The Hirshleifer Effect Revisited: The Case of Disagreement *

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Abstract

We analyze the value of public information in a competitive endowment economy. We provide a global result that an early release of information about the future state of the economy is desired by all agents, or Pareto improving, if agents disagree about the prospect of the economy and asset markets are complete. We further prove that for certain levels of agents' disagreement, all agents prefer an early release of information even if agents cannot trade in asset markets before the information arrives (incomplete asset markets). Thus, the introduction of heterogeneous beliefs reverses the well-known Hirshleifer effect.

JEL-Classification: D80, G14, D51.

Keywords: Hirshleifer effect, disagreement, welfare value of information, incomplete markets.

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1 Introduction

We analyze the social value of public information in a competitive endowment economy. Our main interest is to understand how the timing of public information releases affects risk sharing and ex-ante welfare of economic agents. Our analysis addresses various dimensions of heterogeneity across agents, different levels of asset market perfection, and flexible timing of information releases.

Our main results are twofold, concerning whether agents can trade contingent assets before the expected arrival of public information (complete asset markets) or only after the arrival of information (incomplete asset markets).

First, in complete asset markets, an early release of public information is desired by all agents (information is Pareto improving) if agents have heterogeneous beliefs (disagreement) about the future prospect of the economy. If beliefs are homogeneous, agents are indifferent between early and late public information releases. This result is robust and holds for general additively separable preferences with any level of heterogeneity in agents' time discount rates, risk aversions, beliefs, as well as wealth distributions.

Second, in incomplete asset markets, an early release of public information is desired by all agents if agents have *certain* levels of heterogeneous beliefs about the future prospect of the economy. If beliefs are homogeneous, some agents strictly prefer an early release of public information, and others strictly dislike it. This result is of particular interest because it demonstrates that public information is valuable even when agents cannot trade financial assets before its arrival. This *second* result speaks directly to the Hirshleifer effect.

Hirshleifer (1971) argues that if information is released too early (before agents are able to trade), then it has an adverse impact on risk sharing and reduces welfare. This important insight is known as the *Hirshleifer effect*. Our setting generalizes Hirshleifer (1971)'s model to heterogeneous beliefs and has a surprising result: the introduction of heterogeneous beliefs is able to reverse the Hirshleifer effect and information releases (before asset markets open) are Pareto improving.

Our *first* result that an early release of public information has an unambiguously positive value to welfare in the complete-market heterogeneous-belief setting is intuitive. When agents disagree about the probability distribution of future economic states, they speculate. Each agent believes he can make a profit by betting with other agents on the realization of the state of the economy, and ex-ante every agent feels wealthier. Early resolution of uncertainty allows agents to resolve their disagreement and to capitalize their speculative profits earlier. In contrast, without early information releases, agents not only have to wait longer for the realization of speculative profits, but also cannot borrow against their expected speculative profits precisely because of the disagreement. Therefore, in expectation, early information releases increase consumption, and ex-ante expected utility of every agent.[1]

Our *second* result that an early release of public information has ambiguous implications on welfare in the incomplete-market heterogeneous-belief setting is also intuitive. While agents expect to receive early information, they are not able to trade assets before the information arrives. Once the information is released, on the downside, risk sharing and speculation (based on disagreement) across possible states of the economy is eliminated (Hirshleifer effect). On the upside, because information reveals the future state of the economy, agents are able to concentrate on smoothing consumption over time, and thus increase their utilities with the hindsight conveyed by the public information. Ex-ante, it is not obvious whether smoothing across states or across time is more beneficial. Consequently, ambiguity on the welfare value of information arises. Our analysis identifies sufficient conditions for either cases.

The *second* finding concerning the Hirshleifer effect can also be explained by examining the relative dominance between aggregate and agents' idiosyncratic endowment risks. Our paper builds on the premise that uncertainty in aggregate endowment dominates those in individual agents' endowments. Under this premise, when asset markets are incomplete and heterogeneities across agents are moderate, risk sharing benefits are limited, and the loss in expected utility due to a reduction in risk sharing (by early public information) is modest. As a result, the Hirshleifer effect is subdued and public information is welfare improving whenever the risk sharing motive and capacity are not preeminent.

Our results employ the canonical definition of welfare in the presence of heterogeneous beliefs. We define an improvement in welfare (Pareto improvement) as an ex-ante increase in every agent's expected utility evaluated under the respective agent's belief. Welfare does not necessarily improve under other criteria, which consider speculative trading to be undesirable (Kim, 2012; Gilboa et al., 2014; Gayer et al., 2014; Brunnermeier et al., 2014). But even when speculative trading is not found to be socially beneficial under some welfare criterion, attempting to regulate speculation can be

[1] For additively separable preferences, in the case of early information releases and complete asset markets, there is a trade-off between consumption smoothing across time and consumption smoothing across states. Heterogeneous beliefs affect this trade-off non-trivially.

socially suboptimal because restrictions can leave unintended adverse effects hindering markets proper functioning as Duffie (2014) points out. We do not take a stance on which welfare criterion should be chosen by a benevolent social planner. We simply show under what conditions agents ex-ante prefer an early versus a late release of public information.[2]

Our analysis is robust to many extensions. The derivation is general enough to accommodate imperfect quality of information and an arbitrary number of time periods. Instead of focusing on aggregate uncertainty, our models can be altered to address multiple endowment processes.

The paper is organized as follows. Section 2 introduces the setting of our analysis, and presents a practical example motivating the setting. Section 3 derives a global result on the welfare value of information when asset markets are complete. Section 4 contrasts the welfare value of early versus too-early information releases. Section 5 analyzes the Hirshleifer effect in different belief configurations. Section 6 generalizes our results in several directions. Section 7 concludes.

Related literature

The literature on the value of information arguably goes back to Blackwell (1953) who provides an analytical proof that, in his set-up, early information releases have a positive effect on welfare. Many subsequent papers point out limitations to this result. Most prominently, Hirshleifer (1971) argues that too early information releases reduce risk-sharing and have negative implications on welfare. Orosel (1996) and Schlee (2001) extend the results of Hirshleifer (1971). We show that there exist important limitations to the Hirshleifer effect under both homogeneous and heterogeneous beliefs. In particular, we prove that for some levels of disagreement, an early release of information is Pareto improving even if agents are not allowed to trade before the information is released.

Our paper is closely related to models that analyze re-trading after information releases in an endowment economy. Jaffe (1975), Ng (1975) and Hakansson et al. (1982) show that an unexpected release of information does not induce re-trade, and by revealed preferences, has no value if and only if agents have time additive preferences, homogeneous (prior) beliefs, and markets are complete. Berk (1993) shows that with enough heterogeneity in wealth, there is at least one agent who benefits from generating public information in order to make markets incomplete, but he does not address the welfare aspects of the economy. Blume et al. (2006) and Gottardi and Rahi (2013) analyze necessary and sufficient conditions on the information structure such that an arrival of new

[2] According to our definition, if all agents prefer an early (late) release of information, then we say an early release of information has positive (negative) social value and increases (decreases) welfare.

information causes re-trade in incomplete markets. Gottardi and Rahi (2014) discuss the trade-off between the Blackwell and the Hirshleifer effect in incomplete markets and show that in general a change in information does not lead to a socially optimal allocation. While Gottardi and Rahi (2014) assume homogeneous beliefs and focus on the implications of market incompleteness, we work in various asset market conditions and heterogeneous beliefs.

Most aforementioned papers focus on re-trade and assume that new information releases are unexpected (with the exception of Gottardi and Rahi (2014)). These papers do not analyze ex-ante welfare and do not answer the question whether agents prefer to live in a world with early or late information releases. In contrast, in our setting, agents expect the arrival of information and we provide sufficient conditions which imply that agents have a strict preference for early information releases. An important contribution of our paper is that we relax a strong assumption on the initial endowment distribution. While most papers focus on re-trade and thus, assumes that endowments equal equilibrium consumptions if no new information was released, we start with less restrictive wealth distribution.

Other papers explore the value of information releases in the context of coordination games (see Colombo et al. (2014) for a recent overview), production economies, and information asymmetries (see Goldstein and Yang (2014) for a recent overview of latter two literature). Our focus is on the risk-sharing implications of changes in the timing of public information releases. We do not address information asymmetries, and we intentionally exclude the possibility of real investments and other types of intertemporal transfers of aggregate endowments.

2 The setup

We consider a setting of heterogeneous-agent endowment economy in discrete time. We employ the standard filtered probability space $\{\Omega, \mathcal{F}, \{\mathcal{F}_t\}_t, \mathbb{P}\}$ to model the public information evolution, wherein $\{\mathcal{F}_t\}_t$ is the natural filtration associated with the time-evolution of uncertain future endowments in the economy. We consider a stylized three-period setting, $t \in \{0^-, 0, 1\}$, where $t = 0^-$ denotes the instant right before $t = 0$. The choice of a three-period setting is for conveniences, and can be generalized to a multiple-period setting.[3]

[3]The three-period setting stylizes a period of possible early public information releases (being early with respect to asset market opening), a period of consumption with no aggregate uncertainty, and a future period of consumption with aggregate uncertainty. We can straightforwardly insert more periods in between these three distinct temporal marks.

Endowments: There is a single tree in the economy, which bears deterministic aggregate endowment e_0 in period $t = 0$, and state-contingent aggregate endowment $e_1(s)$ in period $t = 1$, with $s \in \Omega \equiv \{1, \ldots, N\}$ denoting the state at $t = 1$. Time-one endowment $e_1(s)$ is the sole source of uncertainty in the economy. The endowment good is perishable and must be consumed in the same period in which it arrives. There are neither endowments nor consumptions at $t = 0^-$. Asset markets open at time $t = 0^-$. Signals about future endowment $e_1(s)$ arrive either before $t = 0^-$, after $t = 0^-$, or never, depending on the informational setting (economy) considered in the paper.

Agents: There are two agents, A and B, who differ in their risk aversions, time preferences, and beliefs about the distributional prospects of future endowments. There is no information asymmetry, and agents' subjective beliefs may differ because agents agree to disagree in our setting. For tractability, we assume that agents maximize expected utilities with constant relative risk aversions (CRRA) and constant time discount factors. The utility of consumption at time t and state s, discounted to time zero, is,

$$U(c_{It}(s), t) = \beta_I^t u(c_{It}(s)) = \beta_I^t \frac{[c_{It}(s)]^{1-\gamma_I}}{1-\gamma_I}, \qquad \forall I \in \{A, B\}, t \in \{0, 1\}, s \in \Omega. \tag{1}$$

where $u(c)$ denotes period utility (or felicity), and β_I and γ_I respectively agent I's time discount factor and relative risk aversion. We remark that all our results can be generalized to the setting of strictly increasing, strictly concave and continuously differentiable additively separable preferences, as explained in Section 6.

Before state s is realized, agents' beliefs are characterized by two equivalent subjective probability measures,[4]

$$\sum_{s \in \Omega} p_A(s) = \sum_{s \in \Omega} p_B(s) = 1, \qquad p_A(s) \neq 0 \Longleftrightarrow p_B(s) \neq 0.$$

Agent I ($I \in \{A, B\}$) is endowed with e_{It} units of the consumption good at period $t \in \{0, 1\}$. In the aggregate, the consumption good's market clears at each time and state, as quantified in the following resource constraints,[5]

$$\sum_{I \in \{A,B\}} c_{I0} = \sum_{I \in \{A,B\}} e_{I0} = e_0, \qquad \sum_{I \in \{A,B\}} c_{I1}(s) = \sum_{I \in \{A,B\}} e_{I1}(s) = e_1(s), \quad \forall s \in \Omega. \tag{2}$$

[4] Technically, agent I's subjective probability measure $\mathbb{P}_I \equiv \{p_A(s)\}$, $I \in \{A, B\}$ is an ingredient of the corresponding filtered probability space $\{\Omega, \mathcal{F}_I, \{\mathcal{F}_{It}\}_t, \mathbb{P}_I\}$, with $\mathcal{F}_{It} \subset \mathcal{F}_t$, $\forall t$.

[5] Agents' time-zero endowments, e_{A0}, e_{B0}, like their aggregate counterpart e_0, are deterministic.

The conditions for the existence and uniqueness of competitive equilibrium in heterogeneous-agent are discussed in Mas-Colell et al. (1995), and in the recent work by Hugonnier et al. (2012).[6] To place the relevance of public information in perspectives of different asset market settings, we study the three economies specified next. Our road map to assess the welfare value of information is to study the comparative statics of any two of these three economies, which aims to contrast certain aspects of asset markets versus information at a time.

2.1 Three economies

Uninformed economy

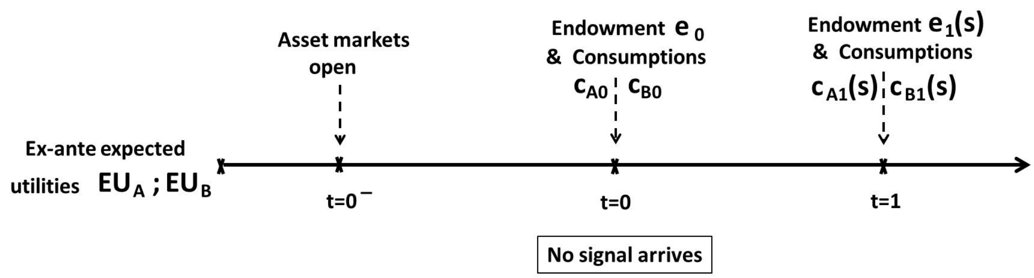

Figure 1: Time scheme of the uninformed economy.

In this economy, no signals on future aggregate endowment e_1 are available before time $t = 1$, but asset markets are complete. At $t = 0^-$, agents can trade a complete set of Arrow-Debreu (AD) contingent claims on future endowments[7] as indicated by Figure 1. Hence, asset trading happens before agents start consuming at $t = 0$.

Consequently, ulterior consumptions at $t = 0, 1$ are endogenous to the ex-ante trading decisions at time $t = 0^-$, when asset markets opens. Because there is no information on the endowment e_1 to be released before $t = 0$ (or ever), agents are content with non-contingent consumptions at $t = 0$. Accordingly, agents trade and choose consumption plans to maximize their expected

[6] Most relevant to our economies are Propositions 4.C.1, 4.C.2, 17.C.1, 17.D.1 and 17.F.2. in the book by Mas-Colell et al. (1995), and Proposition C.1 in online supplement to Hugonnier et al. (2012).

[7] For the economies considered in this paper, whenever asset markets are complete, dynamic trading in multiple-period settings can be reduced to initial trades in AD assets.

utilities[8] subject to their budget constraints,

$$EU_I \equiv \sup_{\{c_{I0}, c_{I1}(s)\}} E^I_{0^-}\left[u(c_{I0}) + \beta_I u(c_{I1}(s))\right], \qquad \forall I \in \{A, B\}, \tag{3}$$

$$\text{s.t. } c_{I0} + \sum_{s\in\Omega} c_{I1}(s)q(s) \le e_{I0} + \sum_{s\in\Omega} e_{I1}(s)q(s),$$

where $q(s)$ is the time-zero price of the s-th AD security paying one unit of consumption good at $t = 1$ when and only when state s realizes, and period utilities $u(c)$ are given in (1). In the aggregate, the resource constraints (2) must hold at each time and state. The optimality in this complete-market economy is achieved when agents perfectly share risks by equalizing their marginal utilities to the AD price in the corresponding state,

$$p_A(s)\beta_A \frac{u'_{A1}(s)}{u'_{A0}} = q(s) = p_B(s)\beta_B \frac{u'_{B1}(s)}{u'_{B0}}, \tag{4}$$

where "prime" denotes the first-order derivative with respect to consumption, $u'_{It}(s) \equiv \frac{\partial u(c_{It}(s))}{\partial c_{It}(s)}$. Substituting the AD prices into (3), we find that the following first order conditions (FOCs) and budget constraint formalize this equilibrium,

$$\text{at } t=0 \begin{cases} u'_{A0} = \lambda u'_{B0}, \\ c_{A0} + c_{B0} = e_0, \end{cases} \qquad \begin{matrix}\text{at } t=1 \\ \forall s \in \Omega\end{matrix} \begin{cases} p_A(s)\beta_A u'_{A1}(s) = \lambda p_B(s)\beta_B u'_{B1}(s), \\ c_{A1}(s) + c_{B1}(s) = e_1(s), \end{cases} \tag{5}$$

$$u'_{A0}c_{A0} + \sum_{s\in\Omega} p_A(s)\beta_A u'_{A1}(s)c_{A1}(s) = u'_{A0}e_{A0} + \sum_{s\in\Omega} p_A(s)\beta_A u'_{A1}(s)e_{A1}(s). \tag{6}$$

where λ is the Pareto weight. For the uninformed economy under consideration, asset markets are complete, thus λ is state-independent. The assumed property of strictly increasing preferences[9] implies that the budget constraint in equilibrium holds at equality as in (6). By Walras' law, the budget constraint for agent B is redundant.[10]

To analyze the equilibrium of the uninformed economy, we first take Pareto weight λ as given. The first (respectively, second) equation system in (5) then indicates how equilibrium consumptions at $t = 0$ (respectively, at $t = 1$) vary with endogenous λ and other exogenous factors, such as beliefs and endowments. Finally, substituting these consumption into the budget constraint (6)

[8] Each agent computes expected utility under his subjective belief ex-ante, before $t = 0$ at which first endowments and consumptions take place.

[9] A weaker assumption of locally non-satiated preferences suffice to bind the budget constraints at the equilibrium.

[10] B's budget constraint is implied from A's budget constraint, the resource constraints, and the FOC.

then indicates how λ varies with the above exogenous factors.[11]

Informed economy

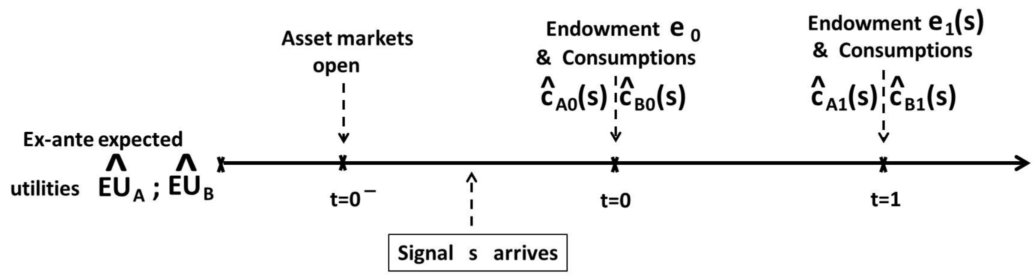

Figure 2: Time scheme of the informed economy.

We employ the sign "hat" to denote quantities associated with the informed economy. In this economy, agents expect that a signal about the future aggregate endowment $e_1(s)$ will be released before $t = 0$, but after $t = 0^-$ (when asset markets opens) as indicated by Figure 2. Asset trading happens before agents start consuming, and ulterior consumptions at $t = 0, 1$ are endogenous to the ex-ante trading decisions at pre-signal time $t = 0^-$. In particular, agents are able to smooth and contract their consumptions at both $t = 0$ and $t = 1$ on the public signal s revealed prior to $t = 0$. Compared to the uninformed economy, agents' optimal consumption plans here possess higher degree of state contingency due to the availability of early public information. For simplicity we assume that the signal is perfect in the sense that its observation clears all uncertainties about future endowments. Section 6 addresses and generalizes our results to a setting of imperfect signals. Consumption smoothing is across both states and times. At pre-signal time $t = 0^-$ (i.e., before the arrival of the signal), agents trade and choose consumption plans to maximize their expected utilities, subject to their budget constraints,

$$\widehat{EU}_I \equiv \sup_{\{\widehat{c}_{I0}(s), \widehat{c}_{I1}(s)\}} E^I_{0^-}\left[u(\widehat{c}_{I0}(s)) + \beta_I u(\widehat{c}_{I1}(s))\right], \qquad \forall I \in \{A, B\}, \tag{7}$$

$$\text{s.t. } \sum_{s\in\Omega} \widehat{c}_{I0}(s)\widehat{q}_0(s) + \sum_{s\in\Omega} \widehat{c}_{I1}(s)\widehat{q}_1(s) \le \sum_{s\in\Omega} e_{I0}\widehat{q}_0(s) + \sum_{s\in\Omega} e_{I1}(s)\widehat{q}(s),$$

[11] Take λ as given, the system at $t = 0$ in (5) has two equations and two unknowns (c_{I0}, for $I \in \{A, B\}$), the system at $t = 1$ in (5) has $2N$ equations and $2N$ unknowns ($c_{I1}(s)$, for $s \in \Omega \equiv \{1, \ldots, N\}$, $I \in \{A, B\}$). The budget constraint (6) is an additional equation to solve for λ. In total, the equilibrium system consists of $2N + 3$ unknowns (including λ) and $2N + 3$ equations.

where $\widehat{q}_0(s)$ and $\widehat{q}_1(s)$ are time-0^- prices of AD securities, paying one unit of consumption respectively at $t = 0$ and $t = 1$ when and only when the signal s arrives. In the aggregate, the resource constraints (2) must hold at each time and state. Asset markets are complete. Consequently, the optimality in this complete-market economy is achieved when agents perfectly share risks by equalizing their marginal utilities to the AD price in the corresponding state and time,

$$\frac{p_A(s)\widehat{u}'_{A0}(s)}{\sum_{s\in\Omega} p_A(s)\widehat{u}'_{A0}(s)} = \widehat{q}_0(s) = \frac{p_B(s)\widehat{u}'_{B0}(s)}{\sum_{s\in\Omega} p_B(s)\widehat{u}'_{B0}(s)}, \tag{8}$$

$$\frac{p_A(s)\beta_A\widehat{u}'_{A1}(s)}{\sum_{s\in\Omega} p_A(s)\widehat{u}'_{A0}(s)} = \widehat{q}_1(s) = \frac{p_B(s)\beta_B\widehat{u}'_{B1}(s)}{\sum_{s\in\Omega} p_B(s)\widehat{u}'_{B0}(s)}, \tag{9}$$

where $\widehat{u}'_{It}(s) \equiv \frac{\partial u(\widehat{c}_{It}(s))}{\partial \widehat{c}_{It}(s)}$. Substituting the AD prices into (7), we find that the following system of FOCs and budget constraint formalizes this equilibrium,

$$\begin{matrix} \text{at } t=0 \\ \forall s\in\Omega \end{matrix} \begin{cases} p_A(s)\widehat{u}'_{A0}(s) = \widehat{\lambda} p_B(s)\widehat{u}'_{B0}(s), \\ \widehat{c}_{A0}(s) + \widehat{c}_{B0}(s) = e_0, \end{cases} \qquad \begin{matrix} \text{at } t=1 \\ \forall s\in\Omega \end{matrix} \begin{cases} p_A(s)\beta_A\widehat{u}'_{A1}(s) = \widehat{\lambda} p_B(s)\beta_B\widehat{u}'_{B1}(s), \\ \widehat{c}_{A1}(s) + \widehat{c}_{B1}(s) = e_1(s), \end{cases} \tag{10}$$

$$\sum_{s\in\Omega} p_A(s)\widehat{u}'_{A0}(s)\widehat{c}_{A0}(s) + \sum_{s\in\Omega} p_A(s)\beta_A\widehat{u}'_{A1}(s)\widehat{c}_{A1}(s) = \sum_{s\in\Omega} p_A(s)\widehat{u}'_{A0}(s)e_{A0} + \sum_{s\in\Omega} p_A(s)\beta_A\widehat{u}'_{A1}(s)e_{A1}(s), \tag{11}$$

where $\widehat{\lambda}$ is the Pareto weight in the current informed economy. $\widehat{\lambda}$ is state-independent because asset markets are complete.

To analyze the equilibrium of this informed economy, we first take Pareto weight $\widehat{\lambda}$ as given. The first (respectively, second) equation system in (10) then indicates how equilibrium consumptions at $t = 0$ (respectively, at $t = 1$) vary with endogenous $\widehat{\lambda}$ and other exogenous factors, such as beliefs and endowments. Substituting these consumption into the budget constraint (11) then indicates how $\widehat{\lambda}$ varies with the above exogenous factors.[12]

Too-early informed economy

We employ the sign "bar" to denote quantities associated with the too-early informed economy. In this economy, a perfect signal[13] about future aggregate endowment $e_1(s)$ is expected to be released before asset markets open at $t = 0^-$. In this sense, the information arrives "too early" and disrupts

[12]In total, the equilibrium system (10)-(11) consists of $4N + 1$ unknowns ($c_{It}(s)$, for $s \in \Omega \equiv \{1, \ldots, N\}$, $t \in \{0, 1\}$, $I \in \{A, B\}$; and λ) and $4N + 1$ equations.

[13]Section 6 addresses and generalizes our results to setting of imperfect signals.

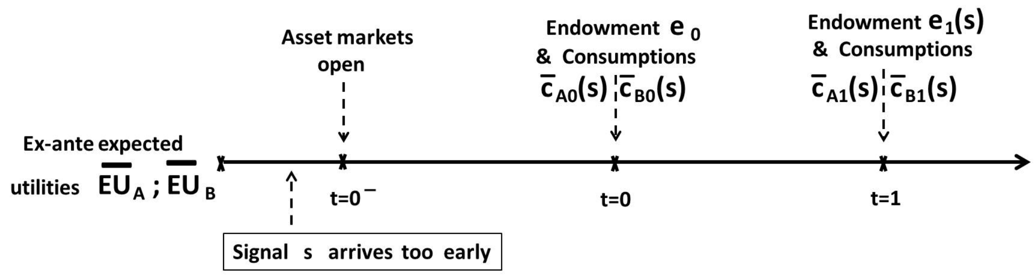

Figure 3: Time scheme of the uninformed economy.

the risk-sharing function of asset markets. After the signal arrives, at $t = 0^-$, agents can trade assets to intertemporally smooth their time-zero and time-one consumptions,[14] as indicated by Figure 3. Consequently, ulterior consumptions at $t = 0, 1$ reflect only the realities of a single state s revealed by the signal arriving just before $t = 0^-$. Given the signal's informational content, agents trade and choose consumption plans to maximize their utilities separately for each state s, subject to their budget constraints,

$$\overline{EU}_I \equiv \sup_{\{\bar{c}_{I0}(s),\bar{c}_{I1}(s)\}} \left[u(\bar{c}_{I0}(s)) + \beta_I u(\bar{c}_{I1}(s))\right], \qquad \forall I \in \{A, B\}, \tag{12}$$

$$\text{s.t. } \bar{c}_{I0}(s) + \bar{c}_{I1}(s)\bar{q}(s) \le e_{I0} + e_{I1}(s)\bar{q}(s), \quad \text{for each } s \in \Omega,$$

where $\bar{q}(s)$ is time-0 price of the state-specific pure-discount security, which pays one unit of consumption at $t = 1$. In the aggregate, the resource constraints (2) must hold at each time and state. For comparative statics study, we always define agents' (ex-ante) welfare as their expected utilities at a pre-signal point in time.

The optimality in this too-early informed economy is achieved when agents perfectly smooth their consumptions intertemporally by equalizing their marginal utilities to the pure-discount security price in each corresponding state s,

$$\beta_A \frac{\overline{u}'_{A1}(s)}{\overline{u}'_{A0}(s)} = \bar{q}(s) = \beta_B \frac{\overline{u}'_{B1}(s)}{\overline{u}'_{B0}(s)}, \tag{13}$$

where $\overline{u}'_c \equiv \frac{\partial u(\bar{c})}{\partial \bar{c}}$. Substituting the AD prices into (12), we find that the following system of FOC

[14]Because the signal already reveals the state, the consumption smoothing is purely intertemporally, but not across the states. Also, since there in no more uncertainty in the market after the signal arrives, a single pure-discount contract suffices to facilitate this consumption smoothing.

and budget constraints formalizes this equilibrium, for each s separately,

$$\begin{array}{ll}\text{at } t=0 \\ \forall s \in \Omega\end{array} \begin{cases} \overline{u}'_{A0}(s) = \overline{\lambda}(s)\overline{u}'_{B0}(s), \\ \overline{c}_{A0}(s) + \overline{c}_{B0}(s) = e_0, \end{cases} \qquad \begin{array}{ll}\text{at } t=1 \\ \forall s \in \Omega\end{array} \begin{cases} \beta_A \overline{u}'_{A1}(s) = \overline{\lambda}(s)\beta_B \overline{u}'_{B1}(s), \\ \overline{c}_{A1}(s) + \overline{c}_{B1}(s) = e_1(s), \end{cases} \tag{14}$$

$$\overline{u}'_{A0}(s)\overline{c}_{A0}(s) + \beta_A \overline{u}'_{A1}(s)\overline{c}_{A1}(s) = \overline{u}'_{A0}(s)e_{A0} + \beta_A \overline{u}'_{A1}(s)e_{A1}(s), \tag{15}$$

where $\overline{\lambda}(s)$ is the state-specific (i.e., post-signal) Pareto weight in the too-early informed economy.

To analyze the equilibrium of this too-early informed economy, we can work with each state s separately. We first take Pareto weight $\overline{\lambda}(s)$ as given. The first (respectively, second) equation system in (10) then indicates how equilibrium consumptions at $t = 0$ (respectively, at $t = 1$) vary with endogenous $\overline{\lambda}(s)$ and exogenous endowments. Substituting these consumption into the budget constraint (11) then indicates how $\overline{\lambda}(s)$ varies with the above exogenous factors.[15] Before proceeding to the detailed comparative analysis of the three economies, we discuss the economic intuitions that motivate their set-ups below.

2.2 A motivating example

A specific example illustrates practical aspects of the three informational settings presented above. Consider the example of dividend payments. The uninformed economy corresponds to a world where dividend payments are announced on the same day as the actual payment is made to investors. In contrast, in the informed economy firms announce their dividend payments well in advance of the dividend-payout date and the time of the announcement is expected by investors. In practice, the dividend declaration date is earlier than (the ex-dividend and) the payout date, and earnings announcements and revisions in analyst forecasts also precede dividend payments. In the too-early informed economy dividends are also announced prior to the payout date. But in contrast to the informed economy the announcement is made earlier than expected and investors find themselves unable to trade in anticipation of a possible stock price change on the announcement date. Instead of an unexpected announcement by the dividend paying firm we can also imagine an unexpected release of an analyst estimate. The assumption that investors are taken by surprise and cannot rebalance their portfolio optimally before the announcement can be motivated by a limited attention story and the fact that many investors only trade at few discrete points in time.

[15] For each state s, the equilibrium system (14)-(15) consists of five unknowns ($c_{It}(s)$, for $t \in \{0, 1\}$, $I \in \{A, B\}$; and $\overline{\lambda}(s)$) and five equations.

Our implications of a change in the timing of information releases on risk-sharing carry over to news releases that also affect real investments. For instance, policy announcements by the Federal Reserve bank, news about bank stress-tests, or changes in a country's sovereign debt rating affect risk-sharing and real investments at the same time. Our results have to be treated with caution under such circumstances. An optimal policy to time information releases with respect to risk-sharing may not coincide with the optimal timing with regard to real investments. It is not clear which of the two mechanisms is quantitatively more important.

There appears to be a strong tendency by the regulator and the general public to demand firms to increase transparency and release information as fast and frequent as possible. In addition, analysts continuously evaluate events of publicly traded firms and produce forecasts about future cash flows of these firms. Though, it is not obvious that an early release of public information is preferred by all investors, and that it leads to better risk-sharing or is Pareto improving. Our model is addressing these questions in the context of complete and incomplete markets.

3 Welfare value of information in complete asset markets

To assess the welfare value of public information when asset markets are complete and frictionless, we compare agents' ex-ante (i.e., before $t = 0^-$) expected utilities in the uninformed and informed economies.[16] We first address the special case of homogeneous beliefs, before tackling the more general case of heterogeneous beliefs.

3.1 Informed vs. uninformed economy: homogeneous beliefs

We assume that agents have identical beliefs, $p_A(s) = p_B(s)$, $\forall s \in \Omega$. Taking the ratio of the FOCs (10), (5) of the two economies at time $t = 0$ yields,

$$\frac{\widehat{u}'_{A0}(s)}{\widehat{u}'_{B0}(s)} = \frac{\widehat{\lambda}}{\lambda} \frac{u'_{A0}}{u'_{B0}},$$

or the ratio of agents' marginal utilities at $t = 0$ is state-independent, $\frac{\widehat{u}'_{A0}(s_1)}{\widehat{u}'_{B0}(s_1)} = \frac{\widehat{u}'_{A0}(s_2)}{\widehat{u}'_{B0}(s_2)}$, $\forall s_1, s_2 \in \Omega$. Because marginal utilities are monotone functions of consumptions, this relationship implies that agents A and B must exhibit the same ordering of time-zero contingent consumptions in the

[16]The asset market is indeed complete for both uninformed and informed economies.

informed economy,

$$\widehat{c}_{A0}(s_1) \geq \widehat{c}_{A0}(s_2) \Longleftrightarrow \widehat{c}_{B0}(s_1) \geq \widehat{c}_{B0}(s_2) \quad \forall s_1, s_2 \in \Omega.$$

Combining this observation with the state-independent resource constraints at $t = 0$ in (10), $\widehat{c}_{A0}(s_1) + \widehat{c}_{B0}(s_1) = e_0 = \widehat{c}_{A0}(s_2) + \widehat{c}_{B0}(s_2)$, it must be that time-zero consumptions of both agents are state-independent in the informed economy,

$$\widehat{c}_{A0}(s) = \widehat{c}_{A0}, \qquad \widehat{c}_{B0}(s) = \widehat{c}_{B0}, \quad \forall s \in \Omega.$$

The equilibrium systems (5) and (10) of respectively uninformed and informed economies are identical when agents have same belief. Under the technical assumption of the equilibrium's uniqueness,[17] we have,

Proposition 1 (Information irrelevance for homogeneous beliefs) *Assume (i) asset markets are complete and (ii) agents have identical beliefs about the prospects of future aggregate outputs. The equilibrium, and thus each agent's optimal consumption plan and expected utility, are identical in the uninformed and informed economies.*

$$\widehat{EU}_A = EU_A, \qquad \widehat{EU}_B = EU_B.$$

Therefore, early public information releases about future endowments have no ex-ante (i.e., before $t = 0^-$) value to welfare in homogeneous-belief complete-market environment.

The intuition behind this result is as follows. Because asset markets are complete, agents ex-ante can optimally share risk by trading full set of contracts contingent on any possible outcomes of the economy. Since agents already agree on the prospects of these future outcomes, the arrival of a signal (informing any specific outcome s) ex-post will not take agents by "surprise" beyond what is covered by their original optimal contingent consumption plans. Thus ex-ante, the possibility of a signal's arrival will not add value to agents' utilities. To place this intuition into perspective, suppose that after agents have traded at $t = 0^-$ to achieve the optimal consumption plans $\{c_{A0}, c_{A1}(s)\}$, $\{c_{B0}, c_{B1}(s)\}$ under the uninformed (no-signal) premise (but before agents make any actual consumptions) agents become aware that the signal s is going to be released before $t = 0$. Agents

[17] Our Assumption 2 below on the proportional endowments ensures uniqueness of the equilibrium. For details, see Mas-Colell et al. (1995).

then are allowed to re-trade if they think that any other feasible and state-specific consumption plans $\{\widehat{c}_{A0}(s), \widehat{c}_{A1}(s)\}$, $\{\widehat{c}_{B0}(s), \widehat{c}_{B1}(s)\}$ deliver higher expected utilities. It turns out that, agents will not re-trade. By virtue of Proposition 1, no new consumption plans can improve agents' utilities compared to the uninformed optimal consumption plans $\{c_{A0}, c_{A1}(s)\}$, $\{c_{B0}, c_{B1}(s)\}$ already in place. This intuition also indicates that the public information irrelevance arises because of neither the lack of uncertainty in time-zero aggregate endowment, nor the perfect quality of the signal. In section 6, we extend the public information irrelevance result to settings in which initial endowments are uncertain and signals are imperfect.

The Proposition 1 is similar to, but goes beyond, the results of Jaffe (1975), Ng (1975), and Hakansson et al. (1982). These authors show that an unexpected arrival of information does not induce re-trade, once agents have already settled on the equilibrium based on their previous expectation that no signal would ever be released prior to $t = 0$. Effectively, their no-trade result arises from endowments readily in equilibrium.[18] The FOCs (and budget constraints) of the no-signal equilibrium hold for each state s, and thus agents do not need to trade away from their positions after observing unexpected perfect signal. In contrast, Proposition 1 asserts the public information irrelevance even when agents expect the information to be released early. As a result, our analysis involves the ex-ante (pre-signal) expected utilities, rather than ex-post utility for each state s separately. In the expectation, signals about future state of the economy do not add value to either agent, compared to the setting in which no signals are ever available before $t = 1$. Proposition 1 applies for general additively separable preferences as well as imperfect signals, as explained in Section 6.

3.2 Informed vs. uninformed economy: heterogeneous beliefs

When agents have different expectations about future endowments, these different beliefs are not canceled in the equilibrium equations at $t = 0$ in (10), and affect equilibrium consumption plans in both periods $t \in \{0, 1\}$.

Expected utilities: period $t = 1$

We concentrate first on the utilities that agents expect to derive from their consumptions at time $t = 1$. We start with the observation that the Pareto weight, in (5) and (10), characterizes the

[18] After reaching the no-signal equilibrium, effectively and contractually, agents are endowed with equilibrium consumptions $\{c_{At}\}$, $\{c_{Bt}\}$, $t \in 0, 1$.

relative importance of agent B in the economy.[19] In equilibrium, the Pareto weight is determined by the marginal utility ratio of the agents A and B. This observation can be quantified further in the following statement on a monotone relationship between equilibrium consumptions and Pareto weight,

Lemma 1 *Assume that agents have strictly increasing and concave utilities of consumption, and aggregate endowments and beliefs are fixed. Agent A's (agent B's) equilibrium consumption unambiguously decreases (increases) with the Pareto weight, at all times and states, in all economies under consideration,*

$$\frac{\partial c_{At}(s)}{\partial \lambda} < 0, \quad \frac{\partial \widehat{c}_{At}(s)}{\partial \widehat{\lambda}} < 0; \qquad \frac{\partial c_{Bt}(s)}{\partial \lambda} > 0, \quad \frac{\partial \widehat{c}_{Bt}(s)}{\partial \widehat{\lambda}} > 0; \qquad \forall t \in \{0,1\}, \forall s \in \Omega.$$

Note that the Pareto weight is an endogenous quantity in the equilibrium. When aggregate endowments and beliefs are keep unchanged, the Pareto weight varies with agents' relative endowments via wealth effect (see (28)). The intuition underlying the above monotone relationships lies with market completeness and fits the above observation in comparative statics sense. All else being equal, a larger equilibrium Pareto weight implies that agent A has higher marginal utility in all states and time. Then agent A must consume relatively less in all states and time.[20] Formally, we derive these monotone relationships by taking the partial derivative with respect to the Pareto weight of the FOCs. For the uninformed economy (5), this operation yields, for all states $s \in \Omega$,

$$\frac{\partial c_{A0}}{\partial \lambda} = \frac{1}{\lambda}\left[\frac{u''_{A0}}{u'_{A0}} + \frac{u''_{B0}}{u'_{B0}}\right]^{-1} < 0, \qquad \frac{\partial c_{A1}(s)}{\partial \lambda} = \frac{1}{\lambda}\left[\frac{u''_{A1}(s)}{u'_{A1}(s)} + \frac{u''_{B1}(s)}{u'_{B1}(s)}\right]^{-1} < 0$$

$$\frac{\partial c_{B0}}{\partial \lambda} = \frac{-1}{\lambda}\left[\frac{u''_{A0}}{u'_{A0}} + \frac{u''_{B0}}{u'_{B0}}\right]^{-1} > 0, \qquad \frac{\partial c_{B1}(s)}{\partial \lambda} = \frac{-1}{\lambda}\left[\frac{u''_{A1}(s)}{u'_{A1}(s)} + \frac{u''_{B1}(s)}{u'_{B1}(s)}\right]^{-1} > 0.$$

Clearly, equilibrium consumptions are decreasing for A, and increasing for B, in λ because agents' utilities are increasing and concave. Similarly, for the informed economy (10), for all states $s \in \Omega$,

$$\frac{\partial \widehat{c}_{A0}(s)}{\partial \widehat{\lambda}} = \frac{1}{\widehat{\lambda}}\left[\frac{\widehat{u}''_{A0}(s)}{\widehat{u}'_{A0}(s)} + \frac{\widehat{u}''_{B0}(s)}{\widehat{u}'_{B0}(s)}\right]^{-1} < 0, \qquad \frac{\partial \widehat{c}_{A1}(s)}{\partial \widehat{\lambda}} = \frac{1}{\widehat{\lambda}}\left[\frac{\widehat{u}''_{A1}(s)}{\widehat{u}'_{A1}(s)} + \frac{\widehat{u}''_{B1}(s)}{\widehat{u}'_{B1}(s)}\right]^{-1} < 0, \qquad (16)$$

[19] Recall that we can arrive at the complete-market FOCs (5) and (10) through the construction of a representative agent, whose utility is $u(c) \sim u_A(c_A) + \lambda u_B(c_B)$. That is, the Pareto weight characterize the agent B's contribution to the representative agent.

[20] We keep aggregate resource in all states and time unchanged. Pareto weight varies as we vary individual agents' endowments.

$$\frac{\partial \widehat{c}_{B0}(s)}{\partial \widehat{\lambda}} = \frac{-1}{\widehat{\lambda}}\left[\frac{\widehat{u}''_{A0}(s)}{\widehat{u}'_{A0}(s)} + \frac{\widehat{u}''_{B0}(s)}{\widehat{u}'_{B0}(s)}\right]^{-1} > 0, \qquad \frac{\partial \widehat{c}_{B1}(s)}{\partial \widehat{\lambda}} = \frac{-1}{\widehat{\lambda}}\left[\frac{\widehat{u}''_{A1}(s)}{\widehat{u}'_{A1}(s)} + \frac{\widehat{u}''_{B1}(s)}{\widehat{u}'_{B1}(s)}\right]^{-1} > 0.$$

Next, we observe that at $t = 1$, the equilibrium structure of the informed and uninformed economies are identical. This is because asset markets are complete and agents perfectly share aggregate endowment risks by devising and contracting their contingent consumptions at $t = 1$ in both economies. This observation is quantified by the fact that the equilibrium equation systems at $t = 1$ are identical up to the Pareto weight in the informed and uninformed economies, as seen in (5) and (10). Lemma (1) indicates that Pareto weights are key to a welfare comparison across different economies, at least for utilities of consumptions at $t = 1$.

Lemma 2 *Assume that agents have strictly increasing and concave utilities of consumption, and aggregate endowments and beliefs are fixed. If Pareto weight is lower (higher) in the uninformed than in the informed economy, then agent A's expected utility of time-one consumption is higher (lower) in the uninformed than in the informed economy. The opposite holds for agent B's utility.*

$$\lambda \leq \widehat{\lambda} \Longrightarrow \begin{cases} EU_{A1} \geq \widehat{EU}_{A1}, \\ EU_{B1} \leq \widehat{EU}_{B1}, \end{cases} \qquad \lambda \geq \widehat{\lambda} \Longrightarrow \begin{cases} EU_{A1} \leq \widehat{EU}_{A1}, \\ EU_{B1} \geq \widehat{EU}_{B1}, \end{cases} \tag{17}$$

where EU_{I1} denotes agent I's expected utility at $t = 0$ of time-one consumption,

$$EU_{I1} \equiv \sum_{s\in\Omega} p_I(s)\beta_I u(c_{I1}(s)), \qquad \widehat{EU}_{I1} \equiv \sum_{s\in\Omega} p_I(s)\beta_I u(\widehat{c}_{I1}(s)). \;\; I \in \{A, B\}. \tag{18}$$

Expected utilities: period $t = 0$

We next concentrate on the utilities that agents expect to derive from their consumptions at time $t = 0$. We observe that at $t = 0$, the equilibrium structure of the informed and uninformed economies are very different. In (10), the arrival of signals in the informed economy fosters optimal state-contingent consumptions already at time $t = 0$, while this contingency is impossible in the uninformed economy (5). Consequently, the result of Lemma 1 can not be directly employed to derive comparative statics for utilities of time-zero consumptions in the uninformed and informed economies.

The key to an unambiguous welfare analysis at $t = 0$ lies with the convexity of agents' utilities of time-zero consumptions. We compute, for each state s, the first and second-order partial derivatives of the informed economy's equilibrium consumption at $t=0$ with respect to the Pareto weight,

$$\frac{\partial \widehat{c}_{A0}(s)}{\partial \widehat{\lambda}(s)} = \frac{-1}{\widehat{\lambda}(s)} \left[\frac{\gamma_A}{\widehat{c}_{A0}(s)} + \frac{\gamma_B}{\widehat{c}_{B0}(s)} \right]^{-1}, \qquad \text{where} \quad \widehat{\lambda}(s) \equiv \frac{p_B(s)}{p_A(s)} \widehat{\lambda},$$

$$\frac{\partial^2 \widehat{c}_{A0}(s)}{\partial \left(\widehat{\lambda}(s)\right)^2} = \frac{1}{\left(\widehat{\lambda}(s)\right)^2} \left[\frac{\gamma_A}{\widehat{c}_{A0}(s)} + \frac{\gamma_B}{\widehat{c}_{B0}(s)} \right]^{-3} \left[\frac{\gamma_A}{\widehat{c}_{A0}(s)} \left(\frac{\gamma_A + 1}{\widehat{c}_{A0}(s)} + \frac{2\gamma_B}{\widehat{c}_{B0}(s)} \right) + \frac{\gamma_B(\gamma_B - 1)}{\widehat{c}^2_{B0}(s)} \right]. \quad (19)$$

Throughout the above derivative analysis, we keep aggregate endowments and beliefs unchanged. The introduction of the state-dependent parameter $\widehat{\lambda}(s)$ simplifies our exposition and is for pure convenience. For each state s, $\widehat{\lambda}(s)$ differs from the Pareto weight $\widehat{\lambda}$ by a non-material multiplicative factor.[21] It is clear from (19) that when $\gamma_B \geq 1$, A's time-zero equilibrium state-contingent consumption $\widehat{c}_{A0}(s)$ is a strictly convex function of the Pareto weight for all states s. Equivalently, consumption good's markets clear at $t=0$ (2), so that $\frac{\partial \widehat{c}_{B0}(s)}{\partial \widehat{\lambda}(s)} = -\frac{\partial \widehat{c}_{A0}(s)}{\partial \widehat{\lambda}(s)}$ and $\frac{\partial^2 \widehat{c}_{B0}(s)}{\partial \left(\widehat{\lambda}(s)\right)^2} = -\frac{\partial^2 \widehat{c}_{A0}(s)}{\partial \left(\widehat{\lambda}(s)\right)^2}$. Therefore,

$$\frac{\partial \widehat{c}_{A0}(s)}{\partial \widehat{\lambda}(s)} > 0; \qquad \text{and} \qquad \frac{\partial \widehat{c}_{B0}(s)}{\partial \widehat{\lambda}(s)} < 0;$$

and $\gamma_B \geq 1$ is a sufficient condition for $\widehat{c}_{B0}(s)$ to be a strictly concave function of the Pareto weight for all state s,

$$\gamma_B \geq 1 \quad \Longrightarrow \quad \frac{\partial^2 \widehat{c}_{A0}(s)}{\partial \left(\widehat{\lambda}(s)\right)^2} > 0; \quad \text{and} \quad \frac{\partial^2 \widehat{c}_{B0}(s)}{\partial \left(\widehat{\lambda}(s)\right)^2} < 0, \quad \forall s \in \Omega.$$

The mechanism underlying this convexity is as follows. All else being equal, a drop in agent B's time-zero consumption means an increase in B's marginal utility, a decrease in A's marginal utility, and hence a surge in the Pareto weight as asserted by Lemma 1. The condition $\gamma_B \geq 1$ suffices for the Pareto weight to increase (weakly) faster than linearly with agent B's time-zero consumption, i.e., the convexity in $\widehat{c}_{B0}(s)$.[22] To put it the other way around, when $\gamma_B \geq 1$, B's time-zero consumption in the informed economy is a strictly concave function of the Pareto weight for all states s. Because the consumption good market clears in each state, A's time-zero consumption is a strictly convex function of the Pareto weight.

Similar to the above argument, when $\gamma_B \geq 1$, for each state s, agent A's indirect utility at $t=0$ is also unambiguously strictly convex in the Pareto weight. Indeed, taking the second derivative of

[21] That is, for each state s, the state-contingent equilibrium consumption $\widehat{c}_{A0}(s)$, (or $\widehat{c}_{B0}(s)$) depends on $\widehat{\lambda}(s)$ and $\widehat{\lambda}$ in identical way. This is because the factor $\frac{p_B(s)}{p_A(s)}$ that sets $\widehat{\lambda}(s)$ and $\widehat{\lambda}$ apart is constant within each state s.

[22] Quantitatively, for each state s, the FOC (10) and resource constraint $\widehat{c}_{A0}(s) + \widehat{c}_{B0}(s) = \widehat{e}_0$ at $t=0$ imply the relationship, $\widehat{\lambda}(s) = \left(\widehat{c}_{B0}(s)\right)^{\gamma_B} \left(e_0 - \widehat{c}_{B0}(s)\right)^{-\gamma_A}$. When $\gamma_B \geq 1$, $\widehat{\lambda}(s)$ is convex in $\widehat{c}_{B0}(s)$.

$u(\widehat{c}_{A0}(s)$ with respect to $\widehat{\lambda}(s)$, using (16) and (19), yields for all s,

$$\frac{\partial^2 u(\widehat{c}_{A0}(s))}{\partial\left(\widehat{\lambda}(s)\right)^2} = \frac{[\widehat{c}_{A0}(s)]^{-\gamma_A}}{\left(\widehat{\lambda}(s)\right)^2}\left[\frac{\gamma_A}{\widehat{c}_{A0}(s)}+\frac{\gamma_B}{\widehat{c}_{B0}(s)}\right]^{-3}\left[\frac{\gamma_A}{\widehat{c}_{A0}(s)}\left(\frac{1}{\widehat{c}_{A0}(s)}+\frac{\gamma_B}{\widehat{c}_{B0}(s)}\right)+\frac{\gamma_B(\gamma_B-1)}{\widehat{c}^2_{B0}(s)}\right]. \tag{20}$$

By a symmetric argument, the dependence of agent B's time-zero consumption $\widehat{c}_{B0}(s)$ and utility $u(\widehat{c}_{B0}(s))$ on $\frac{1}{\widehat{\lambda}(s)} = \frac{p_A(s)}{p_B(s)}\frac{1}{\widehat{\lambda}}$ exactly mirrors the dependence of $\widehat{c}_{A0}(s)$ and $u(\widehat{c}_{A0}(s))$ on $\widehat{\lambda}(s)$.[23] Consequently, we see that $\gamma_A \geq 1$ is a sufficient condition for agent B's time-zero utility to be unambiguously strictly convex in $\frac{1}{\widehat{\lambda}(s)}$ in all states s of the informed economy. We see this convexity explicitly in the following expression, which mirrors (20),

$$\frac{\partial^2 u(\widehat{c}_{B0}(s))}{\partial\left(\frac{1}{\widehat{\lambda}(s)}\right)^2} = \frac{[\widehat{c}_{B0}(s)]^{-\gamma_A}}{\left(\widehat{\lambda}(s)\right)^{-2}}\left[\frac{\gamma_A}{\widehat{c}_{A0}(s)}+\frac{\gamma_B}{\widehat{c}_{B0}(s)}\right]^{-3}\left[\frac{\gamma_B}{\widehat{c}_{B0}(s)}\left(\frac{1}{\widehat{c}_{B0}(s)}+\frac{\gamma_A}{\widehat{c}_{A0}(s)}\right)+\frac{\gamma_A(\gamma_A-1)}{\widehat{c}^2_{A0}(s)}\right]. \tag{21}$$

A direct application of Jensen's inequality on agents A's and B's strictly convex utility functions results in a comparative statics between the informed and uninformed economies in period $t=0$. To facilitate this application, we note that, for each state s, time-zero state-contingent utility $\widehat{u}_{A0}(s)$ is an (indirect) function of $\widehat{\lambda}(s)$.[24] Accordingly, in what follows, we employ notations $\widehat{u}_{A0}(s) = \widehat{u}_{A0}\left(\widehat{\lambda}(s)\right)$ and $\widehat{u}_{B0}(s) = \widehat{u}_{B0}\left(\widehat{\lambda}(s)^{-1}\right)$.

Assuming $\gamma_B \geq 1$ and taking the average of (20) (under agent A's belief) yield Jensen's bound concerning A's expected utilities in period $t=0$, which is the counterpart of (18),

$$\begin{aligned}\widehat{EU}_{A0} &\equiv \textstyle\sum_{s\in\Omega} p_A(s)\widehat{u}_{A0}(s) = \sum_{s\in\Omega} p_A(s)\widehat{u}_{A0}\left(\widehat{\lambda}(s)\right) \\ &> \widehat{u}\left(\textstyle\sum_{s\in\Omega} p_A(s)\widehat{\lambda}(s)\right) = \widehat{u}\left(\sum_{s\in\Omega} p_A(s)\frac{p_B(s)}{p_A(s)}\widehat{\lambda}\right) = \widehat{u}_{A0}\left(\widehat{\lambda}\right) = u\left(c_{A0}\left(\widehat{\lambda}\right)\right).\end{aligned} \tag{22}$$

[23] To see this symmetry, we note that the FOC at time $t=0$ (10) can be written in two equivalent ways, for all s,

$$\widehat{u}_{A0}(s) = \frac{p_B(s)}{p_A(s)}\widehat{\lambda}\widehat{u}_{B0}(s) \equiv \widehat{\lambda}(s)\widehat{u}_{B0}(s) \iff \widehat{u}_{B0}(s) = \frac{p_A(s)}{p_B(s)}\frac{1}{\widehat{\lambda}}\widehat{u}_{A0}(s) \equiv \frac{1}{\widehat{\lambda}(s)}\widehat{u}_{A0}(s).$$

[24] This is because in equilibrium this time-zero utility's only argument $c_{A0}(s)$ is also a function of $\widehat{\lambda}(s)$. A similar observation applies for $\widehat{u}_{B0}(s)$ as an (indirect) function of $\frac{1}{\widehat{\lambda}(s)}$.

Similarly, assuming $\gamma_A \geq 1$ and taking taking the average of (21) (under agent B's belief) yields,

$$\widehat{EU}_{B0} \equiv \sum_{s\in\Omega} p_B(s)\widehat{u}_{B0}(s) = \sum_{s\in\Omega} p_B(s)\widehat{u}_{B0}\left(\frac{1}{\widehat{\lambda}(s)}\right)$$
$$> \widehat{u}\left(\sum_{s\in\Omega} p_B(s)\frac{1}{\widehat{\lambda}(s)}\right) = \widehat{u}\left(\sum_{s\in\Omega} p_B(s)\frac{p_A(s)}{p_B(s)}\frac{1}{\widehat{\lambda}}\right) = \widehat{u}_{B0}\left(\frac{1}{\widehat{\lambda}}\right) = u\left(c_{B0}\left(\widehat{\lambda}\right)\right). \tag{23}$$

As an implication of Lemma 1, when the uninformed economy's Pareto weight is larger, $\lambda \geq \widehat{\lambda}$, agent A's time-zero consumption and hence utility in the informed economy are higher than their counterparts in the uninformed economy,

$$\lambda \geq \widehat{\lambda} \Longrightarrow c_{A0}\left(\widehat{\lambda}\right) \geq c_{A0}\left(\lambda\right) \Longrightarrow u\left(c_{A0}(\widehat{\lambda})\right) \geq u\left(c_{A0}(\lambda)\right). \tag{24}$$

Symmetrically, also as a result of Lemma 1, when the informed economy's Pareto weight is larger, $\widehat{\lambda} \geq \lambda$, agent B's time-zero consumption and hence utility in the informed economy are higher than their counterparts in the uninformed economy,

$$\widehat{\lambda} \geq \lambda \Longrightarrow c_{B0}\left(\widehat{\lambda}\right) \geq c_{B0}\left(\lambda\right) \Longrightarrow u\left(c_{B0}(\widehat{\lambda})\right) \geq u\left(c_{B0}(\lambda)\right). \tag{25}$$

Combining (24) with Jensen's inequality (22), and similarly (25) with Jensen's inequality (23) motivate the following assumption and imply a comparative statics for agents A and B at time $t = 0$.

Assumption 1 (Risk aversions) *Assume that agents have CRRA utilities with relative risk aversions satisfying; $\gamma_A \geq 1$ and $\gamma_B \geq 1$.*

The version of this assumption for general additively separable utilities is reported in Section 6, equation (55).

Lemma 3 *Suppose that (i) asset markets are complete (ii) Assumption 1 holds and (iii) agents have heterogeneous beliefs about the prospect of future aggregate endowments. If the Pareto weight is lower (higher) in the uninformed than in the informed economy, then agent B's (agent A's) expected utility in period $t = 0$ is higher in the informed than in the uninformed economy,*

$$\lambda \leq \widehat{\lambda} \quad \Longrightarrow \quad \widehat{EU}_{B0} > u_{B0}; \qquad \widehat{\lambda} \leq \lambda \quad \Longrightarrow \quad \widehat{EU}_{A0} > u_{A0}.$$

Consequently, when the Pareto weights are the same in the two economies, both A's and B's expected utilities in period $t = 0$ are higher in the informed than in the uninformed economy,

$$\lambda = \widehat{\lambda} \Longrightarrow \quad \widehat{EU}_{A0} > u_{A0}; \quad \text{and} \quad \widehat{EU}_{B0} > u_{B0}.$$

We first note that as a sufficient condition, Assumption (1) can be weakened substantially (see Section 6). It can also be reformulated to accommodate non-CRRA preferences.

The above Lemma aims to extend the comparative statics at $t = 1$ of Lemma 2 to period $t = 0$. Though, the difference in time-zero equilibrium structures of the informed and uninformed economies is profound and goes beyond the difference of λ vs. $\widehat{\lambda}$. As a result, the comparative statics at $t = 0$ arises only under additional assumptions. The gist behind the result reported in Lemma 3 is Jensen's inequality, which applies when agents' preferences are sufficiently non-linear, $\gamma_A, \gamma_B > 1$, our additional assumptions mentioned earlier.[25] Even with these additional assumptions in place, the time-zero comparative statics can be obtained only for a single agent at a time when the two Pareto weights differ, $\lambda \neq \widehat{\lambda}$. The difficulty is not in achieving both (22) and (23), but in having *simultaneously* $c_{A0}\left(\widehat{\lambda}\right) \geq c_{A0}(\lambda)$ *and* $c_{B0}\left(\widehat{\lambda}\right) \geq c_{B0}(\lambda)$ in (24) and (25). Lemma 3 renders this simultaneity unlikely in heterogeneous-belief setting. In fact, only when $\lambda = \widehat{\lambda}$ such unambiguous comparative statics exists. But when agents differ in their beliefs, the equality of Pareto weights in the uninformed and informed economies does not generically hold.

This is an indication of the challenge in establishing the welfare value of public information, unambiguous and comprehensive for entire periods $t \in \{0, 1\}$, when agents disagree about future prospect of the economy. We take up this task next.

Expected utilities: combining periods $t = 0$ and $t = 1$

As we argue above, except in the unlikely case of identical Pareto weights, $\lambda = \widehat{\lambda}$, a unconditional and unambiguous welfare ordering for the uninformed and informed economies does not exist in the setting of heterogeneous beliefs.[26] We establish below a sufficient condition under which both agents' utilities are unambiguously higher in the informed than in the uninformed economy. Our

[25] Note that the assumptions $\gamma_A, \gamma_B \geq 1$ are sufficient, but not necessary, conditions.

[26] We note that the equality $\lambda = \widehat{\lambda}$ holds true in the setting of homogeneous beliefs, in which case the welfare is identical in the uninformed and informed economies as seen in Proposition 1. When agents differ in their beliefs, numerical counterexamples can be easily constructed to show ambiguous implications on welfare, while maintaining conditions $\gamma_A, \gamma_B > 1$. That is, one agent is strictly better off (the other strictly worse off) in the informed economy than in the uninformed economy, as measure by agents' ex-ante expected utilities.

unambiguous comparative statics holds in general setting wherein agents may disagree about future prospect of the economy, and does not require the unlikely identity $\lambda = \widehat{\lambda}$ of Lemma 3.

We start with an remark that for CRRA utilities, there is a close relationship between wealth and expected utilities. This relationship stems from a property of the power function (of marginal utilities), $u'_c = \frac{(1-\gamma_I)u}{c_I}$, and holds for both uninformed and informed economies. Because wealth finances consumption streams, agent I's initial wealth w_I is related to his expected utility EU_I. For the uninformed economy,

$$w_I = c_{I0} + \sum_{s\in\Omega} q(s)c_{I1}(s) = \frac{(1-\gamma_I)\,EU_I}{\sum_{s\in\Omega} p_I(s)u'_{I0}(s)}, \qquad I \in \{A, B\},$$

where we have used expressions (4) for AD prices $q(s)$, and EU_I is agent I's ex-ante expected utility (3), which also quantifies his welfare. Note that $EU_I = u_{I0} + EU_{I1}$; u_0 is the expected utility of time-zero consumption addressed by Lemma 3, and EU_{I1} is the expected utility of time-one consumption addressed by Lemma 2, for the uninformed economy. Combining the wealth ratio of the two agents with the FOC (5) yields the following relationship,

$$\frac{w_A}{w_B} = \frac{1-\gamma_A}{1-\gamma_B}\frac{1}{\lambda}\frac{EU_A}{EU_B}. \tag{26}$$

The intuition is simple. An agent's higher relative wealth translates, in expectation, into his higher relative utility. However, the translation is not necessarily linear, due to the presence of price effects captured by the Pareto weight λ. That is, prices vary endogenously with states; equilibrium consumption goods tend to be cheaper in states of high equilibrium consumptions. For the informed economy, the corresponding relationship between agent I's initial wealth $\widehat{w}_I$, expected utility $\widehat{EU}_I$ (7), AD prices $\widehat{q}_0$ (8), $\widehat{q}_1$ (9) reads,

$$\widehat{w}_I = \sum_{s\in\Omega} [\widehat{q}_0(s)\widehat{c}_{I0}(s) + \widehat{q}_1(s)\widehat{c}_{I1}(s)] = \frac{(1-\gamma_I)\,\widehat{EU}_I}{\sum_{s\in\Omega} p_I(s)\widehat{u}'_{I0}(s)}, \qquad I \in \{A, B\},$$

which implies,

$$\frac{\widehat{w}_A}{\widehat{w}_B} = \frac{1-\gamma_A}{1-\gamma_B}\frac{1}{\widehat{\lambda}}\frac{\widehat{EU}_A}{\widehat{EU}_B}. \tag{27}$$

Again, agent I's ex-ante (pre-signal) expected utility $\widehat{EU}_I = \widehat{EU}_{I0} + \widehat{EU}_{I1}$; $\widehat{EU}_{I0}$ is the expected utility of time-zero consumption addressed by Lemma 3, and EU_{I1} is the expected utility of time-one consumption addressed by Lemma 2, for the informed economy.

Combining (26) with (27) gives a simple cross-economy relationship,

$$\frac{\widehat{EU}_A}{EU_A} = \left(\frac{\widehat{w}_A/w_A}{\widehat{w}_B/w_B}\right) \frac{\widehat{\lambda}}{\lambda} \frac{\widehat{EU}_B}{EU_B} \tag{28}$$

This equation provides us with deeper insight into the limitation of Lemma 3. When $\widehat{\lambda} \geq \lambda$, Lemmas 2 and 3 imply that $\widehat{EU}_B > EU_B$, but this is not enough to assure $\widehat{EU}_A > EU_A$, again because of the endogenous price effect concisely captured by the wealth ratio $\frac{\widehat{w}_A/w_A}{\widehat{w}_B/w_B}$. It could be that, when B is relatively more influential in the informed than in the uninformed economy, (that is, $\widehat{\lambda} \geq \lambda$, and thus $\widehat{EU}_B \geq EU_B$), his relative wealth is even more dominant, $\left(\frac{\widehat{w}_A/w_A}{\widehat{w}_B/w_B} \ll 1\right)$, such that $\widehat{EU}_A \leq EU_A$ by (28). This insight then suggests that one way to obtain an unambiguous welfare result is to control for the variation in agents' relative wealth induced by endogenous equilibrium prices. We thus make the following assumption on agents' endowments.

Assumption 2 (Proportional endowments) *At initial time $t = 0^-$, let agents A and B be endowed respectively with fraction k and $1-k$ of the endowment tree of the economy,*

$$e_{At}(s) = \frac{k}{k+1} e_t(s), \qquad e_{Bt}(s) = \frac{1}{k+1} e_t(s), \quad \forall t \in \{0,1\}, \forall s \in \Omega. \tag{29}$$

for some constant $k > 0$.

Note that proportional endowments limit agents' risk-sharing incentives and benefits. It is also part of a sufficient condition for the uniqueness of the competitive equilibrium in an endowment economy with multiple agents (Mas-Colell et al. (1995), Chapter 17). In section 6 we discuss the merits and alternatives to this assumption.

An agent's wealth is the value of his endowments priced at equilibrium prices, as in budget constraints (3), (7). Thus, (29) implies that the wealth ratio is independent of equilibrium prices,

$$\frac{w_A}{w_B} = \frac{\widehat{w}_A}{\widehat{w}_B} = k.$$

Hence, Assumption 2 eliminates the endogenous price effect on the equilibrium wealth ratio across different economies, rendering $\frac{\widehat{w}_A/w_A}{\widehat{w}_B/w_B} = 1$ in (28), or

$$\frac{\widehat{EU}_A}{EU_A} = \frac{\widehat{\lambda}}{\lambda} \frac{\widehat{EU}_B}{EU_B} \tag{30}$$

holds under Assumption 2. The combination of Lemma 2, Lemma 3, and the identity (30) yields a key unambiguous comparative statics for the uninformed and informed economies,

Proposition 2 (Information relevance for heterogeneous beliefs) *Suppose that (i) asset markets are complete (ii) Assumption 1 holds ($\gamma_A \geq 1$, $\gamma_B \geq 1$), and (iii) Assumption 2 holds ($e_{At}(s)/e_{Bt}(s) = k$, $\forall t, s$). When agents differs in their beliefs about the prospect of future endowments, $p_A \neq p_B$, both agents have higher expected utilities in the informed than in the uninformed economy.*

$$\widehat{EU}_A > EU_A, \qquad \widehat{EU}_B > EU_B.$$

Therefore, early public information releases about future endowments have an unambiguously positive ex-ante (i.e., before $t = 0^-$) value to welfare in heterogeneous-belief complete-market environment.

Evidently the welfare value of public information arises only under additional assumptions on agents' preferences and endowments. In particular, the assumption on proportional endowments, or some weaker version of it, is crucial to deliver this welfare result. Suppose, on the contrary, that agents' endowments, $\{e_{A1}(s)\}$, $\{e_{A1}(s)\}$, are not tightly related, so that aggregate risk does not dominate individual endowment risks. As a result, they introduce higher degree of uncertainties (beyond aggregate uncertainty) into our setting. Even when we maintain asset market completeness, these uncertainties imply arbitrary results through the price effect mentioned above. Indeed, numerical examples can be easily constructed to show that the equilibrium wealth ratio $\frac{\widehat{w}_A/w_A}{\widehat{w}_B/w_B}$ in (28) can either be larger or smaller than unity. In Section 6 we discuss and provide weaker versions of the assumptions underlying Proposition 2 while maintaining unambiguous welfare implications. For now we note that, by contrasting the results of Propositions 1 and 2, it is clear that when asset markets are complete, public information is potentially Pareto improving only when agents hold different beliefs about future aggregate risk. Given sufficient positive correlation of individual endowments, heterogeneous beliefs are the key factor that allows all agents to benefit (in the expectation) from asset trades on the incoming signals of the future economy state.

4 Welfare value of information in incomplete asset markets

To assess the welfare value of public information under various asset market conditions, we compare agents' ex-ante (i.e., pre-signal) expected utilities in the informed and too-early informed economies.[27] To produce a comparative statics, without loss of generality, we fix the belief of agent A and vary that of agent B. We first identify a particular belief of B, under which the equilibria of the two economies are identical. When agent B deviates from that particular belief, moving across these two economies, we show that one agent is strictly better off (the other strictly worse off) as measured by their expected utilities. Consequently, the availability of asset markets plays a key role to render the implication of public information on welfare ambiguous.

Informed vs. too-early informed economy: the convergence

Given agent A's belief $\{p_A(s)\}$, we consider first a particular belief of agent B,

$$p_B^*(s) = \frac{p_A(s)\overline{\lambda}(s)}{\widehat{\lambda}^*}, \qquad \forall s \in \Omega, \tag{31}$$

$$\text{where:} \qquad \widehat{\lambda}^* \equiv \sum_{s\in\Omega} p_A(s)\overline{\lambda}(s). \tag{32}$$

Note that $\{p_B^*(s)\}$ (i) are constructed solely from the state-specific Pareto weights $\{\overline{\lambda}(s)\}$ of the too-early informed economy (14) and agent A's belief, (ii) are all positive and (iii) sum to one.[28] For these heterogeneous beliefs $\{p_A, p_B^*\}$, the informed economy equilibrium system (10) becomes,

$$\widehat{u}'_{A0}(s) = \frac{\widehat{\lambda}}{\widehat{\lambda}^*}\overline{\lambda}(s)\widehat{u}'_{B0}(s), \qquad \widehat{u}'_{A1}(s) = \frac{\widehat{\lambda}}{\widehat{\lambda}^*}\overline{\lambda}(s)\widehat{u}'_{B1}(s). \tag{33}$$

If $\widehat{\lambda}$ equals $\widehat{\lambda}^*$, the above system is identical to the FOC system (14) of the too-early informed economy. Therefore too-early informed economy's equilibrium consumptions $\{\overline{c}_{It}(s)\}$, $t \in \{0,1\}$, $s \in \Omega$, together with $\widehat{\lambda}$ of (32), solve the informed economy's FOCs when agent B has the particular belief $\{p_B^*(s)\}$ (31). The informed economy's resource constraints evidently also hold in all time and states because the aggregate endowments are identical in the two economies.[29] Moreover, taking the weighted average under A's belief of the too-early informed economy's budget constraint (15) yields,

$$\sum_{s\in\Omega} p_A(s)\frac{\partial u\left(\overline{c}_{A0}(s)\right)}{\partial \overline{c}_{A0}(s)}\overline{c}_{A0}(s) + \sum_{s\in\Omega} p_A(s)\beta_A\frac{\partial u\left(\overline{c}_{A1}(s)\right)}{\partial \overline{c}_{A1}(s)}\overline{c}_{A1}(s)$$

[27] Information is expected to be released in both economies. For the too-early informed economy, information arrives before asset markets open, whereas for the informed economy, information arrives after asset markets open.

[28] The Radon-Nikodym derivative characterizing these heterogeneous beliefs is the ratio of Pareto weights in the two economies, $\xi^*(s) \equiv \frac{p_B^*(s)}{p_A(s)} = \frac{\overline{\lambda}(s)}{\widehat{\lambda}^*}$, $\forall s \in \Omega$.

[29] That is, when $\overline{c}_{It}(s)$ is identified with informed economy's equilibrium solution $\widehat{c}_{It}(s)|_{p_B=p_B^*}$, $I \in \{A, B\}$ for particular belief configuration $\{p_A, p_B^*\}$, we have $\overline{c}_{At}(s) + \overline{c}_{Bt}(s) = e_t(s)$.

$$= \sum_{s\in\Omega} p_A(s) \frac{\partial u\left(\bar{c}_{A0}(s)\right)}{\partial \bar{c}_{A0}(s)} e_{A0} + \sum_{s\in\Omega} p_A(s)\beta_A \frac{\partial u\left(\bar{c}_{A1}(s)\right)}{\partial \bar{c}_{A1}(s)} e_{A1}(s).$$

This is the informed economy's budget constraint (11) evaluated at the too-early informed economy's equilibrium consumptions $\{\bar{c}_{It}(s)\}$, $I \in \{A, B\}, t \in \{0, 1\}$. It is worthwhile to observe that the too-early informed economy's equilibrium consumptions $\{\bar{c}_{It}(s)\}$ does not depend on either agents' beliefs. The particular belief p_B^* hence offsets and eliminates the effects of all subjective beliefs on the informed economy's equilibrium. The above analysis shows that the equilibria of the two economies converge in this particular belief configuration.

Lemma 4 *Given the heterogeneous beliefs $\{p_A, p_B^*\}$, where p_B^* is specified in (31), the informed and too-early informed economies have identical equilibrium consumption allocations,*

$$\widehat{c}_{It}(s)\big|_{\{p_B=p_B^*\}} = \bar{c}_{It}(s), \qquad \forall I \in \{A, B\}, t \in \{0, 1\}, s \in \Omega.$$

The Pareto weights in the two economies are related by (32). Consequently, asset market frictions have no impact on agents' welfare, as measured by their ex-ante (pre-signal) expected utilities,

$$\left.\widehat{EU}_A\right|_{\{p_B=p_B^*\}} = \left.\overline{EU}_A\right|_{\{p_B=p_B^*\}}, \qquad \left.\widehat{EU}_B\right|_{\{p_B=p_B^*\}} = \left.\overline{EU}_B\right|_{\{p_B=p_B^*\}}.$$

The intuition behind this asset-market irrelevance is as follows. Generally, agents benefit from trading contingent assets on incoming signals (or other contractible realizations) because they can share the associated risk ex-ante by way of smoothing contingent consumptions across the states to be realized. When asset markets are complete, the risk is perfectly shared and agents' marginal utilities are equalized in all states. However, when agent B's expectation is related to that of A's by (31), B believes that he is able to achieve the perfect risk sharing with A (i.e., equalizing their marginal utilities) without trading endowments across states. As a result, in term of utility, there are no benefits to either agents between (i) trading contingent assets before the arrival of signal and (ii) waiting for the realization of the signal before executing signal-specific trades.

In terms of asset trades, there are differences. In the informed economy with beliefs $\{p_A, p_B^*\}$, agent A (agent B), at time $t = 0^-$, buys $\bar{c}_{A0}(s)$ ($\bar{c}_{B0}(s)$) units of AD securities $\widehat{q}_0(s)$ and $\bar{c}_{A1}(s)$ ($\bar{c}_{B1}(s)$) units of AD securities $\widehat{q}_1(s)$, for each $s \in \Omega$. For each state s, these AD securities holdings $\{\bar{c}_{It}(s)\}$ are financed entirely by the endowments $e_{A0}(s)$, $e_{A1}(s)$ (and $e_{B0}(s)$, $e_{B1}(s)$) of that same state s. These asset trades deliver contingent consumptions $\{\bar{c}_{A0}(s), \bar{c}_{A1}(s)\}$ to A, and

$\{\overline{c}_{A0}(s), \overline{c}_{A1}(s)\}$ to B, which are identical to the optimal consumption plans agents achieve in the too-early informed economy.

By symmetric arguments, given agent B's belief $\{p_B(s)\}$, when A has a particular belief defined by,

$$p_A^*(s) = \frac{p_B(s)\frac{1}{\overline{\lambda}(s)}}{\sum_{s\in\Omega} p_B(s)\frac{1}{\overline{\lambda}(s)}}, \qquad \forall s \in \Omega, \tag{34}$$

the informed and too-early informed economies have identical equilibria. Consequently, asset market frictions have no impact on agents' welfare when their beliefs are $\{p_A^*, p_B\}$.

Obviously, either belief configuration $\{p_A, p_B^*\}$ and $\{p_A^*, p_B\}$ is special cases. We are interested in more generic setting in which B's belief differs from the particular distribution $\{p_B^*(s)\}$ (or A's belief differs from $\{p_A^*(s)\}$).

Informed vs. too-early informed economy: the divergence

To assess the impact of public information on agents' welfare in different asset market settings, we perform a differential analysis around the belief configurations for which the equilibria of informed and too-early informed economies converge. Without loss of generality, we take agent A's belief as given, and vary agent B's around (31),

$$p_B^*(s) \longrightarrow p_B(s) = p_B^*(s) + dp_B(s), \quad \text{s.t.} \quad \sum_{s\in\Omega} dp_B(s) = 0. \tag{35}$$

The last equation assure that new belief configurations $\{p_B(s)\}$ are properly normalized. Without loss of generality, we only vary agent B's belief in our analysis. The variations $\{dp_B(s)\}$ induce changes in the agents' consumptions and welfare along the equilibrium path. Therefore, the equilibrium equation systems (10) and (14) hold at all time. The differential analysis is fit to generate comparative statics for different economies under consideration.[30]

We first note that for each state s, within an economy type, the system of FOCs and resource constraint at $t = 0$ and $t = 1$ are similar.[31] We focus on the differential analysis at each period $t \in \{1, 2\}$ separately. For the informed economy, under the variation of agent B's belief (35), the

[30] A drawback of the differential analysis is that its applicability is limited to local exogenous variations. For the current paper, the differential analysis serves the purpose of demonstrating the relevance or irrelevance of public information unambiguously, albeit locally.

[31] For the informed economy (10), this system involves both $p_A(s)$ and $p_B(s)$ at both periods. For the too-early informed economy (14), this system involves neither $p_A(s)$ nor $p_B(s)$ at both periods.

change in agent B's expected utility in period t reads,

$$d\widehat{EU}_{Bt} = (\beta_B)^t \sum_{s' \in \Omega} \left. \frac{d\left[\sum_{s \in \Omega} p_B(s) u\left(\widehat{c}_{Bt}(s)\right)\right]}{dp_B(s')} \right|_{p_B = p_B^*} dp_B(s')$$

$$= (\beta_B)^t \sum_{s' \in \Omega} \left. \left[u\left(\widehat{c}_{Bt}(s')\right) + \sum_{s \in \Omega} p_B(s) \frac{du\left(\widehat{c}_{Bt}(s)\right)}{dp_B(s')} \right] \right|_{p_B = p_B^*} dp_B(s'),$$

and for the too-early informed economy,

$$d\overline{EU}_{Bt} = (\beta_B)^t \sum_{s' \in \Omega} \left. \frac{d\left[\sum_{s \in \Omega} p_B(s) u\left(\overline{c}_{Bt}(s)\right)\right]}{dp_B(s')} \right|_{p_B = p_B^*} dp_B(s') = (\beta_B)^t \sum_{s' \in \Omega} u\left(\overline{c}_{Bt}(s)\right)]$$

where the last equality arises from the property that the equilibrium consumptions in the too-early informed economy do not depend on agents' beliefs. Using the result from Lemma 4 that at p_B^*, $\widehat{c}_{Bt}(s) = \overline{c}_{Bt}(s)$, $\forall t, s$, the cross-economy change in B's expected utility in period t is,

$$d\widehat{EU}_{Bt} - d\overline{EU}_{Bt} = (\beta_B)^t \sum_{s', s \in \Omega} p_B(s) \left. \frac{du\left(\widehat{c}_{Bt}(s')\right)}{dp_B(s')} \right|_{p_B = p_B^*} dp_B(s'), \quad \forall t \in \{0, 1\}. \tag{36}$$

Similarly, for agent A, the corresponding relationship for the informed economy read,

$$d\widehat{EU}_{At} = (\beta_A)^t \sum_{s' \in \Omega} \left. \frac{d\left[\sum_{s \in \Omega} p_A(s) u\left(\widehat{c}_{At}(s)\right)\right]}{dp_B(s')} \right|_{p_B = p_B^*} dp_B(s') = (\beta_A)^t \sum_{s', s \in \Omega} p_A(s) \left. \frac{du\left(\widehat{c}_{At}(s)\right)}{dp_B(s')} \right|_{p_B = p_B^*} dp_B(s')$$

$$= (\beta_A)^t \sum_{s', s \in \Omega} \left. \left[p_A(s) \widehat{u}'_{At}(s) \frac{d\widehat{c}_{At}(s)}{dp_B(s')} \right] \right|_{p_B = p_B^*} dp_B(s') = -(\beta_B)^t \sum_{s', s \in \Omega} \left. \left[p_B(s) \widehat{\lambda} \widehat{u}'_{Bt}(s) \frac{d\widehat{c}_{Bt}(s)}{dp_B(s')} \right] \right|_{p_B = p_B^*} dp_B(s'), \tag{37}$$

where in the last equality we have used the FOCs (10), and the fact that the aggregate endowments do not vary with variations in agent B's belief, i.e., $d\widehat{c}_{At}(s) = -d\widehat{c}_{Bt}(s)$, $\forall t, s$. For the too-early informed economy, agent A's equilibrium consumptions do not depend on B's belief in any states and times,

$$d\overline{EU}_{At} = 0, \qquad \forall t \in \{0, 1\}, \tag{38}$$

where $\overline{EU}_{At}$ denotes A's expected utility of time-t consumption.

We analyze each agent's welfare, as measured by his pre-signal expected utility, across the informed and too-early informed economies. Using (38), we compare (36) with (37), and note that these equations hold for any t. We obtain the following comparative statics when agent B's belief deviate from (31).

Lemma 5 *Consider the deviation of agents' beliefs locally from the heterogeneous belief configuration* $\{p_A, p_B^*\}$ (31)*. If agent A is strictly better off in one (either informed, or too-early informed) economy, then agent B is strictly worse off in that economy,*

$$\left[d\widehat{EU}_B - d\overline{EU}_B\right]\Big|_{\{p_B=p_B^*\}} = -\left[\widehat{\lambda}\left(d\widehat{EU}_A - d\overline{EU}_A\right)\right]\Big|_{\{p_B=p_B^*\}}. \tag{39}$$

Symmetrically, similar results hold when agents' beliefs deviate locally from the heterogeneous belief configuration $\{p_A^*, p_B\}$ (34),

$$\left[d\widehat{EU}_A - d\overline{EU}_A\right]\Big|_{\{p_A=p_A^*\}} = -\left[\frac{1}{\widehat{\lambda}}\left(d\widehat{EU}_B - d\overline{EU}_B\right)\right]\Big|_{\{p_A=p_A^*\}}.$$

The same key intuition underlies both Lemma 5 and equality (37). By virtue of resource constraint, if the deviations in B's belief increases one agent's equilibrium consumption at time t in state s, they must decrease the other agent's consumption by the same amount at the same (t, s). Because asset markets are complete in the informed economy, agents equalize their marginal utilities along the equilibrium path, up to the Pareto weight $\widehat{\lambda}$. As a result, the increment in one agent's utility of consumption at any (t, s) (thus, his expected utility), is exactly opposite and proportional to the increment in the other agent's utility of consumption at the same (t, s) (thus, his expected utility). This argument applies for equilibrium path within a single (either informed or too-early informed) economy type. But Lemma (4) shows that the two economies' equilibria converge at the starting point $\{p_A, p_B^*\}$ (or $\{p_A^*, p_B\}$) of our differential analysis. As a result, Lemma 5 offers a cross-economy comparative statics, albeit locally. Two further remarks are in order.

First, while the derivation leading to Lemma 5 explicitly involves the variation of only one agent's belief,[32] Lemma 5 holds for any local (possibly simultaneous) deviation of both agents' beliefs $\{p_A, p_B\}$ around $\{p_A, p_B^*\}$ (or $\{p_A^*, p_B\}$). This is because when p_B^* is specified in (31) starting from p_A, the relationship is bi-directional. Given p_B^* in (31), we can invert it to obtain uniquely, $p_A(s) = \frac{p_B^*(s)/\overline{\lambda}(s)}{\sum_s p_B^*(s)/\overline{\lambda}(s)}$. Here $\overline{\lambda}(s)$ is the state-specific Pareto weights for the too-early informed economy and does not depend on either agents' beliefs. Therefore, we can equivalently derive Lemma 5's comparative statics by formally fixing p_B^* while varying p_A. The combination of these two derivation procedures immediately imply yet another derivation procedure in which we vary

[32] We either vary p_B around p_B^* while fixing p_A, or symmetrically we vary p_A around p_A^* while fixing p_B.

both agents' beliefs around the configurations $\{p_A, p_B^*\}$ (or $\{p_A^*, p_B\}$).

Second, if either agent's utility changes by a strictly non-zero amount when agents' beliefs deviate from where the two economies converge, Lemma 5 rules out, at least locally, any unambiguous welfare value of public information, under which both agents would see higher expected utilities in one economy than in the other. Lemma 5 applies virtually for any endowment configuration. Therefore, endowment restrictions similar to Assumption 2 are unlikely to eliminate the ambiguous implication of public information on welfare in the presence of asset market frictions.

Informed vs. too-early informed economy: the unlikely convergence

Lemma 5 still leaves open the possibility that one agent is locally indifferent between the informed and too-early informed economies, when agents' beliefs deviate from the transiting configurations ($\{p_A, p_B^*\}$ or $\{p_A^*, p_B\}$). In this circumstance, the other agent must also be indifferent to the two economies. The following two scenarios,

$$\begin{aligned} \text{scenario (i):} \quad & \left[d\widehat{EU}_B - d\overline{EU}_B\right]\Big|_{\{p_B = p_B^*\}} = \left[d\widehat{EU}_A - d\overline{EU}_A\right]\Big|_{\{p_B = p_B^*\}} = 0, \\ \text{scenario (ii):} \quad & \left[d\widehat{EU}_B - d\overline{EU}_B\right]\Big|_{\{p_A = p_A^*\}} = \left[d\widehat{EU}_A - d\overline{EU}_A\right]\Big|_{\{p_A = p_A^*\}} = 0. \end{aligned} \tag{40}$$

in theory are compatible with Lemma 5.

If either of the two scenarios holds, then public information has an unambiguous welfare value (in conjunction with asset market availability). *Both* agents are better off or worse off, at least locally, in the informed economy (compared to the too-early informed economy) as measured by their pre-signal expected utilities. We next derive a necessary and sufficient condition for the scenarios in (40) to hold. The condition is very restrictive and is satisfied only for a particular and narrow set of agents' heterogeneous preferences. Therefore, the possibility of an unambiguous welfare value of public information in the presence of asset market frictions exists, but is highly unlikely.

Without loss of generality, we work with scenario (i) in (40). Our analysis and results can be immediately established for scenario (ii) by symmetric arguments. We begin with an explicit expression for the variation of agent's A expected utility, obtained from totally differentiating the expected utility $\widehat{EU}_A$ and employing FOC (10),

$$d\widehat{EU}_A = \sum_{s\in\Omega} dp_B(s) \frac{p_A(s)}{p_B(s)} \left(N(s) - \frac{E_A[N]}{E_A[A]} M(s) \right) \tag{41}$$

where $E_A[X] \equiv \sum_{s\in\Omega} p_A(s)X(s)$ denotes the average value of a generic state-contingent quantity X under A's belief. The state-contingent equilibrium quantities $N(s)$ and $M(s)$ respectively are,

$$N(s) \equiv \sum_{t=0}^{1} \frac{-(\beta_A)^t (\widehat{c}_{At}(s))^{-\gamma_A}}{\left(\frac{\gamma_A}{\widehat{c}_{At}(s)} + \frac{\gamma_B}{\widehat{c}_{Bt}(s)} \right)}, \tag{42}$$

$$M(s) \equiv \sum_{t=0}^{1} \frac{-(\beta_A)^t \left[(\widehat{c}_{At}(s))^{-\gamma_A} - \gamma_A (\widehat{c}_{At}(s))^{-\gamma_A-1} (\widehat{c}_{At}(s) - e_{At}(s)) \right]}{\left(\frac{\gamma_A}{\widehat{c}_{At}(s)} + \frac{\gamma_B}{\widehat{c}_{Bt}(s)} \right)}. \tag{43}$$

Note that $N(s)$ and $M(s)$ are always positive for all states s. Because equilibrium in the too-early informed economy does not depend on agents' beliefs (38), the hypothesis $d\widehat{EU}_A - d\overline{EU}_A = 0$ in (40) is equivalent to $d\widehat{EU}_A = 0$, which we now investigate.

Given the normalization constraint $\sum_{s\in\Omega} p_B(s) = 0$ (35) in the variations of B's belief, $d\widehat{EU}_A$ vanishes if and only if the expression associated with each perturbation $dp_B(s)$ in (41) is state-independent,[33]

$$\left. \left[\frac{p_A(s)}{p_B(s)} \left(N(s) - \frac{E_A[N]}{E_A[A]} M(s) \right) \right] \right|_{p_B = p_B^*} = K, \qquad \forall s \in \Omega, \tag{44}$$

where K is constant across states $s \in \Omega$. Taking the average (under distribution $\{p_B^*(s)\}$) of both sides of the above equation determines K unambiguously,

$$\sum_{s\in\Omega} p_B^*(s) \left. \left[\frac{p_A(s)}{p_B(s)} \left(N(s) - \frac{E_A[N]}{E_A[A]} M(s) \right) \right] \right|_{p_B = p_B^*} = \sum_{s\in\Omega} p_B^*(s) K \implies 0 = K.$$

Substituting the value of K into (44) implies that $d\widehat{EU}_A$ (41) vanishes if and only if,

$$\left. \left[N(s) - \frac{E_A[N]}{E_A[M]} M(s) \right] \right|_{p_B = p_B^*} = 0, \qquad \forall s \in \Omega.$$

Since $N(s)$ (42) and $M(s)$ (43) are strictly positive for all $s \in \Omega$, the above condition is equivalent to that the ratio of $N(s)$ and $M(s)$ (at p_B^*) be equal to the ratio of their means (under A's belief), $\left. \frac{N(s)}{M(s)} \right|_{p_B^*} = \frac{E_A[N]}{E_A[M]}$, for all s. We thus have the following necessary and sufficient condition for the

[33] This is a result of a standard Lagrangian optimization subject to an equality constraint.

special case in which Lemma 5 holds at equality.

Lemma 6 *Both agents' expected utility increments (39) stall locally at the belief configuration $\{p_A, p_B^*\}$ if and only if the ratio of $N(s)$ (42) and $M(s)$ (43) is state-independent,*

$$\begin{cases} \left[d\widehat{EU}_A - d\overline{EU}_A\right]\Big|_{\{p_B=p_B^*\}} = 0, \\ \left[d\widehat{EU}_B - d\overline{EU}_B\right]\Big|_{\{p_B=p_B^*\}} = 0 \end{cases} \iff \frac{N(s)}{M(s)}\bigg|_{p_B=p_B^*} = \frac{N(s')}{M(s')}\bigg|_{p_B=p_B^*}, \quad \forall s, s' \in \Omega. \quad (45)$$

We recall from Lemma 4 that exactly at the belief configuration $\{p_A, p_B^*\}$ (or $\{p_A^*, p_B\}$), the equilibria of the informed and the too-early informed economies converge. When agents' beliefs are not exactly at these particular configurations, and across the two economies, most likely only one agent is better off (the other worse off), again by Lemma 4. However, Lemma 6 goes a step further by quantifying the likelihood of the above ambiguity in agents' cross-economy welfare. Both agents have higher expected utilities in the informed than in the too-early informed economy if and only if condition (45) holds.[34] However, this unambiguous comparative statics holds only locally around a particular belief configuration $\{p_A, p_B^*\}$.

We now take a closer look at the necessary and sufficient condition (45). Note that the condition is always evaluated locally at the belief configurations where informed and too-early informed economies' equilibria converge. We can substitute too-early informed equilibrium consumptions $\overline{c}_{It}$ for all consumption quantities in that condition. For CRRA preferences, using expressions for $N(s)$ (42) and $M(s)$ (43), expression (45) is equivalent to the following explicit condition,

$$\frac{1}{\left(\frac{\gamma_A}{\overline{c}_{A0}(s)} + \frac{\gamma_B}{\overline{c}_{B0}(s)}\right)} + \overline{q}(s)\frac{1}{\left(\frac{\gamma_A}{\overline{c}_{A1}(s)} + \frac{\gamma_B}{\overline{c}_{B1}(s)}\right)} \quad (46)$$

$$= L\left[\frac{e_{A0}}{\overline{c}_{A0}(s)}\frac{1}{\left(\frac{\gamma_A}{\overline{c}_{A0}(s)} + \frac{\gamma_B}{\overline{c}_{B0}(s)}\right)} + \overline{q}(s)\frac{e_{A1}(s)}{\overline{c}_{A1}(s)}\frac{1}{\left(\frac{\gamma_A}{\overline{c}_{A1}(s)} + \frac{\gamma_B}{\overline{c}_{B1}(s)}\right)}\right], \quad \forall s \in \Omega,$$

where L is some state-independent coefficient, and $\overline{q}(s) = \beta_A \left(\frac{\overline{c}_{A0}(s)}{\overline{c}_{A1}(s)}\right)^{-\gamma_A}$ are AD prices (13).

To see how restrictive condition (45) is, two observations on its equivalent version (46) are in order. First, condition (45) calls for the equality (46) to hold for each state s, but with a common state-independent L. Second, the too-early informed equilibrium consumptions $\{\overline{c}_{At}(s), \overline{c}_{Bt}(s)\}$ are

[34] The Hessian matrix with respect to the variations $\{dp_B(s)\}$ of the expected utility increment $\widehat{EU}_I - \overline{EU}_I$. $\forall I \in \{A, B\}$ is negative semidefinite at p_B^*. For for both agents, the utility increment thus has a local maximum at $\{p_A, p_B^*\}$.

entirely determined by the FOCs–resource constraint system (14) and the budget constraint (15) for each state s independently. Therefore, when we substitute this equilibrium solution $\{\bar{c}_{At}(s), \bar{c}_{Bt}(s)\}$ into (46) to back out L, we generally obtain a state-dependent coefficient $L(s)$. Hence, condition (45), necessary and sufficient for an unambiguously positive impact of early information releases together with complete asset markets on welfare (Lemma 6) generally does not hold.

The condition (46) though holds for a restrictive set of preferences. In particular, when agents have identical CRRA, $\gamma_A = \gamma_B$, the FOC (14) for the too-early informed economy reduces to the equality of any agent's temporal ratio of consumptions and that of aggregate endowments,

$$\frac{\bar{c}_{A1}(s)}{\bar{c}_{A0}(s)} = \frac{\bar{c}_{B1}(s)}{\bar{c}_{B0}(s)} = \frac{e_{A1}(s)}{\bar{c}_{A0}(s)}, \qquad \forall s \in \Omega. \tag{47}$$

As a result, agent A's budget constraint, $\bar{c}_{A0}(s) - e_{A0}(s) + \bar{q}(s)\left(\bar{c}_{A1}(s) - e_{A1}(s)\right) = 0$ immediately implies (46), which in turn indicates that Lemma 5 holds at equality. Combining this result with the discussion following Lemma 6, we have

Lemma 7 *Assume that agents have identical relative risk aversions $\gamma_A = \gamma_B$. Then $N(s) = M(s)$, $\forall s \in \Omega$, and condition (45) holds. Therefore, both agents have higher expected utilities in the informed than in the too-early informed economy, locally around the belief configurations at which the two economies' equilibria converge.*

$$\gamma_A = \gamma_B \quad \Longrightarrow \quad \left[\widehat{EU}_A \geq \overline{EU}_A\right]\Big|_{\{p_B = p_B^* + \epsilon\}} = 0, \quad \textit{and} \quad \left[\widehat{EU}_B \geq \overline{EU}_B\right]\Big|_{\{p_B = p_B^* + \epsilon\}} = 0.$$

While this result and its derivation appear to adhere explicitly to CRRA preferences (and at the belief configuration $\{p_A, p_B^*\}$), Lemma 7 applies to all additively separable utilities (as well as to the other belief configuration $\{p_A^*, p_B\}$). Neither Assumption 2 of proportional endowments, nor the assumption of homogeneous beliefs alone is sufficient to resurrect condition (45). But assumptions on agents' risk preferences are sufficient to deliver (45), as Lemma 7 demonstrates.[35] Lemma 7 offers an unambiguous welfare value of public information (in conjunction with early asset market availability) only locally around particular belief configuration $\{p_A, p_B^*\}$ (and $\{p_A^*, p_B\}$).

[35] It is interesting to note that, beyond the CRRA class, we can also show that condition (45) holds when agents have constant absolute risk aversion (CARA), for any level of heterogeneity in beliefs.

5 The Hirshleifer effect revisited

To assess the welfare value of public information if information arrives before asset markets open, we compare agents' ex-ante expected utilities in the uninformed and too-early informed economies. In the uninformed economy agents do not have any information before they trade and consume. In the too-early informed economy agents have information before they trade and consume.

The comparative statics of these two economies speaks directly to the Hirshleifer effect. In the uninformed economy, without public information but with complete asset markets, agents can trade to smooth their consumption across states. Consumption smoothing across time is, however, limited by the absence of early public information, which is reflected in the rigid (non-contingent) time-zero consumptions c_{I0} for all agents $I \in \{A, B\}$. In the too-early informed economy, agents smooth consumptions only across time given the hindsight conveyed by the signal, but no trades are allowed to share risk across states. This is reflected in the independence between equilibrium consumptions and agents' beliefs. When agents have additively separable preferences, it is not evident whether the benefit of risk-sharing across states outweights that of temporal consumption smoothing given the hindsight.[36] If both agents expect to have higher utilities in the uninformed economy, then the Hirshleifer effect prevails in the sense that an early release of public information impairs risk-sharing benefits. If at least one agent expects to have lower utilities in the uninformed economy, then the Hirshleifer effect does not prevail. This is possibly because the benefits from risk sharing are not that high a priori, so that the loss of these benefits[37] is not that large to overturn the welfare value of public information.

We focus on premises in which aggregate risks dominate, and accordingly we maintain Assumption 2 of proportional endowments throughout this section. We present an analytical assessment of the Hirshleifer effect in settings of homogeneous beliefs and heterogeneous beliefs next.

The Hirshleifer effect: Homogeneous beliefs

We make a series of simple arguments based on results obtained in previous sections for different sets of economies, to infer a local comparative statics for the uninformed and too-early informed economies. We start with the observation that in the setting of homogeneous risk and time prefer-

[36] Risk sharing benefit depends on agents' risk aversions. Temporal consumption smoothing benefit depends on the elasticities of intertemporal substitution of agents' utility functions. For additively separable preferences, these two characteristics are confounded.

[37] Due to early releases of public information.

ences; $\gamma_A = \gamma_B \equiv \gamma$ and $\beta_A = \beta_B \equiv \beta$, agents have proportional consumptions (47) in equilibrium. Consequently, Assumption 2 implies a no-trade equilibrium in the too-early informed economy (equation (29), for $\gamma_A = \gamma_B$ and $\beta_A = \beta_B$),

$$\bar{c}_{It}(s) = e_{It}(s), \qquad \forall I \in \{A, B\}, t \in \{0, 1\}, s \in \Omega.$$

As a result, Pareto weights $\bar{\lambda}(s)$ are constant by virtue of FOC (14), and (29),

$$\bar{\lambda}(s) = \left(\frac{\bar{c}_{B1}(s)}{\bar{c}_{A1}(s)} \right)^{\gamma} = k^{-\gamma}, \qquad \forall s \in \Omega.$$

From the specification (31) then follows that the particular belief p_B^* is also the homogeneous belief configuration when agents have same time and risk preferences,

$$\left.\begin{array}{rl} \text{Proportional endowments:} & \frac{e_{At}(s)}{e_{Bt}(s)} = k \\ \text{Homogeneous risk preference:} & \gamma_A = \gamma_B \\ \text{Homogeneous time preference:} & \beta_A = \beta_B \end{array}\right\} \implies p_B^*(s) = p_A(s), \;\; \forall s \in \Omega. \tag{48}$$

By analytic continuation on a regular economy (see e.g., Mas-Colell et al. (1995)), this result implies that a slight heterogeneity in risk aversions induces a slight deviation between the homogeneous belief configuration $\{p_A, p_B = p_A\}$ and configuration $\{p_A, p_B^*\}$. Thus, our differential analysis around $\{p_A, p_B^*\}$ (where the equilibria of the informed and too-early informed economies converge, Lemma 4) in Section 4 can be applied to the homogeneous belief configuration $\{p_A, p_B = p_A\}$ (where the equilibria of informed and uninformed economies converge, Proposition 1). We carry out a detailed analysis along this line of reasoning.

We take agents' risk and time preferences $\{\gamma_A, \gamma_B, \beta_A, \beta_B\}$ and agent A's belief p_A as exogenously given. We assume that agents have same time discount factors and strictly different risk aversions, but their difference is sufficiently small. Equivalently, we can also assume that agents' have same risk aversions and strictly different time discount factors.[38] Then relationship (48) implies that belief configuration p_B^* defined in (31) is also strictly different from agent A's belief, but

[38] All we need for our differential analysis is that agents exhibit strict but sufficiently small heterogeneous preferences toward either risk or time dimension.

their difference is sufficiently small, because our economies are regular by construction.

$$\left.\begin{aligned} \gamma_A &\approx \gamma_B \\ \beta_A &= \beta_B \end{aligned}\right\} \Longrightarrow p_B^* \approx p_A. \tag{49}$$

We consider a variation of agent B's belief p_B from p_B^* to p_A along the equilibrium path. Three arguments are in order.

First, because γ_A strictly differs from γ_B, condition (45) does not hold in general, by a similar argument underlying Lemma 7. As a result, our choice of strictly heterogeneous risk aversions, $\gamma_A \neq \gamma_B$, implies the ambiguous implication of information on welfare.[39]

Second, as $p_B^* \approx p_A$ (49), the entire variation path of p_B from p_B^* (the start of the variation) to p_A (the end of the variation) can be considered local around initial configuration p_B^* and Lemma 4 applies. As a result, one agent is strictly better off, the other agent is strictly worse off, in the informed than in the too-early informed economy. Therefore we have two (and only two) mutually exclusive possibilities at $p_B = p_A$ (homogeneous belief configuration), listed below,

$$\text{(I):} \begin{cases} \left.\widehat{EU}_A\right|_{p_B=p_A} > \left.\overline{EU}_A\right|_{p_B=p_A}, \\ \\ \left.\widehat{EU}_B\right|_{p_B=p_A} < \left.\overline{EU}_B\right|_{p_B=p_A}, \end{cases} \qquad \text{(II):} \begin{cases} \left.\widehat{EU}_A\right|_{p_B=p_A} < \left.\overline{EU}_A\right|_{p_B=p_A}, \\ \\ \left.\widehat{EU}_B\right|_{p_B=p_A} > \left.\overline{EU}_B\right|_{p_B=p_A}. \end{cases} \tag{50}$$

Note that all inequalities above are strict inequalities, because γ_A strictly differs from γ_B, and as explained the first argument above.

Third, at the homogeneous belief configuration $p_B = p_A$, Proposition 1 holds. As a result, we have both,

$$\left.\widehat{EU}_A\right|_{p_B=p_A} = \left.EU_A\right|_{p_B=p_A}, \qquad \text{and} \qquad \left.\widehat{EU}_B\right|_{p_B=p_A} = \left.EU_B\right|_{p_B=p_A} \tag{51}$$

But at $p_B = p_A$, the either possibility (I) or (II) in (50) must arise. Combining (51) with possibility (I), or combining (51) with possibility (II), yields two (and only two) possibilities (I') and (II') reported in Proposition 3 below. Altogether, the three preceding arguments imply the following key results.

[39] We assume throughout section 5 that agents have proportional endowments. But as noted in the discussion below Lemma 7, proportional endowments are not sufficient to assure an unambiguous Pareto-improving value of information. Therefore, an ambiguous impact of public information on agents' welfare can arise even under the assumption of proportional endowments maintained in this section.

Proposition 3 (Welfare value of information: Homogeneous beliefs) *Suppose that (i) Assumption 1 holds ($\gamma_A \geq 1$, $\gamma_B \geq 1$), (ii) Assumption 2 holds ($e_{At}(s)/e_{Bt}(s) = k$, $\forall t, s$), (iii) agents' relative risk aversions are close to each other but not identical ($|\gamma_A - \gamma_B| = \epsilon > 0$), and (iv) agents' time discount factors are the same.*[40] *When agent B's belief p_B is identical to agent A's belief p_A, there are two (and only two) mutually exclusive possibilities listed below*

$$\text{(I')}: \begin{cases} EU_A|_{p_B=p_A} > \overline{EU}_A\Big|_{p_B=p_A}, \\ \\ EU_B|_{p_B=p_A} < \overline{EU}_B\Big|_{p_B=p_A}, \end{cases} \qquad \text{(II')}: \begin{cases} EU_A|_{p_B=p_A} < \overline{EU}_A\Big|_{p_B=p_A}, \\ \\ EU_B|_{p_B=p_A} > \overline{EU}_B\Big|_{p_B=p_A}. \end{cases}$$

Therefore, one (and only one) agent is strictly better off, the other agent is strictly worse off in the too-early informed economy than in the uninformed economy.

The Hirshleifer effect categorically captures the notion that early releases of public information impair the benefits of risk sharing to the extent of leaving adverse impacts on welfare. In our setting, the Hirshleifer effect amounts to positive boosts to the agents' utilities in the uninformed economy. Under the assumptions listed above, Proposition 3 shows analytically that the Hirshleifer effect does not prevail. When agents have identical beliefs, the Hirshleifer effect is at best ambiguous in the sense that public information decreases the expected utility of exactly one agent. However, we remark that these analytical results can be demonstrated only locally within the specific premise of Proposition 3.

Intuitively, key assumptions underlying Proposition 3 point to moderate risk sharing demands and benefits. When risk aversions are similar, the risk sharing incentive is mild. When endowments are proportional, the risk sharing capacity is limited. The belief configurations, to which Proposition 3 applies, are where agents have same belief, $p_B = p_A$, or quite similar beliefs, $p_B = p_B^* \approx p_A$ (49). This feature of Proposition 3 actually lends support to the Hirshleifer effect, albeit indirectly. When the expected risk sharing benefits are moderate, the expected loss from forfeiting risk sharing through the means of public information is moderate and likely out-weighted by the benefits of informed consumption smoothing, again through the means of public information. As a result, public information has a welfare value when the risk sharing motive is not preeminent. Technically, the scope of Proposition 3 stems directly from the differential analysis approach that gives rise to

[40] As we noted in Footnote 38, this proposition equally holds when agents' time discount factors differ by small amount, and in that case agents' risk aversions can be either identical or different by small amount.

it. We are making the tradeoff between striving for an unambiguous welfare comparative statics with inevitably local applicability. Given that global unambiguous welfare comparative statics is most likely non-existent, Proposition 3 settles with unambiguous but local characteristics.

The Hirshleifer effect: Heterogeneous beliefs

Our analysis on the Hirshleifer effect when agents disagree about the future prospect of the economy relies on teh global results of the welfare comparison obtained in Proposition 2. When agent B's belief p_B is exactly at p_B^*, Lemma 4 implies that equilibria of the informed and too-early informed economies converge. An application of Proposition 2 then indicates that both agents have higher expected utilities in the too-early informed economy than in the uninformed economy.

Proposition 4 (Welfare value of information: Heterogeneous beliefs) *Suppose that (i) Assumption 1 holds ($\gamma_A \geq 1$, $\gamma_B \geq 1$), (ii) Assumption 2 holds ($e_{At}(s)/e_{Bt}(s) = k$, $\forall t, s$), and (iii) agents have either heterogeneous risk aversions ($\gamma_A \neq \gamma_B$) or heterogeneous time discount factors ($\beta_A \neq \beta_B$). When agent B's belief p_B is given by p_B^* (31), both agents unambiguously have strictly higher expected utilities in the too-early informed economy than in the uninformed economy.*

$$\overline{EU}_A\Big|_{p_B=p_B^*} > EU_A\big|_{p_B=p_B^*}, \qquad \overline{EU}_B\Big|_{p_B=p_B^*} > EU_B\big|_{p_B=p_B^*}.$$

Therefore, in this setting, information is unambiguously Pareto-improving even if no trades are allowed before the information is released.

Under the listed assumptions, Proposition 4 shows analytically that, when agents disagree ($p_B = p_B^* \neq p_A$), the Hirshleifer effect does not prevail. Opposite to that effect, at $p_B = p_B^*$, an early public information release unambiguously benefits both agents in expectation, even when agents cannot trade contingent assets on the incoming signal. All inequalities in Proposition 4 are strict. Then by analytic continuation on regular economies, Proposition 4 holds not only at exactly $p_B = p_B^*$, but also for all B's beliefs p_B sufficiently close to p_B^* in (31).

Compared to the assumptions underlying Proposition 3, neither agents' relative risk aversions nor their time discount factors need to be sufficiently close in Proposition 4.[41] This is because the

[41] The comparative statics result of Proposition 4 is a direct consequence of only Lemma 4 and Proposition 2, so a priori it neither requires any relationship between agents' risk (and time) preferences, nor it relies on differential analysis. But with the assumption of proportional endowments already in place, we need either $\gamma_A \neq \gamma_B$ or $\beta_A \neq \beta_B$. This is because proportional endowments, $\gamma_A = \gamma_B$ and $\beta_A = \beta_B$ together imply that $p_B^* = p_A$. When $p_B = p_B^*$, agents have same beliefs $p_A = p_B$, and Proposition 1 applies instead of Proposition 2.

latter Proposition relies on the global analysis inherited from Proposition 2, while the former relies on differential analysis.

6 Extensions

Our analysis and results on the welfare value of public information can be generalized to broader economic settings and under weaker assumptions. The generalization is important not only for a wider applicability of the results, but also for the better understanding of the economic factors driving these results. This is because different economic factors and their contributions may be confounded when the settings at hand are specific. In this section, we discuss aspects of imperfect information, general additively separable utilities, and possible weakened versions of assumptions previously needed to deliver the main results of the paper.

Imperfect signals

Up to this point, our analysis has focused on the perfect foresight of future endowments, whenever public signals are available. When imperfect signals are observed investors can reduce, but not completely eliminate, the uncertainty about future endowments. We now briefly examine the relevance of imperfect signals on the improvement of risk sharing and welfare of agents.

We consider a parsimonious setting of imperfect signals. In the economies in which such signals are available, at some time before $t = 0$, agents observe perfect signals only on a portion $e_1(s_0)$ of the future aggregate endowment,

$$e_1(s_0, s_1) = e_1(s_0) + \epsilon(s_1),$$

to be realized at $t = 1$. The noise $\epsilon(s_1)$ (or state s_1) intends to model the imperfection of signals s_0, and can only be observed at $t = 1$. Endowments $e_0 = e_{A0} + e_{B0}$ at time $t = 0$ remain constants and known to all agents as before.

Evidently, the imperfect signals do not affect the uninformed economy, in which no signal arrives ever. For the informed economy, before the arrival of signals, complete asset markets allow agents to trade two sets of AD securities $\{\widehat{q}_0(s_0)\}$, $\{\widehat{q}_1(s_0,s_1)\}$ paying off at $t=0$ and $t=1$ respectively.[42] As a result, agents can devise (pre-signal) optimal consumptions $\{\widehat{c}_{I0}(s_0), \widehat{c}_{I0}(s_0,s_1)\}$ contingent on signal s_0 at $t=0$, and on the full state of the economy (s_0,s_1) to be realized at $t=1$. In place of (10)–(11) for perfect signals, the FOC and resource constraints for the current case of imperfect signals read,

$$\begin{array}{l} \text{at } t=0 \\ \forall s_0 \end{array} \begin{cases} p_A(s_0)\widehat{u}'_{A0}(s_0) = \widehat{\lambda} p_B(s_0)\widehat{u}'_{B0}(s_0), \\ \widehat{c}_{A0}(s_0) + \widehat{c}_{B0}(s_0) = e_0, \end{cases}$$

$$\begin{array}{l} \text{at } t=1 \\ \forall s_0, s_1 \end{array} \begin{cases} p_A(s_0,s_1)\beta_A\widehat{u}'_{A1}(s_0,s_1) = \widehat{\lambda} p_B(s_0,s_1)\beta_B\widehat{u}'_{B1}(s_0,s_1), \\ \widehat{c}_{A1}(s_0,s_1) + \widehat{c}_{B1}(s_0,s_1) = e_1(s_0,s_1), \end{cases}$$

and agent A's budget constraint,

$$\sum_{s_0} \widehat{q}_0(s_0)\widehat{c}_{A0}(s_0) + \sum_{s_0,s_1} \widehat{q}_1(s_0,s_1)\widehat{c}_{A1}(s_0,s_1) = \sum_{s_0} \widehat{q}_0(s_0)\widehat{e}_{A0}(s_0) + \sum_{s_0,s_1} \widehat{q}_1(s_0,s_1)\widehat{e}_{A1}(s_0,s_1).$$

where p_I denotes agent I's expectation, $I \in \{A, B\}$.

When agents have homogeneous beliefs, this common belief is canceled out in the FOC at both periods. Therefore, at $t=0$, since there is no aggregate uncertainty, the solution of FOC and resource constraint must be state-independent, $\widehat{c}_{I0}(s_0) = \widehat{c}_{I0}$, $\forall s, I$. By identical arguments leading to Proposition 1, imperfect signals are irrelevant to welfare when agents have homogeneous beliefs and asset markets are complete. Proposition 1 remains valid when public information is imperfect.

When agents have different beliefs, by virtue of the Assumption 2 of proportional endowments, again only aggregate endowments e_0 and $e_1 + \epsilon$ enter the equilibrium equation system in both periods. By identical arguments leading to Proposition 2, imperfect signals make both agents' better off, in the expectation, in the informed economy under the assumption of proportional endowments (and Assumption 1). Proposition 2 remains valid when public information is imperfect.

[42] In place of (8), (9), here the AD prices read,

$$\frac{p_A(s_0)\widehat{u}'_{A0}(s_0)}{\sum_{s_0} p_A(s_0)\widehat{u}'_{A0}(s_0)} = \widehat{q}_0(s_0) = \frac{p_B(s_0)\widehat{u}'_{B0}(s_0)}{\sum_{s_0} p_B(s_0)\widehat{u}'_{B0}(s_0)},$$

$$\frac{p_A(s_0,s_1)\beta_A\widehat{u}'_{A1}(s_0,s_1)}{\sum_{s_0,s_1} p_A(s_0,s_1)\widehat{u}'_{A0}(s_0,s_1)} = \widehat{q}_1(s_0,s_1) = \frac{p_B(s_0,s_1)\beta_B\widehat{u}'_{B1}(s_0,s_1)}{\sum_{s_0,s_1} p_B(s_0,s_1)\widehat{u}'_{B0}(s_0,s_1)}.$$

General additively separable preferences

Most of the results presented earlier, including Assumption 1 ($\gamma_A, \gamma_B \geq 1$), have been derived explicitly for CRRA utilities for ease of exposition. Though our results hold for general additively separable preferences. For illustration, let us derive a sufficient condition for the convexity of agents' equilibrium consumptions in the Pareto weight (19) for general preferences. This condition replaces Assumption 1, and is a key to generalize Proposition 2 to general preferences beyond the CRRA class.

For each time and state, we keep beliefs and aggregate endowments unchanged, and vary the Pareto weight to model the changes in agents' relative endowments in our comparative statics investigation. The analysis is identical for $t = 0$ and $t = 1$. Below we work with period $t = 0$. We begin with totally differentiating the FOC (10) for state s, which yields

$$\left(p_A(s)\widehat{u}''_{A0}(s) + \widehat{\lambda} p_B(s)\widehat{u}''_{B0}(s)\right) \frac{d\widehat{c}_A(s)}{d\widehat{\lambda}} = p_B(s)\widehat{u}'_{B0}(s). \tag{52}$$

Totally differentiating one more time, we obtain the second-order derivative of consumption in the Pareto weight,

$$\begin{aligned} &\left[p_A(s)\widehat{u}''_{A0}(s) + \widehat{\lambda} p_B(s)\widehat{u}''_{B0}(s)\right] \frac{d^2\widehat{c}_A(s)}{d\widehat{\lambda}^2} \\ = &\frac{-d\widehat{c}_A(s)}{d\widehat{\lambda}} \left[2p_B(s)\widehat{u}''_{B0}(s) + \left\{p_A(s)\widehat{u}'''_{A0}(s) - \widehat{\lambda} p_B(s)\widehat{u}'''_{B0}(s)\right\} \frac{d\widehat{c}_A(s)}{d\widehat{\lambda}}\right]. \end{aligned} \tag{53}$$

Following (52), when utilities are increasing and concave in consumption, A's consumption decreases in Pareto weight, $\frac{d\widehat{c}_A(s)}{d\widehat{\lambda}} < 0$. Therefore, A's consumption is convex in Pareto weight when the expression inside square brackets in (53) is negative. Using FOC, and (53), we can express this sufficient condition for $\frac{d^2\widehat{c}_A(s)}{d\widehat{\lambda}^2} > 0$ as,

$$2\frac{\widehat{u}''_{A0}(s)}{\widehat{u}''_{B0}(s)} + \frac{\widehat{u}'_{B0}(s)}{\left(\widehat{u}''_{B0}(s)\right)^2}\widehat{u}'''_{A0}(s) + \widehat{\lambda}\left[2 - \frac{\widehat{u}'_{B0}(s)\widehat{u}'''_{B0}(s)}{\left(\widehat{u}''_{B0}(s)\right)^2}\right] \geq 0. \tag{54}$$

Therefore a sufficient condition for A's consumption to be convex in Pareto weight is

$$\widehat{u}'''_{A0}(s) \geq 0, \qquad \text{and} \qquad \frac{\widehat{u}'_{B0}(s)\widehat{u}'''_{B0}(s)}{\left(\widehat{u}''_{B0}(s)\right)^2} \leq 2. \tag{55}$$

For CRRA utilities, the above sufficient condition reduces to $\gamma_B \geq 1$ in Assumption 1. We remark that, for general additively separable preferences, the above condition involves the third-order derivative of agents' utility functions (or prudence), which is quite obscured in Assumption 1 for the class of CRRA utilities.

Weakening assumptions

Sufficient assumptions, which are key to our earlier results, can also be weakened. Assumption 1, or its version for general preferences (55), can be substantially weakened. This is because the original requirement of the convexity of equilibrium consumptions and indirect utilities in the Pareto weight ((20), (21) or (54)) for the application of Jensen's inequality can be met by contributions of other terms in these expressions.

Assumption 2 of proportional endowments arises when agents are born with shares of the endowment trees, which is plausible in our stylized three-period setting. This assumption's primary role is to make aggregate risks dominant by placing some discipline on the idiosyncratic movements agent-specific components of endowments. If those components are not highly correlated, then either (i) the risk sharing benefits are considerable that public information would have an adverse effect on agents' welfare by means of impairing these benefits (Hirshleifer effect), or (ii) a new dimension of uncertainties (associated with agents' relative wealth) enters the equilibrium dynamics in quite arbitrary way that no unambiguous welfare implication of public information exists. We view the assumption of proportional endowments in this light, and logically any other endowment configurations that do not render large risk-sharing benefits would also fulfill the role of Assumption 2.

7 Conclusion

First, we provide a global result that an early release of information is desired by all agents (information is Pareto improving) if agents have heterogeneous beliefs and if asset markets are complete.[43] If beliefs are homogeneous, agents are indifferent between early and late information releases. This result holds for time additive preferences with any heterogeneity in agent's subjective time discount rates, risk aversions, beliefs, and for any initial wealth distribution.

Second, we prove that for certain levels of disagreement (difference in beliefs), all agents prefer an early release of information (information is Pareto improving) even if agents are not able to trade

[43] Markets are complete in the sense that agents can also contract on the information before it is released.

in asset markets before the information arrives. The set-up is an extension of Hirshleifer (1971)'s model to the setting of heterogeneous beliefs and has a surprising result. Hirshleifer (1971) argues that if information is released too early (before agents are able to trade), then it has an adverse impacts on risk sharing and reduces welfare. Interestingly, the introduction of heterogeneous beliefs is able to reverse the Hirshleifer effect. That is, an early release of information implies an increase in welfare.

Third, for the case of homogeneous beliefs we provide conditions under which the Hirshleifer effect holds. We also show that there exist many instances where one agent prefers information to be released early in time even if no trade is possible before the arrival of the information, while the other agent prefers no information releases.

Fourth, we show that except for some special cases there is no unambiguous preference (i.e., by all agents) for an economy with complete markets and early information releases over an economy with early information releases an no trading before the information arrival.

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