# Optimal Financing and Disclosure*

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**Abstract**

How does a firm's disclosure policy depend on its choice of financing? In this paper, I study a firm that finances a project with uncertain payoffs and jointly chooses its disclosure policy and the security issued. I show that it is optimal to truthfully reveal whether the project's payoffs are above a threshold. This class of threshold policies is optimal for any prior belief, for any security, and any increasing utility function of the entrepreneur. I characterize how the disclosure threshold depends on the underlying security, the prior, and the cost of investment. The security design is indeterminate despite the presence of adverse selection. Among others, the optimum can be implemented with equity, debt, and options.

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*JEL Codes: G32, D82, D83*

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# 1 Introduction

Firms that seek financing have discretion on how much information to disclose to investors. They have access to different communication channels which allow for potentially complex disclosure strategies, such as conference calls, voluntary earnings forecasts, IPO prospectuses, and annual reports. Empirical research has found evidence that all of these channels indeed convey information.[1] Firms also have access to a large set of securities they can issue. How disclosure affects investors' willingness to pay must depend on the security issued and how sensitive it is to the information provided. Likewise, the optimal security must depend on how the firm plans to disclose information. However, the interaction between disclosure and financing choice has so far not been studied.

In this paper, I characterize the joint choice of disclosure policy and security design when a firm finances a project with uncertain payoffs and adverse selection, without putting any a priori restrictions on the set of admissible disclosure strategies.[2] The two main findings are as follows.

First, the optimal strategy is to truthfully reveal whether the project's payoffs are above a threshold, which is chosen to maximize the likelihood the project is financed. This class of *threshold strategies* is optimal for any prior distribution, for any increasing utility function of the entrepreneur, and for any given security issued, even when additional noise is present. Importantly, this result rationalizes empirical findings. For example, Hutton et al. (2003) and Baginski et al. (2004) show that the majority of voluntary earnings forecasts are a range of realizations.

Second, if the firm can choose both the security and the disclosure strategy, the security issued is irrelevant as long as it provides a certain expected payoff to investors. Thus, the Modigliani-Miller irrelevance result holds partially despite adverse selection. The security's information sensitivity, a central concept in the literature on security design under adverse

[1] Bowen et al. (2002) find that analysts participating in conference calls have lower forecast error, Balakrishnan et al. (2014) find earnings guidance increases share liquidity, Hanley and Hoberg (2010) show that IPO prospectuses contain information which helps increase pricing accuracy, and Ball et al. (2014) show that the Management Discussion and Analysis section of a company's annual report contains information beyond that contained in earnings forecasts.

[2] Except measurability.

selection,[3] plays no role and debt is no longer uniquely optimal.

In my model, an entrepreneur designs a disclosure strategy, which is a mapping from the project's realized payoffs to a potentially random message. Investors form posterior beliefs about the payoff using Bayesian updating after observing the message. The price of the security is then determined competitively. The project is financed whenever the amount raised exceeds a fixed cost of investment, which occurs only if investors believe the project's payoffs are sufficiently high. The entrepreneur has a private benefit of control, so that she prefers to finance the project even if the payoffs are low.[4] In equilibrium, the optimal disclosure policy induces two possible posteriors, an "optimistic" and a "pessimistic" one.[5] The project is financed under the first, but not under the second.

A threshold strategy is optimal because the security's payoff is increasing in the project's payoff, which implies that the optimistic posterior must put all mass above a threshold. If its support were disconnected, the entrepreneur could induce a different posterior which moves mass from the lower realizations to fill the gap in the support, which would increase both the entrepreneur's and investors' valuation.

The threshold is chosen so that at the optimistic posterior, the financing constraint binds. Because of Bayesian updating, investors' beliefs must follow a martingale, which implies that inducing a more optimistic posterior must reduce the probability with which it is realized. Thus, there is a tradeoff between convincing investors that the project is good and the likelihood that it is financed. Manipulating the investors' beliefs can never increase the expected payments for the security due to the martingale property. Therefore, it is optimal to induce the least optimistic posterior at which investors are willing to pay enough to cover the cost of investment.

The optimal security is determined by the following tradeoff. Increasing the promised payoff to investors in some state decreases the entrepreneur's realized payoff whenever the project is financed. However, it relaxes the financing constraint, since investors are willing to finance

[3]See e.g. Myers and Majluf (1984), Nachman and Noe (1994), and DeMarzo and Duffie (1999) for the role of information sensitivity in models based on adverse selection. Fulghieri and Lukin (2001) explore the role of information sensitivity when investors can acquire information.

[4]Without this private benefit, full disclosure would be optimal.

[5]I call a posterior "more optimistic" than another if it first-order stochastically dominates it. Thus, under a more optimistic posterior, the expected payoffs to investors from any security are higher.

the project at a less optimistic posterior. Thus, increasing the payoff of the security allows the entrepreneur to choose a disclosure policy that increases the likelihood that the project is financed, which increases the entrepreneur's expected payoff. The second effect always dominates. It is optimal to sell off as much of the project as possible, until either investors hold all claims, or until the disclosure is such that the project is financed whenever it social benefit is positive.

Even though disclosure is imperfect and there is a adverse selection remaining after the message is observed, the particular security issued is irrelevant. The optimum can be implemented by equity, debt, call options, and many other securities. This is because the financing constraint must bind when the project is financed and investors are indifferent between any security that promises the same expected payoffs under the optimistic posterior. This result is in sharp contrast to the literature on security design under asymmetric information (e.g. Nachman and Noe (1994)).

My model builds on the Bayesian persuasion approach of Gentzkow and Kamenica (2011), who study a model in which a sender manipulates the belief of a receiver by revealing information. They show that the problem can be restated as directly maximizing over distributions of posterior beliefs subject to the *Bayes plausibility constraint*, which requires the average posterior to equal the prior. The optimal value is then characterized as the concave closure of the sender's value under any given posterior. In my paper, the state space is an interval and instead of characterizing the concave closure, I decompose the problem into characterizing the posteriors induced by the disclosure policy for a given probability that the project is financed, and then optimizing over the probability. The first problem is an infinite dimensional linear program and its solution can be found by characterizing the dual.

A large literature in economics, finance, and accounting studies optimal disclosure (seminal theoretical work includes Verrecchia (1983), Dye (1985), Fishman and Hagerty (1989), Fishman and Hagerty (1990), and Diamond and Verrecchia (1991)) between firms and investors.[6] My paper is, to the best of my knowledge, the first to characterize the optimal disclosure policy of a firm seeking to finance a project without any a priori restrictions on security

[6]The literature is too large to be surveyed here. Excellent recent reviews of both theoretical and empirical contributions can be found in Healy and Palepu (2001), Holthausen and Watts (2001), and Beyer et al. (2010).

issued, prior distribution, or disclosure strategy.

A recent related paper is Goldstein and Leitner (2015), who study the design of optimal stress tests in the Gentzkow and Kamenica (2011) setting with finitely many states. If banks observe their own type and the ex-ante value of bank capital is low, the optimal policy assigns two scores depending on the value of capital. This result is similar to the threshold strategy in my model, because disclosing whether or not the payoff is above or below the threshold is informationally equivalent to assigning a "high" or "low" score. If banks observe their own type, Goldstein and Leitner show that this result breaks down and the optimal disclosure rule may be non-monotone. In my paper, disclosure is done by the firms themselves and not a regulator so the latter case cannot be studied. Instead, I focus on the interactions between the security issued and the disclosure policy.

My setting also shares some features with Admati and Pfleiderer (2000), who study the disclosure decision of a firm selling an asset with a fixed gain from trade. In their setting, the prior is normal and the firm chooses the variance of a normally distributed signal. In the case of a single firm, the disclosure policy is socially optimal, since the firm internalizes all gains from trade. This is also the case in my model.

Monnet and Quintin (2015) study the effect of information disclosure on liquidity in private equity markets. They model disclosure as choosing a message which truthfully reveals whenever the state is in a given subset of the state space and they allow the sender to choose this subset ex ante. They show that under this restriction, the project is continued whenever it is revealed that its success probability is above a threshold.

My paper is one of the few studying the interaction between disclosure and the choice of financing. Boot and Thakor (2001) study a model in which the firm chooses whether or not to disclose information that is either a complement, independent, or a substitute to information that can be otherwise obtained by investors. The firm can also choose the security to issue, which is modeled as splitting the payoff into a senior and junior claim. The paper then characterizes conditions under which splitting the claims and providing disclosure is indeed an equilibrium for the different cases of information.

In contemporaneous work, Trigilia (2016) also considers a joint problem of security choice and disclosure. In his model, disclosure is costly and the entrepreneur chooses the probability with which the state is truthfully revealed. After investment has taken place, there is a moral hazard problem similar to Gale and Hellwig (1985). The entrepreneur can misreport output and investors can pay a cost to verify it. Depending on the parameters, the optimal incentive compatible contract is either debt or a mix of debt and equity. The likelihood of providing disclosure is interior and trades off the direct cost of disclosure versus the social loss from monitoring. The optimal security is not indeterminate precisely because of the addition of a moral hazard problem after disclosure has taken place.

Recently, there has been considerable empirical interest in the information content of firms' communication. Since information content is not directly measurable, existing papers have used observables such as analyst forecast errors, liquidity measures, or underpricing to quantify the degree of information. In my model, a higher threshold can be interpreted as providing more precise disclosure, because the mean squared error constructed from the investors' posterior and the conditional variance of the payoff are both lower. In this sense, my optimal disclosure policy maps into the approaches found in Bowen et al. (2002), who use analyst forecast errors as a measure of informativeness and Kogan et al. (2010), who use return volatility. If we interpret my model as an IPO, the issue price is the average value of the security under the posterior, since the price is determined competitively. Once the realized payoff becomes known, the price must adjust. For a higher disclosure threshold, the average magnitude of price changes after issuance is lower, which is in line with Hanley and Hoberg (2010), who use the magnitude realized price changes after an IPO to measure the information content of IPO prospectuses.

The paper proceeds as follows. In Section 2, I set up the model. In Section 3, I solve the optimal disclosure policy for the simple case when the state space is binary in order to establish a simple intuition for the tradeoffs involved. Readers familiar with the Bayesian persuasion literature following Gentzkow and Kamenica (2011) may wish to skip ahead to Section 4, where I solve for the optimal disclosure policy with a continuous state space. I derive the optimal security to issue in Section 5. In Section 6, I show that the threshold strategy remains optimal for any increasing, continuous utility function of the entrepreneur and consider the case when the entrepreneur cannot fully reveal the payoff due to exogenous noise. I also show that when the entrepreneur is risk-averse, the optimal security to issue is a call option, due to consumption smoothing. To ease exposition, all proofs are deferred to

Appendix A.

## 2 Model

A risk-neutral entrepreneur is endowed with a project with uncertain payoff $s \in [0,1]$. The project has a cost of investment $I \in (0,1)$, which the entrepreneur must raise by selling a security with payoffs $c(s)$ to a unit mass of risk-neutral investors. Both the investors' payoff $c(s)$ and the entrepreneur's residual $s - c(s)$ are increasing in $s$ and $c(s)$ is continuous.[7] Both parties are protected by limited liability, so that $c(s) \in [0, s]$.

The project's payoff is private information. Ex ante, both entrepreneur and investors have the same prior over $s$, which admits a continuous density $\mu_0(s)$ with support $[0,1]$. Before the state is realized, the entrepreneur designs a public signal $\sigma : [0,1] \to \Delta([0,1])$ which is observable by all investors and which she can condition on the realized state. A signal $\sigma$ sends a potentially random message $m \in [0,1]$ distributed with measure $\sigma(s)$.[8] Upon observing the realization, investors form a posterior belief about the state $\mu$ according to Bayes rule. The entrepreneur can commit to not alter the signal once the state is realized.[9]

Given posterior $\mu$, the price for the security $p(\mu)$ is set competitively via the investors' zero-profit condition

$$p(\mu) = E_\mu c(s). \tag{1}$$

The project is financed whenever the amount raised exceeds the investment cost $I$,[10]

$$p(\mu) \geq I. \tag{2}$$

In addition to the residual payoff $s - c(s)$, the entrepreneur enjoys a private benefit of

[7] This assumption is standard to ensure tractability in the security design literature. See e.g. Innes (1990), Nachman and Noe (1994), and Harris and Raviv (1989). Common securities such as equity, debt, and options satisfy the assumption.

[8] Here, $\Delta(S)$ is the set of all probability distributions over a set $S$. Following Gentzkow and Kamenica (2011), it is without loss of generality to assume that $m \in [0,1]$.

[9] Due to commitment, the unraveling result found in e.g. Grossman and Hart (1980), Grossman (1981), and Milgrom and Roberts (1986) does not hold. Also, it does not matter here whether $s$ is actually observed by the entrepreneur, only that the signal can be conditioned on it.

[10] With a unit mass of investors, $p(\mu)$ is simultaneously the price of the security and the amount raised.

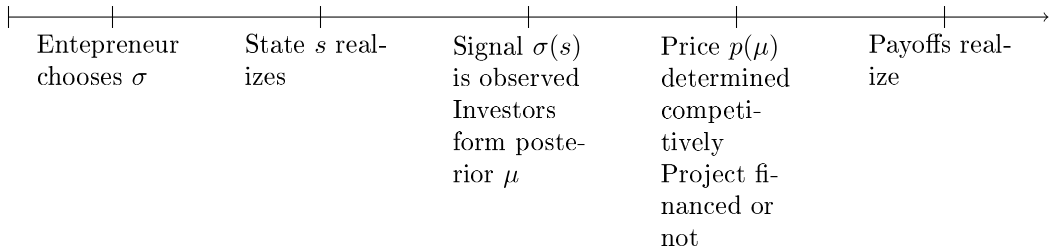


Figure 1: Timeline

control $B > 0$ whenever the project is financed and, in case the amount raised exceeds the cost of investment, she also receives a fraction of the excess cash raised $\lambda\left(E_{\mu}c\left(s\right) - I\right)$, where $\lambda \in [0,1]$ denotes a transaction cost. The entrepreneur's realized payoff is therefore

$$V\left(s,p\right) = s - c\left(s\right) + B + \lambda\left(p - I\right).$$

When $\lambda = 0$, the entrepreneur receives all additional cash while when $\lambda = 1$, the model is equivalent to the security being sold at a fixed price $I$.[11] Because of the private benefit, the entrepreneur would prefer to finance the project even if she knew that the payoffs are zero.[12] Whenever the project is not financed, both entrepreneur and investors receive zero. The timeline is summarized in Figure 1.

Conditional on posterior $\mu$, the expected payoff to the entrepreneur is

$$V\left(\mu\right) = \begin{cases} E_{\mu}\left[s - c\left(s\right) + B\right] + \lambda\left(E_{\mu}c\left(s\right) - I\right) & \text{if } E_{\mu}c\left(s\right) \geq I \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

while each investor's payoff is

$$W\left(\mu\right) = \begin{cases} E_{\mu}c\left(s\right) - I & \text{if } E_{\mu}c\left(s\right) \geq I \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

[11] For example, the entrepreneur might bee seeking commitment from investors to buy the security by offering a fixed price, in which case the investors appropriate the surplus.

[12] This is the reason why truthful disclosure is not optimal in equilibrium.

The entrepreneur's problem is to choose a signal to maximize her expected payoff, subject to the financing constraint (2) and competitive pricing (1). Each signal $\sigma$ induces a distribution over posterior beliefs $q \in \Delta(\Delta([0,1]))$.[13] According to Gentzkow and Kamenica (2011), Prop. 1, the entrepreneur's problem of choosing the optimal message is equivalent to choosing the distribution over posteriors, subject to the *Bayes plausibility constraint*

$$E_q \mu = \mu_0. \tag{5}$$

Intuitively, given any signal $\sigma$, the investors' posterior belief must be a martingale because of Bayesian updating, which implies condition (5). Conversely, it can be shown that for any distribution of posteriors satisfying the condition, there exists a signal inducing it. The entrepreneur's problem is therefore equivalent to

$$\begin{aligned} V^*(\mu_0) &= \max_{q \in \Delta(\Delta([0,1]))} E_q V(\mu) \\ & s.t. \quad E_q \mu = \mu_0. \end{aligned} \tag{6}$$

Throughout the paper, I will say that posterior $\mu'$ is *more optimistic* than posterior $\mu$ if $\mu'$ first-order stochastically dominates $\mu$ and *less optimistic* if the opposite holds. The interpretation is natural since if investors are "more optimistic" about the project in this sense, their expected payoffs from any security must be weakly larger.

In Sections 3 and 4, I assume the entrepreneur takes the security $c(.)$ as given and I characterize the optimal disclosure policy. In Section 5, I allows the entrepreneur to jointly choose the security and disclosure policy.

[13] Intuitively, suppose that $s \in \{0,1\}$ and the prior probability that $s=1$ is $\mu_0$. Then the posterior probability that the state is 1 given message $m$ is

$$\mu(m,\sigma) = \frac{\sigma(m|1)\mu_0}{\sigma(m|1)\mu_0 + \sigma(m|0)(1-\mu_0)},$$

which is a random variable since it depends on $m$. The distribution over posteriors induced by $\sigma$ is

$$q(\mu|\sigma) = \sum_{m:\mu(m,\sigma)=\mu} \sigma(m|1)\mu_0 + \sigma(m|0)(1-\mu_0).$$

# 3 Binary State Space

To gain intuition, consider first the case of a binary state $s \in \{0, 1\}$. The main results developed in this section will carry over to the general case in Section 4.

To save notation, I denote the prior and posterior probability that $s = 1$ with $\mu_0$ and $\mu$ respectively and I write $c(1) = c$, since limited liability implies that $c(0) = 0$. Given posterior $\mu$, the price is now

$$p(\mu) = \mu c$$

and the financing condition is

$$\mu c \geq I. \tag{7}$$

The project is financed whenever the investors' belief that the payoff is high exceeds a certain threshold, which is given by

$$\bar{\mu} = \frac{I}{c}. \tag{8}$$

I assume throughout this section that $I \leq c$ so that $\bar{\mu} \in (0, 1]$.[14] The entrepreneur's utility is

$$V(\mu) \quad = \quad \mu(1 - c + B) + (1 - \mu) B + \lambda(\mu c - I) \tag{9}$$

if $\mu \geq \bar{\mu}$ and zero otherwise.

The entrepreneur's optimization problem becomes

$$\begin{aligned} V^*(\mu_0) \quad &= \quad \max_q E_q \left[ \mathbf{1}\{\mu \geq \bar{\mu}\} \left( \mu(1 - c) + B + \lambda(\mu c - I) \right) \right] \\ s.t. \quad & E_q \mu = \mu_0 \end{aligned} \tag{10}$$

If $\mu_0 \geq \bar{\mu}$, not disclosing any information is optimal, i.e. $\mu = \mu_0$ with probability one. Intuitively, the project can be financed without providing any further information. Since the amount raised from investors is increasing in the posterior, the entrepreneur might provide disclosure in order to raise the expected amount paid for the security. However, such scheme

[14] Since $I > 0$ and $c \leq 1$, $\bar{\mu} > 0$. If $I > c$, we have $\bar{\mu} > 1$ which implies the project is never financed.

can never generate any profit. Under Bayesian updating, the average posterior has to equal the prior, which is reflected by the Bayes plausibility condition (5), and therefore the entrepreneur can never provide a signal that alters the average value of the security from an ex-ante perspective.[15]

If $\mu_0 < \bar{\mu}$, the project is not financed if the entrepreneur does not provide any information. The entrepreneur has to induce a posterior above $\bar{\mu}$, but is constrained by Bayes plausibility. For simplicity, consider a policy that randomizes between two posteriors $\{\mu_l, \mu_h\}$ with $\mu_l < \mu_0 < \bar{\mu} < \mu_h$. Let $q$ the the probability that $\mu_h$ is realized, which is also the probability that the project is financed. The entrepreneur's expected payoff is

$$V = q\left(\mu_h\left(1-c\right) + B + \lambda\left(\mu_h c - I\right)\right) + \left(1-q\right)\cdot 0 \tag{11}$$

and the Bayes plausibility constraint becomes

$$q\mu_h + \left(1-q\right)\mu_l = \mu_0. \tag{12}$$

If the entrepreneur wants to induce a higher posterior belief $\mu_h' > \mu_h$ or $\mu_l' > \mu_l$, the probability that this belief is realized must decrease, otherwise the constraint is violated. Thus, there is an indirect cost of inducing a higher belief, since it decreases the likelihood the project is financed.

Because the entrepreneur's payoff is zero for all $\mu_l < \bar{\mu}$, an optimal policy should choose $\mu_l = 0$ to maximize the probability that the project is financed. Then, constraint (12) becomes

$$q\mu_h = \mu_0$$

and the entrepreneur's value is

$$V = \mu_0\left(1-c\right) + \lambda\left(\mu_0 c - qI\right) + qB.$$

The first terms measure the expected residual payoff for the entrepreneur and the excess cash raised from investors, while the last term measures the expected private benefit of control.

[15]Importantly, this argument extends to the case with a continuous state space. By Bayes plausibility $E_q\mu = \mu_0$ and therefore for any security $c\left(s\right)$, $E_q\left[E_\mu c\left(s\right)\right] = E_{\mu_0} c\left(s\right)$.

Increasing $\mu_h$ does not change the expected residual payoff or the expected amount of money raised from investors due to Bayes plausibility, but it decreases the probability that the project is funded and the expected private benefit.[16] Therefore, the optimal policy chooses the lowest possible $\mu_h$ and induces posteriors $\mu_l = 0$ and $\mu_h = \bar{\mu}$. That is, either investors believe with certainty that the project is bad, or their belief is the lowest one that allows the project to be financed given $c$. Since at $\bar{\mu}$, the financing constraint (7) binds, the entrepreneur receives no cash from investors in excess of the investment cost.

The equilibrium probability that the project is financed is then

$$q = \frac{\mu_0}{\bar{\mu}} = \mu_0 \frac{c}{I},$$

which is increasing in the prior $\mu_0$ and the cash promised to investors, and decreasing in the cost of investment. The expected payoff to the entrepreneur is

$$V^* (\mu_0) \quad = \quad \mu_0 (1 - c) + \frac{\mu_0}{\bar{\mu}} B. \tag{13}$$

In Proposition 1, I verify that this policy is optimal.

**Proposition 1.** *If $\mu_0 \geq \bar{\mu}$, the optimal policy provides no disclosure and the project is financed with probability one. If $\mu_0 < \bar{\mu}$, the optimal value of the entrepreneur's problem (10) is given by equation (13). The optimal policy induces posterior $\bar{\mu}$ with probability $\frac{\mu_0}{\bar{\mu}}$ and 0 with probability $1 - \frac{\mu_0}{\bar{\mu}}$.*

The proof is in Appendix A. Intuitively, it relies on exploiting the fact that expectations are linear in probabilities. Take any distribution over posteriors $q$, which puts mass $q([\bar{\mu}, 1])$ on the range of beliefs for which the project is financed and has conditional expectation $E_q (\mu | \mu \geq \bar{\mu})$. Due to linearity of the expectations operator, this distribution achieves the same payoffs as one that puts point mass $q' = q([\bar{\mu}, 1])$ on posterior $\mu_h = E_q (\mu | \mu \geq \bar{\mu})$. It is then without loss of generality to consider a policy which puts mass on only two posteriors. Above, I have characterized the optimal policy among those putting mass on only two posteriors, which must then be optimal among all policies.

The optimal policy and value are shown in Figure 2. The blue line is $V(\mu)$ while the black

[16] Note that $\mu_h c \geq I$ and $q = \frac{\mu_0}{\mu_h}$ imply $\mu_0 c - qI \geq 0$, so the middle term is positive.

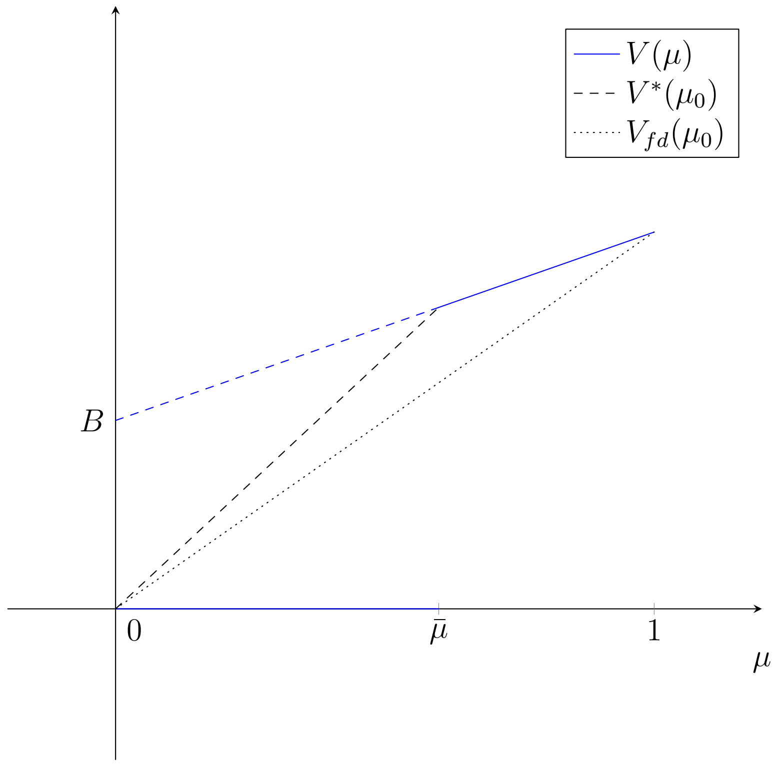


Figure 2: Optimal Value for $B > 0$

dashed line shows the payoff from the optimal policy conditional on the prior. Full disclosure is generally suboptimal. Under full disclosure, the posterior is

$$\mu = \begin{cases} 1 & w.Pr. \quad \mu_0 \\ 0 & w.Pr. \quad 1 - \mu_0 \end{cases}$$

and the entrepreneur's value is

$$V_{fd}(\mu_0) = \mu_0 \left( (1 - c) + B + \lambda (c - I) \right) < V^* (\mu_0) \, .$$

In Figure 2, the black dotted line shows the payoff from full disclosure.

The optimal policy can be implemented by sending as signal $\sigma(m|s)$ with two realizations, $m = H$ and $m = L$, which induce posteriors $\mu_H$ and $\mu_L$. Since the low posterior is zero, the signal must reveal truthfully whenever $s=1$, i.e. $\sigma(H|1)=1$ so that upon observing $L$ investors conclude the payoffs are low. Since the high posterior is below one, the signal may send a false positive when $s=0$ with a certain probability, i.e. $\sigma(H|0)>0$. The exact probability is characterized in the proposition below. The proof is in Appendix A.

**Proposition 2.** *The optimal policy can be implemented by a signal $\sigma(m|s)$ with $m\in\{H,L\}$ such that $\sigma(H|1)=1$ and $\sigma(H|0)=\frac{\mu_0}{\bar{\mu}}\frac{1-\bar{\mu}}{\mu_0}$.*

The optimal disclosure policy and value depend on $c$, the amount promised to investors in the good state. If $c$ increases, the entrepreneur gets a lower payoff if the project is financed. However, investors need to be less optimistic for the project to be financed, since they receive a higher payoff in the case of success.[17]

Formally, we have

$$\frac{dV}{dc}=\mu_0\frac{B-I}{I}.$$

Whenever the private benefit of control is sufficiently large, so that $B>I$, the second effect dominates. It is optimal to sell off as much of the project as possible, until either $c=1$ or $\bar{\mu}=\mu_0$. In the first case, the entrepreneur sells off the entire project and provides imperfect disclosure, while in the second case, she sells off just enough so that the project is financed with certainty without providing any information to investors. The first case occurs whenever $\mu_0<I$ so that at $c=1$ and $\mu=\mu_0$, the financing constraint (7) is violated, while the second case occurs when $I\leq\mu_0$ .

If the private benefit of control is small and $B<I$, the first effect dominates and it is optimal to sell off the smallest possible share such that the project still can be financed for some posteriors. This is achieved by setting $c=I$, so that the financing constraint holds only if $\mu=1$. Thus, the entrepreneur raises as little financing as possible and optimally provides full disclosure.

Finally, when $B\leq 0$, the function $V(\mu)$ is convex. Then, full disclosure is optimal for any $c$ and the optimal financing is to again set $c=I$. Intuitively, if $B>0$ the entrepreneur wants to maximize the probability that the project is financed to get the private benefit while investors only want to finance if the state is good. If $B<0$, the entrepreneur's and

[17] This can be seen from equation (8), which shows that $\bar{\mu}$ is decreasing in $c$, so that the policy that the project is financed, $\frac{\mu_0}{\bar{\mu}}$ is increasing in $c$.

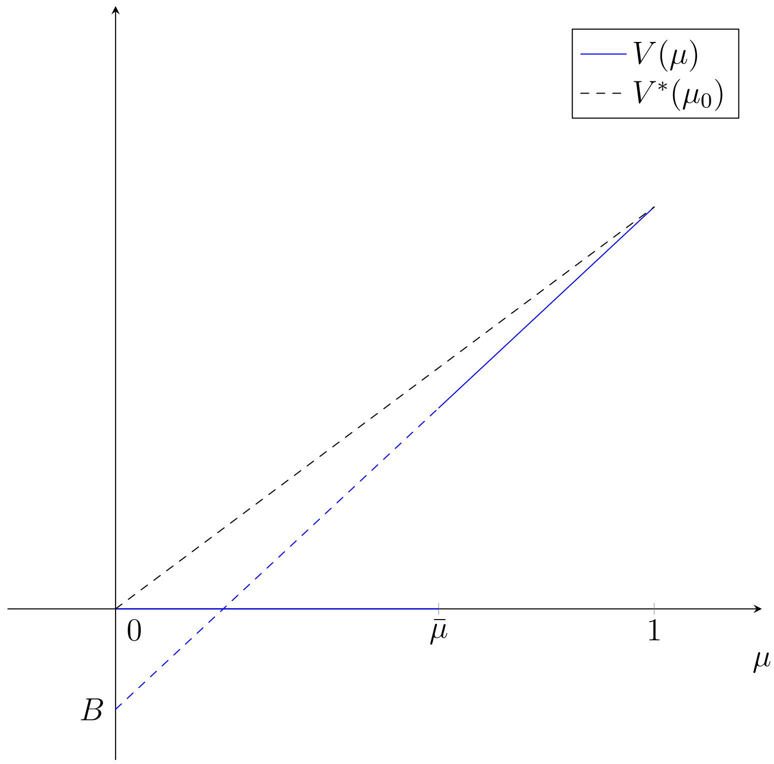


Figure 3: Optimal Value for $B \leq 0$

investors' preferences are aligned, since if the state is bad, the entrepreneur gets a negative payoff. This case is shown in Figure 3.

# 4 Continuous State Space

While the binary case allows for studying the effect of the amount promised to investors in the good state on the entrepreneur's value and the probability that the project is financed, it does not allow to study how to optimally design securities since all securities $c(s)$ are determined by the parameter $c$. In this section, I return to the case $s \in [0, 1]$ and I derive the optimal disclosure policy given any security $c(s)$. In Section 5, I determine the security which maximizes the entrepreneur's value.

As in the binary state case of Section 3, if the project can be financed without disclosure, then not providing any is optimal. I therefore assume $E_{\mu_0} c(s) < I$ throughout this section.

Deriving the optimal policy follows the same logic as in Section 3. In Lemma 3 below, I show that without loss of generality, the optimal policy puts all mass on two posteriors, one such that the project is financed and one such that it is not. I then characterize the optimal posteriors and show that the financing constraint must bind whenever the project is financed. I provide only heuristic arguments in this Section. The full proof is deferred to Appendix A.

**Lemma 3.** *Without loss of generality, the optimal policy $q$ puts all mass on two posteriors $\mu_h$, $\mu_l$ with $E_{\mu_l} c(s) < E_{\mu_0} c(s) < I \leq E_{\mu_h} c(s)$. $\mu_h$ and $\mu_l$ admit a pdf.*

The intuition for the result is the same as in Section 3. To simplify notation, let $q$ denote the probability that $\mu_h$ is realized. In the following, I identify $\mu_0$, $\mu_h$, and $\mu_l$ with their densities $\mu_0(s)$, $\mu_h(s)$, and $\mu_l(s)$. The Bayes plausibility constraint can now be written more intuitively as

$$\mu_0(s) = q\mu_h(s) + (1-q)\mu_l(s) \tag{14}$$

for all $s \in [0,1]$, the financing condition becomes

$$\int_0^1 c(s)\mu_h(s)\,ds \geq I, \tag{15}$$

and the entrepreneur's problem can be written as

$$V = \max_{q,\mu_h(.),\mu_l(.)} q\int_0^1 (s - c(s) + B + \lambda(c(s) - I))\mu_h(s)\,ds \tag{16}$$

subject to the financing condition (15), the Bayes plausibility condition (14), and $\mu_h(s)$ and $\mu_l(s)$ being nonnegative and integrating to one.

As in the binary state case, the principal should maximize the probability that the project is financed, since no Bayes plausible distribution over posteriors can increase the ex-ante expected payoff from investors. However, simply setting $\mu_l(s) = 0$ for all $s$ is no longer optimal with a continuous state, since via equation (14), this would imply that $q = 1$ and $\mu_h(s) = \mu_0(s)$ for all $s$. Then, the project would never be financed, since by assumption $E_{\mu_0} c(s) < I$.

Inspecting the entrepreneur's problem (16), we can see that holding $q$ fixed, choosing $\mu_h$ and $\mu_l$ is an infinite-dimensional linear programming problem. Using the Bayes plausibility constraint, we can substitute

$$\mu_l(s) = \frac{\mu_0(s) - q\mu_h(s)}{1-q}. \tag{17}$$

Then, the problem reduces to choosing $\mu_h(s)$ subject to both $\mu_h$ and $\mu_l$ being probability densities. Since the problem is linear, it is intuitive to conjecture that the optimal policy is bang-bang. Either is $\mu_h(s) = 0$ or $\mu_h(s)$ is set as large as possible. Via condition (17) this implies the optimal policy takes the following form

$$\begin{aligned} \mu_h(s) &= \begin{cases} \frac{\mu_0(s)}{q} & \text{for } s \in \hat{S} \\ 0 & \text{otherwise} \end{cases} \\ \mu_l(s) &= \begin{cases} 0 & \text{for } s \in \hat{S} \\ \frac{\mu_0(s)}{1-q} & \text{otherwise} \end{cases} \\ q &= \int_{\hat{S}} \mu_0(s)\, ds. \end{aligned} \tag{18}$$

Here, $\hat{S} \subset [0,1]$ is the set of states for which $\mu_h(s)$ is nonzero. To determine the form of $\hat{S}$, consider the marginal contribution of including some state $s$ in $\hat{S}$ to the entrepreneur's objective, which is is

$$s - c(s) + B + \lambda(c(s) - I) + \gamma((c(s) - I)). \tag{19}$$

The first term is the contribution to the entrepreneur's value, and the second term is the financing constraint, weighted by a Lagrange multiplier $\gamma > 0$. Since $\lambda \in [0,1]$, and both the payoff to investors $c(s)$ and the entrepreneur's residual $s - c(s)$ are increasing in $s$, the value is increasing in $s$ as well. At the optimum, a state must be included in $\hat{S}$ whenever the above expression is positive, and therefore

$$\hat{S} = [\hat{s}, 1]. \tag{20}$$

Intuitively, if $\mu_h(s)$ puts weight on $[s', s'']$ and $[\hat{s}, 1]$, then we can move the mass from the lower interval and append it to the upper one. Since both the entrepreneur's and investors' values are increasing in $s$, this change improves the objective and relaxes the financing constraint.

Thus, $\mu_h(s)$ puts mass on all states above a cutoff $\hat{s}$ while $\mu_l(s)$ puts mass only on states below the cutoff.

The argument so far has been heuristic. In the proposition below, which is proven in Appendix A, I establish that the solution above is indeed optimal. The argument relies on characterizing the dual to problem (16) and showing that the solutions coincide.

**Proposition 4.** *For a given $q \in (0,1)$, suppose that the problem*

$$V^*(q) = \max_{\mu_h(.),\mu_l(.)} q \int_0^1 (s - c(s) + B + \lambda (c(s) - I)) \mu_h(s) \, ds \tag{21}$$

*subject to financing condition (15), Bayes plausibility constraint (14), and*

$$\begin{aligned} \int_0^1 \mu_h(s) \, ds &= \int_0^1 \mu_l(s) \, ds = 1 \\ \mu_h(s), \mu_l(s) &\geq 0 \; \forall s \in [0,1] \end{aligned}$$

*has a solution. Then, the optimal policy is given by (18) and $\hat{S} = [\hat{s}, 1]$.*

To find the optimal value $V^*$, it remains to optimize over $q$. As in the binary state case, the entrepreneur wants to maximize the likelihood that the project is financed, and $V^*(q)$ is increasing in $q$. There is also again a tradeoff between inducing "high" beliefs and the likelihood that the project is financed, since when $\hat{s}$ increases, i.e. the high posterior only puts weight on higher values, $q$ must decrease.

Given the functional form of $\mu_h$, the financing constraint becomes

$$\int_{\hat{s}}^1 (c(s) - I) \mu_0(s) \, ds \geq 0. \tag{22}$$

Increasing $q$ must decrease $\hat{s}$ at the optimum, which in turn must decrease the expected payoff to investors. The optimal $q$ is therefore the one which makes the financing constraint bind.[18] The following Proposition summarizes the results.

**Proposition 5.** *If $E_{\mu_0} c(s) < I$, the optimal value $V^*$ satisfies*

$$V^* = \int_{\hat{s}}^{1} (s - c(s) + B) \mu_0(s) \, ds, \tag{23}$$

*the optimal policy is given by*

$$\begin{aligned} \mu_h(s) &= \begin{cases} \frac{\mu_0(s)}{q} & \text{for } s \in [\hat{s}, 1] \\ 0 & \text{otherwise} \end{cases} \\ \mu_l(s) &= \begin{cases} 0 & \text{for } s \in [\hat{s}, 1] \\ \frac{\mu_0(s)}{1-q} & \text{otherwise,} \end{cases} \end{aligned} \tag{24}$$

*and $\hat{s}$ and $q$ are determined by*

$$\begin{aligned} 0 &= \int_{\hat{s}}^{1} (c(s) - I) \mu_0(s) \, ds \\ q &= \int_{\hat{s}}^{1} \mu_0(s) \, ds. \end{aligned}$$

*If $E_{\mu_0} c(s) \geq I$, then $\mu_h(s) = \mu_l(s) = \mu_0(s)$ $\forall s$ and*

$$V^* = \int_{0}^{1} (s - c(s) + B + \lambda (c(s) - I)) \mu_0(s) \, ds.$$

The optimal posteriors put either all mass on $s \geq \hat{s}$ or all on $s < \hat{s}$. Since

$$\mu_h(s) = \frac{\mu_0(s)}{\int_{\hat{s}}^{1} \mu_0(s) \, ds}$$

is the posterior probability conditional on $s \geq \hat{s}$ and $\mu_l(s)$ is the posterior probability conditional on $s < \hat{s}$, the optimal disclosure policy can be implemented by truthfully revealing whether the state is in $[\hat{s}, 1]$ or $[0, \hat{s})$.

[18] I show in Appendix A that for any higher values of $q$, problem (16) does not admit a feasible solution since the financing constraint can never by satisfied at $\mu_h$.

**Proposition 6.** *The optimal disclosure policy truthfully reveals whether $s \geq \hat{s}$ or $s < \hat{s}$.*

As in the binary case, full disclosure is generally suboptimal. Under the full disclosure policy, the posterior is $\mu(s') = 1$ if $s' = s$ and zero otherwise. The probability that the project is financed is

$$q_{fd} = \Pr(c(s) \geq I|\mu_0) = \int_{\underline{s}}^{1} \mu_0(s)\, ds$$

where the threshold $\underline{s}$ satisfies $c(\underline{s}) = I$. The distribution over posteriors is simply the ex-ante distribution over states $\mu_0$. Then, the ex-ante value to the entrepreneur is

$$V_{fd} = \int_{\underline{s}}^{1} (s - c(s) + B + \lambda(c(s) - I))\, \mu_0(s)\, ds$$

which is strictly lower than the optimal value, since $\hat{s} < \underline{s}$.

In the following, I illustrate how restricting the disclosure policy ex-ante can lead to vastly different results. In both examples, full disclosure is uniquely optimal for *any* security, even though it is never optimal in the general case.

**Example 7.** Suppose $E_{\mu_0} c(s) < I$ and consider the following disclosure policy. The entrepreneur chooses the probability $q$ with which the state is revealed truthfully. Specifically, the policy sends message $m = s$. With probability $1 - q$, the policy sends an uninformative message, say $m = \emptyset$. The posteriors conditional on message $m = s$ is

$$\mu(s|m) = \begin{cases} 1 & \text{if} \quad m = s \\ 0 & \text{if} \quad m \neq s \\ \mu_0(s) & \text{if} \quad m = \emptyset. \end{cases}$$

The financing constraint conditional on $m$ becomes

$$c(m) \geq I.$$

Thus, the project is financed if and only if the policy sends message $m \geq \hat{s}$, where $\hat{s}$ is such that the above inequality binds. If the policy sends the uninformative message, the project is not financed. Therefore, the entrepreneur's value is

$$V = q \int_{\hat{s}}^{1} (s - c(s) + B) \mu_0(s) \, ds,$$

which is strictly increasing in $q$. Full disclosure is optimal because any other policy reduces the likelihood the project is financed successfully.

**Example 8.** Suppose again that $E_{\mu_0} c(s) < I$ and assume for simplicity that the prior is uniform. Consider the policy which with probability $q$ reveals the state truthfully and with probability $1-q$ sends a random message $m = s'$, which is drawn uniformly from the state space. The posterior conditional on message $m = s$ is

$$\mu(s|m) = \begin{cases} q & \text{if} \quad m = s \\ (1-q) & \text{if} \quad m \neq s. \end{cases}$$

Conditional on message $m$, the financing constraint is

$$qc(m) + (1-q) \int_0^1 c(s) \, ds \geq I.$$

There exists a cutoff $\hat{s}$ at which the financing constraint binds. The project is financed whenever the policy sends a message $m \geq \hat{s}$. The ex-ante distribution over messages is uniform and the entrepreneur's value is

$$V = \int_{\hat{s}}^{1} (s - c(s) + B) \, ds.$$

Since $\hat{s}$ is decreasing in $q$, full disclosure is again optimal. Intuitively, with probability $1-q$, the policy sends a random message under which in expectation, the project is not financed. The optimal policy minimizes that probability.

The optimal disclosure policy depends on the prior $\mu_0$ and the security issued, but it is independent of the particular value of the transaction cost $\lambda$. The following propositions summarize the comparative statics with respect to the prior and the security.

**Proposition 9.** *Consider two priors* $\mu_0'(s)$ *and* $\mu_0(s)$ *such that for both the project cannot be financed without disclosure, i.e.* $E_{\mu_0}c(s) < I$ *and* $E_{\mu_0'}c(s) < I$. *If* $\mu_0'$ *first-order stochastically dominates* $\mu_0$, *then* $\hat{s}' < \hat{s}$. *The equilibrium probability that the project is financed is higher under* $\mu_0'$.

Intuitively, given any prior, the optimal policy discloses truthfully whether $s \geq \hat{s}$ and $\hat{s}$ is chosen to make the financing constraint bind. If $\mu_0'$ first-order stochastically dominates $\mu_0$, the expected payoffs of investors are higher for any threshold $\hat{s}$. The entrepreneur can then optimally provide a signal that induces a less optimistic posterior and increase the probability that the project is financed. In general, second-order stochastic dominance has an ambiguous effect on the optimal disclosure policy and the likelihood of financing.

**Proposition 10.** *Fix the prior* $\mu_0$ *and consider two securities* $c(s)$ *and* $c'(s)$ *with* $E_{\mu_0}c(s) < I$ *and* $E_{\mu_0}c'(s) < I$. *If*

$$\int_{\hat{s}}^{1} \left(c'(s) - c(s)\right)\mu_0(s) > 0$$

*then* $\hat{s}' < \hat{s}$. *The probability that the project is financed is higher under* $c'(s)$.

*Sufficient conditions are (1)* $c'(s) > c(s)$ *for all* $s$ *and (2)* $\mu_0(s)$ *is increasing in* $s$,

$$\int_0^1 c'(s)\,ds \geq \int_0^1 c(s)\,ds,$$

*and for all* $x \in [0,1]$

$$C'(x) \leq C(x)$$

*where* $C'(x) = \frac{1}{\int_0^1 c'(s)ds}\int_0^x c'(s)\,ds$ *and* $C(x) = \frac{1}{\int_0^1 c(s)ds}\int_0^x c(s)\,ds$.

Intuitively, if $c'$ promises higher payoffs to investors, then the financing constraint is relaxed. The optimal posterior therefore has to be less optimistic for the project to be financed, so $\hat{s}' < \hat{s}$ and likelihood that the project is financed increases.

# 5 Optimal Security Design

In the previous section, I have shown that for any given security and any prior distribution, it is optimal to truthfully reveal whether or not the payoff is above a certain threshold, which is chosen such that that financing constraint binds whenever the project is financed. In this section, I allow the entrepreneur to jointly choose the security and the disclosure policy and I show that the optimal security design is indeterminate.

Promising additional payoffs to investors has two effects on the entrepreneur's optimal value. It lowers her realized payoff, but since investors who are promised higher payoffs are willing to finance the project at a less optimistic posterior, it also increases the likelihood that the project is financed. Whether or not it is optimal to promise higher payoffs in a certain state to investors depends on which effect dominates.

Mirroring the results in Section 3, if $B \geq I$, it is optimal to sell off as much of the firm as possible, until either the project gets financed without providing disclosure, or the entrepreneur sells the entire project. Intuitively, if the benefit of control is sufficiently high, it is always worth increasing the payoff of the investors to increase the likelihood the firm gets financed.

If $B < I$, there are two cases. If the cost of investment is sufficiently high, the entrepreneur again sells off the entire project, while when the cost is low, the tradeoff between obtaining financing and giving up payoffs is resolved at an interior point at which the entrepreneur retains some cash flow rights.

Given security $c(s)$, the optimal value is

$$V = \int_{\hat{s}}^{1} (s - c(s) + B) \mu_0(s) \, ds$$

and $\hat{s}$ solves

$$\int_{\hat{s}}^{1} (c(s) - I) \mu_0(s) \, ds = 0. \tag{25}$$

Suppose that $B < I$ and $c(s) < s$ for some $s$. Increasing $c(s)$ decreases the threshold $\hat{s}$, which follows from equation (25). Thus, the optimal disclosure strategy induces a less optimistic posterior and the likelihood that the project is financed increases. The total effect on the entrepreneur's value is[19]

$$\begin{aligned}\frac{dV}{dc(s)} &= -\mu_0(s) - (\hat{s} - c(\hat{s}) + B)\,\mu_0(\hat{s})\,\frac{d\hat{s}}{dc(s)} \\ &= \frac{\mu_0(s)}{-(c(\hat{s}) - I)}(\hat{s} - I + B),\end{aligned}$$

which follows from equation (25) and the implicit function theorem.

If $\hat{s} > I - B$, the entrepreneur's value is increasing in $c(s)$ for all $s$, so it is optimal to increase the payout to investors for all states, until either the entire project is sold, i.e. $c(s) = s$ for all $s \geq \hat{s}$ and $\hat{s} > I - B$, or until the threshold $\hat{s}$ reaches $I - B$. If $\hat{s} < I - B$, then the entrepreneur's value is decreasing in $c(s)$ for all $s$, so it is optimal to decrease the payout until $\hat{s} = I - B$. The following Proposition provides sufficient conditions for the different cases.

**Proposition 11.** *Suppose $B < I$. If selling the project implies it can be financed without disclosure, i.e. $E_{\mu_0}s \geq I$, then $\hat{s} = I - B$ for all $I$. If not, then for any $I$, there exists a $B(I)$ such that for $B > B(I)$, $\hat{s} > I - B$ and $c(s) = s$, and for $B \leq B(I)$, $\hat{s} = I - B$.*

The intuition is as follows. By designing the security, the entrepreneur can extract part of the social surplus of the project. The best possible outcome is to maximize the social surplus, which happens precisely when $\hat{s} = I - B$, i.e. the project is financed if and only if the social surplus is positive, and then extract it. If $E_{\mu_0}s \geq I$, selling off the entire project implies that it can be financed with certainty without providing any disclosure, which guarantees that $\hat{s} = I - B$ is feasible.

If this is not true, and $B$ is large, at $\hat{s} = I - B$, the financing constraint is violated. It is then optimal to sell off as much of the project as possible since the benefit of increasing the likelihood the project gets financed outweighs the loss of promising more money to investors. If $B$ is small, setting $\hat{s} = I - B$ does not violate the financing constraint, so it is again optimal.

Finally, if the private benefit is sufficiently large, so that $B \geq I$, the entrepreneur's payoff is positive for any $s$. Then, it is optimal to promise enough payoff to investors so the project

[19] Notice that $c(\hat{s}) < I$. If $c(\hat{s}) \geq I$, then $c(s) \geq I$ for all $s \in \hat{S}$ and the financing constraint is slack, which cannot be optimal.

can be financed without further disclosure, or, if this is not feasible, to sell off the entire project.

**Proposition 12.** *Suppose $B \geq I$ and there exists a security $c'(s)$such that $E_{\mu_0} c'(s) = I$. Then that security is optimal and $\hat{s} = 0$. If $E_{\mu_0} s < I$, then $c(s) = s$ and $\hat{s} > 0$.*

The result that the optimal security is indeterminate depends crucially on not restricting the set of admissible disclosure policies. If we restrict the policy ex-ante, debt may dominate equity, as the following example illustrates.

**Example 13.** Take the disclosure policy from example 7, i.e. with probability $q$, the state is disclosed truthfully, while with probability $1-q$, the policy sends an uninformative message. I have shown that full disclosure is optimal in this case. Now, I show that for this policy, debt can dominate equity. For simplicity, I assume that the prior is uniform and that $I = \frac{3}{4}$ and $B = \frac{1}{2}$. Also, I set $\lambda = 0$ to ease notation. Consider a contract offering equity share $\alpha \in [0,1]$. The project is financed whenever a state is disclosed such that

$$\alpha s \geq I,$$

which implies $\hat{s} = \frac{I}{\alpha}$. The entrepreneur's value becomes then

$$V_E = \int_{\frac{I}{\alpha}}^{1} \left( (1-\alpha) s + B \right) ds,$$

which is increasing in $\alpha$ given the assumptions on $B$ and $I$. Thus, the optimal equity share is $\alpha = 1$. It is optimal to sell off the entire project to maximize the likelihood the project is financed. To see why debt dominates, consider a debt contract with promised return $I$. The project is financed whenever $s \geq I$, since for $s < I$, investors do not break even. The entrepreneur's value is therefore

$$\begin{aligned} V_D &= \int_I^1 (s - I + B)\, ds \\ &> \int_I^1 B ds = V_E. \end{aligned}$$

Debt dominates equity because it allows the entrepreneur to retain a larger residual share conditional on the same likelihood of financing.

Compare this to the optimal policy, which discloses truthfully whenever $s \geq I - B$. The optimal equity share is characterized by the financing constraint

$$\int_{I-B}^{1} (\alpha s - I)\, ds = 0,$$

which is a version of equation (25), and solves

$$\alpha = 2I \frac{1 - (I - B)}{1 - (I - B)^2}.$$

The optimal debt contract features risky debt, i.e. $I - B < R$, where $R$ is the promised return, and solves

$$\int_{I-B}^{1} \min(s, R)\, ds = 0.$$

The optimal promised return can be computed as

$$R = 1 - \sqrt{1 + (I^2 - B^2) - 2I}$$

and we can verify that the entrepreneur's value is indeed the same for equity and debt.

# 6 Extensions

## 6.1 General Utility Functions

In this section, I show that the threshold strategy of Section 4 remains optimal for any increasing utility function of the entrepreneur, independently of different specifications of the transaction cost.

Specifically, suppose the entrepreneur's utility is increasing and continuous in the residual payoff of the project, and linear in the excess cash raised from issuing the security. In addition, assume that the transaction cost may depend on the amount of excess cash raised, so that with slight abuse of notation,

$$\lambda(x) \in [0, x]$$

for $x \in \mathbb{R}_+$ is the amount of excess cash that can be retained by the entrepreneur, which I assume is increasing in $x$. Her realized payoff now equals[20]

$$V(s, p) = u(s - c(s)) + \lambda(p - I),$$

where $p$ is the amount raised from issuing the security. To capture the private benefit of control, I assume $u(0) > 0$. The entrepreneur's problem is

$$\begin{aligned} V &= \max_q E_q \left[ \mathbf{1}\{E_\mu c(s) \geq I\} \cdot (E_\mu u(s - c(s)) + \lambda(E_\mu c(s) - I)) \right] \\ s.t. \quad & E_q \mu = \mu_0 \end{aligned} \tag{26}$$

Again, without loss of generality, $q$ puts all weight on two posteriors $\mu_h$ and $\mu_l$, which must admit densities. While the entrepreneur's utility function is now nonlinear, her problem is still linear in $\mu_h$ and $\mu_l$. Thus, the approach of Section 4 applies.

**Proposition 14.** *For any security $c(s)$ with $E_{\mu_0} c(s) < I$, any continuous, increasing utility function $u(.)$, and any continuous, increasing transaction cost function $\lambda(x) \in [0, x]$ for $x \in \mathbb{R}_+$, the threshold $\hat{s}$ solves*

$$\int_{\hat{s}}^1 (c(s) - I)\, \mu_0(s)\, ds = 0 \tag{27}$$

*and the optimal value is*

$$V = \int_{\hat{s}}^1 u(s - c(s))\, \mu_0(s)\, ds. \tag{28}$$

Thus, the threshold strategy obtained in the linear case, in Proposition 5, remains optimal. The disclosure threshold is solely determined by the financing condition (27) and therefore depends on the security, the investment cost, and the prior, but not on the utility of the entrepreneur or the particular transaction cost function.

[20] For example, the payoffs of the project may be paid in the future while the excess cash is consumed now and the entrepreneur is risk-averse about future but not current consumption.

## 6.2 Additional Noise

In reality, the entrepreneur may not have perfect information about the project's payoff. However, the threshold strategy of Section 4 remains optimal under natural assumptions.

Suppose the entrepreneur designs a disclosure strategy contingent on the state $s$, which now is correlated with the project payoff $x$ which takes values between zero and one. $x$ and $s$ are jointly distributed with prior $\mu_0(x, s)$. For simplicity, I assume the conditional distribution $x|s$ admits a continuous pdf $\mu_0(x|s)$ for all $s$ with full support. The security maps cash flows into payoffs so that $c(x) \in [0, x]$ and $c(x)$ and the residual $x - c(x)$ are both increasing, analogous to the original setup in Section 2. To capture that higher states imply higher payoffs, I assume that for $s' > s$ $\mu_0(x|s')$ first-order stochastically dominates $\mu_0(x|s)$. Let $\mu$ denote investors' posterior on $s$. The entrepreneur's payoff is

$$V(\mu) = E_\mu \left[ \int_0^1 (x - c(x) + B + \lambda(c(x) - I)) \mu_0(x|s)\, dx \right] \tag{29}$$

if the project is financed, which occurs if

$$E_\mu \int_0^1 c(x) \mu_0(x|s)\, dx \geq I. \tag{30}$$

Defining

$$\begin{aligned} \tilde{s}(s) &= \int_0^1 x\mu_0(x|s)\, ds \\ \tilde{c}(s) &= \int_0^1 c(x) \mu_0(x|s)\, ds \end{aligned}$$

we can rewrite the entrepreneur's value as

$$V(\mu) = E_\mu [\tilde{s}(s) - \tilde{c}(s) + B + \lambda(\tilde{c}(s) - I)] \tag{31}$$

and the financing condition as

$$E_\mu \tilde{c}(s) \geq I. \tag{32}$$

Since $x - c(x)$ and $c(x)$ are increasing and $x|s$ is ordered by first-order stochastic dominance,

$\tilde{s}(s) - \tilde{c}(s)$ and $\tilde{c}(s)$ are both increasing in $s$. Thus, this setting is simply a special case of the one in Proposition 14 and the same result holds. A threshold strategy is optimal.

### 6.3 Risk-Averse Entrepreneur

Suppose that the entrepreneur's utility function is concave, increasing, and twice continuously differentiable. Even though for any given security, the optimal disclosure policy is the same as in the linear case, the optimal security issued must change due to risk-aversion. The entrepreneur prefers the retained payoffs $s - c(s)$ to be constant, which can be achieved by either selling the firm for a fixed price, or selling a call option which is exercised only if the posterior is $\mu_h$.

To see this, consider the Lagrangian associated with maximizing over $c(s)$,[21][22]

$$\mathcal{L} = \int_{\hat{s}}^{1} u(s - c(s))\,\mu_0(s)\,ds - \gamma \int_{\hat{s}}^{1} (c(s) - I)\,\mu_0(s)\,ds. \tag{33}$$

The first-order conditions in $c(s)$ then imply $u'(s - c(s)) = \gamma$. Thus, $c(s)$ must take the form $c(s) = s - K$ all $s \geq \hat{s}$ for some constant $K$. This is exactly the payoff of an option with strike price $K$.[23] The optimal value then takes the form

$$V(K) = u(K) \int_{\hat{s}}^{1} \mu_0(s)\,ds.$$

If the optimal strike price is interior, it can be found via the first-order condition $V'(K) = 0$

[21] I ignore the requirement that $c(s)$ and $s - c(s)$ must be increasing. It will be satisfied at the solution.

[22] The method presented here is heuristic. To study the problem formally, we can formulate it as an optimal control problem where $s$ is interpreted as time, $c(.)$ is the control, and we introduce a state $x(s) = \int_{\hat{s}}^{s} \mu_0(s)(c(t) - I)\,dt$. The initial condition is $x(\hat{s}) = 0$ and condition (27) implies the boundary condition $x(1) = 0$. The boundary $\hat{s}$ is free. Then, Pontryagin's maximum principle can be used to characterize the solution, which is the same as the one derived here. See e.g. Kamien and Schwartz (2012).

[23] Since the entrepreneur's marginal utility must be constant for $s \geq \hat{s}$, we have $\hat{s} \geq K$.

which implies[24]

$$
\begin{aligned}
V'(K) &= u'(K)\int_{\hat{s}}^{1}\mu_0(s)\,ds - u(K)\frac{d\hat{s}}{dK}\mu_0(\hat{s}) \\
&= \int_{\hat{s}}^{1}\mu_0(s)\,ds\cdot\left(u'(K)+u(K)\frac{1}{\hat{s}-K-I}\right) \\
&= 0
\end{aligned}
$$

so that[25]

$$
u'(K) = -\frac{u(K)}{\hat{s}-K-I}. \tag{34}
$$

The left hand side measures the gain in payoff for the entrepreneur from increasing the strike price, while the right hand side is the loss due to the lower likelihood that the project is financed, since investors now have to be more optimistic. Sufficient conditions for the strike price to be interior can be found in Appendix B.1.

[24] $V$ is not necessarily concave in $K$, but it is single-peaked, since $u'(K)+u(K)\frac{1}{\hat{s}-K-I}$ is decreasing in $K$. The first-order condition is thus sufficient for finding an interior maximum.

[25] Note that $\hat{s} < K + I$.

# 7 Conclusion

In this paper, I study the optimal disclosure policy of an entrepreneur who needs to finance a project subject to a fixed cost. I show the optimal policy truthfully reveals whether the project's payoffs are above a threshold. The threshold, and therefore the beliefs of investors, are such that the financing constraint binds whenever the project is financed. The optimal disclosure policy is therefore determined by the security issued, the investment cost, and the prior belief, but it is independent of the particular shape of the entrepreneur's utility function or transaction costs, which may be understood as a standin for liquidity issues unmodeled in this paper. The threshold strategy also remains optimal when additional noise is present.

In disclosure models, the sender's payoff is determined solely by the distribution over induced posterior beliefs. The particular signal used is irrelevant, as long as it induces this a particular distribution. Thus, no such model can provide guidance on which means of communication firms should use, only on which beliefs they should try to induce. Given its robustness, my result hopefully constitutes a useful benchmark for empirical work trying to quantify the degree of information contained in firms' communication.

A higher threshold naturally translates into more precise investor forecasts, be it in the sense of a lower mean squared error, lower conditional variance, and smaller price reversals. The comparative statics I provide in this paper therefore yield testable implications on how the security choice, investment cost, and investors' prior beliefs affect the equilibrium precision of investor information.

In my paper, the optimal security is determined by a novel tradeoff. Promising more cash to investors reduces the entrepreneur's residual payoff, but investors are willing to finance the project at a lower belief. This allows the entrepreneur to choose a disclosure policy which has a higher likelihood of successfully convincing investors that the project is sufficiently profitable. In the absence of further distortions, the optimal security design is indeterminate. The optimum may be implemented with equity, debt, options, and many others. In Section 6.3, I have shown that uniqueness of the optimal security can be restored when the entrepreneur is risk averse. An interesting, but yet unanswered, question is whether the optimality of debt can be restored in a setting with general disclosure policies. This might be done by assuming the entrepreneur can exert unobservable effort after the investment decision is made, but before payoffs are realized, or by loosening the commitment assumption so that firms choose the disclosure policy ex ante and then issue securities to signal their type conditional on the realized signal.

# A Proofs

## A.1 Proof of Proposition 1

Suppose $\mu_0 \geq \bar{\mu}$. For any distribution over posteriors $q$, we have

$$\begin{aligned}\mu_0 (1-c) + B + \lambda(\mu_0 c - I) &\geq E_q \left[\mathbf{1}\{\mu \geq \bar{\mu}\}(\mu(1-c)+B) + \lambda(\mu c - I)\right] \\ &= \int_{\mu<\bar{\mu}} 0 dq(\mu) + \int_{\mu \geq \bar{\mu}} (\mu(1-c) + B + \lambda(\mu c - I)) dq(\mu)\end{aligned}$$

since

$$B \int_{\mu \geq \bar{\mu}} dq(\mu) \leq B$$

and

$$\int_{\mu \geq \bar{\mu}} \mu dq(\mu) \leq E_q \mu = \mu_0.$$

Thus, providing no information is optimal.

Suppose $\mu_0 < \bar{\mu}$. If $q$ puts any mass on $(0, \bar{\mu})$, then there exists a $q'$ that distributes this mass between 0 and $\bar{\mu}$. $q'$ satisfies constraint (5) and admits a higher value. Thus, any optimal $q$ puts zero mass on $(0, \bar{\mu})$. Let $q_0 > 0$ be the mass $q$ puts at zero. The Bayes plausibility constraint becomes

$$\int_{\bar{\mu}}^{1} \mu dq(\mu) + q_0 \cdot 0 = \mu_0$$

and the optimal value must satisfy

$$\begin{aligned}V^* &= \int_{\bar{\mu}}^{1} (\mu(1-c) + \lambda(\mu c - I)) dq(\mu) + B \int_{\bar{\mu}}^{1} dq(\mu) \\ &= \mu_0 (1-c) + \lambda(\mu_0 c - I) + B \int_{\bar{\mu}}^{1} dq(\mu).\end{aligned}$$

We have

$$\int_{\bar{\mu}}^{1} dq(\mu) = (1 - q_0)$$

since $q$ must integrate to one and

$$\int_{\bar{\mu}}^{1} \mu dq(\mu) \geq \bar{\mu} \int_{\bar{\mu}}^{1} dq(\mu) = \bar{\mu}(1 - q_0).$$

If the inequality is strict, we can find an improvement by reducing $q_0$ and having $q$ put point mass $1 - q_0$ on $\bar{\mu}$. Thus, the optimal policy puts mass $q_0$ on 0 and $1 - q_0$ on $\bar{\mu}$ which establishes the result.

## A.2 Proof of Proposition 2

Let $\sigma(H|s)$ denote the probability that the signal sends message $H$ if the state is $s$. Using Bayesian updating, the posteriors upon observing $H$ or $L$ are then

$$\begin{aligned}
\mu_H &= \frac{\mu_0 \sigma(H|1)}{\mu_0 \sigma(H|1) + (1 - \mu_0)\sigma(H|0)} \\
\mu_L &= \frac{\mu_0 \sigma(L|1)}{\mu_0 \sigma(L|1) + (1 - \mu_0)\sigma(H|1)}
\end{aligned}$$

Setting $\sigma(L|1) = 0$ and $\sigma(H|1) = 1$ implies $\mu_L = 0$ and

$$\mu_H = \frac{\mu_0}{\mu_0 + (1 - \mu_0)\sigma(H|0)}.$$

If $\sigma(H|0) = \frac{\mu_0}{\bar{\mu}} \frac{1-\bar{\mu}}{\mu_0}$, then $\mu_H = \bar{\mu}$ and the probability that $H$ is sent is

$$\begin{aligned}
\Pr(\mu = \bar{\mu}) &= \mu_0 \sigma(H|1) + (1 - \mu_0)\sigma(H|0) \\
&= \mu_0 + \frac{1 - \bar{\mu}}{\bar{\mu}} \mu_0 \\
&= \frac{\mu_0}{\bar{\mu}}.
\end{aligned}$$

Thus, this signal implements the optimal disclosure policy.

## A.3 Proof of Lemma 3

Consider an optimal policy $q$. Let $q(E_\mu c(s) < I)$ denote the measure on the set of posteriors $\{\mu : E_\mu c(s) < I\}$, and define $\mu_h = E_q(\mu | E_\mu c(s) \geq I)$ and $\mu_l = E_q(\mu | E_\mu c(s) < I)$. $\mu_h$ and $\mu_l$ are probability measures on $[0,1]$. Then,

$$\begin{aligned} V^* &= q(E_\mu c(s) < I) \cdot 0 \\ &\quad + q(E_\mu c(s) \geq I) \cdot E_q[E_\mu(s - c(s) + B) | E_\mu c(s) \geq I] \\ &= q(E_\mu c(s) \geq I) \cdot E_{\mu_h}(s - c(s) + B) \end{aligned}$$

Consider an alternative policy $q'$ which puts measure $q(E_\mu c(s) < I)$ on posterior $\mu_l$ defined above, measure $1 - q(E_\mu c(s) < I)$ on posterior $\mu_h$, and measure zero everywhere else. Under $q'$ the payoff is the same as above and $q'$ is feasible since

$$\begin{aligned} E_q(\mu) &= q(E_\mu c(s) < I) E_q(\mu | E_\mu c(s) < I) + q(E_\mu c(s) \geq I) E_q(\mu | E_\mu c(s) \geq I) \\ &= q(E_\mu c(s) < I) \mu_l + q(E_\mu c(s) \geq I) \mu_h \end{aligned}$$

and by construction

$$E_q(\mu) = \mu_0.$$

Now, I show that $\mu_h$ and $\mu_l$ admit pdfs. Note that necessarily $q \in (0,1)$. For any set $B \in \mathcal{B}([0,1])$, the Bayes plausibility constraint implies

$$\mu_0(B) = q\mu_h(B) + (1-q)\mu_l(B).$$

Thus, both $\mu_h$ and $\mu_l$ are absolutely continuous with respect to $\mu_0$. Since $\mu_0$ admits a pdf, it is is absolutely continuous with respect to the Lebesgue measure. Because absolute continuity is transitive, $\mu_h$ and $\mu_l$ must also admit a pdf.

## A.4 Proof of Proposition 4

The proof proceeds via a sequence of Lemmas and relies on characterizing the dual problem to (16), which I restate for the convenience of the reader below. Fix a $q \in (0,1)$. We have

$$\begin{aligned} V(q) &= \max_{\mu_h(.),\mu_l(.)} q\int_0^1 (s - c(s) + B + \lambda(c(s) - I))\,\mu_h(s)\,ds \\ s.t. & \quad \int_0^1 c(s)\,\mu_h(s)\,ds \geq I \\ & \quad \mu_0(s) = q\mu_h(s) + (1-q)\,\mu_l(s)\,\forall s \in [0,1] \\ & \quad \int_0^1 \mu_h(s)\,ds = 1 \\ & \quad \int_0^1 \mu_l(s)\,ds = 1 \\ & \quad \mu_h(s) \geq 0\,\forall s \in [0,1] \\ & \quad \mu_l(s) \geq 0\,\forall s \in [0,1] \end{aligned}$$

Substituting the Bayes plausibility condition we can replace the constraints on $\mu_l$ with $\mu_h(s) \in \left[0, \frac{\mu_0(s)}{q}\right]$. Consider the relaxed problem

$$\begin{aligned} V_R(q) &= \max_{\mu_h(.)} q\int_0^1 (s - c(s) + B + \lambda(c(s) - I))\,\mu_h(s)\,ds \qquad (35) \\ s.t. & \quad \int_0^1 c(s)\,\mu_h(s)\,ds \geq I \\ & \quad \int_0^1 \mu_h(s)\,ds \leq 1 \\ & \quad \mu_h(s) \geq 0 \forall\, s \in [0,1] \\ & \quad \mu_h(s) \leq \frac{\mu_0(s)}{q}\,\forall s \in [0,1] \end{aligned}$$

I restrict attention to $\mu_h \in \mathcal{L}^1([0,1])$ which satisfy the following generalization of equicontinuity: For any $\varepsilon > 0$, there exists a $\rho < 0$ such that for all $y \in \mathbb{R}$, $|y| < \rho$ and all feasible $\mu_h$,

$$\int_0^1 |\mu_h(s+y) - \mu_h(s)|\,ds < \varepsilon. \qquad (36)$$

The condition simplifies proving compactness of the set of feasible policies, which I do below in Lemma 15. The solution in Proposition 4 satisfies the additional constraint, so it is without loss of generality. Let $F(q) \subset \mathcal{L}^1([0,1])$ denote the set of set of feasible $\mu_h$, i.e. those satisfying the constraints in Problem (35) and condition (36).

**Lemma 15.** *$F(q)$ is compact and convex.*

*Proof.* Since $\mathcal{L}^1([0,1])$ is a Banach space, $F(q)$ is compact if and only if it is totally bounded, which follows from the Kolmogorov-Reisz Theorem. See Hanche-Olsen and Holden (2010), Theorem 5.[26] Convexity follows trivially since all constraints are linear in $\mu_h$. □

The dual problem to (35) is given by

$$\begin{aligned} V_d(q) &= \min_{\alpha_1(.),\alpha_2,\alpha_3\geq 0} \int_0^1 \alpha_1(s) \frac{\mu_0(s)}{q} ds + \alpha_2 - \alpha_3 I \qquad (37) \\ s.t. \quad & \int_0^1 \alpha_1(s)\mu_h(s)\,ds + \alpha_2 \int_0^1 \mu_h(s)\,ds - \alpha_3 \int_0^1 c(s)\mu_h(s)\,ds \\ & \geq q\int_0^1 (s - c(s) + B + \lambda(c(s) - I))\mu_h(s)\,ds \end{aligned}$$

Since $c(s)$ and $\mu_0(s)$ are continuous, the problem satisfies the conditions of Levinson (1966), Theorem 3, which guarantees that the solution of the dual equals the solution of the primal.[27]

Denote the support of $\mu_h$ as $\hat{S} \subset [0,1]$. A necessary condition for the constraint in the dual problem to hold is that for all $s \in \hat{S}$

$$\alpha_1(s) + \alpha_2 - \alpha_3 c(s) \geq q(s - c(s) + B + \lambda(c(s) - I)).$$

Since the dual problem minimizes over $\alpha_1(s)$, the optimal $\alpha_1$ solves

$$\alpha_1(s) = q(s - c(s) + B + \lambda(c(s) - I)) - \alpha_2 + \alpha_3 c(s)$$

[26] The theorem states that a set $F \subset \mathcal{L}^1(\mathbb{R})$ is totally bounded if and only if (1) $F$ is bounded, (2) for all $\varepsilon > 0$ $\exists R > 0$ such that $\forall \mu \in F$, $\int_{|x|>R} |\mu(x)|\,dx < \varepsilon$, and (3) condition (36) holds. The first two conditions can easily be checked.

[27] In general, infinite dimensional linear programs exhibit positive duality gaps, see e.g. Reiland (1980).

for $s \in \hat{S}$ and $\alpha_1(s) = 0$ otherwise. The objective therefore becomes

$$\begin{aligned} V_d(q) &= \int_{s\in\hat{S}} (s - c(s) + B + \lambda(c(s) - I))\,\mu_0(s)\,ds \\ &\quad + \int_{s\in\hat{S}} (-\alpha_2 + \alpha_3 c(s))\,\frac{\mu_0(s)}{q} ds + \alpha_2 - \alpha_3 I \\ &= \int_{s\in\hat{S}} (s - c(s) + B + \lambda(c(s) - I))\,\mu_0(s)\,ds \\ &\quad + \alpha_3 \left( \int_{s\in\hat{S}} c(s)\,\frac{\mu_0(s)}{q} ds - I \right) + \alpha_2 \left( 1 - \int_{s\in\hat{S}} \frac{\mu_0(s)}{q} ds \right) \end{aligned} \tag{38}$$

At the optimal solution, both the financing and integrability constraints must bind.

**Lemma 16.** *$\int_0^1 \mu_h(s)\,ds \leq 1$ and $\int_0^1 \mu_h(s)\,c(s)\,ds \geq I$ cannot both be slack at the optimal solution.*

*Proof.* Substituting $\mu_h(s)$ and combining the two inequalities implies

$$\int_{\hat{S}} (c(s) - I)\,\mu_0(s) > 0.$$

Since $c(s)$ is continuous there exist states such that $c(s) > I$ which are not part of $\hat{S}$ and have positive mass under $\mu_0$. Then the policy cannot be optimal, since including these states in the support of $\mu_h$ increases the objective without violating any of the constraints.[28] □

The optimal value of the dual problem is therefore

$$V_d(q) = \int_{s\in\hat{S}} (s - c(s) + B + \lambda(c(s) - I))\,\mu_0(s)\,ds. \tag{39}$$

[28]Formally, there exists a set $\tilde{S}$ with $\mu_0\left(\tilde{S}\right) > 0$ such that for all $s \in \tilde{S}$, $c(s) > I$, $\hat{S} \cap \tilde{S} = \emptyset$, and

$$\int_{\tilde{S}\cup\hat{S}} \mu_h(s) \leq 1$$

and

$$\int_{\tilde{S}\cup\hat{S}} \mu_h(s)\,c(s) \geq I.$$

Having $\mu_h(s) = \frac{\mu_0(s)}{q}$ on $\hat{S} \cup \tilde{S}$ strictly improves the objective.

Since the solution of the primal and the dual coincide, this establishes that the policy

$$\begin{aligned} \mu_h(s) &= \begin{cases} \frac{\mu_0(s)}{q} & \text{for } s \in \hat{S} \\ 0 & \text{otherwise} \end{cases} \\ \mu_l(s) &= \begin{cases} 0 & \text{for } s \in \hat{S} \\ \frac{\mu_0(s)}{1-q} & \text{otherwise} \end{cases} \\ q &= \int_{\hat{S}} \mu_0(s)\, ds \end{aligned} \tag{40}$$

is optimal. To prove the proposition, it remains to show that $\hat{S}$ is an interval.

**Lemma 17.** $\hat{S} = [\hat{s}, 1]$.

*Proof.* Substitute $q = \int_{\hat{S}} \mu_0(s)\, ds$ into equation (38), which becomes

$$V_d(q) = \int_{s\in\hat{S}} (s - c(s) + B + \lambda(c(s) - I))\, \mu_0(s)\, ds + \frac{\alpha_3}{q}\left(\int_{s\in\hat{S}} (c(s) - I)\, \mu_0(s)\, ds\right).$$

The contribution of each $s$ to the objective is

$$(s - c(s) + B + \lambda(c(s) - I))\, \mu_0(s) + \frac{\alpha_3}{q}(c(s) - I)\, \mu_0(s)$$

and therefore $s \in \hat{S}$ if and only if

$$(s - c(s) + B + \lambda(c(s) - I)) + \frac{\alpha_3}{q}(c(s) - I) \geq 0.$$

Since both $s - c(s)$ and $c(s)$ are increasing in $s$, this implies $\hat{S} = [\hat{s}, 1]$. [29] □

Since $q = \int_{\hat{s}}^{1} \mu_0(s)\, ds$ at the optimal solution, we must have that for $q' > q$, $\hat{s}' < \hat{s}$ whenever $F(q')$ is nonempty.

[29] We can equivalently define $V_d(q) = \max_{\hat{S}\subset\mathcal{B}([0,1])} \int_{\hat{S}} \mu_0(s)(s - c(s) + B)\, ds$ subject to $\int_{\hat{S}} (c(s) - I)\, \mu_0(s)\, ds \geq 0$. The result is then a direct consequence of the Neyman-Pearson Lemma. See e.g. Dantzig et al. (1951).

## A.5 Proof of Proposition 5

**Lemma 18.** *$V(q)$ increasing in $q$, continuous, and differentiable whenever there exists a neighborhood of $q$ on which $F(q)$ is nonempty.*

*Proof.* Continuity and differentiability follows from the envelope theorem in Milgrom and Segal (2002), Theorem 5, since $F(q)$ is compact and convex-valued, the objective and constraints in the primal problem (35) are continuously differentiable in $q$, and the maximizers are unique.

To see that $V(q)$ is increasing, take $q' > q$ such that $F(q')$ and $F(q)$ are nonempty. Then, $\hat{s}(q') < \hat{s}(q)$ and therefore

$$\begin{aligned} V(q') &= \int_{\hat{s}(q')}^{1} (s - c(s) + B + \lambda (c(s) - I)) \mu_0(s) \, ds \\ &> \int_{\hat{s}(q)}^{1} (s - c(s) + B + \lambda (c(s) - I)) \mu_0(s) \, ds. \end{aligned}$$

□

The following result is a consequence of fact that for $q = 1$, the problem does not admit a solution. Let $\hat{s}(q)$ denote the cutoff such that

$$q = \int_{\hat{s}(q)}^{1} \mu_0(s) \, ds.$$

**Lemma 19.** *Suppose $E_{\mu_0} c(s) < I$. Then there exists a $\bar{q} \in (0,1)$ , such that for $q > \bar{q}$, the problem (35) does not admit a solution. $\hat{s}(\bar{q})$ solves*

$$\int_{\hat{s}(\bar{q})}^{1} (c(s) - I) \mu_0(s) \, ds = 0.$$

*Proof.* Consider the problem of choosing $\mu_h$ to maximize the expected payoff to investors given $q$, which is

$$\begin{aligned} C(q) &= \max_{\mu_h(.)} \int_0^1 c(s)\,\mu_h(s)\,ds \\ s.t. & \quad \int_0^1 \mu_h(s)\,ds \leq 1 \\ & \quad \mu_h(s) \in \left[0, \frac{\mu_0(s)}{q}\right] \end{aligned}$$

This problem has the same solution as problem (35), and $\mu_h(s) = \frac{\mu_0(s)}{q}$ for $s \in [\hat{s}(q), 1]$ and zero otherwise. The value to investors is thus

$$C(q) = \int_{\hat{s}(q)}^1 c(s)\,\frac{\mu_0(s)}{q} ds$$

and for $q > \bar{q}$,

$$C(q) < \int_{\hat{s}(\bar{q})}^1 (c(s))\,\frac{\mu_0(s)}{\bar{q}} ds = I$$

which implies

$$E_{\mu_h} c(s) < I.$$

Thus, for $q > \bar{q}$, $F(q)$ is empty. □

Since $V(q)$ is increasing in $q$, the optimal $q$ is $\bar{q}$, for which the financing constraint binds.

## A.6 Proof of Proposition 9

For both $\mu_0$ and $\mu_0'$, the optimal thresholds $\hat{s}$ and $\hat{s}'$ are determined by

$$\begin{aligned} \int_{\hat{s}}^1 (c(s) - I)\,\mu_0(s)\,ds &= 0 \\ \int_{\hat{s}'}^1 (c(s) - I)\,\mu_0'(s)\,ds &= 0. \end{aligned}$$

If $\mu_0'$ first-order stochastically dominates $\mu_0$, then

$$\int_{\hat{s}}^{1} (c(s) - I)\, \mu_0'(s) > \int_{\hat{s}}^{1} (c(s) - I)\, \mu_0(s) = 0$$

since $c(s)$ is increasing. Then, the financing constraint for $\mu_0'$ binds at $\hat{s}' < \hat{s}$.

## A.7 Proof of Proposition 10

The first sufficient condition is obvious. To prove the second sufficient condition, note that $C'$ and $C$ are both cumulative distribution functions and the condition simply states that $C'$ first-order stochastically dominates $C$. Then, we have

$$\int_{\hat{s}}^{1} \mu_0(s)\, dC'(s) \geq \int_{\hat{s}}^{1} \mu_0(s)\, dC(s)$$

since $\mu_0(s)$ is increasing by assumption, which implies

$$\begin{aligned} \int_{\hat{s}}^{1} c'(s)\, \mu_0(s)\, ds &\geq \frac{\int_0^1 c'(s)\, ds}{\int_0^1 c(s)\, ds} \int_{\hat{s}}^{1} c(s)\, \mu_0(s)\, ds \\ &\geq \int_{\hat{s}}^{1} c(s)\, \mu_0(s)\, ds. \end{aligned}$$

## A.8 Proof of Proposition 11

Substituting the financing constraint into the objective, we have

$$V = \int_{\hat{s}}^{1} (s - I + B)\, \mu_0(s)\, ds \leq \int_{I-B}^{1} (s - I + B)\, \mu_0(s)\, ds.$$

If $E_{\mu_0} s \geq I$, the upper bound is achievable by setting $\hat{s} = I - B$ for any security $c(s)$ such that

$$\int_{I-B}^{1} (c(s) - I)\, \mu_0(s)\, ds = 0.$$

If $E_{\mu_0} s < I$, $\hat{s} = I - B$ is not feasible whenever

$$\int_{I-B}^{1} (s - I)\, \mu_0(s)\, ds < 0$$

and therefore $\hat{s} > I - B$ and $c(s) = s$ at the optimal solution. Letting $B \to I$, the above integral is negative, since it converges to $E_{\mu_0} s - I < 0$, while for $B \to 0$, it converges to

$$\int_{I}^{1} (s - I)\, \mu_0(s)\, ds > 0.$$

Since the integral is decreasing in $B$, there exists a threshold $B(I)$ such that for $B > B(I)$ the integral is negative, and $\hat{s} > I - B$, while for $B \leq B(I)$, it is positive, so $\hat{s} = I - B$. This establishes the result.

## A.9 Proof of Proposition 12

We have for any $c(s)$ with $E_{\mu_0} c(s) < I$,

$$\begin{aligned} V &= \int_{\hat{s}}^{1} (s - c(s) + B)\, \mu_0(s)\, ds \\ &= \int_{\hat{s}}^{1} (s - I + B)\, \mu_0(s)\, ds \\ &\leq \int_{0}^{1} (s - I + B)\, \mu_0(s)\, ds \\ &= \int_{0}^{1} (s - c'(s) + B)\, \mu_0(s)\, ds \end{aligned}$$

Here, the inequality uses $B > I$, which guarantees that the integrand is positive for all $s$. For any $c''(s)$ with $E_{\mu_0} c(s) > I$,

$$\begin{aligned} V &= \int_0^1 (s - c''(s) + B)\,\mu_0(s)\,ds \\ &< \int_0^1 (s - I + B)\,\mu_0(s)\,ds \\ &= \int_0^1 (s - c'(s) + B)\,\mu_0(s)\,ds. \end{aligned}$$

Thus, $c'(s)$ maximizes the entrepreneur's value. If $E_{\mu_0} s < I$, for any $\hat{s} > 0$ and any security with $c(s) < s$ for some $s$, we have $\frac{dV}{dc(s)} > 0$. Thus, $c(s) = s$ is optimal and $\hat{s} > 0$.

# B Additional Results

## B.1 Additional Results with a Risk-Averse Entrepreneur

In the case of Section 6.3, the proposition below finds sufficient conditions for the strike price to be interior. If the utility satisfies INADA and investors are not willing to finance the firm without disclosure even if $K = 0$, then the optimal strike price is characterized by condition *(34)*. Intuitively, INADA guarantees that at $K = 0$, the entrepreneur's value is strictly increasing in $K$. The highest possible price at which the project still gets financed if investors know that $s = 1$ is $\bar{K} = 1 - I$, but as $K \to \bar{K}$, $\hat{s} \to 1$. As the strike price becomes large, the project is financed only if the state is high, which the entrepreneur must truthfully reveal. But since for any $K < \bar{K}$, the financing constraint implies $\hat{s} < K + I$, as $\hat{s} \to 1$ and $K \to \bar{K}$, $V'(K)$ becomes negative. Therefore, the optimal value is interior.

**Proposition 20.** *Suppose that* $\lim_{x \downarrow 0} u'(x) = \infty$. *Then, if* $\int_0^1 (s - I)\,\mu_0(s)\,ds \leq 0$, *the optimal strike price is interior and satisfies equation (34).*

If the project can be financed without disclosure when $K = 0$, which happens when $\int_0^1 (s - I)\,\mu_0(s)\,ds > 0$, whether the strike price is interior depends on the shape of $u$. The corner solution $K_0 = \int_0^1 (s - I)\,\mu_0(s)\,ds$ may be optimal if $V'(K_0) < 0$. This happens when $K_0$ is relatively large.

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