Mutual Fund's Reputation for Information Superiority

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Abstract

Whether a fund has superior information determines its investment outcome, but information superiority is overlooked when we discuss a mutual fund's reputation. We model the fund's reputation as investors' beliefs about whether the fund is informed. The superior information decays but can be endogenously acquired. Thus, we propose a reputation model without exogenous types, which delivers new predictions. In the equilibrium, a fund's previous performance is not a good predictor of its current investment outcome, a fund's size is not continuous in its previous performance, and a fund's most recent performance has the most important impacts on its cash flows.

Keywords: Mutual Fund, Reputation, Information Acquisition, Obsolete Information

JEL Classification Codes: C73, D83, G23

A mutual fund with a higher reputation is believed to perform better, so it attracts more cash flows.[1] Hence, a mutual fund has a strong incentive to manage its reputation and thus works hard for its investors because the mutual fund is paid based on the assets under management rather than its performance (Berk, 2005). However, the concept of reputation is so familiar as to be taken for granted; thus, little attention has been paid to understanding the nature of the reputation in the mutual fund industry. In particular, what exactly do we talk about when we discuss a mutual fund's reputation?

In the literature, a mutual fund's reputation is defined as investors' beliefs over its abilities, such as stock picking and market timing (Kacperczyk, Van Nieuwerburgh, and Veldkamp, 2014). Berk and Green (2004) assume that a mutual fund's ex-ante investment outcome is determined by the fund's ability and the fund's size. Given the fund's size, if the fund's performance is better than expected, the fund is believed to have a higher ability; that is, the fund's reputation increases, leading to cash inflows. In Berk and Green (2004), the fund's ability is unknown to both the fund and the investors, but it will be learned as time goes on. In recent papers about fund managers' career concerns (for example, Dasgupta and Prat (2006, 2008), Dasgupta, Prat, and Verardo (2011), and Malliaris and Yan (2012)), fund managers' reputations are also defined as investors' beliefs over their abilities, but the abilities are fund managers' private information.[2] In these papers, funds' abilities are intrinsic and therefore permanent.

However, besides its ability, we also have to consider a mutual fund's information status when we discuss its reputation, because information superiority is as important as ability to determine a mutual fund's performance, if not more. This stems from the Efficient Market hypothesis. Since stock prices have incorporated all available information in the market, a mutual fund cannot persistently outperform its peers unless it persistently possesses information superiority. Moreover, information superiority has been shown as the reason why mutual funds are endogenously built (García and Vanden (2009) and He (2010)). Recent empirical studies also show the key role of information superiority in determining mutual funds' performance. Cohen, Frazzini, and Malloy (2008) construct connected portfolios and nonconnected portfolios for the same funds and find that connected portfolios perform much better than nonconnected ones. (Here, a fund's connected portfolio consists of stocks of companies whose CEOs have educational connections to the fund manager.) This implies that inside information through educational networks is critical for a

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[1] Reputation is ranked highly among investors' funds picking criteria (see, for example, Capon, Fitzsimons, and Prince (1996) and the Investment Company Institute Survey (1997)). The survey by Jones, Lesseig, and Smythe (2005) show that financial advisers also emphasize the importance of the fund's reputation.

[2] Along this strand of the literature, Guerrieri and Kondor (2012) define fund managers' reputations as investors' beliefs about whether the managers are informed. However, different from our definition below, in Guerrieri and Kondor (2012), whether a manager is informed is exogenous and fixed forever.

fund's good performance even after controlling the fund manager's ability. In addition, superior information from business connections (Tang, 2013), political connections (Gao and Huang, 2014), and industry concentration (Kacperczyk, Sialm, and Zheng, 2007) is also shown to help funds achieve superior performance.

In this paper, we model the reputation of a mutual fund as investors' beliefs about whether the fund has superior information.[3] Two special features of information differentiate our definition of a mutual fund's reputation from those about mutual funds' abilities, as well as those in numerous type reputation models, for example, a borrower's reputation in Diamond (1989).[4] First, since information can be acquired, an uninformed mutual fund can become informed by acquiring information. So a mutual fund's reputation in every period contains investors' beliefs about the fund's information acquisition behavior in that period. Second, information has short-run persistence but will become obsolete in the long run. As a result, conditional on that the fund is informed in the current period, it is possible that the fund is still informed in the next period even without the next period's information acquisition; but conditional on that an uninformed fund never acquires information, the fund's reputation will deteriorate over time. These two features determine that the mutual fund's reputation in our definition is investors' beliefs over a variable that is changing both exogenously and endogenously. As a result, in order to analyze the nature of the mutual fund's reputation under our definition, we develop a "pure" moral hazard model that assumes no exogenous type of a mutual fund.

Our model captures these critical properties of information. Given a fund's *prior reputation*, which is defined as investors' beliefs about whether the fund is currently informed, the uninformed manager makes the information acquisition decision. If she acquires information, she becomes informed; otherwise, she remains uninformed. Investors do not observe the manager's choice but form a belief over her information acquisition behavior. Such a belief, together with the fund's prior reputation, constitutes the fund's *interim reputation*. Investors will then make their purchase decisions based on the interim reputation. The fund's period payoff depends on the number of investors who purchase the fund's shares. Investors will reevaluate whether the fund is informed in the current period after observing the investment outcomes and form a *posterior reputation*:

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[3] We do not consider the agency problem between the fund and the fund manager, which is an optimal contract problem. So in this paper, the fund's reputation and the fund manager's reputation refer to the same thing. We also treat a fund family's reputation and its fund's reputation the same.

[4] There is a huge literature in economics in which an agent's reputation is modeled as investors' beliefs over his/her types. Kreps and Wilson (1982), Milgrom and Roberts (1982), and Fudenberg and Levine (1989) define an agent's reputation as investors' beliefs whether the agent is of a commitment type; Mailath and Samuelson (2001) regard an agent's reputation as investors' beliefs that the agent is not an inept person. Mailath and Samuelson (2013) survey recent works on reputations in repeated games of incomplete information.

because an informed fund manager can generate a good outcome with a higher probability than an uninformed manager does, good outcomes will always promote the fund's posterior reputation. Information may become obsolete, so the fund's posterior reputation in the current period, discounted by the information depreciation, turns into the prior reputation in the next period.

Because the fund's information superiority is privately known to the fund, and the fund's information acquisition behavior is unobservable to investors, moral hazard is inevitably present. Hence, a mutual fund's reputation-building process is an incentive mechanism in our paper. Such an incentive mechanism, together with information's special features, leads to a "work-work-shirk" stationary equilibrium of the model. In the equilibrium, in every period, if an uninformed fund is believed to possess information (high prior reputation), it "shirks" and does not acquire information. When an uninformed fund has a medium or low prior reputation, the fund "works" to acquire information with different probabilities. Hence, the fund's information acquisition decision is neither monotonic nor hump-shaped in its prior reputation.

Our model delivers several predictions about a mutual fund's performance, size, and cash flows, which are different from those in reputation models with exogenous types. First, our equilibrium exhibits an intragroup catch-up property. In every period, mutual funds can be classified into three groups based on their prior reputations: star funds with high prior reputations, medium funds with medium prior reputations, and bad funds with low prior reputations. Consider two medium funds, A and B, where Fund A's prior reputation is higher than that of Fund B. In the "work-work-shirk" equilibrium, Fund A and Fund B will have same expected investment outcomes. Fund B, though less likely to be informed than Fund A at the beginning of the period, will acquire information with a higher probability (if it is uninformed). Then Fund A and Fund B will reach the same interim reputation and thus the same expected investment outcome; they both achieve the ceiling of the expected performance in the group. Such an intragroup catch-up property also holds for bad funds. An empirical implication here is that a fund's history of performance is not the best predictor of the fund's investment outcome, since the fund's current information acquisition decision also matters.

Second, the presence of the intragroup ceiling of interim reputation naturally leads to a discrete intergroup jump of sizes and investment outcomes. Suppose Fund C is on the bottom of the medium funds, and Fund D is on the top of the bad funds. Then Fund C and Fund D will have very similar prior reputations; that is, at the beginning of the period, investors believe that Fund C and Fund D are almost equally likely to be informed. However, in the equilibrium, Fund C will have a much larger size than Fund D, and the expected investment outcome of Fund C will be much better than that of Fund D. This intergroup discrete change rationalizes the discrete differences in fund size generated by morningstar ratings (Reuter and Zitzewitz, 2013). Moreover,

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the discontinuity of the fund's size in the fund's prior reputation distinguishes our model from models with exogenously changing types. If we consider an extension of Berk and Green (2004), where the fund's ability changes exogenously over time, then two funds with very close prior reputations will have very close interim reputation because their abilities will change with the same probability.

Third, our model has different long-run implications about the flow-performance relationship from those in Bayesian learning models with permanent types, for example, Holmstrom (1999) and Berk and Green (2004). In models in which a fund has a permanent type, the fund's flows are barely affected by the recent performance when the fund is sufficiently old. This is due to the convergence result in Bayesian learning models with persistent types: because investors will ultimately learn the fund manager's true ability, the fund's size will not change much because of the fund's recent performance when the fund is old. However, it has been documented in empirical studies that the most recent performance is the best predictor of the fund's flow. For example, Coval and Stafford (2007) find that the most recent performance significantly affects the fund's current flows, even after controlling for a long history of past performance. This empirical fact, however, appears in the "work-work-shirk" equilibrium of our model: a fund's flows are always positively correlated with the fund's recent performance. Because the information superiority has short-run persistence only and the fund's information acquisition decision is unobservable to investors, recent performance is a better indicator of whether the fund is currently informed.

This paper enriches the reputation literature by studying a discrete-time infinite-horizon model in which reputation is defined as investors' beliefs over an endogenous variable. In a recent paper, Board and Meyer-ter-Vehn (2013) study a model in which a firm's reputation is the consumers' belief about the quality of its product. Their model is different from ours in the following aspects. First, in their setting, a shock periodically arrives and resets the quality of the product; upon the arrival of the shock, the firm's current effort determines the new quality.[5] Hence, in their model, the firm controls the quality stochastically, so it plays pure strategy in the equilibrium. In our model, the uninformed fund can directly choose whether to be informed, which naturally leads to mixing in the equilibrium. In addition, in their model, the firm's reputation always responds to good/bad signals. However, in our "work-work-shirk" equilibrium, the fund's reputation changes only if the current signal is different from the previous signal. Consequently, in our equilibrium the marginal effect of a good (or bad) signal is decreasing in an extreme way. Last but not least, in our equilibrium, both the fund's information acquisition effort and its expected performance

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[5]Halac and Prat (2014) adopt a similar setting to study a dynamic inspection game between a worker and a manager.

exhibit the intragroup catch-up property, which is missing in Board and Meyer-ter-Vehn (2013).[6]

The remainder of this paper is organized as follows. In Section 1, we present an infinite-horizon reputation model that captures the two special features of information. Section 2 solves a "work-work-shirk" stationary equilibrium. Section 3 describes the equilibrium properties, which are significantly different from those in reputation models about funds' abilities. Section 4 concludes.

1   The Model

Fund Manager. A fund manager may or may not have superior information for arbitrage. In each period, the uninformed fund decides whether to acquire information. Denote by σ_t the probability that the uninformed manager acquires information. If the uninformed manager chooses not to acquire information, the manager remains uninformed; if the manager acquires information, the manager will become informed in period t, but she needs to pay an effort cost $c > 0$. We assume that the manager knows whether she knows, but this is her private information. The fund manager's information acquisition behavior is unobservable to investors.

Investment Outcomes. In every period, the fund's investment has two possible outcomes: After fees, it either outperforms or underperforms the market. We call the former a "good outcome" and the latter a "bad outcome." The fund manager's information superiority determines the ex-ante distributions of the investment outcomes. If the manager is informed, she will generate a good investment outcome with probability $q \in (1/2, 1)$; if the manager is uninformed, the fund outperforms the market with probability $1 - q$.

Investors. There is a continuum of investors with measure 1. In each period, investors form a public belief about whether the fund is informed or uninformed. We assume that in period t, investors have all records of the fund's past performance, which is denoted by h^{t-1}; that is, $h^{t-1} = \{s_\tau\}_{\tau=1}^{t-1}$, where $s_\tau \in \{B, G\}$ for each $\tau = 1, 2, ...t - 1$.

Reputation. We interpret the public belief that the manager being informed as the manager's *reputation*. Formally, the fund's *prior reputation* in period t is

$$x_t = \Pr(\text{the fund is informed at the beginning of period } t | h^{t-1}).$$

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[6]Dilme (2012) proposed an alternative model of firm reputation in which actions have lasting effects because of switching costs. Bohren (2013) also considers a reputation model in which the reputation of a firm is the persistent quality of its product. However, in her model, there is no asymmetric information. A firm's reputation is an observable quality stock, which can be enhanced through the firm's costly investment.

The law of motion of the reputation within period t is described as follows. At the beginning of each period t, the manager's prior reputation is $x_t \in (0,1)$, and x_1 is exogenously given. Recall that the manager's information acquisition choice is *not* observed by the investors, but the market believes that she acquires information with probability $\hat{\sigma}_t$. Then the manager's *interim reputation* in period t is

$$z_t = x_t + (1 - x_t)\hat{\sigma}_t, \tag{1}$$

where $\hat{\sigma}_t$ is investors' beliefs that the manager acquires information in period t. Once the investment outcome is realized, the market will update the belief about whether the manager is informed in period t again:

$$\begin{cases} z_t^g = \frac{qz_t}{qz_t + (1-q)(1-z_t)}, & \text{if the investment outcome is good;} \\ z_t^b = \frac{(1-q)z_t}{(1-q)z_t + q(1-z_t)}, & \text{if the investment outcome is bad.} \end{cases} \tag{2}$$

Obsolete Information. An informed manager in period t, however, will become uninformed in period $t+1$ with probability $\lambda \in (0,1)$. Given an interim reputation z_t in period t, the prior reputation in period $t+1$ is:

$$x_{t+1} = (1 - \lambda)z_t^s, \tag{3}$$

where $s \in \{b, g\}$.

Payoff. For simplicity, we assume that the manager's flow revenue in period t is z_t, so the manager's discounted continuation payoff at time t is

$$\sum_{\tau=t}^{\infty} \delta^{\tau-1} \left[z_\tau - \mathbb{1}\{\text{the manager acquires information at time } \tau\}c \right], \tag{4}$$

where $\delta \in (0,1)$ is the discount factor and $\mathbb{1}\{\cdot\}$ is the indicator function. We summarize the timing in each period t in Figure 1.
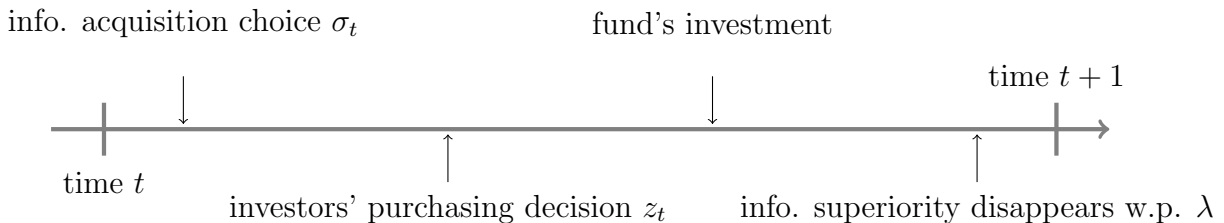

Figure 1: Timing

We are interested in a **Markov perfect equilibrium** in which

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1. the manager's strategy depends on the history through her prior reputation x only. Namely, there exists a function $\sigma : [0,1] \to [0,1]$ such that $\sigma_t = \sigma(x_t)$;

2. the equilibrium strategy $\sigma(x_t)$ maximizes the fund's expected continuation payoff (4);

3. the manager's strategy is consistent with investors' beliefs: $\hat{\sigma}_t = \sigma(x_t)$; and

4. over time, the reputation evolves according to Bayes' rule: equation (1), (2), and (3).

Because we focus on (stationary) Markov strategies, we can ignore the subscript t in the rest of this paper, and we denote x (and z) as the current prior (and interim) reputation and denote x' (and z') as the next period prior (and interim) reputation.

1.1 Discussion of Assumptions

Before analyzing the model, we discuss its special features. The key assumption of our model is $\lambda \in (0,1)$, which captures the two special features of information. On one hand, $\lambda < 1$ implies that information has short-run persistence, so investors may pay attention to the fund's past performance. On the other hand, $\lambda > 0$ suggests that information superiority may disappear quickly in the financial market.

The fund's interim reputation contains the probability of the uninformed fund's information acquisition. Because the fund's information acquisition behavior is not observable to investors, the equilibrium is self-confirming. That is, in the equilibrium, an uninformed fund acquires information with probability σ if and only if the market believes that the probability of information acquisition is σ.

In each period, the fund's size is assumed to be its interim reputation. This reduced form assumption is just for simplicity. It is sufficient to capture the fact that the number of investors is increasing in the fund's interim reputation because, by definition, investors think the fund is more likely to possess the information. However, this assumption can be justified in a number of ways. For example, we can imagine that each individual investor can access a private signal from private research about whether the fund manager is informed. Then given the interim reputation, there exist private signal structures that lead to the equation of the fund's interim reputation and the number of investors purchasing the fund. In fact, our result is robust if we assume that in each period, the size of the fund is an increasing function of the fund's interim reputation.

We assume the fund's period revenue is equal to the size of the fund. This is consistent with the fixed management fees as percentages of the assets under management. We do not consider the possible management fee changes made by the fund. Because the fund has private knowledge,

the fund offering management fees every period will have signaling effects, which make the model rather intractable.

Finally, we assume the fund's productivity has constant returns to scale. In our model, the total capital available in the market is finite, so the fund's revenue is well defined with a constant returns to scale production function. If a fund is a price taker in the financial market, that is, the fund's investment does not affect the assets' prices in the market, the fund's short-run excess rate of return from investment, the alpha, is independent of the volume it trades. Therefore, if the fund has information, the marginal rate of return of investments is constant, which means the fund's productivity has a constant return to scale in the short run. In the literature, the assumption of fund-level decreasing returns to scale is used to explain the fact that the fund's performance is not persistent (Berk and Green, 2004).[7] By assuming constant returns to scale, we show that, in addition to decreasing returns to scale, the fund's incentive to acquire information also contributes to the impersistence of a fund's performance.

2 Equilibrium Characterization

2.1 Preliminary Analysis

One of the most important features of our model is that the information has short-run persistence. That is, if the fund is informed today, it will remain informed with positive probability, so $\lambda \in (0, 1)$. This is important for a fund to have incentives to acquire information. Consider the case of $\lambda = 1$. Because the fund's information acquisition is unobservable to the market, for any given interim reputation in the current period, the fund's prior reputation in the next period is 0. This is because the information has no persistence; therefore, today's information surely becomes useless in the next period. Since the fund will be uninformed at the beginning of the next period, the fund's continuation value is independent of whether the fund is informed in the current period. This argument, together with the fact that the interim reputation is also independent of the fund's information superiority, implies that the reputation premium is 0. Since the effort cost is positive, the unique equilibrium when $\lambda = 1$ is that the fund never acquires information. Therefore, to analyze the reputation effects, we should assume that λ is sufficiently small.

Similarly, for an uninformed fund to have incentives to acquire information, a good investment

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[7]Empirical tests of the fund-level decreasing returns to scale, however, do not reach a conclusive result. Chen, Hong, Huang, and Kubik (2004) and Yan (2008) find negative size-performance relationships, while Reuter and Zitzewitz (2013) find no evidence of fund-level diseconomies of scale after applying a discontinuity approach to U.S. funds. Pástor, Stambaugh, and Taylor (2014) find insignificant negative size-performance relationships after controlling for endogeneity problems.

outcome must be significantly more probable if the fund acquires information. Therefore, we maintain the assumption below in the rest of the paper.

Assumption 1. $(2 - \lambda)q > 1.$

Even with Assumption 1, when the fund's interim reputation is too high, the effect of a good investment outcome on the prior reputation in the next period is dominated by that of the fund manager's information depreciation. The reason is that the fund's information depreciates linearly with probability $1 - \lambda$, while the effect of a good outcome is concave. Specifically, define $\hat{\omega}$ as the interim reputation such that the next period prior reputation following a good investment outcome is just $\hat{\omega}$. Then

$$\hat{\omega} = (1 - \lambda)\hat{\omega}^g = (1 - \lambda)\frac{q\hat{\omega}}{q\hat{\omega} + (1 - q)(1 - \hat{\omega})}.$$

Simple algebra implies that $\hat{\omega} = 1 - \frac{\lambda q}{2q-1} \in (0, 1)$ by Assumption 1. Therefore, conditional on a good investment outcome, when the manager's interim reputation is smaller than $\hat{\omega}$, the manager's prior reputation in the next period will be higher than the current interim reputation. Conversely, the fund's prior reputation in the next period will be smaller than its current interim reputation when its current interim reputation is greater than $\hat{\omega}$. This intuition can be seen from Figure 2 and is summarized in Lemma 1 below.
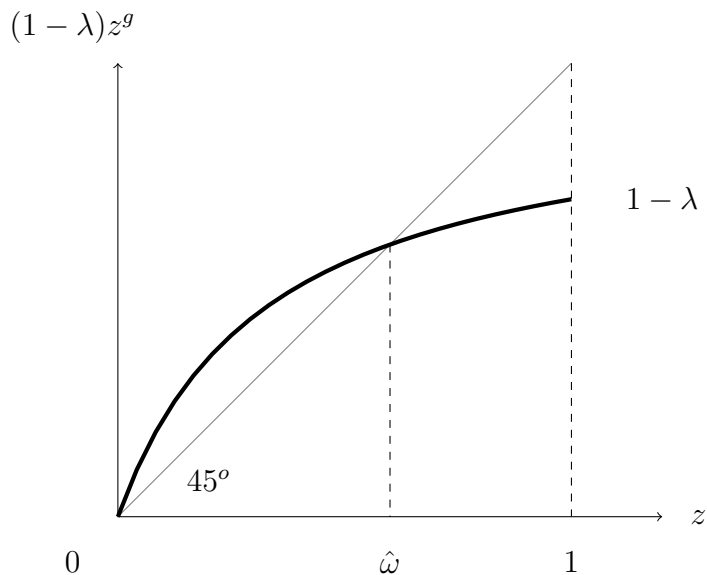

Figure 2: The Determination of $\hat{\omega}$. The thick curve represents the posterior reputation following a good performance by considering the information depreciation.

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Lemma 1. *Under Assumption 1, following a good investment outcome, a fund's prior reputation in the next period is*

$$x' = (1 - \lambda)z^g \begin{cases} < z, & \text{if } z > \hat{\omega}; \\ = z, & \text{if } z = \hat{\omega}; \\ > z, & \text{if } z < \hat{\omega}. \end{cases}$$

Denote by $V_I(x)$ the fund's continuation value if it is currently informed, and denote by $V_U(x)$ the fund's continuation value if it is currently uninformed and chooses to remain uninformed. Then,

$$\begin{aligned} V_I(x) = z \quad &+ \quad \delta q[(1 - \lambda)V_I((1 - \lambda)z^g) + \lambda \max\{V_U((1 - \lambda)z^g), V_I((1 - \lambda)z^g) - c\}] \\ &+ \quad \delta(1 - q)[(1 - \lambda)V_I((1 - \lambda)z^b) + \lambda \max\{V_U(z^b), V_I((1 - \lambda)z^b) - c\}], \end{aligned} \quad (5)$$

and

$$\begin{aligned} V_U(x) = z \quad &+ \quad \delta(1 - q) \max\{V_U((1 - \lambda)z^g), V_I((1 - \lambda)z^g) - c\} \\ &+ \quad \delta q \max\{V_U((1 - \lambda)z^b), V_I((1 - \lambda)z^b) - c\}. \end{aligned} \quad (6)$$

The fund's flow payoff equals the fund's interim reputation $z = x + (1-x)\hat{\sigma}$. Note that the interim reputation is not only affected by the fund's prior reputation but also by investors' beliefs about the uninformed fund's information acquisition. The interim reputation, however, does not depend on the fund's current information acquisition choice or the fact that it is informed or not, since neither its knowledge nor its action is observed by the market. Though an uninformed fund cannot affect its current flow payoff, it can obtain a higher continuation value by acquiring information. There are two benefits from information acquisition. First, the fund can generate a good outcome with a higher probability, so the fund can increase its prior reputation in the next period. Second, if the fund is informed in the current period, with positive probability $1 - \lambda$, the fund is still informed in the next period.

Lemma 2 below shows that in an equilibrium, a fund will not acquire information with probability 1.

Lemma 2. *In any equilibrium, $\sigma(x) < 1$ for each $x \in [0, 1]$.*

The conclusion drawn in Lemma 2 is due to the fact that the fund is forward-looking. Suppose there is an equilibrium in which the uninformed fund with prior reputation x acquires information for certain, i.e., $\sigma(x) = 1$. In such an equilibrium, the market can rationally anticipate the fund's behavior and form the interim belief $z = x + (1-x) \cdot 1 = 1$. Consequently, the posterior reputation

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of the fund is $z^g = z^b = 1$, and the next period prior reputation of the fund is $1 - \lambda$, regardless of the current period's investment outcome. Then the benefit of information acquisition depends only on the difference of continuation values between an informed fund and an uninformed fund at prior reputation $1 - \lambda$. Such a difference of continuation values must be strictly greater than c to support $\sigma(x) = 1$, which implies that an uninformed fund will acquire information for sure if its prior reputation is $1 - \lambda$. Because the fund discounts future values by $\delta < 1$, and the probability that an informed fund becomes uninformed is strictly less than 1, the difference of continuation values between an informed fund and an uninformed fund at the prior reputation $1 - \lambda$ will be strictly less than c, which leads to the contradiction. With Lemma 2, in the rest of the paper, the statement that "the fund acquires information" means the fund acquires information with positive probability.

Note that $V_U(\cdot)$ in Equation (6) is not the value function of the uninformed fund but the continuation value by assuming that the fund chooses to remain uninformed. The value function of an uninformed fund is $\max\{V_U(x), V_I(x) - c\}$. However, because of Lemma 2, in any equilibrium, we have

$$\max\{V_U(x), V_I(x) - c\} = V_U(x)$$

for any $x \in [0, 1]$. Consequently, on any equilibrium, Equations (5) and (6) can be simplified as

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\begin{aligned}
V_I(x) = z \quad &+ \quad \delta q[(1 - \lambda)V_I((1 - \lambda)z^g) + \lambda V_U((1 - \lambda)z^g)] \\
&+ \quad \delta(1 - q)[(1 - \lambda)V_I((1 - \lambda)z^b) + \lambda V_U(z^b)],
\end{aligned}
\tag{7}
$$

and

$$
V_U(x) = z \quad + \quad \delta(1 - q)V_U((1 - \lambda)z^g) + \delta q V_U((1 - \lambda)z^b).
\tag{8}
$$

Given the prior reputation x, we denote $V_I(x) - V_U(x)$ as the *reputation premium* of the fund. Then, the fund's optimal information acquisition rule is

$$
\sigma(x)
\begin{cases}
= 0, & \text{if } V_I(x) - V_U(x) < c; \\
\in [0, 1], & \text{if } V_I(x) - V_U(x) = c; \\
= 1, & \text{if } V_I(x) - V_U(x) > c.
\end{cases}
$$

That is, an uninformed fund acquires information if and only if the reputation premium is greater than the information acquisition cost. Notice that the reputation premium depends on investors' beliefs $\hat\sigma$ via the interim reputation z. Put differently, the fund's incentive to acquire information depends on whether the market believes it will do so, which is the self-confirming property of our equilibrium. Because the reputation premium is bounded in any equilibrium, $\sigma(x) = 0$ for any $x \in [0, 1]$ when c is large. To avoid a trivial case, in the rest of the paper, we assume that c is small.

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2.2 A "Work-Work-Shirk" Equilibrium

In this section, we construct an equilibrium in which the fund plays the following "work-work-shirk" strategy:

$$\sigma(x) = \begin{cases} \frac{\omega_1 - x}{1-x}, & \text{if} \quad x \in [0, \omega_1]; \\ \frac{\omega_0 - x}{1-x}, & \text{if} \quad x \in (\omega_1, \omega_0]; \\ 0, & \text{if} \quad x \in (\omega_0, 1]. \end{cases} \tag{9}$$

Here, $\omega_0 \geq 1 - \lambda$, and

$$\omega_1 = (1-\lambda)\omega_0^b = (1-\lambda)\frac{(1-q)\omega_0}{(1-q)\omega_0 + q(1-\omega_0)} \leq \hat{\omega};$$

that is, if the fund's interim reputation is ω_0 in the current period, ω_1 will be its next period prior reputation if it generates a bad outcome in the current period. This proposed strategy could be represented by Figure 3 below.
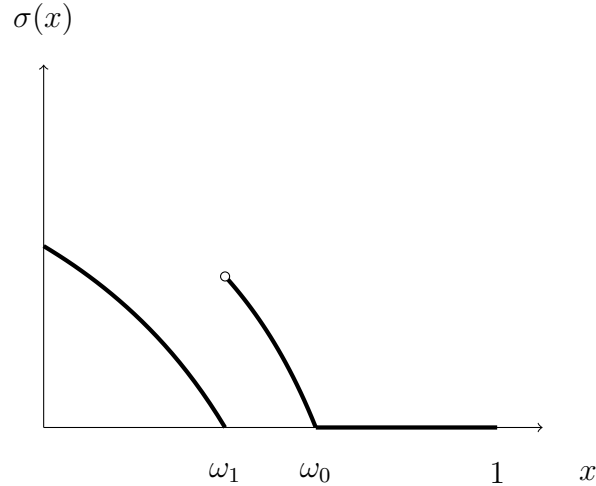

Figure 3: The "Work-Work-Shirk" Strategy

In the strategy profile, when the fund's prior reputation is $x \leq \omega_1$, the fund will acquire information with probability $(\omega_1 - x)/(1-x)$, leading to the interim reputation $z = x + (1-x)\hat{\sigma}(x)$. If the proposed "work-work-shirk" strategy profile is an equilibrium, $\hat{\sigma}(x) = \sigma(x)$, we have $z = \omega_1$. In this case, the next period's prior reputations following a good investment outcome and a bad investment outcome are $(1-\lambda)z^g \in (\omega_1, \omega_0]$ and $(1-\lambda)z^b < \omega_1$, respectively. When the prior reputation is $x \in (\omega_1, \omega_0]$, the fund will acquire information with probability $(\omega_0 - x)/(1-x)$ so that its interim reputation is $z = \omega_0$. Then, the next period's prior reputations following a good

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investment outcome and a bad investment outcome are $(1-\lambda)z^g \in (\omega_1, \omega_0]$ and $(1-\lambda)z^b \in [0, \omega_1)$, respectively. Finally, when the prior reputation is high such that $x > \omega_0$, the fund will certainly not acquire information. So its interim reputation is $z = x$, and its posterior reputations are $(1-\lambda)z^g, (1-\lambda)z^b \in (\omega_1, \omega_0]$. Such a law of motion of the "work-work-shirk" strategy is represented by the automaton in Figure 4 below.
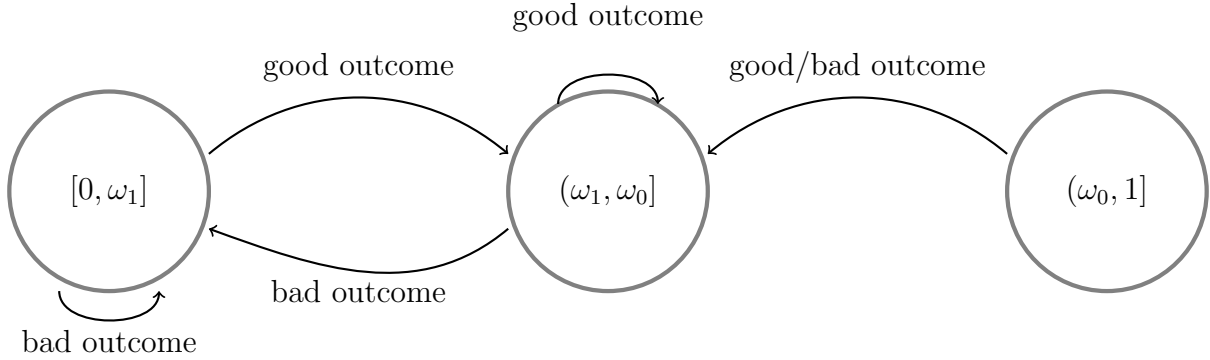

Figure 4: The Automaton Representation of the "Work-Work-Shirk" Equilibrium

Proposition 1. *Suppose that Assumption 1 holds and $\lambda \in [\hat{\lambda}, 1)$ for some $\hat{\lambda} \in (0, 1)$. There exists a pair $(\underline{y}, \bar{y})$ with $0 < \underline{y} < \bar{y}$, such that for each $c \in (\underline{y}, \bar{y})$, there exists a "work-work-shirk" stationary equilibrium described in Equation (9), where ω_0 is pinned down by Equation (15).*

Some remarks about Proposition 1 are in order. First of all, the key equilibrium condition is Equation (13). Because the flow payoffs to the informed fund and the uninformed fund are the same, the difference in values at any prior reputation depends only on continuation values. More important, given the proposed strategy profile, regardless of whether the prior reputation is ω_0 or ω_1, if the fund receives a good outcome, its interim reputation in the next period is ω_0; if the fund receives a bad outcome, its interim reputation in the next period is ω_1. Since the conditional probability of a good outcome is independent of the fund's reputation, the difference between an informed fund's continuation value and an uninformed fund's continuation value is independent of its reputation.

Second, the "work-work-shirk" equilibrium requires medium levels of the effort cost; that is, $c \in (\underline{y}, \bar{y})$. When the effort cost is too large, we cannot have an equilibrium in which the fund acquires information. Because the reputation premium is bounded, the reputation premium will be dominated by the effort cost when the effort cost is too large. On the other hand, when the effort cost is too small, the "work-work-shirk" strategy is not an equilibrium. Because $\omega_0 \geq 1 - \lambda$ and $\omega_1 < \hat{\omega}$, their difference $\omega_0 - \omega_1$ is bounded below by $(1-\lambda) - \hat{\omega}$. From Equation (13), the

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reputation premium at interim reputations ω_0 and ω_1 is determined by $\omega_0 - \omega_1$, so if the effort cost is too small, the fund will acquire information for sure at interim reputations ω_0 and ω_1, which contradicts Lemma 2.

2.3 Other Equilibria

In section 2.2, we characterize a "work-work-shirk" equilibrium in which there are two "work" regimes and one "shirk" regime. In reputation models, there is a "shirk" regime when the agent's reputation is very high so that his incentive is too small to work. However, one may wonder if it is reasonable to focus on the equilibrium with two "work" regimes. In this subsection, we argue that the model has neither a "work-shirk" equilibrium nor an equilibrium with more than two "work" regimes. More details are presented in Appendix B.

In an equilibrium with N "work" regimes ($N \neq 2$),

- there is a decreasing sequence $\{\omega_k\}_{k=0}^{N}$ such that $\omega_N = 0$, and $\omega_0 \leq 1$.

- when the uninformed fund's reputation x is in the kth "work" regime $(\omega_k, \omega_{k-1}]$, it acquires information with probability $\sigma(x) = \frac{\omega_{k-1} - x}{1 - x}$ so that its interim reputation is ω_{k-1}, for $k = 1, 2, ...N - 1$, and a good performance pushes the fund's reputation up to the $(k-1)$th regime, and a bad performance leads it down to the $(k+1)$th regime.

- when the uninformed fund's reputation is $x \in (\omega_0, 1]$, it does not acquire information.

When $N = 1$, it is a "work-shirk" equilibrium in which the fund's interim reputation will be ω_0 after a period in which its posterior reputation is lower than or equal to ω_0. When $N > 2$, the fund's reputation jumps up step by step as long as its investment performance is good, but its reputation is always lower than ω_0.

We show that such equilibria do not exist if (1) $N = 1$ under some mild conditions,[8] or (2) $N > 2$. The reason is as follows. For the equilibrium in which $N = 1$, $x = \omega_0$ is an absorbing state so that the fund's reputation is constant regardless of its investment performance. That implies that the uninformed fund has no incentive to acquire information with positive probability, which violates the equilibrium requirement. For equilibria in which $N > 2$, one has to ensure that the uninformed fund is indifferent between acquiring information or not in N states, which requires N equations holding. We show that it is impossible as long as $N > 2$.

Notice that we do not claim that the "work-work-shirk" equilibrium is the unique Markov perfect equilibrium. The family of equilibria we study is featured with multiple "blocked work

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[8]Without further restrictions, we cannot prove or disprove the existence of a "work-shirk" equilibrium.

regimes": In each state "work" regime, $x \in (\omega_{k+1}, \omega_k]$, the fund's interim reputation is $z = \omega_k$. We focus on such a family of equilibria because it is intuitive and technically tractable. There may exist an equilibrium in which the fund's interim reputation does not equal ω_k. We leave this issue to future research.

3 Reputation Effects

The "work-work-shirk" equilibrium characterized in Proposition 1 shows that a fund's initial reputation has great effects on an uninformed fund's incentives to acquire information. Because a fund's information acquisition decision determines its ex-ante investment outcome, we should expect that the fund's reputation has significant causal effects on the fund's performance, size, and cash flows. In this section, we analyze the reputation effects in the "work-work-shirk" equilibrium and provide several new predictions that do not exist in reputation models about mutual funds' abilities.

In the "work-work-shirk" equilibrium, we can classify funds into three groups based on their prior reputations. We call the funds with prior reputations greater than ω_0 "star" funds, those with prior reputations in $(\omega_1, \omega_0]$ "medium" funds, and those with prior reputations less than or equal to ω_1 "bad" funds.

3.1 intragroup Funds' Performance

Because $\omega_0 > 1 - \lambda$, star funds can exist for one period only. Therefore, we consider intragroup funds' performance for medium funds and bad funds only. Let's consider two medium funds, A and B. Assume $\omega_0 > x_A > x_B > \omega_1$. These two funds are illustrated in Figure 5 below.
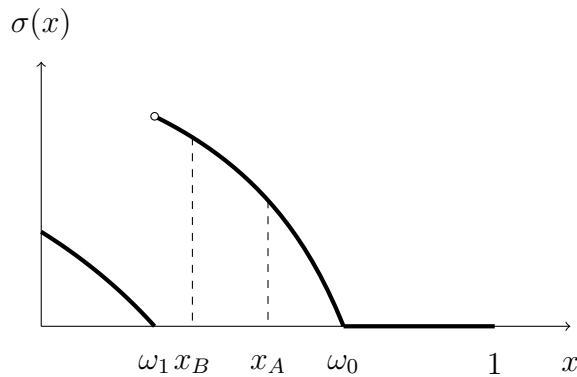

Figure 5: The Intragroup Funds' Performance

Because $x_A > x_B$, investors believe that Fund A is more likely to be informed at the beginning of the period; therefore, one may argue that Fund A should perform better than Fund B. However, this argument overlooks the fund's information acquisition decision. In the equilibrium, since both funds are in the medium group, if they are uninformed, they will acquire information with probability $(\omega_0 - x_A)/(1 - x_A)$ and $(\omega_0 - x_B)/(1 - x_B)$, respectively. Obviously, Fund B acquires information with a higher probability than Fund A does. Then simple algebra leads to the result that $z_A = z_B = \omega_0$. That is, Fund A and Fund B have the same probability to obtain a good investment outcome. The same logic applies to any two bad funds. Such an intragroup "catch-up" property is summarized in Proposition 2 below.

Proposition 2. *In the "work-work-shirk" equilibrium, all medium funds obtain good investment outcomes with a same probability ω_0, and all bad funds obtain good investment outcomes with a same probability ω_1.*

Previous performance is often employed to proxy a fund's prior reputation, which has causal impacts on the fund's current performance. However, Proposition 2 shows that a fund's performance in the previous period is not a good predictor of its current performance because the fund's information acquisition decision in the current period matters a lot for its current investment outcome. In the same group, the fund with a lower prior reputation acquires information with a higher probability, which just compensates its information disadvantage. Hence, all funds in the medium group will have the same interim reputation, which means that all funds in the medium group have the same expected investment outcome.

3.2 intergroup Funds' Sizes

In a recent study, Reuter and Zitzewitz (2013) use a regression discontinuity approach to empirically show that a small change in the fund's previous performance may lead to a discrete jump of cash flows because of its morningstar rating change. This suggests that a mutual fund's size is not continuous in the fund's previous performance.

The intergroup analysis of fund sizes of our model provides a rational way to justify such a discontinuity. Consider Fund C and Fund D, illustrated in Figure 6 below.
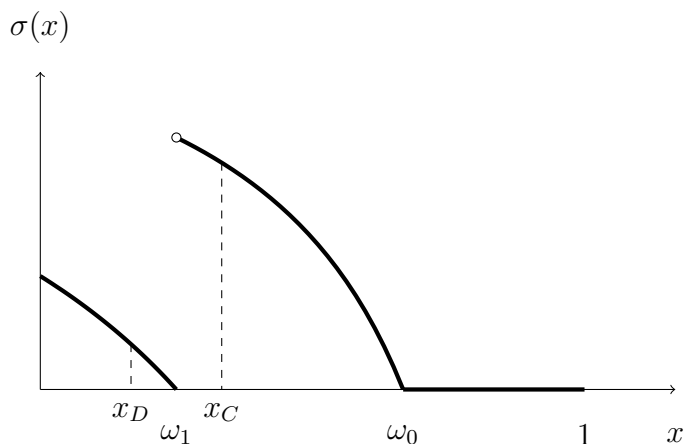
Figure 6: The Intergroup Funds' Sizes

In Figure 6, Fund C has a prior reputation $\omega_1 + \epsilon$, and Fund D has a prior reputation $\omega_1 - \epsilon$, where $\epsilon > 0$. Consider a very small ϵ, then Fund C and Fund D have very similar prior reputations. If we use the fund's previous performance to proxy the fund's prior reputation, Fund C's previous performance is very close to that of Fund D. However, Fund C belongs to the medium group, while Fund D is in the bad group.

In the "work-work-shirk" equilibrium, if neither Fund C nor Fund D is informed, Fund C acquires information with probability $[\omega_0 - (\omega_1 + \epsilon)]/[1 - (\omega_1 + \epsilon)]$, while Fund D acquires information with probability $[\omega_1 - (\omega_1 + \epsilon)]/[1 - (\omega_1 + \epsilon)]$. As a result, Fund C's interim reputation is $z_C = \omega_0$, and Fund D's interim reputation is $z_D = \omega_1$. Since $\omega_0 > \omega_1$, and a fund's size equals its interim reputation in our model, Fund C's size is much larger than that of Fund D. Proposition 3 below summarizes the argument above.

Proposition 3. *The size of a fund is not continuous in its prior reputation. In particular, a fund with a prior reputation slightly greater than ω_1 will have a size ω_0, while a fund with a prior reputation slightly less than ω_1 will have a size ω_1.*

Proposition 3 also distinguishes our model from those in which a mutual fund's ability is changing exogenously over time. Consider an extension of Berk and Green (2004) in which a fund manager's ability changes with positive probability. (With the complement probability, the fund manager's ability stays the same.) In Berk and Green (2004), the fund's size is a continuous function of investors' beliefs over the fund's ability, adding the exogenous type change will not change such a continuity.

3.3 Flow-Performance Relationship

A fund's net flow in any period t is the difference between the fund's size in period t and that in period $t-1$ (ignoring the fund's natural growth). In our model, the fund's size in any period t is just the fund's interim reputation z_t. Therefore, the fund's net flow f_t in period t is:

$$f_t = z_t - z_{t-1}.$$

In this subsection, we discuss how the fund's past performance affects the fund's current flow. Since we assume the fund's investment outcome is either good or bad, we are not trying to explain the cross-sectional convexity of the flow-performance relationship that is first documented by Sirri and Tufano (1998). Instead, we focus on the time series flow-performance relationship for a specific fund.

In the "work-work-shirk" equilibrium characterized by Equation (9), a fund's interim reputation z_t in any period t is determined by the fund's prior reputation in period t. Since the fund's size in period t is assumed to be equal to the fund's interim reputation in period t, the fund's size in period t is also determined by the fund's prior reputation in period t. In the "work-work-shirk" equilibrium, the fund's prior reputation can be pinned down by the fund's performance in the previous period, so we have Lemma 3 below.

Lemma 3. *In the "work-work-shirk" equilibrium described by Equation (9), in any period $t \geq 3$, the fund's size is*

$$z_t = \begin{cases} \omega_0, & if \quad s_{t-1} = G; \\ \omega_1, & if \quad s_{t-1} = B. \end{cases} \tag{10}$$

The conclusion in Lemma 3 that a fund's size is uniquely determined by the fund's performance in the previous period is due to the information's two special features. On one hand, the information superiority disappears with probability λ, so the prior reputation in any period is below ω_0 (since ω_0 by construction is greater than $1 - \lambda$). On the other hand, a bad investment outcome leads to a low prior reputation in the next period that is at most ω_1, while a good investment outcome leads to a prior reputation in the next period that is strictly greater than ω_1. Then in the equilibrium, investors will take the fund's likelihood to acquire information into account so that the interim reputation is ω_0 for any prior reputation between ω_1 and ω_0, and the interim reputation is ω_1 for any prior reputation less than ω_1. Therefore, after initial periods, a fund's size depends only on the fund's performance in the previous period.

The fund's flow in any period t is the net increment of the fund's size from period $t-1$ to period t. By Lemma 3, the fund's size in period t is determined by the fund's performance in

period $t-1$ only, so the fund's flow in period t will depend on the fund's performance in period $t-1$ and that in period $t-2$. Proposition 4 below summarizes the flow-performance relationship in the "work-work-shirk" equilibrium.

Proposition 4. *In the "work-work-shirk" equilibrium described by Equation* (9)*, in any period* $t \geq 4$*, the fund's flow depends on the fund's performance in the two most recent periods. In particular,*

$$f_t = \begin{cases} \omega_1 - \omega_0 < 0, & if \quad s_{t-1} = B \quad and \quad s_{t-2} = G; \\ 0, & if \quad s_{t-1} = s_{t-2} = G \quad or \quad s_{t-1} = s_{t-2} = B; \\ \omega_0 - \omega_1 > 0, & if \quad s_{t-1} = G \quad and \quad s_{t-2} = B. \end{cases} \tag{11}$$

Proposition 4 has important qualitative empirical implications. First of all, after the initial periods, the fund's flow in period t always depends on the fund's performance in recent periods. As a result, no matter how long the fund's record is, the fund's reputation will not converge. The driving forces of this implication are the information depreciation and the unobservable fund's information acquisition. Because the information has only short-run persistence, the fund's recent performance is most important when investors make inferences as to whether the fund is informed at the beginning of the current period. The investors also care about whether the fund, if it is uninformed, acquires information in the current period. If the information acquisition behavior is observable to investors, the fund's performance in a period in which the fund acquires information will be irrelevant to the fund's next period size and thus flow. Corollary 1 below summarizes this argument.

Corollary 1. *Independent of the fund's age, the fund's performance in the recent periods is relevant to predict the fund's flow.*

Corollary 1 is supported by some empirical findings. Chevalier and Ellison (1997) document that even for funds older than 11 years, their current flows are significantly correlated to their performance in recent periods. Coval and Stafford (2007) find that the most recent performance is significantly correlated to the current flow, even after controlling for a long history of performance.

The conclusion drawn in Corollary 1 differs from the convergence result in Bayesian learning models with permanent types, such as Holmstrom (1999) and Berk and Green (2004). In such models, the fund's optimal size depends on the fund's ability, which is the fund's permanent type. Investors can learn the fund's ability by observing the fund's past sizes and performance. Then with a long record, the investors' posterior belief about the fund's true ability will converge to 1. This convergence result implies that after controlling for a long history of performance, the fund's size should not change much by the most recent fund's performance. In other words, in

the Bayesian learning models with persistent types, the fund's performance in recent periods can hardly affect the fund's flow.

Proposition 4 also implies that the fund's most recent performance predicts the fund's current flow best, which has been widely supported by empirical evidence. Sirri and Tufano (1998) conclude that investors respond most strongly to the most recent fund history, based on the observation that investors' reactions to performance are not markedly stronger for extreme performance measured after five years than after one year. Coval and Stafford (2007) also document that the fund's investment return in period $t-1$ has the most important impact on the fund's flow in period t, followed by the fund's investment returns in period $t-2$ and period $t-3$; investment returns before period $t-4$, however, have much smaller economic effects on the current flow. Corollary 2 below summarizes this empirical implication of the model.

Corollary 2. *A fund's performance in most recent periods has the most important impact on its current flow.*

Finally, as noted by Berk and Tonks (2007), in the "work-work-shirk" equilibrium, the correlation of the fund's performance in recent periods also has significant effects on the fund's current flow. As in Berk and Tonks (2007), we define funds with $s_{t-2} = s_{t-1} = B$ as *seasoned* bad funds and those with $s_{t-2} = G$ and $s_{t-1} = B$ as *unseasoned* bad funds. Similarly, funds with $s_{t-2} = s_{t-1} = G$ are called *seasoned* good funds, and funds with $s_{t-2} = B$ and $s_{t-1} = G$ are called *unseasoned* good funds. Proposition 4 then implies that unseasoned funds have more significant flows than seasoned funds do. This implication is mainly due to the self-confirming property of the equilibrium. For example, the seasoned bad funds have a low interim reputation in period $t-1$, so the size is ω_1 in period $t-1$. Then another bad performance $s_{t-1} = B$ makes investors think the fund is more likely to be uninformed, which, together with the information depreciation, leads to a very low prior reputation in period t. That is, $(1-\lambda)\omega_1^b < \omega_1$. At this point, it seems that there should be significant flows out from the fund. However, given that the prior reputation in period t is less than ω_1 in this case, investors will believe that the fund acquires information with probability $(\omega_1 - (1-\lambda)\omega_1^b)/(1 - (1-\lambda)\omega_1^b)$ so that the fund's interim reputation in period t goes back to ω_1 in period t. Therefore, there are no significant cash outflows from seasoned bad funds. The self-confirming property, however, increases the flows into unseasoned good funds. A fund with a bad performance in period $t-2$ has an interim reputation ω_1 in period $t-1$. The fund then generates a good investment outcome in period $t-1$, which promotes the fund's prior reputation to $(1-\lambda)\omega_1^g$. By Lemma 1, because $\omega_1 < \hat{\omega}$, $(1-\lambda)\omega_1^g > \omega_1$. Without considering the probability of the fund's information acquisition, the fund's flow in period t would be $(1-\lambda)\omega_1^g - \omega_1$. However, investors rationally believe that the fund acquires information with

the probability $[\omega_0 - (1-\lambda)\omega_1^g]/[1 - (1-\lambda)\omega_1^g]$ so that the interim reputation of the fund in period t is ω_0. Since ω_0 is strictly greater than $(1-\lambda)\omega_1^g$, the cash inflow is greater. These arguments lead to Corollary 3 below.

Corollary 3. *In the "work-work-shirk" equilibrium, seasoned funds have insignificant flows, while unseasoned funds' flows are more significant. Moreover, whether a fund has outflows or inflows depends on the fund's most recent performance.*

4   Conclusion

While it is widely accepted that information superiority is a necessary condition for a mutual fund to obtain good investment outcomes, information superiority is overlooked when people discuss a mutual fund's reputation. We model a mutual fund's reputation as investors' beliefs about whether the fund has superior information. Because of information's special properties, our definition of a mutual fund reputation is investors' beliefs over a variable that is changing both exogenously and endogenously. Therefore, we propose a "pure" moral hazard discrete time repeated model to analyze the reputation effects on the fund's performance, size, and cash flows.

Our model delivers several predictions that do not exist in reputation models about mutual funds' abilities. We classify mutual funds into three groups based on their prior reputations (or their previous performance) and focus on bad funds and medium funds. First, any two funds within the same group will have a very similar performance because an uninformed fund with a lower prior reputation acquires information with a higher probability. Therefore, past performance is not a good predictor of the fund's current investment outcome. Second, a fund's size is not a continuous function of its prior reputation and thereby is not continuous in its previous performance. Around the cutoff point separating medium funds from bad funds, a slight increase in the fund's prior reputation will generate a discrete jump of the fund's size. Third, a fund's performance in most recent periods is most relevant to predict the fund's cash flows, even when the fund is old.

Appendix A   Omitted Proofs

Proof of Lemma 1. Under Assumption 1, $\hat{\omega} \in (0,1)$. Define function

$$g(z) = (1-\lambda)\frac{zq}{zq + (1-z)(1-q)} - z.$$

By the definition of $\hat{\omega}$, when $z = \hat{\omega}$, $(1-\lambda)z^g = z$. That is, $g(\hat{\omega}) = 0$. Then we just need to show that $g(z)$ is strictly decreasing. Since $(zq)/(zq + (1-z)(1-q))$ is strictly concave, so $g'(z)$ reaches the highest value when $z = 0$. Since $q > 1/2$, $g'(0) = (1-\lambda)\frac{q}{1-q} - 1 < 0$. Therefore, for all z, $g'(z) < 0$. Therefore, $g(z)$ is strictly decreasing. Hence, when $z < \hat{\omega}$, $g(z) > 0$, implying that $(1-\lambda)z^g > z$; when $z > \hat{\omega}$, $g(z) < 0$, implying that $(1-\lambda)z^g < z$.

$\square$

Proof of Lemma 2. Suppose $\sigma(x) = 1$ for some x. Then the manager with the prior reputation x will have the interim reputation $z = x + (1-x)\sigma(x) = 1$. Then the manager's reputation in the next period will be $1 - \lambda$, independent of the investment outcome. To support the equilibrium, we must have $V_I(x) - V_U(x) \geq c$, which implies that

$$V_I(1-\lambda) - \max\{V_I(1-\lambda) - c, V_U(1-\lambda)\} \geq \frac{c}{\delta(1-\lambda)}. \tag{12}$$

If $V_I(1-\lambda) - c > V_U(1-\lambda)$, Equation (12) above becomes $V_I(1-\lambda) - [V_I(1-\lambda) - c] = c \geq \frac{c}{\delta(1-\lambda)}$. Since $\delta < 1$ and $1 - \lambda < 1$, this equation obviously does not hold. In the case that $V_I(1-\lambda) - c \leq V_U(1-\lambda)$, Equation (12) becomes $V_I(1-\lambda) - V_U(1-\lambda) \geq \frac{c}{\delta(1-\lambda)} > c$. As a result, $\sigma(1-\lambda) = 1$ in the equilibrium under consideration. Then $V_I(1-\lambda) - V_U(1-\lambda) = \delta(1-\lambda)[V_I(1-\lambda) - V_U(1-\lambda)]$, which is a contradiction!

$\square$

Proof of Proposition 1. In the proof, we first construct the equilibrium strategy profile and provide the necessary condition under which the "work-work-shirk" equilibrium specified in Proposition 1 holds. The conditions include the restriction on λ and the cutoff reputation ω_0, ω_1 specified in the strategy profile (9). Then, given the constructed equilibrium strategy profile, we verify that no player has the incentive to deviate.

Equilibrium Construction. Given the proposed strategy profile, there are only two relevant interim reputations, ω_0 and ω_1, in our analysis. To save notations, we denote by $V_K^i = V_K(\omega_i)$ the fund's continuation value when the fund's interim reputation is ω_i, and it is in the status $K = I$ (informed) or $K = U$ (uninformed). Let's first consider the fund's continuation value when its prior reputation is $x > \omega_0$. Since investors believe the fund will not acquire information, the fund's

interim reputation is $z = x$.

$$V_I(x) = x + \delta \left[(1-\lambda)V_I^0 + \lambda V_U^0 \right],$$
$$V_U(x) = x + \delta V_U^0.$$

Regardless of whether the fund is informed, its flow payoff will be its interim reputation $z = x$. Because $z > \omega_0$, when the fund gets a good investment outcome, the fund's prior reputation in the next period will be less than $1 - \lambda$, so it will be less than ω_0. This is because the fund's information superiority disappears with probability $1 - \lambda$; as a result, no matter how likely the fund was informed in the previous period, the fund's prior reputation in the next period cannot be greater than $1 - \lambda$. When the fund gets a bad investment outcome, the fund's prior reputation in the next period will be less than $1 - \lambda$ but larger than ω_1, since the fund's current interim reputation is greater than ω_0. Therefore, when the fund's interim reputation is $z > \omega_0$, the fund's prior reputation in the next period is ω_0, which is independent of the fund's performance in the current period. Then, when the fund is informed, the fund will still be informed in the next period with probability $1 - \lambda$; but with probability λ, an informed fund will become uninformed. For an uninformed fund, it will still be uninformed at the beginning of the next period. By Lemma 2, in an equilibrium, keeping uninformed is at least as good as acquiring information to the uninformed fund, so its continuation value is just V_U^0.

When the fund's prior reputation is $x \in (\omega_1, \omega_0]$, the fund's interim reputation will jump to ω_0. So the fund with the prior reputation $x \in (\omega_1, \omega_0]$ has continuation values:

$$V_I(x) = \omega_0 + \delta \left[q(1-\lambda)V_I^0 + q\lambda V_U^0 + (1-q)(1-\lambda)V_I^1 + (1-q)\lambda V_U^1 \right],$$
$$V_U(x) = \omega_0 + \delta \left[(1-q)V_U^0 + qV_U^1 \right].$$

When the fund's prior reputation is $x \in [0, \omega_1]$, the fund's interim reputation will jump to ω_1. In this case, the fund's continuation value is:

$$V_I(x) = \omega_1 + \delta \left[q(1-\lambda)V_I^0 + q\lambda V_U^0 + (1-q)(1-\lambda)V_I^1 + (1-q)\lambda V_U^1 \right],$$
$$V_U(x) = \omega_1 + \delta \left[(1-q)V_U^0 + qV_U^1 \right].$$

If the strategy profile described in Equation (9) is an equilibrium, we must have $\omega_1 < \hat{\omega}$. Suppose $\omega_1 \geq \hat{\omega}$, then when the fund's interim reputation is ω_1, the fund's next period prior reputation will be lower than ω_1, regardless of whether the investment outcome is good or bad, by the definition of $\hat{\omega}$. Then, according to the proposed strategy profile, the fund's next period interim reputation is ω_1, independent of the current period investment outcome. This will break the equilibrium because the fund with a prior reputation less than ω_1 will certainly not acquire

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information. Therefore, to support the existence of ω_0 and ω_1 and thus the equilibrium, we require that the distance between $1 - \lambda$ and $\hat{\omega}$ is small enough, such that when the fund with the current interim reputation $1 - \lambda$ gets a bad outcome, the fund's next period prior reputation is smaller than $\hat{\omega}$. This requirement can be rewritten as $1 < \lambda + q(1 + q)$. By Assumption 1, $q > 1/(2 - \lambda)$. So a sufficient condition is $\frac{1}{2-\lambda}\frac{3-\lambda}{2-\lambda} \geq 1 - \lambda$, which is equivalent to $f(\lambda) = 1 - 7\lambda + 5\lambda^2 - \lambda^3 \leq 0$. We can show that $f(\lambda)$ is strictly decreasing for all $\lambda \in (0, 1)$, $f(0) > 0$, and $f(1) < 0$, so there is a unique $\hat{\lambda} \simeq 0.16$, such that $f(\hat{\lambda}) = 0$. Henceforth, we assume that $\lambda \geq \hat{\lambda}$.

The requirement that $\omega_1 < \hat{\omega}$ also sets an upper bound for us to choose ω_0. That is, the supremum of ω_0 to support the equilibrium will be the interim reputation, with which the fund reaches the next period prior reputation $\hat{\omega}$ after getting a bad outcome. This supremum is the solution to the equation $(1 - \lambda)\frac{(1-q)x}{(1-q)x+q(1-x)} = \hat{\omega}$, which is denoted by

$$\bar{\omega} = \frac{q(2 - \lambda) - 1}{(1 - \lambda(1 - q)(2q - 1) + 2q[q(2 - \lambda) - 1])}.$$

Therefore, we can only choose $\omega_0 \in [1 - \lambda, \bar{\omega})$.

In the strategy profile described in Equation (9), an uninformed fund will be indifferent between acquiring information and remaining uninformed when its interim reputation is ω_0 or ω_1. Hence, if this strategy profile is an equilibrium, we must have

$$V_I^0 - V_U^0 = V_I^1 - V_U^1 = c. \tag{13}$$

In addition, since the fund with the prior reputation greater than ω_0 will certainly not acquire information,

$$V_I(x) - V_U(x) < c. \tag{14}$$

Figure 7 illustrates the fund's continuation value for any prior reputation.
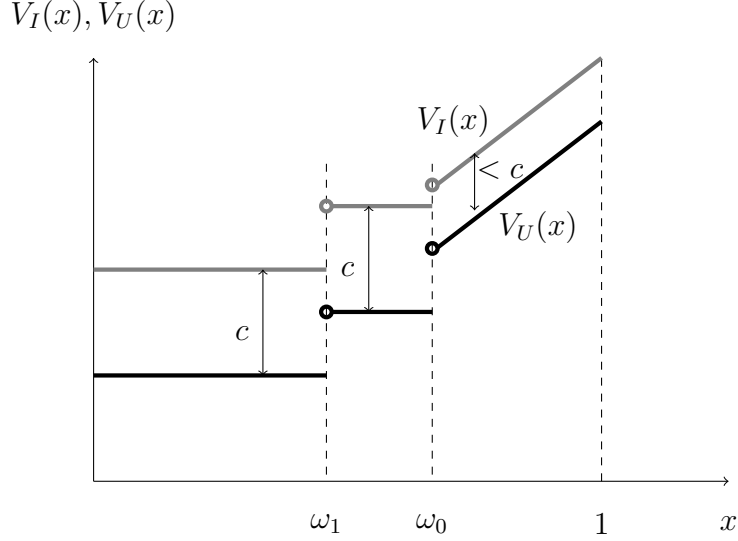
Figure 7: The Equilibrium Continuation Value of the Manager

Note that an informed (uninformed) fund's payoff depends on its interim reputation z instead of its prior reputation x. For an informed (uninformed) fund whose prior reputation is $x \in (0, \omega_1]$, its interim reputation is ω_1 regardless of its prior reputation, so its continuation value is constant in such an interval. The same argument applies in the interval $(\omega_1, \omega_0]$. For a fund whose prior reputation is $x \in (\omega_0, 1)$, investors' beliefs are $\hat{\sigma} = 0$, so its interim reputation is equal to its prior reputation, and its continuation value is a linear function of its prior reputation.

Equation (13) will help to pin down ω_0, which will determine the whole equilibrium construction. Specifically, Equation (13) implies that

$$\frac{\delta(2q-1)}{1-\delta(1-\lambda)} \left[\omega_0 - (1-\lambda)\frac{(1-q)\omega_0}{(1-q)\omega_0 + q(1-\omega_0)} \right] = c. \tag{15}$$

The left-hand side of Equation (15) is a continuous function over $[1-\lambda, \bar{\omega}]$; therefore, the left-hand side of Equation (15) has its maximum and minimum over $[1-\lambda, \bar{\omega}]$, which are denoted by $\bar{y}$ and $\underline{y}$, respectively.

The Optimality of Equilibrium Strategy. From the description of the "work-work-shirk" strategy, we can see that only values of V_I^0, V_U^0, V_I^1, and V_U^1 matter. That is, in order to verify whether the proposed strategy is an equilibrium, we first need to solve these values. Consider the manager with a prior reputation ω_0 first. Since the uninformed manager is believed not to acquire information at this reputation, her interim reputation is the same as the prior reputation. Then

her continuation value is

$$
\begin{aligned}
V_I(\omega_0) &= \omega_0 + \delta\left[q(1-\lambda)V_I^0 + q\lambda V_U^0 + (1-q)(1-\lambda)V_I^1 + (1-q)\lambda V_U^1\right], \\
V_U(\omega_0) &= \omega_0 + \delta\left[(1-q)V_U^0 + qV_U^1\right].
\end{aligned}
$$

Taking the difference, we have

$$
V_I^0 - V_U^0 = \delta\left[q(1-\lambda)\left(V_I^0 - V_U^0\right) + (1-q)(1-\lambda)\left(V_I^1 - V_U^1\right) + (2q-1)\left(V_U^0 - V_U^1\right)\right]. \tag{16}
$$

Similarly, if we calculate the difference between the informed manager's value and the uninformed manager's value at the prior reputation ω_1, we have

$$
V_I^1 - V_U^1 = \delta\left[q(1-\lambda)\left(V_I^0 - V_U^0\right) + (1-q)(1-\lambda)\left(V_I^1 - V_U^1\right) + (2q-1)\left(V_U^0 - V_U^1\right)\right]. \tag{17}
$$

We can see that $V_I^0 - V_U^0 = V_I^1 - V_U^1$. Set $V_I^0 - V_U^0 = V_I^1 - V_U^1 = c$, then the equilibrium requirement Equation (13) is satisfied because the manager will randomize when the prior reputation $x \in [0, \omega_1) \cup (\omega_1, \omega_0)$. Given Equation (13), when the manager's prior reputation $x > \omega_0$, the difference between values of an informed manager and an uninformed manager is $V_I(x) - V_U(x) = \delta(1-\lambda)(V_I^0 - V_U^0) < c$, because $\delta(1-\lambda) < 1$. That is, the equilibrium condition Equation (14) is satisfied. As a result, the uninformed manager with the prior reputation $x > \omega_0$ will choose not to acquire information.

With the equilibrium condition (13), Equation (16) implies

$$
\begin{aligned}
c &= V_I^0 - V_U^0 \\
&= \delta\left[q(1-\lambda)\left(V_I^0 - V_U^0\right) + (1-q)(1-\lambda)\left(V_I^1 - V_U^1\right) + (2q-1)\left(V_U^0 - V_U^1\right)\right] \\
&= \delta\left[(1-\lambda)c + (2q-1)\left(V_U^0 - V_U^1\right)\right],
\end{aligned}
$$

which implies that $V_U^0 - V_U^1 = \omega_0 - \omega_1 = \frac{1-\delta(1-\lambda)}{\delta(2q-1)}c$. By definition, $\omega_1 = (1-\lambda)\frac{(1-q)\omega_0}{(1-q)\omega_0 + q(1-\omega_0)}$, so we have Equation (15): $\omega_0 - (1-\lambda)\frac{(1-q)\omega_0}{(1-q)\omega_0 + q(1-\omega_0)} = \frac{1-\delta(1-\lambda)}{\delta(2q-1)}c$. That is, for a given set of parameters, we can identify ω_0, which is the starting point of the equilibrium construction.

Given the constructed ω_0 and ω_1, we know

$$
\begin{aligned}
V_U^0 &= \omega_0 + \delta\left[(1-q)V_U^0 + qV_U^1\right], \\
V_U^1 &= \omega_1 + \delta\left[(1-q)V_U^0 + qV_U^1\right].
\end{aligned}
$$

Therefore, $V_U^0 - V_U^1 = \omega_0 - \omega_1$. By arranging terms, we can solve $V_U^0 = \frac{(1-\delta q)\omega_0 + \delta q\omega_1}{1-\delta}$ and $V_U^1 = \frac{\delta(1-q)\omega_0 + (1-\delta(1-q))\omega_1}{1-\delta}$. Then we can calculate that

$$
\begin{aligned}
V_I^0 &= c + \frac{(1-\delta q)\omega_0 + \delta q\omega_1}{1-\delta}, \\
V_I^1 &= c + \frac{\delta(1-q)\omega_0 + (1-\delta(1-q))\omega_1}{1-\delta}.
\end{aligned}
$$

Given the derived continuation values, an uninformed fund will be indifferent between acquiring information and remaining uninformed when its prior reputation is below ω_0; the fund will certainly not acquire information if its prior reputation is greater than ω_0. By construction, the fund's strategy is consistent with investors' beliefs. Therefore, the proposed strategy profile is a stationary equilibrium.

$\square$

Proof of Proposition 3. We assume that an informed fund will become uninformed with probability λ, so the prior reputation

$$x_t \leq (1-\lambda)z_t^g \leq 1 - \lambda \leq \omega_0, \forall t \geq 2.$$

Then in the equilibrium, for any $t \geq 2$,

$$z_t = \begin{cases} \omega_0, & \text{if} \quad x \in (\omega_1, \omega_0]; \\ \omega_1, & \text{if} \quad x \in (0, \omega_1]. \end{cases}$$

By construction, $\omega_1 = (1-\lambda)\omega_0^b$ implies that if $z_t = \omega_0$,

$$z_{t+1} = \begin{cases} \omega_0, & \text{if} \quad s_t = G; \\ \omega_1, & \text{if} \quad s_t = B. \end{cases}$$

Because $\omega_1 < \hat{\omega}$, so $(1-\lambda)\omega_1^g > \omega_1$. That is, when the fund's interim reputation is ω_1 in period t, the fund's prior reputation in period $t+1$ is strictly greater than ω_1. Hence, $z_t = \omega_1$ and $s_t = G$ lead to $x_{t+1} \in (\omega_1, \omega_0)$. Then in the equilibrium, $z_{t+1} = \omega_0$. Conversely, $z_t = \omega_1$ and $s_t = B$ will lead to $x_t < \omega_1$. Consequently, $z_{t+1} = \omega_1$ in this case in the equilibrium. Therefore, when $z_t = \omega_1$,

$$z_{t+1} = \begin{cases} \omega_0, & \text{if} \quad s_t = G; \\ \omega_1, & \text{if} \quad s_t = B. \end{cases}$$

Then the interim reputation in period $t+1$ depends on the performance in period t only. Since the argument above holds for any period $t \geq 2$, we have, for any period $t \geq 3$,

$$z_t = \begin{cases} \omega_0, & \text{if} \quad s_{t-1} = G; \\ \omega_1, & \text{if} \quad s_{t-1} = B, \end{cases}$$

which is exactly Equation (10).

$\square$

Proof of Proposition 4. By Lemma 3, if $s_{t-1} = B$ and $s_{t-2} = G$, we have $f_t = z_t - z_{t-1} = \omega_1 - \omega_0 < 0$. If $s_{t-1} = s_{t-2} = B$ or $s_{t-1} = s_{t-2} = G$, $z_t = z_{t-1}$, which implies that $f_t = 0$. Finally, if $s_{t-1} = G$ and $s_{t-2} = B$, we have $f_t = z_t - z_{t-1} = \omega_0 - \omega_1 > 0$.

$\square$

Appendix B   Other Equilibria (Not for publication)

In this section, first we show that there exists no "work-shirk" equilibrium in which

$$
\sigma(x) = \begin{cases} \frac{\omega_0 - x}{1 - x}, & \text{if} \quad x \leq \omega_0; \\ 0, & \text{if} \quad x > \omega_0, \end{cases}
$$

for some $\omega_0 \in [0, 1]$ under a mild condition.

Proposition 5. *There exists no "work-shirk" equilibrium for $\omega_0 \geq (1 - \lambda)z^b(\hat{w})$.*

Proof. Suppose there is a "work-shirk" equilibrium s.t.

$$
\sigma(x) = \begin{cases} \frac{\omega_0 - x}{1 - x}, & \text{if} \quad x \leq \omega_0; \\ 0, & \text{if} \quad x > \omega_0, \end{cases}
$$

for some $\omega_0 \in [0, 1]$. We want to show that such an equilibrium does not exist. There are three cases: (1) $\omega_0 \geq 1 - \lambda$, (2) $\omega_0 \in [\hat{\omega}, 1 - \lambda)$, and (3) $(1 - \lambda)z^b(\hat{w}) \leq \omega_0 < \hat{\omega}$.

Case I. Suppose that $\omega_0 \geq 1 - \lambda$, then for any $x \leq \omega_0$, $\sigma(x) \geq 0$, and $V_I(x) - V_U(x) = c > 0$. However, for any $x \leq \omega_0$, the fund's interim reputation is $z = \omega_0$. Because both $(1 - \lambda)z^g(\omega_0)$ and $(1 - \lambda)z^b(\omega_0)$ are strictly less than $1 - \lambda \leq \omega_0$, the fund's next period interim reputation is ω_0 again regardless of its current performance. Hence, for each $x \leq \omega_0$, we have

$$
V_U(x) = V_U^0 = \omega_0 + \delta V_U^0,
$$

and

$$
V_I(x) = V_I^0 = \omega_0 + \delta(1 - \lambda)V_I^0 + \delta\lambda V_U^0.
$$

Hence, we must have that, for each $x \leq \omega_0$,

$$
V_I(x) - V_U(x) = V_I^0 - V_U^0 = c = \delta(1 - \lambda)[V_I^0 - V_U^0] = \delta(1 - \lambda)c,
$$

which is impossible because $\delta(1 - \lambda) < 1$ and $c > 0$.

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Case II. Suppose that $\omega_0 \in [\hat{\omega}, 1-\lambda)$. Consider a fund with a prior reputation ω_0. To support the equilibrium, $V_I^0 - V_U^0 = c > 0$. However, at ω_0, the next period prior reputations are $(1-\lambda)z^g$ following a good investment outcome or $(1-\lambda)z^b$ following a bad investment outcome. Since $(1-\lambda)z^b < (1-\lambda)z^g \leq \omega_0$, we have $V_I^0 - V_U^0 = c = \delta(1-\lambda)(V_I^0 - V_U^0) = \delta(1-\lambda)c$, according to the proposed strategy profile. However, this cannot be true, since $\delta, \lambda \in (0,1)$ and $c > 0$.

Case III. Suppose that $(1-\lambda)z^b(\hat{\omega}) \leq \omega_0 < \hat{\omega}$. In the proposed strategy profile, for all $x \leq \omega_0$, the fund is randomizing so that the interim reputation is just ω_0. This implies that the fund's reputation premium at ω_0 must be equal to the information acquisition cost; otherwise, the fund with a prior reputation $x \leq \omega_0$ will not randomize. Therefore, we have

$$
\begin{aligned}
V_I^0 - V_U^0 &= \delta q(1-\lambda)(V_I((1-\lambda)z^g(\omega_0)) - V_U^0((1-\lambda)z^g(\omega_0))) \\
&+ \delta(1-q)(1-\lambda)(V_I^0 - V_U^0) \\
&+ \delta(2q-1)(V_U((1-\lambda)z^g(\omega_0)) - V_U^0) \\
&= c.
\end{aligned}
$$

For $x \in [\omega_0, \hat{\omega}]$,

$$
\begin{aligned}
V_I(x) &- V_U(x) \\
&= \delta q(1-\lambda)(V_I((1-\lambda)z^g(x)) - V_U((1-\lambda)z^g(x)) \\
&+ \delta(1-q)(1-\lambda)(V_I((1-\lambda)z^b(x)) - V_U((1-\lambda)z^b(x))) \\
&+ \delta(2q-1)(V_U((1-\lambda)z^g(x)) - V_U^0)
\end{aligned}
\tag{18}
$$

By the strategy profile, we must have $V_I(x) - V_U(x) \leq c$. In the rest of the proof, we are going to show that $V_I(x) - V_U(x) > c$ for $x \in [\omega_0, \hat{\omega}]$, and thus the equilibrium does not exist.

Because $(1-\lambda)z^b(\hat{w}) \leq \omega_0 < \hat{\omega}$, for any $\hat{\omega} \geq x \geq \omega_0$, $\hat{\omega} \geq (1-\lambda)z^g(x) \geq x$ and $(1-\lambda)z^b(x) \leq \omega_0$. Namely, the next period reputation is still lower than $\hat{\omega}$ after a good outcome, whereas it is lower than ω_0 after a bad outcome, and thus $V_K((1-\lambda)z^b(x)) = V_K^0$ for any $x \in [\omega_0, \hat{\omega}]$ and $K = U, I$. Thus, Equation (18) becomes

$$
\begin{aligned}
V_I(x) - V_U(x) &= \delta q(1-\lambda)(V_I((1-\lambda)z^g(x)) - V_U((1-\lambda)z^g(x)) \\
&+ \delta(1-q)(1-\lambda)(V_I^0 - V_U^0) \\
&+ \delta(2q-1)(V_U((1-\lambda)z^g(x)) - V_U^0).
\end{aligned}
\tag{19}
$$

By the strategy profile, for $x \in [\omega_0, \hat{\omega}]$, $\sigma(x) = 0$, and thus

$$
V_U(x) = x + \delta(1-q)V_U[(1-\lambda)z^g(x)] + \delta q V_U[(1-\lambda)z^b(x)],
$$

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given the boundary condition that $V_U(x) = V_U(\omega_0)$ for each $x \le \omega_0$. By using the standard contraction mapping technique in Stokey, Lucas, and Prescott (1989), one can show that $V_U(x)$ is continuous and strictly increasing for $x \in [\omega_0, \hat{\omega}]$. As a result, the third term of the right-hand side of Equation (19), $(V_U((1-\lambda)z^g(x)) - V_U^0)$ is strictly increasing. Again, by applying the standard contraction mapping technique on the functional equation (19), one can show that $V_I(x) - V_U(x)$ is continuous and strictly increasing in $x \in [\omega_0, \hat{\omega}]$. Because $V_I^0 - V_U^0 = c$, $V_I(x) - V_U(x) > c$; thus, the equilibrium does not exist.

$\square$

Next, consider the existence of a "work-shirk" equilibrium when ω_0 is small. Unfortunately, we cannot prove or disprove the existence of a "work-shirk" equilibrium without further restriction. The following proposition shows that a "work-shirk" equilibrium does not exist for a large λ.

Proposition 6. *There exists no "work-shirk" equilibrium if $\lambda \ge 1/2$.*

Proof. Suppose not. Following case III in the proof of 5, for $x \in [\omega_0, \hat{\omega}]$, we have

$$
\begin{aligned}
V_I(x) &- V_U(x) \\
&= \delta q(1-\lambda)(V_I((1-\lambda)z^g(x)) - V_U((1-\lambda)z^g(x)) \\
&\quad + \delta(1-q)(1-\lambda)(V_I((1-\lambda)z^b(x)) - V_U((1-\lambda)z^b(x))) \\
&\quad + \delta(2q-1)(V_U((1-\lambda)z^g(x)) - V_U((1-\lambda)z^b(x))).
\end{aligned}
\tag{20}
$$

We want to show that $V_I(x) - V_U(x) > c$ to disprove the existence of the equilibrium. Unfortunately, because ω_0 is too small, it is not true that $(1-\lambda)z^b(x)$ for any $x \ge \omega_0$, so $V_I(x) - V_U(x)$ may not be monotone in general.

However, for x small enough such that $(1-\lambda)z^b(x) \le \omega_0$, we have

$$
\begin{aligned}
V_I(x) &- V_U(x) - [V_I^0 - V_U^0] \\
&= \delta q(1-\lambda)[V_I((1-\lambda)z^g(x)) - V_I((1-\lambda)z^g(\omega_0))] \\
&= \delta[(1+\lambda)q - 1][V_U((1-\lambda)z^g(x)) - V_U((1-\lambda)z^g(\omega_0))].
\end{aligned}
\tag{21}
$$

The assumption $\lambda \ge 1/2$ and Assumption 1 together imply that $\delta[(1+\lambda)q - 1] \ge 0$. Again, the standard contraction mapping technique implies that $V_I(x), V_U(x)$ are strictly increasing in $x \in [\omega_0, \hat{\omega}]$, so $V_I(x) - V_U(x) - [V_I^0 - V_U^0] > c$ for such a x; thus, the fund has the incentive to deviate. $\square$

When $\lambda < 1/2$, $\delta[(1+\lambda)q - 1] < 0$, the argument above fails. We conjecture that a "work-shirk" equilibrium does not exist as well. However, we cannot prove or disprove its existence.

Last, we show that there exists no "work-work-...-shirk" equilibrium in which

1. there exists a decreasing sequence $\{\omega_i\}_{k=1}^N$, where $N > 2$, $\omega_N = 0$ and $\omega_0 \leq 1$;

2. $\sigma(x) = 0$ for $x \geq \omega_0$;

3. $\sigma(x) = \frac{\omega_k - x}{1 - x}$ for $x \in (\omega_{k+1}, \omega_k]$ and $k \in \{0, 1, ..N - 1\}$; and

4. for $x \in (\omega_{k+1}, \omega_k]$ and $k \in \{0, 1, ..N - 1\}$ and $k < N - 1$, the posterior belief after a good outcome is $(1 - \lambda)z^g \in (\omega_k, \omega_{k-1}]$; while the posterior belief after a bad outcome is $(1 - \lambda)z^b \in (\omega_{k+2}, \omega_{k+1}]$, where $\omega_{N+1} = \omega_N = 0$.

First, we consider the case in which $N = 3$. In Proposition 7, we show that a "work-work-work-shirk" equilibrium does not exist. Then we apply Proposition 7 to show that an equilibrium with $N > 3$ "work" regimes and one "shirk" regime does not exist either.

Proposition 7. *There exists no "work-work-work-shirk" equilibrium.*

Proof. Suppose there exists such an equilibrium s.t.

$$\sigma(x) = \begin{cases} \frac{\omega_2 - x}{1 - x}, & \text{if} \quad x \leq \omega_2; \\ \frac{\omega_1 - x}{1 - x}, & \text{if} \quad x \in (\omega_2, \omega_1]; \\ \frac{\omega_0 - x}{1 - x}, & \text{if} \quad x \in (\omega_1, \omega_0]; \\ 0, & \text{if} \quad x > \omega_0, \end{cases}$$

for some $1 \geq \omega_0 > \omega_1 > \omega_2 \geq 0$. Let $V_U^i = V_U(\omega_i)$ and $V_I^i = V_i(\omega_i)$ for $i = 0, 1, 2$. Then the manager's indifference condition at $x = \omega_0, \omega_1, \omega_2$ implies that

$$V_I^0 - V_U^0 = c > 0,$$

$$V_I^1 - V_U^1 = \delta q(1 - \lambda)(V_I^0 - V_U^0) + \delta(1 - q)(1 - \lambda)(V_I^2 - V_U^2) + \delta(2q - 1)(V_U^0 - V_U^2) = c,$$

and

$$V_U^2 - V_U^2 = \delta q(1 - \lambda)(V_I^1 - V_U^1) + \delta(1 - q)(1 - \lambda)(V_I^2 - V_U^2) + \delta(2q - 1)(V_U^1 - V_U^2) = c.$$

Simple algebra implies that

$$(2q - 1)\delta(V_U^0 - V_U^2) = c[1 - \delta(1 - \lambda)],$$

$$(2q - 1)\delta(V_U^1 - V_U^2) = c[1 - \delta(1 - \lambda)];$$

which further implies that $V_U^1 = V_U^0$, which is a contradiction!

$\square$

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By the same logic, we have the following corollary.

Corollary 4. *There exists no "work-....-work-shirk" equilibrium with N "work" regimes for each $N > 2$.*

Proof. Suppose such an equilibrium exists, then we can apply the proof of Proposition 7 to show the contradiction for the first two "work" regimes: $(\omega_2, \omega_1], (\omega_1, \omega_0]$ to show that such an equilibrium cannot exist. $\qquad\square$

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