Finance Theory Group Summer School 2015

Dynamic Financial Contracting

Part II: Continuous-Time Agency

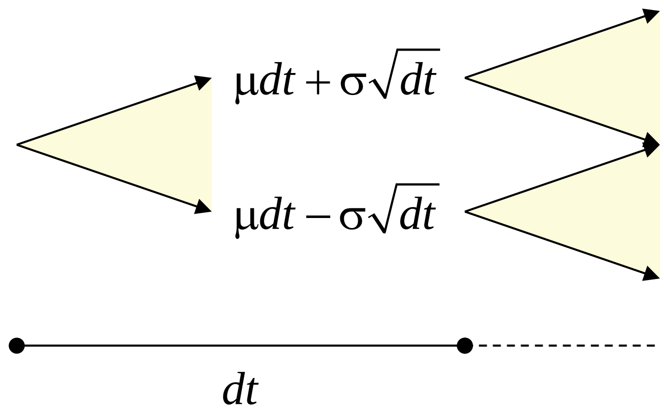
Prof. Peter DeMarzo
Stanford University

Part I Conclusions

- Dynamic Financial Contracting
  - Compensate the agent with cash, or with future promises (“continuation value”)
  - Deferred compensation provides future financial slack
  - Relatively simple capital structure may capture complicated contract dynamics
  - Agency concerns generally lead to capital structure path dependence & positive feedback in investment dynamics
- What’s Next?
  - Not yet sufficiently tractable
  - Continuous-time will simplify & allow for new insights

Binomial Agency

- Discrete time: Binomial Model
  - $E[Y] = \mu\, dt$, $\Delta(Y) = \sigma\, (dt)^{1/2}$



Convergence:
Biais, Mariotti, Plantin, Rochet
RES 2007

Unbounded losses $\Rightarrow$ always risk of termination (FB surplus unattainable in limit)

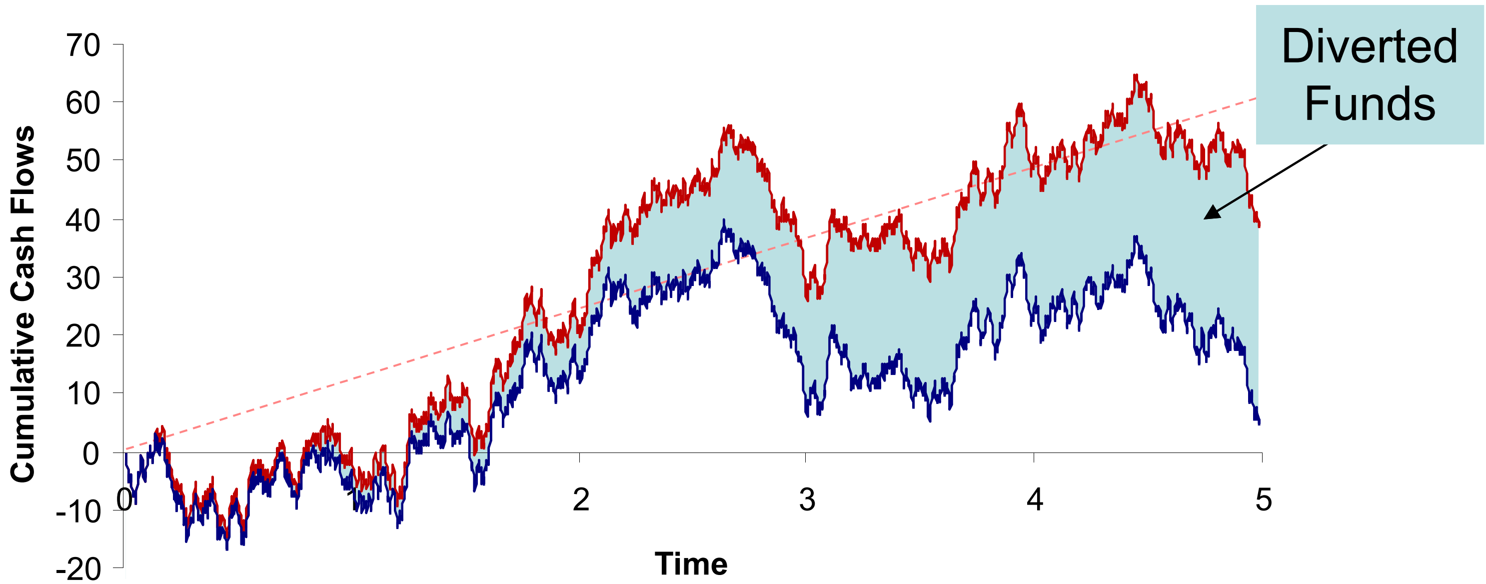
  - Cash Flow Diversion: λ = private benefit per \$ diverted
  - Work / Shirk:
    - Effort determines probability of high outcome
    - λ = private benefit per \$ reduction in mean

Continuous-Time Setting

- Risk-neutral Agent has a profitable business opportunity
  - Initial Investment $K \geq 0$
  - Cash flows accumulate at annual rate $\mu > r K$ and volatility $\sigma > 0$
- Agent requires external capital from risk-neutral investors
  - To fund initial investment, and cover possible operating losses
  - To fund consumption – Agent is impatient (discount rate $\gamma > r$)

The Contracting Problem

- Investors do not observe cash flows / effort
  - Agent can divert or shirk, and consume private benefits (λ/\$)
- How do investors distinguish legitimate operating losses from a diversion of funds?
  - And what can they do about it?

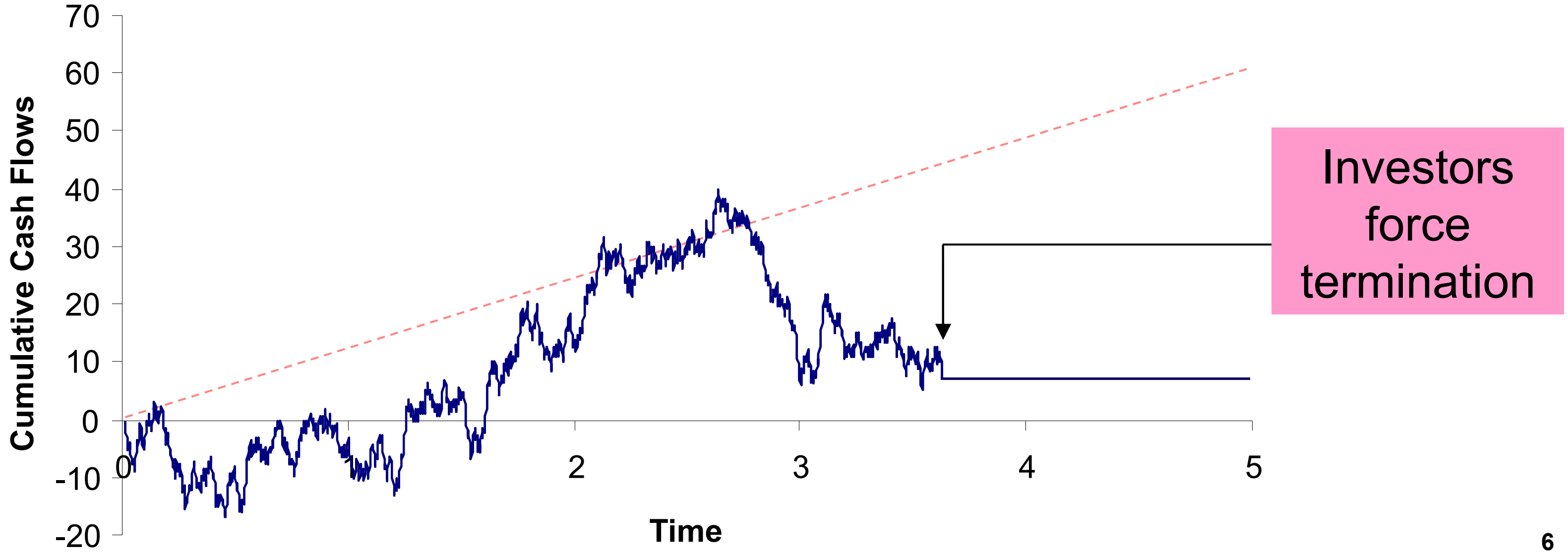


The Contracting Problem

- Investors can control:
  - The agent's compensation
  - The firm's access to capital / credit
  - Termination / Liquidation
    - Liquidation value *L*, Agent's outside option *R*

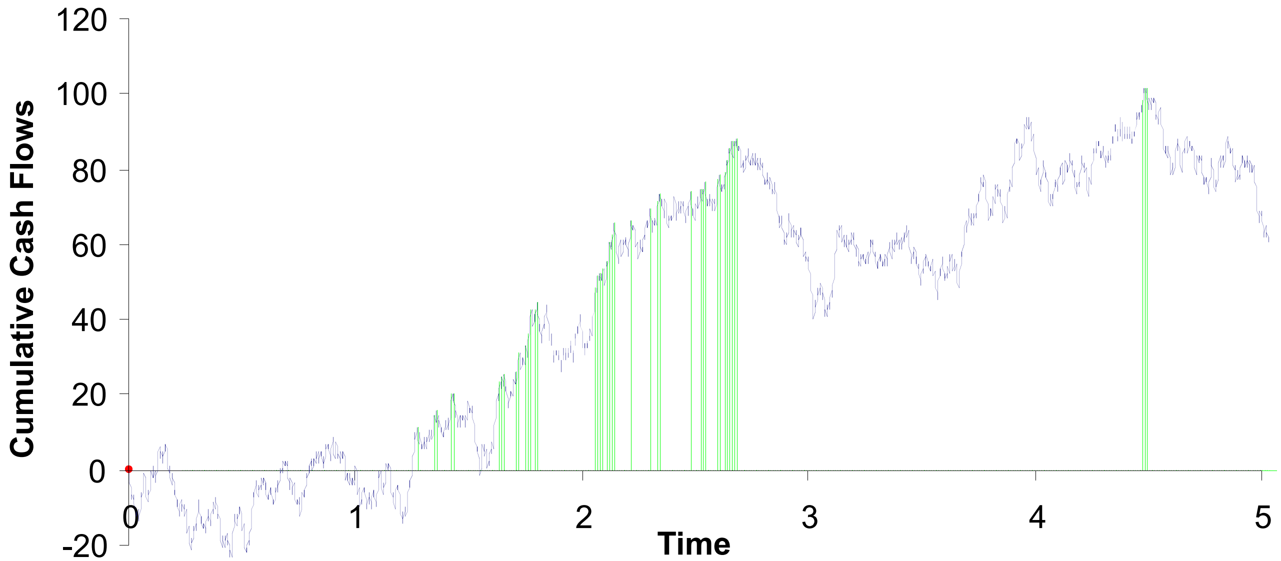
(L,R) may be endogenous…

but inefficient: $r L + \gamma R < \mu$



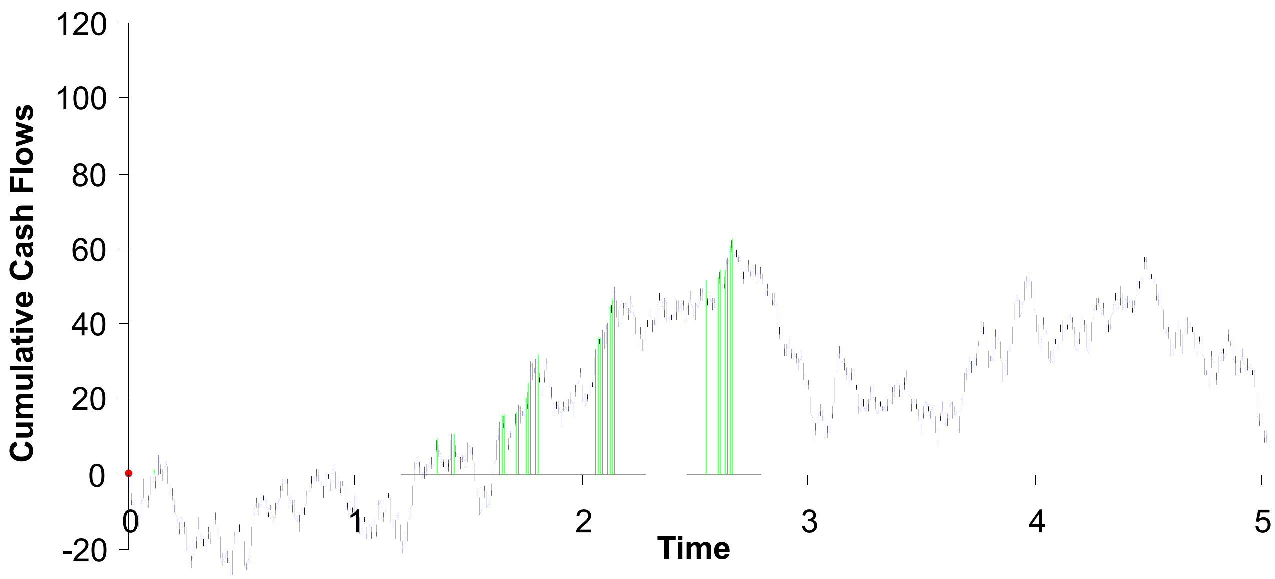
Optimal Contract

- Optimal contract specifies, as a function of history of reported cash flow:
  - Payments to the agent
  - Termination / Liquidation
- Key results
  - History dependent (even though profitability is not)
  - High watermark-like features



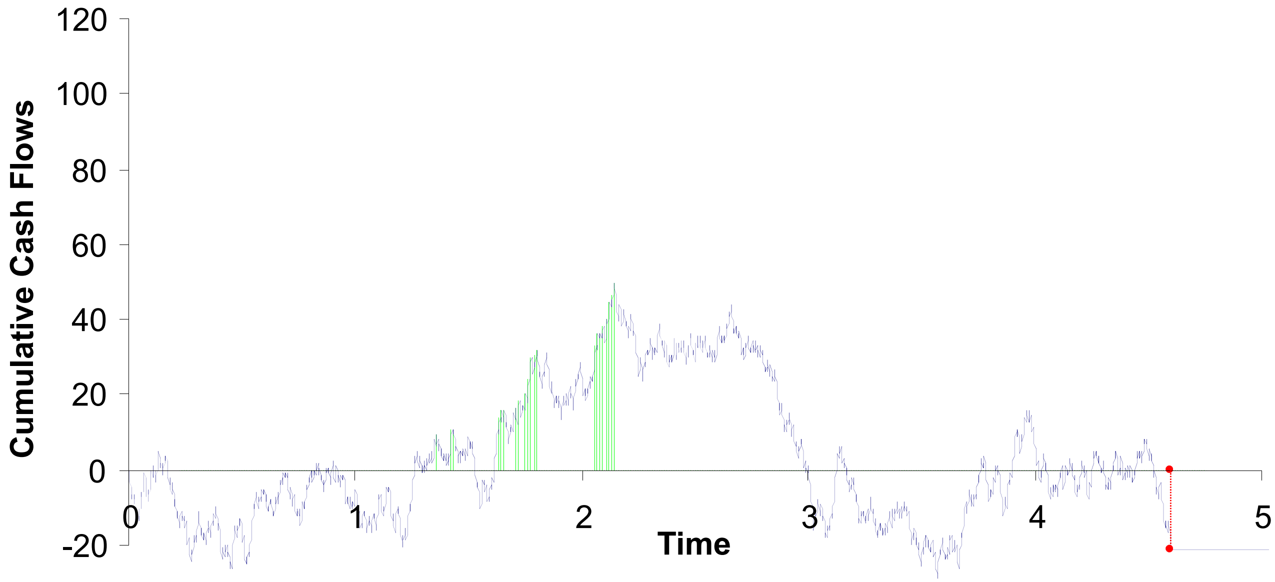
Optimal Contract

- Optimal contract specifies, as a function of history of reported cash flow:
  - Payments to the agent
  - Termination / Liquidation
- Key results
  - History dependent (even though profitability is not)
  - High watermark-like features
  - Forbearance of short term losses



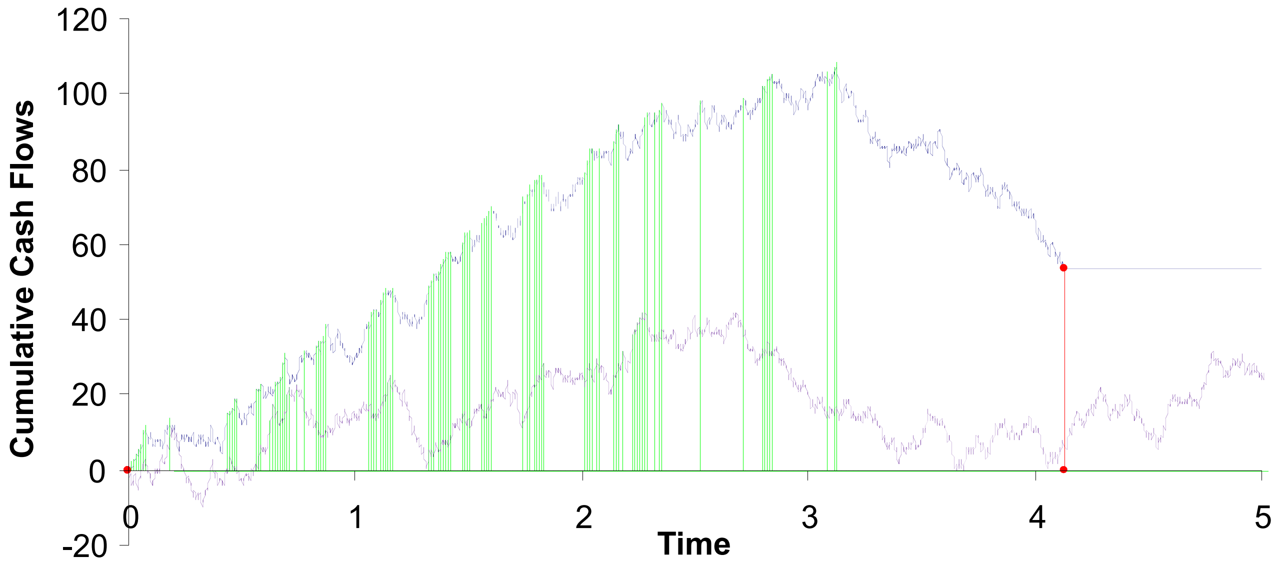
Optimal Contract

- Optimal contract specifies, as a function of history of reported cash flow:
  - Payments to the agent
  - Termination / Liquidation
- Key results
  - History dependent (even though profitability is not)
  - High watermark-like features
  - Forbearance of short term losses (up to a point)



Optimal Contract

- Optimal contract specifies, as a function of history of reported cash flow:
  - Payments to the agent
  - Termination / Liquidation
- Key results
  - History dependent (even though profitability is not)
  - High watermark-like features
  - Forbearance to short term losses (up to a point)

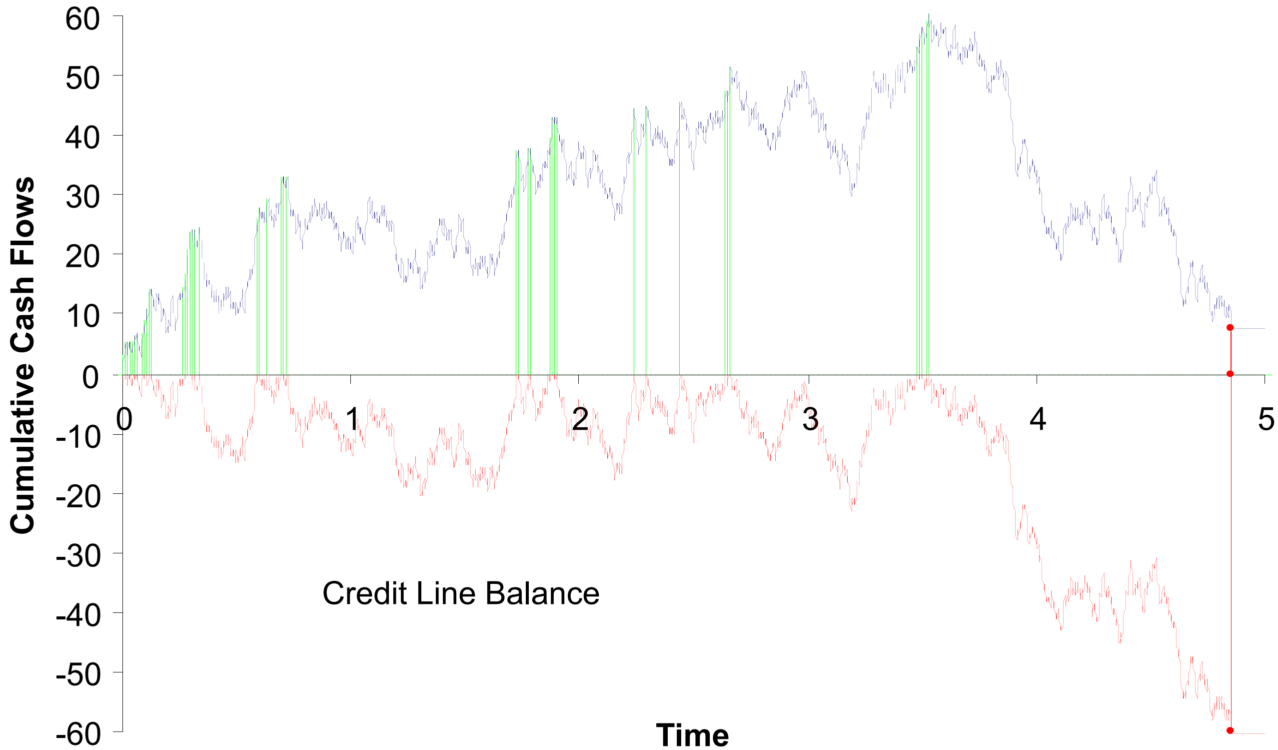


Implementation

- The optimal contract can be implemented via a simple capital structure:
  - LT Debt: Perpetual bond with face value D and coupons rD per period
  - Credit Line: with a fixed interest rate γ and credit limit C^L
  - Equity (inside and outside): dividends paid from excess cash flows once credit line is paid off
  - Default: termination/liquidation of project

Example

- Given: $L = 65$, $R = 0$, $\mu = 10$, $\sigma = 20$, $r = 10\%$, $\gamma = 15\%$, $\lambda = 0.8$
  - LT debt with FV of 100 and 10% coupon rate
  - Credit Line with limit of 60 and 15% interest rate
  - Outside Equity = 20% of firm



The Formal Problem (DS 2006)

- Cash flows
  - Agent "diverts or shirks" at rate $a_t \geq 0$, and reports

$$dY_t = (\mu - a_t)\, dt + \sigma\, dZ_t$$

- Contract $\Pi = (U, \tau)$
  - Principal pays compensation $dU_t \geq 0$
  - Chooses termination (stopping) time τ
- Payoffs
  - Agent: $$w(\Pi) = \max_a \quad E^a\left[\int_0^\tau e^{-\gamma t}\left(dU_t + \lambda a_t dt\right) + e^{-\gamma\tau} R\right]$$
  - Principal: $$p(w) = \max_{\Pi,a} \quad E^a\left[\int_0^\tau e^{-rt}\left(dY_t - dU_t\right) + e^{-r\tau} L\right]$$

$$\text{s.t.} \quad (\Pi, a) \text{ is IC}, \quad w(\Pi) = w_0$$

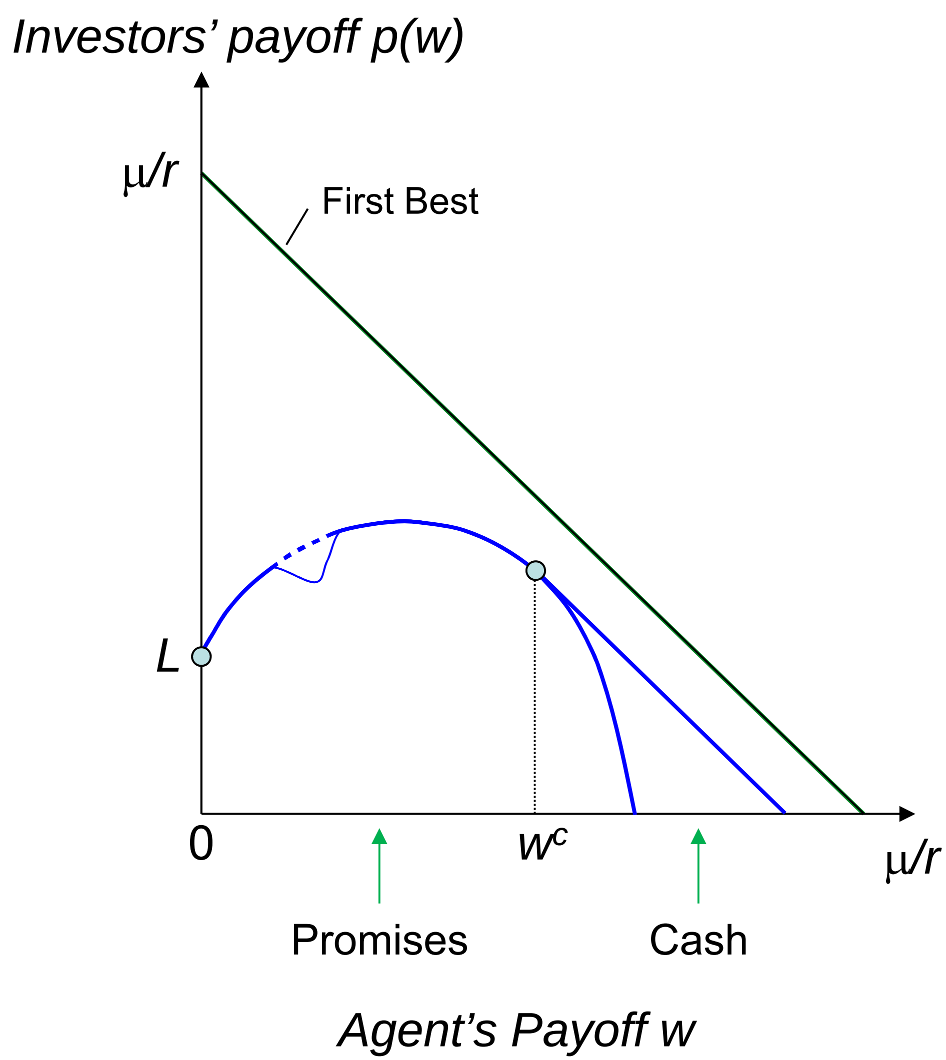
Recursive Contracting (review)

- Methodology: Describe contract recursively using promised future payoffs as state variables
  - Investor's value function:
    - $p(w)$ = max payoff to investor from incentive compatible contract that provides payoff w to the agent
- Optimal Contract
  - Provides expected payoffs $(w_t, p(w_t))$ to agent and investors
  - Based on today's reported cash flows dY_t, contract specifies
    - Transfers between agent and investors
    - Tomorrow's contract: $(w_{t+dt}, p(w_{t+dt}))$
    - Probability of Termination

  } *provide incentives*
- Given the value function p, this is a *static* problem

Solving the Model

- First-Best Value Function
  - $p^{FB}(w) = \mu/r - w$
- Basic Properties
  - Positive payoff from shirking
    $\Rightarrow$ $p(0) = L$
  - Public randomization
    $\Rightarrow$ $p(w)$ is weakly concave
  - Liquidation is inefficient
    $\Rightarrow$ $p(w) + w \leq \mu/r$
- Cash Compensation
  - $\Rightarrow$ $p'(w) \geq -1$
  - Pay cash if $w > w^c$
  - Use promises if $w \leq w^c$



Solving the Model

- Agent's Future Payoff *w*
  - Promise-keeping
    - $E[dw] = \gamma\, w\, dt$
  - Incentive Compatibility
    - $\partial w / \partial y \geq \lambda$

$$\Rightarrow dw = \gamma w dt + \lambda (dy - E[dy])$$
$$= \gamma w dt + \lambda \sigma\, dZ$$

- Investor's Payoff: HJB Equation

$$\underbrace{rp}_{\text{Req. Return}} = \underbrace{\mu}_{E[FCF]} + \underbrace{\gamma\, wp' + \tfrac{1}{2}\lambda^2\sigma^2 p''}_{E[dp]}$$

Boundary Conditions:

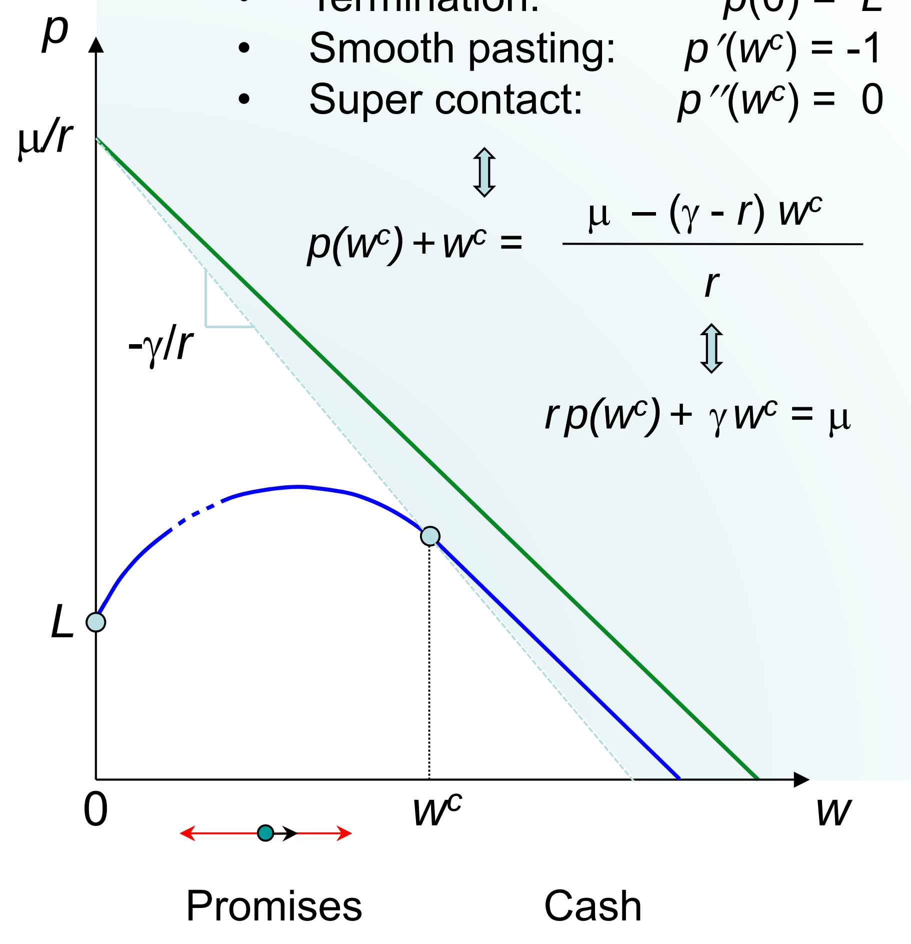
- Termination: $p(0) = L$
- Smooth pasting: $p'(w^c) = -1$
- Super contact: $p''(w^c) = 0$

$\Updownarrow$

$$p(w^c) + w^c = \frac{\mu - (\gamma - r)\, w^c}{r}$$

$\Updownarrow$

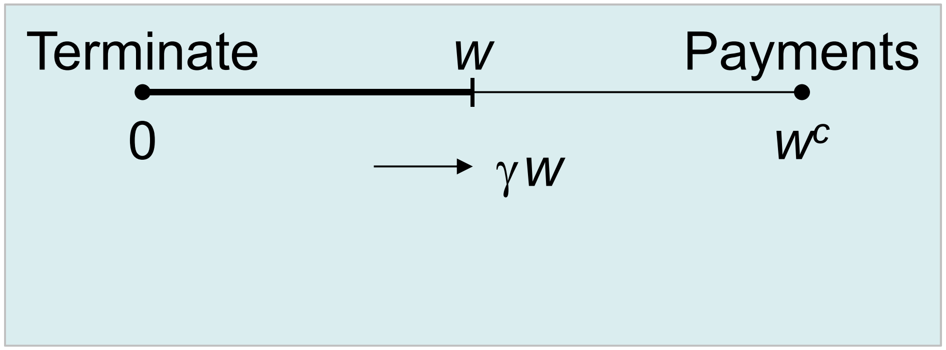
$$r\,p(w^c) + \gamma w^c = \mu$$

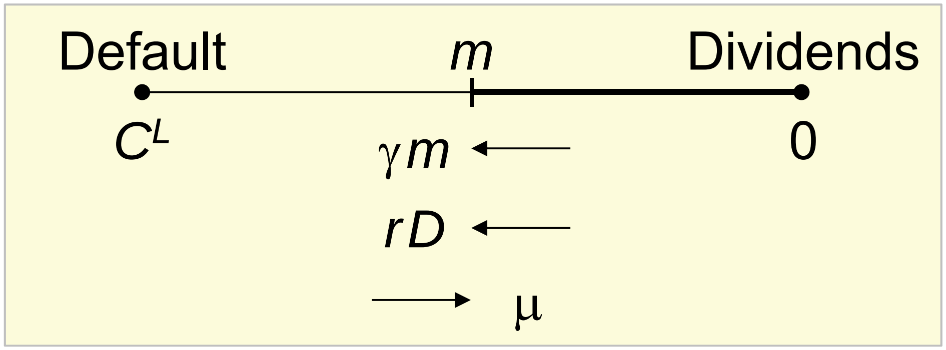


Agent's Payoff w

Implementation

- Dynamics of an Optimal Contract ($\lambda = 1$, $R = 0$)





  - Credit Line: $C^L = w^c$, int. rate = γ
  - LT Debt: $rD = \mu - \gamma\,(m + w) = \mu - \gamma C^L$

- Optimal capital structure
  - $rD + \gamma C^L = \mu$ $\Rightarrow$ payout policy is IC
  - $D = p(w^c)$ $\Rightarrow$ contract is optimal

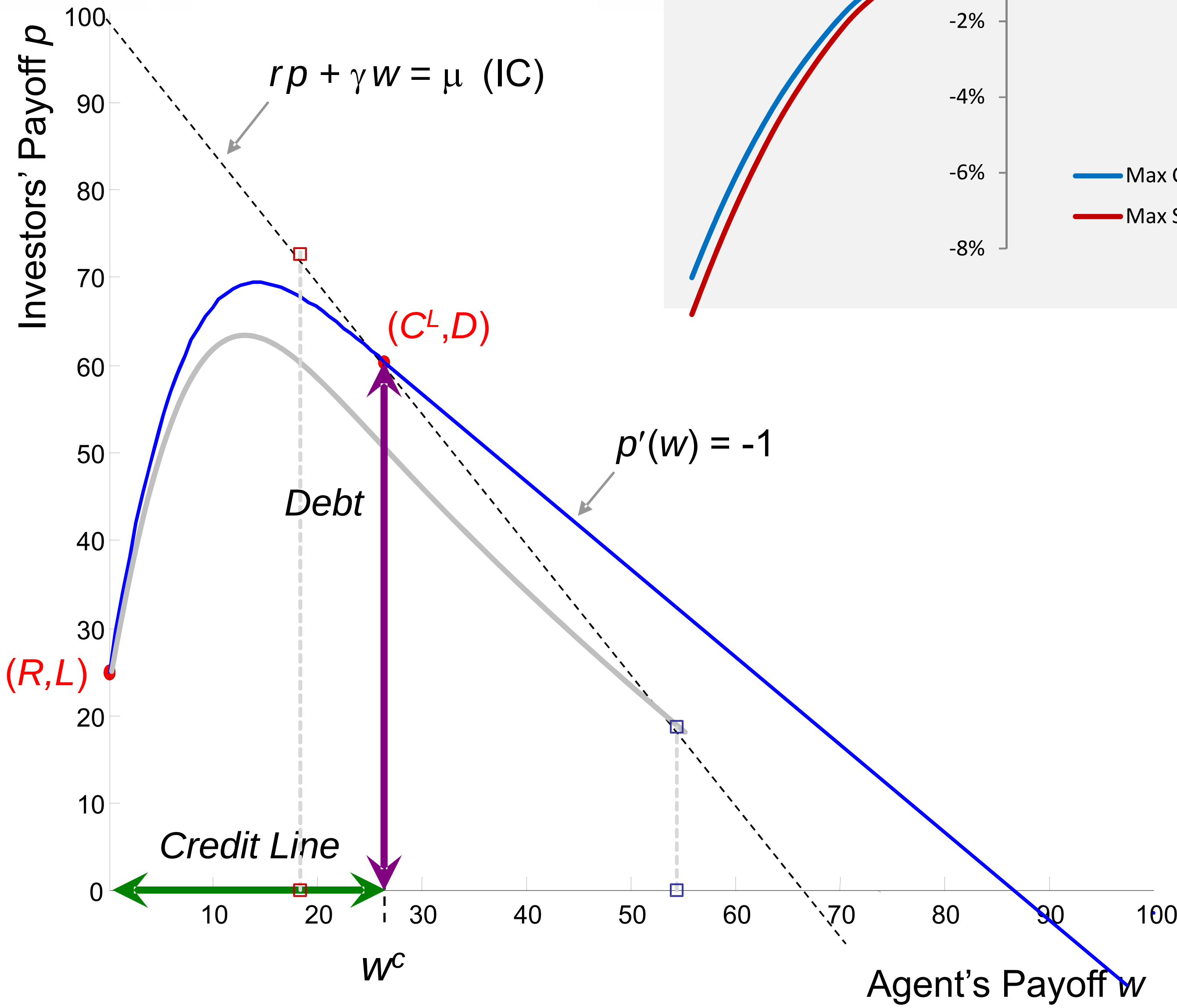
Recall the WACC condition:

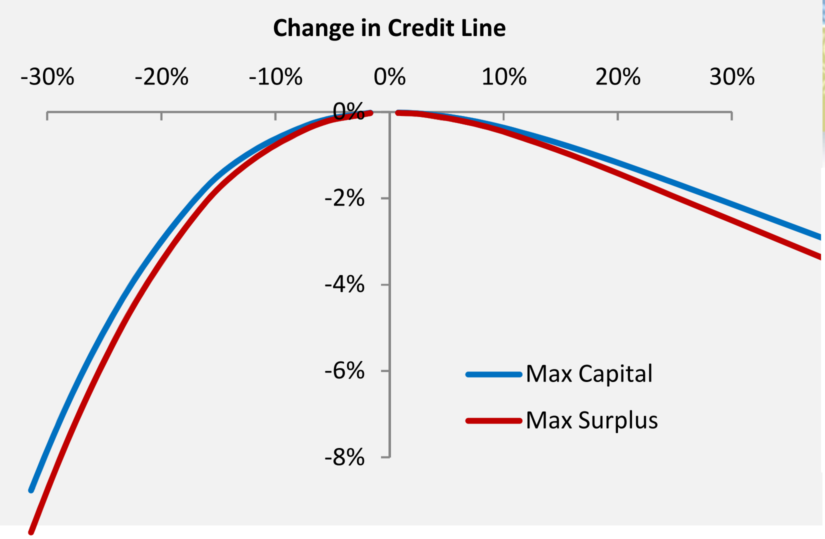
$$r\,p(w^c) + \gamma\,w^c = \mu$$

- Agent's equity share = λ

Example I

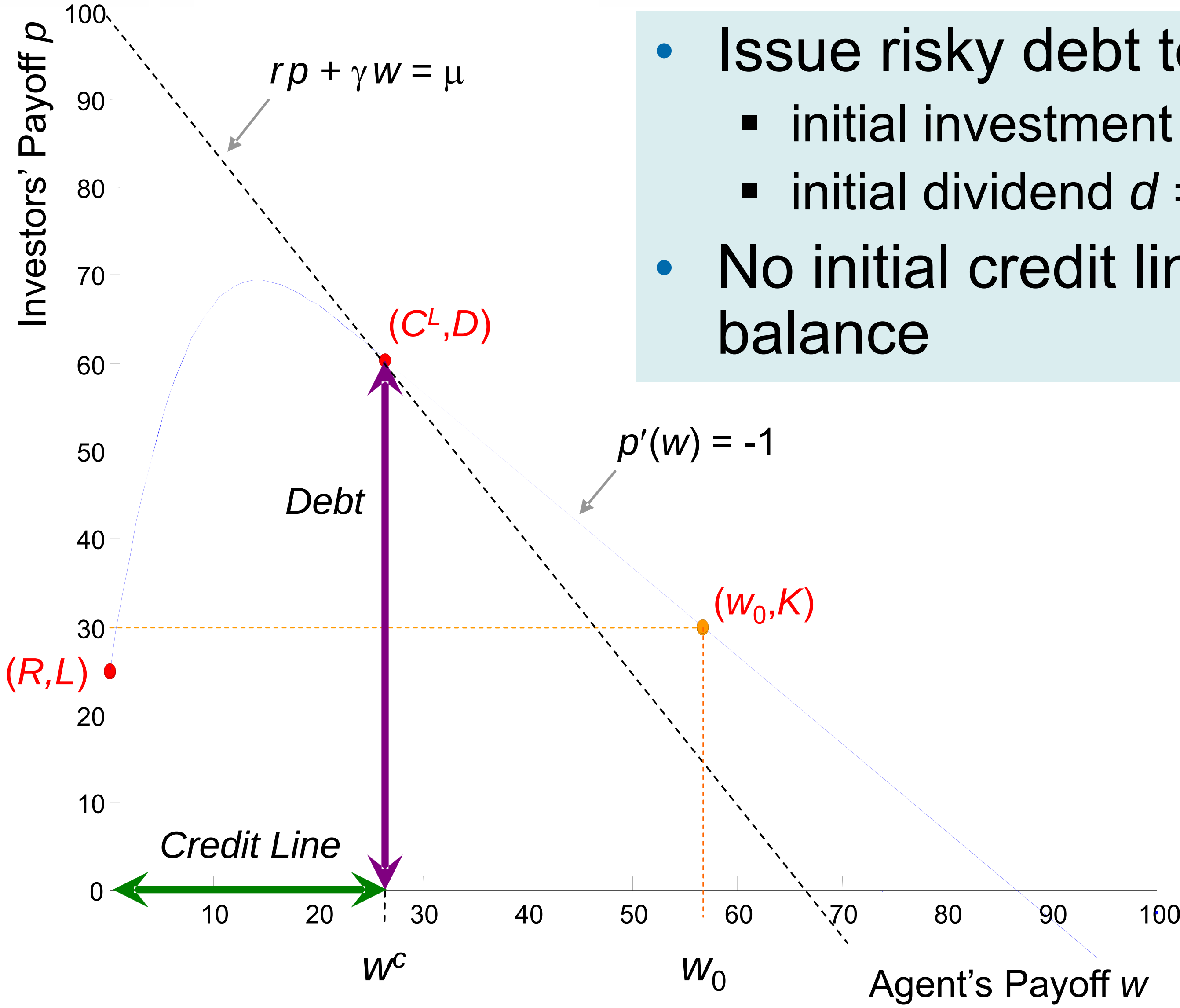
$L = 25$
$R = 0$
$\mu = 10$
$\sigma = 5$
$r = 10\%$
$\gamma = 15\%$
$\lambda = 1$
$K = 30$





Example I

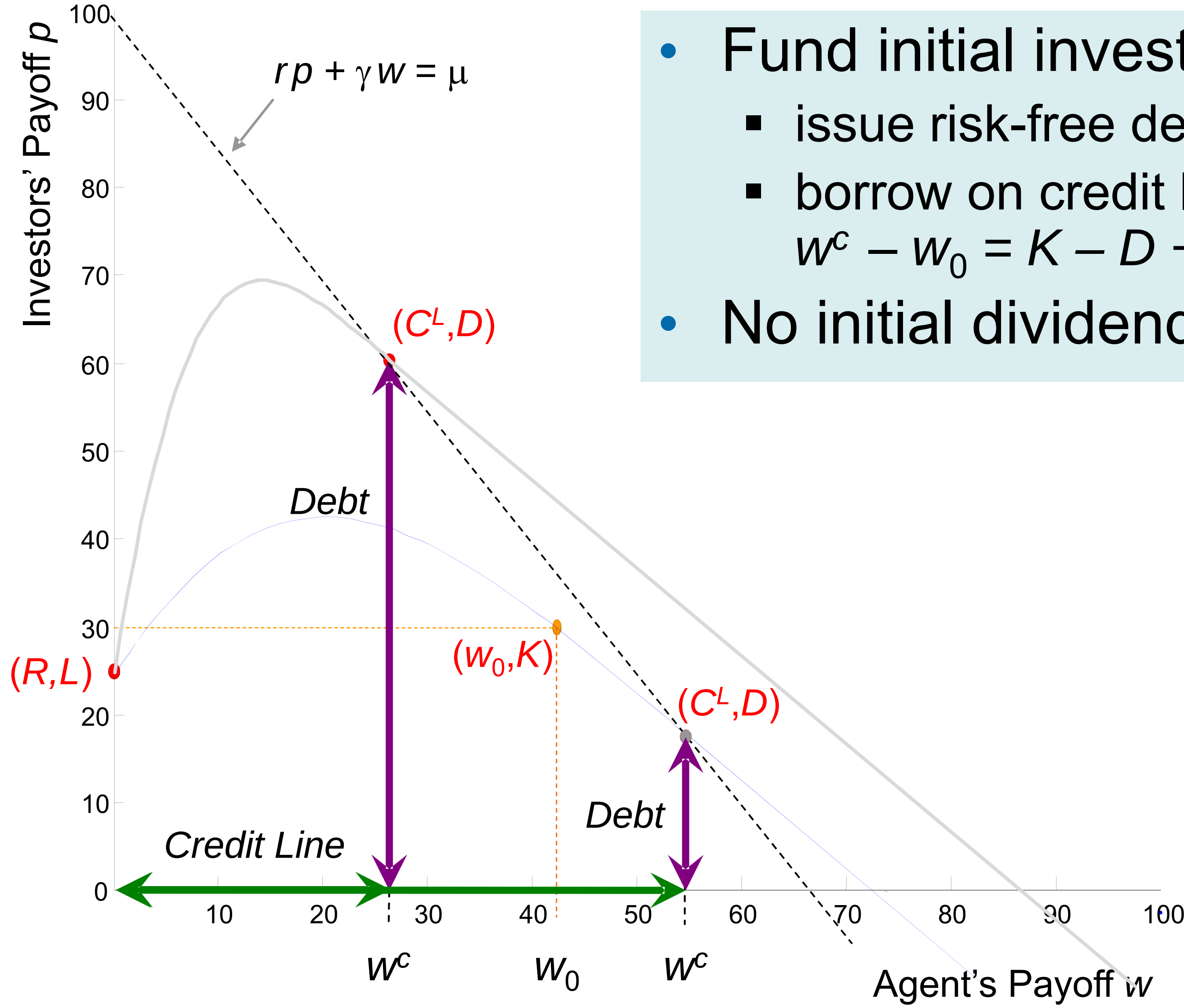
$L = 25$
$R = 0$
$\mu = 10$
$\sigma = 5$
$r = 10\%$
$\gamma = 15\%$
$\lambda = 1$
$K = 30$



- Issue risky debt to fund
  - initial investment K
  - initial dividend $d = w_0 - w^c$
- No initial credit line balance

Example II: Increase Risk ($\sigma = 5 \rightarrow 12.5$)

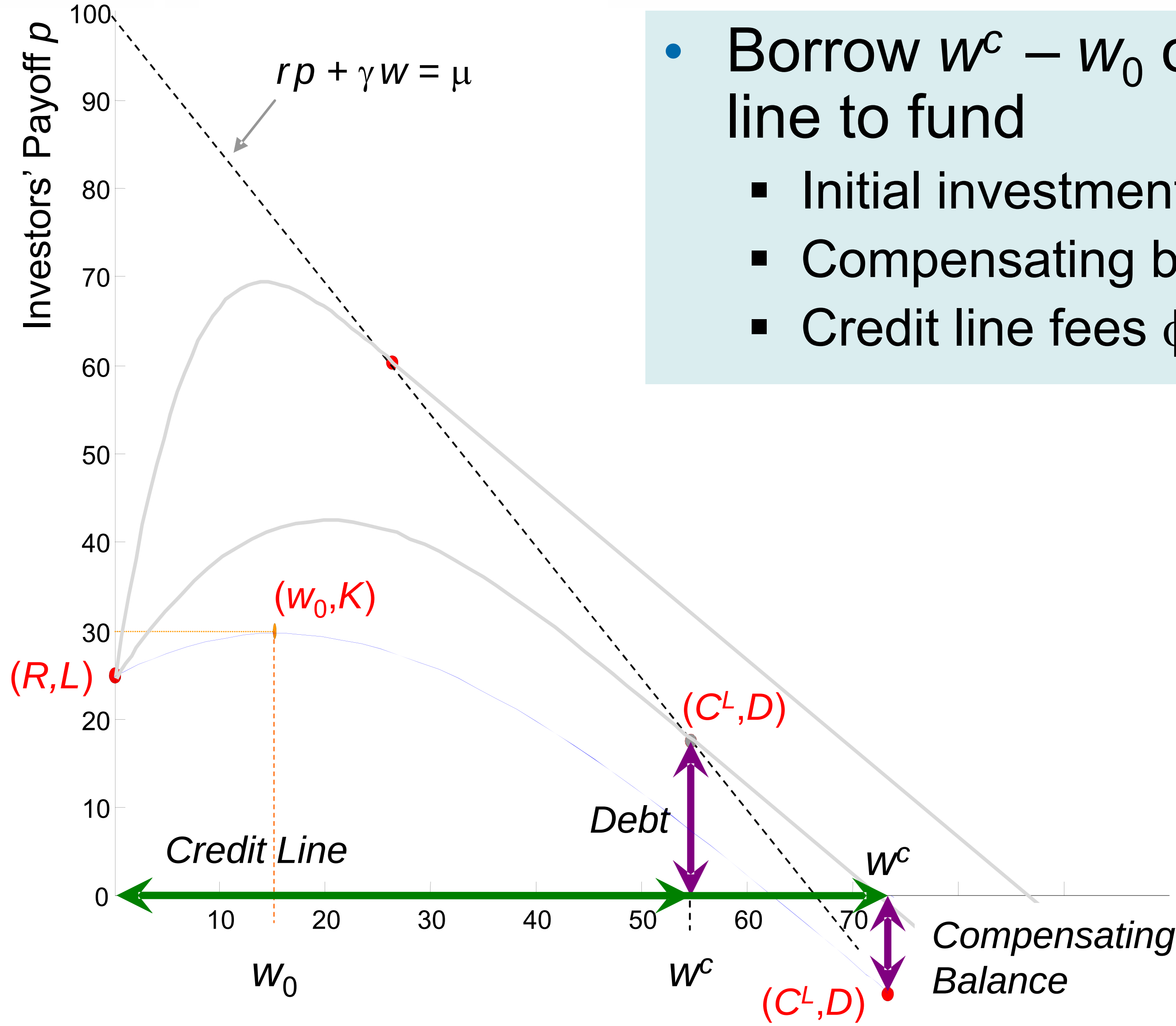
$L = 25$
$R = 0$
$\mu = 10$
$\sigma = 12.5$
$r = 10\%$
$\gamma = 15\%$
$\lambda = 1$
$K = 30$



- Fund initial investment K
  - issue risk-free debt D
  - borrow on credit line $w^c - w_0 = K - D + \phi$
- No initial dividend

Example III: Max Risk (σ = 12.5 $\rightarrow$ 19.1)

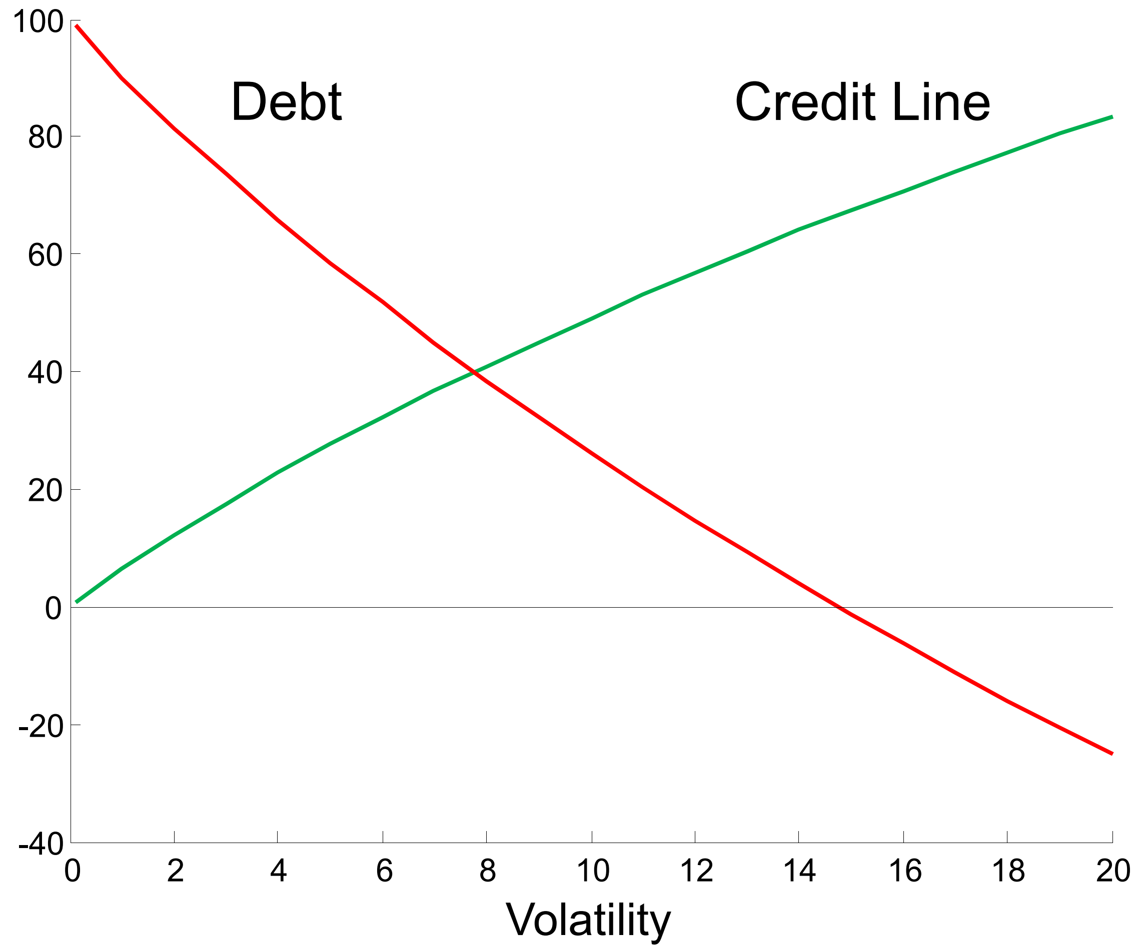
L = 25
R = 0
μ = 10
σ = 19.1
r = 10%
γ = 15%
λ = 1
K = 30



- Borrow $w^c - w_0$ on credit line to fund
  - Initial investment K
  - Compensating balance D
  - Credit line fees ϕ

Comparative Statics

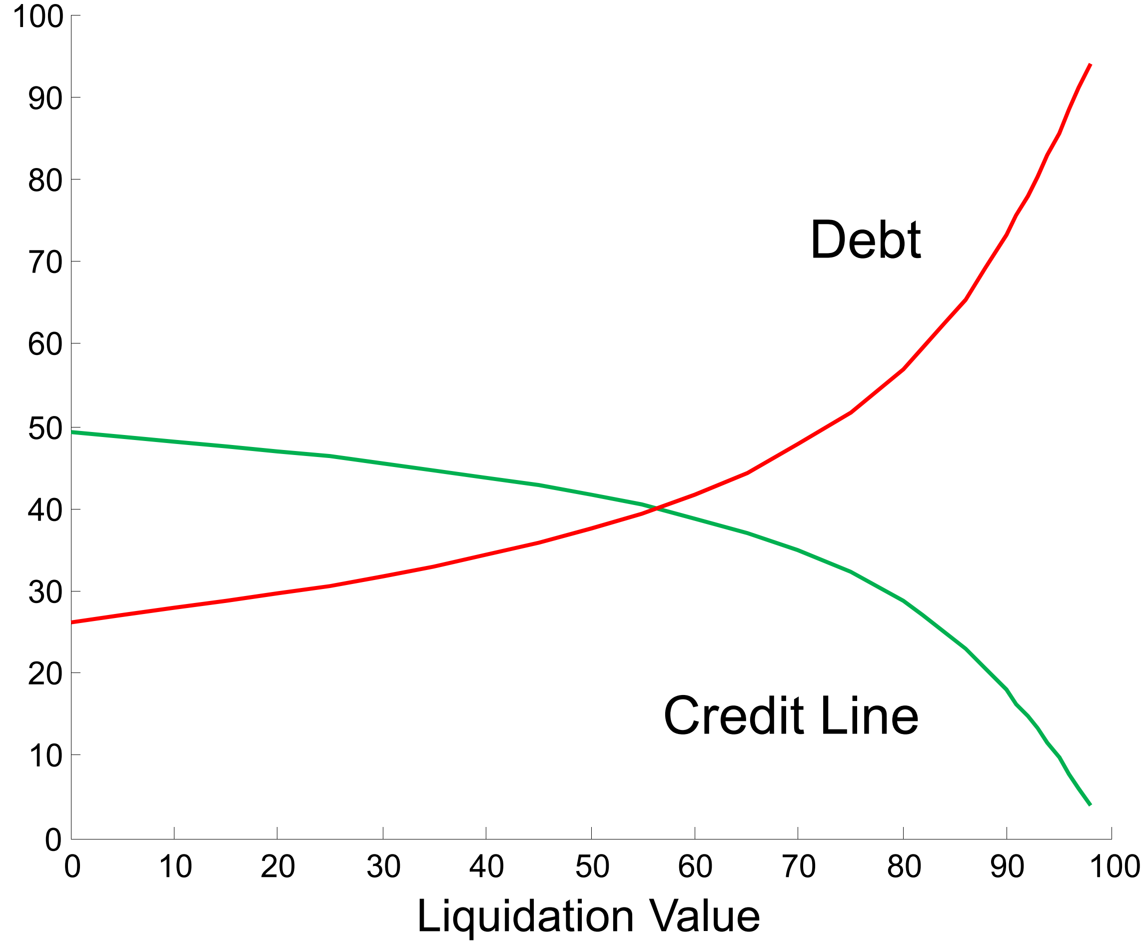
- Volatility
  - With $\sigma \approx 0$, debt approaches FB
  - As volatility increases, debt declines, and financial slack increases
  - To achieve maximal slack, credit line supported with compensating balance



Base: $L = 0$, $R = 0$, $\mu = 10$, $\sigma = 10$, $r = 10\%$, $\gamma = 15\%$, $\lambda = 1$

Comparative Statics

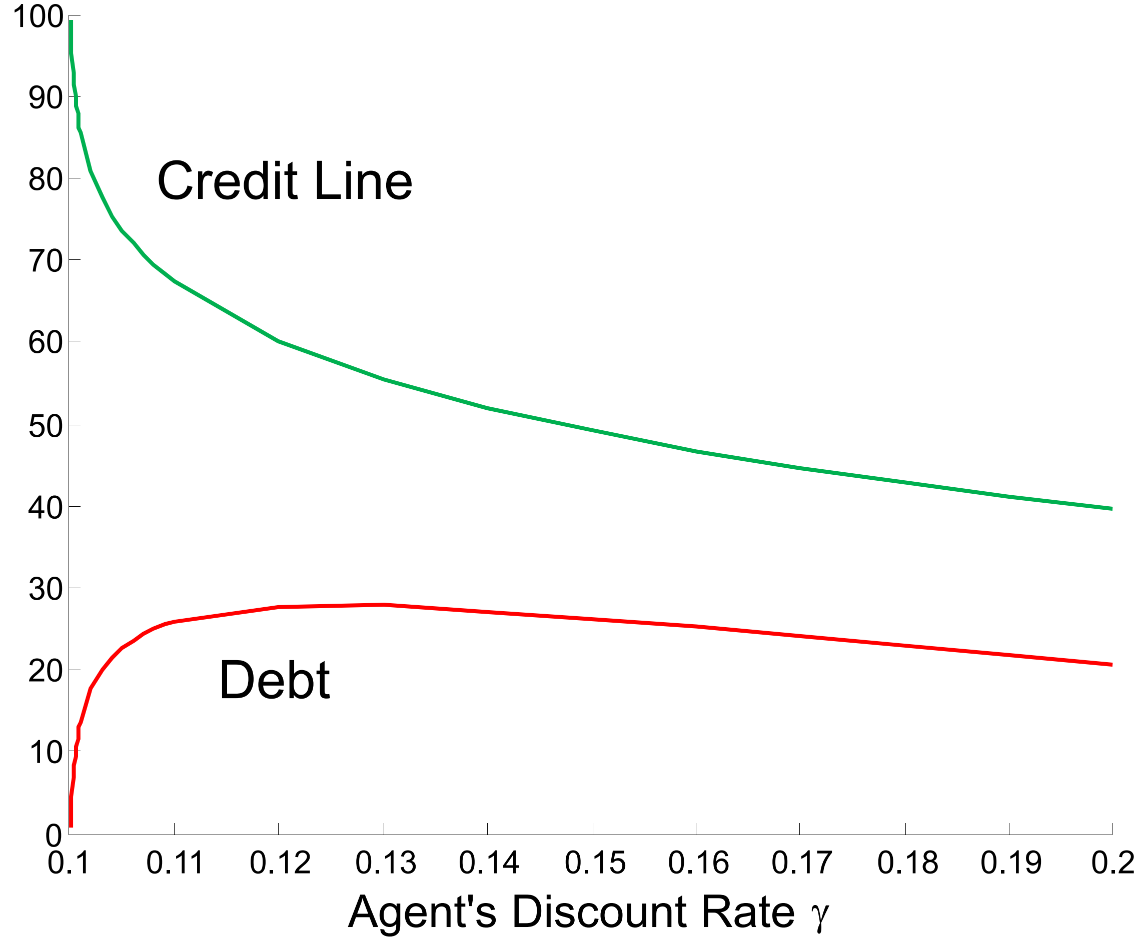
- Liquidation Value
  - With higher liquidation value, the manager is given less financial slack
  - With $L \approx$ FB, debt approaches FB
  - Note that total leverage is relatively insensitive to volatility or liquidation value



Base: $L = 0$, $R = 0$, $\mu = 10$, $\sigma = 10$, $r = 10\%$, $\gamma = 15\%$, $\lambda = 1$

Comparative Statics

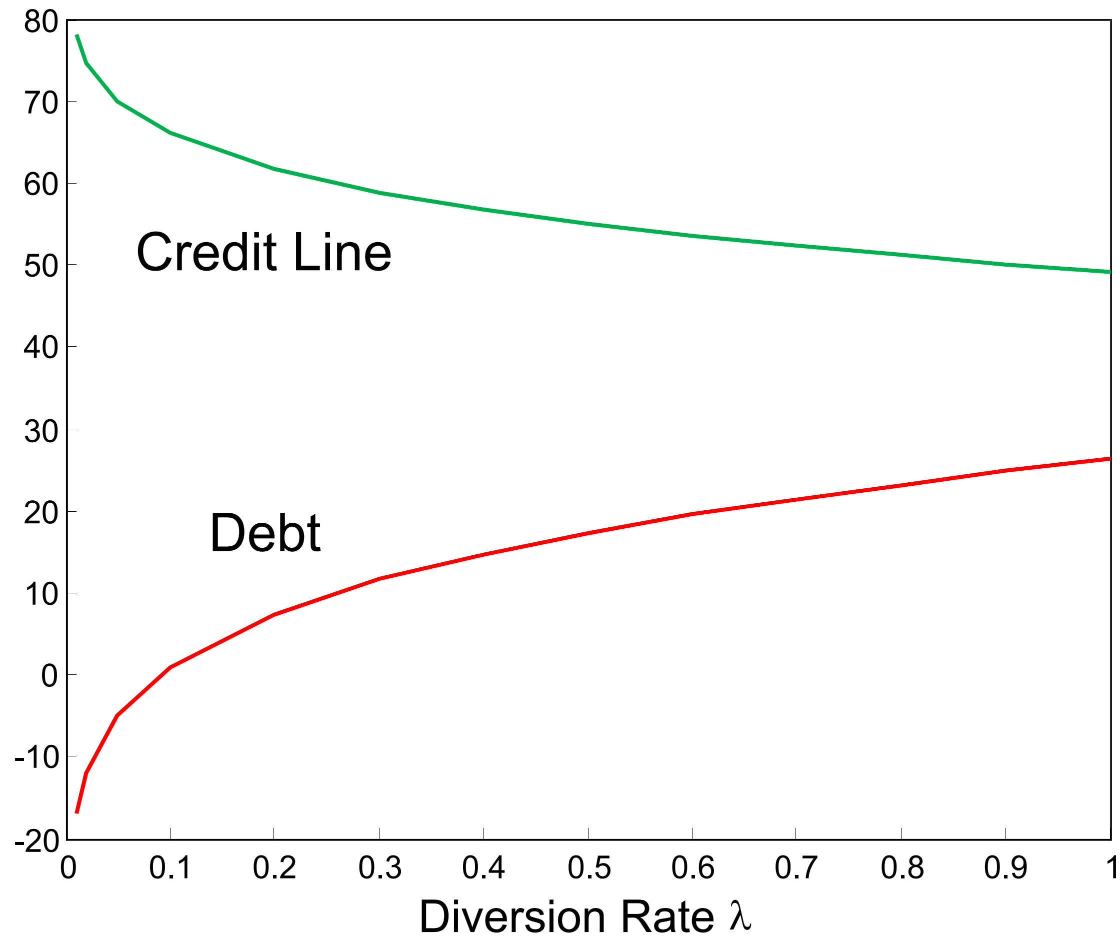
- Impatience
  - As agent's impatience increases, financial slack and total leverage decline
  - For $\gamma \approx r$, total reliance on credit line – no mandatory payouts – but this is not robust



Base: $L = 0$, $R = 0$, $\mu = 10$, $\sigma = 10$, $r = 10\%$, $\gamma = 15\%$, $\lambda = 1$

Comparative Statics

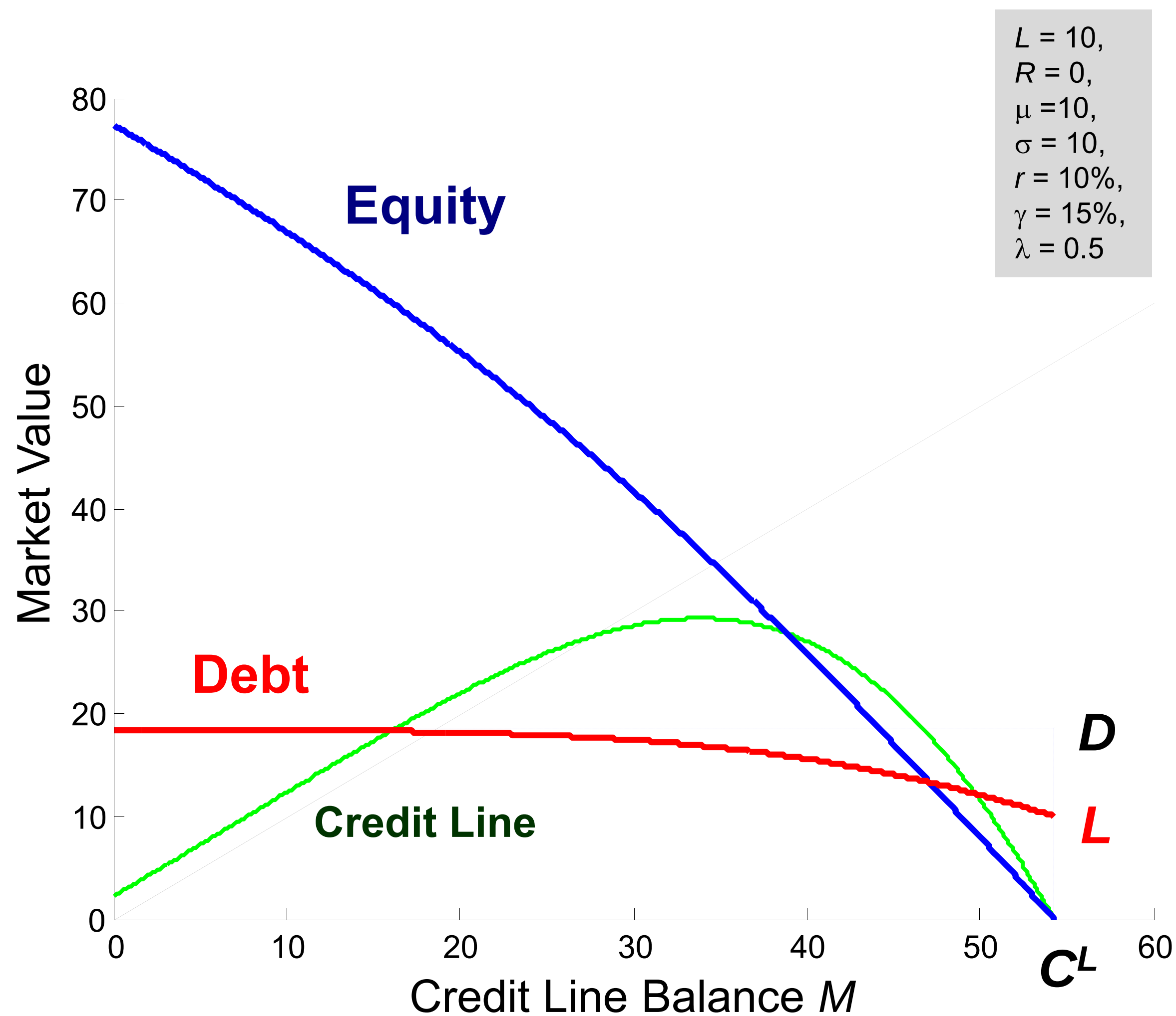
- Private benefits
  - Higher private benefits decrease financial slack
  - With $\lambda \approx 0$, we can provide maximal financial slack
  - Better to overestimate than to underestimate (as with impatience)



Base: $L = 0$, $R = 0$, $\mu = 10$, $\sigma = 10$, $r = 10\%$, $\gamma = 15\%$, $\lambda = 1$

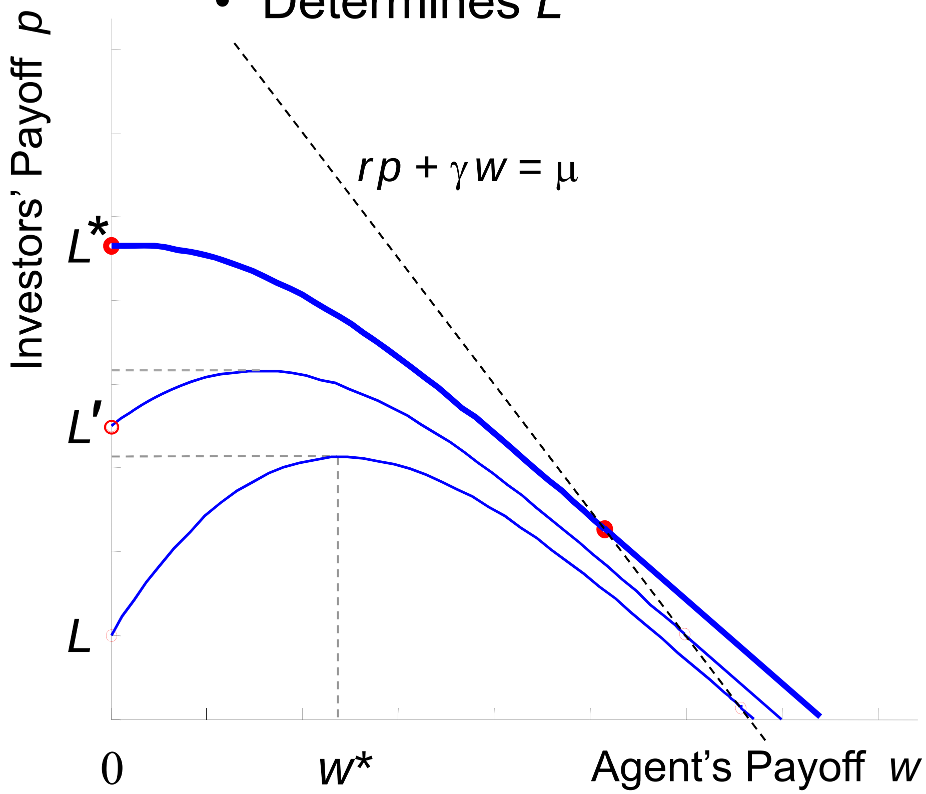
Security Pricing

- Remaining credit measures "distance to default"
  - Assume absolute priority: Debt > Credit Line > Equity
- Note:
  - Values decline as credit line nears limit
  - Credit line is initially profitable, but requires commitment
  - Debt + equity overhang prevents raising equity to avoid default
  - Concavity ⇒
    leverage effect,
    risk management

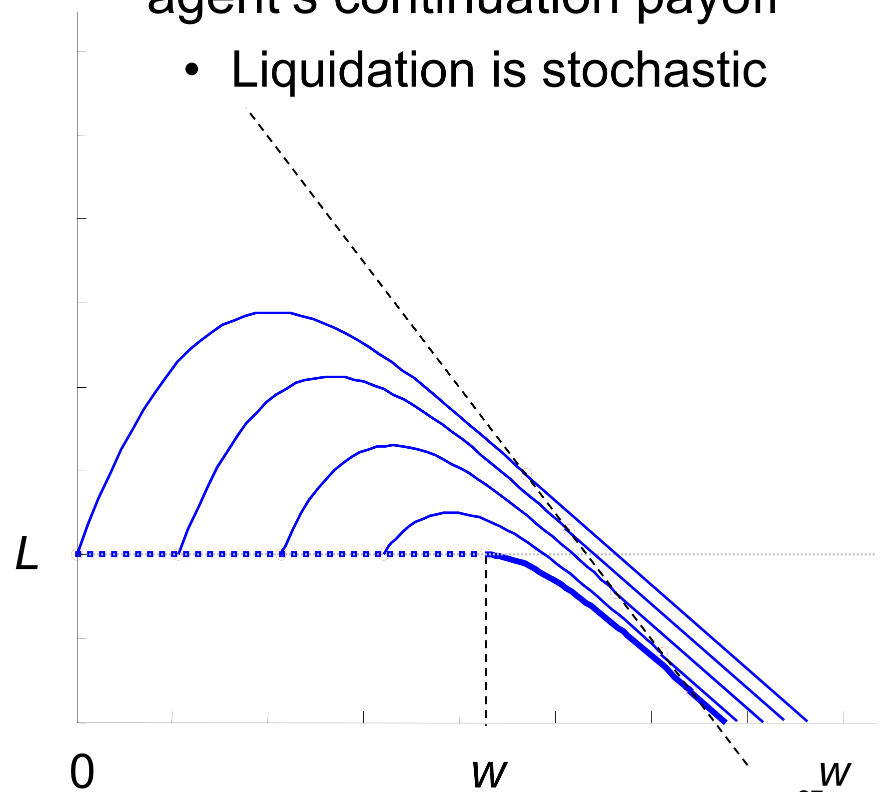


Endogenous Termination / Renegotiation

- Agent is replaceable
  - Termination = Firing & Replacing the Agent
    - Determines L

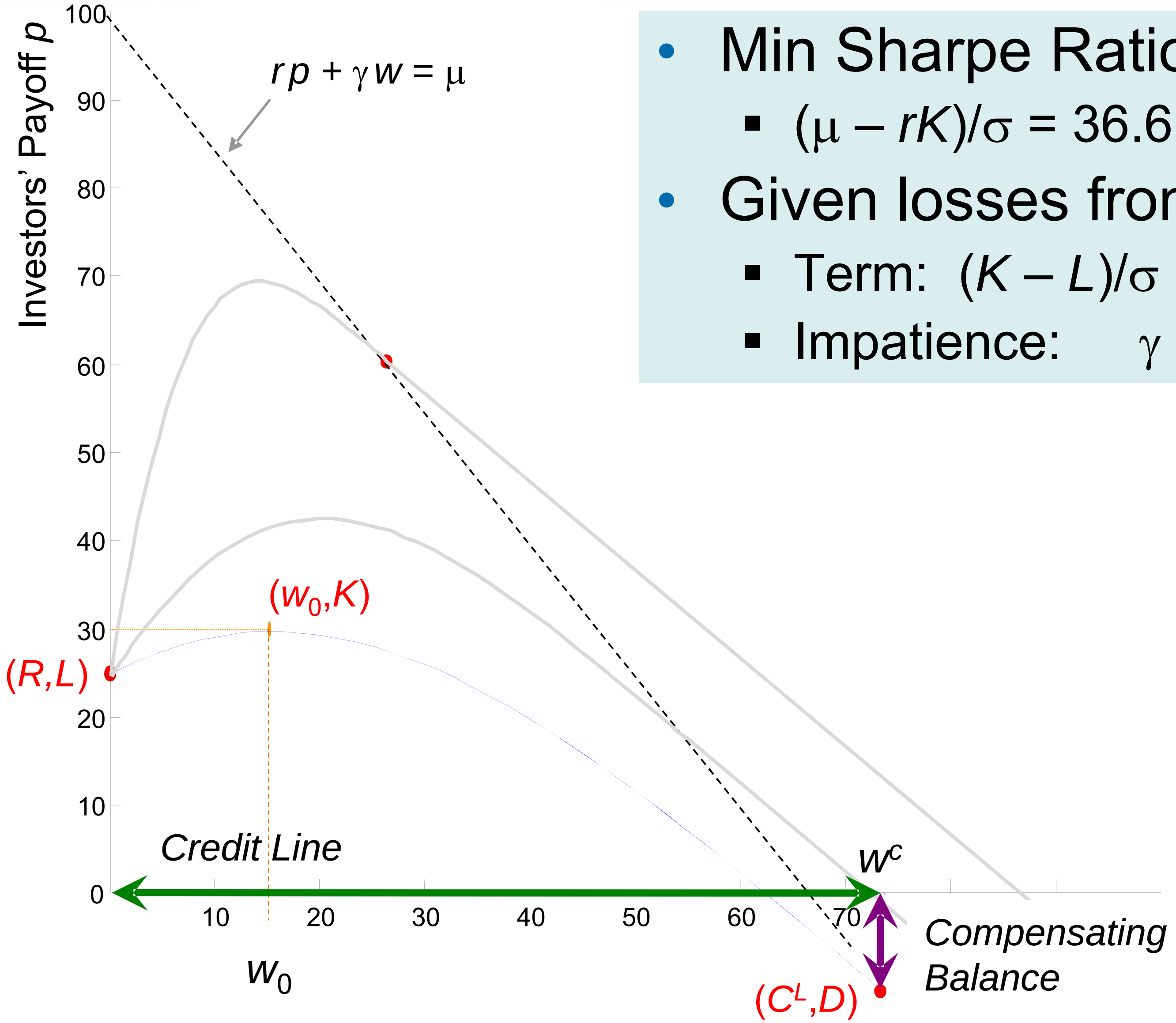


- Agent is vital
  - Agent's ability to renegotiate puts a lower bound on the agent's continuation payoff
    - Liquidation is stochastic



Example III: Minimal Project Profitability

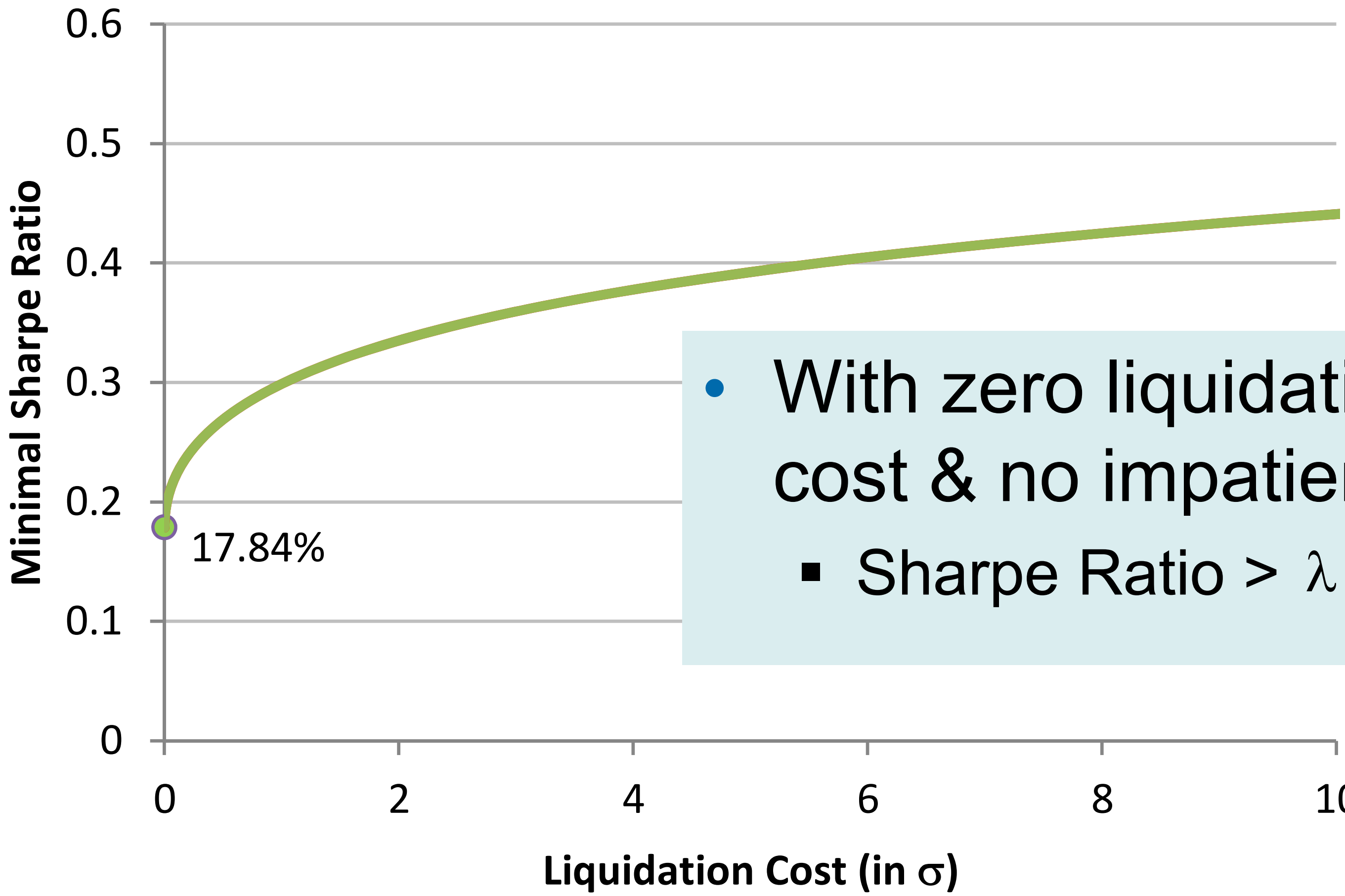
L = 25
R = 0
μ = 10
σ = 19.1
r = 10%
γ = 15%
λ = 1
K = 30



- Min Sharpe Ratio
  - $(\mu - rK)/\sigma$ = 36.6%
- Given losses from
  - Term: $(K - L)/\sigma$ = 26.2%
  - Impatience: $\gamma > r$

Minimum Profitability Threshold

- Suppose $\gamma \approx r$ = 10%, λ = 1



- With zero liquidation cost & no impatience
  - Sharpe Ratio > $\lambda \sqrt{r/\pi}$

Comments and Conclusions

- Discrete time
  - Arbitrary cash flow distributions (bounded below)
  - Non-stationary, finite horizon, $\gamma \geq r$
  - Implementation as equity, debt & credit line
  - Stochastic termination
  - Numerical comp stats
- Continuous time
  - Normal distribution (Brownian motion)
  - Stationary, infinite horizon, $\gamma > r$
  - Same implementation + intuitive conditions for IC of payout policy, and optimality
  - Stopping time (when credit line is exhausted)
  - Analytic comp stats
- Convergence: Binomial $\rightarrow$ Continuous time
  - DS (working paper), Biais et al. (*RES*, 2007)

What's Next?

- Continuous-time model
  - Ideal balance of tractability and richness
  - A workhorse model that provides a useful building block and allows for clear intuition

- New features to add …
  - Monitoring
  - Non-linear effort cost
  - Investment / growth
  - Gambling & tail risk
  - Learning