Competitive Search in Financial Markets

Robert Shimer

Finance Theory Group

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Introduction

- study models of decentralized trading
- standard approach: random search
- here: directed search and price posting ("competitive search")
  - allows for market segmentation
  - allows posted prices to signal information
- why?
  - sometimes easier to solve the model
  - sometimes the model's predictions are different
  - sometimes the model's predictions are stronger (no bargaining)

Homogeneous Assets

Setup

- two periods
- unit measure of risk-neutral investors
  - nonnegative consumption of dividends
  - endowed with one unit of dividends in period 1
  - endowed with one asset that pays dividend δ in period 2
  - heterogeneous discount factors β, distribution G
- trade dividends for assets
  - allow investors to both buy and sell
  - everything is divisible

Search Frictions

- potential "market" at each price $p \in \mathbb{R}_+$
- search is directed to a market
  - buyers bring $d\mu_b(p)$ units of period 1 dividends to market p
  - sellers bring $d\mu_s(p)$ units of assets to market p
- transactions volume is $M(d\mu_b(p), pd\mu_s(p))$ in market p
  - measured in units of period 1 dividends
  - increasing with constant returns to scale
  - special case: $M(b, s) = \min\{b, s\}$
- let $\Theta(p) = d\mu_b(p)/pd\mu_s(p)$ denote the "buyer-seller ratio"
  - buyers trade with probability $\pi_b(\Theta(p)) \equiv M(1, 1/\Theta(p))$
  - sellers trade with probability $\pi_s(\Theta(p)) \equiv M(\Theta(p), 1)$

Definition of Equilibrium

- an equilibrium is three functions (p_s, p_b, Θ) such that
  - sellers set optimal prices
  - buyers set optimal prices
  - markets clear

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  - sellers set optimal prices: for all β

$$p_s(\beta) \in \arg\max_{p \geq \beta\delta} \Big(\pi_s(\Theta(p))p + \big(1 - \pi_s(\Theta(p))\big)\beta\delta\Big)$$

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$$p_b(\beta) \in \arg\max_{p \geq 0} \left(\pi_b(\Theta(p)) \frac{\beta\delta}{p} + (1 - \pi_b(\Theta(p))) \right)$$

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- an equilibrium is three functions (p_s, p_b, Θ) such that
  - sellers set optimal prices
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  - markets clear: for all $p \geq 0$, $d\mu_b(p) = \Theta(p) p d\mu_s(p)$, where $\mu_b(p) = \int_{p_b(\beta) \leq p} dG(\beta)$ and $\mu_s(p) = \int_{p_s(\beta) \leq p} dG(\beta)$

Definition of Equilibrium

- an equilibrium is three functions (p_s, p_b, Θ) such that
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  - markets clear
- note that Θ is defined for all p, even where $d\mu_b(p) = d\mu_s(p) = 0$

Special Case I: No Search Frictions

Equilibrium Characterization

- assume $M(b,s) = \min\{b,s\}$
- equilibrium characterized by p^* solving $p^* G(p^*/\delta) = 1 - G(p^*/\delta)$ and:
  - $$\Theta(p) = \begin{cases} \infty \\ 1 \\ 0 \end{cases} \Leftrightarrow p \begin{matrix} < \\ = \\ > \end{matrix} p^*$$
  - $$\beta\delta \gtrless p^* \Rightarrow \begin{cases} p_b(\beta) = p^* \\ p_b(\beta) < p^* \end{cases} \quad \text{and} \quad \begin{cases} p_s(\beta) > p^* \\ p_s(\beta) = p^* \end{cases}$$
- p^* was set to ensure market clearing
- payoff-equivalent to the competitive equilibrium

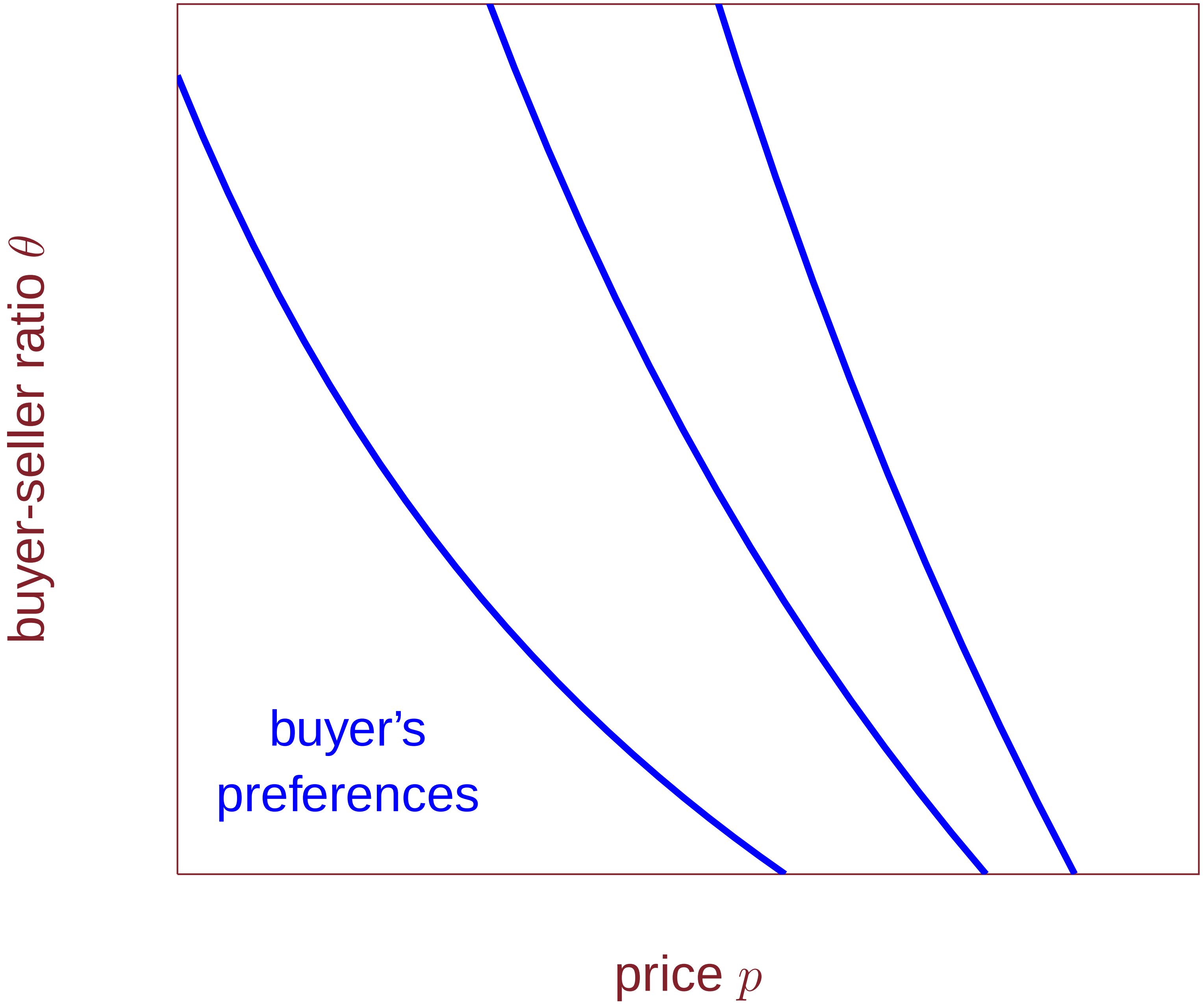
Special Case II: Two Types

Assumption

- again allow for a general matching function
- assume fraction S have $\beta = \underline{\beta} \geq 0$, $B = 1 - S$ have $\beta = \bar{\beta} > \underline{\beta}$
- easy to verify that impatient investors are sellers, patient are buyers
- example: $\bar{\beta} = 1$, $\underline{\beta} = 0.5$, $B = S = \frac{1}{2}$, $M(b, s) = bs/(b + s)$

Indifference Curves

Indifference Curves



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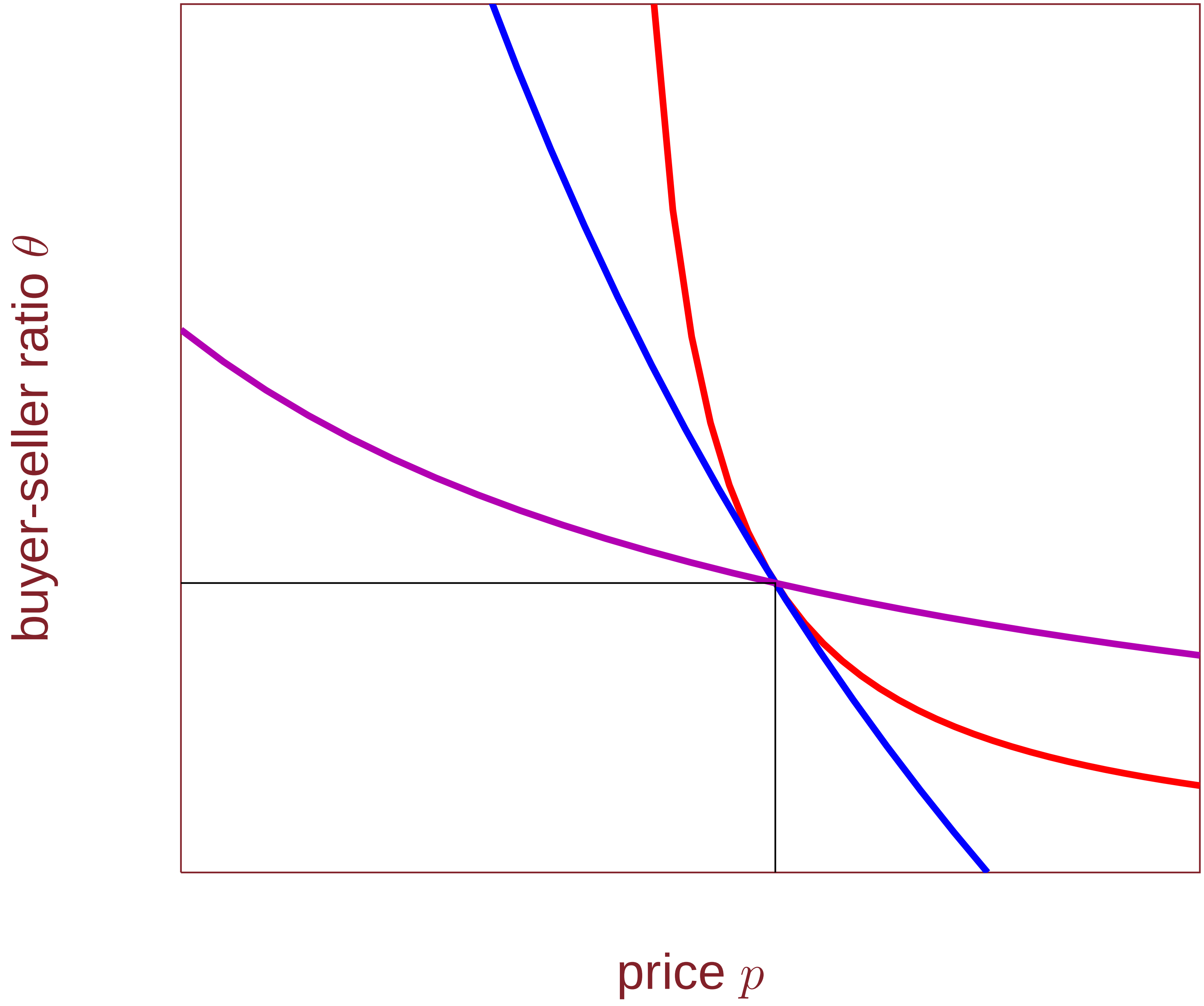

Indifference Curves

Optimization Problem

- seller maximizes utility subject to buyer getting utility v:

$$\max_{p,\theta} \pi_s(\theta)p + (1 - \pi_s(\theta))\underline{\beta}\delta$$

$$\text{s.t. } \pi_b(\theta)\frac{\bar{\beta}\delta}{p} + 1 - \pi_b(\theta) = v$$

$\Rightarrow p(v), \theta(v)$

  - existence and uniqueness of “partial equilibrium”

- markets clear: $\theta(v) = \dfrac{B}{p(v)S}$

  - existence and uniqueness of competitive search equilibrium

Comparison to Random Search

- typically specify a slightly different matching function $M(B, S)$
- ex post bargaining over prices
- competitive search then selects particular bargaining powers
- straightforward to extend both models to a dynamic environment

Search Frictions and Intermediation

Example

- again allow for a general β distribution, set $\delta = 1$
- now assume $M(b, s) = \dfrac{bs}{b + s}$
- standard approach would be to specify a G and find $\Theta \ldots$
- I will backward engineer a G that generates Θ
  - unclear to me how to find Θ given G

Construction

- suppose $\Theta(p) = \dfrac{2-4p}{4p-1}$ for $p \in \left[\frac{1}{4}, \frac{1}{2}\right]$
  - $p_b(\beta) \in \arg\max \dfrac{1}{1+\Theta(p)}\left(\dfrac{\beta}{p} - 1\right) \Rightarrow p_b(\beta) = \frac{1}{2}\sqrt{\beta}$ if $\beta \geq \frac{1}{4}$
  - $p_s(\beta) \in \arg\max \dfrac{\Theta(p)}{1+\Theta(p)}(p-\beta) \Rightarrow p_s(\beta) = \frac{1}{2}\beta + \frac{1}{4}$ if $\beta \leq \frac{1}{2}$

- so $\beta_s \in \left[0, \frac{1}{2}\right]$ sells to $\beta_b = \left(\beta_s + \frac{1}{2}\right)^2 \in \left[\frac{1}{4}, 1\right]$ at price $p = \frac{1}{2}\beta_s + \frac{1}{4}$

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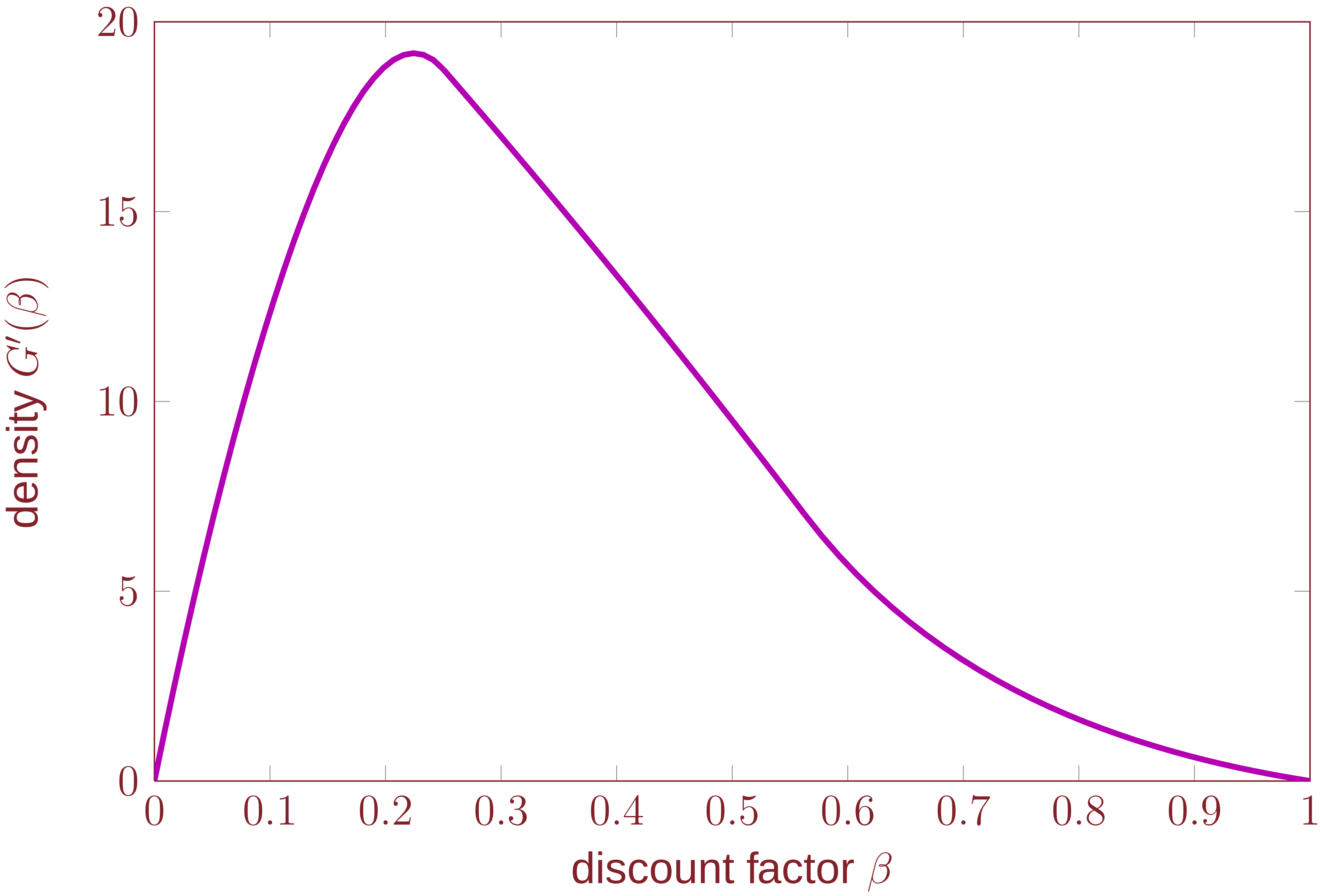
- market clearing: $\Theta(p) = \dfrac{G'(4p^2)}{pG'(2p-1/2)}$
  - straightforward to reverse-engineer many such distributions

Type Density



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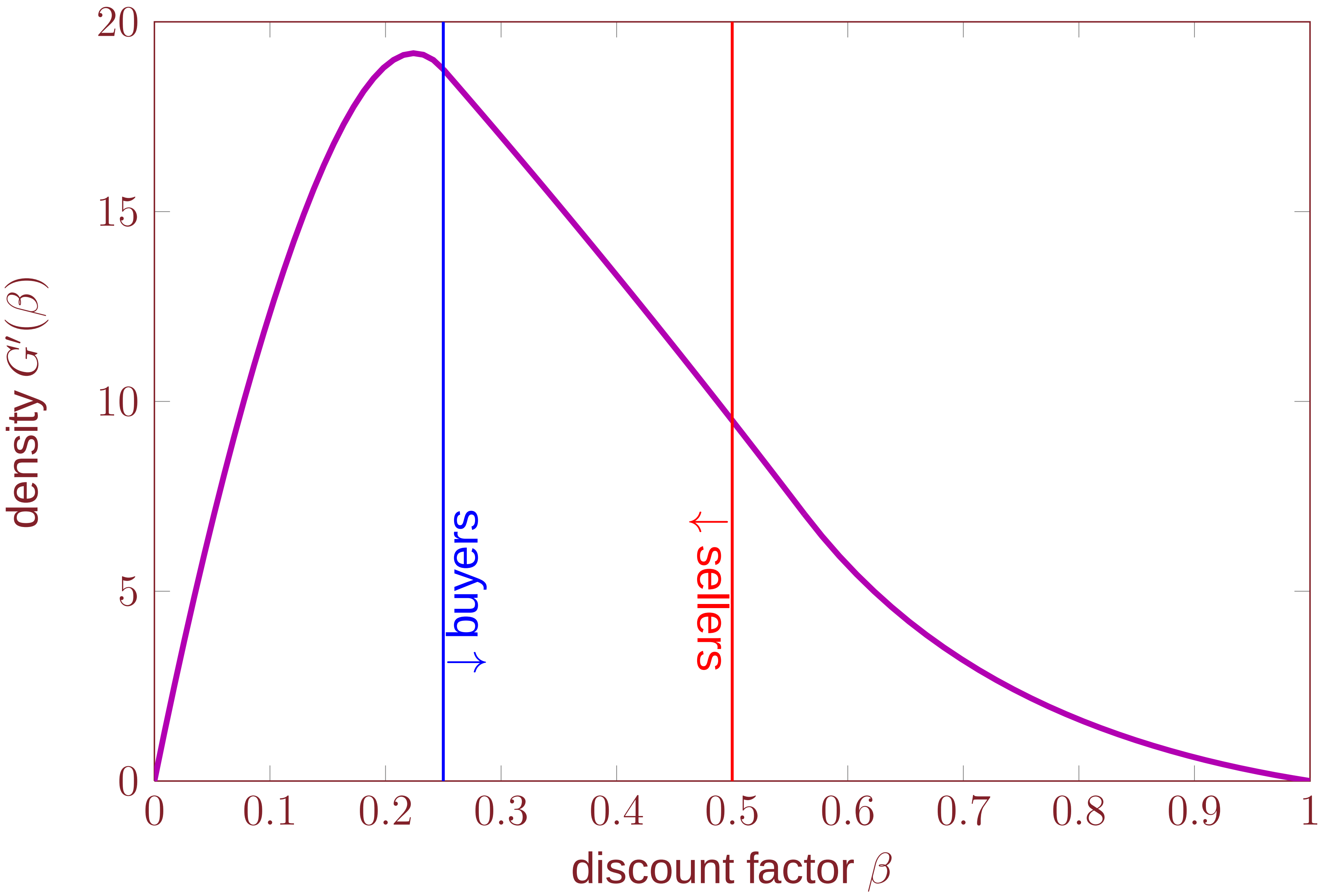
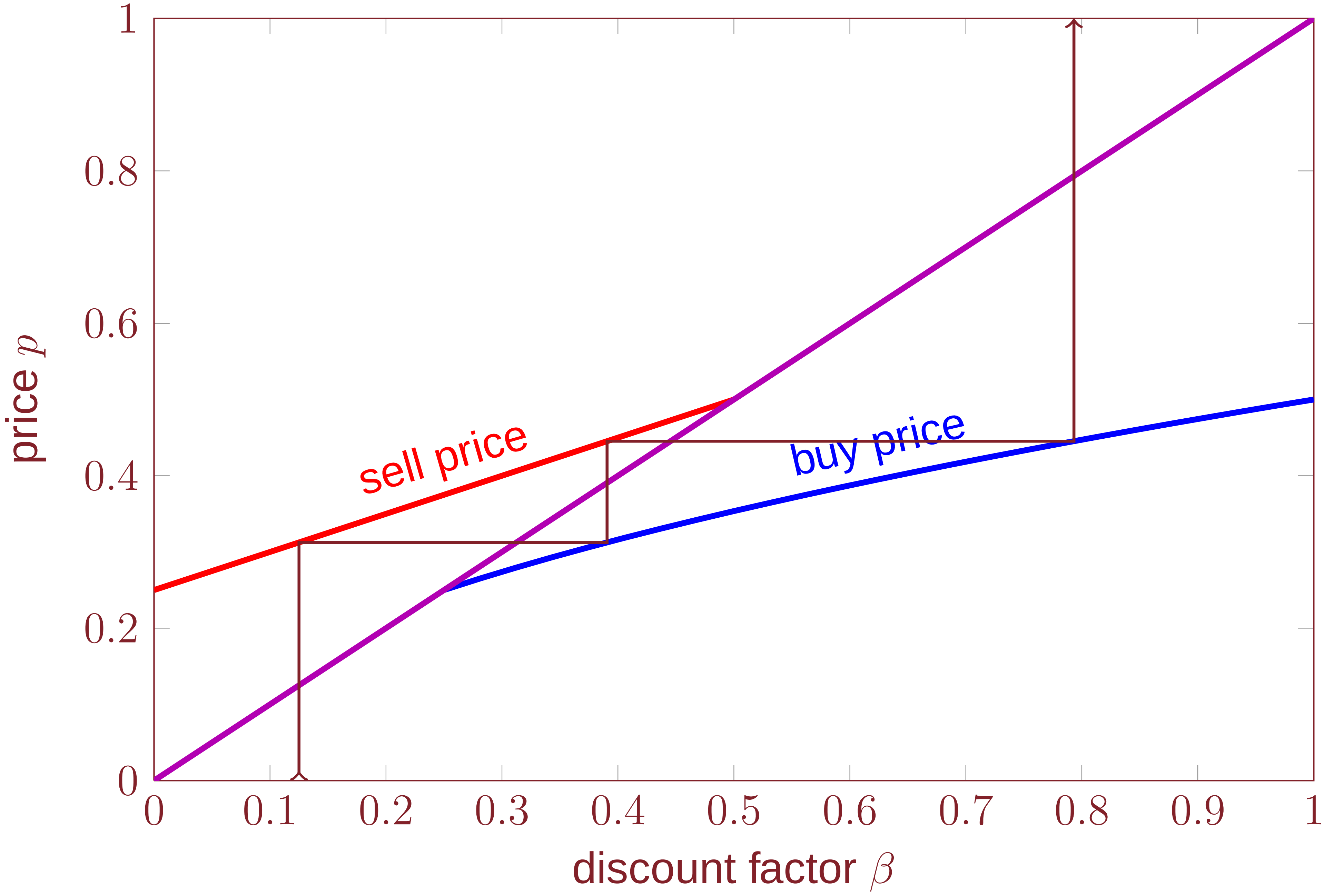


Pricing

1
0.8
0.6
0.4
0.2
0
price p
sell price
buy price
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
discount factor β

Pricing



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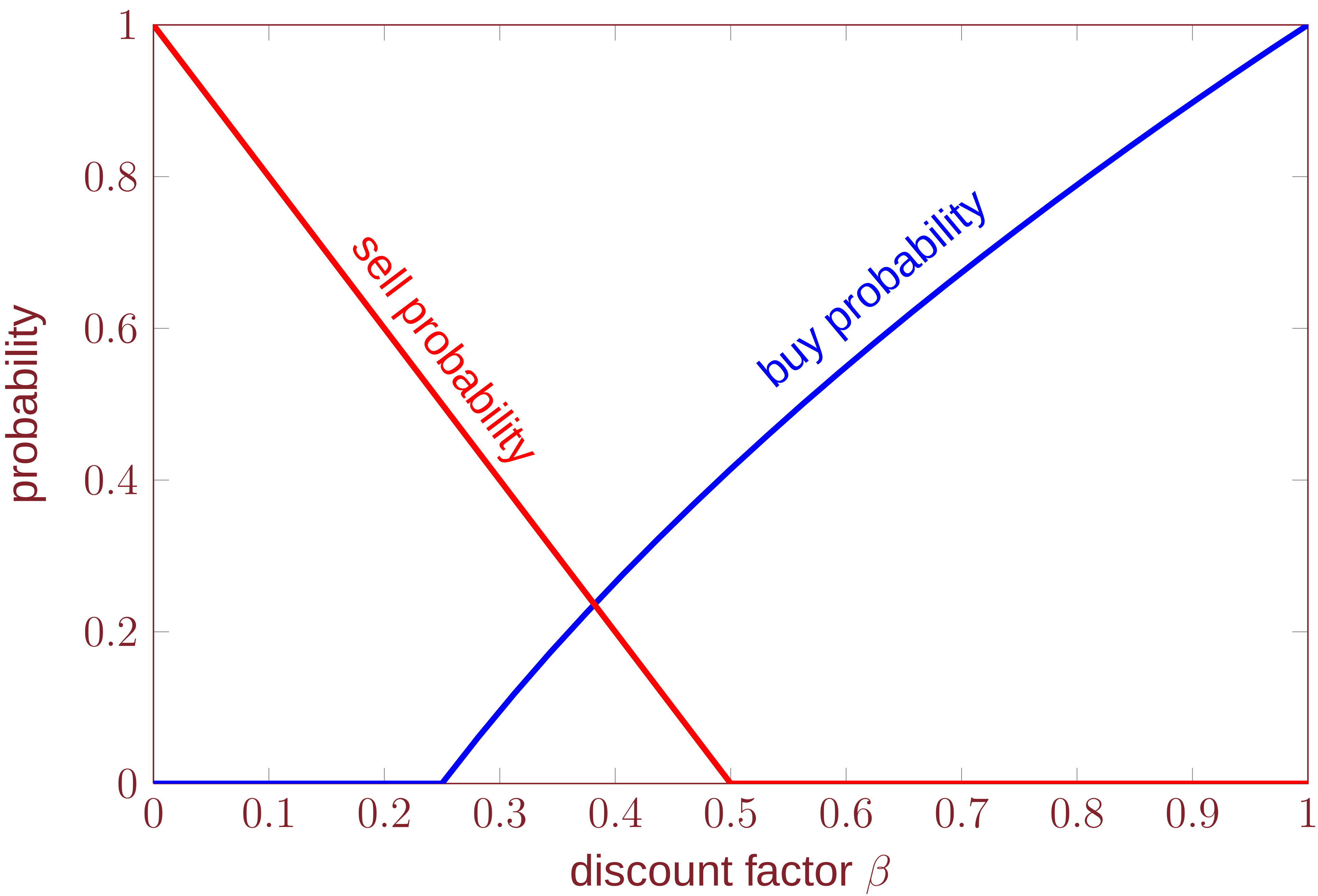
Transaction Probabilities



Summary

- trade occurs at a range of prices, $p \in [0.25, 0.5]$
  - Θ is pinned down over the relevant range
- search frictions create a role for intermediation
  - is it fair to call it this?
  - intermediate investors are intermediaries, buy low and sell high
  - not all trade is intermediated
- decentralized equilibrium is Pareto optimal

Comparison to Random Search

- congestion is an important issue with random search
  - high β sellers make it harder for low β sellers to match
  - low β buyers make it harder for high β buyers to match
  - generally no reason to expect efficiency (Shimer-Smith)
- directed search is no longer a special case of random search

Heterogeneous Assets and Private Information

Heterogeneous Assets

- assets are heterogeneous in terms of δ, joint distribution $G(\beta, \delta)$
  - support $[\underline{\beta}, \bar{\beta}] \times [\underline{\delta}, \bar{\delta}]$
- if δ were observable:
  - all assets would have the same price-dividend ratio
  - nothing important would change
- two remaining cases:
  - observable β: “Dynamic Adverse Selection”
  - unobservable β: “Markets with Multidimensional Private Info”
  - both papers with Veronica Guerrieri
- assume $M(b, s) = \min\{b, s\}$, no search frictions

Observable Preferences

Modeling Issues

- seller (β, δ) sets sale price $p_s(\beta, \delta)$
- buyer (β, δ) sets buy price and type of seller $\{p_b(\beta, \delta), B(\beta, \delta)\}$
- markets are segmented by price and seller's discount factor (p, β)
- sellers are concerned about sale probability $\min\{\Theta(p, \beta), 1\}$
- buyers are concerned about buy probability $\min\{\Theta(p, \beta_s)^{-1}, 1\}$
- buyers are also concerned about average dividend $\Delta(p, \beta_s)$

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- key assumption: asking price is a costly signal of quality
  - analog in random search: delays in bargaining

Definition of Equilibrium

- an equilibrium is five functions $(p_s, p_b, B, \Theta, \Delta)$ such that
  - sellers set optimal prices
  - buyers set optimal prices
  - beliefs are rational
  - markets clear

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$$p_s(\beta,\delta) \in \arg\max_{p \geq \beta\delta} \Big(\min\{\Theta(p,\beta),1\}p + \big(1 - \min\{\Theta(p,\beta),1\}\big)\beta\delta \Big)$$

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$$\{p_b(\beta,\delta), B(\beta,\delta)\} \in \arg\max_{p \geq 0, \beta_s} \Bigg(\min\{\Theta(p,\beta_s)^{-1}, 1\} \frac{\beta \Delta(p,\beta_s)}{p} + \big(1 - \min\{\Theta(p,\beta_s)^{-1}, 1\}\big) \Bigg)$$

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    - if $\exists \delta$ with $p_s(\beta, \delta) = p$, $\Delta(p, \beta) = \mathbb{E}(\delta | p_s(\beta, \delta) = p)$;
    - otherwise $\exists \delta_1 \leq \Delta(p, \beta)$ and $\delta_2 \geq \Delta(p\beta)$ and p is a weakly optimal sale price for (β, δ_1) and (β, δ_2)
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Comments on Rational Beliefs

- pins down $\Theta(p, \beta)$ at prices without trade
  - lower envelope of seller's indifference curves
- no analogous condition with symmetric information
- here needed to rule out "perverse" outcomes
  - nobody finds p an optimal sale price
  - buyers believe $\Delta(p)$ is small so $\Theta(p, \beta) = 0$
  - this rationalizes seller's behavior

Buyer Behavior

- buyer's asset quality δ is obviously irrelevant
- buyer's discount factor matters for behavior
  - $\beta > \hat{\beta}$: indifferent about buying anything
  - $\beta < \hat{\beta}$: do not buy anything
- marginal buyer $\hat{\beta}$ prices everything: $\hat{\beta}\Delta(p, \beta_s) = p$ for all (p, β_s)

Seller Behavior

- seller (β, δ) solves

$$p_s(\beta, \delta) \in \arg\max_{p \geq \beta\delta} \Big(\min\{\Theta(p, \beta), 1\} p + \big(1 - \min\{\Theta(p, \beta), 1\}\big) \beta\delta \Big)$$

  which must deliver $\hat{\beta}\Delta(p_s(\beta, \delta), \beta) = p_s(\beta, \delta)$ for all (β, δ)

- first order condition:

$$\Theta_p(p, \beta)(p - \beta\delta) + \Theta(p, \beta) = 0$$

  and second order condition $\Theta_p < 0$

- combine with buyer's problem, $\delta = \Delta(p, \beta) = p/\hat{\beta}$:

$$\frac{p\Theta_p(p, \beta)}{\Theta(p, \beta)} = \frac{\hat{\beta}}{\beta - \hat{\beta}}$$

  ordinary differential equation in p for each β

Partial Equilibrium

- solve the ordinary differential equation

$$\Theta(p,\beta) = c(\beta) p^{\frac{\hat\beta}{\beta-\hat\beta}}$$

- no distortion at $\delta = \underline{\delta}$ pins down constant $c(\beta)$

$$p_s(\underline{\delta},\beta) = \hat\beta\underline{\delta} \text{ and } \Theta(p_s(\underline{\delta},\beta),\beta) = 1$$

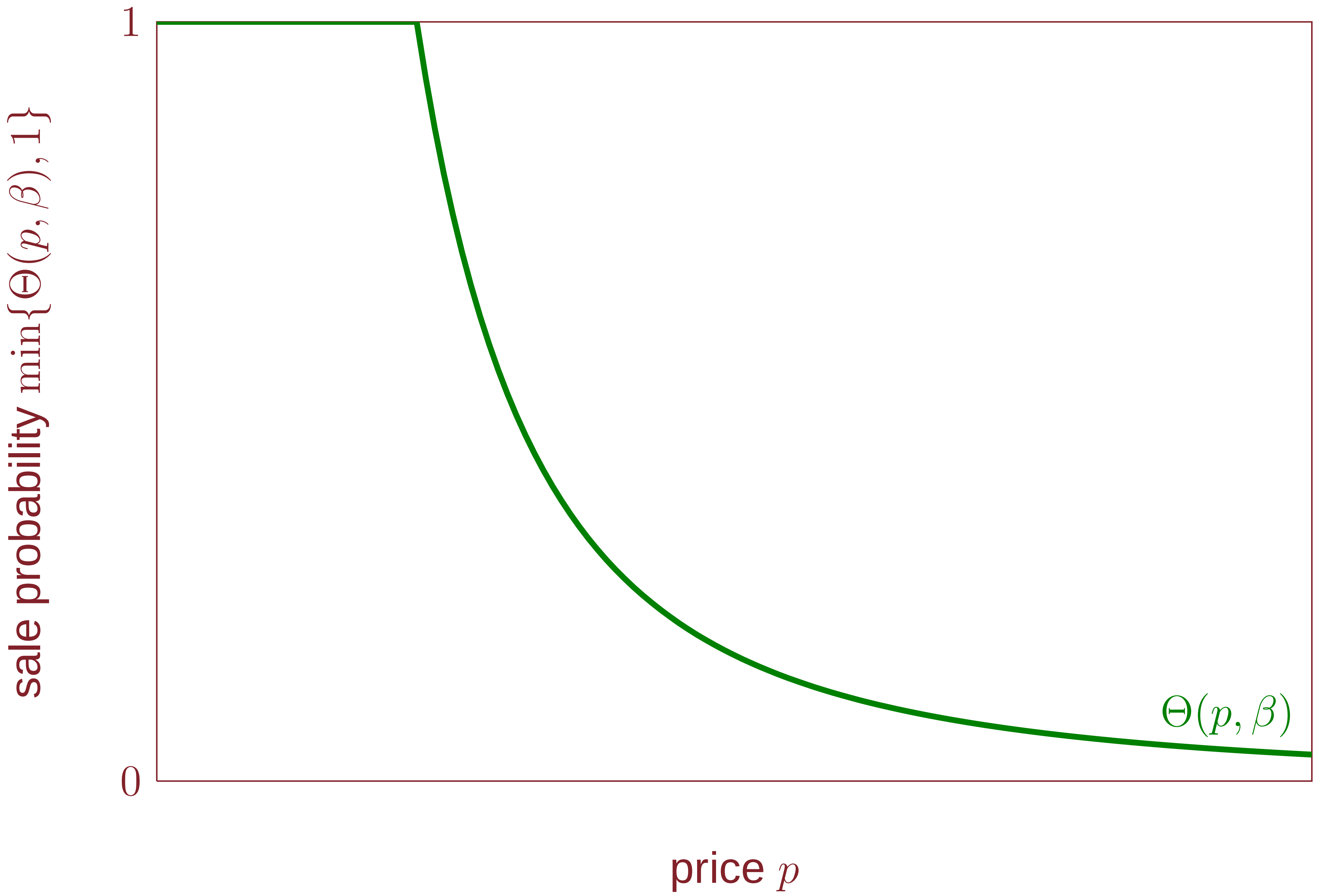
- conclusion:

$$\Theta(p,\beta) = \left(\frac{p}{\hat\beta\underline{\delta}}\right)^{\frac{\hat\beta}{\beta-\hat\beta}}$$

  decreasing if and only if $\beta < \hat\beta$

  - otherwise $\Theta(p,\beta) = 0$

Partial Equilibrium



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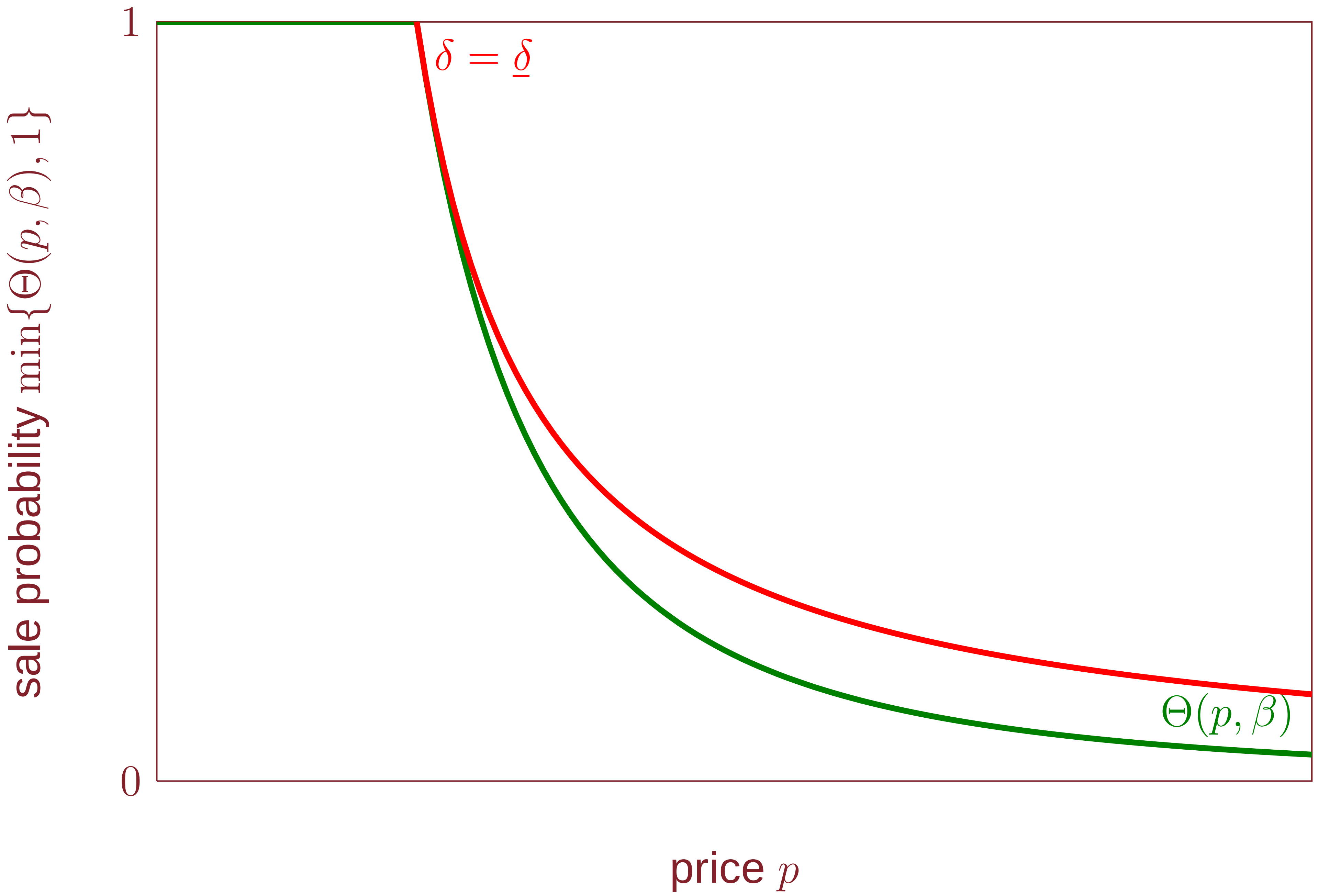

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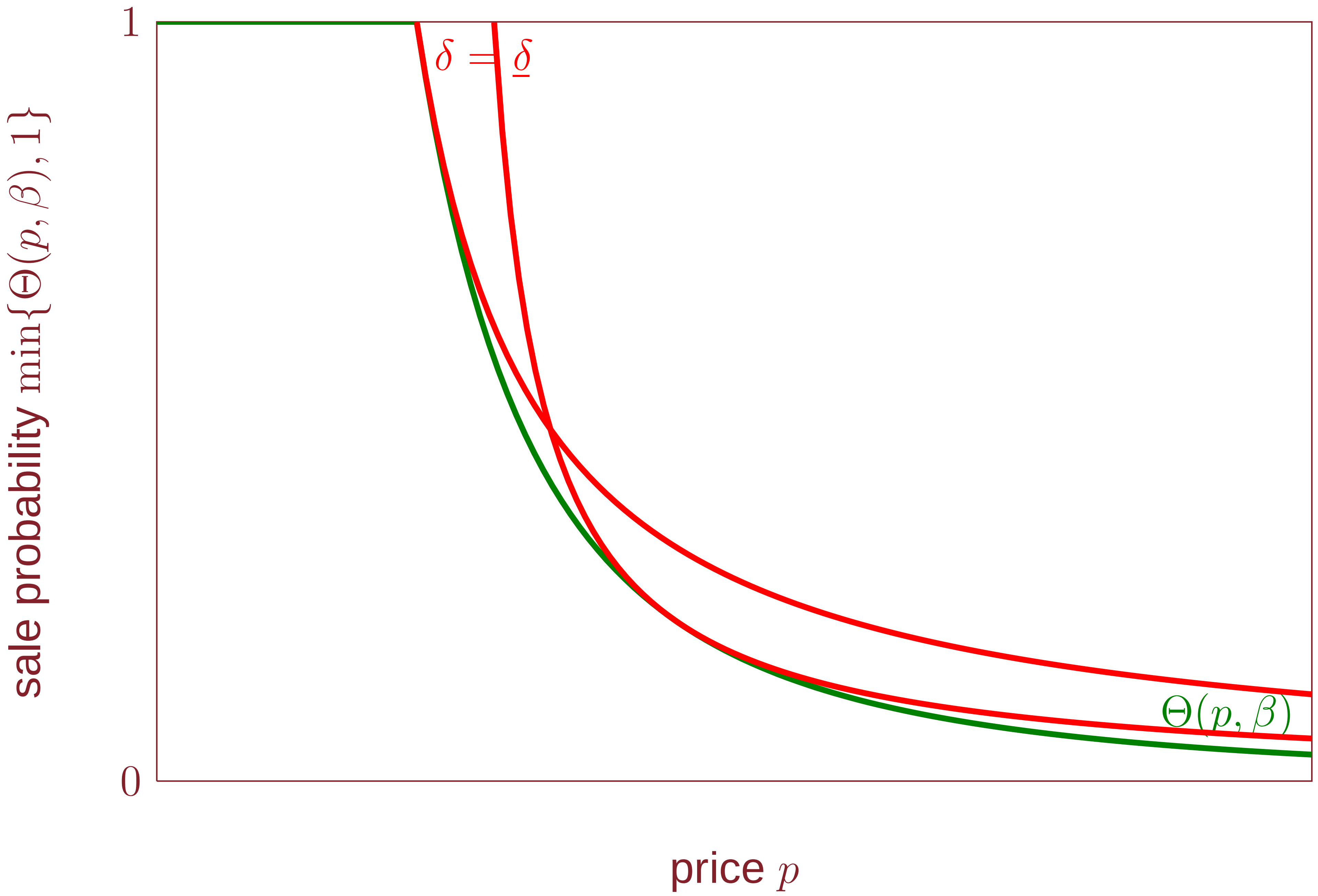

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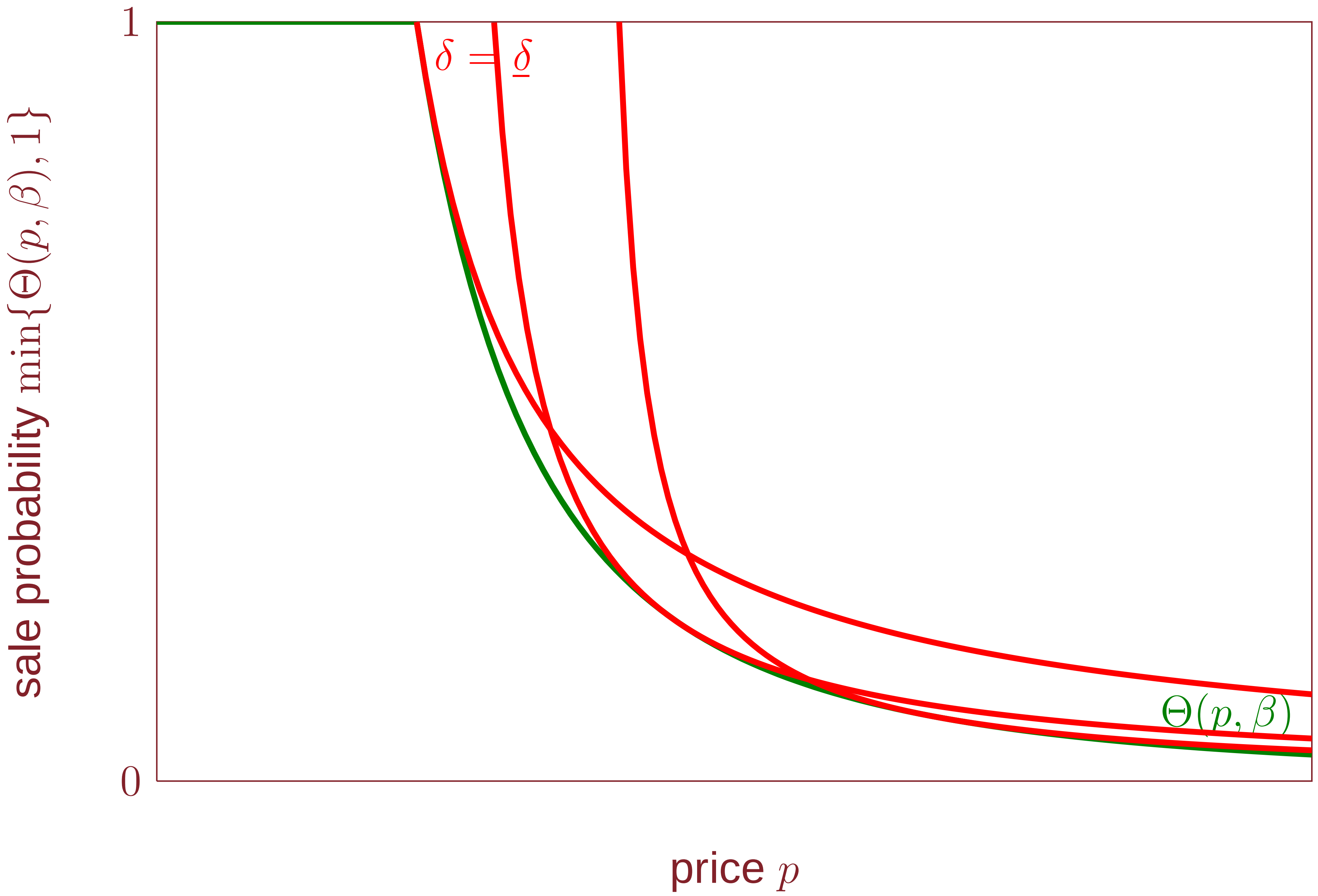

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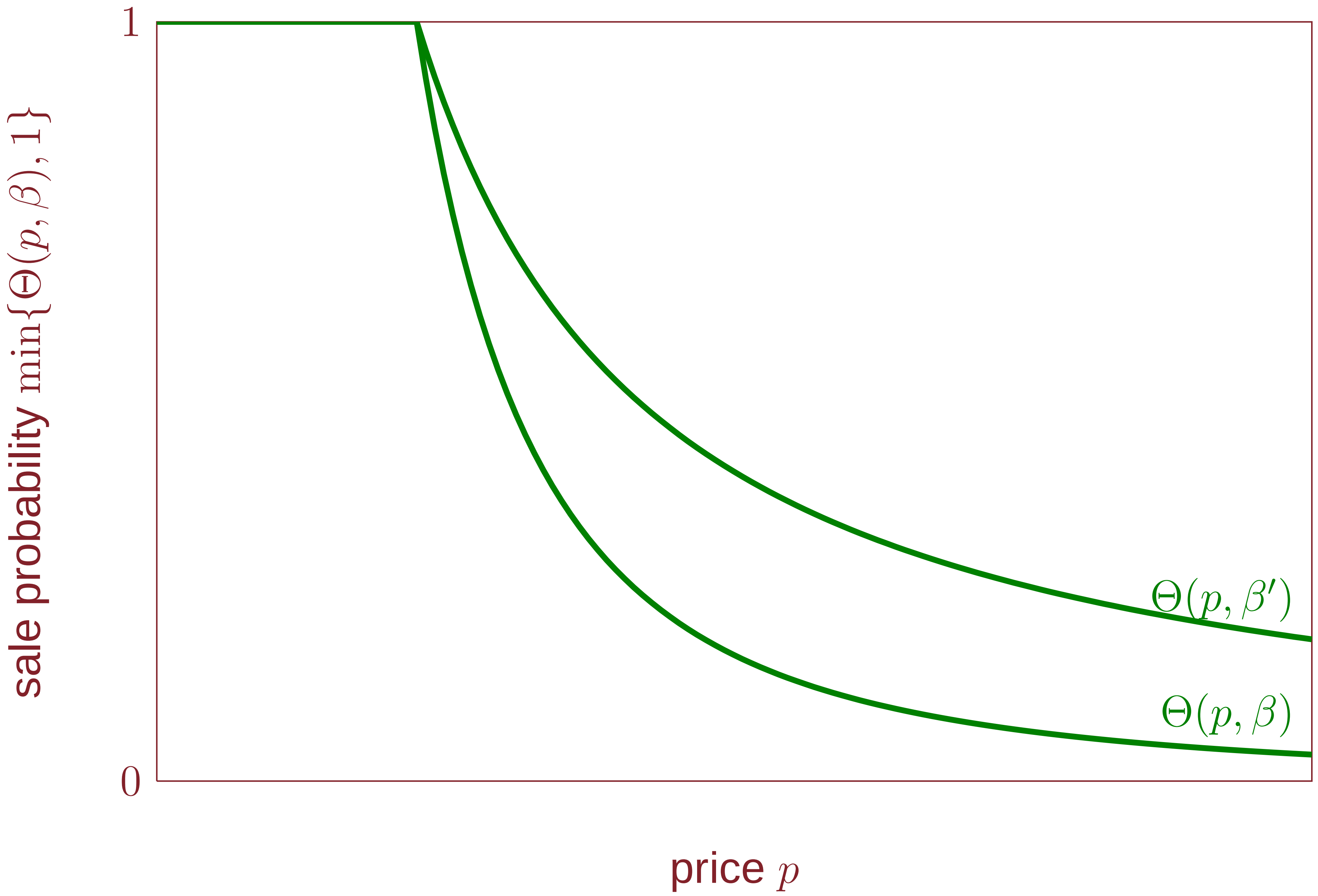

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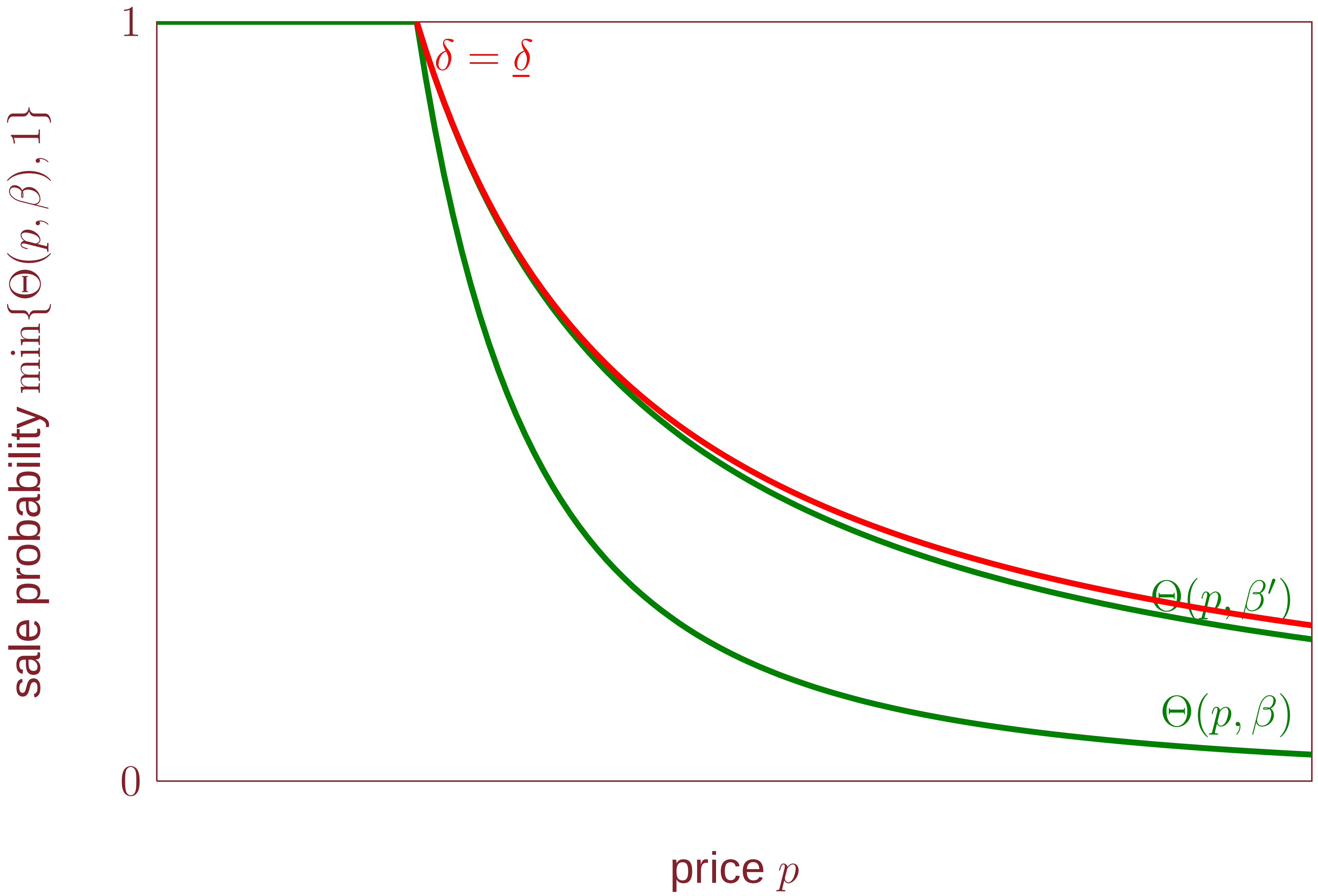

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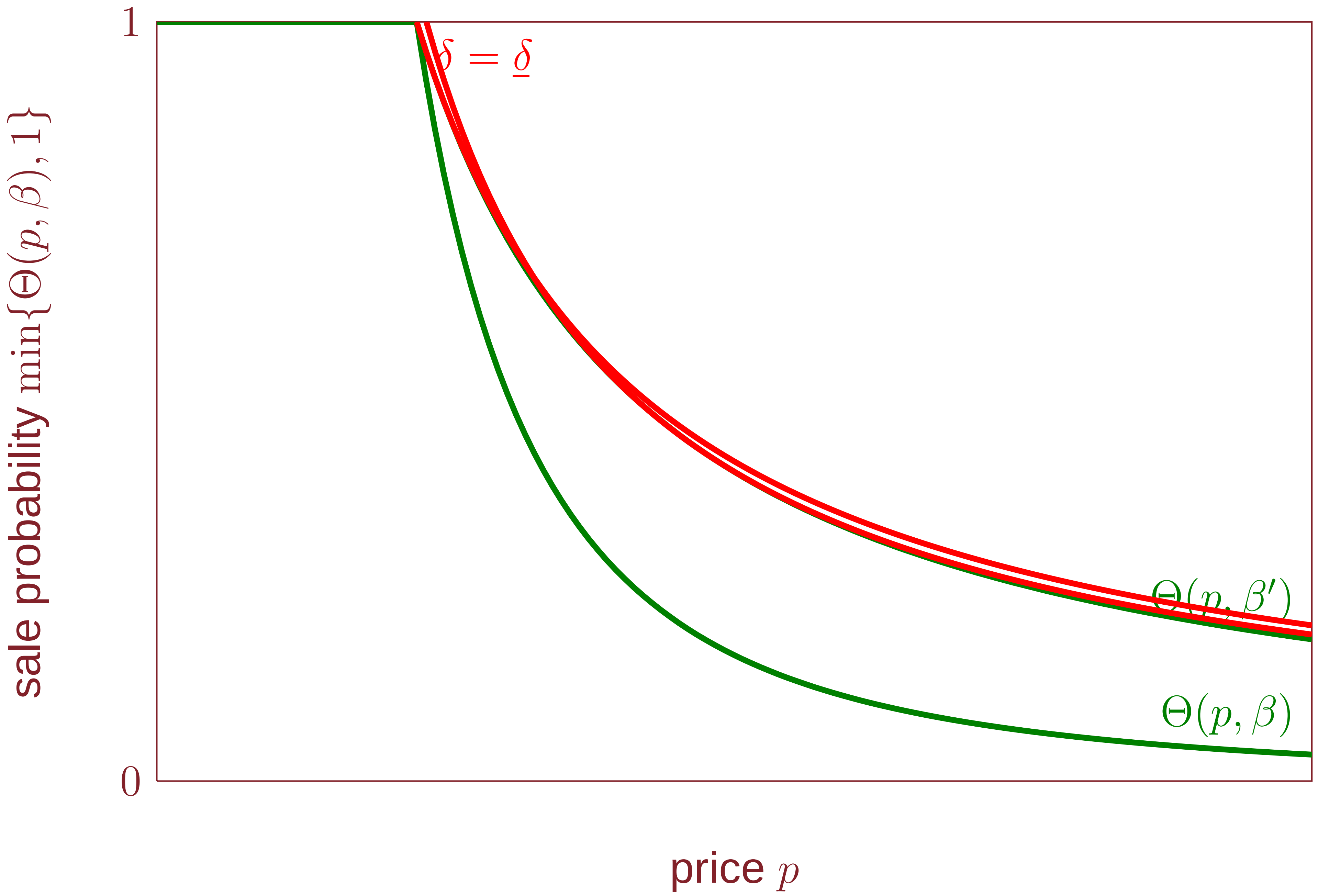

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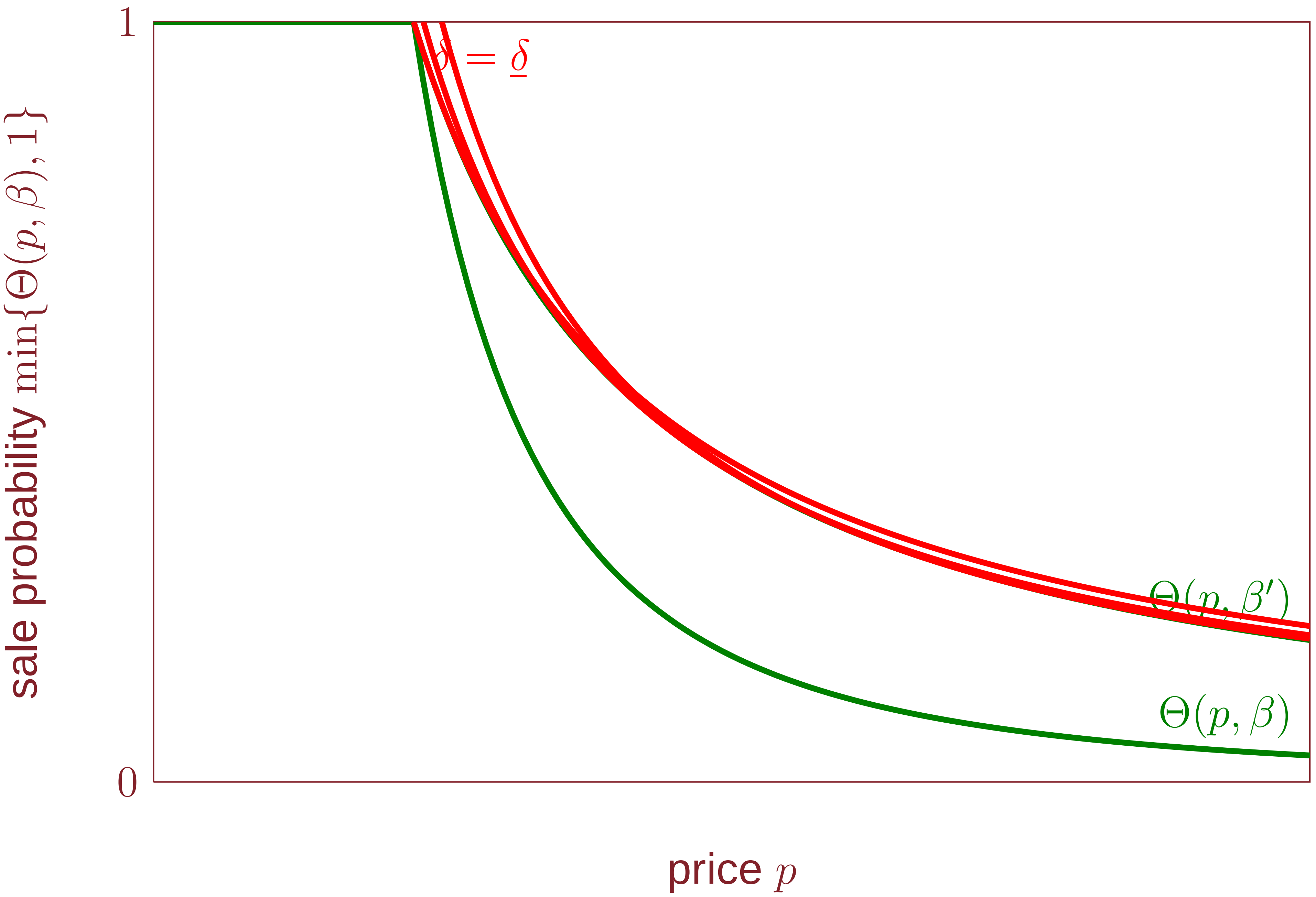


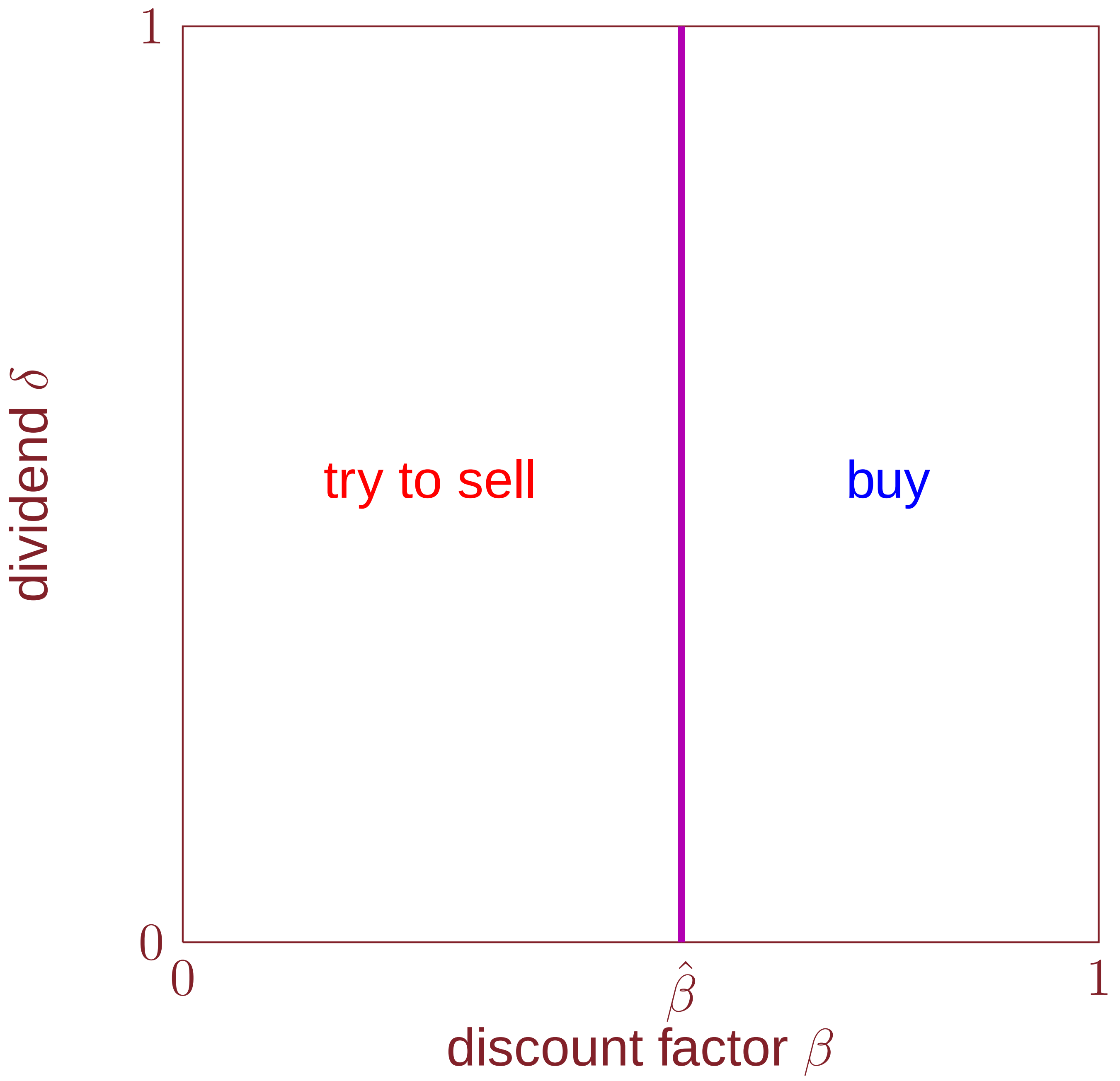
Competitive Equilibrium

- adjust $\hat{\beta}$ so the dividend market clears
- supply of dividends: $\int\int_{\beta>\hat{\beta}} dG(\beta,\delta)$
  - decreasing in $\hat{\beta}$
- demand for dividends:

$$\int\int_{\beta<\hat{\beta}} \hat{\beta}\delta\left(\delta/\underline{\delta}\right)^{\frac{\hat{\beta}}{\beta-\hat{\beta}}} dG(\beta,\delta)$$

  - increasing in $\hat{\beta}$
- existence and uniqueness follow

Equilibrium Behavior



Comments

- almost everyone either buys or tries to sell, not both
- there is no trade if $\underline{\delta} = 0$
  - extreme sensitivity to the lower support of the δ distribution

Unobservable Preferences

Modeling Issues

- seller (β, δ) sets sale price $p_s(\beta, \delta)$
- buyer (β, δ) sets buy price $p_b(\beta, \delta)$
  - can no longer choose type of seller $B(\beta, \delta)$
- markets are segmented by price p
- sellers are concerned about sale probability $\min\{\Theta(p), 1\}$
- buyers are concerned about buy probability $\min\{\Theta(p)^{-1}, 1\}$
- buyers are also concerned about average dividend $\Delta(p)$

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    - if $\exists(\beta, \delta)$ with $p_s(\beta, \delta) = p$, $\Delta(p) = \mathbb{E}(\delta | p_s(\beta, \delta) = p)$;
    - otherwise $\exists$ (β_1, δ_1) with $\delta_1 \leq \Delta(p)$ and (β_2, δ_2) with $\delta_2 \geq \Delta(p)$ for whom p is a weakly optimal sale price
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Parameter Restriction

- for any (β, δ), we call $\beta\delta$ her *continuation value*
- useful definitions:
  - $\underline{v} \equiv \underline{\beta}\underline{\delta}$ is the lowest continuation value
  - $\bar{v} \equiv \bar{\beta}\bar{\delta}$ is the highest continuation value
  - $\Gamma : [\underline{v}, \bar{v}] \mapsto [\underline{\delta}, \bar{\delta}]$ is the expected asset quality

$$\Gamma(v) \equiv \mathbb{E}(\delta | \beta\delta = v) = \frac{\int g\left(\frac{v}{\delta}, \delta\right) d\delta}{\int \frac{1}{\delta} g\left(\frac{v}{\delta}, \delta\right) d\delta}$$

- assume that Γ is continuous and increasing on a convex support

Leading Examples

- $G(\beta,\delta) = (1-\beta^{\alpha_\beta})(1-\delta^{\alpha_\delta})$ on $[1,\infty)^2$ and $\alpha_\beta, \alpha_\delta < 0$ (Pareto)
- $G(\beta,\delta) = \beta^{\alpha_\beta}\delta^{\alpha_\delta}$ and $\alpha_\beta, \alpha_\delta > 0$ on $[0,1]^2$
- in both cases

$$\Gamma(v) = \frac{(\alpha_\delta - \alpha_\beta)(1 - v^{\alpha_\delta - \alpha_\beta + 1})}{(\alpha_\delta - \alpha_\beta + 1)(1 - v^{\alpha_\delta - \alpha_\beta})}$$

  - e.g. $\alpha_\delta = \alpha_\beta + 1$, $\Gamma(v) = \frac{1}{2}(1+v)$
  - $\Gamma(0) = \dfrac{\alpha_\delta - \alpha_\beta}{\alpha_\delta - \alpha_\beta + 1}$ if $\alpha_\delta > \alpha_\beta$, 0 otherwise

Semi-Separating Equilibrium

Buyer Behavior

- buyer's asset quality δ is obviously irrelevant
- buyer's discount factor matters for behavior
  - $\beta > \hat{\beta}$: indifferent about buying anything
  - $\beta < \hat{\beta}$: do not buy anything
- marginal buyer $\hat{\beta}$ prices everything: $\hat{\beta}\Delta(p) = p$ for all p

Seller Behavior

- seller (β, δ) solves

$$p_s(\beta,\delta) \in \arg\max_{p \geq \beta\delta} \Big(\min\{\Theta(p), 1\} p + \big(1 - \min\{\Theta(p), 1\}\big) \beta\delta \Big)$$

- first order condition:

$$\Theta'(p)(p - \beta\delta) + \Theta(p) = 0$$

  and second order condition $\Theta_p < 0$

- semi-separating equilibrium: all sellers with the same $v = \beta\delta$ pool
  - $\Delta(p_s(\beta,\delta)) = \Gamma(\beta\delta)$ for all (β, δ)
  - buyer's problem implies $\hat{\beta}\Gamma(\beta\delta) = p_s(\beta,\delta)$ for all (β, δ)

- conclusion: $\Theta'(p)(p - \Gamma^{-1}(p/\hat{\beta})) + \Theta(p) = 0$

Characterization

- marginal buyer $\hat{\beta} \in (\underline{\beta}, \bar{\beta}]$ (determined from market clearing)
  - $\underline{p} \equiv \hat{\beta}\Gamma(\underline{v})$
  - $\bar{p}$ is the smallest solution to $\bar{p} = \hat{\beta}\Gamma(\bar{p})$ (or $\bar{p} = \infty$)
- buyer-seller ratio satisfies

$$\Theta(p) = \begin{cases} \infty & \text{if} \quad p < \underline{p} \\ \exp\left(-\int_{\underline{p}}^{p} \frac{1}{p' - \Gamma^{-1}(p'/\hat{\beta})} dp'\right) & \text{if} \quad p \in [\underline{p}, \bar{p}] \\ 0 & \text{if} \quad p > \bar{p} \end{cases}$$

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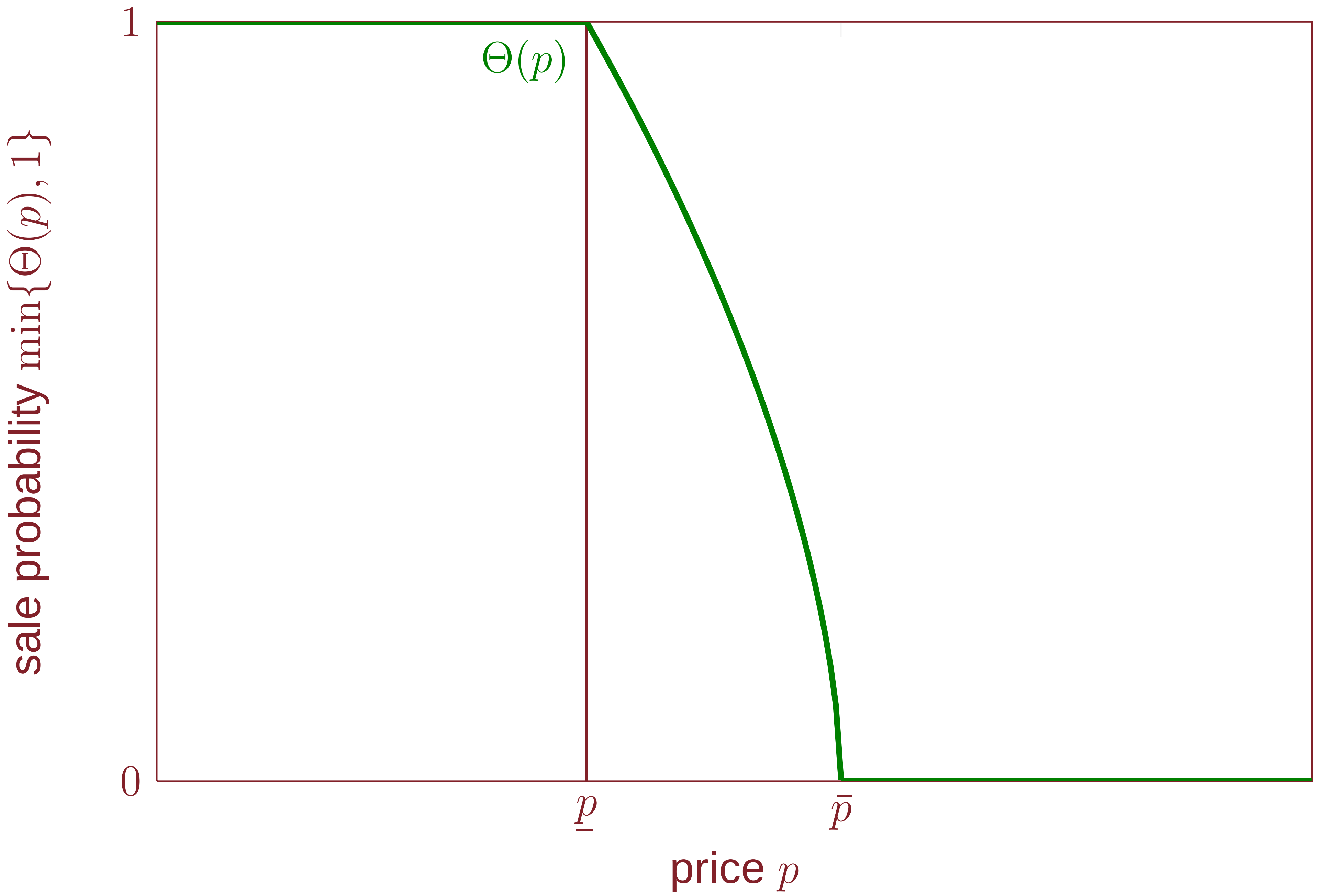
- beliefs satisfy $\Delta(p) = p/\hat{\beta}$ if $p \in [\underline{p}, \bar{p}]$, $\Delta(p) \leq p/\hat{\beta}$ otherwise
  - seller: $p_s(\beta, \delta) = \max\{\hat{\beta}\Gamma(\beta\delta), \beta\delta\}$
  - buyer: $p_b(\beta, \delta) \in [\underline{p}, \bar{p}]$ if $\beta > \hat{\beta}$, $p_b(\beta, \delta) < \underline{p}$ otherwise

A Semi-Separating Equilibrium



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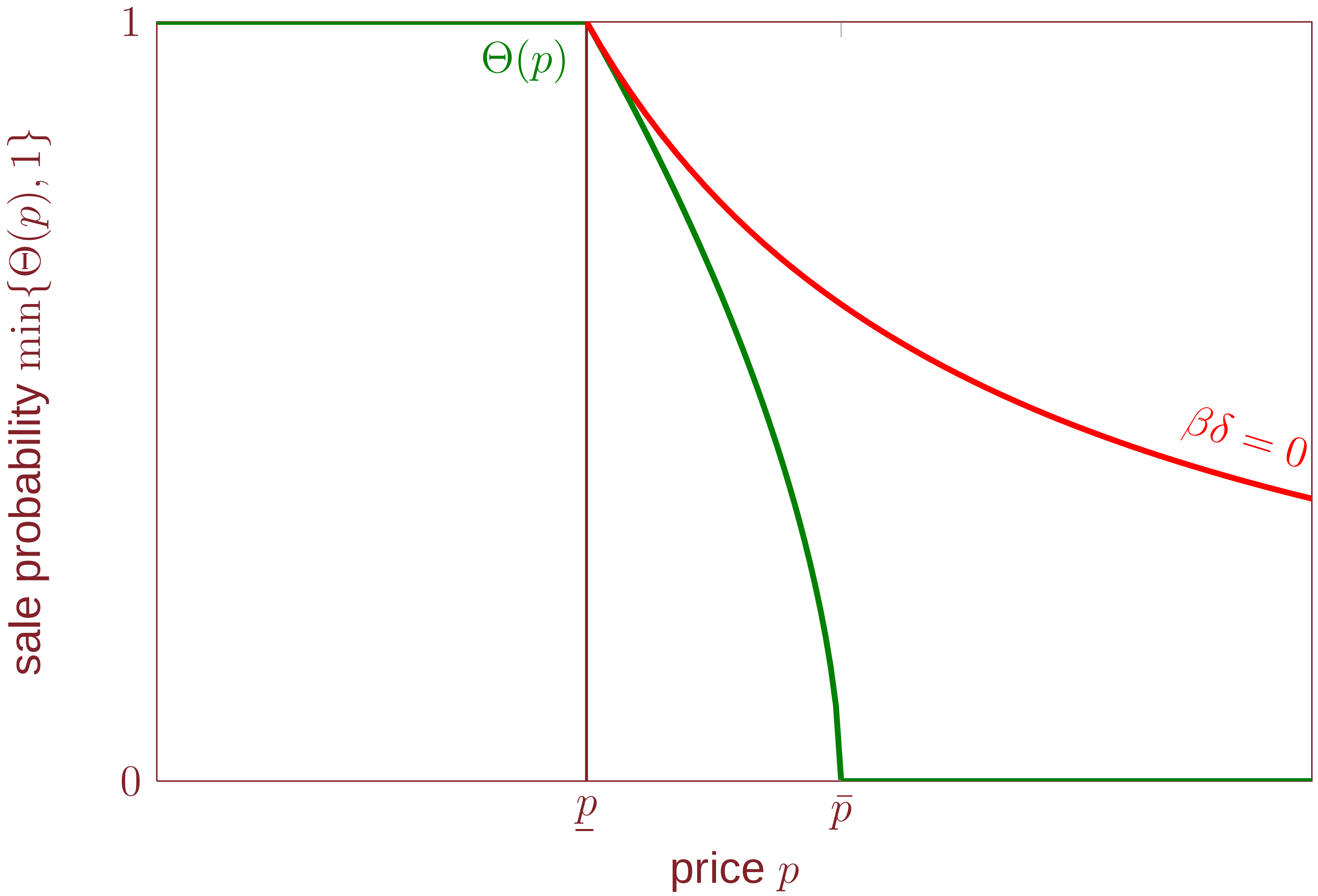

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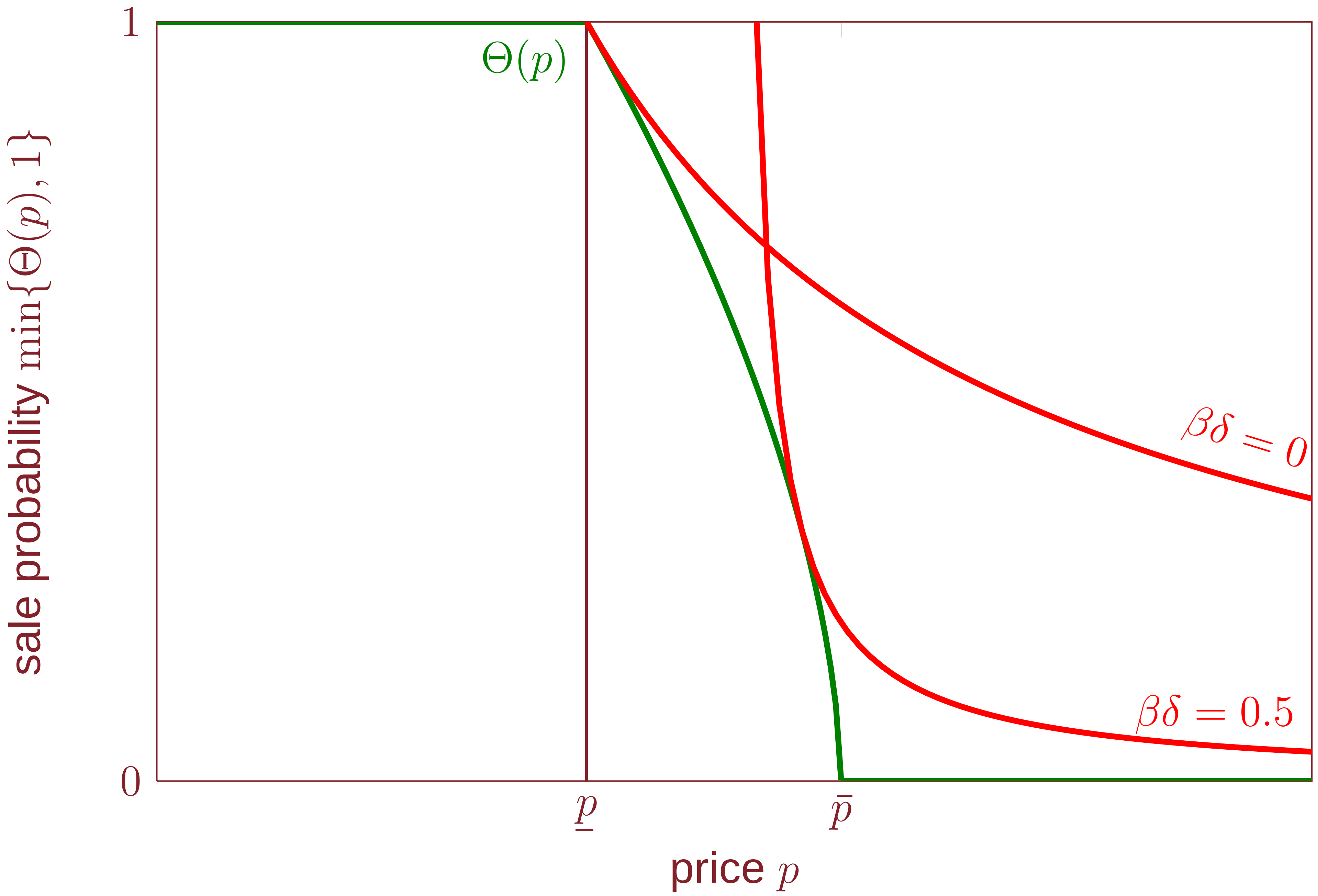

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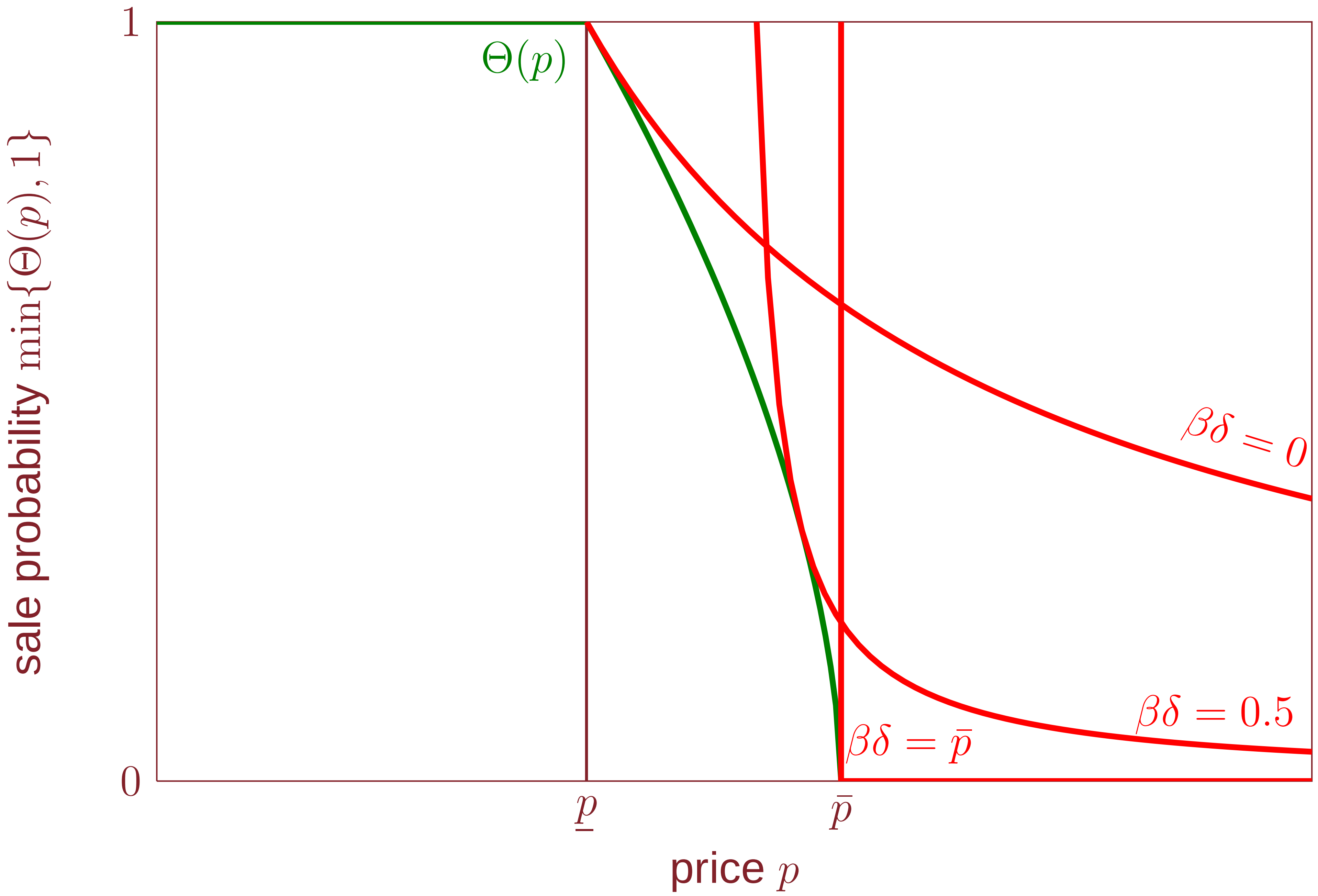
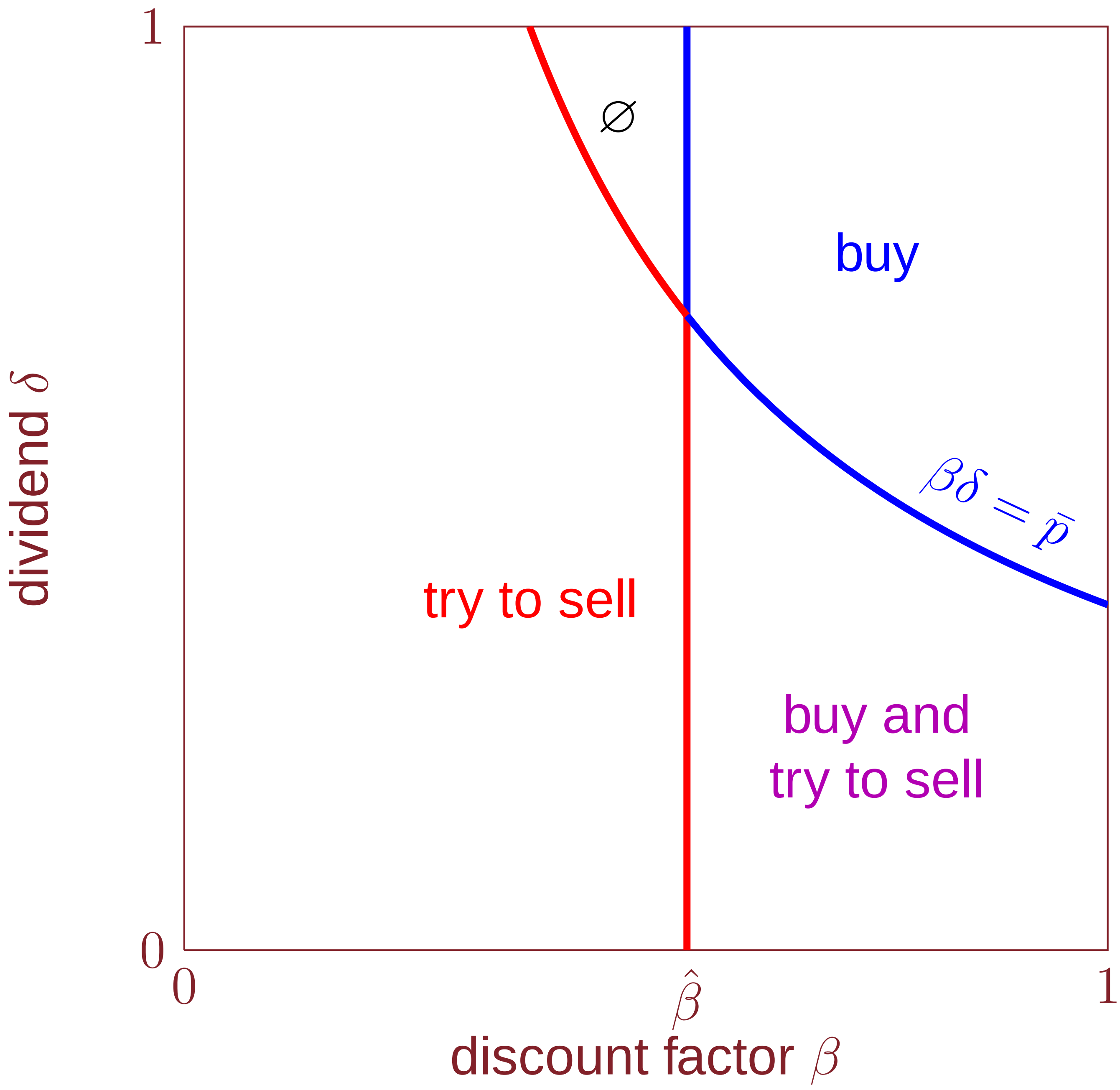


Equilibrium Behavior



One-Price Equilibrium

Standard Definition

- price $\hat{p}$, marginal buyer $\hat{\beta}$ such that:

  1. sellers sell if $\beta\delta < \hat{p}$
  2. buyers buy if $\hat{p} <$ their value of buying stuff for sale

$$\hat{p} = \hat{\beta}\frac{\iint_{\beta\delta<\hat{p}} \delta dG(\beta,\delta)}{\iint_{\beta\delta<\hat{p}} dG(\beta,\delta)}$$

  3. market clearing

$$\iint_{\beta>\hat{\beta}} dG(\beta,\delta) = \hat{p}\iint_{\beta\delta<\hat{p}} dG(\beta,\delta)$$

Our Definition

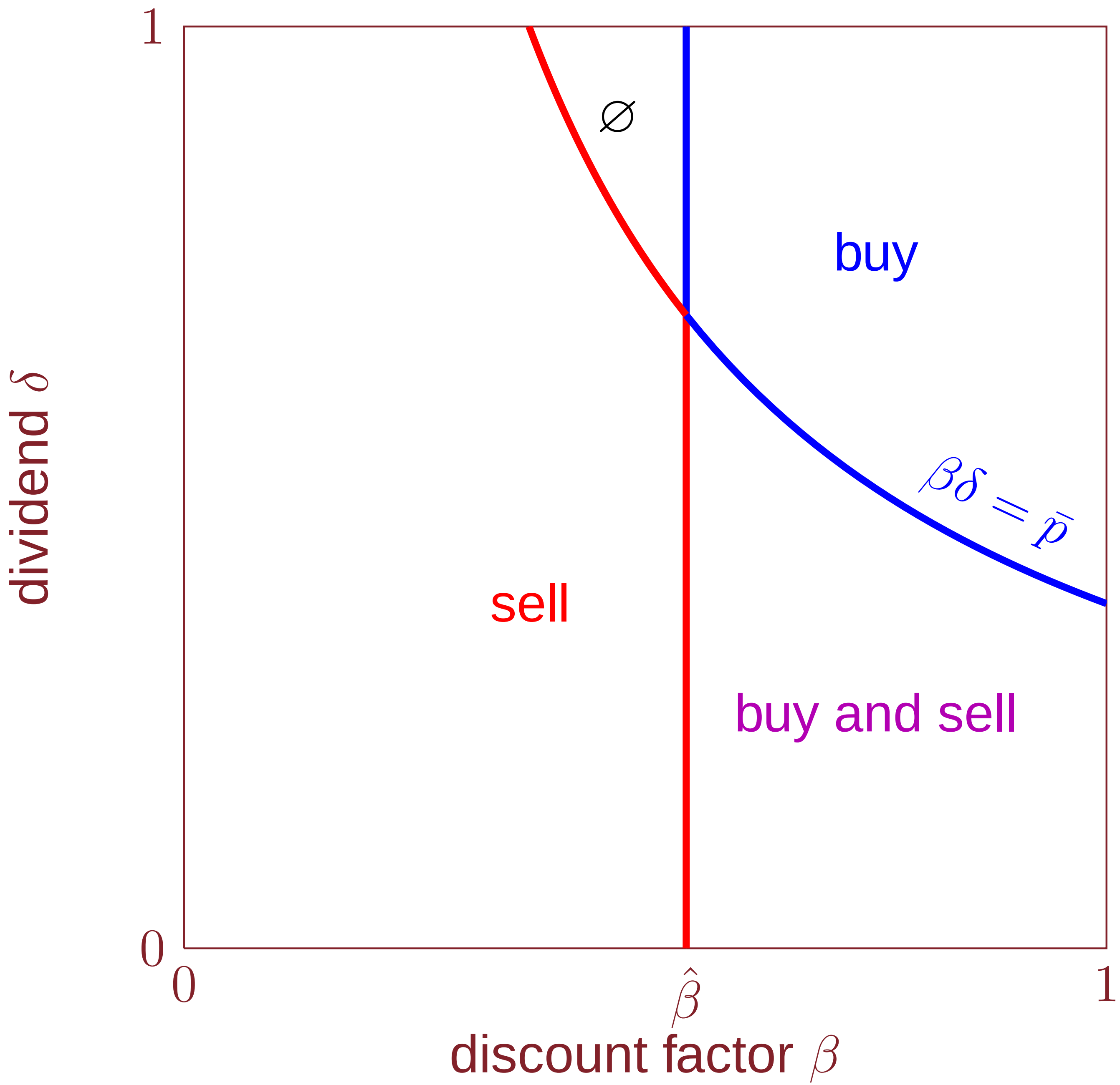
$$p_s(\beta,\delta)\begin{cases} = \hat{p} \\ > \max\{\hat{p},\hat{\beta}\delta\}\end{cases} \Leftrightarrow \beta\delta \lesseqgtr \hat{p}$$

$$p_b(\beta,\delta) = \begin{cases} 0 \\ \hat{p}\end{cases} \Leftrightarrow \beta \lesseqgtr \hat{\beta}$$

$$\Theta(p) = \begin{cases} \infty \\ 1 \\ 0\end{cases} \Leftrightarrow p \begin{matrix} < \\ = \\ > \end{matrix} \hat{p}$$

$$\Delta(p)\begin{cases} = \underline{\delta} \\ = \hat{p}/\hat{\beta} \\ < p/\hat{\beta}\end{cases} \Leftrightarrow p \begin{matrix} < \\ = \\ > \end{matrix} \hat{p}$$

Equilibrium Behavior



Construction

- note that $\mathbb{E}(\delta|\beta\delta = \hat{p}) > \mathbb{E}(\delta|\beta\delta \leq \hat{p})$ whenever $\hat{p} > \underline{v}$
- but buyers expect *lower* quality if they pay $\hat{p} + \varepsilon$
- different sellers with the same preferences behave differently
- this possibility is essential to constructing multiple equilibria
  - continuum of semi-separating equilibria
  - n-dimensional continuum of n-price equilibria
  - equilibria with mixture of mass points and continuous densities

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  - equilibria with mixture of mass points and continuous densities
- multiplicity is not driven by off-the-equilibrium path beliefs

Uniqueness

Additional Assumptions

- there is a $P : [\underline{v}, \bar{v}] \mapsto \mathbb{R}_+$ such that $p_s(\beta, \delta) = P(\beta\delta)$ for all (β, δ)

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- rational beliefs: for any p with $\Theta(p) < \infty$
  - if $\exists\ v$ with $P(v) = p$, $\Delta(p) = \mathbb{E}(\delta | P(v) = p)$;
  - otherwise $\exists\ v_1$ with $\Gamma(v_1) \leq \Delta(p)$ and v_2 with $\Gamma(v_2) \geq \Delta(p)$ for whom p is a weakly optimal sale price

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  - otherwise $\exists\ (\beta_1, \delta_1)$ with $\delta_1 \leq \Delta(p)$ and (β_2, δ_2) with $\delta_2 \geq \Delta(p)$ for whom p is a weakly optimal sale price

Proposition

- only the semi-separating equilibrium survives
  - $\Gamma(\underline{v}) = 0$: no trade
  - $\Gamma(\underline{v}) > 0$: semi-separating, $P(v)$ continuous and increasing

Proposition

- only the semi-separating equilibrium survives
  - ▷ $\Gamma(\underline{v}) = 0$: no trade
  - ▷ $\Gamma(\underline{v}) > 0$: semi-separating, $P(v)$ continuous and increasing
- sketch of proof
  - ▷ $v_1 < v_2 \Rightarrow P(v_1) \leq P(v_2) \Rightarrow \Delta(p)$ nondecreasing
  - ▷ rule out mass of sellers with same price p:
    $\rightsquigarrow$ buyers prefer $p + \varepsilon$
  - ▷ rule out one seller indifferent over range of price $[p_1, p_2]$:
    $\rightsquigarrow$ buyers prefer p_1 over $p_2 + \varepsilon$
  - ▷ conclude that P is one-to-one, use first order conditions to find it

$$P(v) = \arg\max_p \Theta(p)(p - v) \text{ and } P(v) = \arg\max_p \frac{\Gamma(p)}{p}$$

Dynamics

- infinite horizon, trading opportunity in each period
- discount factor follows a Markov process
- look for a stationary distribution $G(\beta, \delta)$
- trading frictions remain even with continuous trading opportunities
  - stock of sellers offering assets
  - flow of dividends only purchases some of the assets
  - looks like a Poisson arrival rate of selling opportunities

Summary

- trade occurs at a range of prices
  - Θ is pinned down everywhere
- private information creates a role for intermediation
  - is it fair to call it this?
  - patient investors with bad assets are intermediaries
  - not all trade is intermediated
- trade breaks down if $\Gamma(\underline{v}) = 0$
  - sensitivity to left tail of joint distribution of (β, δ)
- Pareto optimality???

Competitive Search in Financial Markets

Robert Shimer

Finance Theory Group

May 2, 2014