

Risking Other People's Money: Gambling, Limited Liability, and Optimal Incentives

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Abstract

We consider optimal incentive contracts when managers can, in addition to shirking or diverting funds, increase short term profits by putting the firm at risk of a low probability “disaster.” To avoid such risk-taking, investors must cede additional rents to the manager. In a dynamic context, however, because managerial rents must be reduced following poor performance to prevent shirking, poorly performing managers will take on disaster risk even under an optimal contract. This risk taking can be mitigated if disaster states can be identified ex-post by paying the manager a large bonus if the firm survives. But even in this case, if performance is sufficiently weak the manager will forfeit eligibility for a bonus, and again take on disaster risk. Our model can explain why suboptimal risk-taking can emerge even when investors are fully rational and managers are compensated optimally. We also explore the model's implications for the dynamics of firm growth.

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1 Introduction

Investors who intrust their funds to financial institutions such as investment banks, pension and hedge funds typically have little knowledge and/or understanding about how those institutions operate. In particular, it is extremely hard for investors to observe true realized cash flows of the financial institutions as well as correctly evaluate their risk exposure. This asymmetric information creates an opportunity for managers of the financial institutions to enrich themselves at the expense of the investors, which is aided by their ability to swiftly alter risk profiles of the assets under management. To protect their interest, the investors can try to write a contract with a manager of a financial institution that would align manager's objectives with theirs. The goal of this paper is to investigate whether such a contract exists in a setting in which a manager privately chooses the riskiness of the project *and* can privately divert cash flows for her own consumption.

In particular, we consider a stylized model with two-dimensional moral hazard problem. A risk neutral agent (manager) with limited liability runs a project which cash flows depend on its riskiness. The agent can choose between a low-risk and high-risk projects. Compared to the low risk project, the high-risk project increases the probability of a high cash flow realization, but it also results in high losses in a bad state of nature, which we call "disaster." The low-risk project does not result in high losses in the disaster state. We assume that the low-risk project is the first best. In addition to risk-shifting, the agent can also manipulate the cash flows by diverting the high cash flow for her consumption. Neither the riskiness of the project nor the cash flow realizations are observed by the investors either ex-ante or ex-post unless the "disaster" state occurs.

The analysis of the model generates a number of key insights. First, we find that although it is possible to write a contract that provides the agents with incentives to choose the low-risk project and not to steal the cash flows, it may be too expensive for the investors to break even. This is the case when the contract terms depend only on cash flows reported by the agent and when the probability of the "disaster" state is low. The economic intuition behind this result is pretty transparent. Any incentive compatible contract must reward the agent for reporting the high cash flow, otherwise the agent would steal it. However, conditioning agent's reward on the reported cash flow also creates an incentive for the agent to take the high-risk project. The reason for that is that the agent benefits from the high cash flow and is protected from the losses by the limited liability in the "disaster" state. The contract that

induces the agent to choose the low-risk project without stealing cash flows requires very high payoffs for the agent for the “non-disaster” outcomes. It is so because the agent with limited liability would otherwise ignore the “disaster” state, which occurs with a very low probability, in her calculations of her expected payoff.

Our second finding is that the contract can be much less expensive for the investor to implement if the agent’s payoff can be made conditional directly on the “disaster” state. In the optimal contract the agent should receive a high payoff for the low cash flow in the “disaster” state. This is a cheaper way to provide incentives for the agent to choose the low-risk project, because the high payoffs in “non-disaster” states are no longer required. We suggest that in practice this contract can be implemented by giving the manager out-of-money put options on the companies that are likely to be ruined in the “disaster” state, with a caveat that the manager can collect the payoff from the options only if her company remains in a good shape.

Our third finding is that if it is impossible to write a contract conditional on the “disaster” state, then it may be optimal to gradually increase the scale of the project. An increase in the project size should be used as a reward for the manager for high cash flow realizations. This would make the contract cheaper for the investor, since he would not need to commit to high payoffs to the agent at the beginning.

Finally, we argue that the expected bailout of the company can induce excessive risk taking. Since bailout reduces the investor’s losses in the “disaster” state, he would be more willing to accept the high-risk project, as oppose to provide the agent with costly incentives to choose the low risk project.

Our paper bridges two strands of literature: the literature on moral hazard and/or hidden action and the literature on risk-shifting. The vast literature on moral hazard (see Salanie (1997) for a survey of static models) has focused mostly on the problem of a principal who wants to induce an agent to exert the “optimal” effort. A majority of papers in this literature assume the maximum likelihood property (Milgrom, 1981). This assumption guarantees that the effect of different actions on the expected value of the outcome dominates all other effects, thus eliminating risk-shifting as a candidate for the moral hazard companion. We avoid making this assumption by considering a hidden action setting similar to the one used by DeMarzo and Fishman (2007a,b) in discrete time, and in continuous time by Sannikov (2008), and DeMarzo and Sannikov (2006). Specifically, in our model, both principal and

agent are risk-neutral and the agency problem is that the agent can divert the cash flow for his own private benefit. As such, the (risky) cash flows are observable only by the agent and hence are not directly contractible. Following DeMarzo and Fishman (2007a,b) we also assume that at any time during the life of the project, the project can be terminated. The termination threat is the key to inducing the agent to share the cash flow with investors. We diverge from DeMarzo and Fishman, Sannikov, and DeMarzo and Sannikov by making an additional assumption that the agent is protected by the limited liability.

The risk-shifting was first introduced by Jensen and Meckling (1976) as an agency conflict between equity and debt holders of a levered firm. The agency problem is that the equity would gain all the upside of increasing the risk of the firm's assets-in-place, whereas debt would be responsible for its downside. Recognizing the importance of this agency problem, a large literature studies the impact of nonconcave portions (such as puts and calls) of common executive compensations schemes on the risk-shifting incentives of managers who have access to dynamically complete markets. Contributions include Carpenter (2000), Ross (2004), and Basak, Pavlova, and Shapiro (2007). This literature treats the optimal contract as being exogenous.

The most closely related papers to the static version of our model are Diamond (1998), Palomino and Prat (2003), Hellwig (1994), and Biais and Casamatta (1999). Like us, they study a hidden-action moral hazard problem in which the agent controls both effort and the distribution of the outcome. Diamond (1998) and Palomino and Prat (2003) use delegated portfolio management as the economic motivation. Diamond (1989) asks whether, as the cost of effort shrinks relative to the payoffs, the optimal contract converges to the linear contract. He shows that if the agent has several ways to manipulate the outcome, the principal should offer the simplest possible compensation scheme, that is, the linear contract. While Diamond (1998) considers only three possible outcomes, Palomino and Prat (2003) allow for a continuum of outcomes. Similar to our setting, the agent in their model has limited liability and can sabotage (misreport) the realized return. They show that the optimal contract is simply a bonus contract - the agent is paid a fixed sum if the portfolio return is above a threshold. Also, by using an explicit parametrization of risk, they are able to analyze the sign of inefficiencies in risk taking. Hellwig (1994) and Biais and Casamatta (1999) are interested in the optimal financing of investment projects when managers must exert unobservable effort and can also switch to less profitable riskier ventures. Both papers

find that under some technical conditions optimal financial contracts can be implemented by a combination of debt and equity.

The definition of the risk-shifting is the key difference between our model and this group of papers. In these papers agent can manipulate only the shape of the distribution, but not the set of the outcomes, which is bounded and nonnegative. Contrary to these papers, we define risk-shifting as an increase in the probability of the best outcome at the expense of being exposed to a “disastrous” state with a large loss. This allows for a much more natural assertion of the limited liability - agent is not liable for the losses which are shouldered by the investors.

The most closely related papers to the dynamic version of our model are by Ou-Yang (2003) and Cadenillas, Cvitanic, and Zapatero (2007). Both papers are executed in a continuous time setting where agent controls both the drift (effort choice) and the volatility (project selection) of the underlying payoff process. Unlike us, neither paper allows for the limited liability of the agent. In addition, we allow for the endogenous liquidation before the terminal date of the project. This assumption is critical since it facilitates “all-or-nothing” strategy from the agent. Implementing this assumption in the economic setting of these two papers is technically infeasible since it would superimpose an optimal stopping problem over their current optimization routine. Finally, Makarov and Plantin (2010) develop a dynamic model of active portfolio management in which fund managers may secretly gamble in order to manipulate their reputation and attract more funds. They solve for the optimal contracts that deter this behavior and show that if investors are short-lived, then the manager must leave rents to investors in order to credibly commit not to gamble. If investors are long-lived, any contract that increases but defers expected bonuses after an outstanding performance is optimal. Contrary to our paper, Makarov and Plantin (2010) consider only observable actions by the agent.

The rest of the paper is organized as follows. Static model is presented in Section 2. The dynamic extension is discussed in Section 3. Section 4 concludes.

2 One Period Model

There are two risk-neutral players. First, the principal (investor(s)), owns the company which has value of its assets-in-place equal to A . The principal can add a new project to the company in which case he has to hire the second player, the agent, to run it. The cash flows

from the project, $\tilde{Y}(\sigma)$, depend on the degree of its riskiness, $\sigma \in \{0, 1\}$, which is controlled by the agent. The project has three possible cash-flow realizations

$$\tilde{Y}(\sigma) = \begin{cases} 1, & \text{with probability } p^0 + \sigma p^1 \\ 0, & \text{with probability } 1 - p^0 - \sigma(p^1 + p^2) \\ -L, & \text{with probability } \sigma p^2 \end{cases}, \quad (1)$$

The choice of $\sigma = 0$ corresponds to the “safe” project, in which case only nonnegative cash flows 0 and 1 are possible and its expected value is equal to p^0 . The choice of $\sigma = 1$ corresponds to the “risky” project. In this case the probability of the highest cash flow, 1, is increased by p^1 , but a new negative cash flow $-L$ can be realized with probability $p^2 < p^1$. We assume that the “safe” project has higher expected cash flows than the “risky” project

$$p^1 - p^2 L < 0. \quad (2)$$

L can be interpreted as the direct loss from the operations and it could be quite large, but not greater than the value of assets-in-place, i.e. $L < A$. Under this assumption the principal has limited liability with respect to the company, but not the new project.

The agent can take two actions both unobservable by the principle. First, the agent upon privately observing that the realized cash flow is equal to 1 can report to the principal that the cash flow is 0. By doing so the agent receives a private benefit of $\lambda \in [0, 1]$. We interpret diversion of the firm’s cash flows as stealing. We assume that the agent can secretly transfer money from the firm’s account to his own account using accounting manipulations. However, other hidden activities that benefit the agent at the expense of the principal may fit the setting of the model as well. For instance, the agent can inefficiently use the firm’s cash flows in order to receive non-pecuniary benefits. The fraction $1 - \lambda$ represents the cost of diversion, which can be attributed to different kinds of expenses and inefficiencies associated with the diversion.

Such hidden action setting allows for one-to-one mapping into the hidden effort setting. In this setting the agent chooses a binary effort $e \in \{0, 1\}$ at a cost of λe . Here $e = 1$ stands for “high” (optimal) effort, while $e = 0$ stands for “low” (inferior) effort. The payoffs

conditional on the joint choice of effort and risk are given by

$$\tilde{Y}(\sigma, e) = \begin{cases} 1, & \text{with probability } e(p^0 + \sigma p^1) \\ 0, & \text{with probability } 1 - e(p^0 + \sigma p^1) - \sigma p^2 \\ -L, & \text{with probability } \sigma p^2 \end{cases}. \quad (3)$$

Under the hidden effort interpretation the incentive compatibility constraint enforces the optimal effort.

Privately choosing the “riskiness” of the project, σ , is the second action the agent can take. We also assume that the agent has limited liability. We interpret limited liability as a disallowance of positive transfers from the agent to the principle, i.e. the agent cannot be legally forced to pay back the principal.

The principal does not observe neither the realized cash flow, y , nor the riskiness, σ , implemented by the agent and therefore he must rely wholly on the agent to report the cash flow realizations, \hat{y} . We assume that the monitoring is prohibitively costly. Under these assumptions neither y nor σ is contractable.

If the principal agrees to initiate the project, at the time of initiation the principal and the agent sign a contract that governs their relationship over the life of the project. The contract obligates the agent to report realizations of the cash flows to the principal. Without loss of generality, we assume that the contract requires the agent to pay the reported cash flows to the principal immediately. The contract also specifies transfer payment from the principal to the agent, $a^d(\hat{y}) \geq 0$, which depends on the reported cash flow.

Next we solve for the optimal contract for three different cases: (i) low risk taking; (ii) high risk taking; and (iii) low risk taking under alternative specification.

2.1 Optimal contract implementing the safe project

Here we derive optimal incentive compatible contract that enforces the safe project choice. A contract is *optimal* if it maximizes the principal’s payoff, $b(a; \sigma)$, subject to a certain payoff a for the agent.

DEFINITION 1: *We say that the incentive compatible contract enforcing safe project choice $\phi = \{a^d(0), a^d(1), a^d(-L)\}$ that implements payoffs a and $b(a|\phi)$ for the agent and the principal, respectively, is optimal if there is no other incentive compatible contract $\hat{\phi}$ with the same payoff for the agent a , but with a higher payoff for the principal, $b(a|\hat{\phi}) > b(a|\phi)$.*

This definition is based on the time-zero expected payoffs. The principal's income at the end of the period is given by the difference between the reported cash flow \hat{y} , transfer $a^d(\hat{y})$ and the value of assets-in-place A . Therefore the principal's problem is to choose a contract $\phi = \{a^d(0), a^d(1), a^d(-L)\}$ that maximizes his expected payoff, $b(a; \sigma = 0)$:

$$b(a; 0) = \max_{a^d(\cdot) \geq 0} \mathbb{E} \left[\tilde{Y} - a^d(\hat{y}) + A | \sigma = 0 \right], \quad (4)$$

subject to the promise-keeping (PK thereafter) constraint

$$\text{(PK)} : a = \mathbb{E} \left[a^d(\hat{y}) | \sigma = 0 \right], \quad (5)$$

the incentive compatibility (IC thereafter) constraint

$$\text{(IC)} : a^d(1) \geq a^d(0) + \lambda. \quad (6)$$

and the low-risk-taking (LRT thereafter) constraint

$$\text{(LRT)} : \mathbb{E} \left[a^d(\tilde{Y}(\sigma = 0)) \right] \geq \mathbb{E} \left[a^d(\tilde{Y}(\sigma = 1)) \right]. \quad (7)$$

The function $b(a; 0)$ represents the highest possible payoff attainable by the principal, given any arbitrary payoff a to the agent when the safe project is implemented. The PK (5) constraint implies that the agent's expected payoff is a . The IC constraint (6) ensures that when the cash flow of 1 is realized the agent truthfully reports it. The LRT constraint (7) guarantees that the agent selects safe project over the risky project. Therefore, it is obvious that $a^d(-L) = 0$ which we are going to use throughout this subsection.

Solving for the contracting problem (4), (6), (5), and (7) means finding the optimal *non-negative* transfers $\{a^d(0), a^d(1)\}$. Our solution method relies on the work of DeMarzo and Fishman (2007a,b). In the case of the low risk taking the optimization problem (4) can be written as

$$b(a; 0) = \max_{a^d(\cdot) \geq 0} p^0 \left(1 + A - a^d(1) \right) + (1 - p^0) \left(0 + A - a^d(0) \right), \quad (8)$$

$$\text{s.t. (PK)} : a = p^0 a^d(1) + (1 - p^0) a^d(0), \quad (9)$$

$$\text{(IC)} : a^d(1) \geq a^d(0) + \lambda, \quad (10)$$

$$\text{(LRT)} : (p^1 + p^2) a^d(0) \geq p^1 a^d(1), \quad (11)$$

In rewriting the LRT constraint we have used that

$$\mathbb{E} \left[a^d(\hat{y}) | \sigma \right] = (p^0 + \sigma p^1) a^d(1) + (1 - p^0 - \sigma(p^1 + p^2)) a^d(0). \quad (12)$$

Figure 1 illustrates constraints (9) to (11) in the $(a^d(0), a^d(1))$ plane. The forty-five degree line that starts at λ represents the IC constraint. It is binding along the line and is slack above it. The second straight line that starts at zero represents LRT constraint. It binds along the line and is slack below it. Therefore, the set of all possible transfers which jointly satisfies IC and LRT lies in a wedge between these two lines. The thick dot shows the point of the intersect between the two lines. The coordinates of this points can be found as follows. Combining (10) and (11) we obtain

$$0 \geq p^1 \underbrace{(a^d(1) - a^d(0))}_{\geq \lambda} - p^2 a^d(0) \geq p^1 \lambda - p^2 a^d(0).$$

It implies that constraints (10) and (11) can be jointly satisfied when

$$a^d(0) \geq \frac{p^1}{p^2} \lambda. \quad (13)$$

When used as equality it yields the horizontal coordinate of the intersection point. The vertical coordinate of the intersection point, $a^d(1) = (\frac{p^1}{p^2} + 1)\lambda$, is immediately obtained by substituting (13) into the IC constraint. The dotted line shows the PK constraint. When PK constraint passes through the intersection point the promised utility a is equal to \bar{a}

$$p^0 \underbrace{\left(\frac{p^1}{p^2} + 1\right)\lambda}_{a^d(1)} + (1 - p^0) \underbrace{\frac{p^1}{p^2} \lambda}_{a^d(0)} = \bar{a},$$

from where the cutoff \bar{a} is equal to

$$\bar{a} = \frac{p^1 + p^2 p^0}{p^2} \lambda. \quad (14)$$

For values of a greater or equal than \bar{a} , the PK constraint passes through at least one point in the area where both IC and LRT are jointly satisfied. Formally, it can be written as:

$$a \geq p^0 (a^d(0) + \lambda) + (1 - p^0) a^d(0) = a^d(0) + p^0 \lambda \geq \bar{a}, \quad (15)$$

The intersect point is, however, the “cheapest” contract to the principal. Therefore, the IC binds and the cheapest transfers to the agent are equal to $\left(\frac{p^1}{p^2}\lambda, \left(1 + \frac{p^1}{p^2}\right)\lambda\right)$. Finally, we can substitute the PK constraint (9) into the objective function (8) to obtain

$$\begin{aligned} b(a; 0) &= \max_{a^d(1), a^d(0)} A + p^0 - \underbrace{\left(p^0 a^d(1) + (1 - p^0) a^d(0)\right)}_a \\ &= A + p^0 - a. \end{aligned} \tag{16}$$

Combining (16) and (15) and taking into account that payoffs to the principal are infeasible if the condition (15) is violated yields the complete solution for the second period principal’s continuation payoff, $b(a; 0)$, which is summarized by the Proposition 1.

PROPOSITION 1: *The optimal incentive compatible contract implementing safe project is $\phi = \{a^d(0) = a - p^0\lambda, a^d(1) = a + (1 - p^0)\lambda, a^d(-L) = 0\} \forall a \geq \bar{a}$. The principal’s optimal continuation payoff under ϕ is given by*

$$b(a; 0) = \begin{cases} A + p^0 - a, & a \geq \bar{a} \\ -\infty, & 0 \leq a < \bar{a} \end{cases}. \tag{17}$$

where the minimum expected payoff to the agent \bar{a} is equal to

$$\bar{a} = \frac{p^1 + p^2 p^0}{p^2} \lambda.$$

The highest payoff to the principal is achieved at $a = \bar{a}$. We use the notation $-\infty$ to indicate that for values of $a < \bar{a}$ there exists no incentive compatible contract implementing safe project. Since p^2 can be arbitrarily small (but not necessarily the expected losses since L can be made arbitrarily large), the optimal contract can become too expensive for the principal to implement. This happens since in order to jointly satisfy IC and LRT constraints the agent has to be given high payoffs when cash flows are either 0 or 1. Figure 2 also helps to understand the comparative statics of the optimal contract. As λ increases, the intersection point shifts in the north-east direction, which means that higher transfers to the agents are needed to prevent his misbehavior. When probability of the losses falls, agent has more incentive to gamble and once again the intersection point shifts in the north-east direction.

2.2 Optimal contract implementing the risky project

Here we derive optimal incentive compatible contract that does not enforce the safe project choice.

DEFINITION 2: *We say that the incentive compatible contract $\phi = \{a^d(0), a^d(1), a^d(-L)\}$ that implements payoffs a and $b(a|\phi)$ for the agent and the principal, respectively, is optimal if there is no other incentive compatible contract $\hat{\phi}$ with the same payoff for the agent a , but with a higher payoff for the principal, $b(a|\hat{\phi}) > b(a|\phi)$.*

Just like in the case when only the safe project was implemented, the principal's income at the end of the period is given by the difference between the reported cash flow \hat{y} , transfer $a^d(\hat{y})$ and the value of assets-in-place A . However, in contrast with the previous case, risky project can end either in success, in which case the continuation payoff to the principal is equal to $y + A - a^d$ for any promised payoff to the agent, a^d , or in failure, in which case the continuation payoff to the principal is equal to $A - L$. It is obvious again that the optimal contract should have $a^d(-L) = 0$. The principal's problem is to choose a contract $\phi = \{a^d(0), a^d(1), a^d(-L)\}$ that maximizes his expected payoff, $b(a; \sigma = 1)$:

$$b(a; 1) = \max_{a^d(\cdot) \geq 0} \mathbb{E} \left[\tilde{Y} - a^d(\hat{y}) + A | \sigma = 1 \right], \quad (18)$$

subject to the PK and IC constraints

$$(\text{PK}) : a = \mathbb{E} \left[a^d(\hat{y}) | \sigma = 1 \right], \quad (19)$$

and

$$(\text{IC}) : a^d(1) \geq a^d(0) + \lambda. \quad (20)$$

The problem (18) can be rewritten as

$$b(a; 1) = \max_{a^d} \bar{p} \left(1 + A - a^d(1) \right) + (1 - \bar{p} - p^2) \left(A - a^d(0) \right) + p^2 (A - L), \quad (21)$$

$$\text{s.t. } (\text{PK}) : a = \bar{p} a^d(1) + (1 - \bar{p} - p^2) a^d(0), \quad (\text{IC}) : (6), \quad (22)$$

where we have denoted $\bar{p} \equiv p^0 + p^1$.

Figure 2 illustrates constraints (22) in the $(a^d(0), a^d(1))$ plane. Once again, the forty-five

degree line that starts at λ represents the IC constraint. It is binding along the line and is slack above it. The dotted line shows the PK constraint. It is immediate to realize that the cheapest contract to the principle is given by $\{a^d(0) = 0, a^d(1) = \lambda\}$.

Finally, combining (6) and PK from (22) we obtain the low bound on the promised utility, a :

$$a \geq \bar{p} \left(a^d(0) + \lambda \right) + (1 - \bar{p} - p^2) a^d(0) = \underbrace{a^d(0)}_{\geq 0} + \bar{p}\lambda \geq \bar{p}\lambda. \quad (23)$$

Now we can substitute the PK constraint (22) into the objective function (21) and take into account that payoffs to the principal are infeasible when $a \leq \bar{p}\lambda$ yields the complete solution for the second period principal's continuation payoff, $b(a; 1)$, summarized by Proposition 2.

PROPOSITION 2: *The optimal incentive compatible contract implementing risky project is $\phi = \{a^d(0) = a - \bar{p}\lambda, a^d(1) = a + (1 - \bar{p})\lambda, a^d(-L) = 0\} \forall a \geq \bar{p}\lambda$. The principal's optimal continuation payoff under ϕ is given by*

$$b(a; 1) = \begin{cases} \bar{p} + A - p^2L - a, & a \geq \bar{p}\lambda \\ -\infty, & 0 \leq a < \bar{p}\lambda \end{cases}. \quad (24)$$

Corollaries 1 and 2 highlight our next set of results.

COROLLARY 1: *The lowest agent's compensation under the incentive compatible contract implementing risky project, $\bar{p}\lambda$, is smaller than the lowest compensation under incentive compatible contract implementing safe project, \bar{a} ,*

$$\bar{p}\lambda < \bar{a}. \quad (25)$$

This result follows immediately from the definition of \bar{a} . The optimal incentive compatible contract allowing for the risky project is less expensive than the optimal incentive compatible contract that disallows the risky project because it does not require high levels of the agent's compensation when cash flows are either equal to 0 or 1. At the same time, this contract can result in large losses to the principal.

COROLLARY 2: *When the agent's compensations under the incentive compatible contracts with and without risk taking are equal to their lowest values, the principle is better off allowing risk-taking if*

$$p^2L - p^1 < \frac{1 - p^2}{p^2} p^1 \lambda. \quad (26)$$

This inequality can be satisfied even when the expected losses, p^2L , are high. Note that the right hand side of the inequality (26) goes to infinity when p^2 goes to zero, while L can be made large enough so that the expected losses remain significant. Therefore, while the safe project is strictly better than the risky project, the principal would prefer the risky project or no project at all to the safe project when the loss probability p^2 is low even though the expected losses could be substantial.

2.3 Contracting on the states of nature

In this subsection we allow the principle to contract not only on the reported cash flows, but also on the states of nature. Consider the following incomplete information game between the principal and the agent. At time zero nature makes a draw: With probability p^2 it draws a “disaster” state, and it draws “good” state with probability $1 - p^2$. Ex ante, neither principal nor agent know the exact state of nature, but ex post the true state of nature is a public knowledge. In the “good” state the payoffs conditional on the risk strategy, $\sigma \in \{0, 1\}$, are as follows

$$\tilde{Y}_G(\sigma) = \begin{cases} 1, & \text{with probability } g^0 + \sigma g^1 \\ 0, & \text{with probability } 1 - g^0 - \sigma g^1 \\ -L, & \text{with probability } 0 \end{cases}, \quad (27)$$

while in the “disaster” state the payoffs are

$$\tilde{Y}_D(\sigma) = \begin{cases} 1, & \text{with probability } 0 \\ 0, & \text{with probability } 1 - \sigma \\ -L, & \text{with probability } \sigma \end{cases}. \quad (28)$$

This game can be easily mapped onto the original game by matching the probabilities of payoff of 1:

$$(1 - p^2)(g^0 + \sigma g^1) = p^0 + \sigma p^1, \quad (29)$$

and 0:

$$p^2(1 - \sigma) + (1 - p^2)(1 - g^0 - \sigma g^1) = 1 - p^0 - \sigma(p^1 + p^2). \quad (30)$$

The probability of the disaster payoff $-L$ is matched by construction. We can use either one of these equations to get g^0 and g^1 in terms of p^0 , p^1 , and p^2 . Setting $\sigma = 0$ in (29) yields

$$g^0 = \frac{p^0}{1 - p^2} = 1 - \frac{p^1}{1 - p^2} \leq 1, \quad (31)$$

and setting $\sigma = 1$ in (29) yields

$$g^1 = \frac{p^1}{1-p^2} = 1 - \frac{p^0}{1-p^2} \leq 1. \quad (32)$$

and there exists a non-trivial match between probabilities.

We now write the optimal low-risk incentive compatible contract based on this game. The crucial difference in this case is that the transfer to the agent, $a^d(\hat{y}, n)$, is not a function of just the reported cash flow, \hat{y} , but also depends on the state of nature, $n \in \{G, D\}$. The definition of the optimal incentive compatible contract that enforces the safe project choice is given below. The notion of optimality remains unchanged from previous subsections.

DEFINITION 3: *We say that the incentive compatible contract enforcing safe project choice $\phi = \{a^d(0, G), a^d(1, G), a^d(0, D), a^d(-L, D)\}$ that implements payoffs a and $b(a|\phi)$ for the agent and the principal, respectively, is optimal if there is no other incentive compatible contract $\hat{\phi}$ with the same payoff for the agent a , but with a higher payoff for the principal, $b(a|\hat{\phi}) > b(a|\phi)$.*

The principal's problem is to choose a contract

$$\phi = \{a^d(0, G), a^d(1, G), a^d(0, D), a^d(-L, D)\},$$

that maximizes his expected payoff, $b(a; \sigma = 0)$:

$$b(a; 0) = \max_{a^d(\cdot) \geq 0} \mathbb{E}^{n, \tilde{Y}} \left[\tilde{Y} - a^d(\hat{y}, n) + A|\sigma = 0 \right], \quad (33)$$

subject to the PK constraint

$$(\text{PK}) : a = \mathbb{E}^{n, \tilde{Y}} \left[a^d(\hat{y}, n) | \sigma = 0 \right], \quad (34)$$

the IC constraint

$$(\text{IC}) : a^d(1, G) \geq a^d(0, G) + \lambda. \quad (35)$$

and the LRT constraint

$$(\text{LRT}) : \mathbb{E}^{n, \tilde{Y}} \left[a^d(\tilde{Y}_n(\sigma = 0), n) \right] \geq \mathbb{E}^{n, \tilde{Y}} \left[a^d(\tilde{Y}_n(\sigma = 1), n) \right]. \quad (36)$$

where $\mathbb{E}^{n, \tilde{Y}}[\cdot]$ denotes expectations over the states of nature and payoffs.

The problem of the principal can be written as

$$b(a; 0) = \max_{a^d(\cdot) \geq 0} p^0 \left(1 + A - a^d(1, G) \right) + (1 - p^0 - p^2) \left(0 + A - a^d(0, G) \right) + p^2 (A - a^d(0, D)), \quad (37)$$

$$\text{s.t. (PK) : } a = p^0 a^d(1, G) + (1 - p^0 - p^2) a^d(0, G) + p^2 a(0, D), \quad \text{(IC) : (35),} \quad (38)$$

$$\begin{aligned} \text{(LRT) : } & p^0 a^d(1, G) + (1 - p^0 - p^2) a^d(0, G) + p^2 a^d(0, D) \geq \\ & (p^0 + p^1) a^d(1, G) + (1 - p^0 - p^1 - p^2) a^d(0, G). \end{aligned} \quad (39)$$

The LRT constraint can be simplified to:

$$\text{(LRT) : } p^2 a^d(0, D) \geq p^1 \left(a^d(1, G) - a^d(0, G) \right). \quad (40)$$

Here we have already converted probabilities $g^{0,1}$ into the original probabilities $p^{0,1}$ using relations (31) and (32). Under this specification, the principal's objective function (37) is fully consistent with the program (8) where the conditioning on the states of nature is not allowed. The unconditional probability of cash flow 1 is equal to p^0 and the unconditional probability of cash flow 0 is equal to $1 - p^0$. The key difference between two specifications is that cash flow 0 occurs with probability $1 - p^0 - p^2$ in the “good” state of nature, and with probability p^2 in the “disaster” state when the safe project is implemented.

The immediate observation we can make is that $a^d(-L, D) = 0$. Figure 3 provides graphical exposition of the solution to the rest of this contracting problem. It shows the set of optimal transfers to the agent in the $(a^d(0, G), a^d(1, G))$ plane. The first forty-five degree line that starts at λ represents the IC constraint. It is binding along the line and it is slack above it. The second forty-five degree line that starts at $\frac{p^2}{p^1} a^d(0, D)$ represents the LRT constraint. The LRT constraint is slack below this line and binds along it. Both constraints are satisfied as long as $\frac{p^2}{p^1} a^d(0, D) \geq \lambda$, with the minimum payoff to the agent achieved when this constraint binds to yield

$$a^d(0, D) = \frac{p^1}{p^2} \lambda. \quad (41)$$

In this case the optimal choice of $(a^d(0, G), a^d(1, G))$ satisfying PK constraint is $(0, \lambda)$, meaning that the PK constraint passes through $(0, \lambda)$ point on the plot. This is the cheapest

transfer which satisfies all three constraints. The principal receives

$$\begin{aligned} b(a; 0) &= \max_{a^d(1), a^d(0)} p^0 + A - \underbrace{\left(p^0 a^d(1, G) + (1 - p^0 - p^2) a^d(0, G) + p^2 a^d(0, D) \right)}_a \\ &= p^0 + A - a, \end{aligned} \quad (42)$$

when

$$a \geq p^0 \left(a^d(0, G) + \lambda \right) + (1 - p^0 - p^2) a^d(0, G) + p^1 \lambda \geq \bar{p} \lambda, \quad (43)$$

where we have introduced $\bar{p} \equiv p^0 + p^1$. Proposition 3 summarizes our results.

PROPOSITION 3: *Conditional on the state of nature optimal incentive compatible contract implementing safe project is $\phi = \{a^d(0, G) = a - \bar{p}\lambda, a^d(1, G) = a - \bar{p}\lambda + \lambda, a^d(0, D) = a - \bar{p}\lambda + \frac{p^1}{p^2}\lambda, a^d(-L, D) = 0\} \forall a \geq \bar{p}\lambda$. The principal's optimal continuation payoff under ϕ is given by*

$$b(a; 0) = \begin{cases} A + p^0 - a, & a \geq \bar{p}\lambda, \\ -\infty, & 0 \leq a < \bar{p}\lambda \end{cases}. \quad (44)$$

The minimum expected payoff to the agent, $(p^0 + p^1)\lambda$, is smaller than \bar{a} , and it is much smaller than \bar{a} when p^2 is small. It makes this contract cheaper for the principal to implement. In fact, the cost of implementing this contract to the principal is the same as are the cost of the contract implementing risky project.

The intuition behind this result is transparent and as follows. Depending on the state of nature, cash flow 0 can be either bad or good outcome – it is bad in the “good” state and good in the “disaster” state. When contracting on the states of nature is not allowed as in Propositions 1, the agent has to be given high rewards for both 0 and 1 cash flows in order to implement the safe project, which makes the optimal contract expensive. When contracting on the states of nature is allowed, the contract can differentiate between cash flows 0 and 1, and, therefore, it is not necessary to promise the agent high payoffs for either cash flow in the good state of nature. Instead, it is optimal to give the agent high payoff $a^d(0, D) = a - \bar{p}\lambda + \frac{p^1}{p^2}\lambda$ for cash flow 0 in the disaster state of nature. While this payoff can become unbounded when p^2 goes to zero, the expected payoff $p^2 a^d(0, D)$ remains finite.

2.4 Implementation

In practice, conditioning on states of nature while possible is quite challenging. It is so because states of nature are difficult to categorize and verify. However, our optimal contract requires conditioning only on the “extreme” states of nature, such as the current financial crisis. During this crisis a number of major financial institutions either went bankrupt or their equity suffered extreme losses. Therefore, one practical way to implement our contract is as follows. At the time when the whole economy is doing well, managers who have either authority or ability to change the riskiness of their projects, should be awarded out-of-the-money put options. These options should be written on the companies in the same line of business which would be ruined in the case of disaster. This implementation is relatively inexpensive at the award time since these options would be cheap. However they would result in large payoffs to their holders in the case of the disaster. The necessary caveat is that the manager would get this payoff only if his company stays afloat (payoff 0 in the “disaster” state in our model). Manager would rationally anticipate this large reward at the outset and would implement the safe project.

These contracts are, however, not observed in practice. One argument against a compensation like that is that it provides managers with extra incentives to take down their competitors. Our paper is the first to point out the particular benefits of using put options on other companies as a part of managerial compensations. In light of the recent financial crisis, the benefits of such compensation can outweigh its downside, since losses from excessive risk-taking it helps to prevent can be quite large. A comprehensive cost-benefit analysis of such executive compensation should be executed in a general equilibrium setting which is beyond the scope of this paper and is left for future research.

3 Two-Period Model

This section considers a two-period version of the model, when contracting on the states of nature is not allowed. The purpose of this exercise is to verify that the main insights of the static framework are robust as well as to obtain new insights, specific to the dynamic setting.

We now consider that the project operates for two periods and yields two independent cash flows. To be consistent with the static model we require that the per-period cash flow realizations are equal to exactly half of the cash flow realization in the static setting. These

possible cash-flow realizations, $\tilde{Y}_t(\sigma_t)$, now have a time subscript $t \in \{1, 2\}$ and can be written as

$$\tilde{Y}_t(\sigma_t) = \begin{cases} 0.5, & \text{with probability } p^0 + \sigma_t p^1 \\ 0, & \text{with probability } 1 - p^0 - \sigma_t(p^1 + p^2) \\ -0.5L, & \text{with probability } \sigma_t p^2 \end{cases}. \quad (45)$$

One difference from the static model is that in the two-period setting the project can be liquidated/abandoned by the agent at the end of the first period. We allow for the endogenous stochastic liquidation. The first period liquidation is inefficient, in the sense that $A < A + 0.5p^0$. For the sake of simplicity, we assume that there is no discounting between the periods thus making it optimal to delay all payments to the agent until the end of the second period. The two-period contract is given by $\phi = (\pi(\hat{y}_1), a_2^d(\hat{y}_1, \hat{y}_2))$. Here $1 - \pi(\hat{y}_1)$ is the probability that the project is liquidated at the end of the first period, and $a_2^d(\hat{y}_1, \hat{y}_2)$ is the payment from the principal to the agent at the end of the second period provided the project has not been liquidated at the end of the first period. We consider contracts with full commitment only. No renegotiation of the terms of the contract is allowed.

We consider three possibilities: (i) safe project in both periods; (ii) risky project in the first period and safe project in the second period; and (iii) project size can be changed between the periods.

3.1 Optimal contract implementing the safe project in both periods

In this section we derive optimal contract which implements safe project in both periods. We solve the contracting problem recursively. The principal's income in each period is given by the difference between the reported cash flow \hat{y}_t , transfer a_t^d and the value of assets-in-place A . At the end of the second period the project is liquidated with the probability one and thus the continuation payoff to the principal is equal to $A - a_2^d$ for any promised payoff to the agent, a_2^d . The beginning of period two continuation payoff to the principal when the agent's continuation payoff is equal to a_2 , $b(a_2; 0)$, can be found from

$$b_2(a_2; 0) = \max_{a_2^d(\cdot) \geq 0} \mathbb{E} \left[\tilde{Y}_2 - a_2^d(y_1, \hat{y}_2) + A | \sigma_2 = 0 \right]. \quad (46)$$

Here $a_2^d(y_1, \hat{y}_2)$, which denotes agent's second period continuation payoff just prior the transfer to the agent, satisfies the PK constraint :

$$\text{(PK): } a_2(y_1) = \text{E} \left[a_2^d(y_1, y_2) | \sigma_2 = 0 \right] = p^0 a_2^d(y_1, 1) + (1 - p^0) a_2^d(y_1, 0), \quad (47)$$

IC constraint:

$$\text{(IC): } a_2^d(y_1, 1) \geq a_2^d(y_1, 0) + 0.5\lambda, \quad (48)$$

as well as the LRT constraint:

$$\text{(LRT): } \text{E} \left[a_2^d(y_1, \tilde{Y}_2(\sigma_2 = 0)) \right] \geq \text{E} \left[a_2^d(y_1, \tilde{Y}_2(\sigma_2 = 1)) \right]. \quad (49)$$

The optimization problem (46)-(48) can be easily mapped into Proposition 1 by noting that we just need to substitute $0.5p^0$ for p^0 and 0.5λ for λ in (17) to yield:

$$b_2(a_2(y_1); 0) = \begin{cases} 0.5p^0 + A - a_2(y_1), & a_2 \geq 0.5\bar{a} \\ -\infty, & 0 \leq a_2 < 0.5\bar{a} \end{cases}. \quad (50)$$

Next we turn our attention to the beginning of the first period continuation payoff to the principal, $b_1(a_1; \sigma_1 = 0, \sigma_2 = 0)$. The principal's problem is to choose a contract $\phi = \{\pi, a_2^d\}$ that maximizes $b_1(a_1; 0, 0)$:

$$b_1(a_1; 0, 0) = \max_{a_1^d(\cdot) \geq 0, 0 \leq \pi \leq 1} \text{E} \left[\underbrace{\tilde{Y}_1 + (1 - \pi(\hat{y}_1))A}_{\text{Period 1 payoff}} + \underbrace{\pi(\hat{y}_1) \left(\tilde{Y}_2 - a_1^d(\hat{y}_1) + A \right)}_{\text{Period 2 payoff}} | \sigma_1 = 0, \sigma_2 = 0 \right], \quad (51)$$

subject to the PK constraint

$$\text{s.t. (PK) : } a_1 = \text{E} \left[\pi(\hat{y}_1) a_1^d(\hat{y}_1) | \sigma_1 = 0, \sigma_2 = 0 \right], \quad (52)$$

the IC constraint

$$\text{(IC) : } a_1^d(1) \geq a_1^d(0) + 0.5\lambda. \quad (53)$$

and the LRT constraint

$$\text{(LRT) : } \text{E} \left[a_1^d(\tilde{Y}_1(\sigma_1 = 0)) \right] \geq \text{E} \left[a_1^d(\tilde{Y}_1(\sigma_1 = 1)) \right]. \quad (54)$$

Here a_1 is the beginning of the first period expected payoff to the agent. The incentive compatibility constraint (53) nests the requirement that the agent chooses the safest strategy, so that it has to be satisfied for any other risk strategy (σ_1, σ_2) . Notice that in this case the agent only gets half of the private benefits from misreporting. The function $b_1(a_1; 0, 0)$ represents the possible payoff attainable by the principal, given the promised payoff a_1 for the agent. Proposition 4 summarizes solution to $b_1(a_1; 0, 0)$.

PROPOSITION 4: *The optimal incentive compatible contract implementing safe project in both periods is given by*

$$\phi = \left\{ \pi = \frac{a_1 - 0.5p^0\lambda}{0.5\bar{a}}, a_2^d(0, 0) = 0.5(\bar{a} - p^0\lambda), a_2^d(1, 0) = a_2^d(0, 1) = 0.5(\bar{a} + (1 - p^0)\lambda), \right. \\ \left. a_2^d(1, 1) = 0.5(\bar{a} - p^0\lambda) + \lambda \right\},$$

when $a_1 < 0.5(\bar{a} + p^0\lambda)$, and it is given by

$$\phi = \left\{ \pi = 1, a_2^d(0, 0) = a_1 - p^0\lambda, a_2^d(1, 0) = a_2^d(0, 1) = a_1 - p^0\lambda + 0.5\lambda, a_2^d(1, 1) = a_1 - p^0\lambda + \lambda \right\},$$

when $a_1 \geq 0.5(\bar{a} + p^0\lambda)$. The first period principal's optimum continuation payoff under this contract is given by

$$b_1(a_1; 0, 0) = \begin{cases} p^0 + A - a_1, & a_1 \geq 0.5(\bar{a} + p^0\lambda) \\ p^0 + A - a_1 - \\ 0.5p^0(1 - p^0) \underbrace{\left(1 - \frac{2a_1 - p^0\lambda}{\bar{a}}\right)}_{\text{Prob. of liquidation}}, & 0.5\bar{a} \leq a_1 < 0.5(\bar{a} + p^0\lambda) \\ -\infty, & 0 \leq a_1 < 0.5\bar{a} \end{cases} \quad . \quad (55)$$

Proof is provided in Appendix.

Since $0.5(\bar{a} + p^0\lambda) < \bar{a}$ this contract is cheaper to implement in the no-liquidation region than the one-period optimal contract. The minimum required level of compensation for the agent is going to be almost half its one-period value when p^2 is low in which case $\bar{a} \gg p^0\lambda$. However, since the principal captures all savings from the reduced contracting costs, this contract offers better ex ante payoffs to the principal in the no-liquidation region. These savings result from the fact that the agent can divert per period in two-period setting only half of the total he can divert in one-period model. This makes the per-period agency problem less severe than in one-period case, and, as a result, less costly to mitigate.

Here is an alternative interpretation of Proposition 4. The key assumption made in the two-period setting is that the per-period payoff distributions are the same. As such, the probability of the disaster state (cash flow $-L$) occurring over both periods, $2p^2$, is exactly twice the probability of the disaster state in the one-period setting. One can easily verify that when the probability of the disaster state is equal to $0.5p^2$ in each period, the minimum level of the agent's compensation must be equal to $\bar{a} + 0.5p^0\lambda$ in the no-liquidation region.

In the one-period setting, the minimum payoff to the agent is inversely proportional to the probability of the disaster state. Thus, the result of the two period model is perfectly in line with the finding of the one-period model: The less likely the probability of the disaster outcome – the higher the agent's compensation should be. In the dynamic setting this finding can be restated as follows: The lower the frequency of the disaster outcome – the higher the agent's compensation should be in order to enforce the implementation of the safe project in all periods. The minimum level of agent's compensation is primarily determined by the cumulative probability of the disaster outcome over the lifespan of the project. If the cumulative probability of the disaster outcome over two periods is the same as in the one-period setting, then the minimum agent's compensation in the two-period setting is very close to its one-period counterpart.

4 Optimal Contract with the Project Scale Adjustments

In this Section we assume that the principal can choose the scale $\kappa_t(y_{t-1}) \leq 1$ of the project in each period $t = 1, 2$. The scale can depend on the reported cash flow in the previous period and must be less than or equal to 1 in each period. The project exhibits constant return to scale, i.e., time $t = 1, 2$ cash flow is equal to $\kappa_t \tilde{Y}_t(\sigma_t)$ where

$$\tilde{Y}_t(\sigma_t) = \begin{cases} 1, & \text{with probability } p^0 + \sigma_t p^1 \\ 0, & \text{with probability } 1 - p^0 - \sigma_t(p^1 + p^2) \\ -L, & \text{with probability } \sigma_t p^2 \end{cases} . \quad (56)$$

There is no cost to change the scale of the project. As a result, the liquidation at the end of period 1 becomes redundant and is replaced by the scale reduction. In this setting, we derive the optimal contract that maximizes the principals payoff and implements the safe project in both periods.

Denote $b_2(a_2; \kappa_2(y_1))$ the beginning of period two continuation payoff to the principal

when the agent's continuation payoff is equal to a_2 , given the scale of the project $\kappa_2(y_1)$ for the second period. Equation (50) implies that

$$b_2(a_2; \kappa_2(y_1)) = \begin{cases} \kappa_2(y_1)p^0 + A - a_2, & a_2 \geq \kappa_2(y_1)\bar{a} \\ -\infty, & 0 \leq a_2 < \kappa_2(y_1)\bar{a} \end{cases}, \quad y_1 \in \{0, 1\}. \quad (57)$$

The contracting problem for the beginning of period one takes the following form

$$\begin{aligned} b^* = & \underset{a_1^d(\cdot) \geq 0, 0 \leq \kappa_1, \kappa_2(y_1) \leq 1}{\operatorname{argmax}} \quad \kappa_1 p^0 + p^0 b_2(a_1^d(1); \kappa_2(1)) + (1 - p^0) b_2(a_1^d(0); \kappa_2(0)), \\ & \text{s.t. (IC) : } a_1^d(1) \geq a_1^d(0) + \kappa_1 \lambda, \\ & \text{(LRT) : } (p^1 + p^2) a_1^d(0) \geq p^1 a_1^d(1), \\ & \text{(SC) : } a_1^d(y_1) \geq \kappa_2(y_1) \bar{a}, \quad y_1 \in \{0, 1\}. \end{aligned}$$

The objective function is the expected payoff for the principal. There is no promise keeping condition, because the principal is optimally choosing the agent's expected payoff. Conditions (IC) and (LRT) are the same as before. They imply that in order to implement the safe project in the first period, the agents continuations payoffs must be such that

$$a_1^d(0) \geq \frac{p^1}{p^2} \lambda \kappa_1, \quad (58)$$

$$a_1^d(1) \geq \left(1 + \frac{p^1}{p^2}\right) \lambda \kappa_1. \quad (59)$$

After adjusting for the scale of the project, the above inequalities are the same restrictions on $a_1^d(0)$ and $a_1^d(1)$ as in the one-period model.

Scaling condition (SC) is necessary to induce the agent to implement the safe project in the second period. Thus, the second period scale imposes the second set of restrictions on continuation payoffs $a_1^d(0)$ and $a_1^d(1)$:

$$a_1^d(0) \geq \left(\frac{p^1}{p^2} + p^0\right) \lambda \kappa_2(0), \quad (60)$$

$$a_1^d(1) \geq \left(\frac{p^1}{p^2} + p^0\right) \lambda \kappa_2(1). \quad (61)$$

It is always optimal to set $\kappa_2(1) = 1$ to reward the agent for reporting cash flow 1 in the first period. If it is optimal to keep the scale of the project κ_1 and $\kappa_2(0)$ below 1, then inequalities (58)-(61) must bind. When the inequalities bind, we can solve for the optimal scale of the

project in both periods. The solution is summarized by Proposition 5.

PROPOSITION 5: *When the following inequality holds*

$$\lambda > \frac{2p^2p^0(1-p^0)}{p^1+p^2p^0(2-p^0)}, \quad (62)$$

it is optimal to start in the first period with the following size of the project

$$\kappa_1^* = \frac{p^1+p^2p^0}{p^1+p^2} < 1, \quad (63)$$

and then scale it either up to $\kappa_2^(y_1=1) = 1$ if the first period cash flow is equal to 1, or down to*

$$\kappa_2^*(y_1=0) = \frac{p^1}{p^1+p^2}, \quad (64)$$

if the first period cash flow is equal to 0. Under this contract the expected payoff to the principal is equal to

$$b^* = \kappa_1^*p^0 - (1 - \kappa_2^*(0)) (1 - p^0) p^0 + p^0 + A - \kappa_1^*\bar{a}. \quad (65)$$

Proof is provided in Appendix.

The inequality (62) is obtained by comparing b^* to the highest possible expected payoff to the principal when the safe project is implemented in both periods with maximum scale. This condition is satisfied when either the private diversion parameter, λ , is high or the “disaster” probability, p^2 , is low. Since p^2 can be (made) very small, it is clear that there always exists a set of parameters for which condition (62) holds. It is optimal to vary the scale of the project when the agent’s compensation is expensive to the principal, which is true when either λ is high or p^2 is low.

The agent’s compensation is proportional to the size of the project. By starting with the scale which is less than one, the principal is not only able to motivate the agent to implement the safe project in both periods, but he also saves by committing to less compensation to the agent. He rewards the agent for reporting cash flow 1 by increasing the scale of the project to its maximum level $\kappa_2^*(1) = 1$. If the agent reports cash flow 0, the scale of the project is reduced. Thus the variable project scale is a useful contracting tool which allows the principal both to minimize his risk exposure and reduce the agent’s compensation when the agency problem is severe and the agent’s compensation level is high.

5 Conclusion

This paper studies optimal security design in a setting in which an agent with limited liability privately chooses the riskiness of the project and can privately divert cash flows for consumption. Relative to the low risk project, the high risk project increases the probability of a high cash flow realization, but it also results in high losses in a bad state of nature, named “disaster”. We find that the optimal contract that induces the low-risk taking and truthful reporting may require a very high level of compensation for the agent if the contract terms are contingent only on the reported cash flows. The expected level of the agent’s compensation can be much lower if the agent’s payoff can be conditioned directly on the “disaster” state. It can be implemented by giving the agent out-of-money put options on the companies that are likely to be ruined in the “disaster” state. If it is impossible to write a contract conditional on the “disaster” state, then it may be optimal to gradually increase the scale of the project.

A few comments are in order about our definition of risk-shifting. It is fundamentally different along two dimensions from the definition used in the existing literature. First, the existing literature considers manipulation of the shape of the distribution but not its support. Second, the existing literature considers distributions of payoffs with nonnegative support only. Effectively it means that either shareholders are protected by the limited liability or managers invest only in the positive NPV projects. We have a different story in mind: if an investment bank offers a money-market fund and the top management takes on a risky gamble and loses, money-market investors can get less than \$1 they initially invested. As such, we allow agent/manager to take on negative NPV projects from the principal’s point of view. Due to limited liability, agent/manager in our model would prefer such a high-risk project since it offers a higher probability of the best payoff while the investors are on the hook for losses in the “disaster” state. Therefore in our setting agent/manager can manipulate both the probabilities of outcomes and outcomes which can be negative.

While being quite simple and intuitive, this version of the model provides very useful insights. First, it serves alternative theoretical evidence why the executive compensation has increased dramatically over the last decade. Second, we show that bailout of the agent encourages risk taking since the contracting costs decrease with expected losses.

A Proofs

A.1 Proof of Proposition 4

We start by calculating the end of period one principal's continuation payoff, $b_1^d(a_1^d; 0, 0)$. If at the end of period one the principal needs to implement the continuation payoff to the agent which less than the cutoff $0.5\bar{a}$, she has to liquidate the project with positive probability. Since liquidation is inefficient, the optimal continuation probability is going to be the largest probability that still yields the promised continuation payoff to the agent. The optimization problem for the end of period one principal's continuation payoff, $b_1^d(a_1^d; 0, 0)$, takes the following form in the possible liquidation region

$$b_1^d(a_1^d; 0, 0) = \max_{\pi, a_2} \pi (0.5p^0 + A - a_2) + (1 - \pi)A, \quad (\text{A1})$$

$$\text{s.t. } a_1^d = \pi a_2 + (1 - \pi) \cdot 0, \quad (\text{A2})$$

$$1 \geq \pi \geq 0, \quad a_2 \geq 0.5\bar{a}. \quad (\text{A3})$$

Substituting “promise keeping” condition (A2) into the objective function (A1) yields

$$b_1^d(a_1^d; 0, 0) = \max_{\pi} \pi 0.5p^0 + A - a_1^d, \quad (\text{A4})$$

$$\text{s.t. } \min \left\{ 1, \frac{2a_1^d}{\bar{a}} \right\} \geq \pi \geq 0. \quad (\text{A5})$$

Since $b_1^d(a_1^d; 0, 0)$ is nonnegative and linearly increasing in π , the optimal continuation probability, $\pi^*(a_1^d)$, is given by

$$\pi^*(a_1^d) = \min \left\{ 1, \frac{2a_1^d}{\bar{a}} \right\}. \quad (\text{A6})$$

The expression for the optimal end of period one principal's continuation payoff, $b_1^d(a_1^d; 0, 0)$, follows immediately

$$b_1^d(a_1^d; 0, 0) = \begin{cases} 0.5p^0 + A - a_1^d, & a_1^d \geq 0.5\bar{a} \\ \frac{a_1^d}{\bar{a}} (p^0 - \bar{a}) + A, & 0 \leq a_1^d < 0.5\bar{a} \end{cases}. \quad (\text{A7})$$

Next, we consider the beginning of the first period.

The optimization problem for the beginning of period one principal's continuation payoff

can be rewritten as

$$b_1(a_1; 0, 0) = \max_{a_1^d(\cdot) \geq 0} 0.5p^0 + p^0 b_1^d(a_1^d(1); 0, 0) + (1 - p^0) b_1^d(a_1^d(0); 0, 0), \quad (\text{A8})$$

$$\text{s.t. (PK) : } a_1 = p^0 a_1^d(1) + (1 - p^0) a_1^d(0), \quad (\text{A9})$$

$$\text{(IC) : } a_1^d(1) \geq a_1^d(0) + 0.5\lambda, \quad (\text{A10})$$

$$\text{(LRT) : } (p^1 + p^2) a_1^d(0) \geq p^1 a_1^d(1), \quad (\text{A11})$$

The solution consists of three steps. First, consider the case when the project is never liquidated at the end of the first period, $a_1^d(0) \geq 0.5\bar{a}$.¹ In this case $b_1^d(a_1^d; 0, 0) = 0.5p^0 + A - a_1^d$. Substituting this expression into (A8) we obtain with the help of the “promise keeping” condition (A9) that

$$b_1(a_1; 0, 0) = 0.5p^0 + p^0 \left(0.5p^0 + A - a_1^d(1)\right) + (1 - p^0) \left(0.5p^0 + A - a_1^d(0)\right) = p^0 + A - a_1, \quad (\text{A12})$$

Next, we multiply the no risk-taking inequality (A10) by p^0 and substitute $p^0 a_1^d(1)$ from the PK condition (A9) in it

$$a_1 - (1 - p^0) a_1^d(0) \geq p^0 a_1^d(0) + 0.5p^0 \lambda \implies a_1 \geq a_1^d(0) + 0.5p^0 \lambda. \quad (\text{A13})$$

Since we consider that $a_1^d(0) \geq 0.5\bar{a}$, the above inequality implies the lower bound on a_1 , $a_1 \geq 0.5(\bar{a} + p^0 \lambda)$.

Second, we consider the case when project is always liquidated, $\max \{a_1^d(0), a_1^d(1)\} < 0.5\bar{a}$. In this case the PK condition (A9), no risk-taking inequality (A10), and IC constraint (A10) cannot be satisfied simultaneously.

The third and final part of the proof is provided by Lemma 1.

LEMMA 1: *The first period IC constraint (A10) binds if $a_1^d(0) < 0.5\bar{a}$ and $a_1^d(1) \geq 0.5\bar{a}$.*

Proof. The intuition behind the proof is as follows. The constraint can be relaxed by increasing the agent’s payoff in the good state ($y = 1$) by ε while at the same time decreasing his payoff in the bad state ($y = 0$) by the same ε . However, since the principal’s utility is concave function of agent’s payoff in this case, her marginal utility loss due to increasing the agent’s payoff in the good state is larger than utility gain due to decreasing his payoff in the

¹It immediately follows from the no misreporting condition (A10) that $a_1^d(1) \geq \bar{a}$.

bad state. $a_1^d(0) < 0.5\bar{a}$ guarantees that the probability of liquidation in the first period is positive. It follows from (A7) that in this case

$$\begin{aligned} b_1^d(a_1^d(1); 0, 0) &= 0.5p^0 + A - a_1^d(1), \\ b_1^d(a_1^d(0); 0, 0) &= \frac{2a_1^d(0)}{\bar{a}} (0.5p^0 + A - 0.5\bar{a}) + \left(1 - \frac{2a_1^d(0)}{\bar{a}}\right)A. \end{aligned}$$

Substituting these functions into (A8) and using “promise keeping” constraint (A9) in it yields after some algebra

$$b_1(a_1; 0, 0) = p^0 + A - a_1 - 0.5p^0(1 - p^0)\left(1 - \frac{2a_1^d(0)}{\bar{a}}\right). \quad (\text{A14})$$

Next, we need to find the upper and lower bounds on the range of values a_1 takes in this case. We substitute the PK constraint (A9) into (A10) to get

$$a_1^d(0) \leq a_1 - 0.5p^0\lambda.$$

Since it follows from (A14) that b_1 is increasing in $a_1^d(0)$, the principal must set it to the largest possible value so that the above constraint binds. Also since $a_1^d(0) < 0.5\bar{a}$ we obtain the upper bound on a_1 in this case, $a_1 < 0.5(\bar{a} + p^0\lambda)$. Next, we substitute the equality $a_1^d(0) = a_1 - 0.5p^0\lambda$ into the LRT inequality (A10) to obtain that in this case the lower bound on a_1 is $0.5\bar{a}$. *QED*

A.2 Proof of Proposition 5

In this case the optimization problem for the optimal principal’s payoff, b^* takes the following form

$$b^* = \underset{a_1^d(\cdot) \geq 0, 0 \leq \kappa_2(y_1) \leq 1}{\operatorname{argmax}} \quad \kappa_1 p^0 + p^0 b_2(a_1^d(1); \kappa_2(1)) + (1 - p^0) b_2(a_1^d(0); \kappa_2(0)), \quad (\text{A15})$$

$$\text{s.t. (IC) : } a_1^d(1) \geq a_1^d(0) + \kappa_1\lambda, \quad (\text{A16})$$

$$\text{(LRT) : } (p^1 + p^2) a^d(0) \geq p^1 a^d(1), \quad (\text{A17})$$

$$\text{(SC) : } a_1^d(y_1) \geq \kappa_2(y_1) \bar{a}, y_1 \in \{0, 1\}. \quad (\text{A18})$$

Combining (IC) and (LRT) we obtain

$$0 \geq p^1 \underbrace{\left(a^d(1) - a^d(0) \right)}_{\geq \kappa_1 \lambda} - p^2 a^d(0) \geq p^1 \kappa_1 \lambda - p^2 a^d(0).$$

It implies that constraints (IC) and (LRT) can be jointly satisfied when

$$a_1^d(y_1) \geq \left(y_1 + \frac{p^1}{p^2} \right) \kappa_1 \lambda, \quad y_1 \in \{0, 1\}. \quad (\text{A19})$$

Therefore, it follows from (A19) and (SC) that agent's continuation payoff just prior the transfer to the agent, $a_1^d(y_1)$, is bounded by the following inequality

$$a_1^d(y_1) \geq \max \left\{ \left(y_1 + \frac{p^1}{p^2} \right) \kappa_1 \lambda, \kappa_2(y_1) \bar{a} \right\}, \quad y_1 \in \{0, 1\}. \quad (\text{A20})$$

Lets denote the optimal second period project scale conditional on the “good” first period outcome as $\kappa_2^* \equiv \kappa_2^*(1)$. We can now start from κ_1 low enough that $\left(1 + \frac{p^1}{p^2}\right) \kappa_1 \lambda < \kappa_2^* \bar{a}$. Since $b_1(a_1; \kappa_1)$ is increasing in κ_1 the principal would set the first period scale of the project so that $\left(1 + \frac{p^1}{p^2}\right) \kappa_1^* \lambda = \kappa_2^* \bar{a}$, where

$$\kappa_1^* = \kappa_2^* \frac{p^2}{p^1 + p^2} \frac{\bar{a}}{\lambda} = \frac{p^1 + p^2 p^0}{p^1 + p^2} \kappa_2^* < 1. \quad (\text{A21})$$

After we have pinned down the optimal first period scale, κ_1^* , we can find the optimal second period project scale conditional on the “bad” first period outcome, $\kappa_2^*(0)$. We follow the same reasoning and set $\frac{p^1}{p^2} \kappa_1^* \lambda = \kappa_2^*(0) \bar{a}$, from where it immediately follows that

$$\kappa_2^*(0) = \frac{p^1}{p^2} \kappa_1^* \frac{\lambda}{\bar{a}} = \frac{p^1}{p^1 + p^2 p^0} \kappa_1^* < \kappa_1^*. \quad (\text{A22})$$

Since we do not have any additional conditions which would help us to determine κ_2^* except the fact that $b_2(a_2; \kappa_2)$ increases in κ_2 , we can as well set κ_2^* to 1.

We can now solve for the optimal payoff to the principal, b^* . It is useful to note that as long as $a_1^d(1) \geq \bar{a}$ we never have b_2 to equal to $-\infty$ since this condition immediately implies from the (LRT) that $a_1^d(0) \geq \kappa_2^*(0) \bar{a}$. Now we can substitute $b_2(a_2; \kappa_2^*(y_1))$ into (A15) to

yield

$$\begin{aligned} b^* &= \kappa_1^* p^0 + p^0 (p^0 + A - \bar{a}) + (1 - p^0) (\kappa_2^* (0) p^0 + A - \kappa_2^* (0) \bar{a}) = \\ &= (1 + \kappa_1^*) p^0 - (1 - \kappa_2^* (0)) (1 - p^0) p^0 + A - \kappa_1^* \bar{a}. \end{aligned}$$

Next we are going to check under what condition on the primitives of the model the principal is better off with this strategy than liquidating the project in period 1 in some states. To do that we compare b^* to the lowest possible value of $b_1(a_1; 0, 0)$ under no liquidation, $b_1(\bar{a} + p^0 \lambda; 0, 0)$. $b_1(\bar{a} + p^0 \lambda; 0, 0)$ can be easily found from (55) by setting the payoff in the “good” state of the world to 1 instead of 0.5 to be equal to

$$b_1(\bar{a} + p^0 \lambda; 0, 0) = 2p^0 + A - (\bar{a} + p^0 \lambda). \quad (\text{A23})$$

It follows after simple algebra that $b_1(\kappa_1^* \bar{a}; \kappa_1^*) > b_1(\bar{a} + p^0 \lambda; 0, 0)$ as long as

$$\lambda > \frac{2p^2 p^0 (1 - p^0)}{p^1 + p^2 p^0 (2 - p^0)}. \quad (\text{A24})$$

QED

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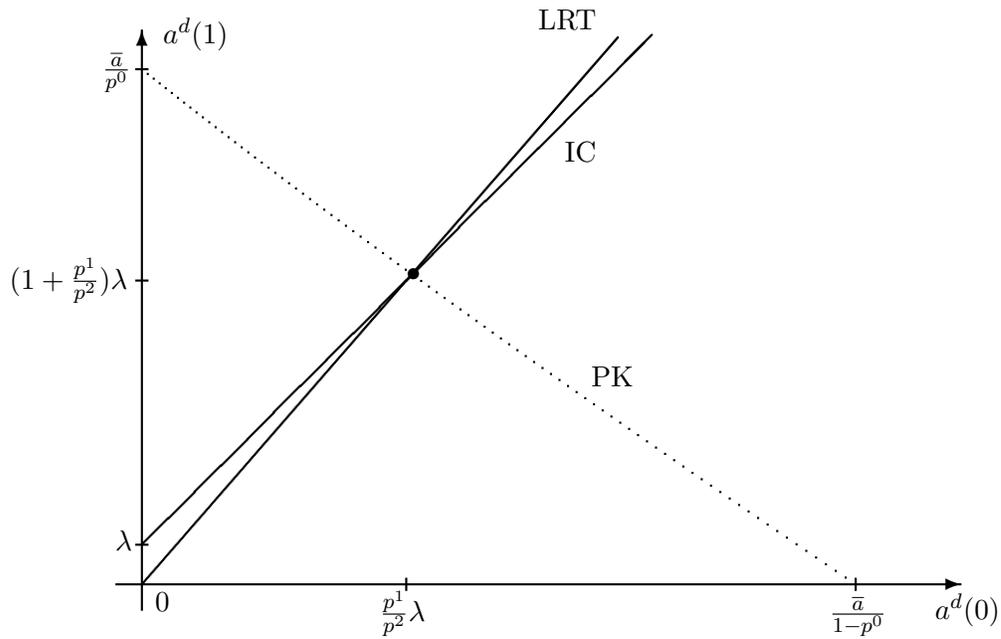


Figure 1. Illustration of the proof of Proposition 1. The plot is shown on the $(a^d(0), a^d(1))$ plane. The IC constraint is satisfied everywhere above the 45-degree straight line starting at λ . The low-risk taking constraint is satisfied below a straight line starting at zero. Both constraints are simultaneously satisfied everywhere between the two lines to the right of their intersection shown by a dot. The cheapest incentive compatible contract is shown by the dot and is given by $(\frac{p^1}{p^2}\lambda, (1 + \frac{p^1}{p^2})\lambda)$. The dotted line represents “promise keeping” constraint where it goes through the cheapest contract.

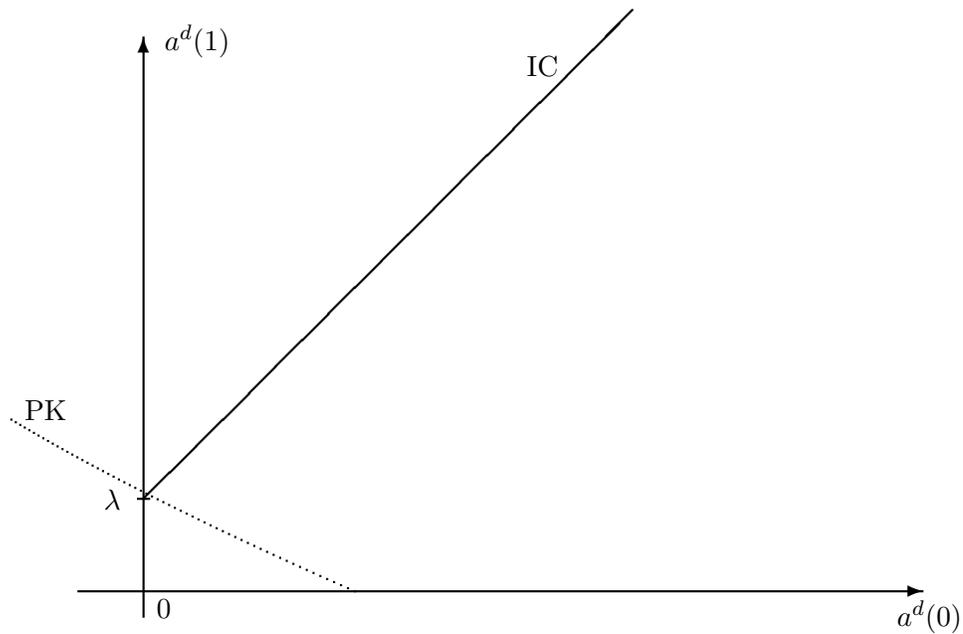


Figure 2. Illustration of the intuition behind the optimal contract with risk-taking. The plot is shown on the $(a^d(0), a^d(1))$ plane. The IC constraint is satisfied everywhere above the 45-degree straight line starting at λ . The dotted line represents “promise keeping” constraint where it goes through the cheapest contract. The cheapest incentive compatible contract is achieved when IC and PK go through $(0, \lambda)$ point. $a^d(-L)$ is equal to zero.

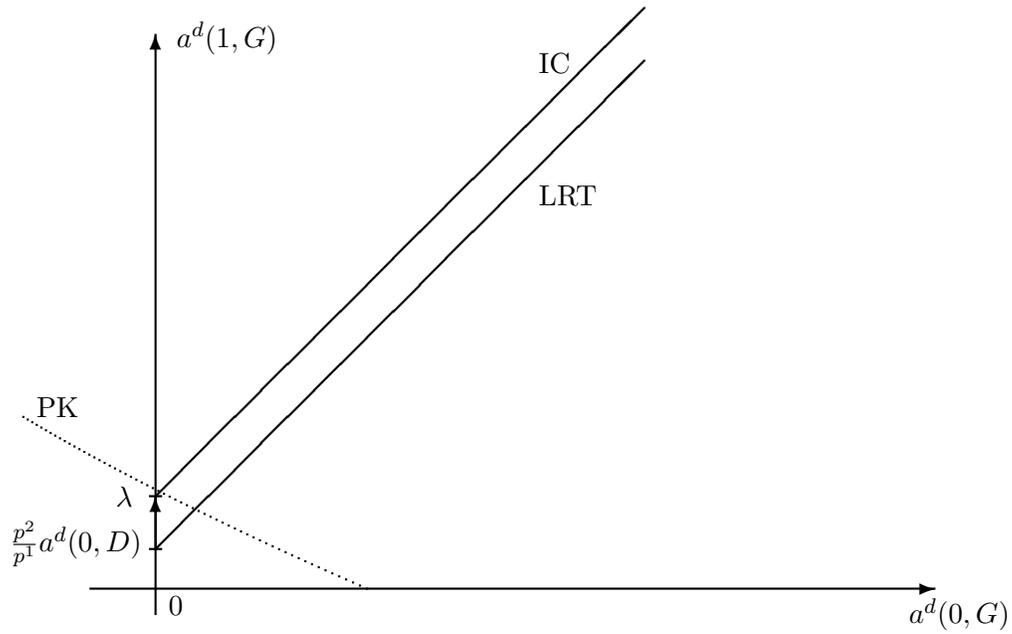


Figure 3. Illustration of the solution to the alternative model. The plot is shown on the $(a^d(0, G), a^d(1, G))$ plane. The IC constraint is satisfied everywhere above the 45-degree straight line starting at λ . The low-risk taking constraint is satisfied below a 45-degree straight line starting at $\frac{p^2}{p^1} a^d(0, D) \geq \lambda$. Both constraints can be simultaneously satisfied only if LRT constraint is shifted above the IC constraint. The cheapest incentive compatible contract is achieved when IC and LRT coincide and the PK constraint goes through $(0, \lambda)$ point.