Sequential Credit Markets

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ABSTRACT

Entrepreneurs who seek financing for projects typically do so in decentralized markets where they need to approach investors sequentially. We study how well such sequential markets allocate resources when investors have expertise in evaluating investment opportunities, and how surplus is split between entrepreneurs and financiers. Contrary to common belief, we show that the introduction of a credit bureau that tracks the application history of a borrower leads to more adverse selection, quicker market break down, and higher rents to investors which are not competed away even as the number of investors grows large. Although sequential search markets lead to substantial investment inefficiencies, they can nevertheless be more efficient than a centralized exchange where excessive competition may impede information aggregation. We also show that investors who rely purely on public information in their lending decisions can out-compete better informed investors with soft information, and that an introduction of interest rate caps can increase the efficiency of the market.

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The main role of primary financial markets is to channel resources to firms with worthwhile projects, a process that requires information about investment opportunities. Investors with expertise in evaluating projects, such as venture capitalists, business angels, or commercial banks, can therefore serve an important role for the productivity and growth of the real economy. Since no single investor usually has all the information for deciding whether a project should be pursued or not, there is a need for financial markets to aggregate information efficiently.

The extent to which markets can aggregate information and allocate resources efficiently depends on how they are organized. At least until very recently, the overwhelming majority of primary capital markets for small- and medium sized firms operate as decentralized search markets in which firms approach potential investors sequentially (one-by-one). This is true whether firms are seeking capital from banks or from equity investors such as business angels and venture capitalists. Historically, transparency of these markets has been limited but advances in technology over the last decades has made these markets more transparent. In particular, most developed markets now have central credit bureaus which not only collect information about credit worthiness of firms and individuals, but also track the application history of borrowers. Recently, innovations in financial technology have even brought some market activity to centralized market places such as peer-to-peer and crowdfunding platforms.

Does the introduction of credit bureaus in a decentralized market lead to better investment decisions and a lower cost of capital for entrepreneurs? Which markets are socially optimal and which markets are better for entrepreneurs?

In this paper, we develop a general but tractable decentralized search model of credit markets to study these questions. We consider a setting in which an entrepreneur with a project idea searches for credit by approaching potential financiers sequentially. We assume that there is uncertainty about whether the project is worthwhile or not. Each investor, if approached, can do due diligence which results in a private signal about the prospects of the project. The search continues until the entrepreneur either finds an investor who is willing to accept her terms for financing the project or runs out of options and abandons the project. Unlike standard search models which focus on the friction introduced by the cost of finding a counterparty, we are interested in the consequences of sequential interactions. We therefore assume that the entrepreneur is infinitely patient and has no search cost, so that all our results are driven by informational frictions.

A credit bureau in our model performs two functions. First, it may produce hard information about credit quality of the project. Second, consistent with practice, we assume that the credit bureau records how many credit checks have been performed on the entrepreneur in the past. This information allows investors to deduce how many times the borrower has applied for financing previously. We refer to the case where the sequence is observable as the "credit bureau" case. In the "no credit bureau" case, a lender does not know how many other lenders an applicant has visited before. This is commonly the case in less developed countries, in informal lending markets, and in non-bank markets such as when an entrepreneur seeks angel- or venture capital financing.

Our first main result is that the introduction of a credit bureau reduces the fraction of surplus captured by the entrepreneur, and often leads to worse lending decisions and a lower total surplus. This result contrasts with the standard economic intuition that revelation of any information which lowers information asymmetry should lead to more efficient outcomes (see, for example, the linkage principle of Milgrom and Weber (1982).)

To understand this result, consider the case with a credit bureau in place. Each time an entrepreneur is rejected, the rejection is recorded in the credit bureau so that remaining investors revise their beliefs about the quality of the project downwards. The impact of a rejection on the beliefs of remaining investors depends on the terms at which they believe the entrepreneur was rejected—if they believe the entrepreneur asked for financing at very favorable terms (a low interest rate), a rejection is not such bad news. Because these terms are not directly observable, the entrepreneur cannot affect the beliefs of investors and improve her prospects in future rounds by asking for more favorable terms in the current round. In equilibrium, this biases her towards asking for less favorable terms.

Asking for less favorable financing terms implies that some rent is left to investors. We show that even when the number of potential investors grows large they continue to earn significant rents even though the entrepreneur has zero search costs and all the bargaining power. The rent can be so high that uninformed lenders who can commit to use only hard information are sometimes able to out compete lenders with private information. The reason is that a hard information lender never makes any rents, which for high credit quality entrepreneurs can make them more attractive despite the lower surplus created.

When there is no credit bureau, an investor cannot verify how many times an applicant has been rejected previously. This is potentially bad for an entrepreneur who has not been rejected, since she might be pooled with rejected entrepreneurs with worse credit quality. A first-time applicant therefore has an incentive to signal her type, and we show that she will always be able to do so by asking for more favorable financing terms (a lower interest-rate loan). This is a credible signal, because a request for more favorable terms has a higher probability of rejection, and rejection is less costly for a first-time applicant who has many investors left to visit. This logic extends to all rounds, leading to a fully separating equilibrium where the entrepreneur asks for slightly less favorable terms with each rejection. Thus, the need for signalling creates a credible way for the entrepreneur to ask for favorable terms early on.

Asking for favorable financing terms leads to more financing rounds relative to the case with a credit bureau because credit quality deteriorates slower with each rejection. In the case of no credit bureau, the entrepreneur can visit all the available investors. In contrast, in the case of a credit bureau, the entrepreneur might get locked out of the market after a single rejection even when there is a large set of potential investors.

The benefits of having extended search depend on the informational content of the signal distribution. The way many financing rounds are sustained is by asking for offers that only the most optimistic investor would accept, while less optimistic information is never incorporated in the financing decision. As a result, extended search is desirable in situations where the informational content of the signal distribution is concentrated towards the top. We show that for these situations, as the number of potential investors grows large, the social surplus without a credit bureau approaches that attained in a large first-price auction, which is also the maximal possible one.

However, extended search can lead to less informative financing decisions in situations where the informational content of the signal distribution is not concentrated towards the top. For these situations, the market with a credit bureau and few financing rounds turns out to be more efficient and can dominate even a centralized auction market. Although we show that a central auction market with an optimally chosen number of investors is always better than a sequential market, it may not always be easy to commit to limit the number of participants in an auction. In the credit bureau market, there is no need for such a commitment—the market breaks down endogenously after a limited set of rounds. Hence, the market with a credit bureau can lead to higher social surplus than a large auction market because it restricts the competition among investors, allowing them to utilize their information more efficiently. Surprisingly, the increased surplus can more than compensate for the higher rent left to investors, so that the entrepreneur can also be better off than in an auction market. In fact, if credit bureaus were to collect information not only on the number of rejections, but also on the terms at which an applicant was rejected, a sequential market would in fact always produce higher surplus and higher entrepreneurial rents than a free entry auction.

We also show that the sequential market with a credit bureau can have multiple equilibria, due to the feedback effect of equilibrium beliefs. When investors believe that rejected borrowers have low credit quality, rejection is more costly for entrepreneurs.

Therefore, entrepreneurs will be more likely to ask for unfavorable financing terms in early rounds to avoid rejection, which means that rejection is a signal of worse quality—a self-fulfilling prophesy. Hence, equilibria with few financing rounds and equilibria with more financing rounds can coexist. The equilibria with few financing rounds are often worse for entrepreneurs because of the unfavorable financing terms, but can be good for social surplus. This gives the surprising implication that social welfare can be improved if the government imposes an interest rate cap. An interest rate cap will eliminate "sub-prime" markets for rejected borrowers, and hence will eliminate the socially inefficient equilibria with many financing rounds.

Our paper is related to several bodies of work. The efficiency of investment decisions in our model depends on the extent to which information is aggregated. Starting with Hayek (1945) and Grossman (1976) there is a large literature that studies information aggregation in financial markets. Sequential decision making has also been studied in the literature on herding and informational cascades. Similar to Bikhchandani, Hirshleifer and Welch (1992) and Welch (1992), we show that investors' sometimes may stop using their signal to screen the entrepreneur after observing that she was rejected by other investors. However, unlike Bikhchandani, Hirshleifer and Welch (1992) and Welch (1992), we allow the entrepreneur to adjust her offers in different rounds. Therefore, herding does not always occur in equilibrium as in Bikhchandani, Hirshleifer and Welch (1992) and Welch (1992), and whether it exists or not depends on the signal distribution.

Similar to us, Bulow and Kelmperer (2009) and Roberts and Sweeting (2013) study relative efficiency of sequential and centralized markets. However, both papers focus on the private-values case, have nonzero costs of information acquisition, and allow free entry of investors. In their setting, having as many potential buyers as possible is always good for a seller, which is not necessarily the case in our setup.

Lauermann and Wolinsky (2016) study the common-value case in a decentralized search setup with a seller searching for buyers. Lauermann and Wolinsky (2016) consider only the case with an infinite number of buyers and assume that the search history is not observable. Lauermann and Wolinsky (2016) analyze only pooling equilibria and conclude that search markets are worse at aggregating information than the centralized markets. We show that with finite but arbitrary large number of buyers there is a separating equilibrium, which can be as efficient as at aggregating information as centralized markets, and we also show that search markets with a credit bureau can be more efficient than centralized markets.

Our work is also related to Zhu (2012) who considers a model of opaque over-the-counter markets. In his model, it is buyers and not the seller who make take-it-or-leave it offers. Similar to Lauermann and Wolinsky (2016), Zhu (2012) also assumes that a search history is not observable and studies only pooling equilibria. Thus, both the focus and analysis of Zhu (2012) are different from ours.

Our paper is also related to the literature on relationship lending started with a seminal paper by Rajan (1992). Papers in this literature have in common with ours the adverse selection created for other borrowers when an informed lender refuses credit, but in the context of an existing borrower rather than a first-time borrower.

We also relate to large literature on search markets. Many papers in this literature focus on the friction introduced by the cost of finding a counter-party in private value environments (see, e.g., Duffie, Garleanu and Pedersen (2005), Lagos and Rocheteau (2009), Vayanos and Weill (2008), Weill (2008)). We differ from this literature by focusing on the consequences of sequential interactions in the common-value environment, where the entrepreneur is infinitely patient and has no search cost.

1. Model

We consider a penniless entrepreneur seeking outside financing for a new project from a set of $N < \infty$ investors. All agents are risk neutral. The project requires one unit of investment, and can be of two types: good (G) and bad (B), where the unconditional probability of the project being good is π. If the project is good it pays $1 + X$. Otherwise, it pays 0. We denote the net present value, or NPV, of the project by V, a random variable that takes value X if the project is good and value -1 if the project is bad.

No one knows the type of the project but investors have access to a screening technology. When an investor makes an investigation, he gets a privately observed informative signal $s \in [0, 1]$ drawn from a distribution $F_G(s)$ with density $f_G(s)$ in case the project is good and from a distribution $F_B(s)$ with density $f_B(s)$ in case the project is bad. We make the following assumption about the signal distribution:

ASSUMPTION 1: *Signals satisfy the monotone likelihood ratio property (MLRP):*

$$\forall s > s', \quad \frac{f_G(s)}{f_B(s)} \geq \frac{f_G(s')}{f_B(s')}.$$

Both $f_G(s)$ and $f_B(s)$ are continuously differentiable at $s = 1$, $f_B(1) > 0$, and $\lambda \equiv f_G(1)/f_B(1) > 1$.

Without loss of generality, we will also assume that $f_G(s)$ and $f_B(s)$ are left-continuous and have right limits everywhere. Assumption 1 ensures that higher signals are at least weakly better news than lower signals. Assuming that densities are continuously differentiable at the top of the signal distribution simplifies our proofs, but is not essential for our results.

We denote the likelihood ratio at the top of the distribution by λ, a quantity that will be important in our asymptotic analysis. Assuming $\lambda > 1$ ensures that MLRP is strict over a set of non-zero measure, which in turn implies that as $N \to \infty$, an observer of all signals would learn the true type with probability one. Therefore, for large enough N, the aggregate market information is valuable for making the right investment decision.

To exclude trivial cases, we assume that the signal of a single investor i can be sufficiently optimistic for the expected value of the project to be positive:

ASSUMPTION 2: $E(V|S_i = 1) > 0$.

The entrepreneur contacts investors sequentially in a random order indexed by $i \in \{1, ..., N\}$. When contacting investor i the entrepreneur makes an exclusive take-it-or-leave-it offer, in which she asks for the loan size of one in exchange for the repayment of $1 + r_i$ in case the project is successful. Based on the signal, the investor decides whether to accept the offer or not. If the offer is rejected the entrepreneur goes to investor $i + 1$. If the offer is accepted, the entrepreneur forfeits the right to contact other investors, so the project is financed and production starts.

We do not allow an entrepreneur to "shop around" an accepted offer by showing it to other investors in the hope of getting better financing terms. This assumption of exclusivity is important. If the entrepreneur could take accepted offers to other investors without losing them the resulting mechanism would be similar to an ascending-price auction, where the entrepreneur gradually reduces the interest rate offers until only one investor is willing to finance. We study the auction case in more detail in Axelson and Makarov (2016) and contrast it with the sequential market case further down.

If the project is financed at interest rate r and is successful, the entrepreneur gets $X - r$ of the project cash flows while the investor gets $1 + r$. If the project is unsuccessful, neither the entrepreneur nor the investor get anything. An important implication of the fact that the entrepreneur earns nothing unless the project is good is that her optimal strategy is independent of her information about the success probability—she will always act to maximize her pay off conditional on the project being successful.

We assume that the entrepreneur commits not to visit the same investor twice. It is clearly in the interest of the entrepreneur to commit not to re-visit the same investor when there is only one investor available. With many investors the situation is less clear. We will show that our main results hold if we allow multiple contacts.

If there is no credit bureau in place, investors do not observe whether the entrepreneur has approached other investors for credit previously, and so rely purely on their own signal and any information volunteered by the entrepreneur when making the credit decision.

If there is a credit bureau in place, investors can access any information collected by the bureau by performing a credit check. A credit bureau in our model performs two functions. First, it may produce hard information about credit quality of the project, which we model as a signal S_0 which satisfies MLRP and is conditionally independent of other signals. Second, consistent with practice, we assume that the credit bureau records how many credit checks have been performed on the entrepreneur in the past. This information allows investors to deduce how many times the borrower has applied for financing previously.

2. Maximal social surplus

In any of the information environments we study, a strategy for the entrepreneur is a set of interest rate offers $\{r_i\}_{i=1}^N$ offered in sequence to investors $i \in \{1, ..., N\}$ until an investor accepts. As a benchmark, we first derive the maximal social surplus achievable by a social planner who can publicly commit to a set of interest rate offers and a sequence in which investors are approached.

We first make the observation that picking a vector of offers $\{r_i\}_{i=1}^N$ is equivalent to picking a set of screening thresholds $\{s_i^*\}_{i=1}^N$ such that the project gets started only if there is an investor i with a signal S_i above the threshold s_i^*. To see this, consider an investor i who is approached with an offer of financing the project at interest rate r_i. The investor conditions on the history Ω_i, which contains the information that each previous investors $j < i$ has rejected the project at interest rate r_j. His expected profit from accepting to finance the project given his own signal $S_i = s$ is then given by

$$\Pr(G|\Omega_i, S_i = s)r_i - \Pr(B|\Omega_i, S_i = s).$$

The investor accepts the offer if and only if

$$r_i \geq \frac{\Pr(B|\Omega_i, S_i = s)}{\Pr(G|\Omega_i, S_i = s)} = \frac{\Pr(B|\Omega_i)}{\Pr(G|\Omega_i)} \frac{f_B(s)}{f_G(s)}, \tag{1}$$

where the last equality follows from Bayes' rule and the independence of signal S_i and history Ω_i conditional on the true state of the project. MLRP implies that the right-hand side decreases in s. Therefore, the project is either rejected for any signal, or there is a unique screening level s_i^* such that the offer is accepted if and only if $S_i \geq s_i^*$.

Define $\mathbf{s^*_{i-1}} = \{s_j^*\}_{j=1}^{i-1}$ as the screening thresholds used prior to round i. Equation (1) then implies that the interest rate offer in round i that implements a screening threshold s_i is given by

$$r_i(s_i, \mathbf{s^*_{i-1}}) = \frac{1-\pi}{\pi} \frac{f_B(s_i)}{f_G(s_i)} \Pi_{j=1}^{i-1} \frac{F_B(s_j^*)}{F_G(s_j^*)}. \tag{2}$$

We will use this relation repeatedly below. We can now write the social planner's surplus maximization problem as a choice of screening thresholds $\{s_i^*\}_{i=1}^N$, which amounts to trading off rejection of good projects versus acceptance of bad projects:

$$\max_{\{s_i^*\}_{i=1}^N} \pi X \left(1 - \prod_{i=1}^N F_G(s_i^*)\right) - (1-\pi)\left(1 - \prod_{i=1}^N F_B(s_i^*)\right). \tag{3}$$

Note that not every choice of screening thresholds $\{s_i^*\}$ is implementable with feasible interest rates $r_i \leq X$, but we show below that the optimal solution to (3) is always implementable:

PROPOSITION 1: *The socially optimal screening policy is to use the same screening threshold $s^* < 1$ for $n \leq N$ rounds and set the screening level at 1 for remaining rounds. The optimal screening threshold is an increasing function of n and is the lowest signal at which investor n breaks even at the maximal interest rate X:*

$$\Pr(G|S_n = s^*, S_1, ..., S_{n-1} \leq s^*) \times X - \Pr(B|S_n = s^*, S_1, ..., S_{n-1} \leq s^*) \geq 0. \tag{4}$$

The social surplus is the same as that generated in a first-price auction where n investors bid with interest rates for the right to finance the entrepreneur.

If $\frac{F_G(s)}{F_B(s)} \frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s then $n = N$ and the expected surplus strictly increases with the number of screenings. If $\frac{F_G(s)}{F_B(s)} \frac{f_B(s)}{f_G(s)}$ is a strictly increasing function for $s \in [s^*, 1]$ then the maximal expected surplus is achieved with no more than n screenings.

Proof: See the Appendix.

Proposition 1 shows that it is optimal to use the same screening threshold for the first $n \leq N$ investors, and completely ignore the rest of the signals. The screening thresholds correspond to a set of interest rate offers as defined in Equation (2) that increase in each round until they reach the maximal feasible rate X in the n^{th} round.

The screening threshold s^* is set such that the project just breaks even when $\max\{s_1, s_2, \ldots, s_n\} = s^*$. The project is financed if and only if the maximal of n

signals is higher than s^*. In Axelson and Makarov (2016) we show that this is also the investment outcome realized in a first-price auction with n bidders. Thus, no sequential credit market can generate higher surplus than a first-price auction if the number of investors in the auction is chosen to maximize social surplus.

Note that the investment outcome is equivalent to the decision of a social planner who observes only the first-order statistic of n signals when making his investment decision. Hence, there is a potentially substantial loss of efficiency relative to the first-best setting where all signals are used in the decision making. If there were no investment mistakes then the expected surplus would be πX. Lemma 1 provides an upper bound on the maximal expected surplus that can be achieved with a screening technology with finite λ. It shows that there is always a loss of at least $(1 - \pi)/\lambda$ compared to the first-best case.

LEMMA 1: *The maximal expected social surplus in a sequential market with a screening technology that satisfies* $f_G(1)/f_B(1) = \lambda$ *is no larger than* $\max(\pi X - (1-\pi)/\lambda), 0)$.

Proof: See the Appendix.

Notice that Proposition 1 and Lemma 1 still hold if the entrepreneur can visit the same investor multiple times. One can show that with repeated visits, the acceptance/rejection decision is still a threshold policy. Define s_i^{*m} as the screening threshold used in the m^{th} contact with investor i. Clearly, for any i the threshold s_i^{*m} can only decrease with m. Therefore, there is a limit $s_i^* = \lim_{m\to\infty} s_i^{*m}$ (the limit is just the last value if the set of repeated contacts is finite). Thus, the project is financed if there is an investor i with a signal S_i above the threshold s_i^*, which leads to the problem (3).

Proposition 1 shows that the social planner may find it optimal to restrict the number of screening rounds—smaller markets can be more efficient than large markets. This surprising result is due to the fact that the investment decision is based only on the information contained in the first-order statistic of signals. For some signal distributions, as outlined in the conditions of the proposition, it is more informative to rely on the highest signal in a small sample rather than a large sample.

The following section will show that a sequential market without a credit bureau will always lead to a maximum number of screenings, which is optimal when the social planner prefers large markets but reduces social surplus when the planner prefers small markets. In Section 4, we show that the introduction of a credit bureau endogenously limits the size of the market, which can increase surplus when the social planner prefers small markets. However, the introduction of a credit bureau will always reduce the fraction of surplus going to the entrepreneur.

3. Equilibrium without a credit bureau

We now turn to the least transparent case in which neither previous offers nor rejections are observed by an investor who is approached for financing. The only information available to an investor in this case is the interest rate he is being offered. However, since it is the entrepreneur who makes the offer, the interest rate she asks may provide useful information about how many times the entrepreneur has been rejected previously and on which terms.

Our main result in this section is to show that under suitable restrictions on out-of-equilibrium beliefs, only fully separating equilibria exist. In any such equilibrium the entrepreneur increases her interest rate offer after each rejection, so the offer perfectly reveals the entrepreneur's application history to the investor. Furthermore, as the number of investors increases, the entrepreneur extracts all the surplus, and the surplus converges to the maximal surplus realized in the social planner's problem when large markets are optimal.

Separation obtains because entrepreneurs with few rejections would like to separate from entrepreneurs with more rejections. They do this by asking for a low interest rate which has a low probability of being accepted by the investor. The low probability of acceptance makes this strategy costly to mimic for an entrepreneur with many rejections who has only few investors left to visit.

Consider a candidate separating equilibrium in which $\{r_i\}_{i=1}^N$, $r_i \neq r_j$ for $i \neq j$ is a set of interest rate offers made by the entrepreneur. In a separating equilibrium, investors will infer how many times the entrepreneur has been rejected from the interest rate offer, and will also correctly conjecture what interest rates where offered in previous rounds. Hence, equilibrium screening thresholds $\{s_i^*\}_{i=1}^N$ must be consistent with Equation (2).

We now formulate the incentive compatibility constraints that must hold so that the entrepreneur will not find it profitable to deviate in round i and ask for interest rate r_j, $j \neq i$. We show that these constraints require the interest rates to increase and screening thresholds to decrease after each rejection.

The entrepreneur maximizes her expected profit conditional on the project being successful. Hence, let V_i denote the expected surplus of the entrepreneur in the beginning of financing round i conditional on the project being good. If the entrepreneur visited $N-1$ investor and was rejected by all of them then he has only one last investor to visit. The offer $r_N(s_N, \mathbf{s}^*_{\mathbf{N-1}})$ is accepted with probability $(1 - F_G(s_N))$ and gives the entrepreneur a payoff of $\left(X - r_N(s_N, \mathbf{s^*_{N-1}})\right)$. Thus,

$$V_N = (1 - F_G(s_N))\left(X - r_N(s_N, \mathbf{s^*_{N-1}})\right).$$

The vector of expected surpluses is then defined recursively as

$$V_i = (1 - F_G(s_i))\left(X - r_i(s_i, \mathbf{s^*_{i-1}})\right) + F_G(s_i)V_{i+1}, \quad i = N-1, ..., 1. \tag{5}$$

To be incentive compatible, a set of interest rate offers must be such that the entrepreneur in financing round i would not be tempted to deviate and quote a different interest rate:

$$V_i \geq (1 - F_G(s_j))\left(X - r_j(s_j, \mathbf{s^*_{j-1}})\right) + F_G(s_j)V_{i+1}, \quad j \neq i. \tag{6}$$

For ease of notation, define U_i as

$$U_i \equiv (1 - F_G(s_i))\left(X - r_i(s_i, \mathbf{s^*_{i-1}})\right).$$

The incentive compatibility constraints (6) imply that for any $i > j$

$$(F_G(s_j) - F_G(s_i))V_{j+1} \geq U_i - U_j \geq (F_G(s_j) - F_G(s_i))V_{i+1}. \tag{7}$$

Since $V_{i+1} < V_{j+1}$, for inequalities (7) to hold it must be that $s_j > s_i$. In other words, the probability of receiving financing must increase with the number of rejections. Because the entrepreneur always prefers lower interest rate for a given probability of being financed, interest rate offers must increase with the number of rejections.

Further inspection of (7) reveals that if the incentive compatibility constraints (6) hold for all adjacent financing rounds i and $j = i + 1$ then they hold for any rounds i and j. Finally, since entrepreneurs with few rejections would like to separate from entrepreneurs with more rejections the entrepreneur in round i is never tempted to ask for the interest rate r_{i+1}. Thus, the only IC constraints that matter are the ones that make sure that the entrepreneur in round $i + 1$ is not tempted to ask for the interest rate r_i. We can now state our main result in this section.

PROPOSITION 2: *Any equilibrium that survives the Cho and Kreps intuitive criterion must be separating. In any separating equilibrium, interest rates strictly increase and screening thresholds strictly decrease with the number of rejections. The screening* *thresholds s_i^* solve*

$$\begin{aligned}
V_N &\equiv \max_{s_N}(1 - F_G(s_N))\left(X - r_N(s_N, \mathbf{s^*_{N-1}})\right), \\
V_i &\equiv \max_{s_i}(1 - F_G(s_i))\left(X - r_i(s_i, \mathbf{s^*_{i-1}})\right) + F_G(s_i)V_{i+1}, \quad i = N-1, ..., 1 \qquad (8) \\
s.t. &\qquad (1 - F_G(s_i))\left(X - r_i(s_i, \mathbf{s^*_{i-1}})\right) + F_G(s_i)V_{i+2} \leq V_{i+1}, \qquad (9)
\end{aligned}$$

where interest rates $r_i(s_i, \mathbf{s^*_{i-1}})$ are given by (2). If MLRP holds strictly then as N goes to infinity the entrepreneur extracts all the surplus and the surplus converges to that generated in a first-price auction with N investors.

Proof: See the Appendix.

We show in Proposition 1 that the maximal surplus is the same as that generated in a first-price auction with optimal number of investors. Therefore, an immediate consequence of Proposition 2 is that in the case without a credit bureau the surplus converges to the social planner's surplus when large markets are optimal, and otherwise is strictly lower.

To prove that the surplus converges to that generated in a first-price auction we show that the interest rate offer in the last round converges to the maximal interest rate X. Suppose this is indeed the case. By giving up all the project's cash flows to the last investor the entrepreneur has no surplus left for himself. Hence, she will offer X in the last round only even an investor with the most optimistic signal cannot break even with an offer less than X. With the strict MLRP this can only happen if the screening threshold in the last round goes to one. Since the last round has the lowest threshold, this proves that screening thresholds in all rounds converge to one. Equation (2) for the interest rate in the last round implies that

$$\pi\lambda X \prod_{i=1}^{N-1} F_G(s_i^*) = (1-\pi) \prod_{i=1}^{N-1} F_B(s_i^*),$$

while Equation (4) for the screening threshold s^* in a first-price auction with N investors implies that

$$\pi\lambda X F_G(s^*)^{N-1} = (1-\pi)F_B(s^*)^{N-1}.$$

The above two equations imply that the probabilities of financing the project in the two setups converge to each other:

$$\prod_{i=1}^{N-1} F_G(s_i^*) \to F_G(s^*)^{N-1} \quad \text{and} \quad \prod_{i=1}^{N-1} F_B(s_i^*)) \to F_B(s^*)^{N-1},$$

which in turn imply the convergence of surplus values.

Suppose now that the last round interest rate does not converge to X. This can only happen if the screening threshold in the last round does not converge to one. Otherwise, the entrepreneur is always better off lowering the screening threshold by offering a slightly higher interest rate. Because after each rejection the perception of the project's quality deteriorates, for the entrepreneur to have a chance of obtaining financing in the last round only a bounded number of screening thresholds can stay away from one as N goes to infinity. Therefore, if the last round interest rate does not converge to X, there will exist an i such that the screening threshold in round i converges to one but the screening threshold in round $i+1$ stays bounded away from one. Notice that the screening threshold in round i can go to one only if the IC constraint (9) binds. Otherwise, the entrepreneur again would be better off lowering the screening threshold by offering a slightly higher interest rate. The binding constraint means that

$$\begin{aligned}&(1-F_G(s_i))\left(X-r_i(s_i,\mathbf{s^*_{i-1}})\right)-(1-F_G(s_{i+1}))\left(X-r_{i+1}(s_{i+1},\mathbf{s^*_i})\right)\\&=(F_G(s_{i+1})-F_G(s_i))V_{i+2}.\end{aligned} \tag{10}$$

The right-hand side of Equation (10) is positive. But the left-hand side is negative since $(X-r_{i+1}(s_{i+1},\mathbf{s^*_i}))$ is bounded away from zero. Thus, the IC constraint cannot bind, which shows that the last round interest rate must converge to X.

Proposition 2 shows that when large markets are optimal the entrepreneur is able to extract the maximal surplus in the limit as N goes to infinity even without repeated contacts with investors. When small markets are optimal the ability to contact the same investor many times does not help restore efficiency because the entrepreneur is always better off making an offer to a new investor than to a previously visited investor. Therefore, she will always make offers to all available investors. But this is the main source of inefficiency in the first place. We now consider the case of a credit bureau.

4. Equilibrium with a credit bureau

With a credit bureau in place investors learn how many times the entrepreneur has been rejected previously but not the terms on which she has been rejected. Therefore, as in the case of no credit bureau studied in Section 3, investors have to form beliefs about the terms at which the entrepreneur has been rejected previously. However, now there is no need for the entrepreneur to signal how many times she was rejected. As a result, there is no reason for an investor to change his beliefs about previous offers if he is offered an out-of-equilibrium interest rate offer.

Proposition 3 characterizes an equilibrium. The equilibrium screening thresholds solve the same maximization problem as in the case without a credit bureau but without the incentive compatibility constraints (9). The proposition shows that without the need to signal her application history the entrepreneur cannot credibly make low interest rate offers and this biases her towards asking for higher interest rates in equilibrium.

PROPOSITION 3: *Suppose rejections are publicly observable. Then equilibrium screening thresholds solve*

$$\begin{aligned} V_N &\equiv \max_{s_N}(1 - F_G(s_N))\left(X - r_N(s_N, \mathbf{s}^*_{\mathbf{N-1}})\right), \\ V_i &\equiv \max_{s_i}(1 - F_G(s_i))\left(X - r_i(s_i, \mathbf{s}^*_{\mathbf{i-1}})\right) + F_G(s_i)V_{i+1}, \quad i = N-1, ..., 1, \end{aligned} \tag{11}$$

where interest rates $r_i(s_i, \mathbf{s}^*_{\mathbf{i-1}})$ are given by (2). If MLRP holds strictly then there is an $\varepsilon > 0$ such that for any number of investors the screening threshold in the first round is less than $1 - \varepsilon$.

Proof: Consider the maximization problem of the entrepreneur in the first round:

$$\max_{s_1}(1 - F_G(s_1))(X - r_1(s_1)) + F_G(s_1)V_2,$$

where V_2 is the entrepreneur's continuation value. Because the entrepreneur cannot affect investors' beliefs with his choice of s_1 she takes V_2 as given. Because $F_G(s)$ is an increasing function of s the equilibrium choice of s_1^* is an increasing function of V_2. Lemma 1 provides an upper bound on the value of V_2. It shows that V_2 is less than $X - (1-\pi)/(\pi\lambda)$. In the proof of Lemma 1 we actually show that if MLRP holds strictly then the bound is strict, that is there exists $\delta > 0$, such that $V_2 = X - (1-\pi)/(\pi\lambda) - \delta$. Using Equation (2) for $r_1(s_1)$ we can rewrite the maximization problem as

$$\max_{s_1}(1 - F_G(s_1))\left(\delta - \frac{1-\pi}{\pi}\left(\frac{f_B(s_1)}{f_G(s_1)} - \frac{1}{\lambda}\right)\right) + V_2.$$

It is clear that the solution s_1^* to the above problem is strictly less than one. ■

With the strict MLRP, the first contacted investor breaks even if he finances the project with the signal equal to s_1^* and makes a positive expected profit if the signal is above s_1^*. Proposition 3 shows that s_1^* does not go to one with N. Therefore, the expected profit does not vanish in the limit as N goes to infinity. We have:

Corollary 1: *Suppose that MLRP holds strictly. Then in equilibrium with a credit bureau investors earn strictly positive rent, which is not competed away even as the number investors grows large.*

The rent collected by investors can be so large that it can outweigh the benefits of informed lending. To illustrate, suppose that $X = 1$, $N = 100$, $f_B(s) \equiv 1$, and $f_G(s) = 2s$. In addition, suppose that a credit bureau may also produce hard information that is relevant for assessing the credit quality of the entrepreneur. We model this as a public signal S_0 which satisfies MLRP and is conditionally independent of other signals. Suppose also that there are investors who can commit not to use any private signals and base their decisions solely on the public signal. Denote the likelihood ratio $\Pr(G|S_0)/\Pr(B|S_0)$ conditional on realization of the public signal as z.

Figure 1 shows the entrepreneur's expected surplus as a function of z in two cases: (1) if she obtains financing from investors that use only public signal (blue line) and (2) if she obtains financing from investors who use both public and private signals (red line). We can see that if the quality of the project after public signal is high enough then the entrepreneur is better off applying to investors who use only public information. Uninformed investors are able to outcompete informed agents because they do not earn rent. While using both public and private information results in better investment decisions the benefits for high quality projects are not sufficient to compensate the rent that must be surrendered to privately informed investors.

The immediate consequence of Proposition 3 and Proposition 2 is that in the case in which it is best to have as large markets as possible the introduction of a credit bureau can actually reduce market efficiency and lead to a lower surplus for the entrepreneur. Consider next the case when there are many potential investors but it is best to have small markets. Proposition 2 shows that without a credit bureau the entrepreneur is never excluded from the market and can visit all available investors, which is inefficient. Proposition 3 shows that with a credit bureau, screening thresholds in the first rounds are lower than they would be if there was no credit bureau. While low interest rate offers mean that some rent is left for investors, they also mean that the negative impact of rejections is stronger and can lead to the exclusion of the entrepreneur from the market. When small markets are efficient restricting the competition among investors and allowing them to utilize their information more efficiently can lead to higher social surplus. The next example shows that the increased surplus can more than compensate for the higher rent left to investors, so that the entrepreneur can also be better off in a sequential market with a credit bureau than in a sequential market without a credit bureau, or even in a centralized auction market with N investors.

Suppose $\pi = 1/2$, $X = 1$, and $f_B(s) = 1$ for all $s \in [0, 1]$, and $f_G(s) = 0$ for

$s \in [0, 1/2]$ and $f_G(s) = 2$ for $s > 1/2$. We show in Axelson and Makarov (2016) that surplus in the first-price auction is maximized with a single investor with a screening threshold set to $1/2$. It is not difficult to see that the same surplus can also be achieved in a sequential market with a credit bureau. Suppose the entrepreneur asks in the first round for the interest rate that corresponds to the threshold $s_1^* = 1/2$. This generates the maximal surplus, and all surplus is captured by the entrepreneur. There will be no second round, because if the project is rejected by the first investor, the updated credit quality is so low that no investor would be willing to finance the project at any interest rate. Thus, the market with a credit bureau creates more social surplus and more profits for the entrepreneur than the market without a credit bureau and the auction market.

Proposition 4 shows that the above example is not a coincidence. It shows that when it is best to have small markets a credit bureau endogenously restricts the number of investors the entrepreneur can visit. This contrasts with the case of no credit credit in which the entrepreneur can always visit all potential investors.

PROPOSITION 4: *If $\frac{F_G(s)}{F_B(s)} \frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s then the entrepreneur can visit all available investors. If $\frac{F_G(s)}{F_B(s)} \frac{f_B(s)^2}{f_G(s)^2}$ is a strictly increasing function at some neighborhood of $s = 1$ then for large N the entrepreneur visits strictly less than N investors.*

Proof: See the Appendix.

We conclude this section by noticing that the negative effect of the credit bureau comes from the fact that it reveals only partial information about the application history of the entrepreneur. By revealing how many times the entrepreneur was rejected but not the interest rates at which she was rejected, a credit bureau eliminates incentives for the entrepreneur to use low interest rates as a signal of her quality and encourages "signal-jamming", in a similar spirit to the papers by Holmstrom (1982) and Stein (1989).

Although it might not be feasible for a credit bureau to record terms on which an entrepreneur is rejected, the next proposition shows that doing so would generally lead to more efficient sequential markets:

PROPOSITION 5: *Suppose a credit bureau registers both rejections and interest rate offers. Then the entrepreneur always prefers the market with a credit bureau over one without. For large enough N, social surplus and the entrepreneur's profit are no less in the sequential market than those in a first-price auction with free entry.*

Proof: If rejections and interest rate offers are publicly observable the entrepreneur in the sequential market with a credit bureau can always replicate surplus and profits generated in the market without by offering the same sequence of interest rates. In the limit as N goes to infinity, all surplus goes to the entrepreneur using this strategy. When using all N rounds does not maximize surplus, the entrepreneur in the market with a credit bureau may be able to earn strictly more profits with a smaller set of investors as shown in the example above. These profits are higher than the surplus in the market without a credit bureau, and hence surplus with a credit bureau (which is always weakly greater than entrepreneurial profits) is strictly higher. ■

5. Multiple equilibria

In this section, we show that there can be multiple equilibria in the case of a credit bureau and provide sufficient conditions for equilibrium existence. Suppose that there are two potential investors and that $X = 1$, $f_B(s) \equiv 1$, and $f_G(s)$ is given by the following equation:

$$f_G(s) = 0.25 + \frac{1}{\exp\left(-100\left(s - \frac{1}{3}\right)\right) + 1} + \frac{0.25}{\exp\left(-100\left(s - \frac{2}{3}\right)\right) + 1}. \qquad (12)$$

Panel (a) of Figure 2 plots densities $f_B(s)$ and $f_G(s)$, which are the continuous versions of the case when investors' signals take three values: low, medium and high as depicted in Figure 2, panel (b). If the project is bad then any of the values is equally likely. If the project is good then the respective probabilities of low, medium and high signals are 1/12, 5/12, 1/2.

Figure 3 plots the expected profit of the entrepreneur as a function of the screening threshold s if there is only one investor. Panels (a), (b), and (c) correspond to the three initial values of the likelihood ratio $z = \pi/(1-\pi)$: $z = 0.9$, $z = 0.95$, and $z = 1$. We can see that two flat areas of $f_G(s)$ lead to two humps in the expected surplus. At high values of z the entrepreneur's profit is maximized at low screening thresholds while at low values of z the profit is maximized at high screening thresholds. There is a value of z (panel (b), $z = 0.95$) at which the same expected surplus is achieved at two different values of s. Even though there is a unique equilibrium in case of a single investor if $z \neq 0.95$ two equilibria can realize when there are two potential investors.

In the first equilibrium, the second investor believes that the entrepreneur asks the first investor for a low screening threshold. This makes it optimal for the entrepreneur to ask for a low screening threshold because the rejection then is very costly for the entrepreneur: If she is rejected she can no longer obtain financing from the second investor even if he receives the most optimistic signal.

In the second equilibrium, the second investor believes that the entrepreneur asks the first investor for a high screening threshold. In this case, the cost of rejection is not so high because even if rejected the entrepreneur has still a chance to obtain financing from the second investor. As a result, it is optimal for the entrepreneur to try for a low interest rate and high screening threshold from the first investor.

For the two equilibria to exist the entrepreneur's choice of screening thresholds must be consistent with investors' beliefs. This happens if the likelihood ratio z is such that $z > 0.95$ and $z(X - V_1) < 0.95$, where V_1 is the expected profit of the entrepreneur in the second equilibrium after she is rejected by the first investor.

If z is just below 0.95 then only the second equilibrium with two screenings exists because even with a single investor the entrepreneur is better off with a high screening threshold. Therefore, no matter what the second investor believes, the entrepreneur asks the first investor for a high screening threshold. If z is just above 1.03 then only the first equilibrium with one screening exists because even if the second investor believes that the screening threshold in the first round is high the entrepreneur finds it profitable to deviate and ask for a low screening threshold. As a result, the entrepreneur can no longer take advantage of two investors and therefore can no longer attain a high expected surplus.

Figure 4, panel (a) plots the entrepreneur's expected profit in the two equilibria as a function of her initial likelihood ratio z. Panel (b) plots social surplus. The blue dashed line corresponds to the first equilibrium with one screening; the red line to the second equilibrium with two screenings. We can see that the entrepreneur is better off in the second equilibrium, in which she can be screened twice. Social surplus, however, is higher in the first equilibrium, in which the entrepreneur is screened only once.

This gives the surprising implication that social welfare can be improved if the government imposes an interest rate cap. Figure 5 shows interest rates in the two equilibria. The blue line shows an interest rate in the first equilibrium with one screening. The red and magenta lines show interest rates in the second equilibrium. Naturally, an interest rate increases if the entrepreneur is rejected by the first investor. If there is an interest rate cap so that the rejected entrepreneur can no longer obtain financing in the second round then the second equilibrium is no longer sustainable. Thus, an interest rate cap can eliminate markets for rejected borrowers, and hence can eliminate the socially inefficient equilibria with many financing rounds.

Panel (a) also illustrates, perhaps surprisingly, that the entrepreneur's profit can be non-monotone in the ex-ante project's quality. This happens because of the entrepreneur's inability to commit to ask for a high screening threshold from the first investor, or in other words, for a low interest rate. As a result, investors get higher rent and the entrepreneur is worse off.

We conclude this section by presenting sufficient conditions for equilibrium existence. The conditions are the same for both cases: with and without a credit bureau.

PROPOSITION 6: *Suppose that $f_B(s)/f_G(s)$ is a continuous function and for any y*

$$(1 - F_G(s))\left(y - \frac{f_B(s)}{f_G(s)}\right) \tag{13}$$

is a quasi-concave function of s. Then there exists a pure-strategy equilibrium in the game with any number of investors with and without a credit bureau.

Proof: First, consider the case of with a credit bureau. We can view the optimization problem in each round i as if it is done by a fictitious agent i. Each fictitious agent i takes decisions of other agents as given and solves (11), which is the same as maximizing (13) with respect to s with an appropriately chosen y. By assumption the payoff of each agent i is quasi-concave in his own action and continuously depends on the actions of other agents. Therefore, by Theorem 1.2 of Fudenberg and Tirole (1991) there exists a pure-strategy equilibrium. The proof is similar if there is no credit bureau. The quasi-concavity of the payoff ensures that the action space of every agent that satisfies the incentive compatibility constraint (9) is a concave set. Therefore, Theorem 1.2 of Fudenberg and Tirole (1991) still applies. ■

6. Conclusion

We have developed a sequential credit market model to analyze the efficiency of primary capital markets for new projects. We compare three regimes of differing level of transparency: A sequential market where lenders have no information about the search history of an entrepreneur, a sequential market where lenders can observe the search history via a credit bureau, and a centralized auction markets. None of these markets lead to first-best investment decisions, even when the number of potential investors grows so large that the aggregate information in the market allows for perfect investment decisions, and even when entrepreneurs are infinitely patient and there are zero search costs. Moving to a more transparent market via the introduction of a credit bureau tends to increase rents to investors at the expense of entrepreneurs, leads to shorter search for financing by the entrepreneur, and has ambiguous effects on the efficiency or resource allocation. A centralized market is more efficient than decentralized markets if the number of investors who participate in the market can be chosen optimally, but may otherwise lead to excessive competition which impedes efficiency relative to decentralized markets.

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Appendix. Proofs

Proof of Proposition 1:

Consider maximization problem (3). Let $n \leq N$ be the largest n such that the expected surplus generated with n screenings is strictly higher than that generated with $n-1$ screenings. Then for all $i > n$, $s_i = 1$ and for all $i \leq n$, s_i satisfy the F.O.C.:

$$\pi X f_G(s_j) \prod_{i \leq n, i \neq j} F_G(s_i) = (1-\pi) f_B(s_j) \prod_{i \leq n, i \neq j} F_B(s_i) \quad j = 1, \ldots, n. \tag{A1}$$

Let $s_* = \min(\{s_i\}_{i=1}^n)$ and $s^* = \max(\{s_i\}_{i=1}^n)$. Suppose that $s_* \neq s^*$. Consider a change Δ in the expected surplus if one changes the threshold s^* (or any if there are multiple s^*) to s_*:

$$\Delta = \prod_{s_i \neq s^*} F_B(s_i) \left[(1-\pi) F_B(s^*) - \pi X F_G(s^*) \prod_{s_i \neq s^*} \frac{F_G(s_i)}{F_B(s_i)} \right] -$$

$$- \prod_{s_i \neq s^*} F_B(s_i) \left[(1-\pi) F_B(s_*) - \pi X F_G(s_*) \prod_{s_i \neq s^*} \frac{F_G(s_i)}{F_B(s_i)} \right] =$$

$$= \prod_{s_i \neq s^*} F_B(s_i) \left(\left[F_B(s^*) - F_G(s^*) \frac{f_B(s^*)}{f_G(s^*)} \right] - \left[F_B(s_*) - F_G(s_*) \frac{f_B(s^*)}{f_G(s^*)} \right] \right),$$

where we use the F.O.C. (A1). Because of the MLRP the function

$$F_B(s) - F_G(s) \frac{f_B(s^*)}{f_G(s^*)}$$

is nondecreasing between s_* and s^*. Thus, the maximum surplus is achieved when all s_i equal to s_*. The F.O.C. (A1) therefore becomes the F.O.C. (4). Equation (4) has a unique solution because of the MLRP.

To prove that the expected surplus strictly increases with the number of screenings if $\frac{F_G(s)}{F_B(s)} \frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s we need to show that for any N the solution to the maximization problem (3) is interior. Suppose on the contrary that at some N it is optimal to set s_N to one. Let N be the lowest number of screenings when this happens. The optimal screening threshold level is the same in all $N-1$ screenings and solves the F.O.C.

$$\pi X f_G(s) F_G(s)^{N-2} = (1-\pi) f_B(s) F_B(s)^{N-2}.$$

Taking the derivative of the surplus with respect to s_N at $s_N = 1$ we have

$$(1-\pi)f_B(1)F_B(s)^{N-q} - \pi X f_G(1)F_G(s)^{N-1} = f_B(1)F_B(s)^{N-1}\left(1 - \lambda\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}\right) < 0,$$

where we have used the F.O.C and where the last inequality follows from the fact that $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a decreasing function of s and therefore takes the lowest value λ^{-1} at $s = 1$. As a result, it is suboptimal to set s_N to 1 and the solution must be indeed interior.

Finally, we prove that the maximal expected surplus can be achieved with no more than n screenings if $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly increasing function for $s \in [s_n^*, 1]$. We prove earlier that Equation (4) has a unique solution, s_n^*, which is strictly increasing in n. We now show that the maximand in (3) is higher for $s_i = s_n^*$, $i = 1, 2, \ldots, n$ and $s_{n+1} = 1$ than for $s_i = s_{n+1}^*$, $i = 1, 2, \ldots, n+1$. To see this, we start at a point s_{n+1}^*, and move the first n screening thresholds s down while moving the $n+1$'s screening threshold s_{n+1} up to hold $F_B(s)^n F_B(s_{n+1})$ constant:

$$\frac{ds}{ds_{n+1}} = -\frac{F_B(s)f_B(s_{n+1})}{nf_B(s)F_B(s_{n+1})}.$$

This changes the maximand with an amount proportional to

$$-F_G(s)^n f_G(s_{n+1}) - nF_G(s)^{n-1}f_G(s)F_G(s_{n+1})\frac{ds_n}{ds},$$

which has the same sign as

$$\frac{F_G(s_{n+1})f_B(s_{n+1})}{F_B(s_{n+1})f_G(s_{n+1})} - \frac{F_G(s)f_B(s)}{F_B(s)f_G(s)}.$$

By the assumption of the Proposition, this change is positive for $s_{n+1} > s_n^*$, and hence the maximand is increased by setting $s_{n+1} = 1$. But at $s_{n+1} = 1$, it is optimal to set all first n screening thresholds to s_n^*.

Q.E.D.

Proof of Lemma 1:

We first observe that the maximal expected surplus respects the order induced by MLPR on the space of signal distributions. Consider two cases of informative signals. Suppose that in both cases if the project is bad the signal is drawn from the same distribution $F_B(s)$. At the same time, if the project is good then in the first case, the signal is drawn from a distribution F_{G_1} with density f_{G_1}, and in the second case, from a distribution F_{G_2} with density f_{G_2}. Suppose that for all $s > s'$

$$\frac{f_{G_1}(s)}{f_{G_2}(s)} \geq \frac{f_{G_1}(s')}{f_{G_2}(s')},$$

then the maximal surplus in the first case is no less than that in the second case. This follows from the fact that MLRP implies the monotone probability ratio (Milgrom (1981)).

Suppose for now that $f_B(s) \equiv 1$. Then given λ, the maximal expected surplus is achieved with $f_G(s) = 0$ for $s \in [0, 1 - \lambda^{-1})$ and $f_G(s) = \lambda$ for $s \in [1 - \lambda^{-1}, 1]$. Setting a screening threshold level to $1 - \lambda^{-1}$ ensures that good projects are always financed and bad projects are financed with probability λ^{-1}. Thus, with a single screening the expected surplus is $\pi X - (1 - \pi)/\lambda$. Direct computations show that $\frac{F_G(s)}{F_B(s)} \frac{f_B(s)}{f_G(s)}$ is an increasing function for $s \in [1 - \lambda^{-1}, 1]$. Thus, by Proposition 1, $\pi X - (1 - \pi)/\lambda$ is in fact the maximal expected surplus. Finally, notice that the assumption that $f_B(s) \equiv 1$ is innocuous. For an arbitrary $f_B(s)$ the maximal surplus is achieved with $f_G(s) = 0$ for $s \in [0, \bar{s})$ and $f_G(s) = \lambda f_B(s)$ for $s \in [\bar{s}, 1]$, where $\bar{s}$ is determined by the condition that $\int_{\bar{s}}^{1} \lambda f_B(s) ds = 1$. Hence, $\int_0^{\bar{s}} f_B(s) ds = 1 - \lambda^{-1}$.

Q.E.D.

Proof of Proposition 2:

First, we show that any equilibrium that survives the Cho and Kreps intuitive criterion must be separating. For this, we need to show that the entrepreneur who has been rejected i times would always like to separate herself from those who have been rejected more than i times. Denote the entrepreneur who has been rejected $i - 1$ times by E_i, and her expected surplus (conditional on the project being good) by V_i, $i = 1, ..., N$. Suppose contrary to the statement of the proposition that there is some pooling in equilibrium. Let i be the first instance such that E_i pools with entrepreneurs rejected more than i times. Let $j = \min\{k > i : E_k \quad \text{pools with} \quad E_i\}$.

Let s^* be a screening threshold asked by E_i and E_j. Let π^* be an investor's belief that the project is good if the entrepreneur asks for a screening threshold s^* before the investor observes his private signal. We have

$$\begin{aligned} V_i &= (1 - F_G(s^*)) (X - r(\pi^*, s^*)) + F_G(s^*) V_{i+1}, \\ V_j &= (1 - F_G(s^*)) (X - r(\pi^*, s^*)) + F_G(s^*) V_{j+1}, \end{aligned}$$

where

$$r(\pi^*, s^*) = \frac{1 - \pi^*}{\pi^*} \frac{f_B(s^*)}{f_G(s^*)}.$$

Let $\hat{\pi}$ be the investor's belief that the project is good if the investor believes that the entrepreneur is of type E_i. Clearly, $\hat{\pi} > \pi^*$. Let $\hat{s}$ be such that

$$V_j = (1 - F_G(\hat{s}))\left(X - r(\hat{\pi}, \hat{s})\right) + F_G(\hat{s})V_{j+1}. \tag{A2}$$

Suppose that investors believe that the entrepreneur is of type E_i if she asks for the screening threshold $\hat{s}$. Then the type E_j entrepreneur is indifferent between asking for s^* and $\hat{s}$. Note that $V_{i+1} > V_{j+1}$ because the type E_i entrepreneur can always follow the strategy of the type E_j entrepreneur. Therefore, Equation (A2) implies that

$$V_i < (1 - F_G(\hat{s}))\left(X - r(\hat{\pi}, \hat{s})\right) + F_G(\hat{s})V_{i+1}.$$

Hence, E_i is better off by deviating and asking a screening threshold, which is slightly above $\hat{s}$. At the same time, E_j is worse off by deviating to this threshold. Thus, no pooling equilibrium survives the Cho-Kreps intuitive criterion.

Next, we prove that if MLRP holds strictly then as N goes to infinity the entrepreneur extracts all the surplus and his surplus converges to that generated in the first-price auction. The proof is done in two steps. First, we show that if $r_N(1, \mathbf{s^*_{N-1}})$ goes to X as N goes to infinity then the entrepreneur's surplus converges to that generated in the first-price auction. Then, we show that in equilibrium $r_N(1, \mathbf{s^*_{N-1}})$ must go to X.

Step 1. Suppose that $\lim_{N\to\infty} r_N(1, \mathbf{s^*_{N-1}}) = X$, where $r_N(1, \mathbf{s^*_{N-1}})$ is given by Equation (2). The expression for social surplus (3) implies that if $\prod_{i=1}^N F_G(s_i^*) \to F_G^N(s^*)$ and $\prod_{i=1}^N F_B(s_i) \to F_B^N(s^*)$, where s^* is a screening threshold in the first-price auction, then surpluses generated in a sequential credit market and in a first-price auction are asymptotically the same.

Using equation (2) for the interest rate $r_N(1, \mathbf{s^*_{N-1}})$ we can see that

$$\lim_{N\to\infty} r_N(1, \mathbf{s^*_{N-1}}) = X \Leftrightarrow \lim_{N\to\infty} \lambda X \frac{\pi}{1-\pi} \prod_{i=1}^{N-1} \frac{F_G(s_i)}{F_B(s_i)} = 1. \tag{A3}$$

If the entrepreneur is rejected $N-1$ times then in the last round she solves

$$V_N = \max_{s_N}(1 - F_G(s_N))\left(X - r_N(s_N, \mathbf{s^*_{N-1}})\right).$$

If $\lim_{N\to\infty} r_N(1, \mathbf{s^*_{N-1}}) = X$ and the strict MLRP holds then $\lim_{N\to\infty} s_N^* = 1$. Proposition 2 shows that $s_i^* > s_N^*$. Therefore for any i, $\lim_{N\to\infty} s_i^* = 1$. Let $\Delta s_i = 1 - s_i^*$.

Taking the Taylor's series of (A3) we have

$$\sum_{i=1}^{N-1} \Delta s_i = a_1 + O(\Delta s_N), \quad a_1 = \frac{\ln(\lambda z X)}{\lambda - 1}. \tag{A4}$$

Therefore,

$$\begin{aligned}\prod_{i=1}^{N} F_G(s_i) &= e^{-\lambda a_1} + O(\Delta s_N),\\ \prod_{i=1}^{N} F_B(s_i) &= e^{-a_1} + O(\Delta s_N).\end{aligned}$$

We show in Axelson and Makarov (2016) that $F_G^N(s^*)$ and $F_B^N(s^*)$ converge to the same corresponding limits.

Step 2. We now show $r_N(1, \mathbf{s^*_{N-1}})$ goes to X in equilibrium. Suppose to the contrary that there exists $\varepsilon > 0$ such that for all N $r_N(1, \mathbf{s^*_{N-1}}) < X - \varepsilon$ for some . Note that only a bounded number of screening thresholds can stay away from one as N goes to infinity. Otherwise, the entrepreneur would not be able to obtain financing in the last round. Let M be the maximal index such that $\limsup_{N\to\infty} s_{N-M} = 1$ but $\limsup_{N\to\infty} s_{N-M+1} < 1$. Consider the problem of the entrepreneur who has been rejected $N - M - 1$ times. She solves problem (8):

$$\begin{aligned}V_{N-M} &\equiv \max_{s_{N-M}} (1 - F_G(s_{N-M}))\left(X - r_i(s_{N-M}, \mathbf{s^*_{N-M-1}})\right) + F_G(s_{N-M})V_{N-M+1},\\ &\text{s.t. } V_{N-M+1} \geq (1 - F_G(s_{N-M}))\left(X - r_i(s_{N-M}, \mathbf{s^*_{N-M-1}})\right) + F_G(s_{N-M})V_{N-M=2}.\end{aligned}$$

As in the proof of Proposition 3 below, one can show that the unconstrained solution to the above problem entails s_{N-M} to be bounded away from one. Since by assumption, s_{N-M} goes to one it must be that the incentive compatibility constraint binds. However, with s_{N-M+1} being away from one, s_{N-M} converging to one, and $r_i(s_{N-M}, \mathbf{s^*_{N-M-1}}) < X - \varepsilon$, the incentive compatibility constraint cannot bind.

Q.E.D.

Proof of Proposition 4:

Consider first the case when $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s. Suppose that there is a round $i < N$ such that $s_i^* < 1$ and the entrepreneur is unable to contact another investor after being rejected in round i, that is $s^*_{i+1} \geq 1$. In this round i, the entrepreneur solves

$$\max_{s_i^*} (1 - F_G(s_i^*))\left(X - r_i(s_i, \hat{\mathbf{s}}_{\mathbf{i-1}})\right), \tag{A5}$$

where $r_i(s_i, \mathbf{\hat{s}_{i-1}})$ is given by (2). Since $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)}{f_G(s)}$ is a strictly decreasing function of s we have

$$\frac{F_B(s_i^*)}{F_G(s_i^*)}\frac{f_G(s_i^*)}{f_B(s_i^*)} < \frac{F_B(1)}{F_G(1)}\frac{f_G(1)}{f_B(1)} = \lambda.$$

Therefore, there exists $s_{i+1}^* < 1$ such that

$$r_{i+1}(s_{i+1}^*, \mathbf{\hat{s}_i}) = r_i(s_i, \mathbf{\hat{s}_{i-1}}) \times \frac{f_G(s_i^*)}{f_B(s_i^*)}\frac{F_B(s_j^*)}{F_G(s_j^*)} \times \frac{f_B(s_{i+1}^*)}{f_G(s_{i+1}^*)} < X.$$

Hence, the entrepreneur has a chance to get financing if she approaches another investor and therefore, round i cannot be the last round.

Suppose now that $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)^2}{f_G(s)^2}$ is a strictly increasing function of s in some neighbourhood of $s = 1$. We first show that this implies a bound on the derivative of the likelihood ratio at $s = 1$. For simplicity, we assume that $f_B(s) \equiv 1$. Note that

$$\left(\frac{F_G(s)}{F_B(s)}\right)'_{s=1} = \lambda - 1.$$

Since

$$\left(\frac{F_G(s)}{F_B(s)}\frac{1}{f_G(s)^2}\right)' = \left(\frac{F_G(s)}{F_B(s)}\right)'\frac{1}{f_G(s)^2} + \frac{F_G(s)}{F_B(s)}\left(\frac{1}{f_G(s)^2}\right)'$$

the fact that $\frac{F_G(s)}{F_B(s)}\frac{f_B(s)^2}{f_G(s)^2}$ is a strictly increasing function at $s = 1$ implies that

$$0 \leq f_G'(1) < \frac{\lambda(\lambda-1)}{2}. \tag{A6}$$

The idea of the proof is to show that relative flatness of the likelihood ratio leads to large screening thresholds. The entrepreneur can contacts all available investors when N goes to infinity only if the number of screening thresholds bounded away from one is uniformly bounded. Suppose for a moment that round i is the last round. The entrepreneur then solves problem (A5). To simplify notation, let

$$z = \frac{\pi}{1-\pi}\Pi_{j=1}^{i-1}\frac{F_G(\hat{s}_j)}{F_B(\hat{s}_j)}.$$

Then

$$r_i(s_i, \mathbf{\hat{s}_{i-1}}) = \frac{1}{zf_G(s_i)}.$$

The F.O.C. to the above problem is

$$-(1 - F_G(s_i))\left(\frac{1}{f_G(s_i)}\right)' = f_G(s_i)zX - 1. \tag{A7}$$

Let $\Delta s = 1 - s_i^*$, where s_i^* is a solution to (A7). Taking the Taylor's series of (A7) at $s_i = 1$ we have

$$\frac{f'_G(1)\Delta s}{\lambda} = \lambda z X - 1 - f'_G(1) z X \Delta s + o(\Delta s).$$

Hence,

$$\Delta s = \frac{\lambda(\lambda z X - 1)}{f'_G(1)(1 + \lambda z X)} + o(\lambda z X - 1). \tag{A8}$$

Therefore,

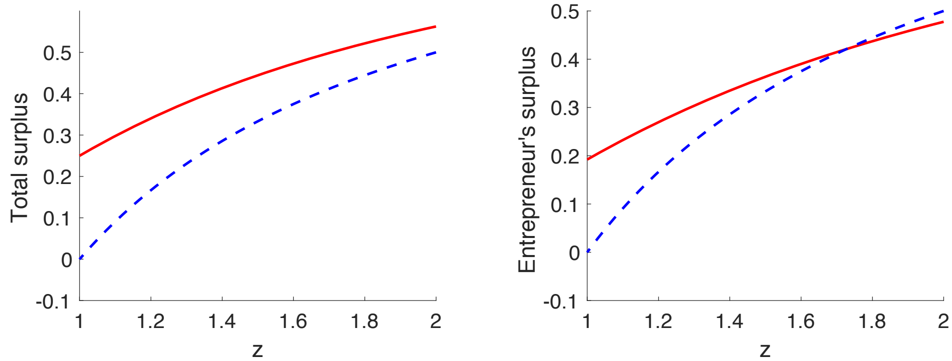
$$F_G(s_i^*)/F_B(s_i^*) = (1 - (\lambda - 1)\Delta s) + o(\lambda z X - 1) = \left(1 - \frac{\lambda(\lambda - 1)(\lambda z X - 1)}{f'_G(1)(1 + \lambda z X)}\right) + o(\lambda z X - 1).$$

Inequality (A6) implies that

$$\left(1 - \frac{\lambda(\lambda - 1)(\lambda z X - 1)}{f'_G(1)(1 + \lambda z X)}\right) < \frac{1}{\lambda z X}.$$

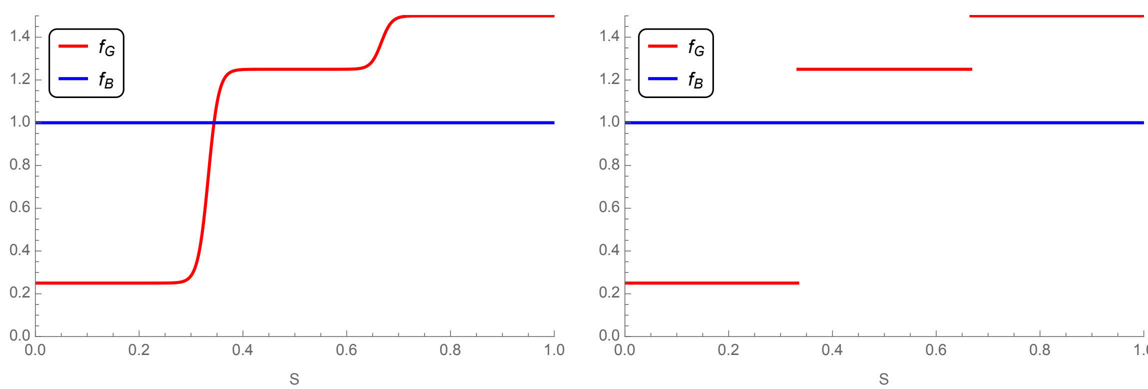
Therefore, if rejected, the entrepreneur is unable to contact another investor.

Q.E.D.



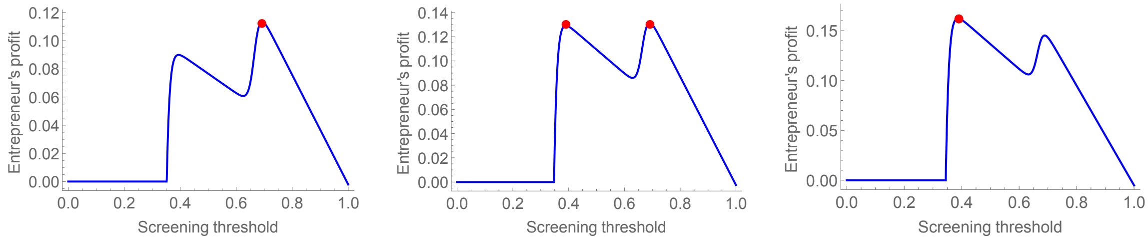
(a) (b)

Figure 1. Hard information screening. The blue dashed line shows the entrepreneur's profit if she obtains financing from investors who use only publicly available information. The red line shows the entrepreneur's profit if she obtains financing from investors who use both public and private information.



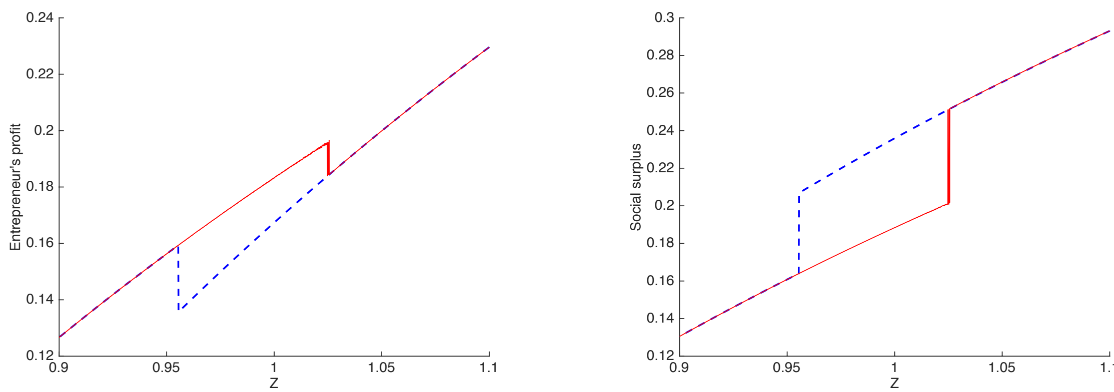
(a) (b)

Figure 2. Multiple equilibria. Signal densities. Figure 2, panel (a) plots densities $f_B(s)$ and $f_G(s)$, where $f_G(s) \equiv 1$ and $f_G(s)$ is defined in equation (12). Densities in panel (a) are smoothed versions of the densities shown in panel (b).



(a) (b) (c)

Figure 3. Entrepreneur's profit. Panels (a), (b), and (c) show the entrepreneur's expected profit when there is only one investor for the three cases: $z = 0.9$, $z = 0.95$, and $z = 1$. Other parameters are as follows: $X = 1$, densities f_B and f_G are displayed in Figure 2, panel (a).



(a) (b)

Figure 4. Multiple equilibria. Entrepreneur's profit and social surplus. Figure 4 Panel (a) plots the entrepreneur's expected profit in the two equilibria described in Section 5 as a function of her initial likelihood ratio z. Panel (b) plots social surplus. The blue dashed line corresponds to the first equilibrium with one screening; the red line - to the second equilibrium with two screenings.

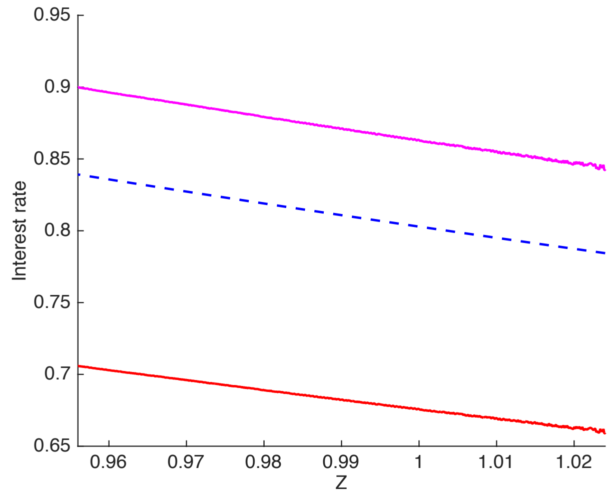


Figure 5. Multiple equilibria. Interest rate. Figure 5 shows interest rate the entrepreneur asks from investors. The blue dashed line shows the interest rate in the equilibrium with single screening. The red and magenta lines show the interest rates in the equilibrium with two screenings, with the highest rate being asked if rejected at the first investor.