

# Multiple Borrowing: Implications for Investment, Misallocation, and Technology Choice

Daniel Green and Ernest Liu\*

August 12, 2015

## Abstract

Falling barriers to credit access have had some unexpected consequences. In the developing world, the rapid entry of lenders into credit markets has led to simultaneous borrowing from multiple lenders, high interest rates and high levels of debt and default. We develop a model that rationalizes these outcomes when borrowers cannot commit to exclusive borrowing from a single lender, and show such commitment problems can also have perverse effects on investment and technology choice. We illustrate a trade-off between the efficiency of production and the degree of commitment (not to borrow further) embedded in it. The most productive investment opportunities may receive low levels of investment, and entrepreneurs may endogenously choose to adopt inferior production technology as long as it is sufficiently concave. These problems are exacerbated when borrowers have access to more lenders, providing an explanation of why increased access to finance does not always improve aggregate outcomes.

---

\*Massachusetts Institute of Technology. We thank Daron Acemoglu, Marios Angeletos, Abhijit Banerjee, Vivek Bhattacharya, Glenn Ellison, Erik Loualiche, Andrey Malenko, Jonathan Parker, Ben Roth, Alp Simsek, Ludwig Straub, and Rob Townsend for guidance and helpful comments, and conference participants at the the University of Chicago (2015), London Business School TADC (2015), and seminar participants at MIT student lunches for helpful comments.

# 1 Introduction

The ability to borrow from multiple lenders is a sign of financial development, yet appears problematic in emerging economies. The quintessential example of this is the spectacular boom and bust of the microfinance industry in the Indian state of Andhra Pradesh. Success of early microfinance initiatives led to the rapid commercialization of microfinance and the entry of thousands of lenders into the industry.<sup>1</sup> Borrowers took out loans from many lenders simultaneously, accumulating large debt balances, and culminating in a default crisis and near-collapse of the industry in 2010 when Andhra Pradesh passed a law limiting the collection tactics used by lenders. A policy report estimates that 84% of rural villagers in Andhra Pradesh had loans from multiple lenders outstanding in 2009.<sup>2</sup> Fear and realization of such multiple borrowing crisis have arisen around the world everywhere microfinance has expanded rapidly through the entry of new lenders.<sup>3</sup>

Why might multiple borrowing be such a problem in the developing world? The Malegam Report (2011), commissioned by the RBI to study issues in microfinance following the Andhra Pradesh crisis, attributes the problems with multiple borrowing to the naivety of borrowers or the deterioration of underwriting standards in response to intense competition. These theories are unappealing because they should also apply to financially developed countries, where multiple borrowing exists, yet does not create problems. It is more likely that the answer lies in the *differences* between the poor and rich worlds—specifically with respect to the quality of the institutions and contracting environments through which borrowing takes place. One potentially important issue is the inability for borrowers and lenders to contractually commit to exclusive relationships. If borrowers are unable to commit to borrowing from a single lender they may have incentives to accumulate additional debt from others, in the process harming their original lender by increasing the likelihood of default. In this paper we develop a model for studying multiple borrowing in the context of such commitment problems and show that it can explain many facts about the nuanced relationship between financial development, resource allocation, and growth.

In the model, an entrepreneur visits multiple lenders to obtain funds for a new in-

---

<sup>1</sup>Wichterich (2012)

<sup>2</sup>Johnson and Meka (2010). MicroSave found at the time of crisis, 51% have loans from  $\geq 3$  MFIs

<sup>3</sup>A set of case studies of microcredit markets in Peru, Guatemala, and Bolivia (de Janvry et al. (2003)) highlight the prevalence of multiple borrowing and high levels of debt that occurs following the rapid entry of lenders in each of these countries. McIntosh et al. (2005) find evidence that lender entry in Uganda led to increased multiple borrowing and default. Policy makers in Bangladesh are concerned that incidence of “overlapping borrowing” across multiple microfinance lenders is on the rise and incidence may be as high as 60%. (Faruquee and Khalily (2011))

vestment opportunity. Crucially, the entrepreneur lacks the ability to commit to exclusive borrowing from a single lender and cannot write loan contracts that are contingent on the terms of contracts subsequently signed with other lenders. The entrepreneur faces a random cost of default that is realized when loans come due, only defaulting if the debt owed exceeds this cost. Thus, the more debt owed the less likely it is to be repaid. This gives rise to a externality between lenders. New lenders willingly provide additional investment that existing lenders would not, because new lenders do not internalize the decreased likelihood of repayment of *existing* debt when pricing new debt contracts. Similarly, such new loans are also attractive to borrowers—they do not internalize the full costs of such borrowing because previous debt is sunk. Rational lenders anticipate this additional borrowing and offer loan terms that compensate them for it, making multiple borrowing undesirable ex-ante, but without commitment unavoidable ex-post. Inability to commit to an exclusive lending relationship is thus a binding constraint and in equilibrium induces distortions in lending and investment outcomes.

We first use this framework to show that increasing access to additional lenders when commitment is limited raises interest rates and entrepreneur's indebtedness inefficiently. This result is a refinement of the results in Bizer and DeMarzo (1992) and is consistent with the observed high levels of debt, high default rates, and high interest rates in micro-finance markets where borrowers can obtain loans from many lenders simultaneously.

We then show that distortions to investment induced by commitment problems depend crucially on the nature of the new investment opportunity the entrepreneur is attempting to finance, and find that more is not always better. More efficient opportunities may receive less investment. The availability of more lenders reduces investment and raises interest rates. The most promising investment opportunities may not be undertaken at all. These results together introduce a novel microfoundation for the relationship between credit market imperfections and aggregate resource misallocation.

At the heart of these results is that lower marginal returns create *endogenous commitment power* not to borrow (much) from additional lenders—the benefits of marginal investment return are not as attractive relative to the cost of higher debt obligations. Since entrepreneurs cannot commit *ex-ante* to limit inefficient future borrowing, properties of their future marginal returns to investment that induce them to limit such borrowing *ex-post* are especially valuable. This induces a trade-off between an investment technology's efficiency and concavity. Low marginal (and hence average) returns are bad for output, but declining marginal returns are good for incentives. When marginal returns are high, borrowers who lack commitment not to continue borrowing may end up on the wrong side of the "debt Laffer curve," meaning they are receiving an amount of investment that

could have been supported by a lower quantity of debt, if only they could have committed to it. Financial constraints are thus endogenously more severe for better projects when they also have sufficiently higher marginal returns.

This relationship between financial constraints and productivity generates a particularly stark form of misallocation: a negative correlation between the *level* of investment in a project and its productivity. In the presence of commitment problems, projects that warrant the most investment end up receiving the least. This prediction of our model is consistent with a growing body of evidence from micro and experimental studies conducted in developing economies that show firms with high marginal products are especially credit constrained (see, for example, McKenzie and Woodruff (2008) for evidence from microenterprise in Mexico and Banerjee and Duflo (2014) for the context of India). In particular, this is a stronger form of misallocation than the failure to equalize marginal returns to capital across entrepreneurs (Hsieh and Klenow (2009)). Our model shows that these issues are exacerbated by increasing the ease with which borrowers can obtain funds from multiple lenders.

As far as we know, our model is unique in generating this empirically relevant form of misallocation. While the failure to equalize marginal returns can easily be explained by credit constraints, *misallocation in levels* can only occur in models in which the degree of credit constraints is influenced by the properties of the productive activities themselves. While forces such as moral hazard and asymmetric information fall into this category, in these examples there is no clear mapping between the degree of the friction and the productive efficiency of the underlying economic activity. A negative correlation between productivity and investment would need to come from an arguably arbitrary parameterization of how these forces vary across productivity levels. In contrast, because the commitment friction is dampened or exacerbated by the shape of the production function, there is a natural mapping in our model between productivity and the degree of credit constraints. Thus, while quite simple in its setup and assumptions, it is capable of generating stark results.

Next, we endogenize project choice by the entrepreneur to reveal the full extent of the misallocative forces in our model. Not only can lack of commitment cause better projects to receive less investment, but it can also cause entrepreneurs to reject the most profitable projects entirely in favor of less efficient endeavors. This can explain why economic activity occurs well inside the technological frontier and why it is difficult to grow out of financial constraints in the long run. When commitment problems are severe, constrained entrepreneurs choose business plans that have low prospects for expansion precisely because these are the easiest to finance. Those who instead choose better projects will receive

low levels of funding at high interest rates, reducing their ability to accumulate equity to finance expansion. In either cases there is limited scope for growth.

Finally, our model has several important implications for credit market policy and regulation. Through commitment problems, increasing the number of lenders in a market *increases* interest rates and decreases borrower surplus, exactly the opposite of the typical effects of entry and competition. This is of particular concern to regulators that use prevailing interest rates to gauge the level of competition in a lending market. From a regulator's perspective, our model says that high interest rates may be a sign of *excessive* competition and care should be taken not to cause further harm by allowing even more entry. In addition, we show that the distortions induced by multiple borrowing can be ameliorated by simple regulatory tools—debt limits and interest rate caps improve efficiency as long as they are not too restrictive. Limiting the number of lenders one can borrow from would also improve outcomes in our model. Intuitively, each of these policies help borrowers obtain commitment power by limiting their ability to borrow in the future and the extent to which the debt held by early lenders can be diluted.

India's policy response to Andhra Pradesh microfinance crisis was consistent with the regulatory prescriptions of our model. In 2011, the Reserve Bank of India imposed new regulations on the banking industry and stated that they were in part meant to address multiple borrowing. A debt limit (USD 790) and interest rate cap (26%) were imposed.<sup>4</sup> The Malegam Report (2011) also recommended borrowers could only have loans from at most two microfinance lenders simultaneously, but this was not implemented in the final regulations. Each of these policies, our model shows, helps limit the impact of lack of commitment in lending markets. These policies are all imperfect, however, and operate by imposing limits that embed in them commitment guarantees that private agents could not themselves enforce.

Our model therefore highlights that commitment problems are especially salient wherever it is difficult to monitor, verify, or enforce agreements between multiple parties. If borrowers could commit, or incentivize themselves to commit through contingent contracting, the externalities that arise in our model would disappear and outcomes would improve, including the efficiency of investment allocation across projects.

In fact, contracting developments in sophisticated financial markets provide further evidence for the empirical relevance of the commitment friction we model. In general, debt covenants that limit the ability of a borrower to raise additional external financing, or require concessions to current investors for doing so, would ameliorate the problems

---

<sup>4</sup>It was also stated that at least 75% of the total loans originated by any regulated MFI should be for the purpose of income generation. (Reserve Bank of India (2011))

highlighted in our model. A specific form of contingent contracting that solves such commitment problems is widely employed in syndicated loan market in the United States. In this market, so called “performance-based pricing” covenants allows interest rates to vary based on changes in observable firm characteristics that occur *after* the loan has been issued. Making interest rates increasing in a firm’s total amount of debt, as is typically observed in performance-based pricing covenants, internalizes the spillovers between lenders and restores the full-commitment lending outcomes, even when borrowers cannot explicitly agree to exclusive borrowing.<sup>5</sup> Such contingent contracting can be difficult and costly to implement, but our model shows it may be important for obtaining efficient allocations of financing to the most productive investment technology.

Theoretically, our model is based on the theme of common agency, which describes environments in which multiple principals with possibly conflicting interests act to influence the behavior of a single agent. In our setup, the principals can be thought of as the multiple lenders, each with a limited ability to control total lending by the borrower, the agent. Early related work on common agency is a collection of papers by Arnott and Stiglitz (1991; 1993, references therein), which study common agency in insurance markets. These papers recognize that additional insurance providers can impose externalities on each other through moral hazard of the buyer. The more insurance the buyer acquires, the more likely adverse events occur for which all insurers are liable. Arnott and Stiglitz (1991) find that Nash equilibria do not always exist in such settings. When a Nash equilibrium does exist, it could take the form of a single insurer providing a level of coverage that discourages the client from purchasing additional insurance contracts.

Bizer and DeMarzo (1992) brings this idea to credit markets.<sup>6</sup> They study a model in which a borrower with a desire to smooth consumption between two periods can sequentially visit lenders to obtain loans backed by the borrower’s stochastic next-period income, which is influenced by non-contractible effort choice. Again as in the Arnott-Stiglitz series of papers, moral hazard generates an externality that new lenders offer loans that harm previous lenders by decreasing their expected profits. The equilibrium in this model corresponds to the single-insurer competitive equilibrium of Arnott and Stiglitz (1991), where a single bank offers a loan that is inefficient but incentivizes the borrower not to obtain additional loans from other lenders. In equilibrium, effort is lower and borrowing amounts and interest rates are higher than they would be if the borrower could commit to visiting only a single lender. As characterized by Arnott and Stiglitz

---

<sup>5</sup>See Asquith et al. (2005) for a detailed description of performance based pricing covenants.

<sup>6</sup>Other papers on the impact of lack of commitment in credit markets include Bisin and Rampini (2006), Kahn and Mookherjee (1998), Parlour and Rajan (2001), Boot and Thakor (1994) and Brunnermeier and Oehmke (2013).

(1991), however, the existence of equilibrium in Bizer and DeMarzo (1992) is not guaranteed in general. Equilibrium existence is achieved in that paper by imposing assumptions on the concavity of the borrower's utility function, ensuring that there is always a loan contract that both guarantees borrowers will not visit additional lenders and provides the borrower some surplus relative to the no-lending allocation.

The model of sequential borrowing in our paper is related to that of Bizer and DeMarzo (1992), with several important differences. We build a dynamic game in which the number of lenders the borrower has access to is stochastic; at any point borrowers face the uncertainty of losing access to new lenders and further borrowing. This generalization allows us to generate comparative statics of the degree of the commitment, with extreme lack of commitment as in Bizer and DeMarzo (1992) and full commitment as the two limiting cases in our model. Second and more substantively, our formulation of multiple borrowing as a stochastic dynamic game with discounting allows us to sidestep issues related to the existence of equilibrium and to achieve an exact analytic solutions to the model. The equilibrium of our model always exists and can be found as the solution to a fixed point dynamic programming problem. Solving the model in closed form allows us to study how the returns to scale of production technology affect equilibrium outcomes in the face of commitment problems, leading to our novel results on misallocation of resources that are particularly salient in the developing world where multiple borrowing is most obviously problematic.

Also related to our paper is the seminal analysis of the effects of competition on relationship lending developed in Petersen and Rajan (1994, 1995). Their theory predicts that competition in lending markets is harmful because it erodes the ability of lenders to establish long term relationships that are necessary to invest in credit constrained firms. Aside from the common prediction that increasing presence of lenders could adversely affects welfare, our model generates novel and distinct implications on investments from very different micro-foundations. In their model, if lenders have limited market power they cannot offer subsidized interest rates early in a firm's life that are necessary to avoid moral hazard and recoup profits later through rent extraction. In contrast, we abstract from market power and focus on a completely different mechanism the inability to refrain from multiple borrowing, which is saliently featured in a wealth of empirical evidence and discussed heavily among policy makers and in the popular press in many developing countries.

The rest of the paper is organized as follows. Section 2 describes our model and section 3 defines and solves for the equilibrium. In section 4 we show the implications of lack of commitment in credit markets with multiple lenders on interest rates and default rates,

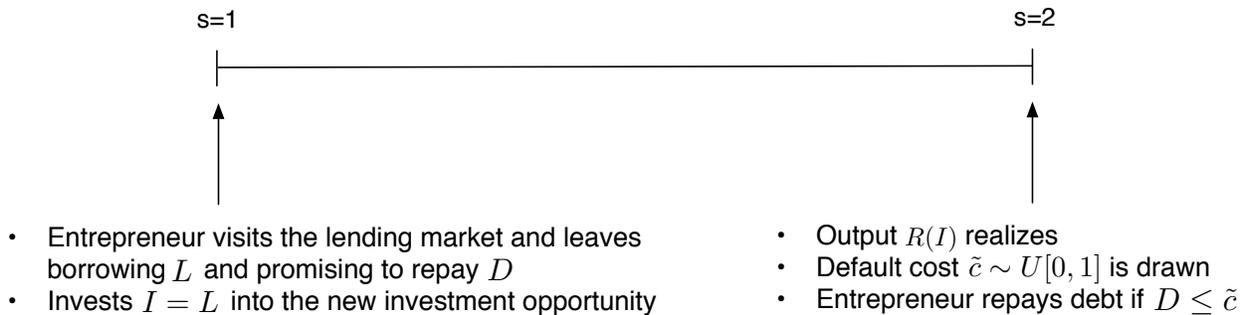


Figure 1: Timing of the Model

and derive our main result that entrepreneurs with more attractive investment opportunities can endogenously face more severe credit constraints and thus invest less than those with inferior projects. We illustrate the implication on inefficient project choice in section 5. We next examine, in section 6, policies adopted by regulatory agencies to address the multiple borrowing problem through the lens of our model. Section 7 concludes. Proofs of all propositions and lemmas are relegated to the Appendix.

## 2 Model

### Overview and Timing

An entrepreneur attempts to finance a new investment opportunity with the constraint that it cannot commit to exclusive borrowing from a single lender and that there is limited enforcement of debt repayment, ie. the borrower only repays if the costs of default are high enough. The model has two stages ( $s = 1, 2$ ) and contains two sets of agents: a single entrepreneur and an infinite sequence of potential lenders. All parties are risk neutral and have no discounting. The entrepreneur is endowed with a *variable-scale* investment opportunity that returns  $R(I)$  deterministically at  $s = 2$  for any  $I \geq 0$  invested at  $s = 1$ . While the returns to this investment opportunity are observable, they are not perfectly pledgeable. After the projects returns are realized, the entrepreneur learns of its cost of defaulting on its debt, and only repays if the default cost exceeds the amount of debt owed. Specifically, we assume default costs  $\tilde{c}$  are drawn from a distribution  $F_{\tilde{c}}$ . Debt  $D$  is repaid if  $\tilde{c} > D$ , and we denote the probability of repayment as  $p(D) \equiv 1 - F_{\tilde{c}}(D)$ , which is of course a decreasing function of  $D$ . Figure 1 summarizes the timing of the model.



cursively and obtain a simple closed form solution. The parameter  $q$  exogenously limits the amount of commitment power borrowers can obtain in the lending market. The closer  $q$  is to zero, the less likely it is that contingencies arise in which early lenders can be exploited by further borrowing. There are several appealing interpretations of this parameter. First,  $q$  can be thought of as inversely related to the difficulty or cost of subverting commitment, and reflects the quality of the contracting environment. As  $q \rightarrow 0$  the contracting environment is able to perfectly enforce contingency in loan terms. When  $q \rightarrow 1$  borrowers can costlessly find new lenders to provide marginal lending. A second view is that  $q$  reflects the composition of search frictions in the lending market and the limited time an entrepreneur has to raise money for an investment opportunity. Under this interpretation one would assume a complete inability to conduct contingent contracting or exclusive borrowing—the severity of the commitment problem is determined by the market structure of lending and time preference for funding.

The assumption of sequential borrowing from a possibly very large number of banks need not be taken literally. Intuitively what drives our results is that lack of commitment to an exclusive lending relationship leaves room for the externalities between lenders and perversely affects equilibrium outcomes. Indeed, our results are exactly the same (but much messier) when the maximum number of lenders a borrower could potentially visit is fixed and finite. In this case, there is a unique subgame perfect equilibrium of the model, and as the maximum number of lenders approaches infinity these equilibria converge to the stationary Markov perfect equilibrium we use to characterize our results.

Further, regarding sequential borrowing, our results hold in a model in which the entrepreneur borrows simultaneously from a finite number of lenders where loan terms cannot be made contingent on the other loans taken by the borrower. However, such a model is subject to the standard critique in the simultaneous contracting literature that the results are sensitive to the specification of agents' off-equilibrium beliefs (Segal and Whinston (2003)). We prefer the sequential formulation adopted in our paper because it delivers the same economic insights while sidestepping the technical modeling issues on the choice of off-equilibrium beliefs and equilibrium existence.

The random default cost formulation of default choice is common to the development and international finance literatures and is often referred to as “limited enforcement” or “limited commitment.” A random default cost simply reflects the fact that forces outside of the model generate ex-ante indeterminacy in the ex-post costs of defaulting on debt. For example, weak institutions are more than capable of generating such variation in default costs. In the Andhra Pradesh default crisis, repayment rates plummeted from near-perfect to near-zero almost overnight because grandstanding local politicians urged

borrowers to stop repaying their debts. Further, we will assume that the distribution of default costs is not a function of the amount invested or debt owed. Thus, our model shows that taking ex-ante debt capacity as given, variation in the returns to new investment opportunities have interesting interactions with multiple borrowing.

Before we move on to characterizing the equilibrium of our model, it is worth pointing out what our model may appear to be but is not. It is not a model of lender competition and it does not speak to how the distribution of market power among lenders affect lending outcomes. Lenders behave competitively and passively in our model as the only decision they make is to accept or reject offers from the entrepreneur.<sup>7</sup> Outside of our model, entry of new lenders creates pro-competitive pressure on incumbents, which could lead to an improvement in the allocation of resources and welfare by lowering markups in interest rate as well as by reducing search cost, or it could tighten credit constraints by compromising long-term relationships as pointed out by Petersen and Rajan (1994, 1995). We choose to abstract away from these forces in the model, and instead we highlight the negative externalities due to lack of commitment to refrain from multiple borrowing. As we have argued in the introduction, we focus on this contracting friction precisely because of its relevance to the empirical evidence in many of the credit markets in which our theory applies.

### 3 Equilibrium

To begin solving the model, we introduce two simplifying assumptions which aide greatly in the exposition of the model and in highlighting the underlying economic forces generating our results.

**Assumption 1.** *The return on the new investment opportunity is linear:*

$$R(I) = \alpha I.$$

**Assumption 2.** *Default costs for the entrepreneur are uniformly distributed between zero and one.*

$$\tilde{c} \sim U[0, 1].$$

Linearity in returns to the investment opportunity means that the level of investment financing obtained from prior lenders is not directly relevant for the objective of the bor-

---

<sup>7</sup>This does not imply that lenders cannot obtain surplus in our model - we can trivially incorporate lender surplus by raising lenders' outside options and setting the risk-free rate to be above one.

rowers or lenders at any stage of the lending game. Debt repayment costs for the borrower are always only a function of the total face value of debt. With linearity, benefits of additional investment from new lenders also do not depend on the level of investment financing previously raised. This simplification will help ground our choice of equilibrium concept (to be introduced below) and allow the borrower's value function at any stage of the game to be written in a simple recursive form.

The assumption on the distribution of default costs represents both an economic restriction and a simplifying functional form choice. As discussed in Section 3, we are assuming that default costs are not a function of properties of the investment opportunity or the terms of the entrepreneur's borrowing, so that in effect the returns from new investment opportunities are not directly pledgeable. The uniform distribution of default costs between 0 and 1 generates simple expressions for properties of debt repayment. The probability of repayment is now simply  $p(D) = 1 - D$  and the expected debt servicing cost (either repayment or strategic default) is  $\mathbb{E}[\min(\tilde{c}, D)] = D - D^2/2$ .

## The Full Commitment Case

We now characterize the solution to the model when borrower can commit to visiting a single lender and use this as a benchmark to illustrate how lack of commitment affects outcomes in our model. The ability of the entrepreneur to commit to borrowing from a single lender can be captured by setting the probability that the entrepreneur retains access to the lending market to zero ( $q = 0$ ), since this ensures it is common knowledge that the borrower will visit exactly one lender.

The above model now simplifies substantially. The entrepreneur makes a take-it-or-leave-it offer  $(D, L)$  to the lender and invests  $I = L$  in the new investment opportunity. The lender accepts any loan offer that is weakly profitable in expectation. The optimal lending contract is found by maximizing the entrepreneur's welfare subject to the lender participation constraint. Denote this problem  $P^{SL}$  for the "single lender" problem:

$$\begin{aligned}
 (P^{SL}) \quad & \max_{D, I} \quad \underbrace{R(I)}_{\text{payoff from new project}} \quad - \quad \underbrace{\mathbb{E}[\min(D, \tilde{c})]}_{\text{expected debt repayment/default costs}} \\
 & \text{s.t. } I \leq p(D) D \quad (\text{lender's participation constraint})
 \end{aligned}$$

**Proposition 1.** *Under Assumptions 1 and 2, the solution to the single lender problem  $P^{SL}$  can be*

characterized by the condition

$$\underbrace{\alpha \times \left( p \left( D^{SL} \right) D^{SL} \right)}_{\text{marg. ret. to new proj.}} \times \underbrace{\left[ p \left( D^{SL} \right) + p' \left( D^{SL} \right) D^{SL} \right]}_{\text{marg. } \Delta \text{ in inv.}} = \underbrace{p \left( D^{SL} \right)}_{\text{marg. } \Delta \text{ in value of resid. claims}} \quad (1)$$

At the optimum, the costs and benefits of pledging an additional dollar of face value of debt must be equalized. The marginal cost is simply the increase in expected debt repayment costs associated with the additional borrowing. The gain from pledging an additional dollar of face value of debt is the marginal return to investment times the marginal investment that can be raised from pledging an additional dollar of face value of debt. Because extra repayment reduces the value of all debt claims (through limited enforcement), the value of *existing* debt falls when new debt is issued. Because in this example there is only a single lender, the lender internalizes the change in value of all existing debt coming from the marginal issuance of new debt, and thus can make zero profits by supplying  $p(D) + p'(D)D$  of investment for a marginal dollar of promised repayment. We now highlight the externality that arises when the entrepreneur can obtain loans from more than one lender.

## Illustrating the Commitment Problem

When  $q > 0$  the borrower cannot obtain the same loan contracts from the first lender that it could when  $q = 0$  because the first lender anticipates the possibility that the borrower will obtain additional debt from future lenders that dilutes the value of its original debt claims. To illustrate why this problem arises, imagine the borrower can borrow sequentially from two lenders, but cannot commit to deal exclusively with one lender or to make terms of either loan conditional on those of the other. Therefore, loan terms with the second lender are arranged to maximize the surplus to the borrower and the second lender taking the terms of the first loan as fixed.

In this example the second lender imposes an externality on the first whenever the first lender has issued a positive amount of debt. To see this, denote the face value of debt promised to the first lender by  $d_1$  and the face value of debt promised to the second lender by  $d_2$  and define  $D \equiv d_1 + d_2$  to be the total face value of debt. Subject to the same regularity conditions as above, taking aggregate debt  $D$  as given the following first order equation characterizes  $d_2$ , the amount of debt issued to the second lender.

$$\alpha \times (p(D)D) \cdot [p(D) + p'(D) \cdot d_2] = p(D). \quad (2)$$

This is identical to Equation 1 except that the term in square brackets is different. Here, the second lender only internalizes the change in the value of its own debt due to its marginal debt issuance at the optimum, not the change in the total value of debt issued. Thus the second lender imposes an externality on the first. When borrowers lack commitment lenders must anticipate future borrowing and charge higher interest rates that compensate them for value they expect to lose. This is exactly why the commitment problem changes lending outcomes in our model and is the key intuition behind the results in Bizer and DeMarzo (1992) that multiple lenders lead to higher interest rates and higher borrowing than would be obtained if commitment were possible. Now that we have clearly illustrated why lack of commitment changes outcomes we turn to solving the model and showing exactly how lack of commitment affects investment outcomes and technology choice.

## Equilibrium without Commitment

To generate our results we focus attention on a Stationary Markov Perfect Equilibrium (SMPE) concept, in which borrower and lender strategies are a function only of a single state variable  $D$ , the cumulative face value of debt the issued so far in the game, and for the lenders of course also a function of the current loan being proposed. This equilibrium concept is stationary in the sense that we restrict all lenders to play the same strategy (as function of  $D$ ). As we discuss further at the end of this section, this choice of equilibrium concept is motivated by the fact that the unique subgame perfect equilibria of finite-lender truncations of this game converge to this equilibrium as the number of potential lenders approaches infinity. We now provide a formal definition of Stationary Markov Perfect Equilibrium of the lending market game.

**Definition 1.** A Stationary Markov Perfect Equilibrium of the lending market game is a set of borrower and lender strategies that are mutual best responses at every subgame when subject to the following constraints. The entrepreneur's strategy when encountering any lender is a mapping from how much it has already pledged to repay,  $D$ , to a simple debt contract  $(L_i, D_i)$ . All lenders' strategies are represented by a (common) function mapping from the state variable  $D$  and the loan contract proposed by the entrepreneur to the lender's decision to either accept or reject the proposal. .

We begin characterizing the equilibrium of this game by exploring the best responses of each player.

## Lender Best Responses

Lenders have rational expectations of future borrowing and thus only accept contracts that yield non-negative expected returns taking the strategies of the borrower and other lenders as given. A lender receiving a loan proposal must, given the state of the game, evaluate the expected profit from the loan, taking into account the strategies of the borrower and future lenders, and the likelihood that the borrower will be able to meet these lenders at all. Specifically, define  $D' \equiv D + D_i$ , and denote  $\tilde{p}_i(D')$  to be lender  $i$ 's perceived probability that the entrepreneur will not default on a loan  $(L_i, D_i)$  to this lender, given the borrower has previously obtained total face value of debt  $D$ . Such a loan is weakly profitable in expectation if  $L_i/D_i \leq \tilde{p}_i(D')$ . Since lenders are risk neutral and do not discount stage 2 cashflows, lender  $i$ 's optimal strategy is to accept the loan offer if and only if it is weakly profitable.

The SMPE restriction to stationary strategies restricts each lender to use the same decision rule as a function of  $D$  and the loan proposal it receives  $(L_i, D_i)$ . Combined with the requirement that lenders only accept profitable loans, this implies that in a SMPE all lenders have the same expectations of repayment as a *function* of aggregate face value of debt conditional on their loan being approved. In other words, there is a function  $\tilde{p}(\cdot)$  such that  $\tilde{p}_i(\cdot) = \tilde{p}(\cdot)$  for all lenders  $i$ . Because of this, we can index potential equilibrium strategies of lenders by  $\tilde{p}(\cdot)$ .

## Borrower Best Response

Conditional on meeting a lender, and given lender strategies indexed by  $\tilde{p}(\cdot)$ , the borrower makes a loan offer that maximizes its expected continuation utility, taking into account the chances of being able to meet more lenders and obtaining further marginal borrowing. Solving for the entrepreneur's best response function thus involves a dynamic optimization problem. Taking lender pricing as given the entrepreneur forms strategies that maximize its continuation utility at each value of the state variable  $D$ .

The borrower knows that lenders will accept any loan they expect to be profitable, so to maximize its own utility it will only offer loans that lenders expect to make exactly zero profits. Thus given the total face value of debt borrowed so far  $D$ , the borrower only proposes loans  $(L_i, D_i)$  that satisfy  $L_i = \tilde{p}(D') D_i$ , where again  $D' \equiv D + D_i$ . Upon meeting any lender, if the entrepreneur has cumulative debt  $D$  and leaves the lender with cumulative debt  $D'$  then the maximum amount of new investment the lender would provide is given by  $\tilde{p}(D') [D' - D]$ . The lender expects the loan to be repaid with probability  $\tilde{p}(D')$ , in which case it will be return  $D' - D$ . Thus, taking loan pricing as given the entrepreneur

solves:

$$V(D) = \max_{D'} \alpha \tilde{p}(D') (D' - D) - (1 - q) \mathbb{E} [\min(D', \tilde{c})] + qV(D') \quad (3)$$

Conditional on arriving at a new lender having already issued face value of debt  $D$  the entrepreneur optimally chooses  $D'$ , the new total face value of debt it will have issued after contracting with this lender. The first term on the right hand side of Equation 3 is the marginal payoff from the new investment opportunity associated with obtaining additional investment  $\tilde{p}(D') (D' - D)$ . With probability  $1 - q$  the entrepreneur loses access to the lending market and either repays debt  $D'$  or defaults and pays the default cost  $\tilde{c}$  if it is lower than the cost of repayment. Finally, with probability  $q$  the entrepreneur does not lose access to the lending market and will receive continuation utility  $V(D')$  from future borrowing. Plugging in for  $\mathbb{E} [\min(D', \tilde{c})]$  given default costs  $\tilde{c}$  are uniform from zero to one gives:

$$V(D) = \max_{D'} \alpha \tilde{p}(D') (D' - D) - (1 - q) \left( D - \frac{D^2}{2} \right) + qV(D') \quad (4)$$

Denote a policy function that solves the dynamic programming problem in Equation 4 by  $g(D)$ . Conditional on arriving to a new lender with aggregate face value of debt  $D$ , the borrower will leave with aggregate face value of debt  $D' = g(D)$ , having proposed a new additional loan  $(\Delta D, \Delta L)$  with  $\Delta D \equiv g(D) - D$  and  $\Delta L \equiv \tilde{p}(g(D)) (g(D) - D)$ . On path, following this strategy generates a sequence of total aggregate face values of debt that have been accumulated up to a given lender, conditional on the lending market progressing that far:  $\{g(0), g(g(0)), g^3(0), \dots\}$ . The aggregate face value of debt obtained in the lending market is thus a random variable  $D^{agg}$  that realizes a particular value of this sequence depending on how many lenders the borrower is able to visit before the lending game ends.

### Equilibrium Characterization

Given the discussion of borrower and lender best responses above, it is clear that a SMPE of the model is equivalent to a solution a fixed point dynamic programming problem. Given lender strategies (captured by  $\tilde{p}(\cdot)$ ), the borrower's optimal strategy is indexed by a policy function  $g(\cdot)$  that solves the dynamic programming problem in Equation 4. Given the distribution of total aggregate face value of debt  $D^{agg}$  induced by  $g(\cdot)$ , each lender forms rational expectations over the probability the borrower will repay, denoted by  $\tilde{p}(D') = \mathbb{E} [p(D^{agg}) | D']$ . A set of strategies forms a SMPE if lender strategies

given by  $\tilde{p}(\cdot)$  are rational given  $g(\cdot)$ , and these borrower strategies are optimal given lender strategies embodied in  $\tilde{p}(\cdot)$ . We now summarize this characterization of an SMPE in the following Theorem.

**Proposition 2.** *A Stationary Markov Perfect Equilibrium (SMPE) of the lending game is characterized by functions  $\tilde{p}(\cdot)$  and  $g(\cdot)$  that map from cumulative debt level  $D \in [0, 1]$  to the interval  $[0, 1]$  such that:*

1.  $g(\cdot)$  is the policy function in the solution to the dynamic programming problem in Equation 4 taking  $\tilde{p}(\cdot)$  as given.
2. Lenders' perceived expected repayment probabilities  $\tilde{p}(\cdot)$  used to form accept/reject strategies are correct taking  $g(\cdot)$  as given.

$$\tilde{p}(D) = \mathbb{E}[p(D^{agg}) | D] = 1 - \mathbb{E}[D^{agg} | D]$$

### Closed Form Solution

We now solve for the SMPE in closed form. Notice that if  $\tilde{p}(D)$  were linear then the dynamic programming problem would have a linear-quadratic form, so we look for an equilibrium where  $\tilde{p}(D)$  and  $g(D)$  are linear functions of  $D$ . The following lemma characterizes the form of the solution to this problem.

**Lemma 1.** *For  $\alpha > 1$  and  $1 > q \geq 0$ , There exists  $\ell^*(\alpha, q) \geq 1$  and  $b^*(\alpha, q) \geq 1$  such that the unique linear SMPE takes the following form:*

$$\begin{aligned} \tilde{p}(D) &= (1 - D) \cdot \frac{1}{\ell^*} \\ 1 - g(D) &= (1 - D) \cdot \frac{1}{b^*} \end{aligned}$$

where  $\ell^*$  and  $b^*$  respectively parameterizes lender and borrower strategy in equilibrium.

There is an intuitive interpretation for both  $\ell^*$  and  $b^*$ . First, a borrower arrives to a lender with a (face value) debt capacity  $1 - D$  since if it acquired additional debt of more than  $1 - D$  it would default on all debt for sure.<sup>8</sup> When leaving this lender the borrower will have pledged a total of  $g(D)$  and thus the borrower will have remaining debt capacity  $1 - g(D)$ . Therefore  $\frac{1}{b^*}$  is the fraction of current borrowing capacity that

---

<sup>8</sup>Recall default costs  $\tilde{c} \sim U[0, 1]$ , so if the aggregate face value of debt equals or exceeds one the debt will never be repaid, as it will be less costly to default on it no matter what realization of default costs occur.

remains after visiting a lender. A higher  $b^*$  (or lower  $\frac{1}{b^*}$ ) corresponds to more aggressive borrowing by the borrower; it will deplete its available debt capacity more rapidly.

Second, recall that with commitment, the repayment probability is exactly  $\tilde{p}(D) = p(D) = 1 - D$ , which corresponds to  $\ell^* = 1$ . Thus  $\ell^* > 1$  corresponds to lower expected repayment probability and higher interest rates. In other words, when  $\ell^*$  is high the lenders makes pricing decisions *as if* they expect the borrower to accumulate substantially more debt from future lenders, which deteriorates the value of the current lender's own claims, and set interest rates to reflect this.

There is also a static interpretation for the dynamic equilibrium. Taking lender's strategy parameterized by  $\ell$  as given (which determines  $\tilde{p}(D)$  and the interest rates), the entrepreneur chooses how much debt to issue when meeting each lender. This specifies a best response  $b = B(\ell)$  for the entrepreneur, which can be thought of as representing a loan demand schedule. Since higher interest rates induce the borrower to take out debt less aggressively, the loan demand schedule is downward-sloping ( $B'(\ell) < 0$ ). On the other hand, given the aggressiveness of entrepreneur's borrowing behavior parameterized by  $b$ , the lenders are able to determine the distribution of the face value of total borrowing. This maps into the distribution of the value of their own debt claims, and lenders set their decision rule such that they only accept loans they expect to be weakly profitable, and thus specifies a best response  $\ell = L(b)$  as the loan supply schedule. The interest rates lenders have to charge in order to break-even increases as the entrepreneur takes out loans more aggressively, hence the loan supply schedule is upward-sloping ( $L'(b) > 0$ ). The unique equilibrium is then the unique intersection of the "loan demand" equation  $B(\ell)$  and the "loan supply" equation  $L(b)$ , which is depicted in Figure 3.

### Choice of Equilibrium Concept

We have been unable to show that the equilibrium of lemma 1 is the unique SMPE. Nevertheless, the equilibrium we study possesses an attractive property: it is the limit of the unique subgame perfect equilibrium of the truncated finite lender game when the maximum number of lenders tends to infinity. Formally, we define the finite  $N$ -lender game by modifying our infinite lending game as follows. Upon meeting the  $i$ -th lender, the borrower gets to meet  $(i + 1)$ -th lender with probability  $q$  if and only if  $i + 1 \leq N$ , and with probability zero otherwise. That is, we truncate the game at a maximum of  $N$  lenders, while keeping the stochastic nature of the lending game unchanged for the first  $N$  lenders. We now state the convergence result that makes our linear equilibrium attractive.

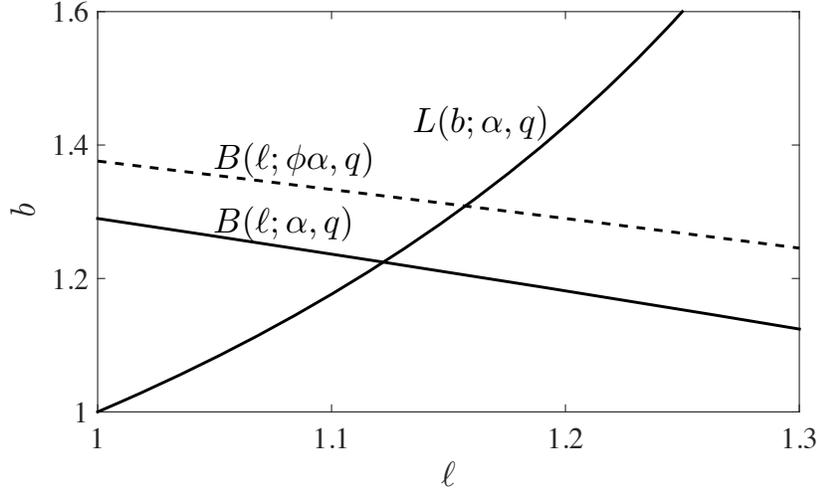


Figure 3: Static Representation of Lending Market Equilibrium.  $\phi > 1$  indicates an increase in marginal returns to new investment. This shifts the borrower's loan aggressiveness best response curve upward but does not change the lender best response curve, which is only a function of  $q$ .

**Proposition 3.** Fix  $i$ . In the finite  $N$ -lender game with  $N > i$ , borrower's policy function upon meeting lender  $i$ ,  $g_i^N(\cdot)$ , and lender  $i$ 's perceived probability of repayment,  $\tilde{p}_i^N(\cdot)$ , converges uniformly to  $g(\cdot)$  and  $\tilde{p}(\cdot)$  respectively as  $N \rightarrow \infty$ .

## 4 Comparative Statics

We now proceed by highlighting the role of commitment in equilibrium investment outcomes. First, because the outcome of the lending game depends on the random number of lenders the borrower meets, we will focus our attention on *expected* outcomes, such as the expected face value of debt, the expected level of investment, the expected aggregate interest rate, and expected welfare. Due to the linear strategies in the game these concepts have simple closed form expressions. We first explore comparative statics of these outcomes to the model parameters  $q$  and  $\alpha$ , and then turn to broader implications outside the model of how these forces can endogenously affect project choice by tightening financial constraints.

## 4.1 Comparative Statics of Commitment

How does the exogenous severity of the commitment problem  $q$  impact equilibrium outcomes? The expected number of lenders a borrower visits is  $\frac{1}{1-q}$  so as  $q \rightarrow 1$  the expected number of lenders the borrower will meet approaches infinity and the commitment problem is most severe. Our comparative static results on  $q$  are consistent with the idea that the commitment problem adversely impacts investment outcomes.

**Proposition 4.** *The following results describe the equilibrium effects of increasing access to additional lenders, as parameterized by  $q$ , for any  $\alpha > 1$  and  $q > 0$  :*

1. *Expected aggregate face value of debt ( $\mathbb{E}[D^{agg}]$ ) increases in  $q$ .*
2. *Expected investment ( $\mathbb{E}[I]$ ) decreases in  $q$ .*
3. *The ex-ante expected interest rate ( $\mathbb{E}[D^{agg}] / \mathbb{E}[I^{agg}]$ ) increases in  $q$ .*
4. *Ex-ante welfare of the entrepreneur decreases in  $q$ , and in the limit as  $q \rightarrow 1$  welfare converges to zero, the level that would be obtained if the entrepreneur does not have access to the lending market at all.*

As  $q$  increases the expected face value of debt raised in equilibrium increases. This is the net effect of two forces. First, for higher  $q$  the borrower's demand aggressiveness curve shifts down, meaning the borrower issues a smaller fraction of its debt capacity at each round of financing for a given interest rate strategy. This is because higher  $q$  means it is more likely that the borrower will be able to exploit the externality future lenders impose and obtain favorable pricing on subsequent loans. In effect, when  $q$  is higher the borrower wants to smooth borrowing over multiple lenders to obtain better aggregate financing terms.

From the lender perspective, a higher  $q$  means it is more likely they will be exploited, so for a given borrowing strategy a higher  $q$  causes lenders to raise interest rates. However, because the spillover between lenders is still present, interest rates do not rise enough to prevent expected face value of debt from rising. In other words, when  $q$  is higher, the spillover between lenders is more severe and will be exploited to a larger extent. The net effect, as summarized in Proposition 4, is that expected face value of debt and effective interest rates rise and expected investment falls. As additional lenders become more accessible, borrowers must pledge more and more debt to raise less and less investment capital under higher and higher interest rates.

In effect, the possibility of sequential banking makes it harder to use available debt capacity to fund new investment. Indeed, the fourth result of Proposition 4 is that welfare of the entrepreneur (equivalent to total surplus) decreases as  $q$  increases.

Now consider the limiting case when  $q \rightarrow 1$ . The commitment problems become so severe that the entrepreneur acquires financing at interest rates that eliminate *all* of the surplus that could have been obtained through investing in the new project. Borrowing and investment still occur as  $q \rightarrow 1$  but at amounts and interest rates such that the borrower is indifferent to not undertaking the new investment at all. At a glance, this result shares striking resemblance to the well-known Coase Conjecture in industrial organization, which states that as consumers become extremely patient, a durable monopolist who cannot commit to future prices would have to sell the durable good at competitive prices instantaneously and is unable to raise any profits (Fudenberg and Tirole (Chapter 10, 1991).) However, it is worth noting that even though in both contexts, the party that lacks commitment (the entrepreneur in our model and the monopolist in Coase Conjecture) loses the ability to capture any surplus, the welfare implication is completely reversed in our model. Under the Coase Conjecture, at the limit of extreme patience the allocation of durable goods is socially efficient: any consumer who values the good above its marginal cost gets to consume it from the initial period onwards. In our model, however, social welfare strictly decreases with  $q$  and at the limit the welfare level is as if the entrepreneur does not have access to a credit market at all.

## 4.2 Comparative Statics of Project Returns

We now turn to the more surprising results about what happens when the marginal returns  $\alpha$  of the investment opportunity increase. When returns to the investment opportunity are higher the borrower has a greater incentive to exploit the externality associated with the commitment problem and take on more new financing at the expense of previous lenders. A central result of our paper is that for projects with higher returns, the effects of the commitment problem can be worsened to the extent that the equilibrium may involve lower levels of investment than would occur with less desirable projects.

To develop this result, first consider the effect of an increase in  $\alpha$  on the equilibrium face value of debt. An increase in  $\alpha$  shifts the borrowing aggressiveness demand equation upward but does not impact the loan supply equation, which is not a function of the parameters of the investment opportunity. Thus borrowing aggressiveness increases in equilibrium, as shown graphically in Figure 3. Because  $q$  is unchanged the expected face value of debt unambiguously increases as the borrower accumulates aggregate debt

issuance more rapidly. The intuition is that for a given interest rate strategy of lenders, the borrower has greater incentive to raise marginal capital for investment from early lenders. At any point in the game the borrower faces the following tradeoff. Taking interest rate strategies as given, the borrower weighs the benefits of borrowing more from the current lender for sure, or taking the gamble that it will be able to meet a new lender from which to borrow marginally at better interest rates that the previous lender would not have offered. When the returns to investment are higher this tradeoff tilts away from a less aggressive borrowing strategy of waiting to try and exploit the externality on the current lender. Of course, more aggressive borrowing means lenders have to charge higher interest rates in anticipation of higher aggregate borrowing. This result is summarized in Proposition 3.

**Proposition 5.** *Expected face value of debt  $\mathbb{E}[D]$  and the effective interest rate  $\mathbb{E}[D] / \mathbb{E}[I]$  are increasing in  $\alpha$ , the returns to scale of the investment technology.*

Now we turn to the main result on the relationship between the equilibrium level of investment  $\mathbb{E}[I]$  and the returns to scale of the new project .

**Proposition 6.** *Fix any  $q \in (0, 1)$*

1. *The equilibrium level of investment  $\mathbb{E}[I]$  is non-monotone in  $\alpha$ . In particular, there exists a cutoff  $\bar{\alpha}(q)$  such that expected investment is increasing in  $\alpha$  below this cutoff, and decreasing in  $\alpha$  for  $\alpha > \bar{\alpha}(q)$ . Formally,*

$$\frac{d\mathbb{E}[I]}{d\alpha} \begin{cases} < 0 & \text{for } 1 \leq \alpha < \bar{\alpha}(q) \\ > 0 & \text{for } \alpha > \bar{\alpha}(q) \end{cases}$$

2. *The stronger the commitment problem in the lending market, the lower is the cutoff level of marginal return  $\bar{\alpha}(q)$  for decreasing investment:*

$$\frac{d\bar{\alpha}(q)}{dq} < 0$$

These results can be visualized in Figure 4, which plots equilibrium expected investment as a function of  $\alpha$  for three different values of  $q$ . For all levels of  $q$ , the level of investment that the entrepreneur gets to raise in expectation first increases in  $\alpha$  and then decreases. Entrepreneurs with better opportunities could be facing tighter constraints. The second part of the proposition shows that such endogenous misallocation of resources is more severe in markets where the commitment problem is worse.

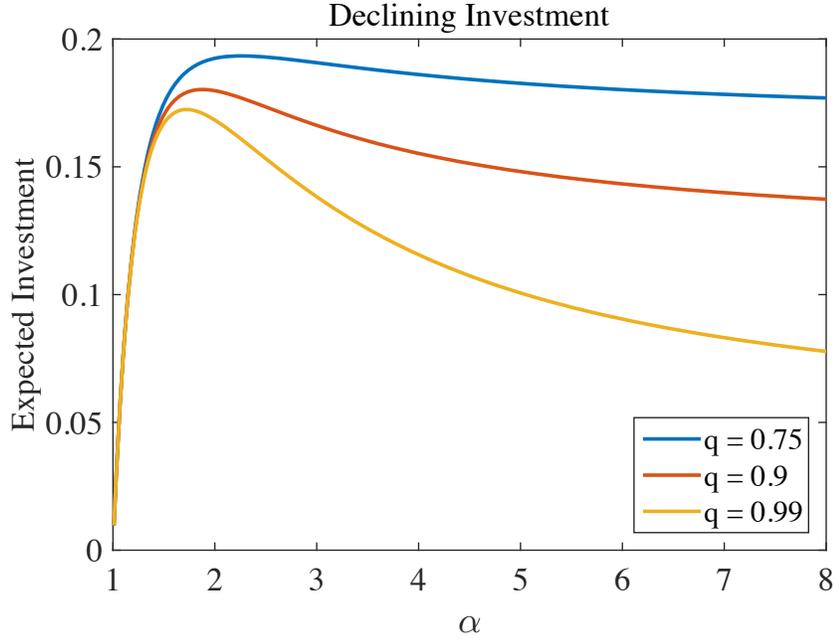


Figure 4: Expected Investment in the Dynamic Lending Game

The seemingly perverse outcome of lower investments in better opportunities arises because when commitment problems are present, it is possible that the equilibrium involves using available debt capacity inefficiently. To better understand the intuition behind this result, recall that the present value of promised repayments to creditors is affected by the face value of these claims in two directions. Holding repayment fixed, higher face value translates into higher present value. However, a higher face value of debt also reduces the probability of repayment. This gives rise to an aggregate debt Laffer curve: the value of debt is initially increasing in the amount of debt pledged starting from zero, but for sufficiently high face values of debt, pledging more debt actually results in a lower present value of debt because the negative force begins to dominate the positive one.

Figure 5 shows precisely how the debt Laffer curve appears in the model. This figure translates the best response curves of Figure 3 into the expected investment vs expected debt space by plotting the values of expected investment and debt that would result if the lenders were optimally responding to arbitrary strategies of the borrowers, and vice-versa. This exercise generates two distinct curves, the (non-zero) intersection of which corresponds to the unique linear equilibrium allocation.<sup>9</sup> The hump-shaped black line

<sup>9</sup>An intersection of the curves in Figure 3 implies an intersection in Figure 5, but the converse is not true. This is because the mapping between  $(b, \ell)$  and  $(\mathbb{E}[D^{agg}], \mathbb{E}[I^{agg}])$  is not one-to-one at  $b = 1$ :  $(b = 1, \ell)$  in the B-L space would generate  $(0, 0)$  in the  $(\mathbb{E}[D^{agg}], \mathbb{E}[I^{agg}])$  for any value of  $\ell$ .

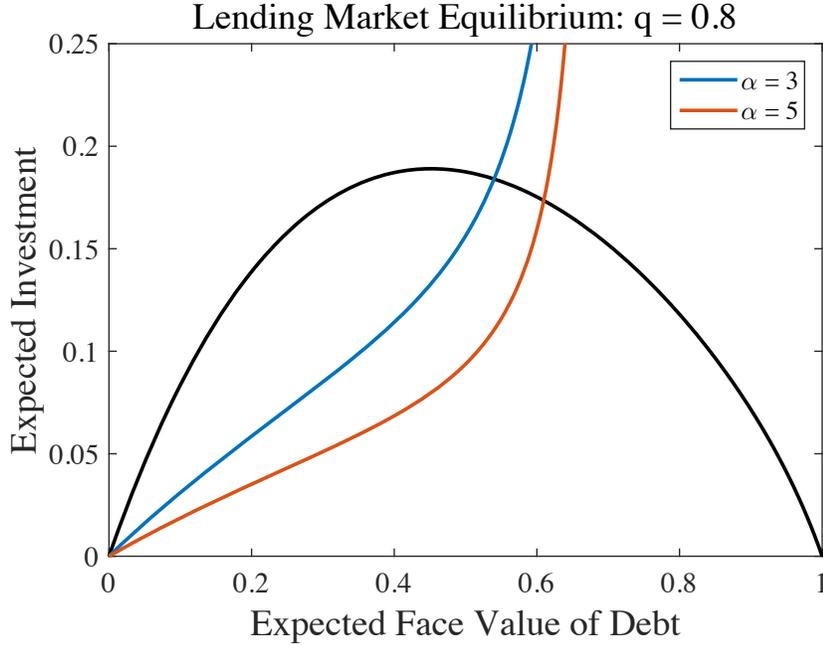


Figure 5: Expected Investment and Expected Debt

embeds lender optimality over a range of responses to all possible borrower strategies  $b \geq 1$ . This is the debt Laffer curve introduced in the previous paragraph. Given a borrowing aggressiveness strategy of the entrepreneur, there is an associated zero-profit level of expected investment.

The upward sloping curves plot values of expected investment and debt if the borrower responds optimally to the range of all possible lender strategies  $\ell \geq 1$ . As illustrated in the graph, for higher  $\alpha$  this curve shifts to the right, representing more aggressive borrowing by the entrepreneur upon meeting each lender. As marginal returns increase, the entrepreneur has more to gain by increasing borrowing (and investment). Unfortunately though, this is also a curse because without commitment, if the entrepreneur gets to meet additional lenders it will want to borrow more from them as well.

We know from Proposition 5 that  $\mathbb{E}[D]$  is monotonically increasing in  $\alpha$ . Without commitment, the equilibrium allocation may end up on the wrong side of the debt Laffer curve, where the entrepreneur is in equilibrium offering such high levels of repayment that it actually receives very little investment. On the wrong side of the curve, increases in  $\alpha$  increase expected debt but decrease expected investment. With the ability to commit to one lender, the borrower would never choose loan terms in this inefficient region.

The results in this section demonstrate that when there are commitment problems, equilibrium investment may be lower for more productive projects. With the opportunity for sequential borrowing, borrowers are able to obtain marginal funds from new

lenders at lower interest rates than previous lenders would have provided, because the new lenders do not internalize the negative impact of their lending on previous lenders. This opportunity is more lucrative for borrowers when marginal returns to additional investment are higher, and in equilibrium borrowers have to tilt their borrowing toward earlier lenders to “buy” commitment power not to borrow as aggressively from subsequent lenders. This results in excessive borrowing and a potentially inefficient use of debt to fund the new investment opportunity.

These results are surprising and run against common intuition that more productive projects induce higher levels of investment despite the presence of financial constraints. In many classical theories of inefficient investment choice, investment occurs if and only if the NPV of a project exceeds a certain threshold, which can be above or below zero. Such models do not generate the stark patterns of resource misallocation that plagues developing countries. The commitment friction we explore in this paper however can generate these distortions. In fact, this force is so strong that it can also distort the influence the choice of investment opportunity chosen by entrepreneurs. In the next section we illustrate this point.

## 5 Endogenous Project Choice

Given the role of high marginal returns in Proposition 6, it is natural to ask what happens when project returns are not linear, but instead concave. In such a setting, borrowers can obtain commitment power by obtaining sufficiently large amounts of capital from early lenders such that the marginal returns to future borrowing, to which they cannot commit to avoid, are lower. In this section, we work with an extreme form of concavity: we specify that investment opportunities have linear returns up to a certain size of investment, and then the projects deliver zero marginal return for any additional investment beyond that point. We call these “linear-flat” projects and we use them to show that even if one investment opportunity is strictly dominated by another in terms of having lower returns for any level of investment, the dominated project could yield higher welfare for the entrepreneur. This result is summarized in Proposition 7.

**Proposition 7.** *There exist pairs of projects  $R_1(\cdot)$  and  $R_2(\cdot)$  such that  $R_2(I) > R_1(I)$  for all  $I$ , but an entrepreneur would prefer to undertake the “worse” project  $R_1(\cdot)$  because doing so delivers higher welfare.*

Formally, a project is linear-flat if the return function can be parameterized by a slope parameter  $\alpha$  and a cutoff parameter  $\bar{L}$  such that  $R(I; \alpha, \bar{L}) = \alpha \min(I, \bar{L})$ . This project

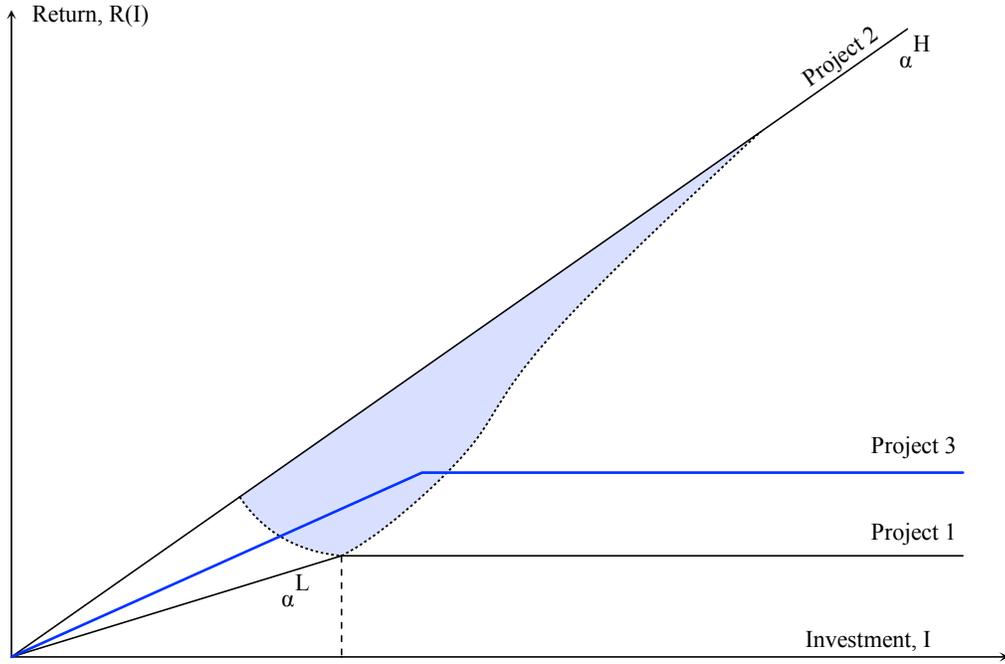


Figure 6: Endogenous Project Choice

returns  $\alpha I$  when  $I \leq L$  but marginal returns are zero for additional investment beyond  $L$ . Define  $L^{SL}(\alpha)$  and  $D^{SL}(\alpha)$  respectively as the market and face value of debt that would be chosen by an entrepreneur who can commit to borrowing from a single lender when the investment opportunity is linear with marginal returns  $\alpha$ . Now consider the linear-then-flat project  $R(I; \alpha, L^{SL}(\alpha))$ , which is linear with slope  $\alpha$  and turns flat after raising an investment level of  $L^{SL}(\alpha)$ . It should be clear that even without commitment power this project achieves the same lending outcome as the linear project with slope  $\alpha$  under commitment: as there are no marginal returns from borrowing beyond  $(D^{SL}(\alpha), L^{SL}(\alpha))$ , the very first lender that the entrepreneur gets to borrow from would be willing to accept a loan at these terms since it will be assured that no future borrowing will occur to deteriorate the value of the claims. Clearly in this extreme example the concavity of investment opportunity effectively solves the commitment problem. We now use this result to illustrate Proposition 7.

Figure 6 shows the payoff functions of several projects. Consider Project 1 to be the linear-flat project with slope  $\alpha^L$  and a threshold for zero marginal return at  $L^{SL}(\alpha^L)$ , i.e.  $R_1(I) = \alpha^L \min(I, L^{SL}(\alpha^L))$ . For a given  $q > 0$ , we can find a fully linear Project 2 with slope  $\alpha^H$  such that the equilibrium welfare obtained by an entrepreneur endowed with either of these two projects would be equal. We must have  $\alpha^H > \alpha^L$  because the

project needs to deliver higher output to compensate for the surplus loss due to the lack of commitment power.

Take any point in the graph and it could be used to represent a project payoff function that is linear from the origin to that point, then turns flat after that. The dotted line plots the set of points that represent projects which give equivalent welfare to the entrepreneur as Project 1 and Project 2. Any project in the shaded region, such as Project 3, gives strictly higher welfare in equilibrium to the entrepreneur than Project 1 does. Any project outside the shaded region and below Project 2 gives strictly lower welfare.

The strength of this result can be seen by comparing Project 3 and Project 2. Despite the fact that viewed in isolation, Project 3 yields lower return than Project 2 for any level of investment, it gives the entrepreneur strictly higher welfare in our model due to the commitment friction. By having zero marginal return beyond a certain level of investment, Project 3 endows the entrepreneur with some endogenous commitment power such that loans can be obtained under better interest rates. As a result, if the entrepreneur gets to choose ex-ante which investment projects to undertake, he may *endogenously* pursue an investment opportunity with lower returns, as long as it is sufficiently concave.

The result in this section highlights that in the presence of the commitment problem there is a powerful trade-off between the average and marginal returns of investment opportunities. Productivity is no longer the sole determinant of investment in economic activities. The commitment friction can be mitigated by undertaking instead in projects that have concave returns and thus embed some degree of commitment. Once these projects interior to the technology frontier are undertaken, they exhibit slower capital accumulation and growth potential. Therefore, this result supports yet another striking feature in the data: firms in developing countries show much less growth in size as they age relative to their counterparts in advanced economies Hsieh and Klenow (2014).

## 6 Policy Implications

We now turn to studying simple regulatory policy tools that can improve outcomes in our model: limiting interest rates, imposing total borrowing limits, and limiting the number of lenders from which a borrower can obtain loans. Conventional arguments suggest that interest rate caps may be helpful in improving allocations when there is a *lack* of competition among lenders as they limit monopoly power. Yet in some scenarios, it seems to have been the *entry* of new lenders into markets and the resulting increase in competition that has driven regulators to consider usury regulations. The microfinance crisis in Andhra Pradesh was precipitated by the rapid entry of thousands of new mi-

crofinance lenders and characterized by over indebtedness of borrowers from multiple lenders. A report commissioned by the Reserve Bank of India to study the causes and potential regulatory responses to the Andhra Pradesh crisis states:

“It has been suggested that with the development of active competition between MFIs there has been a deluge of loan funds available to borrowers which has fuelled excessive borrowing and the emergence of undesirable practices ... Finally, it is believed that in consequence of over-borrowing, default rates have been climbing in some locations but these have not been disclosed because of ever-greening and multiple lending.” Malegam (2011).

Despite highlighting a high degree of competition, the report proceeds to propose regulation that limits interest rates charged to borrowers. Through the lens of our model of multiple borrowing and commitment problems, this type of regulatory response is rational and welfare improving. Interest rate ceilings are more helpful when it becomes easier to borrow from multiple lenders, which can be aided by entry and competitiveness in lending markets.

Interest rate caps have the potential to improve welfare because they embed commitment power. Recall that in the model the more debt the borrower has outstanding, the lower the probability of repayment, and thus the higher interest rates need to be on additional lending. Interest rate caps add commitment power because marginal borrowing at high enough levels of debt would need to violate the interest rate cap for these loans to break even and thus are never issued. Early lenders can then be assured that such future borrowing, which increases the anticipated probability of default, will not occur, and can provide initial loans at interest rates that are closer to those that would prevail if borrowers could fully commit to exclusive borrowing. This better aligns borrower incentives, reducing face values of debt and interest rates in equilibrium. Importantly, interest rate caps also increase investment and improve welfare as long as they are not too severe that they prevent productive investment.

Further, for a given investment opportunity the interest rate cap can be set to fully overcome the inefficiencies induced by lack of commitment. By setting the interest rate cap at exactly the interest rate that would prevail in the full-commitment equilibrium, the borrower can credibly raise exactly the full commitment level of debt, and at the commitment-level interest rate, restoring the full commitment outcomes. By proposing such a loan to the first lender, the lender is assured that any future borrowing would necessarily need to be at interest rates above the allowed limit, and can thus be sure that no such additional borrowing would take place. Thus this loan proposal is expected to

earn zero profits for the lender and is always accepted. By definition this allocation maximizes the entrepreneur's welfare given lenders at least break even, so any rational lender strategy induces this loan proposal as a best response.

In our model a limit on the total face value of debt a borrower can obtain is mechanically equivalent to a particular interest rate cap, and the equivalent debt limit is increasing in the interest rate cap, so all the results above also apply to total debt limits. Both total borrowing limits and interest rate caps were adopted for microfinance loans in India in 2011 (Dr. D. Subbarao (2011)). These policies are equivalent in the model because they both operate by shutting down contingencies of excessive debt accumulation: debt limits directly and interest rate limits indirectly through the fact that the lower bound on interest rates in any equilibrium is increasing in the total cumulative face value of debt. In the contingencies (potentially off-equilibrium) in which either the interest rate or debt limit is binding, the lender zero profit condition implies a unique relationship between interest rates and total face value of debt at this point:  $p(\bar{D}) = (1 + \bar{r})^{-1}$ , where  $\bar{D}$  is a borrowing limit and  $\bar{r}$  is the equivalent interest rate limit.

We now summarize these results in the following proposition.

**Proposition 8.** *Adding an upper bound on interest rates generates the following results:*

1. *For any investment opportunity  $R(I)$ , there is an optimal interest rate cap  $\bar{r}(R)$  that induces the full commitment allocation with the borrower obtaining funding from a single lender. When  $R(I) = \alpha I$ , the optimal interest rate cap is  $\bar{r} \equiv 3 - 2\alpha^{-1}$ .*
2. *If  $\bar{r} < \bar{r}^{SL}$  then single lender borrowing prevails but debt is inefficiently low level, the interest rate cap is too restrictive. Welfare will be lower than the unregulated equilibrium (i.e. with no interest rate cap) if the interest rate cap is sufficiently low.*
3. *If  $\bar{r} > \bar{r}^{SL}$  then the interest rate cap increases expected investment and welfare while lowering expected debt and interest rates relative to the unregulated equilibrium. As  $\bar{r} \rightarrow \infty$  the interest rate cap becomes irrelevant and outcomes converge to the unregulated equilibrium.*
4. *Any interest rate cap  $\bar{r}$  has an associated debt limit  $\bar{D} = 1 - (1 + \bar{r})^{-1}$  that induces the same equilibrium.*

It is important to re-emphasize that in general interest rate caps (and total borrowing limits) have ambiguous implications for welfare, because caps that are too low can restrict productive investment. While there are always welfare improving interest rate limits for a given project, imposing a market-wide policy can have ambiguous effects on welfare if the investment opportunities in the economy are sufficiently heterogeneous. From a

utilitarian perspective, however, “reasonable” interest rate caps can be quite beneficial if many projects in the economy could benefit from them. Further, taking into account the possibility of endogenous project choice, interest rate caps have the potential to “unlock” the best projects available that were previously infeasible due to the endogenous credit constraint induced by lack of commitment.

Finally, a surprising and controversial policy recommendation of Malegam (2011) was to limit borrowers to obtaining loans from at most two micofinance lenders. Limiting the number of lenders from which a borrower can obtain loans will mechanically increase commitment power by limiting the opportunities a borrower has not to commit. While this policy was not ultimately adopted, our model shows that in the face of commitment problems such a policy may actually be quite helpful.

## 7 Conclusion

This paper argues that commitment problems in lending markets can explain emerging empirical evidence that the rapid expansion of credit access can have perverse effects. When borrowers cannot commit to exclusive contracting, increasing the availability of lenders makes markets appear *less* competitive as interest rates rise and entrepreneur investment and welfare fall. Further, commitment problems can result in better projects receiving less investment than worse projects. This force can be so severe that what look like good opportunities are passed over for inferior investment technology. Finally, we show how simple regulatory tools such as interest rate ceilings and debt limits can improve outcomes and ameliorate the misallocative forces we highlight.

The intuition for these result is that the externalities the lenders impose on each other when commitment or contingent contracting is not possible can prevent the borrower from being able to use pledgeable cash flows efficiently. Specifically, through the interaction of lack of commitment to exclusive borrowing and limited pledgeability, the equilibrium value of promised repayment falls when investment opportunities becomes more attractive in the sense that they have higher marginal returns to capital. For such projects there is greater incentive to deviate from commitment by obtaining additional marginal borrowing from some lenders at the expense of others due to the decline in collateral value. In equilibrium borrowing can only occur at interest rates such that borrowers are willing to borrow from future lenders to a degree that makes these interest rates weakly profitable. When projects have higher marginal returns, benefits from further borrowing to increase investment are higher for any interest rate.

Since commitment is less of a problem for projects with lower marginal returns, when

given the choice entrepreneurs will endogenously choose investment opportunities that are everywhere less productive than other available opportunities, as long as they are sufficiently more concave. Thus our model provides a new micro-foundation for the idea that commitment problems in lending markets can induce substantial misallocation in capital investment and can explain observations both of low growth and of economic activity below the technological frontier. Testing the empirical validity these mechanisms and their importance in explaining the failure of increased access to finance to significantly improve outcomes is an important topic for future research.

## References

- Richard Arnott and Joseph Stiglitz. Equilibrium in Competitive Insurance Markets with Moral Hazard. NBER Working Papers 3588, National Bureau of Economic Research, Inc, January 1991.
- Richard Arnott and Joseph Stiglitz. Price Equilibrium, Efficiency, and Decentralizability in Insurance Markets with Moral Hazard. *Working Paper*, 1993.
- Paul Asquith, Anne Beatty, and Joseph Weber. Performance pricing in bank debt contracts. *Journal of Accounting and Economics*, 40(1-3):101–128, December 2005.
- Abhijit Banerjee and Esther Duflo. Do Firms Want to Borrow More: Testing Credit Constraints Using a Targeted Lending Program. *The Review of Economic Studies*, 81(2):572–607, February 2014.
- Alberto Bisin and AA Rampini. Exclusive contracts and the institution of bankruptcy. *Economic Theory*, (847):1–33, 2006.
- DS Bizer and PM DeMarzo. Sequential banking. *Journal of Political Economy*, 100(1):41–61, 1992.
- AWA Boot and AV Thakor. Moral hazard and secured lending in an infinitely repeated credit market game. *International Economic Review*, 35(4):899–920, 1994.
- Markus K. Brunnermeier and Martin Oehmke. The Maturity Rat Race. *Journal of Finance*, 68(2):483–521, April 2013.
- Alain de Janvry, Elisabeth Sadoulet, Craig McIntosh, Bruce Wydick, Jill Luoto, Gustavo Gordillo, and Guilherme Schuetz. Credit bureaus and the rural microfinance sector: Peru, guatemala, and bolivia. Technical report, A joint project between The University of California at Berkeley and The FAO Office for Latin America, December 2003.
- Governor Dr. D. Subbarao. Monetary policy statement 2011-2012 (including developmental and regulatory policies). Report, Reserve Bank of India, May 2011.
- Rashid Faruqee and M. A. Baqui Khalily. Multiple borrowing by mfi clients. Policy paper, Institute of Microfinance (InM), October 2011.
- Drew Fudenberg and Jean Tirole. *Game Theory*. The MIT Press, 1991.

- Chang-Tai Hsieh and Peter J. Klenow. Misallocation and Manufacturing TFP in China and India. *The Quarterly Journal of Economics*, 124(4):1403–1448, November 2009.
- Chang-Tai Hsieh and Peter J. Klenow. The Life Cycle of Plants in India and Mexico. *The Quarterly Journal of Economics*, 129(3):1035–1084, August 2014.
- Doug Johnson and Sushmita Meka. Access to microfinance in andhra pradesh. Report, Institute for Financial Management and Research Centre for Micro Finance, and Centre for Microfinance Research Bankers' Institute of Rural Development, October 2010.
- CM Kahn and D Mookherjee. Competition and incentives with nonexclusive contracts. *The Rand Journal of Economics*, 29(3):443–465, 1998.
- Y.H. Malegam. Report of the sub-committee of the central board of directors of reserve bank of india to study issues and concerns in the mfi sector. Report, Reserve Bank of India, January 2011.
- Craig McIntosh, Alain de Janvry, and Elisabeth Sadoulet. How rising competition among microfinance institutions affects incumbent lenders. *The Economic Journal*, 115:987–1004, October 2005.
- David McKenzie and Christopher Woodruff. Experimental Evidence on Returns to Capital and Access to Finance in Mexico. *World Bank Economic Review*, 22(3):457–482, November 2008.
- CA Parlour and U Rajan. Competition in loan contracts. *American Economic Review*, 2001.
- Mitchell A Petersen and Raghuram G Rajan. The Benefits of Lending Relationships: Evidence from Small Business Data. *Journal of Finance*, 49(1):3–37, March 1994.
- Mitchell A Petersen and Raghuram G Rajan. The Effect of Credit Market Competition on Lending Relationships. *The Quarterly Journal of Economics*, 110(2):407–43, May 1995.
- I Segal and MD Whinston. Robust predictions for bilateral contracting with externalities. *Econometrica*, 71(3):757–791, 2003.
- Christa Wichterich. The Other Financial Crisis: Growth and crash of the microfinance sector in India. *Development*, 55(3):406–412, September 2012.

# Appendix

## Proof of Lemma 1

Borrower's problem can be formulated recursively as:

$$V(D) = \max_{D'} \alpha \tilde{p}(D') (D' - D) + (1 - q) \left[ \frac{(1 - D')^2}{2} - \frac{1}{2} \right] + qV(D')$$

where

$$\tilde{p}(D') \equiv \mathbb{E} [1 - D^{agg} | D']$$

We guess that borrower's policy function  $g(\cdot)$  and lenders' loan pricing function  $\tilde{p}(\cdot)$  both take a linear form and are each characterized by a single endogenous variable,  $b$  and  $\ell$ , respectively:

$$\tilde{p}(D) = \ell^{-1} (1 - D)$$

$$1 - g(D) = b^{-1} (1 - D)$$

To solve for borrower's policy function, we proceed to take first order condition and use the envelope condition for borrower's problem. The first order condition is:

$$-\alpha \ell^{-1} (g(D) - D) + \alpha \ell^{-1} (1 - g(D)) - (1 - q) (1 - g(D)) + qV'(g(D)) = 0$$

and the envelope condition is:

$$V'(D) = -\alpha \ell^{-1} (1 - g(D))$$

Plugging the envelope condition into the first-order condition and after simplifying, we can express the Euler condition as a quadratic function of  $b^{-1}$ :

$$q\alpha \ell^{-1} b^{-2} + (1 - q - 2\alpha \ell^{-1}) b^{-1} + \alpha \ell^{-1} = 0$$

We thus solve for the endogenous parameter  $b^{-1}$  that governs the borrower's policy function as:<sup>10</sup>

$$b^{-1} = \frac{(2\alpha \ell^{-1} - (1 - q)) - \sqrt{(1 - q - 2\alpha \ell^{-1})^2 - 4q\alpha^2 \ell^{-2}}}{2q\alpha \ell^{-1}} \quad (5)$$

---

<sup>10</sup>There are two roots to the quadratic equation, one of which leads to explosive debt accumulation. We choose the other, stable root.

To solve for the lender's loan pricing function, note

$$\begin{aligned}
\tilde{p}(D) &= \mathbb{E}[1 - D^{agg} | D] \\
&= (1 - q) \left[ (1 - D) + q(1 - g(D)) + q^2(1 - g(g(D))) + \dots \right] \\
&= (1 - q) \left[ (1 - D) + qb^{-1}(1 - D) + q^2b^{-2}(1 - D) + \dots \right] \\
&= \frac{1 - q}{1 - qb^{-1}} (1 - D)
\end{aligned}$$

Hence

$$\ell^{-1} = \frac{1 - q}{1 - qb^{-1}} \quad (6)$$

Equation (5) characterizes  $b^{-1}$  as a decreasing function of  $\ell^{-1}$ . On the other hand, equation (6) characterizes  $\ell^{-1}$  as an increasing function of  $b^{-1}$ . The two equations therefore yields a unique solution  $(b^*, \ell^*)$  for each  $q \in [0, 1)$  and  $\alpha \in (1, \infty)$ . In particular, we have

$$(b^*)^{-1} = \frac{\left(2\alpha - 1 - \sqrt{4(1 - q)(\alpha^2 - \alpha) + 1}\right)}{2q(\alpha - 1)}$$

□

### Proof of Proposition 3

Placeholder

□

### Proof of Proposition 4

Before proving the comparative static propositions, it will be useful to derive some expressions for equilibrium objects of interest in terms of parameters  $\alpha$  and  $q$ . Let  $I^{agg}$  denote the aggregate investment and  $D^{agg}$  denote the aggregate debt that have been attained when the lending market game ends. In equilibrium, ex-ante, these are random variables with respect to the number of lenders the borrower will be able to visit.

**Lemma 2.**  $\mathbb{E}[I^{agg}] = \mathbb{E}[p(D^{agg})D^{agg}]$

*Proof.* Let  $N$  denote the random number of lenders the borrower gets to visit before losing access to the lending market game. The random aggregate face value of debt and

aggregate investment can be expressed as:

$$D^{agg} = \sum_{j=1}^{\infty} D_j \mathbf{1}(N \geq j)$$

$$I^{agg} = \sum_{j=1}^{\infty} I_j \mathbf{1}(N \geq j)$$

where  $D_j$  is the amount of debt given by the  $j$ -th lender. Similarly denote  $I_j$  to be the amount of investment capital provided by the  $j$ -th lender. Pick any  $j > 0$ , the zero-profit condition for his loan and investment size is:

$$\mathbb{E} [p(D^{agg}) | N \geq j] D_j \mathbf{1}(N \geq j) = I_j \mathbf{1}(N \geq j)$$

Taking expectation over  $N$  on both sides and applying the law of iterated expectation, we get:

$$\mathbb{E} [p(D^{agg}) D_j \mathbf{1}(N \geq j)] = \mathbb{E} [I_j \mathbf{1}(N \geq j)]$$

We next sum the previous equation over all lenders. By the linearity of the expectations operator, we can bring the sum inside:

$$\mathbb{E} \left[ p(D^{agg}) \sum_{j=1}^{\infty} D_j \mathbf{1}(N \geq j) \right] = \mathbb{E} \left[ \sum_{j=1}^{\infty} I_j \mathbf{1}(N \geq j) \right]$$

Substituting in the definitions of  $D^{agg}$  and  $I^{agg}$ :

$$\mathbb{E} [p(D^{agg}) D^{agg}] = \mathbb{E} [I^{agg}]$$

□

**Lemma 3.** *We can express the expected debt and investment as functions of  $b^{-1}$ :*

$$\mathbb{E} [D^{agg}] = \frac{1 - b^{-1}}{1 - qb^{-1}}$$

$$\mathbb{E} [I^{agg}] = \frac{b^{-1} (1 - b^{-1}) (1 - q)}{(1 - b^{-1}q) (1 - b^{-2}q)}$$

*Proof.* Denote the expected aggregate debt upon leaving a given lender with cumulative debt  $D$  as  $\mathbb{E} [D^{agg} | D]$ . From lender's zero-profit condition, we have

$$\mathbb{E} [D^{agg}|D] = 1 - \tilde{p}(D)$$

The ex-ante expected aggregate debt  $\mathbb{E} [D^{agg}]$  is simply the expected aggregate debt upon leaving a lender with zero outstanding debt, times  $\frac{1}{q}$  (since the borrower meets the first lender with certainty, not probability  $q$ ). Thus we have

$$\begin{aligned} \mathbb{E} [D^{agg}] &= \frac{1}{q} \mathbb{E} [D^{agg}|0] \\ &= \frac{1}{q} \left( 1 - \frac{1-q}{1-qb^{-1}} \right) \\ &= \frac{1-b^{-1}}{1-qb^{-1}} \end{aligned}$$

To get the expression for expected investment:

$$\begin{aligned} \mathbb{E} [I] &= \tilde{p}(g(0))g(0) + q\tilde{p}(g^2(0)) [g^2(0) - g(0)] + q^2\tilde{p}(g^3(0)) [g^3(0) - g^2(0)] + \dots \\ &= \ell^{-1} \left[ (1-g(0))g(0) + q(1-g^2(0)) \left[ (1-g(0)) - (1-g^2(0)) \right] + \dots \right] \\ &= \ell^{-1} \left[ b^{-1}(1-b^{-1}) + qb^{-2} [b^{-1} - b^{-2}] + q^2b^{-3} [b^{-2} - b^{-3}] + \dots \right] \\ &= \ell^{-1}b^{-1}(1-b^{-1}) \left[ 1 + qb^{-2} + q^2b^{-4} + \dots \right] \\ &= \ell^{-1}b^{-1} \frac{1-b^{-1}}{1-qb^{-2}} \\ &= \frac{b^{-1}(1-b^{-1})(1-q)}{(1-b^{-1}q)(1-qb^{-2})} \end{aligned}$$

□

**Lemma 4.** Let  $z \equiv \sqrt{4(1-q)(\alpha^2 - \alpha) + 1}$ . The analytic solution of expected debt, investment, and welfare can be expressed as the following functions of parameters  $q$  and  $\alpha$ :

$$\mathbb{E} [D^{agg}] = \frac{2\alpha - 1 - z}{2q\alpha}$$

$$\mathbb{E} [I^{agg}] = \frac{(\alpha - 1)(z + 1 - 2\alpha(1 - q))}{2\alpha q(2\alpha - 1)}$$

$$V(0) = \frac{1 - 2\alpha(1 - q) - 2q + z}{4q}$$

*Proof.* These expressions can be obtained by substituting the analytic solution of  $b^*$  from lemma 1 into the expressions in lemma 2.  $\square$

Now continuing on, we can express equilibrium  $b^*$  as

$$(b^*)^{-1} = \frac{2\alpha - 1 - z}{2q(\alpha - 1)}$$

Also note that

$$\begin{aligned}\frac{\partial z}{\partial \alpha} &= z^{-1} (1 - q) (4\alpha - 2) \\ \frac{\partial z}{\partial q} &= -2z^{-1} (\alpha^2 - \alpha)\end{aligned}$$

We now proceed to prove Proposition 3 claim by claim.

*Claim 1.*  $\mathbb{E}[D^{agg}]$  is decreasing in  $q$ .

*Proof.* We first express  $\mathbb{E}[D^{agg}]$  as a function of  $\alpha$ ,  $q$ , and  $z$ :

$$\begin{aligned}\mathbb{E}[D^{agg}] &= \frac{1 - \frac{2\alpha - 1 - z}{2q(\alpha - 1)}}{1 - \frac{2\alpha - 1 - z}{2(\alpha - 1)}} \\ &= \frac{2q(\alpha - 1) - 2\alpha + 1 + z}{2q(\alpha - 1) - 2q\alpha + q + qz} \\ &= \frac{(2q\alpha - 2q - 2\alpha + 1 + z)(z + 1)}{q(z - 1)(z + 1)} \\ &= \frac{(2\alpha - 1 - z)(1 - q)(\alpha - 1)}{2\alpha q(1 - q)(\alpha - 1)} \\ &= \frac{2\alpha - 1 - z}{2\alpha q}\end{aligned}$$

Differentiating with respect to  $q$ , we get

$$\frac{d\mathbb{E}[D^{agg}]}{dq} = \frac{-2\alpha q \frac{dz}{dq} - (2\alpha - 1 - z) 2\alpha}{(2q\alpha)^2}$$

which implies

$$\begin{aligned}
\text{sign} \left( \frac{\partial \mathbb{E} [D^{agg}]}{\partial q} \right) &= \text{sign} \left( 2q (\alpha^2 - \alpha) - (2\alpha - 1 - z) z \right) \\
&= \text{sign} \left( 2q (\alpha^2 - \alpha) + 4(1 - q) (\alpha^2 - \alpha) + 1 - (2\alpha - 1) z \right) \\
&= \text{sign} \left( (\alpha^2 - \alpha) (4 - 2q) + 1 - (2\alpha - 1) z \right)
\end{aligned}$$

Let  $RHS \equiv (\alpha^2 - \alpha) (4 - 2q) + 1 - (2\alpha - 1) z$ . The remaining proof consists of three steps: 1) show  $\frac{dRHS}{d\alpha} \geq 0$  for all  $\alpha \geq 1, q \in [0, 1]$ ; 2) show  $\frac{dRHS}{dq} \geq 0$  for all  $\alpha \geq 1, q \in [0, 1]$ , with equality holding only when  $\alpha = 1$  or  $q = 0$ ; 3) RHS evaluated at  $\alpha = 1, q = 0$  is zero, concluding that  $RHS > 0$  for  $\alpha > 1, q > 0$ .

Step 1: show  $\frac{dRHS}{d\alpha} > 0$  for all  $\alpha \geq 1, q \in [0, 1]$ . Differentiating  $RHS$  with respect to  $\alpha$ , we have

$$\begin{aligned}
\frac{dRHS}{d\alpha} &= (2\alpha - 1) (4 - 2q) - 2z - (2\alpha - 1) \frac{\partial z}{\partial \alpha} \\
&= (2\alpha - 1) \left( 4 - 2q - z^{-1} (1 - q) (4\alpha - 2) \right) - 2z \\
&> (2\alpha - 1) \left( 4 - 2q - z^{-1} (1 - q) (4\alpha - 2) \right) \\
&\geq (2\alpha - 1) \left( 4 - \max_{q \in [0, 1]} (2q) - \max_{q \in [0, 1]} z^{-1} (1 - q) (4\alpha - 2) \right) \\
&= (2\alpha - 1) (4 - 2 - 2) \\
&= 0
\end{aligned}$$

Step 2: show  $\frac{dRHS}{dq} > 0$  for all  $\alpha \geq 1, q \in [0, 1]$ , with equality holding only when  $\alpha = 1$  or  $q = 0$ . Differentiating  $RHS$  with respect to  $q$ , we have

$$\begin{aligned}
\frac{dRHS}{dq} &= -2 (\alpha^2 - \alpha) - (2\alpha - 1) \frac{\partial z}{\partial q} \\
&= (\alpha^2 - \alpha) \left( 2z^{-1} (2\alpha - 1) - 2 \right) \\
&= \underbrace{2z^{-1}}_{>0} \underbrace{(\alpha^2 - \alpha)}_{\geq 0} (2\alpha - 1 - z)
\end{aligned}$$

The last term is non-negative and is zero only when  $q = 0$ . To see this, note

$$\begin{aligned} z &= \sqrt{4(1-q)(\alpha^2 - \alpha) + 1} \\ (\text{equal only if } q = 0) &\leq \sqrt{4\alpha^2 - 4\alpha + 1} \\ &= 2\alpha - 1 \end{aligned}$$

Hence we have  $\frac{dRHS}{dq} > 0$ .

Step 3: conclude the proof. Note

$$RHS|_{q=0, \alpha=1} = 0$$

Hence we have, for any  $\alpha > 1$  and  $q > 0$ ,  $RHS > 0$ . Thus  $\frac{\partial \mathbb{E}[D^{agg}]}{\partial q} > 0$ .  $\square$

*Claim 2.*  $\mathbb{E}[I^{agg}]$  is decreasing in  $q$ .

*Proof.* Given the result in lemma (4), we first show that investment is decreasing in  $q$  if and only if the ex ante welfare for the borrower is decreasing in  $q$ . To see this, note

$$\begin{aligned} \mathbb{E}[I^{agg}] &= \frac{(\alpha - 1)(z + 1 - 2\alpha(1 - q))}{2\alpha q(2\alpha - 1)} \\ &= \frac{2(\alpha - 1)}{\alpha(2\alpha - 1)} \cdot \left[ \frac{(z + 1 - 2\alpha(1 - q) - 2q)}{4q} + \frac{1}{2} \right] \\ &= \frac{2(\alpha - 1)}{\alpha(2\alpha - 1)} \cdot \left[ V(0) + \frac{1}{2} \right] \end{aligned}$$

To show  $V(0)$  is decreasing in  $q$ , first note

$$\begin{aligned} V(0) &= \frac{1 - 2\alpha(1 - q) - 2q + z}{4q} \\ \frac{dV(0)}{dq} &= \frac{4q(2\alpha - 2 + z_q) - 4(1 + 2\alpha(q - 1) - 2q + z)}{16q^2} \\ &= \frac{1}{4q^2} [q(2\alpha - 2 + z_q) - 1 + 2\alpha(1 - q) + 2q - z] \\ &= \frac{1}{4q^2} [-2\alpha [(\alpha - 1) \left(\frac{q}{z}\right) - 1] - (1 + z)] \end{aligned}$$

Define  $\Omega \equiv -2\alpha [(\alpha - 1) \left(\frac{q}{z}\right) - 1] - (1 + z)$ , we then have  $\text{sign} \left( \frac{dV(0)}{dq} \right) = \text{sign}(\Omega)$ .

To compute the derivative of  $\Omega$  with respect to  $q$ :

$$\begin{aligned}\frac{d\Omega}{dq} &= -2\alpha(\alpha-1)\frac{z-qzq}{z^2} - zq \\ &= -\frac{1}{z^3}4\alpha^2(\alpha-1)^2q \leq 0\end{aligned}$$

Therefore  $\Omega$  is declining in  $q$  for all  $\alpha > 1$ . This means to check that  $\frac{dV(0)}{dq} < 0$  it is sufficient to check that  $\Omega|_{q=0} \leq 0$ .

$$\begin{aligned}\Omega(q=0) &= (2\alpha-1) - z \\ &= 0\end{aligned}$$

Hence welfare is decreasing in  $q$ . □

*Claim 3.*  $\mathbb{E}[D^{ass}] / \mathbb{E}[I^{ass}]$  is increasing in  $q$ .

*Proof.* Follows directly from claims 1 and 2. □

*Claim 4.*  $\lim_{q \rightarrow 1} V(0) = 0$ .

*Proof.* We can write the ex-ante welfare as

$$V(0) = \frac{1 + 2\alpha(q-1) - 2q + z}{4q}$$

Using the fact that  $\lim_{q \rightarrow 1} z = 1$  and taking limit, the result is immediate. □

## Proof of Proposition 5

Take  $\alpha_1 > \alpha_2$ , and we first show  $b^*(q, \alpha_1) > b^*(q, \alpha_2)$  and  $\ell^*(q, \alpha_1) > \ell^*(q, \alpha_2)$ . From the proof of lemma 1, we have

$$\ell^{-1}(b, q, \alpha) = \frac{1-q}{1-qb^{-1}}$$

hence  $\ell(b, q, \alpha_1) = \ell(b, q, \alpha_2)$ . Moreover,

$$b^{-1}(\ell, q, \alpha) = \frac{(2\alpha\ell^{-1} - (1-q)) - \sqrt{(1-q - 2\alpha\ell^{-1})^2 - 4q\alpha^2\ell^{-2}}}{2q\alpha\ell^{-1}}$$

and it immediately follows that  $b(\ell, q, \alpha)$  is increasing in  $\alpha$ . Since  $\ell_b(b, q, \alpha) > 0$ , it follows that  $b^*(q, \alpha_1) > b^*(q, \alpha_2)$  and  $\ell^*(q, \alpha_1) > \ell^*(q, \alpha_2)$ .

Next, from lemma (3) we have

$$\mathbb{E}[D^{agg}] = \frac{1 - b^{-1}}{1 - qb^{-1}}$$

$$\mathbb{E}[D^{agg}] / \mathbb{E}[I^{agg}] = \frac{1 - b^{-2}q}{b^{-1}(1 - q)}$$

both of which are increasing in  $b$ , which establishes the result that they are also both increasing in  $\alpha$ . □

## Proof of Proposition 6

From lemma (4) we have

$$\mathbb{E}[I^{agg}] = \frac{(\alpha - 1)(z + 1 - 2\alpha(1 - q))}{2\alpha q(2\alpha - 1)}$$

Differentiating with respect to  $\alpha$ , we get

$$\frac{\partial \mathbb{E}[I^{agg}]}{\partial \alpha} = \frac{q(2\alpha - 2\alpha^2 z - 10\alpha^2 + 8\alpha^3) - (z - 6\alpha + 4\alpha^2 z + 12\alpha^2 - 8\alpha^3 - 4\alpha z + 1)}{2\alpha^2 q(2\alpha - 1)^2 z}$$

We use Matlab symbolic toolbox to verify the proposition: for any given  $q \in (0, 1)$ , there exists an unique  $\bar{\alpha}(q) \in (1, \infty)$  such that

$$\frac{d\mathbb{E}[I^{agg}]}{d\alpha} \begin{cases} < 0 & \text{for } 1 \leq \alpha < \bar{\alpha}(q) \\ > 0 & \text{for } \alpha > \bar{\alpha}(q) \end{cases}$$

and

$$\frac{d\bar{\alpha}(q)}{dq} < 0.$$

Proposition 6 can be visualized in figure 7, where we plot  $\mathbb{E}[I^{agg}]$  against  $q$  and  $\alpha$ . The red curve on the graph traces out  $(q, \bar{\alpha}(q), \mathbb{E}[I^{agg}|q, \bar{\alpha}(q)])$ . □

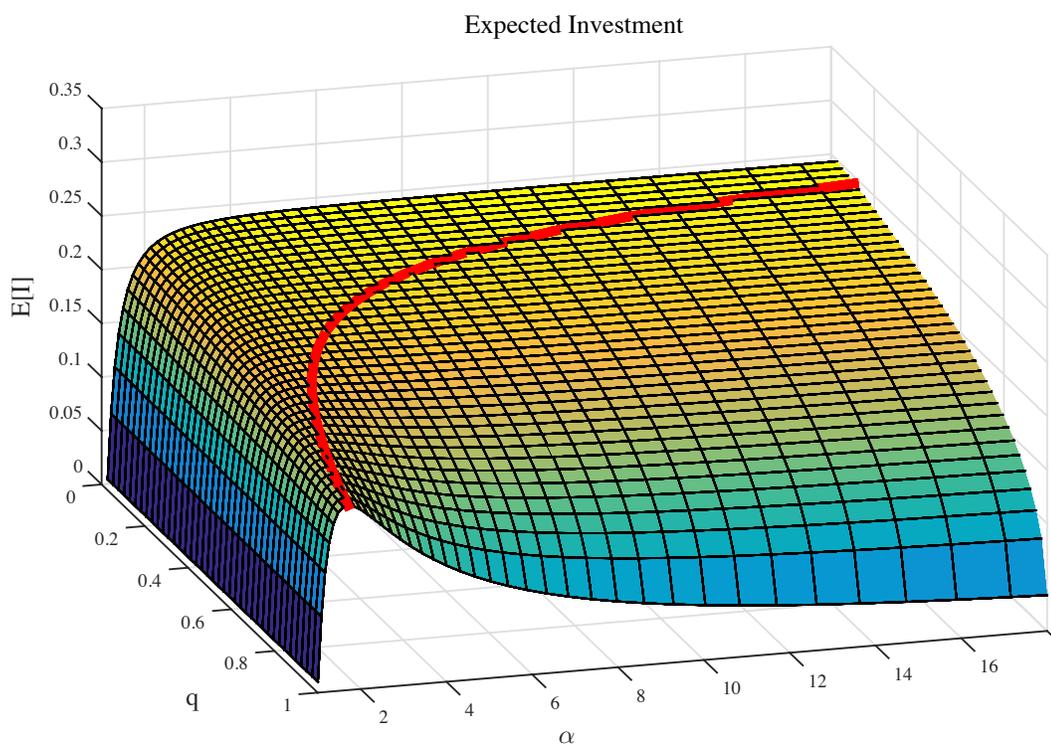


Figure 7: Expected Investment

Note:  $\mathbb{E}[I^{agg}]$  is plotted on the z-axis against  $q$  and  $\alpha$  for  $q \in (0, 1)$  and  $\alpha \in (1, 18)$ . The red curve on the graph traces out  $(q, \bar{\alpha}(q), \mathbb{E}[I^{agg}|q, \bar{\alpha}(q)])$ .