Finance Theory Group Summer School 2015

Dynamic Financial Contracting

Part I: Discrete Time Models

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Agency Models in Corporate Finance

- Static Agency / Contracting models have proved important in Corporate Finance
  - Capital Structure (Jensen and Meckling, 1976)
    - Concentrated equity ownership, Asset substitution
  - Incentive Schemes (Holmstrom, 1979)
    - Monotonicity of payoffs, Informativeness of signals
  - Security Design (Innes, 1990)
    - Debt contracts, inside equity

Why Dynamic Models?

- Key shortcomings of static models
  - Security Design
  - Compensation
  - Capital Structure
  - Shutdown / Termination
  - Investment
- What makes dynamic models challenging?
  - Must balance richness and tractability (HM 87?)
- Capital Structure
  - Firm leverage ratios are not static, but continually changing over time.
  - How do we think about credit lines, which are both a source of debt as well as credit (financial slack)?

The Plan

- Part I: Discrete-Time Models
  - DeMarzo-Fishman (*RFS*, 2007a,b)
    - "Optimal Long-Term Financial Contracting"
    - "Agency and Optimal Investment Dynamics"
  - Methodology, implementation and robust intuitions

- Part II: Continuous-Time Models
  - DeMarzo-Sannikov (*JF*, 2006)
    - "Optimal Security Design and Dynamic Capital Structure in a Continuous-Time Agency Model"
  - Tractability and deeper insights

The Plan

- Part III: Next Steps
  - DeMarzo-Fishman-He-Wang (*JF*, 2012)
    - "Dynamic Agency Theory meets the Q-Theory of Investment"
  - DeMarzo-Livdan-Tchistyi (2014)
    - "Risking Other People's Money: Gambling, Limited Liability, and Optimal Incentives"
  - DeMarzo-Sannikov (2015)
    - "Learning, Termination, and Payout Policy in Dynamic Incentive Contracts"

*"**Begin** at the **beginning**," the King said, very gravely…*

Part I.A:

SOME SIMPLE STATIC MODELS

Static Principal-Agent Problem

- The Problem
  - Risk-neutral and wealthy principal hires an agent to expend effort or take costly actions
  - Effort / action e affects distribution of outcome s
  - Principal chooses incentive scheme $w(s)$ to motivate agent

- Frictions
  - Observed outcome s does not perfectly reveal e
  - Agent has limited liability, may be risk-averse

Compensation Contracts

- With risk aversion, subject to conditions, optimal wage satisfies (Holmstrom 1979):

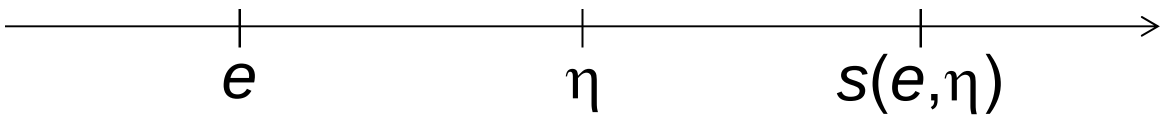
Marginal cost of compensation $\longrightarrow$ $$\frac{1}{u'(w(s))} = a + b\,\frac{f_e(s \mid e)}{f(s \mid e)}$$ $\longleftarrow$ Likelihood ratio of observed outcome

  - MLRP implies increasing wage profile *w*(*s*)

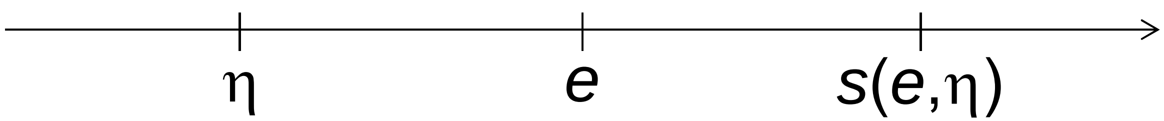
- Risk neutrality with limited liability
  - Bang-bang contracts (Innes 1990)
    - Minimal payoff below a threshold
    - Maximum payoff above threshold
  - Investor monotonicity implies inside *levered equity*

Ex Ante vs. Ex Post Actions

- Output depends on actions and noise: $s(e,\eta)$
  - Standard model: agent chooses action before observing random shocks



  - Ex post action model: agent observes random shock *prior* to choosing action



  - Note:
    - in a general dynamic model both may be true!
    - and in continuous time the distinction may blur altogether…

Ex Ante vs. Ex Post Actions

- IC constraint
  - Ex ante: $\max_e E[u(w(s(e,\eta))) - d(e)]$

    $\Rightarrow E[u'w's_e] = d_e$

    - Many possible IC contracts; shape set to minimize cost

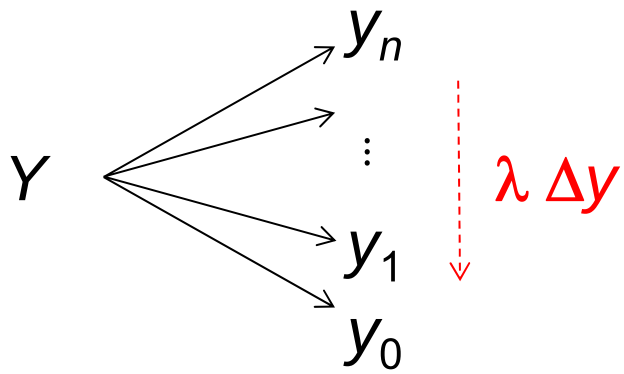
  - Ex post: $\Rightarrow \max_e$ state by state $\Rightarrow u'w's_e = d_e$

    $\Rightarrow w' = d_e / (u's_e)$

    - Shape of contract determined by IC constraint
    - Level determined by participation constraint / limited liability
      - Lacker & Weinberg ('89) "Costly State Falsification"
      - Edmans & Gabaix ('08) "Tractable Incentive Contracts"

Cash Flow Diversion

- Firm hires manager to generate output
  - Output *Y*, stochastic
  - Agency Problem
    - Manager can divert output
    - Private benefit of \$$\lambda$ per \$1 dollar diverted
  - Contract specifies wage $w(Y)$
- How can we provide incentives to prevent diversion?
  - $w'(Y) = \lambda$



Cash Flow Diversion

- Optimal contract
  - Incentive Compatibility:
    - Manager must receive compensation of λ per extra dollar of reported output

$$\Rightarrow \quad w(Y) = \overline{w} + \lambda\,(Y - EY)$$

    - *Linear contract form ("inside equity") is independent of the cash flow distribution or utility function*
  - Limited Liability: $w(y_0) \geq 0$

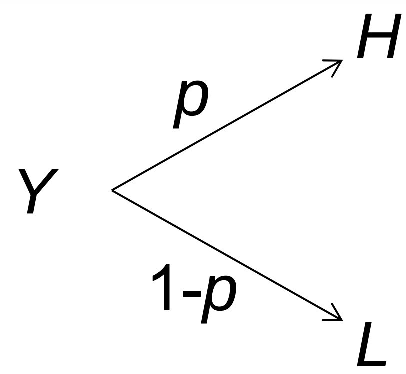
$$\Rightarrow E\,w(Y) \geq \lambda\,(EY - y_0) = \lambda\,\Delta(Y)$$

    - *The manager must be exposed to risk – and receive a minimal level of rents – to provide incentives*

Binary PA Model: Equivalence

- Binary PA setting
  - Risk neutral agent
  - Binary outcomes (H vs. L)
  - Ex ante effort (work/shirk)
    - Working raises probability of high output from **p to $p + \delta$**
    - Working has private cost of **$c(H – L)$**
  - IC constraint: $\delta(w_H – w_L) \geq c(H – L)$

$$\Leftrightarrow \quad \Delta w/\Delta Y \geq c/\delta$$



- Binary PA $\Leftrightarrow$ Cash Flow Diversion with **$\lambda = c/\delta$**
  - Continuous time PA model “looks like” cash flow diversion model

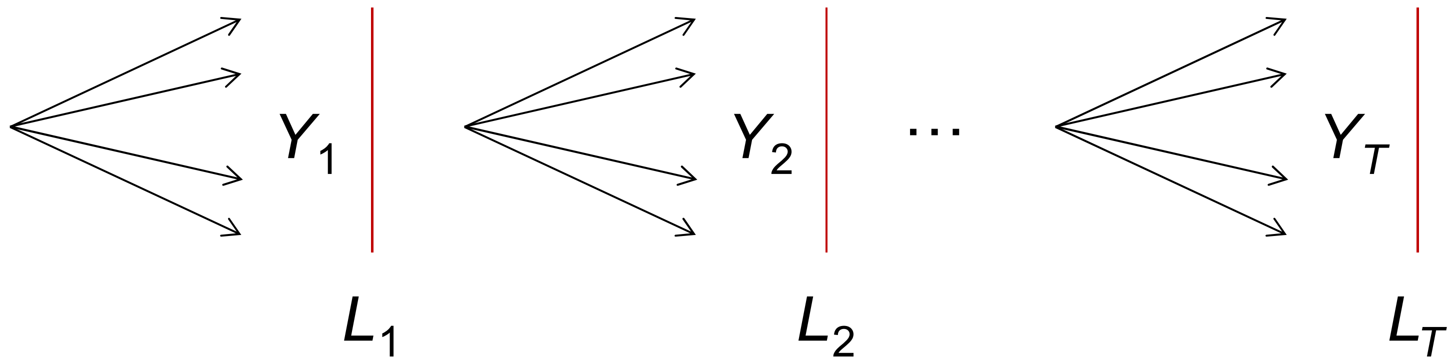
"…and go on till you come to the end…"

Part I.B:

DISCRETE TIME DYNAMIC AGENCY

Multiple Periods

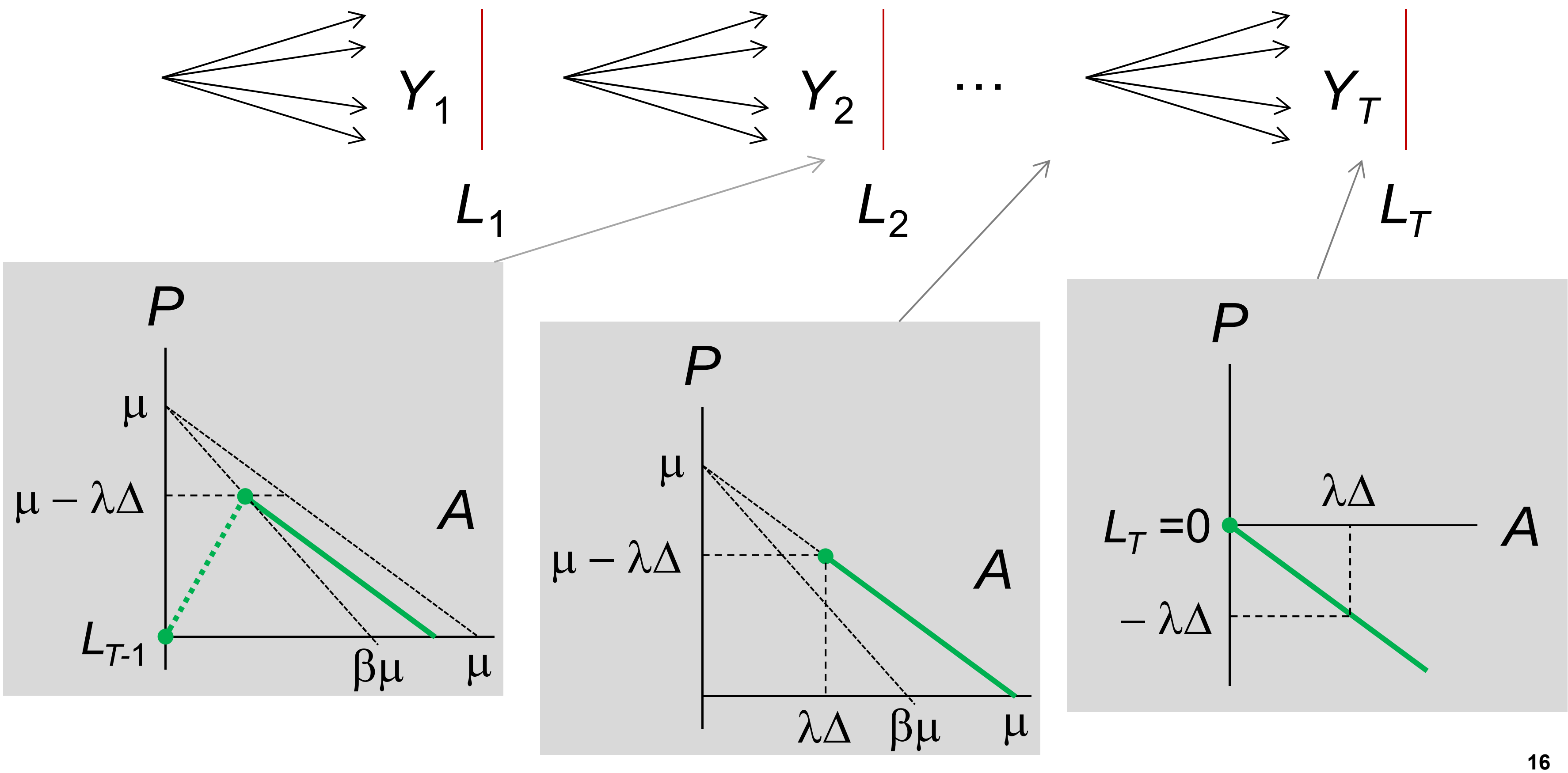
- Discrete time model
  - Independent output Y_t each period
  - Contract sets payment w based on output history
  - Contract may also terminate; liquidation value L_t



- Repeat static contract (IC): $w_t = \lambda\,(Y_t - y_{t0})$
  - Expected rent = λ PV(Δ_t …)

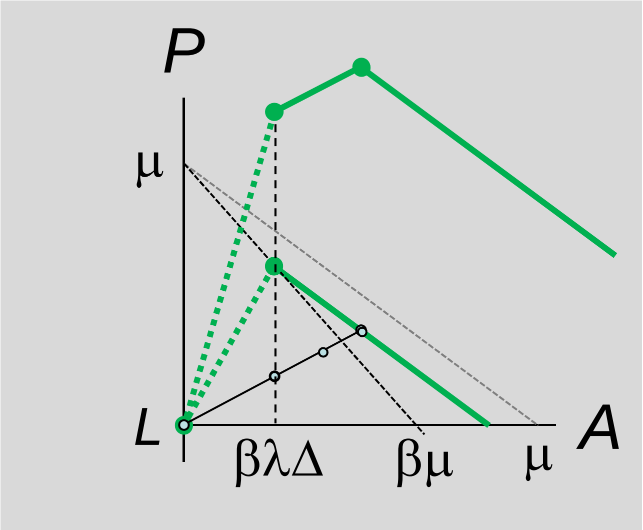
Contract Curve

- What payoff pairs are feasible for a risk-neutral principal and agent?



Dynamic Contract

- Intuition
  - Limited Liability + IC:
    Agent must earn rents
  - Impatience:
    Pay cash beyond some threshold
  - Deferred compensation:
    Use past payments to "buy" future continuation rents



Many Periods (DF 2007)

PROPOSITION 4. (OPTIMAL CONTINUATION FUNCTION)

Given $a^0{}_t$ and b_t concave, the continuation function at $s = t^-$ is given by $a^0{}_s = R_s$ and

$$b_s(a) = \begin{cases} \hat{b}_s(a) & \text{if } a \geq a_s^L \\ L_s + l_s(a - R_s) & \text{if } a_s^0 \leq a < a_s^L \end{cases}, \tag{13}$$

where

$$a_t^1 = \inf\left\{a \geq a_t^0 : b_t'(a) \leq -1\right\}, \tag{14}$$

$$b_t^1(a) = \begin{cases} b_t(a) & \text{for } \; a_t^0 \leq a \leq a_t^1 \\ b_t(a_t^1) - (a - a_t^1) & \text{for } \; a \geq a_t^1 \end{cases}, \tag{15}$$

$$\hat{a}_s^0 = e^{-\gamma(t-s)}\left(\mu_t + a_t^0\right), \tag{16}$$

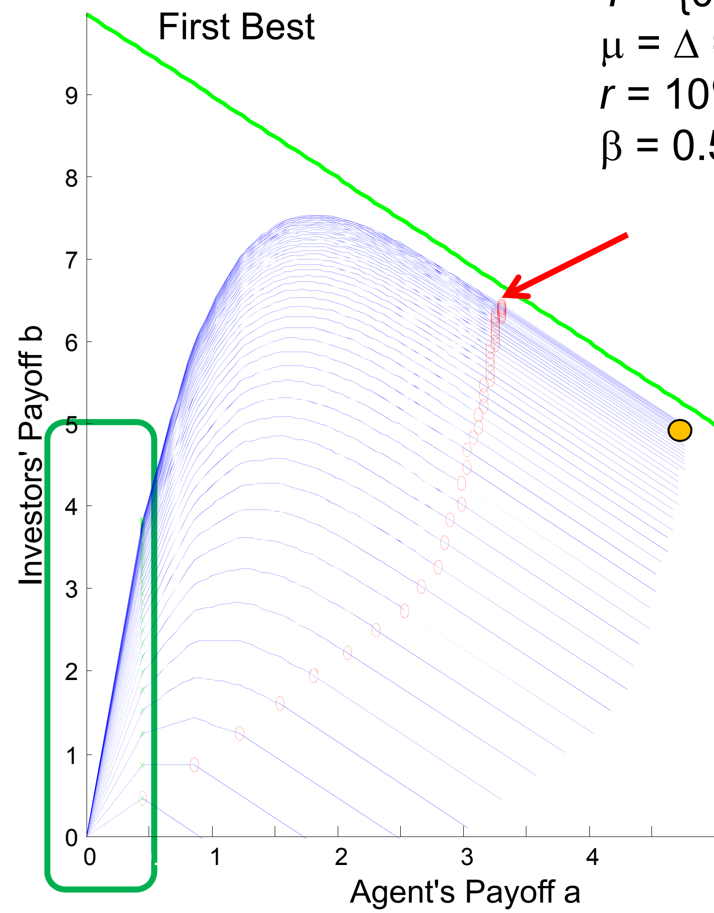
$$\hat{b}_s(a) = e^{-r(t-s)}\left(\mu_t + E\left[b_t^1(e^{\gamma(t-s)}a + Y_t - \mu_t)\right]\right), \tag{17}$$

$$l_s = \sup\left\{\frac{\hat{b}_s(a) - L_s}{a - R_s} : a > \max\left(\hat{a}_s^0, R_s\right)\right\}, \text{ and} \tag{18}$$

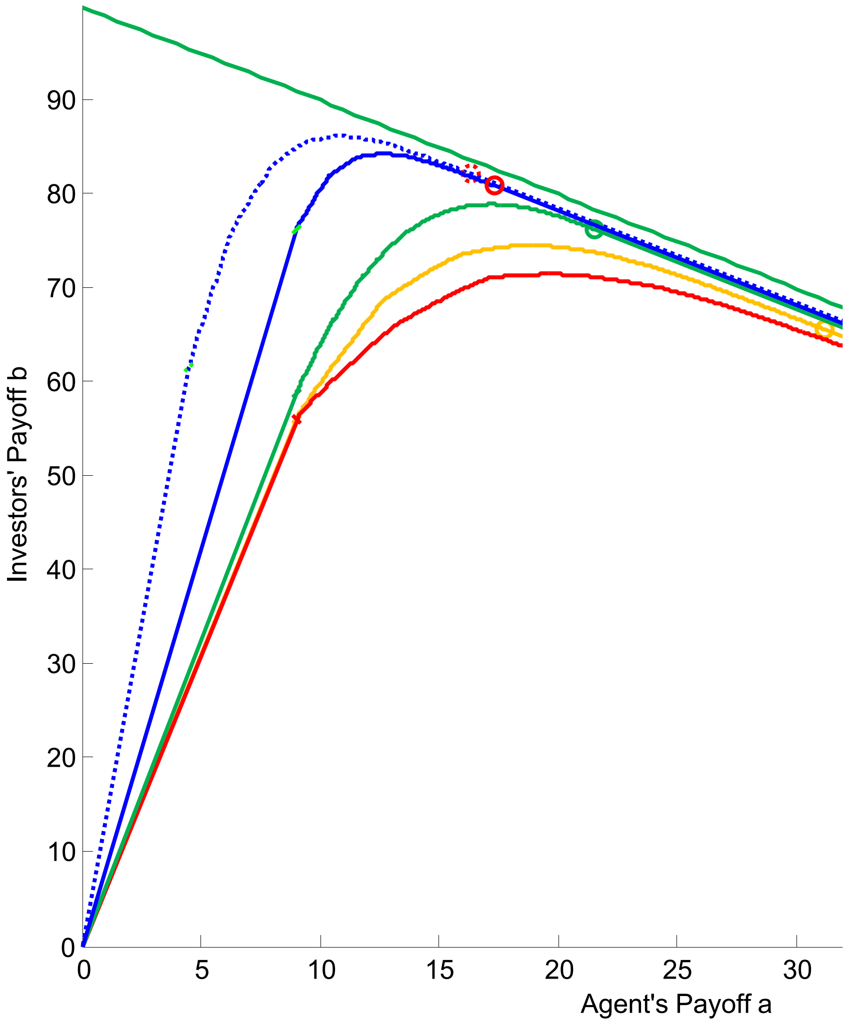
$$a_s^L = \inf\left\{\; a \geq \max\left(\hat{a}_s^0, R_s\right) : \hat{b}_s'(a_s^L) \leq l_s\right\}. \tag{19}$$

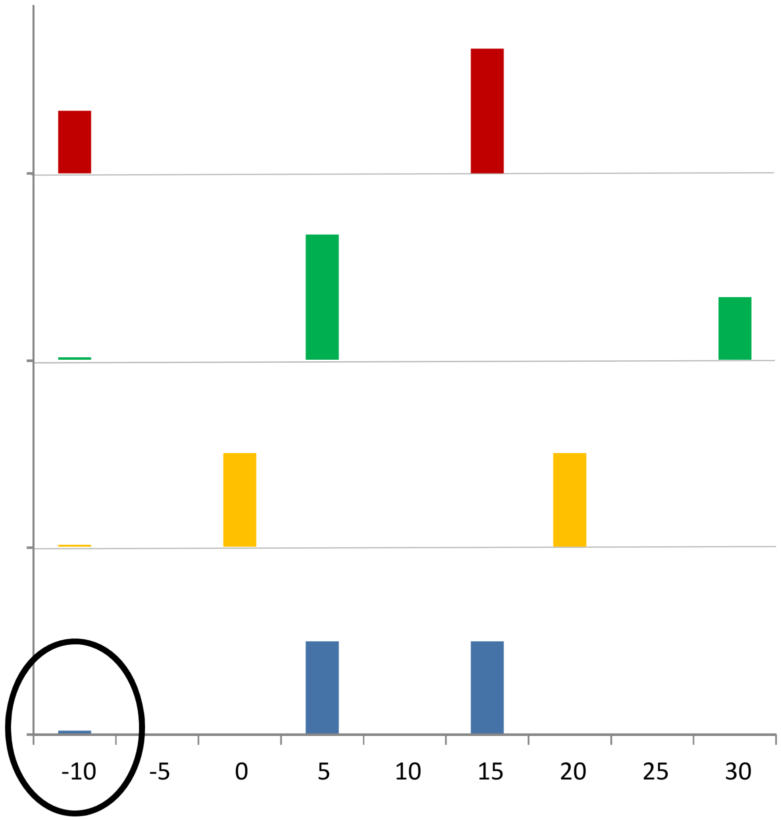
Note finally that b_s is concave.

$\lambda = 0.5$
$Y = \{0,2\}$
$\mu = \Delta = 1$
$r = 10\%$
$\beta = 0.5\%$



Volatility and Skewness





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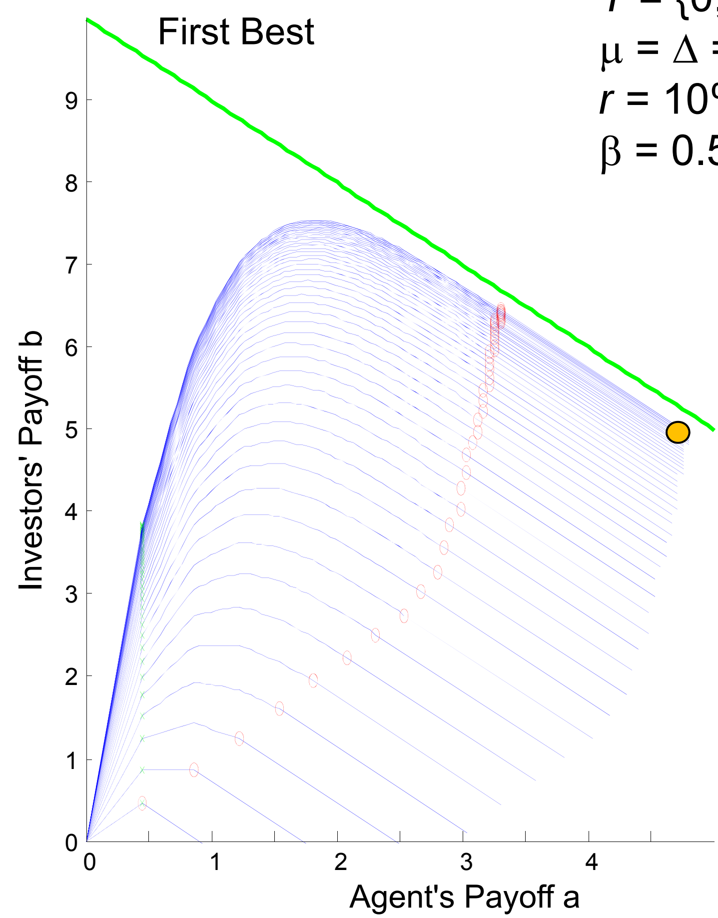


Implementation (DF 2007)

Implementing the Optimal Contract

- **Long Term Debt:** A long term debt contract is characterized by a sequence of fixed payments x_t . If a payment is not made, the firm is in default.

- **Credit Line:** A credit line is characterized by an interest rate $\hat{r}$ and a fixed credit limit $c^L{}_t \geq 0$. No payments need be made on the credit line except for
  - required interest payments once the limit is reached,
  - required payments if the limit is reduced.

  If not paid, the firm is in default.

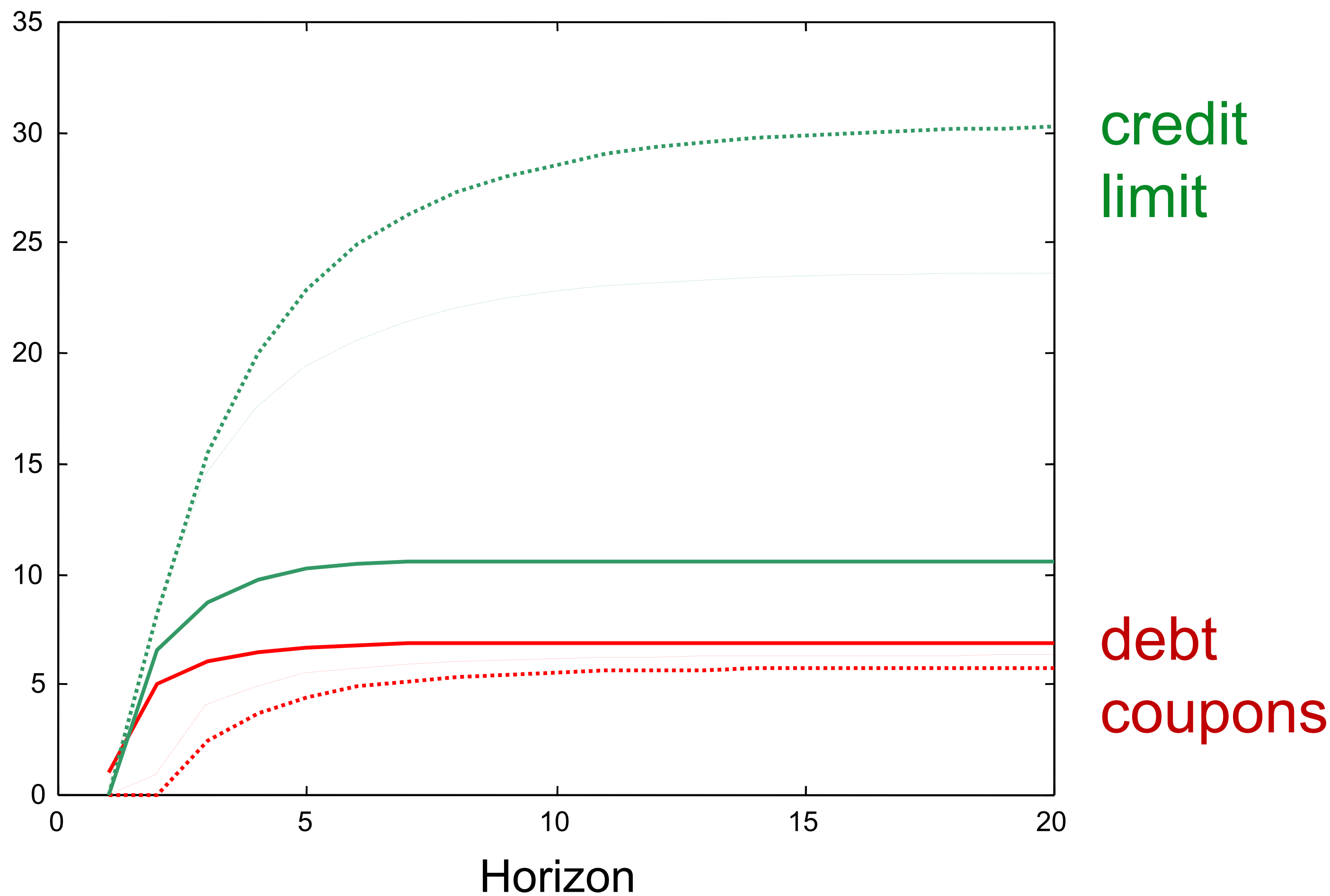
- **Default:** Given default on payments totaling $z > 0$ in period t, the investor terminates with probability

  $$p_t(z) = z / N_t$$

  keeping the proceeds L_t.

Sample Dynamics

- Debt and Credit Limit vs. Horizon (γ = 10.1%, 10.5%, 15%)



"If you don't know where you are going, any road will take you there…"

Part I.C:

RECURSIVE CONTRACTING AND INVESTMENT DYNAMICS

Recursive Contracting

- In a dynamic model, the agent can be compensated in two ways
  - Direct cash payments
  - Promises of future payments ("continuation value")
- Paying via continuation value relaxes future IC constraints
  - If the agent is not impatient, compensation should be maximally deferred
- Dynamic Programming approach to solve for optimal contract
  - Abreu-Pearce-Stacchetti (1986), Spear-Srivastava (1987), Phelan-Townsend (1991), Atkeson (1991), Ljungqvist-Sargent (2000)

Recursive Contracting

- Methodology: Describe contract recursively using promised future payoffs as state variables
  - Investor's value function:
    - $b(w)$ = max payoff to investor from incentive compatible contract that provides payoff w to the agent
- Optimal Contract
  - Provides expected payoffs (w_t, $b(w_t)$) to agent and investors
  - Based on today's reported cash flows Y_t, contract specifies
    - Transfers between agent and investors
    - Tomorrow's contract: (w_{t+dt}, $b(w_{t+dt})$)
    - Probability of Termination

  } *provide incentives*
- Given the value function *b*, this is a *static* problem

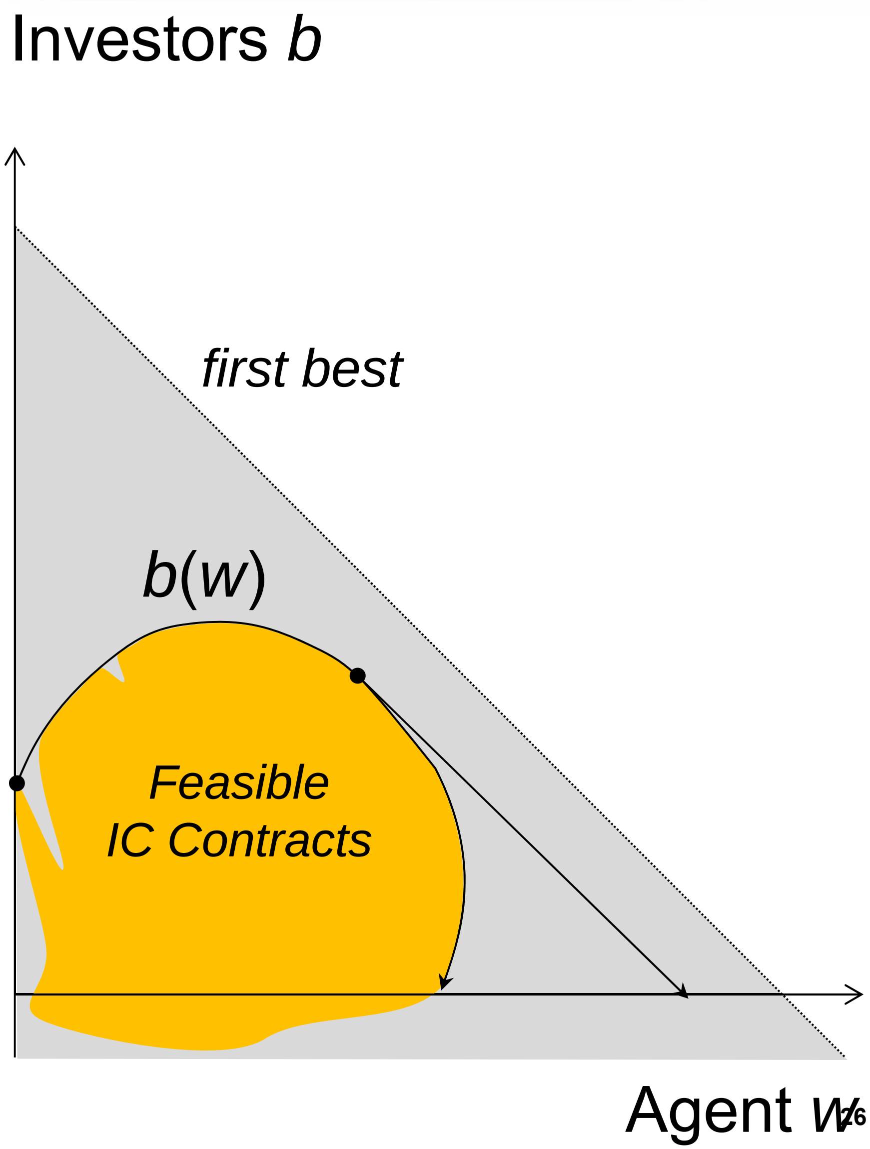
A General Framework

- Investor value $b_t(w)$
  - Concave (randomize)
  - $b'(w) \geq -1$ (pay cash)
- General agency problem
  - Max $E[Y_t + b_t(w_t) \mid e_t]$ *s.t.*

    (IC) $e_t \in \arg\max E[w_t - c_t \mid e_t]$

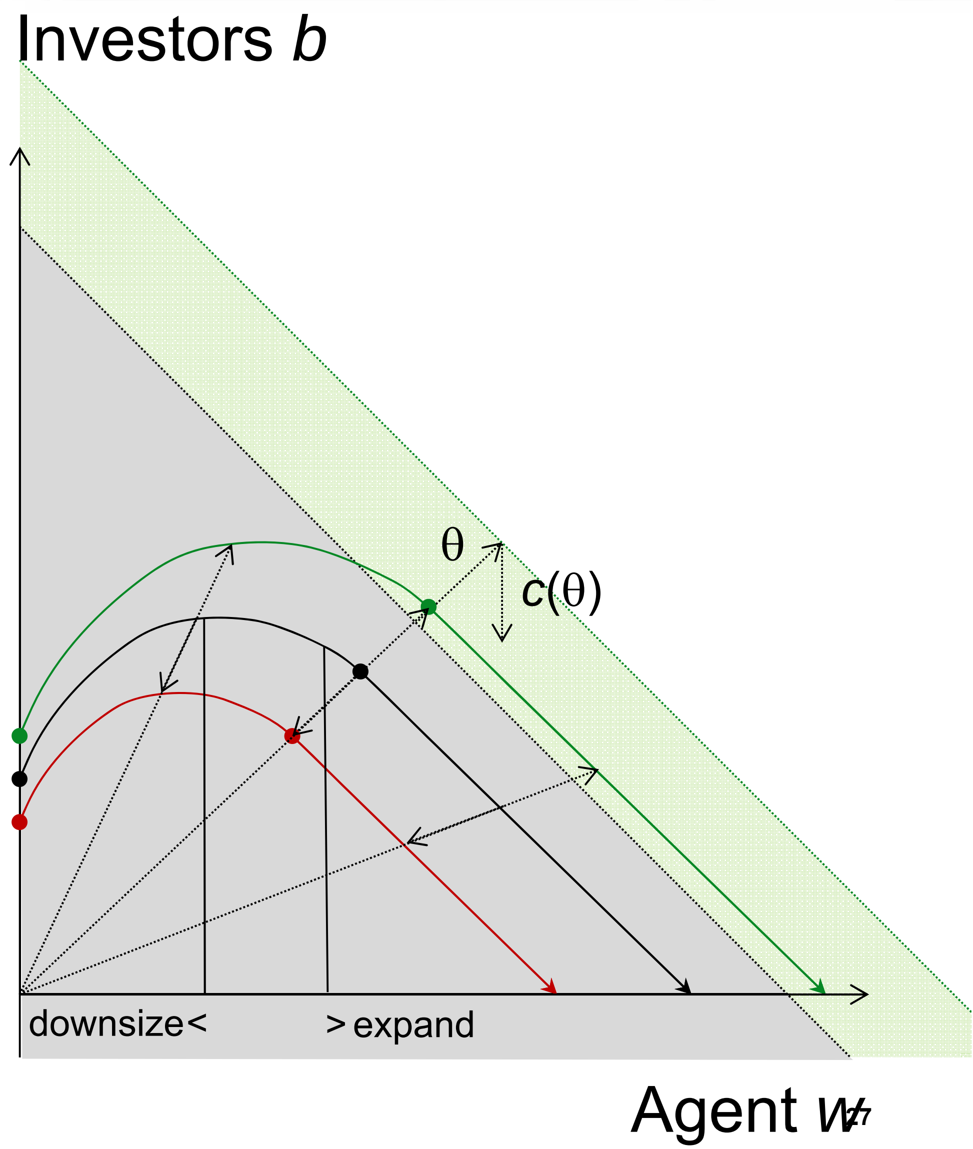
    (PC) $E[w_t - c_t \mid e_t] = w_{t-}$

    (LL) $w_t \geq 0$
- Monotonicity of $w_t(Y_t, w_{t-})$
  - Cash payouts follow high cash flows



Investment Dynamics

- Scalable Technology
  - Investment option
    - Rescales payoffs by θ
    - Capital & adj. cost $c(\theta)$
    - $(w, b) \rightarrow (\theta w, \theta b - c(\theta))$
  - Optimal growth
    - $b^{-}(w) = \max_{\theta} \theta b^{+}(w/\theta) - c(\theta)$
    - b concave $\Rightarrow \theta\uparrow$ with w
- Monotonicity of $w_t(Y_t, w_{t-})$
  - Investment increases with current & past cash flow
  - Investment increases with past investment



Conclusions

- Dynamic Financial Contracting
  - Compensate the agent with cash, or with future promises (“continuation value”)
  - Deferred compensation provides future financial slack
  - Relatively simple capital structure may capture complicated contract dynamics
  - Agency concerns generally lead to capital structure path dependence & positive feedback in investment dynamics
- What’s Next?
  - Not yet sufficiently tractable
  - Continuous-time will simplify & allow for new insights