

Corporate Control Activism

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Abstract

This paper studies the role of blockholders and activist investors in the market for corporate control. We argue that activist investors complement the effort of bidders to acquire companies by relaxing the resistance of incumbent boards to takeovers. Using this insight, we show that there is strategic complementarity between the search of activist investors for firms that are likely to receive a takeover bid, and the search of corporate buyers for targets with which they can create synergies. Among other implications, strategic complementarity implies that the aggregate M&A activity is positively related to the intensity of shareholder activism, and gives rise to unpredictable episodes with high and low volume of transactions. Finally, our model provides empirical predictions on the interaction between shareholder activism and M&A activity, and a framework to study the implications of treatment and selection effects.

KEYWORDS: Takeover, Mergers, Acquisitions, Shareholder Activism, Corporate Governance, Proxy Fight, Search

JEL CLASSIFICATION: D74, D83, G23, G34

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Introduction

This paper studies the role of activist investors in the market for corporate control. The separation of ownership and control in public corporations creates agency conflicts (Berle and Means (1932), Jensen and Heckling (1976)) and opportunities for managers to extract private benefits of control.¹ In order to protect these private benefits, corporate boards may resist takeovers that would otherwise create shareholder value.² Under most jurisdictions, including the Delaware law, merger proposals can be brought to a vote of shareholder approval only by its board of directors. Moreover, corporate boards can use defense measures such as poison pills to block tender offers made directly to their shareholders. The resistance to takeovers can be overcome only if the majority of directors are voted out in a contested election (“proxy fight”), and replaced by a team which is friendly to the takeover. In fact, the power of shareholders to unseat directors is often used by the courts as the basis for allowing boards to block takeovers in the first place (e.g., see Gilson (2001)).

Hostile bidders occasionally challenge target companies by launching a proxy fight.³ However, winning a proxy fight is not trivial, as the challenger must convince the majority of target shareholders that replacing the incumbents with his nominees is in their best interests. Target shareholders, who significantly benefit from takeovers (e.g., see Betton et al. (2008)), may also challenge the incumbent directors if they resist a takeover. For example, in 2014, the board of PetSmart Inc. agreed to be bought out for \$8.7 Billion after facing a months-long pressure from one of its largest shareholders, the activist hedge fund Jana Partners.⁴ Brav et al. (2008) document that activist hedge funds occasionally demand from their portfolio companies to sell all or part of their assets. The idea that activist investors can put firms into play is also consistent with Greenwood and Schor (2009) who show that the positive abnormal returns around 13D filings stems from events in which the target is eventually acquired. Becht et

¹Private benefits from control can include excessive salaries, perquisites, investment in ‘pet’ projects, promotion of relatives, access to private information, pure pleasure of command, prestige, and publicity. See Barely and Wilderness (1989), Nova (2003), and Dyck and Zingales (2004) for related empirical evidence.

²Jenter and Lewellen (2015) provide evidence consistent with managers being reluctant to relinquish control due to career concerns. See also Walkling and Long (1984), Martin and McConnell (1991), Agrawal and Walkling (1994), Hartzel, Ofek, and Yermack (2004), and Wulf and Singh (2011), who show that target CEOs typically suffer poor career prospects following takeovers.

³Mulherin and Poulsen (1998) study 270 proxy contests from the 1979-1994 and find that 43% of the contests were followed by a takeover bid, and half of these firms were eventually acquired. They do not distinguish between cases where the challenger is the acquirer and cases where the challenger is a third party (e.g., an activist investor).

⁴See The New York Times, “Under Pressure From Jana, PetSmart Says It Will Explore Potential Sale”, August 19, 2014, and “Elliott and Jana, Activist Investors, Are Behind 2 Big Buyouts,” December 15, 2014.

al. (2015) extend these results and show that activists' exits via takeovers often involve other governance outcomes during the engagement. Overall, the evidence suggests that activist investors play an important role in the market for corporate control. What are the implications of such interventions? Do they complement or substitute the effort of bidders to acquire companies?

We study these questions by analyzing a bargaining model with the following ingredients. Initially, the bidder tries to negotiate a deal with the target board. The board has private benefits of control, and hence, may reject offers that increase shareholder value. If an agreement is reached then it is brought to a shareholder vote for approval. If the negotiations fail or if the majority of target shareholders reject the agreement, a proxy fight to replace the incumbent directors can be initiated either by the bidder or by an activist investor, if the latter is a shareholder of the target. The proxy fight succeeds if and only if shareholders elect the nominees of the rival team. If the proxy fight succeeds, the winning team obtains control of the target board and a second round of negotiations between the bidder and the newly elected directors takes place. If no proxy fight is launched or if the proxy fight fails, the incumbent board retains control of the target and can use his authority to block the takeover.

Our analysis demonstrates that activist investors complement the effort of bidders to acquire companies by relaxing the resistance of incumbent directors to takeovers.⁵ We identify several channels through which activist investors contribute to an active market for corporate control. We start by arguing that bidders suffer from a fundamental commitment problem that activists do not suffer, or suffer to a lesser extent. A successful proxy fight involves a change in the composition of the target board, but it does not oblige the newly elected directors to execute the takeover at the initial terms. Once his nominees are elected, the bidder might be tempted to abuse the control of the target board to divert resources from the target and lower the takeover premium. Therefore, target shareholders would never elect the bidder's nominees without a commitment to act in their best interests. By contrast, the activist buys a stake in the target with the expectations that the firm will be acquired. Unlike the bidder, the activist has incentives to negotiate the highest takeover premium possible. Since the activist is also sitting on the sell side of the negotiating table, shareholders would trust the activist and elect her nominees to the board, even without a firm commitment to act in their best interests.

⁵Consistent with this view is a speech by a private equity fund manager Thomas H. Lee: "I'd like to thank my friends Carl Icahn, Nelson Peltz, Jana Partners, Third Point," he said. "I'd like to thank these funds for teeing up deals because they're coming in there and shaking up the management and many times these companies are being driven into some form of auction." See The New York Times, "Will Credit Crisis End the Activists' Run?" August 27, 2007.

Taken together, the resistance of incumbents to takeovers can be overcome only if the capacity to disentrench boards is separated from the capacity to take over companies. Our analysis suggests that collaborations between activist investors and bidders are likely to fail.⁶

The result that activist investors complement the effort of bidders to acquire companies continues to hold even when bidders do not suffer from the aforementioned commitment problem. There are several reasons for this result. First, activist investors have a unique governance expertise (e.g., understanding the legal framework of proxy solicitation and lobbying other shareholders), and therefore, might face lower costs of running a proxy fight. Second, a common critique of activist investors is their short-term investment horizon. Short-termism gives the activist stronger incentives to push for the sale of the firm, which is a quick way to exit the investment, thereby making the activist's threat to run a proxy fight more credible than the bidder's. Third, the benefit from the takeover and running a proxy fight depends on the bargaining power of the target. If target shareholders expect to extract most of the value that is created by the takeover, the activist will have stronger incentives than the bidder to challenge the board. Fourth, even if the activist's threat of running a proxy fight is not credible, by exercising her voting rights and lobbying other shareholders, she can help the bidder winning the proxy fight, thereby increasing his credibility and ability to overcome the board's resistance to the takeover. We conclude, activist investors complement the effort of bidders to acquire companies through a variety of channels.

Activist investors not only facilitate takeovers once the offer is on the table, but they also increase the likelihood that companies become a takeover target in the first place. In order to become a target the company must be a good match for potential bidders. Bidders have to search in order to identify with which companies they can create synergies or improve operations. The search process involves carrying out due diligence, hiring advisors, and spending time and corporate resources. Since search is costly, firms will search for targets only if they believe that these companies are also available for sale. Activist investors, who can make potential targets available for sale, also have to search in order to identify companies that are likely to be a good match for bidders, and then buy a stake in these companies. We therefore augment the model with an initial stage in which the activist and the bidder simultaneously search for takeover targets.⁷ If the activist finds the target, she can buy its shares from a

⁶A case in point is the failed acquisition attempt of Allergan by Valeant and Pershing Square in 2014. See Section 3.1 for details.

⁷In Section 5.3 we discuss an extension of the model in which the activist can solicit bids, and show that solicitation can increase the incentives of the bidder to search. Thus, solicitation is yet another channel through which activist investors facilitate takeovers.

market maker and become a shareholder. If the bidder finds the target then he can start negotiating an acquisition agreement with the board, as previously mentioned.

We show that there is strategic complementarity between the search of activist investors for firms that are likely to receive a takeover bid, and the search of bidders for potential takeover targets. Intuitively, since activist investors can relax the opposition of boards to takeovers, bidders have stronger incentives to search for a target if the target is likely to have an activist investor as a shareholder. At the same time, the activist can profit from searching for and buying shares of companies which are a good match for the bidder, only if these companies are likely to receive a takeover bid. Therefore, the incentives of the activist to search increase with the search of the bidders for takeover targets.

Strategic complementarity has three important implications. First, the aggregate volume of M&A activity is positively related to the intensity of shareholder activism, which can be measured by the volume of 13D filings. Second, small changes in the environment have a significant effect on the aggregate volume of M&A. For example, a regulation that eases the access of shareholders to the ballot will directly increase the incentives of activists to search, since challenging incumbents becomes less costly. In anticipation of more shareholder activism, bidders will have stronger incentives to search for potential targets. In turn, with a higher likelihood that the target receives a takeover bid, the activist has even more incentives to search, resulting with an amplified impact on the likelihood of a takeover. Third, strategic complementarity implies that there can be multiple equilibria, characterized by the aggregate volume of M&A transactions. The existence of multiple equilibria suggests that due to effective shareholder activism, the market for corporate assets can experience episodes of high volume of transactions (“hot markets”) and episodes of low volume of transactions (“cold markets”), without any apparent changes in the underlying fundamentals of the economy. In this respect, the extent of M&A activity is self-fulfilling and unpredictable.

Activists investors invest in firms either because they believe they are likely to become a takeover target (selection effect) or because they believe in their ability to facilitate the takeover of these firms (treatment effect). By incorporating financial markets into the analysis, the model provides a framework to study the implications of the treatment and the selection effects, and thereby helps creating identification strategies for empirical research. For example, if only the selection effect is in play, the volume of M&A deals decreases with severity of the agency problems in target firms. This is intuitive, as with more private benefits of control the incumbents are more likely to resist takeover bids. However, this relationship can reverse when the treatment effect is in play. In this case, more resistance of incumbents to takeovers can

result with a *higher* volume of M&A deals. Intuitively, the resistance to takeovers provides activist investors with more opportunities to make profit on their ability to put firms into play. Due to the strategic complementarity, bidders will search more intensively and the aggregate volume of M&A deals will increase. An interesting implication of this result is that the sign of the relationship between the severity of agency problems in the cross section of target firms and the volume of M&A deals can also point the existence of the treatment effect.

Related, the model provides interesting empirical predictions on the interaction between shareholder activism and M&A activity. The model predicts that the average abnormal return around the announcement of an acquisition agreement is smaller when it is preceded by a 13D filing than when it is not. Moreover, the average abnormal returns around the announcement of 13D filing can decrease with factors that increase the credibility of the activist’s threat to run a proxy fight. Last, the model predicts that policies and regulations that undermine shareholder activism will have a negative effect on M&A. For example, in 2014, the Delaware court allowed Sotheby’s to keep a unique two-tier poison pill that was purposely meant to block the activist hedge fund Third Point from increasing its ownership in Sotheby’s above 10%.⁸ Seemingly, poison pills that are designed to undermine the ability of activist investors to intervene should not affect M&A if “standard pills” that prevent unwanted takeovers are already in place. Our analysis suggests that these anti-activism pills can have a large and negative effect on takeovers because they undermine the ability of activist investors to disentrench incumbents, and therefore, deter potential bidders from pursuing acquisition opportunities.

Finally, our analysis identifies two instances in which activist investors *harm* the effort of bidders to acquire companies. First, in management buyouts the incumbents may be too motivated to sell the firm, even if the deal compromises shareholder value. We show that activist investors will challenge the deal by using their influence on target shareholders to either block the transaction or “force” the bidder to sweeten the bid. For example, during the management buyout of Dell in 2013, the pressure of the activist investor Carl Icahn resulted in the increase of the offer price. Second, activist investors may also have the expertise to propose and execute operational, financial, and governance changes that increase the standalone value of the target. We show that by providing a viable alternative to the takeover, the activist can force the bidder to pay a higher takeover premium. While in both cases the bidder may have weaker incentives to search and take over the target, we find that the presence of the activist increases the expected value of target shareholders.

⁸See *THIRD POINT LLC v. Ruprecht*, Del. Court of Chancery 2014.

This paper is organized as follows. The rest of this section reviews the related literature. Section 1 discusses the commitment problem that hostile bidders suffers in takeovers and the related institutional details. Section 2 presents the setup of the model. In Section 3, we analyze the model under the no-commitment assumption, and do comparative statics. In Section 4, we analyze the model under the full-commitment assumption. Section 5 offers several extensions, including the applicability of the model to divestitures and spinoffs. Section 6 concludes. Appendixes A and B give all proofs and results not in the main text.

Related Literature

Our paper connects the literature on blockholders and shareholder activism (for a survey, see Edmans (2014)) with the literature on takeovers (for a survey, see Becht et al. (2003)). Several papers study models where the bidder is also a blockholder of the target prior to the takeover attempt (e.g., Shleifer and Vishny (1986), Hirshleifer and Titman (1990), Kyle and Vila (1991), Burkart (1995), Maug (1998), Singh (1998), and Bulow et al. (1999)). By contrast, in this paper, the activist, who is a shareholder of the target, can pressure the incumbent board to accept a takeover offer, but she cannot take over the target herself. In fact, our analysis emphasizes the benefit from separating the capacity to disentrench boards from the capacity to acquire companies.

Cornelli and Li (2002) study a model in which arbitrageurs accumulate large stakes in the target, and mitigate the free-rider problem of Grossman and Hart (1980) by tendering their shares to the bidder. Gomes (2012) studies a dynamic model of tender offers in which the arbitrageurs, by holding blocks of shares, force the bidder to make a high preemptive bid to counter a credible hold-out. In a contemporaneous and independent work by Burkart and Lee (2015), the activist can relax the free-rider problem by directly negotiating with the bidder. Different from these three studies, we abstract from the free-rider problem, and instead, focus on the agency conflicts between the target board and its shareholders,⁹ and on the search friction in the market for corporate control. We show that activist investors can disentrench corporate boards, a feature which give rise to strategic complementarity between activist's and bidder's search efforts.

Various aspects of proxy fights within and outside the context of takeovers have been

⁹Models in which the target board can resist a takeover offer have also been studied by, for example, Bagnoli, Gordon, and Lipman (1989), Baron (1983), Berkovitch and Khanna (1990), Hirshleifer and Titman (1990), Harris and Raviv (1988), and Ofer and Thakor (1987).

analyzed by several papers (e.g., Shleifer and Vishny (1986), Harris and Raviv (1988), Bhattacharya (1997), Yilmaz (1998), Maug (1999), Bebchuk and Hart (2001), and Gilson and Schwartz (2001)). In none of these papers, however, an activist investor who is not the bidder can launch a proxy fight to replace the incumbent directors of the target. Here, both the bidder and the activist can challenge the board. Our observation that activist investors use proxy fights more effectively than bidders to remove the opposition of incumbents to takeovers is a novel aspect of our analysis.

1 The commitment problem in hostile takeovers

Under existing rules, bidders cannot bring a merger proposal to a shareholder vote without first getting the approval of the target board. A tender offer made directly to shareholders does not require a vote, but it is vulnerable to poison pills. A poison pill is one of the most effective measures available to publicly traded corporations. Most poison pills give the board the option to issue a rights plan that entitle shareholders to dilute the value of the position of a bidder that acquires number of shares of the target firm above pre-specified threshold, typically 15-20%. The dilution makes it virtually impossible for a takeover to succeed. To overcome the resistance of the board, the bidder must first unseat the incumbent directors in a proxy fight and replace them with new directors who are friendly to the takeover. The newly elected directors will redeem the poison pill and permit target shareholders to sell their shares to the bidder.¹⁰

If this line of attack is adopted by the bidder, the proxy fight may be considered a referendum on the takeover offer. However, the proxy fight involves a change in the composition of board without a change of ownership. Shareholders do not vote directly on the terms of the takeover, but rather, on the new composition of the board of directors. Thus, winning a proxy fight does not necessarily compel the bidder to execute the takeover at the initial terms. The bidder can renege on these terms. In fact, since the bidder is the counterparty to the transaction, he will be tempted use the control of the target board to low-ball the takeover premium. For example, the bidder can exploit his privileged access as a board member to the target's proprietary information to claim that the new information he learned does not justify

¹⁰Provisions that make pills nonredeemable such as allowing only continuing directors to redeem the pill or delaying redemption for a specified time after a change in board composition are illegal in most states, including New York and Delaware. On the other hand, a staggered board, which is legal in most states, makes this line of an attack virtually impossible. See Bebchuk et al. (2002) for a discussion.

the initial premium. Moreover, with effective control but no economic ownership, the bidder can divert corporate resources and assets through self-dealing transactions. For example, the bidder may buy assets from the target for less than fair value, or sell assets to the target for more than fair value.¹¹ The fear from diluting actions would pressure target shareholders to accept a takeover deal at unfavorable terms. Therefore, without a commitment not to abuse the power of the board, target shareholders would not surrender the control over the board of directors to the bidder, and the proxy fight initiated by the bidder is doomed to fail.

Can the bidder commit not to abuse the power of the board after winning a proxy fight? Is it in the best interests of the bidder to make this commitment? The extent to which the power of the board can be abused depends on the effectiveness of investor protection laws as well as the enforceability and strength of the legal environment in the country or state where the target firm is incorporated.¹² Generally, the fear from shareholder litigation can hold board members accountable and mitigate the agency problems between firm insiders and its shareholders.¹³

Legal systems provide ways that may allow the bidder to commit. For example, in the US, the bidder can run a proxy fight and at the same time make a tender offer directly to target shareholders that remains pending until after the elections. This way, when the shareholders decide how to cast their vote at the director elections, they know that if the bidder wins they always have the option to tender the shares at the pre-determined offer. Can this “bundle” of proxy fight and a tender offer in fact solve the aforementioned commitment problem? As a general matter the bidder can amend the terms of the tender offer without restriction, at least when any of the conditions to the tender offer remain unsatisfied. Typically, tender offers do have conditions (e.g., ability to finance the offer). In particular, since the tender offer must be commenced prior to the proxy fight, it will have a condition that the offer is valid only if the poison pill is redeemed. Therefore, the newly elected directors can always choose not to redeem the pill, thereby paving the way for the bidder to revise the offer. In order to avoid this situation, the bidder has to make the tender offer without a condition that the position pill is redeemed. This, however, exposes the bidder to the risk that the position pill will be

¹¹See Atanasov et al. (2014) for a discussion on the various forms of tunneling. While the practice of tunneling is more common in developing markets, it also exists in developed markets. For example, see Atanasov et al. (2010), Bates et al. (2006), and Gordon et al. (2004) for some evidence on tunneling in the U.S.

¹²Bebchuk and Hart (2001) propose amending the existing rules governing mergers to allow acquirers to bring a merger proposal directly to a shareholder vote without the approval of the board of directors. Under these rules, the bidder can effectively commit to a certain acquisition price.

¹³Bidder who are active in the market for corporate control (for example, serial acquirers or private equity funds) may have incentives to develop reputation for not expropriating target shareholders. The repeated interaction can help developing a credible commitment.

triggered before he wins the proxy fight. Alternatively, the bidder may condition the tender offer on winning the proxy fight, irrespective of the poison pill redemption. In this case the bidder risks the possibility that the newly elected directors will be disloyal (e.g., see the bid of Air Products for Airgas). For all of these reasons, the ability to commitment may be limited.

Abstracting from the issues above, even if the bidder can commit not to change any of the terms in the tender offer, a commitment of this sort may not be in the bidder's best interests. By committing not to revise the tender offer, the bidder exposes himself to a free-rider problem (Grossman and Hart (1980)). If target shareholders are expected to free-ride, the bidder will have to pay the full post-takeover value of the target in order to convince the majority of them to tender their shares. This strategy will leave the bidder without any profit.

In the analysis below we consider two polar cases. In Section 3, we assume that the bidder is unable or does not have the incentives to commit not to abuse the power of the board upon winning a proxy fight. In Section 4, we assume that the bidder is committing to act in the best interests of target shareholders upon running a proxy fight, and such commitment is feasible. Moreover, in the latter case, the bidder can also overcome the free-rider problem. We argue that in both cases activist investors play a role in the market for corporate control that cannot be perfectly substituted by the bidder, and study the implications of this insight.

2 Setup

Consider an economy with a bidder, an activist investor, and $N \geq 2$ public firms. Initially, each firm is owned by a continuum of passive shareholders and run by its board of directors. We do not distinguish between the manager and other board members, and treat the board as a monolithic entity. We normalize the number of shares of each firm to one. Each share carries one vote. According to the governance rules of the firms, a successful takeover requires at least half of its voting rights. All agents are risk-neutral.

The standalone value of each firm is $q > 0$, where q is a common knowledge. The bidder can add value through acquisition to exactly one of the N firms. We refer to this firm as the *target firm*. The firms are ex-ante symmetric and each firm is equally likely to be the target. If the bidder acquires the target firm, the added value is $\Delta \in [0, \infty)$. The probability density function of Δ is given by f and its cumulative distribution function is given by F . Both are continuous and have full support. If the bidder is a strategic acquirer (e.g., a corporation in a related industry), Δ is the operational or financial synergy with the target. If the bidder is

a financial acquirer (e.g., a private equity firm), Δ is the operational improvement that arises from a going private transaction, or a synergy with one of the bidder's portfolio companies. Δ can also include the bidder's private benefits from acquiring the target. If the bidder acquires a firm which is not the target, the acquisition destroys value. We assume that the value destruction is sufficiently large to deter the bidder from approaching a company without first verifying it is the target.¹⁴

The board of each firm has private benefits of control, which are lost if the firm is acquired by the bidder or if shareholders elect a new board. These private benefits are observed but are non-contractible.¹⁵ Incentive pay cannot fully eliminate the misalignment that is created by these private benefits of control. This assumption is consistent with the evidence provided by Jenter and Lewellen (2015). We denote the incumbent board's private benefits per share (owned by the board) by $b > 0$. Thus, from the perspective of the incumbent board, the standalone value of the target is $q + b$ per share.

A common critique of activist investors is their short-term investment horizon. If the firm is acquired then each of its shareholders immediately receives the proceeds from the acquisition. However, if the firm remains independent, its standalone value is realized in the long-run. The activist is less patient than other shareholders, and discounts payoffs that are received in the long-run by $1 - \gamma \in [0, 1]$. If $\gamma > 0$ then the activist has preferences for early liquidation. Preferences for early liquidation can stem from the activist's unmodeled reputation, higher alternative cost of capital, or the need to meet out-flows from her fund. In this case, γ is the discount the activist incurs when exiting her position if the firm is not acquired.¹⁶

The game has two phases, which are described in details below.

1. Searching for a target and activist's position building:

At the outset, the bidder and the activist, who do not own shares in any firm, have to search in order to learn which firm is the target. Search is costly. We denote by c_B and c_A the search cost of the bidder and the activist, respectively. We assume that c_B and c_A are drawn from

¹⁴If firm i is not the target then its takeover creates a dis-synergy of $\underline{\Delta} < 0$. We assume $\underline{\Delta}$ is sufficiently small (or N is sufficiently large) to ensure that the bidder's expected profit, $q + \mathbb{E}[\Delta] \frac{1}{N} + \underline{\Delta} \frac{N-1}{N}$, is negative.

¹⁵The main results do not change qualitatively if we assume that b is unknown ex-ante (at the search stage), but is revealed once the first round of negotiations starts.

¹⁶The discount can be explained as follows. If the activist's stake is sufficiently large, selling the shares will exert negative pressure on the price. Indeed, the market may realize that the activist is desperate to unload her shares, and will take advantage of the activist's liquidity needs by offering a low share price. Alternatively, as a large shareholder, the activist may learn new private information about the value of the target as a standalone firm. The private information will create adverse selection when the activist attempts to unload her shares, which may result with a negative price impact.

continuous distributions G and H , respectively. Both distributions have full support on $[0, \infty)$, and c_A is independent of c_B . The bidder and the activist privately observe their cost before they decide whether to search. The search decisions are done simultaneously, and they are the bidder's and the activist's private information.¹⁷ By searching, the bidder learns the identity of the target, the value of Δ , and whether the activist is a shareholder of the target. If the bidder does not search, he remains uninformed. If the activist decides to search then she also learns the identity of the target, but she does not learn the value of Δ .

After deciding whether to search, the activist determines the number of shares she would buy in each firm.¹⁸ Short sells are not allowed and the activist does not have enough capital to acquire any firm by herself. This is consistent with Brav et al. (2008), who show that hedge fund activist seldom seek control themselves. The activist trades without knowing whether the bidder searched for a target and has intentions to make an offer. In Section 5.1.1, we discuss the implications of allowing the activist to trade after the negotiations between the bidder and the target become public. The activist trades with a risk-neutral and competitive market maker, who sets the prices equal to the expected value of the firm given the available information. Each firm has a separate market maker. The market maker of firm i privately observes the total order flows for the firm, denoted by $z_i \geq 0$. The order flows of firm i are either generated by the activist or by liquidity traders. The market maker cannot distinguish between the two. Liquidity trades are independent across firms. Specifically, with probability $\frac{1}{2}$ liquidity traders in firm i submit an order to buy $L > 0$ shares, and with probability $\frac{1}{2}$ they do not trade.¹⁹ We denote the share price of firm i by $p_i(z_i)$. After trading, the position of the activist in firm i , denoted by $\alpha_i \geq 0$, is observed by the market maker, the shareholders, and the board of firm i (e.g., by filing schedule 13D). The bidder observes the position of the activist in firm i if and only if he searched. In Section 5.3, we discuss the implications of allowing the activist to solicit bids by informing the bidder about her position.

2. Takeover negotiations and proxy fights:

By assumption, the bidder considers the acquisition of firm i only if he searched and identified it as the target. In all other cases, the firm remains independent and its standalone value

¹⁷The assumptions on the search technology are made for simplicity. The main results continue to hold if instead the search cost is $c(\lambda)$, where $c', c'' > 0$ and λ is the probability the target is identified.

¹⁸The structure above has an alternative interpretation: the activist knows which firm is the target even without searching, but she has limited capital. Therefore, she has to choose between buying shares of the target and exercising her outside option, which is given by c_A .

¹⁹We implicitly assume $L < 0.5$ and that purchasing L shares does not trigger a poison pill if such exists.

is realized. If the bidder identified firm i as the target, he starts negotiating an acquisition agreement with its incumbent board. Our assumption that bidder negotiates with the target board reflects the ability of the latter to veto any takeover attempt. Before the negotiations start, the value of Δ becomes public. The parties negotiate a cash offer for 100% of the shares of the target. There are two rounds of negotiations, indexed by $t \in \{1, 2\}$, which are separated by a proxy fight stage. In each round, the proposer is decided randomly. With probability $s \in (0, 1)$ the proposer is the target board and with probability $1 - s$ the proposer is the bidder. The proposer in each round makes a take-it-or-leave-it offer to the other party. Parameter s can be interpreted as the bargaining power of the target firm.²⁰ We denote the offer made by the proposer in round t by π_t . At the end of each round, if the bidder and the target board reach an agreement then the agreement is brought to a vote of the target shareholders and must receive approval by a majority of them. Apart from the activist investor and the incumbent board, target shareholders are negligible in size and believe that their individual decisions cannot change the outcome of the vote.²¹ We assume that at the voting stage shareholders play undominated strategies, and hence, approve the agreement if and only if the takeover offer is higher than the perceived standalone value of the target. If the agreement is approved by shareholders, each shareholder gets π_t for each share he owns, and the bidder gets $q + \Delta - \pi_t$.

If the bidder and the incumbent board do not reach an agreement in the first round of negotiations, or if shareholders vote down a proposed agreement, a proxy fight to replace the incumbent board can be initiated. A proxy fight can be initiated either by the bidder or by the activist investor. At the proxy fight stage, the bidder and the activist simultaneously decide whether to challenge the incumbents by proposing an alternative slate of directors for election to the target board. The bidder and the activist incur a private fixed cost $\kappa > 0$ if they decide to launch a proxy fight. This cost is not reimbursed by the firm, and captures administrative costs as well as the effort that is exerted by the challenger when campaigning against the incumbent. Once the proxy fight is initiated, shareholders decide whether to vote for the incumbent board or for one of the rival teams. Similar to the vote on the acquisition agreement, shareholders play undominated strategies when they elect directors. The team that receives the largest number of votes, is elected and takes control of the target board.

If the incumbent board wins the proxy fight, it retains control. However, if the incumbent

²⁰The assumptions on the bargaining protocol in each round of negotiations is made for simplicity and can be microfounded using Rubinstein's (1982) model of alternating offers.

²¹The incumbent board and the activist may hold non-trivial number of shares, but we assume it is not large enough to sway the outcome of the vote. We discuss a related extension in Section 5.2.

board loses the proxy fight, it is replaced by the winning team. In this case, the incumbent board loses its private benefits of control and suffers an additional disutility which captures either embarrassment or the lose of directors' reputation. Winning control of the target board has two implications for the rival team. First, it gives the rival the right to negotiate an acquisition agreement with the bidder, and hence, to block the takeover. In particular, if the bidder wins the proxy fight, he would sit on both sides of the negotiating table. Second, the rival takes control the operations of the target, and among other things, it can divert corporate resources as private benefits. We assume that the amount that can be diverted is limited and arbitrarily small.²² We distinguish between two cases. In the first case, which is studied in Section 3, the winning team cannot or has no incentives to commit not to abuse power once elected. In the second case, which is studied in Section 4, the bidder can commit to act in the best interests of the target shareholders upon winning the proxy fight. We defer the exact details of these two assumptions to Section 3 and Section 4, respectively. All other aspects of the model remain identical.

Finally, once the proxy fight stage ends, a second round of negotiations between the bidder and the target board (which may now be populated with the newly elected directors) takes place. The second round of negotiations follows the same protocol as the first round. If no agreement that is approved by target shareholders is reached by the end of the second round, the target remains independent and its standalone value is realized.

3 No commitment

In this section we assume that the bidder and the activist cannot or do not have the incentives to commit to act in the best interests of target shareholders upon winning the proxy fight. Under this assumption, the newly elected directors will maximize the value of the party with which they are affiliated, even if it conflicts with maximizing target shareholder value.

We consider the set of Perfect Bayesian Equilibria in symmetric and pure strategies. We solve the game backward. We start with the negotiations and proxy fights phase, followed by the search and trade phase.

²²The only role that this assumption plays is that if shareholders are indifferent between electing the rival (the bidder or the activist) and retaining the incumbent, they will choose the latter. In an unreported analysis, we show that the results continue to hold if the amount that is being extracted is strictly positive.

3.1 Takeover negotiations and proxy fights

The first result characterizes the outcome of the second round of negotiations.

Lemma 1 *In the second round of negotiations, the target is acquired by the bidder unless the incumbent board retains control and $\Delta < b$. The shareholder expected value is given by*

$$\Pi_{SH}(\Delta) = \begin{cases} q + \mathbf{1}_{\{b \leq \Delta\}} \cdot [s\Delta + (1-s)b] & \text{if the incumbent board retains control,} \\ q + s\Delta & \text{if the activist controls the board,} \\ q & \text{if the bidder controls the board.} \end{cases} \quad (1)$$

Several observations that follow from Lemma 1. First, if the incumbent board is reelected then he would agree to sell the target only if the takeover offer is sufficiently high to compensate him for the loss of the private benefits of control. The bidder can afford to pay a takeover premium of b only if $b \leq \Delta$. In this case, the entrenchment of the incumbent benefits target shareholders (at least ex-post) since it forces the bidder to offer a higher takeover premium than he would have offered otherwise, without endangering the deal. However, if $\Delta < b$ then the bidder would rather walk away from the negotiations. In this case, the entrenchment of the incumbent board results with an inefficient outcome: a value-increasing takeover is rejected.

Second, because of her preferences for early liquidation, the activist is effectively biased toward selling the target. If the activist is elected to the board, she cannot credibly reject offers higher than $q - \gamma q$, even if they are lower than the standalone value of the target. However, since any acquisition agreement must also be approved by the shareholders in a vote, the bidder must offer at least q in order to acquire the target. Moreover, the activist has incentives to maximize the value of her holdings. Therefore, if the activist is the proposer, she would ask for $q + \Delta$, the highest price the bidder would agree pay for the target. Overall, in spite of her bias toward selling the target, if the activist is elected to the board, she would negotiate a “fair” deal in which the bidder pays an expected takeover premium of $s\Delta$.

Third, the bidder has incentives to acquire the target for the lowest price possible. If the bidder wins the proxy fight then he would sit on both sides of the negotiating table, supposedly negotiating on behalf of target shareholders. Without commitment, the bidder will take advantage of his control of the target board to negotiate a deal that offers target shareholders the lowest amount they would accept, which is q .

Taking into account the effect of director elections on the second round of negotiations, the next result characterizes the proxy fight stage.

Lemma 2 *Suppose the first round of negotiations fails. Then:*

(i) *The bidder never runs a proxy fight.*

(ii) *If the activist owns α shares in the target, the activist runs a proxy fight if and only if*

$$\rho(\alpha) \leq \Delta < b, \quad (2)$$

where

$$\rho(\alpha) \equiv \max \left\{ 0, \frac{\kappa/\alpha - \gamma q}{s} \right\}. \quad (3)$$

Whenever the activist runs a proxy fight, she wins.

According to part (i) of Lemma 2, the bidder never runs a proxy fight to replace the target board in any equilibrium of the subgame. This result holds regardless of the gains from the takeover Δ , the cost of running a proxy fight κ , whether or not the activist is also running a proxy fight, and the size of incumbent board's private benefits of control b . The reason is the following. As Lemma 1 suggests, without the ability to commit not to abuse the power of the board, target shareholders are always worse off if they elect the bidder. Indeed, once elected, the bidder will be tempted to divert corporate resources and offer shareholders the lowest price possible. Since shareholders have rational expectations, they would never follow the bidder's request to replace the incumbent board. Since running a proxy fight is both costly and inefficient, the bidder will never run a proxy fight.

To understand part (ii) of Lemma 2, note that shareholders elect the activist if and only if the incumbent board is severely entrenched. According to Lemma 1, if $b \leq \Delta$ and the incumbent board is reelected, he will negotiate a takeover premium of $s\Delta + (1-s)b$. By contrast, if the activist is elected to the board, she will negotiate a smaller premium of $s\Delta$. Therefore, shareholders will reelect the incumbent when $b \leq \Delta$. Anticipating her defeat, the activist will never run a proxy fight. However, if $\Delta < b$ then the incumbent board will block the takeover. If the activist decides not to run a proxy fight, the target would remain independent and the value of the activist's holdings would fall to $\alpha(q - \gamma q)$. However, if the activist is given control of the board, she will negotiate a takeover premium of $s\Delta$. Therefore, shareholders would vote for the activist if she decides to challenge the incumbent. The activist will run a proxy fight if the resulted increase in the value of her stake, $\alpha(s\Delta + \gamma q)$, is higher than the cost of running a proxy fight, κ . This condition holds if and only if $\rho(\alpha) \leq \Delta$. The interval

in (2) is the intersection of the activist’s incentives to run a proxy fight and the incentives of shareholders to support the activist in her challenge.

Part (ii) of Lemma 2 offers an interesting and intuitive comparative statics with of the frequency that the activist runs a proxy fight conditional on the failure of the first round of negotiations. This frequency increases with the target’s bargaining power s , the size of the activist stake in the target firm α , the activist’s preferences of early liquidation γ , and the private benefits of the incumbent board, b . This frequency decreases with the cost of running a proxy fight, κ .

The contrast between parts (i) and (ii) of Lemma 2 emphasizes that even though the bidder and the activist have the same cost of running a proxy fight and the same tendency to divert corporate resources once elected, without a commitment to act in their best interests, the bidder cannot get the needed support from target shareholders to remove the opposition of the incumbent board to the takeover, while the activist can. The lack of trust of target shareholders in the bidder’s motives implies the resistance of incumbents to takeovers can be overcome only if the capacity to disentrench the board is separated from the capacity to take over the firm.

A case in point is the unsolicited bid of Valeant to Allergan in 2014. Valeant teamed up with the hedge fund activist Pershing Square, with the intention that Pershing Square will build a significant toehold in Allergan, and then push for its sale to Valeant. The sophisticated maneuver failed.²³ Our analysis suggests that by teaming up with Valeant, Pershing Square lost its unique ability to relax the opposition of Allergan board to the takeover, since shareholders of Allergan can no longer trust Pershing Square to act in their best interest once elected to the board. Shareholders of Allergan should have been concerned that Pershing Square was advancing the goals of Valeant on their expense. Without the trust of the shareholders of Allergan, Pershing Square was as ineffective as Valeant in relaxing the opposition of Allergan board to the proposed takeover.

The next result characterizes how the threat of running and winning a proxy fight affects the first round of negotiations.

Proposition 1 *Suppose the bidder identifies firm i as a target with a synergy level Δ . Then:*

- (i) *If $b \leq \Delta$ the bidder pays $q + s\Delta + (1 - s)b$ and takes over the target after the first round of negotiations.*

²³Allergan was eventually acquired by Actavis, however, from the perspective of Valeant, the takeover attempt failed. See “The Flaws in Valeant’s Activist Deal Effort” New York Times, November 18, 2014.

(ii) If $\rho(\alpha) \leq \Delta < b$ and the activist owns α shares in the target, the bidder pays $q + s\Delta$ and takes over the target after the first round of negotiations.

(iii) In all other cases the target remains independent under the incumbent board's control.

Once the bidder identifies firm i as the target, there are three cases to consider. First, if $b \leq \Delta$ then whether or not the activist is a shareholder of the target, the incumbent board reaches an agreement in which the bidder pays $q + s\Delta + (1 - s)b$ per share and takes over the target. Second, if $\rho(\alpha) \leq \Delta < b$ then the activist's presence (or lack thereof) affects the first round of negotiations. If the activist is a shareholder of the target, both the bidder and the incumbent board understand that if they fail to reach an agreement, the activist will launch a proxy fight to replace the incumbent, win the support of shareholders, and then use the board to negotiate an agreement in which the bidder pays on average $q + s\Delta$ per share. Therefore, any first round offer below $q + s\Delta$ will be rejected by shareholders, and any offer above $q + s\Delta$ will be rejected by the bidder. The incumbent board understands that the takeover is inevitable, and he will accept the offer $q + s\Delta$ in order to avoid the adverse consequences of losing the proxy fight to the activist. Third, in all other cases, the incumbent board is entrenched but the threat of a proxy fight is not credible. Therefore, the incumbent retains control of the board, maintain his resistance, and successfully block the takeover.

Finally, if the bidder did not identify firm i as a target, by assumption, he does not approach the firm with a takeover offer. Without the possibility of receiving a takeover offer, shareholders are concerned that the activist will abuse her power and divert corporate resources, and hence, they never elect the activist to the board of directors. Therefore, the firm remains independent, the incumbent board retains control, and the standalone value of the target remains q .

3.2 Searching for a target and activist's position building

In this section we characterize the stock price and the activist's decision to buy shares in firm i . We assume that if the activist is indifferent between buying and not buying shares of firm i , the activist does not buy these shares.²⁴

Lemma 3 *Consider an equilibrium in which the bidder and the activist search for a target with probability $\lambda_B \in [0, 1]$ and $\lambda_A \in [0, 1]$, respectively. The activist buys shares of firm i if*

²⁴This assumption is unnecessary as long as $\gamma > 0$. If $\gamma = 0$ and this assumption is relaxed, the equilibrium does not survive the introduction of arbitrarily small transaction costs.

and only if she searches and identifies it as a target, in which case, the activist buys L shares. The share price of firm i in equilibrium is given by,²⁵

$$p_i(z_i; \lambda_A, \lambda_B) = q + \lambda_B \times \begin{cases} \frac{1-\lambda_A}{N-\lambda_A} v(0) & \text{if } z_i = 0 \\ \frac{\lambda_A}{N} v(L) + \frac{1-\lambda_A}{N} v(0) & \text{if } z_i = L \\ v(L) & \text{if } z_i = 2L \end{cases} \quad (4)$$

where for any $\alpha \in [0, 1]$,

$$v(\alpha) = \int_b^\infty [s\Delta + (1-s)b] dF(\Delta) + \int_{\min\{b, \rho(\alpha)\}}^b s\Delta dF(\Delta). \quad (5)$$

The informational advantage of the activist stems from knowing which firm is likely to receive a takeover offer. Therefore, in equilibrium, the activist does not buy shares of any firm unless it first identified it as the target. If the activist searched and identified firm i as the target, then she would buy exactly L shares in order to disguise her trade as a liquidity and uninformed demand.

Lemma 3 also provides share price of firm i in equilibrium. Generally, the value of the firm is its standalone value q plus the expected takeover premium if the bidder identifies the firm as a target. If $z_i = 2L$ then the market maker of firm i knows for sure that the activist purchased L shares of the firm. Since the activist buys shares of firm i only if she identified the firm as a potential takeover target, the market maker infers that firm i is the target, and ascribes probability λ_B that firm will receive a takeover offer. The function $v(\alpha)$ is the expected takeover premium paid by the bidder if the activist owns α shares of the target and conditional on the bidder making an offer. Indeed, based on Proposition 1, if $b \leq \Delta$ the bidder pays $q + s\Delta + (1-s)b$, if $\rho(L) < \Delta < b$ the bidder pays $s\Delta$, and in all other cases the firm remains independent. This explains the term behind $p_i(2L; \lambda_A, \lambda_B)$. By contrast, if $z_i = L$ then the market maker cannot distinguish between events in which firm i is a target and the activist bought L shares, and events in which firm i is not the target and the demand comes from liquidity traders. The term behind $p_i(L; \lambda_A, \lambda_B)$ has the following intuitive interpretation: unconditionally, the market maker believes that firm i is a target with probability $\frac{1}{N}$. Conditional on the firm being a target, he bidder propose a takeover with

²⁵We implicitly assume that if $\lambda_A = 0$ then the market maker's off-equilibrium beliefs when $z_i \neq L$ are that the activist bought L shares, and the activist identified the firm as a target.

probability λ_B . In addition, the market is uncertain whether the activist bought a stake in the firm. With probability λ_A the activist owns L shares of the target, and hence, the fair value of the firm will be $v(L)$. With probability $1 - \lambda_A$ the activist is not a shareholder, and hence, the fair value is $v(0)$. Notice that $p_i(L)$ is also the ex-ante value of the firm. Finally, if $z_i = 0$ then the market maker knows the activist did not buy shares in the firm either because she found out that firm i is not the target, which happens with probability $\lambda_A \frac{N-1}{N}$, or because she did not search, which happens with probability $1 - \lambda_A$. The term behind $p_i(0; \lambda_A, \lambda_B)$ is the weighted average of these two events.

Overall, notice that if $z_i > 0$ the share price increases in λ_B and λ_A . A higher probability that the bidder searches for a target increases the likelihood of a transaction with firm i , and hence, the value of holding shares of firm i . Similarly, a higher probability that the activist searches for a target increases the value of the share of firm i , since if a bidder arrives but the incumbent board of firm i refuses to relinquish control, the presence of the activist can relax the tension and facilitate the transaction. If $z_i = 0$ the share price increases with λ_B but it decreases in λ_A , since $z_i = 0$ is a stronger signal that firm i is not a target when the activist's search intensity is higher.

3.2.1 Activist's decision to search

Consider the activist's decision to search. According to Lemma 3, if the activist does not search, she does not trade and her expected profit is zero. If the activist searches, she will buy L shares of the target firm, provided the expected price is sufficiently low. In Appendix A, we show that if the activist searches, her expected profit net of the search cost and price of buying L shares is given by,

$$\Pi_A(c_A, \lambda_A, \lambda_B) = -c_A + L \times \max \left\{ 0, \frac{1}{2} \lambda_B [v(L) - \frac{\lambda_A}{N} v(L) - \frac{1-\lambda_A}{N} v(0)] - \gamma q [1 - \lambda_B \int_{\min\{b, \rho(L)\}}^{\infty} dF(\Delta)] \right\} \quad (6)$$

The intuition behind (6) is the following. The first line is the activist's profit from trade per share. The activist makes a profit only if she has informational advantage over the market maker and she can camouflage her trade as driven by liquidity demand. The latter occurs with probability $\frac{1}{2}$. The informational advantage of the activist stems from her knowledge which of the N firm is the target and from her knowledge that she is a shareholder of the target. The latter matters because the activist can pressure the incumbent board to accept a future takeover bid. However, both pieces of information are valuable if and only if the bidder

makes a takeover offer to target firm, which happens with probability λ_B . From the activist's perspective, the value of the target is $q + \lambda_B v(L)$. If the activist succeeds in camouflaging her trade, then the share price is $p_i(L; \lambda_A, \lambda_B)$, which is equal to $q + \lambda_B[\frac{\lambda_A}{N}v(L) + \frac{1-\lambda_A}{N}v(0)]$. Since $N \geq 2$ and $v(L) \geq v(0)$, the activist's profit is strictly positive. The second line in (6) is the expected disutility the activist suffers if the target remains independent. The target remains independent if either the bidder did not search, which happens with probability $1 - \lambda_B$, or the bidder identifies firm i as a target but Δ is low relative to the incumbent's private benefits and the incentives of the activist to intervene are weak. If γq is sufficiently large, the activist's expected profit from buying a stake of size L in the target firm can be negative. In this case, the activist will not buy shares in any firm. This explain why the term in the curly brackets in (6) is bounded from below by zero.

In equilibrium, the activist searches for a target if and only if $\Pi_A(c_A, \lambda_A, \lambda_B) \geq 0$. Note that $\Pi_A(c_A, \lambda_A, \lambda_B)$ decreases in c_A . The next lemma follows directly from these observations.

Lemma 4 *In any equilibrium there is a unique threshold $c_A^* \geq 0$ such that the activist searches for a target if and only if $c_A \in [0, c_A^*]$. Moreover:*

- (i) c_A^* satisfies either $\Pi_A(c_A^*, \lambda_A, \lambda_B) = 0$, or $c_A^* = 0$ and $\Pi_A(0, \lambda_A, \lambda_B) < 0$.
- (ii) c_A^* increases in λ_B .

The threshold c_A^* is determined by the indifference of the activist between searching and not searching. If $c_A^* = 0$ and $\Pi_A(0, \lambda_A, \lambda_B) < 0$ then the activist strictly prefers not searching even if search is costless. This case occurs only if γ is sufficiently large. Either way, Lemma 4 implies that in any equilibrium, $\lambda_A = H(c_A^*)$.

Interestingly, $\Pi_A(c_A, \lambda_A, \lambda_B)$ increases in λ_B and decreases in λ_A . Higher market expectations that the activist will buy shares of the target increase the share price, and therefore, reduces the profit of the activist. On the contrary, higher market expectations that the bidder searches for a target increase the profits of the activist. Here, there are two effects. Similar to the effect of λ_A on Π_A , higher λ_B increases the share price, and therefore, negatively affects Π_A . On the other hand, higher λ_B also increases the value of the activist's private information. Since the activist knows which firm is the target but the market maker can only guess, the positive effect of λ_B always dominates. Overall, the activist's benefit from searching increases with the likelihood that the bidder searches. Formally, since $\Pi_A(c_A, \lambda_A, \lambda_B)$ decreases with c_A and increases with λ_B , the threshold c_A^* increases in λ_B .

3.2.2 Bidder's decision to search

Consider the bidder's decision to search. If the bidder does not incur the search cost, he cannot identify which of the N firms is a viable target. Since the expected synergy from the acquisition of an unidentified company is negative, the bidder does not approach any of the N firms. In this case, the bidder's expected profit is zero.

Suppose the bidder identifies firm i as the target, and the activist owns α shares in that firm. Based on Proposition 1, the expected value that is created by the takeover is given by

$$w(\alpha) = \int_{\min\{b, \rho(\alpha)\}}^{\infty} \Delta dF(\Delta). \quad (7)$$

Moreover, conditional on identifying the target firm, the expected takeover premium paid by the bidder is $v(\alpha)$. Therefore, the bidder's expected payoff net of the search cost is

$$\Pi_B(c_B, \lambda_A) = -c_B + \psi(\lambda_A), \quad (8)$$

where

$$\begin{aligned} \psi(\lambda_A) &= \lambda_A (w(L) - v(L)) + (1 - \lambda_A) (w(0) - v(0)) \\ &= (1 - s) \left[\int_b^{\infty} (\Delta - b) dF(\Delta) + \lambda_A \int_{\min\{b, \rho(L)\}}^b \Delta dF(\Delta) \right]. \end{aligned} \quad (9)$$

Intuitively, the profit of the bidder is the surplus generated by the takeover, $w(\alpha)$, less than the expected takeover premium which is given by $v(\alpha)$.

The bidder will exert effort and search for a target if and only if $\Pi_B(c_B, \lambda_A) \geq 0$. Note that $\Pi_B(c_B, \lambda_A)$ decreases in c_B . The next lemma follows directly from this observation.

Lemma 5 *In any equilibrium there is a unique threshold $c_B^* \geq 0$ such that the bidder searches for a target if and only if $c_B \in [0, c_B^*]$. Moreover, $c_B^* = \psi(\lambda_A)$, which is increasing in λ_A .*

The threshold c_B^* is determined by the indifference of the bidder between searching and not searching. Lemma 5 implies that in any equilibrium, $\lambda_B = G(c_B^*)$. Interestingly, $\Pi_B(c_B, \lambda_A)$ increases in λ_A , with a strict monotonicity if and only if $b > \rho(L)$. Intuitively, the higher is the likelihood that the activist is a shareholder of the target firm, the more likely it is the activist will pressure the incumbent board to relinquish control and sell the firm, and hence, the higher is the likelihood that the bidder will consume the synergies from the acquisition.

However, this argument works only if the threat of a proxy fight is credible at times when the resistance of the incumbent board is significant, that is $b > \rho(L)$. Formally, since $\Pi_B(c_B, \lambda_A)$ decreases with c_B and increases with λ_A when $b > \rho(L)$, the threshold c_B^* increases in λ_A .

3.3 Equilibrium

The next result shows that an equilibrium always exists, and characterizes the equilibrium search intensities.

Proposition 2 *An equilibrium always exists. In any equilibrium, $c_B^* = \psi(H(c_A^*)) > 0$ and $c_A^* \geq 0$. Let*

$$\mu(x) \equiv \Pi_A(x, H(x), G(\psi(H(x)))) . \quad (10)$$

If $c_A^ > 0$ then c_A^* is given by the solution of $\mu(x) = 0$, and if $c_A^* = 0$ then $\mu(0) \leq 0$. Moreover, there is $\bar{\gamma} > 0$ such that if $\gamma \in [0, \bar{\gamma}]$ then $c_A^* > 0$ in any equilibrium.*

According to Proposition 2 and expression (9), if $b \leq \rho(L)$ the activist does not affect the takeover process or the incentives of the bidder to search for a target. In this region, the activist's threat to run a proxy fight is not credible enough to relax the resistance of the incumbent board to the takeover. Nevertheless, the activist has incentives to become a shareholder of the target: knowing which firm is likely to be a target gives the activist informational advantage that makes the purchase of the shares of the target a profitable investment. This information is valuable only if the bidder is likely make a takeover offer, and therefore, the gains from the speculative trade and the incentives of the activist to search increase with the bidder's search intensity. In other words, higher M&A activity increases the incentives of activist investors to speculate. In this region, the equilibrium is always unique. We name this region as the "selection region", since the activist invests in firms that are likely to be targets, but her investment has no real effect.

If $b > \rho(L)$ and the activist is a shareholder of the target, she can relax the resistance of the incumbent board to the takeover. Therefore, the bidder has stronger incentives to search for a target if the activist is expected to be a shareholder. This observation implies that the activist affects the takeover process even if ex-post her threat to run a proxy fight is not credible (e.g., $\Delta < \rho(L)$). In turn, and similar to the reasoning in the selection region, the incentives of the activist to search increase with the bidder's search intensity. However, since here the activist affects the takeover process, her informational advantage is more significant,

and consequently, the speculative gains are higher. Therefore, the activist’s incentives to search are more sensitive to the bidder’s effort. Overall, higher intensity of shareholder activism (for example, as reflected by the number of schedule 13D filings) will increase the incentives of bidders to search for companies and approach them with takeover offers, and vice versa. We name this region as the “treatment region”, since the activist invests in firms that are likely to be targets, and by investing in these firms, not only she facilitates the takeover process once the offer is on the table, but she also provides bidders with stronger incentives to make the offer in the first place.

It follows, the game exhibits *strategic complementarity* in the treatment region.²⁶ Strategic complementarity can lead to multiple equilibria. The existence of multiple equilibria suggests that due to effective shareholder activism, the market for corporate assets can experience episodes of high volume of transactions (“hot markets”) and episodes of low volume of transactions (“cold markets”), without any apparent change in to the underlying fundamentals. That is, the activity in the market for corporate control is self-fulfilling and unpredictable.

The multiplicity of equilibria generated by strategic complementarity can be best seen when $N \rightarrow \infty$ and $\gamma \in [0, \bar{\gamma}]$. According to Proposition 2, if $c_A^* > 0$ then it is given by a solution of $\mu(x) = 0$. Since $\mu(0) \geq 0$ and $\lim_{x \rightarrow \infty} \mu(x) = -\infty$, multiple equilibria exist if $\mu(x) = 0$ has a solution $c_A^* \geq 0$ such that $\frac{\partial \mu(x)}{\partial x}|_{x=c_A^*} > 0$. In Appendix B, we show that $\frac{\partial \mu(x)}{\partial x}$ is increasing with $\frac{\partial \Pi_B}{\partial \lambda_A} \frac{\partial \Pi_A}{\partial \lambda_B}$, where

$$\frac{\partial \Pi_B}{\partial \lambda_A} = (1 - s) \int_{\rho(L)}^b \Delta dF(\Delta) \quad (11)$$

and

$$\frac{\partial \Pi_A}{\partial \lambda_B} = L \left(\frac{1}{2} v(L) + \gamma q \int_{\rho(L)}^{\infty} dF(\Delta) \right) \quad (12)$$

are the slopes of the bidder’s and the activist’s best response functions, respectively. Loosely speaking, multiple equilibria is more likely when $\frac{\partial \Pi_B}{\partial \lambda_A} \frac{\partial \Pi_A}{\partial \lambda_B}$ is large. The term $\frac{\partial \Pi_B}{\partial \lambda_A} \frac{\partial \Pi_A}{\partial \lambda_B}$ measures the strength of complementarity between the bidder and the activist search decisions. Simple algebra shows that $\frac{\partial \Pi_B}{\partial \lambda_A} \frac{\partial \Pi_A}{\partial \lambda_B}$ increases in L (market liquidity) and γ (activist’s short-termism), and decreases in κ (the cost of running a proxy fight). Intuitively, these changes in the parameters expand the region in which the activist’s threat to run a proxy fight is credible, $[\rho(L), b]$.

²⁶Strategic complementarity arises when the best response functions are increasing. In our setup, the best response function of the activist is the mapping from c_B^* to c_A^* , assuming that the market price changes with c_B^* , as prescribed by Lemma 3 (note that by accounting for the endogenous market reaction to changes in c_B^* , the sensitivity of the activist to the buyer is attenuated). Similarly, the best response function of the buyer is the mapping from c_A^* to c_B^* .

Moreover, if L increases the activist can better camouflage her trade and increase her profit. If γ increases, the activist's benefit from the bidder's arrival is larger since she can exit her position earlier and without a penalty. For all of these reasons, the sensitivity of one player's search intensity to the other, increases.

The effect of b on the strength of complementarity is more nuanced. On the one hand, higher b expands the treatment region. On the other hand, higher b changes the division of surplus between the bidder and target shareholders. This may have an ambiguous effect on the incentives to search, as we discuss below. However, as $b \rightarrow \infty$, the latter effect is dominated by the former. Intuitively, the incumbent board is completely entrenched, and the target can be acquired only if the activist is a shareholder. In this case, the bidder's incentives to search crucially depend on the activist being a shareholder of the target firm. In Appendix B, we provide sufficient condition under which multiple equilibria exist.

3.4 Comparative statics

In this section we study the key comparative statics of the model. While the equilibrium may not be unique, all equilibria of the game can be ranked by the ex-ante probability that the target firm is acquired by the bidder,

$$\theta^* \equiv G(c_B^*) \left[\int_b^\infty dF(\Delta) + H(c_A^*) \int_{\min\{b, \rho(L)\}}^b dF(\Delta) \right]. \quad (13)$$

Indeed, c_B^* is an increasing function of c_A^* , and θ^* increases in both c_B^* and c_A^* . When multiple equilibria exist, we study the comparative statics of equilibria with the smallest and largest θ^* . We denote these two equilibria by $\underline{\theta}^*$ and $\bar{\theta}^*$, respectively. Focusing on extremal equilibria is common in games of strategic complementarities (e.g., Vives (2005)).²⁷ To ease the exposition, we denote $\rho(L)$ as ρ .

Proposition 3 *Suppose $\gamma \in [0, \bar{\gamma}]$ and θ^* is either $\underline{\theta}^*$ or $\bar{\theta}^*$. Then:*

- (i) *If $b \leq \rho$ then θ^* does not change with L , κ or γ , and it decreases in b and s .*

²⁷Two technical comments are in place. First, we assume that the least and the greatest fixed points always exist. For example, if the function $\hat{\mu}(x) \equiv \mu(x) + x$ is monotonic, then by Tarski's fixed point theorem, it has the least and the greatest fixed points. It is easy to see that for N sufficiently large, $\hat{\mu}(x)$ is monotonic. Second, we focus on local comparative statics, when the equilibrium continues to exist upon a small change in the parameter.

(ii) If $b > \rho$ then θ^* increases in L , decreases in κ , and is ambiguous with respect to γ , b , and s .

In the selection region, where $b \leq \rho$, the activist's threat to run a proxy fight is never credible and the bidder's incentives to search are unaffected by the activist's presence. Therefore, parameters which only affect the incentives of the activist to intervene, do not affect the probability of a takeover. By contrast, the probability of a takeover is decreasing with the private benefits of the incumbent board and the bargaining power of the target. Intuitively, with higher b or s , the bidder has to pay a higher price for the target. Lower profitability decreases the incentives of the bidder to search, and thereby, the probability of a takeover.

In the treatment region, where $b > \rho$, the activist's threat to run a proxy fight is credible. According to (3), if $\rho > 0$ then the credibility of this threat increases with L and decreases with κ . This has two effects. First, it increases the incentives of the bidder to search since an acquisition agreement with the incumbent board is more likely to be reached. Second, the activist's private information of her being a shareholder of the target has a higher value. Moreover, higher L also increases the activist's profits from speculative trade. Therefore, both the activist and the bidder have stronger incentives to search, and the probability of the takeover increases. The effect of γ in the treatment region is more nuanced. Conditional on the bidder identifying the firm as a target, the takeover probability increases in γ . Higher γ increases the credibility of the activist's threat to run a proxy fight (ρ decreases with γ) since she has more to lose if the takeover fails. For the same reasons as above, the activist and the bidder have stronger incentives to search and the probability of a takeover increases. On the other hand, in the event that the target remains independent, the activist suffers a larger disutility when γ is high, and hence, she has weaker incentives to search. This effect reduces the probability of a takeover. In Appendix B, we show that the former effect can dominate the latter, and hence, short-term activists can in fact facilitate value-increasing takeovers by pressuring incumbents to remove their opposition.

Interestingly, the probability of a takeover can increase with b and s in the treatment region. Recall that higher b and s increase the takeover premium paid by the bidder. While this has a negative direct effect on the bidder as was discussed in the selection region, in the treatment region, it also has a positive direct effect on the incentives of the activist to search. Indeed, the activist expects to get a higher premium when the bidder arrives. Moreover, higher b and s increase the credibility of the activist's threat to run a proxy fight (the interval $[\rho, b]$ expands), and as was explained above, this effect increases the activist's incentives to search and buy

shares of the target. Therefore, while higher b and s might reduce c_B^* , it can also increase c_A^* , and thereby increase the probability of a takeover. Thus, contrary to the common wisdom, the probability of a takeover can increase with the resistance of the board to the takeover, as such resistance creates more investment opportunities for the activist.

Due to strategic complementarities in the treatment region, a small change in one of the parameters of the model can have an amplified effect on θ^* . For example, consider a change in regulation that eases the proxy access and decreases κ . There are several effects. First, easier proxy access increases the credibility of the activist's threat to run a proxy fight, and thereby, her ability to relax the incumbent's resistance to the takeover. Second, since the activist's threat is more credible, the bidder has stronger incentives to search since reaching an agreement with the incumbent is more likely. Third, easier proxy access will have an effect on the activist since her informational advantage is more valuable. All of these effects, which are direct consequences of a decrease in κ , increase the activist's and the bidder's search intensity. However, an increase in the activist's and the bidder's search effort feeds back due to strategic complementarities. The activist benefits from the bidder's increased search intensity, and vice versa. Therefore, small changes in the cost of running a proxy fight might have a significant effect on shareholder activism and the volume of M&A transactions. More generally, even small changes in the legal framework of M&A or the regulatory environment (e.g., modifying the rules that govern the filing of schedule 13D) can have a large effect on the market for corporate control.

3.4.1 Analysis of abnormal returns

The model also provides an opportunity to study the abnormal returns around 13D filing of activist investors and announcements of acquisition agreements. We assume that after trade takes place but before the arrival of the bidder becomes public, the activist must file a 13D schedule if and only if she is a shareholder of firm i . According to Lemma 3, the activist buys shares of firm i if and only if she searches and identifies firm i as the target. Therefore, the probability of a 13D filing is λ_A^*/N . If the activist files schedule 13D then the firm's share price jumps to $p_i(2L; \lambda_A^*, \lambda_B^*)$, and otherwise the price drops to $p_i(0; \lambda_A^*, \lambda_B^*)$.²⁸ Therefore, using Lemma 3, we can calculate the average abnormal return around 13D filing. Moreover, using Proposition 1, we can calculate the average abnormal returns around the announcements of

²⁸Note that we invoked the assumption that the market maker of firm i does not observe 13D filings in other firms. As we discuss in the proof of Proposition 4, this assumption is not material.

acquisition agreements conditional on 13D filing or lack thereof.

Proposition 4 *The equilibrium average abnormal returns around the announcement of event χ is:*

$$AR(\chi) = \begin{cases} \frac{\lambda_B^*}{2} \left[v(L) - \frac{\lambda_A^*}{N} v(L) - \frac{1-\lambda_A^*}{N} v(0) \right] & \text{if } \chi \text{ is a 13D filing by an activist, } \chi \equiv 13D \\ \left[\frac{1}{1-F(\min\{b,\rho\})} - \lambda_B^* \right] v(L) & \text{if } \chi \text{ is an acquisition agreement preceded} \\ & \text{by a 13D filing, } \chi \equiv takeover|13D \\ \left[\frac{1}{1-F(b)} - \lambda_B^* \frac{1-\lambda_A^*}{N-\lambda_A^*} \right] v(0) & \text{if } \chi \text{ is an acquisition agreement not preceded} \\ & \text{by a 13D filing, } \chi \equiv takeover|\emptyset. \end{cases} \quad (14)$$

Several observations are in place. First, based on Proposition 4, all events convey positive news on firm value. Second, consider the average abnormal returns around the announcement of 13D filing, $AR(13D)$. This term corresponds to the profit made by the activist from speculative trade. In the selection region, $AR(13D)$ is unaffected by L, κ , and γ . Intuitively, in this region the activist does not affect the takeover process. Therefore, parameters that govern the incentives of the activist to intervene have no effect. By contrast, these parameters affect $AR(13D)$ in the treatment region. Perhaps surprisingly, the abnormal returns around 13D filing can decrease with L and γ , and increase with κ . These changes increase the credibility of the activist's threat to run a proxy fight if the incumbent resist a value-increasing takeover. Therefore, one might expect the market reaction to 13D filing to be stronger in these cases. However, when the activist's threat is credible, share prices will reflect these expectations even before the announcement on the 13D filing takes place, and therefore, the increase in the share price around the announcement can be smaller. Third, the average abnormal return around the announcement of an acquisition agreement is *smaller* when it is preceded by a 13D filing than when it is not, that is, $AR(takeover|13D) < AR(takeover|\emptyset)$. There are two reasons. First, the price prior to the announcement on the takeover is already high if the activist has filed 13D, because 13D filing indicates that the company is likely to be a takeover target, and because the activist can pressure the board to accept a takeover offer. Second, conditional on the announcement of an acquisition agreement, the new price is higher if the activist is *not* a shareholder. Intuitively, without the pressure of the activist, the incumbent board agrees to a takeover only if the premium is sufficiently high to convince him to forgo the private benefits of control.

4 Full commitment

A key assumption behind the main results of Section 3 is the inability or unwillingness of the bidder to commit not to abuse the power of the target board after winning the proxy fight. In this section, we relax this assumption. We assume that the bidder can and will commit to act in the best interests of target shareholders after winning the proxy fight. This assumption has the following implications. First, if the bidder controls the target board, no corporate resources are diverted. Second, the nominees that were appointed by the bidder to the board of directors of the target, are obligated to maximize the value of target shareholders when negotiating a deal with the bidder in the second round. In particular, if the bidder is the proposer, they accept any offer greater than the standalone value of the firm q , and reject any other offer. If the target board is the proposer then they offer the highest price that is acceptable to the bidder, $q + \Delta$. Effectively, with commitment, target shareholders trust the bidder that if they replace the incumbent directors with his nominees, the bidder would offer to buy the firm for its “fair price”, $q + s\Delta$.²⁹

As in Section 3, we solve the game backward. We start with the takeover negotiations and proxy fights phase. Following Lemma 1, shareholder value under the control of the incumbent board is $q + \mathbf{1}_{\{b \leq \Delta\}} \cdot [s\Delta + (1 - s)b]$. Since the bidder can commit to act in the best interests of shareholders, target shareholders are indifferent between giving control to the bidder and the activist, as in both cases the shareholder value is $q + s\Delta$. Therefore, if $b \leq \Delta$ then shareholders always reelect the incumbent regardless of the identity of the rival team. If $\Delta < b$ and a proxy fight is initiated, the challenge to replace the incumbent always succeeds. Since both the activist and the bidder generate the same value when they win the proxy fight, neither of them has incentives to run a proxy fight and incur the associated costs if the other player is also expected to run a proxy fight. The next result uses these observations to characterize the proxy fight stage under the full commitment assumption.

Lemma 6 *Suppose the first round of negotiations fails. Then:*

(i) *The bidder runs a proxy fight if and only if the activist does not run a proxy fight and*

$$\frac{\kappa}{1 - s} \leq \Delta < b. \quad (15)$$

Whenever the bidder runs a proxy fight, she wins.

²⁹Following Lemma 1, whether or not the activist’s commitment problem is relaxed, has no effect on the results.

(ii) *If the activist owns α shares in the target, the activist runs a proxy fight if and only if the bidder does not run a proxy fight and*

$$\rho(\alpha) \leq \Delta < b. \tag{16}$$

Whenever the activist runs a proxy fight, she wins.

Part (ii) of Lemma 6 has the same logic as its counterpart in Lemma 2. Consider part (i). If the bidder runs and wins a proxy fight, his expected payoff is $(1 - s)\Delta - \kappa$. Therefore, the bidder will run a proxy fight only if $\frac{\kappa}{1-s} \leq \Delta < b$. In particular, if $b \leq \frac{\kappa}{1-s}$ then the bidder chooses not to run a proxy fight even if he expects shareholders to vote for him, since the benefit from replacing the incumbent board is not high enough to justify incurring the cost of running a proxy fight. The analysis of this case is identical to the one in Section 3. Indeed, in both cases, the bidder's threat to run a proxy fight is not credible, although for different reasons.

Importantly, if $\rho(L) < b \leq \frac{\kappa}{1-s}$ then the activist can and has incentives to run and win a proxy fight, while the bidder does not. Clearly, this condition holds if the activist faces lower costs of running a proxy fight than the bidder. The activist may have more governance expertise from her experience of challenging entrenched incumbents in other public companies. Moreover, $\rho(L) < \frac{\kappa}{1-s}$ can hold even if the bidder and the activist face the same cost, κ . The reason is that the incentives of the bidder and the activist to run a proxy fight are different. There are two reasons for that. First, the relative bargaining power of the target, captured by s , affects the bidder's benefit from running a proxy fight in an opposite direction that it affects the activist's benefit. Second, if $\gamma > 0$ then the activist has stronger incentives than the bidder to close a deal. In order to secure a quick exit on her investment, the activist would choose to incur the cost of running the proxy fight in circumstances that the bidder would not. In all of these cases, the bidder benefits from the activist's presence since it increases the circumstances under which the target is acquired.

The bidder's threat to run a proxy fight is credible only if

$$\frac{\kappa}{1-s} < b. \tag{17}$$

Hereafter, we maintain this assumption. According to Lemma 6, more than one equilibrium of the subgame that follows the first round of negotiations may exist: one in which the activist runs a proxy fight and one in which the bidder runs a proxy fight. We assume that whenever

there exists an equilibrium of the subgame in which the bidder runs a proxy fight, then this equilibrium is in play. This selection of equilibrium tilts the analysis against our result that the activist has any effect on the outcome of the takeover. Moreover, as the next result shows, when this selection is imposed, the equilibrium of the sub-game which is in play is also the equilibrium that obtains the highest shareholder value in the sub-game.

Proposition 5 *Suppose the bidder identifies firm i as a target with a synergy level Δ . Then:*

- (i) *If $b \leq \Delta$ the bidder pays $q + s\Delta + (1 - s)b$ and takes over the target after the first round of negotiations.*
- (ii) *If $\frac{\kappa}{1-s} \leq \Delta < b$ the bidder pays an expected price of $q + s\Delta + s\kappa$ and takes over the target in the first round of negotiations.*
- (iii) *If $\rho(\alpha) \leq \Delta < \frac{\kappa}{1-s}$ and the activist owns α shares in the target, the bidder pays $q + s\Delta$ and takes over the target after the first round of negotiations.*
- (iv) *In all other cases, the target remains independent under the incumbent board's control.*

The intuition behind Proposition 5 is similar to the intuition behind Proposition 1, with two exceptions. First, if $\frac{\kappa}{1-s} \leq \Delta < b$ then the bidder can credibly threaten to run and win a proxy fight if the first round of negotiations fails. That is, the bidder can overcome the incumbent board's resistance without the activist's assistance. If the bidder runs a proxy fight then he pays shareholders $q + s\Delta$ and takes over the target. Her expected profit is $(1 - s)\Delta - \kappa > 0$. The incumbent board realizes that the takeover is inevitable, and he agrees to sell the firm already in the first round of negotiations. When the incumbent makes the offer to the bidder, he will ask for the highest price the bidder is willing to pay. Since the alternative to an agreement is a proxy fight which costs the bidder κ , the bidder would agree to pay more than $q + s\Delta$ in order to save these costs. For this reason, κ factors into the offer in the first round, an element which was missing from Section 3. Second, the region in which the bidder can reach an agreement with the incumbent board without the activist's pressure is expanded from $[b, \infty)$ to $[\frac{\kappa}{1-s}, \infty)$, and the region in which the activist's pressure is necessary is scaled down from $[\rho(\alpha), b)$ to $[\rho(\alpha), \frac{\kappa}{1-s})$.

In light of Proposition 5, the analysis of the search and position building phase in Section 3.2, does not change when the assumption of full commitment is invoked, with the exception

that if $\frac{\kappa}{1-s} < b$ then the term $v(\alpha)$, given by (5), is everywhere replaced by

$$\hat{v}(\alpha) = \int_b^\infty [s\Delta + (1-s)b] dF(\Delta) + \int_{\frac{\kappa}{1-s}}^b [s\Delta + s\kappa] dF(\Delta) + \int_{\min\{\frac{\kappa}{1-s}, \rho(\alpha)\}}^{\frac{\kappa}{1-s}} s\Delta dF(\Delta), \quad (18)$$

and the term $w(\alpha)$, given by (7), is everywhere replaced by

$$\hat{w}(\alpha) = \int_{\min\{\frac{\kappa}{1-s}, \rho(\alpha)\}}^\infty \Delta dF(\Delta). \quad (19)$$

This observation implies that Lemmas 3, 4, and 5, as well as, Proposition 2, continue to hold.³⁰

Under full commitment, the activist benefits from the bidder's arrival for the same reasons as in Section 3. According to Proposition 5, the takeover premium when $\Delta \in [\frac{\kappa}{1-s}, b]$ is higher under full commitment than under no-commitment, and is the same in all other cases. Therefore, the activist has even stronger incentives to speculate under full commitment. Related, the activist's threat to run a proxy fight is credible only when $\Delta \in [\rho, \frac{\kappa}{1-s}]$. If $\rho < \frac{\kappa}{1-s}$ then $\hat{w}(\alpha) = w(\alpha)$ and $\hat{v}(\alpha) = v(\alpha) + \int_{\frac{\kappa}{1-s}}^b s\kappa dF(\Delta)$. It can be shown that the term $\frac{\partial \Pi_B}{\partial \lambda_A} \frac{\partial \Pi_A}{\partial \lambda_B}$, which governs the strength of the strategic complementarity, has a very similar structure to its counterpart in Section 3.3. Moreover, the key difference in the comparative statics is with respect to κ , the cost of running a proxy fight. Here, κ effects the probability of a takeover also in the selection region, as it affects the credibility of the bidder's threat to run a proxy fight, and the price that is paid when $\Delta \in [\frac{\kappa}{1-s}, b]$.

5 Extensions

In this section we discuss several extensions to the baseline model. Formal results and their proofs are given in Appendix B.

5.1 Activist investors as gatekeepers

The analysis in Sections 3 and 4 relied on the assumption that the incumbent board has private benefits from keeping the firm independent, and hence, is reluctant to relinquish control and sell the firm. However, in some cases, the incumbents might have the opposite bias. Situations in which the target board is too motivated to sell can arise in management buyouts or in

³⁰Another exception is that the integral in the second line in (6) is replaced by $\int_{\min\{\frac{\kappa}{1-s}, \rho(L)\}}^\infty dF(\Delta)$.

transactions where the incumbent board receives from the bidder a large bonus or promises for future employment upon completion of the takeover (Grinstein and Hribar, 2004). In those cases, the concerns that the board may compromise the interests of shareholders by negotiating less favorable buyout terms are particularly high, and activist investors may act as gatekeepers: they will challenge the deal by using their influence on target shareholders to either block the transaction or “force” the bidder to sweeten the bid. For example, during the management buyout of Dell in 2013, the pressure of the activist investor Carl Icahn resulted in the increase of the offer price.³¹ Jiang et al. (2015) provide evidence consistent with activist investors attempting to block announced M&A deals in order to extract better terms.

To study these situations, we modify the baseline model by assuming that $b < 0$. When $b < 0$, the incumbent board has private benefits from selling the firm. Moreover, because the board is motivated to sell the firm, he may not demand the highest price that is acceptable to the bidder when negotiating a deal. To stress this point, we assume that the incumbent board, unless forced otherwise, will sell the firm for the lowest price possible that is acceptable to shareholders, q .³²

Under these assumption, the incumbent board always reaches an acquisition agreement with a zero takeover premium. By contrast, if the activist is elected to the board, she would also reach an acquisition agreement with the bidder, but the takeover premium will be higher, $s\Delta$. Therefore, shareholders always elect the activist to the board whenever she decides to challenge the incumbent in a proxy fight. The activist has incentives to run the proxy fight whenever $\rho(\alpha) < \Delta$. Taken together, the target is always acquired by the bidder. If the activist owns α shares in the target and $\rho(\alpha) \leq \Delta$ then the bidder pays $q + s\Delta$, and in all other cases, the bidder pays q . In light of these observations, the analysis of the search and position building phase in Section 3.2, is similar with the exception that the term $v(\alpha)$ is replaced by $\int_{\rho(\alpha)}^{\infty} s\Delta dF(\Delta)$, and the term $w(\alpha)$ is replaced by $E[\Delta]$. As in Sections 3 and 4, the activist’s incentives to search increase with the likelihood that the bidder will make an offer to the target firm. However, the bidder’s incentives to search *decrease* with the likelihood that the activist is a shareholder of the target. This can be seen by noting that in the modified setup, the counterpart of $\psi(\lambda_A)$ is given by $E[\Delta] - \lambda_A \int_{\rho(L)}^{\infty} s\Delta dF(\Delta)$. Intuitively, the activist increases the expected takeover premium that the bidder is required to pay, without increasing

³¹See Kahan and Rock (2007) for more examples where activist investors block the deal or use pressure to increase the takeover premium.

³²The only assumption that is necessary for the analysis is that the incumbent would negotiate a deal with a takeover premium lower than $s\Delta$.

the likelihood that the incumbent board agrees to sell the firm. Since target shareholders and the activist cannot commit not to negotiate a higher premium, the bidder finds the acquisition less profitable, and his incentives to search decrease. For this reason, in this case the equilibrium is always unique.

When the incumbent is too motivated to sell the firm, the activist benefits shareholders ex-post by acting as a gatekeeper. However, since the bidder has fewer incentives to search for the target, shareholders can be ex-ante worse off. Nevertheless, when the incumbent board is motivated to sell the firm, the board might actively solicit bids, and thereby, reduce the bidder's search costs. At extreme cases as management buyouts, a search by the bidder is not needed at all. Since the need for a gatekeeper arises in situations where the lack of incentives of the bidder to search is not a concern, activists would generally benefit target shareholders ex-ante when acting as gatekeepers.

5.1.1 Arbitrage activism

When the incumbent board agrees to a deal that compromises shareholder value, activist investors may react to the news and buy shares in the target company with the objective of pressuring the board to either cancel the deal or improve its terms. Different from the analysis above, in these situations, the activist buys shares in the company *after* it becomes public that it is a target. From the activist's perspective, she does not need to search, as she already knows which company is the target, and that the bidder intends to take it over ($\lambda_B = 1$). Thus, the activist will have strong incentives to buy shares. On the other hand, the share price already reflects the information that the company is a target, and hence, the activist will have to pay a higher price than if she bought the shares prior to the announcement. This effect will attenuate the incentives of the activist to buy shares. Nevertheless, recall that buying shares of the target and then using her influence to challenge the deal is the private information of the activist. Therefore, the activist can make a profit even if she is buying shares of the target after the announcement.³³

5.2 Influencing voting outcomes

The analyses in Sections 3 and 4 provide a variety of channels through which the activist can affect outcomes in the market for corporate control. Common to all channels is the credible

³³If the activist has a non-zero outside option or she faces transaction costs, there is an equilibrium in which the activist buys shares with a positive probability.

threat of the activist to launch a successful proxy fight to replace the incumbent with her nominees. In this section we highlight a different channel. For this purpose, suppose that the bidder can commit to act in the best interests of target shareholders, and the activist's threat of running a proxy fight is never credible (e.g., $b < \rho(\alpha)$ for all $\alpha \in [0, 1]$). Our key assumption is that the likelihood that shareholders vote for the bidder's nominees at the proxy fight is higher when the activist is a shareholder of the target than when she is not. Intuitively, even though the activist cannot herself run a successful proxy fight, she can use her voting rights and vote for the bidder's nominee. Moreover, the activist can use her influence and lobby other shareholders to support the bidder in his challenge, thereby, pave the way for the bidder's victory.

To illustrate this point, suppose that if the bidder runs a proxy fight then with probability $1 - \varepsilon(\alpha) \in (0, 1)$ target shareholders vote rationally and with probability $\varepsilon(\alpha)$ they vote for the incumbent regardless of the circumstances.³⁴ Diffuse shareholders may abstain or vote blindly for the incumbent because of the false presumption that it is protecting their interest. We also assume that $\varepsilon(\cdot)$ is a decreasing function, and for simplicity, $\varepsilon(L) = 0$ and $\varepsilon(0) > 0$. Intuitively, the noise in the shareholder vote decreases with the number of shares own by sophisticated investors such as the activist. Moreover, if the activist has more skin the game, she may also be able to exercise more influence on other shareholders. Under these assumptions, the bidder is facing a cost of $\frac{\kappa}{1 - \varepsilon(\alpha)}$ per unit of success when running a proxy fight, and this cost is decreasing with the number of shares owned by the activist.³⁵

The analysis of the model under these assumptions is the same as the analysis in Section 4 under the assumption that $b < \rho(L)$, and with the exception that κ is replaced by $\frac{\kappa}{1 - \varepsilon(\alpha)}$. Therefore, if

$$\frac{\kappa}{1 - s} < b \leq \frac{\kappa / (1 - \varepsilon(\alpha))}{1 - s} \quad (20)$$

then according to Lemma 6 the bidder's threat of running a proxy fight is credible if and only if the activist is a shareholder of the target. When the bidder's threat is credible, he can overcome the incumbent's resistance, take over the firm even when $\Delta \in [\frac{\kappa}{1 - s}, b]$, and make an additional profit of $(1 - s) \Delta - \kappa > 0$. Therefore, the bidder has stronger incentives to search.

³⁴For simplicity, we assume that shareholders always vote rationally on acquisition agreements. Indeed, voting on merger proposals are much more predictable (they are almost always approved) than voting in contested director elections.

³⁵Suppose the bidder expects to win if all shareholders are rational (otherwise, she never runs a proxy fight). Also, let Π_{win} be the bidder's payoff if she wins the proxy fight and Π_{lose} if she loses. The bidder will run a proxy fight if and only if $(1 - \varepsilon(\alpha)) \Pi_{win} + \varepsilon(\alpha) \Pi_{lose} - \kappa > \Pi_{lose}$, which holds if and only if $\Pi_{win} - \Pi_{lose} > \frac{\kappa}{1 - \varepsilon(\alpha)}$.

Through this channel the activist can affect outcomes in the market for corporate control, even if her threat to run a proxy fight is never credible.

5.3 Soliciting takeover bids

Our analysis suggests that activist investors have strong incentives to solicit takeover bids once they invest in companies they believe can be a good takeover target. The solicitation can involve meeting with potential bidders or communications with intermediaries such as investment bankers. Alternatively, the activist may attract attention simply by publicly announcing that she invests in the company with the intent of pressuring management to sell the firm. Either way, by actively soliciting bids, the activist can relax the search friction that potential bidders face. The solicitation not only informs the bidder that the firm is a target and its takeover can create value (this effect exists even in the selection region), but it also reassures the bidder that with the activist's pressure the bidder will face a weaker opposition from the incumbent board (this effect exists only in the treatment region).

In the context of our model, suppose that instead of making the search decisions simultaneously, the bidder observes the decision of the activist to search and the firm in which she invested (for example, by following the announcement of a 13D filing) before deciding whether to search. We assume that if the activist invested in firm i , the bidder learns Δ of this specific firm without incurring additional costs. However, in order to learn the value of Δ of all firms in which the activist did not invest, the bidder has to incur the same search costs as in the baseline model. Intuitively, conducting a due diligence on a single firm which is likely to be a target is a second order compared with searching which of the N firm is the target.

Under these assumptions, if the activist does not search then the bidder's problem is the same as in the baseline model, where $\lambda_A = 0$.³⁶ If the activist searched and bought shares in firm i then the bidder would start negotiating an acquisition agreement with the incumbent board of firm i . Effectively, the activist is facing $\lambda_B = 1$ whenever deciding to search. Therefore, with active solicitation, the incentives of the activist to search are stronger, and her effect on the likelihood of a takeover is larger relative to the baseline model. Note that the ex-ante probability of a takeover is not necessarily higher with active solicitation. While a takeover is

³⁶Recall our assumption that without knowing the firm is a target, the expected Δ is negative. Therefore, if N is large and the cost of due diligence is non-zero, the bidder will not investigate firms in which the activist invested if the activist did not initially search. In this case, the bidder will engage in search activist as in the baseline model. Under these assumptions, and for the same reasons as in Lemma 3, the activist will never buy shares in any firm without first identifying it as the target.

more likely when the activist searches, it is less likely when she does not. In the latter case, the incentives of the bidder to search are weaker since the potential target is unlikely to face a pressure from the activist to sell.³⁷ The balance between these two effects depends on the curvature of cdf G . In Appendix B, we give an example where latter effect dominates.

5.4 Corporate restructuring

A common demand of activist investors from firms they target is to divest or spin-off non-core assets. Divestitures and spinoffs can increase shareholder value, but at the same time, force the incumbent board to forego some of its private benefits of control. Therefore, incumbents may resist parting from assets. Our model can be adjusted to study divestitures and spinoffs. Divestitures and spinoffs might involve a smaller loss of private benefits than takeovers, and hence, may trigger a weaker opposition from the incumbent board. This effect is equivalent to having a lower b in the baseline model. However, a key conceptual difference between selling the entire company and selling or spinning off one of its non-core assets is that, in the latter case, under most jurisdictions, the approval of target shareholders is not required (Bebchuk (2005)). Below, we study how this aspect affects the analysis.

5.4.1 Divestitures

Divesting non-core assets (e.g., a division, subsidiary, or product line) creates value if the potential bidder has a more efficient use for the asset. For example, a non-core asset for one company can be a good fit to the main line of business for another corporation. Alternatively, a non-core asset may be inefficiently managed due to the lack of managerial focus or incentives.

To study divestitures, we define the target firm in the baseline model as the firm that owns a non-core asset that has a higher value (of Δ) under the ownership of the bidder. Here, q is the standalone value of the non-core asset, and for simplicity, we normalize the value of all other assets of the firm to zero. Since the bidder is interested in buying only non-core assets but not the entire company, target shareholders would never elect the bidder to the board of directors. Therefore, it is appropriate to assume no commitment, as in Section 3. Different from the baseline model, since the sale of a non-core asset does not require the approval of target shareholders, if the activist controls the board and she has preferences for early liquidation,

³⁷Similar to Section 5.1.1, it is possible that the activist decides to buy shares only after learning that the company is a target for a takeover. Different from Section 5.1.1, here the activist will invest in order to put pressure on the incumbent board to sell.

the bidder can offer to buy the asset for a premium as low as $-\gamma q$, knowing that the offer will be accepted by the activist. The analysis of divestitures is similar to the analysis of takeovers in Section 3, with the exception that under the control of the activist, shareholder value is $q + s\Delta - (1 - s)\gamma q$ instead of $q + s\Delta$. This difference has several implications. Since the activist is too eager to sell assets, shareholders are less likely to elect her to the board of directors (formally, $\rho(\alpha)$ is higher). The inability of the activist to commit not to sell assets at a discount weakens her role in the market for corporate control. Therefore, the bidder and the activist will have weaker incentives to search. On the other hand, since shareholder approval is not required, the bidder expects to pay a lower premium when the activist controls the board. In those cases, the overall effect on the incentives of the bidder to search is ambiguous. We conclude, activist investors play a similar role in the market for corporate assets to the one they play in M&A markets.

5.4.2 Spinoffs

In a spinoff, a public company distributes its equity ownership in a subsidiary to its shareholders. The spinoff involves a complete separation of the two firms, and can create value, for example, by eliminating negative synergies or increasing corporate focus. Unlike divestitures, in most cases there is no third party (a bidder) to which assets are transferred in a spinoff. The model can be adjusted to study spinoffs by replacing each round of negotiations with a unilateral decision of the board whether to spin-off the company. A spinoff will increase shareholder value by Δ , but will also eliminate the incumbent board's private benefits, b . As in the baseline model, the activist has to search for the company that has the opportunity for a value increasing spinoff. The incumbent board might spinoff the company, fearing that his resistance will trigger a costly proxy fight. Therefore, the probability of a spinoff increases with the activist is a shareholder of the target.³⁸

5.5 Non-control activism

While activist investors play a prominent role in putting companies into play or blocking coercive transactions, they may also have the capacity and expertise to propose and execute

³⁸Mathematically, the analysis of spinoffs is equivalent to the model with no commitment under the assumptions that $\lambda_B \equiv 1$ (if the activist finds the target company, a spinoff can always take place) and $s \equiv 1$ (target shareholders do not need to split the gains with a third party). To keep the analogy simple, we implicitly assume that the activist discounts firm value by $1 - \gamma$, and value can be extracted from the firm if and only if the spinoff does not take place.

operational, financial, and governance changes that increase the standalone value of the firm. In this section we investigate how this additional expertise interacts with the activist role in the market for corporate control.

Suppose the activist can propose an action that increases the standalone value of the target firm. If the action is implemented and the firm remains independent, its value increases instantly by $\zeta\Delta$ where $\zeta \in (0, 1]$, and the incumbent board loses his private benefits of control. We assume that without the activist, the incumbent board is either unaware or does not have the expertise to implement the action. The baseline model is a special case where $\zeta = 0$. For simplicity, we focus on the no commitment case.

The activist's ability to increase the standalone value of the firm affects the bidder in two different ways. First, since the standalone value of the target is now higher when the activist is a shareholder, the bidder may have to pay a higher premium. All else equal, the bidder will have fewer incentives to search for the target. Second, because the bidder is expected to pay a higher premium, the activist has stronger incentives to intervene if the incumbent board rejects the takeover offer or refuses to implement the action. As in the baseline model, this effect increases the bidder's incentives to search. In Appendix B, we show that if b is sufficiently large, even if $\zeta = 1$, then the second effect dominates, and the presence of the activist increases the incentives of the bidder to search. Intuitively, when the private benefits of the incumbent board are large, the bidder cannot take over the target unless the activist is there to credibility threaten the incumbent with a proxy fight. Interestingly, we also show that the second effect can be so strong to the extent that the bidder's incentives to search increase with ζ . That is, a takeover is more likely when the activist also has operational expertise. By contrast, if $\zeta > 0$ and b is small then the first effect dominates. Intuitively, in those cases, the bidder is likely to reach an agreement even when the incumbent is not pressured by the activist. The presence of the activist does not improve the likelihood of reaching an agreement, but it forces the bidder to pay a higher premium. In fact, the activist affects firm value even without having a credible threat to intervene. Essentially, the activist provides the incumbent board with an advice how to increase firm value, and when the benefit from the action outweighs the cost of foregoing the private benefits of control, the incumbent will follow the activist's recommendation.³⁹

The ability to increase the standalone value of the firm also affects the incentives of the activist to search for a target. On the one hand, since the expected takeover premium is higher, the activist has stronger incentives to search when the bidder is likely to make an offer.

³⁹See Levit (2013) for analysis of strategic communication between activist investors and firms.

On the other hand, the activist can increase firm value even if the firm remains independent. If the latter effect dominates the former, the activist's incentives to search are less sensitive to presence of a bidder. Overall, depending on the severity of the agency friction in the target firm, the expertise of activist can either attenuate or amplify the complementarities between the search effort of the bidder and the activist.

6 Conclusion

In this paper we study the role of blockholders and activist investors in the market for corporate control. We focus on two key frictions: the need of bidders to search for corporate assets that fit their strategy and are available for sale, and agency problems in public corporations that result with excessive resistance of incumbents to takeovers.

Our analysis demonstrates that activist investors complement the effort of bidders to acquire companies by relaxing the resistance of incumbent directors to takeovers. We identify several channels. First, since bidders cannot commit not to abuse the power of the board and lowball the takeover offer once they are elected to the target board, their ability to win a proxy fight is very weak. By contrast, target shareholders trust the activist since both of them sit on the sell side of the negotiating table. Therefore, unlike bidders, activists can use proxy fights to disentrench boards and facilitate takeovers. Second, we argue that activists have stronger incentives than bidders to intervene due to their greater governance expertise, short-term horizon, and stronger bargaining power of the target company. Third, if bidders can overcome the commitment problem, activists can increase the credibility of the bidder's threat to remove the incumbents by exercising their voting rights and lobbying other shareholders. Last, activists can relax the search friction of bidders by directly soliciting takeover bids. Taken together, the analysis suggests that activist investors play a unique role in securing an active market for corporate control.

We also show that there is strategic complementarity between the search of activists for firms that are likely to receive a takeover bid, and the search of bidders for targets with which they can create synergies. This strategic complementarity has several important implications. First, the aggregate volume of M&A activity is positively related to the intensity of shareholder activism. Second, small changes in the economic environment have a significant effect on the aggregate volume of M&A deals. Third, strategic complementarity can lead to multiple equilibria, giving rise to episodes with high and low volume of transactions without any apparent changes in the

underlying fundamentals of the economy. Finally, the analysis provides empirical predictions on the interaction between shareholder activism and M&A activity, and a framework to study the implications of the treatment and the selection effects of shareholder activism. Among other things, we show that due strategic complementarity, more severe agency problems can result with a higher volume of M&A deals.

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A Proofs of main results

This section contains proofs of the Propositions and Lemmas in the main text. Appendix B contains supplemental results and proofs of results of the extension in Section 5.

Proof of Lemma 1.

Generally, there are three scenarios to consider. The scenarios differ with respect to the composition of the target board after the proxy fight stage. Under all scenarios, target shareholders approve the acquisition agreement if it is brought to a shareholder vote if and only if the takeover offer is higher than the standalone value of the firm, q . Moreover, the bidder will not agree to pay more than $q + \Delta$ for the firm.

In the first scenario, the incumbent board is reelected and retains control of the target. The incumbent board would agree to sell the firm if and only if the bidder offers at least $q + b$ per share. Therefore, if $\Delta < b$ no agreement is reached and the target remains independent under the control of the incumbent. If $\Delta \geq b$ then the incumbent board and the bidder reach an agreement in which the expected takeover premium is $s\Delta + (1 - s)b$: with probability $1 - s$ the bidder proposes to pay $q + b$, which is the lowest price that is acceptable by both the incumbent board and the shareholders, and with probability s the incumbent board propose to receive $q + \Delta$, which is the highest price that the bidder would agree pay for the firm.

In the second scenario, the activist wins the proxy fight and controls the target board. If no agreement is reached with the bidder, the target remains independent, and the activist's payoff per share is $q - \gamma q$, which is the discounted long-term value of the target as a standalone firm. Therefore, the activist would agree to sell the firm if and only if the offer is higher than $q - \gamma q$. Since $\Delta > 0$, the bidder and the activist always reach an acquisition agreement that is also acceptable to target shareholders. With probability $1 - s$ the bidder offers q , which is the lowest price that is acceptable by both the activist and the shareholders, and with probability s the activist offers $q + \Delta$, which is the highest price the bidder would pay for the firm.⁴⁰

In the third scenario, the bidder wins the proxy fight and controls the target board. The argument is given in the main text. ■

Proof of Proposition 1. Consider part (i). Suppose $b \leq \Delta$. Based on Lemma 2, the activist will not run a proxy fight if the first round of negotiations fails. Since $b \leq \Delta$, all

⁴⁰We implicitly assume that the activist is a shareholder of the target, that is, $\alpha > 0$. Below we show that if $\alpha = 0$ the activist will never run a proxy fight.

players expect the takeover to consume in the second round of negotiations, where the price is $q + s\Delta + (1 - s)b$. Therefore, in the first round of negotiations, the incumbent board will reject any offer which is lower, and the bidder will reject any offer which is higher. If there are arbitrarily small waiting costs to either the bidder or the incumbent board, the deal will close in the first round.

Consider part (ii). Based on Lemma 2, if the activist owns α shares of the target and $\rho(\alpha) \leq \Delta < b$, then shareholders would support the activist at the proxy fight if the first round of negotiations fails. Based on Lemma 1, all players expect that once the activist obtains control of the board, she will reach a sale agreement in which the bidder pays in expectations $q + s\Delta$ per share. The bidder realizes that any lower offer will be rejected by shareholders, who expect the activist to negotiate a higher offer at the second round. The bidder can afford to pay $q + s\Delta$. The bidder will not pay more than $q + s\Delta$, since he always has the option to pay that much in the second round when he negotiates with the activist. The incumbent board understands the bidder's incentives. The board also realizes that the takeover of the target is inevitable, and he will lose his private benefits of control. However, by accepting the offer $q + s\Delta$ the board can avoid the costly proxy fight. Therefore, the incumbent and the bidder reach an agreement in the first round where the offer is $q + s\Delta$. This completes part (ii)

Consider part (iii). According to Lemma 2, in all other cases, neither the bidder nor the activist initiate a proxy fight if the first round of negotiations fails. Therefore, the incumbent board retains control. Since in this region $\Delta < b$, based on Lemma 1, the incumbent board and the bidder will not reach an agreement in the second round of negotiations. Therefore, in the first round of negotiations, the incumbent board will reject any offer lower than $q + b$, and the bidder will reject any offer higher than $q + \Delta$. Thus, the parties will not reach an agreement in the first round as well, and the target remains independent. This completes part (iii). ■

Proof of Lemma 6. If the incumbent board retains control of the target in the second round of negotiations, then shareholder value is $q + \mathbf{1}_{\{b \leq \Delta\}} \cdot [s\Delta + (1 - s)b]$. If the activist or the bidder obtains control of the target board, an agreement will be reached in the second round and the expected shareholder value is $q + s\Delta$. Therefore, if $b \leq \Delta$ then neither the bidder nor the activist can win a proxy fight, and hence, they will not initiate one. Suppose $\Delta < b$ and the first round of negotiations failed. Shareholders will support whoever runs a proxy fight, knowing that in both cases an agreement will be reached in the second round of negotiation and the expected shareholder value will be $q + s\Delta$. Therefore, if one player is going to run a proxy fight, the second player has not incentives to run a proxy fight, since by doing

so he will obtain the same profit but will in addition incur the cost κ . Consider the bidder. If the bidder runs a proxy fight then his expected payoff is $(1 - s)\Delta - \kappa$. If neither the bidder nor the activist runs a proxy fight, then the firm will remain independent, and the bidder's profit will be zero. Therefore, the bidder will run a proxy fight if and only if $\frac{\kappa}{1-s} \leq \Delta$. This completes part (i). Consider the activist. If the activist runs a proxy fight then her expected payoff is $\alpha(q + s\Delta) - \kappa$. If neither the bidder nor the activist runs a proxy fight, then the firm will remain independent, and the bidder's profit will be $\alpha q(1 - \gamma)$. Therefore, the activist will run a proxy fight if and only if $\rho(\alpha) \leq \Delta$. This completes part (ii). ■

Proof of Proposition 5. Consider part (i). Based on Lemma 6, if $b \leq \Delta$ then neither the bidder nor the activist will run a proxy fight. Therefore, both the bidder and the incumbent board expect that in the second round of negotiations they will reach an agreement with expected premium of $s\Delta + (1 - s)b$. Therefore, the bidder will not agree to pay more than this amount and the incumbent board will not accept less than this amount. They will reach an agreement in the first round of negotiations, in which the bidder pays a premium of $s\Delta + (1 - s)b$.

Consider part (ii). Recall the assumption that if there is an equilibrium in the subgame that follows the first round of negotiations in which the bidder runs a proxy fight, then this equilibrium is in play. Based on Lemma 6, if the first round of negotiations fails, the bidder will run a proxy fight if and only if $\frac{\kappa}{1-s} \leq \Delta < b$. In this case, the bidder will run and win the proxy fight if the first round of negotiations fails. In the second round, the expected premium is $q + s\Delta$, and the bidder's expected profit is $\Delta(1 - s) - \kappa > 0$. In the first round of negotiations, shareholders would reject any offer lower than $q + s\Delta$, any accept any offer higher than that amount. If the bidder is the proposer, he will offer $q + s\Delta$, and both the board and the shareholders will accept it. If the board is the proposer, he will offer $q + s\Delta + \kappa$, which leaves the bidder with a profit of $\Delta(1 - s) - \kappa > 0$, and hence, the bidder will accept this deal. Overall, the expected takeover premium is $q + s\Delta + s\kappa$, as required.

Consider part (iii). Based on Lemma 6, if the first round of negotiations fails and $\rho(\alpha) \leq \Delta < \frac{\kappa}{1-s}$ then the bidder will not run a proxy fight but the activist will. Therefore, both the bidder and the incumbent board expect that in the second round of negotiations the bidder will negotiate with the activist and they will reach an agreement with expected premium of $s\Delta$. Therefore, the bidder will not agree to pay more than this amount and the incumbent board will not accept less than this amount. They will reach an agreement in the first round of negotiations, in which the bidder pays a premium of $s\Delta$, as required.

Consider part (iv). In all other cases, $\Delta < \min\{\frac{\kappa}{1-s}, b\}$ and either $\Delta < \min\{\rho(\alpha), b\}$ or the activist is not a shareholder of the target. Based on Lemma 6, neither the bidder nor the activist will run a proxy fight. Since $\Delta < b$, the target remains independent under the incumbent board's control, as required. ■

Remark: We prove Lemma 3, Lemma 7, and Proposition 2 for a more general case where there is partial commitment. Partial commitment means the following. There are two states of nature which determine the ability of the bidder to commit with respect to firm i . States are firm specific and independent across firms. Initially, the state in firm i is unknown to the bidder, the activist, shareholders and the market maker. The state becomes public after the search and trade phase, but before the first round of negotiations starts. Intuitively, each M&A situation is different, and hence, it is hard to predict the severity of agency problems, the effectiveness of shareholder litigation, before the identity of the target firm, the bidder, and the activist, are known. Specifically, with probability $1 - \tau \in (0, 1)$, no commitment is possible, and the model is reduced to the case of no-commitment. With probability τ , the bidder can commit. Clearly the cases of full commitment and no commitment are captured by $\tau = 0$ and $\tau = 1$, respectively.

Proof of Lemma 3. Suppose it is a common knowledge that the activist owns $\alpha \geq 0$ shares of firm i and the bidder is expected to make a takeover offer to firm i with probability $\lambda \in [0, 1]$. Based on Propositions 1 and 5, the expected value of the firm (to all shareholders other than the activist) is $V(\alpha, \lambda) = q + \lambda v(\alpha)$, where

$$\begin{aligned} v(\alpha) &= \int_b^\infty [s\Delta + (1-s)b] dF(\Delta) + \tau \int_{\min\{\frac{\kappa}{1-s}, b\}}^b [s\Delta + s\kappa] dF(\Delta) \\ &\quad + (1-\tau) \int_{\min\{b, \rho(\alpha)\}}^b s\Delta dF(\Delta) + \tau \int_{\min\{\frac{\kappa}{1-s}, b, \rho(\alpha)\}}^{\min\{\frac{\kappa}{1-s}, b\}} s\Delta dF(\Delta). \end{aligned} \quad (21)$$

Note that $V(\alpha, \lambda)$ is strictly increasing in λ . Moreover, $\alpha > 0 \Rightarrow V(\alpha, \lambda) \geq V(0, \lambda)$. The expected value of each share for the activist is given by $\Lambda(\alpha, \lambda) = V(\alpha, \lambda) - \Gamma(\alpha, \lambda)$ where

$$\Gamma(\alpha, \lambda) = \gamma q \left[1 - \lambda + \lambda(1-\tau) \int_0^{\min\{b, \rho(\alpha)\}} dF(\Delta) + \lambda\tau \int_0^{\min\{\frac{\kappa}{1-s}, b, \rho(\alpha)\}} dF(\Delta) \right]. \quad (22)$$

Note that $\Gamma(\alpha, \lambda)$ is decreasing in λ . Therefore, $\Lambda(\alpha, \lambda)$ is increasing in λ .

We denote by α^* the number of shares of firm i the activist buys in equilibrium if she identifies firm i as a target, and by α_0 the number of shares of firm i she buys in equilibrium if she does not search. We prove the lemma in several steps.

First, suppose the activist searches and finds that firm i is *not* the target. We argue that the activist buys no shares. Indeed, the activist knows for sure that the bidder will not make an offer to this firm, and the shareholders will never support her at the proxy fight if no bidder has arrived. Therefore, the value of the firm from the activist's perspective is $q(1 - \gamma)$. Since the market maker of firm i does not know for sure that firm i is not a target, the market maker believes that there is a non-negative probability that firm i is the target when it observes a positive order-flow. Therefore, if $z_i > 0$ then $p(z_i) \geq q \geq q(1 - \gamma)$, and the activist's expected profit is non-positive. For this reason, the activist never buys a stake in a firm she identifies not to be a target.

Second, we argue that if $\alpha_0 > 0$ then $\alpha^* > 0$. Suppose on the contrary, $\alpha_0 > 0$ and $\alpha^* = 0$. Let $p(\alpha)$ be the price the activist expects when she submits an order to buy α shares of firm i . Since $\alpha_0 > 0$, by revealed preferences, the activist's expected profit is positive if she buys α_0 shares of firm i without searching. In this case, the activist believes that each firm is equally likely to be the target, and therefore, the bidder makes a takeover offer to firm i with probability λ_B/N . Therefore, $\Lambda(\alpha_0, \lambda_B/N) - p(\alpha_0) \geq 0$. However, if the activist identifies firm i as the target, she expects the bidder to make the firm a takeover offer with probability $\lambda_B > \lambda_B/N$. Since $\Lambda(\alpha, \lambda)$ is strictly increasing in λ , $\Lambda(\alpha_0, \lambda_B) - p(\alpha_0) > 0$. This creates a contradiction, since the activist can make a strictly positive payoff from submitting order to buy $\alpha_0 > 0$ when she identifies firm i as a target.

Third, we argue that if $\alpha_0 > 0$ then $\alpha^* = \alpha_0$. Suppose on the contrary that $\alpha_0 > 0$ and $\alpha^* \neq \alpha_0$, and recall that $\alpha_0 > 0 \Rightarrow \alpha^* > 0$. First we argue that $\alpha^* = L$. Suppose on the contrary $\alpha^* \neq L$. If the activist buys α^* shares of firm i , then the market maker knows for sure that the activist demanded these shares. Since $\alpha^* \neq \alpha_0$, in equilibrium, the market maker infers that the activist identified firm i as a target, and hence, the bidder will make a takeover offer to firm i with probability λ_B . Therefore, $p(\alpha^*) = p(\alpha^* + L) = V(\alpha^*, \lambda_B)$. Since $\Lambda(\alpha^*, \lambda_B) \leq V(\alpha^*, \lambda_B)$, the activist is better off not buying α^* shares, a contradiction.⁴¹

⁴¹Here, we invoke the assumption that if the activist is indifferent she would not buy the shares (note that this assumption is necessary only in the knife edge case where $\gamma = 0$). However, this argument holds even without this assumption. Indeed, since $\Lambda(\alpha^*, \lambda_B) \leq V(\alpha^*, \lambda_B)$, the activist buys $\alpha^* \neq L$ shares of firm i only if $\Lambda(\alpha^*, \lambda_B) = V(\alpha^*, \lambda_B)$, in which case, her profit is zero. If instead the activist buys L shares, then the market maker believes that with probability weakly greater than $\frac{1}{2}$ the demand is from the liquidity

Second, note that if $\alpha_0 \neq \alpha^* = L$ then $\alpha_0 \neq L$. If the activist buys α_0 shares of firm i , then the market maker knows for sure that the activist demanded these shares. Since $\alpha_0 \neq \alpha^* = L$, in equilibrium the market maker infers that the activist did not search, and hence, the bidder will make a takeover offer to firm i with probability λ_B/N . Therefore, $p(\alpha_0) = p(\alpha_0 + L) = V(\alpha_0, \lambda_B/N)$. Since $\Lambda(\alpha_0, \lambda_B/N) \leq V(\alpha_0, \lambda_B/N)$, the activist is better off not buying α^* shares, a contradiction. Overall, if $\alpha^* \neq \alpha_0$, $\alpha_0 > 0$, and $\alpha^* > 0$, then it must be $\alpha^* = \alpha_0 = L$, a contradiction.

Fourth, we argue that $\alpha_0 = 0$. Suppose on the contrary $\alpha_0 > 0$. The previous argument implies $\alpha_0 = \alpha^*$. Suppose the activist does not search. Without searching, the expected value for the activist from each share is $\Lambda(\alpha^*, \lambda_B/N)$. Consider two cases. First, suppose $\alpha^* \neq L$. The market maker of firm i knows for sure that the activist bought α^* shares in firm i . Conditional on this event, the market maker believes that with probability λ_A/N the activist bought firm i because she identified it as the target, and with probability $1 - \lambda_A$ the activist bought firm i without searching and knowing it is the target. Combined, the market maker believes that the probability that the target receives a takeover offer is

$$h(\lambda_B) \equiv \frac{(1 - \lambda_A) \frac{\lambda_B}{N} + \frac{\lambda_A}{N} \lambda_B}{(1 - \lambda_A) + \frac{\lambda_A}{N}}. \quad (23)$$

Therefore, $p(\alpha^*) = p(\alpha^* + L) = V(\alpha^*, h(\lambda_B))$. Since $N \geq 2$ implies $h(\lambda_B) > \lambda_B/N$, $V(\alpha^*, h(\lambda_B)) > V(\alpha^*, \lambda_B/N)$. Since $\Lambda(\alpha^*, \lambda_B/N) \leq V(\alpha^*, \lambda_B/N)$, $\Lambda(\alpha^*, \lambda_B/N) < V(\alpha^*, h(\lambda_B))$ and the activist's expected profit is negative, a contradiction. Second, suppose $\alpha^* = L$. If $z_i = 2L$ then the market maker of firm i knows for sure that the activist bought L shares in firm i . For the same reasons as in the case where $\alpha^* \neq L$, it must be $p_i(2L) = V(L, h(\lambda_B))$. If $z_i = L$ there are three events the market maker considers:

1. With probability $\frac{1}{2}\lambda_A \frac{N-1}{N}$ the activist did not buy a stake because she searched and found that firm i is not the target. In this case, firm value is $V(0, 0) = q$.
2. With probability $\frac{1}{2}\lambda_A \frac{1}{N}$ the activist bought a stake L since she identified firm i as the target. In this case, firm value is $V(L, \lambda_B)$.
3. With probability $\frac{1}{2}(1 - \lambda_A)$ the activist bought a stake L in firm i even though she did not search. In this case, firm value is $V(L, \lambda_B/N)$.

traders and not from the activist. In this event, the value of the firm is $V(0, \lambda_B/N) < V(\alpha^*, \lambda_B)$. Therefore, $p(L) < V(\alpha^*, \lambda_B)$, and the activist can make a strictly positive profit by buying L shares, contradicting the assumption that $\alpha^* \neq L$.

Therefore, the share price is given by

$$p_i(L) = \lambda_A \frac{N-1}{N} V(0,0) + \lambda_A \frac{1}{N} V(L, \lambda_B) + (1 - \lambda_A) V(L, \lambda_B/N). \quad (24)$$

Recall that $V(\alpha, \lambda)$ is linear in λ and equal to $q + \lambda v(\alpha)$. Therefore, the activist's expected profit is negative:

$$\begin{aligned} & \Lambda(L, \lambda_B/N) - \frac{1}{2} p_i(2L) - \frac{1}{2} p_i(L) \\ = & V(L, \lambda_B/N) - \Gamma(L, \lambda_B/N) - \frac{1}{2} V(L, h(\lambda_B)) - \frac{1}{2} \left[\lambda_A \frac{N-1}{N} V(0,0) + \lambda_A \frac{1}{N} V(L, \lambda_B) \right. \\ & \left. + (1 - \lambda_A) V(L, \lambda_B/N) \right] \\ = & -\frac{1}{2} \frac{\lambda_A \lambda_B}{N} \frac{N-1}{(1-\lambda_A)N + \lambda_A} v(L) - \Gamma(L, \lambda_B/N) \\ \leq & 0. \end{aligned}$$

This creates a contradiction. We conclude that the activist does not buy a positive stake of any firm unless she identifies the firm as the target.

Fifth, we show that if $\alpha^* > 0$ then $\alpha^* = L$. Indeed, if $\alpha^* > 0$ but $\alpha^* \neq L$ then the market maker of firm i knows for sure that the activist bought α^* shares in firm i , and that firm i was identified as the target. Therefore, the share price is $V(\alpha^*, \lambda_B)$, while the activist value per share is $\Lambda(\alpha^*, \lambda_B) \leq V(\alpha^*, \lambda_B)$. The activist makes a non-positive profit and hence she is better off not buying a share, yielding a contradiction.

Finally, in any equilibrium, if the activist does not plan on buying shares of the firm she identifies as a potential target, then the activist has no incentives to search for a target in the first place. If $\alpha^* = L$ then $z_i \in \{L, 2L\}$. If $z_i = 2L$ then the market maker of firm i knows for sure that the activist bought L shares in firm i , which is identified by the activist as a potential target. Therefore, the probability that the bidder will make a takeover offer is λ_B , and $p_i(2L) = V(L, \lambda_B)$. Implicitly, we assume that if $\lambda_A = 0$ then the market maker's off-equilibrium beliefs when $z_i \neq L$ are that the activist bought L shares, and the activist identified the firm as a target. Under these beliefs, the price is $V(L, \lambda_B)$. If $z_i = L$ there are three events the market maker considers:

1. With probability $\frac{1}{2} \lambda_A \frac{N-1}{N}$ the activist did not buy a stake because she searched and found that firm i is not the target. In this case, firm value is $V(0,0) = q$.
2. With probability $\frac{1}{2} \lambda_A \frac{1}{N}$ the activist bought a stake L since she identified firm i as the target. In this case, firm value is $V(L, \lambda_B)$.

3. With probability $\frac{1}{2}(1 - \lambda_A)$ the activist did not buy a stake because she did not search for the target. In this case, firm value is $V(0, \lambda_B/N)$.

Combined,

$$\begin{aligned} p_i(L) &= \lambda_A \frac{N-1}{N} V(0, 0) + \lambda_A \frac{1}{N} V(L, \lambda_B) + (1 - \lambda_A) V(0, \lambda_B/N) \\ &= q + \lambda_B \frac{\lambda_A v(L) + (1 - \lambda_A) v(0)}{N}, \end{aligned}$$

as required.

Note that if $z_i = 0$ either case 1 above or case 3 above can take place. Therefore,

$$\begin{aligned} p_i(0) &= \frac{\lambda_A \frac{N-1}{N} V(0, 0) + (1 - \lambda_A) V(0, \lambda_B/N)}{\lambda_A \frac{N-1}{N} + (1 - \lambda_A)} \\ &= q + \frac{1 - \lambda_A}{N - \lambda_A} \lambda_B v(0), \end{aligned}$$

as required. ■

Lemma 7 Consider an equilibrium in which the bidder searches for a target with probability $\lambda_B \in [0, 1]$ and the activist searches for a target with probability $\lambda_A \in [0, 1]$. Conditional on searching, the activist's expected profit net of search cost is

$$\Pi_A(c_A, \lambda_A, \lambda_B) = -c_A + L \times \max \left\{ 0, \frac{1}{2} \lambda_B \left(v(L) - \frac{\lambda_A}{N} v(L) - \frac{1 - \lambda_A}{N} v(0) \right) - \Gamma(L, \lambda_B) \right\} \quad (25)$$

where $\Gamma(\alpha, \lambda_B)$ is given by (22) and $v(\alpha)$ is given by (21). The bidder's expected profit net of search cost is given $\Pi_B(c_B, \lambda_A) = -c_B + \psi(\lambda_A)$ where

$$\psi(\lambda_A) = \lambda_A (w(L) - v(L)) + (1 - \lambda_A) (w(0) - v(0))$$

and

$$w(\alpha) = (1 - \tau) \int_{\min\{b, \rho(\alpha)\}}^{\infty} \Delta dF(\Delta) + \tau \int_{\min\{\frac{\kappa}{1-s}, b, \rho(\alpha)\}}^{\infty} \Delta dF(\Delta) \quad (26)$$

Moreover, $\psi(\lambda_A)$ is an increasing function.

Proof. The activist's expected profit, net of search cost and prices is given by

$$\Pi_A(c_A, \lambda_A, \lambda_B) = -c_A + L \max \left\{ \begin{array}{l} 0, q + \lambda_B v(L) - \Gamma(L, \lambda_B) \\ -\frac{1}{2}p_i(2L; \lambda_A, \lambda_B) - \frac{1}{2}p_i(L; \lambda_A, \lambda_B) \end{array} \right\}.$$

Note that the activist has the advantage of knowing that company i is a target and that she would be a shareholder. This is reflected by the term $q + \lambda_B v(L)$. Using (4) and (22) completes the proof. Similarly, the bidder's expected profit net of search costs is trivially given by the expression in the statement. Note that

$$\begin{aligned} w(\alpha) - v(\alpha) &= (1-s)(1-\tau) \left[\int_b^\infty (\Delta - b) dF(\Delta) + \int_{\min\{b, \rho(\alpha)\}}^b \Delta dF(\Delta) \right] \\ &+ (1-s)\tau \left[\int_b^\infty (\Delta - b) dF(\Delta) + \int_{\min\{\frac{\kappa}{1-s}, b\}}^b (\Delta - \frac{s}{1-s}\kappa) dF(\Delta) \right] \\ &\quad + \int_{\min\{\frac{\kappa}{1-s}, b, \rho(\alpha)\}}^{\min\{\frac{\kappa}{1-s}, b\}} \Delta dF(\Delta) \end{aligned} \quad (27)$$

Since $\rho(\alpha)$ is decreasing with α , $w(\alpha) - v(\alpha)$ is increasing in α . Therefore, $\psi(\cdot)$ is an increasing function. ■

Proof of Proposition 2. In the proof we use the general terms for Π_A and Π_B as given in Lemma 7. Let $\mu(x, \gamma) \equiv \Pi_A(x, H(x), G(\psi(H(x))))$, and note that $\mu(x, \gamma)$ is continuous in x and γ . Also note that $\lim_{x \rightarrow \infty} \mu(x, \gamma) = -\infty$. Based on Lemma 5, since $\psi(\cdot)$ is a one-to-one function, c_B^* is uniquely determined by c_A^* .

We consider two cases. First, suppose $\mu(x, \gamma) < 0$ for all $x \geq 0$. We argue that $(c_A^*, c_B^*) = (0, \psi(0))$ is the unique equilibrium. Suppose on the contrary an equilibrium with $c_A^* > 0$ exists. In this case, $c_B^* = \psi(c_A^*)$, and the activist's profit when his cost is c_A^* is $\mu(c_A^*, \gamma)$. Since $\mu(x, \gamma) < 0$ for all $x \geq 0$, the activist strictly prefer not searching when $c_A = c_A^*$, contradicting the optimality of the threshold strategy c_A^* . Suppose $c_A^* = 0$. In this case, $\lambda_A^* = 0$, $c_B^* = \psi(0)$, and $\lambda_B^* = G(\psi(0))$. The activist's profit is $\Pi_A(c_A, \lambda_A^*, \lambda_B^*)$. Note that $\Pi_A(0, \lambda_A^*, \lambda_B^*) = \mu(0, \gamma)$. Since $\mu(0, \gamma) < 0$ and $\Pi_A(c_A, \lambda_A^*, \lambda_B^*)$ is decreasing c_A , $\Pi_A(c_A, \lambda_A^*, \lambda_B^*) < 0$ for all $c_A \geq 0$. It follows, the activist never searches and $c_A^* = 0$ is the optimal response. We conclude, if $\mu(x, \gamma) < 0$ for all $x \geq 0$ then $(c_A^*, c_B^*) = (0, \psi(0))$ is the unique equilibrium.

Second, suppose there is $\hat{x} \geq 0$ such that $\mu(\hat{x}, \gamma) \geq 0$. Since $\lim_{x \rightarrow \infty} \mu(x, \gamma) = -\infty$, by the intermediate value theorem the equation $\mu(x, \gamma) = 0$ has a non-negative solution. Let the solution be \hat{c}_A . We argue that $(c_A^*, c_B^*) = (\hat{c}_A, \psi(H(\hat{c}_A)))$ is an equilibrium. Indeed, if

$(c_A^*, c_B^*) = (\hat{c}_A, \psi(H(\hat{c}_A)))$ then $\lambda_A^* = H(\hat{c}_A)$ and $\lambda_B^* = G(\psi(H(\hat{c}_A)))$. Since $\mu(\hat{c}_A, \gamma) = 0$ then $\Pi_A(\hat{c}_A, \lambda_A^*, \lambda_B^*) = 0$. Similarly, $\Pi_B(c_B^*, \lambda_A^*) = 0$. Therefore, by construction, \hat{c}_A is the activist's best response to $\psi(H(\hat{c}_A))$, and $\psi(H(\hat{c}_A))$ is the bidder's best response to \hat{c}_A . Therefore, $(c_A^*, c_B^*) = (\hat{c}_A, \psi(H(\hat{c}_A)))$ is an equilibrium, as required.

Last, since $\mu(0, 0) > 0$ (whenever $b < \infty$), from continuity there is $\bar{\gamma} > 0$ such that if $\gamma \in [0, \bar{\gamma}]$ then $\mu(0, \gamma) > 0$. Therefore, in this region, a solution for $\mu(x, \gamma) = 0$ always exists, and any solution is strictly positive. Therefore, if $\gamma \in [0, \bar{\gamma}]$ then in any equilibrium, $c_A^* > 0$.

■

Proof of Proposition 3. Consider part (i), and note that $b < \rho$ implies $\theta^* = G(c_B^*) \int_b^\infty dF(\Delta)$. Since $\int_b^\infty dF(\Delta)$ does not change with respect to L, κ, γ , or s , and decreases with b , the result follows from Lemma 9 part(i).

Consider part (ii), and note that $b \geq \rho$ implies

$$\theta^* = G(c_B^*) \int_b^\infty dF(\Delta) + H(c_A^*) \int_\rho^b dF(\Delta).$$

Since $\frac{\partial \rho}{\partial \kappa} > 0$ and $\frac{\partial \rho}{\partial L} < 0$, the result follows from Lemma 9 part (ii). If $\rho < 0$ then based on Lemma 9 part (iii), θ^* does not change with κ and L . ■

Proof of Proposition 4. We prove that $AR(\chi)$ is given by (14) case by case. First, if $\chi = 13D$ then prior to the announcement the activist bought L shares. Therefore, with probability $\frac{1}{2}$ there was no liquidity demand and the price was $p_i(L)$, and with probability $\frac{1}{2}$ there was a liquidity demand and the price was $p_i(2L)$. Either way, after the announcement the price jumps to $p_i(2L)$. Therefore, conditional on the 13D filing, the average abnormal return is

$$AR(13D) = p_i(2L) - \frac{p_i(2L) + p_i(L)}{2}.$$

Substituting $p_i(z_i)$ with the expressions in (4), gives the result. Second, if $\chi = takeover|13D$ then prior to the announcement on the takeover the price was $p_i(2L)$. Based on Proposition 1, if a merger is announced then $\min\{b, \rho(L)\} \leq \Delta$ and the share price converges to the takeover offer, which is given by $q + s\Delta + \mathbf{1}_{b \leq \Delta} (1-s)b$. Therefore, conditional on a 13D filing, the

average abnormal return is

$$\begin{aligned} AR(\text{takeover}|13D) &= \frac{\int_{\min\{b,\rho(L)\}} [q + s\Delta + \mathbf{1}_{b \leq \Delta} (1-s)b - p_i(2L)] dF(\Delta)}{\int_{\min\{b,\rho(L)\}} dF(\Delta)} \\ &= \left[\frac{1}{1 - F(\min\{b, \rho(L)\})} - \lambda_B \right] v(L), \end{aligned}$$

where we used (5) and substituted $p_i(2L)$ with the expression in (4). Third, if $\chi = \text{takeover}|\emptyset$ then prior to the announcement on the takeover the price was $p_i(0)$.⁴² Based on Proposition 1, if a takeover is announced then $b \leq \Delta$ and the share price converges to the takeover offer, which is given by $q + s\Delta + (1-s)b$. Therefore, conditional on no 13D filing, the average abnormal return is

$$\begin{aligned} AR(\text{takeover}|\emptyset) &= \frac{\int_b [q + s\Delta + (1-s)b - p_i(0)] dF(\Delta)}{\int_b dF(\Delta)} \\ &= \left[\frac{1}{1 - F(b)} - \lambda_B \frac{1 - \lambda_A}{N - \lambda_A} \right] v(0) \end{aligned}$$

where we used (5) and substituted $p_i(0)$ with the expression in (4).

Last, we note that if $b < \rho$ then $AR(13D)$ does not change with L , κ , or γ . In this region, $v(0) = v(L)$, and hence, $AR(13D) = \frac{\lambda_B^*}{2} (1 - 1/N) v(0)$. Note that $v(0)$ is independent of these three parameters. Also note that according to Lemma 9, λ_B^* does not change with these parameters when $b < \rho$. This completes the argument. ■

⁴²Note that we invoked the assumption that the market maker of firm i does not observe 13D filings in other firms. If this assumption is relaxed, the market maker would use the information about other firms as follows. If there was a 13D filing in firm $j \neq i$, then the activist has identified firm j as the target, and a takeover of firm i never takes place. The price of firm i is q . If there was no 13D filing in any other firm, it must imply that the activist did not search. In this case, the price of firm i prior to the announcement of the takeover is $p_i(0, 0, \lambda_B^*)$ instead of $p_i(0, \lambda_A^*, \lambda_B^*)$. Therefore, $AR(\text{takeover}|\emptyset) = [\frac{1}{1-F(b)} - \frac{\lambda_B}{N}]v(0)$, and the observation that $AR(\text{takeover}|13D) < AR(\text{takeover}|\emptyset)$ does not change.

B Supplemental results

Lemma 8 [*Multiple equilibria*] Suppose $N \rightarrow \infty$, $\gamma \in [0, \bar{\gamma}]$, and $b > \rho(L)$. Then:

$$\frac{\partial \mu(x)}{\partial x} = g(\psi(H(x))) h(x) \times \frac{\partial \Pi_B}{\partial \lambda_A} \frac{\partial \Pi_A}{\partial \lambda_B} - 1, \quad (28)$$

where $\frac{\partial \Pi_B}{\partial \lambda_A} \frac{\partial \Pi_A}{\partial \lambda_B}$ is given by the product of (11) and (12). Moreover, if in addition $\gamma = 0$, $b \rightarrow \infty$, and $g(0) h(0) > \frac{1}{\frac{1}{2} L s (1-s) w(L)^2}$ then at least two equilibria exist.

Proof. According to Proposition 2, and expressions (10), (9) and (6), if $N \rightarrow \infty$, $\gamma \in [0, \bar{\gamma}]$,⁴³ and $b > \rho(L)$ then

$$\mu(x) = -x + G(\psi(H(x))) \frac{\partial \Pi_A}{\partial \lambda_B} - L\gamma q$$

and

$$\psi(\lambda_A) = (1-s) \int_b^\infty (\Delta - b) dF(\Delta) + \lambda_A \frac{\partial \Pi_B}{\partial \lambda_A},$$

where $\frac{\partial \Pi_B}{\partial \lambda_A}$ is given by (11) and $\frac{\partial \Pi_A}{\partial \lambda_B}$ is given by (12). Therefore,

$$\begin{aligned} \frac{\partial \mu(x)}{\partial x} &= -1 + \frac{\partial G(\psi(H(x)))}{\partial x} \frac{\partial \Pi_A}{\partial \lambda_B} \\ &= -1 + g(\psi(H(x))) h(x) \frac{\partial \Pi_B}{\partial \lambda_A} \frac{\partial \Pi_A}{\partial \lambda_B}. \end{aligned}$$

This proves the first part. Consider the second statement. Under the stated conditions, any equilibrium c_A^* must satisfy $\mu(c_A^*) = 0$, which is equivalent to

$$c_A^* = G\left(H(c_A^*) \frac{\partial \Pi_B}{\partial \lambda_A}\right) \frac{\partial \Pi_A}{\partial \lambda_B}.$$

Notice that $c_A^* = 0$ always solves this equation. Therefore, it is sufficient to show that $\frac{\partial \mu(x)}{\partial x}|_{x=0} > 0$ to prove that there also exists an equilibrium with $c_A^* > 0$. Based on (28), this condition holds if and only if

$$g(0) h(0) \times \frac{\partial \Pi_B}{\partial \lambda_A} \frac{\partial \Pi_A}{\partial \lambda_B} - 1 > 0 \Leftrightarrow g(0) h(0) > \frac{1}{\frac{1}{2} L s (1-s) w(L)^2}.$$

⁴³We assume γ is sufficiently small such that the activist's profit, including the disutility γq , is non-negative. If $b < \infty$ then $G(\psi(H(x)))$ is bounded away from zero. Therefore, there is $\hat{\gamma} \in (0, \bar{\gamma}]$ such that if $\gamma \in [0, \hat{\gamma}]$ indeed that is the case.

■

Lemma 9 Suppose $\gamma \in [0, \bar{\gamma}]$ and θ^* is either $\underline{\theta}^*$ or $\bar{\theta}^*$. Then:

(i) Suppose $b < \rho$ then:

(a) c_A^* strictly increases in L , strictly decreases in γ , and does not change with respect to κ .

(b) c_B^* does not change with respect to L , κ , or γ , and strictly decreases in b and s .

(ii) If $0 \leq \rho < b$, then both c_A^* and c_B^* strictly increase in L and strictly decrease in κ .

(iii) If $\rho < 0$, then both c_A^* and c_B^* do not change with L and κ .

Proof. By Proposition 2, an equilibrium always exists. Suppose $\gamma \in [0, \bar{\gamma}]$ and consider any parameter $y \in \{L, \kappa, s, b, \gamma\}$. Let $\mu(x, y)$ be the function given by (10), parameterized by y . Recall from the proof of Proposition 2 that $\gamma \in [0, \bar{\gamma}]$ implies that $\mu(x, y)$ can be written as

$$\mu(x, y) = -x + L \left[\begin{array}{c} \frac{1}{2} G(\psi(H(x))) \left[v(L) - \frac{H(x)}{N} v(L) - \frac{1-H(x)}{N} v(0) \right] \\ -\gamma q \left[1 - G(\psi(H(x))) \int_{\min\{b, \rho\}}^{\infty} dF(\Delta) \right] \end{array} \right]^+ \quad (29)$$

where $\psi(\cdot)$ is given by (9). Therefore, if (c_A^*, c_B^*) is an equilibrium then $\mu(c_A^*, y) = 0$ and $c_B^* = \psi(H(c_A^*))$. Moreover, since $\gamma < \bar{\gamma}$ then $c_A^* > 0$. We assume that $\mu(x, y)$, as a function of x , has the least and greatest solution. In Appendix B, we provide regulatory conditions under which this assumption is valid. We denote by $\underline{c}_A^*(y)$ and $\bar{c}_A^*(y)$ the smallest and greatest solutions of $\mu(x, y) = 0$.

We start by proving that if $c_A^* \in \{\bar{c}_A^*, \underline{c}_A^*\}$ then $\frac{\partial \mu(x, y)}{\partial x} \Big|_{x=c_A^*} \leq 0$. To see why, suppose that $\frac{\partial \mu(x, y)}{\partial x} \Big|_{x=\bar{c}_A^*} > 0$. Then there exists $x' > \bar{c}_A^*$ such that $\mu(x', y) > 0$. Since $\lim_{x \rightarrow \infty} \mu(x, y) = -\infty$, there exists $x'' > x'$ such that $\mu(x'', y) < 0$. Hence, by the intermediate value theorem, there exists $x^* \in (x', x'')$ such that $\mu(x^*, y) = 0$. But then x^* is an equilibrium which is strictly greater than \bar{c}_A^* , contradicting the definition of \bar{c}_A^* as the greatest equilibrium. The proof that $\frac{\partial \mu(x, y)}{\partial x} \Big|_{x=\underline{c}_A^*} \leq 0$ is similar. The case $\frac{\partial \mu(x, y)}{\partial x} \Big|_{x=c_A^*} = 0$ is a knife-edge case, in which the function $\mu(\cdot, y)$ is tangent to the x-axis at the equilibrium point c_A^* . Since we focus on local comparative statics, when the equilibrium continues to exist upon a small change in the parameter, this case will be ignored.

Next, consider part (i). Since $b < \rho$ then $v(L) = v(0)$ and

$$\begin{aligned} c_B^* &= (1-s) \int_b^\infty (\Delta - b) dF(\Delta) \\ c_A^* &= L \frac{1}{2} G(c_B^*) (1 - 1/N) \int_b^\infty [s\Delta + (1-s)b] dF(\Delta) \\ &\quad - L\gamma q \left(1 - G(c_B^*) \int_b^\infty dF(\Delta) \right) \end{aligned}$$

which are unique. Part (i.b) follows directly from the functional form above. The effect of L, κ , and γ and on c_A^* also follow directly from the functional form above. The only ambiguity is with respect to b and s . Parameter s also affects c_B^* , but in the opposite direction it affects c_A^* . The sign of the effect depends on the curvature of G . Parameter b suffers from the same ambiguity, but unlike s , it also has an ambiguous direct affect on c_A^* . This completes part (i.a)

Consider parts (ii) and (iii). Applying the implicit function theorem on $\mu(c_A^*, y) = 0$ yields

$$\frac{dc_A^*}{dy} = \frac{\frac{\partial \mu(x,y)}{\partial y} |_{x=c_A^*}}{\frac{\partial \mu(x,y)}{\partial x} |_{x=c_A^*}}.$$

Recall, $\frac{\partial \mu(x,y)}{\partial x} |_{x=c_A^*} < 0$ for $c_A^* \in \{\bar{c}_A^*, \underline{c}_A^*\}$. Therefore, $\text{sign} \left(\frac{dc_A^*}{dy} \right) = \text{sign} \left(\frac{\partial \mu(x,y)}{\partial y} \right)$. Notice that

$$\frac{\partial \mu(x,y)}{\partial \kappa} |_{x=c_A^*} = L \left[\begin{array}{c} \frac{1}{2} g(\psi(\lambda_A^*)) \frac{\partial \psi(\lambda_A^*)}{\partial \kappa} \left[v(L) - \frac{\lambda_A^*}{N} v(L) - \frac{1-\lambda_A^*}{N} v(0) \right] \\ - \frac{1}{2} G(\psi(\lambda_A^*)) \frac{\partial \rho}{\partial \kappa} s \rho f(\rho) \left(1 - \frac{\lambda_A^*}{N} \right) \\ + \gamma q \left(- \frac{\partial \rho}{\partial \kappa} f(\rho) G(\psi(\lambda_A^*)) + g(\psi(\lambda_A^*)) \frac{\partial \psi(\lambda_A^*)}{\partial \kappa} \int_\rho^\infty dF(\Delta) \right) \end{array} \right]$$

Since $c_A^* > 0$ and $0 \leq \rho < b$ then $\frac{\partial \psi(\lambda_A^*)}{\partial \kappa} < 0$. Also, based on (3) $\frac{\partial \rho}{\partial \kappa} > 0$. Therefore, $\frac{\partial \mu(x,y)}{\partial \kappa} |_{x=c_A^*} < 0$. We conclude that $\frac{dc_A^*}{d\kappa} < 0$. Notice that if $\rho < 0$ then $\frac{\partial \psi(\lambda_A^*)}{\partial \kappa} = 0$ and $\frac{\partial \rho}{\partial \kappa} = 0$, and hence, $\frac{dc_A^*}{d\kappa} = 0$. Similarly,

$$\frac{\partial \mu(x,y)}{\partial L} |_{x=c_A^*} = \frac{\mu(c_A^*, y)}{L} + L \left[\begin{array}{c} \frac{1}{2} g(\psi(\lambda_A^*)) \frac{\partial \psi(\lambda_A^*)}{\partial L} \left[v(L) - \frac{\lambda_A^*}{N} v(L) - \frac{1-\lambda_A^*}{N} v(0) \right] \\ - \frac{1}{2} G(\psi(\lambda_A^*)) \frac{\partial \rho}{\partial L} s \rho f(\rho) \left(1 - \frac{\lambda_A^*}{N} \right) \\ \gamma q \left(- \frac{\partial \rho}{\partial L} f(\rho) G(\psi(\lambda_A^*)) + g(\psi(\lambda_A^*)) \frac{\partial \psi(\lambda_A^*)}{\partial L} \int_\rho^\infty dF(\Delta) \right) \end{array} \right]$$

Since $c_A^* > 0$ and $0 \leq \rho < b$, $\frac{\partial \psi(\lambda_A^*)}{\partial L} > 0$. Also, based on (3) $\frac{\partial \rho}{\partial L} > 0$. Moreover, since c_A^* is an equilibrium, $\mu(c_A^*, y) = 0$. Therefore, $\frac{\partial \mu(x, y)}{\partial L}|_{x=c_A^*} > 0$. We conclude that $\frac{dc_A^*}{dL} > 0$. Notice that if $\rho < 0$ then $\frac{\partial \psi(\lambda_A^*)}{\partial L} = 0$ and $\frac{\partial \rho}{\partial L} = 0$, and hence, $\frac{dc_A^*}{dL} = 0$.

Finally, to complete the proof, we note that $c_B^* = \psi(H(c_A^*))$. Therefore, $\frac{dc_B^*}{dy} = \frac{\partial \psi(\lambda_A^*)}{\partial y} + \frac{\partial \psi(\lambda_A^*)}{\partial \lambda_A^*} \frac{d\lambda_A^*}{dy}$. Notice that $0 \leq \rho < b$ implies $\frac{\partial \psi(\lambda_A^*)}{\partial c_A^*} > 0$. Since $\frac{\partial \psi(\lambda_A^*)}{\partial \kappa} < 0$ and $\frac{dc_A^*}{d\kappa} < 0$, we conclude $\frac{dc_B^*}{d\kappa} < 0$. Since $\frac{\partial \psi(\lambda_A^*)}{\partial L} > 0$ and $\frac{dc_A^*}{dL} > 0$, we conclude $\frac{dc_B^*}{dL} > 0$. If $\rho < 0$ then $\frac{\partial \psi(\lambda_A^*)}{\partial c_A^*} = 0$. Since $\frac{\partial \psi(\lambda_A^*)}{\partial c_A^*} = \frac{\partial \psi(\lambda_A^*)}{\partial L} = \frac{dc_A^*}{d\kappa} = \frac{dc_A^*}{dL} = 0$, we conclude $\frac{dc_B^*}{dL} = \frac{dc_B^*}{d\kappa} = 0$ as well. ■

Supplemental material for the proof of Lemma 9. If $b \leq \rho$ then $v(L) = v(0)$ and $\mu(x)$, as given by (10) is reduced to

$$\mu(x) = -x + L \left[\frac{1}{2} G(\psi(0)) (1 - 1/N) v(0) - \gamma q \left(1 - G(\psi(0)) \int_b^\infty dF(\Delta) \right) \right]^+, \quad (30)$$

which is strictly decreasing in x , and hence, the equilibrium is unique. Suppose $b > \rho$. We prove that \underline{c}_A^* and \bar{c}_A^* exist if N is sufficiently large. Let

$$M(x) \equiv L \left[\begin{array}{l} \frac{1}{2} G(\psi(H(x))) \left[v(L) - \frac{H(x)}{N} v(L) - \frac{1-H(x)}{N} v(0) \right] \\ - \gamma q \left[1 - G(\psi(H(x))) \int_{\min\{b, \rho\}}^\infty dF(\Delta) \right] \end{array} \right] \quad (31)$$

and $\hat{\mu}(x) \equiv \max\{M(x), 0\}$. Note that $\hat{\mu}(x) \equiv \mu(x) + x$. It can be seen from (10), (6), and (5) that $\hat{\mu}(x)$ is bounded. Therefore, there exists B such that $\hat{\mu}(x) \in [0, B]$ for all x . Note that

$$\frac{\partial \hat{\mu}(x)}{\partial x} = \mathbf{1}_{\{M(x) > 0\}} \cdot \frac{\partial M(x)}{\partial x}. \quad (32)$$

Moreover,

$$\lim_{N \rightarrow \infty} \frac{\partial M(x)}{\partial x} = g(\psi(H(x))) h(x) \frac{\partial \Pi_B}{\partial \lambda_A} \frac{\partial \Pi_A}{\partial \lambda_B}$$

where $\frac{\partial \Pi_B}{\partial \lambda_A} > 0$ is given by (11) and $\frac{\partial \Pi_A}{\partial \lambda_B} > 0$ is given by (12). Therefore, from the continuity of $M(x)$ there is $N_0 \in (0, \infty)$ such that if $N > N_0$ then $\frac{\partial M(x)}{\partial x} > 0$, and therefore, $\frac{\partial \hat{\mu}(x)}{\partial x} \geq 0$. By Tarski's Fixed Point Theorem, $\hat{\mu}(x)$ has the least and greatest fixed points \underline{c}_A^* and \bar{c}_A^* on $[0, B]$. ■

Proposition 6 [Non-control activism] Suppose an activist with a stake of size L is expected to buy a stake in the target firm with probability λ_A . For any $\zeta \in [0, 1]$, the bidder's expected profit is given by $\Pi_B(c_B, \lambda_A, \zeta) = -c_B + \psi(\lambda_A, \zeta)$ where

$$\begin{aligned} \psi(\lambda_A, \zeta) &= (1-s) \int_b^\infty (\Delta - b) dF(\Delta) \\ &\quad + (1-s) \lambda_A \left(\int_{b/\zeta}^\infty (b - \Delta\zeta) dF(\Delta) + \int_{\min\{b, \rho(\alpha, \zeta)\}}^b \Delta(1-\zeta) dF(\Delta) \right) \end{aligned}$$

and

$$\rho(L, \zeta) = \max \left\{ 0, \frac{\kappa/L - \gamma q}{s + \zeta(1-s)} \right\}.$$

Moreover:

- (i) For all $\zeta \in [0, 1]$ there is $\bar{b} < \infty$ such that if $b > \bar{b}$ then $\frac{\partial}{\partial \lambda_A} \psi(\lambda_A, \zeta) > 0$.
- (ii) For all $\zeta \in (0, 1]$ there is $\underline{b} > 0$ such that if $b < \underline{b}$ then $\frac{\partial}{\partial \lambda_A} \psi(\lambda_A, \zeta) < 0$.
- (iii) If $\gamma = 0$, $\kappa/\alpha > \frac{[s+\zeta(1-s)]^2}{(1-s)(1-\zeta)}$, $b > \frac{\kappa/\alpha}{s+\zeta(1-s)}$, and $f\left(\frac{\kappa/\alpha}{s+\zeta(1-s)}\right) \rightarrow 1$ then $\frac{\partial^2}{\partial \lambda_A \partial \zeta} \psi(\lambda_A, \zeta) > 0$.

Proof. Suppose the first round of negotiations fails and the activist owns a stake of size α in the target firm. We start by arguing that if $b \leq \Delta$ then the activist never runs a proxy fight. To see why, notice that with the activist presence, the incumbent learns about the action than can increase firm value by $\zeta\Delta$. Since $b \leq \Delta$, under the incumbent board control, an agreement in which the bidder pays a premium of $s\Delta + (1-s) \max\{\zeta\Delta, b\}$ is always reached. Once the activist obtains control of the board, the standalone value of the firm increases by $\zeta\Delta$, and hence, the activist will accept a takeover offer if and only if the premium is greater than $-\gamma q + \zeta\Delta$. However, shareholders would reject any offer with a premium smaller than $\zeta\Delta$. Therefore, if an agreement between the activist and the bidder is reached, the expected takeover premium is $s\Delta + (1-s)\zeta\Delta$. So the activist has no incentives to run a proxy fight. We conclude, if $b \leq \Delta$ then the activist never runs a proxy fight, and the incumbent board reach an agreement in which the bidder pays a premium of $s\Delta + (1-s) \max\{\zeta\Delta, b\}$.

Second, suppose $\Delta < b$. We argue the activist runs a proxy fight if and only if $\rho(\alpha, \zeta) \leq \Delta$. Moreover, we argue that in the former case the activists reaches an agreement in which bidder pays a premium of $s\Delta + (1-s)\zeta\Delta$, and in the latter case the firm remains independent under the activist's control. Since $\Delta < b$, the incumbent will refuse the sell the firm or implement

the action proposed by the activist. Therefore, firm value is q . Shareholders always elect the activist. The activist has incentives to run a proxy fight if and only if

$$\begin{aligned} \alpha [q + s\Delta + (1-s)\zeta\Delta] - \kappa &> \alpha(1-\gamma)q \Leftrightarrow \\ \rho(\alpha, \zeta) &\leq \Delta, \end{aligned}$$

as required.

Given the first two steps,

$$\begin{aligned} \psi(\lambda_A, \zeta) &= (1-s)(1-\lambda_A) \int_b^\infty (\Delta - b) dF(\Delta) \\ &+ (1-s)\lambda_A \left(\int_{b/\zeta}^\infty \Delta(1-\zeta) dF(\Delta) + \int_b^{b/\zeta} (\Delta - b) dF(\Delta) \right) \\ &\quad + \int_{\min\{b, \rho(\alpha, \zeta)\}}^b \Delta(1-\zeta) dF(\Delta). \end{aligned}$$

Simple algebra shows that the expression above is equivalent to the one in the statement of the proposition.

Consider part (i). Note that

$$\frac{\partial}{\partial \lambda_A} \psi(\lambda_A, \zeta) = (1-s) \left[\int_{\min\{b, \rho(L, \zeta)\}}^b \Delta(1-\zeta) dF(\Delta) - \int_{b/\zeta}^\infty (\Delta\zeta - b) dF(\Delta) \right]$$

and

$$\lim_{b \rightarrow \infty} \frac{\partial}{\partial \lambda_A} \psi(\lambda_A, \zeta) = (1-s) \int_{\rho(L, \zeta)}^\infty \Delta(1-\zeta) dF(\Delta) > 0.$$

This completes part (i).

Consider part (ii), and suppose $\zeta > 0$. Then

$$\lim_{b \rightarrow 0} \frac{\partial}{\partial \lambda_A} \psi(\lambda_A, \zeta) = -(1-s)\zeta \int_0^\infty \Delta dF(\Delta) < 0,$$

which completes part (ii).

Finally, note that under the assumptions in part (iii),

$$\frac{\partial}{\partial \lambda_A} \psi(\lambda_A, \zeta) = (1-s) \left[\int_{\frac{\kappa/L}{s+\zeta(1-s)}}^b \Delta(1-\zeta) dF(\Delta) - \int_{b/\zeta}^\infty (\Delta\zeta - b) dF(\Delta) \right].$$

Therefore,

$$\frac{\partial^2}{\partial \lambda_A \partial \zeta} \psi(\lambda_A, \zeta) = (1-s) \left[\begin{array}{c} \frac{\kappa/L}{[s+\zeta(1-s)]^2} (1-s) \frac{\kappa/L}{s+\zeta(1-s)} f\left(\frac{\kappa/L}{s+\zeta(1-s)}\right) (1-\zeta) \\ - \int_{\frac{\kappa/L}{s+\zeta(1-s)}}^b \Delta dF(\Delta) - \int_{b/\zeta}^{\infty} \Delta dF(\Delta) \end{array} \right].$$

Since $f\left(\frac{\kappa/\alpha}{s+\zeta(1-s)}\right) \rightarrow 1$ then

$$\frac{\partial^2}{\partial \lambda_A \partial \zeta} \psi(\lambda_A, \zeta) \rightarrow (1-s) f\left(\frac{\kappa/L}{s+\zeta(1-s)}\right) \frac{\kappa/L}{s+\zeta(1-s)} \left[\frac{\kappa/L}{[s+\zeta(1-s)]^2} (1-s)(1-\zeta) - 1 \right].$$

and since $\kappa/\alpha - \gamma q > \frac{[s+\zeta(1-s)]^2}{(1-s)(1-\zeta)}$, we have $\frac{\partial^2}{\partial \lambda_A \partial \zeta} \psi(\lambda_A, \zeta) > 0$, as required. ■

Example for Section 5.3. We show that the probability of a takeover in baseline model under no commitment is higher than the probability of a takeover under the assumptions of Section 5.3. Suppose $\rho(L) < b$ and $G(x) \approx \mathbf{1}_{\{x=\psi(0)+\varepsilon\}}$ for some arbitrarily small $\varepsilon > 0$. Let $\bar{\lambda}_A$ be the probability the activist searches if she expects the bidder to arrive with probability one. Under these assumptions, the probability of a takeover in the baseline model is given by

$$\int_b^{\infty} dF(\Delta) + \bar{\lambda}_A \int_{\rho(L)}^b dF(\Delta).$$

Indeed, as long as $\lambda_A > 0$, the bidder will search with probability arbitrarily close to one. In equilibrium, it must be $\lambda_B^* = 1$ and $\lambda_A^* = \bar{\lambda}_A$. By contrast, with active solicitation, the probability of a takeover is given by

$$\bar{\lambda}_A \int_{\rho(L)}^{\infty} dF(\Delta) + (1 - \bar{\lambda}_A) G(\psi(0)) \int_b^{\infty} dF(\Delta) = \bar{\lambda}_A \int_{\rho(L)}^{\infty} dF(\Delta).$$

Indeed, if the activist searches then the bidder always starts negotiating an agreement. In equilibrium, the activist searches with probability $\bar{\lambda}_A$. However, if the activist does not search then $c_B^* = \psi(0)$. Therefore, the bidder will search with probability zero. As can be seen from the two expressions above, the expected probability of a takeover is higher in the baseline model. ■

Examples for Section 3.4.

We first provide the example of the ex-ante probability of takeover decreases with b , the

incumbent's private benefits of control. We set the parameters as follows: $L = 0.10$, $N = 100$, $\gamma = 0$, $s = 0.95$, $\kappa = 0.03$, $F(\Delta) = 1 - e^{-\Delta}$, and $G(c) = H(c) = 1 - e^{-50c}$. In this example, the probability of takeover decreases from 91.79% to 47.85% as b increases from zero to 1.5, but then *increases* to 54.5%.

Next we provide an example where the ex-ante probability of takeover increases with γ , the activist's bias for early liquidation. We set the parameters as follows: $L = 0.05$, $N = 100$, $s = 0.9$, $\kappa = 0.06$, $b = 3$, $q = 3$, $F(\Delta) = 1 - e^{-\Delta}$ if $\Delta \geq 0$, and $G(c) = H(c) = 1 - e^{-100c}$. In this example, the probability of takeover increases from around 27% to 90% as γ increases from $\gamma = 0$ to $\gamma = 0.35$.

Finally, we provide an example where the average abnormal returns to announcement of 13D filing of the activist, $AR(13D)$, increase as with κ . We set the parameters as follows: $L = 0.05$, $N = 10$, $s = 0.75$, $\gamma = 0.2$, $b = 75$, $q = 100$, and chose lognormal distributions, specifically $F(\Delta)$ following a lognormal distribution whose corresponding normal has mean 3.85 and standard deviation 0.7, and $G(c) = H(c)$ following a lognormal distribution whose corresponding normal has mean 0.1 and standard deviation 0.01. In this example, $AR(13D)$ increases from 17.64% to 18.35% if κ increases from $\kappa = 0$ to $\kappa = 1.65$, but decreases for larger κ . ■