# Market Run-Ups, Market Freezes, Inventories, and Leverage[*]

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## Abstract

We study trade between a buyer and a seller when both may have existing inventories of assets similar to those being traded. We analyze how these inventories affect trade, information dissemination, and price formation. We show that when the buyer's and seller's initial leverage is moderate, inventories increase price and trade volume (a market "run up"), but when leverage is high, trade may become impossible (a market "freeze"). Our analysis predicts a pattern of trade in which prices and trade volume first increase, and then markets break down. Under many circumstances, the presence of competing buyers amplifies the increased-price effect. We use our model to discuss implications for regulatory intervention in illiquid markets.

# 1 Introduction

Consider the sale of mortgages by a loan originator to a buyer. As widely noted, the originator has a natural information advantage and knows more about the quality of the underlying assets than other market participants. One consequence, which has been much discussed, is that he will attempt to sell only the worst mortgages.[1] However, a second important feature of this transaction has received much less attention. Both the buyer and the seller may hold significant inventories of mortgages similar to those being sold, and they may care about the market valuation of these inventories, which affects how much leverage they can take. Consequently, they may care about the dissemination of any information that affects market valuations of their inventories. In this paper, we analyze how inventories affect trade, information dissemination, and price formation. Our setting applies to the sale of mortgage-related products, but more broadly, to situations in which the seller has more information about the value of the asset.

Our main result is that the effect of inventories on trade depends on the buyer's and seller's initial leverage. When leverage is moderate, inventories increase price and trade volume (a market "run up"); but when leverage is high, trade may become impossible (a market "freeze"), and information dissemination ceases. In a dynamic extension, our model predicts a pattern of trade in which prices and trade volume first increase, and then markets break down.

The intuition is as follows. Since the seller has an information advantage, a sale reveals information about the value of the traded asset. This information may be used to reassess the value of inventoried assets and the amount of leverage that the buyer and seller can take. To ensure neither agent violates his capital constraint (i.e., that the market value of each agent's assets is greater than the value of his liabilities), the buyer may need to offer a higher price, which increases the market's posterior about the value of the asset, and hence of inventories. The higher price also increases the probability that the seller will accept the offer. Hence, both prices and expected trade volume may increase. However, when the capital constraint

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[1]See, for example, Ashcraft and Schuermann (2008); and Downing, Jaffee and Wallace (2009).

is too tight, the buyer can no longer increase the price without losing money. At this point, trade collapses and market participants learn nothing about asset values.

Inventories and capital constraints are both crucial for the market freeze result above. In a standard lemons problem in which the gains from trade are common knowledge but without inventories and capital constraints, trade does not break down completely, since the gains from trade compensate the buyer for the expected losses to the informed seller. In this standard problem it is always optimal for the buyer to make an offer that there is a positive probability of the seller accepting, and so the seller's response to this offer reveals information about the asset value. In contrast, when the buyer or seller are capital constrained and are concerned about the effect of information revelation on the value of their inventories, trade may break down completely, in the sense that there is no offer that the buyer can profitably make that there is a positive probability of the seller accepting.[2]

The notion that market participants adjust their trading behavior with an eye to influencing the dissemination of information is most natural when markets are "thin." Accordingly, in our baseline model we assume that there is a single potential buyer, who hence enjoys a monopoly position. Interestingly, our results continue to hold even when markets become "thicker," in the sense that the number of potential buyers increase. Some of the results are actually strengthened. In particular, a buyer may be forced to raise his bid not only because he is leveraged, but also because a *competing* buyer is leveraged; and he may be even forced to acquire assets at a *loss-making* price, just to make sure that a competing buyer does not acquire them at a lower price. The key insight here is that a purchase by one buyer may lead to release of information which causes a violation of the capital constraint of a competing buyer, and this may force the competing buyer to increase the price. In each of the cases above, competition and capital constraints combine to push the price strictly higher than would be the case under either competition alone, or a single buyer with a capital constraint. However, when all buyers are highly leveraged, concerns about inventory valuation again lead to a market freeze and prevent the dissemination of information about asset quality, just as

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[2] An alternative explanation for a market freeze in situations in which there are gains from trade involves Knightian utility; see, e.g., Easley and O'Hara (2010).

in the single-buyer case.

Our baseline results, which are obtained in a static setting with a single round of trade, are suggestive of a dynamic process in which buyers increase leverage and prices until the market breaks down eventually. In a dynamic extension of our basic framework we model this process explicitly and show that a market freeze may be preceded by a run up in prices. This result is interesting because the run up in prices occurs even though (by assumption) the underlying fundamentals remain unchanged. In this sense, the run up shares features of a "bubble." In our model, this result reflects the fact that increasing inventories force the buyer to increase his bid. In particular, when the buyer adds assets to his balance sheet, he reduces the market value of his existing assets and increases his leverage. This forces him to bid a higher price in the next trade, or else not bid at all.

We use our model to discuss implications for regulatory intervention in illiquid markets. On the buyer side, our analysis highlights the potential role of a large investor unencumbered by existing inventories (the government, for example); one implication is that by purchasing assets, the government may impose a cost on potential buyers who choose not to trade. On the seller side, our analysis suggests potential limitations to the standard prescription that sellers should retain a stake in the assets they sell. We also relate the model's predictions to the freeze in the market for mortgage-backed securities during the recent financial crisis and provide some new testable implications regarding the relationship between dealers' inventories and prices.

As a technical contribution, we show that out of all possible trading mechanisms, the one that maximizes the payoff of a monopolist buyer is the simple mechanism in which the buyer makes a take-it-or-leave-it offer to buy up to $q$ units of the asset at a price per unit $p$ (the buyer selects both $p$ and $q$). This result generalizes a classic result of Samuelson (1984), who analyzes essentially the same setting in the absence of capital constraints and inventories. Moreover, this result implies there is no loss in focusing on linear price schedules.

*Related literature.* Our paper relates to the literature on bargaining under asymmetric information (e.g., Samuelson, 1984), in which the seller is better informed and the gains from

trade are common knowledge. As noted earlier, absent inventories and capital constraints, the market may *partially* break down in the sense that there is a positive probability that efficient trade does not take place; however, the market does not break down *completely*, as in our paper.[3]

Two recent papers obtain time intervals of no trade in a dynamic lemons problem. To do so they add the assumption that some noisy information about the asset quality is revealed (exogenously). In Kremer and Skrzypacz (2007) information is revealed at some future point in time and trade ceases just before that point. In Daley and Green (forthcoming) information is revealed gradually. Instead, we obtain a no trade result by adding inventories and capital constraints to a standard lemons problem. We also show that concerns about the value of inventories can increase the probability of trade, as potential buyers are induced to increase the price. In our setting, trade is always efficient and so increasing the price increases welfare. In this sense, our paper differs from papers in which price manipulation creates distortions that are suboptimal from a social point of view.[4]

Our paper also relates to the literature that explores the link between leverages and trade. For example, Shleifer and Vishny (1992) show that high leverage may force firms to sell assets at fire-sale prices, while Diamond and Rajan (2011) show that the prospect of fire sales may lead to a market freeze. In their model, banks do not sell their assets because the gains from selling are captured by the bank's creditors rather than the bank's equity holders. Other papers explore feedback effects between asset prices and leverage: Low prices reduce borrowing capacity, and hence asset holdings and prices also; see, e.g., Kiyotaki and Moore (1997). In contrast, we model a situation in which firms can meet their financial needs by staying with the status quo. Therefore, there is no need for fire sales or "cash in the market" pricing, as in Allen and Gale (1994). The only motive for trade in our paper is that the buyer values the asset more than the seller does, and both agents know this.

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[3]In a recent paper, Glode, Green, and Lowery (forthcoming) endogenize the extent of adverse selection in a standard lemons problem, showing that firms may overinvest in financial expertise. The outcome of this is that if uncertainty increases, the probability of efficient trade is reduced.

[4]Examples includes Allen and Gale (1992); Brunnermeier and Pedersen (2005); Goldstein and Guembel (2008).

In a contemporaneous paper, Milbradt (2010) shows that a trader who is subject to a mark-to-market capital constraint (i.e., one based on the last trade price rather than on the actual expected value of the asset) may suspend trade so that losses are not revealed. While the general idea relates to ours, there are big differences, including the following. First, in Milbradt (2010) the price is exogenous, whereas in our setting, the price is endogenous. This allows us to obtain predictions regarding the relationship between prices, inventories, and leverage, both when the buyer is a monopolist and when the buyer faces competition. Second, in Milbradt (2010), trade suspension is bad news, whereas in our setting, a market freeze is neither good news nor bad news. The key difference is that in Milbradt (2010), the agent who acts strategically is the informed agent, whereas in our setting, the agent who acts strategically (the buyer) is uninformed. Third, our results do not depend on a specific accounting or regulatory regime. Instead, the market value of existing assets is derived from Bayes' rule. Our main results continue to hold, however, even if we assume marking to market.

The idea that inventory holdings affect price-setting behavior is found in market microstructure papers that study the effect of market-maker inventories; see, e.g., Amihud and Mendelson (1980); Ho and Stoll (1981,1983). These papers assume symmetric information and are therefore silent with respect to our main results. Moreover, these papers predict that as inventories increase, prices fall—a prediction that seems inconsistent with the empirical findings in Manaster and Mann (1996). In contrast, by interpreting buyers in our model as market-makers, our framework naturally delivers exactly this prediction: when market-makers care about the market value of their inventories, higher inventories may lead to higher prices, once we control for leverage. The reason for the two opposite predictions is as follows. In the classic inventories models, the dealer wants to reduce the price when he has more inventories because he is either risk averse and concerned about future price movements, or else he is not allowed to carry too much inventories. In contrast, in our setting, inventories serve as collateral and so when the dealer has more inventories, he may choose to offer a higher price in order to increase the borrowing capacity of his inventoried

assets. Our model also provides a new testable hypothesis, namely that a price offered by one dealer may increase when *other* dealers hold more inventories.

Finally, our paper relates to the literature on equity issuance, in which the issuing firm cares about the market valuation of its remaining equity.[5] However, we do not focus on signaling. Instead, we show how leverage affects the buyer's bidding strategy and the probability of trade.

*Paper outline.* The paper proceeds as follows. Section 2 describes the model and in Section 3, we solve the monopolist buyer case. We illustrate our results both for the case in which the buyer is capital constrained and for the case in which the seller is capital constrained. We also relate the results to the freeze in the market for mortgage-backed securities during the recent financial crisis. In Section 4, we extend our setting to a two-period model and show that a market freeze may be preceded by increased prices and increased trading volume. In Section 5, we analyze the effects of competition between multiple buyers. In Section 6, we discuss policy and empirical implications. In Section 7, we discuss extensions and robustness. Section 8 concludes and the appendix contains proofs and other omitted details.

## 2    The Model

In the basic model, there is a risk-neutral buyer and a risk-neutral seller. The value of an asset is $v$ to the seller and $v + \Delta$ to the buyer, where $\Delta > 0$ denotes the gains from trade. It is common knowledge that $v$ is drawn from a uniform distribution on $[0, 1]$. The seller knows $v$. Everyone else is uncertain about the value of $v$. Consequently, trade affects posterior beliefs about $v$, and hence the market value of each unit of asset. Since $\Delta > 0$, trade is always efficient.

In one interpretation, the seller is a loan originator. The gains from trade reflect the fact that the seller has better skills in originating loans whereas the buyer has better skills in managing loans. The buyer may also have a lower cost than the seller of retaining risky

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[5]See, for example, Allen and Faulhaber (1989), Grinblatt and Hwang (1989), and Welch (1989).

assets on his balance sheet because of lower borrowing costs or less stringent regulation. We can also think of the buyer as a broker dealer, who helps with the matching process between the seller and other investors, who have higher valuations for the asset.

The seller owns $x$ units of the asset for sale. The buyer has an inventory of $M$ units of the asset, which he acquired earlier. The buyer also has a short-term debt liability $L$, which he can either pay back or roll over, but only if the value of his assets is high enough relative to the value of his liabilities. We refer to the latter constraint as the capital constraint. Specifically, suppose the buyer purchases $q$ additional units at a price per unit $p$, and let $h$ denote the market valuation of each unit (that is, the market valuation of $v$). Assume, for simplicity, that the buyer holds only the asset traded and that the purchase is financed by additional short-term borrowing (i.e., the buyer holds no cash). Then the buyer's capital constraint is

$$(h + \Delta)(M + q) \geq L + pq, \tag{1}$$

where $M+q$ is the buyer's total inventory of assets net of trade, and $L+pq$ is the buyer's total liabilities, net of trade. In Section 7.2 and in the appendix, we discuss the generalization in which the lefthand side of (1) is $\alpha(h + \gamma\Delta)(M + q)$, reflecting constraints on the buyer's ability to pledge all his future cash flows.

As we explain in the next section, the asset's market value is conditional on the trading outcome and is derived using Bayes' rule. Consequently, when choosing a trading strategy, the buyer must consider not only the effect of trade on expected profits from purchasing new units of asset, but also the effect of trade on the market value of inventoried assets. In Section 7.3, we also analyze the case in which the capital constraint is based on marking to market.

If the buyer violates his capital constraint, he incurs a cost $F$, which can represent loss of growth opportunities due to bankruptcy or closure by the regulator. We focus on the case in which the capital constraint is satisfied before trading begins. This assumption allows us to focus on the question of how the buyer changes his behavior to avoid violating the capital constraint, rather than on the much-studied fire sales that follow when the constraints are

violated. We also assume that the cost $F$ is sufficiently high so that the buyer's first priority is to satisfy his capital constraint.[6]  Hence, the buyer's objective is to maximize the expected value of his assets subject to not violating his capital constraint. Finally, we assume that the quantity of the asset available for trade is small relative to the buyer's existing asset holdings; that is,

**Assumption 1** $x < M$

Assumption 1 implies that when the buyer purchases more assets, his capital constraint is tightened. It also ensures that increasing the bid loosens the buyer's capital constraint (see Section 3.2).

To determine the outcome of trade, we use a variant of the seminal Glosten and Milgrom (1985) model of price-setting in markets with asymmetric information, in which the uninformed party (the market-maker in their model, here the buyer) posts a "bid" price at which he is prepared to buy. We depart from Glosten and Milgrom by first analyzing the case in which the buyer is a monopolist, and then modeling the effects of competition between multiple strategic buyers; in both cases, we assume that the seller is not subject to any capital constraints. In Section 3.3, we also analyze the case in which the seller is capital constrained and must retain some fraction of his assets on his balance sheet.

# 3    A Monopolist Buyer

The monopolist buyer makes a take-it-or-leave-it offer to buy $q \in [0, x]$ units of the asset at a price per unit $p$. The seller can either accept or reject the offer. If he is indifferent, he accepts.

Making the take–it-or-leave-it offer above is equivalent to offering a linear price schedule according to which the buyer offers to buy up to $q$ units at a price per unit $p$, since whenever the seller chooses to sell, he sells as many units as he can. More generally, the buyer can

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[6]#p# In particular, for all results relating to a single buyer (i.e., everything except Section 5), the assumption $F > x(1 + \Delta)$ is enough to ensure that the disutility from violating the capital constraint is larger than the profit gained by doing so.

offer a price schedule $p(\cdot)$, which is not necessarily linear, where $p(q)$ denotes the per-unit price that the buyer is willing to pay for $q$ units. However, the next proposition (proved in Appendix B) shows that the monopolist buyer cannot gain by doing so, thereby generalizing the classic result of Samuelson (1984) to a setting with inventories and capital constraints:[7]

**Proposition 1** *A monopolist buyer cannot gain by offering a nonlinear price schedule.*

We start with the benchmark case $M = 0$, in which the buyer has no inventories. Then we analyze the main case, $M > 0$.

## 3.1   Buyer Does Not Have Inventories

In the benchmark case, the buyer offers a pair $(p, q)$ to maximize his expected profits subject to $q \leq x$. To ensure that the seller's acceptance decision is nontrivial, we assume that the gains from trade are not too high, $\Delta < \frac{1}{2}$, so that the buyer always offers to pay $p \leq 1$. The seller accepts the offer if and only if $v \leq p$, which happens with probability $p$ (since $v$ is uniform on $[0, 1]$). Conditional on the seller accepting the offer, the expected value of the asset to the buyer is $\frac{1}{2}p + \Delta$, and since the buyer pays $p$, his expected profit per unit bought is $\Delta - \frac{1}{2}p$. Taking into account the probability of trade and the quantity traded, the buyer's expected profit is $\pi(p, q) \equiv pq(\Delta - \frac{1}{2}p)$.

The buyer's profit-maximizing bid is to buy everything, $q = x$, for a price $p = \Delta$. Thus, the probability of trade, $p$, increases when the gain from trade is higher. The gains from trade are split equally between the buyer and seller. The seller obtains rents because of his private information. The buyer obtains rents because he is the one making the offer.

**Proposition 2** *In the benchmark case, the buyer offers to buy $x$ units at a price per unit $\Delta$. The seller accepts this offer if and only if $v \leq \Delta$.*

For use below, observe that any $p \in [0, 2\Delta]$ provides the buyer with nonnegative profits.

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[7]Although Samuelson's analysis is formulated in mechanism design terms, one can equivalently analyze non-linear price schedules: a (direct-revelation) mechanism specifying a transfer of $q(v)$ units of the asset in exchange for a monetary payment of $p(v) q(v)$ is equivalent to giving the seller the choice of quantity-price pairs in the menu $\{(q(v), p(v)) : v \in [0, 1]\}$.

Consequently, the buyer has room to increase his bid beyond the benchmark price $\Delta$ while still maintaining positive profits.

## 3.2 Buyer Cares About the Value of His Inventory

As before, the seller accepts the buyer's offer if and only if $v \leq p$. Accepted offers reduce the market value of the asset and hence of existing inventories. However, the purchase of new units of asset may generate a profit. On net, these two forces tighten the capital constraint, since, by assumption 1, inventories are large relative to new trades.

**Lemma 1** *The acceptance of an offer tightens the capital constraint.*

In contrast, the rejection of an offer relaxes the capital constraint since it increases the market value of the asset. Hence, it is enough to ensure that the capital constraint is satisfied only after an offer is accepted.

Since $v$ is uniform on $[0, 1]$, we obtain from Bayes' rule that if the seller accepts an offer with $q > 0$, the market value of the asset drops to $h = \frac{1}{2}p$, which is the expected value of $v$ conditional on $v \in [0, p]$. Substituting $h$ into the buyer's capital constraint yields

$$(\frac{1}{2}p + \Delta)(M + q) - pq \geq L. \tag{2}$$

Note that if the seller rejects the offer, the market value of the asset rises to $\frac{1}{2}(1+p)$, i.e., the expected value of $v$ conditional on $v \in [p, 1]$. Consequently, the buyer seeks to maximize, by choice of $(p, q)$ and subject to satisfying the capital constraint, the expected value of his asset position,

$$p\left((\frac{1}{2}p + \Delta)(M + q) - pq\right) + (1 - p)M(\frac{1 + p}{2} + \Delta). \tag{3}$$

The first term corresponds to the probability $p$ event that the offer is accepted, while the second term corresponds to the probability $1 - p$ event that the offer is rejected; $\frac{p}{2}$ and $\frac{1+p}{2}$ are the market valuations conditional on these two events. By straightforward algebra, this expression simplifies to

$$(\frac{1}{2} + \Delta)M + \pi(p, q). \tag{4}$$

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Economically, by the law of iterated expectations the expected value of the buyer's existing inventories is simply its prior, $\frac{1}{2}+\Delta$, and is unaffected by the buyer's strategy. So the buyer's problem reduces to maximizing the expected profit from his trade, $\pi(p,q)$.

Hence, the buyer's problem is to choose a bid $(p,q) \in [0,1] \times [0,x]$ to maximize expected profits $\pi(p,q)$, subject to his capital constraint. In cases of indifference, we assume that the buyer makes the bid associated with the highest quantity $q$, thereby maximizing social welfare.

Define $\delta \equiv \frac{L}{(\frac{1}{2}+\Delta)M}$, a measure of the buyer's initial leverage.(i.e., the ratio of his net liabilities to the initial market valuation of his assets). Since $q \leq x < M$ (Assumption 1), the buyer's capital constraint can be rewritten as

$$p \geq \frac{\delta + 2\Delta(\delta - 1 - \frac{q}{M})}{1 - \frac{q}{M}} \equiv p(q), \tag{5}$$

where $p(q)$ is the minimum price that the buyer must offer in order to keep his capital constraint satisfied if the seller accepts the offer.

Equation (5) implies that increasing $p$ loosens the capital constraint. Increasing the price increases the perceived value of existing inventories, which helps loosen the capital constraint, but it also increases the amount the buyer pays for the additional units he purchases, which tightens the capital constraint. When the amount of inventories is large relative to the amount for sale (Assumption 1), the first effect dominates.

Since the buyer can borrow against the full value of his assets, and since he makes nonnegative profits, buying more assets also loosens the capital constraint. Formally, one can see that from (2): since the buyer makes nonnegative profits, we know $p \leq 2\Delta$, and so the right-hand side of the inequality is increasing in $q$. Consequently, if the buyer finds it worthwhile to bid at all, he bids for the entire quantity available, $q = x$. Bidding for a lower quantity not only lowers the buyer's profits, but it also makes it harder to satisfy his capital constraint. In Section 7.2 and in the appendix, we show that if the buyer can borrow only against a fraction of the market value of his assets, it may not longer be the case that increasing $q$ loosens the capital constraint. In this case, we may obtain an interior solution in which the buyer offers to buy less than the full amount.

The buyer's problem reduces to choosing $p$ to maximize his expected profits $\pi(p, x)$, such that $p \geq p(x)$, so that the capital constraint is satisfied. Since the buyer loses money from bids $p > 2\Delta$, trade is impossible if $p(q) > 2\Delta$, which reduces to $\delta > \frac{4\Delta}{1+2\Delta}$. If instead $\delta \leq \frac{4\Delta}{1+2\Delta}$, the buyer bids as close to his benchmark bid of $\Delta$ as possible; that is, $p = \max\{\Delta, p(x)\}$

**Proposition 3** *When the buyer cares about the value of his inventories, trade can happen if and only if $\delta \leq \frac{4\Delta}{1+2\Delta}$. In this case, the buyer offers to buy $x$ units at a price per unit $\max\{\Delta, p(x)\}$, and the seller accepts this offer if and only if $p(x) \leq v$.*

When the buyer's initial leverage is low, the price and the probability of trade are the same as in the benchmark case because the buyer has enough slack to satisfy his capital constraint even though trade reduces the perceived value of his inventoried assets. When leverage increases, so that the buyer has less slack, the buyer must increase his bid to ensure that his capital constraint is satisfied if the seller accepts the offer. Since a higher bid increases the probability that the seller will accept the offer, the probability of trade increases. Finally, if leverage is too high, the market breaks down because any bid that is high enough to satisfy the buyer's capital constraint provides him with negative expected profits.

We focus on an extreme case in which the buyer has all of the bargaining power, but the nature of the result remains even if the buyer has only some of the bargaining power. In particular, the result that for some parameter values, the price increases in leverage depends on the fact that in the benchmark case the buyer can capture some of the surplus, and so when his leverage increases, he can increase the price, while still maintaining positive profits.

An immediate corollary to Proposition 4 concerns the effect of high leverage and the corresponding market breakdown on the revelation of the seller's information about asset values:

**Corollary 1** *If initial leverage is high, $\delta > \frac{4\Delta}{1+2\Delta}$, market participants learn nothing about the value $v$ of the asset.*

## 3.3 Seller Cares About the Value of His Inventory

The analysis can extend to the case in which the seller is subject to a capital constraint. The interesting case is when the seller is forced to retain some assets on his balance sheet. For example, suppose that the seller has $M_s$ units of the asset, but can sell at most $x$. In this case, the seller cares about the affect of trade on the market value of the remaining $M_s - x$ units. For ease of exposition, we analyze the case in which the seller is constrained and the buyer is not. We denote the seller's liabilities by $L_s$ and define $\delta_s \equiv \frac{L_s}{\frac{1}{2}M_s}$, which is a measure of the seller's initial leverage.

As before, we assume that initially the capital constraint is satisfied and that the cost of violating the constraint is very high. The relevant constraint is when the seller accepts the buyer's offer. In this case, the market learns that $E(v) = \frac{1}{2}p$, and the capital constraint becomes

$$\frac{1}{2}p(M_s - q) + pq \geq L_s, \tag{6}$$

where $M_s - q$ is the seller's total inventory net of trade and $pq$ is the sale proceeds.

The seller's capital constraint can be rewritten as

$$p \geq \frac{\delta_s}{1 + \frac{q}{M_s}}. \tag{7}$$

Holding the offer price fixed, it is easier to satisfy the seller's capital constraint when $q$ is higher, and therefore it is optimal for the buyer to offer either $q = 0$ or $q = x$. Intuitively, since the market valuation of the asset is less than the sale price (for standard adverse selection reasons), replacing assets with cash relaxes the capital constraint. As in the buyer's case, trade can happen only if the seller's initial leverage is sufficiently low so that the capital constraint is satisfied if the seller accepts the offer; that is, if $\delta_s \leq p(1 + \frac{q}{M_s})$. If trade happens, the buyer chooses $p = \max\{\Delta, \frac{\delta_s}{1 + \frac{q}{M_s}}\}$, and the seller accepts the offer if and only if $v \leq p$.

**Proposition 4** *When only the seller cares about the value of his inventory, trade can occur if and only if $\delta_s \leq 2\Delta(1 + \frac{x}{M_s})$. In this case, the buyer offers to buy $x$ units at a price per unit $\max\{\Delta, \frac{\delta_s}{1 + \frac{x}{M_s}}\}$, and the seller accepts if and only if $v \leq p$.*

The relationship between leverage, prices and probability of trade, is similar to the relationship we obtained earlier for the case in which the buyer was capital constrained.

The seller's case also provides some interesting predictions regarding $x$, the maximum amount that can be sold. Trade can happen only if $x$ is sufficiently large. However, once trade happens, a further increase in $x$ reduces the probability of trade. Intuitively, when $x$ increases, the seller retains less assets on his balance sheet and so the market value of remaining assets place a less important role. This means that the market is less likely to break down, but it also means that if trade happens, the buyer does not need to increase the price as much to satisfy the seller's capital constraint, and so the probability of trade is reduced.

## 3.4 Discussion

Our model implies that socially efficient trade can completely break down ("freeze") if the seller has an information advantage and if either the buyer or the seller is both highly leveraged and holds significant inventories of similar assets. This implication is consistent with the freeze in the markets for mortgage-backed securities during the recent financial crisis. Adrian and Shin (2010) document a sharp increase in dealers' leverage, while many market observers expressed the view that concerns about the value of inventories induced firms not to sell their assets. For example, an analyst was quoted in *American Banker*[8] as saying that "Other [companies] may be wary of selling assets for fear of establishing a market-clearing price that could force them to mark down the carrying value of their nonperforming portfolio." Also related is the view expressed in Lewis' book (2010) that dealers who sold credit default swaps on subprime mortgage bonds did not make a market in these securities so that the bad information is not revealed and their positions do not lose money. Moreover, and consistent with our results in Section 4, Lewis suggests that prior to the crisis, prices increased in a way not supported by fundamentals.[9]

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[8] "Nonperformance Space: Risky Assets Find Market" (*American Banker*, August 19, 2009).

[9] For example, on page 184, Lewis writes that "Burry [an investor who bought credit default swaps on subprime mortgage bonds] sent his list of credit default swaps to Goldman and Bank of America and Morgan Stanley with the idea that they would show it to possible buyers, so he might get some idea of

# 4  Run-ups and break-downs

The static model is suggestive of a dynamic process in which the buyer increases leverage and prices until the market breaks down eventually. To model this explicitly, we extend our single-period model to a two-period model in which the monopolist buyer trades sequentially with two potential sellers. In this case the buyer's leverage changes endogenously because the outcome of trade with the first seller affects the value of the buyer's assets, and hence his leverage, before the second trade. We focus on the case in which the buyer is capital constrained but the sellers are not and characterize the buyer's optimal bidding strategy. One of the results is that a market freeze may be preceded by a run-up in prices and increased trade volume.

Each seller sells a different asset; seller $i$ $(i = 1, 2)$ sells asset $i$. The value (per unit) of asset $i$ is $v_i$ to the seller and $v_i + \Delta$ to the buyer, where $v_1, v_2$ are independent random variables drawn from a uniform distribution on [0,1]. Each seller can sell at most $x$ units. Before trading begins, the buyer has inventories of $M$ units of asset 1 and $M$ units of asset 2 (as before, $x < M$ and $\Delta \in (0, 1/2)$). Since the values of the two assets are independent, one cannot infer anything about the value of one asset by observing trade in the other asset. This allows us to focus only on the effect of leverage. For simplicity, we assume that the discount rate equals zero.

In the first period, the buyer makes a take-it-or-leave-it-offer $(p_1, q_1)$ to the first seller, who can either accept or reject the offer. In the second period, the buyer makes a take-it-or-leave-it-offer $(p_2, q_2)$ to the second seller, who can also either accept or reject it. Assume that $q_i \in \{0\} \cup [\underline{q}, x]$; that is, if the buyer offers to buy something, he must buy at least $\underline{q} > 0$ units. The parameter $\underline{q}$ can be made arbitrarily small; as we explain below, this assumption is made to avoid an open set problem. Note that adding this assumption has no substantive effect on the results in the previous sections.

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the market price. That, after all, was the dealer's stated function: middleman. Market-makers. That is not the function they served, however. 'It seemed the dealers were just sitting on my lists and bidding extremely opportunistically themselves,' said Burry. The data from the mortgage servicers was worse every month...and yet the price of insuring those loans, they said, was falling." On page 185, he adds that "The firms always claimed that they had no position themselves...but their behavior told him otherwise."

As before, we assume that initially the capital constraint is satisfied and that the cost of violating the capital constraint is sufficiently high to outweigh any profit gains obtained from doing so. We also assume there is a positive probability that the second-period trade opportunity exogenously disappears. Consequently, the buyer must ensure the capital constraint is satisfied *both* after the first period and after the second period.

The buyer's problem is to choose a sequence of offers $(p_i, q_i)_{i=1,2}$ to maximize his expected profits, subject to the capital constraint being satisfied after each trade. As before, in cases of indifference, we assume the buyer makes the bid associated with the highest quantity, thereby maximizing social welfare.

Since the parameters in each round are the same, it is suboptimal to delay offers; if it is suboptimal to make an offer in the first round, it is also suboptimal to make an offer in the second round. Thus, a bidding strategy can be summarized by $(p_1, q_1; p_a, q_a; p_r, q_r)$, where $(p_1, q_1)$ denotes the offer to the first seller, and $(p_a, q_q), (p_r, q_r)$ denote the offer to the second seller given that the first seller accepted or rejected the offer, respectively.

As in the previous section, accepted offers tighten the capital constraint, while rejected offers relax the constraint. The potentially binding constraints are hence as follows. The first-period capital constraint must be satisfied if the first seller accepts the offer. In this case, the market value of the first asset is $\frac{1}{2}p_1$, while the market value of the second asset equals its prior, $\frac{1}{2}$, so the capital constraint is

$$(\frac{1}{2}p_1 + \Delta)(M + q_1) + (\frac{1}{2} + \Delta)M \geq L + p_1 q_1. \tag{8}$$

The second-period capital constraints must be satisfied when the second seller accepts the buyer's offer. For the case in which the buyer's first offer is accepted, the capital constraint is

$$(\frac{1}{2}p_1 + \Delta)(M + q_1) + (\frac{1}{2}p_a + \Delta)(M + q_a) \geq L + p_1 q_1 + p_a q_a, \tag{9}$$

while for the case in which the buyer's first offer is rejected, the capital constraint is

$$(\frac{1}{2} + \frac{1}{2}p_1 + \Delta)(M) + (\frac{1}{2}p_r + \Delta)(M + q_r) \geq L + p_r q_r \tag{10}$$

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Since the first seller accepts his offer with probability $p_1$, the buyer's expected utility is

$$\pi(p_1, q_1) + p_1 \pi(p_a, q_a) + (1 - p_1)\pi(p_r, q_r). \tag{11}$$

The problem reduces to finding a bidding strategy that maximizes the buyer's expected utility such that equations (8), (9), and (10) are satisfied.

It turns out that whenever the buyer's first-period bid is rejected, his capital constraint becomes sufficiently slack that he can make his unconstrained optimal bid of $(\Delta, x)$ in the second period. The intuition is that an offer $p_1$ satisfies the capital constraint only if either the bid price $p_1$ is high — in which case a rejected bid results in a substantial slackening of the capital constraint; or the capital constraint is very slack to begin with.

**Lemma 2** If $p_1 > 0$, then $(p_r, q_r) = (\Delta, x)$.

Hence, it remains to characterize $(p_1, q_1)$ and $(p_a, q_a)$. It follows from Proposition 3 that in the second period the buyer offers to buy either everything or nothing and makes nonnegative profits; hence, we can assume, without loss of generality, that $q_a = x$, with the interpretation that $p_a = 0$ corresponds to not making an offer.

In contrast, it is sometimes in the buyer's interest to make a loss-making offer of a very high price $p_1$ in the first period. The advantage of doing so is that in the event that this high offer is rejected, the market valuation of the buyer's inventory rises to a commensurately high level, relaxing the buyer's capital constraint. This then allows the buyer to make a highly profitable offer in the second period. The buyer would like the loss-making first-period offer to be for the *smallest* quantity that still gives rise to the increase in market valuation—this quantity is $\underline{q}$ in our notation.[10]

Our main result in this section is:

**Proposition 5** *Trade can happen (i.e., $p_1 > 0$) if and only if the buyer's initial leverage is not too high.*

*(i) When leverage is low, the buyer makes the benchmark bid $(\Delta, x)$ in both periods.*

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[10] For more details, see Lemma A-1 in the appendix.

*(ii) When leverage is intermediate, the buyer offers to pay strictly more than the bench-mark in the first period, and if the first offer is accepted, the buyer offers to pay even more (i.e., $p_a > p_1 > \Delta$); in both periods the buyer bids for the maximum amount $x$.*

*(iii) When leverage is high, the buyer withdraws from the market in the second period if his first offer is accepted. The buyer's initial bid ($p_1$) is increasing in leverage. In particular, when initial leverage is sufficiently high, the buyer initially bids more than the benchmark; that is, the market freeze is preceded by high prices. The quantity the buyer bids for in the first period is decreasing in leverage.*

Proposition 5 captures a few aspects of a dynamic behavior. If initial leverage is relatively moderate, the buyer has enough slack in his capital constraint to make two rounds of offers. But unless leverage is very low, the buyer still needs to consider his capital constraint, and this leads him to bid more than the benchmark price in both periods. If his first bid is accepted, his capital constraint is tightened, forcing him to bid even more in the second period. In other words, the price at which trade occurs rises with successful trades.

If instead initial leverage is high, the buyer has insufficient slack to have two bids accepted. Thus, there must be a period in which trade does not occur. In particular, if the buyer's first period offer is accepted, his capital constraint is too tight to make a bid in the second period and the market freezes. The proposition also sheds light on the price path leading up to this market freeze. When initial leverage is very high, the capital constraint is binding, forcing the buyer to make a high bid. Thus, the market freeze may be preceded by a run-up in prices.

The quantity the buyer offers to buy in the first period is decreasing in leverage. If leverage is low, the buyer offers to buy the full amount to maximize his expected profits in the first period. If leverage is high, the buyer offers to buy the lowest amount possible, since as we explained earlier, if the offer is accepted, he loses money and the only purpose of the offer is to relax his capital constraint if the offer is rejected.

# 5   Competition Among Buyers

Up to now we have focused on the case of a single buyer. As we have shown, concerns about preserving the market value of existing asset inventories affect the pattern of trade. When the buyer is very leveraged and so his capital constraint has little slack, such concerns lead to a trade breakdown, and prevent the dissemination of information about asset quality. However, if instead the buyer is only moderately leveraged, these same concerns drive up both prices and trade volumes.

A natural question is how these results would be affected by the presence of multiple competing buyers. In particular, one might conjecture that when multiple buyers are present, it is hard for any individual buyer to prevent the dissemination of bad news about asset values. In this section, we show that this conjecture is only partially correct. When all competing buyers are very leveraged, concerns about the market value of inventories again lead to a trade breakdown and prevent the dissemination of information about asset quality. However, under some circumstances in which one buyer is more leveraged than another, competition does indeed force trade and price dissemination to occur, even though it is against the most leveraged buyer's interests. In this sense, competition actually strengthens our previous finding that inventories may drive up prices: now, inventories of one buyer drive up the price offered by a second buyer.

In detail, we analyze the effects of competition between two strategic buyers, who are both subject to capital constraints.[11] Buyer $i$ has an inventory of $M_i$ units of asset and a debt liability $L_i$. The gain from trade with buyer $i$ is $\Delta_i$. The seller has $x$ units for sale, and he is not subject to a capital constraint. Everything is common knowledge, except for the true value of the asset ($v$), which is private information to the seller. As before, $\Delta_i < \frac{1}{2}$, $x < M_i$, the capital constraint for each buyer is initially satisfied and the cost for violating the constraint is large.

Both buyers make offers simultaneously. Buyer $i$ offers a price and quantity $(p_i, q_i)$,

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[11]The assumption of two buyers is for expositional clarity; qualitively, our results would extend to the case of more than two buyers.

meaning that he is willing to buy up to $q_i$ units at a price per unit $p_i$. Posting a price of zero is equivalent to not making an offer, and we assume, without loss of generality, that $p_i > 0$ if and only if $q_i > 0$. The seller selects quantities $q_i' \leq q_i$ to sell to each of buyers $i = 1, 2$ so as to maximize his profits $\sum_{i=1}^{2}(p_i - v)q_i'$, subject $q_1' + q_2' \leq x$. Note that we allow the seller to trade with both buyers. In the case that prices coincide, $p_1 = p_2$, the seller splits any trade between the buyers in proportion to the quantities $q_1$ and $q_2$, i.e., the seller's trade with buyer 2 is a fraction $\frac{q_2}{q_1}$ of his trade with buyer 1.

If $v > \max\{p_1, p_2\}$, the seller rejects both offers and trade does not take place. Otherwise, the seller accepts the offer with the highest price; and if the seller still has remaining assets to sell, he also accepts the lowest price offer if $v \leq \min\{p_1, p_2\}$.

As in the single-buyer case, denote buyer $i$'s leverage by $\delta_i \equiv \frac{L_i}{(\frac{1}{2}+\Delta_i)M_i}$, and denote by $p_i(q)$ the minimum price that buyer $i$ must offer if he were the only buyer and offered to purchase $q$ units. The expression for $p_i(q)$ follows from equation (5) and is given by

$$p_i(q) = \frac{\delta_i + 2\Delta_i(\delta_i - 1 - \frac{q}{M_i})}{1 - \frac{q}{M_i}}. \tag{12}$$

From Proposition 3, if $p_i(x) \leq 2\Delta$, a monopolist buyer offers to buy the full amount $x$ at a price per unit $\max\{p_i(x), \Delta\}$, which is the price that maximizes his expected profits, subject to satisfying his capital constraint. For use below, it is convenient to define $r_i \equiv \max\{p_i(x), \Delta\}$. Observe that $r_i$ is a monotone increasing transformation of our leverage measure $\delta_i$.

As in Section 4, to avoid technical issues we assume that the minimum quantity a buyer can offer to buy is $\underline{q}$, i.e., $q_i \in \{0\} \cap [\underline{q}, x]$. We also assume that the price space is finite, and the values $\{r_i, r_i - \varepsilon, r_i + \varepsilon, 2\Delta_i, 2\Delta_i - \varepsilon, 2\Delta_i + \varepsilon\}_{i \in \{1,2\}}$ lie within this space. The "tick" size $\varepsilon$ is assumed to be close to zero and for clarity, we exclude it from the statements of the results.

We characterize Nash equilibria of the bidding game. We focus on equilibria that survive the following iterated process of elimination of weakly dominated strategies. In the first stage we eliminate all strategies that are weakly dominated. In the second stage, we consider the game remaining after the first stage and eliminate strategies that are weakly dominated

in this new game. And so on. Our first result characterizes offers that survive the first elimination round.

**Lemma 3 (First elimination round)** *(A) If $r_i < 2\Delta_i$, an offer $(p_i, q_i)$ survives the first round of elimination of weakly dominated strategies if and only if $p_i \in [r_i, 2\Delta_i)$ and $q_i = x$.*

*(B) If $r_i = 2\Delta_i$, the unique offer to survive the first round of elimination of weakly dominated strategies is $(p_i, q_i) = (r_i, x)$.*

*(C) If $r_i > 2\Delta_i$, the offers $p_i = 0$ and $(p_i, q_i) = (r_i, x)$ survive the first round of elimination of weakly dominated strategies. In contrast, any offer $(p_i, q_i)$ with $p_i \geq r_i'$ and $p_i \neq r_i$ is eliminated.*

Part (A) says that when a buyer has a profitable trade, he always tries to exploit it by making an offer that yields positive profits and does not violate his capital constraint. This behavior is similar to the single-buyer case previously analyzed. Part (C) reflects the fact that, with competition, the buyer may also offer to buy the asset when it is not profitable to him. The buyer makes this "preemptive" bid to ensure that his capital constraint is not violated should the other buyer make an offer at a low price.

## 5.1 Equilibrium When Inventories Do Not Matter

In the benchmark case, in which neither buyer is subject to a capital constraint, the equilibrium price is $\min\{2\Delta_1, 2\Delta_2\}$. This is the standard outcome for settings with public buyer valuations. The buyer with the highest valuation acquires the asset at a price determined by the buyer with the second-highest valuation.[12] This result generalizes easily to the case in which both buyers have low leverage, so that their capital constraints are not binding in equilibrium.

**Lemma 4** *If $\max\{r_1, r_2\} \leq \min\{2\Delta_1, 2\Delta_2\}$, the seller sells everything to the buyer with the higher valuation for a price $\min\{2\Delta_1, 2\Delta_2\}$.*

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[12]See, for example, Ho and Stoll (1983).

## 5.2 Equilibrium When Inventories Matter

We start with the case in which at least one of the two buyers has a low-enough leverage such that he would acquire the asset if he were the only buyer. Without loss of generality, let this buyer be buyer 1; formally, $r_1 \leq 2\Delta_1$. From Lemma 3, we know that buyer 1 bids for the full amount $x$.

The key observation is that if buyer 1 acquires all $x$ units of the asset for less than $p_2(0)$, the information released by this trade causes buyer 2's capital constraint to be violated. Consequently, when buyer 2 is highly leveraged, so $p_2(0)$ is high, buyer 2 may bid more aggressively to ensure that the seller does not accept a lower bid from buyer 1. Parallel to $r_i$, it is useful to define $r'_2 \equiv \max\{p_2(0), \Delta\}$ to keep track of this effect.

Our main result is:

**Proposition 6** *Assume $r_1 \leq 2\Delta_1$. Then the only equilibrium outcome that survives iterated elimination of weakly dominated strategies is as follows:*

*(A) If buyer 2 has low leverage relative to buyer 1 (i.e., $r'_2 \leq r_1$), the seller sells everything for price $\max\{r_1, \min\{2\Delta_1, 2\Delta_2\}\}$. Whenever $r_1 > \min\{2\Delta_1, 2\Delta_2\}$, buyer 1 acquires the asset at price $r_1$, which depends on his leverage.*

*(B) If buyer 2 has high leverage relative to buyer 1 (i.e., $r'_2 > r_1$), the seller sells everything for price $\max\{r_2, \min\{2\Delta_1, 2\Delta_2\}\}$. In particular, if $r_2 \in (2\Delta_2, 2\Delta_1)$, the seller sells everything to buyer 1 at price $r_2$; and if $r_2 \geq \max\{2\Delta_1, 2\Delta_2\}$, the seller sells everything to buyer 2 who makes negative profits.*

Recall that in the benchmark case without capital constraints, the equilibrium price is $\min\{2\Delta_1, 2\Delta_2\}$. In part (A), capital constraints interact with competition in a straightforward way. The important capital constraint is buyer 1's, and sometimes forces buyer 1 to increase his offer to $r_1$.

In part (B), in contrast, the interaction between capital constraints and competition is less straightforward, and can lead to a form of "spillover" of capital constraints. That is, if buyer 2's leverage is relatively high so that $r_2 \in (2\Delta_2, 2\Delta_1)$, buyer 2's capital constraint leads

him to compete more aggressively with buyer 1, and consequently buyer 1 ends up paying an amount $r_2$ that is determined by buyer 2's capital constraint. If $r_2 \geq \max\{2\Delta_1, 2\Delta_2\}$, buyer 1 can no longer compete, and so buyer 2 acquires everything at a price $r_2$. In the latter case, buyer 2 makes negative profits, even though he would not bid at all if he were the only buyer. Buyer 2 is forced to make this bid, since otherwise the seller will trade with buyer 1 and the capital constraint of buyer 2 is violated.

Finally, we consider the case in which both buyers are so leveraged, that, if bidding individually, trade collapses. Clearly, no trade is an equilibrium that survives iterated elimination of weakly dominated strategies, since given that one buyer is not willing to acquire the asset, the unique best response for the other buyer is also not to acquire. Moreover, no trade is the only outcome to survive iterated elimination of weakly dominated strategies when $r_1 \neq r_2$.[13]

**Proposition 7** *If both buyers are highly leveraged (i.e., $r_i > 2\Delta_i$ for $i \in \{1, 2\}$), then a no-trade equilibrium survives iterated elimination of weakly dominated strategies. When $r_1 \neq r_2$, this is the unique equilibrium that survives iterated elimination.*

Proposition 7 shows that when both buyers have tight capital constraints, the conclusions of the single buyer case still hold: trade collapses, and price dissemination stops. Indeed, the condition $r_i > 2\Delta_i$ in Proposition 7 is equivalent to the condition for no trade in Proposition 3, i.e., $\delta_i > \frac{4\Delta_i}{1+2\Delta_i}$.

# 6 Policy and Empirical Implications

Our analysis has implications for government attempts to defrost markets and for regulatory proposals aimed at improving market functioning. It also provides some empirical implications regarding the relationship between dealers' inventories and prices.

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[13]In the special case $r_1' = r_2' < r_1 = r_2$, we are unable to rule out other equilibria in which one buyer makes a "latent" offer, knowing that it will not be accepted in equilibrium, and the second buyer makes a "loss making" offer to rule out a situation in which the seller trades with the first buyer and the capital constraint of the second buyer is violated.

## 6.1 Defrosting Frozen Markets

Consider the case in which only the buyer cares about inventory values and in which trade has completely broken down; that is, $\delta > \frac{4\Delta}{1+2\Delta}$ (Proposition 3). The discussion can easily be extended to the case of more than one buyer, as in Proposition 7.

One option open to a government is to offer to buy the seller's assets. Formally, suppose the government's valuation of the asset is $v + \Delta_g$, and as before $v$ is private information to the seller. Also suppose that the buyer is a more efficient holder of the asset than the government, so $\Delta_g < \Delta$. In line with commonly voiced concerns, a general problem with voluntary government purchase schemes is that sellers part with only their worst assets: If the government offers to pay $p$, the seller sells only if $v \leq p$ (the same as with a private buyer) and makes an expected profit of $\frac{p^2}{2}$.

A central question is whether the government purchase scheme can succeed without taxpayer subsidies (in expectation). Our model has two implications in this respect. First, observe that because of the rent the seller makes from his informational advantage, a subsidy-free purchase scheme is possible *only if* the asset is worth more to the government than to the seller ($\Delta_g > 0$); for example, if the government is a more efficient holder of risk than the asset seller. Second, even if this condition is satisfied, a subsidy-free purchase scheme imposes a cost on the original potential buyer. Recall that this buyer does not purchase the asset himself because doing so violates his capital constraint. However, the same is true when the government buys the asset at unsubsidized terms.[14] A similar issue arises if the government subsidizes a second private buyer to purchase the asset. Consequently, if either the asset is worth less to the government than to the seller, $\Delta_g \leq 0$, or if the government wishes to avoid hurting the original buyer, a taxpayer subsidy is required to defrost the frozen market.

Another option is to remove assets from the buyer's balance sheet, that is to replace assets with cash. If the buyer can borrow against the full value of the asset, as assumed in our

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[14]Formally, since the market has broken down, we know that $p(q) > 2\Delta$, for every $q \in (0, x]$; and since $\Delta_g < \Delta$, it follows that any unsubsidized offer $p \leq 2\Delta_g$ satisfies $p < 2\Delta$. Therefore, $p < p(q)$ for every $q \in (0, x]$, and any unsubsidized offer violates the buyer's capital constraint.

analysis so far, purchasing assets from the buyer can relax his capital constraint only if the purchase involves a taxpayer subsidy. If instead, the buyer has limited borrowing capacity, as in Section 7.2, purchasing assets from the buyer might relax his capital constraint even if the purchase does not involve a taxpayer subsidy.[15]

## 6.2 Should Regulation Mandate Some Retention of the Asset by the Seller?

A commonly voiced regulatory proposal is that sellers of assets subject to asymmetric information problems, such as issuers of asset-backed securities, should be required to retain some stake in the assets they sell.[16] Our analysis identifies a potential cost to this proposal, namely, that under some circumstances it leads to a market breakdown. To see this, reinterpret the parameter $x$ in our model as stemming from a regulation mandating that the seller retain a fraction $\frac{M_s - x}{M_s}$ of the asset he is selling. From Proposition 4, whenever $x$ is sufficiently low, trade is impossible because the seller cares too much about the market's perception of the value of the assets he is forced to retain. Moreover, notice that this case arises more easily when the seller is highly leveraged (measured by $\delta_s$).

The goal that regulators appear to have in mind with this regulation is to reduce moral hazard on the part of asset sellers; for example, to discourage loan originators from making bad loans and/or shirking on monitoring later on. Our analysis does not speak to this issue, and it seems likely that the regulation will have its intended effect in this regard. Our point here is instead to draw attention to a potentially significant cost of this regulation, namely, that it can lead to the breakdown of socially efficient trade.

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[15]In particular, if the government plans to spend $Z_g$ dollars, it can buy $Z_g/(\frac{1}{2} + \Delta_g)$ units without a taxpayer subsidy. Since each of these units allows the buyer to borrow $a(\frac{1}{2} + \Delta)$ dollars, the government purchase loosens the buyer's capital constraint only if the sale proceeds exceed the assets' borrowing capacity; that is, only if $Z_g > \alpha(\frac{1}{2} + \Delta)\frac{Z_g}{\frac{1}{2} + \Delta_g}$. This condition reduces to $\Delta_g > \alpha\Delta - \frac{1}{2}(1 - \alpha)$ and can be satisfied even if $\Delta_g < \Delta$.

[16]See, for example, section 15G of the Investor Protection and Securities Reform Act of 2010.

## 6.3   Dealers' Inventories and Prices

Holding initial leverage constant, our model predicts a positive relationship between dealers' inventories and the prices they offer to pay.

To see this, start with the single buyer case, and assume the buyer is a broker-dealer. Consider an increase in the buyer's inventories $M$ and a proportional increase in the buyer's liabilities, so that our measure of the buyer's initial leverage $\delta$ remains unchanged. The condition for trade $\delta < \frac{4\Delta}{1+2\Delta}$ implies that the increase in $M$ causes $p(x)$ to increase as well. Hence, as $M$ increases, the price $\max\{\Delta, p(x)\}$ weakly increases, and it does so strictly when $\delta$ is sufficiently large that $p(x) > \Delta$.

The positive relationship between inventories and prices continues to hold when there are multiple competing dealers. In particular, part (A) in Proposition 6 implies that when a dealer's inventories increase (and his initial leverage remains unchanged), he may offer a higher price. Moreover, part (B) shows that the price offered by one dealer may increase when a competing dealer has more inventories.

As noted in the introduction, the predictions above, which are different from the ones produced by existing market microstructure inventory models, are consistent with the empirical findings in Manaster and Mann (1996). We also provide a new testable hypothesis in this direction. For example, to our knowledge, the prediction that bid prices quoted by one dealer may increase when other dealers have more inventories has not been tested yet.[17]

# 7   Extensions and Robustness

## 7.1   What Can the Market Observe?

So far, we have assumed that market participants observe the terms of the buyer's offer, regardless of whether the offer is accepted or rejected. What if, instead, market participants observe the offer terms only if the offer is accepted and trade actually occurs? Perhaps surprisingly, the equilibrium outcomes are exactly the same under this alternative information

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[17]Biais, Glosten, and Spatt (2005) provide an excellent survey.

assumption.

To see this, consider first the one-period monopolist buyer case. As discussed above, rejection of *any* offer increases the market's expectation about the asset value. Consequently, if the market observes no trade, the capital constraint is at least weakly slackened. Hence the important capital constraint is the one following an accepted offer, which is observed. Consequently, the equilibrium outcomes are the same as in the main case studied, where rejected offers are observed. An identical argument applies in the competing-buyer case and in the dynamic case.[18]

## 7.2   Limited Borrowing Capacity

What if the buyer and seller can borrow against only a fraction of the market value of their assets, as opposed to the full value as we have thus far assumed? The analysis of the one-period case is easily extended to capture this. Replace $h$ in the capital constraint with $\alpha h$, where $\alpha$ is a constant, such that $\alpha \in (0, 1]$. The parameter $\alpha$ represents "haircuts" set by a regulator or by potential lenders to account for the asset's risk. Alternatively, $\alpha < 1$ represents limitations on the ability of potential lenders to seize borrowers' assets.

One can show (both in the seller's case and the buyer's case) that increasing $\alpha$ increases the region in which trade can happen.[19] Thus, a regulator might be able to defrost the market by increasing $\alpha$ by, for example, reducing capital requirements or providing loan guarantees. However, once $\alpha$ is large enough so that trade can occur, but not too large so that the benchmark solution is not achieved, a further increase in $\alpha$ reduces the bid price and the probability of trade. Intuitively, a higher $\alpha$ increases the market value of existing assets and therefore has a similar effect to that of reducing initial leverage.

The case $\alpha < 1$ also provides some new results in the buyer's case. First, the buyer may offer to buy less than the full amount. In fact, the quantity offered is continuous in leverage and as leverage increases, the quantity gradually drops from the full amount $x$ to

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[18]Recall that in the dynamic case, Lemma 2 tells us that the capital constraint is always slack enough after rejection to allow the buyer to make the benchmark offer $(p, q) = (\Delta, x)$. So even when the terms of rejected offers are observed, what happens after an offer is rejected plays no role in the maximization problem.

[19]Details are in the appendix.

zero. Second, expected volume, $q \Pr(v \leq p)$, is continuous in leverage. It first rises and then drops gradually to zero. The initial increase in expected volume occurs because at moderate levels of leverage, the buyer increases the price but keeps the quantity unchanged, at $q = x$. As leverage increase further, the buyer continues to increase the price, but he also reduces the quantity until it reaches zero. We provide full details in the appendix.[20]

## 7.3 Marking to Market

In our main analysis above, we assume that the market value of assets is derived using all available information; that is, using Bayes' rule. However, one obtains qualitatively similar results if instead assets are "marked to market"; that is, valued at the most recent transaction price.

Denote by $p_0 \in (0, 1]$ the price of the last offer accepted. Under marking to market with a borrowing capacity $\hat{\alpha} \in (0, 1]$, the initial borrowing capacity of each unit of asset is $\hat{\alpha} p_0$, and if an offer $(p, q)$ is accepted, the borrowing capacity changes to $\hat{\alpha} p$. A rejected offer has no effect on borrowing capacity. Hence, the relevant capital constraint for the buyer is $\hat{\alpha} p(M + q) \geq L + pq$. Assume that the buyer cannot gain in the short-run from an increase in the mark to market value of his assets. Hence, the objective remains to maximize the expected value of his assets conditional on all available information.

For ease of exposition consider the case $\hat{\alpha} = 1$, so the capital constraint reduces to $pM \geq L$.[21] Denoting the buyer's initial leverage by $\delta' \equiv \frac{L}{p_0 M}$, the capital constraint becomes $p \geq \delta' p_0$. If leverage is low or intermediate, i.e., $2\Delta \geq \delta' p_0$, the buyer offers to buy $x$ units at a price per unit $\max\{\Delta, \delta' p_0\}$. Otherwise, the buyer does not make any offer. The results for competing buyers are also similar to those in our previous analysis.[22]

For the two-period case, the relevant capital constraints, which are the analogues of equations (8), (9), and (10), are $p_1 M + p_0 M \geq L$, $p_1 M + p_a M \geq L$, and $p_0 M + p_r M \geq L$.

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[20]The appendix contains a more general case in which the buyer's capital constraint is $\alpha(h + \gamma\Delta)(M + q)$ for some $\alpha \in (0, 2)$ and $\gamma \in [0, 1]$.

[21]The case $\hat{\alpha} < 1$ is a special case of our general analysis with $\alpha_i = 2\hat{\alpha}$ and $\gamma = 0$, described in footnote 20.

[22]Denote the initial leverage of buyer $i$ by $\delta'_i \equiv \frac{L_i}{p_0 M_i}$ and redefine $r_i$ and $r'_i$ as $r_i = r'_i = \max(\Delta_i, \delta'_i p_0)$.

Our previous analysis of the dynamic case revealed that the buyer sometimes makes a high loss-making bid early on (i.e., date 1) in order to raise market beliefs about the asset value, and relax his capital constraint. In the case of marking-to-market, the feasibility of this strategy depends on whether the last price $p_0$ was high or low, i.e., on whether market conditions are worsening or improving.

Suppose first that market conditions are worsening, so $p_0$ is high compared with current valuations, and in particular, $p_0 \geq 1$. Then the buyer cannot relax the capital constraint by bidding a high price. Instead, any accepted offer tightens the constraint. The analysis is very similar to the main model. If the buyer's initial leverage is low, the benchmark solution is achieved. Otherwise, the buyer must decide whether to try to buy in two periods, or just one. When he tries to buy in two periods, the first bid is higher than the benchmark, and if accepted, the second bid is even higher.[23] In addition, the buyer always bids for the maximum amount and makes nonnegative profits in each period.[24]

In contrast, if market conditions are improving, so $p_0$ is *low* compared with current valuations, i.e., $p_0 < 1$, the buyer may try to boost the market by making a small trade at a high price on which he expects to lose money. As in the main model, losing money in the first period allows the buyer to make a profitable trade in the second period so that on net, the buyer's expected profits are nonnegative.

# 8  Summary

We analyze how existing stocks of assets — inventories — affect trade, information dissemination, and price formation. When market participants are close to their maximal leverage, concerns about the revelation of bad news prevent socially beneficial trade and information dissemination. However, when market participants are further from their maximal leverage, inventories lead to overbidding (in the sense that the buyer pays more than he would like),

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[23]To see this, define $H = \frac{L}{M}$ and $\sigma = p_0$ in the proof of Proposition 5.

[24]Note, however, that since rejected offers do not increase the market value of inventories, Lemma 2's conclusion that the buyer makes the bnechmark bid $\Delta$ after a rejected offer no longer holds; instead, the capital constraint implies that $p_r = \max\{\Delta, \frac{L}{M} - p_0\}$.

which stimulates socially beneficial trade. Because trade increases buyer inventories and often increases buyer leverage, these predictions imply that prices and trade volumes may first increase before collapsing. Our results continue to hold when buyers compete with one another, and some of the results are even strengthened. We use our model to comment on several prominent policy questions and we also derive several new empirical predictions. As a technical contribution, we generalize a classic result of Samuelson (1984) to cover buyers with capital constraints and inventories.

# Appendix A

We use the notation $\pi(p) \equiv p(\Delta - \frac{1}{2}p)$ to denote the buyer's expected profits per unit.

## Part 1: Static Monopolist

**Proof of Lemma 1.** Suppose the buyer offers $(p, q)$. The acceptance of this offer has two effects: First, the value of existing assets falls from $(\frac{1}{2} + \Delta)M$ to $(\frac{1}{2}p + \Delta)M$, with a net effect $\frac{1}{2}(1 - p)M > \frac{1}{2}(1 - p)x$. Second, the buyer adds $q$ units, each with a borrowing capacity of $\frac{1}{2}p + \Delta$, but he also pays $p$ per unit. If profits $\frac{1}{2}p + \Delta - p_i$ are negative, this second effect is also negative, and the proof is complete. Otherwise, the added borrowing capacity from this is $(\Delta - \frac{1}{2}p)q$, which is at most $1(1 - p)x$, since $q \leq x$ and $\Delta < \frac{1}{2}$. Combining the two effects, it follows that the overall effect is negative and the capital constraint is tightened. Q.E.D.

**Buyer has limited borrowing capacity**

Below we solve a more general case in which equation (2) is replaced with

$$\alpha(\frac{1}{2}p + \gamma\Delta)(M + q) - pq \geq L, \tag{A-1}$$

where $\alpha \in (0, 2)$ and $\gamma \in [0, 1]$. The case $\alpha > 1$ does not reflect borrowing constraints, but it is useful when we discuss marking to market in Section 7.3. We replace Assumption 1 with $x < \frac{\alpha}{2-\alpha}M$ and define leverage as $\delta = \frac{L}{(\frac{1}{2}+\gamma\Delta)M}$, which generalizes the definition in the text. We also generalize the definition of $p(q)$ in equation (5), so that $p(q) \equiv \frac{2\theta - cq}{1 - \beta q}$, where $\theta \equiv \frac{L}{\alpha M} - \gamma\Delta$, $\beta \equiv \frac{2-\alpha}{\alpha M}$, and $c \equiv \frac{2\gamma\Delta}{M}$.

**Proposition A-1** *(i) If $\frac{\alpha\gamma}{2-\alpha} < 1$, trade can happen if and only if the buyer's initial leverage satisfies $\delta < \frac{2(1+\gamma)\Delta}{1+2\gamma\Delta}$. If leverage is sufficiently low, the capital constraint does not bind, and the buyer offers to buy the entire quantity $x$ for a price $\Delta$. As leverage increases, the buyer increases the price; and as leverage increases further, the buyer also reduces the quantity he offers to buy. Both price and quantity are continuous in leverage. As leverage approaches $\frac{2(1+\gamma)\Delta}{1+2\gamma\Delta}$, the price approaches $2\Delta$ and the quantity approaches zero. Expected volume is also continuous in leverage; it first increases and then drops to zero.*

*(ii) If $\frac{\alpha\gamma}{2-\alpha} \geq 1$, trade is possible if and only if initial leverage is sufficiently low. If trade happens, the buyer offers to buy the entire quantity $x$ for a price per unit $\max\{\Delta, p(x)\}$, which is weakly increasing in initial leverage $\delta$.*

**Proof of Proposition A-1.**

For use below, note that $\theta = \delta\frac{1+2\gamma\Delta}{2\alpha} - \gamma\Delta$. Observe that $1 - \beta q > 0$, since $x < \frac{\alpha}{2-\alpha}M$. Hence, the capital constraint can be written as $p \geq p(q)$. Observe that $p'(q) = \frac{2\beta\left(\theta - \frac{c}{2\beta}\right)}{(1-\beta q)^2} = \frac{2\beta\left(\theta - \frac{\alpha\gamma\Delta}{2-\alpha}\right)}{(1-\beta q)^2}$. Thus,

$$\min_{q\in[0,x]} p(q) = \begin{cases} p(0) & \text{if } \theta \geq \frac{\alpha\gamma\Delta}{2-\alpha} \\ p(x) & \text{if } \theta \leq \frac{\alpha\gamma\Delta}{2-\alpha}. \end{cases} \tag{A-2}$$

Trade is possible if and only if there exists a quantity $q \in (0, x]$, such that $p(q) \leq 2\Delta$, so that the buyer makes nonnegative profits; that is, if either $p(x) \leq 2\Delta$ or $p(0) = 2\theta < 2\Delta$. Hence, trade is possible if and only if $\theta$ falls in some lower interval.

*Case 1:* $\frac{\alpha\gamma}{2-\alpha} < 1$.

At $\theta = \Delta$, $p(x) > p(0) = 2\Delta$. Thus, trade is impossible for $\theta \geq \Delta$ but is possible for all $\theta < \Delta$, or equivalently, $\delta < \frac{2\alpha(1+\gamma)\Delta}{1+2\gamma\Delta}$. Next, we characterize the buyer's offer when $\theta < \Delta$. If $p(x) \leq \Delta$, which is equivalent to $\theta \leq \frac{1}{2}(\Delta - \beta\Delta x + cx)$, the capital constraint does not bind and the buyer makes the benchmark offer $(x, \Delta)$. If $\theta \leq \frac{\alpha\gamma}{2-\alpha}\Delta$, increasing $q$ relaxes the capital constraint, and since $\theta < \Delta$ we know the buyer has a strictly profitable trade. Consequently, the buyer bids for the entire amount $x$ available and chooses a price $\max\{\Delta, p(x)\}$. This price is weakly increasing in $\theta$, and hence in initial leverage $\delta$.

The remainder of the proof of this case deals with the open interval of $\theta$ values above $\max\left\{\frac{\alpha\gamma}{2-\alpha}\Delta, \frac{1}{2}\left(\Delta - \beta\Delta x + cx\right)\right\}$ but below $\Delta$. Since $\theta < \Delta$, we know that the buyer has a strictly profitable trade. Moreover, any strictly profitable trade in which the capital constraint is slack is strictly dominated by one in which it binds: either $q < x$, in which case $q$ can be increased, or $q = x$ and $p > p(x) > \Delta$, in which case $p$ can be decreased. Consequently, the buyer's best offer is the solution to the more constrained maximization problem in which he must keep the capital constraint binding; that is,

$$\max_{q \in [0,x]} q\pi\left(p\left(q\right)\right). \tag{A-3}$$

Observe that $\frac{\partial}{\partial q}q\pi\left(p\left(q\right)\right) = \pi\left(p\left(q\right)\right) + qp'\left(q\right)\pi'\left(p\left(q\right)\right)$, and recall $p'\left(q\right) > 0$ in the interval under consideration. Hence, (A-3) has a unique solution, as follows: If $q\pi\left(p\left(q\right)\right) > 0$ and $\frac{\partial}{\partial q}q\pi\left(p\left(q\right)\right) \le 0$ for some $q$, then $\pi'\left(p\left(q\right)\right) < 0$, and so by the strict concavity of $\pi$ and the strict convexity of $p$, it follows that $\frac{\partial}{\partial q}q\pi\left(p\left(q\right)\right) < 0$ for all higher $q$. Moreover, the maximizer of (A-3) must be such that $\pi'\left(p\left(q\right)\right) < 0$ (if the maximizer is the corner $q = x$, this follows from $p\left(x\right) > \Delta$). Hence, in the interval from which the maximizer of (A-3) is drawn, $\frac{\partial}{\partial q}q\pi\left(p\left(q\right)\right)$ strictly decreases in $\theta$. Hence, the buyer's choice of $q$ weakly decreases as $\theta$ (and hence initial leverage $\delta$) increases. Note also that if $q$ is strictly decreasing at any $\theta$, the same is true for all higher $\theta$.

For the effect of $\theta$ (and hence leverage) on the price offered, note first that if the buyer offers to buy everything $(q = x)$ for price $p\left(x\right)$, it follows immediately that the price increases. If instead the buyer offers to buy $q < x$, the price satisfies $p = p(q)$, and the optimal $p$ solves $\max_{p \in [0,1]} q(p)\pi(p)$, where $q(p) = \frac{p-2\theta}{\beta p - c}$ is the inverse function of $p(q)$. Observe that $\frac{\partial}{\partial p}q(p)\pi(p) = q'(p)\pi(p) + q(p)\pi'(p)$; $q'(p) = \frac{2\theta\beta - c}{(\beta p - c)^2} > 0$; and recall that the optimal $p$ satisfies $\pi'(p) < 0$. Hence, in the interval in which the (unique) optimal $p$ is drawn, $\frac{\partial}{\partial p}q(p)\pi(p)$ strictly increases in $\theta$ and strictly decreases in $p$ (the last part follows from the concavity of $\pi$ and $q$). Hence, the optimal $p$ increases in $\theta$.

From the analysis above, the buyer's offer is continuous as a function of $\theta$. Finally, as $\theta$ approaches $\Delta$, only offers with $q$ close to 0 can satisfy the capital constraint (with a price below $2\Delta$). It follows easily that as $\theta$ approaches $\Delta$, the buyer's offer converges to

$(q, p) = (0, 2\Delta)$. The expected volume $(pq)$ converges to 0.

To show that expected volume first increases in leverage, it is enough to show that there exists some interval to the right of $\frac{1}{2}(\Delta - \beta\Delta x + cx)$ such that when $\theta$ falls in this interval the buyer offers to buy everything, $q = x$. If $\frac{\alpha\gamma}{2-\alpha}\Delta > \frac{1}{2}(\Delta - \beta\Delta x + cx)$, this is immediate from the analysis above. Otherwise, note that at $\theta = \frac{1}{2}(\Delta - \beta\Delta x + cx)$, we know $p(x) = \Delta$; thus, $\frac{\partial}{\partial q}q\pi\left(p\left(q\right)\right)\Big|_{q=x} = \pi\left(\Delta\right) + xp'\left(x\right)\pi'\left(\Delta\right) = \pi\left(\Delta\right) > 0$. By continuity, it follows that $\frac{\partial}{\partial q}q\pi\left(p\left(q\right)\right)\Big|_{q=x} > 0$ over some interval to the right of $\frac{1}{2}(\Delta - \beta\Delta x + cx)$, implying that the buyer offers to buy $q = x$ in this interval.

From the analysis above, $q$ must eventually be strictly decreasing in $\theta$, (and if it is strictly decreasing at some $\theta$, the same is true for all higher $\theta$ up to $\Delta$, when trade becomes impossible). In this case, expected volume changes by $\frac{\partial(qp(q))}{\partial\theta} = \frac{\partial q}{\partial\theta}p(q) + qp'(q)\frac{\partial q}{\partial\theta} = \frac{\partial q}{\partial\theta}[p(q) + qp'(q)]$, which is strictly negative in the interval under consideration.

*Case 2:* $\frac{\alpha\gamma}{2-\alpha} \geq 1$ (contains Proposition 3 as a special case)

In this case, at $\theta = \Delta$, $p\left(x\right) \leq p\left(0\right) = 2\theta$, and so $p\left(x\right) \leq 2\Delta$. Hence, trade is certainly possible up to $\theta = \Delta$. Hence, trade is possible for all $\theta$ weakly below the cutoff value of $\theta$ such that $p\left(x\right) = 2\Delta$. The characterization of the buyer's offer for $\theta$ below this cutoff is the same as for the first part of the case $\frac{\alpha\gamma}{2-\alpha} < 1$.

Changes in $\alpha$: From the capital constraint $\alpha(\frac{1}{2}p + \gamma\Delta)(M + q) - pq \geq L$, it follows that if trade is possible when $\alpha = \alpha'$, it is also possible when $\alpha \geq \alpha'$. Also observe that $\beta$, $p\left(q\right)$, and $p'(q)$ strictly decrease in $\alpha$. Hence, following similar steps as above, one can show that increasing $\alpha$ has a similar effect on the price and quantity as reducing $\theta$.   Q.E.D.

## Seller has limited borrowing capacity

Below we solve the generalization in which equation (6) is replaced with $\frac{1}{2}\alpha p(M_s - q) + pq \geq L_s$, where $\alpha \in (0, 2)$. Equation (7) becomes $p \geq \frac{\delta_s}{\alpha + (2-\alpha)\frac{q}{M_s}}$. Trade can occur if and only if $\delta \leq 2\Delta[\alpha + (2 - \alpha)\frac{x}{M_s}]$, and if trade occurs, the buyer offers to buy $x$ units at a price per unit $\max\{\Delta, \frac{\delta_s}{\alpha + (2-\alpha)\frac{x}{M_s}}\}$.

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## Part 2: Dynamic Monopolist

**Lemma A-1** *The second-period offer satisfies $p_a, p_r \leq 2\Delta$ and $q_a, q_r \in \{0, x\}$; that is, the buyer offers to buy either everything or nothing, and he makes nonnegative profits. The first-period offer satisfies $q_1 \in \{0, \underline{q}, x\}$. If $q_1 = \underline{q}$, expected profits in the first period are negative; that is, $p_1 > 2\Delta$. If $q_1 = x$, expected profits in the first period are nonnegative; that is, $p_1 \leq 2\Delta$.*

**Proof of Lemma A-1.** Since the capital constraint is satisfied at the start of the second period (as $F$ is sufficiently large), it follows from Proposition 3 that $q_a, q_r \in \{0, x\}$. For the first period, note that the quantity $q_1$ enters the capital constraint with a coefficient $(\frac{1}{2}p_1 + \Delta) - p_1$, which is the expected value of the asset acquired minus the price paid. This expression has the same sign as the per-unit profit $\pi(p_1)$. Consequently, if $\pi(p_1) \geq 0$, the buyer offers to buy the maximum amount $x$, since doing so relaxes the capital constraint and increases profits; while if $\pi(p_1) < 0$, the buyer offers to buy the minimum amount $\underline{q}$ or nothing. Offering $\underline{q}$ might be optimal because if the offer is rejected, the capital constraint is loosened and the buyer starts the second period with a lower leverage. Q.E.D.

**Proof of Lemma 2.** Since choosing $(p_r, q_r) = (\Delta, x)$ maximizes second-period profits, it is enough to show that the second-period capital constraint is not violated after choosing this pair; that is, we need to show that $(p_r, q_r) = (\Delta, x)$ satisfies equation (10):

$$(\frac{1}{2} + \frac{1}{2}p_1 + \Delta)M + (\frac{3}{2}\Delta)(M + x) \geq L + \Delta x. \tag{A-4}$$

Since $q_1 \leq x$, it is enough to show that

$$(\frac{1}{2} + \frac{1}{2}p_1 + \Delta)M + (\frac{3}{2}\Delta)(M + q_1) \geq L + \Delta q_1. \tag{A-5}$$

To do so, we use the fact that the offer $(p_1, q_1)$ satisfies the first-period capital constraint (equation (8)). That is,

$$(\frac{1}{2}p_1 + \Delta)(M + q_1) + (\frac{1}{2} + \Delta)M \geq L + p_1 q_1. \tag{A-6}$$

Equation (A-5) can be rewritten as

$$(\frac{1}{2}p_1 + \Delta)(M + q_1) + (\frac{1}{2} + \Delta)M + \frac{1}{2}\Delta(M - q_1) + \frac{1}{2}p_1 q_1 \geq L + p_1 q_1. \tag{A-7}$$

Since $M > x \geq q_1$, equation (A-6) implies equation (A-7). Q.E.D.

**Proof of Proposition 5.** Define $H \equiv \frac{L - Z - 2\Delta(M + x)}{\frac{1}{2}(M - x)}$ and $\sigma \equiv \frac{\frac{1}{2}M - \Delta x}{\frac{1}{2}(M - x)}$. Observe that $\sigma > 1$, since $x < M$ and $\Delta < \frac{1}{2}$. In addition, $H$ is a monotone transformation of the buyer's initial leverage, defined as $\delta = \frac{L - Z}{(\frac{1}{2} + \Delta)2M}$. For use below, note that when $q_1 = x$, the first-period capital constraint, (8), is equivalent to $p_1 \geq H - \sigma$.

*Case 1: $H \leq 2\Delta + \sigma$.* We first show that in this case $q_1 = x$. Define $\overline{p} = \max\{\Delta, H - \sigma\}$. Since the offer $(p_1, q_1) = (\overline{p}, x)$ satisfies the first-period capital constraint and provides nonnegative profits (since $H - \sigma \leq 2\Delta$), trade can always happen. Thus, by Lemma A-1, it is enough to show that it is suboptimal to choose $q_1 = \underline{q}$. The proof is by contradiction. Suppose to the contrary that the optimal bidding strategy is $(p_1, q_1; p_a, q_a; p_r, q_r)$ with $q_1 = \underline{q}$. From Lemmas A-1 and 2, $p_1 > 2\Delta$; $(p_r, q_r) = (\Delta, x)$; and we can assume, without loss of generality, that either (i) $p_a = q_a = 0$, or (ii) $p_a \in (0, 2\Delta]$ and $q_a = x$. We obtain a contradiction as follows: If $p_a = 0$, we can increase the buyer's expected utility by choosing the strategy $(\widetilde{p}_1, \widetilde{q}_1; \widetilde{p}_a, \widetilde{q}_a; \widetilde{p}_r, \widetilde{q}_r) = (\overline{p}, x; p_a, q_a; p_r, q_r)$, as follows:

$$U(\widetilde{p}_1, \widetilde{q}_1; \widetilde{p}_a, \widetilde{q}_a; \widetilde{p}_r, \widetilde{q}_r) = x\pi(\overline{p}) + (1 - \overline{p})x\pi(\Delta) \tag{A-8}$$
$$> q_1\pi(p_1) + (1 - p_1)x\pi(\Delta) = U(p_1, q_1; p_a, q_a; p_r, q_r).$$

The inequality follows since $H - \sigma \leq 2\Delta < p_1$. If instead $p_a > 0$, we can increase the buyer's expected utility by choosing the strategy $(\widetilde{p}_1, \widetilde{q}_1; \widetilde{p}_a, \widetilde{q}_a; \widetilde{p}_r, \widetilde{q}_r) = (p_a, q_a; p_1, q_1; p_r, q_r)$, as follows:

$$U(\widetilde{p}_1, \widetilde{q}_1; \widetilde{p}_a, \widetilde{q}_a; \widetilde{p}_r, \widetilde{q}_r) = q_a\pi(p_a) + p_a q_1\pi(p_1) + (1 - p_a)x\pi(\Delta) \tag{A-9}$$
$$> p_1 q_a\pi(p_a) + q_1\pi(p_1) + (1 - p_1)x\pi(\Delta) = U(p_1, q_1; p_a, q_a; p_r, q_r).$$

The first equality follows since the strategy $(\widetilde{p}_1, \widetilde{q}_1; \widetilde{p}_a, \widetilde{q}_a; \widetilde{p}_r, \widetilde{q}_r)$ satisfies capital constraints; in particular, since the capital constraint is satisfied after the offers $(p_1, q_1)$ and $(p_a, q_a)$ are accepted, it follows from Lemma 1 that it is also satisfied if the buyer makes only one of these offers. The inequality follows since $p_a < p_1 \leq 1$, $\pi(p_1) < 0$, and $\pi(p_a) \geq 0$.

So far, we have established $q_1 = x$. For strategies in which $p_a > 0$, the second-period capital constraint, (9), reduces to $p_1 + p_a \geq H$. By Lemma 1, equation (8) implies equation (8). Thus, if $H \leq 2\Delta$, the benchmark solution, $p_1 = p_a = \Delta$, is achieved.

The remainder of Case 1 deals with $H \in (2\Delta, 2\Delta + \sigma]$. Define $V(p_1, p_a) \equiv \pi(p_1) + p_1 \pi(p_a) + (1 - p_1)\pi(\Delta)$, which is profits from $(p_1, p_a)$ divided by the bid size $x$. Observe that if a solution has $p_a > 0$, then $p_1 + p_a = H$: Otherwise, $p_1 + p_a > H > 2\Delta$, and so either $p_1 > \Delta$ and $\frac{\partial V}{\partial p_1} = \pi'(p_1) + \pi(p_a) - \pi(\Delta) < 0$, which is a contradiction; or else $p_a > \Delta$, which is clearly suboptimal. Thus, the problem reduces to choosing $p_1$ to maximize $\max\{V(p_1, 0), V(p_1, H - p_1)\}$ subject to the first-period constraint $p_1 \geq H - \sigma$. Define $R_1(H) \equiv \max_{p_1 \geq H-\sigma} V(p_1, 0)$ and $R_2(H) \equiv \max_{p_1 \geq H-\sigma} V(p_1, H - p_1)$. Both $R_1$ and $R_2$ are continuous. Observe that $R_2(2\Delta) > R_1(2\Delta)$, and $4\Delta - \sigma < 2\Delta$ (since $\Delta < 1/2$). For $H \in (4\Delta, 2\Delta + \sigma)$, if both $p_1, p_a > 0$, then at least one exceeds $2\Delta$, which we know is suboptimal (Lemma A-1); and since the pair $p_1 < 2\Delta$, $p_a = 0$ gives strictly positive profits, $R_2(H) < R_1(H)$ in this range. Hence, there exist $H_1, H_2 \in (2\Delta, 4\Delta)$, such that $R_2(H) > R_1(H)$ whenever $H < H_1$; and $R_1(H) > R_2(H)$ whenever $H > H_2$. When $H \in (2\Delta, H_1)$, the buyer chooses his first-period bid to make sure he can bid in the second period even if his first-period bid is accepted. When $H \in (H_2, 2\Delta + \sigma)$, the buyer withdraws from the market if his first offer is accepted.

Next, we show that if $H \in (2\Delta, H_1)$, then $p_a > p_1 > \Delta$, that is, $p_1 \in (\Delta, \frac{H}{2})$. In this case, $V(p, H - p)$ is a cubic in $p$, and the coefficient on the cubic term is negative. Thus, the result follows if $\frac{d}{dp_1} V(p_1, H - p_1)\big|_{p_1 = \Delta} > 0 > \frac{d}{dp_1} V(p_1, H - p_1)\big|_{p_1 = H/2}$. Evaluating, $\frac{d}{dp_1} V(p_1, H - p_1) = \pi'(p_1) + \pi(H - p_1) - p_1 \pi'(H - p_1) - \pi(\Delta)$. Since $\pi$ is a quadratic with its maximum at $\Delta$, for any $p$, $\pi(p) = \pi(\Delta) + \frac{1}{2}(p - \Delta)\pi'(p)$. Given this,

$$\frac{d}{dp_1} V(p_1, H - p_1)\bigg|_{p_1 = \Delta} = \pi(H - \Delta) - \Delta \pi'(H - \Delta) - \pi(\Delta) \qquad \text{(A-10)}$$

$$= \left(\frac{1}{2}(H - \Delta - \Delta) - \Delta\right) \pi'(H - \Delta),$$

which is positive, since $H \in (2\Delta, H_1)$ and $H_1 < 4\Delta$. Similarly,

$$
\begin{aligned}
\frac{d}{dp_1} V(p_1, H - p_1) \Big|_{p_1 = H/2} &= (1 - \frac{H}{2})\pi'(\frac{H}{2}) + \pi(\frac{H}{2}) - \pi(\Delta) \qquad\qquad \text{(A-11)}\\
&= (1 - \frac{H}{2} + \frac{1}{2}(\frac{H}{2} - \Delta))\pi'(\frac{H}{2})\\
&= (1 - H/4 - \Delta/2)\pi'(H/2),
\end{aligned}
$$

which is negative, since $H \in (2\Delta, H_1)$, $H_1 < 4\Delta$, and $\Delta < 1/2$.

Finally, we characterize the optimal offer when $H \in (H_2, 2\Delta + \sigma)$. The optimal $p_1$ maximizes $V(p_1, 0)$, subject to $p_1 \geq H - \sigma$. Observe that $V(p_1, 0)$ is quadratic and that the unconstrained solution is $\widetilde{p}_1 = \Delta - \frac{1}{2}\Delta^2$. Thus, the optimal solution is $p_1 = \max\{\widetilde{p}_1, H - \sigma\}$, which is increasing in $H$ (and hence, in leverage). In addition, $p_1 > \Delta$ when $H > \Delta + \sigma$.

*Case 2:* $H > 2\Delta + \sigma$. We first claim that if there is trade, $p_1 > 2\Delta$. To establish this, suppose to the contrary that $p_1 \leq 2\Delta$, so from Lemma A-1, $q_1 = x$ is optimal. But then $(p_1, q_1)$ violates the first-period capital constraint, giving a contradiction. It then follows from Lemma A-1 that if trade occurs, $q_1 = \underline{q}$; and from Lemma 2, we know that $(p_r, q_r) = (x, \Delta)$.

Next, we show that if the first offer is accepted, it is optimal not to make any offer in the second period. Recall that accepted offers reduce value (Lemma 1), so if the buyer makes an offer in the first period and the offer is accepted, he starts the second period with a capital constraint that is tighter than the one he had at the start of the first period. Hence, the fact that the buyer had to choose $p_1 > 2\Delta$ implies that either $p_a = 0$ or $p_a > 2\Delta$. Since choosing $p_a > 2\Delta$ is suboptimal, we must have $p_a = 0$.

The buyer's expected utility reduces to $\underline{q}\pi(p_1) + (1 - p_1)x\pi(\Delta)$. This expression is strictly decreasing in $p_1$ when $p_1 > 2\Delta$. Thus, if trade occurs, the optimal $p_1$ satisfies the first-period capital constraint with equality, and it follows that $p_1 = \underline{H}$, where $\underline{H} \equiv \frac{L - Z - 2\Delta(M + \underline{q})}{\frac{1}{2}(M - \underline{q})} - \frac{\frac{1}{2}M - \Delta\underline{q}}{\frac{1}{2}(M - \underline{q})}$. Observe that $\underline{H}$ is a monotone transformation of the buyer's initial leverage. Thus, if there is trade, the initial bid is increasing and continuous in leverage. To establish that trade can happen if and only if leverage is sufficiently low, observe that at $\underline{H} = 2\Delta$, the buyer's expected utility is strictly positive, whereas at $\underline{H} = 1$, the buyer's expected utility is strictly negative. Q.E.D.

## Part 3: Static Competition

**Lemma A-2** *For every $i \in \{1, 2\}$, one of the following is true: (i) $r_i = r'_i = 2\Delta_i$; (ii) $r_i > r'_i > 2\Delta_i$; or (iii) $r_i \leq r'_i < 2\Delta_i$.*

**Proof of Lemma A-2.** The capital constraint (2) can be written as $(\frac{1}{2}p + \Delta_i)M_i + q_i(\Delta_i - \frac{1}{2}p) \geq 0$. By definition, $p_i(0)$ and $p_i(x)$ are the minimum solutions in $p$ for $q_i = 0$ and $q_i = x$, respectively. (Note that by Assumption 1, the left-hand side is increasing in $p$.) Hence, if (i) $p_i(0) = 2\Delta_i$, then also $p_i(x) = 2\Delta_i$; (ii) if $p_i(0) > 2\Delta_i$, we must have $p_i(x) > p_i(0)$; and (iii) if $p_i(x) < 2\Delta_i$, then $p_i(0) > p_i(x)$, which combined with (ii) gives $p_i(x) < p_i(0) < 2\Delta_i$. The result then follows. Q.E.D.

**Proof of Lemma 3.** As a preliminary, consider an offer $(p_i, q_i)$ by buyer $i$. If buyer $-i$ offers $p_{-i} > p_i$ and $q_{-i} < x$, the seller sells a quantity $q'_i = \min\{q_i, x - q_{-i}\}$ to buyer $i$ if and only if $v \leq p_i$, and the expectation of $v$ conditional on this event is $\frac{1}{2}p_i$. If buyer $-i$ offers $p_{-i} < p_i$, the seller sells a quantity $q'_i = q_i$ to buyer $i$ if and only if $v \leq p_i$, and the expectation of $v$ conditional on this event is $\frac{1}{2}p_i$. Finally, if buyer $-i$ offers $p_{-i} = p_i$, the seller sells a quantity $q'_i = \min\left\{q_i, \frac{q_i x}{q_i + q_{-i}}\right\}$ to buyer $i$, and the expectation of $v$ conditional on this event is $\frac{1}{2}p_i$.

*Part (A):* From Lemma A-2, $r_i < r'_i < 2\Delta_i$. Any offer $(p_i, q_i)$ that satisfies $p_i \in [r_i, 2\Delta_i)$ and $q_i = x$ is not weakly dominated (and hence survives the first stage of elimination) since it is a unique best response for buyer $i$ when buyer $-i$ bids $p_{-i} = p_i - \varepsilon$. All other offers are weakly dominated as follows.

(i) Any offer $(p_i, q_i)$ with $p_i \geq 2\Delta_i$ is weakly dominated by the offer $(r'_i, x)$. Specifically, the offer $(r'_i, x)$ produces positive profits whenever it is accepted (e.g., if $p_{-i} = 0$) and it also guarantees that buyer $i$'s capital constraint is satisfied, regardless of $-i$'s offer; the last point follows since $p_i(q'_i) \in [r_i, r'_i]$ for any $q'_i \in [0, x]$. In contrast, offering $p_i \geq 2\Delta_i$ never leads to positive profits.

(ii) Any offer $(p_i, q_i)$ with $p_i < r_i$ is also weakly dominated by the offer $(r'_i, x)$. To see that note that the only case in which an acceptance of $(p_i, q_i)$ may not lead to a violation

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of $i$'s capital constraint is when buyer $-i$ offers $(p_{-i}, q_{-i})$, such that $p_{-i} \in (0, p_i)$, $q_i < x$, and the seller accepts only $i$'s offer; in this case the market value of the asset becomes $\frac{1}{2}(p_i + p_{-i})$. However if the seller accepts both offers (which happens with probability $p_{-i}$), buyer $i$'s capital constraint is again violated. So provided the cost $F$ of violating the capital constraint is sufficiently large, buyer $i$'s offer is weakly dominated by offering $(r_i', x)$.

(iii) Finally, any offer $(p_i, q_i)$ that satisfies $p_i \in [r_i, 2\Delta_i)$ and $q_i < x$ is weakly dominated. This is because raising the quantity to $x$ weakly increases both buyer $i$'s profits and set of circumstances under which his capital constraint is satisfied, and does so strictly if $-i$'s offer is lower (e.g., if $p_{-i} = 0$).

*Part (B):* From Lemma A-2, $r_i = r_i' = 2\Delta_i$. The offer $(2\Delta_i, x)$ for buyer $i$ weakly dominates any other offer $(p_i, q_i)$, as follows. Offering $(2\Delta_i, x)$ produces zero profits and guarantees that buyer $i$'s capital constraint is satisfied, regardless of $-i$'s offer. In contrast, if $p_i > 2\Delta_i$, the offer $(p_i, q_i)$ produces negative profits whenever it is accepted (e.g., if $p_{-i} = 0$), and if $p_i < 2\Delta_i$, the offer $(p_i, q_i)$ leads to violation of buyer $i$'s capital constraint, as in case (ii) of Part (A) above.

*Part (C):* From Lemma A-2, $r_i > r_i' > 2\Delta_i$. The offer $p_i = 0$ is not weakly dominated since it is the unique best response for buyer $i$ if buyer $-i$ bids $p_{-i} = 0$: If buyer $i$ bids $p_i = 0$, he obtains zero profits and his capital constraint is satisfied. If he bids $p_i > 0$, then whenever the seller accepts his offer, he either makes negative profits or his capital constraint is violated.

The offer $(p_i, q_i) = (r_i, x)$ is not weakly dominated since it is buyer $i$'s unique best response when buyer $-i$ bids $p_{-i} \in (0, r_i' - \varepsilon)$. Specifically, the offer $(p_i, q_i) = (r_i, x)$ guarantees that buyer $i$'s capital constraint is satisfied. In contrast, if buyer $i$ bids $p_i < r_i$ or $q_i < x$, his capital constraint is violated with probability at least $p_{-i}$, and if he bids $p_i > r_i$, his expected profits are reduced.

Any offer $(p_i, q_i)$ with $p_i > r_i$ is weakly dominated by the alternative offer $(r_i, q_i)$, as follows. The alternative offer raises buyer $i$'s profits/ reduces expected losses. If $p_{-i} \leq r_i < p_i$, either $v \leq p_{-i}$ and buyer $i$'s capital constraint is relaxed by the alternative offer (in

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particular, the market valuation of each unit remains unchanged, but his profits increase/ he loses less money); or $v > p_{-i}$, and buyer $i$'s capital constraint is satisfied under the alternative offer. If instead $p_{-i} > r_i$, one can easily show the buyer $i$'s capital constraint is always satisfied by considering in turn the cases $v \leq r_i$, $v \in (r_i, p_{-i}]$ and $v > p_{-i}$.

Finally, any offer $(p_i, q_i)$ such that $p_i \in [r'_i, r_i)$ is weakly dominated by $\tilde{p}_i = 0$, as follows. If $p_{-i} \geq r'_i$, the offer $\tilde{p}_i = 0$ provides buyer $i$ zero profits and guarantees that his capital constraint is satisfied. In contrast, since $r_i > r'_i > 2\Delta_i$, the offer $(p_i, q_i)$ provides negative profits whenever it is accepted (e.g., if, $p_{-i} = r'_i$). If $p_{-i} < r'_i$, the offer $\tilde{p}_i = 0$ weakly reduces the probability that buyer $i$'s capital constraint will be violated, since under $\tilde{p}_i = 0$, the constraint is violated with probability $p_{-i}$, while under $(p_i, q_i)$, it is violated with probability $p_i$ if $q_i = x$ and a probability of at least $p_{-i}$, otherwise. Q.E.D.

**Proof of Lemma 4.** First, consider the case $\Delta_1 \neq \Delta_2$, and assume, without loss of generality, that $\Delta_1 > \Delta_2$, so in equilibrium the highest bid is posed by buyer 1. From Lemma 3 and standard competition arguments, if $r_2 < 2\Delta_2$, the highest bid is $2\Delta_2$ (in particular, $p_2 = 2\Delta_2$ is weakly dominated, and so $p_2 = 2\Delta_2 - \varepsilon$ and $p_1 = 2\Delta_2$), and if $r_2 = 2\Delta_2$, the highest bid is $2\Delta_2 + \varepsilon$. Since $r_1 \leq \min\{2\Delta_1, 2\Delta_2\}$, the capital constraint of buyer 1 is satisfied. The capital constraint of buyer 2 is also satisfied, since by Lemma A-2, $r'_2$ is less than the equilibrium price. Next, consider the case $\Delta_1 = \Delta_2 = \Delta$. If $\max\{r_1, r_2\} = 2\Delta$, then by Lemma 3, the equilibrium price is $2\Delta$ and is posted by the buyer with the highest $r_i$. From Lemma A-2, both capital constraints are satisfied. If $\max\{r_1, r_2\} < 2\Delta$, Lemma A-2 implies that $\max\{r_1, r_2\} < \max\{r'_1, r'_2\} \leq 2\Delta - \varepsilon$. Lemma 3 and competition then imply that both buyers post the price $2\Delta - \varepsilon$ for the full amount. Clearly, both capital constraints are satisfied given the equilibrium price. Q.E.D.

**Lemma A-3 (Second elimination round)** *Suppose $2\Delta_1 \geq r_1$ and $r_2 > 2\Delta_2$.*

*(i) If $r'_2 \leq r_1 \leq r_2$, no offer $(p_2, q_2)$ with $p_2 = r_2$ survives two rounds of elimination of weakly dominated strategies.*

*(ii) If $r_2' > r_1$, the only offer $(p_2, q_2)$ with $p_2 > \max\{2\Delta_2, r_1\}$ to survive two rounds of elimination of weakly dominated strategies is $(p_2, q_2) = (r_2, x)$.*

**Proof of Lemma A-3.**

From the first round of elimination (Lemma 3), we know that buyer 1 bids $p_1 \in [r_1, 2\Delta_1]$ and $q_1 = x$; buyer 2 bids either $p_2 = r_2$ or $p_2 < r_2'$; and the offers $p_2 = 0$ and $(p_2, q_2) = (r_2, x)$ survived. Also, from Lemma A-2, $r_2 > r_2' > 2\Delta_2$.

*Part (i):* Since $p_1 \geq r_1 \geq r_2'$, the offer $\tilde{p}_2 = 0$ guarantees that buyer 2 makes zero profits and his capital constraint is satisfied, regardless of buyer 1's exact choice of $p_1 \in [r_1, 2\Delta_1]$. In contrast, offering $p_2 = r_2$ can never lead to positive profits and it leads to negative profits if buyer 1 bids $p_1 = r_1$.

*Part (ii):*

The offer $(p_2, q_2) = (r_2, x)$ survives the second elimination round since it is a unique best response for buyer 2 if buyer 1 offers $(p_1, q_1) = (r_1, x)$. Specifically, under $(r_2, x)$ buyer $2's$ capital constraint is satisfied, while under any other offer that survives the first round, the capital constraint is violated with a positive probability.

Any offer $(p_2, q_2)$ remaining after the first elimination round, such that $p_2 \in (\max\{2\Delta_2, r_1\}, r_2)$, or $p_2 = r_2$ and $q_2 < x$, is weakly dominated by $\tilde{p}_2 = 0$, as follows. If $p_1 > p_2$, buyer 2 makes zero profits under both offers, and the probability his capital constraint is violated is also the same (namely $p_1$ if $p_1 < r_2'$, and 0 if $p_1 \geq r_2'$). If $p_1 \leq p_2$ and $p_1 < r_2'$, the offer $\tilde{p}_2$ gives zero profits and buyer 2's capital constraint is violated with probability $p_1$; while the offer $(p_2, q_2)$ gives buyer 2 strictly negative profits, and violates his capital constraint with probability $p_2 \geq p_1$ if $q_2 = x$, and a probability of at least $p_1$ if $q_2 < x$. Finally, if $p_1 \leq p_2$ and $p_1 \geq r_2'$, the offer $\tilde{p}_2$ gives buyer 2 zero profits and satisfies his capital constraint, while the offer $(p_2, q_2)$ gives buyer 2 strictly negative profits (and may or may not violate his capital constraint). Q.E.D.

**Proof of Proposition 6:**

From the first round of elimination (Lemma 3), we know that buyer 1 bids for the entire

amount and so whenever the seller accepts buyer 1's offer, the market value of the asset drops to $\frac{1}{2}p_1$, regardless of buyer 2's offer. In addition, $p_1 \in [r_1, 2\Delta_1]$.

Part (A), $r_2' \leq r_1$:

First, consider the case $2\Delta_2 \geq r_1$. Since $2\Delta_2 \geq r_2'$, Lemma A-2 implies that $2\Delta_2 \geq r_2' \geq r_2$. Moreover, $2\Delta_1 \geq r_1 \geq r_2' \geq r_2$. Hence, we can apply Lemma 4, to complete the proof of this case.

Second, consider the case $2\Delta_2 < r_1$, in which case $\max\{r_1, \min\{2\Delta_1, 2\Delta_2\}\} = r_1$. If $2\Delta_2 \geq r_2$, then from the first round of elimination, buyer 2 bids at most $2\Delta_2$, and since $2\Delta_2 < r_1$, buyer 1 bids $(p_1, q_1) = (r_1, x)$, which is his unique best response. If instead $2\Delta_2 < r_2$, then any offer $(p_2, q_2)$ with $p_2 \geq r_2'$ and $p_2 \neq r_2$ is eliminated in the first stage, while the offer $\tilde{p}_2 = 0$ remains. If $r_2 < r_1$, then since $p_2 \leq r_2$, the unique best response for buyer 1 is to offer $(p_1, q_1) = (r_1, x)$. If instead $r_2 \geq r_1$, a second round of elimination (Lemma A-3) implies that buyer 2 offers $p_2 < r_2'$, and the unique best response for buyer 1 is to offer $(p_1, q_1) = (r_1, x)$.

Part (B), $r_2' > r_1$:

Start with the case $r_2 \leq 2\Delta_2$. If $r_2 > 2\Delta_1$, the first round of elimination implies that buyer 2 offers $(p_2, q_2) = (r_2, x)$, which is his unique best response when buyer 1 bids $p_1 \in [r_1, 2\Delta_1]$. The seller then sells everything to buyer 2 at price $r_2$. If instead $r_2 \leq 2\Delta_1$, then since $r_2' \leq 2\Delta_2$ (from Lemma A-2), it follows that $r_1 \leq 2\Delta_2$, and we can apply Lemma 4 to complete the the proof.

The remainder of the proof deals with the case $r_2 > 2\Delta_2$, in which case $\max\{r_2, \min\{2\Delta_1, 2\Delta_2\}\} = r_2$. From the first elimination round, the offer of buyer 2 must satisfy $p_2 \leq r_2$. From Lemma A-2, $\max\{2\Delta_2, r_1\} < r_2' < r_2$.

There is no equilibrium in which buyer 2 bids $p_2 \leq \max\{2\Delta_2, r_1\}$ and buyer 1 bids $p_1 > p_2$, since any such equilibrium would have $q_1 = x$ and $p_1 = \max\{r_1, p_2 + \varepsilon\} < r_2'$, implying that buyer 2's capital constraint is violated whenever $v \leq p_1$; but then buyer 2 would deviate to offer $(p_2, q_2) = (r_2, x)$. Nor is there an equilibrium in which buyer 2 bids $p_2 \leq \max\{2\Delta_2, r_1\}$ and buyer 1 bids $p_1 \in [r_1, p_2] < r_2'$, since buyer 2's capital constraint would be violated

whenever $v \leq p_1$, and again buyer 2 would deviate to offer $(p_2, q_2) = (r_2, x)$.

Consequently, the second elimination round (Lemma A-3) implies that $(p_2, q_2) = (r_2, x)$ in any candidate equilibrium. Hence, if $r_2 < 2\Delta_1$, the unique equilibrium outcome is that buyer 2 bids $(p_2, q_2) = (r_2, x)$ and buyer 1 bids $(p_1, q_1) = (r_2 + \varepsilon, x)$. If instead, $r_2 \geq 2\Delta_1$, the unique equilibrium outcome is that buyer 2 bids $(p_2, q_2) = (r_2, x)$, and the equilibrium price is $r_2$. Q.E.D.

**Proof of Proposition 7.**

From Lemma A-2, $r_i > r'_i > 2\Delta_i$ for $i \in \{1, 2\}$. From the first round of elimination (Lemma 3), either $p_i = r_i$, or $p_i < r'_i$, for $i \in \{1, 2\}$. In addition, we know that for $i \in \{1, 2\}$, the offers $p_i = 0$ and $(p_i, q_i) = (r_i, x)$ survive the first round. In fact, the offers $p_1 = 0$ and $p_2 = 0$ can never be eliminated during the process of iterated elimination since each one is the unique best response against the other. Hence, the no trade equilibrium survives the elimination process.

Start with the case $r'_1 > r_2$. In the second elimination round, the offer $(p_1, q_1) = (r_1, x)$ survives since it is a unique best response if buyer 2 bids $(p_2, q_2) = (r_2, x)$. However, any offer $(p_1, q_1)$ with $p_1 \in (0, r'_1)$ is eliminated, since it is strictly dominated by the offer $(r_1, x)$, as follows. If buyer 1 bids $p_1 \in (0, r'_1)$, his capital constraint is violated with positive probability. In contrast, if he offers $(p_1, q_1) = (r_1, x)$, his capital constraint is never violated. So provided the cost of violating the capital constraint is sufficiently large, buyer 1 strictly prefers $(r_1, x)$. Hence, from the second elimination round, buyer 1 bids either $p_1 = 0$ or $(p_1, q_1) = (r_1, x)$. Hence, in the third elimination round, buyer 2 weakly prefers $p_2 = 0$ to any other remaining offer, as follows. The offer $p_2 = 0$ gives buyer 2 zero profits and guarantees that his capital constraint is satisfied (since $r_1 > r'_2$). Any other remaining offer gives zero profits if $p_1 = r_1$ and leads to negative profits and/or violates buyer 2's capital constraint, if $p_1 = 0$. Hence, the unique equilibrium that survives the third elimination round is that both buyers bid nothing.

Next, consider the case $r'_1 \leq r_2$. If $r'_2 > r_1$, we can apply the same logic as in the case $r'_1 > r_2$ to show that the unique equilibrium that survives the third elimination round is that

both buyers bid nothing.

The rest of the proof focuses on the case $\max\{r_1', r_2'\} \leq \min\{r_1, r_2\}$. If all offers $(p_2, q_2)$, such that $p_2 \in (0, r_1')$ were eliminated in the first round, then for the first buyer only the offer $p_1 = 0$ survives the second round, since it is a unique best response for buyer 1 if $p_2 = 0$ and does not do worse than any other offer if $p_2 \geq r_1'$. Hence, the unique equilibrium is no trade. Similarly, if all offers $(p_1, q_1)$, such that $p_1 \in (0, r_2')$ were eliminated in the first round, then for the second buyer only the offer $p_2 = 0$ survives the second round and the unique equilibrium is no trade.

It remains to consider the case in which for buyer $i \in \{1, 2\}$, there exists an offer $(\bar{p}_i, \bar{q}_i)$ that survived the first round such that $\bar{p}_i \in (0, r_{-i}')$. Then the offer $(r_i, x)$ for buyer $i \in \{1, 2\}$ must survive the second round, since it is a unique best response for buyer $i$ when the other buyer bids $(\bar{p}_{-i}, \bar{q}_{-i})$.

If $r_1' < r_2'$, then any offer $(p_2, q_2)$, such that $p_2 \in [r_1', r_2')$ and $q_2 = x$ is eliminated in the second round since it is weakly dominated by the offer $\tilde{p}_2 = 0$, as follows. If $(p_1, q_1) = (r_1, x)$, buyer 2 is indifferent between $\tilde{p}_2 = 0$ and $(p_2, q_2)$, since in both cases he obtains zero profits and his capital constraint is satisfied. If $p_1 = 0$, or if $p_1 = r_1$ and $q_1 < x$, buyer 2 strictly prefers $\tilde{p}_2 = 0$. It remains to show that buyer 2 weakly prefers $\tilde{p}_2 = 0$ to $(p_2, q_2)$, given all other potential offers from buyer 1 that survive the first round; any such offer would have $p_1 \in (0, r_1')$. Given such offer, if buyer 2 offers $\tilde{p}_2 = 0$, his capital constraint is violated with probability $p_1$, but if he chooses $p_2 \in [r_1', r_2')$ and $q_2 = x$, his capital constraint is violated with probability $p_2$.

Similarly, we can show that if $r_2' < r_1'$, then any offer $(p_2, q_2)$, such that $p_2 \in [r_2', r_1')$ and $q_2 = x$ is eliminated. Hence, we showed that at the end of the second round either the only equilibrium outcome is no trade or else, the offer $(r_i, x)$ for buyer $i \in \{1, 2\}$ survives and if there is a remaining offer $(p_i, q_i)$ such that $p_i \in [\min\{r_1', r_2'\}, \max\{r_1', r_2'\})$, we must have $q_i < x$.

If $r_1 < r_2$, then in the third elimination round any remaining offer $(p_1, q_1)$, such that $p_1 \in (0, r_1')$ is eliminated since it is weakly dominated by the offer $(r_1, x)$, as follows. If

$(p_2, q_2) = (r_2, x)$, both offers provide the same utility. Otherwise, under the offer $(p_1, q_1)$, the capital constraint of buyer 1 is violated with a positive probability, whereas under the offer $(r_1, x)$, it is never violated. Hence, buyer 1 bids either $p_1 = 0$ or $p_1 = r_1$. If $p_1 = r_1$ is eliminated in the third round, we are done and the unique equilibrium is not trade; otherwise, the only offer for buyer 2 that survives the fourth round of elimination is $p_2 = 0$, and the unique equilibrium is not trade. The case $r_1 > r_2$ is similar.

The argument above does not apply for the case $r_1 = r_2$. The potential equilibria that remain in this case other than the no trade equilibrium are an equilibrium in which buyer 1 bids $p_1 \in (0, r'_2)$ and buyer 2 bids $(p_2, q_2) = (r_2, x)$, and in equilibrium in which buyer 2 bids $p_2 \in (0, r'_1)$ and buyer 1 bids $(p_1, q_1) = (r_1, x)$. However, the last two equilibria do not survive the iteration process if the asset is nondivisble, or if the seller can select at most one buyer. In this case, $q_i \in \{0, x\}$, and any offer $(p_i, q_i)$ such that $p_i \in (0, r'_i)$ is eliminated by the end of the third round since it is weakly dominated by the offer $\tilde{p}_i = 0$, as follows. If $p_{-i} > p_i$, buyer $i$ is indifferent, since in both cases his offer is not accepted and does not effect the market value of his asset when the seller accepts $-i$'s offer. If $p_{-i} = p_i$, the buyer is indifferent between the two offers, since in both cases, his capital constraint is violated with probability $p_i$. (#Here we need the assumption that the buyer ends up with nothing if he violates the constraint.#). Finally, if $p_{-i} < p_i$, the buyer strictly prefers $\tilde{p}_i$, since under $\tilde{p}_i$, his capital constraint is violated with probability $p_{-i}$, while under $p_i$, his capital constraint is violated with probability $p_i$. Hence, from the third elimination round we know that buyer $i$ bids either $p_i = 0$ or $(p_i, q_i) = (r_i, x)$. The next elimination round then implies $p_i = r_i$ is weakly dominated by $p_i = 0$. Hence, the only equilibrium that remains is no trade. Q.E.D.

## Appendix B

**Proof of Proposition 1:** We prove the proposition for the case in which the asset is divisible into $N$ individual units, where $N$ is sufficiently large. We write $x_N \equiv \frac{x}{N}$. A price schedule $p(\cdot)$ is a sequence of prices $(p_1, p_2, ..., p_N)$, where $p_i x_N$ denotes the price paid for the $i$'s unit bought. That is, if the seller sells $n$ units (i.e., a quantity $n x_N$ of the asset),

the buyer pays $x_N \sum_{i=1}^{n} p_i$. A price schedule weakly dominates another schedule if it weakly increases the buyer's expected utility, which is the expected value of existing assets minus the expected cost for violating the capital constraint. From the law of iterated expectations, the expected value of existing inventories equals its prior, independent of the price schedule. Hence, the buyer's problem is to find a price schedule that maximizes his expected profits minus the expected cost of violating his capital constraint. The proof has three steps:

(1) We first establish that for any price schedule $p(\cdot)$, there exists an alternate price schedule $\tilde{p}(\cdot)$ that satisfies $\tilde{p}_1 \geq \tilde{p}_2 \geq ... \geq \tilde{p}_N$ and leaves all payoffs and capital constraints unchanged. The proof is as follows: Suppose there exists $k$, such that $p_k < p_{k+1}$. Let $\underline{k}$ be the smallest integer such that $p_i = p_k$ for every integer $i \in [\underline{k}, k]$, and let $\overline{k}$ be the largest integer such that $p_i = p_{k+1}$ for every integer $i \in [k+1, \overline{k}]$. Define $\overline{p} = \frac{1}{\overline{k}-\underline{k}+1} \sum_{i=\underline{k}}^{\overline{k}} p_i$, and define a new price schedule by $\tilde{p}_i = \overline{p}$ for $i \in [\underline{k}, \overline{k}]$, and $\tilde{p}_i = p_i$ otherwise. [Assume that if the seller is indifferent between selling and not selling, he chooses to sell.] Note that if the seller chooses to sell only $j$ units of asset even though the buyer is willing to buy more than $j$ units, we must have $p_{j+1} < v \leq \frac{1}{j-j'+1} \sum_{i=j'}^{j} p_i$ for every $j' \leq j$. It then follows that under both schedules, the seller never sells any quantity $q \in [\underline{k}x_N, (\overline{k}-1)x_N]$. In addition, by construction, if he sells any other quantity, he gets the same payoff under both schedules. Consequently, the seller's response to and payoff from the two schedules is exactly the same. Hence, the buyer's profits are also the same, and since the seller's behavior is the same, the information revealed in equilibrium is the same, and so the capital constraints are the same. Iteration of the argument above completes the proof of the claim. The iteration process ends in a finite number of steps since under the new price schedule, the number of prices $p_i$ that are different from their follower $p_{i+1}$ is lower than that in the original schedule; that is, $\sum_{i=1}^{N-1} \hat{t}_i < \sum_{i=1}^{N-1} t$, where $t_i = \begin{cases} 1 & \text{if } p_i \neq p_{i+1} \\ 0 & \text{otherwise} \end{cases}$ and $\hat{t}_i = \begin{cases} 1 & \text{if } \hat{p}_i \neq \hat{p}_{i+1} \\ 0 & \text{otherwise} \end{cases}$.

(2) Next, we show that if $N$ is sufficiently large, then for any price schedule $p(\cdot)$ that satisfies $p_1 \geq p_2 \geq ... \geq p_N$, there exists $\hat{n}$, such that $p(\cdot)$ is weakly dominated by an alternate price schedule $\tilde{p}(\cdot)$ in which the buyer offers to buy at most $\tilde{n}$ units and the capital constraint associated with the sale of $\tilde{n}$ units is satisfied. The proof is as follow: Under

$p(\cdot)$, if $v \leq p_N$, the seller sells everything $q = Nx_N = x$. If $v \in (p_i, p_{i-1}]$, the seller sells $q = (i-1)x_N$. If $v > p_1$, the seller sells nothing, $q = 0$. Hence, the seller sells the $i$'s unit if and only if $v \leq p_i$, and the expected profits from selling the $i$'s units are $p_i x_N (\Delta + \frac{1}{2}p_i - p_i)$, which is the probability of the $i$'s unit being sold multiplied by profits conditional on it being sold. Hence, the buyer's expected profits are $x_N \sum_{i=1}^{N} p_i (\Delta - \frac{1}{2}p_i)$.

If the original schedule $p(\cdot)$ violates the capital constraint given any amount that the seller chooses to sell, then $p(\cdot)$ is dominated by the degenerate schedule of making no offer, since by the assumption, the penalty $F$ associated with violating the capital constraint exceeds maximal attainable profits. Otherwise, let $n$ be the highest quantity for which the capital constraint is satisfied. If $n$ is also the highest quantity that the buyer offers to buy (i.e., $p_i = 0$ for $i \geq n+1$), there is nothing to prove. Otherwise, a sale of $n$ units indicates that $v \in (p_{n+1}, p_n]$, and so the capital constraint associated with this sale is

$$(M + nx_N)(\gamma\Delta + \frac{1}{2}p_n + \frac{1}{2}p_{n+1}) - x_N(p_n + p_{n-1} + ... + p_1) \geq L. \qquad \text{(A-12)}$$

If there is enough slack in equation (A-12) so that it is also satisfied when $p_{n+1} = 0$, then the original price schedule is dominated by a price schedule $\tilde{p}(\cdot)$ under which the buyer offers to buy at most $n$ units and $\tilde{p}_i = p_i$ for $i \leq n$. In particular, the new schedule satisfies all relevant constraints and increases the buyer's expected utility by $\sum_{i=n+1}^{N}[p_i F - x_N p_i(\Delta - \frac{1}{2}p_i)]$. Since $F > x(1 + \Delta)$ (i.e., the penalty associated with violating the capital constraints exceeds maximal attainable profits), it follows that $F > x_N(\Delta - \frac{1}{2}p_i)$, and so the new schedule also increases the buyer's expected utility. Otherwise, consider a price schedule $\tilde{p}(\cdot)$ under which the buyer offers to buy at most $n$ units and the prices are $\tilde{p}_i = p_i$ for $i \leq n-1$, and $\tilde{p}_n$ solves

$$(M + nx_N)(\gamma\Delta + \frac{1}{2}\tilde{p}_n) - x_N(\tilde{p}_n + \tilde{p}_{n-1} + ... + \tilde{p}_1) \qquad \text{(A-13)}$$
$$= (M + nx_N)(\gamma\Delta + \frac{1}{2}p_n + \frac{1}{2}p_{n+1}) - x_N(p_n + p_{n-1} + ... + p_1).$$

Rearranging terms in (A-13), we obtain

$$\frac{1}{2}[M + (n-2)nx_N]\tilde{p}_n = \frac{1}{2}[M + (n-2)x_N]p_n + \frac{1}{2}(M + nx_N)p_{n+1} \qquad \text{(A-14)}$$

Hence,

$$\tilde{p}_n = p_n + \frac{M + nx_N}{M + (n-2)x_N}p_{n+1} \tag{A-15}$$

From equations (A-13) and (A-12), it follows that the new schedule satisfies the capital constraint associated with the sale of $n$ units. The only other constraint in which $\tilde{p}_n$ appears is the constraint associated with the sale of $n-1$ units. When the seller chooses to sell $n-1$ units, one learns that $E(v) = \frac{1}{2}\tilde{p}_n + \frac{1}{2}\tilde{p}_{n-1}$, and so $\tilde{p}_n$ enters the constraint with a positive coefficient. Hence, the new schedule either relaxes the constraint (compared to the original schedule) or keeps it unchanged. All other relevant constraints remain unchanged under the new schedule.

To complete the proof of the second claim, we need to show that the new schedule $\tilde{p}\,(\cdot)$ increases the buyer's expected utility. As we show below, this result would follow when the size of each unit is low, so the loss of profits from increasing $p_n$ is small relative to the cost of violating the constraint. Formally, as before, since $F > x(1 + \Delta)$, it follows that $F > x_N(\Delta - \frac{1}{2}p_i)$, and so $\sum_{i>n+1}[x_N p_i(\Delta - \frac{1}{2}p_i) - p_i F) < 0$. Hence, the change in the buyer's expected utility resulting from the new schedule is more than

$$x_N\tilde{p}_n(\Delta - \frac{1}{2}\tilde{p}_n) - x_N p_n(\Delta - \frac{1}{2}p_n) - x_N p_{n+1}(\Delta - \frac{1}{2}p_{n+1}) + p_{n+1}F \tag{A-16}$$

Hence, using the identity

$$p_n(\Delta - \frac{1}{2}p_n) - \tilde{p}_n(\Delta - \frac{1}{2}\hat{p}_n) = \frac{1}{2}(\tilde{p}_n - p_n)^2 + (\tilde{p}_n - p_n)(p_n - \Delta), \tag{A-17}$$

we need to show that

$$\frac{1}{x_N}p_{n+1}F > \frac{1}{2}(\tilde{p}_n - p_n)^2 + p_{n+1}(\Delta - \frac{1}{2}p_{n+1}) + (\tilde{p}_n - p_n)(p_n - \Delta) \tag{A-18}$$

Using equation (A-15), and dividing both sides by $p_{n+1}$, the equation above reduces to

$$\frac{1}{x_N}F > \frac{1}{2}(\frac{M + nx_N}{M + (n-2)x_N})^2 p_{n+1} + (\Delta - \frac{1}{2}p_{n+1}) + (\frac{M + nx_N}{M + (n-2)x_N})(p_n - \Delta) \tag{A-19}$$

Observe that since $n \geq 1$ and $x < M$,

$$\frac{M + nx_N}{M + (n-2)x_N} = \frac{NM + nx}{NM + (n-2)x} < \frac{NM + nM}{NM + (n-2)M} < \frac{N+1}{N-1} \quad \text{(A-20)}$$

Hence, since $p_i \in [0, 1]$, it is sufficient to show that

$$\frac{1}{x_N}F > \frac{1}{2}(\frac{N+1}{N-1})^2 + \Delta + (\frac{N+1}{N-1})(1-\Delta) \quad \text{(A-21)}$$

Both sides in equation (A-21) increase in $N$ (since $N > 0$). When $N$ approaches infinity, the left hand side approaches infinity while the right hand side approaches $3/2$. Hence, the condition is satisfied when $N$ is sufficiently large, i.e., when the size of each unit is small.

(3) We are now ready to prove the main result that for any price schedule $p(\cdot)$ there exists $\tilde{n}$, such that $p(\cdot)$ is weakly dominated by an alternate linear price schedule in which the buyer offers to buy at most $\tilde{n}$ units. The proof is as follows. From the previous two results, we can assume without loss of generality that there exists $n$ such that the buyer offers to buy at most $n$ at prices $p_1 \geq p_2 \geq ... \geq p_n$, and that the capital constraint associated with the sale of all $n$ units is satisfied under the original schedule, i.e.,

$$(M + nx_N)(\Delta + \frac{1}{2}p_n) - x_N(p_1 + ... + p_n) \geq L.$$

Since $p_n$ is the lowest price, the capital constraint would also be satisfied under the linear schedule in which the buyer offers to buy up to $nx_N$ units at a price per unit $p_n$, i.e.,

$$(M + nx_N)(\Delta + \frac{1}{2}p_n) - x_N np_n \geq L.$$

Since $M \geq x$, the capital constraint would also be satisfied under any linear schedule in which the buyer offers to buy up to $nx_N$ units at a price per unit of $p \geq p_n$. In particular, the capital constraint is satisfied by the linear schedule with unit price $p = \arg\max_{\tilde{p} \in \{p_1, ..., p_n\}} \tilde{p}\left(\Delta - \frac{1}{2}\tilde{p}\right)$, which by definition also increases the buyer's expected profits. Q.E.D.

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