The Maturity Structure of Inside Money*

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Abstract

We study risk and maturity transformation when bank liabilities facilitate trade in goods markets and households face aggregate liquidity shocks. Banks' balance sheets transform aggregate investment risk providing a stable source of liquidity to households. When investments are sufficiently risky, bank liabilities transform risk and maturity: liabilities are less risky and have shorter term payoffs than banks' real investments. When maturity transformation is socially efficient, aggregate long-term liquidity is scarce raising the relative price of long-term bank issuances. In the competitive equilibrium banks provide too little maturity transformation relative to the social optimum.

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1 Introduction

Banks engage in maturity and risk transformation: their liabilities and assets have difference maturity and risk structures. A standard rationale for such bank balance sheet transformation is that banks provide insurance against households' idiosyncratic liquidity risk as in Diamond and Dybvig (1983). Whether banks provide insurance against idiosyncratic liquidity risk efficiently—and, therefore, transform their balance sheets efficiently—depends on the frictions prevalent in the economy. For example, in the presence of private trading markets, Jacklin (1987) and Farhi et al. (2009) find that banks engage in too little maturity transformation while in the presence of fire sale externalities Stein (2012) finds that banks engage in too much maturity transformation. We develop a new rationale for bank balance sheet transformation when banks provide insurance against aggregate liquidity risk and show that banks tend to engage in too little maturity transformation.

We study an economy in which households purchase and use bank liabilities to facilitate trade in goods markets. Banks use the proceeds from liability issuance along with their inside equity to finance real investments subject to aggregate risk. Those real investments and the bank's inside equity back the time- and state-contingent liabilities endogenously chosen by the bank. Their liquidity depends on households' exogenous willingness to accept bank liabilities in trade and on the endogenous and state-contingent cash flows backing them. Households face aggregate liquidity risk when the value of aggregate bank liabilities face aggregate risk. We examine how banks transform aggregate, real investment risk into their chosen liability structure.

Trading constraints in goods markets cause households to be more risk averse to bank liabilities' coupon payments than otherwise. Such risk aversion leads banks to issue liabilities with less risk than their underlying real investments: banks engage in risk transformation by committing to transfer part of the returns to their inside equity to households in low return states and retain part of the returns to externally financed investments in high return states. When banks cannot fully commit to long-term promises, as in Calomiris and Kahn (1991), they cannot engage in efficient risk transformation. In such cases, banks may issue liabilities with shorter-term payoffs than their underlying investments: banks engage in maturity transformation to improve risk transformation. In contrast to models with fire sales, the competitive equilibrium features less maturity transformation than socially optimal.

In our finite-horizon economy, heterogeneous households trade in frictional decentralized markets following Lagos and Wright (2005). As in Kocherlakota (1998), anonymity of households and inability to enforce private credit arrangements leads to household demand sources of liquidity. That liquidity comes from inside money issued by risk neutral agents we refer to as banks following Cavalcanti and Wallace (1999) and Gu et al. (2013). Decentralized trade is facilitated by inside money partially backed by the banks' underlying real investments with stochastic cash flows and partially backed by banks' own endowments.

Following Kiyotaki and Wright (1989) and recent contributions such as Rocheteau (2011), Lester et al. (2012), and Nosal and Rocheteau (2013), liquidity premia may arise in environments with exogenous asset specific liquidity constraints, informational asymmetries, or asset liquidation costs. Scarcity of real assets provides incentives to households to hold non-interest bearing assets to facilitate decentralized trade. We assume a scarcity of non-interest bearing assets in our model so that bank liabilities are the only payment instrument used to facilitate trade.

Payoffs associated with bank liabilities may differ in both timing and risk profile from those associated with banks' real investments. Banks face a limited commitment problem because they must retain sufficient equity in on-going investments. We allow for the possibility of costly early liquidation that changes the timing of the risky cash flows. Liquidation of long-term assets transfers cash flows from the future into the present allowing banks to deliver on liabilities with shorter-term payoffs than their investments.

When each banks' long-term investment cash flows are high enough in all histories, a pass-through claim where banks pay out coupons equal to the return on any investments financed through liability issuance is socially efficient and coincides with competitive equilibrium. The pass-through claim features neither risk nor maturity transformation because the pass-through claim is valuable enough to support socially efficient trade in all periods. There is no liquidity premium in the pricing of inside money.

When long-term cash flows are low enough in some histories so that the pass-through claim would lead to inefficient trade in decentralized markets, both socially efficient and competitive equilibrium allocations feature risk transformation and, in some cases, maturity transformation. When the pass-through claim leads to inefficient trade in decentralized markets, consumers who use bank liabilities as a medium of exchange face binding liquidity constraints in decentralized markets; their portfolio value is not sufficient to lead sellers in decentralized markets to produce enough goods to maximize the gains from trade. Binding liquidity constraints cause households to be relatively more risk averse with respect to the cash flows from the bank liabilities they purchase. As a result, it is efficient for banks to insure households against investment risk by issuing liabilities with less risky payoffs than the banks' underlying investments: banks perform risk transformation.

In the absence of any commitment problems, it is efficient for banks to provide such insurance only in late periods by promising to transfer their own equity to consumers in states where investments yield low returns and to retain a portion of the returns on household financed investments in states where investments yield high returns.

Banks have incentives to back-load liability payoffs and provide insurance only in late periods for three reasons. First, by assumption, issuing liabilities with early payoffs requires costly liquidation of long-term assets. Second, reducing future expected cash flows reduces the usefulness of inside money as a medium of exchange in future periods. Third, because the value of inside money is forward looking, the reduction in future value implies that a reduction in future cash flows also makes a claim to future cash flows less useful in facilitating current decentralized trade. As a result, in the absence of commitment problems, even if early liquidation of long-term assets did not reduce the present discounted value of the assets, banks would not engage in maturity transformation.

In the presence of commitment problems, shortening bank liabilities—and, therefore, bank assets—may be beneficial for decentralized trade. If in some histories investment returns are sufficiently low, then risk transformation with no maturity shortening may require banks to promise to transfer more of their equity than they can credibly commit to do. In this case, early liquidation of long-term assets relaxes the banks' commitment problem and allows banks to provide better insurance to consumers through their liability issues. In the presence of commitment problems, maturity transformation may be an efficient means of transforming risk associated with inside money.

We examine the efficiency properties of competitive equilibrium. The socially optimal maturity and risk profile internalizes how changes in bank maturity impact the liquidity premium associated with the liabilities issued by banks. Although banks understand that their liabilities may command a liquidity premium, they do not internalize the impact of their issuance decisions on equilibrium liquidity premia. When early liquidation is socially efficient in histories with low investment returns, the long-term aggregate liquidity premium is high because the value of aggregate bank liabilities is low. This high aggregate liquidity premium raises the relative price of liabilities promising long term cash flows in the low investment return state and incentivizes each individual bank to issue such long-term liabilities. Each bank attempting to free-ride on the high long-term liquidity premium results in an equilibrium where banks issue liabilities with too little liquidation and too little maturity transformation.

Jacklin (1987) and Farhi et al. (2009) obtain a similar finding and show that when households may trade bank liabilities, banks under-provide maturity transformation in the presence of idiosyncratic liquidity risk—indeed, banks engage in no maturity transformation in the absence of policy. In our model, however, banks provide no insurance against idiosyncratic risk; rather, the role of banks is to insure consumers against aggregate investment risk.

Our results also stand in contrast to theories of banking subject to fire sale externalities. In such papers, because bank assets are priced ex post in spot markets, a pecuniary externality which manifests on the asset side of the bank balance sheet causes banks either to issue too much short-term debt as in Stein (2012), or too much total credit as in Lorenzoni (2008). While our work features exogenously costly early liquidation of bank assets which can be interpreted as a fire sale at an exogenous price, early liquidation itself is not a source of constrained inefficiency in our model. Instead, the inefficiency arises because banks do not internalize the impact of their issuance choices on aggregate liquidity premia and the externality manifests on the liability side of the bank balance sheet.

Our finding that banks issue too little short-term debt also differs from Brunnermeier and Oehmke (2013) who find that when borrowers—interpreted as banks in their model—cannot commit to a debt maturity structure, they issue too much short-term debt. The borrower in their model, banks, play no role in providing insurance to creditors against either idiosyncratic or aggregate risk. Lastly, our paper is related to recent contributions by DeAngelo and Stulz (2015) and Gale and Gottardi (2017) who examine the efficiency of bank *leverage* in a model where bank liabilities serve as inside money. Our findings differ because we focus on the efficiency of the maturity of bank liabilities holding fixed bank leverage while these recent contributions focus of the level of bank equity.

When maturity transformation is a feature of constrained efficient allocations in our model, policies which require banks to issue liabilities with minimum expected short term payout can improve upon competitive equilibrium allocations. Such a policy conflicts with the types of liquidity management policies implemented in the wake of Basel

III. For example, the liquidity coverage ratio introduced by U.S. financial regulators in 2014 requires that banks hold sufficient liquid assets to cover expected short-term net outlays during a 30-day stress period. Such policies incentive banks to minimize expected short term outlays. Our finding that banks must be incentivized to issue liabilities with large enough short term payouts shows a novel cost associated with the liquidity management policies implemented after Basel III.

2 The Model

The model has three periods denoted 0, 1 and 2. Period 1 and period 2 are split into two sub-periods, a decentralized market sub-period followed by a centralized market sub-period. Period 0 features only a centralized market sub-period. The economy is populated by banks and two types of households: buyers and sellers.

At the beginning of period 1 a public signal $\omega \in \{\omega_l, \omega_h\} \equiv \Omega$ is observed by all. The signal determines the cash flows paid by the real investments in the economy with

$$\gamma(\omega_i) \equiv \text{Prob}\left(\omega = \omega_i\right). \tag{1}$$

2.1 Banks

There is a large number of identical banks, each having access to an investment technology which converts capital goods in period 0 into consumption goods in periods 1 and 2. To fund purchases of capital goods, banks may issue liabilities to the returns on their investments. Each risk neutral bank values consumption of goods during the centralized market sub-periods at time $t = 1, 2$ with a zero discount rate between periods. $c_t^B(\omega)$ is the bank's consumption in period t in state ω and $c^B \equiv \{c_t^B(\omega)\}$ is the bank's consumption plan.

The bank has initial endowment of $K^B \geq 0$ units of capital good and has access to a constant return to scale investment project that returns stochastic quantities of a general good in period 2. The bank can liquidate part of the project at time t, with $L(\omega) \in [0, 1]$ the fraction of the project liquidated in the first period at state ω, and κ a constant satisfying $0 < \kappa < 1$ with $1 - \kappa$ measuring the liquidation cost. The bank can walk away from the bank, or abscond with the bank's capital between time 0 and time 1 or between time 1 and time 2.[1]

Suppose that the bank does not walk away. For an initial investment of $I > 0$ in the project and a liquidation level $L(\omega) \in [0, 1]$ the project has stochastic cash flows of

$$\begin{aligned} &\text{time 1,} \quad I\left[\kappa L(\omega) z(\omega)\right], \\ &\text{time 2,} \quad I[1 - L(\omega)] z(\omega), \end{aligned} \tag{2}$$

where $z(\omega_h) \geq z(\omega_l) \geq 0$ are the stochastic time 2 cash flows per unit of investment.

If the bank walks away with I units of capital between time 0 and time 1, then the bank the bank will simply consume the cash flows from the tree and receive an expected payoff of

$$\xi I \sum_{\omega \in \Omega} \gamma(\omega) z(\omega), \tag{3}$$

where the parameter $0 \leq \xi \leq 1$ measures the productivity of the capital after the bank walks away. If the bank walks away with I units of capital between time 1 and time 2, then the bank receives second period payoffs of

$$I[1 - L(\omega)] z(\omega) \xi. \tag{4}$$

If the bank walks away with the capital at any time, then he cannot be forced to make any remaining coupon payments.

The bank issues liabilities with coupon payments backed by the cash flows generated by the investment project. We normalize the number of liabilities issued to one. Let $d_t(\omega) \geq 0$ be the promised coupon payoffs per liability at time t. The bank can consume whatever cash flows that remain after the coupons are paid in each period. Define a liabilities issue D as the coupon payments issued by the bank:

$$D \equiv \{D(\omega_l), D(\omega_h)\} \equiv \{d_1(\omega_l), d_2(\omega_l), d_1(\omega_h), d_2(\omega_h)\}. \tag{5}$$

The function $p_0(D)$ is the initial price of the bank liabilities in terms of the general good and $p_t(D(\omega))$ is the ex-coupon liability price at time $t = 1, 2$ with $p_t(D(\omega)) + d_t(\omega)$ the time-t cum-coupon liability price. For an investment and liquidation decision, ex-coupon liability prices depend only on the coupon process D. p_0^k is the price of capital goods in terms of the general good.

[1]One could equivalently interpret what the bank receives from walking away as a cost the bank must pay in units of utility to operate the investment technology.

The representative bank purchases capital goods, decides on a liquidation strategy, and a state and date contingent coupon payment plan. The bank can improve upon allocations obtained by households because the bank has unique access to the investment technology so that capital goods are more valuable when in possession of the bank than in possession of households and because households may prefer to hold liabilities with different stochastic payoffs than the underlying investment technology.

In principle, each bank may issue liabilities with different payoff structures. Each bank can commit to a feasible investment, liquidation and liability issuance strategy. The liability offered by any bank is indexed by the feasible coupon strategy chosen by the issuing bank. Each type of liability is perfectly observably by households. The model therefore admits aggregation of banks; in equilibrium the set of types of issued bank liabilities is degenerate and we focus on a representative bank.

In period 0, a representative bank chooses a consumption plan c^B, an investment scale, I, and a liquidation and liability coupon policy, (L, D) taking into account how its liabilities issue impacts the liabilities' price and taking the price function $p_0(D)$ as given. The bank solves:

$$\max_{I,L,D,c^B} \sum_{\omega\in\Omega} \gamma(\omega) \left[c_1^B(\omega) + c_2^B(\omega) \right], \tag{6}$$

subject to:

$$\begin{aligned}
p_0^k I &\leq p_0^k K^B + p_0(D), &(7)\\
c_1^B(\omega) + d_1(\omega) &= \kappa L(\omega) I z(\omega), &(8)\\
c_2^B(\omega) + d_2(\omega) &= [1 - L(\omega)] I z(\omega), &(9)\\
c_t^B(\omega) &\geq 0, \quad t = 0, 1, 2, &(10)\\
c_2^B(\omega) &\geq [1 - L] I z(\omega) \xi, &(11)\\
\sum_{\omega\in\Omega} \gamma(\omega) \left[c_1^B(\omega) + c_2^B(\omega) \right] &\geq \xi I \sum_{\omega\in\Omega} \gamma(\omega) z(\omega), &(12)\\
\sum_{\omega\in\Omega} \gamma(\omega) \left[c_1^B(\omega) + c_2^B(\omega) \right] &\geq K^B \sum_{\omega\in\Omega} \gamma(\omega) z(\omega). &(13)
\end{aligned}$$

Inequality (7) is the bank's time 0 budget constraint. Equations (8) and (9) are resource constraints and inequalities (10) are limited liability constraints. Inequalities (11) and (12) are limited commitment constraints ensuring that the bank does not to walk away with the capital between period 1 and period 2 in any state ω, and between period 0 and period 1. Inequality (13) is the bank's ex-ante participation constraint. Define $\mathcal{D}$

as the set of coupons that can be issued by the bank satisfying the feasibility conditions in Equations (8)–(13).

2.2 Households

Households produce and consume general goods in centralized markets and engage in trade of a special good in decentralized markets subject to trading frictions. Households may purchase portfolios of liabilities issued by banks and they may wish to do so to facilitate trade in decentralized markets.

There are two types of households: buyers and sellers. Each household knows if it is a buyer or a seller with their type fixed over time as in Rocheteau and Wright (2005).[2] The superscript b denotes buyers and s denotes sellers. There is measure 1 of buyers and measure $n \geq 0$ of sellers. Each type $i \in \{b, s\}$ household is initially endowed with k^i units of capital goods so the aggregate stock of capital goods held by households K^H is

$$K^H \equiv k^b + nk^s. \tag{14}$$

Let q_t denote goods produced or consumed in decentralized sub-period t, x_t goods consumed in centralized sub-period t, and y_t production of goods in centralized sub-period t. Buyers have period t preferences

$$U_t^b\left(q_t, x_t, y_t\right) = u(q_t) + \left[v(x_t) - y_t\right], \tag{15}$$

and sellers have period t preferences

$$U_t^s\left(q_t, x_t, y_t\right) = -c(q_t) + \left[v(x_t) - y_t\right]. \tag{16}$$

Buyers' and sellers' have concave utility v from consuming the centralized market goods, they have linear disutility of labor in the centralized market, and they do not discount utility over time.[3] Buyers enjoy utility of $u(q_t)$ from consuming q_t and sellers pay utility cost of $c(q_t)$ from producing q_t in the decentralized market. The gains from

[2] Alternatively, we could allow household types to vary as in Lagos and Wright (2005). In such a model, equilibrium asset prices would reflect similar liquidity premia to our model and yield similar results on liquidity and risk transformation.

[3] Rocheteau and Wright (2005) allow a discount rate of β_d between the centralized and decentralized sub-periods as well as discounting over time. For simplicity, we abstract from all discounting in our finite horizon model.

trade are $u(q_t) - c(q_t)$.

Buyers and sellers face matching fractions in decentralized markets. Let $\alpha(n) > 0$ denote the probability that a buyer meets a seller so $\alpha(n)/n$ is the probability a seller meets a buyer.[4] When $\alpha(n) > 0$ and a buyer and a seller meet in a decentralized market, they engage in proportional bargaining to determine the terms of trade. The probability $\alpha(n)$ is the exogenous component of bank liabilities' liquidity. If $\alpha(n) = 0$ then bank liabilities are not accepted in decentralized trade; liquidity is zero for any possible coupons. If $\alpha(n) > 0$ then bank liabilities are accepted in decentralized trade; liquidity depends on the coupons. In order to reduce notational complexity, we will write drop the n argument from $\alpha(n)$ wherever possible.

Since the state ω is revealed at the beginning of period 1, there is no residual uncertainty about the liability payoffs after the beginning of period 1. As a result, the relevant aggregate state for a household is $D(\omega)$, the coupons associated with the liability issued by the representative bank. The idiosyncratic state of a household upon entering the centralized market in period $t \in \{1, 2\}$ is the number of the representative bank's liabilities the household owns, a, with cum-dividend value of $a\,[p_t(\omega) + d_t(\omega)]$.

A household of type $i \in \{b, s\}$ solves[5]

$$W_t^i(a; D(\omega)) = \max_{x,y,a'} v(x) - y + V_{t+1}^i(a'; D(\omega)), \tag{17}$$

subject to:

$$x + a' p_t(D(\omega)) \leq y + a[p_t(D(\omega)) + d_t(\omega)], \tag{18}$$

where $V_{t+1}^i(a'; D(\omega))$ is the household value function upon entering the decentralized market in period $t + 1$. The notation reflects that the equilibrium cum-coupon liability price depends on the history. Since buyers and sellers are symmetric in the centralized market, the decision problem is the same for both types of households.

In period $t = 0$ before ω is realized, the household sells capital and purchases liabilities from the representative bank, and the relevant aggregate state for a household is the

[4]The function $\alpha(n)$ satisfies: $\alpha'(n) > 0, \alpha''(n) < 0, \alpha(n) \leq \min\{1, n\}, \alpha(0) = 0, \alpha'(0) = 1, \alpha(\infty) = 1$.

[5]Households could also be allowed to purchase capital goods in the centralized markets occurring in periods 0, 1 or 2. Since we assume that households' direct claims to capital goods may not be used to facilitate trade in decentralized markets, it is without loss of generality to focus on equilibria where households do not purchase capital goods.

total vector of liability coupon payments, D. The household's problem is

$$W_0^i(D) = \max_{x,y,a'} v(x) - y + \sum_{\omega \in \Omega} \gamma(\omega) V_1^i(a'; D(\omega)), \tag{19}$$

subject to:

$$x + a' p_0(D) \leq y + p_0^k k^i.$$

Let $q_t(a^b, a^s; D(\omega))$ and $m_t(a^b, a^s; D(\omega))$ be the terms of trade in a meeting between a buyer and seller in the period t decentralized market when the buyer owns a^b units of liabilities and the seller owns a^s units of liabilities. The function q_t is the amount produced for the buyer and the function m_t is the amount of liabilities transferred from the buyer to the seller. Let Ψ_t^i denote the period t distribution over liabilities held by type $i \in \{b, s\}$ households at the start of the period t decentralized market. For periods $t = 1, 2$, a buyer with a units of liabilities upon entering the period t decentralized market has value function

$$V_t^b(a; D(\omega)) = \alpha \int_{a^s} \Big\{ u[q_t(a, a^s; D(\omega))] + W_t^b(a - m_t(a, a^s; D(\omega)); D(\omega)) \Big\} \, d\Psi_t^s(a^s) + (1 - \alpha) \, W_t^b(a; D(\omega)), \tag{20}$$

and a seller with a units of liabilities has value function

$$V_t^s(a; D(\omega)) = \frac{\alpha}{n} \int_{a^b} \Big\{ -c[q_t(a^b, a; D(\omega))] + W_t^s(a + m_t(a^b, a; D(\omega)); D(\omega)) \Big\} \, d\Psi_t^b(a^b) + \left(1 - \frac{\alpha}{n}\right) W_t^s(a; D(\omega)). \tag{21}$$

In period $t = 3$, $V_3^i(a; D(\omega)) = 0$ for both buyers and sellers.

We determine the terms of trade in decentralized meetings through proportional bargaining. Under proportional bargaining, the buyer's surplus from a match equals $\eta/(1-\eta)$ times the seller's surplus, with $\eta \in [0, 1]$ summarizing the buyer's bargaining power. In a match between a buyer and seller with liabilities $\left(a^b, a^s\right)$ and in history ω, the terms of trade (q_t, m_t) solve

$$\max_{q_t, m_t} u(q_t) + \left[W_t^b(a^b - m_t; D(\omega)) - W_t^b(a^b; D(\omega)) \right], \tag{22}$$

subject to:

$$m_t \leq a^b, \tag{23}$$

$$u(q_t) + \left[W_t^b(a^b - m_t; D(\omega)) - W_t^b(a^b; D(\omega))\right] = \frac{\eta}{1-\eta}\left[-c(q_t) + (W_t^s(a^s + m_t; D(\omega)) - W_t^s(a^s; D(\omega))\right]. \tag{24}$$

The trading constraint (23) assumes that the only medium of exchange available to households is their holdings of bank liabilities. Implicitly, (23) assumes that households may not use their own holdings of capital goods to facilitate trade in decentralized markets; this assumption can be interpreted as a form of limited commitment to deliver on assets pledged in decentralized markets.

In the absence of the trading constraint (23), sellers would always produce the efficient level of output q^* in decentralized meetings satisfying $u'(q^*) = c'(q^*)$. If buyers do not bring sufficient bank liabilities into meetings with sellers—or these bank liabilities are not valuable enough—then sellers may produce less than q^* units of output.

Figure 1 summarizes the model's timeline.

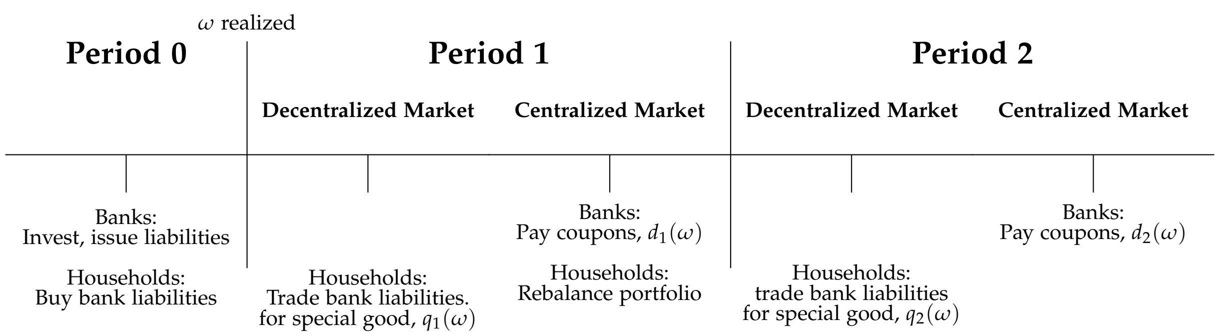

Figure 1: Timeline

Definition 1 (Equilibrium). *An* equilibrium *consists of an allocation for the bank* (I, L, D, c^B), *households' value functions* $\{(W_t^i)_{t=0,1,2}, (V_t^i)_{t=1,2}\}_{i=s,b}$ *and policy functions* $\{(x_t^i, y_t^i, a_t^i)_{t=0,1,2}\}_{i=s,b}$, *terms of trade,* $\{(q_t, m_t)_{t=1,2}\}$, *and prices* $\{p_0^k, p_0(D), (p_t(D(\omega)))_{t=1,2})\}$ *such that*

1. *The bank's allocation solves the bank's problem (6) subject to (7)—(13);*

2. *Given prices and value functions, the policy functions are optimal;*

3. *Given prices and policy functions, the value functions satisfies Equations (17), (20), and (21);*

4. *The terms of trade are the proportional bargaining solutions in Equations (22).*

5. *Goods, capital, and liabilities markets clear:*

$$x_0^b + nx_0^s = y_0^b + ny_0^s, \tag{25}$$

$$x_t^b(\omega) + nx_t^s(\omega) = y_t^b(\omega) + ny_t^s(\omega) + d_t(\omega), \forall t, \omega, \tag{26}$$

$$I = K^B + K^H, \tag{27}$$

$$a_t^b(\omega) + na_t^s(\omega) = 1, \forall t, \omega. \tag{28}$$

3 Pareto Optimal Outcomes

The social planner chooses liability issuance, and allocates resources to buyers and sellers in centralized and decentralized markets subject to the decentralized trading frictions. The proportional bargaining constraints in the decentralized markets depend on the price of bank liabilities. Those liability prices themselves depend on the planner's choice of liabilities.

Given any liability issuance, the remaining equilibrium prices and allocations resemble those in standard search-theoretic monetary economies as in Lagos and Wright (2005) and Rocheteau and Wright (2005). Quasi-linearity of preferences ensures that in any centralized market, a household's optimal choice of liabilities to purchase is independent of the liabilities they bring into the centralized market so that the equilibrium distributions of liability holdings for buyers and sellers are degenerate. Following Rocheteau and Wright (2005), we characterize equilibrium in which in each centralized market the buyers purchase all of the bank's liabilities and use these liabilities to facilitate trade in the subsequent decentralized market. Since the measure of buyers is the same as the measure of bank liabilities, each buyer holds 1 bank liability in any equilibrium. Moreover, the buyers' marginal decision to hold bank liabilities determines the equilibrium price of these liabilities in each period and after every history. Since equilibrium outcomes associated with a given liability issuance are standard for this class of models, Appendix A provides such a characterization, and instead we focus on the of terms of trade and asset prices.

Recall that q^* is the level of output that maximizes the static surplus in a meeting between a buyer and a seller—the efficient level of trade in a decentralized meeting. Let d^* be

$$d^* = (1-\eta)u(q^*) + \eta c(q^*). \tag{29}$$

The threshold d^* is the value of a bank liability sufficient to support efficient trade in decentralized markets when each buyer holds 1 unit of the liability. Define

$$\hat{q}_t\left(D(\omega)\right) \equiv \left(q \,|(1-\eta)u(q) + \eta c(q) = p_t(D(\omega)) + d_t\left(\omega\right)\right), \tag{30}$$

where $p_2(D(\omega)) = 0$. From the the quasi-linear specification of preferences, equilibrium production is

$$q_t^{eq}(D(\omega)) = \begin{cases} q^*, & \text{if } p_t(D(\omega)) + d_t\left(\omega\right) \geq d^*, \\ \hat{q}_t\left(D(\omega)\right), & \text{else.} \end{cases} \tag{31}$$

Production in each period is efficient when cum-dividend liability price is high enough and constrained when the cum-dividend liability price is low enough. $q_2^{eq}\left(D\left(\omega\right)\right)$ depends only on the period 2 coupon $d_2(\omega)$ while $q_1^{eq}\left(D\left(\omega\right)\right)$ depends directly on the period 1 coupon $d_1\left(\omega\right)$ and indirectly on the period 2 coupon $d_2\left(\omega\right)$ which influences the period 1 ex-coupon price.

The first period ex-coupon asset price is determined by the buyer's marginal decision to purchase bank liabilities in period 1. In a given history, ω, the price at which buyers are willing to purchase 1 unit of bank liabilities is

$$p_1\left(D(\omega)\right) = d_2\left(\omega\right) + \alpha\eta d_2\left(\omega\right) \frac{u'\left(q_2^{eq}\left(D(\omega)\right)\right) - c'\left(q_2^{eq}\left(D(\omega)\right)\right)}{(1-\eta)u'\left(q_2^{eq}\left(D(\omega)\right)\right) + \eta c'\left(q_2^{eq}\left(D(\omega)\right)\right)}, \tag{32}$$

with $a^b = 1$ and $a^s = 0$. The liability price is the liabilities' discounted expected coupon plus the discounted liquidity premium. The discounted liquidity premium is strictly positive only when decentralized trade is constrained, which occurs when $d_2(\omega) < d^*$ so that $q_2^{eq}\left(D(\omega)\right) < q^*$.

Equation (32) is familiar in models with no decentralized trade and risk-neutral agents in which the asset price is equal to the discounted expected value of the coupons. Equation (32) is also familiar in monetary models where asset prices reflect not only their coupons but also their usefulness in relaxing trading frictions—see Lagos (2010) for example. By similar reasoning, the period 0 price of bank liabilities satisfies

$$p_0(D) = \sum_{\omega} \gamma(\omega) \left[d_1(\omega) + p_1(D(\omega))\right] \\ \times \left[1 + \alpha\eta \frac{u'\left(q_1^{eq}\left(D(\omega)\right)\right) - c'\left(q_1^{eq}\left(D(\omega)\right)\right)}{(1-\eta)u'\left(q_1^{eq}\left(D(\omega)\right)\right) + \eta c'\left(q_1^{eq}\left(D(\omega)\right)\right)}\right], \quad (33)$$

where $p_1(D(\omega))$ satisfies (32).

In Appendix A, we show that welfare obtained by the social planner associated with any feasible liability issue D is

$$W_0^P(D) = (1+n)\,\bar{v} + \sum_{\omega} \gamma\,(\omega) \left(U_1^P(D(\omega)) + U_2^P(D(\omega))\right), \quad (34)$$

where

$$U_t^P\left(D\left(\omega\right)\right) = (1+n)\bar{v} + d_t\left(\omega\right) + \alpha\left[u\left(q_t^{eq}\left(D\left(\omega\right)\right)\right) - c\left(q_t^{eq}\left(D\left(\omega\right)\right)\right)\right], \quad (35)$$

with $\bar{v} = \max_x v\,(x) - x$. Here, $U_t^P\left(D\left(\omega\right)\right)$ is the planner's period t indirect welfare function. The welfare function aggregates the sum of buyers' and sellers' utilities in the centralized and decentralized markets with $(1+n)\,\bar{v} + d_t\,(\omega)$ the households' welfare in the centralized market and the final term the households' welfare in decentralized markets. Recall that meetings in this market occur at rate α.

The second period indirect welfare function $U_2^P(D\,(\omega))$ defined in Equation (35) depends only on $d_2\,(\omega)$ and is concave in $d_2\,(\omega)$ near d^*. For values of d_2 below d^*, decentralized trade is constrained. As a result, surplus between buyers and sellers in decentralized markets is increasing in d_2 for such values. When $d_2 \geq d^*$, decentralized trade is efficient so surplus in decentralized meetings is independent of d_2 in this region. As a result, $U_2^P\left(D\left(\omega\right)\right)$ increases at a decreasing rate around d^*. The planner's period welfare function exhibits additional risk aversion with respect to coupon payments if in some states $d_2\,(\omega) < d^*$ relative to cases in which $d_2(\omega) \geq d^*$ for all ω.

If the liability payoff in state ω satisfies $d_2\,(\omega) < d^*$, then trade is constrained in period 2 decentralized markets, implying that the liquidity premium in the liability price is strictly positive in that state. Any change in the period 2 cash flow in history ω will then impact the liquidity premium and, therefore, bank liability prices. Changing the liabilities' coupons to increase in the liability price through liquidation may prove useful for the planner to relax constraints on decentralized trade in period 1.

The planner chooses the stochastic coupons D backing the liabilities to solve

$$\max_{D \in \mathcal{D}} W_0^P(D). \tag{36}$$

3.1 Efficient Asset Transformation

The stochastic return on households' initial capital goods is $z(\omega) K^H$. The planner may transform risk by issuing liabilities with less volatile coupon payments than the households' returns and may transform maturity by issuing liabilities with period 1 coupon payments.

Definition 2 (Risk Transformation). *An allocation features* risk transformation *if the liability payoffs satisfy*

$$z(\omega_l) K^H < d_1(\omega_l) + d_2(\omega_l) \leq d_1(\omega_h) + d_2(\omega_h) < z(\omega_h) K^H. \tag{37}$$

Risk transformation occurs when liability payoffs are less volatile than the return on households' initial capital goods.

Definition 3 (Maturity Transformation). *An allocation features* maturity transformation *if for some ω, $d_1(\omega) > 0$.*

Maturity transformation occurs when bank liability payoffs are larger than bank assets absent any liquidation of their underlying assets. In the absence of liquidation, households' initial capital goods yield no returns in period 1. If a bank engages in liquidation, then in period 0, their balance sheets show higher shorter term payoffs than their assets—they perform maturity transformation.

We characterize conditions under which risk and maturity transformation are efficient. For each case, a complete characterization of efficient allocations is in Appendix B.

Assumption 1. *Endowments K^H, K^B and the parameter ξ satisfy*

$$\frac{K^B}{K^H + K^B} \geq \xi. \tag{38}$$

Assumption 1 is a minimum capital requirement for the bank relative to households. We maintain Assumption 1 for the remainder of the paper. From (34) and (35), when

$\alpha = 0$ so that there is no decentralized trade, then the planner has no incentive to transform assets since households are risk-neutral with respect to coupon payments. Under Assumption 1, the no-transformation allocation satisfies the bank's limited commitment constraints and ensures that any allocation which satisfies the bank's participation constraint in equation (13) also satisfies the bank's limited commitment constraint from period 0 to period 1 in equation (12).

We show in Appendix B that if the banks' investment opportunities are good enough, or if banks are well capitalized enough, or if bank claims are not used to back trade in goods markets, then the constrained optimal allocation features neither maturity nor risk transformation. The planner uses the bank as a pass-through operation. The efficient liability issue satisfies $d_1(\omega) = 0$ and $d_2(\omega) = z(\omega) K^H$ and we call the resulting liability the *pass-through claim*. With the pass-through claim, the bank receives the households' initial stock of capital goods, invests these capital goods into the investment project, and pays out coupons equal to the realized return on the household's capital in the absence of any liquidation in each state.

When there is intermediate risk associated with bank investment opportunities and if banks are sufficiently well capitalized, then liabilities backed by bank investments do not provide sufficient liquidity to households who use those liabilities as a medium of exchange. The planner wants to smooth the coupon process relative to the underlying investments. We call the resulting liability the *insurance-only claim*. The banks receives the households' initial stock of capital goods, invests them, and pays out coupons only in period 2, with the coupons less volatile than the underlying investment returns. In the high state, the bank retains a portion of the return to households' initial capital goods for private consumption while in the low state, the bank pays a portion of the return to its own initial capital goods to households. In this sense, the planner directs the bank to provide insurance to households. Since the insurance-only claim features no period 1 coupons, the efficient allocation features no maturity transformation.

With intermediate liquidity, investments are sufficiently productive in the low state to allow the planner to smooth coupon payments without violating the bank's limited commitment constraint even in the absence of any liquidation. In this case, there is risk transformation but no maturity transformation. When bank investments are unproductive enough, the bank's limited liability constraint is violated in the absence of liquidation. In this case, this is both risk and maturity transformation.

Proposition 1 (Both risk and maturity transformation). *There exists* $\underline{\xi} < K^B/(K^B + K^H)$,

$\underline{\kappa} > 0$ and $\underline{z}$ such that if $\xi \geq \underline{\xi}$, $\kappa \geq \underline{\kappa}$, and $z(\omega_l) < \underline{z}$, then efficient allocations feature both risk and maturity transformation.

The conditions of the proposition describe a case of insufficient liquidity. The efficient liability issue in this case satisfies $d_1(\omega_l) > 0$, $d_1(\omega_h) = 0$ so that $L(\omega_l) > 0$ and $L(\omega_h) = 0$. Moreover, the bank's limited commitment constraint binds in the low state so that

$$d_2(\omega_l) = \left(K^H + K^B\right)(1 - L(\omega_l))(1 - \xi)z_2(\omega_l). \tag{39}$$

The coupon in state ω_h is determined from the feasibility constraint of the liability in state ω_h along with the bank's ex ante participation constraint holding with equality. We call the resulting liability the liquidation claim. The liquidation claim corresponds to the bank receiving and investing the households' initial stock of capital goods. In the low state, the bank liquidates a portion of its total investment and pays out all the liquidation proceeds to liability holders in period 1. Then, in period 2, the bank retains just enough of the returns to satisfy its limited commitment constraint and pays the remainder out to liability holders. In the high state, the bank does not liquidate any of its investment in period 1 and in period 2, the bank retains the return to its own initial capital goods as well as a portion of the return to households' initial capital goods for private consumption.

To see why strictly positive liquidation is a feature of Pareto optimal allocations, consider the best allocation the planner may attain without liquidation. The best allocation necessarily satisfies the bank's limited commitment constraint with equality in state ω_l yielding period 2 coupon payments of

$$d_2(\omega_l) = \left(K^H + K^B\right)(1 - \xi)z_2(\omega_l). \tag{40}$$

If the bank's limited commitment constraint were slack, $d_2(\omega_l) < \left(K^H + K^B\right)(1 - \xi)z_2(\omega_l)$, then the planner could increase $d_2(\omega_l)$, decrease $d_2(\omega_h)$—allowing the planner to continue to satisfy the bank's ex ante participation constraint—and strictly raise households' welfare through an improvement in decentralized terms of trade. When $d_2(\omega_l)$ satisfies (40), $d_2(\omega_h)$ in the best allocation without liquidation may be obtained from the bank's ex ante participation constraint holding with equality when $d_1(\omega) = c_1^B(\omega) = 0$. Denote this value of $d_2(\omega_h)$ by $\bar{d}_{2h}$.

Consider a perturbation from the best allocation without liquidation with $L(\omega_l) = \varepsilon > 0$ in which the bank's limited commitment constraint in period 2 state ω_l continues to hold with equality. By construction, the perturbation reduces the bank's consumption $c_2^B(\omega_l)$ by $\varepsilon\xi\left(K^H+K^B\right)z_2(\omega_l)$ to satisfy the bank's participation constraint, we raise $c_2^B(\omega_h)$ by $\gamma(\omega_l)\,\varepsilon\xi\left(K_0^H+K_0^B\right)z_2(\omega_l)\,/\,\gamma(\omega_h)$. Under the conditions of Proposition 1, the perturbation does not reduce $d_2(\omega_h)$ below d^*.

The perturbed allocations are

$$\begin{aligned} d_1(\omega_l,\varepsilon) &= \varepsilon\kappa\left(K^H+K^B\right)z_2(\omega_l)\,,\;\; d_1(\omega_h,\varepsilon)=0,\\ d_2(\omega_l,\varepsilon) &= (1-\varepsilon)(1-\xi)\left(K^H+K^B\right)z_2(\omega_l)\,,\\ d_2(\omega_h,\varepsilon) &= \bar{d}_{2h}-\varepsilon\xi\left(K^H+K^B\right)z_2(\omega_l)\frac{\gamma(\omega_l)}{\gamma(\omega_h)}. \end{aligned} \tag{41}$$

The marginal impact on welfare arising from the perturbation is

$$\begin{aligned} &\left(K^H+K^B\right)z_2(\omega_l)\,\gamma(\omega_l)\left\{U^P_{1,1_l}\kappa-\left(U^P_{1,2_l}+U^P_{2,2_l}\right)(1-\xi)\right\}\\ &\qquad -\left(K^H+K^B\right)z_2(\omega_l)\,\gamma(\omega_h)\,\xi\frac{\gamma(\omega_l)}{\gamma(\omega_h)}\left\{U^P_{1,2_h}+U^P_{2,2_h}\right\}, \end{aligned} \tag{42}$$

where U^P_{t,i_j} is the derivative of U^P_t with respect to $d_i(\omega_j)$ for $j=l,h$.

The perturbation raises period 1 liability payouts in the low state in period 1 but lowers all other liability payouts. The marginal adjustments to liability payouts all occur at rate $\left(K^H+K^B\right)z_2(\omega_l)$. The first line of (42) is the net impact of changes in liability payouts in the low state. The perturbation increases the period 1 coupon at rate κ with marginal benefit $U^P_{1,d_1}>1$.

The marginal benefit is larger than one in the low state because $d_1(\omega_l,\varepsilon)+p_1(\omega_l,\varepsilon)<d^*$ and decentralized terms of trade are improved. The perturbation decreases the period 2 coupon at rate $(1-\xi)$ with marginal cost $U^P_{1,2_l}+U^P_{2,2_l}>1$. The marginal cost is larger than 1 because $d_2(\omega_l,\varepsilon)<1$ and the reduction in $d_2(\omega_l,\varepsilon)$ reduces both second period decentralized terms of trade as well as first period terms of trade through its impact on period 1 liability price. When ξ is large, the perturbation does not reduce period 2 coupon payments significantly.

The second line of (42) is the net impact of changes in liability payouts in the high state. The perturbation reduces the period 2 coupon at rate $\xi\gamma(\omega_l)\,/\,\gamma(\omega_h)$ with marginal cost $U^P_{1,2_h}+U^P_{2,2_h}=1$. The marginal cost of this reduction is 1 because $d_2(\omega_h,\varepsilon)>d^*$. The perturbation reduces the period 2 coupon in order to compensate the bank for receiving lower consumption in period 2 in the low state. The planner is able to compensate the bank in the high state where households' marginal value of coupon payments is low, therefore allowing for the possibility that liquidation may be optimal. Simplifying (42), the marginal benefit of the perturbation is

$$\left(K^H + K^B\right) z_2\left(\omega_l\right) \gamma\left(\omega_l\right) \left\{U^P_{1,1_l}\kappa - \left(U^P_{1,2_l} + U^P_{2,2_l}\right)(1-\xi) - \xi\right\}. \tag{43}$$

In Appendix B, we show that when the conditions of Proposition 1 hold, the perturbation yields a Pareto improvement—efficient liabilities feature liquidation and, therefore, maturity transformation.

For maturity transformation to be optimal, broadly speaking, three conditions need to be satisfied. First, bank liabilities must circulate as a medium of exchange—that is, $\alpha > 0$. Households are risk-averse to bank liability payouts only to the extent that liabilities serve as a medium of exchange. Since the marginal benefit of liquidation is an improvement in smoothing payouts, households do not value this benefit when bank liabilities do not circulate. Second, the costs of liquidation cannot be too large—κ must be close enough to 1. An increase in κ directly reduces the costs of liquidation making it a more attractive option. Third, ξ must be large enough for the bank's limited commitment to be sufficiently binding. Maturity transformation in our model is only (constrained) efficient to the extent that it relaxes banks' limited commitment constraints and allows for an improvement in risk-sharing.

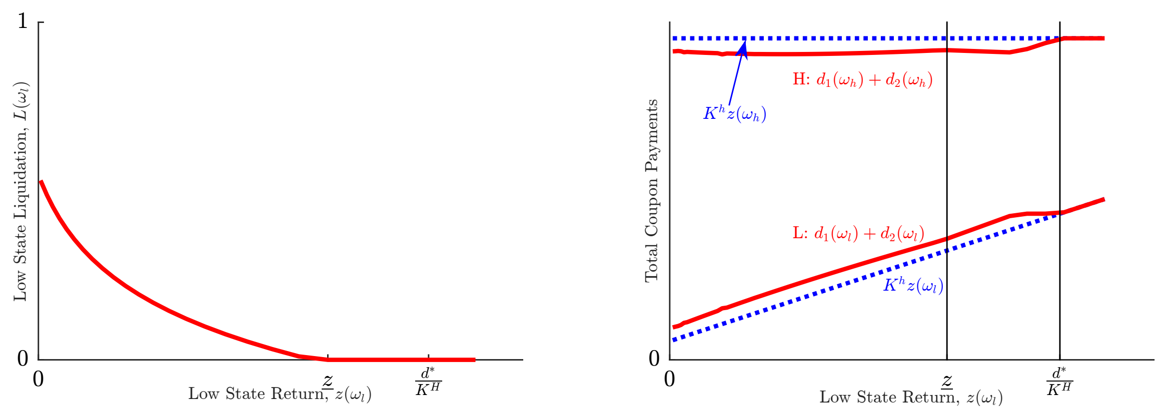

Figure 2: Constrained efficient liquidation in state ω_l (left-panel) and total state-contingent coupon payments (right-panel) for various values of $z(\omega_l)$ in a numerical example. The parameter assumptions are: $\alpha(n) = n, u(q) = [(q+z)^{(1-a)} - z^{(1-a)}]/(1-a)$ where $z = 10^{-5}$ and $a = 0.5$, $c(q) = q, \eta = 0.5, n = 0.5, \gamma(\omega_h) = \gamma(\omega_l) = 0.5$, $K^h = 20, K^B = 43.3, \xi = 0.65, \kappa = 0.98$.

Figure 2 illustrates features of constrained efficient allocations from a numerical example. In the left panel of Figure 2 we plot the constrained efficient liquidation level when the state is ω_l. In the right panel of Figure 2 the solid lines depict total coupon payments $d_1(\omega) + d_2(\omega)$ for each state for various values of $z(\omega_l)$. The dashed lines depict the present discounted value of households' endowments of capital goods for various values of $z(\omega_l)$.

When $z(\omega_l) \geq \underline{z}$, efficient allocations feature no liquidation and when $z(\omega_l) < \underline{z}$ liquidation is strictly positive. Notice, however, that once $z(\omega_l)$ falls below d^*/K^H, efficient allocations smooth coupon payments relative to the value of households' endowments so that $d_1(\omega_l) + d_2(\omega_l) > K^h z(\omega_l)$ and $d_1(\omega_h) + d_2(\omega_h) < K^h z(\omega_h)$. Figure 2 shows that as investments become less productive in state ω_l, efficiency first calls for banks to engage in risk-transformation. As bank investments become even less productive, efficiency calls for banks to engage in maturity transformation to provide risk transformation.

Proposition 1 states that if there is enough risk associated with bank investment opportunities and if banks' limited commitment problem is severe enough, then liabilities which only feature risk transformation do not provide sufficient liquidity to households using those liabilities as a medium of exchange. In this case, risk transformation is impeded by the bank's limited commitment constraint. Maturity transformation relaxes the bank's commitment constraint and allows for an improvement in risk transformation, making the households better off. Given the direct costs of liquidation and the severity of the bank's commitment problem, maturity transformation is socially optimal only when bank assets are risky enough.

Figure 3 illustrates the threshold $\underline{z}$ for various values of the match rate between buyers and sellers, α for high and low values of bank capital. To the left of each threshold, constrained efficient allocations feature maturity transformation while to the right of each threshold constrained efficient allocations feature no maturity transformation. Independent of the bank's capital level, the figure illustrates that maturity transformation is more likely to be a feature of constrained efficient allocations when α is high so that households expect to match in decentralized markets frequently. In such a case, the motive for the planner to smooth liquidity distortions is high leading to maturity transformation.

Figure 3 also demonstrates that maturity transformation is more likely to be a feature of constrained efficient allocations when bank capital is low (so that the region of

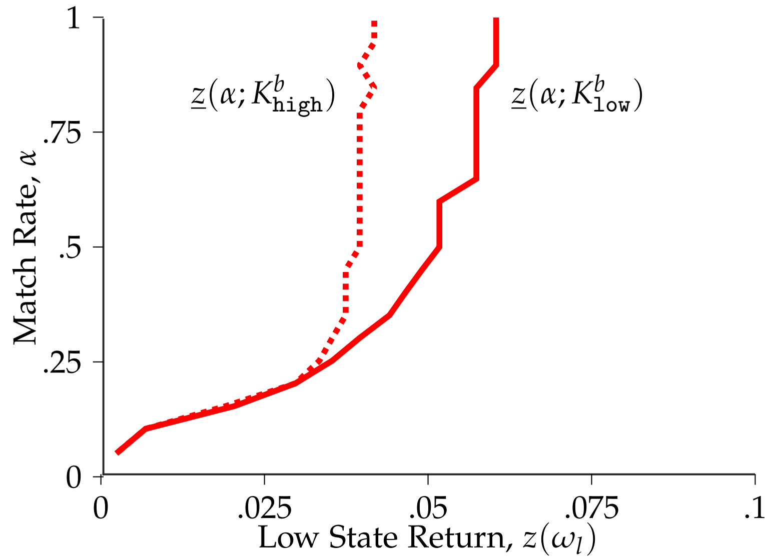

Figure 3: Numerical illustration of the threshold, $\underline{z}$, as a function of the match rate α for two given values of bank capital. The parameter assumptions are the same as in Figure 2 with $\alpha(n) = \alpha n$ for all $\alpha \in (0, 1)$, $K^B_{\texttt{low}} = 43.3$ and $K^B_{\texttt{high}} = 52$.

maturity transformation is larger). This result follows immediately from the fact that the limited commitment constraints are more severe for banks with less capital and these binding commitment constraints, interacting with the desire to smooth liquidity distortions induces maturity transformation.

4 Competitive Equilibrium Outcomes

We now describe the competitive outcomes, and show that they do not coincide with Pareto Optimal outcomes with maturity transformation. Appendix A describes the competitive equilibrium that obtains when all banks issue arbitrary liabilities $D \in \mathcal{D}$. Once banks issue their liabilities, equilibrium prices are determined by the trading decisions of the households. We consider sequential and symmetric subgame perfect equilibrium where each bank takes aggregate liability issues as given.

Computing a bank's optimization problem for alternative liability issues requires banks to know what proceeds they will receive from any possible liability issue. Let D be the aggregate liability issue, D^i the liability issue that the i^{th} bank is considering and $p_0(D^i; D)$ bank i's conjectured liability pricing for coupon vector D^i given an aggregate liability issue D. To construct $p_0(D^i; D)$, note that in the symmetric equilibrium, each bank will issue the same liability. From (33) and (32), the symmetric subgame equilibrium liability price is

$$\begin{aligned} p_0(D;D) = \sum_{\omega} \gamma(\omega) \Bigg\{ d_1(\omega) \left[1 + \alpha\eta \frac{u'\left(q_1^{eq}\left(D(\omega)\right)\right) - c'\left(q_1^{eq}\left(D(\omega)\right)\right)}{(1-\eta)u'\left(q_1^{eq}\left(D(\omega)\right)\right) + \eta c'\left(q_1^{eq}\left(D(\omega)\right)\right)} \right] \\ + d_2(\omega) \left[1 + \alpha\eta \frac{u'\left(q_1^{eq}\left(D(\omega)\right)\right) - c'\left(q_1^{eq}\left(D(\omega)\right)\right)}{(1-\eta)u'\left(q_1^{eq}\left(D(\omega)\right)\right) + \eta c'\left(q_1^{eq}\left(D(\omega)\right)\right)} \right] \\ \times \left[1 + \alpha\eta \frac{u'\left(q_2^{eq}\left(D(\omega)\right)\right) - c'\left(q_2^{eq}\left(D(\omega)\right)\right)}{(1-\eta)u'\left(q_2^{eq}\left(D(\omega)\right)\right) + \eta c'\left(q_2^{eq}\left(D(\omega)\right)\right)} \right] \Bigg\}. \end{aligned} \tag{44}$$

We re-write the period 0 price as a linear combination of the state-contingent coupon plans with weights resembling Arrow-Debreu prices.[6] Let $\pi_t(\omega; D)$ be defined by

$$p_0(D;D) = \sum_t \sum_{\omega} \pi_t(\omega; D) d_t(\omega). \tag{45}$$

For any alternative liability issue D^i, we assume that the liability price is

$$p_0(D^i;D) = \sum_t \sum_{\omega} \pi_t(\omega; D) d_t^i(\omega). \tag{46}$$

Equation (46) reveals that banks understand that issuing liabilities with larger coupon payments will raise their revenues from issuance but they do not perceive that their issuance decisions will impact households' willingness to purchase liabilities.[7] In this sense, banks are price-takers.

Each competitive bank solves (6) using the conjectured pricing function in Equation (46) taking the Arrow-Debreu prices as given, subject to the bank's budget constraints in equations (7) to (11) and the bank's participation constraint (13). To reduce notation, we let D^* be the equilibrium liability issue, and let $p_0(D^*)$ be the pricing function computed using the Arrow-Debreu prices evaluated at the equilibrium liability issue:

$$p_0(D^*) \equiv p_0(D^*; D^*). \tag{47}$$

[6] When equilibrium allocations feature liquidity premia, the equilibrium period 0 price is not a linear function function of equilibrium coupon payments.

[7] This formulation of prices is consistent with the limiting case of an economy with N banks and N buyer-type households as the number of households and banks tends to infinity and where prices are determined in an equilibrium in which each of the buyer-type households purchases a share of the liabilities issued by each bank.

When the planner's optimal allocation features no risk or maturity transformation, the competitive issue also features no risk or maturity transformation. In such situations, the equilibrium Arrow-Debreu prices satisfy $\pi_t(\omega; D) = \gamma(\omega)$: the economy has risk-neutral pricing and the planner's allocation is a competitive equilibrium. With risk-neutral pricing, there are no liquidity premia in liability prices. If the planner's optimal allocation features risk transformation, but no maturity transformation, then liability prices do contain liquidity premia. Nonetheless, the planner's allocation is a competitive equilibrium.

Lemma 2. *If the conditions of Lemma 4 or Lemma 5 hold then the efficient allocation is a competitive equilibrium.*

Suppose instead that the conditions of Proposition 1 are satisfied so that the planner's optimal allocation features maturity transformation in the low state. Let D^* denote the planner's optimal coupon issuance noting that in this case, the planner liquidates only in the low state. Let $\pi_t(\omega; D^*)$ denote the Arrow-Debreu prices which would obtain in an equilibrium consistent with the planner's allocation where, since liquidity is scarce in the low state, these prices feature a strictly positive liquidity premium in the low state so that $\pi_t(\omega_l; D^*) > \gamma(\omega)$.

We first argue that the planner's allocation cannot coincide with competitive equilibrium allocations because banks would be able to increase their payoffs by deviating to an alternative allocation. Bank optimality, much like the planner, requires the bank to enjoy no consumption in period 1 and, since the conjectured equilibrium prices feature a strictly positive liquidity premium in the low state, the bank's commitment constraint in the low state must bind. If the commitment constraint were not binding, the bank would be able to increase the coupon payment in the low state, decrease its coupon payment in the high state, and in doing so, would increase its total revenues from coupon issuance. With these binding constraints, the bank's optimality condition for liquidation satisfies

$$\kappa \pi_1(\omega_l; D^*) - (1 - \xi)\pi_2(\omega_l; D^*) - \xi\gamma(\omega_l) = 0. \tag{48}$$

The optimality condition in (48) reflects the impact of a marginal increase in liquidation on revenues raised through liability issuance of the bank less the forgone consumption in period 2 in the low state. A marginal increase in $L(\omega_l)$ allows the bank to pay $I\kappa$ more coupons in period 1 in the low state increasing revenues by $I\kappa\pi_1(\omega_l; D^*)$. The marginal increase requires the bank to pay $I(1 - \xi)$ fewer coupons in period 2 in the low state because the binding commitment constraint reduces issuance revenues by $I(1-\xi)\pi_2(\omega_l; D^*)$. The resulting marginal increase in liquidation requires an expected reduction in bank consumption in period 2 in the low state of $I\xi\gamma(\omega_l)$.

Next, compare (48) to the optimality condition of liquidation in the low state for the planner. Using the Arrow-Debreu, (43) may be re-written as

$$\kappa\pi_1(\omega_l; D^*) - (1-\xi)\pi_2(\omega_l; D^*) - \xi\gamma(\omega_l) = \\ -\gamma(\omega_l)(1-\eta)(1-\kappa) + \\ \frac{d\pi_2(\omega_l; D^*)}{dq_2^{eq}} \frac{(1-\xi)d_2^*(\omega_l)\left[\pi_1(\omega_l; D^*) - \gamma(\omega_l)\right]}{\pi_1(\omega_l; D^*)\left[(1-\eta)u'\left(q_2^{eq}\left(D^*(\omega_l)\right)\right) + \eta c'\left(q_2^{eq}\left(D^*(\omega_l)\right)\right)\right]}. \quad (49)$$

When q_2^{eq} lies below q^*, an increase in q_2^{eq} reduces the liquidity premium associated with bank liabilities and therefore decreases the implicit Arrow-Debreu price, $\pi_2(\omega_l, D^*)$: $d\pi_2(\omega_l; D^*)/dq_2^{eq} < 0$. Under the conditions of Proposition 1 $\pi_1(\omega_l; D^*) > \gamma(\omega_l)$, the right hand side of (49) is strictly negative. It follows then that the efficient allocation does not satisfy bank optimality (48) since the bank would strictly prefer to reduce $L(\omega_l)$.

The difference between optimal liquidation for the bank in Equation (48) and efficient liquidation in Equation (49) shows why equilibrium allocations are inefficient. The Arrow-Debreu price in period 2 in the planner's allocation in the low state is too high in the competitive equilibrium since it provides incentives for a single bank to increase period 2 coupon payments and increase issuance revenues and expected consumption faster than the resulting losses from revenues from the concomitant period 1 low state coupon issue.

The Arrow-Debreu price $\pi_2(\omega_l; D^*)$ is too high for two reasons. The first source of inefficiency is captured by the first term on the right hand side of (43) and is proportional to $1-\eta$. The inefficiency is a standard bargaining inefficiency. Since bank liabilities are priced by buyer-type households, these buyers do not internalize the fact that by bringing more liabilities into decentralized markets they generate more surplus for the seller-type household they meet. This first source of inefficiency resembles those that arise in most models with bargaining (see Hosios (1990)) and as $\eta \to 1$, the first source of inefficiency declines.

The second source of inefficiency is captured by the second term on the right hand side of (43) and reflects a distinct pecuniary externality novel to our environment. While the planner internalizes how a change in liquidation impacts the liquidity premium and, therefore, the implicit Arrow-Debreu price reflected by $d\pi_2(\omega_l; D^*)/dq_2^{eq}$, an individual bank does not internalize this effect. An individual bank is able to free-ride on the high liquidity premium associated with period 2 coupon issues in the low state in the efficient allocation; the bank does not internalize that if all banks were to issue more period 2 coupon issuances, they would ultimately reduce the liquidity premium associated with period 2 coupons. In any competitive equilibrium, banks liquidate *less* than the efficient amount in the low state.

Proposition 3. *Suppose the efficient allocation satisfies $L(\omega_l) > 0$. Then the efficient allocation cannot be implemented as a competitive equilibrium and the competitive equilibrium features less liquidation than the efficient allocation.*

Figure 4 illustrates Proposition 3 for the same numerical example depicted in Figure 2. The dashed red line is constrained efficient liquidation in state ω_l, and the solid green line is liquidation in the competitive equilibrium.

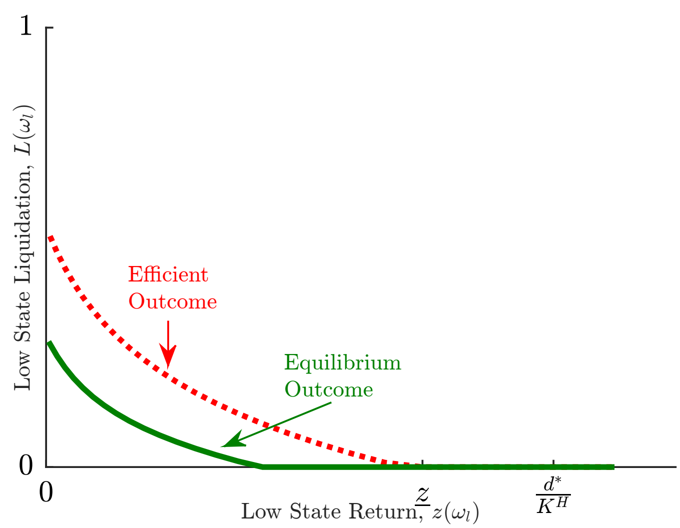

Figure 4: Constrained efficient liquidation and Equilibrium liquidation in state ω_l in a numerical example.

Proposition 3 shows that there is a role for regulative policy when $\alpha > 0$ and efficient allocations require banks to perform maturity transformation. In this case, in the absence of policy, banks issue liabilities which promise too many cash flows in period 2 and too few cash flows in period 1 in low-return states. Inside money issued by banks in the unregulated competitive equilibrium features too much risk in the sense that the variance of expected discounted cash flows is larger than that in the efficient allocation.

The simplest policy which implements the constrained efficient allocations is a state-contingent liquidation floor. The bank's liquidation in state ω must satisfy $L(\omega) \geq \bar{L}(\omega)$, where $\bar{L}(\omega_h) = 0$, $\bar{L}(\omega_l) = L^*(\omega_l)$ with $L^*(\omega_l)$ the constrained efficient liquidation level in state ω_l. Since banks have private incentives to reduce $L(\omega_l)$ below the threshold, the binding constraint in state ω_l ensures banks implement the constrained efficient liquidation policy and, therefore, the constrained efficient level of maturity transformation.

We interpret the policy as a minimum expected short term payout policy for banks from the perspective of period 0. Such an optimal policy conflicts with liquidity policies implemented in the wake of Basel III.[8] The liquidity coverage ratio introduced by U.S. financial regulators in 2014 requires that banks hold sufficient liquid assets to cover expected short-term net outlays during a 30-day stress period. Since banks may wish to minimize holdings of liquid assets generally with low returns, such policies incentive banks to minimize expected short term outlays. Our finding that banks must be incentivized to issue liabilities with large enough short term payouts suggests that the liquidity coverage ratio may impede on banks ability to create a stable, low-risk source of liquidity or means of payments to households.

5 Conclusions

We develop a theory linking the usefulness of banks' liabilities as a medium of exchange to risk and maturity transformation in the presence of aggregate liquidity risk. Shortening the maturity of banks' liabilities only increases social surplus if such shortening also reduces the riskiness of long-term liabilities and banks face a binding commitment problem. When maturity transformation is socially efficient, aggregate long-term liquidity is scarce raising the relative price of long-term bank issuances. In the competitive equilibrium, banks issue too many long-term liabilities and perform too little maturity transformation relative to the social optimum. In our model, bank liabilities are backed by real assets–there is no maturity mismatch between the assets and liabilities. But even in the absence of roll-over risk, there is a social incentive for risk and maturity transformation.

[8]See Ennis et al. (2011) for a discussion of revisions to capital requirements contained in Basel III and House et al. (2016) for a specific discussion of liquidity coverage ratios implemented in 2014 in the U.S.

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A Equilibrium Characterization Given a Liability Issue

In this Appendix, we characterize equilibrium outcomes and asset prices for a given coupon issue. We proceed by backward induction. The ex-dividend price of liabilities in the centralized market of period 2 is zero: $p_2(D(\omega)) = 0$. Hence, the value functions for both buyers and sellers satisfy

$$W_2^i(a; D(\omega)) = ad_2(\omega) + \bar{v}, \tag{A1}$$

where $\bar{v} \equiv \max_x v(x) - x$.

In the decentralized market in period 2, in any match between a buyer and seller, the terms of trade, $q_2(a_2^b, a_2^s; D(\omega)), m_2(a_2^b, a_2^s; D(\omega))$ solve the proportional bargaining problem. Using the value function in equation (A1), note that for either a buyer or a seller, and for any number of liabilities exchanged, m, the net continuation surplus for the consumer is

$$W_2^i(a+m; D(\omega)) - W_2^i(a; D(\omega)) = (a+m)\,d_2(\omega) + \bar{v} - ad_2(\omega) - \bar{v} = md_2(\omega). \tag{A2}$$

Requiring buyers to receive total surplus equal to a fraction of the surplus of the seller is equivalent to requiring

$$u(q_2) - m_2 d_2(\omega) = \frac{\eta}{1-\eta}\left[-c(q_2) + m_2 d_2(\omega)\right], \tag{A3}$$

or

$$(1-\eta)u(q_2) + \eta c(q_2) = m_2 d_2(\omega). \tag{A4}$$

Hence, for a given amount of production q_2, the number of liabilities that must be transferred from the buyer to the seller is

$$m_2 = \frac{(1-\eta)u(q_2) + \eta c(q_2)}{d_2(\omega)}, \tag{A5}$$

Substituting this amount of liabilities exchanged into the surplus of the buyer, the production choice q_2 satisfies

$$\max_{q_2} \eta \left[u(q_2) - c(q_2)\right], \tag{A6}$$

subject to

$$(1-\eta)u(q_2) + \eta c(q_2) \leq d_2(\omega)\, a_2^b. \tag{A7}$$

q_2 and, therefore, m_2 is determined independently of a_2^s. Thus, the seller's asset holdings have no impact on the terms of trade,

$$q_2(a_2^b, a_2^s; D(\omega)) = q_2\left(a_2^b; D(\omega)\right), \text{and } m_2\left(a_2^b, a_2^s; D(\omega)\right) = m_2(a_2^b; D(\omega)). \tag{A8}$$

We now determine q_2. Recall that q^* satisfies $u'(q^*) = c'(q^*)$. In a match between a buyer and a seller where the buyer has assets a_2^b such that

$$a_2^b \geq \frac{1}{d_2(\omega)} \left[(1-\eta)u(q^*) + \eta c(q^*)\right], \tag{A9}$$

then $q_2(a_2^b; D(\omega)) = q^*$. Otherwise, the constraint in equation (A7) binds so that q_2 is determined by equation (A7) holding with equality. It also follows that the value functions V_2^b and V_2^s satisfy

$$V_2^b(a_2^b; D(\omega)) = \alpha\eta \left[u(q_2\left(a_2^b; D(\omega)\right)\right) - c(q_2\left(a_2^b; D(\omega)\right)\Big] + \left[a_2^b d_2(\omega) + \bar{v}\right], \tag{A10}$$

and

$$V_2^s(a_2^s; D(\omega)) = \frac{\alpha}{n}(1-\eta) \int_{a_2^b} \left[u(q_2\left(a_2^b; D(\omega)\right)\right) - c(q_2\left(a_2^b; D(\omega)\right)\Big] d\Psi_2^b(a_2^b) \\ + \left[a_2^s d_2(\omega) + \bar{v}\right]. \tag{A11}$$

Next, we determine the value functions and asset price in the period 1 centralized market. Given the quasi-linearity of preferences in the centralized market, the problem of choosing asset holdings to carry into period 2 is independent of the number and value of the liabilities the consumer brings into the centralized market. The value function for either type of consumer is

$$W_1^i(a; D(\omega)) = \left(p_1(D(\omega)) + d_1(\omega)\right) a + \bar{v} + \max_{a'} -p_1(D(\omega))a' + V_2^i(a'; D(\omega)). \tag{A12}$$

From (A11), the seller's value function V_2^s is linear in a' implying that the seller's optimal choice of a' is bounded only if

$$p_1(D(\omega)) \geq d_2(\omega). \tag{A13}$$

Inequality (A13) holds in equilibrium with strict inequality so that all sellers choose $a_2^s = 0$ for all ω. Consider the optimal choice of a' for a buyer. Assuming an interior solution, the optimal choice for a buyer satisfies:

$$p_1(D(\omega)) = d_2(\omega) + \alpha\eta \left[u'(q_2(a'; D(\omega)) - c'(q_2(a', D(\omega))\right] \frac{dq_2(a'; D(\omega))}{da'} \tag{A14}$$

where

$$\frac{dq_2(a'; D(\omega))}{da'} = \frac{d_2(\omega)}{(1-\eta)u'(q_2(a'; D(\omega))) + \eta c'(q_2(a'; D(\omega)))}. \tag{A15}$$

Under conditions on preferences and bargaining weights, $V_2^b(a_2^b, D(\omega))$ is strictly concave for $a_2^b \leq a^*$ where a^* satisfies inequality (A9) with equality. This ensures a unique optimal choice of a' for buyers so that $\Psi_2^b(a_2^b)$ is degenerate. We focus on equilibrium in which $a_2^b = 1$ implying that the asset price is

$$p_1(D(\omega)) = d_2(\omega)\left[1 + \alpha\eta\frac{u'(q_2(1;D(\omega)) - c'(q_2(1;D(\omega))}{(1-\eta)u'(q_2(1;D(\omega))) + \eta c'(q_2(1;D(\omega)))}\right]. \tag{A16}$$

We proceed iteratively to determine the period 1 decentralized market value functions as well as the period 0 centralized market value functions and the asset price p_0. The terms of trade are independent of the seller's holdings of liabilities and satisfy

$$q_1(a_1^b; D(\omega)) = \begin{cases} q^* & \text{if } a_1^b \geq a_1^* = [(1-\eta)u(q_1) + \eta c(q_1)] \,/\, (p_1(D(\omega)) + d_1(\omega)) \\ \hat{q}(a_1^b; D(\omega)) & \text{otherwise} \end{cases} \tag{A17}$$

where $\hat{q}(a_1^b; D(\omega))$ is the value of q that satisfies

$$(1-\eta)u(q) + \eta c(q) = (p_1(D(\omega)) + d_1(\omega))\, a_1^b. \tag{A18}$$

Moreover, $m_1(a_1^b; D(\omega))$ is

$$m_1(a_1^b; D(\omega)) = \frac{(1-\eta)u(q_1(a_1^b; D(\omega))) + \eta c(q_1(a_1^b; D(\omega)))}{(p_1(\omega) + d_1(\omega))}. \tag{A19}$$

These terms of trade imply the value functions for buyers and sellers in the period 1 decentralized market are:

$$V_1^b(a_1^b; D(\omega)) = \alpha\eta\left[u\left(q_1\left(a_1^b; D(\omega)\right)\right) - c\left(q_1\left(a_1^b; D(\omega)\right)\right)\right] + W_1^b(a_1^b; D(\omega)), \tag{A20}$$

$$V_1^s(a_1^s; D(\omega)) = \frac{\alpha}{n}(1-\eta)\int_{a_1^b}\left[u\left(q_1\left(a_1^b; D(\omega)\right)\right) - c\left(q_1\left(a_1^b; D(\omega)\right)\right)\right] d\Omega_1^b(a_1^b) + W_1^s(a_1^s; D(\omega)). \tag{A21}$$

Buyers and sellers problems in the period 0 centralized market are

$$W_0^i(a) = p_0^k k^i + \bar{v} + \max_{a'} -p_0(D)a' + \sum_{\omega}\gamma(\omega)V_1^i(a'; D(\omega)). \tag{A22}$$

To determine the period 0 asset price, note that the seller's demand for the asset is finite, when

$$p_0 \geq \sum_{s_1}(p_1(\omega) + d_1(\omega)), \tag{A23}$$

and at an interior solution for the buyer, we require that

$$p_0 = \sum_{\omega}\gamma(\omega)\frac{dV_1^b(a'; D(\omega))}{da'}. \tag{A24}$$

B Efficiency Proofs

Welfare for a given coupon issue is

$$W_0^P(D) = (1+n)\,\bar{v} + \sum_{\omega} \gamma(\omega) \left(U_1^P(D(\omega)) + U_2^P(D(\omega)) \right), \tag{B1}$$

with

$$U_t^P(D(\omega)) = (1+n)\bar{v} + d_t(\omega) + \alpha \left[u\left(q_t^{eq}(D(\omega))\right) - c\left(q_t^{eq}(D(\omega))\right) \right]. \tag{B2}$$

Before proving the proposition, we report two preliminary lemmas.

Lemma 4 (No asset transformation). *If $z(\omega_l) \geq \frac{d^*}{K^H}$, then efficient allocations feature neither risk nor maturity transformation.*

Proof of Lemma 4. The unconstrained optimal level of trade in decentralized markets satisfies $q_t^{eq}(D(\omega)) = q^*$. If this level of decentralized trade can be attained by a coupon issue which satisfies the planner's constraints and minimizes payments to the bank, that is,

$$\sum_{\omega} \gamma(\omega) c_t^B(\omega) = K^B \sum_{\omega} \gamma(\omega) z(\omega) \tag{B3}$$

then the allocation must be an efficient allocation.

Under the assumption of the lemma, the pass-through claim satisfies this property. By assumption, if $d_2(\omega) = K^H z(\omega)$, then $d_2(\omega) \geq d^*$ for $\omega = \omega_l, \omega_h$. Hence, the pass-through claim, $D(\omega) = \{0, K^H z(\omega_l), 0, K^H z(\omega_h)\}$ satisfies $q_t^{eq}(D(\omega)) = q^*$. Moreover, the commitment constraint in each state is satisfied since

$$c_2^B(\omega) = \left(K^H + K^B\right) z(\omega) - d_2(\omega) = K^B z(\omega) \geq \xi \left(K^H + K^B\right) z(\omega), \tag{B4}$$

where the final inequality follows from Assumption 1. ■

Lemma 5 (Only Risk Transformation). *There exists a threshold $\underline{z}_r \leq \frac{d^*}{K^H}$ such that if $\frac{d^*}{K^H} > z(\omega_l) \geq \underline{z}_R$, and $E_0[z(\omega)] \geq \frac{d^*}{K^H}$, then efficient allocations feature risk transformation and feature no maturity transformation.*

Proof of Lemma 5. We construct $d_t(\omega)$ such that $q_t^{eq}(D(\omega)) = q^*$ and

$$\sum_{\omega} \gamma(\omega) c_t^B(\omega) = K^B \sum_{\omega} \gamma(\omega) z(\omega). \tag{B5}$$

Since such an allocation attains the maximum of welfare subject to the resource feasibility and bank's participation constraints, the allocation is constrained efficient as long as it also satisfies the bank's limited commitment constraints. The pass-through claim does not attain this value since $d^* > K^H z(\omega_l)$.

Consider an allocation satisfying $d_2(\omega_l) = d^*$ and

$$d_2(\omega_h) = K^H z(\omega_h) - \frac{\gamma(\omega_l)}{\gamma(\omega_h)} \left[d^* - K^H z(\omega_l) \right]. \tag{B6}$$

Under the assumptions of the Lemma, it follows that $d^* \leq d_2(\omega_h) < K^H z(\omega_h)$. By construction, the allocation satisfies the bank's participation constraint with equality, or $\sum_\omega \gamma(\omega) c_2^B(\omega) = K^B \sum_\omega \gamma(\omega) z(\omega)$. Moreover, it is straightforward to show that under the assumptions of the Lemma along with Assumption 1 that the commitment constraints of the bank are satisfied. ■

Proof of Proposition 1. Suppose

$$d^* > \left(K^H + K^B \right) (1 - \xi) z_{2l}. \tag{B7}$$

We guess and then verify that the commitment constraint is slack in the high state but binds in the low state. In this case, it is not commitment-feasible for the bank to choose $d_2(\omega_l) \geq d^*$. We start by characterizing the optimum taking $L(\omega) = 0$. Then we see if an increase in $L(\omega_l)$ can improve outcomes.

When $L(\omega_l) = 0$, it is immediate $d_2(\omega_l) = \left(K^H + K^B\right)(1 - \xi) z(\omega_l)$. To see this, suppose $d_2(\omega_l) < \left(K^H + K^B\right)(1 - \xi) z(\omega_l)$. Consider perturbing $d_2(\omega_l)$ to $d_2(\omega_l) + \varepsilon$ and $c_2^B(\omega_l)$ to $c_2^B(\omega_l) - \varepsilon$. Since

$$\begin{aligned} c_2^B(\omega_l) &= \left(K^H + K^B \right) z(\omega_l) - d_2(\omega_l) \\ &> \left(K^H + K^B \right) z(\omega_l) \xi \end{aligned} \tag{B8}$$

as long as

$$\varepsilon < c_2^B(\omega_l) - \left(K^H + K^B \right) z(\omega_l) \xi \tag{B9}$$

this perturbation will continue to satisfy the limited commitment constraint of the bank. Further, increase $c_2^B(\omega_h)$ by $\gamma(\omega_l)\varepsilon / \gamma(\omega_h)$ to ensure the bank's ex ante participation constraint is satisfied. This increase requires reducing $d_2(\omega_h)$ by the same amount.

To show this is feasible without reducing $d_2(\omega_h)$ below d^*, suppose that

$$K^H \sum_\omega \gamma(\omega) z(\omega) - \gamma(\omega_l) \left(K^H + K^B \right) (1 - \xi) z(\omega_l) > \gamma(\omega) d^*. \tag{B10}$$

Then for any allocation with $d_2(\omega_l) < \left(K^H + K^B\right)(1 - \xi) z(\omega_l)$ and

$$\sum_\omega \gamma(\omega) c_2^B(\omega) = K^B \sum_\omega \gamma(\omega) z(\omega), \tag{B11}$$

it must be the case that $d_2(\omega_h) > d^*$. As a consequence, the perturbation is feasible.

Now, consider the impact of this perturbation on welfare. Since $d_2(\omega_h) \geq d^*$, we have

$$U_{2,d_2}(\omega_h) = 1, U_{1,d_2}(\omega_h) = 0. \tag{B12}$$

Hence, the impact on ex ante welfare from this decrease in period 2 coupons is

$$-\gamma(\omega_h)\frac{\gamma(\omega_l)}{\gamma(\omega_h)}\varepsilon = -\gamma(\omega_l)\varepsilon. \tag{B13}$$

However, since $d_2(\omega_l) < d^*$,

$$U_{2,d_2}(\omega_l) > 1, U_{1,d_2}(\omega_l) > 0. \tag{B14}$$

Hence, the impact on ex ante welfare from the increase in period 2 coupon payments is

$$\gamma(\omega_l)\varepsilon\left(U_{2,d_2}(\omega_l) + U_{1,d_2}(\omega_l)\right) > \gamma(\omega_l)\varepsilon. \tag{B15}$$

So the overall effect of this perturbation must increase ex ante welfare. This proves that when $L(\omega_l) = 0$, $d_2(\omega_l) = \left(K^H + K^B\right)(1-\xi)z(\omega_l)$.

We now show that an allocation with $L(\omega_l) > 0$ improves welfare relative to the best allocation without liquidation. Consider a perturbed allocation with $L(\omega_l) = \epsilon$. Define the coupon payments in the perturbed allocation as

$$\begin{aligned} d_1(\omega_l;\varepsilon) &= \kappa\left(K^H + K^B\right)z(\omega_l)\varepsilon & \text{(B16)}\\ d_1(\omega_h;\varepsilon) &= 0 & \text{(B17)}\\ d_2(\omega_l;\varepsilon) &= (1-\varepsilon)(1-\xi)\left(K^H + K^B\right)z(\omega_l) & \text{(B18)}\\ d_2(\omega_h;\varepsilon) &= d_2(\omega_h) - \varepsilon\xi\left(K^H + K^B\right)z(\omega_l)\frac{\gamma(\omega_l)}{\gamma(\omega_h)} & \text{(B19)} \end{aligned}$$

By construction, this perturbed allocation leaves the bank's expected consumption unchanged, and, as long as $z(\omega_h)$ is sufficiently large, this perturbation will not reduce $d_2(\omega_h)$ below d^*.

For any ε, welfare satisfies

$$\sum_{\omega}\gamma(\omega)\left[U_1\left(d_1(\omega;\varepsilon), d_2(\omega;\varepsilon)\right) + U_2(d_2(\omega;\varepsilon))\right]. \tag{B20}$$

Hence, the impact of this perturbation is

$$\sum_{\omega}\gamma(\omega)\left[U_{1,d_1}(\omega)\frac{dd_1(\omega;\varepsilon)}{d\varepsilon} + U_{1,d_2}(\omega)\frac{dd_2(\omega;\varepsilon)}{d\varepsilon} + U_{2,d_2}(\omega)\frac{dd_2(\omega;\varepsilon)}{d\varepsilon}\right]. \tag{B21}$$

Because $U_{t,d_t}(\omega_h) = 1$ and $U_{1,d_2}(\omega_h) = 0$, we simplify the impact of this perturbation as

$$\gamma(\omega_l) \left(K^H + K^B\right) z(\omega_l) \left[U_{1,d_1}(\omega_l)\kappa - \left(U_{1,d_2}(\omega_l) + U_{2,d_2}(\omega_l)\right)(1-\xi) - \xi\right] \tag{B22}$$

We show that there exist thresholds $\underline{\kappa}$ and $\underline{\xi}$ such that as $z(\omega_l) \to 0$, the impact of this perturbation is strictly positive.

Consider the term in brackets in (B22). With a slight abuse of notation, let $q_t^{eq}(\varepsilon) = q_t^{eq}(d_1(\omega_l;\varepsilon), d_2(\omega_l;\varepsilon))$ and $p_1(d_1(\omega_l;\varepsilon), d_2(\omega_l;\varepsilon)) = p(\epsilon)$. Then, since

$$U_{1,d_1}(\omega_l) = 1 + \alpha \left[u'\left(q_1^{eq}(\varepsilon)\right) - c'\left(q_1^{eq}(\varepsilon)\right)\right] \frac{dq_1^{eq}(\varepsilon)}{dd_1(\omega_l;\varepsilon)} \tag{B23}$$

$$U_{2,d_1}(\omega_l) = \alpha \left[u'\left(q_1^{eq}(\varepsilon)\right) - c'\left(q_1^{eq}(\varepsilon)\right)\right] \frac{dq_1^{eq}(\varepsilon)}{dd_2(\omega_l;\varepsilon)} \tag{B24}$$

$$U_{2,d_2}(\omega_l) = 1 + \alpha \left[u'\left(q_2^{eq}(\varepsilon)\right) - c'\left(q_2^{eq}(\varepsilon)\right)\right] \frac{dq_2^{eq}(\varepsilon)}{dd_2(\omega_l;\varepsilon)} \tag{B25}$$

and

$$\frac{dq_1^{eq}(\varepsilon)}{dd_2(\omega_l;\varepsilon)} = \frac{dq_1^{eq}(\varepsilon)}{dd_1(\omega_l;\varepsilon)} \frac{dp_1(\varepsilon)}{dd_2(\omega_l;\varepsilon)}, \tag{B26}$$

this term in brackets simplifies to

$$\begin{aligned} &\alpha \left[u'\left(q_1^{eq}(\varepsilon)\right) - c'\left(q_1^{eq}(\varepsilon)\right)\right] \left[\kappa - (1-\xi)\frac{dp_1(\varepsilon)}{dd_2(\omega_l;\varepsilon)}\right] \frac{dq_1^{eq}(\varepsilon)}{dd_1(\omega_l;\varepsilon)} \\ &\qquad - \alpha \left[u'\left(q_2^{eq}(\varepsilon)\right) - c'\left(q_2^{eq}(\varepsilon)\right)\right] \frac{dq_2^{eq}(\varepsilon)}{dd_2(\omega_l;\varepsilon)}(1-\xi) - (1-\kappa). \end{aligned} \tag{B27}$$

Since

$$\frac{dq_t^{eq}(\varepsilon)}{dd_t(\omega_l;\varepsilon)} = \frac{1}{(1-\eta)u'\left(q_t^{eq}(\varepsilon)\right) + \eta c'\left(q_t^{eq}(\varepsilon)\right)}, \tag{B28}$$

and $\lim_{z(\omega_l)\to 0} \lim_{\varepsilon \to 0} q_t^{eq}(\varepsilon) = 0$, it follows that

$$\begin{aligned} &\lim_{z(\omega_l)\to 0} \lim_{\varepsilon\to 0} \alpha \left[u'\left(q_t^{eq}(\varepsilon)\right) - c'\left(q_t^{eq}(\varepsilon)\right)\right] \frac{dq_t^{eq}(\varepsilon)}{dd_t(\omega_l;\varepsilon)} \\ &= \lim_{z(\omega_l)\to 0} \lim_{\varepsilon\to 0} \alpha \frac{u'\left(q_t^{eq}(\varepsilon)\right) - c'\left(q_t^{eq}(\varepsilon)\right)}{(1-\eta)u'\left(q_t^{eq}(\varepsilon)\right) + \eta c'\left(q_t^{eq}(\varepsilon)\right)} \\ &= \frac{\alpha}{1-\eta}. \end{aligned} \tag{B29}$$

Similarly,

$$\lim_{z(\omega_l)\to 0}\lim_{\varepsilon\to 0}\frac{dp_1(\varepsilon)}{dd_2(\omega_l;\epsilon)} = 1+\frac{\alpha\eta}{1-\eta} \tag{B30}$$

Then,

$$\begin{aligned}&\lim_{z_{2l}\to 0}\lim_{\varepsilon\to 0} U_{1,d_1}(\omega_l)\kappa - \left(U_{1,d_2}(\omega_l)+U_{2,d_2}(\omega_l)\right)(1-\xi)-\xi\\ &=\frac{\alpha}{1-\eta}\left[\kappa-(1-\xi)\left(2+\frac{\alpha\eta}{1-\eta}\right)\right]-(1-\kappa)\end{aligned} \tag{B31}$$

If

$$\xi \geq \frac{1-\eta+\alpha\eta}{2(1-\eta)+\alpha\eta} = \underline{\xi}, \tag{B32}$$

where for Assumption 1 to be satisfied requires

$$\frac{K^B}{K^H+K^B} > \underline{\xi}, \tag{B33}$$

then there exists $\kappa < 1$ s.t. (B31) is strictly positive. Indeed, $\underline{\kappa}$ satisfies

$$\underline{\kappa}(\xi) \geq \frac{1-\eta}{1-\eta+\alpha}\left[1+(1-\xi)\left(2+\frac{\alpha\eta}{1-\eta}\right)\right]. \tag{B34}$$

Hence, if $\xi \geq \underline{\xi}$ and $\kappa \geq \underline{\kappa}(\xi)$, since (B31) is strictly positive, there must exist a threshold $\underline{z}$ such that for $z(\omega_l) < \underline{z}$, this perturbation strictly raises the value of the social planner implying that strictly positive liquidation—that is, $L(\omega_l) > 0$—is efficient. ■

C Equilibrium Proofs

Proof of Lemma 2. From the proof of Lemma 4, the pass-through claim satisfies all the bank's constraints and mimimizes the bank's liability payoffs. At the resulting allocation, the liability has no liquidity premium. Using the Arrow-Debreu prices,

$$\pi_t(\omega_i; D) = \gamma(\omega_i),\ \omega_i \in \Omega,\ t \in \{1,2\}. \tag{C1}$$

the pass-through claim is optimal for the bank so that the pass-through claim is a competitive equilibrium. A similar argument applies for the allocation in Lemma 5. ■

Proof of Proposition 3. The proof that an efficient allocation with $L(\omega_l) > 0$ is not an equilibrium is by contradiction. We begin by constructing the liability issue associated the efficient outcome when $L(\omega_l) > 0$. Since there is sufficient liquidity in the high state ($z(\omega_h)$ is sufficiently large), $L(\omega_h) = 0$ and it is immediate that $L(\omega_l) > 0$ only if the commitment constraint in the low state binds. For a given choice of liquidation, then, the efficient liability issue satisfies

$$d_1(\omega_l) = (K^H + K^B)\kappa z(\omega)L(\omega_l), \tag{C2}$$

$$d_1(\omega_h) = 0, \tag{C3}$$

$$d_2(\omega_l) = (K^H + K^B)(1-\xi)z(\omega_l)(1-L(\omega_l)), \tag{C4}$$

$$d_2(\omega_h) = (K^H + K^B)z(\omega_h) - \frac{1}{\gamma(\omega_h)}\left[K^B \sum_\omega \gamma(\omega)z(\omega) - \gamma(\omega_l)\xi(1-L(\omega_l))(K^H+K^B)z(\omega_l)\right], \tag{C5}$$

where the last equality results from the bank's participation constraint holding with equality.

The efficient level of $L(\omega_l)$ satisfies

$$\sum_\omega \gamma(\omega)\left[\begin{array}{c} U_{1,d_1}(d_1(\omega), d_2(\omega))\frac{dd_1(\omega)}{dL(\omega_l)} + U_{1,d_2}(d_1(\omega), d_2(\omega))\frac{dd_2(\omega)}{dL(\omega_l)} \\ +U_{2,d_2}(d_2(\omega))\frac{dd_2(\omega)}{dL(\omega_l)} \end{array}\right] = 0. \tag{C6}$$

Since there is sufficient liquidity in the high state, Conditions (C6) is

$$0 = -\xi + U_{1,d_1}(d_1(\omega_l), d_2(\omega_l))\kappa - (1-\xi)\left(U_{1,d_2}(d_1(\omega_l), d_2(\omega_l)) + U_{2,d_2}(d_2(\omega_l))\right). \tag{C7}$$

Let D^* be the coupon issue defined by (C2)-(C5) when $L(\omega)$ satisfies (C7).

Define the function $H(q)$ as

$$H(q) \equiv \frac{u'(q) - c'(q)}{(1-\eta)u'(q) + \eta c'(q)}. \tag{C8}$$

Given $D^*(\omega)$, the market for liabilities in period 0 clears when the price of liabilities satisfy

$$\pi_1(\omega_h; D^*) = \gamma(\omega_h), \tag{C9}$$

$$\pi_2(\omega_h; D^*) = \gamma(\omega_h) \tag{C10}$$

$$\pi_1(\omega_l; D^*) = \gamma(\omega_l)\left[1 + \alpha\eta H(q_1^{eq}(d_1^*(\omega_l), d_2^*(\omega_l)))\right], \tag{C11}$$

$$\pi_2(\omega_l; D^*) = \gamma(\omega_l)\left[1 + \alpha\eta H(q_1^{eq}(d_1^*(\omega_l), d_2^*(\omega_l)))\right] \times \left[1 + \alpha\eta H(q_2^{eq}(d_1^*(\omega_l), d_2^*(\omega_l)))\right]. \tag{C12}$$

The period 0 budget constraint of a bank, then, is

$$I \leq K^B + \frac{1}{p_0^k}\sum_t \sum_\omega \pi_t(\omega; D^*)d_t(\omega). \tag{C13}$$

We construct a strictly profitable deviation for the bank from the efficient liability issue.

Take the Pareto allocation and consider the following perturbation:

$$\hat{L}(\omega_l) = L^*(\omega_l) - \epsilon, \tag{C14}$$

$$\hat{d}_1(\omega_l) = (K^H + K^B)\kappa z(\omega_l)\hat{L}(\omega_l), \tag{C15}$$

$$\hat{d}_2(\omega_l) = (K^H + K^B)(1-\xi)z(\omega_l)(1-\hat{L}(\omega_l)), \tag{C16}$$

$$\hat{d}_1(\omega_h) = 0 \tag{C17}$$

$$\hat{d}_2(\omega_h) = d_2(\omega_h) + (K^H + K^B)\frac{\gamma_l}{\gamma_h}\xi\epsilon z(\omega_l). \tag{C18}$$

By construction, this perturbation has no impact on the bank's expected consumption since

$$\begin{aligned}\sum_\omega \gamma(\omega)[\hat{c}_1(\omega) + \hat{c}_2(\omega)] &= \sum_\omega \gamma(\omega)[c_1(\omega) + c_2(\omega)] \\ &\quad -\gamma_h(K^H + K^B)\frac{\gamma_l}{\gamma_h}\xi\epsilon z(\omega_l) + \gamma_l(K^H + K^B)\xi\epsilon z(\omega_l) \\ &= \sum_\omega \gamma(\omega)[c_1(\omega) + c_2(\omega)]. \end{aligned} \tag{C19}$$

However, consider how this perturbation impacts the revenues raised from issuing liabilities in the initial period. Revenues raised from the perturbed liability issuance are

$$\begin{aligned}\frac{1}{p_0^k}\sum_t\sum_\omega \pi_t^b(\omega; D^*)\hat{d}_t(\omega) &= \frac{1}{p_0^k}\sum_t\sum_\omega \pi_t^b(\omega; D^*)d_t^*(\omega) \\ &\quad + \frac{1}{p_0^k}\epsilon\gamma_l(K^H + K^B)z(\omega_l) \\ &\quad \times \left(\xi - [1+\alpha\eta H(q_1^*)]\,\kappa + [1+\alpha\eta H(q_2^*)]\,[1+\alpha\eta H(q_1^*)]\,(1-\xi)\right), \end{aligned} \tag{C20}$$

where with a slight abuse of notation we write $q_t^* = q_t^{eq}(d_1^*(\omega_l), d_2^*(\omega_l))$.

We now argue that

$$\xi - [1+\alpha\eta H(q_1^*)]\,\kappa + [1+\alpha\eta H(q_2^*)]\,[1+\alpha\eta H(q_1^*)]\,(1-\xi) > 0. \tag{C21}$$

First, note that the left hand side of (C21) can be rewritten as

$$\begin{aligned}&\xi - [1+\alpha\eta H(q_1^*)]\,\kappa + [1+\alpha\eta H(q_2^*)]\,[1+\alpha\eta H(q_1^*)]\,(1-\xi) \\ &= \alpha\eta\left[\frac{1-\kappa}{\alpha\eta} - \kappa H(q_1^*) + (1-\xi)\,[H(q_1^*) + H(q_2^*) + \alpha\eta H(q_1^*)H(q_2^*)]\right]. \end{aligned} \tag{C22}$$

Next, (C7) which determines the efficient level of liquidation can be rewritten as

$$\xi = \kappa \left(1 + \alpha H(q_1^*)\right) \\ - (1-\xi) \left[\alpha H(q_1^*) \left[1 + \alpha\eta H(q_2^*) + d_2^*(\omega_l)\alpha\eta G(q_2^*)\right] + 1 + \alpha H(q_2^*)\right], \quad \text{(C23)}$$

where $d_2^*(\omega_l)$ is the efficient coupon and

$$G(q) \equiv \frac{c'(q)u''(q) - u'(q)c''(q)}{[(1-\eta)u'(q) + \eta c'(q)]^3}. \quad \text{(C24)}$$

Using (C23) to substitute for ξ into the left-hand side of (C22) and re-arranging terms,

$$\xi - [1 + \alpha\eta H(q_1^*)]\,\kappa + [1 + \alpha\eta H(q_2^*)]\,[1 + \alpha\eta H(q_1^*)]\,(1-\xi) \\ = \alpha(1-\eta)\left[\kappa H(q_1^*) - (1-\xi)\left[H(q_1^*) + H(q_2^*) + \alpha\eta H(q_1^*)H(q_2^*)\right]\right] \\ - (1-\xi)\alpha^2\eta H(q_1^*)d_2^*(\omega_l)G(q_2). \quad \text{(C25)}$$

Combining (C22) and (C25) implies

$$\kappa H(q_1^*) - (1-\xi)\left[H(q_1^*) + H(q_2^*) + \alpha\eta H(q_1^*)H(q_2^*)\right] \\ = \frac{1-\kappa}{\alpha} + (1-\xi)\alpha\eta H(q_1^*)d_2^*(\omega_l)G(q_2). \quad \text{(C26)}$$

It follows that

$$\xi - [1 + \alpha\eta H(q_1^*)]\,\kappa + [1 + \alpha\eta H(q_2^*)]\,[1 + \alpha\eta H(q_1^*)]\,(1-\xi) \\ = \alpha\eta\left[\frac{(1-\kappa)(1-\eta)}{\alpha\eta} - (1-\xi)\alpha\eta H(q_1^*)d_2^*(\omega_l)G(q_2^*)\right] > 0. \quad \text{(C27)}$$

where the inequality follows because $H(q) \geq 0$ when $q < q^*$ and $G(q) \leq 0$. ■