Credit Rating Inflation and Firms' Investments

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Abstract

We analyze credit ratings' effects on firms' investments in a debt-rollover game that features a feedback loop. We show that the credit rating agency's (CRA) ex-ante real effects may be negative, especially when high-risk high-return projects are available to the firm. Potentially inflated credit ratings acting as new informative signals always have positive effects; hence, CRAs' adverse ex-ante real effects arise from the feedback between credit ratings and firms' investments, through which CRAs allows more firms to gamble for resurrection. Laxer rating standards may not be accompanied by higher rating inflation, suggesting new empirical implications.

Key Words: Credit rating agency, rating inflation, real effect, feedback effect, global game

JEL Classification: D82, D83, G24, G32

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1 Introduction

Since the collapse of Enron in 2002, many observers have been questioning the role of credit rating agencies (CRAs) in promoting the financial market's efficiency, criticizing CRAs for issuing overgenerous credit ratings to debt-issuing firms.[1] Later, after the $2007-2009$ financial crisis, the Financial Crisis Inquiry Report even concludes that "the failures of credit rating agencies were essential cogs in the wheel of financial destruction." Several empirical studies, such as Jiang, Stanford, and Xie (2012), Strobl and Xia (2012), and Cornaggia and Cornaggia (2013), formally document the credit rating inflation. They further show that credit rating inflation is attributed to the conflicts of interest caused by the "issuer-pays" business model prevailing in the credit rating industry: because CRAs are paid by issuers they are assessing, they will have strong incentives to assign overgenerous ratings, in order to charge higher fees and attract new clients.

It then follows from these criticisms that inflated credit ratings may mislead investors, make some bad firms funded, and thus have negative real effects. However, to mislead investors, credit ratings must be new informative signals; otherwise, rational and well-informed investors will not even look at them, and thus, CRAs should have no effect. On the other hand, if CRAs are providing investors with informative (though potentially biased) signals, they should be able to promote the market efficiency and thus guide, rather than mislead, investors. Hence, it seems that CRAs who potentially issue inflated credit ratings always have positive ex-ante effects (relative to the world without a CRA), even though they do not help to reach the first-best scenario where they always provide accurate information.

Then, when investors are perfectly rational and well-informed, how do potentially inflated credit ratings affect firms' investments? Will the CRA have positive or negative ex-ante real effects? How do credit ratings acting as new informative signals have negative real effects, if they do have any?

In this paper, we address these questions. We show that potentially inflated ratings can exclude extremely bad firms from creditors' belief supports and thus are positive signals about better-rated firms. Such positive signals then make creditors more optimistic, reduce firms' financial costs, and change certain firms' investment decisions. In particular, with high ratings and thus lower financial costs, some firms take risks instead of default efficiently, while some other firms switch from risky projects to safe projects. The CRA's ex-ante real effects, how-

[1] The overgenerous credit ratings are usually called *inflated* credit ratings, as they stand for better credit qualities than issuers actually have.

ever, change from positive to negative, as the upside returns of risky projects increase beyond a threshold. Potentially inflated credit ratings that act as informative signals correctly guide creditors and lead some firms to invest in socially optimal projects; hence, such so-called "informational effects" of the CRA are always positive. The CRA's negative real effects then arise from its "feedback effects," a component missing in the literature on credit rating inflation and credit ratings' real effects. Specifically, when issuing ratings, the CRA takes into account the creditors' and the firm's best responses to the ratings, hence it strategically assigns high ratings to more firms, allowing them to gamble for resurrection.[2]

The above conclusions rely on two critical features of credit ratings. First, because some of the firm's decisions are observable and verifiable, credit ratings are subject to a *partial verifiability constraint*. Specifically, we assume that after observing the financial cost, which is determined by the firm's fundamentals and the aggregation of creditors' debt-investment decisions, the firm may strategically opt for an early default, or invest either in a viable project (VP) with a high success probability and a positive expected NPV, or a risky project (HR) with a negative expected NPV due to a low probability of success. Importantly, whereas the firm's choice of VP or HR is hidden and unverifiable, the public can observe its early default. Therefore, the firm's early default will be the evidence against the CRA in any lawsuit or cause huge reputation costs to the CRA, if the CRA assigns a high rating to the firm. Hence, the CRA would never assign a high rating when it knows that the firm will default early. Put differently, if the CRA assigns a high rating, creditors will learn that the firm's fundamentals are not extremely bad, although a high rating does not warrant an investment in VP.

Second, a firm's debt market features coordination among creditors, as emphasized by Boot, Milbourn, and Schmeits (2006). In addition, Morris and Shin (2004) point out that the creditors may have heterogeneous private signals. Therefore, any individual creditor needs to consider the impact of credit ratings not only on his own belief but also on other creditors' decisions.

Our model, which captures the above two features of credit ratings, has a unique equilibrium. In the equilibrium, the conflicts of interest imply that the CRA always employs a lax

[2]The assumption that the CRA will consider the ratings' effects on the firm and its creditors is not moot. All credit rating agencies claim that their ratings are forward looking, emphasizing that they will assess the potential impact of foreseeable future events that include the impacts of the ratings themselves. In particular, Moody's, in a document that explains its rating process, explicitly states that "Moody's will proceed with issuing or changing a rating, notwithstanding the effect of the rating action on the issuer, including the possible effect on issuer's market access or conditional obligations. The level of rating that Moody's assigns to an issuer that might experience potential changes in market access or conditional obligations will reflect Moody's assessment of the issuer's creditworthiness, including such considerations."

rating strategy that assigns all firms, which invest in HR, the rating standing for the VP's credit quality.[3] However, a high credit rating generated by the lax rating strategy is not a cheap talk as in Crawford and Sobel (1982). Due to the partial verifiability constraint, the high rating provides creditors with a public signal about the firm. Such a public signal is endogenous and takes a different form from that in Morris and Shin (2002): it truncates the supports of creditors' interim beliefs from below. Hence, the high credit rating will shift up creditors' beliefs about the firm's fundamentals.

Such a belief shift, then, leads to a non-trivial increase in the amount of creditors who buy the firm's debt.[4] Because the creditors receive heterogeneous private signals, the inflated credit rating, which truncates their interim belief supports from below, will lead some of them to buy the debt. Such a direct effect is reinforced by the creditors' coordination incentives; that is, any individual creditor has stronger incentives to buy the debt when more other creditors are doing so.

Therefore, an inflated credit rating reduces the firm's financial costs and thus changes the firm's investment decisions. The CRA's real effects then vary by the firm's fundamentals. In particular, firms with fundamentals just above the threshold of the high rating will invest in HR, instead of defaulting early when there is no CRA, to gamble for resurrection; hence, for such firms, the CRA's real effects are negative, since HR leads to the lowest ex-ante social welfare. On the other hand, the high rating leads some other firms that invest in HR in the scenario without a CRA to switch to VP, implying positive real effects of the CRA.

Whether the ex-ante real effects of the CRA are positive or negative, however, depends on the exogenous economic environment. Specifically, the CRA's ex-ante real effects are negative, when the upside return of HR is beyond a threshold (but constrained by the assumption that HR leads to the lowest social welfare). To see this, we first notice that there are two effects of an increase in the upside return of HR. On the one hand, the CRA is able to assign the high rating

[3] In the core model, the equilibrium rating can be either the highest possible rating or the lowest possible one. This is so because in the core model, the CRA's rating behavior is constrained only by the partial verifiability constraint. In Section 7.1, we generalize the partial verifiability constraint to a scheme of rating-dependent reputation costs conditional on the failure of the firm. The analysis there implies that when the firm has several mutually exclusive investment projects with different success probabilities, medium ratings may be assigned in an equilibrium and be potentially inflated.

[4] In Section 7.2, we consider the case where there is only one creditor with a precise public signal; hence, there is no coordination among creditors. Then, the high rating can barely shift the creditor's belief about the firm's fundamentals, and thus cannot change her belief about the firm's investment decision. Consequently, in such a case, the high credit rating does not have any real effects on the firm's investment, showing the role of the coordination among creditors in our core model.

to firms with relatively bad fundamentals, allowing them to gamble for resurrection; on the other hand, some firms will choose HR over VP, because HR becomes more profitable. Therefore, as the upside return of HR increases, more firms pick the project with the lowest social welfare, and ultimately, the CRA's ex-ante real effects become negative. This provides a potential explanation for a long-lasting puzzle: the conflicts of interest caused by the issuer-pays business model in the credit rating industry have been recognized for a long time, but they did not attract much attention until the recent subprime crisis. Our argument implies that because the high-return high-risk financial products, such as MBSs, are available to the financial institutions, CRAs' ex-ante real effects were negative during the subprime crisis, putting them in criticisms.

The channel through which the CRA affects the firm's investment decisions implies that the CRA's real effects can be decomposed into two parts: its informational effects and its feedback effects. Indeed, the CRA's informational effects are equivalent to the real effects of a CRA who commits to assign the high rating to firms that do not default early in the scenario where there is no CRA. We refer to such a CRA as a "reflecting CRA." We show that though the rating strategy that the reflecting CRA commits to is likely inflated, a high rating assinged by the reflecting CRA will lead more firms to invest in VP than in the case without a CRA, because it provides creditors with a positive informative signal and thus lower the firm's financial costs. Therefore, the CRA's informational effects are always positive; that is, inflated credit ratings acting as new informative signals promote, rather than hurt, social welfare.

Then, the negative real effects of the CRA must arise from its feedback effects. Indeed, the CRA knows that it can assign the high rating to firms with relatively bad fundamentals, because with a high rating, such firms will invest in HR, instead of default early. Such a feedback between credit ratings and firms' investments leads to the CRA's adverse real effects, since it allows more firms to gamble for resurrection, and such adverse real effects are exacerbated when the upside return of HR increases.

Our theory has several empirical implications. Importantly, rating standards and rating inflation are two different endogenous terms, and laxer rating standards are not necessarily accompanied by higher rating inflation. We show this claim in several comparative static analyses, which also provide us with new empirical predictions about CRAs' credit rating standards and credit rating inflation. In particular, a decrease in the firm's transparency,[5] an increase in upside returns of risky projects, and an increase in the market liquidity will all lead to laxer

[5]This shares some similar spirits with Fong, Hong, Kacperczyk, and Kubik (2014), who find that higher analyst coverages provide investors with more information about firms, leading to stricter rating strategies.

rating standards (assigning high ratings to more firms). However, these changes of economic environments do not necessarily cause higher rating inflation. Specifically, a decrease in the firm's transparency has an ambiguous effect on rating inflation, an increase in upside returns of risky projects will cause higher rating inflation, and an increase in the market liquidity leads to lower rating inflation.

Our paper is closely related to the works about the feedback between credit ratings and the firm's behavior. Boot, Milbourn, and Schmeits (2006) uncover the CRA's coordination role when the firm has moral hazard problems. In their model, a credit rating sets a focal point for equilibrium selection when there exist multiple equilibria in the game between the firm and investors. Manso (2013) analyzes the feedback effect between the firm's endogenous default decision and the CRA's ratings. Holden, Natvik, and Vigier (2016) study how credit ratings reinforce the likelihood of investors' coordination failure and reveals a pro-cyclical impact of credit rating. In these models, the CRA always wants to provide accurate credit ratings, so there are no conflicts of interest between CRAs and investors, and there is no credit rating inflation. In this paper, we aim to address the question why credit ratings provide informative signals and thus promote the market efficiency but may still have negative ex-ante real effects. Hence, without incorporating information asymmetry, the feedback models cited above are not able to answer our question. Indeed, in this paper, we demonstrate how the CRA takes advantage of the information asymmetry and the feedback between credit ratings and firms' investments to allow more firms to gamble for resurrection, which leads to the CRA's adverse real effects. Therefore, it further develops the research agenda of CRAs' feedback effects.

Our paper also contributes to the literature on CRAs' rating inflation. This literature attributes credit rating inflation to investors' imperfect rationality (Bolton, Freixas, and Shapiro 2012; Skreta and Veldkamp 2009), or regulations tied to ratings (Opp, Opp, and Harris 2013).[6] However, in these models, inflated credit ratings are not informative signals to the investors who are naïve or have regulatory motives; therefore, they may not help address our main research question. We analyze CRAs' roles in an environment where investors are perfectly rational; hence, in our model, inflated credit ratings are informative signals to all creditors. Then, we are able to study the real effects of credit ratings that provide informative signals and promote the market efficiency.

[6]One exception is Frenkel (2015) who shows credit rating inflation may be generated by CRAs' "double reputation." However, one necessary condition in Frenkel (2015) is that the CRA is possibly of behavioral types, "honest" or "corrupt." In contrast, in our paper, the conflicts of interest caused by the issuer-pays business model are commonly known by all creditors, which is a better description of the real credit market, especially after the subprime crisis.

In our model, creditors play a global game (Carlsson and van Damme 1993; Morris and Shin 2003). Our model differs from the traditional global games mainly in that the creditors' dominant region of not buying the debt is endogenous, which is determined by the CRA's rating strategy. Our model's inclusion of endogenous information provided by the CRA relates it to the growing literature of global games with endogenous information and information manipulation. Angeletos, Hellwig, and Pavan (2006) and Angeletos and Pavan (2013) model the signaling effects of the government's preemptive defending policies, which pools very strong governments and very weak ones together. Edmond (2013) discusses a dictator's costly private information manipulation, but all revolutionaries' interim beliefs have full supports. Hence, the belief updating in these models differs from that in our model. In fact, the belief updating in our model is closer to that in Angeletos, Hellwig, and Pavan (2007) and Huang (2017). Nevertheless, our model has a unique equilibrium, because a high credit rating, the endogenous information, does not completely remove creditors' dominant region of not investing. Finally, our paper is also related to Goldstein and Huang (2016) who show how the government persuades depositors not to run a commercial bank by committing to a monitoring policy. In our current model, the CRA cannot commit to a rating strategy, and the firm has severe moral hazard issue; therefore, unlike the government in Goldstein and Huang (2016), the CRA in the current paper may have negative ex-ante real effects.

Our paper also contributes to the literature on experts' disclosure or persuasion. Lizzeri (1999) considers the information manipulation of perfectly informed information intermediaries. He shows that a monopoly intermediary's optimal disclosure choice is to reveal that the quality remains above a minimum standard. We model a CRA as a certified expert who discloses information about the firm to creditors. Our conclusion shares some spirit with Lizzeri (1999): a high rating does not mean the firm will invest in VP, but implies that the firm will not default early. However, the lowest type of the firm that receives a high rating is endogenously determined by how the creditors interpret the inflated rating. In addition, our model differs from those in the literature on Bayesian persuasion, such as Kamenica and Gentzkow (2011), in that the CRA cannot commit to a rating rule and thus make no information design.

We organize the rest of the paper as follows. In Section 2, we describe our model. Section 3 establishes a benchmark without a CRA. In Section 4, we analyze credit ratings' informativeness and the CRA's equilibrium rating strategy. Section 5 compares the firm's equilibrium investments in the model with the CRA to those in the benchmark model without a CRA to derive the CRA's real effects. It then further study the CRA's informational effects and feedback effects. Section 6 presents some empirical implications, and Section 7 discusses two important model assumptions. Section 8 concludes. All proofs appear in the Appendix.

2 A Model of Corporate Credit Ratings

We study a model of a CRA that is assigning credit ratings to a firm who needs to roll over its maturing debt. There are three dates, $t = 0, 1, 2$. At the beginning of date 0, the firm's existing debt is mature, and so it has to repay \$1 to the existing debt holders. To finance such \$1, the firm can issue new debt (with relatively low costs) or borrow from a predetermined bank credit line (with relatively high costs). At date 0, the CRA assigns credit ratings to the firm.[7] Observing the rating, new creditors in the debt market (with the measure $1 - \gamma$) simultaneously decide whether to buy the newly issued debt or not. At date 1, depending on the financial cost, the firm may choose to default or to continue investing. In the latter case, the cash flow is realized at date 2, and, if possible, creditors are paid in full.

2.1 Firm Investment and Social Welfare

Following Boot, Milbourn, and Schmeits (2006), we assume that if the firm fully repays the existing debt, it can continue investments either in a viable (i.e., low-risk) project VP or a high-risk alternative HR at date 1. VP generates a cash flow $V > 0$ with probability $p \in (0, 1)$; however, it fails with probability $1 - p$. Similarly, HR generates a cash flow $H > V$ with probability $q \in (0, p)$ but fails with probability $1 - q$. The firm will receive a zero cash flow if the project fails. Since both VP and HR fail with positive probabilities, the firm's investment choice between VP and HR is unobservable and unverifiable.[8]

At date 1, instead of investing in VP or HR, the firm may choose to default. In such a case, the firm will not borrow from the bank credit line, and its liquidation value is $B \in (0, \gamma]$. We assume that the liquidation value and the funds from the newly issued debt are used to repay the exiting debt, since the existing debt is senior.[9] If the firm defaults at date 1, the game ends, and thus its early default decision is publicly observable and verifiable.

[7]In the real financial market, CRAs assign credit ratings to both "issuers" and specific "issues." Although these two types of ratings are consistent in our model, we focus mainly on the issuer credit ratings in this paper.

[8]In practice, creditors may know the name of the project the firm invests in, but they usually lack the professional knowledge to judge whether the project is HR or VP. Therefore, the choice between VP and HR is unverifiable even ex post.

[9] We assume $B \leq \gamma$ for simplicity. By this assumption, when the firm defaults at date 1, the amount of funds available is at most \$1. Hence, any creditor who buys the newly issued debt will get nothing.

Social welfare is ranked by the firm's decisions.[10] When the firm invests in VP, the social welfare is $pV - 1$; when the firm invests in HR, the social welfare is $qH - 1$; and if the firm defaults at date 1, the social welfare is $B - 1$. We assume that it is very unlikely for HR to generate a positive cash flow; specifically,

$$pV > 1 > B > qH. \tag{1}$$

Therefore, VP has a positive expected NPV and is the social optimal investment project. Inequality (1) also implies that whereas both HR and early default lead to negative social welfare, an investment in HR leads to an even lower social welfare than the early default does.

2.2 Financing

There is a continuum of creditors with measure $1 - \gamma$ in the debt market, each having \$1. Here, γ measures the liquidity of the debt market, with a larger γ meaning a lower liquidity level of the debt market. We assume that $\gamma \in (0, 1)$, and so it is impossible for the firm to finance all the \$1 by just issuing new debt.

The new debt is a zero-coupon bond with the face value $F > 1$. It matures at date 2. So long as the firm does not default either endogenously at date 1 or exogenously at date 2, the creditors who buy the new debt will get full repayment.[11]

We assume $pF > 1$, and so if any creditor i knows that the firm will invest in VP, he will buy the new debt. On the other hand, the probability that HR is successful is so low ($qF < qH < B < 1$) that creditor i will not buy the new debt, if he knows that the firm will surely invest in HR. Obviously, if creditor i knows that the firm will default early, he does not buy the new debt either. We denote by $a_i \in \{0, 1\}$ creditor i's debt-investment decision, where $a_i = 1$ means creditor i buys the new debt, while $a_i = 0$ means creditor i does not buy.

[10]This notion of social welfare is indeed the sum of all agents' ex-ante payoffs at date 0, because the firm's financial cost or liquidation value is paid to the bank, the new debt holders, or the existing debt holders. Note that the financial cost may be higher than the expected cash flows generated by a project; in such a case, the bank is earning a rent.

[11]To focus on the role of the credit rating agency, we take the new debt contract as exogenously given, as in He and Xiong (2012). One might argue that a higher promised payment F' can attract more creditors, and so the firm can reduce its financial cost by offering F'. However, an endogenously promised payment $F' > F$ will have signaling effects. Because it immediately rules out all θ's such that $f(\theta) < F'$, a higher promised payment F' may be self-defeated, and so its effect is ambiguous. Furthermore, there exists a pooling equilibrium, in which the firm will always offer the promised payment F, with the off-equilibrium path belief that any firm choosing $F' \neq F$ will invest in HR.

We denote by W the measure of creditors who buy the new debt, and so the firm needs to finance $1 - W$ from the bank credit line. The firm can withdraw up to \$1 from the credit line with the constant marginal cost $f(\theta)$. Here, θ represents the firm's capacity to manage liquidity and is drawn by nature from the real line $\mathbb{R}$, according to a common improper uniform distribution. (We also call θ the fundamentals of the firm, and call a firm with the fundamentals θ the "θ-firm.") The function $f(\cdot)$ is differentiable and strictly decreasing. When the firm's fundamentals are extremely good, the marginal cost of the credit line financing approaches the payment of the new short-term debt; that is, $\lim_{\theta \to +\infty} f(\theta) = F$. However, when the firm's fundamentals are extremely bad, borrowing from the credit line is extremely expensive, so $\lim_{\theta \to -\infty} f(\theta) = +\infty$. Therefore, if the firm decides to invest in either VP or HR, the firm's financial cost is

$$K(\theta) = WF + (1 - W) f(\theta). \tag{2}$$

One important feature of our model is that the firm's fundamentals determine the marginal cost of the firm's non-debt financing (e.g., the bank credit line) rather than the qualities of the investment projects. The reasons of this modeling choice are twofold. First, because the firm will make the investment choices in an analysis of the real effects of CRAs, assuming that the uncertain fundamentals do not also affect the investment projects' qualities can largely simplify the analysis. In addition, as it will become clear later in Subsection 4.1, if the firm's uncertain fundamentals determine the investment projects' qualities only, the firm will never choose to default at date 1, which makes a high credit rating completely uninformative in our model.

Second, and more importantly, the firm's liquidity management is a key criterion CRAs consider when assigning ratings but is missing in most of the literature on CRAs. Hence, one contribution of our model is to link the firm's investments, its liquidity management, and CRAs, by showing that the firm's liquidity management will affect the firm's investments not only directly through the fund-providing channel but also indirectly through the credit ratings it will be assigned. While we model the firm's ability to manage liquidity as the marginal cost of the predetermined bank credit line, it may refer to other factors of liquidity management, such as the collateral value of the firm's real estate and the derivatives to hedge (Almeida, Campello, Cunha, and Weisbach 2014).

This feature of our model indeed matches the fall of Bear Stearns well.[12] Right before Bear Stearns collapsed, executives and regulators believed that the firm was solvent. However, its

[12] CRAs played an important role in the fall of Bear Stearns. About one month before the firm's collapse, CRAs all assigned strong ratings to Bear Stearns. But as its liquidity was revealed to be weak, CRAs began to downgrade Bear Stearns, further exacerbating its liquidity problem.

liquidity dropped by \$16 billion in the four days before collapse. In addition, Bear Stearns called JP Morgan to request a credit line but was turned down. Therefore, as concluded by the Financial Crisis Inquiry Report (2011), the fall of Bear Stearns was partly due to its insufficient liquidity.

2.3 Firm's Payoff

The firm has limited liability. If it defaults, whether endogenously at date 1 or exogenously at date 2 (when the project fails), its payoff is zero. If the firm generates a positive cash flow at date 2, the firm needs to repay the creditors according to the new debt contract. Therefore, the firm's payoff U depends on its own investment choice and its financial cost:

$$U = \begin{cases} 0, & \text{if the firm defaults at date 1;} \\ p\left[V - WF - (1-W)f(\theta)\right], & \text{if the firm invests in VP;} \\ q\left[H - WF - (1-W)f(\theta)\right], & \text{if the firm invests in HR.} \end{cases} \tag{3}$$

2.4 Information Structure

The firm's liquidity management ability, θ, is the firm's private information, which remains unknown to creditors. Before deciding whether to buy the new debt, each creditor i observes a private signal $x_i = \theta + \xi_i$, where $\xi_i \sim \mathcal{N}(0, \beta^{-1})$ is independent of θ and independent across all creditors. In our model, creditors are well-informed if and only if β is sufficiently large, and we aim to analyze credit ratings' effects on rational, well-informed creditors, and so in this paper, we focus on the case when β is sufficiently large. Besides their private signals, creditors will also observe a public credit rating by a credit rating agency (CRA).

2.5 Credit Rating Agency

The CRA assigns the firm a credit rating R.[13] We restrict the space of ratings to $R \in \{0, q, p\}$, because these are the only possible credit qualities of the firm: early default at date 1 means the firm will certainly default, and thus the firm's credit quality is 0; similarly, the firm investing in HR has a credit quality q, and the firm investing in VP has a credit quality p.

We assume that the CRA knows θ, so that we can separate credit rating bias due to the conflicts of interest from that caused by the CRA's capacity to acquire precise information.

[13] This may also be interpreted as a revision of the previous credit rating.

Because the CRA knows θ, there is no aggregate shock to the CRA. In addition, we consider pure strategies, and so the CRA can perfectly predict the firm's choice and its corresponding default probability at date 0. Hence, our model captures an important feature of credit ratings — forward-looking.

Due to the "issuer-pays" business model, the CRA always has incentives to assign the firm a high credit rating, in order to please issuers. The CRA's incentives to assign high credit ratings may also come from issuers' rating shopping (Bolton, Freixas, and Shapiro 2012), or the CRA's reputation for being nice to issuers (Frenkel 2015). Therefore, for simplicity, we assume that in the core model, for each θ, the CRA wants to maximize the nominal rating. We formally model the CRA's payoff in section 7.1, where the CRA's revenue and costs (conditional on the failure of the firm) are both strictly increasing in the nominal rating it assigns.

A partial verifiability condition constrains the CRA's rating. The CRA must beware of lawsuits resulting from verifiable frauds; that is, the CRA never wants to be caught lying. Consequently, for any given θ, the CRA wants to assign the firm the highest possible rating, provided that it cannot be verified as wrong (White 2013).[14]

2.6 Timeline and Equilibrium

We summarize the model's timeline in Figure 1 below. The CRA's rating strategy, denoted by $\mathcal{R}$, maps the firm's fundamentals to the rating space $\{0, q, p\}$; the firm's strategy maps its fundamentals, the CRA's rating, and the measure of creditors investing in the new debt to project choices; and creditors' strategies map their own private signals and the CRA's rating to their debt-investment decisions.

[14]Credit ratings are viewed as CRAs' "free speech." So, protected by the First Amendment, CRAs are not liable for any losses incurred by the inaccuracy of their ratings, unless it is proven that they know the ratings were false.

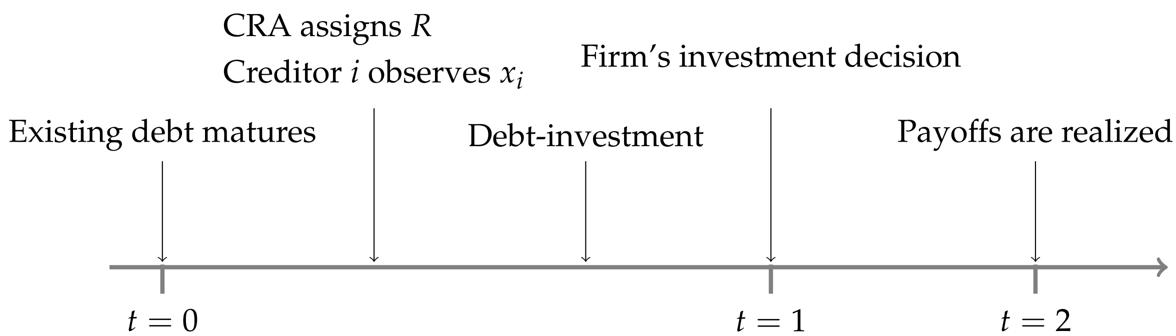

Figure 1: Timing

We are interested in monotone equilibria.

Definition 1 *The CRA's rating strategy, the firm's investment strategy, and creditors' debt-investment strategies constitute a monotone equilibrium, if*

1. *given the firm's investment strategy and creditors' debt-investment strategies, the CRA maximizes the nominal rating $\mathcal{R}(\theta)$ for all $\theta \in \mathbb{R}$ subject to the partial verifiability constraint;*
2. *given financial costs in equation (2), the firm's investment strategy will maximize the firm's expected profits;*
3. *given the CRA's rating strategy, the firm's investment strategy, and other creditors' strategies, any creditor i's strategy is monotonic in his private signal x_i and will maximize his expected payoff;*
4. *and, creditors use Bayes' rule to update their beliefs.*

3 The Benchmark: No CRA

In order to analyze the CRA's real effects, we set up a benchmark that excludes the CRA. In such a benchmark, when deciding whether to roll over the short-term debt, all patient creditors make choices solely based on their own private information. After observing the measure of creditors who roll over, the firm makes its investment choice. The model is similar to the debt-run model by Morris and Shin (2004), with the key difference being that the firm's choice is not binary.

Let's first analyze the firm's behavior in such a benchmark model. Because of the law of large numbers, given the creditors' strategies, the measure of creditors who roll over the debt is a deterministic function $W(\theta)$. Hence, any θ-firm's financial cost is deterministic:

$$W(\theta)F + (1 - W(\theta))\, f(\theta).$$

Since $H > V$, the θ-firm will default early, if and only if,[15]

$$W(\theta)F + (1 - W(\theta)) f(\theta) > H. \tag{4}$$

Conditional on that the θ-firm decides to continue investing, it invests in VP rather than HR, if and only if,

$$\begin{aligned} & p\,[V - W(\theta)F - (1 - W(\theta))f(\theta)] \geq q\,[H - W(\theta)F - (1 - W(\theta))f(\theta)] \\ \Rightarrow \quad & W(\theta)F + (1 - W(\theta))f(\theta) \leq \frac{pV - qH}{p - q}. \end{aligned} \tag{5}$$

The firm's choice between VP and HR is the same as in Boot, Milbourn, and Schmeits (2006). To enhance the model's interest, we assume that if all creditors, patient and impatient, roll over the debt, and so the firm can reach its lowest possible financial cost F, the firm will choose VP. That is, we maintain the assumption that

$$F < \frac{pV - qH}{p - q}.$$

As a result, given the creditors' strategies, the θ-firm's optimal investment strategy is

$$\begin{cases} \text{early default,} & \text{if} \quad W(\theta)F + (1 - W(\theta))f(\theta) > H; \\ \text{HR,} & \text{if} \quad W(\theta)F + (1 - W(\theta))f(\theta) \in \left(\frac{pV - qH}{p - q}, H\right]; \\ \text{VP,} & \text{if} \quad W(\theta)F + (1 - W(\theta))f(\theta) \leq \frac{pV - qH}{p - q}. \end{cases} \tag{6}$$

Since impatient creditors with measure $\gamma > 0$ will not roll over the debt for exogenous liquidity reasons, the firm has to withdraw some funds from the credit line, if it decides to invest in either VP or HR. Then, from the properties of the function $f(\cdot)$, we know that when the firm's fundamentals are extremely good ($\theta \to +\infty$), $f(\theta)$ is very close to the par value of the

[15] We assume that the firm will default at date 1, if its financial cost is larger than the highest possible cash flow the firm can generate. Because the firm's investment outcome and debt repayments are both publicly observable at date 2, if the firm's investment is successful, but it still defaults, the firm's manager will receive litigation punishments or incur a reputation loss.

firm's debt, F; then $WF + (1-W)f(\theta)$ is strictly less than $(pV - qH)/(p-q)$, implying that the firm will invest in VP. When the firm's fundamentals are extremely bad ($\theta \to -\infty$), $f(\theta)$ is extremely large, so that $WF + (1-W)f(\theta) > H$, implying that the firm will default early.

Hence, as shown in the global game literature, in such a benchmark model, all patient creditors have both the *dominant regions of rolling over and running*. That is, when creditor i's private signal x_i is extremely negative, he believes that the firm's fundamentals are weak, so that even if all other creditors roll over the debt, the firm's financial cost of investing in one project is beyond its highest possible cash flow H, and thus the firm will default at date 1. Therefore, creditor i will run, even when all other patient creditors roll over the debt. This establishes creditors' dominant region of running. Conversely, creditors also have a dominant region of rollover. If creditor i's private signal x_i is extremely positive, he believes that the firm's fundamentals are extremely good, and thus the firm will choose VP; as a result, creditor i will roll over the debt, even when all other patient creditors run. Therefore, as in other global game models, in a monotone equilibrium, any patient creditor will employ a cutoff strategy with the threshold $\tilde{x}$, such that he rolls over the debt, if and only if $x_i \geq \tilde{x}$.

Given θ and the creditors' cutoff strategy, the measure of creditors who roll over is

$$W(\theta) = (1-\gamma)\Pr(x \geq \tilde{x}|\theta) = (1-\gamma)\left\{1 - \Phi\left[\sqrt{\beta}(\tilde{x}-\theta)\right]\right\},$$

where $\Phi(\cdot)$ is the CDF of the standard normal distribution. Then, the θ-firm's financial cost is

$$\begin{aligned} K(\theta) &= (1-\gamma)\left\{1 - \Phi\left[\sqrt{\beta}(\tilde{x}-\theta)\right]\right\}F + \left[\gamma + (1-\gamma)\Phi[\sqrt{\beta}(\tilde{x}-\theta)]\right]f(\theta) \\ &= \left[(1-\gamma)F + \gamma f(\theta)\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(\tilde{x}-\theta)\right](f(\theta) - F). \end{aligned} \tag{7}$$

The first term in equation (7) is the financial cost resulting from the exogenous liquidity shocks to creditors, whereas the second term in equation (7) is the endogenous financial cost resulting from creditors' strategic complementarities.

As θ increases, that is, as the firm's fundamentals improve, the cost of withdrawing funds from the credit line decreases (since $f(\theta)$ is strictly decreasing), and the measure of patient creditors who roll over increases. Therefore, given creditors' cutoff strategies, the firm's financial cost strictly decreases in its fundamentals. In contrast with classical global games, in this benchmark model the firm has two indifference conditions. First, given the creditors' strategies, the firm will choose to default early if and only if $\theta < \tilde{\theta}_1$. This implies that

$$K(\tilde{\theta}_1) = \left[(1-\gamma)F + \gamma f(\tilde{\theta}_1)\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(\tilde{x}-\tilde{\theta}_1)\right]\left(f(\tilde{\theta}_1) - F\right) = H. \tag{8}$$

Because $K(\theta)$ is strictly decreasing, for any $\theta < \tilde{\theta}_1$, the firm's financial cost will be greater than H, the upside cash flow of HR; as a result, the firm would default at date 1. But if $\theta \geq \tilde{\theta}_1$, the firm can at least choose HR in order to receive a non-negative expected payoff due to its limited liability, and thus the firm will not default early.

When $\theta \geq \tilde{\theta}_1$, the firm needs to choose between VP and HR. From equation (6) and the fact that $K(\theta)$ is strictly decreasing in θ, there must be a $\tilde{\theta}_2 > \tilde{\theta}_1$, such that the firm will choose VP if and only if $\theta \geq \tilde{\theta}_2$. Hence,

$$K(\tilde{\theta}_2) = \left[(1-\gamma)F + \gamma f(\tilde{\theta}_2)\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(\tilde{x} - \tilde{\theta}_2)\right]\left(f(\tilde{\theta}_2) - F\right) = \frac{pV - qH}{p-q}. \tag{9}$$

Following the above arguments, in a monotone equilibrium, the firm will default early if $\theta < \tilde{\theta}_1$, invest in HR if $\theta \in \left[\tilde{\theta}_1, \tilde{\theta}_2\right)$, and invest in VP if $\theta \geq \tilde{\theta}_2$.

Any creditor i, receiving a private signal x_i about θ, first updates his belief about θ according to Bayes' rule:

$$\theta | x_i \sim \mathcal{N}(x_i, \frac{1}{\beta}).$$

Then, given the firm's strategy described above, creditor i calculates his return from rolling over the debt:

$$\left\{\Phi\left[\sqrt{\beta}(\tilde{\theta}_2 - x_i)\right] - \Phi\left[\sqrt{\beta}(\tilde{\theta}_1 - x_i)\right]\right\} qF + \left\{1 - \Phi\left[\sqrt{\beta}(\tilde{\theta}_2 - x_i)\right]\right\} pF.$$

Because any creditor will receive the payoff 1 if he runs, the creditor with private signal $\tilde{x}$ would be the marginal creditor who is indifferent about running or rolling over. As a result, the creditor's indifference condition is

$$\left\{\Phi\left[\sqrt{\beta}(\tilde{\theta}_2 - \tilde{x})\right] - \Phi\left[\sqrt{\beta}(\tilde{\theta}_1 - \tilde{x})\right]\right\} qF + \left\{1 - \Phi\left[\sqrt{\beta}(\tilde{\theta}_2 - \tilde{x})\right]\right\} pF = 1. \tag{10}$$

Proposition 1 below characterizes the equilibrium of the benchmark model.

Proposition 1 (The Unique Equilibrium in the Benchmark Model) *There exists a $\tilde{\beta} > 0$, such that for all*[16] $\beta > \tilde{\beta}$*, the benchmark model without a CRA has a unique equilibrium described by* $\left(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{x}\right)$*, where* $\tilde{\theta}_1 < \tilde{\theta}_2$*. In particular,*

[16] In this paper, we focus on the case that β is sufficiently large. There are two reasons. First, as in the global game literature, sufficiently precise private signals can guarantee equilibrium uniqueness, which is critical for the equilibrium analysis. More importantly, the aim of this paper is to study the credit ratings' effects when creditors have very precise private information, which refers to sufficiently large β in the model.

1. *the firm's investment strategy is*

$$\begin{cases} VP, & \text{if } \theta \geq \tilde{\theta}_2; \\ HR, & \text{if } \theta \in \left[\tilde{\theta}_1, \tilde{\theta}_2\right); \\ \text{defaults early}, & \text{if } \theta < \tilde{\theta}_1; \end{cases}$$

2. *and, any creditor i will roll over the firm's short-term debt if and only if $x_i \geq \tilde{x}$.*

4 Credit Rating Inflation

We now consider our core model where the CRA strategically designs the rating rule. As a step to solve an equilibrium, we first discuss the informativeness of credit ratings. Although the CRA always has incentives to assign overgenerous ratings, its rating strategy is subject to the partial verifiability constraint: the event of the firm's early default is publicly observable and thus verifiable. As a result, in the equilibrium, the CRA will assign a high rating if and only if the firm does not default at date 1. We will show that this partial verifiability constraint plays a critical role in determining the informativeness of the CRA's ratings.

We then solve the CRA's equilibrium rating strategy. Importantly, when assigning credit ratings, the CRA will take into account the effects of the ratings on the creditors' rollover decisions and thus the firm's investment choices. As we show later in Section 5, this feature, as well as the informativeness of credit ratings, is the key to understand the credit ratings' real effects.

4.1 Informativeness of a Rating Strategy

We first argue that an equilibrium rating strategy must be monotonic. Consider a rating strategy $\mathcal{R}(\theta)$ that assigns the rating p to θ'-firm and the rating q to θ''-firm, where $\theta'' > \theta'$. Because creditors' strategies are decreasing in their private signals in a monotone equilibrium, among firms that receive the same rating, the ones with better fundamentals have lower financial costs. This is because firms with better fundamentals have both lower non-debt financing costs and more creditors rolling over. Hence, if the CRA deviates to assign θ''-firm the rating p, θ''-firm will have a lower financial cost than θ'-firm. Because of the partial verifiability constraint, for the rating strategy $\mathcal{R}(\theta)$ being an equilibrium rating strategy, θ'-firm will not default at date 1. Then, θ''-firm will not default at date 1 either, if it receives the rating p. As a result, such a deviation is profitable, and thus, the rating strategy $\mathcal{R}(\theta)$ under consideration cannot be part of an equilibrium. Similarly, in an equilibrium, the CRA will not assign the rating 0 to θ-firm, if it assigns the rating p to θ'-firm with $\theta' < \theta$. Therefore, an equilibrium rating strategy is increasing in the firm's fundamentals.

Now, suppose that given the rating strategy $\mathcal{R}(\theta)$, firms receiving the rating q have worse fundamentals than firms receiving the rating p. Then, more creditors will roll over if they observe the rating p. As a result, if the CRA deviates to assign the rating p to firms that receive the rating q under $\mathcal{R}(\theta)$, these firms' financial costs decrease. Since such firms do not default at date 1 when assigned the rating q, with lower financial costs after receiving the rating p, they will not default at date 1 either. Therefore, the CRA's deviation is also profitable, which implies that the CRA will not assign the rating q in an equilibrium.[17]

These arguments lead to Lemma 1 below, which characterizes all possible equilibrium rating strategies and simplifies our analysis much.

Lemma 1 (Cutoff Rating Strategy) *In an equilibrium (if any exists), the CRA's rating strategy can be described by a threshold θ_1^*, such that*

$$\mathcal{R}(\theta) = \begin{cases} 0, & \text{if} \quad \theta < \theta_1^*; \\ p, & \text{if} \quad \theta \geq \theta_1^*. \end{cases} \tag{11}$$

From Lemma 1, when θ_1^* decreases, the CRA assigns more firms with the high rating p. So for two rating strategies $\mathcal{R}_1$ with the threshold θ_1^* and $\mathcal{R}_2$ with the threshold θ_2^*, we say that the rating strategy $\mathcal{R}_2$ is *laxer* than the rating strategy $\mathcal{R}_1$ if and only if $\theta_2^* < \theta_1^*$. However, the laxer rating strategy $\mathcal{R}_2$ may not lead to higher *credit rating inflation*, which refers to the fact that the nominal rating is strictly higher than the real credit quality. Formally:

Definition 2 *A credit rating assigned to a θ-firm is inflated, if in an equilibrium, the θ-firm chooses HR and thus has the credit quality q, but the CRA assigns the rating p. In addition, a rating strategy is inflated, if credit ratings assigned according to the rating strategy are inflated for a non-negligible subset of fundamentals; and a credit rating strategy is more inflated, if for a larger measure of fundamentals, credit ratings assigned according to the rating strategy are inflated.*

In an equilibrium, the firm receiving the rating p does not default at date 1, due to the partial verifiability constraint. However, the rating p cannot guarantee that the firm will invest

[17] On the off-equilibrium path following $R = q$, the creditors form the belief that the firm will choose to continue to invest in HR. In Section 7.1, we analyze a self-disciplined CRA, where the rating $R = q$ may appear in some equilibria.

in VP. Indeed, if all patient creditors believe that the firm with the rating p will surely invest in VP, they will all roll over, leading to the lowest possible financial cost to any θ-firm. Then, the assumption that the θ_1^*-firm will invest in VP implies that θ_1^*-firm's financial cost is strictly less than H. Then, the firm with θ less than but very close to θ_1^* will not default early, if it receives the rating p, providing the CRA with incentives to deviate to assign the rating p to such a firm. Therefore, in an equilibrium, some firms with the rating p will invest in HR, implying credit rating inflation in an equilibrium. Formally:

Lemma 2 (No Equilibrium without Rating Inflation) *There is no equilibrium in which the θ-firm invests in VP for all $\theta \in \{\theta' : \mathcal{R}(\theta') = p\}$.*

While rating inflation inevitably appears in an equilibrium, credit ratings are still informative to creditors. Lemma 1 implies that if $R = p$, all patient creditors know that $\theta \geq \theta_1^*$. So the rating p guarantees creditors that the firm's fundamentals are not extremely bad.

Corollary 1 (Creditors' belief supports following $R = p$) *Following the credit rating $R = p$, regardless of his private signal x_i, the support of any creditor i's interim belief about θ is truncated from below by θ_1^*.*

4.2 Firm's Investment after the Rating p

As shown in Lemma 1, only the rating 0 and the rating p may appear in an equilibrium. Because the CRA tries to maximize the nominal rating, it will assign the rating 0 only if it knows that the firm will default early even with the rating p. Therefore, when creditors observe the rating 0, they all believe that the firm will default early, and so they will run. Hence, following $R = 0$, there is a unique equilibrium, in which all creditors run, and the firm defaults at date 1. Since the rating strategy assigns the rating 0 to the firm if and only if $\theta < \theta_1^*$, we must have $K(\theta) = f(\theta) > H$, $\forall \theta < \theta_1^*$. Then, by the continuity of $f(\cdot)$, we have the first condition for the equilibrium below:

$$f(\theta_1^*) \geq H. \tag{12}$$

We now focus on the subgame following the rating p. Given the rating strategy, after observing the rating p, all creditors believe that the firm's true fundamentals are above θ_1^*. However, as shown in Lemma 2, creditors are not sure whether the firm will invest in HR or VP. In particular, because the firm will invest in HR when θ is greater than but very close to θ_1^*, the creditors with extremely negative signals believe that the firm will invest in HR and thus choose to run as a dominant action. As a result, in a monotone equilibrium, given the belief about the CRA's rating strategy described by θ_1^*, after observing the rating p, any creditor i will roll over the debt if and only if x_i lands above some threshold x^*. Then, as in the benchmark model, because the firm's financial cost strictly decreases in its fundamentals, the firm will invest in HR if and only if θ is less than a threshold θ_2^*. Hence, given a possible equilibrium rating strategy θ_1^*, a monotone equilibrium following the rating p could be described by (x^*, θ_2^*), such that

1. $\theta_2^* > \theta_1^*$;
2. creditor i rolls over the short-term debt if and only if $x_i \geq x^*$; and
3. θ-firm chooses VP if $\theta \in [\theta_2^*, +\infty)$, and it chooses HR if $\theta \in [\theta_1^*, \theta_2^*)$.

Given the creditors' cutoff strategy with the threshold x^*, $(1-\gamma)\left[1-\Phi(\sqrt{\beta}(x^*-\theta))\right]$ measure of creditors will roll over the debt, for any $\theta \geq \theta_1^*$. Consequently, if the θ-firm decides to invest in either VP or HR, its financial cost is

$$\begin{aligned} & K(\theta) \\ = \; & (1-\gamma)\left[1-\Phi(\sqrt{\beta}(x^*-\theta))\right]F + \left[\gamma + (1-\gamma)\Phi(\sqrt{\beta}(x^*-\theta))\right]f(\theta) \\ = \; & [(1-\gamma)F + \gamma f(\theta)] + (1-\gamma)\Phi(\sqrt{\beta}(x^*-\theta))(f(\theta)-F). \end{aligned}$$

This is precisely the same as equation (7). Because the firm invests in VP if and only if $K(\theta) \geq (pV - qH)/(p-q)$, and θ_2^*-firm is indifferent between HR and VP, the firm's indifference condition, given the creditors' strategy, is

$$(1-\gamma)\left[1-\Phi(\sqrt{\beta}(x^*-\theta_2^*))\right]F + \left[\gamma + (1-\gamma)\Phi(\sqrt{\beta}(x^*-\theta_2^*))\right]f(\theta_2^*) = \frac{pV-qH}{p-q}. \tag{13}$$

Let's consider a creditor i's decision. With his private signal x_i, creditor i's interim belief about θ given the CRA's rating strategy θ_1^* would be a normal distribution with mean x_i and precision β, truncated below by θ_1^*. This truncation is due to creditors' belief about the CRA's rating strategy that $\mathcal{R}(\theta) = p$ if and only if $\theta \geq \theta_1^*$. Then, given the firm's strategy, creditor i's expected payoff from rolling over is

$$\frac{\Phi[\sqrt{\beta}(\theta_2^*-x_i)] - \Phi[\sqrt{\beta}(\theta_1^*-x_i)]}{1-\Phi[\sqrt{\beta}(\theta_1^*-x_i)]}qF + \frac{1-\Phi[\sqrt{\beta}(\theta_2^*-x_i)]}{1-\Phi[\sqrt{\beta}(\theta_1^*-x_i)]}pF.$$

Because refraining from rolling over the short-term debt always brings a creditor a payoff 1, a marginal creditor with the private signal x^* must have

$$\frac{\Phi[\sqrt{\beta}(\theta_2^*-x^*)] - \Phi[\sqrt{\beta}(\theta_1^*-x^*)]}{1-\Phi[\sqrt{\beta}(\theta_1^*-x^*)]}qF + \frac{1-\Phi[\sqrt{\beta}(\theta_2^*-x^*)]}{1-\Phi[\sqrt{\beta}(\theta_1^*-x^*)]}pF = 1. \tag{14}$$

Lemma 3 (Debt Financing Following $R = p$) *There exists a $\beta^* > 0$, such that for any $\beta > \beta^*$, if an equilibrium exists, given the CRA's rating strategy θ_1^*, following the rating p, there is a unique solution (θ_2^*, x^*) with $\theta_2^* > \theta_1^*$ to equation (13) and equation (14).*

In the analysis of the interaction between the firm and the creditors above, the CRA's rating strategy θ_1^* is given. Lemma 4 below shows how the CRA's rating strategy affects the creditors' debt rollover decisions and the firm's moral hazard.

Lemma 4 (Laxer Rating Strategy) *For any $\beta > \beta^*$, both x^* and θ_2^* are strictly decreasing in θ_1^*.*

When θ_1^* is lower, the CRA's rating strategy is laxer. In this scenario, creditors discount the good rating by increasing their debt rollover threshold. Because more creditors refrain from rolling over the debt, the firm's financial cost is higher for any θ; as a result, the threshold that the firm chooses VP is also higher.

4.3 Equilibrium Rating Strategy

Lemma 3 shows that creditors' belief about the CRA's rating strategy θ_1^* determines the measure of creditors rolling over and thus any θ-firm's financial cost. One the other hand, when the CRA assigns the rating to θ-firm, the CRA, based on the knowledge of θ and creditors' responses to the ratings, can perfectly predict whether the firm will default early or not. Hence, in an equilibrium, the θ_1^*-firm must be indifferent between early default and HR. Because of the firm's limited liability, the firm will choose to default early only if the financial cost is higher than H, the upside return from investing in HR. Therefore, the θ_1^*-firm's indifference condition implies

$$(1-\gamma)\left[1-\Phi(\sqrt{\beta}(x^*-\theta_1^*))\right]F + \left[\gamma + (1-\gamma)\Phi(\sqrt{\beta}(x^*-\theta_1^*))\right]f(\theta_1^*) = H. \tag{15}$$

Proposition 2 below shows that the model has a unique equilibrium, in which the CRA's rating, the firm's investment decision, and the creditors' rollover decisions interact with one another.

Proposition 2 (Unique Equilibrium) *There is a $\beta^* > 0$, such that when $\beta > \beta^*$, the model has a unique equilibrium. The equilibrium is characterized by $(\theta_1^*, \theta_2^*, x^*)$ with $\theta_2^* > \theta_1^*$, such that*

1. *the CRA will assign a rating $R = p$, if the firm's fundamentals $\theta \in [\theta_1^*, +\infty)$; and it will assign a rating $R = 0$, if the firm's fundamentals $\theta < \theta_1^*$;*

2. *if $R = 0$, all creditors run, and the firm defaults at date 1;*

3. *if $R = p$, a patient creditor rolls over the debt if and only if his private signal lands above x^*, and the firm will choose HR if $\theta \in [\theta_1^*, \theta_2^*)$ and VP if $\theta \in [\theta_2^*, +\infty)$; and*

4. *$(\theta_1^*, \theta_2^*, x^*)$ solves equations (13), (14), and (15).*

The equilibrium uniqueness first arises from creditors' new dominant region of running, generated by the credit rating p. From Lemma 2, because the CRA aims to maximize the nominal rating, it will assign the rating p to the firm that has the fundamentals just above θ_1^* and thus will invest in HR. Consequently, when creditors receive very negative signals, they believe that the firm has fundamentals landing within this region and thus invests in HR, so they will run. Hence, it is impossible for all patient creditors to roll over in an equilibrium, and thus, creditors have a unique best response to the rating p.

The equilibrium uniqueness also arises from the monotonic effect of creditors' belief about the CRA's rating strategy θ_1^* on their best responses. Lemma 4 shows that if creditors believe that the CRA employs a rating strategy with a higher θ_1^*, they best respond by decreasing the rollover threshold, and any θ-firm's financial cost decreases. This is because a tighter rating strategy is more informative and thus is discounted less by creditors. Since the firm's non-debt financing cost decreases in its fundamentals, θ_1^*-firm's financial cost is strictly decreasing as θ_1^* increases. As a result, there will be a unique θ_1^*, with which the firm is just indifferent between early default and HR, which, together with creditors' unique best response to the rating p, implies the equilibrium uniqueness.

Proposition 2 provides us with a clear measure of equilibrium rating inflation. When $\theta < \theta_1^*$, the CRA will assign the rating 0 to the firm. Since the firm will default early, the credit rating truly reflects the firm's credit quality. When $\theta \geq \theta_2^*$, the firm's fundamentals are sufficiently good, so it will invest in VP. So, the credit rating p also equal the firm's credit quality in this case. However, when $\theta \in [\theta_1^*, \theta_2^*)$, the firm invests in HR and thus has the credit quality q, but it receives the high rating p. So the credit ratings assigned to such firms are inflated. Hence, the rating inflation can be measured by $\theta_2^* - \theta_1^*$.

5 The CRA's Real Effects

We are now in a position to analyze the CRA's real effects. For a given θ-firm, if the assigned credit rating changes its investment (comparing to its investment in the benchmark model without a CRA), we say that the CRA has *real effects* on the θ-firm. Such effects are *positive*, if the CRA leads the θ-firm to choose an investment with higher social welfare; conversely, if the CRA leads the θ-firm to choose an investment with lower social welfare, the CRA's real effects on the θ-firm are *negative*. The CRA's ex-ante real effects are then measured by the total change of the ex-ante social welfare.[18] Hence, the ex-ante real effects of the CRA are positive (negative) if the social welfare is higher (lower) with the CRA.

From Proposition 2, we can see that the CRA affects a firm's investment decision through two interacting channels. On the one hand, by assigning the rating $R = p$, the CRA separates firms with $\theta \geq \theta_1^*$ from those with $\theta < \theta_1^*$. Hence, the rating $R = p$ provides creditors with new information about the firm's fundamentals. Such new information affects creditors rollover decisions, and thus the firm's financial cost and investment. We call such effects the CRA's "informational effects."

On the other hand, the CRA strategically chooses θ_1^* to pool firms investing in HR with those investing in VP. Hence, the set of types of the firm that invest in either HR or VP may differ in cases with and without a CRA. This also affects firms' investment decisions. We call such effects the CRA's "feedback effects," since the CRA, when choosing θ_1^*, takes into account the creditors' and the firm's best responses to the ratings.

In this section, we first compare the firm's equilibrium investment decision in the model with the CRA to those in the benchmark model. Such a comparison shows the CRA's real effects. And then, we decompose the CRA's real effects into its information effects and its feedback effects to get a full understanding of its real effects.

5.1 The CRA's Real Effects

Lemma 5 below shows that, with the CRA, both the firm's early default threshold and VP-investing threshold are lower than those in the benchmark model without a CRA.

Lemma 5 *Comparing the equilibrium of the model with a CRA (described in Proposition 2) to that of the benchmark model without a CRA (described in Proposition 1), we have $\theta_1^* < \tilde{\theta}_1$, $\theta_2^* < \tilde{\theta}_2$, and $x^* < \tilde{x}$. However, the sign of $\theta_2^* - \tilde{\theta}_1$ is undetermined.*

[18] We only consider the real effects of the CRA when its rating changes the firm's investment decisions. In our model, it means we focus on the real effects of the CRA's inflated ratings. When the CRA assigns the low ratings, the firm's investment decision (early default) does not change. The CRA, however, always has positive real effects in this case, because the low ratings can perfectly reveal the firm's early default decision, lead to efficient runs of investors, and thus reduce social loss.

When $\theta_2^* > \tilde{\theta}_1$, there are two cases of the CRA's real effects. First, when $\theta \in \left[\theta_1^*, \tilde{\theta}_1\right)$, without the CRA, the firm's financial cost is so high that it will default early; but when the CRA is present, it will assign the firm the inflated rating p, leading to lower financial costs to the firm. Such a decrease in the financial costs encourages the firm to gamble for resurrection, rather than default early, which shows the CRA's negative real effects. Second, when $\theta \in \left[\theta_2^*, \tilde{\theta}_2\right)$, because the high rating p helps the firm reduce financial costs, the firm switches from HR to VP, which implies positive real effects.

The CRA's real effects in the case with $\theta_2^* > \tilde{\theta}_1$ is illustrated in Figure 2 below.

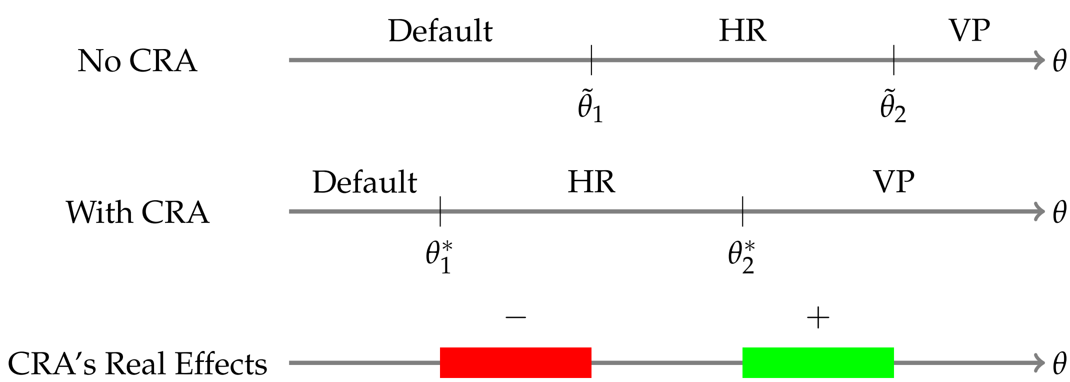

Figure 2: CRA's real effects when $\theta_2^* > \tilde{\theta}_1$.

When $\theta_2^* \leq \tilde{\theta}_1$, the CRA's real effects are similar. Proposition 3 below formally summarizes the CRA's real effects.

Proposition 3 *There are two cases of the analysis of the CRA's real effects.*

1. *If $\theta_2^* > \tilde{\theta}_1$, the CRA has positive real effects when $\theta \in \left[\theta_2^*, \tilde{\theta}_2\right)$ and has negative real effects when $\theta \in \left[\theta_2^*, \tilde{\theta}_2\right)$; hence, the CRA's net real effects are*

$$(\tilde{\theta}_2 - \theta_2^*)(pV - qH) + \int_{\theta_1^*}^{\tilde{\theta}_1} (qH - \gamma - (1-\gamma)\Phi(\sqrt{\beta}(\tilde{x} - \theta)))\mathrm{d}\theta.$$

2. *If $\theta_2^* \leq \tilde{\theta}_1$, the CRA has positive real effects when $\theta \in \left[\theta_2^*, \tilde{\theta}_2\right)$ and has negative real effects when $\theta \in [\theta_1^*, \theta_2^*)$; hence, the CRA's net real effects are*

$$(\tilde{\theta}_2 - \theta_2^*)(pV - qH) + \int_{\theta_1^*}^{\theta_2^*} (qH - \gamma - (1-\gamma)\Phi(\sqrt{\beta}(\tilde{x} - \theta)))\mathrm{d}\theta.$$

Importantly, Proposition 3 shows that the CRA who employs an inflated rating strategy in the equilibrium may have positive or negative real effects, depending on the firm's fundamentals. The CRA's net ex-ante real effects, then, depend on the model's parameters. In Figure 3 below, we numerically show that the CRA's net ex-ante real effects as a function of the upside return of the risky project, H.

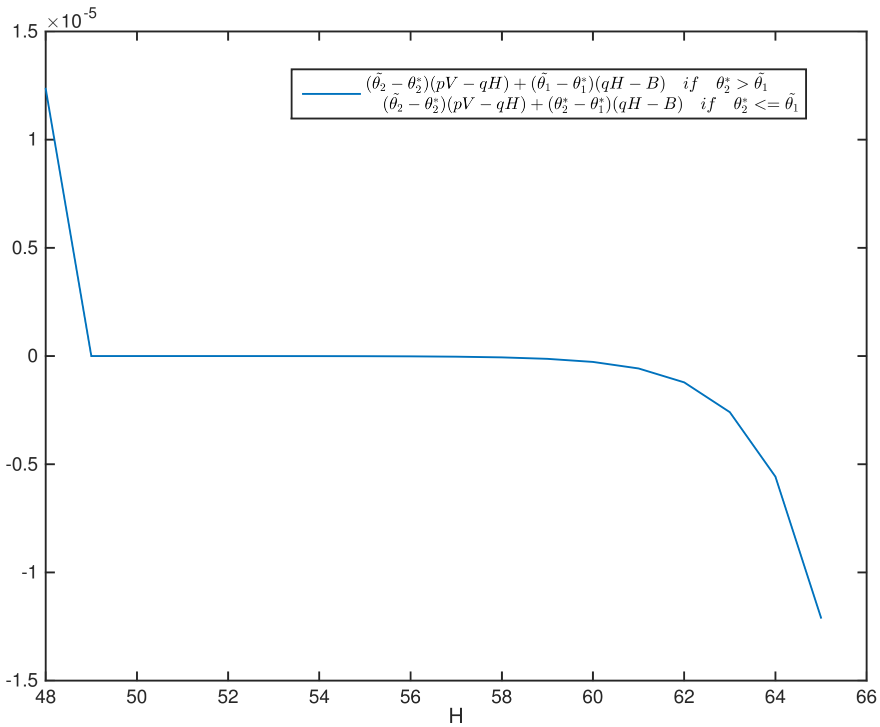

Figure 3: The CRA's Real Effects as a Function of H

Figure 3 shows that when the upside return of the risky project is relatively high, the CRA's real effects are negative. This is because, as shown in Proposition 6, when H is large, the firm has stronger incentives to take risks by investing in HR and thus is less likely to default early. The CRA then will assign more firms the high rating $R = p$, which allows those firms to gamble for resurrection, and so have negative real effects. When H is relatively small, the CRA encourages more firms to switch from HR to VP and thus has positive real effects.

The fact that as the upside return of the risky project increases beyond a threshold, the CRA's ex-ante net real effects become negative provides a potential explanation to a long-lasting puzzle. Indeed, the conflicts of interest caused by the issuer-pays business model have been recognized since early 1970s, but such an issue did not attract much attention until the recent subprime crisis. Our theory implies that it is the high-risk high-return financial products, such as mortgage-backed securities, that exacerbate the conflicts of interest in the credit rating industry, leading to the CRA's adverse ex-ante net real effects.

5.2 Informational Effects and Feedback Effects

As we have argued, the CRA's real effects can be decomposed into two components: the informational effects, because the CRA provides creditors with new information, and the feedback effects, because the CRA strategically chooses its rating rule, taking advantage of the feedback between credit ratings and firms' investments. We are now analyzing how these two effects interact each other and determine the CRA's real effects.

We first analyze the CRA's informational effects. Let's consider the case in which the CRA commits to the following rating strategy:

$$\mathcal{R}(\theta) = \begin{cases} 0, & \text{if } \theta < \hat{\theta}_1 \equiv \tilde{\theta}_1; \\ p, & \text{if } \theta \geq \hat{\theta}_1. \end{cases} \tag{16}$$

Here, $\hat{\theta}_1$ equals $\tilde{\theta}_1$, which is part of the equilibrium characterized in Proposition 1.

In such a case, the CRA does not behave strategically, though such a rating strategy may still be inflated. Indeed, the committed rating strategy just reflects the firm's investment decision when there is no CRA. So, for ease of exposition, we call such a CRA a "reflecting CRA" and the CRA analyzed in Section 4 a "strategic CRA." Importantly, a reflecting CRA does not have feedback effects, because it does not take into account its effects on the firm's investment decision when committing to its rating strategy. Therefore, the real effects of the reflecting CRA is purely the information effects. Then, by comparing the strategic CRA's real effects and the reflecting CRA's real effects, we can get the strategic CRA's feedback effects.

Proposition 4 shows the firm's equilibrium investment decision in the case with the reflecting CRA.

Proposition 4 *Given the committed rating strategy in equation* (16)*, the generated credit ratings lead to two subgames. In particular,*

1. *in the subgame following $R = 0$, there is a unique equilibrium in which the firm defaults at date 1; and,*

2. *in the subgame following $R = p$, in any equilibrium, the θ-firm invests in VP if $\theta \geq \hat{\theta}_2$ and invests in HR if $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$, where $\hat{\theta}_2 \geq \hat{\theta}_1$.*

Now, let's analyze the reflecting CRA's real effects, which are also the strategic CRA's informational effects. Proposition 4 shows that with the rating p assigned by a reflecting CRA, if $\theta > \tilde{\theta}_2$ (which is strictly greater than $\hat{\theta}_2$), the firm invests in VP, in both the case with a reflecting CRA and the case without a CRA. Therefore, for any $\theta > \tilde{\theta}_2$, the reflecting CRA does not have real effects. Same arguments show that the reflecting CRA does not have any real effects when $\theta \in [\hat{\theta}_1, \hat{\theta}_2)$

However, the reflecting CRA will change the firm's investment decision when $\theta \in [\hat{\theta}_2, \tilde{\theta}_2)$. In particular, without a CRA, the firm invests in HR, but with a reflecting CRA, the firm will invest in VP. Therefore, the reflecting CRA has positive real effects, which are measured by $(\tilde{\theta}_2 - \hat{\theta}_2)(pV - qH)$. That is, the strategic CRA's informational effects are always positive, precisely because its rating $R = p$, though potentially inflated, provides creditors with an informative signal and thus correctly guides creditors' rollover decisions and the firm's investments.

Finally, let's investigate the strategic CRA's feedback effects. Similarly to Proposition 3, there are two cases: $\theta_2^* \geq \hat{\theta}_1$ and $\theta_2^* < \hat{\theta}_1$. In both cases, the strategic CRA's feedback effects have a negative component. Because the strategic CRA knows that when it assigns the rating $R = p$, more creditors will roll over and the firm's financial cost decreases, it can assign more types of the firm the high rating $R = p$. That is, in an equilibrium, the strategic CRA will employ the rating strategy with the threshold $\theta_1^* < \min\{\hat{\theta}_1, \theta_2^*\}$. Such a manipulation leads firms with $\theta \in [\theta_1^*, \min\{\hat{\theta}_1, \theta_2^*\})$ to gamble for resurrection, and thus leads to adverse real effects.

In the case with $\theta_2^* \geq \hat{\theta}_1$, the strategic CRA's real effects have another negative component. Because $\theta_1^* < \hat{\theta}_1$, the rating $R = p$ assigned by the strategic CRA is less informative than the rating $R = p$ assigned by the reflecting CRA. So, with the strategic CRA, after the rating $R = p$, less creditors roll over (comparing to the case with the reflecting CRA), the firm's financial cost increases, and thus less types of firms (measured by $\hat{\theta}_2 - \theta_2^*$) switch from HR to VP. That is, in the case with $\theta_2^* > \hat{\theta}_1$, the strategic CRA's rating strategy will weaken its information effects.

In the other case with $\theta_2^* < \hat{\theta}_1$, the second component of the strategic CRA's feedback effects is positive. This is because by assigning the rating $R = p$ to the firm with $\theta \in [\theta_1^*, \hat{\theta}_1)$, it is possible for the firm to invest in VP. Indeed, when $\theta \in [\theta_2^*, \hat{\theta}_2)$, the firm invests in VP, implying that the strategic CRA has positive feedback effects.

The above arguments are summarized in Proposition 5 below.

Proposition 5 *The strategic CRA's real effects can be decomposed into its informational effects and its feedback effects. The strategic CRA's informational effects, which are measure by $(\tilde{\theta}_2 - \hat{\theta}_2)(pV - qH)$, are always positive. When the set of parameters are such that $\theta_2^* \geq \hat{\theta}_1$, the strategic CRA's feedback effects, measured by*

$$\int_{\theta_1^*}^{\tilde{\theta}_1} (qH - \gamma - (1-\gamma)\Phi(\sqrt{\beta}(\tilde{x} - \theta)))\mathrm{d}\theta + (\theta_2^* - \hat{\theta}_2)(qH - pV),$$

are negative; but given the set of parameters such that $\theta_2^* < \hat{\theta}_1$, the strategic CRA's feedback effects are measured by

$$\int_{\theta_1^*}^{\theta_2^*} (qH - \gamma - (1-\gamma)\Phi(\sqrt{\beta}(\tilde{x} - \theta)))\mathrm{d}\theta. + (\hat{\theta}_2 - \theta_2^*)(pV - qH),$$

whose sign is undetermined.

Proposition 5 implies that credit rating inflation itself does not necessarily lead to negative real effects. Because inflated ratings are still informative signals, which can increase the market's efficiency and lead to positive real effects. The negative real effects, however, arise from the CRA's feedback effects. Because the CRA knows that the rating will reduce the firm's financial costs and default likelihood, it will assign more firms the high rating, providing them the opportunities to gamble for resurrection.

6 Empirical Predictions

The theory we develop in this paper provides several new empirical predictions about CRAs' rating strategies and credit rating inflation. In this section, we analyze how a CRA's rating strategy and the rating inflation vary when the economic environment changes. That is, we perform comparative static analysis to provide empirical predictions about CRAs' rating strategies and credit rating inflation.

From these comparative static analysis, we show that laxer credit rating strategies are not necessarily accompanied by higher rating inflation. Intuitively, in our model, both the CRA's rating strategy (measured by θ_1^*) and the credit rating inflation (measured by $\theta_2^* - \theta_1^*$) are endogenously determined. Then, an exogenous economic environment changes may lead to a laxer rating strategy and a lower financial cost to the firm at the same time. While the former effect may increase the rating inflation, the latter effect encourages the firm to invest in VP, which reduces the rating inflation. Therefore, whether a laxer rating strategy is accompanied by higher rating inflation depends on which effect dominates. This is related to recent empirical findings. Indeed, Alp (2013) and Baghai, Servaes, and Tamayo (2014) find that CRAs become more conservative by using stricter rating standards (strategies); however, the stricter rating strategies do not reduce credit rating inflation, as shown by Strobl and Xia (2012).

Proposition 6 *When β is sufficiently large,*[19] *a decrease in β, an increase in H, and a decrease in γ will all lead to a decrease in θ_1^*. However, a decrease in β has ambiguous effects on $\theta_2^* - \theta_1^*$, an increase in H increases $\theta_2^* - \theta_1^*$, and a decrease in γ decreases $\theta_2^* - \theta_1^*$.*

First, β is the precision of creditors' private signals, so it measures the firm's transparency. Proposition 6 shows that for more opaque firms, the CRA employs laxer rating strategies. By the properties of the truncated normal random variable's mean, when creditors' private signals become less precise, they believe that the firm is more likely to invest in VP. As a result, more creditors roll over, and the firm's financial cost decreases, which allows the CRA to employ a laxer rating strategy. Equivalently, this comparative static analysis shows that when creditors' private signals are more precise, the CRA will employ a stricter rating standard. This is consistent with a recent empirical finding in Fong, Hong, Kacperczyk, and Kubik (2014) that security analysts can discipline CRAs by providing creditors with more information.

While Proposition 6 implies that CRAs employ laxer rating strategies for more opaque firms, it does not imply that credit ratings assigned to more opaque firms are more inflated. Since creditors will decrease their rollover threshold when the firm is more opaque, the firm's financial cost is lower, causing a smaller θ_2^*, which is the firm's VP investment threshold. Consequently, when creditors' signals are less precise, the CRA is more likely to assign the rating p to the firm, but the firm with a high rating is more likely to invest in VP. As a result, whether the rating inflation for a more opaque firm is higher or lower depends on which of these two effects dominate. This in turn depends on other parameters of the model.

Second, cross-sectionally, firms differ in the upside returns of their available projects. Equation (15) suggests that the highest upside return among all available projects may determine the credit rating assigned to the firm. Hence, it is interesting to consider how the firm's upside return from HR affects the CRA's rating strategy. An increase in H does not directly affect creditors' behavior, because creditors' payoffs are solely determined by the debt contract, which does not involve the cash flow to the firm, conditional on the success of the investment. Yet, H has direct effects on both the firm's investment and the CRA's rating strategy. On the one hand, an increase in H increases the firm's incentives to invest in HR, because the expected re-

[19] This is the condition for the equilibrium uniqueness, which is critical for comparative static analysis.

turn from the HR is higher. On the other hand, an increase in H decreases the firm's incentives to default early, because the firm has limited liability. As a result, for fixed creditors' strategies, when H increases, the CRA's rating strategy will be laxer, and the firm is more likely to invest in HR rather than VP, resulting in higher credit rating inflation.

Finally, suppose the measure of impatient creditors, γ, decreases. The direct effect of a decrease in γ is that the firm's financial cost will surely decrease, because the firm needs to finance less money from expensive non-debt sources. In addition, a decrease in γ will lead to more patient creditors rolling over the debt, due to the strategic complementarities among patient creditors. This further reduces the firm's financial cost. Then, the firm's threshold of investing in VP will decrease, implying that less firms will invest in HR given the CRA's credit rating strategy. In the meanwhile, the lower financial cost of the firm means that less firms may default early. As a result, the CRA would want to employ a laxer rating strategy. Furthermore, as γ decreases, the measure of firms that shift from HR to VP due to the reduced financial cost is greater than the measure of firms that gamble for resurrection because of the high credit rating, leading to lower credit rating inflation.

7 Discussions

In our model, the CRA maximizes the nominal rating for each θ-firm, and the credit ratings are only regulated by the partial verifiability constraint. That is, unless the firm defaults early, the CRA that assigns a positive rating to the firm will not be punished. However, people may argue that conditional on the firm's investment's failure, the CRA incurs a higher reputation cost when assigning the rating p than when assigning the rating q. In this section, we first analyze how credit ratings' real effects change, when the CRA that assigns the rating p incurs a higher reputation cost, if the firm's investment fails. This analysis not only generalizes our results in the core model, but also provides an important potential policy to regulate the credit rating industry.

We have also shown that the coordination and heterogeneous information among creditors play critical roles in the inflated credit rating's real effects. In this section, we further study the role of coordination and heterogeneous information among creditors by assuming that all patient creditors do not have private signals but receive a common precise public signal. We find that when the public signal is sufficiently precise, the CRA will have zero real effect, demonstrating the important role of creditors' coordination in the CRA's real effects.

7.1 Reputation Costs when the Firm's Investment Fails

In this subsection, we consider the CRA's reputation cost conditional on the failure of the firm's investment. We assume that the CRA's benefits by providing the rating services are increasing in the ratings, due to the conflicts of interest. Therefore, $V^p > V^q > V^0 = 0$, where V^R is the CRA's benefit from assigning the rating R, and we normalize the benefit from assigning the rating $R = 0$ to be zero. If the firm does not default (either at date 1 or at date 2), the CRA will not incur any cost. However, if the firm defaults early, the CRA will incur a cost greater than V^p, because the early default is verifiable.[20] Importantly, if the firm invests, and the investment fails at date 2, the CRA incurs a cost C^R, if it assigns the rating R. Here, $C^p > C^q > 0$.

Given creditors' belief about the CRA's rating strategy and their best responses to credit ratings, the CRA can perfectly predict the firm's investment. Lemma 6 below shows that, the CRA's equilibrium rating strategy depends on the ratio of the benefit increment to the cost increment due to the rating upgrading from q to p.

Lemma 6 *The CRA's equilibrium rating strategy depends on the ratio $(V^p - V^q) / (C^p - C^q)$. There are three cases.*

1. *if $\frac{V^p - V^q}{C^p - C^q} \geq 1 - q$, the equilibrium rating strategy is in the form*

$$\mathcal{R}(\theta) = \begin{cases} p, & \text{if } \theta \geq \theta'; \\ 0, & \text{if } \theta < \theta'. \end{cases} \tag{17}$$

2. *if $\frac{V^p - V^q}{C^p - C^q} \leq 1 - p$, the equilibrium rating strategy is in the form*

$$\mathcal{R}(\theta) = \begin{cases} q, & \text{if } \theta \geq \theta'; \\ 0, & \text{if } \theta < \theta'. \end{cases} \tag{18}$$

3. *if $\frac{V^p - V^q}{C^p - C^q} \in (1 - p, 1 - q)$, the equilibrium rating strategy is in the form*

$$\mathcal{R}(\theta) = \begin{cases} p, & \text{if } \theta \geq \theta'; \\ q, & \text{if } \theta \in [\theta'', \theta')\,; \\ 0, & \text{if } \theta < \theta'', \end{cases} \tag{19}$$

[20] This is a general form of the partial verifiability constraint assumed in the core model. More importantly, the extension analyzed in this section shows that our analysis of the core model can be extended to the case with several possible projects and multiple rating categories, in which credit rating inflation may occur at some specific rating categories but not at others.

where $\theta'' \leq \theta'$.

Lemma 6 shows that if the benefit of upgrading the rating from q to p is high enough (relative to the change of the cost), the CRA will employ an inflated rating strategy in an equilibrium. This is actually the case in our core model, so the equilibrium will be the same as described in Proposition 2, and credit ratings' real effects are the same as in Proposition 3.

When the benefit of upgrading the rating from q to p (relative to the cost) is low enough, the CRA will employ a *deflated* rating strategy in an equilibrium. Since such a rating strategy will not change creditors' belief (following the rating q), the equilibrium characterization is very similar to that in Proposition 2, and credit ratings' real effects are the same as in Proposition 3.

When the ratio of the benefit increment to the cost increment due to the rating upgrading from q to p is between $1-p$ and $1-q$, the CRA will assign a rating, which reflects the firm's true investment choice. In this case, the CRA is "self-disciplined." We now analyze a self-disciplined CRA's equilibrium rating behavior in more details.

Suppose that the CRA's rating is $R = 0$. Since the CRA is self-disciplined, all creditors believe that the firm will surely default early. Then, after $R = 0$, no creditor will roll over the debt, leading to the firm's financial cost $K(\theta) = f(\theta)$. Thus, the firm chooses to default early following $R = 0$ if and only if

$$f(\theta) \geq H. \tag{20}$$

Let z_0 be the solution to equation (20). Then, following the rating $R = 0$, the firm will default early if $\theta \leq z_0$.

Suppose $R = q$; then, in the equilibrium, all creditors believe that the firm will invest in HR. Because $qF < 1$, no creditor invests in the debt. Thus, we also have the financial cost $K(\theta) = f(\theta)$. For the firm to choose HR, we must have

$$\frac{pV - qH}{p - q} \leq f(\theta) < H. \tag{21}$$

Denote the solution to the equation $\frac{pV-qH}{p-q} = f(\theta)$ by z_q. Because $f(\theta)$ is decreasing, and $H > \frac{pV-qH}{p-q}$, we have $z_q > z_0$. Then, the firm invests in HR following the rating $R = q$, if and only if $\theta \in \left(z_0, z_q\right]$.

Finally, suppose $R = p$; then, in the equilibrium, all creditors invest in the debt, because they believe that the firm will invest in VP. This implies that the firm's financial cost is

$$K(\theta) = (1-\gamma)F + \gamma f(\theta).$$

Denote the solution to the equation $(1-\gamma)F + \gamma f(\theta) = \frac{pV-qH}{p-q}$ by z_p. Then, the firm invests in VP following the rating $R = p$, if and only if $\theta \geq z_p$. Note that $(1-\gamma)F + \gamma f(\theta) < f(\theta)$ for all θ. Therefore, $z_q > z_p$.

The arguments above prove Proposition 7 below, which characterizes the equilibrium of the model with a self-disciplined CRA.

Proposition 7 *When $\frac{V^p - V^q}{C^p - C^q} \in (1-p, 1-q)$, the model has a unique equilibrium. In particular, if $z_q > z_p > z_0$, the equilibrium rating strategy is*

$$\mathcal{R}(\theta) = \begin{cases} 0, & \textit{if} \quad \theta \leq z_0; \\ q, & \textit{if} \quad \theta \in (z_0, z_p]; \\ p, & \textit{if} \quad \theta \in (z_p, +\infty). \end{cases} \tag{22}$$

If $z_p < z_0 < z_q$, the equilibrium rating strategy is

$$\mathcal{R}(\theta) = \begin{cases} 0, & \textit{if} \quad \theta \leq z_p; \\ p, & \textit{if} \quad \theta > z_p. \end{cases} \tag{23}$$

Proposition 7 has great policy implications. Because both the credit rating and the firm's investment outcome are verifiable, the government can design a cost scheme (C^p, C^q), such that $\frac{V^p - V^q}{C^p - C^q} \in (1-p, 1-q)$, to punish the CRA when the firm's investment fails. Such a cost scheme can make the CRA "self-disciplined" and lead to unbiased credit ratings. Specifically, equation (23) shows that when $z_p < z_0$, the CRA can eliminate all inefficiencies: for weak firms, no creditor rolls over the debt and there is zero social loss, while all other firms will choose the socially optimal investment project. Even if $z_p > z_0$, except $\theta \in (z_0, z_p)$ when the firm invests in HR in any event, given all other θ's, firms attain social optimal allocations.

However, to design the appropriate cost scheme (C^p, C^q), the government has to know the CRA's benefits from ratings, (V^p, V^q). While such information may not be available to the government, Lemma 6 at least shows that imposing too high costs is not a correct policy, because this may lead the CRA to employ a deflated rating strategy, which will have the same real effects as those of an inflated rating strategy.

7.2 Coordination's Role and Public Signal Policy

In this section, we show how creditors' coordination incentives and belief dispersion play a role in determining the CRA's real effects. We analyze an environment with a precise public signal about θ, which are publicly observable. We further assume that creditors do not receive private signals.[21]

To formalize the idea, let's assume that the public signal leads to the belief $\theta \sim \mathcal{N}(\theta_s, \alpha^{-1})$. We will consider the case when α is sufficiently large. We maintain the assumption that the measure of patient creditors is $1-\gamma$ and consider symmetric equilibria. Because patient creditors will employ a symmetric strategy in an equilibrium, the firm's financial cost of investing in either HR or VP will be

$$K(\theta) = \begin{cases} (1-\gamma)F + \gamma f(\theta), & \text{if patient creditors roll over the debt;} \\ f(\theta), & \text{if patient creditors run.} \end{cases}$$

When creditors choose to roll over the debt, the firm's optimal investment choice is

$$\begin{cases} \text{Default early,} & \text{if} \quad (1-\gamma)F + \gamma f(\theta) > H; \\ \text{HR,} & \text{if} \quad (1-\gamma)F + \gamma f(\theta) \in \left(\frac{pV-qH}{p-q}, H\right]; \\ \text{VP,} & \text{if} \quad (1-\gamma)F + \gamma f(\theta) \leq \frac{pV-qH}{p-q}. \end{cases}$$

Denote the solution to the equation $(1-\gamma)F + \gamma f(\theta) = H$ by y_1 and that to the equation $(1-\gamma)F + \gamma f(\theta) = (pV - qH)/(p-q)$ by y_2. The firm's optimal investment choice when patient creditors roll over the debt can be written as

$$\begin{cases} \text{Default early,} & \text{if} \quad \theta < y_1; \\ \text{HR,} & \text{if} \quad \theta \in [y_1, y_2); \\ \text{VP,} & \text{if} \quad \theta \geq y_2. \end{cases}$$

Similarly, we denote the solution to the equation $f(\theta) = H$ by y_1' and that to the equation $f(\theta) = (pV - qH)/(p-q)$ by y_2'. The firm's optimal investment choice when patient creditors run can be written as

$$\begin{cases} \text{Default early,} & \text{if} \quad \theta < y_1'; \\ \text{HR,} & \text{if} \quad \theta \in [y_1', y_2'); \\ \text{VP,} & \text{if} \quad \theta \geq y_2'. \end{cases}$$

[21] In the core model, we assume for simplicity an improper uniform prior. This assumption is without loss of generality, because we consider the case in which investors' private signals are sufficiently precise. Adding a normal prior belief or a normal public signal to the core model will not change the results, but then the ex-ante economic effects of the CRA depend on the prior mean of the firm's fundamentals. In this subsection about the policy of providing public signals, also without loss of generality, we ignore investors' private signals, because we assume the public signal is much more precise than investors' private signals.

Because $f(\theta) > (1-\gamma)F + \gamma f(\theta)$ for any θ, we have $y_1 < y_1'$ and $y_2 < y_2'$. When α is sufficiently large, creditors will mainly rely on the public signal to make the debt rollover decision. Because creditors' behavior determines the firm's investment choice, the public signal and the creditors' behavior determine the CRA's credit rating. Proposition 8 below shows the equilibrium credit rating strategy when all patient creditors receive a homogeneous, precise public signal.

Proposition 8 (Credit Ratings Affected by Public Signals) *There exists $\bar{\alpha} > 0$, such that for all $\alpha > \bar{\alpha}$, the public signal determines the CRA's equilibrium rating strategy. Specifically,*

1. *when $\theta_s \geq y_2'$, the CRA will employ the rating strategy $\theta_1^* = y_1$;*
2. *when $\theta_s < y_2$, the CRA will employ the rating strategy $\theta_1^* = y_1'$; and*
3. *when $\theta_s \in [y_2, y_2')$, the CRA will set $\theta_1^* = y_1$ if patient creditors roll over the debt after $R = p$, while the CRA will set $\theta_1^* = y_1'$ if patient creditors run after $R = p$.*

Significantly, we observe in Proposition 8 that, when the public signal is very positive ($\theta_s \geq y_2'$), the CRA employs a laxer rating strategy, meaning that the good rating is a less positive signal. When the public signal is very negative ($\theta_s < y_2'$), the CRA employs a stricter rating strategy, meaning that the good rating is a more positive signal. Such a "substitution" results from creditors relying more heavily on the public signal when making decisions. When the public signal is in the medium range, there will be multiple equilibria: if the creditors roll over the debt, the CRA will employ a more inflated credit rating strategy; and if the creditors refrain from rolling over the debt, the CRA will employ a more conservative rating strategy.

It follows from Proposition 8 that, when creditors do not have dispersed beliefs and the public signal is sufficiently precise, the CRA does not have real effects, because creditors will ignore the information extracted from the credit ratings. This shows the importance of creditors' coordination in our core model, where the CRA has significant real effects.

8 Conclusion

We study credit rating agencies' effects on firms' investments. We show that high credit ratings, though commonly known to be potentially inflated, exclude extremely bad firms from creditors' belief supports. Therefore, high ratings make creditors more optimistic, reduce firms' financial costs, and thus change firms' investments. That is, even in an environment with perfectly rational and well-informed creditors, inflated ratings still have significant real effects.

Such real effects, however, could be positive or negative. With the high ratings, some firms take risky projects instead of default efficiently, implying CRAs' adverse real effects; but some other firms will switch from risky investments to safe investments, showing CRAs' positive real effects. CRAs' ex-ante real effects, then, depend on the economic environment. Specifically, when the upside return of the risky project is high, CRAs' ex-ante real effects are negative. This provides a potential explanation to a long-lasting puzzle: while the conflicts of interest are recognized for a long time, they didn't attract much attention until the 2007-2009 subprime crisis. Our theory implies that because of the high-risk high-return financial products, such as MBSs, CRAs' ex-ante real effects are negative, putting them in criticisms.

We further decompose CRAs' real effects into their informational effects and their feedback effects. We show that credit ratings that act as new informative signals do positively affect firms' investments. Hence, CRAs' negative real effects arise from their feedback effects. Indeed, they take advantage of the feedback between credit ratings and firms' investments to assign high ratings to more firms, providing chances for those firms to gamble for resurrection. Such a manipulation leads to negative real effects.

We emphasize that credit rating standards and credit rating inflation are two different concepts, and they are both endogenously determined. Therefore, changes of economic environments that lead to laxer rating strategies do not necessarily cause higher rating inflation.

Our paper offers applied and theoretical contributions. From the applied perspective, we provide a rational framework, enabling us to analyze credit rating inflation and credit rating agencies' real effects. While we focus on the credit ratings assigned to a firm in the paper, our model can also be applied to the sovereign ratings. In fact, in our opinion, the assumption of the credit ratings' partial verifiability constraints is more appropriate in the scenario of sovereign ratings: because there are fewer data points of sovereign ratings, the inflated ratings are harder to be detected.

Our model also generates several testable empirical predictions and some reasonable policy suggestions. The partial verifiability constraint can be applied to many other scenarios, such as financial advising, auditing, marketing, and academic recommendation.

From the theoretical perspective, we also analyze an expert information disclosure model with multiple audiences, who have coordination incentives and dispersed beliefs. More importantly, the expert's message will endogenously affect the fundamentals signaled by the message, which may motivate new research on general theoretical disclosure models.

A Proofs of lemmas and propositions

Proof of Proposition 1:

To show there is a unique equilibrium in this benchmark model, we only need to show that there is a unique solution $(\tilde{\theta}_1, \tilde{\theta}_2, \tilde{x})$ to equations (8), (9), and (10).

We first solve $\tilde{x}$ from equation (9). Define

$$\tilde{\Delta} = \frac{\frac{pV-qH}{p-q} - \left[(1-\gamma)F + \gamma f(\tilde{\theta}_2)\right]}{(1-\gamma)\left(f(\tilde{\theta}_2) - F\right)}.$$

Because $f(\theta)$ is strictly decreasing, $\tilde{\Delta}$ is strictly increasing in $\tilde{\theta}_2$. Then we have

$$\tilde{x} = \tilde{\theta}_2 + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\tilde{\Delta}), \tag{24}$$

and $\tilde{x}$ is strictly increasing in $\tilde{\theta}_2$.

Plugging $\tilde{x}$ as a function of $\tilde{\theta}_2$ into equation (10), we have

$$\tilde{\Delta}(pF - qF) + \Phi\left[\sqrt{\beta}\left(\tilde{\theta}_2 - \tilde{\theta}_1 + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\tilde{\Delta})\right)\right] qF = 1. \tag{25}$$

The left-hand side of equation (25) strictly increases in $\tilde{\theta}_2$ and strictly decreases in $\tilde{\theta}_1$. So, we have $\partial\tilde{\theta}_2/\partial\tilde{\theta}_1 > 0$ and $\partial\tilde{x}/\partial\tilde{\theta}_1 > 0$.

Let's finally consider equation (8). The derivative of the left-hand side of equation (8) is

$$\frac{\partial K}{\partial \tilde{\theta}_1} + \frac{\partial K}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial \tilde{\theta}_1},$$

where

$$\begin{aligned}
\frac{\partial K}{\partial \tilde{\theta}_1} &= \left[\gamma + (1-\gamma)\Phi\left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right)\right] f'(\tilde{\theta}_1) \\
&\qquad - (1-\gamma)\sqrt{\beta}\phi\left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right)\left(f(\tilde{\theta}_1) - F\right) < 0; \\
\frac{\partial K}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial \tilde{\theta}_1} &= (1-\gamma)\sqrt{\beta}\phi\left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right)\left(f(\tilde{\theta}_1) - F\right)\frac{\partial \tilde{x}}{\partial \tilde{\theta}_1} > 0.
\end{aligned}$$

Note that $\tilde{\Delta}$ is between 0 and 1. From equation (25), we have $\beta \to +\infty$, $\tilde{\Delta}$ is bounded away from both 0 and 1. To see this, suppose $\tilde{\Delta} \to 1$ first. Then, the left-hand side of equation (25) goes to pF, which is greater than 1, the right-hand side of equation (25). Similarly, if $\tilde{\Delta} \to 0$, the left-hand side of equation (25) is strictly less than 1.

Hence, from equation (24), $\tilde{x} \to \tilde{\theta}_2$. In addition, as $\beta \to +\infty$, $\tilde{\theta}_2$ cannot converge to $\tilde{\theta}_1$; otherwise, equation (8) and equation (9) cannot hold at the same time. Therefore, as $\beta \to +\infty$, $\tilde{x} - \tilde{\theta}_1$ is bounded away from 0. This implies that

$$\lim_{\beta \to +\infty} \sqrt{\beta}\phi\left(\sqrt{\beta}(\tilde{x} - \tilde{\theta}_1)\right) = 0.$$

Therefore, though $\frac{\partial K}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial \tilde{\theta}_1} > 0$, when β is large enough, $\frac{\partial K}{\partial \tilde{x}}\frac{\partial \tilde{x}}{\partial \tilde{\theta}_1}$ is very close to 0. For the term of $\frac{\partial K}{\partial \tilde{\theta}_1}$, it will not go to 0 as β goes to ∞, because $f'(\tilde{\theta}_1) < 0$. Therefore, there is a $\tilde{\beta} > 0$, such that for all $\beta > \tilde{\beta}$, the left-hand side of equation (8) strictly decreases in $\tilde{\theta}_1$. The left-hand side of equation (8) converges to F when $\tilde{\theta}_1$ goes to $+\infty$ and diverges to $+\infty$ when $\tilde{\theta}_1$ goes to $-\infty$. Then by the continuity of function $f(\cdot)$, there is a unique $\tilde{\theta}_1$. Then, there is a unique solution to equation (8), (9), and (10).

Q.E.D.

Proof of Lemma 1:

We first show that the CRA will not assign the rating q in an equilibrium. Suppose there is an equilibrium, in which the CRA assigns the rating q to θ-firm when $\theta \in (\theta_1, \theta_2)$. There are two cases. In the first case, there is no $\theta < \theta_1$ such that θ-firm receives the rating p. Then, observing the rating p, creditors are more optimistic about the firm's fundamentals. Since f is strictly decreasing in θ, if the CRA deviates to assign the rating p to θ-firm when $\theta \in (\theta_1, \theta_2)$, more creditors will roll over, reducing θ-firm's financial cost. Hence, because θ-firm does not default early following the rating q, it does not default early following the rating p either. So, a rating strategy that assigns the rating q to θ-firm for $\theta \in (\theta_1, \theta_2)$ but does not assign the rating p to a positive measure of firms with θ below θ_1 cannot be an equilibrium rating strategy.

Now, consider the second case in which the rating strategy specifies $\mathcal{R}(\theta) = p$ when $\theta \in [\theta_3, \theta_4)$ and $\mathcal{R}(\theta) = p$ when $\theta \in (\theta_1, \theta_2)$, where $\theta_3 < \theta_4 < \theta_1$. Due to the partial verifiability constraint, θ_3-firm does not default early. Then, if the CRA assigns the rating p to θ-firm for $\theta \in (\theta_1, \theta_2)$, θ-firm's financial cost is lower than θ_3-firm's. Therefore, θ-firm does not default early, and the deviation is profitable. These arguments show that the CRA will not assign the rating q in an equilibrium.

Suppose there is an equilibrium, in which θ-firm does not default early. So the CRA will assign the rating $\mathcal{R}(\theta) = p$, because the CRA wants to maximize the nominal rating $\mathcal{R}(\theta)$. Let $W(p)$ be the measure of creditors who choose to roll over, after observing the credit rating $\mathcal{R}(\theta)$

and their own private signals. Then the assumption that θ-firm does not default early implies

$$K(\theta) = W(p,\theta)F + (1 - W(p,\theta))\, f(\theta) < H.$$

Now, let's consider any θ'-firm with $\theta' > \theta$. Again, because the CRA wants to maximize the nominal rating $R(\theta')$, if and only if the θ'-firm does not default early, the CRA will assign $R(\theta') = p$. In a monotone equilibrium, any creditor i's strategy is monotonic in his private signal x_i, and any creditor's private signal conditional on θ' First-order Stochastic dominates that conditional on θ. So $W(p,\theta') > W(p,\theta)$. Recalling that $f(\theta) > F$ for all θ, we have

$$\begin{aligned} K(\theta') &= W(p,\theta')F + \left(1 - W(p,\theta')\right) f(\theta') \\ &< W(p,\theta)F + (1 - W(p,\theta))\, f(\theta') \\ &< W(p,\theta)F + (1 - W(p,\theta))\, f(\theta) \\ &< H. \end{aligned}$$

Therefore, if θ-firm does not default early, θ'-firm does not default early either, implying that in the equilibrium, $R(\theta') = p$.

Furthermore, independent of creditors' decisions, when θ is very negative, the firm will default early, and when θ is very positive, the firm will not default early. As a result, in any equilibrium (if exists), the CRA's rating strategy must be in the form described by equation (11).

Q.E.D.

Proof of Lemma 2:

Suppose there is an equilibrium in which the firm invests in VP for all θ such that $\mathcal{R}(\theta) = p$. All patient creditors will roll over the debt, leading to the firm's financial cost

$$\gamma f(\theta) + (1 - \gamma)F.$$

For the firm to choose VP if and only if $\theta \geq \theta_1^*$, we must have

$$(1-\gamma)F + \gamma f(\theta_1^*) \leq \frac{pV - qH}{p - q} < H.$$

But because $f(\dot{})$ is continuous and strictly decreasing, there exists $\hat{\theta}_1^* < \theta_1^*$ such that,

$$\frac{pV - qH}{p - q} < \gamma f(\hat{\theta}_1^*) + (1 - \gamma)F < H.$$

That is, there is a positive measure subset of θ's that are greater than $\hat{\theta}_1^*$ but very close to $\hat{\theta}_1^*$, the firm will invest in HR. Since the firm's investment choice HR is unverifiable, a deviation to the rating strategy with $\hat{\theta}_1^*$ is profitable to the CRA. Therefore, the rating strategy with θ_1^* such that $(1-\gamma)F + \gamma f(\theta_1^*) \leq \frac{pV-qH}{p-q}$ cannot be part of an equilibrium. Therefore, if an equilibrium exists, in an equilibrium, the rating strategy must be inflated.

Q.E.D.

Proof of Lemma 3:

For a given $x^* \in \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, the left-hand side of equation (13) is strictly decreasing in θ. When $\theta \to +\infty$, the LHS of equation (13) goes to F, which is assumed strictly less than $\frac{pV-qH}{p-q}$. However, if when $\theta = \theta_1^*$, the LHS is still greater than $\frac{pV-qH}{p-q}$, the firm will always choose VP after the rating $R = p$. This contradicts Lemma 2. Therefore, for a given $x^* \in \{-\infty\} \cup \mathbb{R} \cup \{+\infty\}$, there is a unique $\theta_2^* > \theta_1^*$, such that equation (13) holds. Then we can solve for x^* from equation (13)

$$x^* = \theta_2^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}\left[\frac{\frac{pV-qH}{p-q} - ((1-\gamma)F + \gamma f(\theta_2^*))}{(1-\gamma)\left[f(\theta_2^*) - F\right]}\right].$$

Denote

$$\Delta = \frac{\frac{pV-qH}{p-q} - ((1-\gamma)F + \gamma f(\theta_2^*))}{(1-\gamma)\left[f(\theta_2^*) - F\right]},$$

so $x^* = \theta_2^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)$. Because $f(\cdot)$ is strictly decreasing, Δ is strictly increasing in θ_2^*, and thus x^* is strictly increasing in θ_2^*.

Then, plugging x^* as a function of θ_2^* into equation (14), we have

$$\frac{\Delta}{\Phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]}(pF - qF) = 1 - qF. \tag{26}$$

Differentiating the left-hand side of equation (26), the sign of this derivative would be the same as the sign of

$$\frac{\partial \Delta}{\partial \theta_2^*} \frac{\Phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]}{\Delta}$$
$$-\phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right](\sqrt{\beta} + \frac{1}{\phi(\Delta)}\frac{\partial \Delta}{\partial \theta_2^*})$$
$$= \frac{\partial \Delta}{\partial \theta_2^*}\frac{pF - qF}{1 - qF} - \phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right](\sqrt{\beta} + \frac{1}{\phi(\Delta)}\frac{\partial \Delta}{\partial \theta_2^*}).$$

The first term is positive for any β. The second, though is negative, will converge to 0 as $\beta \to +\infty$. This is because $\phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]$ will converge to 0 higher order faster than $\sqrt{\beta}$. We need to consider three cases to prove this argument. First, as $\beta \to +\infty$, $\Delta \to 1$. In this case, it is trivially that $\phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]\sqrt{\beta}$ converges to 0. Second, as $\beta \to +\infty$, Δ is bounded away from both 0 and 1. Then $x^* - \theta_2^* \to 0$. But because $\theta_2^* - \theta_1^*$ is positive and bounded away from 0, $\phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]\sqrt{\beta} = \phi\left[\sqrt{\beta}(x^* - \theta_1^*)\right]\sqrt{\beta}$ must converge to 0. Finally, as $\beta \to +\infty$, $\Delta \to 0$. Then from equation (26), we must have $\Phi\left[\sqrt{\beta}(x^* - \theta_1^*)\right] \to 0$ and thus $\sqrt{\beta}(x^* - \theta_1^*) \to -\infty$ as $\beta \to +\infty$. By L'Hôpital's rule, we have

$$\lim_{\beta \to +\infty} \frac{1}{\sqrt{\beta}(x^* - \theta_1^*)} = \lim_{\beta \to +\infty} \frac{\beta^{-\frac{1}{2}}}{(x^* - \theta_1^*)} = \lim_{\beta \to +\infty} \frac{1}{2\beta^{\frac{3}{2}}\frac{\partial x^*}{\partial \beta}} = 0$$

Therefore, $\lim_{\beta \to +\infty} 2\beta^{\frac{3}{2}}\frac{\partial x^*}{\partial \beta} = +\infty$. Then, simple algebra can lead to the result that $\phi\left[\sqrt{\beta}\left(\theta_2^* - \theta_1^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}(\Delta)\right)\right]\sqrt{\beta}$ converges to 0, as $\beta \to +\infty$. Therefore, there is a β^* such that when $\beta > \beta^*$, the left-hand side of equation (26) is strictly increasing in θ_2^*.

Note by definition, Δ must be a number in $[0, 1]$. Therefore, there are $\bar{\theta}$ and $\underline{\theta}$ such that, $\theta_1^* < \underline{\theta} < \bar{\theta} < +\infty$, $\Delta(\bar{\theta}) = 1$, and $\Delta(\underline{\theta}) = 0$. Then when $\theta_2^* \to \bar{\theta}$, the left hand side of equation (26) is strictly greater than $1 - qF$; when $\theta_2^* \to \underline{\theta}$, the left hand side of equation (26) is close to 0 and thus strictly smaller than $1 - qF$.

Therefore, there is a unique θ_2^*, and thus there is a unique x^*.

Q.E.D.

Proof of Lemma 4:

The left-hand side of equation (26) is strictly increasing in θ_1^*, fixing θ_2^*. Combined with the fact that the left-hand side of equation (26) is strictly increasing in θ_2^*, the Implicit Function Theorem implies that θ_2^* is strictly decreasing in θ_1^*. Since x^* is strictly increasing θ_2^*, x^* is strictly decreasing in θ_1^* (given θ_2^*, x^* is determined by equation $x^* = \theta_2^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}[\Delta]$).

Q.E.D.

Proof of Proposition 2:

When β is sufficiently large, Proposition 3 shows that, for a fixed θ_1^*, there is a unique solution, (x^*, θ_2^*), to equation (13) and equation (14) following $R = p$. Then, by Lemma 4, we only need to show that there is a unique θ_1^* such that equation (15) holds, given x^* as a function of θ_1^*. When β is sufficiently large, by Lemma 4, x^* is strictly decreasing in θ_1^*. Then, the derivative of the left hand side of equation (15) with respect to θ is

$$\left[\gamma + (1-\gamma)\Phi[\sqrt{\beta}(x^* - \theta)]\right] f'(\theta)$$
$$-(1-\gamma)\phi[\sqrt{\beta}(x^* - \theta)]\sqrt{\beta}(f(\theta) - F) + (1-\gamma)\phi[\sqrt{\beta}(x^* - \theta)]\sqrt{\beta}\frac{\partial x^*}{\partial \theta} < 0.$$

We know that when $\theta \to +\infty$, $f(\theta) \to F$, the left hand side of equation (15) converges to F, which is less than H; when $\theta \to -\infty$, $f(\theta) \to +\infty$, the left hand side of equation (15) diverges to $+\infty$, which is greater than H. Therefore, the solution to equation (15) exists and is unique.

Because $H > \frac{pV - qH}{p-q}$, equation (15) and equation (13) imply that $\theta_2^* > \theta_1^*$. In addition, equation (15) also implies that $f(\theta_1^*) > H$, because $f(\theta) > F$ for all $\theta \in \mathbb{R}$. These complete the proof of the uniqueness of the equilibrium of the model.

Q.E.D.

Proof of Lemma 5:

Recall that the three equations determining the equilibrium of the benchmark model are

$$[(1-\gamma)F + \gamma f(\theta_1)] + (1-\gamma)\Phi\left[\sqrt{\beta}(x - \theta_1)\right](f(\theta_1) - F) = H \quad (27)$$
$$[(1-\gamma)F + \gamma f(\theta_2)] + (1-\gamma)\Phi\left[\sqrt{\beta}(x - \theta_2)\right](f(\theta_2) - F) = \frac{pV - qH}{p-q} \quad (28)$$
$$\left\{\Phi\left[\sqrt{\beta}(\theta_2 - x)\right] - \Phi\left[\sqrt{\beta}(\theta_1 - x)\right]\right\} qF + \left\{1 - \Phi\left[\sqrt{\beta}(\theta_2 - x)\right]\right\} pF = 1; \quad (29)$$

and the three equations determining the equilibrium of the model with the CRA are

$$
\begin{aligned}
[(1-\gamma)F + \gamma f(\theta_1)] + (1-\gamma)\Phi\left[\sqrt{\beta}(x-\theta_1)\right](f(\theta_1)-F) &= H \qquad (30)\\
[(1-\gamma)F + \gamma f(\theta_2)] + (1-\gamma)\Phi\left[\sqrt{\beta}(x-\theta_2)\right](f(\theta_2)-F) &= \frac{pV-qH}{p-q} \qquad (31)\\
\frac{\Phi[\sqrt{\beta}(\theta_2-x)]-\Phi[\sqrt{\beta}(\theta_1-x)]}{1-\Phi[\sqrt{\beta}(\theta_1-x)]}qF + \frac{1-\Phi[\sqrt{\beta}(\theta_2-x)]}{1-\Phi[\sqrt{\beta}(\theta_1-x)]}pF &= 1 \qquad (32)
\end{aligned}
$$

The difference between the equilibrium in the benchmark model and that in the model with the CRA stems from the difference between equation (29) and equation (32). That is, the creditors' indifference conditions differ. If we change equation (29) by dividing both sides by the term $1-\Phi[\sqrt{\beta}(\theta_1-x)]$, we have

$$
\begin{aligned}
& \frac{\Phi[\sqrt{\beta}(\theta_2-x)]-\Phi[\sqrt{\beta}(\theta_1-x)]}{1-\Phi[\sqrt{\beta}(\theta_1-x)]}qF + \frac{1-\Phi[\sqrt{\beta}(\theta_2-x)]}{1-\Phi[\sqrt{\beta}(\theta_1-x)]}pF \\
= & \frac{1}{1-\Phi[\sqrt{\beta}(\theta_1-x)]} \qquad (33)\\
> & 1.
\end{aligned}
$$

Solve x as a function of θ_2 from equation (28) or equation (31), and plug it into equation (33) and equation (32). Then, for a same θ_1, θ_2 in equation (33) is greater than that in equation (32), because the left-hand sides of these two equations are strictly increasing in θ_2. Hence, x in equation (33) is greater than x in equation (32). Furthermore, because θ_1 in the benchmark model is positively correlated to θ_2, while θ_1 in the model with the CRA is negatively correlated to θ_2, we know $\theta_1^* < \tilde{\theta}_1$. Moreover, we have $\tilde{\theta}_2 > \theta_2^*$ and $\tilde{x} > x^*$.

However, the sign of $\theta_2^* - \tilde{\theta}_1$ is undetermined. Consider equation (27) and equation (31). Both $\tilde{\theta}_1$ and θ_2^* are strictly increasing functions of x. While we have shown that $\tilde{x} > x^*$, the right-hand side of equation (27) is greater than that of equation (31). Therefore, without specifying parameters' values, we cannot determine the sign of $\theta_2^* - \tilde{\theta}_1$.

Q.E.D.

Proof of Proposition 3:

Suppose the CRA assigns the rating p. When $\theta \geq \tilde{\theta}_2$, $\Omega = \tilde{\Omega} = pV - 1$, so the CRA has no effect on the social welfare.

In case 1 where $\theta_2^* > \tilde{\theta}_1$, for $\theta \in [\theta_1^*, \tilde{\theta}_1)$, $\Omega = qH - 1 < \gamma + (1-\gamma)\Phi(\sqrt{\beta}(\tilde{x}-\theta)) - 1$, because qH is assumed to be less than γ. For $\theta \in [\theta_2^*, \tilde{\theta}_2)$, $\Omega = pV - 1 > qH - 1 = \tilde{\Omega}$. For all other θ's, the firm's investment choice is the same in both the benchmark model and the model with the CRA; thus, CRA has no effect on the social welfare for such firms. Therefore, in such a case, the CRA's ex-ante real effects are

$$(\tilde{\theta}_2 - \theta_2^*)(pV - qH) + \int_{\theta_1^*}^{\tilde{\theta}_1} (qH - \gamma - (1-\gamma)\Phi(\sqrt{\beta}(\tilde{x} - \theta)))\mathrm{d}\theta.$$

In case 2 where $\theta_2^* \leq \tilde{\theta}_1$, for $\theta \in [\theta_1^*, \theta_2^*)$, $\Omega = qH - 1 < \gamma + (1-\gamma)\Phi(\sqrt{\beta}(\tilde{x} - \theta)) - 1$, because qH is assumed to be less than γ. For $\theta \in \left[\theta_2^*, \tilde{\theta}_2\right)$, $\Omega = pV - 1 > \gamma + (1-\gamma)\Phi(\sqrt{\beta}(\tilde{x} - \theta)) - 1 > qH - 1 = \tilde{\Omega}$. For all other θ's, the firm's investment choice is same in both the benchmark model and the model with the CRA; thus, CRA has no effect on the social welfare for such firms. Therefore, in this case, the CRA's ex-ante real effects are

$$(\tilde{\theta}_2 - \theta_2^*)(pV - qH) + \int_{\theta_1^*}^{\tilde{\theta}_1} (qH - \gamma - (1-\gamma)\Phi(\sqrt{\beta}(\tilde{x} - \theta)))\mathrm{d}\theta.$$

Q.E.D.

Proof of Proposition 4:

Part 1: We first consider the subgame following the rating $R = 0$. It then follows from equation (16) that $\theta < \hat{\theta}_1 = \tilde{\theta}_1$. Suppose that all creditors run, then the θ-firm's financial cost is

$$\begin{aligned}
K(\theta) &= f(\theta) \\
&> [(1-\gamma)F + \gamma f(\theta)] + (1-\gamma)\Phi\left(\sqrt{\beta}(x^* - \theta)\right)(f(\theta) - F) \\
&> \left[(1-\gamma)F + \gamma f(\hat{\theta}_1)\right] + (1-\gamma)\Phi\left(\sqrt{\beta}(x^* - \hat{\theta}_1)\right)\left(f(\hat{\theta}_1) - F\right) \\
&> \left[(1-\gamma)F + \gamma f(\tilde{\theta}_1)\right] + (1-\gamma)\Phi\left(\sqrt{\beta}(x^* - \tilde{\theta}_1)\right)\left(f(\tilde{\theta}_1) - F\right) \\
&= H.
\end{aligned}$$

Hence, if all creditors run, the θ-firm will default early. On the other hand, given that any θ-firm will default early, no creditor will roll over, implying that there is an equilibrium in which the firm will default early when receiving the rating $R = 0$ assigned by the reflecting CRA.

We now show that the subgame following $R = 0$ has no equilibrium in which the firm will continue to invest in either HR or VP. Suppose there is a (monotone) equilibrium in which a creditor with the private signal x_i rolls over if and only if $x_i \geq x'$, when the rating is $R = 0$. Here, $x' \in \mathbb{R}$. Since some creditors are willing to roll over, they must believe that any θ-firm will invest in VP if $\theta \in \left[\theta_2', \hat{\theta}_1\right)$, and that by the continuity of the firm's financial cost, any θ-firm will invest in HR if $\theta \in [\theta_1', \theta_2')$, where $\theta_1', \theta_2' \in \mathbb{R}$ and $\theta_1' < \theta_2' < \hat{\theta}_1$. Therefore, such an equilibrium can be characterized by the following system of equations

$$\left[(1-\gamma)F + \gamma f(\theta_1')\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(x' - \theta_1')\right]\left(f(\theta_1') - F\right) = H \tag{34}$$

$$\left[(1-\gamma)F + \gamma f(\theta_2')\right] + (1-\gamma)\Phi\left[\sqrt{\beta}(x' - \theta_2')\right]\left(f(\theta_2') - F\right) = \frac{pV - qH}{p-q} \tag{35}$$

$$\frac{\Phi[\sqrt{\beta}(\theta_2' - x)] - \Phi[\sqrt{\beta}(\theta_1' - x')]}{\Phi[\sqrt{\beta}(\hat{\theta} - x')]} qF + \frac{\Phi[\sqrt{\beta}(\hat{\theta} - x')] - \Phi[\sqrt{\beta}(\theta_2' - x')]}{\Phi[\sqrt{\beta}(\hat{\theta} - x')]} pF = 1 \tag{36}$$

Comparing equation (35) with equation (9), we can see that since $\theta_2' < \hat{\theta}_1 = \tilde{\theta}_1 < \tilde{\theta}_2$, x' must be strictly less than $\hat{x}$. Solving x' as a function of θ_2' from equation (35) and substituting it into equation (34) and equation (36), we get

$$\begin{aligned}(1-\gamma)\Phi\left(\Psi\right)\left(f(\theta_1') - F\right) + \gamma f(\theta_1') &= H - (1-\gamma)F \\ \Delta(pF - qF) + qF\Phi\left(\Psi\right) - (pF-1)\Phi\left(\Psi'\right) &= 1,\end{aligned}$$

where $\Delta = \frac{\frac{pV-qH}{p-q} - \left[(1-\gamma)F + \gamma f(\theta_2')\right]}{(1-\gamma)\left(f(\theta_2') - F\right)}$, $\Psi = \sqrt{\beta}(\theta_2' - \theta_1') + \Phi^{-1}(\Delta)$, and $\Psi' = \sqrt{\beta}(\theta_2' - \hat{\theta}_1) + \Phi^{-1}(\Delta)$.

Totally differentiating the above system of equations, we get

$$A \begin{bmatrix} \frac{\partial \theta_1'}{\partial \hat{\theta}_1} \\ \frac{\partial \theta_2'}{\partial \hat{\theta}_1} \end{bmatrix} = \begin{bmatrix} 0 \\ (pF-1)\phi(\Psi')\frac{\partial \Psi'}{\partial \hat{\theta}_1} \end{bmatrix}, \tag{37}$$

where

$$A = \begin{bmatrix} [(1-\gamma)\Phi(\Psi)+\gamma]f'(\theta_1') + (1-\gamma)\left(f(\theta_1') - F\right)\phi(\Psi)\frac{\partial \Psi}{\partial \theta_1'} & (1-\gamma)\left(f(\theta_1') - F\right)\phi(\Psi)\frac{\partial \Psi}{\partial \theta_2'} \\ qF\phi(\Psi)\frac{\partial \Psi}{\partial \theta_1'} & (p-q)F\frac{\partial \Delta}{\partial \theta_2'} + qF\phi(\Psi)\frac{\partial \Psi}{\partial \theta_2'} - (pF-1)\phi(\Psi')\frac{\partial \Psi'}{\partial \theta_2'}. \end{bmatrix} \tag{38}$$

Note that, when β is sufficiently large, $|A| < 0$. We show that $\partial\theta_1'/\partial\hat{\theta}_1 < 0$ and $\partial\theta_2'/\partial\hat{\theta}_1 < 0$. Therefore, when $\hat{\theta}$ goes to $+\infty$, the equilibrium converges to an equilibrium with $x' < \hat{x}$. However, when $\hat{\theta}$ goes to $+\infty$, equation (36) goes to equation (10). This implies that the benchmark model has another equilibrium with $x' < \hat{x}$. This contradicts the conclusion in Proposition 1 that the benchmark model has a unique equilibrium. Therefore, in the case with a reflecting CRA, the subgame following $R = 0$ has a unique equilibrium in which the firm defaults early and all creditors run.

Part 2: Similarly to the proof of Part 1, in the subgame following $R = p$, there cannot be an equilibrium in which a positive measure of types of the firm defaults early. Otherwise, when $\hat{\theta}_1$ goes to $-\infty$, we show that the benchmark model will have two different equilibria, violating the equilibrium uniqueness result.

Now, suppose that there is $\hat{\theta}_2 \in \left[\hat{\theta}_1, +\infty\right)$. It follows from Lemma 4 that $\hat{\theta}_2$ and $\hat{x}$ are both strictly decreasing in $\hat{\theta}_1$. If $\hat{\theta}_2 > \theta_2^*$ is part of an equilibrium, when the reflecting CRA has the rating strategy $\hat{\theta}_1 = \theta_1^*$, there is an equilibrium in which $\hat{\theta}_2 > \theta_2^*$, violating the equilibrium uniqueness conclusion in Proposition 2. Therefore, in an equilibrium of the subgame following $R = p$, $\hat{\theta}_2 < \theta_2^*$.

Q.E.D.

Proof of Proposition 6:

We first show the comparative static analysis with respect to β. Recall that

$$x^* = \theta_2^* + \frac{1}{\sqrt{\beta}}\Phi^{-1}\left[\Delta\right],$$

where

$$\Delta = \frac{\frac{pV-qH}{p-q} - \left[(1-\gamma)F + \gamma f(\theta_2^*)\right]}{(1-\gamma)\left[f(\theta_2^*) - F\right]},$$

so $\partial\Delta/\partial\theta_2^* > 0$.

Substitute x^* as a function of θ_2^* into equation (14) and equation (15), and denote $\sqrt{\beta}(\theta_2^* - \theta_1^*) + \Phi^{-1}(\Delta) = \Psi$ for simplicity, we have

$$\begin{aligned} \Delta(pF - qF) - \Phi\left[\sqrt{\beta}(\theta_2^* - \theta_1^*) + \Phi^{-1}(\Delta)\right](1 - qF) &= 0 && (39) \\ (1-\gamma)\Phi\left[\sqrt{\beta}(\theta_2^* - \theta_1^*) + \Phi^{-1}(\Delta)\right]\left[f(\theta_1^*) - F\right] + \gamma f(\theta_1^*) &= H - (1-\gamma)F && (40) \end{aligned}$$

Total differentiation of the above two equations with respect to θ_2^*, θ_1^*, and β, we have

$$A\begin{bmatrix} \frac{\partial\theta_2^*}{\partial\beta} \\ \frac{\theta_1^*}{\partial\beta} \end{bmatrix} = \begin{bmatrix} \phi(\Psi)(1-qF)\frac{\theta_2^*-\theta_1^*}{2\sqrt{\beta}} \\ -(1-\gamma)\phi(\Psi)\frac{\theta_2^*-\theta_1^*}{2\sqrt{\beta}}\left[f(\theta_1^*) - F\right] \end{bmatrix}, \quad (41)$$

where

$$A = \begin{bmatrix} \frac{\partial\Delta}{\partial\theta_2^*}(pF-qF) - \phi(\Psi)\left(\sqrt{\beta} + \frac{1}{\phi(\Delta)}\frac{\partial\Delta}{\partial\theta_2^*}\right)(1-qF) & -\phi(\Psi)\sqrt{\beta}(1-qF) \\ (1-\gamma)\phi(\Psi)\left[f(\theta_1^*) - F\right]\left(\sqrt{\beta} + \frac{1}{\phi(\Delta)}\frac{\partial\Delta}{\partial\theta_2^*}\right) & -(1-\gamma)\phi(\Psi)\left[f(\theta_1^*) - F\right]\sqrt{\beta} + \left[\gamma + (1-\gamma)\Phi(\Psi)\right]f'(\theta_1^*) \end{bmatrix} \quad (42)$$

As we have shown in the proofs of Lemma 3, when β is large enough, $\phi(\Psi)\sqrt{\beta}$ is very close to 0. Therefore, when β is sufficiently large, the determinant of the matrix A is close to

$$\frac{\partial \Delta}{\partial \theta_2^*}(pF - qF)\left[\gamma + (1-\gamma)\Phi(\Psi)\right] f'(\theta_1^*) < 0,$$

because $f'(\theta_1^*) < 0$.

Then further algebra shows that when β is sufficiently large, the sign of $\partial\theta_1^*/\partial\beta$ is the same as that of

$$\frac{\partial \Delta}{\partial \theta_2^*}\frac{pF - qF}{1 - qF},$$

which is positive. Therefore, θ_1^* is strictly increasing in β.

Now, let us consider the comparative static analysis with respect to H. Total differentiation of equation (39) and equation (40) with respect to θ_2^*, θ_1^*, and H, we have

$$A\begin{bmatrix}\frac{\partial\theta_2^*}{\partial H}\\ \frac{\theta_1^*}{\partial H}\end{bmatrix} = \begin{bmatrix}-(pF-qF)\frac{\partial\Delta}{\partial H} + \phi(\Psi)\frac{1}{\phi(\Delta)}\frac{\partial\Delta}{\partial H}(1-qF)\\ 1-(1-\gamma)\phi(\Psi)\frac{1}{\phi(\Delta)}\frac{\partial\Delta}{\partial H}\left[f(\theta_1^*)-F\right]\end{bmatrix},$$

where A is defined in equation (42).

Note that $\phi(\Psi)\sqrt{\beta}$ is very close to 0 when β is sufficiently large, we have

$$\text{sign}\begin{bmatrix}\frac{\partial\theta_2^*}{\partial H}\\ \frac{\theta_1^*}{\partial H}\end{bmatrix} = \text{sign}\begin{bmatrix}\left[\gamma+(1-\gamma)\Phi(\Psi)\right]f'(\theta_1^*)\frac{\partial\Delta}{\partial H}\\ -\frac{\partial\Delta}{\partial\theta_2^*}\end{bmatrix}.$$

Because $f'(\theta_1^*) < 0$, $\partial\Delta/\partial H < 0$, and $\partial\Delta/\partial\theta_2^* > 0$, we have $\partial\theta_1^*/\partial H < 0$ and $\partial\theta_2^*/\partial H > 0$. Therefore, $\partial(\theta_2^* - \theta_1^*)/\partial H > 0$.

We finally show the comparative static analysis with respect to γ. Similar to that about H, the total differentiation of of equation (39) and equation (40) with respect to θ_2^*, θ_1^*, and γ, we have

$$A\begin{bmatrix}\frac{\partial\theta_2^*}{\partial H}\\ \frac{\theta_1^*}{\partial H}\end{bmatrix} = \begin{bmatrix}-(pF-qF)\frac{\partial\Delta}{\partial\gamma} + \phi(\Psi)\frac{1}{\phi(\Delta)}\frac{\partial\Delta}{\partial\gamma}(1-qF)\\ -\left[1-\Phi(\Psi)-(1-\gamma)\phi(\Psi)\frac{1}{\phi(\Delta)}\frac{\partial\Delta}{\partial\gamma}\right](f(\theta_1^*)-F)\end{bmatrix},$$

where A is defined in equation (42).

Note that $\phi(\Psi)\sqrt{\beta}$ is very close to 0 when β is sufficiently large. In addition, $(1-\Phi(\Psi))/\phi(\Psi) \to 0$ when $\beta \to +\infty$ and $\partial\Delta/\partial\gamma > 0$. Then simple algebra will show that $\partial\theta_1^*/\partial\gamma > 0$ and $\partial\theta_2^*/\partial\gamma > 0$. Furthermore, because when β is sufficiently large, $\partial\theta_1^*/\partial\gamma$ is close to 0, while $\partial\theta_2^*/\partial\gamma > 0$ is bounded away from 0, we have $\partial(\theta_2^* - \theta_1^*)/\partial\gamma > 0$.

Q.E.D.

Proof of Lemma 6:

We prove each part of this lemma.

1. The case $\frac{V^p - V^q}{C^p - C^q} \geq 1 - q$.

   Because $p > q$, this also implies

   $$\frac{V^p - V^q}{C^p - C^q} \geq 1 - q > 1 - p. \tag{43}$$

   Suppose there is an equilibrium in which the rating strategy $\mathcal{R}(\theta)$ assigns the rating q when $\theta \in (\theta_1, \theta_2)$. Then, as in the proof of Lemma 1, if the CRA deviates to assign the rating p to θ-firm for all $\theta \in (\theta_1, \theta_2)$, the firm will not default early.

   Now, consider any θ-firm with $\mathcal{R}(\theta) > 0$. equation (43) implies that

   $$V^p - (1-p)C^p \quad > \quad V^q - (1-p)C^q, \tag{44}$$
   $$V^p - (1-q)C^p \quad > \quad V^q - (1-q)C^q. \tag{45}$$

   equation (44) implies that if θ-firm invests in VP whether it receives the rating p or q, the CRA wants to assign the rating p; equation (45) implies that if θ-firm invests in HR whether it receives the rating p or q, the CRA wants to assign the rating p; and equation (44) and equation (45) together imply that if θ-firm invests in VP after receiving the rating p and invests in HR after receiving the rating q, the CRA wants to assign the rating p. These arguments also suggest that θ-firm has a lower financial cost after receiving the rating p than after receiving the rating q, so it is impossible for the firm to invest in HR following the rating p and to invest in VP following the rating q.

   Therefore, the CRA's deviation of assigning the rating p to θ-firm for all $\theta \in (\theta_1, \theta_2)$ is profitable. As a result, if the firm does not default early, the CRA will assign the rating p. Then, because the firm's financial cost is monotonic in θ, the equilibrium rating strategy must be in the form of equation (17).

2. The case $\frac{V^p - V^q}{C^p - C^q} \leq 1 - p$.

   This case is same as the previous one, except that we replace the rating p by the rating q.

3. The case $\frac{V^p - V^q}{C^p - C^q} \in (1-p, 1-q)$.

   In this case, we have

   $$V^p - (1-p)C^p > V^q - (1-p)C^q > V^q - (1-q)C^q > V^p - (1-q)C^p.$$

   Therefore, when θ-firm chooses VP, no matter the rating is p or q, the CRA will assign the rating p; when θ-firm chooses HR, no matter the rating is p or q, the CRA will assign the rating q; when θ-firm chooses VP following the rating p and HR following the rating q, the CRA will assign the rating p. Furthermore, because for any particular θ-firm, the financial cost following the rating p is lower than the financial cost following the rating q, if the firm invest in HR following the rating p, it will invest in HR following the rating q.

   Hence, the rating strategy must be in the form as in equation (19). Whether the rating q can appear in the equilibrium depends on the model's parameters, so θ'' could be equal to θ', in which case the CRA will not assign the rating q in an equilibrium.

Q.E.D.

References

Almeida, H., M. Campello, I. Cunha, and M. S. Weisbach (2014). Corporate Liquidity Management: A Conceptual Framework and Survey. *Annual Review of Financial Economics 6*(1), 135–162.

Alp, A. (2013, December). Structural Shifts in Credit Rating Standards. *Journal of Finance 68*(6), 2435–2470.

Angeletos, G.-M., C. Hellwig, and A. Pavan (2006, June). Signaling in a Global Game: Coordination and Policy Traps. *Journal of Political Economy 114*(3), 452–484.

Angeletos, G.-M., C. Hellwig, and A. Pavan (2007, May). Dynamic Global Games of Regime Change: Learning, Multiplicity, and the Timing of Attacks. *Econometrica 75*(3), 711–756.

Angeletos, G.-M. and A. Pavan (2013, September). Selection-free predictions in global games with endogenous information and multiple equilibria. *Theoretical Economics 8*(3), 883–938.

Baghai, R. P., H. Servaes, and A. Tamayo (2014, October). Have Rating Agencies Become More Conservative? Implications for Capital Structure and Debt Pricing. *Journal of Finance 69*(5), 1961–2005.

Bolton, P., X. Freixas, and J. Shapiro (2012, January). The credit ratings game. *Journal of Finance 67*(1), 85–111.

Boot, A. W. A., T. T. Milbourn, and A. Schmeits (2006). Credit Ratings as Coordination Mechanisms. *Review of Financial Studies 19*(1), 81–118.

Carlsson, H. and E. van Damme (1993, September). Global Games and Equilibrium Selection. *Econometrica 61*(5), 989–1018.

Cornaggia, J. and K. J. Cornaggia (2013). Estimating the Costs of Issuer-Paid Credit Ratings. *Review of Financial Studies 26*(9), 2229–2269.

Crawford, V. and J. Sobel (1982, November). Strategic information transmission. *Econometrica 50*(6), 1431–1451.

Edmond, C. (2013, May). Information Manipulation, Coordination, and Regime Change. *Review of Economic Studies 80*(4), 1422–1458.

Fong, K. Y. L., H. G. Hong, M. T. Kacperczyk, and J. D. Kubik (2014, August). Do Security Analysts Discipline Credit Rating Agencies? Working paper, University of New South Wales.

Frenkel, S. (2015, February). Repeated Interaction and Rating Inflation: A Model of Double Reputation. *American Economic Journal: Microeconomics 7*(1), 250–280.

Goldstein, I. and C. Huang (2016, May). Bayesian persuasion in coordination games. *American Economic Review Papers and Proceedings 106*(5), 592–596.

He, Z. and W. Xiong (2012, February). Dynamic Debt Runs. *Review of Financial Studies 25*(6), 1799–1843.

Holden, S., G. J. Natvik, and A. Vigier (2016). Credit rating and debt crises. *International Economic Review*. forthcoming.

Huang, C. (2017). Defending against Speculative Attacks: The Policy Maker's Reputation. *Journal of Economic Theory*. forthcoming.

Jiang, J. X., M. H. Stanford, and Y. Xie (2012, September). Does It Matter Who Pays for Bond Ratings? Historical Evidence. *Journal of Financial Economics 105*(3), 607–621.

Kamenica, E. and M. Gentzkow (2011, October). Bayesian Persuasion. *American Economic Review 101*(6), 2590–2615.

Lizzeri, A. (1999, Summer). Information Revelation and Certification Intermediaries. *The RAND Journal of Economics 30*(2), 214–231.

Manso, G. (2013, August). Feedback Effects of Credit Ratings. *Journal of Financial Economics 109*(2), 535–548.

Morris, S. and H. S. Shin (2002, December). Social Value of Public Information. *American Economic Review 92*(5), 1521–1534.

Morris, S. and H. S. Shin (2003, June). Global Games: Theory and Applications. In *Advances in Economics and Econometrics: Theory and Applications*, Volume 1, pp. 57–114. Cambridge University Press.

Morris, S. and H. S. Shin (2004). Coordination Risk and the Price of Debt. *European Economic Review 48*, 133–153.

Opp, C., M. Opp, and M. Harris (2013, April). Rating agencies in the face of regulation. *Journal of Financial Economics 108*(1), 46–61.

Skreta, V. and L. Veldkamp (2009, July). Ratings Shopping and Asset Complexity: A Theory of Ratings Inflation. *Journal of Monetary Economics 56*(5), 678–695.

Strobl, G. and H. Xia (2012, February). The Issuer-Pays Rating Model and Ratings Inflation: Evidence from Corporate Credit Ratings. Working paper, University of Texas at Dallas.

White, L. (2013, November). Credit Rating Agencies: An Overview. *Annual Review of Financial Economics 5*, 93–122.