

Collusion in Markets with Syndication*

John William Hatfield
McCombs School of Business
University of Texas at Austin

Scott Duke Kominers
Society of Fellows
Harvard University

Richard Lowery
Tepper School of Business
Carnegie Mellon University

November 15, 2016

Abstract

Markets for IPOs and debt issuances are syndicated, in the sense that a bidder who wins a contract may invite losing bidders to join a syndicate that together fulfills the contract. We show that in markets with syndication, standard intuitions from industrial organization can be reversed: Collusion may become easier as market concentration falls, and market entry may in fact facilitate collusion. In particular, price collusion can be sustained by a strategy in which firms refuse to join the syndicate of any firm that deviates from the collusive price. Our results thus can rationalize the apparently contradictory empirical facts that the market for IPO underwriting exhibits seemingly collusive pricing despite its low level of market concentration.

JEL Classification: D43, L13, G24, L4

Keywords: Collusion, Antitrust, IPO underwriting, Syndication, Repeated games

*The authors thank Lauren Cohen, Leslie Marx, Ehud I. Ronn, Alex White, Lucy White and seminar participants at University of Western Ontario for helpful comments. Kominers gratefully acknowledges the support of the National Science Foundation (grants CCF-1216095, SciSIP-1535813, and SES-1459912), the Harvard Milton Fund, and the Ng Fund of the Harvard Center of Mathematical Sciences and Applications. Lowery gratefully acknowledges the hospitality of the Tepper School of Business at Carnegie Mellon University, which hosted him during parts of this research. Any comments or suggestions are welcome and may be emailed to richard.lowery@mcombs.utexas.edu.

1 Introduction

The spread investment banks collect for initial public offerings (IPOs) strongly suggests collusive behavior, with investment banks apparently coordinating on spreads of 7% for moderately-sized IPOs (Chen and Ritter, 2000). At the same time, the number of investment banks running moderately-sized IPOs is quite large, and there appears to be a nontrivial amount of entry and exit in the market (Hansen, 2001); this presents a puzzle, as standard industrial organization intuitions would therefore suggest that pricing should be competitive.

However, the market for running IPOs is syndicated; once the bid to run an IPO is accepted, the winning investment bank must then organize a syndicate to complete the IPO. In this paper, we offer syndication as an explanation for how collusion may be maintained in the presence of small firms. We show that the presence of syndication can reverse the standard intuition regarding the effect of market concentration: as a market with syndication becomes less concentrated, the scope for price collusion may *increase*. Because syndication follows the pricing stage, investment banks can punish a bank that undercuts the collusive price by refusing to participate in that bank’s syndicate; this type of punishment becomes more powerful in unconcentrated markets, as, in such markets, the involvement of other firms in the syndicate is more important for ensuring a successful IPO.

Figure 1 shows the behavior of pricing spreads over the last forty years. In the late 1970s, spreads for IPOs tended to be quite high, exceeding 7%. In the early 1980s, spreads for IPOs in excess of \$20 million in proceeds began to fall below 7%, but over the course of the late 1980s and particularly through the 1990s spreads for IPOs with proceeds between \$20 and \$100 million became increasingly clustered at 7%. This clustering continues in the 2000s; notably, as the IPO market largely ceased to operate following the 2007-2008 financial crisis, very few IPOs took place but those that did still paid the 7% spread. However, as first documented by Hansen (2001), the market for IPOs since the 1990s appears “competitive,” in the sense that many firms were active in the market; indeed, the largest four firms together

Figure 1: This animation shows the relationship between IPO proceeds and spreads over time for IPOs with proceeds less than \$100 million. Proceeds (in millions of dollars) are plotted on the x -axis and spreads (i.e., the percent of the proceeds that go to the underwriter) are plotted on the y -axis. The movie shows a rolling twelve-month window of data. Over time, spreads become increasingly concentrated at exactly 7%. Click anywhere on the figure to start the animation.

only make up between 40% and 50% of the market in this period, as depicted in Figure 2.

We model a market with syndication as a repeated extensive form game: In each period, firms compete on price for the opportunity to complete a single project and, upon being selected, the chosen firm may invite additional firms to join in the production process. Recruiting additional firms is valuable because production costs are convex in the proportion of a project completed by a single firm. Each invited firm then decides whether to join the syndicate. The project is then completed, payoffs are realized, and play proceeds to the next period.

We show that, in markets with syndication, less concentrated markets may have prices that are farther from the marginal cost of production. In particular, the highest sustainable price in any equilibrium is a U-shaped function of market concentration: When markets are very concentrated, collusion can be sustained by punishing a firm that undercuts on price in

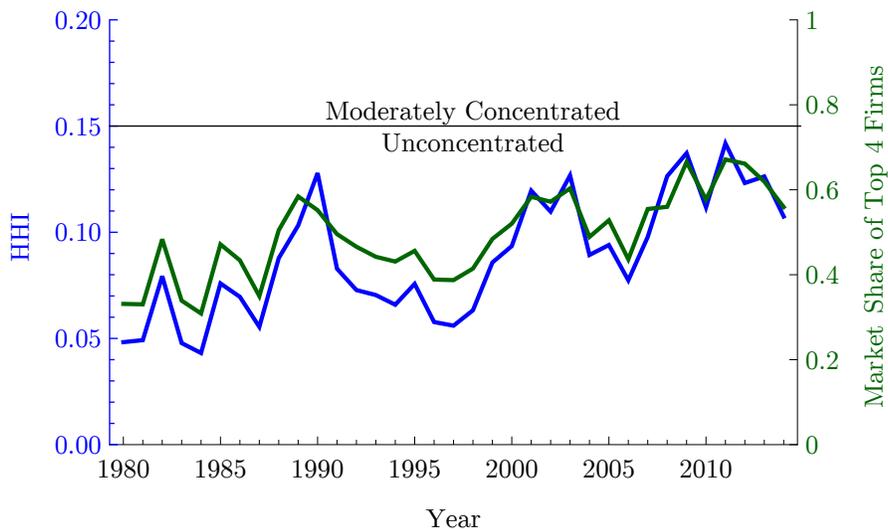


Figure 2: The Herfindahl-Hirschman index (HHI) and the market share of the largest four firms in the market for IPOs. The U.S. Department of Justice defines an industry with an HHI of less than .15 to be an “unconcentrated market” (Department of Justice and Federal Trade Commission, 2010, p. 19).

future periods, as in many standard industrial organization contexts.¹ However, when many small firms are present, collusion can be sustained by in-period punishments: after a firm undercuts on price, other firms can punish the undercutting firm in the same period by the refusing to join its syndicate. Of course, such behavior by other firms must itself be incentive compatible. Thus, after a period in which firms reject syndication offers from the firm that undercut on price, the firms who rejected the offers of syndication must be rewarded for their behavior.

In repeated normal form games, punishments can be enforced using the the *simple penal codes* of Abreu (1986), in which any firm which deviates from its equilibrium strategy is punished as harshly as possible. However, as noted by Mailath et al. (2016), in the analysis of repeated extensive form games it is necessary to consider more nuanced punishments after

¹See, for instance, Tirole (1988).

deviations.² In particular, in our setting, it is key that firms punish a price undercutter *in-period* by refusing the undercutter’s offers of syndication; to do this, we must construct strategies that simultaneously punish a firm that undercuts on price and reward firms which refuse to join a price undercutter’s syndicate.

Our baseline model considers the case of symmetric firms, but our results extend to markets with heterogenous firms. As in the case with symmetric firms, collusion can be maintained even when the market is very fragmented; indeed, heterogeneity itself can increase the ability of firms to collude. Moreover, the entry of small firms can enhance the scope for collusion in markets with syndication, again counter to the standard results in the theory of industrial organization.

Whether spreads on IPOs are set in a competitive or collusive manner has been debated in the finance literature since [Chen and Ritter \(2000\)](#) first documented the clustering of IPO spreads at 7%. [Abrahamson et al. \(2011\)](#) documented that the spreads for IPOs are significantly higher in the United States than in Europe, and cited this as evidence that pricing in this market is collusive. [Kang and Lowery \(2014\)](#) presented and estimated a formal model of why collusion would lead to clustering on spreads, combining insights on collusive behavior from [Rotemberg and Saloner \(1986\)](#) and [Athey et al. \(2004\)](#). By contrast, [Hansen \(2001\)](#) claims that the clustering of IPO spreads is likely to be the result of efficient contracting, documenting the apparent relative ease of entry and lack of concentration in the market.³ Our work helps reconcile the apparently conflicting evidence: we show that collusion in IPO markets is possible despite—and in fact facilitated by—a lack of market concentration.

There also is a related debate over whether the pricing of the IPO securities themselves is collusive. IPO shares generally gain about 15% on the first day of trading, suggesting that issuers are “leaving money on the table” ([Loughran and Ritter, 2004](#)). Some authors argue that underpricing of initial offerings can be desired by issuers and thus can occur even if underwriters

²[Nocke and White \(2007\)](#) were the first to use the theory of repeated extensive form games to study collusion, showing that vertical mergers can facilitate collusion under certain circumstances.

³[Torstila \(2003\)](#) documents the clustering of spreads in countries other than the United States at lower levels, arguing that this provides evidence that clustering does not imply collusive behavior.

compete aggressively (Rock, 1986; Allen and Faulhaber, 1989; Benveniste and Spindt, 1989; Chemmanur, 1993; Lowry and Shu, 2002). Others view underpricing as a means for underwriters to extract rents from issuers—likely a feature of an uncompetitive market (Biais et al., 2002; Cliff and Denis, 2004; Loughran and Ritter, 2004; Kang and Lowery, 2014). While our work does not address the issue of underpricing directly, it does show that collusion can be maintained in the market for IPOs, even though the market is highly fragmented.

The remainder of the paper is organized as follows: Section 2 introduces our model of a market with syndicated production. Section 3 characterizes the highest price sustainable via collusion in such markets. Section 4 considers how the the highest price sustainable via collusion depends on market conditions. Section 5 extends the model to allow for contracting over production shares. Section 6 explores the effect of firm heterogeneity and market entry on the highest sustainable price. Section 7 concludes.

2 Model

We introduce a model of price competition in markets with syndication. There is a finite set of long-lived identical *firms* F and an infinite sequence of short-lived identical *buyers* $\{b_t\}_{t \in \mathbb{N}}$; we let $\varphi \equiv \frac{1}{|F|}$ be the *market concentration*. Time is discrete and infinite; firms discount the future at the rate $\delta \in (0, 1)$.

Each firm f is endowed with a production technology with a *cost function* $e(s, m)$, where s is the fraction of production done by firm f and m is the mass of the productive capacity controlled by the firm. We assume that the *total productive capacity* is given by k and is evenly divided among the firms. We assume that the cost function is homogenous of degree one, strictly increasing and strictly convex in the production done by the firm, strictly decreasing in the productive capacity of the firm, and that $e(0, m) = 0$ for all m . Note that these assumptions imply that the cost function is also strictly convex in the productive capacity of the firm.

In each period t , the firms and the buyer b_t play the following extensive-form stage game:

1. Each firm $f \in F$ simultaneously makes a *price offer* $p_t^f \in [0, \infty)$. All offers to the buyer are immediately and publicly observed.
2. The buyer accepts at most one offer; the buyer's action is immediately and publicly observed. If no offer is accepted, the stage game ends.
3. If the offer from firm f is accepted, firm f becomes the (*syndicate*) *leader*; firm f then simultaneously offers each other firm $g \in F \setminus \{f\}$ a *fee* w_t^g . These offers are immediately and publicly observed.⁴
4. Each firm $g \in F \setminus \{f\}$ either accepts or rejects the fee w_t^g from f . We call the set of firms that accept f 's offer, along with the firm f , the *syndicate* G_t . At the end of the period, all agents observe the syndicate⁵

The buyer b_t has a fixed *value* of v for the finished product. Thus, the payoff to the buyer b_t is $v - p_t^f$ if he accepts the price offer from firm f and 0 if he does not accept any offer. If the buyer b_t does not accept any offer, then each firm $f \in F$ obtains a payoff of 0. If the buyer b_t accepts the offer of firm f , then production is performed efficiently *ex post* by the members of f 's syndicate, and so each member of the syndicate performs an equal share of the task. Thus, the stage game payoffs for the firms after a successful offer to the buyer from firm f are as follows:

1. The payoff for f is $p_t^f - e\left(\frac{1}{|G_t|}, \varphi k\right) - \sum_{g \in G_t} w_t^g$, i.e., the price paid by the buyer less the cost of f 's production less the cost of the fees paid to other firms.
2. The payoff for $g \in G_t \setminus \{f\}$ is $w_t^g - e\left(\frac{1}{|G_t|}, \varphi k\right)$, i.e., the fee paid to g less the cost of g 's production.
3. The payoff for $h \in F \setminus G_t$ is 0.

⁴In Section 5, we consider the case where an offer specifies not only a fee but also the proportion of production done by the firm.

⁵Consequently, all agents know which syndication offers were accepted.

3 Optimal Collusion

We now characterize the highest price sustainable via collusion in markets with syndication. When the market is very concentrated, i.e., there are a small number of firms, any price (less than or equal to v) can be sustained by “grim trigger” strategies in which deviations from the collusive price are punished in subsequent periods by play in which every firm obtains 0 profits. This type of equilibrium is standard in the analysis of markets with Bertrand competition; in such markets, however, once there are enough firms in the market, no price above the cost of production can be sustained.

In markets with syndication, as in Bertrand competition markets, grim trigger strategies lose their bite as the number of firms in the market grows. However, unlike in Bertrand competition, markets with syndication admit a second method of punishing collusion: if a firm bids lower than the price mandated by the collusive equilibrium, i.e., if a firm becomes a *price deviator*, other firms can punish that firm “in period” by refusing any offers of syndication, thus raising the cost of production for that firm (as the firm must now complete the project on its own instead of engaging in efficient syndication). However, in order to incentivize firms to not join the price deviator’s syndicate, we need to promise them rewards in future periods. For this reason, reverting to “perfect competition” in periods after a period in which a firm bids lower than the price mandated by the collusive equilibrium is not the best continuation plan to sustain collusion. Instead, we choose a continuation plan that simultaneously rewards firms for refusing offers of syndication while punishing the price deviator. In particular, “the reward should fit the temptation” (Mailath et al., 2016)—the better the syndication offer a price deviator makes, the higher the continuation payoff needed to induce rejection of that offer of syndication. It is also important to punish a firm if it joins the syndicate formed by the deviating firm: to do this, we *do* revert to perfect competition, as it punishes as harshly as possible *both* the initial deviator and any firm which joins the syndicate.

Unlike grim trigger strategies, syndicate punishment strategies become *more* powerful as

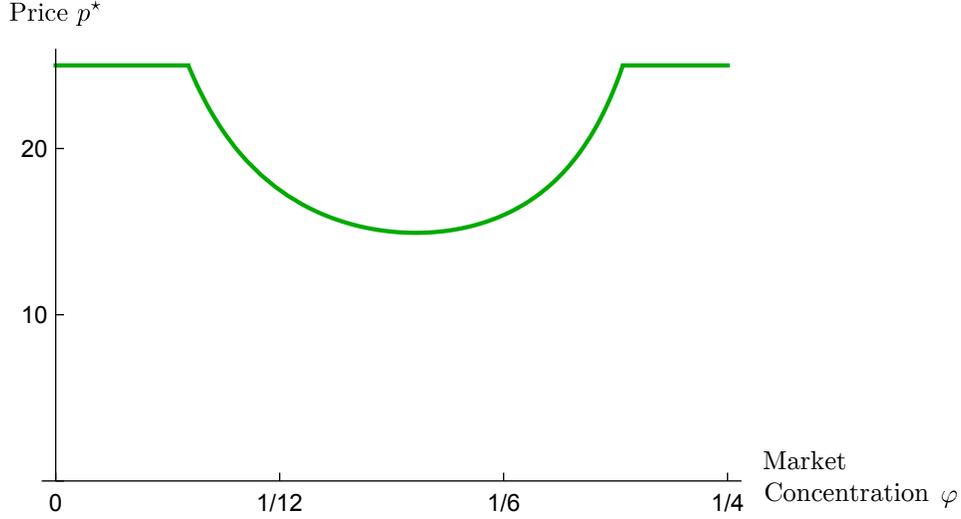


Figure 3: The highest sustainable price p^* as a function of market concentration φ . Here, $e(s, m) = \frac{s^2}{m}$, $k = 1$, $\delta = \frac{3}{4}$, and the maximum price that the buyer is willing to pay is $v = 25$. For sufficiently concentrated industries, the monopoly price can be sustained through grim trigger strategies. For intermediate industry concentration levels the highest sustainable price is lower, but as market concentration goes to 0 the highest sustainable price reaches the buyer's value v . The marginal cost of production (when the syndicate includes all firms) is 1 for all market concentrations φ .

the market becomes less concentrated, as the cost of completing the project alone becomes increasingly expensive. Consequently, the preceding observations imply that the highest sustainable price is *not* monotone in market concentration; indeed, as market concentration goes to zero, syndicate punishments enable collusion on the monopolistic price.

Theorem 1. *For $\delta \geq \frac{1}{2}$, the highest price sustainable in a subgame-perfect Nash equilibrium, p^* , is given by⁶*

$$p^* = \begin{cases} v & \varphi \in [1 - \delta, 1] \\ \min\left\{\frac{(1-\delta)e(1,k\varphi) - \varphi e(1,k)}{1-\delta-\varphi}, v\right\} & \varphi \in (0, 1 - \delta). \end{cases}$$

Moreover, p^* is quasiconvex in φ and $\lim_{\varphi \rightarrow 0} p^* = v$.

Figure 3 plots the highest sustainable price p^* as a function of φ .

⁶We require that $\delta \geq \frac{1}{2}$ in order to ensure that the future is sufficiently valuable; when this condition is satisfied, there does not exist a set of offers of syndication which simultaneously make the price deviator better off (when those offers are accepted) and are optimal for the other firms to accept. As we discuss in Appendix A, our result also obtains when the discount factor δ is slightly less than $\frac{1}{2}$.

In the rest of this section, we show that the p^* defined in Theorem 1 can be sustained as a subgame-perfect Nash equilibrium of the game defined in Section 2 and, moreover, p^* is the highest price that can be sustained. Finally, for ease of exposition, we set $k = 1$ throughout the rest of this section.

3.1 Bertrand Reversion Nash Equilibrium

We first describe the *Bertrand reversion (subgame-perfect) Nash equilibrium* of the stage game, i.e., the equilibrium in which all firms make zero profits and the buyer obtains the good at the lowest possible cost of production. In this equilibrium, each firm f offers a price $p^f = e(1, 1)$, which is exactly the cost of producing the good under full participation in the syndicate. The buyer then chooses each firm as syndicate leader with equal probability. The syndicate leader then offers each other firm g a fee $w_i^g = e(\varphi, \varphi)$ equal to g 's cost of production (assuming all syndication offers are accepted). Each firm $g \in F \setminus \{f\}$ accepts this offer (and any higher offer). Under this behavior, each firm in the syndicate other than f then incurs production costs of $e(\varphi, \varphi)$ and thus breaks even. Moreover, the syndicate leader breaks even as he obtains $e(1, 1) = |F|e(\varphi, \varphi)$ from the buyer, he incurs production costs of $e(\varphi, \varphi)$, and he pays $(|F| - 1)e(\varphi, \varphi)$ in total to the syndicate.⁷ If any firm makes an offer other than $e(1, 1)$ to the buyer, the buyer chooses among all the lowest offers with equal probability; the behavior of firms with respect to accepting syndication offers does not depend on the set of offers made to the buyer. If the syndicate leader offers a fee other than $e(\varphi, \varphi)$ to any firm, then within period continuation play follows any profile of actions for the other firms $g \neq f$ that constitutes a Nash equilibrium of the within period continuation game.⁸ Note, however, that regardless of the equilibrium play after a fee other than $e(\varphi, \varphi)$ has been offered to some firm, the syndicate leader f 's profits are no greater than $p^f - e(\varphi, \varphi) - (|F| - 1)e(\varphi, \varphi) \leq e(1, 1) - |F|e(\varphi, \varphi) = 0$, as no offer greater than $e(1, 1)$

⁷Recall that $e(\cdot, \cdot)$ is homogenous of degree one.

⁸Note that there be multiple such Nash equilibria, as whether a syndication offer is profitable for an agent may depend on whether other agents accept their syndication offers.

will be accepted by the buyer, and no firm will accept a syndication offer of less than $e(\varphi, \varphi)$.

Our first result shows that the Bertrand reversion Nash equilibrium strategies just described in fact constitute a subgame-perfect Nash equilibrium of the stage game in which each firm obtains its lowest individually rational payoff.

Proposition 1. *There exists a subgame-perfect Nash equilibrium of the stage game, i.e., the Bertrand reversion Nash equilibrium, in which each firm obtains a payoff of 0, its lowest individually rational payoff.*

In the analysis of repeated normal form games, reverting to the stage game equilibrium described in Proposition 1 would be sufficient to punish any off-equilibrium behavior. That is, the Bertrand reversion Nash equilibrium can be used to implement the simple penal codes of [Abreu \(1986\)](#). However, as noted by [Mailath et al. \(2016\)](#), simple penal codes are insufficient to characterize the set of equilibrium payoffs in repeated extensive form games. Nevertheless, the Bertrand reversion equilibrium is a key component in constructing the equilibrium that supports the highest sustainable price.

3.2 Maintaining Collusion with Grim Trigger Strategies When the Market Is Concentrated

We first show that, when firms are patient, the monopoly price v is sustainable when the number of firms is sufficiently small. Moreover, under these conditions, collusion can be sustained via “grim trigger” strategies: after a deviation in either step of the stage game, play in all subsequent periods reverts to the Bertrand reversion Nash equilibrium described in Section 3.1.

Proposition 2. *If the discount factor is sufficiently high, i.e., $\delta \geq 1 - \varphi$, then there exists a subgame-perfect Nash equilibrium where every firm offers the monopoly price, i.e., $p_t^f = v$ for any $v \geq e(1, 1)$, for all $f \in F$ and for all t .*

To prove Proposition 2, we construct an equilibrium in which, in every period, each firm bids the monopoly price v ; the short-lived buyer then accepts one such offer (choosing each offer with equal probability). If the offer from firm f is accepted, f offers a fee $w_t^g = e(\varphi, \varphi)$ to each other firm $g \in F \setminus \{f\}$; each other firm g then accepts and joins the syndicate.

If a firm deviates on price in the first step, i.e., becomes a *price deviator*, every other firm accepts the offer of syndication if the price deviator offers $e(\varphi, \varphi)$ to each firm.⁹ However, in every subsequent period following such a deviation, play reverts to the Bertrand reversion Nash equilibrium described in Section 3.1. Finally, if any firm chooses to not accept an offer of syndication with fee $e(\varphi, \varphi)$, play also reverts to the Bertrand reversion Nash equilibrium. Thus, in each period, the syndicate leader has profits of

$$v - e(\varphi, \varphi) - (|F| - 1)e(\varphi, \varphi) = v - |F|\varphi e(1, 1) = v - e(1, 1)$$

and each other member of the syndicate has profits of

$$e(\varphi, \varphi) - e(\varphi, \varphi) = 0.$$

Given this proposed equilibrium structure, it is clear that no firm will reject an offer of syndication with fee $e(\varphi, \varphi)$. Thus, to ascertain whether this is equilibrium, we need only check whether a firm is willing to offer the monopoly price in the first step, or would rather offer an infinitesimally lower price and win with certainty. The expected discounted value of the current payoff and all future payoffs from following the equilibrium is

$$\sum_{t=0}^{\infty} \delta^t \varphi (v - e(1, 1)) = \frac{\varphi}{1 - \delta} (v - e(1, 1)),$$

which is greater than $v - e(1, 1)$ so long as $\varphi > 1 - \delta$; meanwhile, the expected discounted

⁹If the syndicate leader offers a fee other than $e(\varphi, \varphi)$ to any firm, then within period continuation play follows any profile of actions for the other firms $g \neq f$ that constitutes a Nash equilibrium of the within period continuation game.

value of all future payoffs from offering an infinitesimally lower price is

$$v - e(1, 1).$$

Proposition 2 is the analogue in our setting to the familiar result that collusion at any price can be maintained by grim trigger strategies in models of Bertrand competition when the industry is sufficiently concentrated. However, in the standard model of Bertrand competition, collusion cannot be maintained at any price when $\delta < 1 - \varphi$; in the next section, we show that this is *not* true in our setting.

3.3 Maintaining Collusion with Syndicate Punishments

In this section, we first provide an intuitive description of an equilibrium which supports the price p^* defined in Theorem 1. We then give a formal construction of the strategy profile, and show that the strategy profile constitutes a subgame-perfect Nash equilibrium. Finally, we show that no subgame-perfect Nash equilibrium can sustain a price higher than p^* .

The key idea is to construct strategies that exploit syndicate boycotting to enforce higher prices. Play begins in the *cooperation phase*, in which each firm offers the same price p^* and a firm, upon having its offer accepted, engages in efficient syndication. Play continues in the cooperation phase so long as no one deviates. However, some firm may deviate in the first step, i.e., offer a lower price to the buyer in order to guarantee that it wins the bid; we call such a firm a *price deviator*. Intuitively, other firms can punish a price deviator “within period” by refusing to join its syndicate, thus raising the cost of production for the price deviator. However, in order to incentivize firms to not join a price deviator’s syndicate, we need to promise them rewards in future periods. For this reason, Bertrand reversion in periods after a period with a price deviation is not necessarily the best continuation plan to sustain collusion. In fact, it is important to reward firms for not joining the syndicate (by promising rewards in future periods). Moreover, it is also important to punish a firm if it joins a price

deviator’s syndicate; to do this, we *do* use Bertrand reversion, as it punishes as harshly as possible *both* the initial deviator and any firm which joins the syndicate. Thus, whenever any firm deviates with respect to accepting or rejecting offers of syndication, play enters the *Bertrand reversion phase*, where firms play the Bertrand reversion Nash equilibrium each period.

After a period in which a firm f is a price deviator, but no firm joins its syndicate, we enter a *collusive punishment phase* which both punishes the price deviator and rewards those who refused to join its syndicate. But behavior during such a collusive punishment phase must itself be subgame-perfect. The lower the price is in the collusive punishment phase, the lower the total reward that other firms receive for not joining the syndicate. If the price is too high in the collusive punishment phase, the collusive punishment phase will itself not be subgame-perfect, as the price deviator or another firm may wish to price deviate in this phase as well.

Moreover, the continuation payoff to a firm other than the price deviator during a collusive punishment phase may depend on the offer that was made to that firm by the price deviator. In particular, “the reward should fit the temptation” (Mailath et al., 2016)—the larger the fee offered to the firm by the price deviator, the greater the continuation payoff offered to that firm to induce it to reject the offer of syndication.

Thus, to characterize the highest sustainable price, we specify a subgame-perfect Nash equilibrium that exploits the possibility of in-period punishments. This equilibrium is composed of three types of phases: In the *cooperation phase*, each firm offers p to the short-lived buyer, who then chooses each firm with equal probability; afterwards, an efficient syndicate is formed. If any firm f price-deviates, but no other firm joins its syndicate, then we enter a *collusive punishment phase with continuation values ψ* , in which the continuation values are determined by the syndication offers. In a collusive punishment phase with continuation values ψ , each firm offers a specific price q to the short-lived buyer, who then chooses each firm with equal probability; we call q the *collusive punishment price*. The

winning bidder then efficiently syndicates production; in so doing, it offers each firm g a fee equal to its assigned continuation value, ψ^g , plus g 's production cost, $e(\varphi, \varphi)$. Finally, if any firm deviates from equilibrium play with respect to accepting or rejecting offers of syndication, play enters the *Bertrand reversion phase*, in which firms play the Bertrand reversion Nash equilibrium each period.

By making future play conditional on offers of syndication, firms are incentivized to punish price deviators in period, by refusing to join their syndicates. This then reduces the incentive for agents to deviate on price, since each firm is aware that after such a deviation, the firm will have to engage in lone production. Since lone production becomes costlier as the market becomes more fragmented, reducing market concentration makes it easier to sustain collusion at a given price.

We now give a formal construction of the strategy profile that supports p^* . We first construct an equilibrium that supports this price for $p^* < v$. We then extend that construction to cover the low φ cases in which $p^* = v$. (For the high φ range where $p^* = v$, the result extends immediately.) The equilibrium is constructed as follows:

- There are three phases of equilibrium play:
 1. In the *cooperation phase*,
 - every firm submits the same bid $p = p^*$,
 - the short-lived buyer then accepts one offer at $p = p^*$, choosing each with equal probability,
 - every firm, if it becomes the syndicate leader, offers every other firm $e(\varphi, \varphi)$ to join the syndicate, and
 - every other firm accepts this offer.
 2. In the *collusive punishment phase with continuation values ψ* ,
 - every firm submits the same bid¹⁰ $q = \min\{e(1, \varphi), v\}$,

¹⁰Note that when $1 - \delta - \varphi > 0$, i.e., a high price can not be maintained purely through Bertrand reversion, so we have that $q < p^*$ so as long as $p^* < v$.

- the short-lived buyer then accepts one such offer of q , and chooses each offer with equal probability,
- every firm $g \in F$, if it becomes the syndicate leader, offers $e(\varphi, \varphi) + \psi^h$ to every firm $h \in F \setminus \{g\}$,
- and every other firm accepts the offer by the syndicate leader g to join the syndicate.

3. In the *Bertrand reversion phase*, agents play the Bertrand reversion Nash equilibrium.

- Under equilibrium play, play continues in the same phase. If, in the cooperation phase or a collusive punishment phase, any firm f deviates in the first step or deviates with respect to the prescribed set of offers, then we calculate the sum across the other firms of the (positive) differences between the syndication offer and the costs of doing φ of the project for each firm as $\sum_{g \in F \setminus \{f\}} (w^g - e(\varphi, \varphi))^+$.¹¹ We categorize the set of offers made by a deviating firm f into three categories: *uniformly low offers*, *insufficient offers*, and *sufficient offers*.

Uniformly Low Offers: $\sum_{g \in F \setminus \{f\}} (w^g - e(\varphi, \varphi))^+ = 0$. In this case, all syndication offers are insufficient to induce any other firm to accept the offer of syndication (as the offer is weakly less than the cost of production). Thus, every firm rejects the offer of syndication and play enters the Bertrand reversion phase.

Insufficient Offers: $0 < \sum_{g \in F \setminus \{f\}} (w^g - e(\varphi, \varphi))^+ \leq \frac{\delta}{1-\delta} (q - e(1, 1))$ In this case, absent dynamic rewards and punishments, some firms would be tempted to accept the offer of syndication. All firms *do* reject the syndication offers and play proceeds

¹¹Here, $(x)^+ \equiv \max\{0, x\}$.

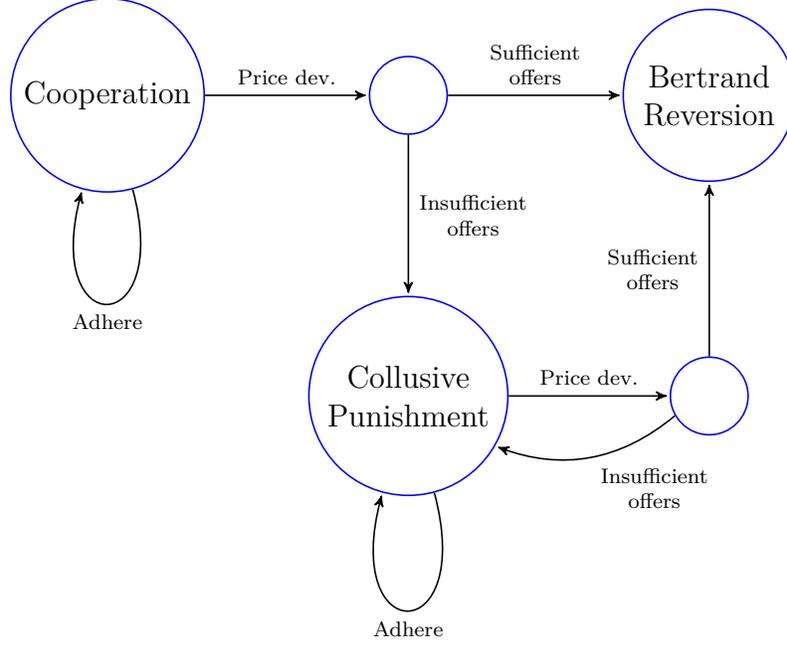


Figure 4: Automaton representation of the class of equilibria we consider. Labeled nodes are phases; unlabeled nodes are intermediate phases, which represent the branching of transitions based on behavior in the second step of the game.

going forward in a collusive punishment phase with

$$\psi^h = \begin{cases} \frac{(w^h - e(\varphi, \varphi))^+}{\sum_{g \in F \setminus \{f\}} (w^g - e(\varphi, \varphi))^+} (q - e(1, 1)) & h \neq f \\ 0 & h = f. \end{cases}$$

Sufficient Offers: $\sum_{g \in F \setminus \{f\}} (w^g - e(\varphi, \varphi))^+ > \frac{\delta}{1-\delta} (q - e(1, 1))$. In this case, play enters the Bertrand reversion phase in the next period; in period, each firm h accepts if and only if $w^h \geq \bar{w}$, where $\bar{w} = e\left(\sum_{g \in F \setminus \{f\}} \mathbb{1}_{\{w^g \geq \bar{w}\}}, \varphi\right)$, i.e., each firm accepts or rejects the offer so as to maximize its in period payoff given the actions of other firms.

Finally, if any firm accepts or rejects a syndication offer contrary to the prescribed play, we proceed to the Bertrand reversion phase.

Figure 4 provides an automaton representation of the subgame-perfect Nash equilibrium

described here.

It is immediate that the conjectured equilibrium delivers a price of p^* in each period. Thus, all that remains is to verify that the prescribed strategies constitute a subgame-perfect Nash equilibrium. We first show that the prescribed actions regarding accepting or rejecting syndication offers are best responses. It is immediate that, after equilibrium play in either the cooperation phase or a collusive punishment phase, it is a best response for each agent to accept the offer of syndication.¹² It is also immediate that, in the case of uniformly low offers, it is a best response for each firm to reject the offer of syndication.¹³ Finally, it is immediate that, in the case of sufficient offers, each firm plays a best response as the firm only accepts if accepting provides a non-negative payoff in this period, and play continues to the Bertrand reversion phase regardless of the firm's actions.

To show that, in the case of insufficient offers, it is a best response for each firm to reject the offer of syndication, we calculate the total payoff for h from accepting the offer as

$$w^h - e\left(\frac{1}{2}, \varphi\right) < w^h - e(\varphi, \varphi),$$

as play reverts to the Bertrand reversion phase if h accepts the offer (assuming other firms do not accept their syndication offers).¹⁴ Meanwhile, the total payoff for h in the continuation game from rejecting the offer is

$$\begin{aligned} \frac{\delta}{1-\delta}\psi^h &= \frac{\delta}{1-\delta} \left(\frac{(w^h - e(\varphi, \varphi))^+}{\sum_{g \in F \setminus \{f\}} (w^g - e(\varphi, \varphi))^+} (q - e(1, 1)) \right), \\ &\geq w^h - e(\varphi, \varphi) \end{aligned}$$

¹²This follows as each offer of syndication provides the firm with non-negative surplus and, if the firm rejects the syndication offer, play continues to the Bertrand reversion phase, and so the firm's future payoffs are 0.

¹³This follows as each offer of syndication provides the firm with non-positive surplus and play continues to the Bertrand reversion phase regardless of the firm's actions.

¹⁴Note that, since each other firm rejects its offer of syndication, h expects that it will be the only firm to join the syndicate if it accepts its offer of syndication, and thus expects its costs to be $e(\frac{1}{2}, \varphi)$. Since $e(\frac{1}{2}, \varphi) > e(\varphi, \varphi)$, there is slack in the incentive constraint, and so for some discount factors less than $\frac{1}{2}$, the price p^* can still be maintained; see Appendix A for the derivation of the tight bound on the discount factor.

where the inequality follows from the fact that $\sum_{g \in F \setminus \{f\}} (w^g - e(\varphi, \varphi))^+ \leq \frac{\delta}{1-\delta}(q - e(1, 1))$, as we are in the insufficient offers case.

It remains to verify whether any firm has an incentive to deviate on price or not follow the prescribed syndication offers during either the cooperation phase or a collusive punishment phase. During a collusive punishment phase, the payoff to a firm f who undercuts on price or who does not follow the prescribed syndication offers, and makes uniformly low or insufficient offers (and thus does not successfully recruit any syndicate members), is at most $q - e(1, \varphi) \leq e(1, \varphi) - e(1, \varphi) = 0$. Meanwhile, firm f enjoys a continuation value $\psi^f \geq 0$ by not deviating. During a collusive punishment phase, the payoff to a firm f who undercuts on price or who does not follow the prescribed syndication offers, but makes just sufficient offers, is at most

$$\underbrace{q}_{\text{Price}} - \underbrace{e(1, 1)}_{\text{Cost of completing the task with everyone}} - \underbrace{\frac{\delta}{1-\delta}(q - e(1, 1))}_{\text{Dynamic compensation to other firms}} = \left(1 - \frac{\delta}{1-\delta}\right)(q - e(1, 1)) \leq 0$$

as $\delta \geq \frac{1}{2}$.

Finally, we verify that no firm has an incentive to deviate on price or not follow the prescribed syndication offers during the cooperation phase. During the cooperation phase, there is no incentive for a firm to deviate (by either undercutting on price or not following the prescribed syndication offers) and make insufficient offers (and thus not successfully recruit any syndicate members), so long as

$$\frac{1}{1-\delta} \varphi(p^* - e(1, 1)) \geq p^* - e(1, \varphi),$$

which holds so long as $p^* \leq \frac{(1-\delta)e(1, \varphi) - \varphi e(1, 1)}{1-\delta-\varphi}$. There is also no incentive for any firm to deviate (by either undercutting on price or not following the prescribed syndication offers) and make

sufficient offers so long as

$$\frac{1}{1-\delta}\varphi(p^* - e(1,1)) \geq p^* - e(1,1) - \frac{\delta}{1-\delta}(q - e(1,1)),$$

which reduces to

$$p^* \leq \frac{(1-\delta)e(1,1) + \delta(q - e(1,1)) - \varphi e(1,1)}{1-\delta-\varphi}.$$

There are now two cases to consider: In the first case, $q = e(1,\varphi)$ which implies that $p^* \leq \frac{(1-\delta)e(1,\varphi) - \varphi e(1,1)}{1-\delta-\varphi}$. Thus, there is no incentive for a firm to deviate by making sufficient offers so long as

$$\begin{aligned} \frac{(1-\delta)e(1,\varphi) - \varphi e(1,1)}{1-\delta-\varphi} &\leq \frac{(1-\delta)e(1,1) + \delta(e(1,\varphi) - e(1,1)) - \varphi e(1,1)}{1-\delta-\varphi} \\ (1-\delta)e(1,\varphi) &\leq (1-\delta)e(1,1) + \delta(e(1,\varphi) - e(1,1)) \\ (2\delta-1)e(1,1) &\leq (2\delta-1)e(1,\varphi), \end{aligned}$$

which holds since $\delta \geq \frac{1}{2}$ and $e(1,1) < e(1,\varphi) = q$.

In the second case, $q = v$, which implies that $p^* = v$. Thus, there is no incentive for a firm to deviate by making sufficient offers so long as

$$\begin{aligned} v &\leq \frac{(1-\delta)e(1,1) + \delta(v - e(1,1)) - \varphi e(1,1)}{1-\delta-\varphi} \\ (1-\delta-\varphi)v &\leq \delta v + (1-2\delta-\varphi)e(1,1) \\ (2\delta-1)e(1,1) &\leq (2\delta-1)v, \end{aligned}$$

which holds since $\delta \geq \frac{1}{2}$ and $v \geq e(1,1)$.

Thus, for $\delta \geq \frac{1}{2}$, p^* can be sustained. It now remains to show that no price higher than p^* can be sustained. But, this result is by construction: If the equilibrium price exceeded p^* , a firm could profitably deviate by undercutting infinitesimally on price and completing the project without recruiting any syndicate members; such a deviation would be profitable even

though the firm would receive its lowest individually rational payoff in all subsequent periods. Thus, we have established the highest sustainable price stated in the theorem for cases where $p^* \leq v$.

Since p^* is quasiconvex, the only regions where p^* may equal v are low and high values of φ . That is, there will be (at most) a single threshold value of φ above which $p^* = v$ and a single threshold value of φ below which $p^* = v$. For φ above the higher threshold, it is immediate that no firm will deviate from $p = v$ in expectation of completing the project alone for $q = e(1, \varphi)$; the payoff for such a deviation is strictly lower than if the price were $p = \frac{(1-\delta)e(1,\varphi) - \varphi e(1,1)}{1-\delta-\varphi}$ as all payoffs in future periods are 0 while the current payoff from the deviation is $v - e(1, \varphi) < \frac{(1-\delta)e(1,\varphi) - \varphi e(1,1)}{1-\delta-\varphi} - e(1, \varphi)$. The analysis for preventing firms from joining a deviator's syndicate is identical to the case where $p^* \leq v$.

In the region where φ is small, the analysis is slightly different. When $p^* = v$ but $e(1, \varphi) < v$, the analysis does not change. When $e(1, \varphi) > v$, it is trivial to enforce v by refusing to join a syndicate after a price deviation from v , even when the price in future periods remains v . Preventing a deviation to join a price deviator's syndicate is identical to the lower φ case, except the threshold for sufficient offers and the continuation payoffs for rejecting a syndicate offer are determined by $v - e(1, 1)$ rather than $e(1, \varphi) - e(1, 1)$.

4 Prices, Profits, and Capacity

We now consider the question of how the highest sustainable price and industry profits vary as a function of productive capacity k . In standard industrial organization models, industry profits are increasing in the productive efficiency of firms. However, in our setting, this is not necessarily the case: for a large class of cost functions, industry profits are strictly *decreasing* in the productive capacity k .

Proposition 3. *Suppose that $e(s, \varphi) - e(s, 1)$ is convex in s for all $\varphi \in (0, 1 - \delta)$.¹⁵ Then*

¹⁵For instance, all cost functions of the form $e(s, m) = s\left(\frac{s}{m}\right)^\alpha$, where $\alpha > 0$, satisfy our condition.

industry profits and p^* are decreasing in industry capacity k .

Intuitively, increasing the productive capacity k should enhance industry profits as it lowers the cost of production $e(1, k)$. However, increasing the productive capacity also reduces the cost of cost of completing the project alone. When the difference in cost of doing the project by yourself as opposed to in collaboration with all other firms is increasing and convex in the size of the project, the second effect dominates. Consequently, the highest sustainable price p^* decreases with productive capacity so quickly that the gains from increased productivity in the industry as a whole are insufficient to offset the decrease.

5 Contracting over Production Shares

We now consider where a syndication contract offered to g additionally specifies the share of production to be completed by g : under this form of contracting, in step 3 of the extensive form stage game, the syndicate leader f offers each other firm a contract (s_t^g, w_t^g) , where s_t^g is the *production share* that will be required of g and w_t^g is (as before) the *fee* that f pays to g if he accepts the offer of syndication. The stage game payoffs in this case (where, as before, the set of firms who accept the offer of syndication is denoted by G_t) are given by

1. The payoff for f is $p_t^f - e\left(1 - \sum_{g \in G_t} s_t^g, \varphi k\right) - \sum_{g \in G_t} w_t^g$, i.e., the price paid by the buyer less the cost of f 's production less the cost of the fees paid to other firms.
2. The payoff for $g \in G_t \setminus \{f\}$ is $w_t^g - e(s_t^g, \varphi k)$, i.e., the fee paid to g less the cost of g 's production.
3. The payoff for $h \in F \setminus G_t$ is 0.

Surprisingly, the highest sustainable price in this game is the same as in the case described in Theorem 1, where firms are unable to contract over production shares.

Theorem 2. *For $\delta \geq \frac{1}{2}$, the highest price sustainable in a subgame-perfect Nash equilibrium is given by p^* , as defined in Theorem 1. Moreover, p^* is quasiconvex in φ and $\lim_{\varphi \rightarrow 0} p^* = v$.*

The construction of the equilibrium that supports p^* is very similar to that of the construction of the equilibrium in Section 3; see Appendix C for details.

6 Heterogenous Firms

We now consider the case where firms may differ by their productive capacity in a setting where a contract specifies both the fee and the production share.¹⁶ Thus, for each $f \in F$, let κ^f be the *productive capacity controlled by firm f* . It will be helpful to define κ^{\max} as the largest share of productive capacity controlled by a single firm, i.e., $\kappa^{\max} \equiv \max_{f \in F} \{\kappa^f\}$. Moreover, the total productive capacity is given by $k = \sum_{f \in F} \kappa^f$.

6.1 Equilibrium Characterization

We now characterize the highest sustainable price as a function of the productive capacities of each firm by constructing a subgame-perfect Nash equilibrium that supports said price. In such an equilibrium, if a firm f is small enough, it is allocated no surplus in the cooperation phase; if a firm is small enough, the highest sustainable price will be less than f 's cost of production, and so no surplus is needed to disincentivize this firm from completing the project on its own. Larger firms will be allocated positive surplus; we call firms that obtain positive surplus in an equilibrium supporting the highest sustainable price *collusion beneficiaries* and denote the set of collusion beneficiaries as \hat{F} .

To prevent a collusion beneficiary f from undercutting on price and engaging in lone production, f 's portion of the surplus must be large enough that f prefers to adhere to the equilibrium. Let r^f denote the portion of surplus allocated to f and consider an equilibrium to sustain the price p . Then, for f to not engage in lone production, we must have

$$\frac{1}{1-\delta} r^f (p - e(1, k)) \geq p - e(1, \kappa^f); \quad (1)$$

¹⁶Here, the assumption that a contract specifies the production share is natural, since efficient production requires firms with different productive capacities to perform differing production shares.

maximizing price subject to this constraint for each collusion beneficiary yields the highest sustainable price, as expressed in Theorem 3.

Theorem 3. *The highest price sustainable in a subgame-perfect Nash equilibrium, \hat{p}^* , is given by the solution to*

$$\hat{p}^*(\kappa; \delta) = \begin{cases} v & \varphi \in [1 - \delta, 1] \\ \min\left\{\frac{(1-\delta)\hat{\varphi} \sum_{f \in \hat{F}} e(1, \kappa^f) - \hat{\varphi} e(1, k)}{1 - \delta - \hat{\varphi}}, v\right\} & \varphi \in (0, 1 - \delta). \end{cases}$$

$$\hat{F}(\kappa; \delta) = \left\{f \in F : \hat{p}^*(\kappa; \delta) \geq e(1, \kappa^f)\right\} \text{ with } \hat{\varphi} = \frac{1}{|\hat{F}|}$$

with the maximal $\hat{p}^*(\kappa; \delta)$ so long as $\delta \geq \hat{\delta}(\kappa; \delta) \equiv \frac{\hat{p}^*(\kappa; \delta) - e(1, k)}{\hat{p}^*(\kappa; \delta) - e(1, k) + \min\{\hat{p}^*(\kappa; \delta), e(1, \kappa^{\max})\} - e(1, k)} \in [\frac{1}{2}, 1)$.

To support the price $\hat{p}^*(\kappa; \delta)$, we construct an equilibrium similar to the construction in Section 3. In the cooperation phase, each firm submits a bid of $\hat{p}^*(\kappa; \delta)$. However, the amount of surplus received by each firm via the syndication offer now depends on the productive capacity of that firm. Larger firms, i.e., firms with a larger productive capacity, receive a greater share of surplus, as the cost of lone production is lower for a larger firm. After a deviation, we again enter a collusive punishment phase if everyone rejects the deviator's offers of syndication. The price in the collusive punishment phase is given by $\min\{\min_{g \in F}\{e(1, \kappa^g)\}, v\}$ to ensure that no firm has an incentive to deviate and engage in lone production (as the cost of lone production will be no less than the price). Finally, there is also a Bertrand reversion phase, in which the price is just the cost of production $e(1, k)$; we enter this stage whenever any firm deviates with respect to accepting or rejecting offers of syndication.

The highest sustainable price depends on the average cost for lone production *among the collusion beneficiaries*. Suppose the productive capacity of a collusion beneficiary f decreases, increasing f 's cost of lone production; then constraint (1) slackens, and so by reducing the portion r^f allocated to f , we can raise the highest sustainable price. Since this is true for

every collusive beneficiary the highest sustainable price depends on the average cost of lone production. The restriction on the discount factor $\hat{\delta}(\kappa; \delta)$ ensures that undercutting on price and recruiting a syndicate is not profitable, analogously to the $\frac{1}{2}$ threshold for δ in the symmetric case.¹⁷

6.2 Effects of Heterogeneity

Using Theorem 3, we can now characterize the effects of a small degree of heterogeneity.

Proposition 4. *For any total productive capacity k , there exists an $\epsilon > 0$ such that, for any distribution of productive capacities κ with total productive capacity k such that $|\kappa^f - \varphi k| < \epsilon$ for all $f \in F$, the highest sustainable price is strictly higher under κ than under the homogenous distribution of productive capacities, i.e.,*

$$\hat{p}^*(\kappa; \delta) > \hat{p}^*((\varphi k)_{f \in F}; \delta)$$

for all $\delta > \hat{\delta}((\varphi k)_{f \in F}; \delta)$ so long as $\hat{p}^*((\varphi k)_{f \in F}; \delta) < v$.

Proposition 4 is illustrated in Figure 5. When firms are nearly homogenous, each firm is a collusion beneficiary and so the highest sustainable price is linearly increasing in the average cost of lone production across all firms, $\varphi \sum_{f \in F} e(1, \kappa^f)$; moreover, since the cost of lone production by a firm is convex in that firm's productive capacity, this sum is increasing in the degree of heterogeneity. However, when some firms are very small, they no longer receive positive surplus and the relevant average cost is the average cost of the larger firms. Thus, in Figure 5, when the smaller firms have a productive capacity less than $\frac{1}{24}$ each, those smaller firms no longer receive positive surplus—the relevant average cost then becomes the average cost across the six large firms, which is decreasing in the degree of heterogeneity. Thus, as the degree of heterogeneity increases beyond $\frac{1}{24}$, the price is now decreasing in the

¹⁷The expression for $\hat{\delta}$ does not immediately reduce to $\frac{1}{2}$ for the case of symmetric firms, as the expression is derived allowing for the possibility that there is at least one firm obtaining no surplus.

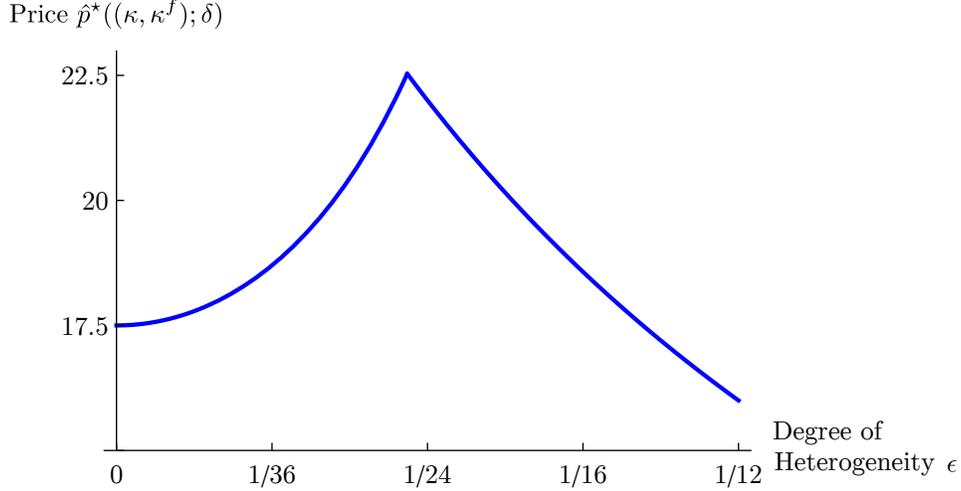


Figure 5: The highest sustainable price $\hat{p}^*(\kappa; \delta)$ as a function of the degree of heterogeneity ϵ . Here, $e(s, m) = \frac{s^2}{m}$, $\delta = \frac{3}{4}$, and there are 8 incumbent firms; half of the incumbent firms have productive capacity $\frac{1}{8} + \epsilon$, and half of the incumbent firms have productive capacity $\frac{1}{8} - \epsilon$.

degree of heterogeneity.

6.3 Market Entry

We now consider the effect of entry by a small firm on the highest sustainable price. When a firm enters the market, there are two effects: First, that firm may undercut the current price and engage in lone production. Second, collusion at the current price becomes more profitable since the cost of joint production goes down due to the additional productive capacity of the entrant. For a small enough entrant, the first effect does not arise, since the cost of lone production is higher than the highest sustainable price when the entrant is not present. However, the latter effect always has bite, and so higher prices can be sustained.

Proposition 5. *Consider any distribution of productive capacities κ , and suppose that $\lim_{m \rightarrow 0} e(s, m) = \infty$ for all $s > 0$. Then there exists an $\epsilon > 0$ such that entry by a firm f with productive capacity less than ϵ will increase the highest sustainable price, i.e.,*

$$\hat{p}^*((\kappa, \kappa^f); \delta) > \hat{p}^*(\kappa; \delta)$$

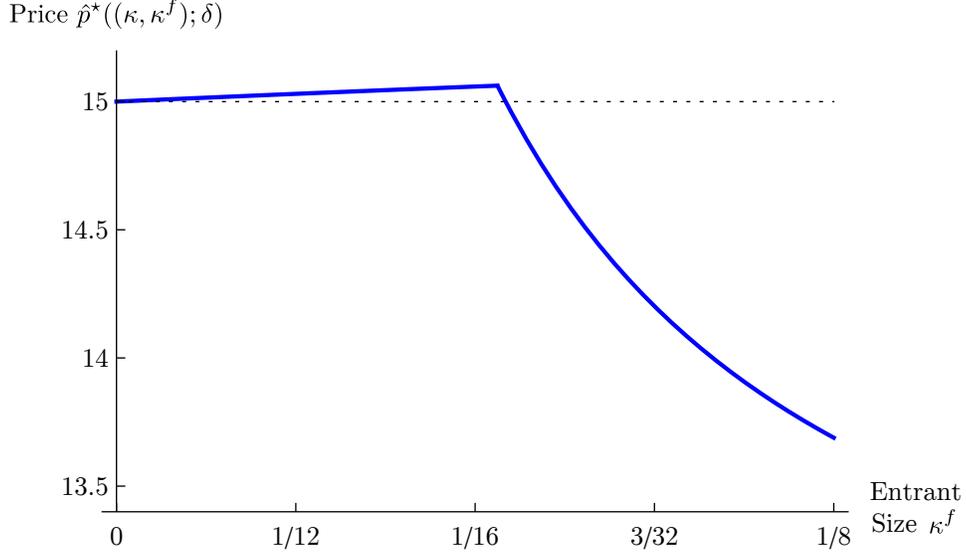


Figure 6: The highest sustainable price $\hat{p}^*((\kappa, \kappa^f); \delta)$ as a function of entrant size κ^f . Here, $e(s, m) = \frac{s^2}{m}$, $\delta = \frac{3}{4}$, and there are 8 incumbent firms each with productive capacity $\frac{1}{8}$. The dashed line denotes the highest sustainable price without entry.

for all $\delta > \hat{\delta}(\kappa; \delta)$ so long as $\hat{p}^*(\kappa; \delta) < v$.

Figure 6 depicts the highest sustainable price for a typical economy as a function of the size of the entrant. When no entrant is present, the highest sustainable price is 15; however, for small entrants, the highest sustainable price is (slightly) higher than 15. This happens because an entrant of size less than $\frac{1}{16}$ does not have the productive capacity to profitably undercut the collusive price and complete the task itself. Moreover, the entrant's capacity makes collusion more profitable for the incumbent firms, as it decreases the cost of joint production. Thus, entry by a sufficiently small firm will facilitate collusion as opposed to hampering it.

However, for a sufficiently large entrant, collusion will become more difficult, since the entrant can profitably undercut the collusive price and complete the task itself; this occurs when κ^f becomes approximately $\frac{1}{16}$ in Figure 6.¹⁸ Thus, when the entrant is sufficiently large, some industry profits must be allocated to the entrant in order to forestall the entrant from deviating. This leaves fewer industry profits for the other firms, making collusion more

¹⁸In fact, the highest sustainable price is maximized at $\frac{1}{16}$.

difficult and reducing the highest sustainable price.

7 Conclusion

Our results show that in markets with syndication classical industrial organization intuitions are not always valid: Decreasing market concentration can *raise* prices, as it allows the threat of stronger punishments in-period by refusing offers of syndication.¹⁹ Moreover, entry by a small firm can also *raise* prices, as a small firm can not threaten to engage in lone production. Thus, our analysis suggests that standard antitrust remedies are at best of questionable use in markets where syndication plays a significant role and where collusion appears to be a problem. However, we have shown that increasing capacity throughout the industry will decrease the highest sustainable price. Unfortunately, such an expansion of capacity, while it does lower the total costs of production on the equilibrium path, will likely reduce joint industry profits. Thus, it is unlikely that the industry will take coordinated efforts to raise capacity. This analysis is particularly important in the financial sector, where regulatory barriers routinely restrict participation by investors in certain types of investments based on requirements for net worth or financial sophistication. Our analysis suggests that the financial sector may actively support such restrictions since this policy can facilitate collusion.

Our work also highlights the importance of the full extensive form of firm interactions in industrial organization settings. Many industries are characterized by repeated, complex interactions that are best modeled as repeated extensive form games: financial markets with syndication, municipal auctions followed by horizontal subcontracting between bidders, and real estate transactions with agent selection. Further exploring repeated extensive form games is thus crucial to understanding subtle but important strategic interactions in markets.

¹⁹Although here we work in a complete information environment, in ongoing work, we show that our conclusions are largely robust to relaxing our assumption that syndication offers are public.

References

- Abrahamson, M., T. Jenkinson, and H. Jones (2011). Why don't U.S. issuers demand european fees for IPOs? *Journal of Finance* 66(6), 2055–2082.
- Abreu, D. (1986). Extremal equilibria of oligopolistic supergames. *Journal of Economic Theory* 39(1), 191–225.
- Allen, F. and G. R. Faulhaber (1989). Signalling by underpricing in the IPO market. *Journal of Financial Economics* 23(2), 303–323.
- Athey, S., K. Bagwell, and C. Sanchirico (2004). Collusion and price rigidity. *Review of Economic Studies* 71(2), 317–349.
- Benveniste, L. M. and P. A. Spindt (1989). How investment bankers determine the offer price and allocation of new issues. *Journal of Financial Economics* 23(3), 343–361.
- Biais, B., P. Bossaerts, and J.-C. Rochet (2002). An optimal IPO mechanism. *Review of Economic Studies* 69(1), 117–146.
- Chemmanur, T. J. (1993). The pricing of initial public offerings: a dynamic model with information production. *Journal of Finance* 48(1), 285–304.
- Chen, H.-C. and J. Ritter (2000). The seven percent solution. *Journal of Finance* 55(3), 1105–1131.
- Cliff, M. T. and D. J. Denis (2004). Do IPO firms purchase analyst coverage with underpricing? *Journal of Finance* 59(6), 2871–2091.
- Department of Justice and Federal Trade Commission (2010). Horizontal merger guidelines. <https://www.justice.gov/atr/horizontal-merger-guidelines-08192010>.
- Hansen, R. S. (2001, March). Do investment banks compete in IPOs?: The advent of the '7% plus contract'. *Journal of Financial Economics* 59(3), 313–346.

- Kang, A. and R. Lowery (2014). The pricing of IPO services and issues: Theory and estimation. *Review of Corporate Finance Studies* 2(2), 188–234.
- Loughran, T. and J. Ritter (2004). Why has IPO underpricing changed over time? *Financial Management* 33(3), 5–37.
- Lowry, M. and S. Shu (2002). Litigation risk and IPO underpricing. *Journal of Financial Economics* 65(3), 309–355.
- Mailath, G. J., V. Nocke, and L. White (Forthcoming, 2016). When and how the punishment must fit the crime. *International Economic Review*.
- Nocke, V. and L. White (2007). Do vertical mergers facilitate upstream collusion? *American Economic Review* 97(4), 1321–1339.
- Rock, K. (1986). Why new issues are underpriced. *Journal of Financial Economics* 15(1-2), 187–212.
- Rotemberg, J. and G. Saloner (1986). A supergame-theoretical model of price wars during booms. *American Economic Review* 76(3), 390–407.
- Sundaram, R. K. (1996). *A first course in optimization theory*. Cambridge university press.
- Tirole, J. (1988). *The Theory of Industrial Organization*. MIT Press.
- Torstila, S. (2003, September). The clustering of IPO gross spreads: International evidence. *Journal of Financial and Quantitative Analysis* 38(3), 673–694.

A Perfect Information Equilibria for Low Discount Factors

Theorem A.1. For $\delta \geq \frac{e(1,\varphi)-e(1,1)-E}{2(e(1,\varphi)-e(1,1))-E}$, where $E \equiv e(\varphi, \varphi) + \frac{1-\varphi}{\varphi}e\left(\frac{1}{2}, \varphi\right) - e(1, 1) > 0$, the highest price sustainable in a subgame-perfect Nash equilibrium, is given by p^* , the price

derived in Theorem 1.

Proof. The proof is virtually identical to the case with $\delta > \frac{1}{2}$, with the sole modification being that the check that no player has an incentive to deviate to accept the syndicate offer when offers are insufficient compares

$$(1 - \delta) \left(w^h - e\left(\frac{1}{2}, \varphi\right) \right) \leq \delta \frac{w^h - e\left(\frac{1}{2}, \varphi\right)}{\sum_{g \in F \setminus f} (w^g - e\left(\frac{1}{2}, \varphi\right))} (e(1, \varphi) - e(1, 1)).$$

That is, no player has an incentive to deviate to accept the offer if he expects to complete the project alone. Then, we need only check that there is no incentive to deviate from a price of p^* when making just sufficient offers:

$$\varphi(p - e(1, 1)) \leq (1 - \delta) \left(p - e(\varphi, \varphi) - \frac{1 - \varphi}{\varphi} e\left(\frac{1}{2}, \varphi\right) - \frac{\delta}{1 - \delta} (e(1, \varphi) - e(1, 1)) \right) \quad (2)$$

$$p \leq \frac{(1 - \delta) \left(e(\varphi, \varphi) + \frac{1 - \varphi}{\varphi} e\left(\frac{1}{2}, \varphi\right) \right) + \delta(e(1, \varphi) - \varphi e(1, 1))}{1 - \delta - \varphi} \quad (3)$$

We now define $E \equiv e(\varphi, \varphi) + \frac{1 - \varphi}{\varphi} e\left(\frac{1}{2}, \varphi\right) - e(1, 1)$ and thus express

$$p \leq \frac{(1 - \delta)(e(1, 1) + E) + \delta(e(1, \varphi) - e(1, 1)) - \varphi e(1, 1)}{1 - \delta - \varphi}.$$

But, again, $p^* = \frac{(1 - \delta)e(1, \varphi) - \varphi e(1, 1)}{1 - \delta - \varphi}$, so the posited equilibrium holds as long as

$$(1 - \delta)e(1, \varphi) \leq (1 - \delta)e(1, 1) + \delta(e(1, \varphi) - e(1, 1)) + (1 - \delta)E$$

$$(2\delta - 1)e(1, 1) \leq (2\delta - 1)e(1, \varphi) + (1 - \delta)E$$

which is clearly true for $\delta \geq \frac{1}{2}$ since E is greater than 0. To see that E is greater than 0, observe that E is the costs of completing an equal share of the project, plus the costs of paying each other firm to complete half the project, minus the cost of completing the project with all available capacity.

If $\delta < \frac{1}{2}$, the condition holds as long as

$$e(1, \varphi) \leq e(1, 1) + \underbrace{\frac{1 - \delta}{1 - 2\delta} E}_{>0}$$

or,

$$\delta \geq \frac{e(1, \varphi) - e(1, 1) - E}{2(e(1, \varphi) - e(1, 1)) - E}.$$

□

The condition here relates the discount factor to the convexity of costs. The minimal discount factor that continues to support the price determined by a deviation to complete the project alone is determined by the costs associated with compensating each firm as if they were the only firm to join the syndicate. In order to recruit any player to a syndicate, that player must view the deviation as profitable even if he expects all other players to refuse to join. This conjecture makes the firm in question pivotal to whether the game proceeds to the Bertrand Nash reversion equilibrium or continues in an equilibrium with price given by $e(1, \varphi)$, consistent with the posited condition for there to be no profitable deviation. Further, each player expects to complete the project in a syndicate of two, thus needing to run half the task. This expectation, driven by the conjecture that other players reject the offer, is what makes deviations that were profitable in the above conjectured equilibrium when $\delta < \frac{1}{2}$ no longer profitable.

B Proof of Proposition 3

It is easy to verify that price is now given by:

$$p^*(k) = \frac{(1 - \delta)e(1, k\varphi) - \varphi e(1, k)}{1 - \delta - \varphi}.$$

Industry profits per period are thus

$$\Pi \equiv \frac{(1-\delta)e(1, k\varphi) - \varphi e(1, k)}{1-\delta-\varphi} - e(1, k) = \frac{1-\delta}{1-\delta-\varphi} k \left(e\left(\frac{1}{k}, \varphi\right) - e\left(\frac{1}{k}, 1\right) \right).$$

where the equality follows from the fact that the cost function is homogenous of degree 1.

Differentiating profits with respect to k , and then multiplying by $\frac{1-\delta-\varphi}{1-\delta}$ gives

$$\frac{1-\delta-\varphi}{1-\delta} \frac{\partial \Pi}{\partial k} = \left(e\left(\frac{1}{k}, \varphi\right) - e\left(\frac{1}{k}, 1\right) \right) - \frac{1}{k} \left(e_s\left(\frac{1}{k}, \varphi\right) - e_s\left(\frac{1}{k}, 1\right) \right)$$

Letting $g(x) = e(x, \varphi) - e(x, 1)$ and $x = \frac{1}{k}$, we have that

$$\begin{aligned} \frac{1-\delta-\varphi}{1-\delta} \frac{\partial \Pi}{\partial k} &= g(x) - xg'(x) \\ &= g(x) - g(0) - (x-0)g'(x) \\ &< 0, \end{aligned}$$

where the second equality follows from the fact that $e(0, y) = 0$ for all $y \geq 0$, and the inequality follows from the convexity assumption of the theorem.

C Proof of Theorem 2

To show that p^* is the highest sustainable price, we construct an equilibrium of the following form:

1. In the *cooperation phase*,

- every firm submits the same bid $p = p^*$,
- the short-lived buyer then accepts one such offer, and chooses each offer with equal probability,
- every firm, if it becomes the syndicate leader, offers every other firm a fee of $e(\varphi, \varphi)$

for agreeing to perform φ of production,

- and every other firm accepts this offer.

2. In the *collusive punishment phase with continuation values ψ* ,

- every firm submits the same bid $q = e(1, \varphi)$,
- the short-lived buyer then accepts one such offer of q , and chooses each offer with equal probability,
- every firm $g \in F$, if it becomes the syndicate leader, offers $e(\varphi, \varphi) + \psi^h$ to every firm $h \in F \setminus \{g\}$ for agreeing to perform φ of production,
- and every other firm accepts the offer by the syndicate leader g to join the syndicate.

3. In the *Bertrand reversion phase*, agents play the Bertrand reversion Nash equilibrium (where play after bidding is now characterized by the winning firm offering $e(\varphi, \varphi)$ to each firm for agreeing to perform φ of production (and every firm accepting)).

4. Under equilibrium play, play continues in the same phase. If, in the cooperation phase or a collusive punishment phase, any firm f deviates in the first step or deviates with respect to the prescribed set of offers, then we calculate the sum across the other firms of the (positive) differences between the syndication offer and the costs of doing φ of the project for each firm as $\sum_{g \in F \setminus \{f\}} (w^g - e(s^g, \varphi))^+$.

Uniformly Low Offers: $\sum_{g \in F \setminus \{f\}} (w^g - e(s^g, \varphi))^+ = 0$. In this case, all syndication offers are insufficient to induce any other firm to accept the offer of syndication (as the offer is weakly less than the cost of production). Thus, every firm rejects the offer of syndication and play enters the Bertrand reversion phase.

Insufficient Offers: $0 < \sum_{g \in F \setminus \{f\}} (w^g - e(s^g, \varphi))^+ \leq \frac{\delta}{1-\delta} (e(1, \varphi) - e(1, 1))$. In this case, absent dynamic rewards and punishments, some firms would be tempted to

accept the offer of syndication. All firms *do* reject the syndication offers and play proceeds going forward in a collusive punishment phase with

$$\psi^h = \begin{cases} \frac{(w^h - e(s^h, \varphi))^+}{\sum_{g \in F \setminus \{f\}} (w^g - e(s^g, \varphi))^+} (e(1, \varphi) - e(1, 1)) & h \neq f \\ 0 & h = f. \end{cases}$$

Sufficient Offers: $\sum_{g \in F \setminus \{f\}} (w^g - e(s^g, \varphi))^+ > \frac{\delta}{1-\delta} (e(1, \varphi) - e(1, 1))$. In this case, play enters the Bertrand reversion phase in the next period; in period, each firm h accepts if and only if $w^h \geq e(s^g, \varphi)$.

Finally, if any firm accepts or rejects a syndication offer contrary to the prescribed play, we proceed to the Bertrand reversion phase.

The proof that this strategy profile is a subgame-perfect Nash equilibrium and that it attain the highest sustainable price of any subgame-perfect Nash equilibrium then follows as in the discussion following Theorem 1.

D Proofs from Section 6

D.1 Proof of Theorem 3

To find $\hat{\varphi}$, we solve the problem

$$\max_{p,r} \{p\} \tag{4}$$

subject to the constraints

$$\begin{aligned} \frac{1}{1-\delta} r^f (p - e(1, k)) &\geq p - e(1, \kappa^f) && \text{for all } f \in F \\ r^f &\geq 0 && \text{for all } f \in F \\ \sum_{f \in F} r^f &= 1. \end{aligned}$$

We transform this problem by letting $\pi^f = r^f (p - e(1, 1))$ be the continuation value for f from adhering to the equilibrium strategy in the cooperation phase, and so obtain the problem

$$\max_{\pi} \left\{ \sum_{f \in F} \pi^f \right\}$$

subject to the constraints

$$\begin{aligned} \frac{1}{1-\delta} \pi^f &\geq \sum_{g \in F} \pi^g + e(1, k) - e(1, \kappa^f) && \text{for all } f \in F \\ \pi^f &\geq 0 && \text{for all } f \in F. \end{aligned}$$

The first constraint is the *no lone deviation constraint*. This is a convex optimization problem, and moreover is it immediate that it satisfies Slater's condition. Thus, by Theorem 7.16 of [Sundaram \(1996\)](#), there exists a vector of continuation payoffs $\hat{\pi}$ and Lagrangian multipliers λ and μ that satisfy the Kuhn-Tucker conditions, i.e., for all $f \in F$,

$$\begin{aligned} 1 + \frac{\delta}{1-\delta} \lambda^f - \sum_{g \in F} \lambda^g + \mu^f &= 0 \\ \lambda^f \geq 0 \text{ and } \lambda^f \left(\frac{1}{1-\delta} \pi^f - \sum_{g \in F} \pi^g - e(1, k) + e(1, \kappa^f) \right) &= 0 \\ \mu^f \geq 0 \text{ and } \mu^f \pi^f &= 0. \end{aligned}$$

Let the set of firms for which $\lambda^f \neq 0$ be denoted \hat{F} ; thus, for each $f \in \hat{F}$, we have that

$$\frac{1}{1-\delta}\pi^f - \sum_{g \in F} \pi^g - e(1, k) + e(1, \kappa^f) = 0.$$

Summing over firms in \hat{F} , we obtain

$$\begin{aligned} \frac{1}{1-\delta} \sum_{f \in \hat{F}} \pi^f &= \sum_{f \in \hat{F}} \left(\sum_{g \in F} \pi^g + e(1, k) - e(1, \kappa^f) \right) \\ \frac{1}{1-\delta} (p - e(1, k)) &= \sum_{f \in \hat{F}} \left(p - e(1, k) + e(1, k) - e(1, \kappa^f) \right) \\ p - e(1, k) &= (1-\delta)|\hat{F}|p - \sum_{f \in \hat{F}} e(1, \kappa^f) \\ p &= \frac{(1-\delta)\hat{\varphi} \sum_{f \in \hat{F}} e(1, \kappa^f) - \hat{\varphi}e(1, k)}{1-\delta-\hat{\varphi}} \end{aligned}$$

where $\hat{\varphi} = \frac{1}{|\hat{F}|}$. Note that if $\lambda^f \neq 0$, then we can rewrite $\frac{1}{1-\delta}\pi^f - \sum_{g \in F} \pi^g - e(1, k) + e(1, \kappa^f) = 0$ as $\frac{1}{1-\delta}\pi^f = p - e(1, \kappa^f)$; thus, $\hat{F} = \{f \in F : p \geq e(1, \kappa^f)\}$.

It is immediate that we can not construct an equilibrium with a price higher than $p^* = \frac{(1-\delta)\hat{\varphi} \sum_{f \in \hat{F}} e(1, \kappa^f) - \hat{\varphi}e(1, k)}{1-\delta-\hat{\varphi}}$, since under any such price, some firm will have an incentive to slightly underprice and engage in lone production.

To show that \hat{p}^* is the highest sustainable price, we construct an equilibrium of the following form:

1. In the *cooperation phase*,

- every firm submits the same bid $p = \hat{p}^*$,
- the short-lived buyer then accepts one such offer, and chooses each offer with equal probability,
- every firm, if it becomes the syndicate leader, offers every other firm a fee of $e(\varphi^g, \kappa^g) + r^g$ for agreeing to perform φ^g of production, where $\varphi^g \equiv \frac{\kappa^g}{k}$.
- and every other firm accepts this offer.

2. In the *collusive punishment phase with continuation values* ψ ,

- every firm submits the same bid $q = \min\{e(1, \kappa^{\max}), \hat{p}^*\}$,
- the short-lived buyer then accepts one such offer of q , and chooses each offer with equal probability,
- every firm $g \in F$, if it becomes the syndicate leader, offers $e(\varphi^h, \kappa^h) + \psi^h$ to every firm $h \in F \setminus \{g\}$ for agreeing to perform φ^h of production,
- and every other firm accepts the offer by the syndicate leader g to join the syndicate.

3. In the *Bertrand reversion phase*, agents play the Bertrand reversion Nash equilibrium (where play after bidding is now characterized by the winning firm offering $e(\varphi^g, \kappa^g)$ to each firm g for agreeing to perform φ^g of production (and every firm accepting).

4. Under equilibrium play, play continues in the same phase. If, in the cooperation phase or a collusive punishment phase, any firm f deviates in the first step or deviates with respect to the prescribed set of offers, then we calculate the sum across the other firms of the (positive) differences between the syndication offer and the costs of doing φ^g of the project for each firm g as $\sum_{g \in F \setminus \{f\}} (w^g - e(s^g, \kappa^g))^+$.

Uniformly Low Offers: $\sum_{g \in F \setminus \{f\}} (w^g - e(s^g, \kappa^g))^+ = 0$. In this case, all syndication offers are insufficient to induce any other firm to accept the offer of syndication (as the offer is weakly less than the cost of production). Thus, every firm rejects the offer of syndication and play enters the Bertrand reversion phase.

Insufficient Offers: $0 < \sum_{g \in F \setminus \{f\}} (w^g - e(s^g, \kappa^g))^+ \leq \frac{\delta}{1-\delta}(q - e(1, k))$. In this case, absent dynamic rewards and punishments, some firms would be tempted to accept the offer of syndication. All firms *do* reject the syndication offers and play proceeds

going forward in a collusive punishment phase with

$$\psi^h = \begin{cases} \frac{(w^h - e(s^h, \varphi^h))^+}{\sum_{g \in F \setminus \{f\}} (w^g - e(s^g, \varphi^g))^+} (q - e(1, k)) & h \neq f \\ 0 & h = f. \end{cases}$$

Sufficient Offers: $\sum_{g \in F \setminus \{f\}} (w^g - e(s^g, \kappa^g))^+ > \frac{\delta}{1-\delta} (e(1, \kappa^g) - e(1, k))$. In this case, play enters the Bertrand reversion phase in the next period; in period, each firm h accepts if and only if $w^h \geq e(s^g, \kappa^h)$.

Finally, if any firm accepts or rejects a syndication offer contrary to the prescribed play, we proceed to the Bertrand reversion phase.

It is immediate that the conjectured equilibrium delivers a price of \hat{p}^* in each period. Thus, all that remains is to verify that the prescribed strategies constitute a subgame-perfect Nash equilibrium. We first show that the prescribed actions regarding accepting or rejecting syndication offers are best responses. It is immediate that, after equilibrium play in either the cooperation phase or a collusive punishment phase, it is a best response for each agent to accept the offer of syndication.²⁰ It is also immediate that, in the case of uniformly low offers, it is a best response for each firm to reject the offer of syndication.²¹ Finally, it is immediate that, in the case of sufficient offers, each firm plays a best response as the firm only accepts if accepting provides a non-negative payoff in this period and play continues to the Bertrand reversion phase regardless of the firm's actions.

To show that, in the case of insufficient offers, it is a best response for each firm to reject the offer of syndication, we calculate the total payoff for h from accepting the offer as

$$w^h - e(s^h, \kappa^h),$$

²⁰This follows as each offer of syndication provides the firm with non-negative surplus and, if the firm rejects the syndication offer, play continues to the Bertrand reversion phase, and so the firm's future payoffs are 0.

²¹This follows as each offer of syndication provides the firm with non-positive surplus and play continues to the Bertrand reversion phase regardless of the firm's actions.

as play reverts to the Bertrand reversion phase if h accepts the offer. Meanwhile, the total payoff for h in the continuation game from rejecting the offer is

$$\begin{aligned} \frac{\delta}{1-\delta}\psi^h &= \frac{\delta}{1-\delta} \left(\frac{(w^h - e(s^h, \kappa^h))^+}{\sum_{g \in F \setminus \{f\}} (w^g - e(s^g, \kappa^g))^+} (q - e(1, k)) \right) \\ &\geq w^h - e(s^h, \kappa^h) \end{aligned}$$

where the inequality follows from the fact that $\sum_{g \in F \setminus \{f\}} (w^g - e(s^g, \kappa^g))^+ \leq \frac{\delta}{1-\delta} (q - e(1, k))$, as we are in the insufficient offers case.

It remains to verify whether any firm has an incentive to deviate on price or not follow the prescribed syndication offers during either the cooperation phase or a collusive punishment phase. During a collusive punishment phase, the payoff to a firm f who undercuts on price or who does not follow the prescribed syndication offers, and makes uniformly low or insufficient offers (and thus does not successfully recruit any syndicate members), is at most $q - e(1, \kappa^f) = \min\{e(1, \kappa^{\max}), \hat{p}^*\} - e(1, \kappa^f) \leq 0$. Meanwhile, firm f enjoys a continuation value $\psi^f \geq 0$ by not deviating. During a collusive punishment phase, the payoff to a firm f who undercuts on price or who does not follow the prescribed syndication offers, but makes just sufficient offers, is at most

$$\underbrace{q}_{\text{Price}} - \underbrace{e(1, k)}_{\text{Cost of completing the task with everyone}} - \underbrace{\frac{\delta}{1-\delta}(q - e(1, k))}_{\text{Dynamic compensation to other firms}} = \left(1 - \frac{\delta}{1-\delta}\right)(q - e(1, k)) \leq 0$$

as $\delta \geq \hat{\delta}(\kappa; \delta) \geq \frac{1}{2}$.

Finally, we verify that no firm has an incentive to deviate on price or not follow the prescribed syndication offers during the cooperation phase. During the cooperation phase, there is no incentive for a firm to deviate (by either undercutting on price or not following the prescribed syndication offers) and make insufficient offers (and thus not successfully recruit

any syndicate members), so long as

$$\frac{1}{1-\delta}r^f(\hat{p}^* - e(1, k)) \geq p^* - e(1, \kappa^f);$$

but this constraint is satisfied by the construction of \hat{p}^* —see (4).

There is also no incentive for any firm to deviate (by either undercutting on price or not following the prescribed syndication offers) and make sufficient offers so long as

$$\frac{1}{1-\delta}r^f(\hat{p}^* - e(1, k)) \geq p^* - e(1, k) - \frac{\delta}{1-\delta}(q - e(1, k)).$$

First, note that, for a small enough firm f , we could have $r^f = 0$. Thus, we must have δ large enough to that

$$0 \geq p^* - e(1, k) - \frac{\delta}{1-\delta}(q - e(1, k)).$$

Thus, solving for δ , we have

$$\delta \geq \frac{p^* - e(1, k)}{(p^* - e(1, k)) + (q - e(1, k))},$$

which will be satisfied since $q = \min\{e(1, \kappa^{\max}), \hat{p}^*\}$

D.2 Proof of Proposition 4

First, note that $\hat{F}(\kappa; \delta) = F$ for all κ when ϵ is sufficiently small. Moreover, $\delta(\kappa; \delta)$ is continuous in κ , and so, for ϵ sufficiently small, we have that $\delta > \delta(\kappa; \delta)$ since $\delta > \delta((\varphi k)_{f \in F}; \delta)$. If $\hat{p}^*(\kappa; \delta) = v$, we are done, since $\hat{p}^*((\varphi k)_{f \in F}; \delta) < v$ by assumption. Thus, when $\hat{p}^*(\kappa; \delta) < v$, we can write

$$\hat{p}^*(\kappa; \delta) - \hat{p}^*((\varphi k)_{f \in F}; \delta) = (1-\delta)\hat{\varphi} \frac{\sum_{f \in \hat{F}} e(1, \kappa^f) - \sum_{f \in \hat{F}} e(1, \varphi k)}{1-\delta-\hat{\varphi}} > 0$$

where the inequality follows from the strict convexity of $e(s, m)$ with respect to m .

D.3 Proof of Proposition 5

Let ϵ be small enough so that $e(1, \epsilon) > v$. Note that such an ϵ must exist, as $e(1, \epsilon) \rightarrow \infty$ as $\epsilon \rightarrow 0$. Solving for the highest sustainable price when f is present, i.e., solving the problem given in (4), we obtain

$$\hat{p}^*((\kappa, \kappa^f); \delta) = \min \left\{ \frac{(1 - \delta)\hat{\varphi} \sum_{f \in \hat{F}} e(1, \kappa^f) - \hat{\varphi} e(1, k + \kappa^f)}{1 - \delta - \hat{\varphi}}, v \right\}.$$

Note that ϵ has been chosen to ensure that $f \notin \hat{F}$. Thus,

$$\hat{p}^*((\kappa, \kappa^f); \delta) - \hat{p}^*(\kappa; \delta) = \min \left\{ \hat{\varphi} \frac{e(1, k) - e(1, k + \kappa^f)}{1 - \delta - \hat{\varphi}}, v - \hat{p}^*(\kappa; \delta) \right\} > 0.$$

Finally, note that $\hat{\delta}((\kappa, \kappa^f); \delta) \geq \hat{\delta}(\kappa; \delta)$ as $\hat{p}^*((\kappa, \kappa^f); \delta) > \hat{p}^*(\kappa; \delta)$.