Finance Theory Group Summer School 2015

Dynamic Financial Contracting

Part III:
Using the Model

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A Quick Review

- Cash flows
  - Agent "diverts or shirks" at rate $a_t \geq 0$, and reports

$$dY_t = (\mu - a_t)\, dt + \sigma\, dZ_t$$

- Contract $\Pi = (U, \tau)$
  - Principal pays compensation $dU_t \geq 0$
  - Chooses termination (stopping) time τ
- Payoffs
  - Agent: $w(\Pi) = \max_a \quad E^a\left[\int_0^\tau e^{-\gamma t}\left(dU_t + \lambda a_t dt\right) + e^{-\gamma\tau} R\right]$
  - Principal: $p(w) = \max_{\Pi,a} \quad E^a\left[\int_0^\tau e^{-rt}\left(dY_t - dU_t\right) + e^{-r\tau} L\right]$

s.t. (Π, a) is IC, $w(\Pi) = w_0$

Solving the Model

- Agent’s Future Payoff w
  - Promise-keeping
    - $E[dw] = \gamma\, w\, dt$
  - Incentive Compatibility
    - $\partial w / \partial y \geq \lambda$

$$\Rightarrow dw = \gamma w\, dt + \lambda(dy - E[dy])$$

$$= \gamma w\, dt + \lambda \sigma\, dZ$$

- Investor’s Payoff: HJB Equation

$$\underbrace{rp}_{\text{Req. Return}} = \underbrace{\mu}_{E[FCF]} + \underbrace{\gamma\, wp' + \tfrac{1}{2}\lambda^2\sigma^2 p''}_{E[dp]}$$

Boundary Conditions:

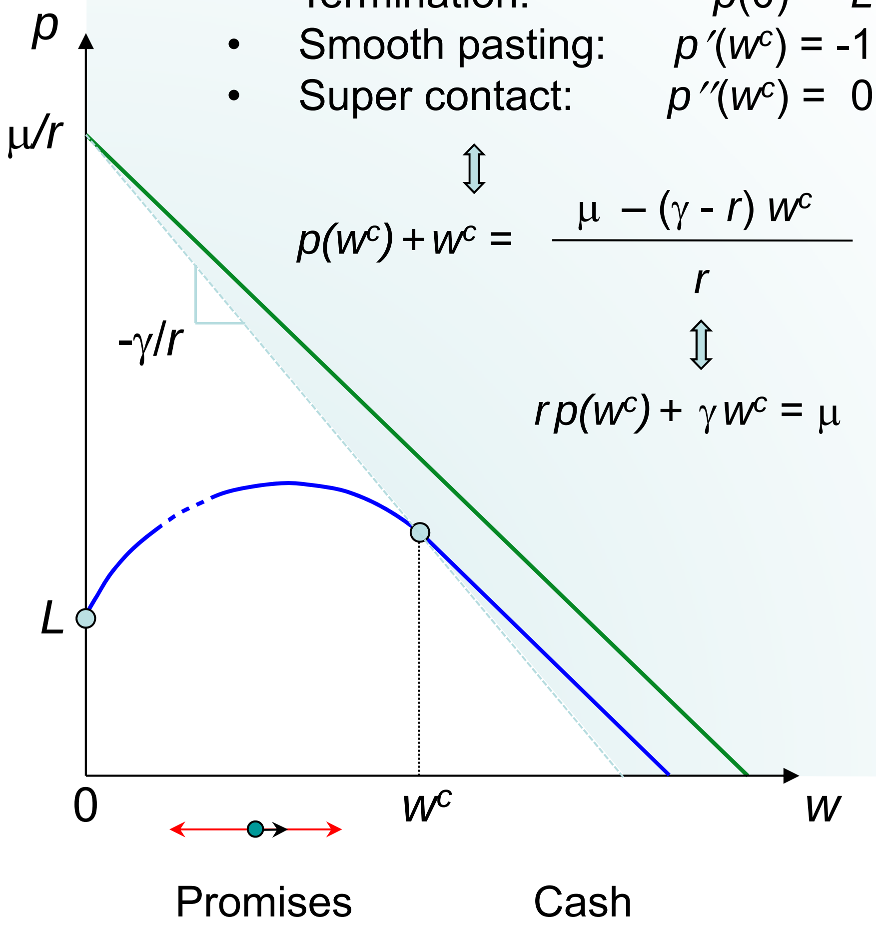
- Termination: $p(0) = L$
- Smooth pasting: $p'(w^c) = -1$
- Super contact: $p''(w^c) = 0$

$$\Updownarrow$$

$$p(w^c) + w^c = \frac{\mu - (\gamma - r)\, w^c}{r}$$

$$\Updownarrow$$

$$r\, p(w^c) + \gamma w^c = \mu$$



Agent’s Payoff w

Part III.A:

COSTLY MONITORING, RISK TAKING & EFFORT DYNAMICS

Costly Monitoring

- Suppose the principal can engage in costly monitoring to
    - Make it more difficult for the agent to divert cash flows
    - Reduce the private benefits from “shirking”

- Specifically, suppose
    - Monitoring reduces profitability to $\mu_m < \mu$
    - Monitoring lowers private benefit to $\lambda_m < \lambda$

- When should the principal monitor?

Costly Monitoring

- Recall: Investor's Payoff determined by HJB equation

$$rp = \underbrace{\mu}_{\text{Profitability}} + \underbrace{\gamma\, wp'}_{\text{Compensation / promise-keeping}} + \underbrace{\tfrac{1}{2}\lambda^2\sigma^2 p''}_{\text{Incentives / risk-bearing}}$$

- When the principal can choose "regimes," choose the one that maximizes the value function:

$$\boxed{rp = \max_{\Omega}\ \mu + \gamma\, wp' + \tfrac{1}{2}\lambda^2\sigma^2 p''}$$

Costly Monitoring

$$rp = \max_{\Omega} \; \mu + \gamma\, wp' + \tfrac{1}{2}\lambda^2\sigma^2 p''$$

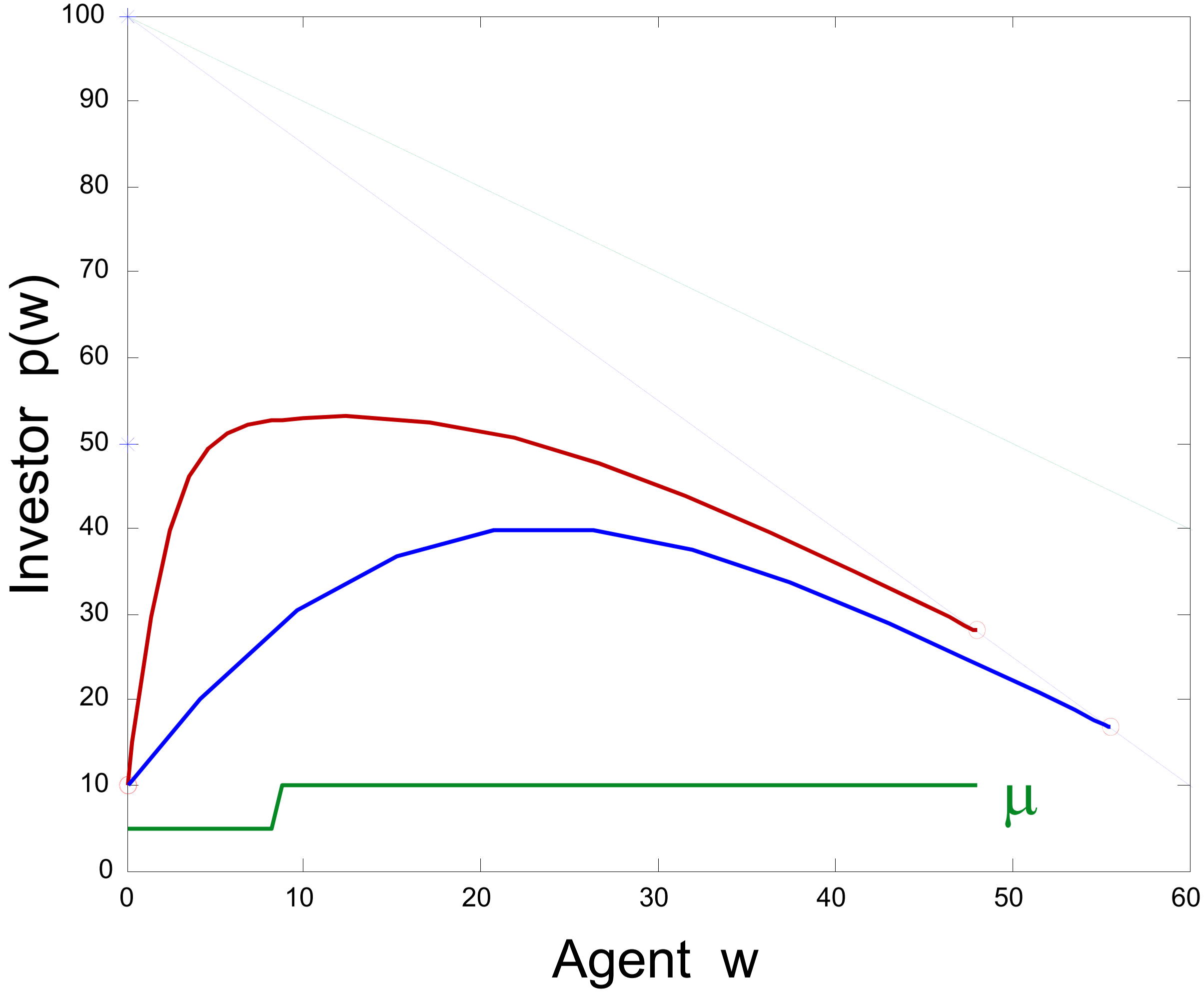
- In this case:
  - Cost = reduced profitability, $\Delta\mu$
  - Benefit = reduced incentive cost, $\tfrac{1}{2}\Delta(\lambda^2)\,\sigma^2 p''$
  - Tradeoff depends on p''
  - $\Rightarrow$ Monitor if p'' sufficiently negative

- Optimal monitoring
  - As w approaches the payout boundary, $p'' \to 0$
  - Typically, monitor only for w sufficiently low

Costly Monitoring

- Example: μ: 10 → 5, λ: 0.8 → 0.1

L = 10
R = 0
μ = 5,10
σ = 15
r = 10%
γ = 15%
λ = .1,.8



Risk Choice

- Note the value function only depends on the product, $\lambda\ \sigma$:

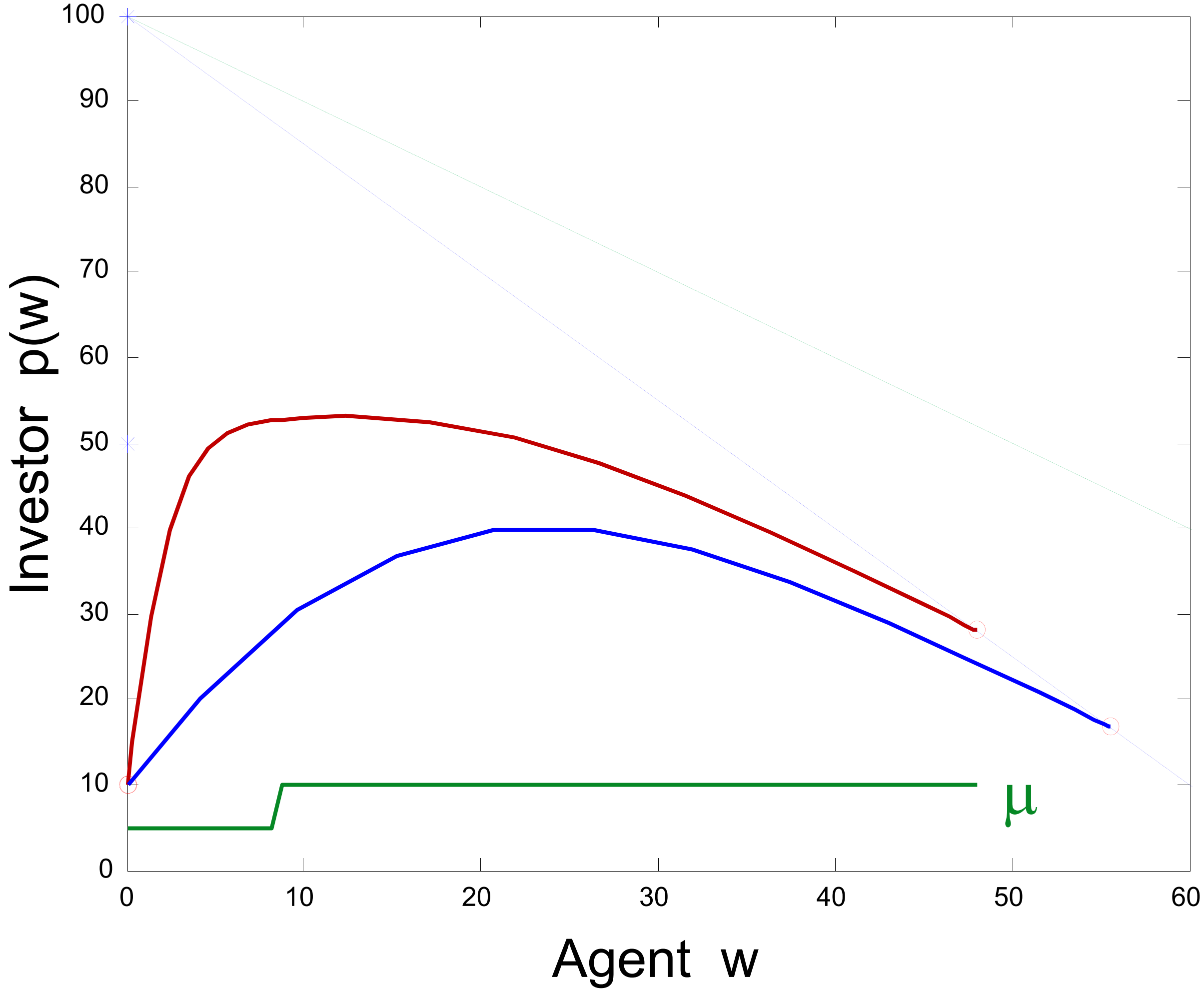
$$rp = \underbrace{\mu}_{\text{Profitability}} + \underbrace{\gamma\, wp'}_{\substack{\text{Compensation /}\\ \text{promise-keeping}}} + \underbrace{\tfrac{1}{2}\lambda^2\sigma^2 p''}_{\substack{\text{Incentives /}\\ \text{risk-bearing}}}$$

- Suppose we can choose the risk of the project from some risk-return frontier

$\Rightarrow$ *Reduce risk as w falls*

Risk Taking

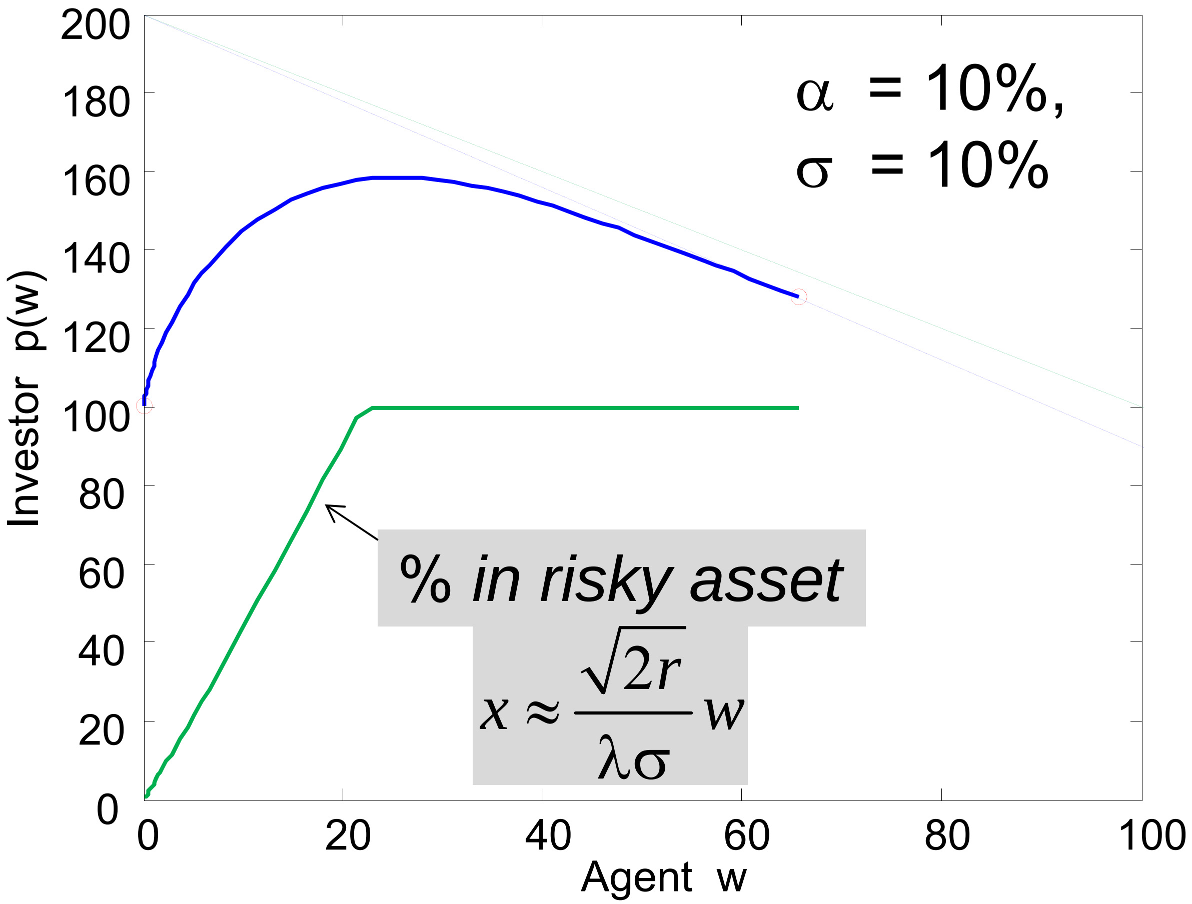
- Example: μ: 10 → 5, σ: 24 → 3



L = 10
R = 0
μ = 5,10
σ = 3,24
r = 10%
γ = 15%
λ = 0.50

Risk Taking

- Investment in risky (vs. risk-free) asset
  - Invest x *(%)*, Return: $\mu = (r_f + x\alpha)L$, vol = $x\sigma$

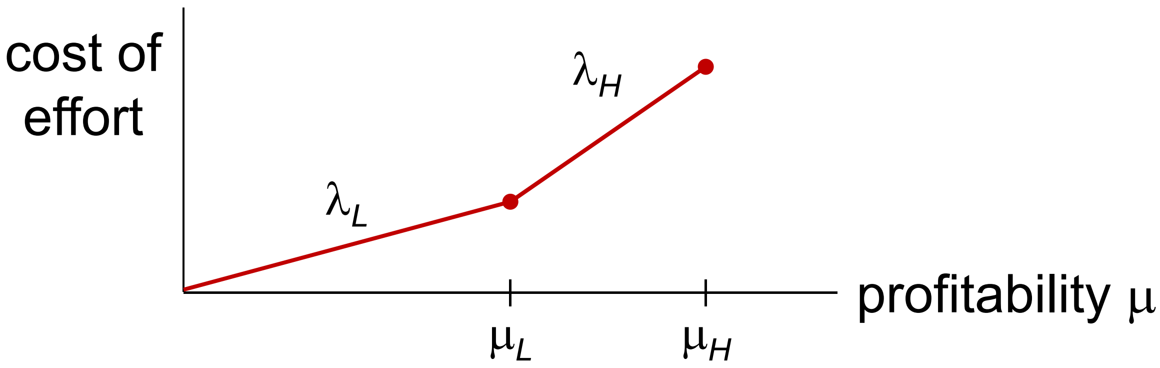


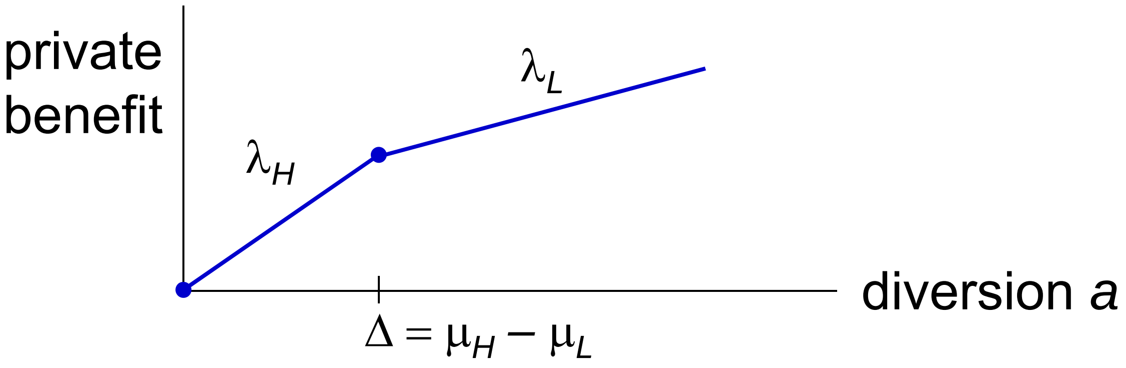
$$x \approx \frac{\sqrt{2r}}{\lambda\sigma} w$$

L = 100
R = 0
μ = 10-20
σ = 0-10
r = 10%
γ = 11%
λ = 1

Easy-Life Contracts

- Suppose the cost of effort is increasing:





When might the optimal contract benefit by offering low power incentives?

Easy-Life Contracts

- Maximizing the value function:

$$rp = \max \begin{cases} \mu + \gamma w p' + \tfrac{1}{2}\lambda_H^2\sigma^2 p'' \\ \mu - \Delta + (\gamma w - \lambda_H \Delta) p' + \tfrac{1}{2}\lambda_L^2\sigma^2 p'' \end{cases}$$

promise-keeping

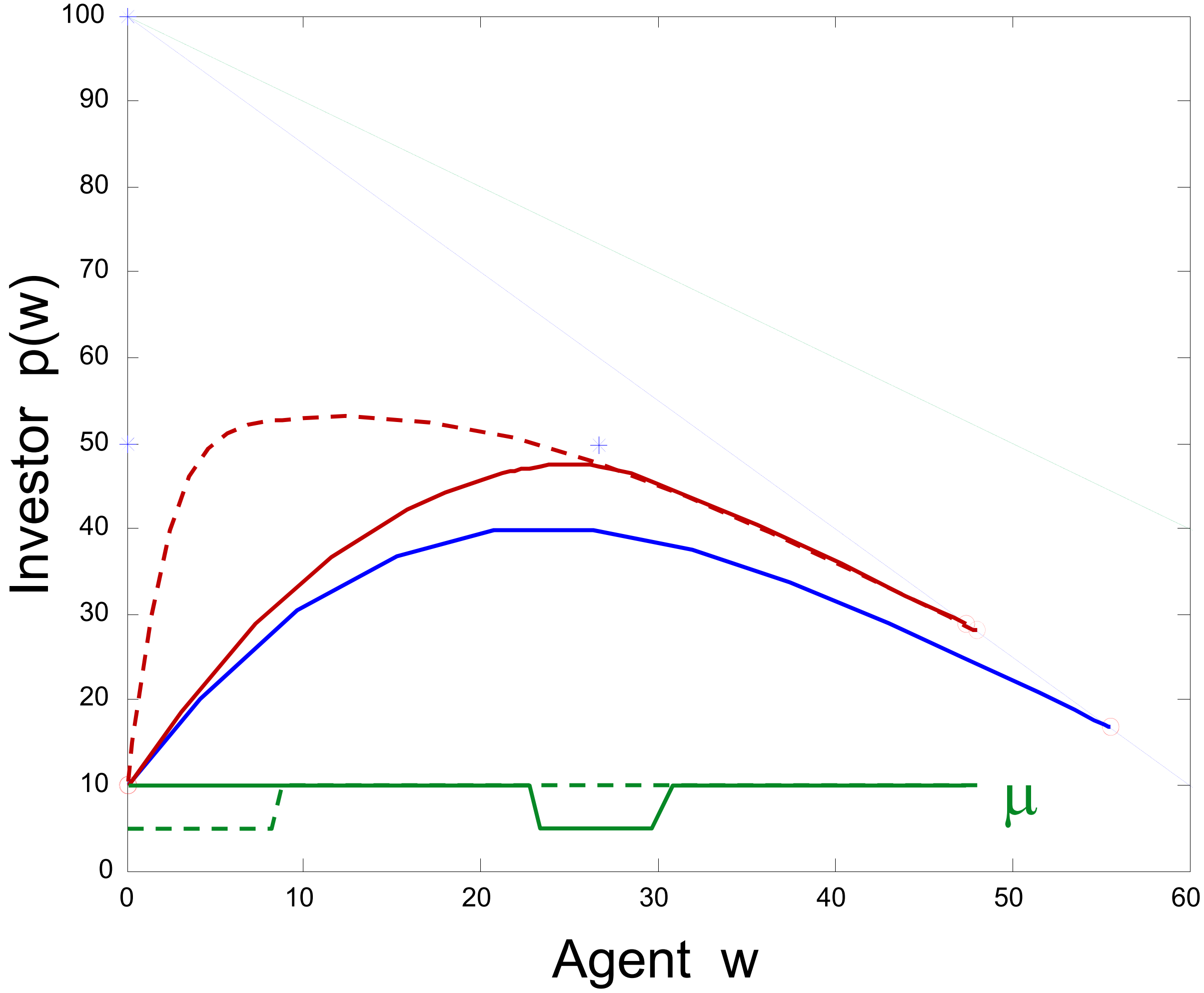
Decrease drift – highest benefit when is p' is most negative (w high)

incentive compatibility

Decrease risk – highest benefit when is p'' is most negative (w low)

Easy-Life Contracts

- Example: μ: 10 → 5, λ: 0.8 → 0.1



L = 10
R = 0
μ = 5,10
σ = 15
r = 10%
γ = 15%
λ = .1,.8

General Convex Effort Cost

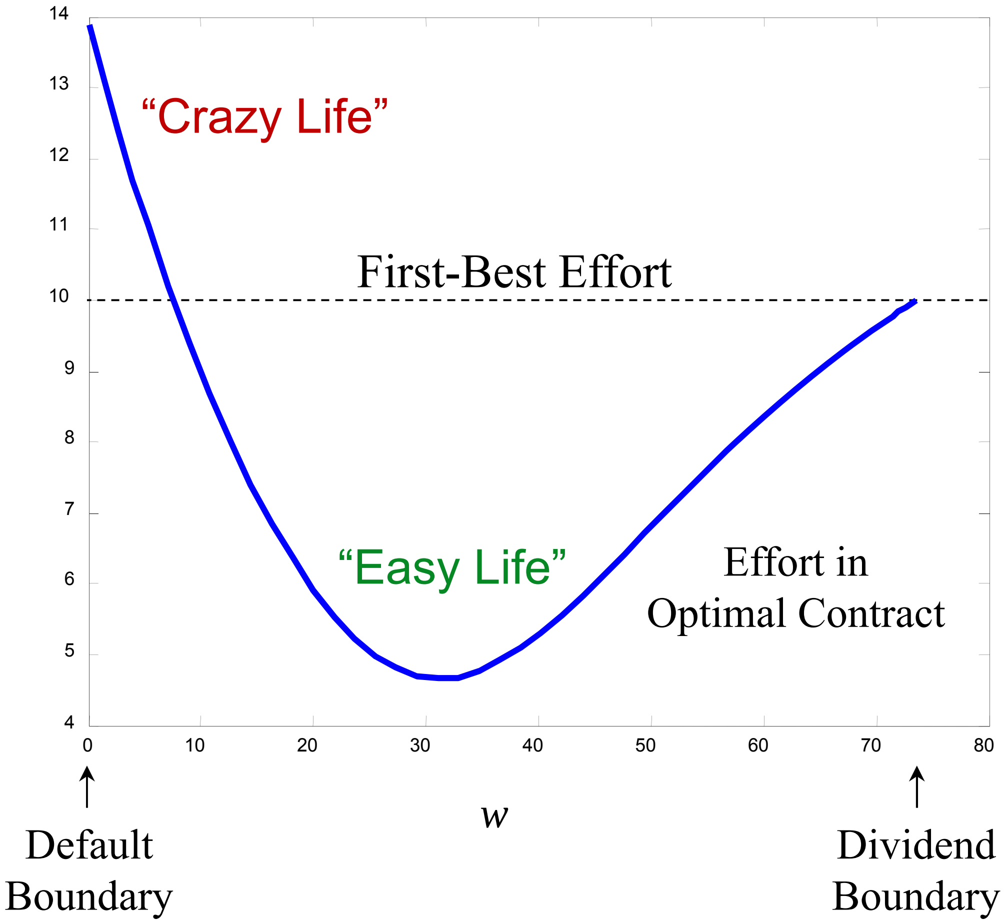
- Agent chooses drift μ
    - Private cost $c(\mu)$
    - Incentive compatibility $\Rightarrow dw = \gamma w dt + c'(\mu)(dy - E[dy])$

- Optimal contract
    - $rp = \max_{\mu} \; \mu + (\gamma w + c(\mu))p' + \frac{1}{2} c'(\mu)^2 \sigma^2 p''$
    - At payout boundary: $p' = -1$, $p'' = 0$

      $\Rightarrow \; \max_{\mu} \; \mu - c(\mu) \; \Rightarrow c'(\mu) = 1$ “First Best”
    - Elsewhere?

General Convex Effort Cost

- Typical outcome

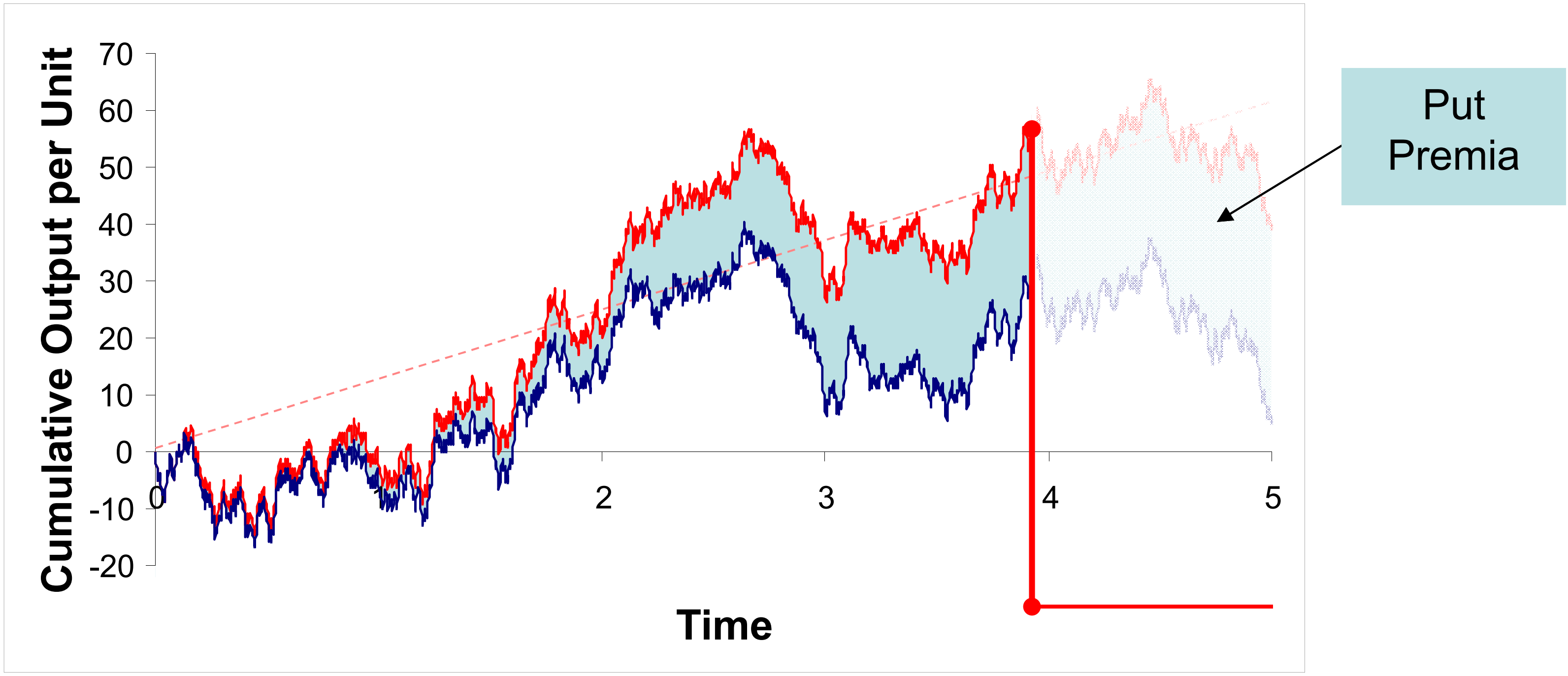


Part III.B:

GAMBLING & TAIL RISK

The Gambling Problem

- Agent may increase profits by taking on tail risk
  - E.g. selling disaster insurance / deep OTM puts – earn $\rho\ dt$
  - Risk of disaster that wipes out franchise – arrival rate $\delta\ dt$, loss D



Put Premia

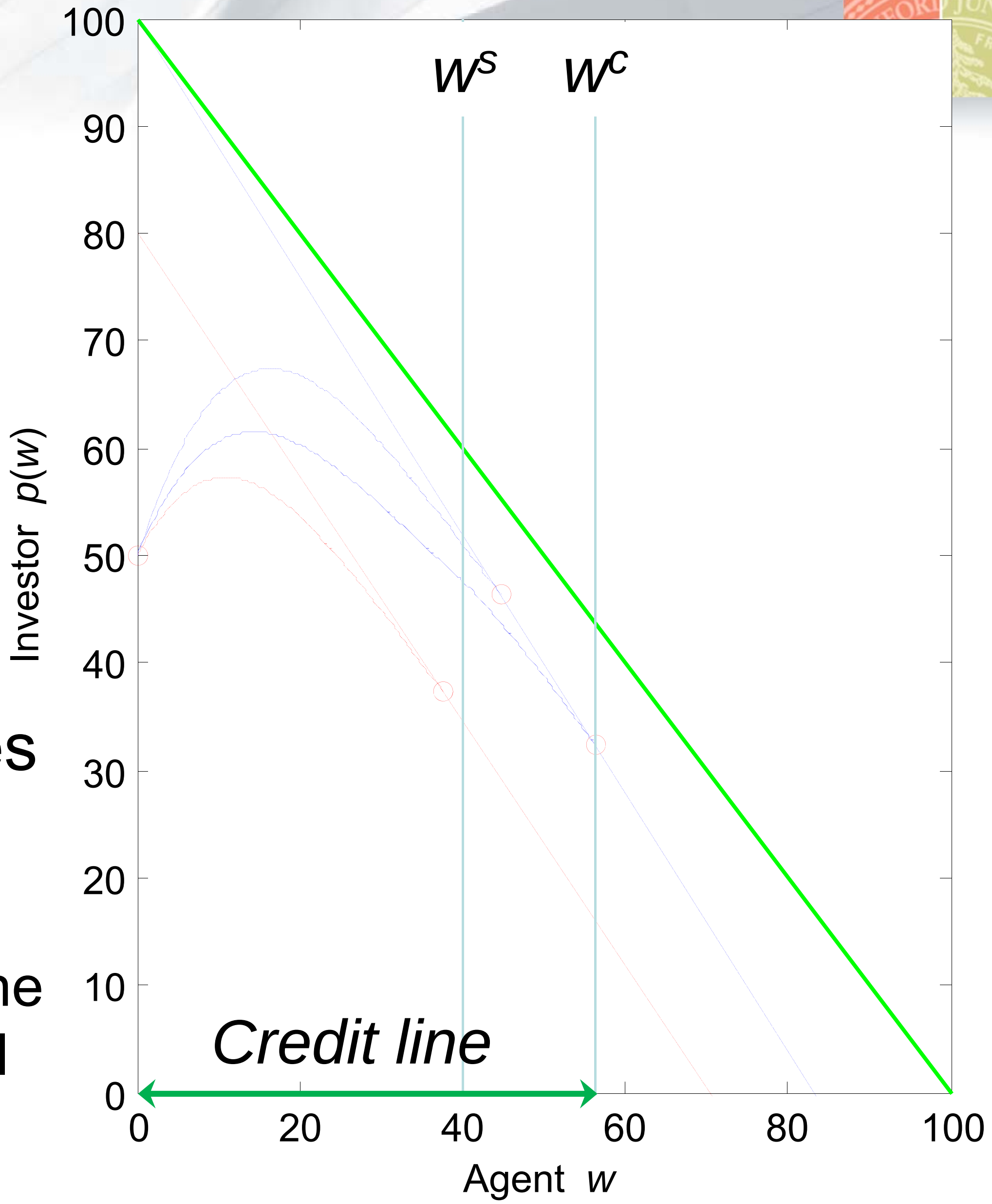
The Gambling Problem

- Agent's incentives
  - Gain from gambling: $\lambda \rho \, dt$
  - Potential loss: w_t, with probability $\delta \, dt$
  - Agent will gamble if $\lambda \rho < \delta w_t$ or $w_t < w^s \equiv \lambda \rho / \delta$
  - Agent will gamble if not enough "skin in the game"

- Gambling region
  - Contract dynamics: $dw = (\gamma + \delta) w \, dt + \lambda (dy - E[dy])$
  - Value function: $(r + \delta) p^g = (\mu + \rho - \delta D) + (\gamma + \delta) w p^{g\prime} + \tfrac{1}{2} \lambda^2 \sigma^2 p^{g\prime\prime}$
    - Increased impatience, investors' cash flows may be higher or lower
  - Smooth pasting: $p(w^s) = p^g(w^s)$, $p'(w^s) = p^{g\prime}(w^s)$

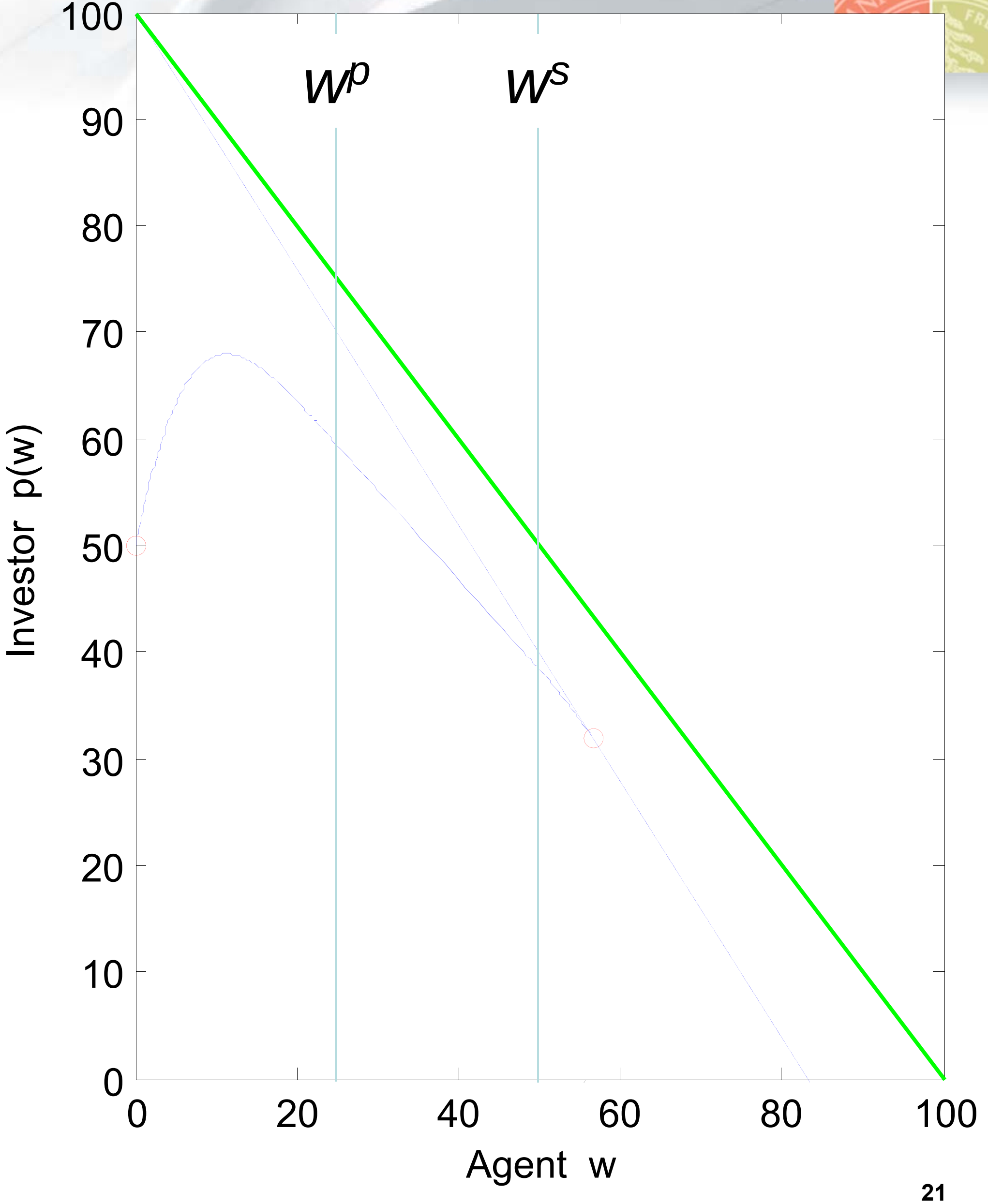
Example

- First Best = 100
  - $\mu = 10$, $r = 10\%$, $\gamma = 12\%$, $\sigma = 8\%$, $L = 50$, $\lambda = 1$
- Cash if $w > 56$
  - $w^c = 56$
- Gamble if $w < 40$
  - $\rho = 2$, $\delta = 5\%$, $w^s = 40$, $D = 0$
- Compare to pure cases
  - Longer deferral of compensation
  - Greater use of credit line vs. debt (more financial slack)

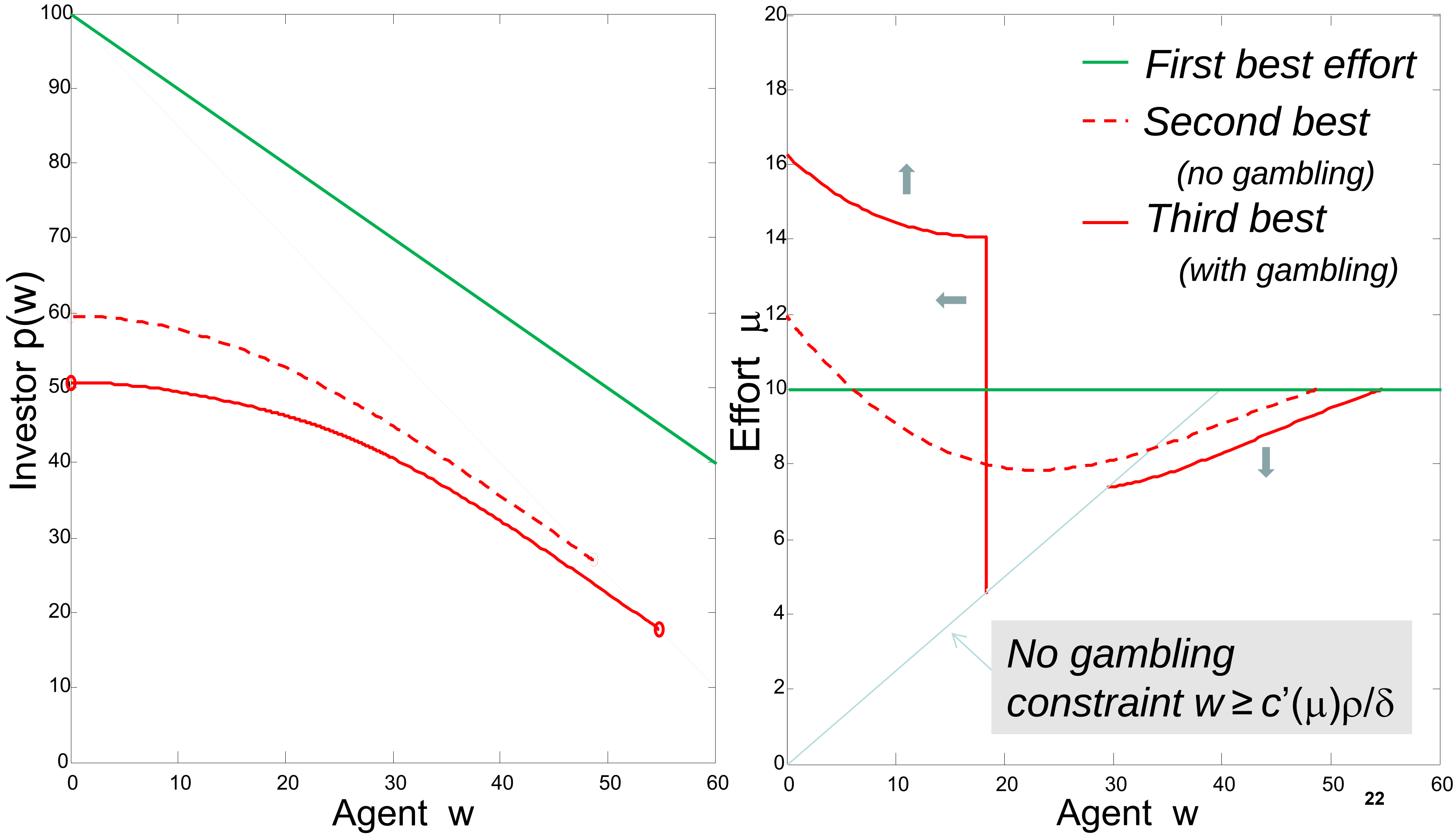


Harsh Penalties

- Example
  - $\sigma = 5\%$, $\rho = 2.5$
- Possible jump if w_t drops below w^s
  - Forgive and reset to $w^s = 50$
  - Punish and reset to $w^p = 25$
  - $w^p \to 0$ as cost of disaster rises



Convex Effort Cost (w/replacement)



No gambling constraint $w \geq c'(\mu)\rho/\delta$

Part III.C:

INVESTMENT DYNAMICS

Key Intuition

- For incentive reasons, agent is rewarded (penalized) when profits are high (low).

- In a dynamic model, reward agent with future rents
  - Higher firm profits $\Rightarrow$ higher agent rents
  - Higher agent rents $\Rightarrow$ lower future agency costs
  - Lower agency costs $\Rightarrow$ higher return to investment

- These effects are persistent $\Rightarrow$ q is history-dependent and investment is positively serially correlated

Production Technology

- Firm generates stochastic cash flows:
  - $dA_t = \mu \, dt + \sigma \, dZ$ <u>per unit of capital</u>
- Firm can invest to increase capital stock:
  - $dK_t = (i_t - \delta) \, K_t \, dt$
- Investment entails convex adjustment costs:
  - $c(i_t) \, dt$ <u>per unit of capital</u>, $c'' > 0$
- Total free cash flow:

$$dY_t = K_t \, dA_t - c(i_t) \, K_t \, dt$$

Neoclassical Case (Hayashi, 1982)

- Investors
  - Risk neutral, Unlimited capital, interest rate r
- CRT Technology ("AK"), Homogeneous Adj. Costs: $c(i)\,K$
- First Best Investment:

$$\max \mathrm{E}[\ PV\big(dY_t\big)\ ] = \mathrm{E}[\ PV\big(K_t\,[\mu\,dt + \sigma\,dZ_t - c(i_t)\,dt\,]\big)\ ]$$

- Marginal q = Average Q = value per unit of capital

$$q^* = Q^* = \max_i \ \frac{\mu - c(i)}{r - (i - \delta)}$$

- Optimal Investment: $c'(i^*) = q^*$
- Quadratic Adj. costs: $q^* = 1 + \theta\, i^*$

Note:
- Restrict $\mu < c(r + \delta)$
- Volatility plays no role
- Can add add'l state variables

The Contracting Environment

- What can Investors contract on?
  - Firm size K_t and cash flow dY_t are verifiable
  - Thus, so is the history of firm investment I_t and productivity dA_t
- What can they control?
  - The agent's compensation $dU_t \geq 0$
  - The firm's access to external capital
  - Termination / Liquidation
    - Agent's outside option = 0
    - Investors receive $L\, K_t$
- Contract specifies, as a function of past output
  - Investment, I_t
  - Agent's cumulative compensation, U_t (increasing)
  - Termination time, τ_t

Solving the Model

- Agent’s Future Payoff *w*
  - Promise-keeping
    - $E[dw] = (\gamma - i)\, w\, dt$
  - Incentive Compatibility
    - $\partial w / \partial y \geq \lambda$

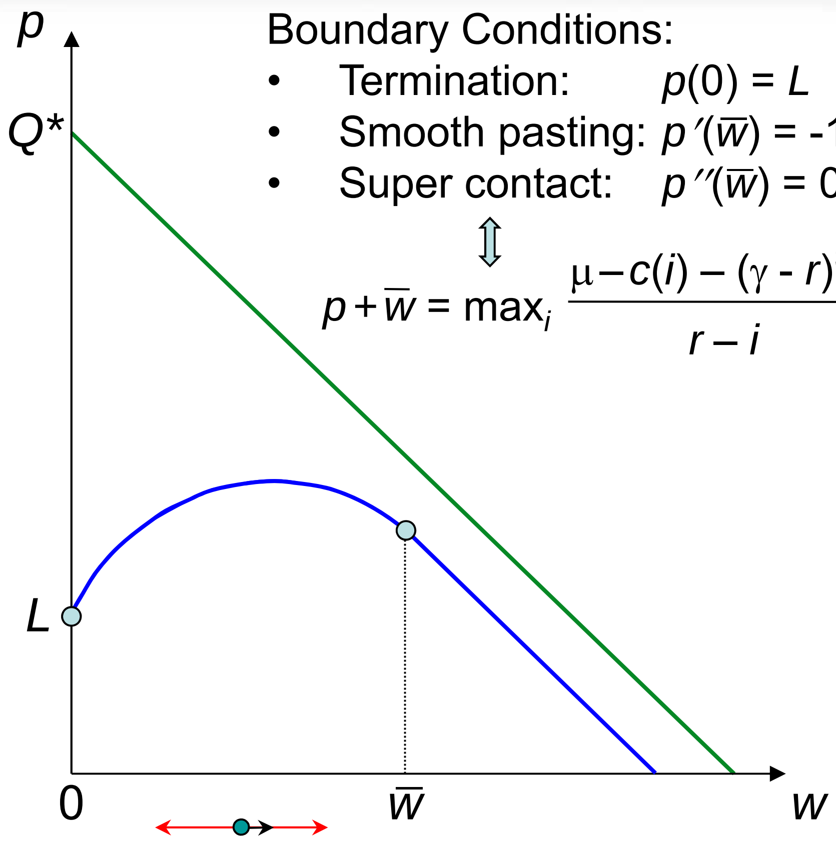
$$\Rightarrow dw = (\gamma - i) w dt + \lambda (dy - E[dy])$$

$$= (\gamma - i) w dt + \lambda \sigma \, dZ$$

- HJB Equation:

$$rp = \sup_i \underbrace{(\mu - c(i))}_{\text{FCF}} + \underbrace{ip}_{\text{growth}} + \underbrace{(\gamma - i)wp' + ½\lambda^2\sigma^2 p''}_{E[dp]}$$

  - FOC: $c'(i) = p(w) - w\,p'(w)$



Boundary Conditions:

- Termination: $p(0) = L$
- Smooth pasting: $p'(\bar{w}) = -1$
- Super contact: $p''(\bar{w}) = 0$

$$\Updownarrow$$

$$p + \bar{w} = \max_i \frac{\mu - c(i) - (\gamma - r)\bar{w}}{r - i}$$

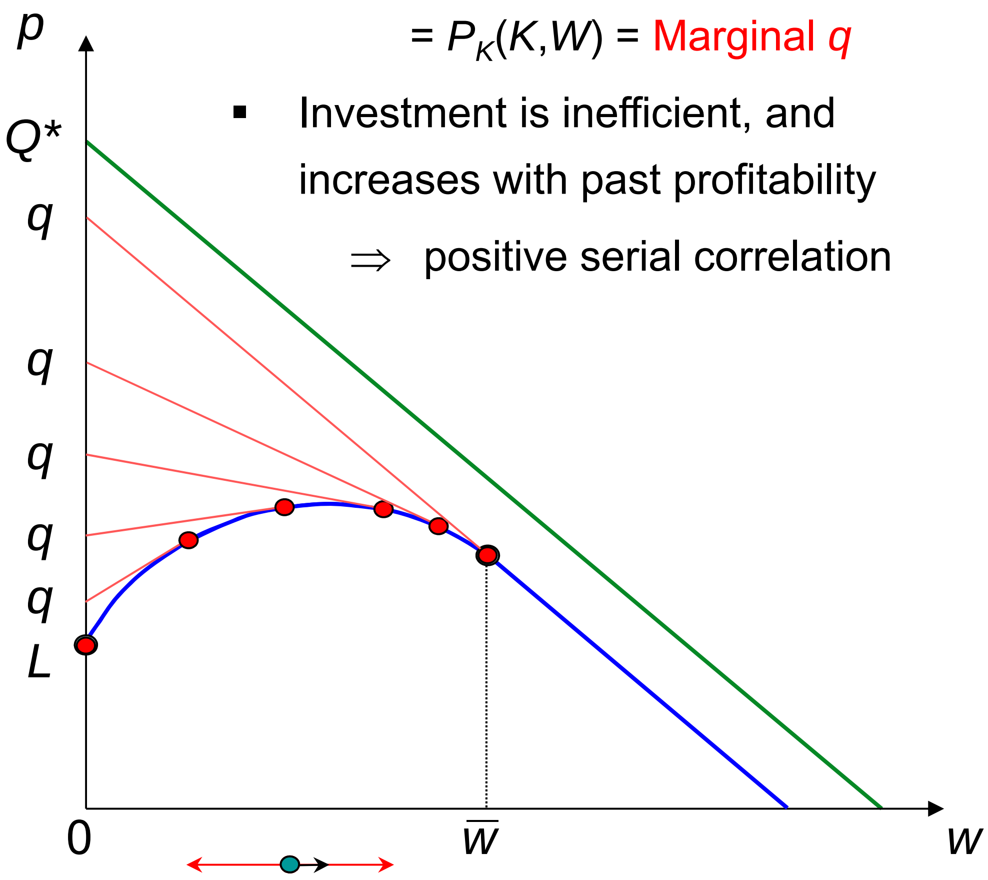
$$(r - i)\, p = (\mu - c(i)) + (\gamma - i) wp' + ½\, \lambda^2 \sigma^2 p''$$

$$c'(i) = p(w) - wp'(w)$$

$$p(0) = L,\ p'(\bar{w}) = -1,\ p''(\bar{w}) = 0$$

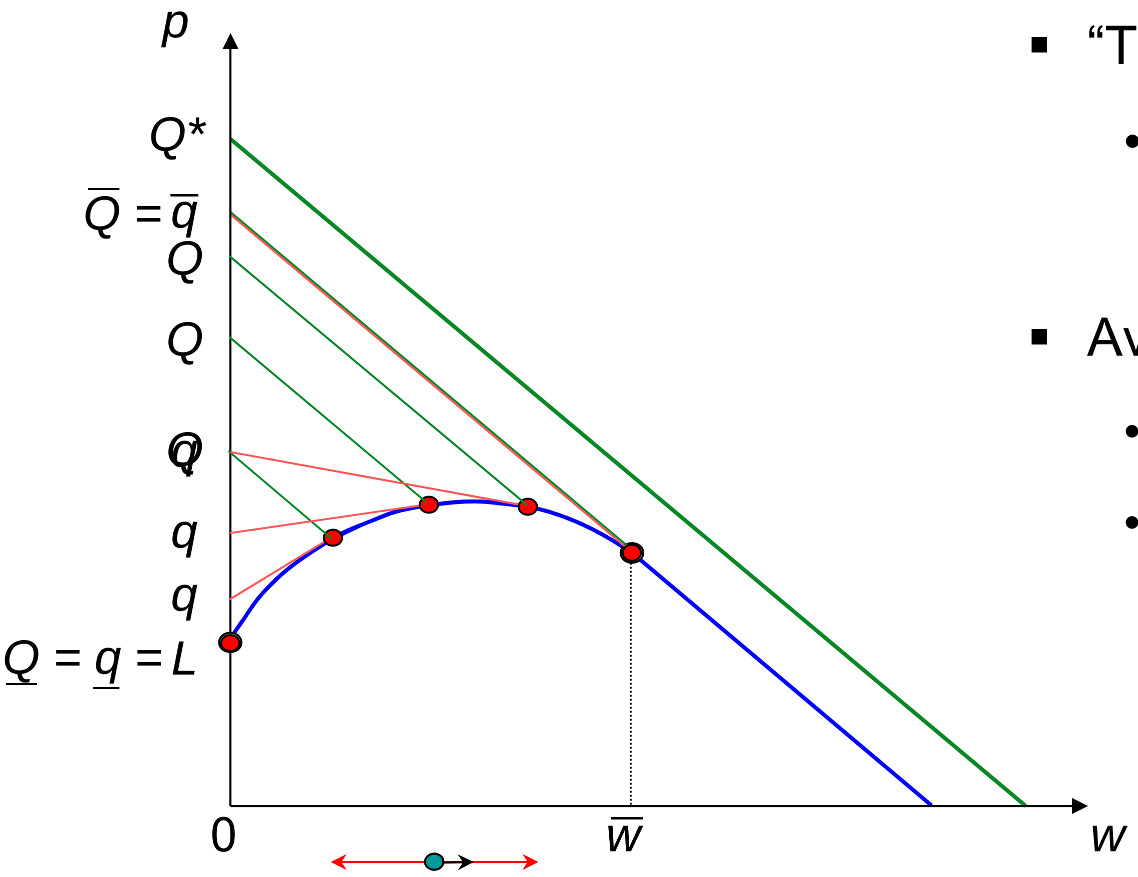
Investment Dynamics

- Investment Dynamics
  - $c'(i) = p(w) - w\,p'(w)$
    $= P_K(K,W) =$ Marginal q
  - Investment is inefficient, and increases with past profitability
    - ⇒ positive serial correlation



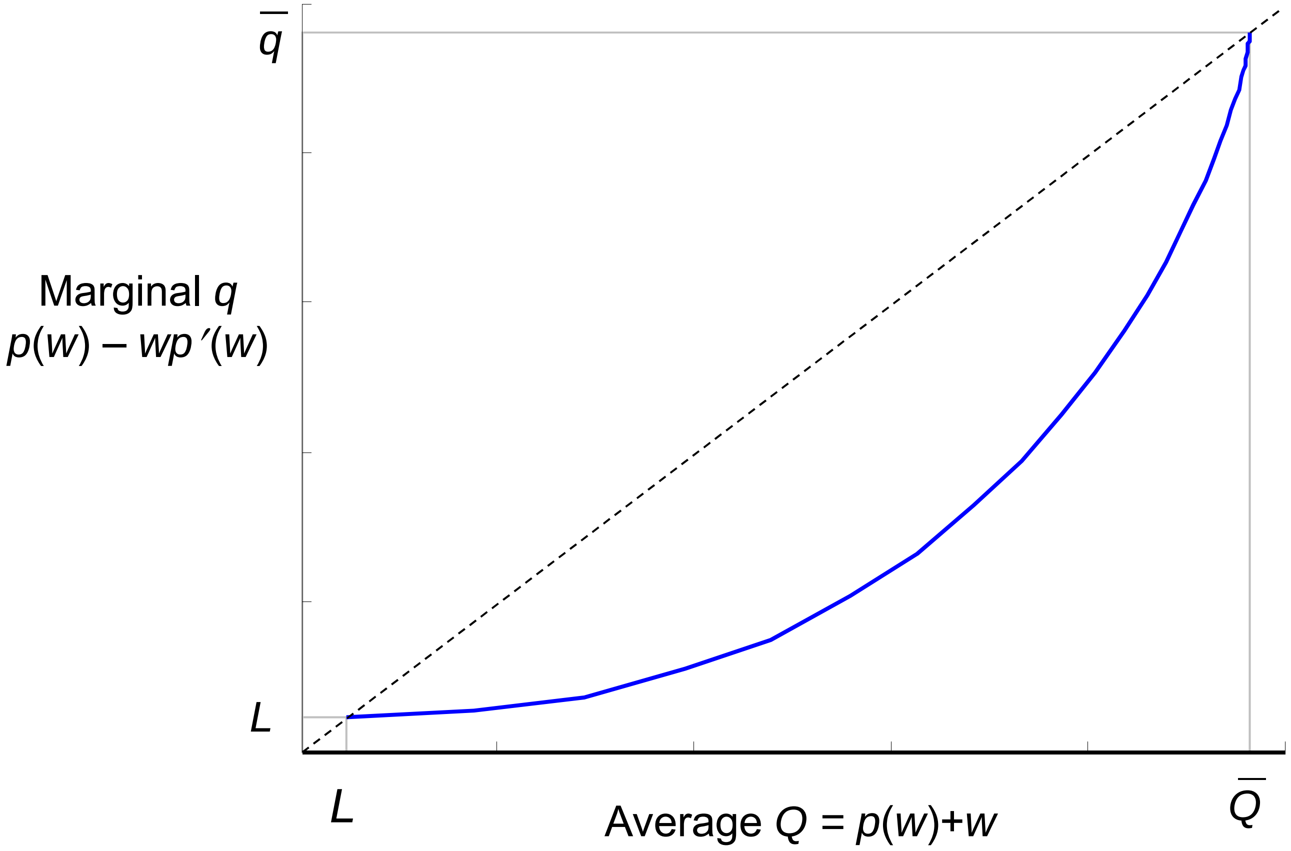
- While investment opportunities are stationary, *q* is history dependent
  - When profits are high…
  - ⇒ Agent is rewarded with a larger share of future cash flows
  - ⇒ Agency costs decrease
  - ⇒ Returns to investment increase

Marginal vs. Average Q

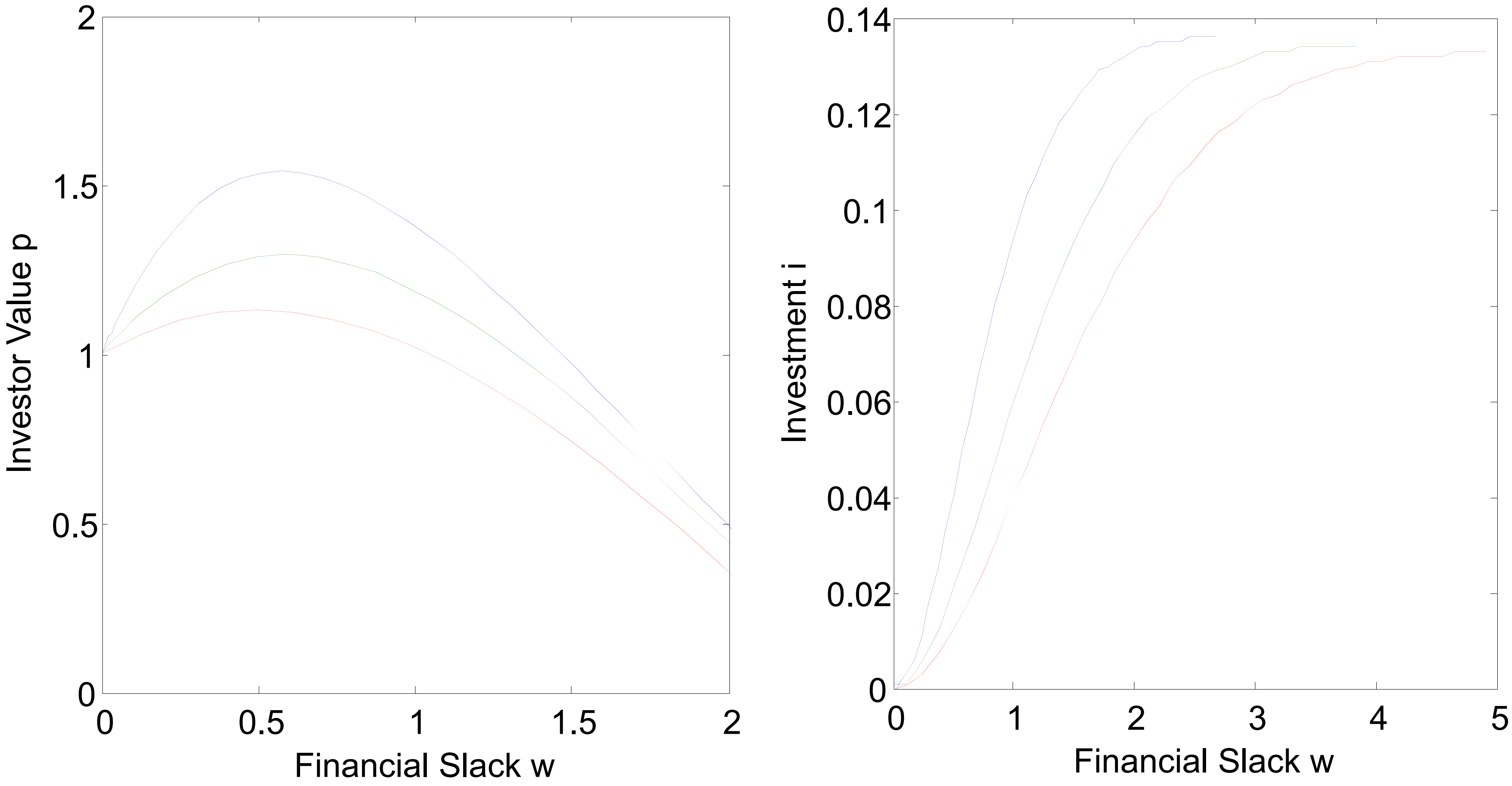


- Average Q
  - "Total Rent" per unit of capital:
    - $Q = p(w) + w$
  - Ave. Q $\geq$ Marginal q
    - Equal at boundaries
    - Gap is not monotone in financial constraints

Marginal vs. Average *Q*



Volatility and Investment



Vs. Transaction Cost Model

“Inventory” Model	**Agency Model**
Cost of holding cash	Manager impatience
Fixed cost of financing	Cost of replacing manager
Proportional cost of financing	Manager’s equity share
Realized FCF contributes to cash balances	***Unexpected*** FCF contributes to cash balances -- Net of cash used for (contractual) investment

Part III.D:

PERSISTENT SHOCKS & PAYING FOR LUCK

Observable Price Shocks

- So far, the only source of risk has been transitory, hidden productivity shocks
  - Tied to the agency problem ⇒ affect compensation
  - Though the shocks are completely transitory, they impact future investment through the agency problem
- Suppose in addition there are persistent, observable shocks to the firm's output price, π_t :

$$dy_t = \pi_t \, dA_t - c(i_t) \, dt$$

  - Assume π_t is independent of firm actions or productivity (i.e., a competitive product market), and can be contracted on / hedged
  - Will these shocks affect compensation?
  - Do they affect the relation between Tobin's Q and investment?

Some “Intuition”

- In a standard static principal-agent setting, optimal wage satisfies (Holmstrom, 1979):

Marginal cost of compensation →

$$\frac{1}{u'(w(s))} = a + b\,\frac{f_e(s\,|\,e)}{f(s\,|\,e)}$$

← Likelihood ratio of observed outcome

⇒ If $f(s, \pi | e) = f_1(s\,|\,e)\, f_2(\pi)$, then π has no impact on the wage

  - Optimal compensation will not depend on observable and thus “hedgeable” exogenous risks
  - Managers should be compensated based on relative rather than absolute performance
  - But this prediction seems to fail empirically …

Compensation with Persistent Shocks

- In a dynamic model:

$$\text{Marginal cost of compensation} = p_w$$

  - Again, in an optimal contract p_w should be independent of market risk, π, and only related to information relevant to the manager’s action, s :

$$p_w = \phi(s)$$

  - However, if the market shock is persistent, it is likely to impact the value function:

$$p_w(w, \pi) = \phi(s)$$

  - Thus, the optimal wage w will depend on exogenous shocks *that affect marginal compensation costs*

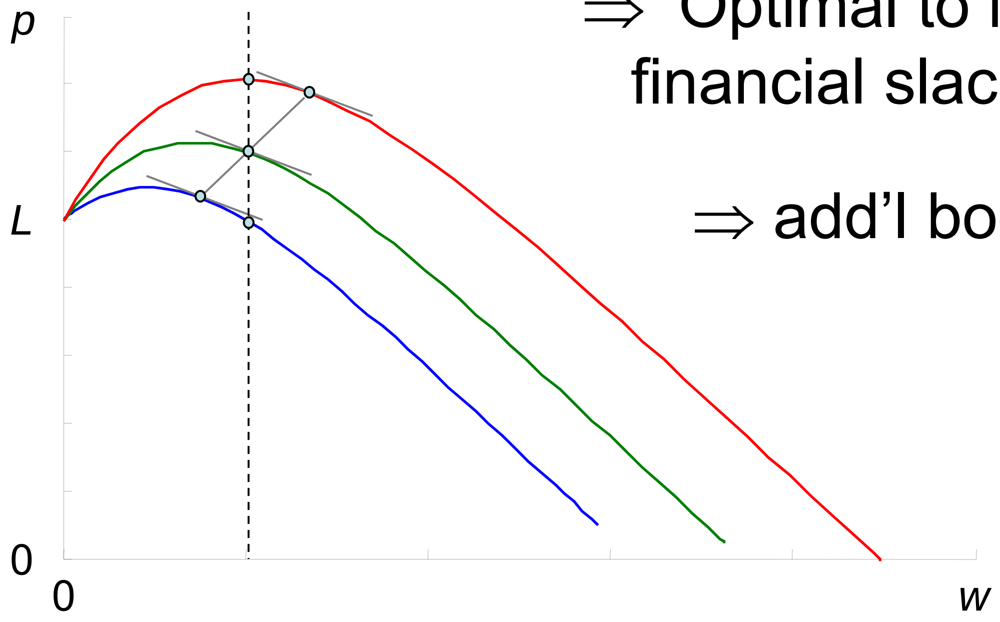
Impact of Output Price Shocks

- When output price fluctuates
  - Firm becomes more / less profitable
    - When price is higher, termination is more costly

⇒ Marginal cost of compensation declines

⇒ Optimal to increase compensation / financial slack with the output price

⇒ add'l boost to investment

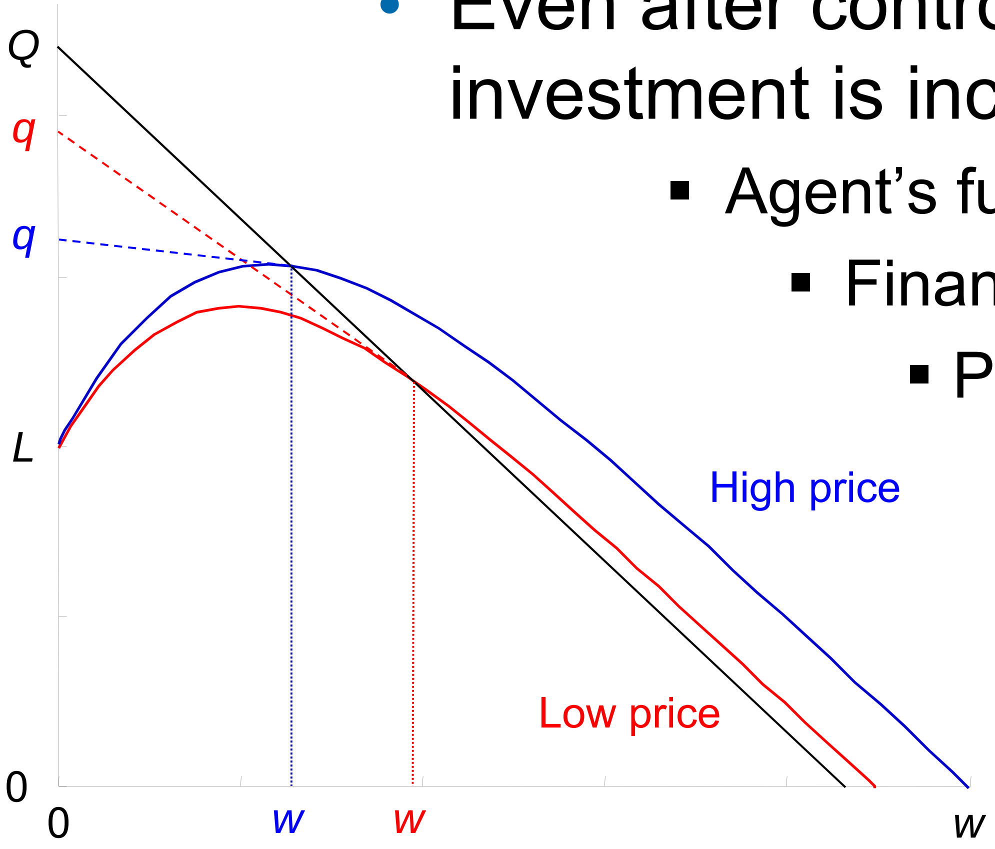


If *w* is sufficiently low:

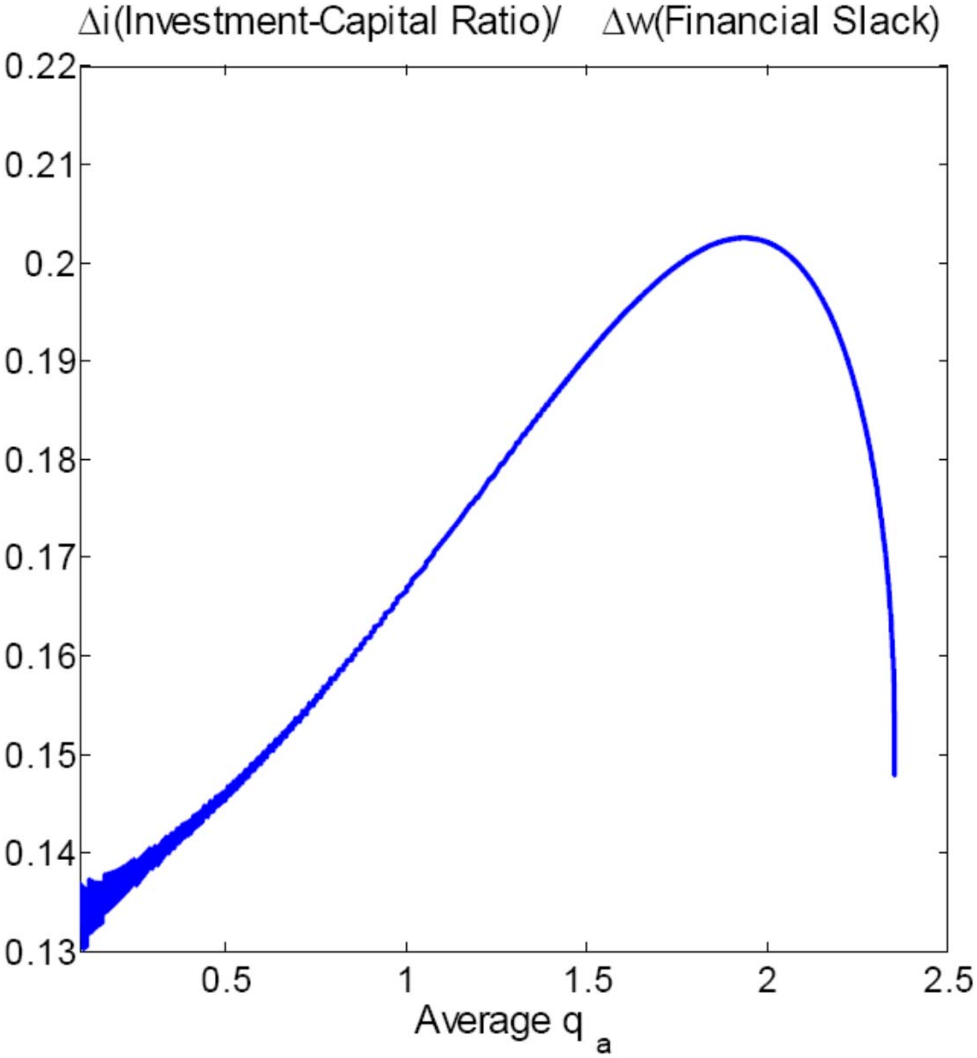
- Manager will be fired after a price decline

Tobin’s Q and Investment

- Tobin’s Q driven by profitability _and_ financial slack
- Even after controlling for Tobin’s Q, investment is increasing with
    - Agent’s future payoff share
        - Financial slack
            - Past profitability



Investment Sensitivity to Financial Slack



Learning about Profitability

- Suppose project profitability μ is volatile and unknown
  - Agent & principal learn about μ from cash flows (Kalman Filter)
  - Efficient Termination: Firm has option to abandon given low μ
  - First Best: need sufficient slack to survive until opt. abandonment
- Incentives
  - Shirking $\Rightarrow$ immediate private benefit λ
  - Cash shortfall $\Rightarrow$ reduced expectations (ν)
  - Total gain: $\lambda + \lambda\nu/r\ \ [1 - e^{-r\tau}]$ “information rent”
- Optimal Contract
  - Early stage: no payouts, termination threshold relaxed over time
  - Mature stage: smooth payouts (exp. earnings), fixed termination
- Permanent Distortions:
  - Poor early performance $\Rightarrow$ higher long-run termination threshold

Conclusions

- Dynamic agency and contracting is a rich area of research, with potential to link micro-theory to broader issues in both corporate finance and asset pricing
- Simple work-horse model combines both the tractability of the continuous-time setting with the ability to consider a broad array of potential frictions and interactions
- Thanks for your time and attention!