

# Sentiment, Liquidity and Asset Prices

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## Abstract

We study a dynamic market for durable assets, in which asset owners are privately informed about the quality of their assets and experience occasional productivity shocks that generate gains from trade. An important feature of our environment is that asset buyers must worry not only about the quality of assets they are buying, but also about the prices at which they can re-sell the assets in the future. We show that this interaction between adverse selection and resale concerns generates an inter-temporal coordination problem and gives rise to multiple self-fulfilling equilibria. We find that there is a rich set of sentiment driven equilibria, in which sunspots generate large fluctuations in asset prices, output and welfare, resembling what one may refer to as “bubbles.”

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# 1 Introduction

In a frictionless market, all gains from trade are realized, and durable assets or securities always end up being held by the parties that value them the most. As a result, asset prices reflect not only the current but also all expected future gains from trade. In contrast, in the presence of frictions, some gains from trade may remain unrealized and, thus, asset prices may be depressed. In such an environment, there is a close connection between *liquidity* – the ease with which assets are re-allocated, – and asset prices. Moreover, as we show in this paper, if the frictions result from *information asymmetries*, there can be multiple self-fulfilling equilibria. Even though all agents are fully rational and asset prices reflect fundamentals, the mix of assets that is traded can depend on *sentiments* – the agents’ expectations about future market conditions. We show that there is a rich set of sentiment driven equilibria, in which sunspots generate large fluctuations in asset prices, output and welfare, resembling what one may be tempted to refer to as “bubbles.”

We consider a dynamic market for assets (Lucas trees), in which asset owners are privately informed about the quality of their assets (the fruit to be harvested). Gains from trade arise stochastically over time because the current asset owners may experience “productivity” or “liquidity” shocks that depress their value of holding or employing assets relative to other agents in the economy. Buyers compete for assets, but they may face a lemons problem as in Akerlof (1970), since they do not observe the quality of the owners’ assets nor the motive for their sale. The buyers who purchase assets in any given period become asset owners in the next period. The important feature of our environment is that the buyers must worry not only about the quality of the assets for which they currently bid, but also about market prices were they to resell the assets in the future.

If information is symmetric, all asset owners with (productivity) shocks immediately sell their assets and, in the unique equilibrium, asset prices are equal the expected discounted value of asset cash-flows at their most efficient allocation (Proposition 1). Furthermore, this economy features no aggregate fluctuations in asset prices, output or welfare.

Instead, when information is asymmetric, the owners of low quality assets want to mimick the owners of high quality assets, and their presence in the market depresses the buyers’ willingness to pay. Absent resale considerations (or in a static setting), the buyers only care about the flow payoff that they expect to receive from holding the asset in the current period. As a result, when the proportion of high quality assets is sufficiently low, the buyers’ willingness to pay drops below the reservation value of shocked owners of high quality assets. In this case, only low quality assets trade in equilibrium, asset allocation is inefficient, and asset prices are depressed

to reflect this. On the other hand, when the proportion of high quality assets is sufficiently high, the buyers' willingness to pay remains above the reservation value of the shocked owners of high quality assets. In this case, all shocked asset owners trade, asset allocation is efficient and asset prices are high to reflect this. Therefore, depending on parameters, there can be two possible equilibria but, more importantly, the equilibrium is unique.

Our main result is that the interaction between information frictions and resale concerns generates an inter-temporal coordination problem and gives rise to multiple equilibria (Theorem 1). The reason is that, when buyers anticipate that they might be shocked and need to sell the assets in the future, their willingness to pay for these assets depends also on their beliefs about future market conditions. If buyers believe that the market will be liquid (illiquid) and asset prices will be high (low) tomorrow, they will bid more (less) aggressively for assets today, and thus able to attract a better (worse) quality pool of assets today. To show how these concerns about future market conditions can generate multiplicity of equilibria, we first construct two types of what we term *constant price equilibria*. A defining property of these equilibria is that both asset prices and asset allocation among different owners types are fixed over time.

We construct an *efficient trade* equilibrium, in which all shocked asset owners trade their assets immediately and, as a result, asset prices, output and welfare are permanently high. We show that there exists a lower bound  $\bar{\pi}_{ET}$  on the proportion of high quality assets, such that the *efficient trade* equilibrium exists when the actual proportion  $\pi$  of high quality assets is greater than  $\bar{\pi}_{ET}$ . Then, we construct an *inefficient trade* equilibrium, in which only low quality asset owners trade and, as a result, asset prices, output and welfare are permanently low. We show that there exists an upper bound  $\bar{\pi}_{IT}$ , such that the *inefficient trade* equilibrium exists when  $\pi$  is smaller than  $\bar{\pi}_{IT}$ . Importantly, however, we show that these thresholds are ranked as  $\bar{\pi}_{ET} < \bar{\pi}_{IT}$  and, therefore, the two equilibria coexist when  $\pi \in (\bar{\pi}_{ET}, \bar{\pi}_{IT})$ .

Next, we capture the notion of *sentiment equilibria* as coordinated beliefs about future market conditions. To do so, we introduce a binary sunspot variable  $z_t \in \{B, G\}$  and we look for equilibria in which agents coordinate on efficient (inefficient) trade whenever  $z_t = G$  ( $z_t = B$ ). To facilitate inter-temporal coordination, the sunspot must be sufficiently persistent: when  $z_t = G$ , the agents at time  $t$  believe it is more likely that efficient trade will be played in the future, whereas when  $z_t = B$ , the agents believe it is less likely. Indeed, the equilibrium with constant prices are special cases in which the sunspot becomes perfectly persistent. It turns out that the coexistence of multiple constant price equilibria is necessary and sufficient condition for the existence of sentiment equilibria, provided that the sunspot driving the equilibrium play is sufficiently persistent (Theorem 2). In these equilibria, asset prices always reflect fundamentals

and there is no aggregate fundamental uncertainty. Nevertheless, asset prices, output and welfare can display large fluctuations due to sunspots.

Our model shows that sentiments can actually affect the fundamental value of assets by changing the mix of assets that are traded and, therefore, the extent to which gains from trade are realized. Thus, market sentiments cannot be separated from fundamental value, and both are essential in determining asset valuations and asset allocations. In particular, even when there is no intrinsic information about changes in the characteristics of the assets, sentiments can lead to large price swings. Thus, our model can provide a fully rational explanation for the documented excess volatility in asset prices.<sup>1</sup> Our model also illustrates that sentiments can be an important source of macroeconomic volatility. Although measured total factor productivity (TFP) may fluctuate, it may not be the driver of output volatility, as both can instead be driven by shocks to expectations. Thus, an econometrician, who cannot directly observe sentiments, must be cautious when estimating and interpreting measures of TFP, so as to avoid over-estimating the role of technology shocks.

## 1.1 Related Literature

Our paper naturally relates to the recent and growing literature that embeds adverse selection in a macro-finance context.<sup>2</sup> Daley and Green (2016) and Fuchs et al. (2016) explicitly model re-trade considerations. Unlike the setting of this paper, those papers allow for time-on-the-market to serve as a signal of quality. Thus, given current beliefs, the equilibria are essentially unique in both papers. Although both papers can generate time varying liquidity, a fact that is particularly stressed in Daley and Green (2016), this variation is not driven by inter-temporal coordination and expectations of future market liquidity. Instead, it is driven by whether the current beliefs about the asset quality are above or below the critical threshold at which pooling is an equilibrium.

Some of the recent work in this area considers a search environment rather than competitive markets. The closest papers within this literature are Chiu and Koepl (2016), Maurin (2016) and Mäkinen and Palazzo (2017). They share with our model the feature that assets can be of high or low quality and that buyers are concerned both about current adverse selection and future resale conditions, since eventually they have a reason to sell their assets. Unlike us, these papers assume that there is persistence in the shocks that motivate holders to sell their assets.<sup>3</sup>

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<sup>1</sup>See, for example, LeRoy (2004) and Shiller (2005).

<sup>2</sup>See, for example, Eisfeldt (2004), Martin (2005), Kurlat (2013), Bigio (2015), Chari et al. (2010), Gorton and Ordoñez (2014, 2016), Daley and Green (2016) and Fuchs et al. (2016).

<sup>3</sup>Chiu and Koepl (2016) consider the case of full persistence.

This persistence introduces a backward looking aspect of liquidity that is absent in our baseline model. Namely, if the market is liquid today, assets are allocated more efficiently today and, thus, there will be fewer gains from trade to be realized tomorrow. This makes the adverse selection problem endogenously more severe tomorrow, leading to a drop in future liquidity.

In addition to the differences in market structure, these papers have a very different focus from ours. The main consideration in Chiu and Koepl (2016) is the interaction between adverse selection and search frictions, and it is largely motivated by the recent financial crisis: they mainly discuss policy interventions when the fraction of low quality assets in the market is so large that there would be no trade absent an intervention. Although Maurin (2016) notes that there is a possibility for multiple equilibria, his main contribution is the construction of equilibria with cycles. Unlike our sentiment equilibria, these equilibria are deterministic and are not driven by inter-temporal coordination, but rather by the backward looking aspect of liquidity arising from the persistence of the shocks. Finally, Mäkinen and Palazzo (2017) have a more general search and matching technology that allows for congestion externalities. Their focus is on the additional negative effect (and policies to overcome it) from the fact that traders who are not shocked are in the market trying to trade away their lemons and creating congestion externalities for shocked sellers.

The inter-temporal aspect of the coordination leading to multiplicity of equilibria relates our work with the broad literature on money and rational bubbles.<sup>4</sup> There is an important difference between our work and most of that literature. In our setting, the value of assets is always pinned down by fundamentals and we do not rely on a violation of the “No-Ponzi games” condition for assets to have positive prices. Closer to our model is the contemporaneous work of Donaldson and Piacentino (2017), who motivate potential runs on banks as arising from failures of coordination in the re-trading of “money-like” bank obligations. In their setting, trading frictions are exogenous, there is no adverse selection and trade completely breaks down, whereas adverse selection is the source of the endogenous frictions in our model. Furthermore, there is always some trade in our model.

The papers by Plantin (2009) and Malherbe (2014) are also related to our work, although the strategic considerations in their papers are contemporaneous rather than dynamic. In Malherbe (2014), firms must make a portfolio choice decision between holding cash versus assets with privately known quality. He shows that multiple equilibria are possible due to complementarities in firms’ cash-holding decisions. If a firm decides to increase its cash-holdings in the first period, then if that firm trades in the second period, it is less likely that the trade is the result of a

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<sup>4</sup>See, for example, Samuelson (1958), Tirole (1985), Weil (1987), Santos and Woodford (1997), and Martin and Ventura (2012).

liquidity shock. As a result, there are less gains from trade in the second period and there is more adverse selection in the market. This in turn makes it more attractive for other firms to also hoard cash. Thus, there can be two equilibria, one in which firms expect other firms not to hoard cash and the second period market to work well, and another in which firms expect other firms to hoard cash and, as a result, the second period market dries-up. A similar mechanism is present in Plantin (2009). Although there is no cash-hoarding by firms in his setting, the number of investors who decide to buy the bond in the first period affects the potential market size for the bonds and hence their price in the future. As in Malherbe (2014), this contemporaneous complementarity can lead to self-fulfilling market failures. It is important to highlight that equilibrium multiplicity in these papers arises due to static coordination failures. Indeed, as in the global games literature, Plantin (2009) is able to obtain uniqueness of equilibrium by introducing a noisy private signal about the probabilities of default of the bonds.

Finally, there has been an increased interest among macroeconomists to understand how *sentiments* – in the form of correlated shocks to agents’ information sets, – can be drivers of aggregate fluctuations. Some recent papers include Lorenzoni (2009), Hassan and Mertens (2011), Angeletos and La’O (2013), and Benhabib et al. (2015). In this literature, the dispersion of information among agents about aggregate economic states is an essential ingredient. We contribute to this literature by showing that, in the presence of adverse selection, sentiments which coordinate agents’ expectations about future market conditions can generate aggregate fluctuations even when the information about aggregate variables is common to all economic agents at all times.

The rest of the paper is organized as follows. In Section 2, we present the setup of the model. In Section 3, we characterize the equilibria of the model and conduct comparative statics. In Section 4, we consider several extensions, and we conclude in Section 5. All proofs are relegated to the Appendix.

## 2 The Model

Time is infinite and discrete, indexed by  $t \in \{0, 1, \dots\}$ . There is a mass of indivisible assets or Lucas trees, indexed by  $i \in [0, 1]$ , which are identical in every respect except their quality. These trees live forever<sup>5</sup> and each tree can either be of high or low quality, which we denote by  $\theta_i \in \{L, H\}$ . A tree of quality  $\theta_i$  can potentially produce  $x_{\theta_i}$  units of output per period, where  $x_H > x_L > 0$ . The probability that a given tree is of good quality is  $\mathbb{P}(\theta_i = H) = \pi \in (0, 1)$ ,

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<sup>5</sup>In Section 4.3, we allow trees to depreciate or to mature stochastically.

which is also assumed to be the fraction of good quality trees in the economy. For expositional simplicity, we suppose that asset quality is permanent; we extend our analysis to transitory quality in Section 4.3.

There is a large mass  $M$  of ex-ante identical risk-neutral agents, indexed by  $j \in [0, M]$ , who discount payoffs with a factor  $\delta \in (0, 1)$ . Each of these agents can operate only one unit of the Lucas tree, and we refer to those who currently operate assets as *owners* and to the rest as potential *buyers*. We introduce gains from asset trade by supposing that owners experience occasional productivity shocks that depress their asset valuation relative to the potential buyers. In particular, each period owner  $j$  can have two possible productivities, denoted by  $\omega_j \in \{\chi, 1\}$ , which implies that she can produce  $\omega_j x_{\theta_i}$  units of output by operating a tree of quality  $\theta_i$ , where  $\chi \in (0, 1)$  and  $\chi x_H \geq x_L$ .<sup>6</sup> When  $\omega_j = \chi$ , we say that “owner  $j$  is shocked,” and we assume that each period an owner is shocked with probability  $\mathbb{P}(\omega_j = \chi) = \lambda \in (0, 1)$ . All potential buyers are assumed to be unshocked.<sup>7</sup> An owner’s productivity status is assumed to be independent of the quality of the tree she operates and of her productivity status in the past; we extend our analysis to persistent productivity shocks in Section 4.2.

**Remark 1** *We can alternatively interpret the differences in  $\omega$ ’s as representing heterogeneity in agents’ cashflow valuations arising due to liquidity/borrowing constraints or hedging demands.*

The market for assets is competitive - in each period, at least two buyers are randomly matched with an owner, and they compete for the owner’s tree a la Bertrand. When an owner receives offers from the buyers, she decides which if any offer to accept. If the owner rejects all offers, then she continues to be an owner in the next period and is rematched with a new set of buyers. Instead, if the owner accepts an offer, then she sells her tree and enters the pool of potential buyers.<sup>8</sup> A buyer whose offer is rejected continues to be a buyer in the next period, whereas a buyer whose offer is accepted, buys the tree and becomes an owner of that tree in the next period.<sup>9</sup>

Trade in our economy may be hindered by the presence of asymmetric information. In particular, we assume that the quality of an owner’s tree  $\theta$  and her productivity status  $\omega$  are both that owner’s private information.

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<sup>6</sup>This assumption ensures that adverse selection is sufficiently severe. Although inessential for our results, it reduces the number of cases that we need to consider.

<sup>7</sup>This assumption is without loss of generality as long as there are at least two unshocked buyers competing for each owner’s tree.

<sup>8</sup>Since there is a continuum of trees and matching is random, the probability that an owner who sells her tree is rematched to bid for that same tree is zero.

<sup>9</sup>We implicitly assume that all buyers have sufficiently large endowments in each period, so that their budget constraints do not bind during asset trade. It is sufficient that each agent have a per period endowment of  $e > (1 - \delta)^{-1} \cdot x_H$  of the output good.

We suppose that the time- $t$  information set of a buyer includes aggregate histories (e.g., aggregate output, aggregate trading volume), but not the trading history of the individual asset for which he bids.<sup>10</sup> The strategy of each buyer is a mapping from his information set to a probability distribution over offers. An owner's information set includes the quality  $\theta$  of her asset, her productivity status  $\omega$ , and the buyers' information set. The strategy of each owner is a mapping from her information set to a probability of acceptance.

We use Perfect Bayesian Equilibrium (PBE) as our equilibrium concept. This has the following implications. First, each owner's acceptance rule must maximize her expected payoff taking as given the buyers' strategies (*Owner Optimality*). Second, any offer in the support of a buyer's strategy must maximize his expected payoff given his beliefs, the owner's and the other buyers' strategies (*Buyer Optimality*). Third, given their information set, buyers' beliefs are updated using Bayes' rule whenever possible (*Belief Consistency*).

### 3 Equilibrium

In this section, we characterize the set of equilibria of our model. Let us briefly outline how we will proceed. We start by analyzing a benchmark economy in which asset qualities are observable (Section 3.1). We show that in the unique equilibrium of this economy, there are no aggregate fluctuations and all assets are allocated efficiently (Proposition 1). We then proceed to our main analysis. We first focus on equilibria that do not feature aggregate fluctuations (Section 3.2), and we show that multiple equilibria can arise and be ranked in terms of asset prices, output and welfare (Theorem 1). We then consider sentiment equilibria (Section 3.3), and we provide conditions under which these equilibria exist and feature sunspot driven fluctuations in asset prices, output and welfare (Theorem 2).

#### 3.1 Benchmark without information frictions

Before we proceed to the analysis, we consider a useful benchmark economy in which the qualities of the Lucas trees are public information. It turns out that observability of asset qualities suffices to ensure that the asset allocations are efficient. The following proposition characterizes the unique equilibrium of this benchmark. Let  $E\{\cdot\}$  denote the expectations operator, then:

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<sup>10</sup>The primary role of this assumption is to eliminate signaling considerations which would complicate our analysis considerably. If trading history of individual trees were observable, owners may reject certain offers and engage in costly delay in order to signal their types. Our qualitative results would extend to a setting where such signaling is possible as long as trading history provides an imperfect signal of asset quality.

**Proposition 1 (Observable Quality)** *If asset qualities are publicly observable, then the equilibrium is unique, in it all assets are efficiently allocated and, for all  $t$ , the price of  $\theta$ -quality assets is  $p_\theta^{FB} = (1 - \delta)^{-1}x_\theta$ , and the output and welfare are  $Y^{FB} = E\{x_\theta\}$  and  $W^{FB} = (1 - \delta)^{-1} \cdot Y^{FB}$  respectively.*

For any given (observable) quality, buyers value the trees weakly more than the owners (strictly so if owners are shocked). Thus, in equilibrium, all trees must be reallocated from shocked owners to the buyers, i.e., asset allocation is efficient. As a result, the aggregate output is the output of all trees at their most efficient allocation, and welfare is simply the present discounted value of this output. Finally, because markets are competitive, all trees are priced at the present discounted value of their output.

We next study how these results change in the presence of information frictions.

### 3.2 Constant price equilibrium

We begin our analysis by considering a simple class of stationary equilibria which allow us to clearly illustrate the link between asset prices and market liquidity.

From now on, we will refer to an owner with productivity status  $\omega$  and an asset of quality  $\theta$  as a  $(\theta, \omega)$ -type owner. Let  $p_t$  denote the (common) asset price that prevails in equilibrium at time  $t$ .<sup>11</sup> Then,

**Definition 1** *We say that a PBE is a **constant price equilibrium** if the asset price and the distribution of owner types is the same in every period.*

In what follows, we will drop the time superscripts and we will superscript the future variables by primes. Consider the problem of a  $(\theta, \omega)$ -type owner, who must decide whether to trade her tree or to hold on to it. Let  $V(\theta, \omega)$  denote the value to this asset owner, and let  $p^*$  denote the (constant price) equilibrium price. In equilibrium, for any  $(\theta, \omega)$ , it must be that:

$$V(\theta, \omega) = \max \{p^*, E\{\omega x_\theta + \delta V(\theta', \omega') | \theta, \omega\}\} \quad (1)$$

If the owner sells her tree today, then she gets the price  $p^*$ . Instead, if she does not, then she produces the output  $\omega x_\theta$  today plus she gets the expected discounted value from owning the tree tomorrow, where expectations are conditional on the holder knowing her type today.<sup>12</sup>

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<sup>11</sup>The price of an asset is defined to be the maximal bid of the buyers for that asset, and it must be common to all assets since all assets appear identical to the buyers at any time  $t$ .

<sup>12</sup>Although in our baseline model current productivity status is uninformative about the future, we include it in the agents' information sets for completeness.

Owner optimality requires that the owner's equilibrium value be the maximum of the expected payoffs from either trading the asset or holding on to it.

Adverse selection can arise whenever the quality of assets traded depends (adverseley) on the asset price itself. Suppose that at time  $t$  an owner were to receive a maximal offer  $p$ , which may or may not equal the equilibrium price  $p^*$ . The owner would accept such an offer if and only if it exceeded  $V$ . In particular, the set of owner types who accept a maximal offer  $p$  is:

$$\Gamma(p) = \{(\theta, \omega) : V(\theta, \omega) \leq p\}. \quad (2)$$

Because  $V(\cdot, \cdot)$  will be different for owners of different quality assets, the set  $\Gamma(p)$  will depend non-trivially on  $p$ . Also, note that, because today's offers for an asset are unobserved by the buyers of that asset in the future, the value  $V$  depends on the equilibrium price  $p^*$  and not on offers made off-equilibrium.

Consider the problem of the buyers who are bidding for an owner's tree. Because at least two buyers compete for the owner's tree, in any equilibrium the buyers' expected profits must be zero. Therefore, in equilibrium, the asset price must satisfy:

$$p^* = E\{x_\theta + \delta V(\theta', \tilde{\omega}') | (\theta, \omega) \in \Gamma(p^*), \tilde{\omega} = 1\}, \quad (3)$$

where tilde on  $\tilde{\omega}$  indicates that it is the productivity status of the buyer. Rationality and belief consistency require that buyers understand the potential adverse selection problem and condition their expectations of asset quality on the set of owner types who accept their offers.<sup>13</sup> Since a buyer who gets the tree in the current period becomes an owner in the next, the equilibrium asset price depends on the expected value from being an asset owner.

Finally, buyer optimality requires that in equilibrium no buyer can profitably deviate by making an offer to the owner that strictly exceeds the equilibrium price, i.e., it must be that:

$$\hat{p} \geq E\{x_\theta + \delta V(\theta', \tilde{\omega}') | (\theta, \omega) \in \Gamma(\hat{p}), \tilde{\omega} = 1\} \quad (4)$$

for all offers  $\hat{p}$  strictly greater than the equilibrium price  $p^*$ .

By inspecting of (1)-(4), we can already see the feedback between asset prices and liquidity that is at the heart of our paper. First, the owners' value is increasing in the asset price because, conditional on trading the asset, the payoff to the owner is higher. As a result, when asset prices are higher the set of owner types who trade becomes less adversely selected, and the market

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<sup>13</sup>If  $\Gamma(p) = \emptyset$ , we set without loss of generality  $p^* = E\{x_\theta + \delta V(\theta', \tilde{\omega}') | \tilde{\omega} = 1\}$ .

becomes more liquid. Second, the asset price is increasing in the value from being an asset owner because, when making their offers, buyers are forward-looking and care not only about the quality of the tree they buy but also about the resale value of the tree in the future. As we will see, this dynamic feedback between asset prices and valuations plays a crucial role in determining the set of equilibria of our economy.

The following proposition puts further structure on the possible constant price equilibria by showing that in any such equilibria the owners with low quality assets always trade whereas the owners with high quality assets who are unshocked never do.

**Proposition 2** *Any constant price equilibrium is characterized by a value function  $V$  and asset price  $p^*$  satisfying (1)-(4). In any such equilibrium,  $V(L, \chi) = V(L, 1) = p^* \leq V(H, \chi) < V(H, 1)$ . Thus, the low quality assets always trade, whereas the high quality assets held by unshocked owners never trade.*

First, because the flow payoff to a shocked owner is lower than to an unshocked owner, the values can be ranked according to the productivity status,  $V(\theta, 1) \geq V(\theta, \chi)$  for  $\theta \in \{L, H\}$ . Second, because buyers are unshocked and can at least guarantee themselves a low quality asset, their asset valuation is higher than that of low quality asset owners. Hence, it must be that all owners with low quality assets trade and  $V(L, 1) = p^*$ . Finally, since all low quality assets trade, the unconditional value  $E\{V(\theta, 1)\}$  is an upper bound on the payoff that any buyer can attain by purchasing an asset. Thus, the buyers will never be able to attract the  $(H, 1)$ -type owner to trade without making losses in expectation.

From Proposition 2, it follows that there can be two types of constant price equilibria, depending on whether the  $(H, \chi)$ -type owner trades in equilibrium. We adopt the following definition in order to distinguish among them.

**Definition 2** *We say that a constant price equilibrium features **efficient trade** if in it both high and low quality assets trade. Otherwise, if only low quality assets trade, we say that it features **inefficient trade**.*

In the efficient trade equilibrium, all shocked owners trade and the trees are efficiently allocated. Instead, in the inefficient trade equilibrium, the allocation is inefficient because the high types who are shocked stay out of the market.

The following theorem states our main result. It provides the conditions under which each type of equilibrium exists and under which they coexist. Importantly, when the two equilibria coexist, they can be ranked in terms of asset prices, output and welfare.

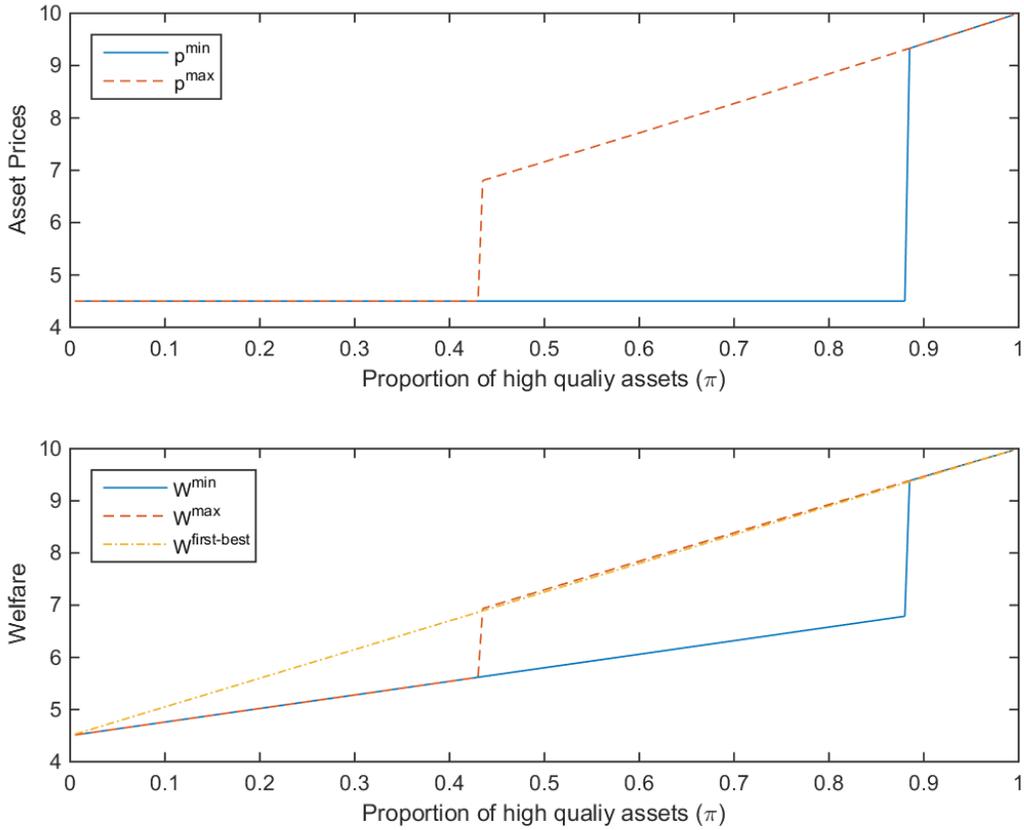


Figure 1: **Asset Prices and Welfare.** Unless stated otherwise, the parameters used are:  $\delta = 0.9$ ,  $\lambda = 0.6$ ,  $\chi = 0.5$ ,  $x_H = 1$  and  $x_L = 0.45$ .

**Theorem 1 (Characterization and Multiplicity)** *A constant price equilibrium exists. There exist thresholds  $0 < \bar{\pi}_{ET} < \bar{\pi}_{IT} < 1$  on the proportion of high quality assets such that:*

1. Efficient trade. *There is at most one efficient trade equilibrium, which exists if and only if  $\pi \geq \bar{\pi}_{ET}$ ,*
2. Inefficient trade. *There is at most one inefficient trade equilibrium, which exists if and only if  $\pi \leq \bar{\pi}_{IT}$ .*

*Thus, the two equilibria coexist when  $\pi \in (\bar{\pi}^{ET}, \bar{\pi}^{IT})$ . Furthermore, the asset prices, output and welfare are higher in the efficient trade than in the inefficient trade equilibrium.*

Figure 1 illustrates this result graphically. The top (bottom) panel depicts the maximal and minimal asset price (welfare) that is possible in equilibrium as a function of the proportion

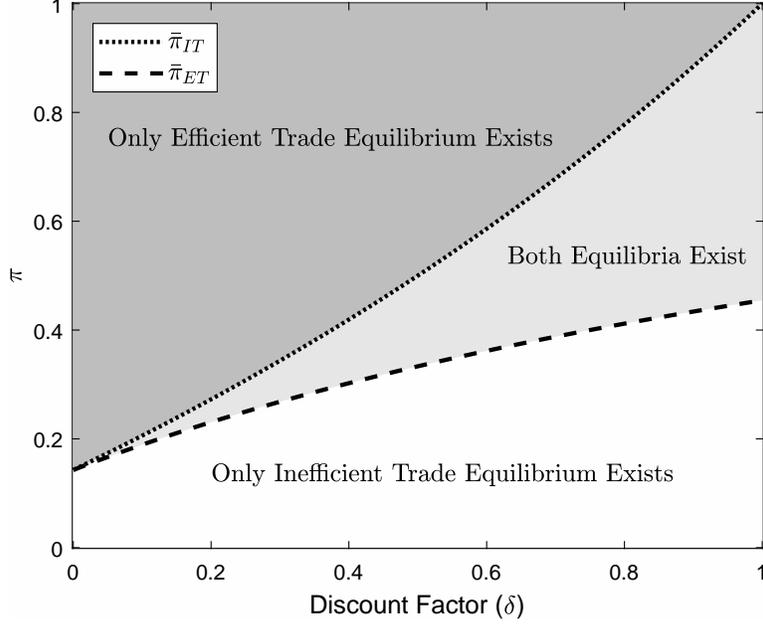


Figure 2: **Equilibrium Set and Role of Dynamics.** Unless stated otherwise, the parameters used are:  $\delta = 0.9$ ,  $\lambda = 0.6$ ,  $\chi = 0.5$ ,  $x_H = 1$  and  $x_L = 0.45$ .

$\pi$  of high quality assets. The maximal and minimal asset price (welfare) coincide when the equilibrium is unique. In constant price equilibria, output and welfare are isomorphic, so we omit output for brevity. When  $\pi$  is sufficiently low, asset prices and welfare are low since the unique equilibrium features inefficient trade. Instead, when  $\pi$  is sufficiently high, asset prices and welfare are high since the unique equilibrium features efficient trade. It is for intermediate values of  $\pi$  where multiple constant price equilibria exist and, thus, where asset allocation can either be efficient or inefficient, and asset prices and welfare can either be high or low depending on which equilibrium is played.

Figure 2 illustrates that dynamic considerations are essential for our result. We can see that as the economy becomes “static,” i.e. as  $\delta \rightarrow 0$ , the region of  $\pi$ 's where multiple equilibria arise vanishes; instead, as agents care more and more about the future, i.e. as  $\delta$  increases, the region where multiple equilibria exist expands.

In what follows, we show explicitly how to construct the constant price equilibria and we provide the intuition for when and why each type of equilibrium exists. To this end, it is convenient to index the equilibrium price and owner values by the type (i.e., efficient vs inefficient) of the equilibrium:  $\{(p^{*j}, V^j)\}_{j \in \{ET, IT\}}$ . We begin with the efficient trade equilibrium.

### 3.2.1 Efficient trade

In the efficient trade equilibrium, all owners except for the  $(H, 1)$ -type trade at price  $p^{*ET}$ . Therefore, their values are given by:

$$V^{ET}(L, \chi) = V^{ET}(L, 1) = V^{ET}(H, \chi) = p^{*ET}, \quad (5)$$

whereas the value of the  $(H, 1)$ -type owner is:

$$V^{ET}(H, 1) = x_H + \delta (\lambda p^{*ET} + (1 - \lambda)V^{ET}(H, 1)), \quad (6)$$

i.e., this owner consumes the output this period, and in the next period she is either shocked (w.p.  $\lambda$ ) in which case she sells her tree, or she remains unshocked (w.p.  $1 - \lambda$ ) and holds on to it. The equilibrium price is:

$$p^{*ET} = \hat{\pi}x_H + (1 - \hat{\pi})x_L + \delta ((1 - \hat{\pi}(1 - \lambda))p^{*ET} + \hat{\pi}(1 - \lambda)V^{ET}(H, 1)). \quad (7)$$

where  $\hat{\pi} \equiv \frac{\lambda\pi}{\lambda\pi + 1 - \pi}$  is the probability that the tree is of high quality, conditional on being sold.<sup>14</sup> To see why, note that a buyer who gets the tree today expects to consume its output, and he will keep the tree tomorrow only if *both* it is of high quality and he remains unshocked (w.p.  $\hat{\pi}(1 - \lambda)$ ). Importantly, the buyer understands that due to adverse selection the tree is of high quality with probability strictly smaller than  $\pi$ . We can combine (6) and (8) to further simplify and get an analytical expression for the asset price:

$$p^{*ET} = (1 - \delta)^{-1} \left( \hat{\pi}x_H + (1 - \hat{\pi})x_L + \delta(1 - \hat{\pi})(1 - \lambda) \frac{\hat{\pi}(x_H - x_L)}{1 - \delta(1 - \hat{\pi})(1 - \lambda)} \right). \quad (8)$$

In this equilibrium, all owners except the  $(H, 1)$ -type trade their trees. As a result, the asset allocation is efficient and the output and welfare, as in our benchmark economy of Section 3.1, are respectively given by:

$$Y^{ET} = E\{x_\theta\} \quad \text{and} \quad W^{ET} = (1 - \delta)^{-1} \cdot Y^{ET}. \quad (9)$$

For existence of such an equilibrium, we must rule out profitable deviations for the owners and the buyers. It is clear that there are no deviations for the buyers, since any such deviation would need to attract the  $(H, 1)$ -type, which is impossible without the buyers making losses in

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<sup>14</sup>In the stationary distribution of owner types, the probability that an owner is an  $(H, \chi)$ -type is  $\lambda\pi$  and the probability that she is a low type is  $1 - \pi$ .

expectation. For the owners, it is sufficient to check that the  $(H, \chi)$ -type gets a lower payoff if she were to keep the asset for one period rather than sell it:

$$\chi x_H + \delta (\lambda p^{*ET} + (1 - \lambda)V^{ET}(H, 1)) \leq p^{*ET}. \quad (10)$$

Using the equilibrium conditions (5)-(8), we can re-express condition (10) as:

$$(\chi x_H - \hat{\pi} x_H - (1 - \hat{\pi}) x_L) + \delta (1 - \hat{\pi}) (1 - \lambda) \frac{(1 - \hat{\pi})(x_H - x_L)}{1 - \delta(1 - \hat{\pi})(1 - \lambda)} \leq 0. \quad (11)$$

The threshold  $\bar{\pi}_{ET}$  in Theorem 1 is the value of  $\pi$  at which condition (11) holds with equality, and it is straightforward to show that this threshold is interior and unique.

Next, we consider the inefficient trade equilibrium.

### 3.2.2 Inefficient trade

In the inefficient trade equilibrium, only owners of low quality assets trade. Therefore, their values are given by:

$$V^{IT}(L, \chi) = V^{IT}(L, 1) = p^{*IT}, \quad (12)$$

whereas the values of the owners of the high quality assets are:

$$V^{IT}(H, \omega) = \omega x_H + \delta (\lambda V^{IT}(H, \chi) + (1 - \lambda)V^{IT}(H, 1)), \quad (13)$$

for  $\omega \in \{\chi, 1\}$ , as these owners both consume the output of their trees today and expect to do so in the future. The equilibrium price is:

$$p^{*IT} = (1 - \delta)^{-1} x_L, \quad (14)$$

since buyers understand that due to adverse selection only low quality trees trade.

In this equilibrium, not all gains from trade are realized. Since all  $(H, \chi)$ -types (mass  $\pi\lambda$  of owners) keep their trees and produce with productivity  $\chi$  rather than 1, the output and welfare are respectively given by:

$$Y^{IT} = E\{x_\theta\} - (1 - \chi)\pi\lambda x_H \quad \text{and} \quad W^{IT} = (1 - \delta)^{-1} \cdot Y^{IT}, \quad (15)$$

which are lower than their counterparts in the efficient trade equilibrium.

For existence of such an equilibrium, we must rule out profitable deviations for the owners

and the buyers. It is clear that there are no deviations for the owners, since the high types strictly prefer keep their trees (recall that  $\chi x_H \geq x_L$ ), whereas the low types prefer to trade. To rule out deviations for the buyers, it suffices to check that the buyers' profits are non-positive at any offer that attracts the  $(H, \chi)$ -type:

$$\hat{\pi}(x_H + \delta (\lambda V^{IT}(H, \chi) + (1 - \lambda)V^{IT}(H, 1))) + (1 - \hat{\pi})(x_L + \delta p^{*IT}) \leq V^{IT}(H, \chi), \quad (16)$$

where as before  $\hat{\pi} \equiv \frac{\lambda\pi}{\lambda\pi+1-\pi}$  is the probability that the tree is of high quality, conditional on being sold. The left-hand side of condition (16) is the expected payoff to a buyer if he were able to attract the  $(H, \chi)$ -type, whereas the right-hand side is the lowest bid a buyer needs to make to attract that type.

Using the equilibrium conditions (12)-(14), we can re-express condition (16) as:

$$0 \leq (\chi x_H - \hat{\pi} x_H - (1 - \hat{\pi}) x_L) + \delta (1 - \hat{\pi}) \frac{(1 - \lambda + \lambda\chi) x_H - x_L}{1 - \delta}. \quad (17)$$

The threshold  $\bar{\pi}_{IT}$  in Theorem 1 is the value of  $\pi$  at which condition (17) holds with equality, and it is straightforward to show that this threshold interior and unique.

We have characterized the conditions for the existence of each type of equilibrium. We next illustrate why multiple equilibria arise in our setting.

### 3.2.3 Equilibrium multiplicity

Using the equilibrium properties above, we can re-express the conditions (10) and (16) for existence of each type of equilibrium as follows. An efficient trade equilibrium exists whenever:

$$\chi x_H + \delta E\{V^{ET}(H, \omega')\} \leq \hat{\pi} (x_H + \delta E\{V^{ET}(H, \omega')\}) + (1 - \hat{\pi}) (x_L + \delta p^{ET}) \quad (18)$$

In contrast, an inefficient trade equilibrium exists whenever:

$$\chi x_H + \delta E\{V^{IT}(H, \omega')\} \geq \hat{\pi} (x_H + \delta E\{V^{IT}(H, \omega')\}) + (1 - \hat{\pi}) (x_L + \delta p^{IT}) \quad (19)$$

The inequality (18) states that the  $(H, \chi)$ -type prefers to trade, rather than keep her tree, at the pooling price that prevails in the efficient trade equilibrium. The inequality (19) states that the buyers cannot attract the  $(H, \chi)$ -type without making losses in expectation, if they expect equilibrium to feature inefficient trade in the future.

Note the importance of dynamics and re-sale concerns. It is clear that, as  $\delta \rightarrow 0$ , generically

only one of the two equilibria exists. The reason is that the agents only care about the flow payoffs of the trees; thus, whether the equilibrium features efficient or inefficient trade depends only on how the flow payoff of the pool  $\hat{\pi}x_H + (1 - \hat{\pi})x_L$  compares to the payoff  $\chi x_H$  to the  $(H, \chi)$ -type.

Next, note that because asset prices are higher in the efficient trade equilibrium, i.e.  $p^{*ET} > p^{*IT}$ , so are the  $H$ -type owners' expected values if they were to hold to the asset till tomorrow, i.e.  $E\{V^{ET}(H, \omega')\} > E\{V^{IT}(H, \omega')\}$ . But importantly  $H$ -type owners' values adjust less than one for one with asset prices, as they keep the assets sometimes. As a result, the buyers' payoff from owning the asset is more sensitive to the asset price than the owners', as reflected respectively in the right-hand side and the left-hand side of inequalities (18) and (19). This allows the two inequalities can be satisfied at the same time. Algebraically, this is equivalent to the ranking of thresholds  $\bar{\pi}_{ET} < \bar{\pi}_{IT}$  in Theorem 1.

Thus far, we considered equilibria in which asset prices and asset allocations do not change over time. In the next section, we ask whether our economy can also feature fluctuations driven by *sentiments*; that is, sunspots that coordinate agents' beliefs but may be unrelated to economic fundamentals.

### 3.3 Sentiment equilibrium

The empirical asset pricing literature has pointed out that one of the features of data that are difficult to reconcile with standard asset pricing models is that there appears to be excess volatility in asset prices relative to the volatility of the cash-flows that assets generate or to the volatility in discount rates. Given the possibility of multiple equilibria, our model has the potential to generate substantial volatility in asset prices even if there is no volatility in cash-flows nor market interest rates. To illustrate this point, we will next introduce a sunspot random variable, to which we will refer as market 'sentiment,' and we will allow traders to coordinate on its realizations.

Consider a sunspot-random variable  $z_t$  which takes values in some finite set  $Z$  and follows a Markov process with transition probability  $P_Z(z'|z)$  for  $z, z' \in Z$ . Assume that the realization of the random variable is public information. Then,

**Definition 3** *We say a PBE is a **sentiment equilibrium** with sunspot  $z_t$  if equilibrium asset prices and asset allocations depend on the realizations of the sunspot.*

Together with the transition probability  $P_Z$  for the sunspot, all sentiment equilibria are

characterized by a equilibrium prices  $p^*$  and owner values  $V$  which satisfy:

$$\begin{aligned}
V(\theta, \omega, z) &= \max_{s \in \{0,1\}} s \cdot p^*(z) + (1 - s) \cdot E\{\omega x_\theta + \delta V(\theta', \omega', z') | (\theta, \omega, z)\} \quad \forall (\theta, \omega, z), \\
\Gamma(p, z) &= \{(\theta, \omega) : V(\theta, \omega, z) \leq p\} \quad \forall (p, z), \\
p^*(z) &= E\{x_\theta + \delta V(\theta', \tilde{\omega}', z') | (\theta, \omega) \in \Gamma(p^*(z), z), z, \tilde{\omega} = 1\} \quad \forall z, \quad \text{and} \\
\hat{p} &\geq E\{x_\theta + \delta V(\theta', \tilde{\omega}', z') | (\theta, \omega) \in \Gamma(\hat{p}, z), z, \tilde{\omega} = 1\} \quad \forall \hat{p} > p^*(z).
\end{aligned}$$

Note that these equations are simply the sunspot-contingent analogues of the equations (1), (2), (3), and (4) in Section 3.2. In fact, all constant price equilibria are solutions to the above system under the additional restriction that asset prices satisfy  $p^*(z) = p^*(\hat{z})$  for all  $z, \hat{z} \in Z$ .

It is straightforward to show that the results in Proposition 2 extend to sunspot equilibria as well. It must that the low quality assets trade at all times whereas the high quality assets held by unshocked owners never do. Thus, the only role of the sunspot is to shift equilibrium play from one in which the  $(H, \chi)$ -type owner trades to one where she does not. Thus, we can without loss of generality restrict attention to binary sunspot processes. In particular, we suppose that  $Z \equiv \{Bad, Good\}$  and at time  $t$  (if possible) the market coordinates on inefficient trade (i.e.,  $(H, \chi)$ -type does not trade) when  $z_t = B$ ; otherwise, it coordinates on efficient trade (i.e.,  $(H, \chi)$ -type trades). The sunspot is assumed to follow a symmetric first-order Markov process parameterized by  $\rho \equiv P(z_{t+1} = G | z_t = G) = P(z_{t+1} = B | z_t = B)$ .<sup>15</sup>

The following theorem provides necessary and sufficient conditions for the existence of sentiment equilibria with sunspot  $z_t$ .

**Theorem 2 (Sentiments)** *A sentiment equilibrium with sunspot  $z_t$  exists if and only if  $\rho$  is sufficiently large and  $\pi \in (\bar{\pi}_{ET}, \bar{\pi}_{IT})$ , where  $\bar{\pi}_j$ 's are as in Theorem 1.*

This result emphasizes the role of inter-temporal (and not static) coordination for the existence of multiple equilibria in our setting. The realization of the sunspot today not only must signal to the agents equilibrium today, but it must also be informative about how the equilibrium play will proceed in the future. These two objectives are accomplished precisely by a sunspot process that is sufficiently persistent.

In Figure 3, we depict the evolution of asset prices and welfare in the economy for a simulation of the sunspot process. The solid line depicts the case where the sunspot is very persistent ( $\rho = 0.95$ ), whereas the dashed line depicts a less persistent process ( $\rho = 0.8$ ). In both cases, the sunspot is sufficiently persistent so that a sentiment equilibrium with that sunspot exists.

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<sup>15</sup>It is straightforward to extend the analysis to more general sunspot processes.

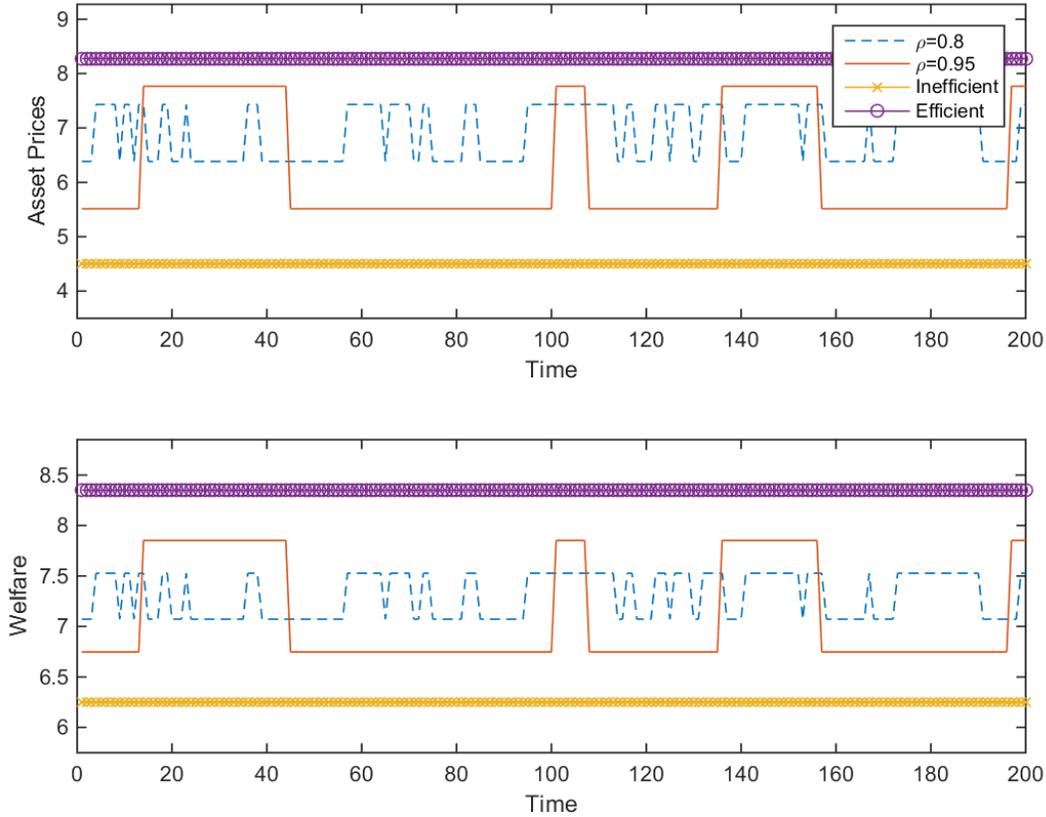


Figure 3: **Asset Prices and Welfare in a Sentiment Equilibrium.** The parameters used are:  $\pi = 0.7$ ,  $\delta = 0.9$ ,  $\lambda = 0.6$ ,  $\chi = 0.5$ ,  $x_H = 1$  and  $x_L = 0.1$ . The solid line depicts a simulation with  $\rho = 0.95$ , whereas the dashed line depicts a simulation with  $\rho = 0.8$ .

Note that the more persistent is the sunspot process, the less frequent are asset prices fluctuations and they are of larger size. This is intuitive because asset prices are forward looking and incorporate the future transitions of the economy. In the *Bad* state, more persistence implies that the assets will be allocated inefficiently and produce less output for a longer period of time. Conversely, in the *Good* state, more persistence implies that the assets will be allocated efficiently and produce more output for a longer period of time. Importantly, since different states correspond to different asset allocation and output, the fluctuations in asset prices are mirrored by fluctuations in the agents' welfare.

Finally, the figure also depicts the asset prices and welfare in the constant price equilibria of Section 3.2, which provide upper and lower bounds on asset prices and welfare that can be attained in any sentiment equilibrium.

## 4 Extensions

In this section, we consider some extensions of our baseline model. In Section 4.1, we consider shocks to fundamentals. In Section 4.2, we extend our analysis to persistent productivity shocks. In Section 4.3, we consider transitory asset quality and asset duration.

### 4.1 Fundamental shocks

As discussed in Manuelli and Peck (1992), “the early sunspot literature was motivated by the idea that small shocks to fundamentals are not very different from sunspots.” They show that, in an overlapping generations endowment economy with money, small shocks to fundamentals can serve as the coordination device for different monetary equilibria. Furthermore, in the limit, as the underlying shocks have no direct effect on endowments, for every equilibrium of the pure sunspot economy with no shocks to endowments, there is a sequence of equilibria of the economy with risky endowments that converges to it. Our baseline economy can also be extended to allow for aggregate shocks which, even when small, can have large effects by serving as a coordination device for agents’ expectations regarding the future market conditions.

To illustrate this point, suppose that the output of the Lucas trees is also a function of some aggregate state  $z_t \in \{G, B\}$ , which follows a persistent and observable two state markov process. Concretely, consider the case where in state  $z_t = G$  the payoff or output of a tree of quality  $\theta$  in the hands of a holder with liquidity or productivity status  $\omega$  is  $(1 + \varepsilon) \cdot \omega \cdot \theta$ , whereas in state  $z_t = B$  the output is  $(1 - \varepsilon) \cdot \omega \cdot \theta$ , for some  $\varepsilon \in [0, 1)$ . Note that when  $\varepsilon = 0$ , we are back to our baseline setup, where the aggregate state is a pure sunspot and has no direct impact on any given tree’s output, but can still serve as a coordination device. It is therefore straightforward to see that, for  $\varepsilon$  small but positive, we have the potential for the amplification of fundamental shocks. The equilibrium features amplification in the sense that the shocks have a negligible effect on the cashflow of any given tree. But, as shown in Section 3.3, these news can change market expectations about future, change the pool of assets that are traded in equilibrium, and thus have a large impact on equilibrium asset prices, market liquidity and welfare.

If we think of agent specific shock  $\omega$  as the productivity with which that agent can employ the tree, then the aggregate TFP in the economy at time  $t$  would be:

$$TFP_t = \begin{cases} (1 + \varepsilon) \cdot (\pi \cdot x_H + (1 - \pi) \cdot x_L) & \text{if } z_t = G \\ (1 - \varepsilon) \cdot (\pi (1 - \lambda + \lambda\chi) \cdot x_H + (1 - \pi) \cdot x_L) & \text{if } z_t = B \end{cases}$$

Since  $\chi < 1$ , the measured TFP can display large fluctuations even if the extrinsic shock to productivity is small, i.e.,  $\varepsilon \approx 0$ .

## 4.2 Gains from trade and history dependence

In this section, we extend our baseline setup to the case with persistent productivity shocks.

As before, the unconditional probability of owner  $j$  being shocked at time  $t$  is  $\mathbb{P}(\omega_{j,t} = \chi) = \lambda$ , but we now assume that  $\mathbb{P}(\omega_{j,t+1} = \chi | \omega_{j,t} = \chi) = \rho^\omega$ . As in Section 3.2, it is straightforward to show that equilibrium play must feature one of the following two possibilities. In a given period  $t$ , either only low type owners trade or all owners except the  $(H, 1)$ -types trade. In contrast to our baseline model, however, the distribution of owner types is now endogenous to equilibrium play. To see this, let  $s_t \in \{0, 1\}$  denote the indicator that equals 1 if and only if the  $(H, \chi)$ -type trades at  $t$ . Also, let  $\mathbb{P}_t(\theta, \omega)$  denote the fraction of owners who are of type  $(\theta, \omega)$  at the beginning of period  $t$ . Since low quality trees are always on the market, the fraction  $\mathbb{P}_t(H, \chi)$  is crucial in determining the potential pool quality and possible equilibrium play at any time  $t$ . It can be shown to evolve as follows:

$$\mathbb{P}_t(H, \chi) = \pi \cdot \lambda \cdot \frac{1 - \rho^\omega}{1 - \lambda} + \left( \rho^\omega - \lambda \cdot \frac{1 - \rho^\omega}{1 - \lambda} \right) \cdot (1 - s_{t-1}) \cdot \mathbb{P}_{t-1}(H, \chi). \quad (20)$$

If the buyers were able to attract the  $(H, \chi)$ -type owners to trade, then the quality of an average tree in the pool would be  $\hat{\pi}_t = \frac{\mathbb{P}_t(H, \chi)}{\mathbb{P}_t(H, \chi) + 1 - \pi}$ , which evolves over time. For example, the more trade there is at  $t - 1$  (i.e., higher  $s_{t-1}$ ), the more efficiently are assets allocated at  $t$  (i.e., lower  $\mathbb{P}_t(H, \chi)$ ), and therefore the worse is the quality of an average tree in the pool (i.e., lower  $\hat{\pi}_t$ ). Thus, with positively correlated productivity shocks, past market liquidity reduces market liquidity today. As we show next, though our main results remain unchanged, history dependence in market liquidity can introduce a new type of equilibria, in which the economy undergoes endogenous deterministic cycles.

**Definition 4** *We say that an equilibrium features cyclical trade if both high and low quality assets trade every  $T > 1$  periods, but only low quality assets trade otherwise.*

In the following proposition, we summarize our findings.

**Proposition 3** *If  $\rho^\omega > \lambda$ , there exist thresholds  $0 < \tilde{\pi}_{ET} < \tilde{\pi}_{ICT} < 1$  such that: (i) efficient trade equilibrium exist when  $\pi \geq \tilde{\pi}_{ET}$ , (ii) either inefficient or cyclical trade equilibrium exists when  $\pi < \tilde{\pi}_{ICT}$ , and (iii) multiple equilibria exist when  $\pi \in (\tilde{\pi}_{ET}, \tilde{\pi}_{ICT})$ .*

The fraction of  $(H, \chi)$ -types at the beginning of period  $t$  is given by those who were  $(H, 1)$ -type at the end of period  $t - 1$  but became shocked at  $t$ :  $\frac{(1-\rho)\cdot\lambda}{1-\lambda} \cdot (\mathbb{P}_{t-1}(H, 1) + s_{t-1} \cdot \mathbb{P}_{t-1}(H, \chi))$ ; and by those who were  $(H, \chi)$ -type at the end of period  $t - 1$  and remained shocked at  $t$ :  $\rho \cdot (1 - s_{t-1}) \cdot \mathbb{P}_{t-1}(H, \chi)$ .<sup>16</sup>

As in Section 3.2, we can construct candidate constant price equilibria, in which the asset prices and distribution of owner types is the same in every period. From (20), the stationary fraction of  $(H, \chi)$ -types in any such equilibrium is given by:

$$\mathbb{P}_\infty(H, \chi) = \pi \cdot \lambda \cdot \frac{1 - \rho^\omega}{1 - \lambda} \cdot \frac{1}{1 - (1 - s_\infty) \cdot \left( \rho^\omega - \frac{(1-\rho^\omega)\cdot\lambda}{1-\lambda} \right)},$$

where  $s_\infty$  is the indicator that the  $(H, \chi)$ -type trades in a given constant price equilibrium.

Importantly,  $\mathbb{P}_\infty(H, \chi)$  is lower in the efficient trade than in the inefficient trade equilibrium. As a result, it is possible that neither of the two constant price equilibria exist: (i) in the efficient trade, the quality of the pool  $\hat{\pi}_\infty$  may be endogenously too low, implying low willingness to pay for the buyers and that the  $(H, \chi)$ -type would rather prefer to keep her tree; (ii) in the inefficient trade, the quality of the pool  $\hat{\pi}_\infty$  may be endogenously too high, implying that the buyers may be willing to bid aggressively and attract the  $(H, \chi)$ -type to trade. However, we can show that in this scenario there always exists a cyclical equilibrium with some period  $T > 1$ . In such an equilibrium, once the  $(H, \chi)$ -types trade, the pool quality is low enough so that in the next period only low quality trees trade in the market. But, as time evolves, from 20 we see that  $\mathbb{P}_t(H, \chi)$  and  $\hat{\pi}_t$  increase, so that the pool quality gradually improves eventually allowing for the  $(H, \chi)$ -types to be attracted to trade again. The precise length  $T$  of the cycle is determined by finding the first period in which the buyers' valuation of the tree exceeds the reservation value of the  $(H, \chi)$ -type owner.

### 4.3 Transitory quality and duration

In this section, we extend our baseline setup to the cases in which asset quality is transitory and asset duration is less than infinite.

First, we denote the quality of asset  $i$  at time  $t$  by  $\theta_{i,t}$ . As in our baseline setup, we assume that the unconditional probability of an asset being high quality is  $\mathbb{P}(\theta_{i,t} = H) = \pi$ ; but we now assume that  $\mathbb{P}(\theta_{i,t+1} = H | \theta_{i,t} = H) = \rho^\theta \in [\pi, 1]$ . The shocks to asset quality are assumed to be independent across assets, so the fraction of good quality assets at any point in time is

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<sup>16</sup>Notice that assets traded by  $(H, \chi)$ -types in period  $t - 1$  are held by  $(H, 1)$ -types at the end of the period.

also given by  $\pi$ . Second, we assume that at any point in time an asset matures and no longer delivers cash-flows with probability  $1 - \xi \in [0, 1]$ . Whether an asset matures is realized at the beginning of the period. When  $\xi < 1$ , the duration of any given asset is given by  $(1 - \xi)^{-1}$ ; otherwise, the duration is infinite.

The following proposition states that multiple equilibria, as given in Theorem 1, can arise as long as assets are not short-lived and there is some persistence in asset quality.

**Proposition 4** *If  $\rho^\theta > \pi$  and  $\xi > 0$ , there exist thresholds  $0 < \tilde{\pi}_{ET} < \tilde{\pi}_{IT} < 1$  such that: (i) efficient trade equilibrium exists when  $\pi \geq \tilde{\pi}_{ET}$ , (ii) inefficient trade equilibrium exists when  $\pi \leq \tilde{\pi}_{IT}$ , and (iii) multiple equilibria exist when  $\pi \in (\tilde{\pi}_{ET}, \tilde{\pi}_{IT})$ . Moreover, these thresholds depend on the parameters  $\delta$ ,  $\xi$  and  $\rho^\theta$  only through  $\hat{\delta} \equiv \delta \cdot \xi \cdot \frac{\rho^\theta - \pi}{1 - \pi}$ .*

## 5 Conclusions

We have presented a parsimonious model, which illustrates that when valuing assets we cannot separate sentiments, liquidity and fundamentals. Even in the absence of any changes to underlying asset fundamentals and with asset prices always corresponding to discounted cash flows, it is possible to generate volatility in asset prices and market liquidity as a result of changes in agents' expectations about future market conditions.

From a macroeconomic point of view, our model also illustrates that sentiments can generate substantial fluctuations in aggregate (or sectoral) output. Even though measured TFP may fluctuate, it is not the cause of output volatility; both are instead driven by shocks to expectations. These expectations could in turn be driven by pure sunspots or be connected to exogenous changes to fundamentals. In the latter case, sentiments would serve as an amplification mechanism. Thus, an outside observer, who cannot directly measure sentiments, may overestimate the effect of such fundamental shocks. One must therefore be careful when estimating and interpreting measures of TFP shocks.

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## Appendix A - Proofs for Section 3

**Proof of Proposition 1.** See text. ■

**Proof of Proposition 2.** Note that the equilibrium price satisfies:

$$p^* = E\{x_\theta + \delta V(\theta', \omega') | (\theta, \omega) \in \Gamma(p^*), \tilde{\omega} = 1\} \geq E\{x_\theta + \delta V(\theta', \omega') | \theta = L\} \quad (21)$$

where the right-hand side is equal to the value of the  $(L, 1)$ -type if in equilibrium she holds on to the asset; otherwise,  $V(L, 1) = p^*$ . Thus, in equilibrium the  $(L, 1)$ -type always trades and  $V(L, 1) = p^*$ . On the other hand  $(L, \chi)$ -type has a weakly lower value than the  $(L, 1)$ -type since the quality of her asset is the same, but her flow payoff is lower. Hence, in equilibrium she also trades and  $V(L, \chi) = p^*$ . Finally,  $V(H, \chi) \geq p^*$  holds trivially since the holder always has the option to trade at price  $p^*$ , and  $V(H, 1) > p^*$  follows from the fact that all low types trade and therefore:

$$p^* = E\{x_\theta + \delta V(\theta', \omega') | (\theta, \omega) \in \Gamma(p^*), \tilde{\omega} = 1\} < E\{x_\theta + \delta V(\theta', \omega') | \theta = H\} = V(H, 1), \quad (22)$$

which implies that buyers cannot attract the  $(H, 1)$ -type to trade without making losses in expectation. Thus, indeed, all low quality asset trade at all times, but the high quality assets held by unshocked owners never do. ■

**Proof of Theorem 1.** That there can at most be two types of constant price equilibria follows from Proposition 2, which shows that there are only two possibilities depending on whether the  $(H, \chi)$ -type trades or not.

Efficient trade. The equations (5), (6), and (8) characterize the equilibrium owner values  $V^{HT}$  and asset price  $p^{*,HT}$  in candidate efficient trade equilibria. Since this is a system of linear equations, if an efficient trade equilibrium exists, it is unique. Moreover, this equilibrium exists if and only if inequality (10) is satisfied. Combining (5) - (10), we have that the efficient trade equilibrium exists if and only if:

$$(\chi x_H - \hat{\pi} x_H - (1 - \hat{\pi}) x_L) + \delta (1 - \hat{\pi}) \frac{(1 - \lambda)(1 - \hat{\pi})(x_H - x_L)}{1 - \delta(1 - \hat{\pi})(1 - \lambda)} \leq 0, \quad (23)$$

where  $\hat{\pi} \equiv \frac{\lambda\pi}{\lambda\pi+1-\pi}$ . Note that the left-hand side is strictly decreasing in  $\pi$ , positive at  $\pi = 0$  and negative at  $\pi = 1$ . Hence, the threshold  $\bar{\pi}_{ET} \in (0, 1)$  exists, is unique, and the efficient trade equilibrium exists if and only if  $\pi \geq \bar{\pi}_{ET}$ .

Inefficient trade. The equations (12), (13), and (14) characterize the equilibrium owner values

$V^{IT}$  and asset price  $p^{*,IT}$  in candidate inefficient trade equilibria. Since this is a system of linear equations, if an inefficient trade equilibrium exists, it is unique. Moreover, this equilibrium exists if and only if inequality (16) is satisfied. Combining (12) - (16), we have that the inefficient trade equilibrium exists if and only if:

$$0 \leq (\chi x_H - \hat{\pi} x_H - (1 - \hat{\pi}) x_L) + \delta (1 - \hat{\pi}) \frac{(1 - \lambda + \lambda \chi) x_H - x_L}{1 - \delta}, \quad (24)$$

where  $\hat{\pi} \equiv \frac{\lambda \pi}{\lambda \pi + 1 - \pi}$ . Note that the right-hand side is strictly decreasing in the belief  $\pi$ , positive when  $\pi = 0$  and negative when  $\pi = 1$ . Hence, the threshold  $\bar{\pi}_{IT} \in (0, 1)$  exists, is unique, and the inefficient trade equilibrium exists if and only if  $\pi \leq \bar{\pi}_{IT}$ .

Existence and Multiplicity. Next, we show that  $\bar{\pi}_{ET} < \bar{\pi}_{IT}$ , which will establish that an equilibrium exists and that the two equilibria coexist whenever  $\pi \in (\bar{\pi}_{ET}, \bar{\pi}_{IT})$ . From (23) and (44), we have that  $\bar{\pi}_{ET} < \bar{\pi}_{IT}$  if and only if:

$$\frac{(1 - \lambda)(1 - \hat{\pi})(x_H - x_L)}{1 - \delta(1 - \hat{\pi})(1 - \lambda)} \Big|_{\pi = \bar{\pi}_{ET}} < \frac{(1 - \lambda + \lambda \chi)x_H - x_L}{1 - \delta}. \quad (25)$$

But the latter inequality must hold because for any  $\pi < 1$ , we have:

$$\begin{aligned} \frac{(1 - \lambda)(1 - \hat{\pi})(x_H - x_L)}{1 - \delta(1 - \hat{\pi})(1 - \lambda)} &\leq \frac{(1 - \lambda)(x_H - x_L)}{1 - \delta(1 - \lambda)} \\ &< \frac{(1 - \lambda)(x_H - x_L) + \lambda(\chi x_H - x_L)}{1 - \delta} \\ &= \frac{(1 - \lambda + \lambda \chi)x_H - x_L}{1 - \delta}, \end{aligned}$$

where we used the fact that  $\chi x_H \geq x_L$ .

Finally, from equations (8), (9), (14), and (15) we see that asset prices, output and welfare are strictly higher in the efficient trade than in the inefficient trade equilibrium. ■

**Proof of Theorem 2.** Suppose that  $\pi \in (\bar{\pi}_{ET}, \bar{\pi}_{IT})$ . The system of equations that characterizes sentiment equilibrium with sunspot  $z_t$  is given by:

$$V(H, 1, z) = x_H + \delta E \{V(H, \omega', z') | z\} \quad (26)$$

for  $z \in \{B, G\}$ ,

$$V(H, \chi, z) = \begin{cases} p^*(G) & \text{if } z = G \\ \chi \cdot x_H + \delta E \{V(H, \omega', z') | z = B\} & \text{if } z = B \end{cases}, \quad (27)$$

and

$$V(L, \omega, z) = p^*(z), \quad (28)$$

for  $\omega \in \{\chi, 1\}$  and  $z \in \{B, G\}$ . The equilibrium asset prices are given by:

$$p^*(z) = \begin{cases} \hat{\pi} \cdot V(H, 1, G) + (1 - \hat{\pi}) \cdot (x_L + \delta \cdot E\{p^*(z') | z = G\}) & \text{if } z = G \\ x_L + \delta \cdot E\{p^*(z') | z = B\} & \text{if } z = B \end{cases} \quad (29)$$

Moreover, there are no profitable deviations for the buyers and for the owners if and only if the equilibrium prices and values satisfy:

$$p^*(G) \geq \chi \cdot x_H + \delta E\{V(H, \omega', z') | z = G\}, \quad (30)$$

and

$$\hat{\pi}V(H, 1, B) + (1 - \hat{\pi})(x_L + \delta \cdot E\{p^*(z') | z = B\}) \leq V(H, \chi, B). \quad (31)$$

In (26) through (29), we have a system of linear equations that determine the equilibrium asset prices and values  $\{p^*(z), V(\theta, \omega, z)\}$  in a candidate sentiment equilibrium. Furthermore, these prices and values are continuous in  $\rho$ .

Next, note that as  $\rho \rightarrow 1$ , we have that (i)  $V(\theta, \omega, G) \rightarrow V^{ET}(\theta, \omega)$  and  $V(\theta, \omega, B) \rightarrow V^{IT}(\theta, \omega)$  for all  $(\theta, \omega)$ , and (ii)  $p^*(G) \rightarrow p^{*ET}$  and  $p^*(B) \rightarrow p^{*IT}$ . Therefore, by continuity, if  $\pi \in (\bar{\pi}_{ET}, \bar{\pi}_{IT})$ , then a sentiment equilibrium must exist provided that  $\rho$  is sufficiently large. In contrast, as  $\rho \rightarrow \frac{1}{2}$ , then the left-hand sides and the right-hand sides of inequalities (30) and (31) coincide. Therefore, by continuity, generically, a sentiment equilibrium cannot exist when  $\rho$  is sufficiently small. ■

## Appendix B - Proofs for Section 4

**Proof of Proposition 3.** Using the same arguments as in the construction of the efficient trade equilibrium in Section 3.2, we can show that the efficient trade equilibrium exists if and only if:

$$\chi x_H - \hat{\pi} x_H - (1 - \hat{\pi}) x_L + \delta (1 - \hat{\rho}^\omega) (1 - \hat{\pi}) \cdot \frac{\left(\frac{1 - \rho^\omega}{1 - \hat{\rho}^\omega} - \hat{\pi}\right) (x_H - x_L)}{1 - \delta (1 - \hat{\rho}^\omega) (1 - \hat{\pi})} \leq 0, \quad (32)$$

where  $\hat{\rho} = \frac{1-\rho}{1-\lambda}$  and  $\hat{\pi} = \frac{\pi\hat{\rho}^\omega}{\pi\hat{\rho}^\omega+1-\pi}$ . Note that this inequality becomes the same as (11) when  $\rho^\omega \rightarrow \lambda$ . The left-hand side is strictly decreasing in  $\pi$ , positive at  $\pi = 0$  and negative at  $\pi = 1$ . Hence, the threshold  $\tilde{\pi}_{ET} \in (0, 1)$  that sets this inequality to an equality exists, is unique, and the efficient trade equilibrium exists if and only if  $\pi \geq \tilde{\pi}_{ET}$ .

Analogously, we can show that the inefficient trade equilibrium exists if and only if:

$$0 \leq \chi x_H - \hat{\pi} x_H - (1 - \hat{\pi}) x_L + \delta (1 - \hat{\pi}) \frac{\left( \chi + \frac{1-\rho^\omega}{1-\delta(\rho^\omega-\hat{\rho}^\omega)} (1 - \chi) \right) x_H - x_L}{1 - \delta}, \quad (33)$$

where  $\hat{\rho}^\omega = \frac{1-\rho^\omega}{1-\lambda}$  and  $\hat{\pi} = \frac{\pi\lambda}{\pi\lambda+1-\pi}$ . Note that this inequality becomes the same as (17) when  $\rho^\omega \rightarrow \lambda$ . The right-hand side is strictly decreasing in  $\pi$ , positive at  $\pi = 0$  and negative at  $\pi = 1$ . Hence, the threshold  $\tilde{\pi}_{IT} \in (0, 1)$  that sets this inequality to an equality exists, is unique, and the inefficient trade equilibrium exists if and only if  $\pi \leq \tilde{\pi}_{ET}$ .

Define threshold  $\tilde{\pi}_{ITC}$  to be the value of  $\pi$  that sets inequality (33) to equality, but where  $\hat{\pi} = \frac{\pi\hat{\rho}^\omega}{\pi\hat{\rho}^\omega+1-\pi}$ . Below, we will show that a cyclical equilibrium of period  $T > 1$  exists when  $\pi \in (\tilde{\pi}_{IT}, \tilde{\pi}_{ITC})$ , which is non-empty since  $\rho^\omega > \lambda$  implies  $\tilde{\pi}_{IT} < \tilde{\pi}_{ITC}$ . Next, we establish that  $\tilde{\pi}_{ET} < \tilde{\pi}_{ITC}$ , which proves our result that when  $\pi \in (\tilde{\pi}_{ET}, \tilde{\pi}_{ITC})$ , the efficient trade equilibrium coexists with either the inefficient or the cyclical trade equilibrium. But the latter inequality holds if and only if:

$$\frac{(1 - \hat{\rho}^\omega) \left( \frac{1-\rho^\omega}{1-\hat{\rho}^\omega} - \hat{\pi} \right) (x_H - x_L)}{1 - \delta (1 - \hat{\rho}^\omega) (1 - \hat{\pi})} \Big|_{\pi=\tilde{\pi}_{ET}} < \frac{\left( \chi + \frac{1-\rho^\omega}{1-\delta(\rho^\omega-\hat{\rho}^\omega)} (1 - \chi) \right) x_H - x_L}{1 - \delta}, \quad (34)$$

which follows from the fact that for any  $\pi < 1$ ,

$$\begin{aligned} \frac{(1 - \hat{\rho}^\omega) \left( \frac{1-\rho^\omega}{1-\hat{\rho}^\omega} - \hat{\pi} \right) (x_H - x_L)}{1 - \delta (1 - \hat{\rho}^\omega) (1 - \hat{\pi})} \Big|_{\pi=\tilde{\pi}_{ET}} &\leq \frac{(1 - \rho^\omega) (x_H - x_L)}{1 - \delta (1 - \hat{\rho}^\omega)} \\ &< \frac{(1 - \rho^\omega) (x_H - x_L) + \rho^\omega (\chi x_H - x_L)}{1 - \delta} \\ &= \frac{(1 - \rho^\omega + \rho^\omega \chi) x_H - x_L}{1 - \delta} \\ &\leq \frac{\left( \chi + \frac{1-\rho^\omega}{1-\delta(\rho^\omega-\hat{\rho}^\omega)} (1 - \chi) \right) x_H - x_L}{1 - \delta}, \end{aligned}$$

where we used that  $\chi x_H \geq x_L$  and  $\rho^\omega > \hat{\rho}^\omega$ .

We now show that when  $\pi \in (\tilde{\pi}_{IT}, \tilde{\pi}_{ITC})$ , then there exists a cyclical equilibrium of length  $T > 1$ . Thus, consider a candidate equilibrium with cycle length  $T$ . Let  $\tau \in \{1, \dots, T\}$  denote

the time that has passed since the  $(H, \chi)$ -types traded the last time. The stationary distribution of pool quality is then given by:

$$\hat{\pi}_\tau = \frac{\mathbb{P}_\tau(H, \chi)}{\mathbb{P}_\tau(H, \chi) + 1 - \pi} = \frac{\pi \hat{\rho} \cdot \frac{1 - (\rho - \hat{\rho}^\omega)^\tau}{1 - (\rho - \hat{\rho}^\omega)}}{\pi \hat{\rho} \cdot \frac{1 - (\rho - \hat{\rho}^\omega)^\tau}{1 - (\rho - \hat{\rho}^\omega)} + 1 - \pi}, \quad (35)$$

where  $\hat{\pi}_\tau$  is strictly increasing in  $\tau$  with  $\hat{\pi}_1 = \frac{\pi \cdot \hat{\rho}^\omega}{\pi \cdot \hat{\rho}^\omega + 1 - \pi} < \frac{\pi \cdot \lambda}{\pi \cdot \lambda + 1 - \pi} = \lim_{\tau \rightarrow \infty} \hat{\pi}$ .

Let  $V_\tau$  and  $p_\tau$  denote the equilibrium owner values and asset prices, as they depend on  $\tau$ . Then,

$$p_\tau^* = \begin{cases} x_L + \delta \cdot p_{\tau+1}^* & \text{if } \tau < T \\ \hat{\pi}_T \cdot V_T(H, 1) + (1 - \hat{\pi}_T) \cdot (x_L + \delta \cdot p_1^*) & \text{if } \tau = T \end{cases}, \quad (36)$$

and the values are  $V_\tau(L, \chi) = V_\tau(L, 1) = p_\tau^*$ ,

$$V_\tau(H, 1) = x_H + \delta \cdot (\hat{\rho}^\omega \cdot V_{\tau+1}(H, \chi) + (1 - \hat{\rho}^\omega) \cdot V_{\tau+1}(H, 1)), \quad (37)$$

$$V_\tau(H, \chi) = \begin{cases} \chi \cdot x_H + \delta \cdot (\rho \cdot V_{\tau+1}(H, \chi) + (1 - \rho) \cdot V_{\tau+1}(H, 1)) & \text{if } \tau < T \\ p_T^* & \text{if } \tau = T \end{cases}. \quad (38)$$

To show that such an equilibrium exists, we must check that neither the buyers nor the owners want to deviate, i.e., the buyers cannot attract the  $(H, \chi)$ -type in periods  $\tau \neq T$ :

$$V_\tau(H, \chi) \geq \hat{\pi}_\tau \cdot V_\tau(H, 1) + (1 - \hat{\pi}_\tau) \cdot (x_L + \delta \cdot p_{\tau+1}^*), \quad (39)$$

and the  $(H, \chi)$ -type prefers to trade rather than keep her asset in period  $T$ :

$$p_T^* \geq \chi \cdot x_H + \delta \cdot (\rho \cdot V_1(H, \chi) + (1 - \rho) \cdot V_1(H, 1)). \quad (40)$$

Notice that the efficient trade equilibrium is a special case of a cyclical equilibrium, in which the cycle length is  $T = 1$ , whereas the inefficient trade equilibrium has cycle  $T = \infty$ . Therefore, when  $\pi \in (\tilde{\pi}_{IT}, \tilde{\pi}_{ICT})$ , neither  $T = 1$  nor  $T = \infty$  can be an equilibrium. In the former case, the  $(H, \chi)$ -type owner wants to deviate and keep her asset. In the latter case, the buyers want to deviate and attract the  $(H, \chi)$ -type to trade. Next, we show that there exists a  $0 < T < \infty$  such that neither the buyers nor the owners want to deviate, thus establishing the result.

These no-deviation conditions can be expressed compactly as follows:

$$\hat{\pi}_\tau \cdot V_\tau(H, 1) + (1 - \hat{\pi}_\tau) \cdot (x_L + \delta \cdot p_{\tau+1}^*) \begin{cases} \leq \chi \cdot x_H + \delta \cdot (\rho \cdot V_{\tau+1}(H, \chi) + (1 - \rho) \cdot V_{\tau+1}(H, 1)) & \text{if } \tau < T \\ \geq \chi \cdot x_H + \delta \cdot (\rho \cdot V_1(H, \chi) + (1 - \rho) \cdot V_1(H, 1)) & \text{if } \tau = T \end{cases} \quad (41)$$

In search of a contradiction, suppose that for all  $T$ :

$$\hat{\pi}_\tau \cdot V_\tau(H, 1) + (1 - \hat{\pi}_\tau) \cdot (x_L + \delta \cdot p_{\tau+1}^*) \leq \chi \cdot x_H + \delta \cdot (\rho \cdot V_{\tau+1}(H, \chi) + (1 - \rho) \cdot V_{\tau+1}(H, 1)). \quad (42)$$

Fix  $\tau$  and note that  $\lim_{T \rightarrow \infty} V_{\tau+1}(H, \omega) = V^{IT}(H, \omega)$  and  $\lim_{T \rightarrow \infty} p_{\tau+1}^* = p^{IT}$ . But then as  $T$  grows large, (42) becomes the same as (33), which defines threshold  $\tilde{\pi}_{IT}$ , except that the pool quality  $\hat{\pi} = \frac{\pi\lambda}{\pi\lambda+1-\pi}$  is replaced with  $\hat{\pi}_\tau$ . Since  $\pi > \tilde{\pi}_{IT}$ , (33) is violated. Because  $\hat{\pi}_\tau \rightarrow \hat{\pi}$ , there exists a finite  $\tau$  such that (42) is violated as well. ■

**Proof of Proposition 4.** Using the same arguments as in the construction of the efficient trade equilibrium in Section 3.2, we can show that the efficient trade equilibrium exists if and only if:

$$(\chi x_H - \hat{\pi} x_H - (1 - \hat{\pi}) x_L + \hat{\delta} (1 - \hat{\pi}) \frac{(1 - \lambda)(1 - \hat{\pi})(x_H - x_L)}{1 - \hat{\delta}(1 - \lambda)(1 - \hat{\pi})}) \leq 0, \quad (43)$$

where  $\hat{\delta} \equiv \delta \xi \frac{\rho^\theta - \pi}{1 - \pi}$  and  $\hat{\pi} = \frac{\pi\lambda}{\pi\lambda+1-\pi}$ . This condition is the same as equation (11) that determines the existence of efficient trade equilibrium in our baseline model, with the only exception of  $\hat{\delta}$  replacing the discount factor  $\delta$ . Therefore, the efficient trade equilibrium exists under the same conditions as in the baseline economy, but with the discount factor adjusted to  $\hat{\delta}$ .

Analogously, we can show that the inefficient trade equilibrium exists if and only if:

$$0 \leq (\chi x_H - \hat{\pi} x_H - (1 - \hat{\pi}) x_L) + \hat{\delta} (1 - \hat{\pi}) \frac{(1 - \lambda + \lambda\chi) x_H - x_L}{1 - \hat{\delta}}, \quad (44)$$

which is the same as equation (17) that determines the existence of inefficient trade equilibrium in our baseline model, with the only exception of  $\hat{\delta}$  replacing the discount factor  $\delta$ . Therefore, the inefficient trade equilibrium exists under the same conditions as in the baseline economy, but with the discount factor adjusted to  $\hat{\delta}$ .

Therefore, we conclude that the region for multiple equilibria exists, i.e.,  $0 < \tilde{\pi}_{ET} < \tilde{\pi}_{IT} < 1$ , since by assumption  $\hat{\delta} \in (0, 1)$ . Furthermore, note that the parameters  $\delta$ ,  $\xi$ , and  $\rho^\theta$  affect thresholds  $\tilde{\pi}_{ET}$  and  $\tilde{\pi}_{IT}$  only through  $\hat{\delta}$ . ■