

Fragile New Economy

Ye Li*

Columbia University

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Abstract

This paper provides a dynamic general equilibrium model to explain the secular upward trends in corporate liquidity holding and financial-sector risk-taking in the past few decades. The structural transformation to an intangible-intensive "new economy" results in increased corporate liquidity demand, because intangible-intensive investments require internal liquidity due to limited pledgeability. By issuing safe debt held by the non-financial sector, the financial sector supplies liquidity and benefits from the liquidity premium, which leads to cheap leverage and enlarged risk-taking capacity. As the financial sector bids up asset price (Tobin's q), the non-financial sector wants to invest more and chooses to hold more liquidity as internal funds, driving up the liquidity premium and pushing down the financial sector's leverage cost even further. This feedback mechanism leads to fragility through dramatic increase in financial-sector risk-taking in boom and drastic evaporation of financial-sector net worth in crisis.

1 Introduction

From 1952 to 2010, the assets of the financial sector have risen from about 25 percent of the total assets in the economy (one times GDP) to about 40% (four times GDP) (Gorton, Lewellen, and Metrick (2012)).¹ The growth of the financial sector is primarily fueled by

*Correspondence to: Finance and Economics Division, Columbia Business School, New York, NY, 10025, United States. E-mail: yli17@gsb.columbia.edu. I would like to thank Patrick Bolton, Guojun Chen, Gur Huberman, Michael Jiang, Siyuan Liu, Tano Santos, Aleksey Semenov, Suresh Sundaresan, and Neng Wang for helpful discussions and comments. I am also grateful to seminar participants at Columbia Financial Economics Colloquium and PhD Student Seminar at Columbia Business School.

¹Just banking sector alone, Schularick and Taylor (2012) find that its assets-to-GDP ratio have increased from about 50% in 1950s to 200% in 2000s.

high leverage, especially through repurchase agreements (repo), commercial papers (CP), and other money market securities that are safe and short-term.²

Meanwhile, U.S. non-financial corporations hold more liquidity. According to Bates, Kahle, and Stulz (2009), their cash-to-total asset ratios have doubled since 1980. In fact, the non-financial corporations' saving has become the largest component of national saving (source: NIPA). This secular trend poses a challenge to corporate finance theories (Riddick and Whited (2009)).

What drives these two trends? What is the connection between them? What are the implications on capital allocation, asset pricing, and economic (in)stability? This paper aims to answer these questions in a unified framework of liquidity supply from the financial sector and liquidity demand of the non-financial sector.

In the past few decades, the US economy is transforming from a manufacturing-based economy to a service-based economy. This structural change has profound impact on firms' liquidity management. When the incremental capital is increasingly biased towards intangibles, such as product design, employee training, and organizational capital, investment has to be largely financed by firms' internal funds because intangible capital cannot be easily verified or liquidated and therefore is subject to limited pledgeability.

This investment-driven liquidity hoarding in the "new economy" drives up the real (non-financial) sector's demand for safe assets. The demand is satisfied by the financial sector's issuance of safe debt, especially when the Federal debt issuance has been stagnant until the great recession. The non-financial sector's demand of liquidity has fueled the financial sector's leverage, because as the liquidity demand outpaces supply, a large liquidity premium is attached to safe debt, which lowers the financial sector's debt cost.³ As pointed out by Pozsar (2011), corporate treasuries have become a prominent component of "institutional cash pools" that supply financial-sector leverage, especially through money markets.

Cheap leverage enlarges the risk-taking capacity of the financial sector, which in turn pushes up asset price and Tobin's q . A higher Tobin's q incentivizes firms to invest more. But in order to invest more, firms have to hold even more internal liquidity, as the external financing capacity of the new investment opportunities is limited. Thereby, the

²See, e.g. Adrian and Shin (2010), Coval, Jurek, and Stafford (2009a), Gorton and Metrick (2010, 2012), and Pozsar, Adrian, Ashcraft, and Boesky (2010).

³Therefore, a low interest rate environment could be fully driven by the domestic demand of liquidity due to increased limited pledgeability of investment projects and a accommodative (passive) monetary policy that makes sure the financial sector's leverage is not constrained by, for example, required reserves ratio. This view contrasts with the view that the central bank has been actively pushing down interest rate, and the view that low interest rate results from foreign demand of US debt.

investment-driven liquidity demand in the non-financial sector increases along Tobin's q , which pushes the liquidity premium attached to financial debt even higher. The financial sector's leverage cost is further reduced, accelerating financial risk-taking and elevating asset price.

This paper builds a dynamic general equilibrium model to capture the aforementioned feedback mechanism that results in the simultaneous increase of non-financial sector's liquidity holding and financial sector's risk-taking. Increasingly high leverage makes the financial sector vulnerable to small adverse shocks. This positive feedback mechanism accumulates endogenous risk when an adverse shock has not yet arrived, and it amplifies the economy's reaction to shocks through the drastic evaporation of financial-sector net worth. The interplay of liquidity demand and supply characterizes the fragile new economy.

Evidence and Related Theories

Liquidity supply. In this paper, "liquidity" is safe debt. Being information-insensitive, risk-free debt is immune from adverse selection, because agents do not have the incentive to generate private information (Dang, Gorton, and Holmtröm (2011)). Therefore, safe debt facilitates trade by serving as the "medium of exchange", and earns liquidity premium (Rocheteau and Wright (2010)). Safe debt is also a "store of value". As pointed out by Hart and Zingales (2014), the financial sector is special because people in need of liquidity invest in its liabilities, with bank deposits as the quintessential example.

By issuing risk-free liabilities held by the non-financial sector, the financial sector creates value by supplying liquidity.⁴ Thereby, it captures the liquidity premium and enjoys a lower debt cost when they expand risk-taking through high leverage (DeAngelo and Stulz (2013)).⁵

A driving force of liquidity supply in the past decades is the leverage of investment banks through money markets. Mean leverage of primary dealers has more than doubled from mid-1980s to the onset of the recent financial crisis, especially through the exponential growth of repo and financial CP (Adrian and Shin (2010)). The growth of assets held by securities broker and dealers dwarfs the other sectors, signalling heightened risk-taking.

⁴The idea that liquidity production is intrinsic to financial intermediation is discussed extensively in the literature by, among others, Diamond and Dybvig (1983), Diamond and Rajan (2001), Gorton (2010), Gorton and Pennachi (1990), and Holmstrom and Tirole (1998, 2011).

⁵The literature on aggregate liquidity (see e.g. Woodford (1990), Holmstrom and Tirole (1998), Farhi and Tirole (2012)) also emphasizes the role of governments in providing (possibly contingent) liquidity as stores of value. The public provision of liquidity is beyond the scope of this paper.

Liquidity demand in the new economy. At the heart of this paper is the investment-driven liquidity demand from the non-financial sector. First of all, let us pay attention to another more fundamental trend in the economy. From 1970 to 2010, intangible capital ratio increases from below 3% of net total assets (tangible) to above 90% for the non-financial firms (Falato, Kadyrzhanova, and Sim (2013)). Corrado and Hulten (2010) estimate a total intangible investment of \$1.6 trillion in 2007 (11.3% of GDP).

The economy is moving away from physical capacities and moving towards a new economy that relies heavily on intangibles, such as product design, marketing and customer support, human capital, and organizational development. As a result, the collateralizable share of new investment projects shrinks, and firms need to hold more liquidity as internal funding in anticipation of future investment opportunities.⁶

Opler, Pinkowitz, Stulz, and Williamson (1999) find firms with strong growth opportunities and limited access to capital markets hold relatively high ratios of cash to total assets. Duchin (2010) finds cross-section evidence that firms who are more likely to experience financing shortfall when investment opportunities arrive tend to hold more liquidity.

Falato, Kadyrzhanova, and Sim (2013) find time-series evidence that firm cash holdings are positively related with intangible capital. And in the cross section, firms with greater intangible capital have stronger investment-cash holding sensitivity. Peters and Taylor (2014) provide evidence that intangible investments are more responsive to firms' internal cash and Tobin's q than tangible investments.

Theoretically, in the presence of cash flow shocks and capital adjustment cost, firms accumulate liquidity either for payout smoothing or as internal funds for future investment projects subject to limited pledgeability (Bolton, Chen, and Wang (2011) among others). The latter aspect, investment-driven demand for liquidity, has always been an essential ingredient in the theories of corporate liquidity management (e.g. Froot, Scharfstein, and Stein (1993)).

This paper considers a closed economy, so it ignores the foreign demand for liquidity, which is arguably another source of cheap leverage (Bernanke, Bertaut, DeMarco, and Kamin (2011)). While acknowledging the importance of this channel, this paper focuses on the domestic driver of liquidity demand that results from the irreversible and structural transformation to the new economy.

⁶It is well-established that tangible asset support more external financing. Please refer to Shleifer and Vishny (1992) and Rampini and Viswanathan (2010) for theoretical arguments, and Sibilkov (2009) for empirical evidence.

Liquidity premium. Under the growing reliance on intangible capital, the non-financial sector demands increasingly large amount of liquid assets as internal funding for future investment opportunities, which drives up the liquidity premium. Nagel (2014) shows four waves of ascending liquidity premium, 1976-1981, 1982-1988, 1993-2000, and 2001-2008.⁷ These waves coincide with the booms of intangible investments identified by Peters and Taylor (2014).⁸

While the liquidity premium correlates with Fed fund rate (the banks' opportunity cost of holding liquidity), which is the focus of Nagel (2014), the comovement breaks down in 1980s and the later half of 1990s, which calls for an alternative explanation and favors the investment-driven liquidity demand story.

Moreover, the liquidity premium exhibits cyclical comovement with the intangible-included Tobin's q in Peters and Taylor (2014), and it collapses following the slump of Tobin's q . This corroborates the aforementioned model mechanism: non-financial sector's investment-driven liquidity demand follows the profitability of investment opportunities.

A direct support for the investment-driven liquidity demand comes from Eisfeldt and Rampini (2009). They use a similar proxy for liquidity premium as Nagel (2014),⁹ and find that at the aggregate level, the liquidity premium is predicted by financing shortfalls, the difference between investment and operation cash flow.

Financial fragility and macroeconomics. This paper contributes to the macro literature by providing a new feedback mechanism. Fueled by the liquidity premium from the non-financial sector's investment-driven liquidity demand, high leverage boosts the risk-taking capacity of the financial sector, which pushes down price of risk and drives up the non-financial sector's liquidity demand even further through elevated Tobin's q . The boom period is characterized by simultaneous increase in non-financial liquidity holding and financial risk-taking, with the rising asset price at the center of the interplay.

This mechanism emphasizes the liability side of the financial sector's balance sheet, i.e. its ability to supply liquidity. This "liability channel" contrasts with the typical "asset channel", such as "credit or lending channel", where financial intermediaries' role is to channel funds from savers to investors (reviewed by Gertler and Kiyotaki (2011)), and

⁷Nagel (2014), Figure 2: Liquidity premium is proxied by the spread between three-month CD (certificate of deposit) rate and three-month Treasury Bill yield.

⁸Peters and Taylor (2014), Figure 2. Note that the last episode of ascending liquidity premium in 2000s might be more likely due to the liquidity demand from foreign countries (Caballero and Krishnamurthy (2009)).

⁹The liquidity premium is measured by the residual from a regression of the spread between three-month commercial paper and the three-month Treasury Bill on the Baa-Aaa spread. The regression is to remove the very little default risk in commercial papers.

the "fire sale channel", where the financial sector holds risky assets as "natural buyers" or "specialists" (reviewed by Brunnermeier, Eisenbach, and Sannikov (2012)).

Existing macro and asset pricing models with an active liquidity supplier mostly have the liquidity demand driven by consumption shocks.¹⁰ This tradition goes back to the cash-in-utility specification by Sidrauski (1967). However, the calibrated liquidity demand from consumption-smoothing is very low (Eisfeldt (2007)). With corporate savings being the largest component of national saving, it is also more realistic to focus on the investment-driven demand of liquidity.

Quadrini (2014) studies a macroeconomic model where banks actively supply liquidity by issuing debt and firms demand liquidity due to uninsurable shocks to operating profits. My paper differs from it by focusing on investment opportunity shocks, and introduces a new feedback mechanism based on the interaction between liquidity demand and supply with Tobin's q and liquidity premium at the center stage.

The organization of the paper is as follows: Section 2 describes the model and discusses the mechanism. Section 3 is devoted to the macro and asset pricing implications of the calibrated model. Section 4 concludes.

2 Model

2.1 Standard setup

In this section, I describe an infinite-horizon economy in continuous time. There are two sectors, the financial sector ("Financial") and the non-financial sector ("Non-financial"). Each sector has a continuum of representative agents with measure equal to one. Both representative agents are active, dynamic choice makers.

Utility. Both have the same time-separable expected logarithm utility:

$$V_t^i = E_t \left[\int_{s=t}^{\infty} e^{-\rho(s-t)} \log(c_s^i) dt \right], \quad i \in \{f, nf\} \quad (1)$$

Throughout this paper, superscripts denote sectors, with $i = f$ being the financial sector and $i = nf$ being the non-financial sector. V_t^f is the financial agent's life-time utility or the value function at time t , and c_s^{nf} is the non-financial agent's consumption at time s .

¹⁰See, for example, Diamond and Dybvig (1983), Aiyagari and Gertler (1991), Heaton and Lucas (1996), Vayanos (1998), Vayanos and Vila (1999), Huang (2003), Allen and Gale (2004), Eisfeldt (2004), Gârleanu, and Pedersen (2004), Kiyotaki and Moore (2005), and more recently, Krishnamurthy and Vissing-Jorgensen (2012), Nagel (2014), and Moreira and Savov (2014).

Both sectors have the same time-discount rate ρ .

Assets. Agents can hold their wealth in two assets, risk-free debt and risky asset ("capital"). At any time t , investing in risk-free debt is short-term (mature at $t + dt$), unsecured lending, and it earns a competitive-market return of r_t from t to $t + dt$.

The risky asset is the productive capital in the economy. From t to $t + dt$, K units capital can produce $aKdt$ units non-durable output, which can be either consumed ("consumption goods") or invested ("investment goods"). Risky assets can be freely exchanged for consumption goods in a competitive market at price q_t .

Risk. Holding capital is risky. Upon a Poisson arrival (with intensity equal to λ), a certain fraction of capital will be lost. The random arrival of capital destruction is the only source of aggregate uncertainty in this economy ("macro shock"). To interpret this shock, we can regard the CRS production technology as a project that rents capital to produce goods. Some projects fail when hit by the macro shock.

Financial-sector expertise. When Financial owns capital, $\underline{\phi}$ fraction of capital is lost upon the macro shock, which is smaller than $\bar{\phi}$, the fraction lost when capital is held by Non-financial. There are several reasons for $\underline{\phi} < \bar{\phi}$. The financial sector may possess a better monitoring technology (Diamond (1984)). Financial agents can also minimize inefficiency in financial distress through renegotiation (Bolton and Freixas (2000)). Therefore, the financial sector has comparative advantage in holding capital. It is the natural buyer of the risky asset, and levers up their expertise by borrowing from the non-financial sector in equilibrium.¹¹

Budget constraint. Let n_t^i denote the net worth of a representative agent in sector i . Before the arrival of macro shock, the flow of funds constraint (i.e. the evolution of wealth) is

$$\mu_t^{n,i} = \frac{dn_t^i/dt}{n_t^i} = r_t - \frac{c_t^i}{n_t^i} + x_{t,k}^i \left(\frac{a}{q_t} + \mu_t^q - r_t \right), \quad i \in \{f, nf\} \quad (2)$$

where $x_{t,k}^i$ is the fraction of wealth invested in the risky asset and $\mu_t^q = \frac{dq_t/dt}{q_t}$ is the growth rate of asset price q_t . The pre-shock excess return comprises dividend gain $\frac{a}{q_t}$ and capital gain μ_t^q . In equilibrium, Financial borrows from Non-financial, i.e. $x_{t,k}^f > 1$ and $x_{t,k}^{nf} < 1$.

Aggregate state variable. The economy is scale-free, so the total capital stock is not a state variable. The only state variable in this economy is the fraction of wealth held

¹¹Other specifications of financial-sector "specialness" include lower risk aversion (Longstaff and Wang (2012)), impatience (Brunnermeier and Sannikov (2014)), their abilities to collateralize loans (Rampini and Viswanathan (2010)), to invest in risky securities (He and Krishnamurthy (2012a)), to monitor projects (Holmstrom and Tirole (1997)), to mitigate information asymmetry (Bigio (2014), and Fiore and Uhlig (2014)), or to reduce search frictions (Duffie and Strulovici (2012)).

by the financial sector η_t .

$$\eta_t = \frac{n_t^f}{n_t^f + n_t^{nf}}$$

This is in line with the literature of heterogeneous-agent models, such as Basak and Cuoco (1998), Brunnermeier and Sannikov (2014a), and Longstaff and Wang (2012) among others. Aggregate variables are all functions of η , and their dynamics are given by Ito lemma, for example

$$\frac{dq_t/dt}{q_t} = \mu_t^q = \frac{q'(\eta_t)}{q(\eta_t)} \mu_t^\eta \eta_t \quad (3)$$

Endogenous risk. When the shock arrives, both sectors lose a chunk of wealth. Denote the post-shock net worth as \tilde{n}_t^i . Throughout the paper, " \sim " hat denotes the post-shock value of an endogenous variable. Financial wealth changes from n_t^f to \tilde{n}_t^f :

$$\tilde{n}_t^f = \left[\frac{\tilde{q}_t}{q_t} (1 - \phi) x_{t,k}^f + (1 - x_{t,k}^f) \right] n_t^f \quad (4)$$

Non-financial wealth changes from n_t^{nf} to \tilde{n}_t^{nf} :

$$\tilde{n}_t^{nf} = \left[\frac{\tilde{q}_t}{q_t} (1 - \bar{\phi}) x_{t,k}^{nf} + (1 - x_{t,k}^{nf}) \right] n_t^{nf} \quad (5)$$

$1 - \phi$ fraction of risky asset holdings remain and get re-evaluated at the post-shock price \tilde{q}_t . The rest of wealth, invested in the risk-free debt is intact, because debt is always valued at par. Since the financial sector levers up, it loses more net worth than the non-financial sector, and η_t decreases ($\tilde{\eta}_t < \eta_t$).

Financial agents' net worth is hit from three channels. First, ϕ fraction of capital is lost. This risk is exogenous. Second, the remaining capital is re-evaluated downward, because $\tilde{q}_t < q_t$ in equilibrium due to the contraction of natural buyers' capacity ($q'(\eta) > 0$). This risk is endogenous. Third, the value of liabilities does not change, so net worth is wiped out drastically due to leverage.¹²

High financial leverage leads to significant decrease in the state variable and capital valuation. For financial agents, the ratio of total loss to exogenous loss per unit of risky investment represents the strength of the shock amplification mechanism, and the

¹²Brunnermeier and Sannikov (2014b) allow the intermediary to borrow in long-term risk-free debt that creates an additional shock amplification mechanism, because as the price of long-term debt increases when the economy is hit by an adverse shock ("flight-to-safety"). For the sake of model transparency, I do not consider this channel in this paper.

contribution of endogenous risk

$$\frac{1 - \frac{\tilde{q}_t}{q_t} (1 - \phi)}{\phi} > 1 \tag{6}$$

2.2 Investment-driven liquidity demand

Investment opportunity. The non-financial sector has opportunities to create new capital. The arrival of investment opportunity follows a Poisson process with intensity equal to λ_I . It is independent across non-financial agents and independent from the macro shock. Even though this investment shock is idiosyncratic and it does not contribute to the aggregate uncertainty, the shock is uninsurable, so it affects non-financial agents choice.

This transient opportunity allows agents to convert one unit of general output into one unit of capital instantaneously, i.e. creating new risky asset that is worth q_t .¹³ The investment technology is scalable, so when $q_t > 1$, this is essentially an arbitrage opportunity, and the agents will choose to invest as much as possible.

Capital illiquidity. Following Holmstrom and Tirole (1997) and Kiyotaki and Moore (2012), I assume that only $\theta < 1$ fraction of the newly created capital can be pledged for external funds that is raised at prevailing market price. In other words, for one unit of investment, the investing agents can sell up to θ fraction and raise θq_t amount of investment goods. The investing agents capture all the surplus, and optimally chooses to maximize external financing. The return on internal funds (*ROI*) is

$$ROI_t = \frac{(1 - \theta) q_t}{1 - \theta q_t} \tag{7}$$

It has two components: the unlevered return $(1 - \theta) q_t$ (numerator) and the leverage $\frac{1}{1 - \theta q_t}$ (denominator). Both components increase in asset price q_t .

Internal funds come from two sources: the liquidation of risk-free and risky investments. Investing agents can freely exchange their holdings of risk-free debt for investment goods. This is due to the information-insensitivity feature of risk-free debt. Immune from information asymmetry, financial-sector debt can act as "money", a safe store of value and medium of exchange.

In contrast, only $\zeta < 1$ fraction of the holdings of existing risky assets can be exchanged for investment goods. The illiquidity of capital can be motivated by information

¹³Technically, one unit investment creates one unit capital at t out of thin air, which is installed instantaneously and produces adt from t to $t + dt$. The new capital requires the input of one unit goods from t to $t + dt$, which is interpreted as the one-time fixed cost.

asymmetry. Coming together with the investment opportunity is another technology that enables the investing agents to create worthless garbage at zero cost that is indistinguishable from productive capital. The outside buyers can only verify the quality of capital up to a certain fraction of the inventory (θ for new capital and ζ for existing capital).¹⁴ It is reasonable to assume that θ is smaller than ζ , which reflects the higher illiquidity of new capital. As capital matures from t to $t + dt$, its liquidity increases from θ to ζ .

The assumption of capital illiquidity is motivated by the economy's structural transformation to the intangible-intensive new economy in the past few decades. According to Corrado and Hulten (2010), intangible capital had overtaken tangible capital to become the largest systematic source of growth. However, intangible capital is hard to measure.¹⁵ Eisfeldt and Papanikolaou (2014) point out that a large fraction of intangible assets, "organization capital" does not show up in market values.¹⁶ The difficulty to evaluate the quality of intangibles is captured by the aforementioned limited verification capacity of outside capital buyers, which leads to illiquidity.

Source of liquidity premium. The garbage-creating technology is not available to non-investing agents, so holdings of existing capital can be freely exchanged for consumption goods at price q_t . In other words, risky asset is illiquid for investment purpose, while risk-free asset is liquid for both consumption and investment purposes. The investment-driven liquidity demand biases the portfolio choice of non-financial agents towards the risk-free asset and induces a liquidity premium attached to safe financial debt.

The benefit of holding liquidity is reflected in the jump of investing agents' net worth upon the investment shock. Given the total internal funds

$$\left(1 - x_{t,k}^{nf} + \zeta x_{t,k}^{nf}\right) n_t^{nf}$$

, which is fully invested, the scale of investment is

$$i_t = \frac{1}{1 - \theta q_t} \left(1 - x_{t,k}^{nf} + \zeta x_{t,k}^{nf}\right) n_t^{nf} \quad (8)$$

¹⁴Theoretically, θ and ζ can be endogenous and optimally chosen by the outside buyers, i.e. the financial agents and the non-investing non-financial agents. Bigio (2012, 2014), and Eisfeldt (2004) study the endogenous asset liquidity. In this paper, I assume these parameters being exogenous, so that the model mechanism is only about the non-financial sector's investment-driven demand of financial-sector debt as internal liquidity.

¹⁵See the important contributions in Corrado, Haltiwanger, and Sichel (2005); Atkeson and Kehoe (2005); McGrattan and Prescott (2010); and Corrado, Hulten, and Sichel (2009).

¹⁶According to Corrado, Hulten, and Sichel (2009), this type of capital is the single largest category of business intangible capital, accounting for about 30 percent of all intangible assets in the United States.

It creates i_t units of capital worth $q_t i_t$. The logarithm of i_t is

$$\ln n_t^{nf} \left(1 - x_{t,k}^{nf} + \zeta x_{t,k}^{nf} \right) - \ln (1 - \theta q_t) \quad (9)$$

which increases in internal funds and Tobin's q , empirically the most important elements that affect investment (Peters and Taylor (2014)).

The investing agent's wealth jumps up to

$$\widehat{n}_t^{nf} = \left(ROI_t \left(1 - x_{t,k}^{nf} + \zeta x_{t,k}^{nf} \right) + (1 - \zeta) x_{t,k}^{nf} \right) n_t^{nf} \quad (10)$$

where \widehat{n}_t^{nf} denotes the post-investment-shock value of non-financial agent's net worth. More internal funds lead to larger upward jump in net worth through larger investment.¹⁷ Therefore, the non-financial sector is willing to pay a premium to invest in the financial sector's debt, which lowers the leverage cost for financial agents.

The meaning of "liquidity" varies across scenarios. In this paper, I define "liquidity" as an asset that can be exchanged for goods at any time for any purpose. The empirical counterpart is corporate holdings of liquidity, i.e. "cash and cash equivalents".¹⁸ A large fraction of corporate liquidity is invested in financial-sector debt through money markets, in anticipation of financially constrained investment opportunities (Duchin (2010)).

The theme of this paper is to put corporate liquidity demand and the financial-sector leverage (liquidity supply) into a unified framework. In this model, the economy's shift towards intangible capital is exogenous, encoded in θ and ζ . The illiquidity of capital leads to the interplay between endogenous investment-driven liquidity demand and financial-sector leverage.

2.3 Individual optimality and equilibrium

The financial sector. Define the financial agent's value function as $V^f(n, \eta)$, a function of net worth n and aggregate state variable η . The Hamilton–Jacobi–Bellman equation

¹⁷Empirical studies such as Lamont (1997) and Bakke and Whited (2011), have shown that capital expenditures react positively to exogenous shocks to firms' internal liquidity.

¹⁸"Cash and cash equivalents" include checking, time, and savings deposits, bankers' acceptances, certificates of deposit, money market instruments, commercial papers, and Treasuries. In this paper, I do not consider the supply of public liquidity, i.e. Treasury securities, by the government, but will discuss its effect on the results.

(HJB) is

$$\rho V^f(n, \eta) = \max_{c^f, x_k^f} \ln c^f + V_n^f \mu^{n,f} n + V_\eta^f \mu^\eta \eta + \lambda [V^f(\tilde{n}, \tilde{\eta}) - V^f(n, \eta)]$$

subject to the budget constraint (2). The relation between pre-macro shock wealth n and post-macro-shock wealth \tilde{n} is given by equation (4). The log utility implies the functional form of $V^f(n, \eta)$ is

$$V^f(n, \eta) = \frac{1}{\rho} \ln n + U^f(\eta)$$

After substituting this function into HJB, the first order condition (F.O.C.) implies the optimal consumption and portfolio choice

$$c^f = \rho n$$

$$\frac{a}{q(\eta)} + \mu^q(\eta) - r(\eta) = \lambda \left(\frac{1 - \frac{q(\tilde{\eta})}{q(\eta)} (1 - \underline{\phi})}{1 - \left[1 - \frac{q(\tilde{\eta})}{q(\eta)} (1 - \underline{\phi}) \right] x_k^f} \right) \quad (11)$$

$1 - \frac{q(\tilde{\eta})}{q(\eta)} (1 - \underline{\phi})$ is the total loss per unit of risky asset holding when hit by a macro shock. Holding constant the pre-shock excess return, x_k^f decreases in the $1 - \frac{q(\tilde{\eta})}{q(\eta)} (1 - \underline{\phi})$. Risk has two components, the exogenous risk $\underline{\phi}$ (the fraction of capital lost) and the endogenous risk $\frac{q(\tilde{\eta})}{q(\eta)}$ (the re-evaluation of capital). The larger the ratio of total risk to exogenous risk (equation (6)) is, the stronger the shock amplification mechanism in this economy.

The non-financial sector. Next define the non-financial agent's value function as $V^{nf}(n, \eta)$, a function of net worth n and aggregate state variable η . HJB is

$$\rho V^{nf}(n, \eta) = \max_{c^{nf}, x_k^{nf}} \ln c^{nf} + V_n^{nf} \mu^{n,nf} n + V_\eta^{nf} \mu^\eta \eta + \lambda [V^{nf}(\tilde{n}, \tilde{\eta}) - V^{nf}(n, \eta)]$$

$$+ \lambda_I [V^{nf}(\hat{n}, \eta) - V^{nf}(n, \eta)]$$

subject to the budget constraint (2) and short-sale constraint ($x_k^{nf} \geq 0$).¹⁹ The relation between pre-macro shock wealth n and post-macro-shock wealth \tilde{n} is given by equation (5). The log utility implies the functional form of $V^f(n, \eta)$ is $\frac{1}{\rho} \ln n + U^{nf}(\eta)$, and consumption is proportional to permanent income $c^{nf} = \rho n$.

¹⁹The interpretation of risky asset as productive capital intends to capture a broad range of contingent claims on the cash flow of non-financial corporations. In reality, except for indices, equities and through derivatives, short sale is difficult. Since $x_k^f > 0$, the short-sale constraint for financial agents never binds.

The last term in the HJB equation reflects the jump in value function when the net worth of investing households jumps from n to \hat{n} . The relation between n and \hat{n} is given by equation (10). Value function jumps up upon the arrival of an investment opportunity. The size of the jump increases in internal funds, particularly the holdings of liquidity. This is what drives the liquidity premium that the non-financial sector attaches to the financial sector's debt.

The optimal portfolio choice gives

$$\begin{aligned} \frac{a}{q(\eta)} + \mu^q(\eta) - r(\eta) \leq \lambda \left(\frac{1 - \frac{q(\hat{\eta})}{q(\eta)} (1 - \bar{\phi})}{1 - \left[1 - \frac{q(\hat{\eta})}{q(\eta)} (1 - \bar{\phi})\right] x_k^{nf}} \right) \\ + \lambda_I \left(\frac{ROI(\eta) (1 - \zeta) - (1 - \zeta)}{ROI(\eta) (1 - x_k^{nf} + \zeta x_k^{nf}) + (1 - \zeta) x_k^{nf}} \right) \end{aligned} \quad (12)$$

The last term captures the compensation required by non-financial agents for capital illiquidity, which increases in the return on internal funds and asset price (Tobin's q).²⁰ The more profitable the investment opportunity is, the more demand for liquidity and the higher compensation the non-financial agents require for holding illiquid capital.

Market clearing. So far, we have only discussed the individual optimization problems of financial agents and non-investing non-financial agents. Investing agents create $i_t \lambda_I dt$ amount of new capital, liquidate ζ fraction of their risky asset holdings and all of their investment in risk-free debt. However, they only have a measure of $\lambda_I dt$ and their wealth is $\eta \lambda_I dt$ fraction of total wealth, so the resulting change in the aggregate supply of capital and debt is of magnitude dt , and therefore, can be ignored. Capital market clears:

$$x_k^f(\eta) \eta + x_k^{nf}(\eta) (1 - \eta) = 1 \quad (13)$$

The debt market clears automatically by Walras' law.

The measure of investing agents is $\lambda_I dt$, and each of them creates i_t units of new capital, so the aggregate investment is

$$(\lambda_I dt) i_t = (\lambda_I dt) \frac{1}{1 - \theta q_t} \left(1 - x_{t,k}^{nf} + \zeta x_{t,k}^{nf} \right) n_t^{nf}$$

where i_t is given by equation (8). Denote the total capital stock as K_t . The goods market

²⁰Take derivative of the last term in (12) w.r.t. ROI , we have $(1 - \zeta) ROI^{-2} \left(\hat{n}_t^{nf} / n_t^{nf} \right) > 0$.

clearing implies:²¹

$$aK_t dt = (\lambda_I dt) i_t + \rho \left(n_t^f + n_t^{nf} \right) dt$$

Because the aggregate wealth equals to the total value of productive capital (i.e. $q_t K = n_t^f + n_t^{nf}$), the equation above can be further simplified (ignoring time subscript):²²

$$a - \rho q(\eta) = \lambda_I \left(\frac{1 - x_k^{nf}(\eta) + \zeta x_k^{nf}(\eta)}{1 - \theta q(\eta)} \right) (1 - \eta) q(\eta) \quad (14)$$

K is cancelled out due to CRS production technology, which confirms the conjecture that the economy is scale-free.

State variable dynamics. The economy is driven by the dynamics of η_t : the pre-shock drift of η_t is

$$\begin{aligned} \mu_t^\eta &= \frac{d\eta_t/dt}{\eta_t} = \frac{n_t^{nf}}{(n_t^f + n_t^{nf})} \left(\frac{dn_t^f/dt}{n_t^f} - \frac{dn_t^{nf}/dt}{n_t^{nf}} \right) \\ &= (1 - \eta_t) \left\{ \left(\frac{a}{q(\eta_t)} + \mu^q(\eta_t) - r(\eta_t) \right) (x_{t,k}^f - x_{t,k}^{nf}) - \right. \\ &\quad \left. \lambda_I \left[ROI_t \left(1 - x_{t,k}^{nf} + \zeta x_{t,k}^{nf} \right) + (1 - \zeta) x_{t,k}^{nf} - 1 \right] \right\} \end{aligned} \quad (15)$$

which is also a univariate function of η_t , confirming the conjecture that η is the only state variable. The financial sector outgrows the non-financial sector by loading more on the pre-shock excess return ($x_{t,k}^f > x_{t,k}^{nf}$), while the non-financial sector has investment opportunities that lead to jump in net worth for $\lambda_I dt$ measure of agents. When the two forces balance each other ($\mu_t^\eta = 0$), the economy reaches its steady state.

2.4 Feedback mechanism

When asset price increases, return on internal liquidity increases ($\partial ROI / \partial q > 0$), which leads to higher required compensation for illiquidity (equivalently, higher premium attached to liquidity), and the non-financial sector's portfolio choice is more biased towards risk-free financial debt. Increased liquidity demand lowers the leverage cost for the financial sector, which boosts its risk-taking capacity and drives up asset price even further.

²¹The aggregate output and flow of goods is of magnitude dt , so the investing agent's choice of optimal investment affects the goods market equilibrium. The aggregate consumption of investing households from t to $t + dt$ is $(\rho n_t^{nf} dt) (\lambda dt)$ at the order of $(dt)^2$, which can be ignored.

²² K is cancelled out due to CRS production technology, which confirms the conjecture that η is the only state variable and the economy is scale-free.

Capital illiquidity leads to a feedback loop.

This increases the risk-taking by the financial sector through three channels. First, increased liquidity demand pushes down the leverage cost. Second, when the non-financial sector is actively participating in the capital market, i.e. (12) holds in equality, increased illiquidity compensation boosts pre-shock excess return, which incentivizes the financial sector to hold more risky assets. The first two channels are through income statement. Third, asset price increase directly boosts financial-sector net worth (amplified by leverage), which is the balance-sheet channel. Increased financial risk-taking capacity pushes asset price up even further, as the "natural buyer" of capital, increases its demand. This is the static multiplier effect.

The dynamic multiplier effect works through the financial-sector's retained earnings. Financial net worth accumulates at faster pace due to increased risk-taking in a boom period (pre-shock period), taking advantage of the high pre-shock excess return. This implies higher financial net worth in the future. Higher future risk-taking capacity of asset natural buyers further elevates the current price of risky assets through rational expectation.

The positive feedback loop accumulates endogenous risk. In the process, the financial sector absorbs more and more risky asset on its balance sheet by issuing risk-free debt to the non-financial sector. They become more vulnerable to the macro shock. When the shock arrives (exogenous risk), the net worth of the financial sector is wiped out drastically, which leads to the asset price slump (endogenous risk) that further impairs the financial-sector balance sheet.

The next section shows graphically the model's properties, and particularly the aforementioned feedback mechanism. Before that, please let me discuss several assumptions and simplifications in this model.

2.5 Discussion

Equity issuance constraint. Both sectors do not issue equity. The financial intermediary is better at holding capital than the household, so it would be socially optimally for the financial intermediary to hold all the risky asset. Under the assumption of log-utility, the intermediary needs to make sure the post-shock net worth is positive, which caps its total exposure to the risky asset ($\tilde{n}_t^b > 0$):

$$x_{t,k}^b < \frac{1}{1 - \frac{\tilde{q}_t}{q_t} (1 - \underline{\phi})} \quad (16)$$

which gives the upper bound of total units of risky asset held by the financial sector

$$\frac{x_{t,k}^b n_t^b}{q_t} < \left(\frac{1}{1 - \frac{\tilde{q}_t}{q_t} (1 - \underline{\phi})} \right) \left(\frac{n_t^b}{q_t} \right)$$

The upper bound increases with intermediary equity n_t^b , so if intermediary can raise equity capital from household, it can increase the asset under management. He and Krishnamurthy (2011) study an economy where the financial intermediary has exclusive access to investing in risky assets, and it raises equity capital from the household subject to an incentive constraint. In this paper, I assume both household and intermediary can invest in risky assets, but make an extreme assumption that the financial intermediary cannot issue equity at all.²³

Under this assumption, after the macro shock, intermediary becomes relatively under-capitalized and has to accumulate equity gradually through retained earnings. This assumption links the wealth distribution (Financial vs. Non-financial) to asset price and price of risk, and generates the persistent effect of macro shock.

Risk-free debt. The current model assumes that financial intermediaries cannot issue equity but can freely borrow at risk-free rate on an unsecured basis.²⁴ Under log-utility, optimal consumption is proportional to wealth, and the Inada condition implies infinite marginal utility when wealth equals to zero. Therefore, the agents always try to avoid default on debt. In this model, I do not impose any debt constraint, so at any time t , they can borrow freely and repay all of their debt at $t + dt$.

Value added of the financial sector. On the asset side, financial intermediaries resolve incentive problems by exerting monitoring effort, and in financial distress, they help avoid inefficient liquidation by providing financial flexibility and engaging into renegotiation. These functions of financial intermediation are captured by a small value of $\underline{\phi}$ ($< \bar{\phi}$).

On the liability side, financial intermediaries' debt is a major component in the supply of liquidity. The liquidity transformation functionality of financial intermediation has been strongly emphasized in the literature, such as Gorton and Pennacchi (1990). Liquidity provision earns the financial sector a liquidity premium, which lowers its leverage cost.

²³This constraint can be motivated by assuming that a low $\underline{\phi}$, loss upon shock, results from costly monitoring effort of the intermediary. The issuance of outside equity will dilute intermediary's incentive to monitor.

²⁴Debt constraints, such as value-at-risk constraint and margin requirements, are important to generate pro-cyclical leverage. In this paper, financial-sector leverage is counter-cyclical (with cyclicity defined by asset price). This is due to the fact that equity is always marked-to-market, so when asset price increases, equity grows much faster than debt, which is always priced at par.

In this paper, the financial sector provides liquidity by issuing risk-free debt that can be freely exchanged for investment goods by the non-financial sector.

It is worth noting that even when we set $\underline{\phi} = \bar{\phi}$, the financial sector would still be the natural buyer of capital, because it can borrow cheaply thanks to the liquidity premium attached to its debt and therefore discounts future cash flow at a lower rate than the non-financial sector.

Scalability of investment project. In the current model, I assume that the non-financial sector's investment opportunity is fully scalable and the marginal cost of investment is fixed at one. Therefore, when asset price increases, investment opportunities always become more profitable, and the non-financial sector demands more liquidity. Although it would be more realistic to consider convex variable cost, fixed cost, and shocks to investment technology,²⁵ this paper assumes scalability to make transparent the feedback mechanism that features simultaneous increase in asset price, investment-driven liquidity demand, liquidity premium, and financial-sector risk-taking.

3 Analysis

In this section, I solve the dynamic equilibrium numerically, and provide a graphical demonstration of the model mechanism. In the numeric solution, one unit of time from t to $t + 1$, corresponds to a month. I follow Kiyotaki and Moore (2012) and Del Negro, Eggertsson, Ferrero, and Kiyotaki (2014) setting the arrival rate of investment opportunity λ_I equal to 0.05 per quarter (i.e. 0.0125 per month). λ is $\frac{1}{60}$, i.e. the macro shock arrives every five years, consistent with the NBER business cycle frequency in the post-war period. I set $\underline{\phi} = 0.025$ and $\bar{\phi} = 0.05$, which is broadly consistent with the estimates of default loss in Andrade and Kaplan (1998) and Chen (2010). ρ is set to 0.0025 (annualized to 0.03), and a set to 0.0167 (annualized to 0.2) (Huang (2014)),

ζ is set to 0.9 in the following benchmark calibration. It is the key parameter in this model, as it measures the fraction of existing capital that can be pledged for external financing when an investment opportunity arrives. Lower value of ζ corresponds higher illiquidity of capital. As the economy is becoming more and more intangible-intensive, ζ becomes smaller. We can fix the other parameters, and think of the steady states as indexed by ζ , which is exogenously given by the evolution of production technology in the economy. Finally, the external financing capacity of investment projects, θ is set to 10%.

²⁵For example, in order to generate the non-monotonicity relation between internal funds and investment (Kaplan and Zingales (1997)).

3.1 Equilibrium properties

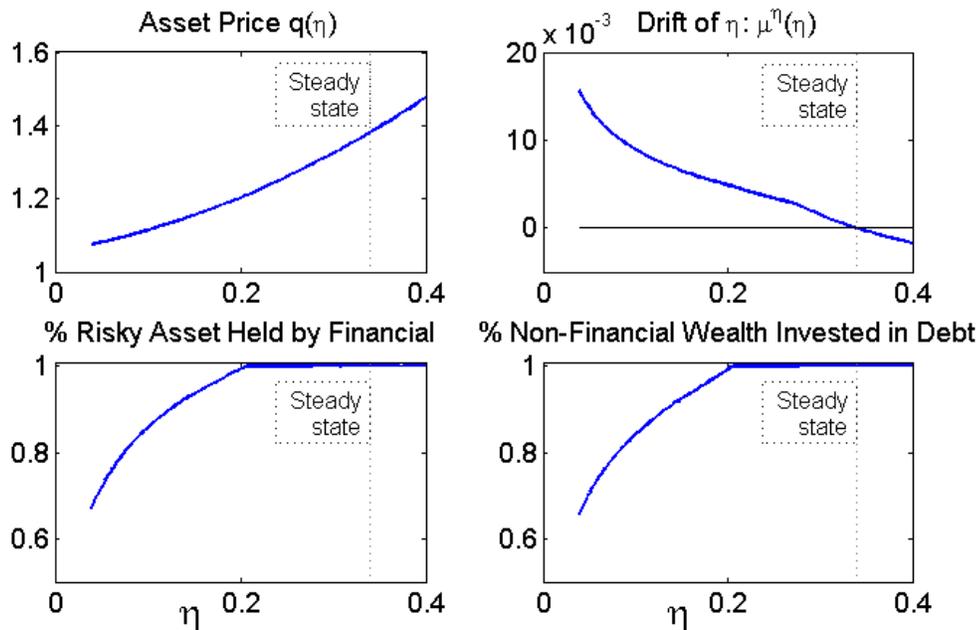


Figure 1: Equilibrium Asset Price, Steady State, and Capital Allocation

Asset price. The top left panel of Figure 1 shows the equilibrium asset price as a function of intermediaries' wealth share η . The financial sector is the natural buyer of risky assets for two reasons: First, when hit by the macro shock, it suffers less loss than the non-financial sector ($\underline{\phi} < \bar{\phi}$, the asset-side specialness); second, intermediaries' debt can be converted to investment goods freely, so households attach a liquidity premium to it in anticipation of future investment opportunities, which lowers the cost of leverage for intermediaries (the liability-side specialness). When the financial sector shrinks, as represented by a smaller value of η , asset price decreases.

Steady state. The drift of state variable η is shown in the top right panel of Figure 1. There are two messages from this graph. First, the stochastic steady state exists around $\eta = 0.34$. The steady state is marked by the dotted vertical lines in the four panels. When the financial sector's wealth share reaches 34 percent, the drift of state variable is equal to zero, and the economy will stay there until hit by a macro shock. This steady state is stable, because to the left $\mu^\eta > 0$ and to the right $\mu^\eta < 0$. Second, the growth rate is large where η is small. This is due to high risk premium. Through levered positions on the risky asset, the financial sector earns a higher return when the risk premium is higher.

We can think of the past few decades as part of the transition dynamics towards the steady state. According to simulation results, when the economy starts at $\eta = 5\%$, it

takes 125 years to reach the steady state $\eta = 34\%$.²⁶ During the process, we observe the expansion of the financial sector (η), increasing asset price (q), and increasing share of risky assets being held by the financial sector ($x_k^f \eta$).

Capital allocation. When the financial sector is still small, the non-financial sector have to hold risky assets ($\eta < 0.2$), and when they do, the expected return on risky asset must compensate for the illiquidity (equation (12)). Because of $\underline{\phi} < \bar{\phi}$, it is inefficient for the non-financial sector to hold existing capital, but it has to when the natural buyer's risk-taking capacity is limited. Around the steady state, all the existing capital is held by the financial sector, while the non-financial sector specializes in creating new capital and invests all of its wealth in liquidity.

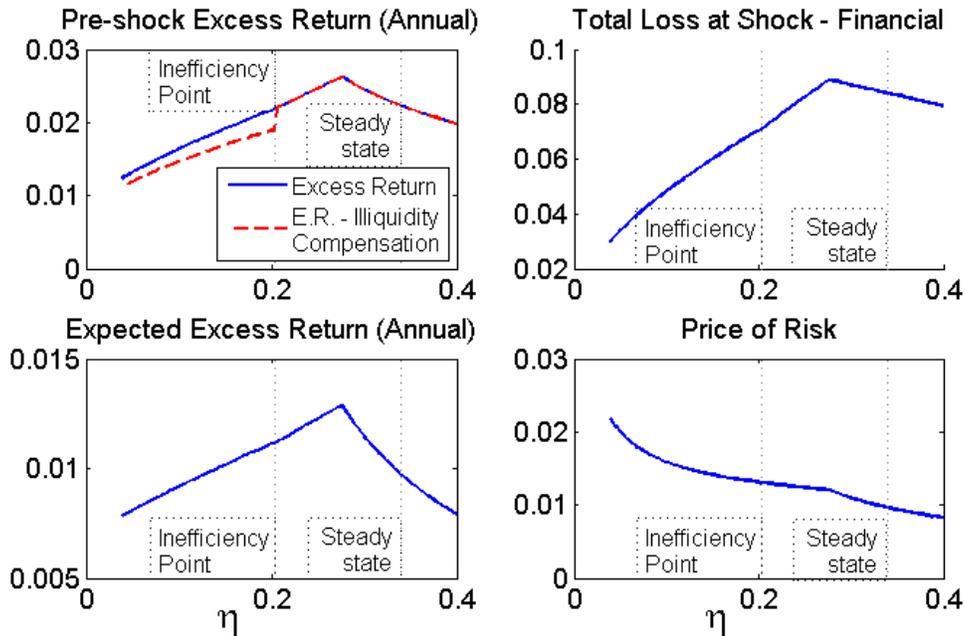


Figure 2: Risk Premium

Risk premium and price of risk. Steady state is marked by the dotted vertical line in Figure 2. Also marked is the inefficiency point, below which the financial sector is still small and the non-financial sector has to hold risky asset to clear the capital market.

The top left panel shows the pre-shock excess return, while the top right panel shows the total loss faced by the financial sector (per unit of risky investment), $1 - \frac{\tilde{q}}{q} (1 - \underline{\phi})$. Here I use the total loss faced by the financial sector, because it is always the marginal investor, while the non-financial sector refrains from holding capital when η is large enough. As

²⁶If the financial sector is allowed to issue equity, we should expect the process speeds up.

the financial sector grows and the economy moves towards the steady state, endogenous risk accumulates. The financial sector's total loss increases from around 2.5% (almost no endogenous risk) to around 9%. As a consequence, it requires higher pre-shock excess return to compensate for the increasing risk.

When non-financial agents hold risky asset, equation (12) holds in equality and decomposes the pre-shock excess return into two components: the macro risk compensation

$$\lambda \left(\frac{1 - \frac{q(\tilde{\eta})}{q(\eta)} (1 - \bar{\phi})}{1 - \left[1 - \frac{q(\tilde{\eta})}{q(\eta)} (1 - \bar{\phi}) \right] x_k^{nf}} \right)$$

and the illiquidity compensation

$$\lambda_I \left(\frac{ROI(\eta) (1 - \zeta) - (1 - \zeta)}{ROI(\eta) (1 - x_k^{nf} + \zeta x_k^{nf}) + (1 - \zeta) x_k^{nf}} \right) \quad (17)$$

The first component compensates households for bearing the aggregate uncertainty, and the second component for capital illiquidity, the limited conversion to investment goods upon the arrival of an investment opportunity. Because we only have two assets, the risky asset and the risk-free asset debt, the illiquidity compensation also reflects the liquidity premium that the non-financial sector attaches to risk-free debt.

The red dotted line in the top left panel shows the pre-shock excess return minus the illiquidity compensation. The gap between the two lines widens as η grows and asset price increases, because higher asset price leads to higher return on internal liquidity, which increases the non-financial sector's required compensation for capital illiquidity.

Risk premium, i.e. expected excess return, is the pre-shock excess return adjusted by the expected loss

$$rp(\eta) = \frac{a}{q(\eta)} + \mu^q(\eta) - r(\eta) + \left(\frac{q(\tilde{\eta})}{q(\eta)} - 1 \right) (1 - \underline{\phi}) \lambda + (-1) \underline{\phi} \lambda$$

The bottom left panel shows risk premium as a function of η . It spikes where the endogenous risk is highest (top right panel).

Shown in the bottom right panel is the the price of risk, defined as the ratio of risk premium divided by total loss per unit of investment

$$\frac{rp(\eta)}{1 - \frac{q(\tilde{\eta})}{q(\eta)} (1 - \underline{\phi})}$$

It decreases as the natural buyers' wealth share increases. The price of risk shows the effective risk aversion in the economy. The model economy generates counter-cyclical price of risk, a fact well documented by the empirical asset pricing literature (for example, Lettau and Ludvigson, 2010).

Shown by the top right panel, the model exhibits the accumulation of endogenous risk, as the economy is moving towards a steady state characterized by capital illiquidity ($\zeta < 1$). As the financial sector grows, asset price increases and the return on the non-financial sector's internal liquidity (ROI) increases. This leads to higher demand of liquidity, and higher liquidity premium attached to financial debt. Reduced leverage cost pushes the financial sector to hold more risky assets, making its balance sheet more vulnerable to adverse shocks. Figure 3 analyzes this endogenous risk in detail.

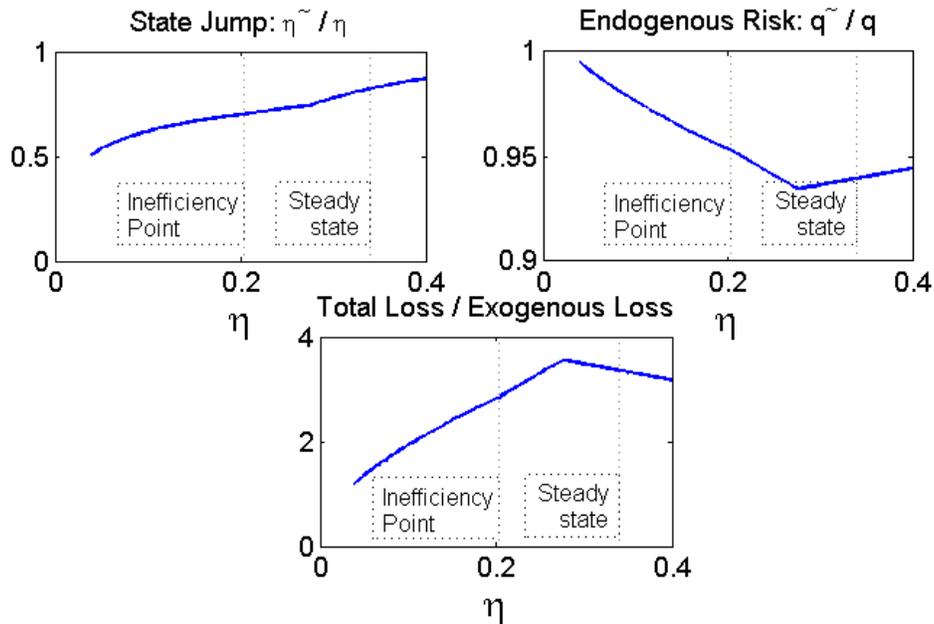


Figure 3: Endogenous Risk

Endogenous risk. The top left panel shows the ratio of post-shock state to the pre-shock state, $\tilde{\eta}/\eta$. The ratio increases as η increases. The financial sector becomes more robust as it grows, but this does not mean that the endogenous risk decreases. The top right panel shows the ratio of post-shock asset price to pre-shock asset price, \tilde{q}/q . As this ratio decreases, the downward jump of asset price becomes larger, reflecting higher endogenous risk. As the financial sector grows, taking on leverage and supplying liquidity to the non-financial sector, the system accumulates endogenous risk.

The bottom panel in Figure 3 shows the strength of shock amplification in the model

economy (equation (6)). The ratio of total loss $(1 - \frac{q(\bar{\eta})}{q(\eta)} (1 - \phi))$ to exogenous loss (ϕ) increases to above 3.5, and then decreases because the financial sector's net worth is large enough to endure macro shocks. However, even at the steady state, the endogenous risk is still significantly larger than where the transition starts (for example, $\eta = 0.05$).

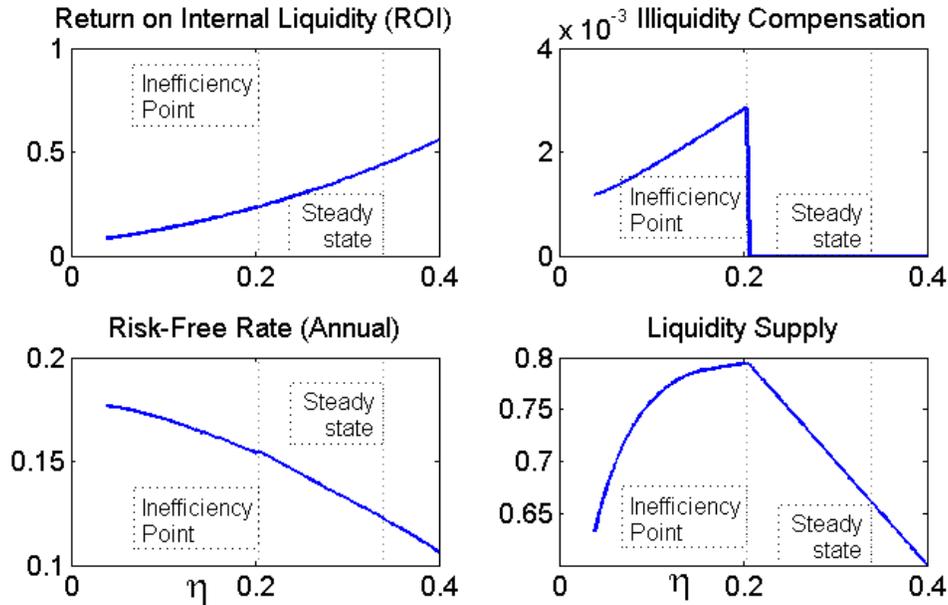


Figure 4: Liquidity Demand and Supply

Liquidity demand and supply. The top left panel shows the non-financial sector's return on internal liquidity. As η grows, asset price grows accordingly, which leads to higher ROI (equation (7)) and therefore higher required illiquidity compensation (equation (17)) in the top right panel (annualized). Note that after the inefficiency point, the non-financial sector does not hold the risky asset, so the plot stops of illiquidity compensation.

Higher demand for liquidity pushes down the risk-free rate, which reduces the leverage cost for the financial sector. The bottom right panel plots $(1 - \eta) (1 - x_k^{nf})$, i.e. the total debt to total net worth ratio. $(1 - \eta)$ share of wealth is in the non-financial sector, of which $(1 - x_k^{nf})$ fraction is invested in risk-free debt. When the financial sector is small, its liquidity provision capacity is restricted, because it does not have enough net worth to buffer the macro shock. The total liquidity, and the total indebtedness of the economy, increases as the financial sector grows and the leverage cost decreases due to the increasing

liquidity premium.²⁷ After the inefficiency point, the financial sector already holds all the risky asset, so the any additional growth of financial net worth reduces its indebtedness.

Before the inefficiency point, we can see the simultaneous increase in financial debt, the non-financial sector’s return on liquidity holding, liquidity premium (reflected in the illiquidity compensation, and decreasing risk-free rate), along with the increasing asset price. At the same time, an increasing share of risky assets is held by the financial sector. Therefore, the model captures the empirical patterns that financial-sector risk-taking and liquidity demand from the non-financial sector increase simultaneously in the transition towards an intangible-intensive economy (the steady state indexed by $\zeta < 1$).

Simulation. Figure 5 paints a more vivid picture of the model economy by simulating one path of the economy for 30 years (360 months).

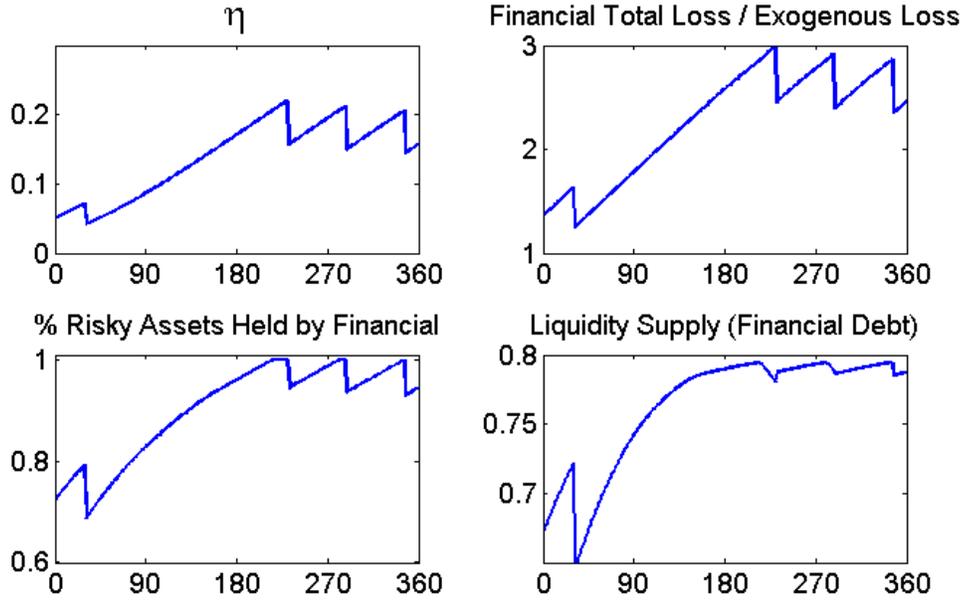


Figure 4: Simulation

The economy starts from $\eta = 5\%$. After an initial hit in month 40, the economy experiences a prolonged period of boom. The financial sector’s wealth share grows to more than 20%. The financial sector increases risk-taking (bottom left panel, i.e. ηx_k^f) by providing liquidity to the non-financial sector (bottom right panel, i.e. $(1 - \eta) (1 - x_k^{nf})$). Along with it, the economy accumulates endogenous risk (top right panel, equation (6)). The

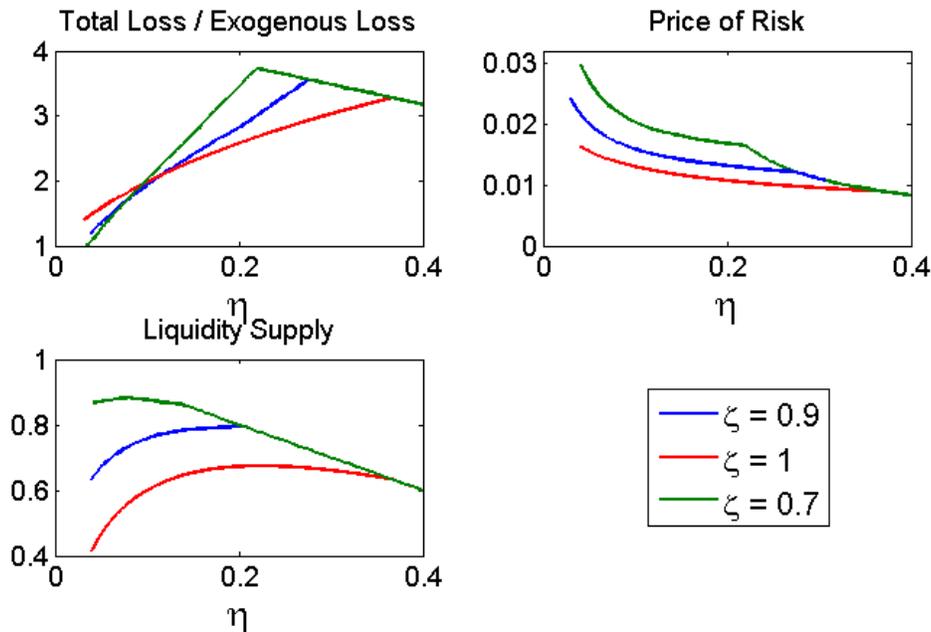
²⁷Because effectively the risk-free debt acts as money in this economy, the total indebtedness can be interpreted as a measure total money issuance. As the economy grows, it requires an increasingly large amount of money, because the increased demand for liquidity from investing agents due to more profitable investment opportunities (higher Tobin’s q).

boom is followed by three recessions. Each recession lasts for three to four years, during which the financial sector cannot hold all the risky asset and the economy moves away from the efficiency region.

Comparative statics of ζ . When $\zeta = 1$, the liquidity premium mechanism is shut down, because upon the arrival of an investment opportunity, risk-free debt and risky asset are equally liquid and both can be freely exchanged for investment goods. The investing households can invest all of their wealth in creating new capital.²⁸

$\zeta < 1$ means that liquidity is valuable because it can be readily used for internal financing of investment projects. This assumptions resemble the assumptions in the dynamic corporate finance literature, where firms' productive capital is subject to adjustment cost (technologically illiquidity) and external financing is more costly than internal financing via retained earnings (for example, Bolton, Chen, and Wang , 2011).

Figure 6 compares the economy with different level of capital illiquidity ($\zeta = 0.9, 1, 0.7$). An economy indexed by a lower value of ζ can be interpreted more intangible-intensive than an economy with a higher value of ζ .



Moving from $\zeta = 1$ to $\zeta = 0.9$ and 0.7 , the shock amplification mechanism is strengthened (top left panel). When capital becomes more illiquid, the non-financial sector attaches higher liquidity premium to risk-free debt, which encourages the financial sector

²⁸They are still financially constrained, in the sense that the creation of new capital needs internal funding ($\theta < 1$).

to take more risks via leverage, as shown by the financial-sector indebtness in the bottom left panel. A stronger the feedback mechanism (as discussed in section 2) leads to higher and more non-linear price of risk.

As the economy is shifting towards more intangible-intensive, the limited pledgeability issue becomes more severe. This drives up the real sector's demand for liquidity in anticipation of financially constrained investment opportunities, and pushes down the leverage cost for the financial sector awarding it for liquidity provision. This mechanism leads to endogenous instability, and results in simultaneous increase in financial-sector risk-taking and non-financial-sector liquidity holding.

4 Conclusion

This paper aims to explain the secular upward trends in corporate liquidity holding and financial-sector risk-taking in the past few decades. The model features the interaction between the financial sector (liquidity supply) and the non-financial sector (liquidity demand). The non-financial sector holds risk-free financial debt as internal liquidity in anticipation of investment opportunities to create new capital, because the holding of existing capital is subject to limited pledgeability. The financial sector enjoys reduced cost of leverage and enlarges risk-taking capacity by issuing risk-free debt.

When the financial sector is more willing to take risks, asset price (i.e. Tobin's q) increases. Investment opportunities become more profitable, and to invest more, the non-financial sector demands more liquidity and attaches higher liquidity premium to financial debt, which further reduces leverage cost. Increased risk-taking and indebtness makes the financial sector vulnerable to adverse shocks. Endogenous risk accumulates. When the shock arrives, its net worth is wiped out drastically, resulting in asset price slump and contraction of real activities.

The model ignores the active provision of public liquidity through central bank's balance sheet expansion and government debt issuance. The existing literature debates on whether public liquidity and private liquidity are substitutes or complements. The conclusion has to be conditional on boom vs. crisis, but more importantly on the specific mechanism that leads to financial instability. It would be fruitful line of future research to incorporate a government sector into the current model, and to study the interplay between public and private liquidity provision.

Appendix - Numerical Solution

The model is solved in a backward fashion in the state space, starting from $\eta = 0.99$. When η is higher, $x_k^{nf} = 0$. In this case the goods market equilibrium implies a quadratic equation for q :

$$(a - \rho q)(1 - \theta q) = \lambda_I q(1 - \eta) \quad (18)$$

for the calibrated parameters, there exists one root that is larger than 1 and smaller than $\frac{1}{\theta}$. This equation links q directly to η via the assumption 4, the limited plegeability of investment opportunities. The analytical solution of asset price when η is large (and $x_k^{nf} = 0$) contrasts with most papers in the literature of heterogeneous-agent equilibrium models that have to solve numerically an ordinary differential equation and the entire path of $q(\eta)$.

The reason to start backward is that $q(\eta)$ is given by this equation when $x_k^{nf} = 0$, and the equilibrium that we are interested has the following feature: there exists a threshold $\bar{\eta}$, above which $x_k^{nf} = 0$ and $x_k^f = \frac{1}{\eta}$ (financial agents hold all risky asset), and below which, $x_k^{nf} > 0$.

Note that when $q < 1$, aggregate investment is zero. Goods market clearing implies $q = \frac{g}{\rho}$, which is larger than one under the calibration leading to a contradiction. Therefore, $q \geq 1$. q has to be small than $\frac{1}{\theta}$. Otherwise, the investment project becomes self-financing, in which case the scale of investment is positive infinite.

Given the q implied by $x_k^{nf} = 0$ and $x_k^f = \frac{1}{\eta}$, I solve $\tilde{\eta}$ by equation (??) assuming that $x_k^{nf}(\tilde{\eta}) = 0$ and $q(\tilde{\eta})$ is given by equation (18) with η replaced by $\tilde{\eta}$. Substitute $\frac{q(\tilde{\eta})}{q(\eta)}$ into financial-sector and non-financial-sector F.O.C. Consider the following scenarios:

If the non-financial sector's F.O.C. is met, then we have solved $q(\tilde{\eta})$. Then we move to $\tilde{\eta}$ and repeat this calculation. If the non-financial sector's F.O.C. is violated, in which case, it is profitable to have $x_k^{nf} > 0$. In this case, substitute equation (11) into Non-financial F.O.C., which holds in equality now, and solve $q(\tilde{\eta})$. Substitute $\frac{q(\tilde{\eta})}{q(\eta)}$ into equation (??), and following the jump, we move to the next value of state variable.

The calculation ends when $\frac{q(\tilde{\eta})}{q(\eta)} = 1$, in which case endogenous risk is eliminated at a very small value of η . Therefore, I solve exactly a path of η following the jumps. By perturbing the starting point, I can calculate multiple paths filling in all the values of $q(\eta)$ for all $\eta < 1$.

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