$ is the value of the bank to shareholders, p_0 represents the systemic cost borne by the regulator in default, and p_1 represents the flow cost of debt to the regulator.

Figure 7: Effect of the cash-flow parameters



Notes. This figure plots the net present value to shareholders as a function of the coupon level for different values of the ratio $\frac{T}{D}$ (upper left panel), the leverage at the target level of liquid reserves $b_T^*(c_L)$ and at the refinancing point T as functions of the coupon level (upper right panel), and the net present value to the regulator as a function of the coupon level for different values of the cost parameters $p = (p_0, p_1)$ (lower panels). On the horizontal axis $c_L^{80} = 0.0471$ indicates the coupon level where the leverage at refinancing equals 80%, $c_L^*(0) = 0.1149$ indicates the coupon level that is optimal for the shareholders of the bank given any liquidity requirement $T \leq D$, and $\hat{c}_L = 0.1843$ indicates the coupon level where where the bank switches to a no-refinancing strategy.

The bottom panels of Figure 7 plot the social net present value for different systemic costs of default (bottom left panel) as well as different flow costs of debt (bottom right panel) as functions of the selected coupon payment. As shown by the figure, systemic default costs have little effect on the coupon payment that maximizes the social net present value. For example, increasing systemic default costs from $p_0 = 0$ (no systemic costs) to $p_0 = 10$ (systemic costs corresponding to 150% of the total bank value) only changes the range of leverage ratios from $[91.89\%; 93.49\%]$ to $[89.78\%; 91.32\%]$. This result is due to the fact that the default probability is very low at the equity value-maximizing leverage ratio so that increasing systemic default costs only has a marginal effect on the coupon payment maximizing the social value of the bank.

Similarly, reducing the tax benefit of debt to the regulator only has a large effect if the reduction is drastic, i.e. if p_1 is very large. For example, increasing the flow cost of debt to the regulator from $p_1 = 0$ to $p_1 = 0.5$ (i.e. reducing the tax benefit of debt to the regulator by 50%) only changes the range of leverage ratios from $[91.89\%; 93.49\%]$ to $[91.26\%; 92.84\%]$. Overall, our analysis therefore suggests that while constraining leverage ratios clearly reduces insolvency risk, it seems difficult to justify leverage ratios in the range of 80%, unless one significantly reduces the tax benefits of debt.

V. Conclusion

We develop a dynamic of model of banking in which banks face taxation, flotation costs of securities, and default costs and are financed with equity, insured deposits, and risky debt. In this model, liquidity management, capital structure policies, and default decisions are jointly and endogenously determined. Shareholders have limited liability and banks' policies maximize shareholder value.

Using this model, we show that when raising outside funds is costly, inside and outside equity are not perfect substitutes and banks find it optimal to hold buffers of liquid assets.

To reduce the risk of default and save on recapitalization costs, banks manage their liquidity buffers dynamically by adjusting their dividend payments to shareholders. Banks facing higher cost of outside financing are less levered, pay less dividends, hold more liquid reserves, and default earlier. In addition, we show that equity value is concave, implying that shareholders in levered banks have no incentives to increase risk.

Turning to regulation, we show that liquidity requirements constraining banks to hold a minimum amount of liquid reserves have no long-run effects on default risk or bank risk-taking but may increase default risk in the short run. We also show that leverage requirements, indicating how much equity capital banks should have relative to their total assets, increase banks' *ex post* franchise value and reduce default risk. *Ex ante*, however, such requirements may reduce significantly total bank value by increasing the bank's cost of capital. In our base environment for example, we find that increasing equity capital from the equity value-maximizing level to 20%, as suggested by Admati and Hellwig (2013), reduces total bank value by 6.25%.

While the framework developed in this paper is based on a number of simplifying assumptions, we believe that it is a natural starting point to think about bank equity capital and liquid reserves and to examine the effects of prudential regulation on bank behavior and insolvency risk. In future research, we plan to extend this framework to examine additional issues related to bank optimal capital structures and asset choices.

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Appendix to:

Bank Capital, Liquid Reserves, and Insolvency Risk

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Contents

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A. The auxiliary problem

A.1. Notation

To facilitate the presentation we start by fixing some notation that will be of repeated use throughout the appendix. A strategy is a pair $\pi = (P^\pi, R^\pi)$ of adapted, non decreasing, *left continuous and right limited processes* with initial value zero such that

$$R_t^\pi = \sum_{n=1}^{\infty} 1_{\{t>\xi_n\}} r_n$$

for some increasing sequence of stopping times $(\xi_n)_{n=1}^{\infty}$ and some sequence of nonnegative random variables $(r_n)_{n=1}^{\infty}$ such that r_n is measurable with respect to $\mathcal{F}_{\xi_n}$. The liquid reserves process and liquidation time associated with the use of a given payout and financing strategy are defined by

$$S_t^\pi = s + C_t - P_t^\pi + R_t^\pi = s + \bar{\mu}t + \sigma B_t - \sum_{n=1}^{N_t} Y_n - P_t^\pi + R_t^\pi$$

and

$$\tau_\pi = \inf\left\{ t \geq 0 : S_{t+}^\pi = \lim_{u \downarrow t+} S_u^\pi \leq 0 \right\}.$$

with the strictly positive constant $\bar{\mu} = (1-\theta)(\mu - c)$. The set $\Pi(s)$ of strategies that are admissible starting from $s \in \mathbb{R}$ is defined as the set of strategies such that

$$\Delta^+ P_t^\pi \leq S_t^\pi + \Delta^+ R_t^\pi \tag{A1}$$

and

$$\mathbb{E}_s \left[\int_0^{\tau_\pi} e^{-\rho t} (dP_t^\pi + d\Phi_t(R^\pi)) \right] < \infty. \tag{A2}$$

where the nondecreasing process

$$\Phi_t(R^\pi) = R_t^\pi + \sum_{n=1}^{\infty} 1_{\{t>\xi_n\}} \phi$$

represents the total contribution of shareholders to the bank, and

$$\Delta^+ Z_t = Z_{t+} - Z_t = \lim_{u \downarrow t} Z_u - Z_t$$

denotes the jump that occurs immediately after time $t \geq 0$. The inequality constraint imposed by (A1) prevents shareholders from distributing dividends that exceed the available liquid reserves and is necessary to guarantee that the optimization problems (2), (5) are well-defined. Otherwise the bank would be able to generate infinite value by simply paying out amounts that it does not hold.

A.2. Immediate liquidation

LEMMA A.1: *Denote by $v^* > 0$ the unique solution to*

$$z(v) = -(\rho + \lambda)v + \bar{\mu} + \lambda \mathbb{E}\left[(v - Y_1)^+\right] = 0$$

If the liquidation value is such that $\alpha \geq v^*$ then it is optimal for shareholders to shut down the bank immediately.

PROOF. The function $z(v)$ is continuous, non increasing, starts out from $\bar{\mu} > 0$ at the origin and satisfies

$$\lim_{v \to \infty} z(v) = -\infty.$$

Therefore it crosses the horizontal axis at a unique point $v^* > 0$. Given the result of Lemma A.8 below it now suffices to show that the equity value function $u_0(s) = (\alpha + s)^+$ associated with the strategy of immediate liquidation satisfies

$$\mathcal{H}u(s) = \max\{1 - u'(s), \mathcal{D}u(s)\} \leq 0, \qquad s > 0, \tag{A3}$$

with the integro-differential operator defined by

$$\mathcal{D}u(s) = -\rho u(s) + \bar{\mu} u'(s) + \frac{1}{2}\sigma^2 u''(s) + \lambda \mathbb{E}\left[u(s - Y_1) - u(s)\right]. \tag{A4}$$

Substituting the function $u_0(s)$ into the right hand side of (A3) gives

$$\mathcal{H}u_0(s) = \max\left(0, z(\alpha + s)\right), \qquad s > 0.$$

Since the term inside the bracket is decreasing we have that $\mathcal{H}u_0(s) \leq 0$ for all $s > 0$ if and only if $\mathcal{H}u_0(0) = 0$. This is equivalent to $z(\alpha) \leq 0$ and the required result follows from the definition of the constant v^*. ■

A.3. Value of a barrier strategy

In order to compute the equity value of the auxiliary bank under a given barrier strategy let us first start by fixing some notation. Let

$$B_1 < -\beta < B_2 < 0 < B_3 \tag{A5}$$

denote the three real roots of the cubic equation

$$\rho = B_i \left(\bar{\mu} + B_i \frac{\sigma^2}{2} - \frac{\lambda}{\beta + B_i} \right), \tag{A6}$$

set

$$A(\alpha) = \lambda \mathbb{E}\left[(\alpha - Y_1)^+\right] = \lambda \left(\alpha - \frac{F(\alpha)}{\beta} \right) \in [0, \alpha\lambda]$$

and define

$$W(x) = \sum_{i=1}^{3} 1_{\{x \geq 0\}} \frac{2(\beta + B_i)}{\sigma^2 \prod_{k \neq i}(B_i - B_k)} e^{B_i x}. \tag{A7}$$

The function $W(x)$ is referred to as the ρ−scale function of the uncontrolled liquid reserves process (see Kuznetsov, Kyprianou, and Rivero (2013) for a comprehensive survey of the theory of scale functions) and the following result shows that this function can be used as a building block to derive explicit expressions for the functions appearing on the right hand side of equation (6).

Lemma A.2: *We have*

$$w(s;0;b) = \mathbb{E}_s\left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b\right] = (s-b)^+ + \frac{W(s\wedge b)}{W'(b)} \tag{A8}$$

and

$$\psi(s;\alpha) = \sum_{i=1}^{2} a_i(\alpha) e^{B_i s}, \qquad s \geq 0, \tag{A9}$$

with the coefficients defined by

$$a_i(\alpha) = (-1)^i \left[\frac{2A(\alpha) + \alpha(\beta+B_i)(\beta+B_3)\sigma^2}{(B_2-B_1)(\beta+B_3)\sigma^2}\right] > 0 \qquad i = 1, 2.$$

The function $\psi(s;\alpha)$ is completely monotone with respect to $s \geq 0$ and therefore decreasing and convex.

Proof. The first part follows directly from Avram, Palmowski, and Pistorius (2007, Proposition 1). To establish the second part we start by decomposing the function into two components according to whether the uncontrolled process enters the negative real line continuously (first term), or through a jump (second term):

$$\psi(x;\alpha) = \alpha\mathbb{E}_x\left[e^{-\rho\zeta_0} 1_{\{\Delta X_{\zeta_0}=0\}}\right] + \mathbb{E}_x\left[e^{-\rho\zeta_0}(\alpha + X_{\zeta_0})^+ 1_{\{\Delta X_{\zeta_0}\neq 0\}}\right]. \tag{A10}$$

Combining Corollary 2 and Equation (6) of Bertoin (1997) shows that the first term can be computed in terms of the scale function as

$$\alpha\mathbb{E}_x\left[e^{-\rho\zeta_0} 1_{\{\Delta X_{\zeta_0}=0\}}\right] = \frac{\alpha\sigma^2}{2}\left(W'(x) - B_3 W(x)\right). \tag{A11}$$

On the other hand, Bertoin (1997, Corollary 2) shows that the potential measure of the uncontrolled liquid reserves process killed at ζ_0 is given by

$$U(x,dy) = \mathbb{E}_x\left[\int_0^\infty e^{-\rho t} 1_{\{\zeta_0 > t\}\cap\{X_t\in dy\}} dt\right] = \left(e^{-B_3 y} W(x) - W(x-y)\right) dy \tag{A12}$$

and it thus follows from the compensation formula for point processes that second term can be calculated as

$$\mathbb{E}_x\left[e^{-\rho\zeta_0}(\alpha+X_{\zeta_0})^+1_{\{\Delta X_{\zeta_0}\neq 0\}}\right]=\int_0^\infty \lambda U(x,dy)\int_y^\infty (\alpha+y-u)^+dF(u). \tag{A13}$$

Substituting (A11) and (A13) into (A10), calculating the integral, and simplifying the result gives the formula reported in the statement. The nonnegativity of $A(\alpha)$ and (A5) imply that $a_2(\alpha)>0$. On the other hand, using (A5) and

$$\prod_{i=1}^{3}(\beta+B_i)+\frac{2\lambda\beta}{\sigma^2}=0<A(\alpha)-\lambda\alpha$$

we deduce that

$$a_1(\alpha)=\frac{2}{(B_1-B_2)(\beta+B_3)}\left(A(\alpha)-\frac{\lambda\beta\alpha}{\beta+B_2}\right)>0$$

and the required complete monotonicity of the function $\psi(s;\alpha)$ now follows from the fact that the constants B_1 and B_2 are negative. ■

A.4. The optimal barrier

In the earnings retention region the value of a barrier strategy depends on the barrier level only through the function

$$H(b;\alpha)=\frac{1-\psi'(b;\alpha)}{W'(b)}.$$

Therefore, since the scale function is strictly positive by Lemma A.6 below, we have that the existence and uniqueness of an optimal barrier amount to the existence and uniqueness of a global maximizer for this function.

Lemma A.3: *For any $\alpha\in[0,v^*)$ the function $H(b;\alpha)$ achieves its supremum over the positive half line at a unique point b^*.*

Proof. Since the case $\alpha=0$ follows directly from the result of Lemma A.6 we will assume that $\alpha\in(0,v^*)$. Using the definition of the functions $z(\alpha)$ and $A(\alpha)$ in conjunction with the fact that the roots of (A6) satisfy

$$\beta + \sum_{i=1}^{3} B_i + \frac{2\bar{\mu}}{\sigma^2} = \prod_{i=1}^{3} B_i - \frac{2\beta\rho}{\sigma^2} = 0,$$

we deduce that

$$H'(0;\alpha) = \bar{\mu} + A(\alpha) - \alpha(\rho + \lambda) = z(\alpha) > 0, \tag{A14}$$

where the inequality follows from the fact that $\alpha < v^*$ if and only if $z(\alpha) > 0$ as established in the proof of Lemma A.1. To study the behavior of the derivative away from the origin consider the function

$$G(b;\alpha) = -\frac{\psi''(b;\alpha)}{W''(b)}.$$

A direct calculation using the definition of the function $H(b;\alpha)$ shows that

$$H'(b;\alpha) = \left(G(b;\alpha) - H(b;\alpha)\right)\frac{W''(b)}{W'(b)} \tag{A15}$$

and it thus follows from Lemma A.6 that we have

$$H'(b;\alpha) \geq 0 \iff (x^* - b)(H(b,\alpha) - G(b,\alpha)) \geq 0 \tag{A16}$$

for any $b \neq x^*$ where the constant $x^* > 0$ is the unique solution to $W''(x^*) = 0$ provided by Lemma A.6 below. Now consider the threshold defined by

$$b^* = b^*(\alpha) \equiv \inf\{b \geq 0 : H(b;\alpha) \leq G(b;\alpha)\}.$$

Since $G(x^*,\alpha) = \infty$ by construction and $H(0;\alpha) > G(0;\alpha)$ due to (A14) and (A16), we have that this threshold lies in $(0, x^*)$ and satisfies

$$H'(b^*;\alpha) = H(b^*;\alpha) - G(b^*;\alpha) = 0 \leq H'(b;\alpha), \qquad 0 \leq b \leq b^*. \tag{A17}$$

Therefore, the proof will be complete once we show that function $H(b;\alpha)$ is decreasing on the interval $[b^*,\infty)$. Combining Lemmas A.2 and A.6 we deduce that

$$G(b;\alpha) \leq 0 \leq H(b;\alpha), \qquad b > x^*,$$

and it thus follows from (A16) that $H(b;\alpha)$ is decreasing over (x^*,∞). On the other hand, the fact that $b^* > 0$ implies that we have $G'(b^*;\alpha) > 0$ and therefore

$$G'(b;\alpha) > 0, \qquad b \geq b^*, \tag{A18}$$

by Lemma A.7. Combining this property with (A17) shows that there exists an $\epsilon_0 > 0$ such that the function

$$\Phi(b;\alpha) = H(b;\alpha) - G(b;\alpha)$$

is strictly negative on the interval $N_0 = (b^*, b^* + \epsilon_0]$ and it now follows from (A16) that we have $H'(b;\alpha) < 0$ on N_0. Repeating the same argument at $b^* + \epsilon_0$ then allows to propagate this property to the whole interval $(b^*, x^*]$ and completes the proof. ■

Intuitively we expect that the incentives of the bank to retain earnings decrease as the liquidation value of assets increases. The next result confirms this intuition by showing that the optimal barrier is decreasing in the liquidation value of assets.

LEMMA A.4: *The function $b^*(\alpha)$ is strictly decreasing on the interval $[0,v^*]$ with $b^*(0) = x^*$ and $b^*(v^*) = 0$.*

PROOF. As shown in the proof of Lemma A.3 we have that for any $\alpha \in [0,v^*)$ the optimal barrier satisfies

$$H(b^*(\alpha);\alpha) - G(b^*(\alpha);\alpha) = 0.$$

Therefore it follows from the implicit function theorem that the function $b^*(\alpha)$ is once continuously differentiable with

$$\frac{db^*(\alpha)}{d\alpha} = -\frac{G_\alpha(b^*(\alpha);\alpha) - H_\alpha(b^*(\alpha);\alpha)}{G_b(b^*(\alpha);\alpha) - H_b(b^*(\alpha);\alpha)} = \frac{H_\alpha(b^*(\alpha);\alpha) - G_\alpha(b^*(\alpha);\alpha)}{G_b(b^*(\alpha);\alpha)}$$

where the second equality follows from (A17). By (A18) we have that the denominator is strictly positive and so it only remains to show that the numerator is negative on $[0, v^*)$. To this end it suffices to show that the function

$$K(b; \alpha) = \frac{1}{4}\sigma^4 W'(b)W''(b)(H_\alpha(b; \alpha) - G_\alpha(b; \alpha))$$

is strictly positive. A direct calculation shows that this function can be decomposed into a sum of exponentials as

$$K(b; \alpha) = c_{12}(\alpha)e^{(B_1+B_2)b} + c_{13}(\alpha)e^{(B_1+B_3)b} + c_{23}(\alpha)e^{(B_2+B_3)b}$$

with the coefficients

$$c_{ij}(\alpha) = -\frac{B_i B_j((\beta + B_i)(\beta + B_j)\sigma^2 + 2A'(\alpha))}{2(B_i - B_{-ij})(B_i - B_{-ij})}, \qquad i \neq j \in \{1, 2, 3\}.$$

The increase of the function $A(\alpha)$ and (A5) imply that $c_{23}(\alpha)$ is strictly positive. On the other hand, using (A5) in conjunction with

$$\prod_{i=1}^{3}(\beta + B_i) + \frac{2\lambda\beta}{\sigma^2} = 0 < A'(\alpha) - \lambda \tag{A19}$$

we deduce that

$$c_{13}(\alpha) = \frac{-2B_1B_3}{(B_1 - B_2)(B_3 - B_2)}\left(A'(\alpha) - \frac{\lambda\beta}{(\beta + B_2)}\right) > 0.$$

If the remaining coefficient is also nonnegative then $K(b; \alpha) > 0$ for all $b \geq 0$ and the proof is complete. If instead the remaining coefficient is strictly negative then it follows from (A5) and the result of Lemma A.5 that there exist a constant γ such that $K(b; \alpha) > 0$ if and only if $b > \gamma$. Since

$$K(0; \alpha) = c_{12}(0) + c_{13}(0) + c_{23}(0) = \rho + \lambda - A'(\alpha) > 0.$$

by (A19) we have that $\gamma < 0$ and required result follows. The limit value at zero follows from the definition of the constant x^*. On the other hand, (A14), (A15) and the definition of the constant v^* imply that

$$\lim_{\alpha \to v^*} H'(0;\alpha) = \lim_{\alpha \to v^*} (H(0;\alpha) - G(0;\alpha)) = 0$$

and the result now follows from the fact that, as shown in the proof of Lemma A.3, the function $H(b;\alpha)$ is decreasing in b after the point where it crosses $G(b;\alpha)$. ■

LEMMA A.5: *Let $a_1 < 0 < a_3$ and $b_1 < b_2 < b_3$ be constants. Then there exists a unique constant γ^* such that the function*

$$f(x) = \sum_{i=1}^{3} a_i e^{b_i x}$$

is positive if and only if $x \geq \gamma^*$.

PROOF. Under the conditions of the statement we have that

$$k(x) = e^{-b_1 x} f(x) = a_1 + a_2 e^{(b_2 - b_1)x} + a_3 e^{(b_3 - b_1)x}$$

tends to $a_1 < 0$ as $x \to -\infty$ and to ∞ as $x \to \infty$. If a_2 is nonnegative, then this function is nondecreasing and therefore crosses the origin only once. On the other hand, if a_2 is negative then $k(x)$ is ∪-shaped and therefore attains a minimum at the unique point where its derivative equals zero. In either case, the equation $k(x) = 0$ admits a unique solution at which $k'(x) > 0$ and the desired result follows. ■

LEMMA A.6: *The scale function is strictly increasing on $[0, \infty)$ and there exists a constant $x^* > 0$ such that $W''(x) \leq 0$ if and only if $x \leq x^*$.*

PROOF. Differentiating (A8) and using (A5) shows that for any $x \geq 0$ the derivative of the scale function is given by

$$W'(x) = \sum_{i=1}^{3} c_i e^{B_i x}$$

with the coefficients

$$c_i = \frac{2(\beta + B_i)B_i}{\sigma^2 \prod_{k \neq i}(B_i - B_k)} > 0, \qquad i = 1, 2, 3.$$

This shows that the scale function is strictly increasing over the positive real line and hence positive since $W(0) = 0$. The second part follows from an application of Lemma A.5 and the fact that

$$W''(0) = \frac{2}{\sigma^2}(B_1 + B_2 + B_3 + \beta) = -\frac{4\bar{\mu}}{\sigma^4}$$

is strictly negative. ∎

Lemma A.7: *There exists an $x_0 \in [0, x^*)$ such that $G(x;\alpha)$ is decreasing on $[0, x_0]$ and increasing on (x_0, x^*).*

Proof. Consider the function defined by

$$K(x;\alpha) = \frac{1}{2}(\sigma W''(x))^2 G'(x;\alpha) \tag{A20}$$

A direct calculation using (A9) and the definition of the scale function shows that this function is explicitly given by

$$K(x;\alpha) = a_{13}(\alpha)e^{(B_1+B_3)x} + a_{23}(\alpha)e^{(B_2+B_3)x} + a_{12}(\alpha)e^{(B_1+B_2)x}$$

with the coefficients

$$a_{13}(\alpha) = \frac{B_1^2 B_3^2(\beta + B_3)}{B_3 - B_2} a_1(\alpha)$$
$$a_{23}(\alpha) = \frac{B_2^2 B_3^2(\beta + B_3)}{B_3 - B_1} a_2(\alpha)$$

and

$$a_{12}(\alpha) = \frac{B_1^2 B_2^2(2A(\alpha) + \alpha\sigma^2(\beta + B_1)(\beta + B_2))}{(B_3 - B_1)(B_3 - B_2)\sigma^2}.$$

As shown in the proof of Lemma A.2, we have that $a_1(\alpha), a_2(\alpha) > 0$ and combining this with (A5) shows that $a_{13}(\alpha), a_{23}(\alpha) > 0$.

To complete the proof we distinguish two cases depending on the sign of the last coefficient. If we have that $a_{12}(\alpha) \geq 0$ then $K(x;\alpha) \geq 0$ and the required result holds with the constant $x_0 = 0$. On the contrary, if we have that $a_{12}(\alpha) < 0$ then it follows from (A5), (A20) and Lemma A.5 that there exists an a such that

$$x \leq a \Longleftrightarrow G'(x;\alpha) \geq 0.$$

In this case we let $x_0 = a^+$ and it now only remains to show that $x_0 < x^*$. Since

$$\lim_{x \uparrow x^*} G(x;\alpha) = \lim_{x \uparrow x^*} -\frac{\psi''(x;\alpha)}{W''(x)} = \infty$$

by definition of x^*, we have that $G'(x;\alpha) > 0$ in a left neighborhood of x^* and the desired result now follows from the definition of the constant a. ■

A.5. Verification

By construction we have that the barrier strategy at $b^*(\alpha)$ is optimal in the restricted class of barrier strategies. To show that this strategy is in fact optimal among all admissible strategies we will rely on the following verification result.

LEMMA A.8: *Assume that $u : \mathbb{R} \to \mathbb{R}$ is a continuous function that is twice continuously differentiable on $(0, \infty)$ and such that*

$$u(s) - (\alpha + s)^+ = 0, \quad s \leq 0, \tag{A21}$$

$$\max\{1 - u'(s), \mathcal{D}u(s)\} \leq 0, \quad s > 0. \tag{A22}$$

where the operator $\mathcal{D}$ is defined in (A4). Then $u(s) \geq w(s;\alpha)$ for all $s \in \mathbb{R}$.

PROOF. Assume that the function $u(\cdot)$ satisfies the conditions of the statement, fix a strategy $\pi \in \Pi_0(s)$ and consider the nonnegative process defined by

$$Y_t = e^{-\rho t \wedge \tau_\pi} u(S_{t \wedge \tau_\pi}) + \int_0^{t \wedge \tau_\pi} e^{-\rho s} dP^\pi_s.$$

Applying Itô's formula for semimartingales (see for example Dellacherie and Meyer (1980, Theorem VIII.25)) shows that there is a local martingale M_t such that

$$
\begin{aligned}
Y_t - M_{t\wedge\tau_\pi} = u(s) &+ \sum_{0\leq s<t\wedge\tau_\pi} e^{-\rho s}\left(\Delta^+ P^\pi_s + \Delta^+ u(S^\pi_s)\right) \\
&+ \int_0^{t\wedge\tau_\pi} e^{-\rho s}\mathcal{D}u(S^\pi_{s-})ds + \int_0^{t\wedge\tau_\pi} e^{-\rho s}(1-u'(S_{s-}))dP^{\pi,c}_s
\end{aligned} \tag{A23}
$$

where $P^{\pi,c}_t$ denotes the continuous part of the cumulative payout process. Using the definition of the liquidation time together with the fact that the function $u(s)$ satisfies (A21) and (A22), we deduce that

$$
\begin{aligned}
&1_{\{s<\tau_\pi\}}\left(\Delta^+ P^\pi_s + \Delta^+ u(S^\pi_s) + \mathcal{D}u(S^\pi_{s-})ds + (1-u'(S_{s-}))dP^{\pi,c}_s\right) \\
&\quad\leq 1_{\{s<\tau_\pi\}}\left(\Delta^+ P^\pi_s + \Delta^+ u(S^\pi_s)\right) \\
&\quad= 1_{\{s<\tau_\pi\}}\left(u(S^\pi_s - \Delta^+ P^\pi_s) + \Delta^+ P^\pi_s - u(S^\pi_s)\right) \leq 0
\end{aligned}
$$

and it follows that the process on the right hand side of (A23) is decreasing. This in turn implies that Y_t is local supermartingale and hence a supermartingale since it is nonnegative. In particular, combining (A21) with the the optional sampling theorem for nonnegative supermartingales shows that

$$
\begin{aligned}
u(s) = Y_0 \geq \mathbb{E}_s\left[Y_{\tau_\pi}\right] &= \mathbb{E}_s\left[e^{-\rho\tau_\pi}u(S^\pi_{\tau_\pi}) + \int_0^{\tau_\pi} e^{-\rho s}dP^\pi_s\right] \\
&= \mathbb{E}_s\left[e^{-\rho\tau_\pi}(\alpha + S^\pi_{\tau_\pi})^+ + \int_0^{\tau_\pi} e^{-\rho s}dP^\pi_s\right]
\end{aligned}
$$

and the required result now follows from the arbitrariness of the strategy by taking the supremum over $\pi \in \Pi_0(s)$ on both sides. ■

Lemma A.9: *The continuous function*

$$
u(s;\alpha) = w(s;\alpha, b^*(\alpha)), \qquad s \in \mathbb{R}, \tag{A24}
$$

is concave and twice continuously differentiable on $(0,\infty)$ *with* $u'(s;\alpha) \geq 1$ *for all* $s > 0$.

Proof. By construction we have that $u(s;\alpha)$ is twice continuously differentiable on the set $(0,\infty)\backslash b^*(\alpha)$ and so it suffices to establish the required smoothness at the barrier.

Differentiating on both sides of (A8) we deduce that

$$\lim_{s\uparrow b^*(\alpha)} u'(s;\alpha) = \lim_{s\uparrow b^*(\alpha)} \left(\psi'(s;\alpha) + W'(s)H(b^*(\alpha);\alpha)\right) = 1$$

and it now follows from (6) that the function $u(s;\alpha)$ is continuously differentiable across the barrier. Similarly, differentiating twice with respect to s on both sides of (A8) and using the definition of the optimal barrier gives

$$u''(s;\alpha) = W''(s)\left(G(b^*(\alpha);\alpha) - G(s;\alpha)\right), \qquad s < b^*(\alpha), \tag{A25}$$

and the required smoothness now follows from (**??**) and the continuity of $G(b;\alpha)$. By Lemmas A.3 and A.6 we have

$$W''(b) \leq 0 \leq G(b;\alpha) \leq H(b;\alpha) \leq H(b^*(\alpha);\alpha) = G(b^*(\alpha),\alpha)$$

for all $b \leq b^*(\alpha) \leq x^*$. Therefore, it follows from (A25) that $u(s;\alpha)$ is concave on $[0, b^*(\alpha))$ and hence on the positive half line since it is linear outside of this interval. Finally, using this concavity and the fact that $u'(s;\alpha) = 1$ above the point $b^*(\alpha)$, we deduce that $u'(s;\alpha) \geq 1$ for all $s \geq 0$ and the proof is complete. ■

The next result establishes the global optimality of the barrier strategy at $b^*(\alpha)$ for the auxiliary problem (5) and concludes the proof of Proposition 3.

Lemma A.10: *For any $\alpha \in [0, v^*)$ the value function of (5) is given by $w(s;\alpha) = u(s;\alpha)$ and the optimal strategy is a barrier strategy at $b^*(\alpha)$.*

Proof. Since by construction $u(s;\alpha) \leq w(s;\alpha)$, it suffices to show that $u(s;\alpha)$ satisfies the conditions of Lemma A.8. By Lemma A.9 we have that this function is continuous on $\mathbb{R}$, twice continuously differentiable on $(0,\infty)$ and satisfies

$$u'(s;\alpha) \geq 1, \qquad s > 0,$$

as well as (A21). Therefore it only remains to show that $\mathcal{D}u(s;\alpha) \leq 0$ for all $s > 0$. A direct calculation using (A6) shows that we have

$$\mathcal{D}\psi(s;\alpha) = \mathcal{D}W(s) = 0, \qquad s > 0. \tag{A26}$$

Combining this identity with (6) and (A24) we conclude that

$$\mathcal{D}u(s;\alpha) = \mathcal{D}\psi(s;\alpha) + H(b^*(\alpha);\alpha)\mathcal{D}W(s) = 0, \qquad s < b^*(\alpha).$$

On the other hand, using (A26) in conjunction with (6), the result of Lemma A.9 and the definition of $\mathcal{D}$ shows that for $s \geq b^*(\alpha)$

$$\begin{aligned}\mathcal{D}u(s;\alpha) &= \mathcal{D}u(s;\alpha) - \mathcal{D}\psi(s;\alpha) - H(s;\alpha)\mathcal{D}W(s) \\ &= \rho\left(w(s;\alpha,s) - u(s;\alpha)\right) - \frac{\sigma^2}{2}\left(\psi''(s;\alpha) + H(s;\alpha)W''(s)\right) \\ &+ \lambda\int_0^\infty \left[(u(s-y;\alpha) - w(s-y;\alpha,s)) - (u(s;\alpha) - w(s;\alpha,s))\right] dF(y).\end{aligned} \tag{A27}$$

Fix an arbitrary $s \geq b^*(\alpha)$ and consider the function defined by

$$\varphi(x;\alpha,s) = w(x;\alpha,s) - u(x;\alpha)$$

Using (6) together with (A24) we deduce that

$$\varphi'(x;\alpha,s) = W'(x)\left(H(s;\alpha) - H(b^*(\alpha);\alpha)\right) \leq 0, \qquad x \in (0, b^*(\alpha)) \tag{A28}$$

where the inequality follows from Lemmas A.6 and A.3. On the other hand, using (6) together with (A24) we deduce that

$$\begin{aligned}\varphi'(x;\alpha,s) &= \psi'(x;\alpha) + W'(x)H(s;\alpha) - 1, \qquad b^*(\alpha) \leq x \leq s \\ &= W'(x)\left(H(s;\alpha) - H(x;\alpha)\right) \leq 0\end{aligned} \tag{A29}$$

where the inequality follows from Lemma A.6 and the fact that, as established in the proof of Lemma A.3, the function $H(x;\alpha)$ is decreasing over $[b^*(\alpha), \infty)$. Combining (A28) and (A29) shows that we have

$$\varphi'(x;\alpha,s) \leq 0, \qquad 0 \leq x \leq s,$$

and since

$$\varphi(x;\alpha,s) = w(x;\alpha,s) - u(x;\alpha) = 0, \qquad x \leq 0,$$

because of (6) we conclude that the function $\varphi(x;\alpha,s)$ is non positive on $(-\infty,s]$. This implies that the first and last term in (A27) are non positive. Finally, it follows from the definition of the function $H(s;\alpha)$ that the second term satisfies

$$-\frac{\sigma^2}{2}\left(\psi''(s;\alpha) + H(s;\alpha)W''(s)\right) = \frac{\sigma^2}{2}W'(s)H'(s;\alpha)$$

and the desired conclusion follows from Lemma A.6 and the fact that $H(x;\alpha)$ is decreasing over the interval $[b^*(\alpha),\infty)$ as shown in the proof of Lemma A.3. ■

B. Proofs

B.1. The first best problem

In order to prove Proposition 1 we consider the parametrized family of optimal stopping problems defined by

$$p(\Lambda) = \sup_{\tau\in\mathcal{S}} \mathbb{E}\left[\int_0^\tau e^{-\rho s} dC_s + e^{-\rho\tau}\max(\Lambda, -\Delta C_\tau)\right], \qquad \Lambda \geq 0, \tag{B1}$$

where $\mathcal{S}$ denotes the set of all stopping times and the cumulative cash flow process evolves according to

$$C_t = A_t - (1-\theta)ct = \bar{\mu}t + \sigma B_t - \sum_{n=1}^{N_t} Y_n.$$

The following result provides a closed-form solution to this family of problems and allows to recover the conclusion of Proposition 1 by setting $\Lambda = (-D-L)^+ = 0$.

Proposition B.11: *The value function and the optimal stopping time for problem* (B1) *are explicitly given by* $p(\Lambda) = \max(\Lambda, v^*)$ *and*

$$\tau^*(\Lambda) = 1_{\{\Lambda\leq v^*\}}\inf\{t \geq 0 : v^* + \Delta C_t \leq 0\}$$

where the strictly positive constant v^* is the unique solution to

$$\rho v^* = \bar{\mu} - \lambda \mathbb{E}\left[\min(v^*, Y_1)\right]. \tag{B2}$$

In particular, the value function of the first best problem is given by $p(0) = v^*$ and the optimal strategy is to liquidate the bank the first time that the absolute value of a jump of the cash flow process exceeds v^*.

Before proving the above proposition, we start by establishing a verification result for the HJB equation associated with (B1).

LEMMA B.12: *Fix an arbitrary $\Lambda \geq 0$ and assume that $q \in \mathbb{R}$ satisfies*

$$\max\left(\Lambda - q; -\rho q + \bar{\mu} + \lambda \mathbb{E}\left[\max(0, q - Y_1, \Lambda - Y_1) - q\right]\right) = 0 \tag{B3}$$

Then we have $q \geq p(\Lambda)$.

PROOF. Assume that q satisfies the assumption of the statement, fix an arbitrary stopping time $\zeta \in \mathcal{S}$ and consider the process

$$w_t = e^{-\rho t} q 1_{\{\zeta > t\}} + e^{-\rho \zeta} \max(\Lambda; -\Delta C_\zeta) 1_{\{\zeta \leq t\}} + \int_0^{t \wedge \zeta} e^{-\rho s} dC_s.$$

Since q satisfies (B3) we have that $q \geq \Lambda$ and

$$\rho q \geq \bar{\mu} + \lambda \mathbb{E}\left[\max(0, q - Y_1, \Lambda - Y_1) - q\right] = \bar{\mu} - \lambda \mathbb{E}\left[\min(v^*, Y_1)\right].$$

Combining these properties with an application of Itô's lemma shows that

$$\begin{aligned}
dw_t &= 1_{\{\zeta > t\}} e^{-\rho t} \left(\sigma dB_t + \Delta C_t + (\bar{\mu} - \rho q) dt\right) + 1_{\{\zeta = t\}} e^{-\rho t} \left(\max(0, \Lambda + \Delta C_t) - q\right) \\
&\leq 1_{\{\zeta > t\}} e^{-\rho t} \left(\sigma dB_t + \Delta C_t + (\bar{\mu} - \rho q) dt\right) \\
&\leq 1_{\{\zeta > t\}} e^{-\rho t} \left(\sigma dB_t - Y_{N_t} dN_t + \lambda \mathbb{E}\left[q - \max(0, q - Y_1, \Lambda - Y_1)\right] dt\right) \\
&= 1_{\{\zeta > t\}} e^{-\rho t} \left(\sigma dB_t - Y_{N_t} dN_t + \lambda \mathbb{E}\left[Y_1\right] dt + \lambda \mathbb{E}\left[\min(0, q - Y_1)\right] dt\right) \\
&\leq 1_{\{\zeta > t\}} e^{-\rho t} \left(\sigma dB_t - Y_{N_t} dN_t + \lambda \mathbb{E}\left[Y_1\right] dt\right)
\end{aligned}$$

Integrating on both sides and using the fact that $q \geq \Lambda$ then gives

$$w_t - q \leq \int_0^t 1_{\{\zeta > s\}} e^{-\rho s} \left(\sigma dB_s - Y_{N_s} dN_s + \lambda \mathbb{E}\left[Y_1\right] ds\right) \equiv m_t. \tag{B4}$$

By construction, we have that the process m_t is a local martingale and since its quadratic variation

$$[m]_t = \int_0^t 1_{\{\zeta > s\}} e^{-2\rho s} \left(\sigma^2 ds + Y_{N_s}^2 dN_s\right)$$

satisfies

$$\mathbb{E}[m]_\zeta \leq \mathbb{E}\left[\int_0^\infty e^{-2\rho s} \left(\sigma^2 ds + Y_{N_s}^2 dN_s\right)\right] = \frac{\sigma^2 + \lambda \mathbb{E}\left[Y_1^2\right]}{2\rho} < \infty.$$

it follows from the Burkholder-Davis-Gundy inequality that it is a true martingale on the interval $[\![0, \zeta]\!]$. Combining this property with (B4) then shows that

$$q \geq \mathbb{E}\left[w_\zeta - m_\zeta\right] = \mathbb{E}\left[w_\zeta\right] = \mathbb{E}\left[e^{-\rho\zeta} \max(\Lambda; -\Delta C_\zeta) + \int_0^\zeta e^{-\rho s} dC_s\right]$$

and the desired result now follows from the arbitrariness of $\zeta \in \mathcal{S}$ by taking the supremum on both sides of this inequality. ■

Proof of Proposition B.11. Consider the function defined by

$$z(v) = \bar{\mu} + \lambda \mathbb{E}\left[(v - Y_1)^+\right] - (\rho + \lambda) v$$

and observe that (B2) is equivalent to $z(v^*) = 0$. The function $z(v)$ is continuous, non increasing, starts out from $\bar{\mu} > 0$ at the origin and satisfies

$$\lim_{v \to \infty} z(v) = -\infty.$$

Therefore it crosses the horizontal axis at a unique point $v^* > 0$ and it follows that (B2) admits a unique strictly positive solution. A direct calculation using the definition of this solution shows that

$$q^* = \max(\Lambda, v^*)$$

is a solution (B3). Therefore, it follows from Lemma B.12 that we have $q^* \geq p(\Lambda)$. To establish the reverse inequality denote by

$$\bar{q} = \mathbb{E}\left[e^{-\rho\tau^*(\Lambda)}\max(\Lambda; -\Delta C_{\tau^*(\Lambda)}) + \int_0^{\tau^*(\Lambda)} e^{-\rho s} dC_s\right]$$

the value associated with the stopping time $\tau^*(\Lambda)$. By definition we have $\bar{q} \leq p(\Lambda)$ and we claim that $\bar{q} = q^*$. If $\Lambda \geq v^*$ then the claim immediately follows from the definition of the stopping time. On the contrary, if $\Lambda < v^*$ then it follows from the law of iterated expectations and the definition of the cash flow process that

$$c(\bar{q}) \equiv -\rho\bar{q} + \bar{\mu} + \lambda\mathbb{E}\left[(\bar{q} - Y_1)1_{\{Y_1 \leq v^*\}} - \bar{q}\right] = 0.$$

As is easily seen the continuous function $c(q)$ is decreasing in q and therefore crosses the horizontal axis at most once. Since

$$\begin{aligned} c(v^*) &= -\rho v^* + \bar{\mu} + \lambda\mathbb{E}\left[(v^* - Y_1)1_{\{Y_1 \leq v^*\}} - v^*\right] \\ &= -\rho v^* + \bar{\mu} - \lambda\mathbb{E}\left[\min(v^*, Y_1)\right] = 0 \end{aligned}$$

we have that this crossing point is uniquely given by $\bar{q} = v^*$. ■

B.2. Value of an unregulated bank

Consider the function defined by

$$h(\alpha; b) = w(b; \alpha; b) - b - \phi.$$

and let $\hat{h}(\alpha; b) = h(\alpha; b)^+$. The following lemma will be used in the construction of the optimal strategy for the bank subject to refinancing costs.

LEMMA B.13: *The functions $\hat{h}(\alpha; b)$ and $\hat{h}(\alpha; b^*(\alpha))$ each admit a unique fixed point and both of these fixed points are located in the semiopen interval $[0, v^*)$.*

PROOF. Fix a barrier level $b > 0$ and two liquidation values $0 \leq \alpha_2 < \alpha_1$. By definition, we have that the corresponding values satisfy

$$\begin{aligned} w(s;\alpha_2;b) \leq w(s;\alpha_1;b) &= \mathbb{E}_s\left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho\tau_{\pi_b}}(\alpha_1 + S_{\tau_{\pi_b}}^{\pi_b})^+\right] \\ &\leq \alpha_1 - \alpha_2 + \mathbb{E}_s\left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho\tau_{\pi_b}}(\alpha_2 + S_{\tau_{\pi_b}}^{\pi_b})^+\right] \\ &= \alpha_1 - \alpha_2 + w(s;\alpha_2;b) \end{aligned}$$

and it follows that

$$\frac{\partial w(s;\alpha;b)}{\partial \alpha} \in [0,1], \qquad (s,\alpha,b) \in \mathbb{R} \times [0,v^*) \times \mathbb{R}_+. \tag{B5}$$

On the other hand, using (6) together with Lemma A.3 we get

$$\left.\frac{\partial w(s;\alpha;b)}{\partial b}\right|_{s=b^*(\alpha)} = 1 - \left.\frac{\partial w(s;\alpha;b^*(\alpha))}{\partial s}\right|_{s=b^*(\alpha)} = 0$$

and combining this with (B5) we conclude that the functions $h(\alpha;b)$ and $h(\alpha;b^*(\alpha))$ are both increasing and satisfy

$$\max\left\{\frac{dh(\alpha;b)}{d\alpha};\frac{dh(\alpha;b^*(\alpha))}{d\alpha}\right\} \leq 1, \qquad \alpha \geq 0.$$

If we have that $h(0;b) \leq 0$ then this implies that $\hat{h}(\alpha;b)$ admits a unique fixed point that is located at the origin. On the contrary, if we have that $h(0;b) > 0$ then the decreasing function $h(\alpha;b) - \alpha$ is strictly positive at $\alpha = 0$ and since

$$\begin{aligned} h(v^*;b) - v^* &= w(b;v^*,b) - b - \phi - \alpha \\ &\leq w(b;v^*,b^*(v^*)) - b - \phi - v^* = -\phi < 0 \end{aligned}$$

by Lemmas A.3 and A.4, we conclude that the function $\hat{h}(\alpha;b)$ admits a fixed point that lies in the interval $(0,v^*)$. The proof of the corresponding property for $\hat{h}(\alpha;b^*(\alpha))$ is entirely analogous. We omit the details. ■

PROOF OF PROPOSITION 4. Consider the strategy $\hat{\pi}_b$ that consists in distributing dividends to maintain liquid reserves at or below $b > 0$ and in raising funds back to b whenever liquid reserves become negative if that is profitable. Denote by

$$v(s;b) = \mathbb{E}_s \left[\int_0^{\tau_{\hat{\pi}_b}} e^{-\rho t} (dP_t^{\hat{\pi}_b} - d\Phi_t(R^{\hat{\pi}_b})) \right] \tag{B6}$$

the value of the bank under this strategy. By definition we have that

$$v(s;b) = (v(b;b) - b + s - \phi)^+, \qquad s \leq 0,$$

and

$$R_t^{\hat{\pi}_b} = P_t^{\hat{\pi}_b} - P_t^b = 0, \qquad 0 \leq t \leq \tau_{\pi_b}$$

where the stopping time τ_{π_b} denotes the stochastic liquidation time associated with the barrier strategy for dividends at level b. Combining these properties with (B6) and the law of iterated expectations gives

$$\begin{aligned} v(s;b) &= \mathbb{E}_s \left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho \tau_{\pi_b}} v(S_{\tau_{\pi_b}}^{\pi_b}; b) \right] \\ &= \mathbb{E}_s \left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho \tau_{\pi_b}} (v(b;b) - b + S_{\tau_{\pi_b}}^{\pi_b} - \phi)^+ \right] \\ &= \mathbb{E}_s \left[\int_0^{\tau_{\pi_b}} e^{-\rho t} dP_t^b + e^{-\rho \tau_0} (\hat{\alpha}(b) + S_{\tau_{\pi_b}}^{\pi_b})^+ \right] = w(s; \hat{\alpha}(b); b) \end{aligned} \tag{B7}$$

where $\hat{\alpha}(b)$ denotes the unique fixed point of $\hat{h}(\alpha; b)$ provided by Lemma B.13. Consider now the barrier $b_0^* = b^*(\alpha_0^*) > 0$ where α_0^* denotes the unique fixed point of $g(\alpha) = \hat{h}(\alpha; b^*(\alpha))$ also provided by Lemma B.13. By uniqueness we have $\alpha_0^* = \hat{\alpha}(b_0^*)$ and combining this with (B7) shows that the value of the associated strategy satisfies

$$v(s; b_0^*) = w(s; \alpha_0^*; b_0^*) = w(s; \alpha_0^*), \qquad s \in \mathbb{R}.$$

As shown in the proof of Lemma A.10, we have that

$$\max\{1 - w'(s; \alpha_0^*); \mathcal{D}w(s; \alpha_0^*)\} \leq 0, \qquad s > 0,$$

and the result will follow from Lemma B.15 below once we show that $w(s; \alpha_0^*)$ satisfies (B8). By application of Lemma A.9, we have that this function is concave and twice continuously differentiable on $(0,\infty)$ with $w'(s;\alpha_0^*) = 1$ for $s \geq b_0^*$. Therefore

$$b_0^* \in \operatorname*{argmax}_{b\geq 0} \left(-b + w(b;\alpha_0^*)\right)$$

and combining this with (5) gives

$$w(s;\alpha_0^*) = (s+\alpha_0^*)^+ = \max_{b\geq 0} \left(w(b;\alpha_0^*) - b + s - \phi\right)^+, \qquad s \leq 0,$$

which is the required boundary condition. ■

Corollary B.14: *The bank raises funds if and only if $\phi < \phi^* = w(x^*;0) - x^*$.*

Proof. By Proposition 4 we have that the bank raises funds if and only if $\alpha_0^* = v(0) > 0$ and a direct calculation shows that this is equivalent to $\phi \geq \phi^*$. ■

Lemma B.15: *Assume that $u : \mathbb{R} \to \mathbb{R}$ is a continuous function that is twice continuously differentiable on $(0,\infty)$ and such that*

$$u(s) - \sup_{b\geq 0} \left(u(b) - b + s - \phi\right)^+ = 0, \qquad s \leq 0, \tag{B8}$$

$$\max\{1 - u'(s), \mathcal{D}u(s)\} \leq 0, \qquad s > 0. \tag{B9}$$

Then we have $u(s) \geq v(s)$ for all $s \in \mathbb{R}$.

Proof. Assume that the function $u(s)$ satisfies the conditions of the statement, fix an admissible strategy $\pi \in \Pi(s)$ and consider the process

$$Y_t = e^{-\rho t\wedge\tau_\pi} u(S^\pi_{t\wedge\tau_\pi}) + \int_0^{t\wedge\tau_\pi} e^{-\rho s}(dP^\pi_s - d\Phi_s(R^\pi)).$$

Applying Itô's formula for semimartingales (see for example Dellacherie and Meyer (1980, Theorem VIII.25)) shows that

$$Y_t - M_{t\wedge\tau_\pi} = u(S^\pi_0) + \sum_{0\leq s<t\wedge\tau_\pi} e^{-\rho s}\left(\Delta^+ u(S^\pi_s) + \Delta^+(P^\pi_s - \Phi_s(R^\pi))\right) \tag{B10}$$
$$+ \int_0^{t\wedge\tau_\pi} e^{-\rho s}\mathcal{D}u(S^\pi_{s-})ds + \int_0^{t\wedge\tau_\pi} e^{-\rho s}(1 - u'(S_{s-}))dP^{\pi,c}_s$$

for some local martingale M_t. Using the fact that $u(s)$ satisfies (B8) and (B9), together with the definition of the liquidation time, we deduce that

$$
\begin{aligned}
&1_{\{s<\tau_\pi\}}\left(\Delta^+ u(S_s^\pi) + \Delta^+(P_s^\pi - \Phi_s(R^\pi)) + \mathcal{D}u(S_{s-}^\pi)ds + (1-u'(S_{s-}))dP_s^{\pi,c}\right) \\
&\qquad \le 1_{\{s<\tau_\pi\}}\left(\Delta^+ u(S_s^\pi) + \Delta^+(P_s^\pi - \Phi_s(R^\pi))\right) \\
&\qquad = 1_{\{s<\tau_\pi\}}\left(u(S_s^\pi - \Delta^+(P_s^\pi - R_s^\pi)) + \Delta^+(P_s^\pi - \Phi_s(R^\pi)) - u(S_s^\pi)\right) \\
&\qquad \le 1_{\{s<\tau_\pi\}}\left(u(S_s^\pi + \Delta^+ R_s^\pi) - \Delta^+\Phi_s(R^\pi) - u(S_s^\pi)\right) \\
&\qquad = 1_{\{s<\tau_\pi\}\cap\{\Delta^+ R_s^\pi>0\}}\left(u(S_s^\pi + \Delta^+ R_s^\pi) - \Delta^+ R_s^\pi - \phi - u(S_s^\pi)\right). \qquad \text{(B11)}
\end{aligned}
$$

On the other hand, (B8) and (B9) jointly imply that

$$
\begin{aligned}
u(s) &= 1_{\{s\le 0\}}u(s) + 1_{\{s>0\}}\left(u(0) + \int_0^s u'(x)dx\right) \\
&\ge 1_{\{s\le 0\}}u(s) + 1_{\{s>0\}}\left(u(0) + s\right) \\
&\ge 1_{\{s\le 0\}}\sup_{b\ge 0}\left(u(b) - b + s - \phi\right)^+ + 1_{\{s>0\}}\left(s + \sup_{b\ge 0}\left(u(b) - b - \phi\right)^+\right) \\
&\ge \sup_{b\ge 0}\left(u(b) - b + s - \phi\right)^+
\end{aligned}
$$

and combining this inequality with (B11) shows that that the process on the right hand side of (B10) is decreasing. This in turn implies that the process Y_t is a local supermartingale and it follows that there exists an increasing sequence of stopping times $(\theta_n)_{n=1}^\infty$ such that $\lim_n \theta_n = \infty$ and

$$
\begin{aligned}
u(s) \ge \mathbb{E}_s\left[Y_{\theta_n}\right] &= \mathbb{E}_s\left[\int_0^{\theta_n\wedge\tau_\pi} e^{-\rho t}(dP_t^\pi - d\Phi_t(R^\pi)) + e^{-\rho\theta_n\wedge\tau_\pi}u(S_{\theta_n\wedge\tau_\pi}^\pi)\right] \\
&\ge \mathbb{E}_s\left[\int_0^{\theta_n\wedge\tau_\pi} e^{-\rho t}(dP_t^\pi - d\Phi_t(R^\pi))\right]
\end{aligned}
$$

where the last inequality follows from the fact that the function $u(s)$ is nonnegative as a result of (B8) and (B9). Letting $n\to\infty$ and using (A2) in conjunction with the dominated convergence theorem then gives

$$
u(s) \ge \mathbb{E}_s\left[\int_0^{\tau_\pi} e^{-\rho t}(dP_t^\pi - d\Phi_t(R^\pi))\right]
$$

and the desired conclusion now follows from the arbitrariness of the strategy by taking the supremum over $\pi \in \Pi(s)$ on both sides. ■

B.3. *Value of a bank subject to a liquidity requirement*

Let us now turn to the case of a bank that is subject to a minimal cash holding requirement at some level $T \geq 0$. The equity value of such a bank is given by

$$v(s;T) = \sup_{\pi \in \Pi(s,T)} \mathbb{E}_s \left[\int_0^{\tau_{\pi,T}} e^{-\rho t}(dP_t^\pi - d\Phi_t(R^\pi)) + e^{-\rho \tau_{\pi,T}}(S^\pi_{\tau_{\pi,T}} - D - L)^+ \right]$$

where the stopping time

$$\tau_{\pi,T} = \inf\{t \geq 0 : S^\pi_{t+} \leq T\}$$

denotes the liquidation time associated with the use of a strategy π in the presence of a cash holding requirement at level T, and $\Pi(s,T)$ denotes the set of payout and financing strategies such that

$$\Delta^+ P_t^\pi \leq S_t^\pi - T + \Delta^+ R_t^\pi, \qquad t \geq 0.$$

and (A2) holds. As a first step towards the proof of Proposition 5 the following lemma establishes a verification result for this optimization problem.

Lemma B.16: *Assume that $u : \mathbb{R} \to \mathbb{R}$ is a continuous function that is twice continuously differentiable on (T, ∞) and such that*

$$u(s) - \sup_{b \geq T} (\ell(s) \vee (u(b) - b + s - \phi)) = 0, \qquad s \leq T, \tag{B12}$$

$$\max\{1 - u'(s), \mathcal{D}u(s)\} \leq 0, \qquad s > T. \tag{B13}$$

Then $u(s) \geq v(s;T)$ for all $s \in \mathbb{R}$.

Proof. The proof is similar to that of Lemma B.15 and therefore is omitted. ■

Proposition B.17: *Assume that $\ell(T) \leq \alpha_0^*$. Then the equity value of a regulated bank is*

$$v(s;T) = v(s-T;0), \qquad s \in \mathbb{R},$$

and the optimal strategy consists in paying dividends to maintain liquid reserves below $T+b_0^*$, raising funds to move to $T+b_0^*$ whenever liquid reserves fall below T with a shortfall less than α_0^*, and liquidating otherwise.

Proof. Assume that $\ell(T) \leq \alpha_0^*$ and let

$$u(s) = v(s-T;0) = w(s-T;\alpha_0^*;b_0^*) = w(s-T;\alpha_0^*)$$

denote the candidate value function. From the proof of Lemma A.9, we know that this function satisfies (B13). On the other hand, the definition of the auxiliary problem and the assumption of the statement imply that for all $s \leq T$:

$$\begin{aligned} u(s) = w(s-T;\alpha_0^*) &= (w(0;\alpha_0^*) + s - T)^+ \\ &= (\alpha_0^* + s - T)^+ \geq (\ell(T) + s - T)^+ \geq \ell(s). \end{aligned}$$

Combining this inequality with the fact that the candidate value function satisfies (B8) then implies that for all $s \leq T$

$$\begin{aligned} u(s) &= \max\{\ell(s), u(s)\} \\ &= \max\left\{\ell(s), \sup_{b\geq 0}\left(v(b,0) - b + s - T - \phi\right)\right\} \\ &= \max\left\{\ell(s), \sup_{b\geq 0}\left(u(b+T) - b + s - T - \phi\right)\right\} \\ &= \max\left\{\ell(s), \sup_{q\geq T}\left(u(q) - q + s - \phi\right)\right\}. \end{aligned}$$

This shows that the candidate value function satisfies (B12) and it now follows from Lemma B.16 that we have $u(s) \geq v(s,T)$ for all $s \in \mathbb{R}$. To establish the reverse inequality consider the strategy that pays dividends to maintain liquid reserves at or below $T+b$, raises outside funds to move to $T+b$ whenever they become smaller than T with a shortfall less than a, and liquidating otherwise. Let

$$Y_t(s,T,a,b) = [S_t, P_t, R_t]\,(s,T,a,b), \qquad t \geq 0,$$

with

$$S_t(s,T,a,b) = s + C_t - P_t(s,T,a,b) + R_t(s,T,a,b)$$

and

$$R_0(s,T,a,b) = P_0(s,T,a,b) = 0$$

stand for the liquid reserves, cumulative dividend and cumulative financing processes associated with this strategy under the assumption that the bank starts out holding $s \in \mathbb{R}$ in cash reserves. In addition, let

$$\tau(s,T,a,b) = \inf\{t \geq 0 : S_{t+}(s,T,a,b) \leq T\} = \inf\{t \geq 0 : S_t(s,T,a,b) \leq T - a\}$$

give the corresponding liquidation time, and denote by

$$\vartheta_t(s,T,a,b) = \sup\{0 \leq u \leq t : T - a < S_u(s,T,a,b) \leq T\},$$

with the convention that $\sup \emptyset = 0$, the last time before $t \geq 0$ that the bank raises outside funds. With this notation, we have that the cumulative financing and dividend processes associated with the strategy satisfy

$$\begin{aligned} R_t(s,T,a,b) &= \sum_{0 \leq u < t} 1_{\{T-a<S_u(s,T,a,b)\leq T\}} \left(T + b - S_u(s,T,a,b)\right) \\ P_t(s,T,a,b) &= P_{\vartheta_t(s,T,a,b)}(s,T,a,b) \\ &\quad + \max_{\vartheta_t(s,T,a,b) \leq u < t} \left(S_{\vartheta_t(s,T,a,b)+}(s,T,a,b) + C_u - C_{\vartheta_t(s,T,a,b)} - (T+b)\right)^+ \end{aligned}$$

for all $t > 0$, and a straightforward calculation using the dynamics of the liquid reserves process then shows that we have the almost sure identities

$$Y_t(s,T,a,b) = (T,0,0) + Y_t(s-T,0,a,b)$$

and

$$\tau(s,T,a,b) = \tau(s-T,0,a,b). \tag{B14}$$

between the state variables, controls and liquidation times with and without a liquidity requirement. As a result, the equity value function

$$v(s,T,a,b) = \mathbb{E}\Bigg[\int_0^{\tau(s,T,a,b)} e^{-\rho u}\left(dP_u(s,T,a,b) - d\Phi_u(R(s,T,a,b))\right) + e^{-\rho\tau(s,T,a,b)}\left(S_{\tau(s,T,a,b)}(s,T,a,b) - D - L\right)^+\Bigg]$$

generated by the use of the strategy associated with the triple $(T,a,b) \in \mathbb{R}^3_+$ starting from the initial cash holding $s \in \mathbb{R}$ satisfies

$$v(s,T,a,b) = v(s-T,0,a,b), \qquad (s,T,a,b) \in \mathbb{R}\times\mathbb{R}^3_+. \tag{B15}$$

Coming back to the proof, we have that the candidate for the optimal strategy described in the statement corresponds to the triple (T, α_0^*, b_0^*). As shown in the proof of Proposition 2, we have

$$\alpha_0^* = v(0) = v(0, b_0^*) = \hat{\alpha}(b_0^*)$$

and, since $\ell(T) \leq \alpha_0^*$ by assumption, it follows that the associated liquidation payment to shareholders is given by:

$$\begin{aligned}
&\left(S_{\tau(s,T,\alpha_0^*,b_0^*)}(s,T,\alpha_0^*,b_0^*) - D - L\right)^+ \\
&\qquad = \left(S_{\tau(s-T,0,\alpha_0^*,b_0^*)}(s-T,0,\alpha_0^*,b_0^*) + T - D - L\right)^+ \\
&\qquad \leq \left(S_{\tau(s-T,0,\alpha_0^*,b_0^*)}(s-T,0,\alpha_0^*,b_0^*) + \ell(T)\right)^+ \\
&\qquad \leq \left(\alpha_0^* - \ell(T)\right)^+ = 0.
\end{aligned}$$

Using these properties in conjunction with (B7) and (B15) we finally obtain that

$$\begin{aligned}
v(s,T,\alpha_0^*,b_0^*) &= v(s-T,0,\alpha_0^*,b_0^*) \\
&= v(s-T,b_0^*) = w(s-T,\hat{\alpha}(b_0^*),b_0^*) \\
&= w(s-T,\alpha_0^*,b_0^*) = u(s)
\end{aligned}$$

and the proof is complete. ■

PROPOSITION B.18: *Assume that $\ell(T) > \alpha_0^*$. Then the equity value of a regulated bank is*

$$v(s;T) = w(s-T;\ell(T))$$

and the optimal strategy consists in paying dividends to maintain liquid reserves at or below the level $T + b^*(\ell(T) \wedge v^*)$, and liquidating the first time that they fall below T.

PROOF. Assume that $\ell(T) > \alpha_0^*$ and let

$$u(s) = w(s-T;\ell(T)) = w\left(s-T;\ell(T);b^*(\ell(T) \wedge v^*)\right)$$

denote the candidate value function. By application of Lemma A.9 we have that this function satisfies (B13), and using the definition of $w(s;\alpha)$, we obtain

$$u(s) = (w(0;\ell(T)) + s - T)^+ = (\ell(T) + s - T)^+ = \ell(s), \qquad s \leq T. \tag{B16}$$

On the other hand, the same lemma shows that $w(s;\ell(T))$ is increasing and concave on $(0,\infty)$ with $w'(s,\ell(T)) = 1$ for $s > b^*(\ell(T) \wedge v^*)$. In particular, we have

$$b^*(\ell(T) \wedge v^*) \in \operatorname*{argmax}_{b \geq 0} \left(-b + w(b;\ell(T))\right)$$

and combining this property with the definition of the function $u(s)$ then shows that for any cash holding $s \leq T$ we have

$$\begin{aligned}
\mathcal{T}(u(s)) &\equiv \sup_{q \geq T} \left(u(q) - q + s - \phi\right) \\
&= \sup_{b \geq 0} \left(w(b,\ell(T)) - b + s - T - \phi\right) \\
&= w(b^*(\ell(T) \wedge v^*), \ell(T)) - b^*(\ell(T) \wedge v^*) + s - T - \phi \\
&\leq \ell(s) + 1_{\{v^* > \ell(T)\}} \left(h(\ell(T); b^*(\ell(T))) - \ell(T)\right).
\end{aligned} \tag{B17}$$

As shown in the proof of Lemma B.13 we have that the function $h(a, b^*(a)) - a$ is monotone decreasing and equal to zero at the point $a = \alpha_0^*$. Therefore, it follows from (B17) and the assumption of the statement that we have

$$\mathcal{T}(u(s)) \leq \ell(s), \qquad s \leq T,$$

and combining this inequality with (B16) finally gives

$$u(s) = \ell(s) = \max\{\ell(s), \mathcal{T}(u(s))\}, \qquad s \leq T.$$

This shows that the candidate value function satisfies (B12) and it now follows from Lemma B.16 that we have $u(s) \geq v(s,T)$ for all $s \in \mathbb{R}$. To establish the reverse inequality consider the candidate optimal strategy described in the statement. Since this strategy does not involve any refinancing it follows from (B15), and the definition of $\ell(s)$ that the corresponding value satisfies

$$\begin{aligned} v(s,T,0,b^*(\ell(T) \wedge v^*)) &= v(s-T,0,0,b^*(\ell(T) \wedge v^*))) \\ &= w(s-T,\ell(T),b^*(\ell(T) \wedge v^*)) = u(s) \end{aligned}$$

and the proof is complete. ■

B.4. Debt issuance

Proof of Proposition 7. Assume that $L \geq N(c_L,T)$. Combining this assumption with the fact that $\phi \leq \Phi$ and the definition of $N(c_L,T)$ gives

$$\begin{aligned} T - D - L &\leq v(b_0^*(c_L)|c_L) - b_0^*(c_L) - \Phi \\ &\leq v(b_0^*(c_L)|c_L) - b_0^*(c_L) - \phi \\ &\leq \sup_{b \geq 0} (v(b|c_L) - b - \phi)^+ = v(0|c_L) = \alpha_0^*(c_L). \end{aligned}$$

Therefore it follows from Proposition B.17 that the barrier strategy $\Theta^*(c_L)$ defined in the statement is optimal for shareholders and that the equity value function of the constrained bank is given by

$$v(s,T|c_L,L) = v(s-T,0|c_L,L) = v(s-T|c_L).$$

By Lemma A.9 and Proposition 2, we know that the function on the right hand side of this identity is increasing and concave on $(0, \infty)$ with $v'(x|c_L) = 1$ for $x \geq b_0^*(c_L)$. In particular, we have

$$T + b_0^*(c_L) = \underset{z \geq T}{\operatorname{argmax}} \left(-z + v(z, T|c_L, L)\right).$$

and it immediately follows that

$$\begin{aligned}\eta_s(c_L, L, T) &= v(T + b_0^*(c_L), T|c_L, L) - (b_0^*(c_L) + T) + D + L - \Psi \\ &= v(b_0^*(c_L)|c_L) - (b_0^*(c_L) + T) + D + L - \Psi \\ &= L - N(c_L, T) \geq 0.\end{aligned}$$

To establish the converse implication, assume that $L < N(c_L, T)$ and let us show that the present value to shareholders is negative. If we have

$$\ell = T - D - L \leq v(0|c_L) = \alpha_0^*(c_L)$$

then the same arguments as above imply that the present value to shareholders is given by (7) which is strictly negative. On the contrary, if we have $\ell > \alpha_0^*(c_L)$ then it follows from Proposition B.18 that the shareholders' optimal strategy and the corresponding equity value function are given by $(T, 0, b)$ and

$$v(s, T|c_L, L) = w(s - T, \ell|c_L) = w(s - T, \ell; b|c_L)$$

for some nonnegative dividend barrier b. In particular, refinancing the bank is then never optimal from the point of view of shareholders and combining this observation with the fact that $\phi \leq \Psi$ gives

$$\begin{aligned}0 = w(0, \ell) - \ell &= v(T, T|c_L, L) - \ell \\ &\geq \sup_{x \geq 0} \left(-x - \phi + v(T + x, T|c_L, L)\right) - \ell \geq \eta_s(c_L, L, T)\end{aligned}$$

which is the desired conclusion. ■

Fix a coupon rate c_L and a face value L, let $W(x) = W(x|c_L)$ stand for the $\rho-$scale function of the corresponding uncontrolled liquid reserves process, and denote by

$$\Delta(x, \Theta|L) = \mathbb{E}\left[\delta\left(x - Y_1, \Theta|L\right)\right]$$

with

$$\delta(x, \Theta|L) = 1_{\{x+a \leq 0\}} \min(x + T - D, L)^+$$

the average payment that creditors receive under the strategy Θ if liquidation occurs following the arrival of a jump at a point in time where the liquid reserves of the bank exceed the minimal required level by some amount $x \in \mathbb{R}$.

LEMMA B.19: *Fix a barrier strategy $\Theta = (T, a, b)$. For any $s \geq T$ the market value of the bank's debt can be computed as*

$$d\left(s, \Theta|c_L, L\right) = \sum_{i=1}^{2} \left[\varphi_i(s - T, \Theta|c_L, L) + \varphi_i(b, \Theta|c_L, L)\frac{\gamma(s - T, \Theta|c_L)}{1 - \gamma(b, \Theta|c_L)}\right] \tag{B18}$$

with the functions defined by

$$\varphi_1(x, \Theta|c_L, L) = c_L \left[\frac{W(b)W(x \wedge b)}{W'(b)} - \int_0^{x \wedge b} W(z)dz\right] \tag{B19}$$

$$\begin{aligned}\varphi_2(x, \Theta|c_L, L) = {} & (\sigma^2/2)\left[W'(x \wedge b) - \frac{W(x \wedge b)W''(b)}{W'(b)}\right]\delta(0, \Theta|L) \\ & + \int_0^b \lambda\Delta\left(z, \Theta|L\right)\left[\frac{W(x \wedge b)W'(b - z)}{W'(b)} - W(x \wedge b - z)\right]dz\end{aligned} \tag{B20}$$

and

$$\begin{aligned}\gamma(x, \Theta|c_L) = {} & (\sigma^2/2)\left[W'(x \wedge b) - \frac{W(x \wedge b)W''(b)}{W'(b)}\right]1_{\{a>0\}} \\ & + \int_0^b \lambda F(a)(1 - F(z))\left[\frac{W(x \wedge b)W'(b - z)}{W'(b)} - W(x \wedge b - z)\right]dz.\end{aligned} \tag{B21}$$

The creditors' present value is strictly decreasing in the face value argument and admits a unique root $L^*(c_L, T)$ that lies in the interval $(0, c_L/\rho]$.

PROOF. Fix a strategy $\Theta = (T, a, b)$ and let $\Theta_b = (0, 0, b)$. The definition of Θ implies that the debt value satisfies the value matching condition

$$d(s, \Theta|c_L, L) = d(T + b, \Theta|c_L, L), \qquad s \in (T - a, T] \cup [T + b, \infty).$$

On the other hand, (B14) and the fact that the strategy Θ_b does not involve refinancing implies that we have

$$\tau(s, \Theta) = \tau(s - T, 0, a, b) \geq \tau(s - T, \Theta_b), \qquad s \in \mathbb{R}.$$

Combining theses identities with the definition of the debt value function and the law of iterated expectations then shows that for $s \geq T$ we have

$$d(s, \Theta|c_L, L) = \sum_{i=1}^{2} \varphi_i(s - T, \Theta|c_L, L) + \gamma(s - T, \Theta|c_L)d(T + b, \Theta|c_L, L), \tag{B22}$$

with the functions defined by

$$\varphi_1(x, \Theta|c_L, L) = \mathbb{E}\left[\int_0^{\tau(x,\Theta_b)} e^{-\rho t} c_L dt\right] = (c_L/\rho)\left(1 - \mathbb{E}\left[e^{-\rho\tau(x,\Theta_b)}\right]\right),$$
$$\varphi_2(x, \Theta|c_L, L) = \mathbb{E}\left[e^{-\rho\tau(x,\Theta_b)}\delta(S_{\tau(x,\Theta_b)}(x, \Theta_b), \Theta|L)\right],$$

and

$$\gamma(x, \Theta|c_L) = \mathbb{E}\left[e^{-\rho\tau(x,\Theta_b)} 1_{\{S_{\tau(x,\Theta_b)}(x,\Theta_b)+a>0\}}\right].$$

Evaluating (B22) at the point $s = T + b$, solving the resulting equation for the value of the debt at the dividend barrier and substituting the solution back into (B22) shows that (B18) holds and it now remains to show that (B19), (B20), and (B21) are also satisfied. The required result for $\varphi_1(x, \Theta|c_L, L)$ follows from Kuznetsov et al. (2013, Theorem 2.8.ii). To compute the other two functions we proceed as in the proof of Lemma A.2: Applying the dividend/penalty identity (see Gerber, Lin, and Yang (2006)) and using the same notation as in Section A.A.3 we find that

$$\varphi_2(x,\Theta|c_L,L) = \sum_{i=1}^{2}\left[\varphi_{2,i}(x\wedge b,\Theta|c_L,L) - \varphi_{2,i}'(b,\Theta|c_L,L)\frac{W(x\wedge b)}{W'(b)}\right]$$

with the auxiliary functions defined by

$$\begin{aligned}\varphi_{2,1}(x,\Theta|c_L,L) &= \mathbb{E}_x\left[e^{-\rho\zeta_0}1_{\{\Delta X_{\zeta_0}=0\}}\delta(0,\Theta|L)\right] \\ \varphi_{2,2}(x,\Theta|c_L,L) &= \mathbb{E}_x\left[e^{-\rho\zeta_0}1_{\{\Delta X_{\zeta_0}\neq 0\}}\delta(X_{\zeta_0},\Theta|L)\right].\end{aligned} \tag{B23}$$

By (A11) we know that the first of these auxiliary functions can be computed explicitly in terms of the generalized scale function as

$$\varphi_{2,1}(x,\Theta|c_L,L) = \frac{\sigma^2}{2}\left(W'(x) - B_3W(x)\right)\delta(0,\Theta|L) \tag{B24}$$

where the constant $B_3 \equiv B_3(c_L)$ is the strictly positive root of the cubic equation (A6). On the other hand, using the compensation formula for point processes and the potential density given in (A12) we obtain that

$$\varphi_{2,2}(x,\Theta|c_L,L) = \int_0^\infty \lambda\left(e^{-B_3z}W(x) - W(x-z)\right)\Delta(z,\Theta|L)dz. \tag{B25}$$

Differentiating (B24) and (B25), substituting into (B23), and simplifying shows that (B20) is satisfied. A similar argument shows that the function $\gamma(x,\Theta|L)$ can be computed as indicated in (B21) and it now remains to establish the last part of the statement.

Using the dominated convergence theorem in conjunction with the fact that the function $\delta(x,\Theta|L)$ is uniformly Lipschitz continuous in L, we deduce that the creditors' present value is continuously differentiable with

$$\frac{\partial\eta_c(c_L,L,T)}{\partial L} \leq \mathbb{E}\left[e^{-\rho\tau(s,\Theta^*(c_L))}\right] - 1,$$

and the required strict monotonicity follows from the fact that, since the optimal dividend barrier $b_0^*(c_L)$ is strictly positive by Lemmas A.4 and B.13, we have

$$\mathbb{P}[\tau(T + b_0^*(c_L),\Theta^*(c_L)) > 0] = 1. \tag{B26}$$

To complete the proof it now suffices to observe that the present value of creditors is continuous and strictly decreasing with

$$\eta_c(c_L, c_L/\rho, T) \leq 0$$

and

$$\eta_c(c_L, 0, T) = \frac{c_L}{\rho}\left(1 - \mathbb{E}\left[e^{-\rho\tau(T+b_0^*(c_L),\Theta^*(c_L))}\right]\right) > 0$$

where the strict inequality follows from (B26). ■

REMARK B.20: *The integrals in the definition of the functions* $\gamma(x, \Theta|c_L)$ *and* $\varphi_i(x, \Theta|c_L, L)$ *are left unevaluated to simplify the presentation but can easily be expressed as combinations of exponentials by using the explicit expression of the scale function in* (A7) *and the fact the jumps of the cash flow process are exponentially distributed.*

PROOF OF PROPOSITION 8. If $T \leq D$ then the second term in the definition of the debt value is equal to zero. As result we have that $L^*(c, T) = L^*(c, 0)$ for all $c \geq 0$ and combining this observation with the definition of the shareholder's present value shows that

$$L^*(c, T) - N(c, T) = L^*(c, 0) - N(c, 0) - T, \qquad c \geq 0.$$

This in turn implies that the set $\mathcal{C}(T)$ is non empty if and only if it contains the coupon rate $c_L^*(0) \in \mathcal{C}(0)$ that is optimal in the absence of a liquidity requirement, and the desired result now follows from the definition of the objective function. ■

B.5. Default probability and the distribution of default losses

To quantify the effect of liquidity requirements on the default risk of the bank we need to calculate the probability

$$f(s, y, t, \Theta) = \mathbb{P}[\{\tau(s, \Theta) \leq t\} \cap \{S_{\tau(s,\Theta)}(s, \Theta) \leq T - y\}]$$

that the bank is liquidated prior to a fixed horizon in a state where the shortfall of its liquid reserves relative to the required level exceeds a given amount $y \geq 0$. Unfortunately, this probability cannot be computed in closed form due to the time dependence induced by the presence of a fixed horizon. To circumvent this difficulty we consider instead the Laplace transform

$$\widehat{f}(s,y,k,\Theta) = \int_0^\infty e^{-kt} f(s,y,t,\Theta)dt = (1/k)\,\mathbb{E}\left[e^{-k\tau(s,\Theta)} 1_{\{S_{\tau(s,\Theta)}(s,\Theta)\leq T-y\}}\right].$$

Relying on arguments similar to those we used in the computation of the debt value function allows to obtain this Laplace transform in closed form. In order to state the result let $W_k(x)$ be defined as in (A7) but with k replacing ρ denote the scale function of the uncontrolled liquid reserves process associated with the discount rate $k \geq 0$.

LEMMA B.21: *For any $s \geq T$ the Laplace transform of the bank's default probability can be computed as*

$$\widehat{f}(s,y,k,\Theta) = (1/k)\left[\kappa(s-T,y,k,\Theta) + \kappa(b,y,k,\Theta)\frac{\gamma(s-T,\Theta)}{1-\gamma(b,\Theta)}\right]$$

where we have set

$$\kappa(x,y,k,\Theta) = (\sigma^2/2)\left[W_k'(x\wedge b) - \frac{W_k(x\wedge b)W_k''(b)}{W_k'(b)}\right] 1_{\{y=a=0\}}$$
$$+ \int_0^b \lambda(1-F(z+a\vee y))\left[\frac{W_k(x\wedge b)W_k'(b-z)}{W_k'(b)} - W_k(x\wedge b - z)\right]dz$$

and the function $\gamma_k(x,\Theta)$ is defined as in (B21) *but with $W_k(x)$ instead of $W(x)$.*

PROOF. The computation follows the same steps as that of the debt value function in the proof Lemma B.19 and therefore is omitted. ■

To obtain the default probability we will numerically invert the Laplace transform using the Gaver-Stehfest formula (Gaver (1966), Stehfest (1970)):

$$f(s,y,t,\Theta) \approx \sum_{n=1}^{N} \omega_n(t,N)\widehat{f}\left(s,y,n\frac{\log 2}{t},\Theta\right)$$

where $N \in 2\mathbb{N}$ is an even constant chosen to insure the convergence of the approximation, and the weights are defined by

$$\omega_n(t,N) = \sum_{m=[\![(n+1)/2]\!]}^{n\wedge N/2} \frac{(-1)^{n+\frac{N}{2}}(\log 2)t^{-1}m^{N/2}(2m)!}{m!(m-1)!(n-m)!(2m-n)!(N/2-m)!}.$$

The main advantage of this method is that it does not require the evaluation of the Laplace transform in the complex plane and therefore allows to avoid solving (A6) at complex values of the transform parameter $\rho = k$. Its main disadvantage is that requires a high accuracy to deal with the fact that the weights and the successive approximations both include factorials and alternating signs. This however is not a problem in software packages such as Mathematica®. For example, in our implementation we achieve a precision of 6 digits by using $N = 18$ and an accuracy of 100 digits in the computations.

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