Sentiment, Liquidity and Asset Prices

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Abstract

We study a dynamic market for durable assets, in which asset owners are privately informed about the quality of their assets and experience occasional (productivity or liquidity) shocks that give rise to gains from trade. An important feature of our environment is that asset buyers must worry not only about the quality of assets for which they currently bid, but also about the prices at which they can resell the assets in the future. We show that the interaction between adverse selection and resale concerns generates an inter-temporal coordination problem between agents at different points in time and gives rise to multiple self-fulfilling equilibria. Although agents are fully rational and asset prices are always equal to *fundamentals* – the present discounted value of asset cashflows, – the mix of assets that is traded in the market depends on *sentiments* – the agents' expectations about future market conditions. Indeed, there is a rich set of sentiment driven equilibria, in which sunspots generate large fluctuations in asset prices, market liquidity, output and welfare, resembling what one may refer to as "bubbles."

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1 Introduction

In a frictionless market, all gains from trade are realized, and durable assets or securities always end up being held by the parties that value them the most. Thus, asset prices reflect not only the current but also all expected future gains from trade. In contrast, in the presence of frictions, some gains from trade may remain unrealized and, as a result, asset prices may be depressed. In such an environment, there is a close connection between market *liquidity* – the ease with which assets are re-allocated,– and asset prices. Moreover, as we show in this paper, if the frictions result from *information asymmetries*, there can be multiple self-fulfilling equilibria. Although agents are fully rational and asset prices are always equal to fundamentals, the mix of assets that is traded in the market depends on *sentiments* – the agents' expectations about future market conditions. We show that there is a rich set of sentiment driven equilibria, in which sunspots generate large fluctuations in asset prices, market liquidity, output and welfare, resembling what one may refer to as "bubbles."

We consider a dynamic market for assets (Lucas trees), in which asset owners are privately informed about the quality of their assets (the fruit to be harvested). Gains from trade arise stochastically over time because the current asset owners may experience "liquidity" or productivity shocks that depress their value of holding or employing assets vis-a-vis the potential buyers in the market. Buyers compete for assets, but they may face a lemons problem as in Akerlof (1970), since they do not observe the quality of the owners' assets nor the motive for their sale. The buyers who purchase assets in any given period become asset owners in the next period. The important feature of our environment is that the buyers must worry not only about the quality of the assets for which they currently bid, but also about market prices were they to resell the assets in the future.

If information were symmetric, all asset owners with liquidity shocks would immediately sell their assets and, in the unique equilibrium, asset prices would equal the expected discounted value of asset cash-flows at their most efficient allocation (Proposition 1). Furthermore, as we suppose that the proportion of good quality assets is constant over time, this economy would feature no aggregate fluctuations in asset prices, market liquidity or welfare.

Instead, when information is asymmetric, the owners of low quality assets want to mimick the owners of high quality assets with liquidity shocks, and their presence in the market depresses the buyers' willingness to pay. Absent resale considerations (or in a static setting), the potential buyers only care about the flow payoffs that they expect to receive from holding the asset in the current period. As a result, when the proportion of low quality assets is sufficiently large, the buyers willingness to pay for the assets drops below the reservation value of liquidity shocked owners of high quality assets. In this case, the only assets traded in equilibrium are the low quality assets and asset prices are depressed to reflect this. Instead, when the proportion of high quality assets is sufficiently high, the buyers' willingness to pay remains above the reservation value of the liquidity shocked owners of high quality assets, asset prices are high and trade is efficient. Thus, depending on parameters, there can be two possible equilibria but, more importantly, the equilibrium is unique.

Our main result is that the interaction between information frictions and resale concerns generates a dynamic feedback between market liquidity and asset prices, which gives rise to multiple equilibria (Theorem 1). The reason is that, when buyers anticipate that they might need to sell the assets in the future, their willingness to pay for these assets depends also on their beliefs about market liquidity and asset prices in the future. If they believe that the market will be liquid and asset prices will be high, they will be willing to offer higher prices to asset sellers today. The converse is true when they expect an illiquid market with low prices in the future. We show that these concerns about future market conditions generate multiple self-fulfilling equilibria and, to do so, we first construct two types of *constant price equilibria*.

We construct a *high trade* equilibrium, in which all liquidity shocked asset owners trade their assets immediately and, as a result, asset prices, market liquidity and welfare are permanently high. Then, we construct a *low trade* equilibrium, in which only low quality asset owners trade and, as a result, asset prices, market liquidity and welfare are permanently low. We show that there exists a lower bound $\bar{\pi}_{HT}$ on the proportion of high quality assets, such that the *high trade* equilibrium exists when the actual proportion π of high quality assets is greater than $\bar{\pi}_{HT}$. Analogously, we show that there exists an upper bound $\bar{\pi}_{LT}$, such that the *low trade* equilibrium exists when π is smaller than $\bar{\pi}_{LT}$. Importantly, however, we show that $\bar{\pi}_{HT} < \bar{\pi}_{LT}$ and, thus, the two equilibria coexist when $\pi \in [\bar{\pi}_{HT}, \bar{\pi}_{LT}]$. Moreover, the difference $\bar{\pi}_{LT} - \bar{\pi}_{HT}$ is increasing in the agents' discount factor, making multiple equilibria more likely when dynamic considerations are more important.

Next, we capture the notion of *sentiment equilibria* as coordinated beliefs about future market conditions. To do so, we introduce a binary sunspot variable $z_t \in \{B, G\}$ and we look for equilibria in which agents coordinate on high trade (low trade) whenever $z_t = G$ ($z_t = B$). To faciliate inter-temporal coordination, the sunspot must be sufficiently persistent: when $z_t = G$, the agents at time t believe it is more likely that high trade will be played in the future, whereas when $z_t = B$, the agents believe it is less likely. Indeed, the equilibrium with constant prices are special cases in which the sunspot becomes perfectly persistent. It turns out that the coexistence of multiple constant price equilibria, i.e., if $\pi \in (\pi_{HT}, \pi_{LT})$, is a sufficient condition for the existence of *sentiment equilibria*, provided that the sunspot driving the equilibrium play is sufficiently persistent (Proposition 5). In these equilibria, asset prices are always equal to fundamentals and there is no aggregate fundamental uncertainty. Yet, asset prices, market liquidity and welfare can display large fluctuations due to sunspots. We show that the magnitude of these fluctuations increases in the persistence of the sunspot process, which strengthens the agents' ability to coordinate inter-temporally.

Our model shows that *sentiments* can actually affect the fundamental value or performance of the assets by changing the mix of assets that get traded and, therefore, the extent to which gains from trade are realized. Thus, market *sentiment* cannot be separated from fundamental value, and both are essential in determining asset valuations. Also, even when there is no intrinsic information about changes in the characteristics of the assets, *sentiments* can lead to large price swings. Thus, our model can provide a fully rational explanation for the documented excess volatility in asset prices.[1]

1.1 Related Literature

Our paper naturally relates to the recent and growing literature that embeds adverse selection in a macro-finance context.[2]. Daley and Green (2016) and Fuchs et al. (2016) explicitly model re-trade considerations. Unlike the setting of this paper, those papers allow for time-on-the-market to serve as a signal of quality. Thus, given current beliefs, the equilibria are essentially unique in both papers. Although both papers allow for time varying liquidity, a fact that is particularly stressed in Daley and Green (2016), this variation is not driven by inter-temporal coordination and expectations of future market liquidity. Instead, it is driven by whether the current beliefs about the asset quality are above or below the critical threshold at which pooling is an equilibrium.[3]

Some of the recent work in this area considers a search environment rather than competitive markets. The closest papers within this literature are Chiu and Koeppl (2016), Maurin (2016) and Mäkinen and Palazzo (2017). They share with our model the feature that assets can be of high or low quality and that buyers are concerned both about current adverse selection and future resale conditions, since eventually they might be inclined to sell their assets. Unlike us, these papers assume that there is persistence in the shocks that motivate holders to sell

[1]See, for example, LeRoy (2004) and Shiller (2005).

[2]See, for example, Eisfeldt (2004), Martin (2005), Bigio (2015), Chari et al. (2010), Gorton and Ordoñez (2014, 2016), Daley and Green (2016) and Fuchs et al. (2016).

[3]In Fuchs et al. (2016), even though the unique equilibrium is fully separating, there also could be time varying liquidity if the distribution of types is non-uniform or if the gains from trade are type-dependent.

their assets.[4] This persistence introduces a backward looking aspect of liquidity that is absent in our benchmark model. Namely, if the market is liquid today, asset are allocated more efficiently today and, thus, there will be fewer gains from trade to be realized tomorrow. This makes the adverse selection problem endogenously more severe tomorrow, leading to a drop in future liquidity.

In addition to the differences in market structure, these papers have a very different focus from ours. The main consideration in Chiu and Koeppl (2016) is the interaction between adverse selection and search frictions, and it is largely motivated by the recent financial crisis: they mainly discuss policy interventions when the fraction of low quality assets in the market is so large that there would be no trade absent an intervention. Although Maurin (2016) notes that there is a possibility for multiple equilibria, his main contribution is the construction of equilibria with cycles. Unlike our sentiment equilibria, these equilibria are deterministic and are not driven by inter-temporal coordination, but rather by the backward looking aspect of liquidity arising from the persistence of the shocks. Finally, Mäkinen and Palazzo (2017) have a more general search and matching technolodgy that allows for congestion externalities. Their focus is on the additional negative effect (and policies to overcome it) from the fact that traders who are not shocked are in the market trying to trade away their lemons and creating congestion externalities for shocked sellers.

The inter-temporal aspect of the coordination leading to multiplicity of equilibria relates our work with the broad literature on money and rational bubbles.[5] There is an important difference between our work and most of that literature. In our setting, the value of assets is always pinned down by fundamentals and we do not rely on a violation of the transversality condition for assets to have positive prices. Closer to our model is the contemporaneous work of Donaldson and Piacentino (2017), who motivate potential runs on banks as arising from failures of coordination in the re-trading of"money-like" bank obligations. In their setting, trading frictions are exogenous, there is no adverse selection and trade completely breaks down, whereas adverse selection is the source of the endogenous frictions in our model. Furthermore, there is always some trade in our model.

The papers by Plantin (2009) and Malherbe (2014) are also related to our work, although the strategic considerations in their papers are contemporaneous rather than dynamic. In Malherbe (2014), firms must make a portfolio choice decision between holding cash versus assets with privately known quality. He shows that multiple equilibria are possible due to

[4]Chiu and Koeppl (2016) consider the case of full persistence.

[5]See, for example, Samuelson (1958), Tirole (1985), Weil (1987), Santos and Woodford (1997), and Martin and Ventura (2012).

complementarities in firms' cash-holding decisions. If a firm decides to increase its cash-holdings in the first period, then if that firm trades in the second period, it is less likely that the trade is the result of a liquidity shock. As a result, there are less gains from trade in the second period and there is more adverse selection in the market. This in turn makes it more attractive for other firms to also hoard cash. Thus, there can be two equilibria, one in which firms expect other firms not to hoard cash and the second period market to work well, and another in which firms expect other firms to hoard cash and, as a result, the second period market dries-up. A similar mechanism is present in Plantin (2009). Although there is no cash-hoarding by firms in his setting, the number of investors who decide to buy the bond in the first period affects the potential market size for the bonds and hence their price in the future. As in Malherbe (2014), this contemporaneous complementarity can lead to self-fulfilling market failures. It is important to highlight that equilibrium multiplicity in these papers arises due to static coordination failures. Indeed, as in the global games literature, Plantin (2009) is able to obtain uniqueness of equilibrium by introducing a noisy private signal about the probabilities of default of the bonds.

Finally, there has been an increased interest among macroeconomists to understand how *sentiments* – in the form of correlated shocks to agents' information sets,– can be drivers of aggregate fluctuations. Some recent papers include Lorenzoni (2009), Hassan and Mertens (2011), Angeletos and La'O (2013), and Benhabib et al. (2015). In this literature, the dispersion of information among agents about aggregate economic states is an essential ingredient. We contribute to this literature by showing that, in the presence of adverse selection, *sentiments* which coordinate agents' expectations about future market conditions can generate aggregate fluctuations even when the information about aggregate variables is common to all economic agents at all times.

The rest of the paper is organized as follows. In Section 2, we present the setup of the model. In Section 3, we characterize the equilibria of the model and conduct comparative statics. In Section 4, we consider several extensions, and we conclude in Section 5. All proofs are relegated to the Appendix.

2 The Model

Time is infinite and discrete, indexed by $t \in \{0, 1, ...\}$. There is a mass of indivisible assets, indexed by $i \in [0, 1]$, which are identical in every respect except their quality. Assets can be of either high or low quality, which we denote by $\theta_i \in \{L, H\}$. An asset of quality θ_i delivers a per period cash-flow of x_{θ_i}, where $x_H > x_L > 0$. The probability that an asset is of good quality is $\mathbb{P}(\theta_i = H) = \pi \in (0,1)$, which is also assumed to be the fraction of good quality assets in the market. For expositional simplicity, we suppose that asset quality is permanent; we extend our analysis to the case of transitory quality in Section 4.3.

There is a large mass of ex-ante identical risk-neutral agents who discount payoffs with a factor $\delta \in (0,1)$. Each of these agents can hold at most one unit of the asset, and we refer to those who currently hold assets as *holders* and to the rest as potential *buyers*. We introduce gains from trade by supposing that holder j values the cash-flow of quality θ_i asset at $\omega_j x_{\theta_i}$, where $\omega_j \in \{\chi, 1\}$ with $\chi \in (0,1)$ and $\chi x_H \geq x_L$.[6] All potential buyers value the cash-flow of quality θ_i asset at x_{θ_i}. For concreteness, we refer to ω_j as agent j's liquidity status and we refer to an agent with $\omega_j = \chi$ as an agent who has liquidity needs. We suppose that each period a holder experiences a liquidity need with probability $\lambda \in (0,1)$. Thus, a holder's liquidity status is independent both of her asset quality and her liquidity status in the past; we extend our analysis to the case of persistent liquidity needs in Section 4.2.[7]

The following are two possible interpretations for our trading environment. The traders are firms, and the assets they trade are units of capital which can be of heterogeneous quality (θ). The heterogeneity in firms' asset valuations (ω) corresponds to idiosyncratic productivity differences across firms in utilizing these assets in production. In this interpretation, a more efficient allocation of assets results in higher aggregate output. Alternatively, the traders are consumers faced with idiosyncratic shocks to their marginal utilities of consumption (ω), and due to borrowing constraints they need to sell (real or financial) assets of heterogeneous quality (θ) to satisfy consumption needs. In this interpration, a more efficient allocation of assets results in higher aggregate utility. Naturally, the predictions of the theory will depend on the application one has in mind and which interpretation fits it better.

The market for assets is competitive - in each period, at least two buyers are randomly matched with a holder, and they compete for the holder's asset a la Bertrand. When a holder receives offers from the buyers, she decides whether and which offer to accept. If she rejects all offers, then she continues to be a holder in the next period and is rematched with a new set of buyers. Instead, if she accepts an offer, then she sells her asset and enters the pool of potential buyers.[8] A buyer whose offer is rejected continues to be a buyer in the next period,

[6] This assumption ensures that adverse selection is sufficiently severe. Although inessential for our results, it reduces the number of cases that we need to consider.

[7] As long as there are at least two buyers without liquidity needs competing for each holder's asset, it is without loss of generailty to suppose that buyers do not have liquidity needs.

[8] Since there is a continuum of assets and matching is random, the probability that an agent who sells her asset is rematched to bid for that same asset is zero.

whereas a buyer whose offer is accepted, buys the asset and becomes a holder of that asset in the next period.

In a frictionless market, all assets would be reallocated from agents with liquidity needs (e.g., holders) to those without (e.g., buyers). As we will see, however, trade in our economy may be hindered by the presence of asymmetric information. In particular, we will assume that the quality of a holder's asset and her liquidity status are both that holder's private information.

We suppose that the time-t information set of a buyer includes the aggregate history of trades (e.g., aggregate volume), but not the individual trading history of the asset for which he bids.[9] The strategy of each buyer is a mapping from his information set to a probability distribution over offers. A holder's information set includes the quality θ of her asset, her liquidity status ω, and the buyers' information set. The strategy of each holder is a mapping from her information set to a probability of acceptance.

We use Perfect Bayesian Equilibrium (PBE) as our equilibrium concept. This has the following implications. First, each holder's acceptance rule must maximize her expected payoff taking as given the buyers' strategies (*Holder Optimality*). Second, any offer in the support of a buyer's strategy must maximize his expected payoff given his beliefs, the seller's and the other buyers' strategies (*Buyer Optimality*). Third, given their information set, buyers' beliefs are updated using Bayes' rule whenever possible (*Belief Consistency*).

3 Equilibrium

In this section, we characterize the set of equilibria of our model. Let us briefly outline how we will proceed. We start by analyzing a benchmark economy in which asset qualities are observable (Section 3.1). We show that in the unique equilibrium of this economy, there are no aggregate fluctuations and all assets are allocated efficiently (Proposition 1). We then proceed to our main analysis. We first focus on equilibria that do not feature aggregate fluctuations (Section 3.2), and we show that multiple equilibria, ranked both in terms of asset prices and welfare, can arise (Theorem 1). Second, we conduct comparative statics on the equilibrium set, asset prices and welfare (Section 3.3). Finally, we consider sentiment driven stochastic equilibria, in which asset prices and welfare fluctuate due to sunspots (Section 3.4).

[9] The primary role of this assumption is to eliminate signaling considerations which would complicate our analysis considerably. If trading history of individual assets were observable, holders may reject certain offers and engage in costly delay in order to signal their types. Our qualitative results would extend to a setting where such signaling is possible as long as trading history provides an imperfect signal of asset quality.

3.1 Benchmark without asymmetric information

Let us consider an economy in which asset qualities are publicly observable.[10] As we show next, in such an environment, the equilibrium is unique and in it both asset prices correctly reflect asset qualities and all assets are allocated efficiently.

Proposition 1 (Observable Quality) *If asset quality is observable, then the equilibrium is unique, in it all assets are efficiently allocated and, for all t, the price of θ-quality assets is $p_\theta^{fb} = (1-\delta)^{-1}x_\theta$, and the expected total surplus is $W^{fb} = (1-\delta)^{-1}E\{x_\theta\}$.*

For any given quality, buyers value the assets more than the holders; thus, in any equilibrium, all assets must be reallocated from holders with liquidity needs to the buyers who do not have liquidity needs; hence, the efficiency of asset allocation. Because buyers compete, in equilibrium all assets must be priced at the expected discounted value of their cash-flows, which depend on quality. Finally, expected total surplus is simply the expected value of the holders, which in this case is given by the expected prices of assets that they hold.

3.2 Constant price equilibrium

We begin our analysis by considering a simple class of stationary equilibria which allow us to highlight the link between asset prices and market liquidity – the efficiency with which assets are allocated in the market.

From now on, we will refer to a holder with liquidity status ω and an asset of quality θ as a (θ, ω)-type holder. Let p_t denote the (common) asset price that prevails in equilibrium at time t.[11] Then,

Definition 1 *We say that a PBE is a* **constant price equilibrium** *if the asset price and the distribution of holder types is the same in every period.*

In what follows, we will drop the time superscripts and we will superscript the future variables by primes. Consider the problem of a (θ, ω)-type asset holder, who must decide whether to trade her asset or to hold on to it. Let $V(\theta, \omega)$ denote the value to this asset holder, and let p^* denote the (constant price) equiilibrium asset price. In equilibrium, for any

[10] With observable quality, whether holders' liquidity status is observable is irrelevant for equilibrium prices and allocations.

[11] The price of an asset is defined to be the maximal bid of the buyers for that asset, and it must be common to all assets since all assets look identical to the buyers at any time t.

(θ, ω), it must be that:

$$V(\theta, \omega) = \max_{s \in \{0,1\}} s \cdot p^* + (1 - s) \cdot E\{\omega x_\theta + \delta V(\theta', \omega')|\theta, \omega\} \tag{1}$$

where $s \in \{0, 1\}$ is an indicator for whether the holder trades the asset or not. If the holder sells the asset today (i.e., $s = 1$), then her payoff is the asset price. Instead, if she does not (i.e, $s = 0$), then she gets the flow payoff ωx_θ from the asset today plus the expected value from holding the asset tomorrow, where expectations are conditional on the holder knowing her type today. Holder optimality requires that the holder's equilibrium value be the maximum of the expected payoffs from either trading the asset or holding on to the asset.

Adverse selection can arise whenever the quality of assets traded depends on the asset price itself. Suppose that at time t an asset holder were to receive a maximal offer p, which may or may not equal the equilibrium price p^*. The holder would accept such an offer if and only if it exceeds V. In particular, the set of holder types who accept a maximal offer p is:

$$\Gamma(p) = \{(\theta, \omega) : V(\theta, \omega) \leq p\}. \tag{2}$$

Because $V(\cdot, \cdot)$ will be different for holders of different quality assets, the set $\Gamma(p)$ will depend non-trivially on p. Also, note that, because today's offers for an asset are unobserved by the buyers of that asset in the future, the holder's value V depends on the equilibrium price p^* and not on offers made off-equilibrium.

Consider the problem of the buyers who are bidding for a holder's asset. Because at least two buyers compete for the holder's asset, in any equilibrium the buyers' expected profits must be zero. Therefore, in equilibrium the asset price must satisfy:

$$p^* = E\{x_\theta + \delta V(\theta', \tilde{\omega}')|(\theta, \omega) \in \Gamma(p^*), \tilde{\omega} = 1\}, \tag{3}$$

where tilde on $\tilde{\omega}$ indicates that this is the liquidity status of the buyer. Rationality and belief consistency require that buyers understand the potential adverse selection problem and condition their expectations of asset quality on the set of types who accept their offers.[12] Since a buyer who gets the asset in the current period becomes a holder in the next, the equilibrium asset price depends on the expected value from being an asset holder.

Finally, buyer optimality requires that in equilibrium no buyer can profitably deviate by

[12]If $\Gamma(p) = \emptyset$, we set without loss of generality $p^* = E\{x_\theta + \delta V(\theta', \tilde{\omega}')|\tilde{\omega} = 1\}$.

making an offer to the holder that strictly exceeds the equilibrium price, i.e., it must be that:

$$\hat{p} \geq E\{x_\theta + \delta V(\theta', \tilde{\omega}') | (\theta, \omega) \in \Gamma(\hat{p}), \tilde{\omega} = 1\} \tag{4}$$

for all offers $\forall \hat{p}$ strictly greater than the equilibrium price p^*.

All constant price equilibria are fully characterized by a price-value function pair (p^*, V) that satisfies conditions (1)-(4). In fact, we can already see the feedback between asset prices and market liquidity that is at the heart of our paper. First, the holders' value is increasing in the asset price because, conditional on trading the asset, the payoff to the holder is higher. As a result, the set of holder types who trade will be less adversely selected, and the market will be more liquid, when asset prices are higher. Second, the asset price is increasing in the value from being an asset holder because, when making their offers, buyers are forward-looking and care about the resale value of the asset in the future. As we will see, this feedback between asset prices and their valuations plays an important role in determining the set of equilibria.

The following proposition puts further structure on the possible constant price equilibria by stating that, in any such equilibria, the holders with low quality assets always trade whereas the holders with high quality assets who do not have liqudity needs never do.

Proposition 2 *Any constant price equilibrium is fully characterized by a value function V and asset price p^* satisfying (1)-(4). In any such equilibrium, $V(L, \chi) = V(L, 1) = p^* \leq V(H, \chi) < V(H, 1)$. Thus, the low quality assets always trade whereas the high quality assets owned by holders without liquidity needs never trade.*

First, because the flow payoff to a holder with liquidity needs is lower than to a holder without, the values can be ranked according to the liquidity status, $V(\theta, 1) \geq V(\theta, \chi)$ for $\theta \in \{L, H\}$. Second, because buyers do not have liquidity needs and they can at least guarantee themselves a low quality asset, it must be that all holders with low quality assets trade and $V(L, 1) = p^*$. Finally, since all low quality assets trade, the unconditional value $E\{V(\theta, 1)\}$ is an upper bound on the payoff that the buyers can get; hence, the buyers can never attract the $(H, 1)$-type holder without making losses in expectation.

From Proposition 2, it follows that there can be two types of constant price equilibria, depending on whether the (H, χ)-type holder trades in equilibrium. We adopt the following definition in order to distinguish among them.

Definition 2 *We say that there is* **high trade** *if in equilibrium both high and low quality assets trade. Otherwise, if only low quality assets trade, we say that there is* **low trade**.

In the high trade equilibrium, all holders with liquidity needs trade and the assets are efficiently allocated. Instead, in the low equilibrium, the allocation is inefficient because the high types with liquidity needs stay out of the market.

The following theorem states our main result. It provides the conditions under which each type of equilibrium exists and under which they coexist. Importantly, when the two equilibria coexist, they can be strictly ranked in the level of asset prices and welfare.

Theorem 1 (Characterization and Multiplicity) *A constant price equilibrium exists, and it either features high trade or low trade. There are thresholds* $0 < \bar{\pi}_{HT} < \bar{\pi}_{LT} < 1$ *on the beliefs about asset quality such that:*

(i) There is at most one high trade equilibrium, and it exists if and only if $\pi \geq \bar{\pi}_{HT}$*, and*

(ii) There is at most one low trade equilibrium, and it exists if and only if $\pi \leq \bar{\pi}_{LT}$*.*

Thus, the two equilibria coexist when $\pi \in [\bar{\pi}_{HT}, \bar{\pi}_{LT}]$*. When the two equilibria coexist, the asset prices and welfare are higher in the high trade than in the low trade equilibrium.*

We now show how to construct the two types of equilibria and the conditions for their existence.[13] We begin with the high trade equilibrium.

High trade. In such an equilibrium, all asset holders except for the $(H, 1)$-type trade at price p^* and their values are given by:

$$V(L, \chi) = V(L, 1) = V(H, \chi) = p^*, \tag{5}$$

whereas the value of the $(H, 1)$-type holder is:

$$V(H, 1) = x_H + \delta \left(\lambda p^* + (1 - \lambda)V(H, 1)\right), \tag{6}$$

i.e., this holder consumes the flow payoff this period, and in the next period she either becomes liquidity hit (w.p. λ) in which case she sells her asset, or she remains non-liquidity hit (w.p. $1 - \lambda$) and holds on to her asset. The equilibrium price is:

$$p^* = \hat{\pi} x_H + (1 - \hat{\pi})x_L + \delta \left((1 - \hat{\pi}(1 - \lambda))p^* + \hat{\pi}(1 - \lambda)V(H, 1)\right). \tag{7}$$

where $\hat{\pi} \equiv \frac{\lambda\pi}{\lambda\pi+1-\pi}$ is the probability that the asset is of high quality, conditional on being

[13] A formal proof of Theorem 1 is in the Appendix.

traded.[14] To see this, note that a buyer who gets the asset today expects to consume the flow payoff of the asset and that he will keep the asset tomorrow if *both* it is of high quality and he does not experience liquidity needs. Importantly, the buyer understands that due to adverse selection the asset is of high quality with probability $\hat{\pi}$ that is strictly smaller than π.

To show that the high trade equilibrium exists, we must rule out profitable deviations for the holders and the buyers. It is clear that there are no deviations for the buyers, since any such deviation would need to attract the $(H, 1)$-type, which is impossible without the buyers making losses in expectation. For the holders, it is sufficient to check that the (H, χ)-type gets a lower payoff if she were to hold on to the asset for one period rather than sell it at the equilibrium price:

$$\chi x_H + \delta \left(\lambda p^* + (1 - \lambda)V(H, 1)\right) \leq p^*. \tag{8}$$

Using the equilibrium conditions (5)-(7), we can re-express condition (8) as:

$$\left(\chi x_H - \hat{\pi} x_H - (1 - \hat{\pi})\, x_L\right) + \delta\, (1 - \hat{\pi}) \frac{(1 - \lambda)\,(1 - \hat{\pi})\,(x_H - x_L)}{1 - \delta\,(1 - \hat{\pi})\,(1 - \lambda)} \leq 0. \tag{9}$$

The threshold belief $\bar{\pi}_{HT}$ stated in Theorem 1 is the belief π about asset quality at which condition (9) holds with equality, and it is straightforward to show that this threshold is interior and unique. This result is intuitive; when the belief π about asset quality is high, buyers' do not need to worry as much about adverse selection and efficient trade is more likely.

Next, let us consider the low trade equilibrium.

Low trade. In such an equilibrium, only low quality asset holders trade and, thus, their values are given by:

$$V(L, \chi) = V(L, 1) = p^*, \tag{10}$$

whereas the values of the high quality asset holders are:

$$V(H, \omega) = \omega x_H + \delta \left(\lambda V(H, \chi) + (1 - \lambda)V(H, 1)\right) \text{ for } \omega \in \{\chi, 1\}, \tag{11}$$

since these holders both consume the flow payoffs of their assets today and expect to do so in the future. The equilibrium price is:

$$p^* = (1 - \delta)^{-1} x_L, \tag{12}$$

[14] In the stationary distribution of holder types, the probability that the holder is an (H, χ)-type is $\lambda\pi$ and the probability that she is a low type is $1 - \pi$.

since buyers understand that due to adverse selection only low quality assets trade.

To show that the low trade equilibrium exists, we are left to rule out profitable deviations for the holders and the buyers. It is clear that there are no deviations for asset holders, since the high types strictly prefer to hold on to their assets (recall that $\chi x_H \geq x_L$), whereas the low type holders prefer to trade. To rule out deviations for the buyers, it suffices to check that the buyers' profits are non-positive at any offer that attracts the (H, χ)-type:

$$\hat{\pi} x_H + (1-\hat{\pi}) x_L + \delta \left((1-\hat{\pi}) p^* + \hat{\pi} \lambda V(H, \chi) + \hat{\pi}(1-\lambda) V(H, 1) \right) \leq V(H, \chi), \qquad (13)$$

where as before $\hat{\pi} \equiv \frac{\lambda\pi}{\lambda\pi+1-\pi}$ is the probability that the asset is of high quality, conditional on being traded. Note that the expression on the left-hand side of condition (13) is the expected payoff to a buyer if he were able to attract the (H, χ) type, whereas the right-hand side is the lowest bid a buyern needs to make to attract that type.

Using the equilibrium conditions (10)-(12), we can re-express condition (13) as:

$$0 \leq \left(\chi x_H - \hat{\pi} x_H - (1-\hat{\pi}) x_L \right) + \delta (1-\hat{\pi}) \frac{(1-\lambda+\lambda\chi) x_H - x_L}{1-\delta}. \qquad (14)$$

The threshold belief $\bar{\pi}_{LT}$ stated in Theorem 1 is the belief π about asset quality at which condition (14) holds with equality, and it is straightforward to show that this threshold is interior and unique. This result is also intuitive; when the belief π about asset quality is low, buyers' are more worried about adverse selection and trade is more likely to be inefficient.

We have characterized the conditions for the existence of each type of equilibrium. We next consider the conditions for their coexistence.

Coexistence. It is convenient to index the equilibrium price and holder values by the type (i.e., high vs low trade) of the equilibrium: $\{(p^{*j}, V^j)\}_{j \in \{LT, HT\}}$. Using the equilibrium properties that we described above, we can re-express the conditions (8) and (13) for existence of each type of equilibrium as follows: high trade equilibrium exists if and only if:

$$\hat{\pi} x_H + (1-\hat{\pi}) x_L - \chi x_H - \delta(1-\hat{\pi}) E\{V^{HT}(H, \omega') - V^{HT}(L, \omega')\} \geq 0, \qquad (15)$$

whereas low trade equilibrium exists if and only if:

$$\hat{\pi} x_H + (1-\hat{\pi}) x_L - \chi x_H - \delta(1-\hat{\pi}) E\{V^{LT}(H, \omega') - V^{LT}(L, \omega')\} \leq 0. \qquad (16)$$

The first inequality states that the (H, χ)-type prefers to trade at the pooling price that prevails in the high trade equilibrium. The second inequality states that the buyers cannot attract the (H, χ)-type without making expected losses, if they expect equilibrium play to be low trade. Note that, though the two conditions rule out deviations by different agents, i.e., by buyers vs. by sellers, they are strikingly similar. The reason is that it is the incentives of the (H, χ)-type to accept bids (on- or off-equilibrium) that are essential to determine whether either type of equilibrium exists.

More importantly, however, although the expressions in (15) and (16) look similar, there is a critical difference; namely, the values of asset holders are endogenous to the equilibrium played. In particular, we show in the proof that:

$$E\{V^{HT}(H,\omega') - V^{HT}(L,\omega')\} < E\{V^{LT}(H,\omega') - V^{LT}(L,\omega')\},$$

which in turn implies that $\bar{\pi}_{LT} > \bar{\pi}_{HT}$ and the two equilibria can coexist.

The intuition for this result is as follows. Since a buyer today becomes a holder tomorrow, when deciding how aggressively to bid for the asset today, the buyer cares about the resale value of the asset in the future. If the buyers expect to be able to resell assets at higher prices in the future (as in the high trade equilibrium), they bid for assets more aggressively today and are able to attract the (H, χ)-type to trade, thus giving rise to a high trade equilibrium. Conversely, when resale prices are expected to be low (as in the low trade equilibrium), the buyers are less aggressive when bidding and cannot attract the (H, χ)-type, thus giving rise to a low trade equilibrium.[15]

3.3 Comparative statics

In this section, we conduct comparative statics on the equilibrium set, asset prices and welfare with respect some of the key parameters of the model. First, we ask under what conditions does each type of equilibrium (high vs. low trade) exist and when do the two equilibria coexist. To answer this question, we study how the threshold beliefs $\bar{\pi}_{HT}$ and $\bar{\pi}_{LT}$ change with the model parameters. Second, we ask how are asset prices and welfare affected by changes in the belief π about asset quality. This can give us a sense as to how the market can react to small changes in the fundamental π, when these are interacted with information frictions.

[15]But do not higher asset prices in the future also increase the outside option of the holders from holding on to the assets? Although this is indeed the case, it turns out that as long as there is some persistence in asset quality, the buyers' willingness to pay for the assets is always more sensitive to expected resale prices than the outside option of the holders (see Section 4).

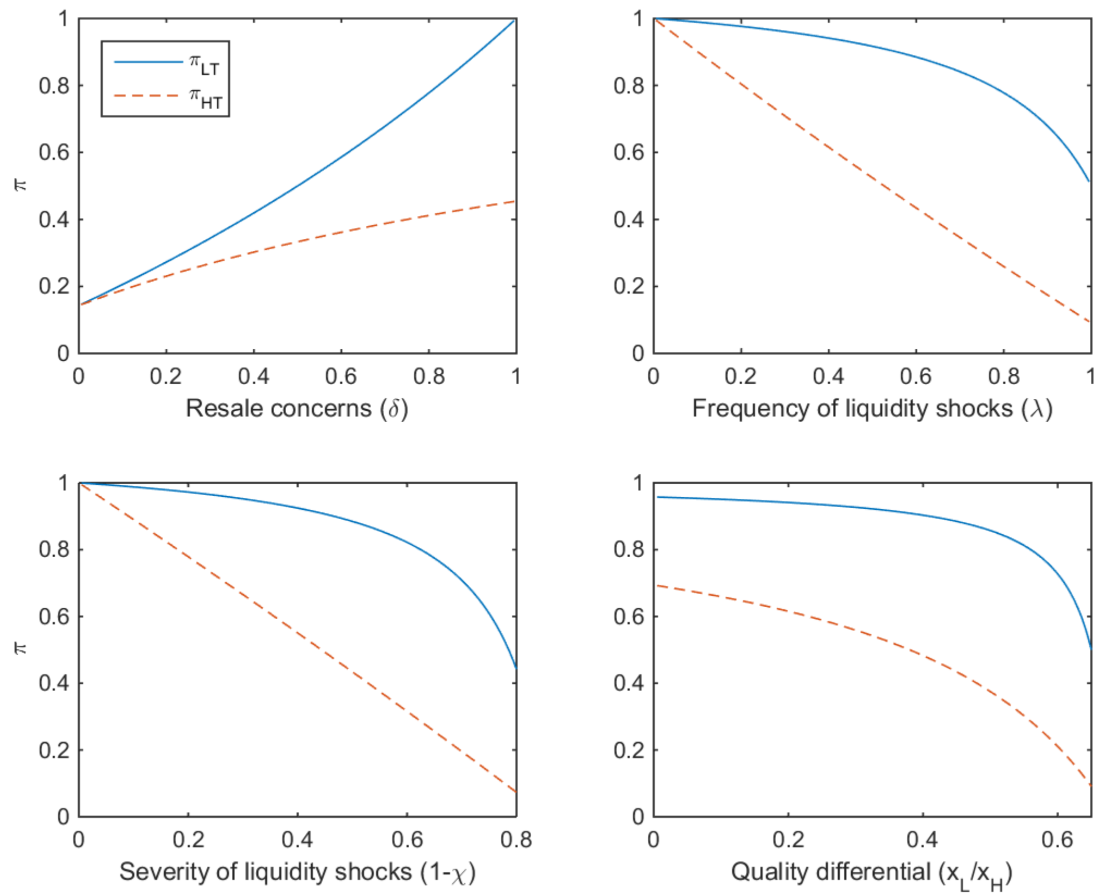

Figure 1: **Comparative Statics on Equilibrium Set**. Unless stated otherwise, the parameters used are: $\pi = 0.5$, $\delta = 0.9$, $\lambda = 0.6$, $\chi = 0.5$, $x_H = 1$ and $x_L = 0.45$.

The following proposition states our first result regarding comparative statics on the equilibrium set.

Proposition 3 (Equilibrium Set) *The belief thresholds $\bar{\pi}_{HT}$ and $\bar{\pi}_{LT}$ are increasing in δ, and they are decreasing in λ, $1-\chi$ and $\frac{x_L}{x_H}$.*

Moreover, the difference $\bar{\pi}_{LT} - \bar{\pi}_{HT}$ is increasing in δ, i.e., multiple equilibria are more likely to arise when resale considerations are more important.

Figure 1 illustrates this result graphically. In each panel, the blue solid line depicts the comparative statics on the threshold belief $\bar{\pi}_{LT}$, whereas the red dashed line depicts the comparative statics on the threshold belief $\bar{\pi}_{HT}$.

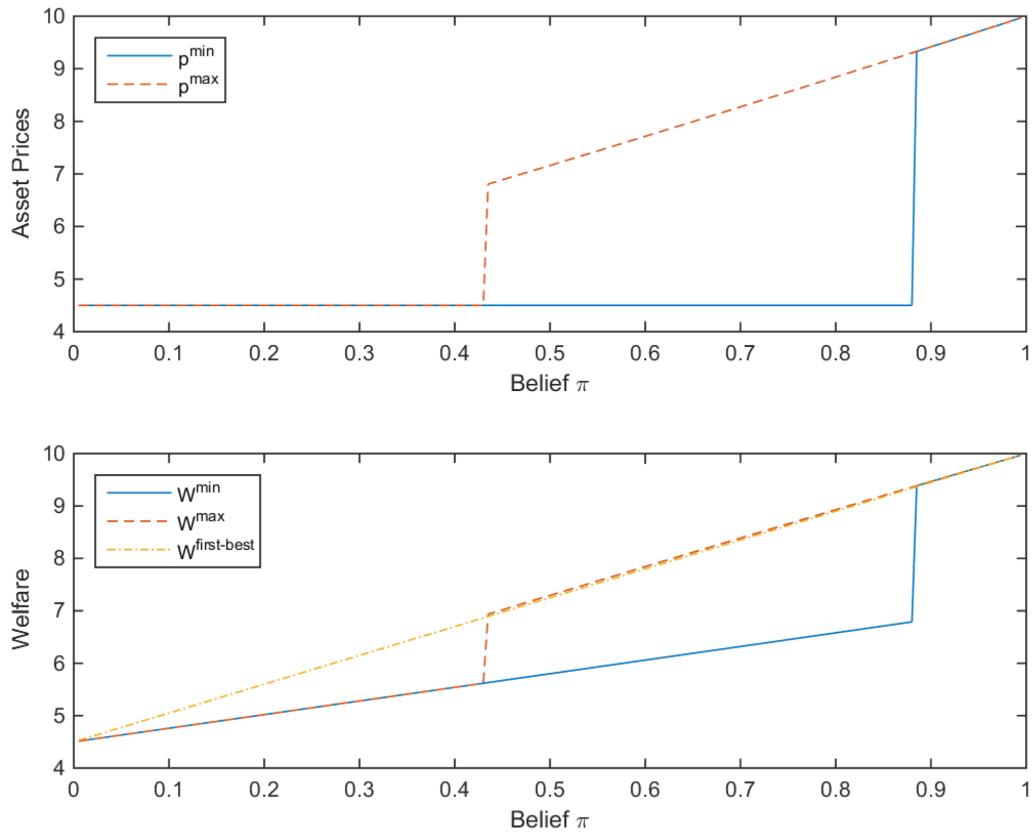

Figure 2: **Comparative Statics on Asset Prices and Welfare**. Unless stated otherwise, the parameters used are: $\delta = 0.9$, $\lambda = 0.6$, $\chi = 0.5$, $x_H = 1$ and $x_L = 0.45$.

Proposition 4 (Asset Prices and Welfare) *In the low trade equilibrium, the asset prices are independent of the belief π whereas welfare is increasing in π. In the high trade equilibrium, the asset prices and welfare are increasing in π.*

Figure 2 illustrates this result graphically. The top (bottom) panel depicts the maximal and minimal asset price (welfare) that is possible in equilibrium as a function of the belief π, for our baseline parameterization. Welfare is defined as the ex-ante expected value of the holders (recall that buyers break-even). We can see that when beliefs are sufficiently pessimistic, asset prices and welfare are low since the equilibrium features inefficient trade. Instead, when beliefs are sufficiently optimistic, asset prices and welfare are high since the equilibrium features high trade. It is for intermediate beliefs where multiple constant price equilibria exist, and where asset prices and welfare can either be high or low, and asset allocation can either be efficient or inefficient, depending on which equilibrium agents' coordinate upon.

In the next section, we show that there can also exist *sentiment equilibria*, in which asset prices and welfare fluctuate due to sunspots unrelated to fundamentals.

3.4 Sentiment equilibrium

The empirical asset pricing literature has pointed out that one of the features of data that are difficult to reconcile with standard asset pricing models is that there appears to be excess volatility in asset prices relative to the volatility of the cash-flows that assets generate or to the volatility in market interest rates. Given the possibility of multiple equilibria, our model has the potential to generate substantial volatility in asset prices even if there is no volatility in cash-flows nor market interest rates. To illustrate this point, we will next introduce a sunspot random variable, to which we will refer as market 'sentiment,' and we will allow traders to coordinate on its realizations.

Consider a sunspot-random variable z_t which takes values in some finite set Z and follows a Markov process with transition probability $P_Z(z'|z)$ for $z, z' \in Z$.[16] Assume that the realization of the random variable is common knowledge. Then,

Definition 3 *We say a PBE is a* **sentiment equilibrium** *with sunspot z_t if equilibrium asset prices and asset allocations depend on the realizations of the sunspot.*

Together with the transition probability P_Z for the sunspot, all sentiment equilibria are characterized by a equilibrium prices p^* and holder values V which satisfy:

$$V(\theta, \omega, z) = \max_{s \in \{0,1\}} s \cdot p^*(z) + (1-s) \cdot E\{\omega x_\theta + \delta V(\theta', \omega', z')|(\theta, \omega, z)\} \quad \forall(\theta, \omega, z), \tag{17}$$

$$\Gamma(p, z) = \{(\theta, \omega) : V(\theta, \omega, z) \leq p\} \quad \forall(p, z), \tag{18}$$

$$p^*(z) = \mathbb{E}\{x_\theta + \delta V(\theta', \omega', z')|(\theta, \omega) \in \Gamma(p^*(z), z), z\} \quad \forall z, \quad \text{and} \tag{19}$$

$$\hat{p} \geq \mathbb{E}\{x_\theta + \delta V(\theta', \omega', z')|(\theta, \omega) \in \Gamma(\hat{p}, z), z\} \quad \forall \hat{p} > p^*(z). \tag{20}$$

Note that these equations are simply the sunspot-contingent analogues of the equations (1), (2), (3), and (4) in Section 3.2. In fact, all constant price equilibria are solutions to the above system under the additional restriction that asset prices satisfy $p^*(z) = p^*(\hat{z})$ for all $z, \hat{z} \in Z$.

It is straightforward to show that our results in Proposition 2 extend to sunspot equilibria as well. In particular, in any such equilibrium, it must that the low quality assets trade at all times whereas the high quality assets held by holders without liquidity needs never do. Thus,

[16] It is straightforward to extend the analysis to more general stationary sunspot processes.

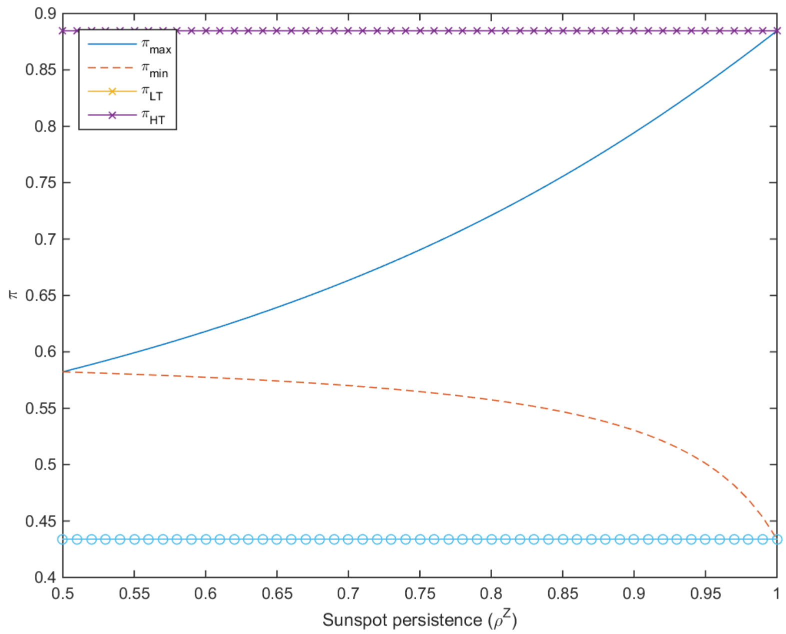

Figure 3: **Sentiment Persistence and Equilibrium Existence Set**. The top and bottom horizontal lines are the threshold beliefs $\bar{\pi}_{LT}$ and $\bar{\pi}_{HT}$ from Theorem 1. The region between the solid blue line and the dashed red line is the set of beliefs for which a sentiment equilibrium exists. The parameters used are: $\delta = 0.9$, $\lambda = 0.6$, $\chi = 0.5$, $x_H = 1$, $x_L = 0.1$, and $\gamma^Z = 0.5$.

the only role of the sunspot is to shift equilibrium play from one in which the (H, χ)-type holder trades to one where this holder type does not trade. Given these observations, we can without loss of generality restrict our attention to sunspot processes that take two values. In particular, we suppose that $Z \equiv \{B, G\}$ and suppose that at time t, if possible, the market coordinates on inefficient trade (i.e., (H, χ)-type does not trade) when $z_t = B$; otherwise, it coodinates on efficient trade (i.e., (H, χ)-type trades). Let $\gamma^Z \equiv P(z_t = G) \in (0, 1)$ and $\rho^Z \equiv P(z_{t+1} = G | z_t = G)$.

The following proposition shows that sunspot-driven equilibria can indeed exist and characterizes some of their key properties.

Proposition 5 (Sentiments) *Fix $\gamma^Z \in (0, 1)$ and suppose that the belief about asset quality satisfies $\pi \in (\bar{\pi}_{HT}, \bar{\pi}_{LT})$, where $\bar{\pi}_j$'s are as in Theorem 1. Then, a sentiment equilibrium with sunspot z_t exists if and only if ρ^Z is sufficiently large. Moreover, the difference in asset prices and welfare across the two states is increasing continuously in ρ^Z.*

These results further emphasize that the coordination failure that gives rise to multiple equilibria in our environment is of inter-temporal nature. The realization of the sunspot

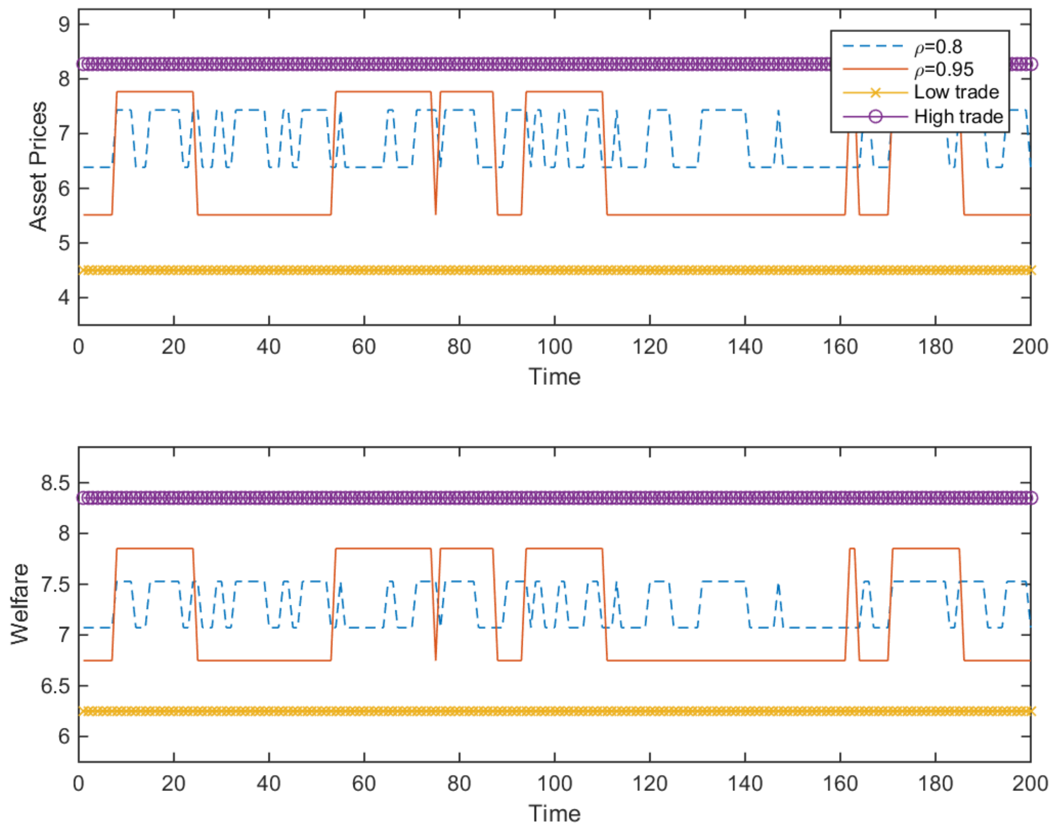

Figure 4: **Asset Prices and Welfare in a Sentiment Equilibrium**. The parameters used are: $\pi = 0.7$, $\delta = 0.9$, $\lambda = 0.6$, $\chi = 0.5$, $x_H = 1$, $x_L = 0.1$, and $\gamma^Z = 0.5$. The solid line depicts a simulation with $\rho^Z = 0.95$, whereas the dashed line depicts a simulation with $\rho^Z = 0.8$.

today must not only 'tell' the agents what equilibrium to play today, but it must also signal to them how the equilibrium play will proceed in the future. These two goals are accomplished precisely by a sufficiently persistent sunspot process.

Figure 3 illustrates this result graphically, by depicting the set of beliefs about asset quality for which sentiment equilibria are possible as a function of the persistence parameter ρ^Z. The upper and the lower bound on the beliefs are drawn as a solid blue and a dashed red line respectively. We can see the set of beliefs for which sentiment equilibria can arise expands with the persistence of the process. Furthermore, note that as the sunspot becomes arbitrarily persistent, the bounds converge continuously to the threshold beliefs $\bar{\pi}_{LT}$ and $\bar{\pi}_{HT}$, which define the existence regions for the high and the low trade constant price equilibria.

In Figure 4, we illustrate the evolution of the asset prices and welfare in the economy for a particular simulation of the sunspot process. We keep all model parameters the same as in Figure 3, and we set the belief about asset quality to $\pi = 0.7$, so that indeed a sentiment equilibrium exists if the sunpsot is sufficiently persistent. The simulation with a solid line depicts the case where the sunspot is very persistent ($\rho^Z = 0.95$), whereas the one with a dashed line depics a less persistent process ($\rho^Z = 0.8$). As stated in Propositon 5, the less persistent process implies more frequent fluctuations, but of smaller size. Importantly, since different states correspond to different asset allocation, we see that the fluctuations in asset prices are mirrored by fluctuations in the expected welfare of the traders. Finally, the figure also depics the asset prices and welfare in the high trade and low trade constant price equilibria. These equilibria provide the bounds on the prices and welfare that can be attained in sentiment equilibrium (as $\rho^Z \to 1$).

4 Extensions

In this section, we consider some extensions of our baseline model. In Section 4.1, we consider shocks to fundamentals. In Section 4.2, we extend our analysis to persistent liquidity needs. In Section 4.3, we consider transitory asset quality and asset duration.

4.1 Fundamental shocks

As discussed in Manuelli and Peck (1992), "the early sunspot literature was motivated by the idea that small shocks to fundamentals are not very different from sunspots." They show that, in an overlapping generations endowment economy with money, small shocks to fundamentals can serve as the coordination device for different monetary equilibria. Furthermore, in the limit, as the undelying shocks have no direct effect on endowments, for every equilibrium of the pure sunspot economy with no shocks to endowments, there is a sequence of equilibria of the economy with risky endowments that converges to it. Our baseline economy can also be extended to allow for aggregate shocks which, even when small, can have large effects by serving as a coordination device for agents' expectations regarding the future market conditions.

To illustrate this point, suppose that the output of the Lucas trees is also a function of some aggregate state $z_t \in \{G, B\}$, which follows a persistent and observable two state markov process. Concretely, consider the case where in state $z_t = G$ the payoff or output of a tree of quality θ in the hands of a holder with liquidity or productivity status ω is $(1 + \varepsilon) \cdot \omega \cdot \theta$, whereas in state $z_t = B$ the output is $(1 - \varepsilon) \cdot \omega \cdot \theta$, for some $\varepsilon \in [0, 1)$. Note that when $\varepsilon = 0$, we are back to our baseline setup, where the aggregate state is a pure sunspot and has no direct impact on any given tree's output, but can still serve as a coordination device. It is therefore straightforward to see that, for ε small but positive, we have the potential for the amplification of fundamental shocks. The equilibrium features amplification in the sense that the shocks have a negligible effect on the cashflow of any given tree. But, as shown in Section 3.4, these news can change market expectations about future, change the pool of assets that are traded in equilibrium, and thus have a large impact on equilibrium asset prices, market liquidity and welfare.

If we think of agent specific shock ω as the productivity with which that agent can employ the tree, then the aggregate TFP in the economy at time t would be:

$$TFP_t = \begin{cases} (1+\varepsilon)\cdot(\pi \cdot x_H + (1-\pi)\cdot x_L) & \text{if} \quad z_t = G \\ (1-\varepsilon)\cdot(\pi\,(1-\lambda+\lambda\chi)\cdot x_H + (1-\pi)\cdot x_L) & \text{if} \quad z_t = B \end{cases}$$

Since $\chi < 1$, the measured TFP can display large fluctuations even if the extrinsic shock to productivity is small, i.e., $\varepsilon \approx 0$.

4.2 Gains from trade and history dependence

In this section, we extend our baseline setup to the case where an agent's liquidity status is non-iid over time. As before, we assume that the unconditional probability of holder j being shocked at time t is $\mathbb{P}(\omega_{j,t} = \chi) = \lambda$; but we now assume that $\mathbb{P}(\omega_{j,t+1} = \chi|\omega_{j,t} = \chi) = \rho^{\omega}$, and we allow correlation to be either positive ($\rho^{\omega} > \lambda$) or negative ($\rho^{\omega} < \lambda$). Our baseline setup corresponds to the special case with $\rho^{\omega} = \lambda$.

As in Section 3.2, it is straightforward to show that equilibrium play must feature one of the following two possibilities. In a given period t, either only low type holders trade or all holders except the $(H, 1)$-types trade. In contrast to Section 3.2, however, the distribution of types may be endogenous to equilibrium play. To see this, let $s_t \in \{0, 1\}$ denote the indicator that equals 1 if and only if the (H, χ)-type trades at t. Also, let $\mathbb{P}_t(\theta, \omega)$ denote the fraction of asset holders who are of type (θ, ω) at the beginning of period t. Since low quality assts are always on the market, the fraction $\mathbb{P}_t(H, \chi)$ is crucial in determining equilibrium play at any time t, and it can be shown to evolve as follows:

$$\mathbb{P}_t(H,\chi) = \pi \cdot \lambda \cdot \frac{1-\rho^{\omega}}{1-\lambda} + \left(\rho^{\omega} - \lambda \cdot \frac{1-\rho^{\omega}}{1-\lambda}\right) \cdot (1-s_{t-1}) \cdot \mathbb{P}_{t-1}(H,\chi). \qquad (21)$$

If $\rho^{\omega} > \lambda$, then past trade worsens the mix of assets on the market, i.e., $\mathbb{P}_t(H, \chi) < \pi \cdot \lambda$. If

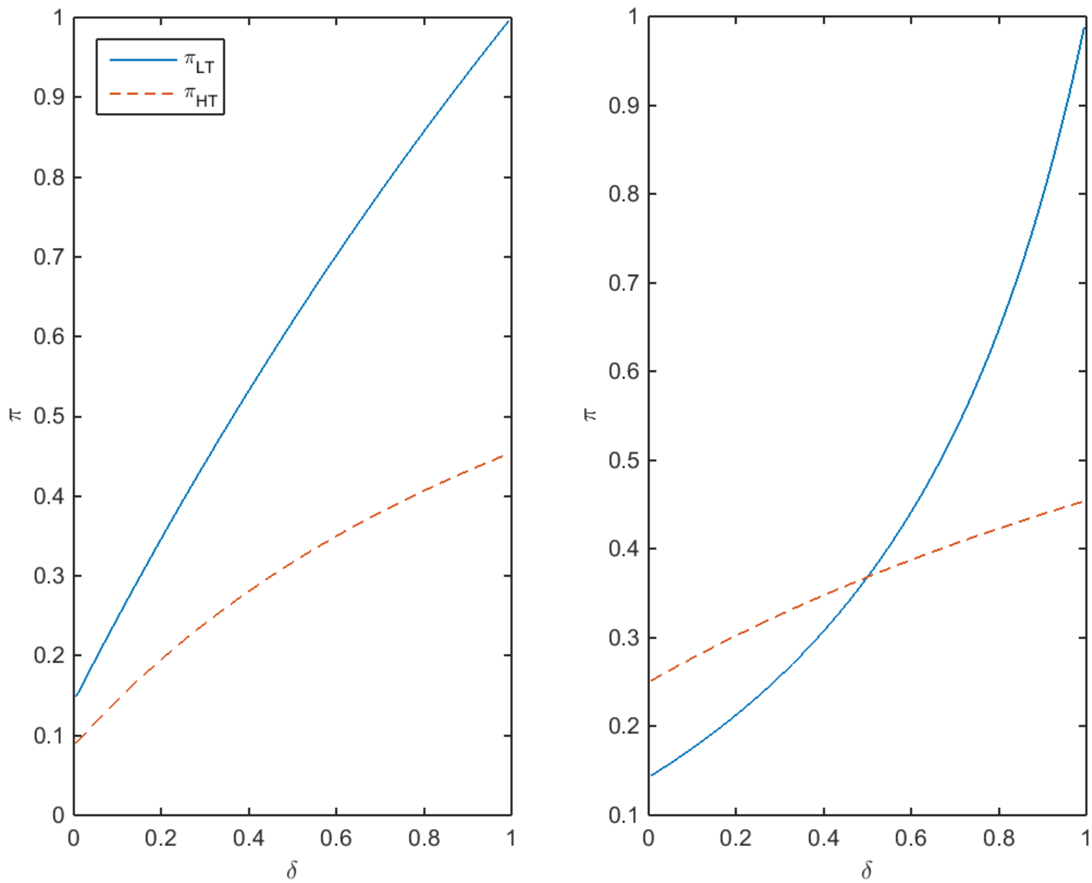

Figure 5: **Comparative Statics on Equilibrium Set and Correlation of Liquidity Shocks**. Unless stated otherwise, the parameters used are: $\pi = 0.5$, $\delta = 0.9$, $\lambda = 0.6$, $\chi = 0.5$, $x_H = 1$ and $x_L = 0.45$. The left panel depicts the case of negatively correlated liquidity shocks, with $\rho = 0.7$. The right panel depicts the case of positively correlated liquidity shocks, with $\rho = 0.3$.

$\rho^\omega > \lambda$, then past trade improves the mix of assets on the market, i.e., $\mathbb{P}_t\left(H,\chi\right) > \pi\cdot\lambda$. Finally, note that when $\rho^\omega = \lambda$, as in our baseline setup, the distribution of types is independent of trading history, i.e., $\mathbb{P}_t\left(H,\chi\right) = \pi\cdot\lambda$ for all t.

We next construct constant price equilibria, in which the asset prices and distribution of holder types is the same in every period. From (21), the stationary fraction of $\mathbb{P}_\infty(H,\chi)$-types in any such equilibrium is given by:

$$\mathbb{P}_\infty\left(H,\chi\right) = \pi\cdot\lambda\cdot\frac{1-\rho^\omega}{1-\lambda}\cdot\frac{1}{1-(1-s_\infty)\cdot\left(\rho^\omega - \frac{(1-\rho^\omega)\cdot\lambda}{1-\lambda}\right)}, \tag{22}$$

where s_∞ is the indicator that the (H, χ)-type trades in a given constant price equilibrium.

Figure (5) plots the analogues of thresholds $\bar{\pi}_{HT}$ and $\bar{\pi}_{LT}$ in Theorem 1 for different values of the discount factor δ, where the high trade equilibrium exists when $\pi \geq \bar{\pi}_{HT}$ and the low trade equilibrium exists when $\pi \leq \bar{\pi}_{LT}$. The left panel depicts the case where liquidity shocks are correlated negatively over time ($\rho^\omega < \lambda$). The key difference from our baseline model (as illustrated in top left panel of Figure 1) is that now multiple constant price equilibria are possible even in the absence of re-sale considerations, i.e., even as $\delta \to 0$. To see this point simply, note that when $\delta \to 0$, dynamic considerations become unimportant and agents care only about the flow payoff of the asset. Therefore, the condition for the existence of the high trade equilibrium reduces to:

$$\chi \cdot x_H \leq p^* = \frac{\pi \cdot \lambda \cdot \frac{1-\rho^\omega}{1-\lambda}}{\pi \cdot \lambda \cdot \frac{1-\rho^\omega}{1-\lambda} + 1 - \pi} \cdot x_H + \left(1 - \frac{\pi \cdot \lambda \cdot \frac{1-\rho^\omega}{1-\lambda}}{\pi \cdot \lambda \cdot \frac{1-\rho^\omega}{1-\lambda} + 1 - \pi}\right) \cdot x_L, \tag{23}$$

i.e., it must be that the flow payoff to the (H, χ)-type from holding the asset is lower than the equilibrium price if all types except the $(H, 1)$-type were to trade. On the other hand, the low trade equilibrium exists if:

$$\chi \cdot x_H > \hat{p} = \frac{\pi \cdot \lambda}{\pi \cdot \lambda + 1 - \pi} \cdot x_H + \left(1 - \frac{\pi \cdot \lambda}{\pi \cdot \lambda + 1 - \pi}\right) \cdot x_L, \tag{24}$$

i.e., it must be that the flow payoff to the (H, χ)-type from holding the asset exceeds the buyers' value for the asset if they could attract all types other than $(H, 1)$. By inspection, we see that $\rho^\omega < \lambda$ implies that $p^* > \hat{p}$ and, thus, multiple constant price equilibria can exist.

In contrast, the right panel depicts the case where liquidity shocks are correlated positively over time ($\rho^\omega > \lambda$). We can see that now, as re-sale considerations become unimportant, constant price equilibria may even cease to exist. To see this point simply, note that from equations (23) and (25), when $\rho^\omega > \lambda$, then $p^* < \hat{p}$ and thus it is possible that neither high trade nor low trade equilibrium exist. Nevertheless, we can construct stationary equilibria which feature deterministic cycles of the following form: all gains from trade are realized every T periods and in all remaining periods only low quality assets trade in the market. The reason is that right after the (H, χ)-type trades, the fraction of agents who have high quality assets and liquidity shocks in next period is small when ρ^ω is large. But as time passes, the mass of these types grows and eventually pooling trade can again be sustained. Going back to our example with $\delta \to 0$, the period T can explicitly be computed to be the minimal time τ at which:

$$\chi \cdot x_H \leq \hat{p}_\tau = \hat{\pi}_\tau \cdot x_H + (1 - \hat{\pi}_\tau) \cdot x_L, i.e., \tag{25}$$

where $\pi \cdot \lambda \cdot \frac{1-\rho^\omega}{1-\lambda} \cdot \frac{1-(\rho^\omega - \lambda \cdot \frac{1-\rho^\omega}{1-\lambda})^{\tau+1}}{1-(\rho^\omega - \lambda \cdot \frac{1-\rho^\omega}{1-\lambda})}$ denotes the fraction of (H, χ)-types after τ periods in which only low quality asset trade. Thus, $\hat{p}_\tau$ is the buyers' valuation of the asset if they were able to attract all types except the $(H, 1)$-type. That such an equilibrium exists follows from the fact that (i) $\hat{\pi}_\tau$ is monotonically increasing in τ, (ii) $\hat{\pi}_0 = p^*$ given in equation (23), and (iii) $\lim_{\tau \to \infty} \hat{\pi}_\tau = \hat{p}$ given in equation (25). The possibility of such deterministic cycles in markets with adverse selection has recently been pointed out by Maurin (2016). Though our setting are substantially different, the forces that give rise to such equilibria are closely related.

4.3 Transitory quality and duration

In this section, we extend our baseline setup to the cases in which asset quality is transitory and asset duration is less than infinite.

First, we denote the quality of asset i at time t by $\theta_{i,t}$. As in our baseline setup, we assume that the unconditional probability of an asset being high quality is $\mathbb{P}(\theta_{i,t} = H) = \pi$; but we now assume that $\mathbb{P}(\theta_{i,t+1} = H | \theta_{i,t} = H) = \rho^\theta$. The shocks to asset quality are assumed to be independent across assets, so the fraction of good quality assets at any point in time is also given by π. Second, we assume that at any point in time an asset matures and no longer delivers cash-flows with probability $1 - \xi \in [0, 1]$. Whether an asset matures is realized at the beginning of the period. When $\xi < 1$, the duration of any given asset is given by $(1 - \xi)^{-1}$; otherwise, the duration is infinite. The maturity dates are assumed to be independent across assets, so the fraction of assets maturing at each date is also $1 - \xi$.

The following proposition states that multiple constant price equilibria and stochastic sentiment equilibria, as given in Theorem 1 and Proposition 5, can arise as long as assets are not short-lived and there is some persistence in asset quality.

Proposition 6 *If $\rho^\theta > \pi$ and $\xi > 0$, then there exist thresholds $0 < \tilde{\pi}_{HT} < \tilde{\pi}_{LT} < 1$ on the belief about asset quality, such that the high and the low trade constant price equilibria coexist if and only if $\pi \in [\tilde{\pi}_{HT}, \tilde{\pi}_{LT}]$. Furthermore, for any $\pi \in (\tilde{\pi}_{HT}, \tilde{\pi}_{LT})$, a sentiment equilibrium with sunspot z_t as in Proposition 5 exists if and only if it is sufficiently persistent.*

5 Conclusions

We study a dynamic market for durable assets in which asset owners are privately informed about the quality of their assets and may experience occasional liquidity or productivity shocks that depress their asset valuations vis a vis potential buyers. We find that the interaction of adverse selection with resale concerns on behalf of the buyers gives rise to multiple self-fulfilling equilibria that are ranked in terms of asset prices, market liquidity and welfare. Although asset prices are always equal to fundamentals, they can display large fluctuations due to "sentiments" – sunspots that are unrelated to fundamentals,– resembling what one may refer to as "bubbles."

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Appendix A - Proofs for Section 3

Proof of Proposition 2. For the relative ranking of holder values, note that the equilibrium price satisfies:

$$p^* = E\{x_\theta + \delta V(\theta', \omega') | \theta \in \Theta(p^*)\} \geq E\{x_\theta + \delta V(\theta', \omega') | \theta = L\} \tag{26}$$

where the right-hand side is equal to the value of the $(L, 1)$-type if in equilibrium she holds on to the asset; otherwise, $V(L, 1) = p^*$. Thus, in equilibrium the $(L, 1)$ type always trades and $V(L, 1) = p^*$. On the other hand (L, χ)-type has a weakly lower value than the $(L, 1)$ type since the quality of her asset is the same, but her flow payoff is lower. Hence, in equilibrium she also trades and $V(L, \chi) = p^*$. Finally, $V(H, \chi) \geq p^*$ holds trivially since the holder always has the option to trade at price p, and $V(H, 1) > p^*$ follows from the fact that all low types trade and therefore:

$$p^* = E\{x_\theta + \delta V(\theta', \omega') | \theta \in \Theta(p^*)\} < E\{x_\theta + \delta V(\theta', \omega') | \theta = H\} = V(H, 1), \tag{27}$$

which implies that buyers cannot attract the $(H, 1)$-type to trade without making losses in expectation. ■

Proof of Theorem 1. That there can at most be two types of constant price equilibria follows from Proposition 2, which states that which equilibrium arises depends on whether the (H, χ) type trades or not.

High trade. The equations (5), (6), and (7) characterize the equilibrium holder values V^{HT} and asset price $p^{*,HT}$ in candidate high trade equilibria. Since this is a system of linear equations, if a high trade equilibrium exists, it is unique. Moreover, this equilibrium exists if and only if inequality (8) is satisfied. Combining (5) - (8), we have that the high trade equilibrium exists if and only if:

$$(\chi x_H - \hat{\pi} x_H - (1 - \hat{\pi}) x_L) + \delta (1 - \hat{\pi}) \frac{(1 - \lambda)(1 - \hat{\pi})(x_H - x_L)}{1 - \delta (1 - \hat{\pi})(1 - \lambda)} \leq 0, \tag{28}$$

where $\hat{\pi} \equiv \frac{\lambda \pi}{\lambda \pi + 1 - \pi}$. Note that the left-hand side is strictly decreasing in π, positive at $\pi = 0$ and negative at $\pi = 1$. Hence, the threshold $\bar{\pi}_{HT} \in (0, 1)$ exists, is unique, and the high trade equilibrium exists if and only if $\pi \geq \bar{\pi}_{HT}$.

Low trade. The equations (10), (11), and (12) characterize the equilibrium holder values V^{LT} and asset price $p^{*,LT}$ in candidate low trade equilibria. Since this is a system of linear equations, if a low trade equilibrium exists, it is unique. Moreover, this equilibrium exists if and only if inequality (13) is satisfied. Combining (10) - (13), we have that the low trade equilibrium exists if and only if:

$$0 \leq (\chi x_H - \hat{\pi} x_H - (1-\hat{\pi}) x_L) + \delta (1-\hat{\pi}) \frac{(1-\lambda+\lambda\chi) x_H - x_L}{1-\delta}, \tag{29}$$

where $\hat{\pi} \equiv \frac{\lambda\pi}{\lambda\pi+1-\pi}$. Note that the right-hand side is strictly decreasing in the belief π, positive when $\pi = 0$ and negative when $\pi = 1$. Hence, the threshold $\bar{\pi}_{LT} \in (0, 1)$ exists, is unique, and the low trade equilibrium exists if and only if $\pi \leq \bar{\pi}_{LT}$.

Existence and Multiplicity. Next, we show that $\bar{\pi}_{HT} < \bar{\pi}_{LT}$, which will establish that an equilibrium exists and that the two equilibria coexist whenever $\pi \in [\bar{\pi}_{HT}, \bar{\pi}_{LT}]$. From (28) and (29), we have that $\bar{\pi}_{HT} < \bar{\pi}_{LT}$ if and only if:

$$\frac{(1-\lambda)(1-\hat{\pi})(x_H - x_L)}{1-\delta(1-\hat{\pi})(1-\lambda)}|_{\pi=\bar{\pi}_{HT}} < \frac{(1-\lambda+\lambda\chi) x_H - x_L}{1-\delta}. \tag{30}$$

But the latter inequality must hold because for any $\pi < 1$, we have:

$$\begin{aligned} \frac{(1-\lambda)(1-\hat{\pi})(x_H - x_L)}{1-\delta(1-\hat{\pi})(1-\lambda)} &\leq \frac{(1-\lambda)(x_H - x_L)}{1-\delta(1-\lambda)} \\ &< \frac{(1-\lambda)(x_H - x_L) + \lambda(\chi x_H - x_L)}{1-\delta} \\ &= \frac{(1-\lambda+\lambda\chi) x_H - x_L}{1-\delta}, \end{aligned}$$

where we used the fact that $\chi x_H \geq x_L$.

Finally, it is clear that asset prices and welfare are strictly higher in the high trade equilibrium than in the low trade equilibrium. ■

Proof of Proposition 3. Consider first the expression defining the theshold $\bar{\pi}_{LT}$:

$$(\chi x_H - \hat{\pi} x_H - (1-\hat{\pi}) x_L) + \delta (1-\hat{\pi}) \frac{(1-\lambda+\lambda\chi) x_H - x_L}{1-\delta}|_{\pi=\bar{\pi}_{LT}} = 0, \tag{31}$$

where the left-hand side is decreasing in π. The comparative statics results then follow by noting that the left-hand side is also increasing in δ and χ, and it is decreasing in λ and $\frac{x_L}{x_H}$.

Next, consider the expression defining the threshold $\bar{\pi}_{HT}$:

$$(\chi x_H - \hat{\pi} x_H - (1-\hat{\pi}) x_L) + \delta (1-\hat{\pi}) \frac{(1-\lambda)(1-\hat{\pi})(x_H - x_L)}{1-\delta(1-\hat{\pi})(1-\lambda)}|_{\pi=\bar{\pi}_{HT}} = 0, \tag{32}$$

where the left-hand side is decreasing in π. The comparative statics results then follow by noting that the left-hand side is increasing in δ and χ, and it is decreasing in λ and $\frac{x_H}{x_L}$.

Finally, because $\frac{(1-\lambda)(1-\hat{\pi})(x_H-x_L)}{1-\delta(1-\hat{\pi})(1-\lambda)}|_{\pi=\bar{\pi}_{HT}} < \frac{(1-\lambda+\lambda\chi)x_H-x_L}{1-\delta}$ (see proof of Theorem 1) and because the term $\frac{(1-\lambda)(1-\hat{\pi})(x_H-x_L)}{1-\delta(1-\hat{\pi})(1-\lambda)}$ is decreasing in $\hat{\pi}$, it follows that $\bar{\pi}_{LT}$ increases faster in δ than $\bar{\pi}_{HT}$. Thus, the difference $\bar{\pi}_{LT} - \bar{\pi}_{HT}$ is increasing in δ. ■

Proof of Proposition 4. Fix γ^Z and suppose $\pi \in (\bar{\pi}_{HT}, \bar{\pi}_{LT})$. The system of equations that characterizes sentiment equilibrium with sunspot z_t is given by:

$$V(H,1,z) = x_H + \delta E\{V(H,\omega',z')|z\} \tag{33}$$

for $z \in \{B, G\}$,

$$V(H,\chi,z) = \begin{cases} p(G) & \text{if } z = G \\ \chi \cdot x_H + \delta E\{V(H,\omega',z')|z = B\} & \text{if } z = B \end{cases} \tag{34}$$

and $V(L,\omega,z) = p^*(z)$, where equilibrium asset prices are given by:

$$p^*(z) = \begin{cases} \hat{\pi} \cdot V(H,1,G) + (1-\hat{\pi}) \cdot (x_L + \delta \cdot E\{p^*(z')|z = G\}) & \text{if } z = G \\ x_L + \delta \cdot E\{p^*(z')|z = B\} & \text{if } z = B \end{cases} \tag{35}$$

Moreover, there are no profitable deviations for the buyers and for the asset holders if and only if the equilibrium prices and values satisfy:

$$\hat{\pi} \cdot V(H,1,B) + (1-\hat{\pi}) \cdot (x_L + \delta \cdot E\{p^*(z')|z = B\}) \leq V(H,\chi,B), \tag{36}$$

and

$$\chi \cdot x_H + \delta E\{V(H,\omega',z')|z = G\} \leq p^*(G). \tag{37}$$

For $z \in \{B, G\}$ and $\omega \in \{\chi, 1\}$, define $\alpha_{z,\omega} \equiv V(H,\omega,z) - p^*(z)$. Then, from equations (33)-(35), we have that:

$$\alpha_{G,1} = (1-\hat{\pi}) \cdot \left(x_H - x_L + \delta \cdot \left(\rho^Z \cdot (1-\lambda) \cdot \alpha_{G,1} + \left(1-\rho^Z\right) \cdot ((1-\lambda) \cdot \alpha_{B,1} + \lambda \cdot \alpha_{B,\chi})\right)\right) \tag{38}$$

$$\alpha_{B,1} = (1-\hat{\pi}) \cdot \left(x_H - x_L + \delta \cdot \left(\gamma^Z \cdot \frac{1-\rho^Z}{1-\gamma^Z} \cdot (1-\lambda) \cdot \alpha_{G,1} + \left(1 - \gamma^Z \cdot \frac{1-\rho^Z}{1-\gamma^Z}\right) \cdot ((1-\lambda) \cdot \alpha_{B,1} + \lambda \cdot \alpha_{B,\chi})\right)\right) \tag{39}$$

$$\alpha_{B,\chi} = \chi \cdot x_H - x_L + \delta \cdot \left(\gamma^Z \cdot \frac{1-\rho^Z}{1-\gamma^Z} \cdot (1-\lambda) \cdot \alpha_{G,1} + \left(1 - \gamma^Z \cdot \frac{1-\rho^Z}{1-\gamma^Z}\right) \cdot ((1-\lambda) \cdot \alpha_{B,1} + \lambda \cdot \alpha_{B,\chi}) \right) \tag{40}$$

This is linear system of equations in three unknowns that can be used to explicitly compute any candidate sentiment equilibrium. Furthermore, note that the candidate equilibrium values for $\alpha_{z,\omega}$'s are continuous in ρ^Z. Next, the no deviations conditions are can be expressed as:

$$\gamma^Z \cdot \frac{1-\rho^Z}{1-\gamma^Z} \cdot (1-\lambda) \cdot \alpha_{G,1} + \left(1 - \gamma^Z \cdot \frac{1-\rho^Z}{1-\gamma^Z}\right) \cdot ((1-\lambda) \cdot \alpha_{B,1} + \lambda \cdot \alpha_{B,\chi}) \geq \frac{\hat{\pi} x_H + (1-\hat{\pi}) x_L - \chi \cdot x_H}{\delta \cdot (1-\hat{\pi})} \tag{41}$$

$$\rho^Z \cdot (1-\lambda) \cdot \alpha_{G,1} + \left(1-\rho^Z\right) \cdot ((1-\lambda) \cdot \alpha_{B,1} + \lambda \cdot \alpha_{B,\chi}) \leq \frac{\hat{\pi} x_H + (1-\hat{\pi}) x_L - \chi \cdot x_H}{\delta \cdot (1-\hat{\pi})}. \tag{42}$$

Thus, if the $\alpha_{z,\omega}$'s given by (38)-(40) satisfy the inequalities (41) and (42), the candidate sentiment equilibrium is indeed an equilibrium. We now argue that this is the case if and only if ρ^Z is sufficiently large.

First, note that as $\rho^Z \to 1$, we have: $\alpha_{G,1} \to V^{HT}(H,1) - p^{HT}$, $\alpha_{B,1} \to V^{LT}(H,1) - p^{LT}$ and $\alpha_{B,\chi} \to V^{LT}(H,\chi) - p^{LT}$, i.e., the values and equilibrium prices converge to their constant price equilibrium counterparts. In turn, the conditions for the existence of sentiment equilibrium converge to the conditions for the coexistence of a high trade and a low trade equilibrium. By continuity and because $\pi \in (\bar{\pi}_{HT}, \bar{\pi}_{LT})$, a sentiment equilibrium exists if ρ^Z sufficiently large.

Second, note that as $\rho^Z \to \gamma^Z$, the left-hand sides of (41) and (42) become equal and, thus, generically these two inequalities cannot hold at the same time. By continuity, generically, a sentiment equilibrium will not exist if ρ^Z is sufficiently small. ■