Inventory Management, Dealers' Connections, and Prices in OTC Markets*

Jean-Edouard Colliard† Thierry Foucault‡ Peter Hoffmann§

October 5, 2018

Abstract

We propose a new model of interdealer trading. Dealers trade together to reduce their inventory holding costs. Core dealers share these costs efficiently and provide liquidity to peripheral dealers, who have heterogeneous access to core dealers. We derive predictions about the effects of peripheral dealers' connectedness to core dealers and the allocation of aggregate inventories between core and peripheral dealers on the distribution of interdealer prices, the efficiency of interdealer trades, and trading costs for the dealers' clients. For instance, the dispersion of interdealer prices is higher when fewer peripheral dealers are connected to core dealers or when their aggregate inventory is higher.

Keywords: OTC markets, Interdealer trading, Inventory management.

*This is a major revision of a paper previously entitled "Trading Frictions in the Interbank Market, and the Central Bank." We are grateful to Ana Babus, Vincent Bignon, Regis Breton, Briana Chang, Hans Degryse, Hughes Dastarac, Rod Garratt, Co-Pierre Georg, Denis Gromb, Cornelia Holthausen, Cyril Monnet, Artem Neklyudov, Miklos Vari, Mao Ye, and audiences at the 2017 AFA Meetings, the 2017 FIRS Conference, the EIEF 1st Junior Finance Conference, the 12th Annual Central Bank Workshop on the Microstructure of Financial Markets, the conference on Monetary Policy Implementation in the Long Run, University of Sydney, University of Technology Sydney, Banque de France, Bundesbank, European Central Bank, KU Leuven, Paris Dauphine, INSEAD, Ecole Polytechnique, and the University of Illinois at Urbana-Champaign, for helpful comments and suggestions. We are extremely grateful to Yapei Zhang for excellent research assistance. We acknowledge financial support from the Investissements d'Avenir Labex (ANR-11-IDEX-0003/Labex Ecodec/ANR-11-LABX-0047). The views expressed in this paper are the authors' and do not necessarily reflect those of the European Central Bank or the Eurosystem.

†HEC Paris. 1 rue de la Libération, 78351 Jouy-en-Josas, France. E-mail: colliard@hec.fr.

‡HEC Paris. 1 rue de la Libération, 78351 Jouy-en-Josas, France. E-mail: foucault@hec.fr.

§European Central Bank. Sonnemannstrasse 20, 60314 Frankfurt am Main, Germany. E-mail: peter.hoffmann@ecb.int.

1 Introduction

Many assets (fixed income securities, interest rate swaps, credit default swaps, securitized assets, currencies etc.) trade in over-the-counter (OTC) markets. Given their size and key role for risk-sharing, understanding prices and allocations in these markets is important. Liquidity in OTC markets stems from dealers' willingness to absorb temporary imbalances between end-investors' supply and demand and therefore their ability to manage their inventories (Duffie (2012b)). In particular, dealers can offer better prices to their clients when they bear smaller inventory holding costs.

Interdealer trading plays a central role for managing these costs.[1] As interdealer trading is often decentralized, a dealer's ability to manage his inventory depends on his relationships (connections) with other dealers. On this dimension, dealers are not all equal. Typically, a small set of ("core") dealers provides liquidity to other ("peripheral") dealers.[2] In this context, it is natural to ask how peripheral dealers' access to core dealers affects their ability to manage their inventory, the terms of trades in interdealer markets, and ultimately trading costs for the dealers' clients.

To address this question, we consider a new model of interdealer trading. Dealers enter the interdealer market with inventory positions stemming from past trades with their clients and bear a cost when their end-of-day inventory deviates from their target inventory. Dealers use the interdealer market to minimize this cost (e.g., dealers with a long position sell the asset to dealers with a short position).

The interdealer market features three groups of dealers that we label core dealers, connected peripheral dealers, and unconnected peripheral dealers. We assume that the trading process among core dealers is efficient: they reach the same allocation of inventory holding costs as that which would be obtained in a competitive centralized market. Connected pe-

[1] See, for instance, Madhavan and Smidt (1993), Fleming (2007), or Schultz (2017) for evidence on dealers' inventory management. See also Reiss and Werner (1998), Dunne, Hau, and Moore (2015), Li and Schuerhoff (2017), or Collin-Dufresne, Junge, and Trolle (2016), for evidence that interdealer trades account for a significant fraction of total trading volume in OTC markets. For instance, Schultz (2017) reports that about half of the corporate bonds trades in his sample is due to interdealer trading. He also shows that these trades mainly serve to reduce inventories for both parties.

[2] In the Online Appendix B.1, we give four examples of assets (corporate bonds, foreign exchange, interest rate swaps, and banks' reserves) that trade in markets with a structure similar to that assumed in our model.

ripheral dealers can trade with core dealers while unconnected peripheral dealers cannot. Moreover, due to the sparsity of connections among peripheral dealers, each peripheral dealer has only few opportunities to trade with other peripheral dealers. Specifically, each peripheral dealer can only receive one offer or make one offer (if he rejects the offer he receives) to another peripheral dealer. Thus, a peripheral dealer has local market power when he makes an offer.

We solve for the equilibrium of the model, i.e., equilibrium allocations and prices for all possible matches (e.g., connected buyer - unconnected seller, connecter buyer - connected seller, etc.). In particular, in the peripheral market, we characterize (i) the offer made by each type of dealer (connected or unconnected) given his inventory position and (ii) the likelihood that this offer is accepted by another peripheral dealer.

As usual in models with inventory effects (e.g., Grossman and Miller (1988)), an increase in the aggregate position of dealers in one segment depresses prices in this segment. For instance, peripheral buyers obtain a larger fraction of gains from trade when peripheral dealers' aggregate inventory increases (so that the ratio of sellers to buyers in the interdealer market increases). The prices in a given segment are also affected by aggregate inventories in the other segment, but to a lesser extent. Beyond these standard effects, the model generates several new testable predictions that all stem from the lack of full connectedness between peripheral and core dealers and are therefore specific to our framework.[3]

First, heterogeneity in transaction prices among peripheral dealers is explained by whether dealers involved in the transaction are unconnected or connected. In particular, consistent with empirical findings in Di Maggio, Kermani, and Song (2017), connected buyers (resp., sellers) trade on average at (weakly) better prices than unconnected buyers (resp., sellers) because connected dealers have a better outside option: they can trade with core dealers when their offers to other peripheral dealers are rejected.

Second, the spread between the prices at which transactions take place between peripheral dealers and the price in the core market (the "core-periphery spread") should have a sign opposite to that of peripheral dealers' aggregate inventory. For instance, when peripheral dealers' *aggregate* inventory is negative (they are short relative to their target

[3]When all peripheral dealers are connected to core dealers, equilibrium prices and allocations are identical to those obtained in a competitive centralized market.

inventory), dealers with a long position (sellers) have a strong bargaining position and even more so if they are connected. Indeed, they are on the "uncrowded side" of the market. Thus, if they reject an offer, they have less risk of failing to find a counterparty. In this case, transaction prices among peripheral dealers are higher than the core market price (the core-periphery spread is positive on average).

Third, an increase in *core* dealers' aggregate inventory (e.g., due to a new issuance) lowers transaction prices among peripheral dealers. It reduces the price at which core dealers are willing to buy the asset and therefore the outside option for connected peripheral dealers with a long position (sellers). Peripheral dealers with a short position (buyers) can then purchase the asset at lower prices from connected sellers, which in turn induces unconnected sellers to offer better prices. This means that the negative shock to core dealers' price is passed through to peripheral dealers' prices even though their inventory does not change. However, this pass-through is less than one-for-one for prices posted by connected peripheral dealers on the "uncrowded side" of the market. As a result, an increase in core dealers' aggregate inventory raises (resp., reduces) the dispersion of prices in the interdealer market (measured by the difference between the highest and lowest possible transaction prices in equilibrium) if peripheral dealers have a long (resp., short) position in aggregate.

Fourth, a sudden loss of access to core dealers by some peripheral dealers increases the dispersion of transaction prices among peripheral dealers.[4] The reason is that the remaining connected peripheral dealers have more market power and can demand larger markups (resp., discounts) when selling (resp., buying) the asset.

More generally, a decrease in connectedness significantly distorts the distribution of equilibrium prices in the peripheral market relative to the distribution of prices that would be observed if trading were centralized. In particular, the distribution of prices in the peripheral market is shifted to the right (left) if peripheral dealers' aggregate inventory position is negative (positive). This shift occurs because unconnected peripheral dealers trade above the core market price in this case. Moreover, for the same reason the distribu-

[4] In reality, there might be various reasons for such a shock. For instance, increases in concerns about counterparty risk might lead some core dealers to severe their relationships with other dealers in some markets (e.g., interbank or OTC derivative markets). Another possibility is the bankruptcy of a core dealer, as considered in Di Maggio, Kermani, and Song (2017).

tion of prices becomes multimodal, the additional modes reflecting the transaction prices of dealers with worse bargaining positions.

Under limited connectedness, inventory allocations among dealers are inefficient for two reasons. First, rent-seeking by unconnected dealers can induce them to make offers that have a small chance of being accepted, which raises the risk of carrying excess inventories when the market closes. We show that this problem arises when core dealers' aggregate inventory has a sign opposite to that of peripheral dealers and is large (in absolute value) relative to peripheral dealers' aggregate inventory. For instance, suppose that core dealers' aggregate inventory experiences a large increase. The price in the core market declines significantly and therefore connected peripheral buyers' outside option improves. Consequently, peripheral sellers must either post more attractive offers (accepted by all buyers, connected or not) *or* post less attractive offers, accepted only by unconnected buyers, at the risk of not finding such buyers. The second choice is optimal if the ratio of buyers to sellers among peripheral dealers is high enough, i.e., if peripheral dealers' aggregate position is sufficiently negative. One testable implication of this mechanism is that unconnected dealers on the uncrowded side of the market are more likely to carry out excess inventories overnight on days in which core dealers' positions are large and of sign opposite to peripheral dealers' aggregate positions.

A second source of inefficiency stems from connected dealers' participation decision to the core and peripheral markets. In our model this decision is endogenous in the sense that connected dealers can choose to exclusively trade with core dealers. Now, participation of connected dealers on the crowded side of the peripheral market exerts a *negative externality* on unconnected dealers. Indeed, when they trade with an unconnected dealer on the uncrowded side of the market, they shrink the pool of potential liquidity suppliers to other unconnected dealers on the crowded side of the market. This is inefficient because unconnected dealers do not have access to liquidity providers in the core, unlike connected dealers. Thus, allocative efficiency requires that connected dealers on the crowded side of the market trade with core dealers only.

Finally, we analyze how dealers' connectedness and the efficiency of interdealer transactions affect trading costs for dealers' clients (end-investors). To this end, we introduce

a trading round between dealers and clients before interdealer trading. We show that connected peripheral dealers charge smaller bid-ask spreads than unconnected dealers because they obtain a higher expected profit on interdealer transactions, both because they can manage inventory costs more efficiently and obtain larger rents. Moreover, the difference between the bid-ask spreads charged by unconnected and connected dealers increases when more dealers become unconnected, which suggests that dealers' connectedness is a determinant of the dispersion in bid-ask spreads observed in OTC markets. Relatedly, the difference between equilibrium transaction costs for dealers' clients and those that would be obtained if trades were conducted efficiently among dealers (the first best allocation of inventories among dealers) is inversely related to the level of connectedness between peripheral and core dealers.

The paper is organized as follows. The next section positions our contribution in the theoretical literature on OTC markets and inventory management. We describe the model in Section 3 and derive its equilibrium in Section 4. In Section 5, we analyze the testable implications of the model while in Section 6, we study the causes of allocational inefficiencies in the interdealer market and their impact on trading costs. Section 7 concludes. Proofs of the main results are in the Appendix.

2 Contribution to the Literature

A key motivation for our analysis is the growing evidence that dealers' inventory holding costs matter for understanding asset prices and liquidity in OTC markets (see, for instance, Schultz (2017), Friewald and Nagler (2017), Randall (2015a), or Fleming (2007)). Moreover, there are concerns that bond markets have become less liquid because dealers' inventory holding costs have increased due to regulatory changes following the 2007-2008 crisis (see Duffie (2012b), Bessembinder *et al.* (2018), or Adrian, Boyarchenko, and Shachar (2017)).

Yet, the two strands of the literature that are necessary to understand these phenomena have remained somewhat separated. The theoretical literature on inventory management (e.g., Stoll (1978), Ho and Stoll (1983), Grossman and Miller (1988), or Biais (1993),

Hendershott and Menkveld (2014)) has not considered how inventory holding costs affect asset prices when interdealer trading is decentralized. Conversely, the existing theoretical literature on OTC markets remains largely silent on dealers' inventory management and the effects of dealers' aggregate inventories on transaction prices.[5] Our paper fills a gap in the literature by featuring a decentralized interdealer market in a setup with inventory costs.

More precisely, the theoretical literature on OTC markets mostly focuses on decentralized trading among customers and between customers and dealers, but usually assumes that interdealer trading is centralized (e.g., Duffie, Garleanu, and Pedersen (2005), Lagos and Rocheteau (2009), Lester, Rocheteau, and Weill (2015), Babus and Parlatore (2018)).[6] Thus, the distribution of dealer inventories between various segments of the interdealer market (e.g., core and peripheral dealers) plays no role. For instance, Randall (2015b) derives results on the impact of inventory costs, but the interdealer market is competitive so that only the aggregate inventory matters (as when all dealers are connected in our model), not how it is split between peripheral and core dealers. In contrast, we focus on how the distribution of inventories affects asset prices when frictions prevent an efficient allocation of inventories. Frictions in our model do not arise from search costs (see Footnote 10) but from (i) the lack of connections between core and peripheral dealers (segmentation) and (ii) rent-seeking.[7]

Some recent papers have proposed theories to endogenize the existing structure of OTC markets, and in particular three features: (i) The presence of intermediaries (dealers) and the existence of both dealer-to-customer and dealer-to-dealer trades (Atkeson, Eisfeldt, and

[5]Inventory costs play an explicit role in other recent papers such as Randall (2015b), Babus and Parlatore (2018), or Wang (2017), as explained below. More generally, differences in traders' private valuations in models of OTC trading can always be interpreted as reflecting heterogeneity in inventory positions and corresponding holding costs. However, the literature has not explicitly linked prices observed in various segments of the interdealer market to the distribution of dealers' aggregate inventories across segments.

[6]Afonso and Lagos (2015) is an exception in that they consider a decentralized interdealer market (the Fed funds market). However, there is no segmentation between various groups of dealers (core/peripheral) as in our model. Hence, the distribution of dealers' inventories across various groups play no role in their model.

[7]Duffie (2012a) highlights these as two key research issues regarding OTC markets and writes (Chapter 1, p.1): "*Some of the key research and policy issues regarding OTC markets include: [...] (ii) the manner in which the price negotiated on a particular trade reflects the relative degree of connectedness of the buyer and seller with the rest of the market [...] (iv) the influence of market structure on the cross-sectional dispersion of prices negotiated at a particular time [...]*".

Weill (2015), Hugonnier, Lester, and Weill (2016), Farboodi, Jarosh, and Shimer (2017), Chang and Zhang (2016)); (ii) The core-periphery structure of the dealer market (Neklyudov (2014), Neklyudov and Sambalaibat (2017), Wang (2017)); and (iii) the fragmentation of investors across various dealers (Babus and Parlatore (2018)). Our goal here is different. We take the market structure as given and focus on the role of core and peripheral dealers' aggregate inventory positions in determining prices observed in the core market and in the peripheral market. The resulting predictions are, to our knowledge, new and can help empiricists to specify empirical models relating prices to dealers' aggregate inventories (e.g., as in Friewald and Nagler (2017)).

3 Model

3.1 Market participants and timing

We consider an interdealer market with two types of participants: (i) a continuum of "peripheral" dealers and (ii) a continuum of "core" dealers. The mass of peripheral dealers is normalized to 1, and the mass of core dealers is denoted κ. Dealers trade an asset with final payoff v. We normalize the expected payoff to zero to simplify notations.

[Insert Figure 1]

There are four dates $t \in \{0, 1, 2, 3\}$ (see Figure 1). In each period, dealer i has a position z_{it} in the asset and cash holdings m_{it}. We set $m_{i0} = 0$, as dealers' initial cash holdings play no role in the analysis. Dealer i's initial asset endowment, z_{i0}, reflects trades with his clients prior to date 0 (we model these trades explicitly in Section 6.2). A fraction α^{pe} of peripheral dealers and a fraction α^{co} of core dealers have a long position $z_{i0} = +1$ while the other dealers have a short position, $z_{i0} = -1$. Thus, the aggregate net inventory positions of peripheral and core dealers, respectively, are:

$$z_0^{pe} = (2\alpha^{pe} - 1), z_0^{co} = \kappa(2\alpha^{co} - 1). \tag{1}$$

Dealers trade together at dates 1 and 2 (see Section 3.2), and we denote by q_{it} their net trading at date $t \in \{1, 2\}$. In addition, each dealer i receives an additional inventory

shock ϵ_i (independent of v) between dates 2 and 3. This shock represents variations in the dealer's inventory due to "end-of-day" transactions with his clients. Alternatively, it can be interpreted as a dealer's exposure to another source of risk, realized at date 3, correlated with the payoff of the asset. For core dealers, ϵ_i has a continuous and cumulative probability distribution $\Phi(.)$, with support over $\mathbb{R}$ and symmetric around 0 ($\Phi(x) + \Phi(-x) = 1$). For peripheral dealers, ϵ_i is set to zero (we explain the role of these assumptions in Section 4.1). The final position of dealer i at date 3 is:

$$z_{i3} = z_{i0} + q_{i1} + q_{i2} + \epsilon_i. \tag{2}$$

The asset payoff is realized at date 3. Dealers incur a per unit cost of $C^s > 0$ (resp. C^b) holding a long (resp., short) inventory position at date 3. These costs capture, in reduced form, dealers' limited tolerance for risk taking (as in Stoll (1978)) or limited ability to commit capital for the funding of long and short positions (e.g., as in Wang (2017)). The final payoff of dealer i is:

$$\Pi_i = \begin{cases} v \cdot z_{i3} - C^b|z_{i3}| + m_{i3} & if \quad z_{i3} < 0, \\ v \cdot z_{i3} - C^s|z_{i3}| + m_{i3} & if \quad z_{i3} > 0. \end{cases} \tag{3}$$

The heterogeneity in dealers' inventory positions generates gains from trade. For instance, consider two dealers i and j with a long and a short position, respectively. As the expected payoff of the asset is zero, if i sells the asset to j at price p, he increases his expected payoff by $(p + C^s)$, while j increases his expected payoff by $(-p + C^b)$. Thus, gains from trade are equal to $(C^s + C^b)$ per unit traded. As shown below, equilibrium prices will be in $(-C^s, C^b)$. Thus, in equilibrium, a dealer with a short position buys the asset and a dealer with a long position sells it. For brevity, we refer to the former as buyers and the latter as sellers (indexed by b and s, respectively).

3.2 Market structure

Interdealer trading takes place in two stages. At date 1, peripheral dealers trade together in the "peripheral market." Then, at date 2, trading takes place among peripheral and core

dealers in the "core market." A proportion $(1-\lambda)$ of peripheral dealers are "connected" and can trade in the core market. The other peripheral dealers are "unconnected" and can only trade in the peripheral market. We assume $0 < \lambda \leq \frac{1}{2}$.[8] It is convenient for the exposition to describe the trading process in the peripheral and core markets as if they were taking place sequentially at dates 1 and 2. However, results would be identical if all decisions in each market were made simultaneously.

The peripheral market. Peripheral dealers trade bilaterally. Each peripheral dealer is matched with his "predecessor" and "successor" in the continuum of dealers. If he has received an offer to buy or sell one unit of the the asset from his predecessor, a dealer can either accept or reject this offer. If he accepts, both dealers exit the market. If he rejects the offer or if he receives no offer, the dealer can make an offer to his successor and so on and so forth (see Figure 2).[9] If a peripheral dealer trades neither with his predecessor nor his successor then he can trade in the core market (at date 2) if he is connected.

If dealer j is a seller, his successor is a buyer with probability $\pi_b(\alpha^{pe})$. Conversely, for a buyer the successor is a seller with probability $\pi_s(\alpha^{pe})$. We assume that:

$$\pi_b(\alpha^{pe}) = \zeta \min\left(\frac{1-\alpha^{pe}}{\alpha^{pe}}, 1\right), \pi_s(\alpha^{pe}) = \zeta \min\left(\frac{\alpha^{pe}}{1-\alpha^{pe}}, 1\right). \tag{4}$$

Thus, π_b weakly decreases with the mass of sellers, α^{pe}, while π_s weakly increases with this mass (for brevity, henceforth, we omit the argument of π_b and π_s). Thus, finding a counterparty is more difficult for dealers on the crowded side of the market (i.e., with an inventory position of the same sign as peripheral dealers' aggregate inventory). When the peripheral market is balanced (i.e., there are as many buyers as sellers), each seller is matched with a buyer with probability ζ ($\pi_s(1/2) = \pi_b(1/2) = \zeta$). For simplicity, we assume in the rest of the paper that $\zeta = 1$.[10]

[8] This assumption reduces the number of cases to analyze in the model; see Footnote 15.

[9] We exclude by assumption the possibility of intermediation trades. Otherwise, unconnected dealers could "pass" their inventory to each other until they reach a connected dealer, who would offload the accumulated inventory in the core. Such "intermediation chains" are beyond the scope of this paper. Importantly, they would not suppress the effects of segmentation in our model as long as peripheral dealers' inventories cannot be offloaded in the core with probability one, for instance due to asymmetric information between dealers (see e.g. Colliard and Demange (2017) for a model along these lines). With intermediation ruled out, we show in Section 4.1 that it is indeed optimal for a peripheral dealer to fully unwind their inventory, i.e., to trade exactly one unit: $q_{i1} = -z_{i0}$.

[10] The matching technology used in our model implies that the number of matches (per capita) is

[**Insert Figure 2**]

The core market. The core market operates as a centralized Walrasian market. Each dealer operating in this market submits his demand for the asset and behaves competitively (i.e., takes the core market price p^{co} as given). The equilibrium price in the core market is such that this market clears. The difference in market structure between the peripheral and core market captures the idea that core dealers are much more efficient at finding counterparties than peripheral dealers.

In sum, our model is purposely built to study the effects of two (related) frictions: (i) the lack of full connectedness between core and peripheral dealers and (ii) peripheral dealers' strategic behavior in accepting and making offers. Our goal is to analyze how the equilibrium prices and allocations are affected by (i) these frictions and (ii) the relative aggregate inventories of core and peripheral dealers (z_0^{co} and z_0^{pe}).

4 Equilibrium

To solve for the equilibrium of the model, we proceed as follows. First, in Section 4.1, we derive the equilibrium in the core market taking as given the masses of connected peripheral buyers and sellers who eventually trade with core dealers. Then, in Section 4.2, we solve for the equilibrium of the peripheral market, taking as given the price in the core market. Finally, in Section 4.3, we solve for the equilibrium of the entire market by imposing the condition that the equilibrium of the core and the peripheral markets must be consistent with each other.

$\alpha^{pe}\pi_b + (1-\alpha^{pe})\pi_s = 2\min(\alpha^{pe}, 1-\alpha^{pe})$. This specification corresponds to a directed search technology yielding a Leontief matching function $m(s,b) = 2\min(s,b)$, where s is the number of sellers and b the number of buyers. In contrast, most of the literature on OTC markets has focused on random matching, resulting in the matching function $m(s,b) = Csb$, where C is a constant. Our specification satisfies two important properties: (i) All mutually beneficial trades can be achieved because dealers on the crowded side have a probability one of meeting a dealer on the other side (as $\zeta = 1$). Hence, inefficiencies in our model do not stem from a search friction. (ii) The unconditional probability that dealer j is a seller is $\frac{\pi_s}{\pi_b+\pi_s} = \alpha^{pe}$, so that the matching probabilities are consistent with the proportion of buyers and sellers in the peripheral market.

4.1 Equilibrium in the Core Market

Trading in the core market involves core dealers and connected peripheral dealers. First, we derive the asset demand at date 2 for core dealer i at price p^{co}. If she trades q_{i2} units, using (3) and the assumption that $\mathbb{E}(v) = 0$, her expected payoff is:

$$\mathbb{E}(\Pi_i) = \Pr(z_{i3} < 0)\mathbb{E}[z_{i3}|z_{i3} < 0]C^b - \Pr(z_{i3} > 0)\mathbb{E}[z_{i3}|z_{i3} > 0]C^s - q_{i2}p^{co}, \tag{5}$$

where $\Pr(z_{i3} < 0) = \Pr(z_{i0} + q_{i2} + \epsilon_i < 0) = \Phi(-(q_{i2} + z_{i0}))$ is the probability that core dealer i ends the day with a short position. Core dealer i's optimal demand for the asset at date 2, q_{i2}^{co*}, maximizes (5) and satisfies the following first-order condition:[11]

$$C^b\Phi(-q_{i2}^{co*} - z_{i0}) - C^s[1 - \Phi(-q_{i2}^{co*} - z_{i0})] = p^{co}. \tag{6}$$

To understand this condition, consider a marginal increase in dealer i's demand for the asset for dealer i when her demand is q_{i2}. If the dealer ends up with a short position, she saves the inventory holding cost C^b on the additional purchased units. This event happens with probability $\Phi(-q_{i2} - z_{i0})$. If instead the dealer ends up with a long position, she incurs an extra inventory holding cost C^s on the additional purchased units. Thus, Condition (6) states that dealer i's optimal demand for the asset equates the marginal expected utility of buying an extra unit of the asset to its price in the core market. Using this condition, we obtain the following demand function for core dealer i:

$$q_{i2}^{co*}(p^{co}, z_{i0}) = -z_{i0} - \Phi^{-1}\left(\frac{p^{co} + C^s}{C^s + C^b}\right), \text{ for } p^{co} \in (-C^s, C^b), \tag{7}$$

A core dealer's demand decreases with the price of the asset in the core market and her initial inventory (as usual in models with inventory holding costs). We deduce from (7) that core dealers' *aggregate* demand for the asset at date 2 is:

$$q^{co*}(p^{co}, z_0^{co}) = -z_0^{co} - \kappa\Phi^{-1}\left(\frac{p^{co} + C^s}{C^s + C^b}\right), \text{ for } p^{co} \in (-C^s, C^b). \tag{8}$$

[11] The second order condition for a maximum is always satisfied because $C^s + C^b > 0$. Dealer i's demand is thus uniquely defined.

We now derive the demand $q_{i2}^{pe*}(p^{co}, z_{i1})$ of a connected peripheral dealer. A peripheral dealer does not receive an additional shock to his inventory after trading at date 2. Thus, his trade at date 2 determines his final position with certainty ($z_{i3} = z_{i1} + q_{i2}^{pe}$). If $z_{i3} < 0$ and dealer i marginally increases his demand for the asset, he reduces his total inventory holding cost by C^b and pays p^{co}. He is therefore strictly better off if $p^{co} < C^b$. Symmetrically, for $z_{i3} > 0$, dealer i is strictly better off selling the asset if $p^{co} > -C^s$. We deduce that the optimal demand of the connected peripheral dealer i is:

$$q_{i2}^{pe*}(p^{co}, z_{i1}) = -z_{i1}, \text{ for } p^{co} \in (-C^s, C^b). \tag{9}$$

This demand is inelastic to the price in the core market because peripheral dealers face no uncertainty on their final position after trading at date 2, in contrast to core dealers.[12] Equation (9) shows that connected peripheral dealers who traded with other peripheral dealers (at date 1) have no reason to trade in the core market since, for these dealers, $z_{i1} = 0$. Thus, the only connected peripheral dealers who optimally trade with core dealers are those who did not trade with other peripheral dealers (for these dealers, $z_{i1} = z_{i0}$). Let Δ be the net aggregate inventory of these dealers. We deduce from (9) that the aggregate net demand of peripheral dealers in the core market is:

$$q^{pe*} = -\Delta, \text{ for } p^{co} \in (-C^s, C^b). \tag{10}$$

The equilibrium price in the core market, p^{co*}, is such that the net demand in this market is zero, so that:

$$q^{co*}(p^{co*}, z_0^{co}) = \Delta. \tag{11}$$

Thus, in equilibrium, connected peripheral dealers who did not trade with other peripheral dealers transfer their aggregate inventory to core dealers at price p^{co*}. In this sense, core

[12] If core dealers did not receive a final shock to their inventory as well, their demand for the asset would be inelastic to p^{co} for $p^{co} \in (-C^s, C^b)$, and any price in this interval could be an equilibrium. Any level of uncertainty on their final inventory, no matter how small, solves this indeterminacy (see Lemma 1). An unbounded shock additionally ensures that p^{co} is strictly between $-C^s$ and C^b, which simplifies the analysis. Conversely, assuming that peripheral dealers also have a final inventory shock would complexify the analysis of the bargaining game between peripheral dealers, as their demand would become elastic to the price.

dealers provide liquidity to peripheral dealers. Henceforth, we refer to Δ as the *order flow* (or order imbalance) from peripheral dealers in the core market (this is the difference between sell and buy orders from connected peripheral dealers in the core market). Combining equations (8) and (11), we obtain the following result.

Lemma 1. *Let $z^* = \Delta + z_0^{co}$. The equilibrium price in the core market is:*

$$p^{co*}(\alpha^{co}, \Delta) = \Phi(-\kappa^{-1}z^*)C^b - (1 - \Phi(-\kappa^{-1}z^*))C^s. \tag{12}$$

It decreases with core dealers' initial aggregate inventory, z_0^{co}, and the order flow from peripheral dealers, Δ.

After trading, core dealers' aggregate inventory position is z^*. The equilibrium price in the core market decreases with this position, as usual in models with inventory holding costs (e.g., Ho and Stoll (1983) or Hendershott and Menkveld (2014)). When the relative mass of core dealers (κ), becomes infinite, the impact of peripheral dealers' order flow on the core market price becomes negligible and this price is only determined by the mass of core dealers with a long position, α^{co} (since $\kappa^{-1}z^*$ goes to $(2\alpha^{co} - 1)$ when κ becomes infinite). We shall refer to this case as the *"thick core market"* case.

Centralized benchmark. Consider the case in which all peripheral and core dealers trade in a centralized market. In this case, one can derive the equilibrium price, denoted p^{bench*}, exactly as we did in Lemma 1, by setting $\Delta = z_0^{pe}$. Therefore:

$$p^{bench*} = \Phi(-\kappa^{-1}z_0)C^b - (1 - \Phi(-\kappa^{-1}z_0)C^s, \tag{13}$$

where $z_0 = z_0^{pe} + z_0^{co}$ is dealers' aggregate inventory. We will use this (Walrasian) price as a benchmark to assess how frictions in our model affect equilibrium prices and allocations.

4.2 Equilibrium in the Peripheral Market

We now analyze the equilibrium in the peripheral market, taking the core market price $p^{co} \in (-C^s, C^b)$ as given.

Equilibrium strategies. We use subscripts b and s to index buyers' and sellers'

actions, and superscripts U and C to index unconnected and connected peripheral dealers' actions, respectively. Thus, there are four possible types of dealers: $(k, i) \in \{b, s\} \times \{U, C\}$. The offer received by a dealer can be summarized by a quantity $q \in \{-1, 0, 1\}$ ($q = 0$ meaning no offer; $q = 1$ an offer to buy; and $q = -1$ an offer to sell), and a price p. A dealer's strategy specifies whether he accepts the offer (q, p) and, if he does not, his new offer. We focus on Markov perfect equilibria, that is, each dealer's decision is contingent on his type and the offer he receives, but not on the full history of the game. Moreover, we assume that dealers do not observe whether their counterparty is connected or not.[13] Thus, their strategy cannot depend on this characteristic.

We must solve for the equilibrium strategy of each of the four types of dealers conditionally on each offer (q, p). The following remarks (i) to (iv) help to substantially reduce the dimensionality of the problem.

(i) There are no gains from trade between dealers with the same position. Hence, a buyer (resp., seller) rejects the offer from a buyer (resp., seller).

(ii) Consider a dealer of type (k, i). In a Markov perfect equilibrium, the maximum payoff he can obtain if he rejects an offer (q, p) is independent of the history of the game. Hence, conditionally on *rejecting* (or not receiving) an offer, we can define the dealer's continuation value V_k^{i*} as a function of his type only. Let $\varphi_b^*(p_b)$ (resp., $\varphi_s^*(p_s)$) be the equilibrium probability that an offer $(1, p_b)$ (resp., $(-1, p_s)$) is accepted by a seller (resp., buyer). Using (3), for unconnected dealers we have:

$$V_b^{U*} = \max_{p_b \in (-C^s, C^b)} -\varphi_b^*(p_b)p_b - [1 - \varphi_b^*(p_b)]C^b \tag{14}$$

$$V_s^{U*} = \max_{p_s \in (-C^s, C^b)} \varphi_s^*(p_s)p_s - [1 - \varphi_s^*(p_s)]C^s. \tag{15}$$

When a connected dealer's offer is rejected, he is better off trading in the core market at price p^{co} rather than bearing the inventory holding cost C^k. Hence, the continuation value

[13] As in Caballero and Simsek (2013), this assumption models that dealers do not know with certainty who are the counterparties of their own counterparties, e.g., whether they have access to core dealers.

of a connected dealer if he rejects an offer, V_k^{C*}, is:

$$V_b^{C*} = \max_{p_b \in (-C^s, C^b)} -\varphi_b^*(p_b)p_b - [1 - \varphi_b^*(p_b)]p^{co} \tag{16}$$

$$V_s^{C*} = \max_{p_s \in (-C^s, C^b)} \varphi_s^*(p_s)p_s + [1 - \varphi_s^*(p_s)]p^{co}. \tag{17}$$

Note that a connected dealer can always choose to only trade with core dealers by making an offer with a zero probability of being accepted (e.g., $p_b = -C^s$ or $p_s = C^b$). If this happens with probability one, we say that the connected dealer is *inactive* in the peripheral market. From these definitions, we immediately deduce that a buyer of type $i \in \{U, C\}$ accepts an offer to sell at price p_s if and only if $p_s \leq -V_b^{i*}$, and a seller accepts an offer to buy at price p_b if and only if $p_b \geq V_s^{i*}$.

(iii) It immediately follows from (14)-(17) that $V_k^{C*} \geq V_k^{U*}$: Connected dealers are (weakly) better off because they have the option of trading with core dealers. Using remark (i), we deduce the equilibrium values of the acceptance probabilities φ_b^* and φ_s^*:

$$\varphi_b^*(p_b) = \begin{cases} 0 & \text{if } p_b < V_s^{U*} \\ \lambda\pi_s & \text{if } p_b \in [V_s^{U*}, V_s^{C*}) \\ \pi_s & \text{if } p_b \geq V_s^{C*} \end{cases}, \varphi_s^*(p_s) = \begin{cases} 0 & \text{if } p_s > -V_b^{U*} \\ \lambda\pi_b & \text{if } p_s \in (-V_b^{C*}, -V_b^{U*}] \\ \pi_b & \text{if } p_s \leq -V_b^{C*} \end{cases} \tag{18}$$

It follows that equilibrium offers to buy are only made at price $p_b \in \{V_s^{U*}, V_s^{C*}\}$. When peripheral buyers choose an offer, they face a trade-off between an offer at a high price $p_b = V_s^{C*}$ and high probability of acceptance π_s, and an offer at a low price $p_b = V_s^{U*}$ and a low probability of acceptance $\lambda\pi_s$. Symmetrically, sellers optimally choose $p_s \in \{-V_b^{U*}, -V_b^{C*}\}$ according to the same trade-off between price and likelihood of trading.

(iv) A connected dealer always has the possibility to trade at the core market price instead of accepting an offer, so that $V_s^{C*} \geq p^{co}$ for connected sellers, and $V_b^{C*} \geq -p^{co}$ for buyers. Thus, connected sellers (resp., buyers) only accept prices above (resp., below) p^{co} and therefore connected dealers never trade together. The buyers only make offers at price $p_b = V_s^{U*}$, and the sellers at price $p_s = -V_b^{U*}$.

Equilibrium characterization. From the previous remarks, we deduce the following:

Lemma 2. *Any Markov-perfect equilibrium of the peripheral market is fully characterized by a strategy profile* $\Sigma^*(p^{co}, \alpha^{pe}, \lambda) = (\theta_s^*, \theta_b^*, \gamma_s^*, \gamma_b^*)$ *such that:*

- A seller (resp., buyer) of type $i \in \{U, C\}$ *accepts an offer* $(1, p_b)$ *if* $p_b \geq V_s^{i*}$ *(resp., an offer* $(-1, p_s)$ *if* $p_s \leq -V_b^{i*}$*).*

- A connected dealer of type $k \in \{b, s\}$ *who does not accept an offer trades in the core market with probability* $1-\gamma_k^*$*. With probability* γ_k^**, the connected dealer makes a new offer at price* $p_s^{C*} = -V_b^{U*}$ *if* $k = s$*, or* $p_b^{C*} = V_s^{U*}$ *if* $k = b$*.*

- An unconnected dealer of type $k \in \{b, s\}$ *who does not accept an offer always makes a new offer. With probability* θ_k^**, the price is* $p_s^{U*} = -V_b^{C*}$ *for* $k = s$*, or* $p_b^{U*} = V_s^{C*}$ *for* $k = b$*. With probability* $1-\theta_k^*$*, the price is* $p_s^{U*} = -V_b^{U*}$ *for* $k = s$*, or* $p_b^{U*} = V_s^{U*}$ *for* $k = b$*.*

Thus, solving for the equilibrium of the peripheral market amounts to solving for the equilibrium strategy profile Σ^*. The equilibrium is in pure strategies if θ_k^* and γ_k^* are all either zero or one. Otherwise, the equilibrium is in mixed strategies.[14] We show in Lemma 3 that there are only three pure strategy profiles that arise in equilibrium, namely:[15]

- *Active Connected Dealers (ACD):* $\Sigma^{ACD} = (1, 1, 1, 1)$. Unconnected dealers make offers that are accepted by all dealers with an opposite trading need. Connected dealers' offers are only accepted by unconnected dealers with an opposite trading need. When they reject an offer, connected dealers make a new one.

- *Inactive Connected Sellers (ICS):* $\Sigma^{ICS} = (1, 0, 0, 1)$. Unconnected buyers make offers that are only accepted by unconnected sellers, while unconnected sellers make offers that are accepted by all buyers. When they reject an offer, connected buyers make offers that are accepted by unconnected sellers only. Connected sellers trade in the core market only, and are thus inactive in the periphery.

- *Inactive Connected Buyers (ICB):* $\Sigma^{ICB} = (0, 1, 1, 0)$. Unconnected sellers make offers that are accepted by unconnected buyers only, while unconnected buyers make offers that

[14] In equilibrium, dealers can play mixed strategies over the offer they make (θ_k^*) or over their decision whether to make an offer (γ_k^*). There is no equilibrium in which dealers play a mixed strategy over the rejection or acceptance of an offer. Indeed, this would require dealers receiving an offer to be indifferent between these two options. However, the dealer making the offer can break this indifference by improving his offer by an infinitesimal amount and be strictly better off.

[15] When $\lambda \geq \frac{1}{2}$, there are other pure-strategy equilibria. In particular, there is an equilibrium in which unconnected dealers only trade with unconnected dealers and connected dealers only trade with core dealers ($\Sigma = (0, 0, 0, 0)$). However, the main new insights of the model are already conveyed when $\lambda \leq \frac{1}{2}$. Thus, for brevity, we focus on this case.

are accepted by all sellers. When they reject an offer, connected sellers make offers that are accepted by unconnected buyers only. Connected buyers trade in the core market only, and are thus inactive in the periphery.

For brevity, we refer to an equilibrium in which peripheral dealers' strategy profile is, say, of type ACD as an "ACD equilibrium."

Lemma 3. *Let* $\omega_i = \frac{\lambda(1-\pi_i)}{1-\pi_i\lambda(2-\lambda)}$*, for* $i \in \{b, s\}$*. When* p^{co} *is different from* $(1-\omega_b)C^b - \omega_b C^s$ *and* $\omega_s C^b - (1-\omega_s)C^s$*, the unique equilibrium regime is:*

$$\Sigma^*(p^{co}, \alpha^{pe}, \lambda) = \begin{cases} \Sigma^{ICS} & \text{if } p^{co} > (1-\omega_b)C^b - \omega_b C^s \\ \Sigma^{ACD} & \text{if } \omega_s C^b - (1-\omega_s)C^s < p^{co} < (1-\omega_b)C^b - \omega_b C^s \\ \Sigma^{ICB} & \text{if } p^{co} < \omega_s C^b - (1-\omega_s)C^s. \end{cases} \tag{19}$$

If $p^{co} = (1-\omega_b)C^b - \omega_b C^s$ *(resp.,* $p^{co} = \omega_s C^b - (1-\omega_s)C^s$*) then either an ICS (resp. ICB) or an ACD equilibrium obtains, or a mixed equilibrium.*

The intuition for this result is as follows. Suppose, for instance, that $\alpha^{pe} < 1/2$. In this case, buyers are on the crowded side of the peripheral market ($\pi_b = 1$ and $\pi_s < 1$). Therefore, connected sellers have a strong bargaining position and can trade at prices above the core market price in the peripheral market. Because they are better off trading in the peripheral market, the ICS equilibrium cannot be obtained.[16] Hence the equilibrium is either ACD or ICB. Which equilibrium is obtained depends on whether unconnected sellers make offers that are attractive to both types of buyers (the ACD equilibrium) or only unconnected buyers (the ICB equilibrium). If an unconnected seller's offer is rejected, he cannot trade in the core. To avoid this outcome, the seller can offer a price low enough to be acceptable by connected buyers, i.e., a price below the core price. This option becomes relatively *less* attractive for the seller when (i) the core market price decreases or (ii) the price that an unconnected buyer is willing to accept increases, that is, when the likelihood of finding a counterparty for an unconnected buyer–if he rejects an offer–decreases (i.e., α^{pe} decreases). Thus, the ICS equilibrium obtains for a sufficiently low core market price

[16] Formally, if $\pi_b = 1$ then $\omega_b = 0$ and therefore the condition $p^{co} > (1-\omega_b)C^b - \omega_b C^s$ required for an ICS equilibrium can never be satisfied since $p^{co} \in (-C^s, C^b)$.

or a sufficiently small α^{pe} (so that the condition $p^{co} < (1-\omega_s)C^b - \omega_s C^s$ is satisfied). Otherwise, the ACD equilibrium obtains.

Lemma 3 shows that there are two values of the core market price, p^{co}, for which a mixed strategy equilibrium can be obtained in the peripheral market. We come back to this issue in Section 4.3. Otherwise, there is a unique equilibrium and the equilibrium is in pure strategies.

Equilibrium order flow to the core. We can now compute the equilibrium value of the order flow of peripheral dealers to the core market. Let $\mu_s^{co}(\alpha^{pe}, \lambda, \Sigma)$ and $\mu_b^{co}(\alpha^{pe}, \lambda, \Sigma)$ be, respectively, the masses of connected sellers and buyers who trade in the *core market*. We derive these quantities in closed-form for $\Sigma \in \{\Sigma^{ACD}, \Sigma^{ICS}, \Sigma^{ICB}\}$ in the Online Appendix B.3. As explained in Section 4.1, connected sellers (resp., buyers) who trade in the core market sell (resp., buy) one unit at any price in $(-C^s, C^b)$. Thus, the order flow is:

$$\Delta(\alpha^{pe}, \lambda, \Sigma) = [\mu_s^{co}(\alpha^{pe}, \lambda, \Sigma) - \mu_b^{co}(\alpha^{pe}, \lambda, \Sigma)]. \tag{20}$$

The next lemma provides a closed-form solution for the order flow and highlights some of its properties.

Lemma 4. *For $\alpha^{pe} \geq \frac{1}{2}$, peripheral dealers' order flow in the core market is:*

$$\Delta(\alpha^{pe}, \lambda, \Sigma) = \begin{cases} \frac{(1-\lambda)(1-\pi_b\lambda)}{1-\lambda\pi_b[2-\lambda-\lambda\pi_b(1-\lambda)^2]} \times (2\alpha^{pe}-1) & \text{if } \Sigma = \Sigma^{ACD}, \\ \frac{(1-\lambda)(1-\pi_b(1-\lambda(1-\lambda))}{(1-\pi_b)(1-\pi_b\lambda)} \times (2\alpha^{pe}-1) & \text{if } \Sigma = \Sigma^{ICS}. \end{cases}$$

For $\alpha^{pe} < 1/2$, we have $\Delta(\alpha^{pe}, \lambda, \Sigma^{ACD}) = -\Delta(1-\alpha^{pe}, \lambda, \Sigma^{ACD})$ and $\Delta(\alpha^{pe}, \lambda, \Sigma^{ICB}) = -\Delta(1-\alpha^{pe}, \lambda, \Sigma^{ICS})$. In addition, $\Delta(\alpha^{pe}, \lambda, \Sigma^*)$ has the following properties:

(i) It is positive if $\alpha^{pe} > 1/2$, negative if $\alpha^{pe} < 1/2$, and equal to zero if $\alpha^{pe} = 1/2$.

(ii) Holding Σ^* constant, it increases with α^{pe}.

(iii) Holding α^{pe} constant, it is higher in absolute value when connected dealers on one side only trade in the core market, i.e., $\Delta(\alpha^{pe}, \lambda, \Sigma^{ICS}) > \Delta(\alpha^{pe}, \lambda, \Sigma^{ACD})$ for $\alpha^{pe} > 1/2$ and $\Delta(\alpha^{pe}, \lambda, \Sigma^{ICB}) < \Delta(\alpha^{pe}, \lambda, \Sigma^{ACD})$ for $\alpha^{pe} < 1/2$.

These properties are intuitive. For instance, when there are more sellers than buyers in the peripheral market, connected buyers are more likely to find a counterparty than

connected sellers. Thus, more connected sellers than connected buyers trade in the core, which results in more sells than buys from peripheral dealers in the core ($\Delta > 0$; point (i)). A larger number of sellers makes this order flow even more negative (point (ii)). Finally, in an ICS equilibrium, connected sellers only trade in the core market, which further increases the magnitude of the order flow from peripheral dealers (point (iii)).

4.3 Full Equilibrium

Lemma 3 shows that the strategy profile chosen by peripheral dealers in equilibrium depends on the core market price. Conversely, Lemma 1 shows that the equilibrium price in the core market depends on the peripheral dealers' order flow, which is a function of the peripheral dealers' strategy profile (Lemma 4). We say that the market is in a *full equilibrium* when the equilibrium of the peripheral market and the equilibrium price of the core market are mutually consistent.

Definition 1. *A full equilibrium of the market is (i) a price in the core market p^{co*} and (ii) a strategy profile Σ^* for peripheral dealers, such that $p^{co*} = p^{co*}(\alpha^{co}, \Delta(\alpha^{pe}, \lambda, \Sigma^*))$ (given by (12)) and $\Sigma^* = \Sigma^*(p^{co*}, \alpha^{pe}, \lambda)$ (given by (19)), where $\Delta(\alpha^{pe}, \lambda, \Sigma^*)$ is given by (20) .*

In the next proposition, we characterize the equilibrium strategy profile of peripheral dealers in a full equilibrium in terms of the exogenous parameters of the model.

Proposition 1. *There exist four thresholds $\alpha^+_{ACD}, \alpha^+_{ICB}, \alpha^-_{ACD}, \alpha^-_{ICS}$, given in the Appendix, such that $\alpha^+_{ICB} \geq \alpha^+_{ACD} > \frac{1}{2} > \alpha^-_{ACD} \geq \alpha^-_{ICS}$.*

- For $\alpha^{pe} \geq \frac{1}{2}$, the unique full equilibrium strategy profile is $\Sigma^* = \Sigma^{ICS}$ if $\alpha^{co} < \alpha^-_{ICS}$ and $\Sigma^* = \Sigma^{ACD}$ if $\alpha^{co} > \alpha^-_{ACD}$.

- For $\alpha^{pe} \leq \frac{1}{2}$, the unique full equilibrium strategy profile is $\Sigma^* = \Sigma^{ACD}$ if $\alpha^{co} < \alpha^+_{ACD}$ and $\Sigma^* = \Sigma^{ICB}$ if $\alpha^{co} > \alpha^+_{ICB}$.

To understand this proposition, it is useful to first consider the polar case in which the core dealer market is "thick" relative to the peripheral market, i.e., κ goes to infinity. In this case, the peripheral dealers' order flow in the core market, Δ, is negligible relative to the core dealers' initial position κz_0^{co}, and has therefore no effect on the core market price. As a result, the aggregate inventory in the core market influences the equilibrium outcome

in the peripheral market (through p^{co}), but the reverse is not true. For this reason, the thresholds α^+ and α^- do not depend on the peripheral dealers' strategy profile. In fact, in this case, we obtain (see the proof of Proposition 1):

$$\alpha^+_{ACD} = \alpha^+_{ICS} = \frac{1}{2}(1 - \Phi^{-1}(\omega_s)),\ \alpha^-_{ACD} = \alpha^-_{ICB} = \frac{1}{2}(1 - \Phi^{-1}(1 - \omega_b)). \tag{21}$$

Accordingly, when $\kappa \to \infty$, Proposition 1 describes the equilibrium outcome for *all* possible values of α^{co}, α^{pe}, and λ. Figure 3 (Panel A) shows the equilibrium map in this case in the $(\alpha^{pe}, \alpha^{co})$ space. In particular, it highlights that the ICB and ICS equilibria obtain only when the aggregate inventory positions of core and peripheral dealers are of opposite signs. For instance, the ICB equilibrium, in which connected buyers do not make offers to peripheral sellers, obtains only if (a) there is an excess of buyers in the peripheral market ($\alpha^{pe} < \frac{1}{2}$) and (b) there is a significant excess of sellers in the core market ($\alpha^{co} > \frac{1}{2}$). Intuitively, in this case, the price in the core market is low because sellers dominate in this market while prices in the peripheral market are relatively high because buyers dominate in this market. Hence, a connected buyer prefers to directly trade in the core market. Moreover, as connected buyers' outside option is attractive, unconnected sellers are better off making offers at a relatively high price (accepted only by unconnected buyers), even if this raises the likelihood of not rebalancing their inventory.

[Insert Figure 3]

Figure 3 (Panel B) illustrates Proposition 1 in the general case, $\kappa < \infty$. The effects of α^{pe} and α^{co} are identical to those in the thick core market case, and the same economic intuition applies. However, in this case there are *two-way interactions* between the core and the peripheral markets. In particular, the influence of the peripheral dealers' order flow on the core market price implies that the transition from the ACD strategy profile to other profiles (ICS or ICB) is more complex than in the thick core market case.

For instance, suppose that $\alpha^{pe} < \frac{1}{2}$ so that there is no ICS equilibrium. Moreover, suppose that α^{co} is just slightly below α^+_{ACD} so that an ACD equilibrium obtains. Now consider a small increase in the mass of core dealers who need to sell the asset so that α^{co} becomes slightly larger than α^+_{ACD}. Other things equal, this marginal increase in α^{co} pushes

the price in the core market downward and the drop in the core market price implies that the ACD equilibrium cannot be sustained anymore. Indeed, at the new core market price, all connected buyers are better off trading in the core market. If the core market is thick, this inflow of buy orders in the core market has no effect on the core market price and the ICB equilibrium obtains. In contrast, if the core market is not thick, this inflow is so strong that it triggers a discrete positive jump in the core market price, back to a level such that all connected buyers are better off not contacting directly core dealers, so that the ICB equilibrium cannot be obtained. We end up in a situation in which neither the ACD, nor the ICB equilibrium can be obtained for $\alpha^{co} \in [\alpha^{+}_{ACD}, \alpha^{+}_{ICB}]$. Hence, the only possibility is a mixed strategy equilibrium so that only a fraction of connected buyers directly contact core dealers, thereby smoothing the impact of their orders on the core market price. We provide a full characterization of the equilibrium in this case in the Online Appendix B.2. The range of values for α^{co} such that a mixed strategy equilibrium obtains shrinks as κ increases.

The next proposition completes the characterization of the pure strategy equilibrium in the peripheral market with a closed-form solution for all prices:

Proposition 2. *For $k \in \{b, s\}$, denote $\rho^C_k = \frac{1-\pi_k}{1-\pi_b\pi_s\lambda}$ and $\rho^U_k = -\lambda\pi_k\rho^C_{-k}$. In a full equilibrium (Σ^*, p^{co*}), transactions among peripheral dealers occur at the following prices:*

$$\text{- If } \Sigma^* = \Sigma^{ACD}: \quad p^{U*}_s = p^{co*} + \rho^U_s(C^s + p^{co*}), \quad p^{C*}_s = p^{co*} + \rho^C_s(C^b - p^{co*}) \tag{22}$$

$$p^{U*}_b = p^{co*} - \rho^U_b(C^b - p^{co*}), \quad p^{C*}_b = p^{co*} - \rho^C_b(C^s + p^{co*}) \tag{23}$$

$$\text{- If } \Sigma^* = \Sigma^{ICS}: \quad p^{U*}_s = p^{co*} + \rho^U_s(C^s + p^{co*}), \quad p^{C*}_s \text{ is not observed} \tag{24}$$

$$p^{U*}_b = p^{co*} - \rho^C_b(C^s + p^{co*}), \quad p^{C*}_b = p^{co*} - \rho^C_b(C^s + p^{co*}) \tag{25}$$

$$\text{- If } \Sigma^* = \Sigma^{ICB}: \quad p^{U*}_s = p^{co*} + \rho^C_s(C^b - p^{co*}), \quad p^{C*}_s = p^{co*} + \rho^C_s(C^b - p^{co*}) \tag{26}$$

$$p^{U*}_b = p^{co*} - \rho^U_b(C^b - p^{co*}), \quad p^{C*}_b \text{ is not observed.} \tag{27}$$

In Proposition 2, we have chosen to express prices offered in the peripheral market relative to the equilibrium price in the core market (p^{co*}). This formulation is convenient because it immediately reveals how prices in the peripheral market are positioned relative to the core market price in each possible equilibrium regime (see Figure 4). This is interesting

as, for empirical purposes, this price is likely to serve as natural benchmark for transaction prices among peripheral dealers.

[Insert Figure 4]

An extreme case of particular interest is when $\lambda \to 0$ (no segmentation). Then price dispersion vanishes and all trades take place at a single price, which is identical to the price that would be obtained if trading were centralized (the Walrasian price).

Proposition 3. *When $\lambda \to 0$, the only possible equilibrium strategy profile in the peripheral market is Σ^{ACD}. Moreover, in equilibrium, the core market price and unconnected peripheral dealers' offers converge to the Walrasian price p^{bench*}.*

5 Empirical Implications

Our model has three distinguishing features relative to other models of OTC markets: (i) two distinct groups of dealers (core and peripheral), (ii) prices are explicitly related to dealers' *aggregate* inventory positions in each group, and (iii) bargaining between dealers might not lead to a trade, even when gains from trade exists. We derive testable implications that follow from these features. A common theme is that the breakdown of dealers' aggregate inventories between core and peripheral dealers is important.

5.1 Connectedness and Price Heterogeneity.

Holding the price in the core market constant, our model predicts that the price observed for a given transaction among peripheral dealers depends on their connection to core dealers. To analyze this point, let $\bar{p}^{i,j}$ be the average price at which a seller of type $i \in \{C, U\}$ trades with a buyer of type $j \in \{C, U\}$ in equilibrium. For instance, $\bar{p}^{U,U}$ is the weighted average of p_s^{U*} and p_b^{U*} (weighted by the frequencies at which transactions occur at each price in equilibrium). The first price is observed when an unconnected seller makes the offer to an unconnected buyer while the second is observed when an unconnected buyer makes the offer to an unconnected seller.[17] Furthermore, let $M^{i,j} = \bar{p}^{i,j} - p^{co*}$ be

[17] The side who makes an offer is typically not reported on datasets on OTC trades. This is the reason why we develop predictions for the average price at which a given pair of types trades in equilibrium.

the average "markup" *relative to the core price*. In the model, two connected dealers may trade with each other only in the core market at price p^{co*}.[18] Thus, it is natural to set $M^{C,C} = 0$. We obtain the following testable implication.

Corollary 1. *(i) When peripheral dealers have an aggregate long position ($z_0^{pe} > 0$), markups in the peripheral market are negative and such that $M^{U,C} \leq M^{U,U} \leq M^{C,U} \leq M^{C,C}$; (ii) When peripheral dealers have an aggregate short position ($z_0^{pe} < 0$), markups in the peripheral market are positive and such that $M^{C,U} \geq M^{U,U} \geq M^{U,C} \geq M^{C,C}$.*

Corollary 1 makes two distinct predictions. First, it implies that the sign of the mark-ups for transactions among peripheral dealers depends on the sign of the peripheral dealers' aggregate position, z_0^{pe}. For instance, if $z_0^{pe} > 0$, there are more peripheral sellers than buyers so that the former have less market power. Hence, buyers trade with unconnected sellers at a price below the core price ($M^{U,C} < 0$ and $M^{U,U} < 0$). Moreover, connected sellers trade at their reservation price ($M^{C,U} = 0$). The case in which $z_0^{pe} < 0$ is symmetric. To our knowledge, this prediction regarding the effect of peripheral dealers' aggregate inventory on the relative position of prices in the peripheral and core markets is new.

Second, controlling for the direction of the trade, a connected dealer always obtains a better price than an unconnected dealer when peripheral dealers' aggregate inventory is not zero ($z_0^{pe} \neq 0$). For instance, the average price obtained by connected sellers is above the average price obtained by unconnected sellers, holding the price in the core market constant (i.e., controlling for core dealers' aggregate inventory). Intuitively, connected dealers have more market power than unconnected dealers because they have a better outside option.

Interestingly, the ranking of mark-ups predicted by the model is similar to that found by Di Maggio, Kermani, and Song (2017) (see their Table 3), with two differences. First, they classify dealers in a transaction according to whether they are core or peripheral dealers, while we classify transactions among peripheral dealers according to whether they are *connected or not*. Second, in Di Maggio, Kermani, and Song (2017), all markups

[18] For instance, suppose that there are two peripheral sellers and one peripheral buyer in the core market. At the core market price, one peripheral seller is matched with the peripheral buyer and the other seller is matched with a core dealer. Thus, there is one transaction between two connected peripheral dealers.

are positive. Our model makes the sharp prediction that this should be the case only if peripheral dealers of corporate bonds have a short position in aggregate.[19]

One could test this novel prediction of our model by regressing mark-ups on dummy variables for the types of dealers involved in transactions, interacted with peripheral dealers' aggregate inventory. Our model implies that (i) coefficients on these interaction terms should be negative and (ii) the coefficient for each dummy variable should rank in a way consistent with Corollary 1.[20]

5.2 Price effects of shocks to core dealers' aggregate inventory

Dealers' aggregate inventory positions affect asset prices. Specifically, when their aggregate position increases (e.g., following a block sale) then the asset price drops initially and reverts subsequently. While this relationship has been widely documented for the equity market (see, e.g., Comerton-Forde *et al.* (2010), Chordia *et al.* (2002), or Chordia and Subrahmanyam (2004)), similar evidence has also been obtained for assets that trade OTC such as treasuries (Fleming (2007)) and corporate bonds (Friewald and Nagler (2017)).

In line with these findings, the core market price, p^{co*}, decreases when core dealers' aggregate inventory position, z_0^{co}, increases, for a given equilibrium strategy profile, Σ^*, for peripheral dealers. In addition, our model predicts that prices in the peripheral market should be negatively affected as well, *even for transactions between unconnected dealers.*

Corollary 2. *Consider a positive shock to core dealers' aggregate inventory that leaves peripheral dealers' equilibrium strategy profile, Σ^*, unchanged. Following this shock:*

- The equilibrium price in the core market drops (i.e., $\frac{\partial p^{co*}}{\partial z_0^{co}} < 0$).

- Equilibrium prices in the peripheral market drop as well (i.e., $\frac{\partial p_k^{i*}}{\partial z_0^{co}} < 0$ for $(k,i) \in \{b,s\} \times \{C,U\}$) but by less than the core market price on average.

[19] Dealers do take significant short positions in the U.S. corporate bond markets (see, for instance, Figure 3 in Duffie (2012b)).

[20] More formally, one could estimate the following regression:

$$\begin{aligned} M_t^{i,j} &= \alpha^{C,U} D[C,U] + \alpha^{U,U} D[U,U] + \alpha^{U,C} D[U,C] + \gamma z_t^{pe} \\ &+ \beta^{C,U} * D[C,U] * z_t^{pe} + \beta^{U,U} * D[U,U] * z_t^{pe} + \beta^{U,C} * D[U,C] * z_t^{pe} + \eta_{ijt}, \end{aligned}$$

where z_t^{pe} is peripheral dealers' aggregate inventory position on day t and $D[i',j']$ is a dummy variable equal to 1 if $(i,j) = (i',j')$. Corollary 1 predicts that $\beta^{U,C} \leq \beta^{U,U} \leq \beta^{C,U} \leq 0$.

When core dealers' aggregate inventory increases, their demand for the asset shifts down and, as a result, the equilibrium price in the core market decreases (first part of Corollary 2). Accordingly, connected sellers' outside option deteriorates while connected buyers' outside option improves. For this reason, transaction prices among peripheral dealers drop as well. In other words, changes in core dealers' prices due to a shock to their aggregate inventory position are passed through to peripheral dealers, holding the latter's aggregate inventory position constant. However, when $\lambda > 0$, this pass-trough is not one-for-one so that the average price observed in the peripheral market is less sensitive to shocks to core dealers' inventories than the core market price (second part of Corollary 2).

This implication could be tested by using shocks to core dealers' inventory in a given asset, e.g., due to bond fire sales by institutional investors (to the extent that these sales mainly affect core dealers)[21] or new bond issuances. For instance, Fleming (2007) or Fleming and Rosenberg (2008) find that primary dealers absorb a significant fraction of Treasury issues (about 71% according to Fleming (2007)) and keep a significant fraction of Treasury issues until redemption. Thus, new issuances (redemptions) are positive (negative) shocks to their aggregate inventory. According to the model, the price effects of these shocks should be reflected in peripheral dealers' quotes as well, but to a lesser extent, even if the latter do not participate to new issues (or are not affected by redemptions). Similarly, open market operations of the Federal Reserve also change primary dealers' aggregate inventory positions and could therefore be used to test Corollary 2. Interestingly, the last part of Corollary 2 suggests that the effectiveness of these operations might depend on the connectedness between primary dealers and other dealers in the Treasury market.

Finally, we study the impact of core dealers' aggregate inventory on price dispersion, which we measure by the spread between the highest and the lowest prices observed in the peripheral market, denoted *Spread*. Using Proposition 2, we obtain that:

$$Spread = \begin{cases} \Phi(-\kappa^{-1}z^*)(\frac{1-\pi_b}{1-\pi_b\lambda})(C^b + C^s) \;\text{ if }\; z_0^{pe} > 0, \\ (1-\Phi(-\kappa^{-1}z^*))(\frac{1-\pi_s}{1-\pi_s\lambda})(C^b + C^s) \;\text{ if }\; z_0^{pe} < 0. \end{cases} \tag{28}$$

[21] Consistent with this possibility, Li and Schuerhoff (2017) find that core dealers in the U.S. municipal bond markets are more likely to buy bonds that experience large mutual funds outflows ("fire sales").

Corollary 3. *The dispersion of prices in the interdealer market increases (resp., decreases) in the core dealers' aggregate inventory z_0^{co} if $z_0^{pe} < 0$ (resp. if $z_0^{pe} > 0$).*

Corollary 3 implies (i) that supply shocks in the core market should affect price dispersion in the interdealer market and (ii) that the sign of this effect depends on whether peripheral dealers have a positive or a negative position in the asset.[22] To our knowledge, these predictions are new.

[Insert Figure 5]

Figure 5 illustrates Corollary 3 when $z_0^{pe} < 0$. In this case, all trades in the interdealer market take place at prices above or equal to the core market. Thus, the core market price is the lowest price observed in equilibrium, while the highest price is connected sellers' offer, p_s^{C*} (see Figure 4). This price falls by less than the core market price when core dealers' aggregate inventory increases (see Corollary 2). As a result, when $z_0^{pe} < 0$, price dispersion increases with core dealers' aggregate inventory, z_0^{co}. In contrast, when $z_0^{pe} > 0$, all trades in the interdealer market take place at prices below or equal to the core market price. The lowest price in this case is that offered by connected buyers, p_b^{C*}. As this price falls by less than the core market price when core dealers' aggregate inventory increases, price dispersion decreases in this case.

5.3 Relationships Breakdowns

In this section we analyze the effect of an increase in the share of unconnected peripheral dealers, λ. Such an increase might be due to the default of a core dealer, as in Di Maggio, Kermani, and Song (2017), an increase in the cost of maintaining existing relationships, or a crisis episode (see the discussion of the interbank market for central bank reserves in the Online Appendix B.1). Our model makes several predictions about the effect of such shocks on prices and trading volume.

Our first prediction regards the effect of an increase in λ on the core market price.

[22] See Randall (2015a) for evidence of dispersion in interdealer transaction prices.

Using (12), we deduce that, for a given equilibrium strategy profile Σ^*:

$$\frac{\partial p^{co*}}{\partial \lambda} = -\Phi'(-\kappa^{-1}z^*)\frac{\partial \Delta}{\partial \lambda}\frac{(C^b + C^s)}{\kappa}. \tag{29}$$

As $\Phi'(-\kappa^{-1}z^*) > 0$, the sign of the effect of λ on the core market price depends on the sign of its effect on peripheral dealers' order flow (i.e., the sign of $\frac{\partial \Delta}{\partial \lambda}$) in the core market. We show in the Online Appendix B.5 that, in the ACD equilibrium, the absolute value of Δ, decreases with λ. Intuitively, a smaller fraction of peripheral dealers' aggregate inventory finds its way to the core market when fewer dealers are connected to the core. The next implication follows from this result and equation (29).[23]

Corollary 4. *When an ACD equilibrium obtains, a marginal increase in the fraction of unconnected dealers has a positive (resp., negative) effect on the core market price if peripheral dealers' aggregate position z_0^{pe} is positive (resp., negative).*

For instance, if peripheral dealers in bond markets have a short aggregate position, our model predicts that an increase in the mass of unconnected dealers should decrease the average price at which core dealers trade.

Corollary 5. *Other things equal, in the ACD equilibrium, the dispersion of prices in the interdealer market increases when fewer dealers are connected to core dealers ($\frac{\partial Spread}{\partial \lambda} > 0$).*

To understand this result, suppose that $z_0^{pe} < 0$. In this case, all trades in the interdealer market take place at prices above or equal to the core market price (see Figure 4). Thus, the core market price is the lowest price observed in equilibrium while the highest is the price offered by peripheral sellers, p_s^{C*}. Price dispersion is therefore equal to the markup required by connected sellers when they make an offer, i.e., $p_s^{C*} - p^{co*} = \rho_s^C(C^b - p^{co*})$ (Proposition 2). A higher λ increases this markup and therefore price dispersion for two distinct reasons. First, as shown in Corollary 4, it lowers the core market price and thus raises gains from trade $(C^b - p^{co*})$ between connected sellers and unconnected buyers. Second, it raises the fraction ρ_s^C of gains from trade captured by connected sellers.

[23] When $\Sigma^* \neq \Sigma^{ACD}$, Δ is not always monotonic in λ. However, cases in which Δ increases in λ in absolute value only arise for extreme values of α^{pe}. Thus, Corollaries 4 and 5 focus on the most relevant case.

5.4 Connectedness and the distribution of prices

In this section, we show how the various implications derived so far combine to affect the distribution of transaction prices in the interdealer market. In practice, an empiricist would estimate this distribution by collecting data on trades in the interdealer market over multiple days with potentially different levels of liquidity in the two market segments. To make the distribution derived from the model more comparable to what an empiricist would observe, we consider a large number of trading days and assume that the fraction of sellers on both segments vary across days (over time). We set $\alpha^{pe} = \overline{\alpha}^{pe} + \eta^{pe}$ and $\alpha^{co} = \overline{\alpha}^{co} + \eta^{co}$, where η^{co} and η^{co} follow independent normal distributions with zero mean and the same variance σ_η^2. We interpret (η^{pe}, η^{pe}) as shocks to the mass of peripheral and core dealers with long positions in a given day, while $(\overline{\alpha}^{pe}, \overline{\alpha}^{co})$ are the average masses of dealers with long positions in each segment of the interdealer market (e.g., when $\overline{\alpha}^{co} > 0.5$, core dealers are structurally sellers of the asset because their aggregate position is long on average).[24]

We then proceed through numerical simulations. We draw one million realizations of the pair (η^{pe}, η^{co}). For each realization, we compute equilibrium prices of trades among peripheral dealers as given by Proposition 2 and the likelihood that each price is observed in equilibrium, based on the stationary probability distributions derived in the Online Appendix B.3.[25] Figure 6 depicts the resulting histogram (see the caption for more details on the procedure).

In Panel A, we set $\bar{\alpha}^{pe} = \bar{\alpha}^{co} = 0.5$, so that, on average, the aggregate positions of peripheral and core dealers are zero. We plot the histogram of prices for $\lambda = 0$ (red curve) and $\lambda = 0.4$ (blue curve). When $\lambda = 0$, the histogram of prices is also the histogram of core and Walrasian prices for all values of λ, due to the assumption that κ is infinite (core

[24] Dealers might structurally have long or short positions in some assets for two reasons. First, liquidity shocks from their clients can be one-sided. For instance, Schultz (2012) shows that dealers in the municipal bonds market buy bonds at issuance and then resell them predominantly to institutional investors likely to hold the bonds until maturity. Thus, on average, dealers participating to new corporate bond issues tend to have long positions. If they are core dealers, this naturally implies $\overline{\alpha}^{co} > 0.5$. Second, dealers need time to unwind their positions. In corporate bond markets, Schultz (2017) and Friewald and Nagler (2017) find that the half-life of dealers' inventories varies between 4 to 10 weeks.

[25] For instance, the likelihood of observing a transaction at price p_s^U is the likelihood that an unconnected seller makes an offer (μ_4^* in the Online Appendix B.3) and that this offer is accepted (π_b or $\lambda\pi_b$ depending on the equilibrium type).

prices do not depend on λ) and Proposition 3 (peripheral and core prices converge to the Walrasian price when λ goes to zero). For all values of λ, variations in the core price only stem from daily changes in core dealers' inventories (η^{pe} and η^{co}). For peripheral prices, an additional source of variation comes from the dispersion across different matches for a given core market price. For this reason, the distribution of prices for $\lambda = 0.4$ has a higher variance and fatter tails than for $\lambda = 0$.

In Panel B, we set $\bar{\alpha}^{pe} = 0.2, \bar{\alpha}^{co} = 0.8$ so that peripheral dealers are structurally short and core dealers structurally long. For $\lambda = 0$, the distribution of prices has the same shape as in the previous case, but is shifted to the left, reflecting the fact that core dealers have, in aggregate, a long position in the asset (and therefore a relatively low marginal valuation for the asset). For $\lambda = 0.4$, the distribution of prices has a very different shape, as it features two other modes. The reason is that there are relatively few sellers among peripheral dealers when $\alpha^{pe} << 0.5$. Hence, they have market power and, as a result, the prices offered by unconnected buyers (p_b^U) or connected sellers (p_s^C) are strictly above the core market price (see Figure 4). Interestingly, this implies that when $\lambda > 0$ the *average* price in the periphery is higher in this example than the core market price.

Table 1 confirms this observation by reporting the average and standard deviation of the distributions of core and periphery prices in these two examples, as well as for additional values of λ. The table also shows that, in this example, the fact that price dispersion increases in λ (Corollary 5) also implies that the volatility of prices in the interdealer market increases when peripheral dealers become less connected to core dealers.

[Insert Figure 6 and Table 1]

To summarize, the model suggests that variations in (i) the level of connectedness between core and peripheral dealers and (ii) peripheral dealers' average aggregate position are important determinants of the distribution of prices in the interdealer market. In particular, this distribution has a second and a third mode and a different mean relative to the distribution of the frictionless price of the asset. If peripheral dealers have a structural short position in the asset the second and third mode are to the right of the distribution and the average price for peripheral transactions is higher than the Walrasian price. The opposite prediction obtains if peripheral dealers have a structurally long position.

6 Efficiency of Inventory Management and Client Trading Costs

So far we have studied the impact of shocks to core or peripheral dealers' aggregate inventories and connectedness on transaction prices. We now study how these shocks affect the allocative efficiency of interdealer transactions and show how inefficiencies in the interdealer market have a bearing on the trading costs borne by dealers' clients.

6.1 Sources of inefficiencies in the interdealer market

We measure the efficiency of equilibrium allocations by computing the expected gains from interdealer trades. Obviously, the most efficient allocation of inventories among dealers is obtained when trading is centralized. However, our assumptions on the market structure prevents dealers from achieving this outcome when $\lambda > 0$. The interesting question is whether and why the equilibrium allocation of inventories deviates from the efficient allocation, taking the market structure *as given* (in particular, the sequence in which peripheral dealers are matched and their connections to core dealers).

To study this question, we focus on the thick core market case ($\kappa \to +\infty$) for tractability. As all core dealers bear identical inventory costs, the optimal allocation is to split core dealers' aggregate inventory equally among them. This is indeed the allocation obtained in equilibrium (see Lemma 1). With this allocation, the expected inventory costs of core dealers after trading are equalized and equal to the equilibrium price in the core market: $p^{co} = \Phi(-(2\alpha^{co} - 1))C^b - (1 - \Phi(-(2\alpha^{co} - 1))C^s$.

Hence, there is no inefficiency in the allocation of inventories among core dealers. Thus, the social planner's problem boils down to organizing trades among peripheral dealers. The social planner observes the type $(k, i) \in \{b, s\} \times \{U, C\}$ of each peripheral dealer sequentially and must decide whether to match him with his predecessor (if the latter is unmatched) or his successor, whose type is yet *unknown to the planner*.[26] In the latter case, the predecessor is either matched with a core dealer if he is connected or remains unmatched

[26]Thus, the social planner has the same information as that available to peripheral dealers when they make their decisions. Hence, gains in efficiency for the planner do not come from the fact that he has more information than a peripheral dealer when the latter chooses to make an offer.

if he is unconnected. Each match (trade) between two peripheral dealers generates a surplus of $(C^s + C^b)$. In addition, a trade between a connected seller and a core dealer generates a surplus equal to $(C^s + p^{co})$, and a trade between a connected buyer and a core dealer generates a surplus equal to $(C^b - p^{co})$.

The goal of the social planner is to find the "matching plan" that maximizes peripheral dealers' expected gain from trade per period, denoted $W(\Sigma)$. We formally define the social planner's problem and $W(\Sigma)$ in the Online Appendix B.6. We say that a strategy profile Σ is *efficient* if the expected gains from trade per period with this profile are equal to those achieved by the social planner and *inefficient* otherwise.

Proposition 4. *If $\alpha^{pe} \neq 1/2$, then any strategy profile $\Sigma \in \{ACD, ICB, ICS\}$ is inefficient. The profile $\Sigma^{FB-} = (1, 1, 1, 0)$ (resp., $\Sigma^{FB+} = (1, 1, 0, 1)$) is efficient when $\alpha^{pe} < 1/2$ (resp., $\alpha^{pe} > 1/2$).*

Thus, the equilibrium regime of the peripheral market (Σ^*) is always inefficient, except in the knife-edge case $\alpha^{pe} = 1/2$. Two types of inefficiencies arise in equilibrium. First, rent-seeking behavior by unconnected dealers can induce them to choose offers with a relatively low chance of acceptance (i.e., offers accepted by unconnected dealers only). This outcome is inefficient because it raises the likelihood that these dealers hold inventory when the market closes (at date 3). We refer to this source of inefficiency as a *rent-extraction inefficiency*. With the efficient strategy profile, this inefficiency does not arise because unconnected peripheral dealers always make offers that are accepted by both connected and unconnected dealers ($\theta_b = \theta_s = 1$).

Interestingly, the rent-extraction inefficiency is present only in the ICB and ICS profiles (since in the ACD regime, $\theta_b^* = \theta_s^* = 1$). For instance, in the ICB regime, an unconnected seller makes offers that are rejected by all connected dealers, whether buyers or sellers. Now, when connected buyers reject an offer, they trade in the core market where their trade generates a surplus of $(C^b - p^{co})$ instead of $(C^b + C^s)$ if they had traded trade with the unconnected seller, as with the efficient strategy (Σ^{FB-} in Proposition 4). Thus, the realized welfare loss relative to the first best is $[(C^b + C^s) - (C^b - p^{co})] = (C^s + p^{co})$ and the expected welfare loss is therefore $(C^s + p^{co})$ times the likelihood that an unconnected seller makes an offer to a connected buyer. In the Online Appendix B.6, we show that the

expected welfare loss due to this inefficiency is:

$$W(\Sigma^{FB^-}) - W(\Sigma^{ICB}) = \mu_3^*(1-\lambda)(C^s + p^{co}), \tag{30}$$

where $\mu_3^*(1-\lambda)$ is the stationary probability that an unconnected seller trades with a connected buyer in equilibrium (see the Online Appendix B.3).

The second source of inefficiency stems from a negative externality exerted by connected dealers on unconnected dealers. Indeed, when a connected dealer makes an offer accepted by an unconnected dealer, it depletes the peripheral market from a potential liquidity provider (the matched unconnected dealer) for other unconnected dealers. This raises the likelihood that unconnected dealers do not find a counterparty because, in contrast to connected dealers, they cannot use core dealers as liquidity providers of last resort. As connected dealers do not internalize this negative externality when they decide whether to trade with other peripheral dealers, there can be too much trading between connected and unconnected dealers in equilibrium. We call this a *match displacement inefficiency*.

For instance, suppose that $\alpha^{pe} < 1/2$ and consider the following sequence of three arrivals: a connected buyer, an unconnected seller, and an unconnected buyer. In the ACD regime, the connected buyer makes an offer that is accepted by the unconnected seller. This trade generates a surplus equal to $(C^b + C^s)$. However, it prevents the third dealer (unconnected buyer) from being matched with the second dealer (unconnected seller). Yet, this match would be more efficient because (i) the unconnected buyer is not sure to find a counterparty since the market is crowded on the buyer's side ($\alpha^{pe} < 1/2$), (ii) the match between an unconnected seller and a buyer generates the same gains from trade $(C^b + C^s)$ whether the buyer is connected or not and (iii) the first connected buyer can trade with certainty with core dealers. In sum, it is efficient to match connected dealers on the crowded side with core dealers and to keep the liquidity available from connected dealers on the uncrowded side for unconnected dealers. This is the reason why an efficient strategy profile sets $\gamma_b = 0$ or $\gamma_s = 0$ (depending on which side of the market is crowded).

Interestingly, the match displacement inefficiency only arises in the ACD regime (because in this case $\gamma_b^* = \gamma_s^* = 1$) but not in the other regimes. In the Online Appendix B.6,

we show that the magnitude of the welfare loss due to this inefficiency for $\alpha^{pe} < 1/2$ is:

$$W(\Sigma^{FB^-}) - W(\Sigma^{ACD}) = \frac{\mu_5^* \lambda^2 \pi_s (1-\pi_s)(C^b - p^{co})}{1 - \lambda \pi_s}. \tag{31}$$

As explained previously, the rent-extraction inefficiency arises only in the ICS or ICB regimes. Thus, when these regimes are obtained in equilibrium, unconnected peripheral dealers on the uncrowded side of the market (e.g., dealers with short positions when peripheral dealers' aggregate inventory is positive) are more likely to hold inventories when the market closes. We highlight this testable implication of the model in the next corollary.

Corollary 6. *Unconnected dealers on the uncrowded side of the market are more likely to hold inventories at the end of the trading session when core dealers' aggregate inventory is large and of sign opposite to peripheral dealers' aggregate inventory.*

6.2 Implications for trading costs in OTC markets

Inefficiencies in the interdealer market are important because they raise dealers' expected inventory holding costs and, therefore, should result in larger trading costs for end-investors. To illustrate this point, we consider a simple extension of the model in which clients are randomly matched with dealers at date 0. In this extension, clients' trades determine dealers' starting inventories z_{i0}. Clients trade at ask and bid prices determined by Nash bargaining, assuming that all dealers have the same bargaining power relative to their clients. Thus, these prices depend on dealers' expected equilibrium payoff in the interdealer market. We derive closed-form expressions for the ask and bid prices charged by dealers to their clients, given their type (connected or unconnected), in the Appendix (Section A.3.15).[27] We also compute average transaction costs for peripheral dealers' clients, defined as the difference between the average price at which buy orders execute and the average price at which sell orders execute.

[27] Our analysis in this section is related to Dunne, Hau, and Moore (2015) who study a model with inventory limits in which dealers' quotes to their clients and dealers' quotes in the interdealer market are jointly determined in equilibrium. In contrast to them, we do not assume that the interdealer market is competitive, and the dispersion of bid-ask spreads charged to customers comes from the heterogeneity in dealers' connectedness.

This extension delivers two insights. First, we show that unconnected (connected) dealers always charge a higher (lower) bid-ask spread than core dealers. Indeed, unconnected dealers do not have access to the core market and trade at unfavorable prices in the peripheral market if they are on the crowded side. Conversely, connected dealers can exploit their bargaining power over unconnected dealers to obtain a better price in the peripheral market than in the core market and unload their inventories on core dealers if they do not find a counterparty among peripheral dealers. Thus, overall, connected dealers get a higher payoff from acquiring inventory positions than other dealers. Moreover, numerical simulations (Fig. 7) show that the difference in bid-ask spreads charged by connected and unconnected dealers increases when fewer dealers are connected to core dealers. Thus, dealers' connectedness is a determinant of price dispersion both in the interdealer market and for trades between dealers and their customers.

Second, inefficiencies in the allocation of inventory costs among dealers (see the previous section) lead to higher transaction costs for dealers' clients relative to the first best allocation of inventories among dealers. Moreover, these transaction costs increase with λ because inefficiencies in the interdealer market become more serious as more peripheral dealer lose access to core dealers. Figure 8 illustrates these points by showing transaction costs for peripheral dealers' clients in (i) equilibrium and (ii) the first best allocation given in Proposition 4. For $\lambda = 1/2$, equilibrium transaction costs for clients are almost twice as high as what is achievable under the first-best.

[Insert Figures 7 and 8]

7 Conclusion

We analyze a model of interdealer trading in which dealers trade to reduce their inventory holding costs. Our model features two types of dealers: core dealers and peripheral dealers. Core dealers act as liquidity providers to peripheral dealers but not all peripheral dealers have established relationships (connections) with core dealers. We show that equilibrium prices and allocations are determined by (i) the level of connectedness between core and peripheral dealers and (ii) the breakdown of aggregate inventories between highly connected

(core) dealers and less connected (peripheral) dealers.

In particular, peripheral dealers trade bilaterally at different prices if (i) their aggregate position is not zero and (ii) not all peripheral dealers are connected to core dealers. Connected dealers obtain better prices than unconnected dealers because they have access to core dealers' liquidity. Moreover, the average price at which trades take place among peripheral dealers is above (below) the price charged by core dealers if peripheral dealers' aggregate inventory is negative (positive) and the dispersion of prices in the interdealer market increases as fewer dealers are connected to core dealers. We also show that limited connectedness leads to inefficiencies in the allocation of inventory holding costs among dealers, which translate into larger transaction costs for dealers' clients.

In sum, our model suggests new avenues for empirical research on prices in OTC markets. In studying the determinants of these prices, empiricists should account for (cross-asset and time) variations in the level of connectedness between core and peripheral dealers and the distribution of dealers' aggregate inventories between core and peripheral dealers.

A Appendix

A.1 Main Notations

Variable	Definition
v	Asset payoff
α^{pe}	Fraction of peripheral dealers with a long position
α^{co}	Fraction of core dealers with long position
κ	Mass of core dealers
λ	Fraction of peripheral dealers connected to core dealers
z_0^{pe}	Peripheral dealers aggregate inventory at date 0 $(= (2\alpha^{pe} - 1))$
z_0^{co}	Core dealers aggregate inventory at date 0 $(= \kappa(2\alpha^{co} - 1))$
$\overline{z}_0$	Dealers' aggregate inventory at date 0 $(= z_0^{pe} + z_0^{co})$
z_{it}	Dealer i's position at date t
C^b	Cost (per unit) of a short position at date 3
C^s	Cost (per unit) of a long position at date 3
q_{it}	Trade of dealer i at date t
π_b	Probability that a peripheral seller finds a peripheral buyer $(= min\{\frac{1-\alpha^{pe}}{\alpha^{pe}}, 1\})$
π_s	Probability that a peripheral buyer finds a peripheral seller $(= min\{\frac{\alpha^{pe}}{1-\alpha^{pe}}, 1\})$
ϵ_i	Final inventory shock of core dealer i
$\Phi(.)$	Cumulative probability distribution of ϵ_i
p^{co}	Price in the core market
θ_i	Likelihood that a dealer of side $i \in \{b, s\}$ makes an offer at price $p_i = V_{-i}^{C*}$
γ_i	Likelihood that a dealer of side $i \in \{b, s\}$ does **not** directly trade in the core market
Σ	A strategy profile in the peripheral market $(\Sigma = (\theta_s, \theta_b, \gamma_s, \gamma_b))$
ω_i	$= \frac{\lambda(1-\pi_i)}{1-\pi_i\lambda(2-\lambda)}$ for $i \in \{b, s\}$
μ_s^{co}	Mass of peripheral dealers with a long position trading in the core market
μ_b^{co}	Mass of peripheral dealers with a short position trading in the core market
$\Delta(\alpha^{pe}, \lambda, \Sigma)$	Order flow (sales - buys) of connected peripheral dealers trading in the core market

A.2 Figures

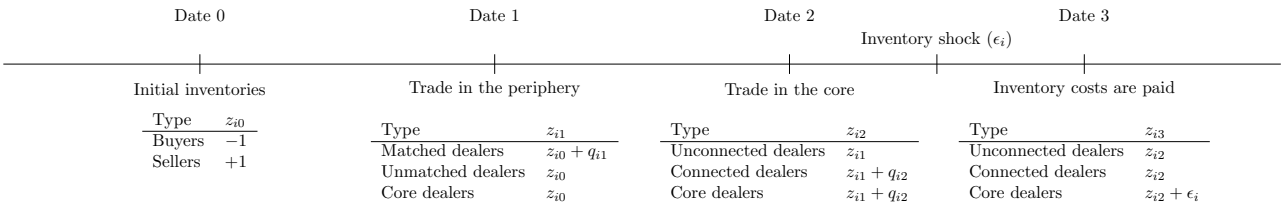


Figure 1: Timeline.

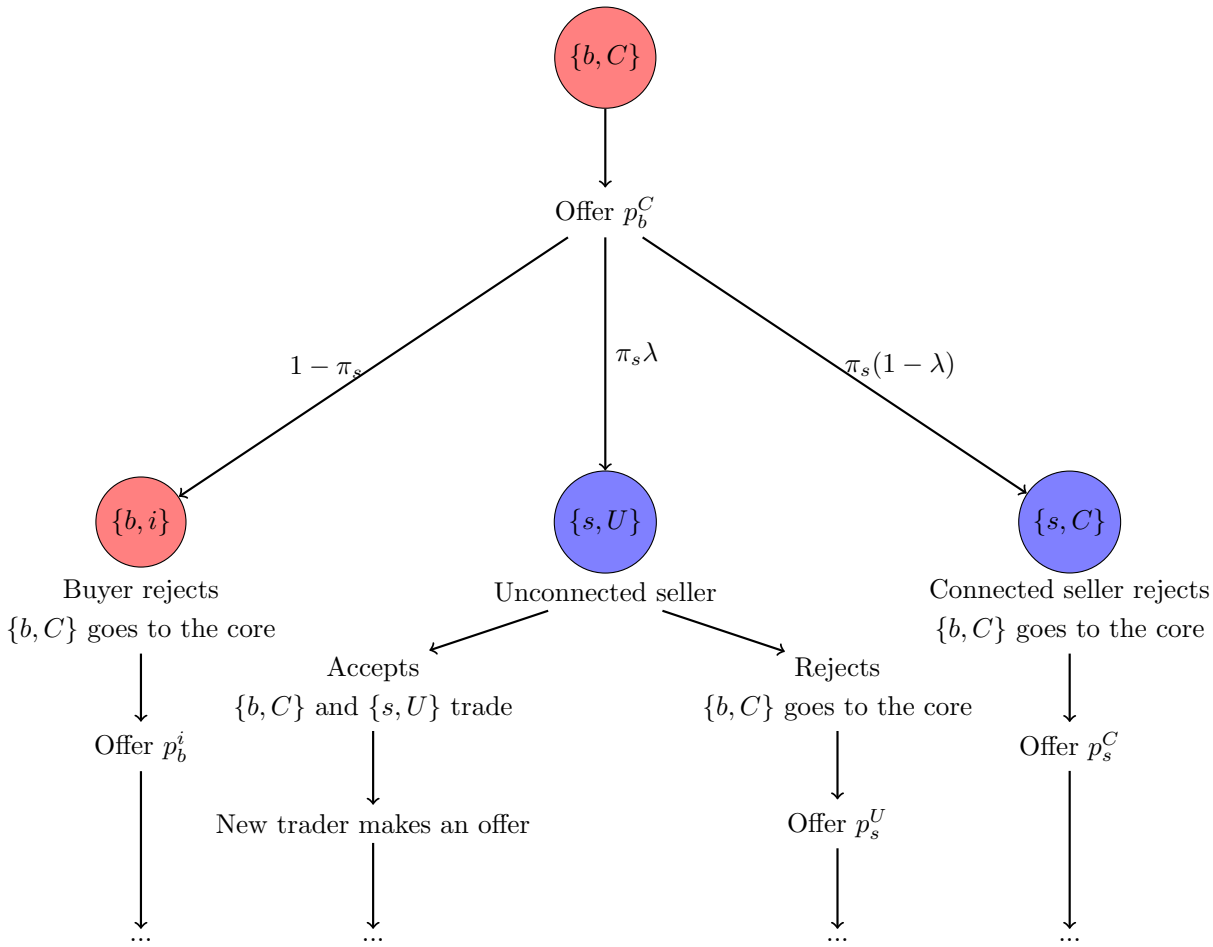
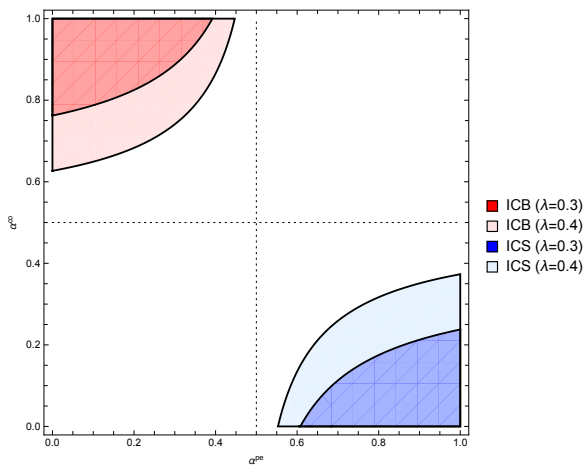


Figure 2: Trading in the periphery. This figure depicts the trading process in the peripheral market for a particular sequence of arrivals. Red circles designate buyers while blue circles designate sellers. We denote by p_b^j, the offer made by a buyer of type $j \in \{C, U\}$ and by p_s^j, the offer made by a seller of type $j \in \{C, U\}$.

Figure 3: Equilibrium type as a function of α^{pe} and α^{co}.

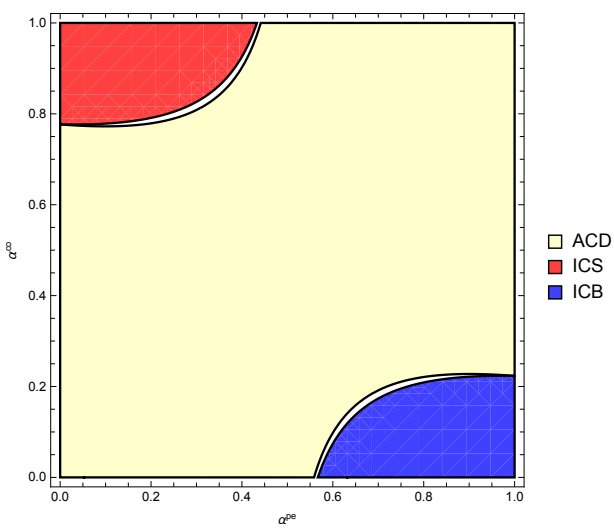
Panel A: $\kappa \to \infty$, $\lambda = 0.3$ and $\lambda = 0.4$.

For each pair $(\alpha^{pe}, \alpha^{co})$, Panel A provides the corresponding strategy profile obtained in equilibrium in the thick core market case. The colored areas represent the $(\alpha^{pe}, \alpha^{co})$ pairs for which an ICS (for $\alpha^{pe} > \frac{1}{2}$) or an ICB (for $\alpha^{pe} < \frac{1}{2}$) equilibrium is obtained, for two values of λ (0.3 and 0.4). An ACD equilibrium obtains otherwise (white area).



Panel B: $\kappa = 2$, $\lambda = 0.4$.

For each pair $(\alpha^{pe}, \alpha^{co})$, Panel B provides the corresponding strategy profile obtained in equilibrium when the core market is not thick. The colored areas represent the $(\alpha^{pe}, \alpha^{co})$ pairs for which an ACD, ICS, or ICB equilibrium is obtained. A mixed equilibrium obtains otherwise.



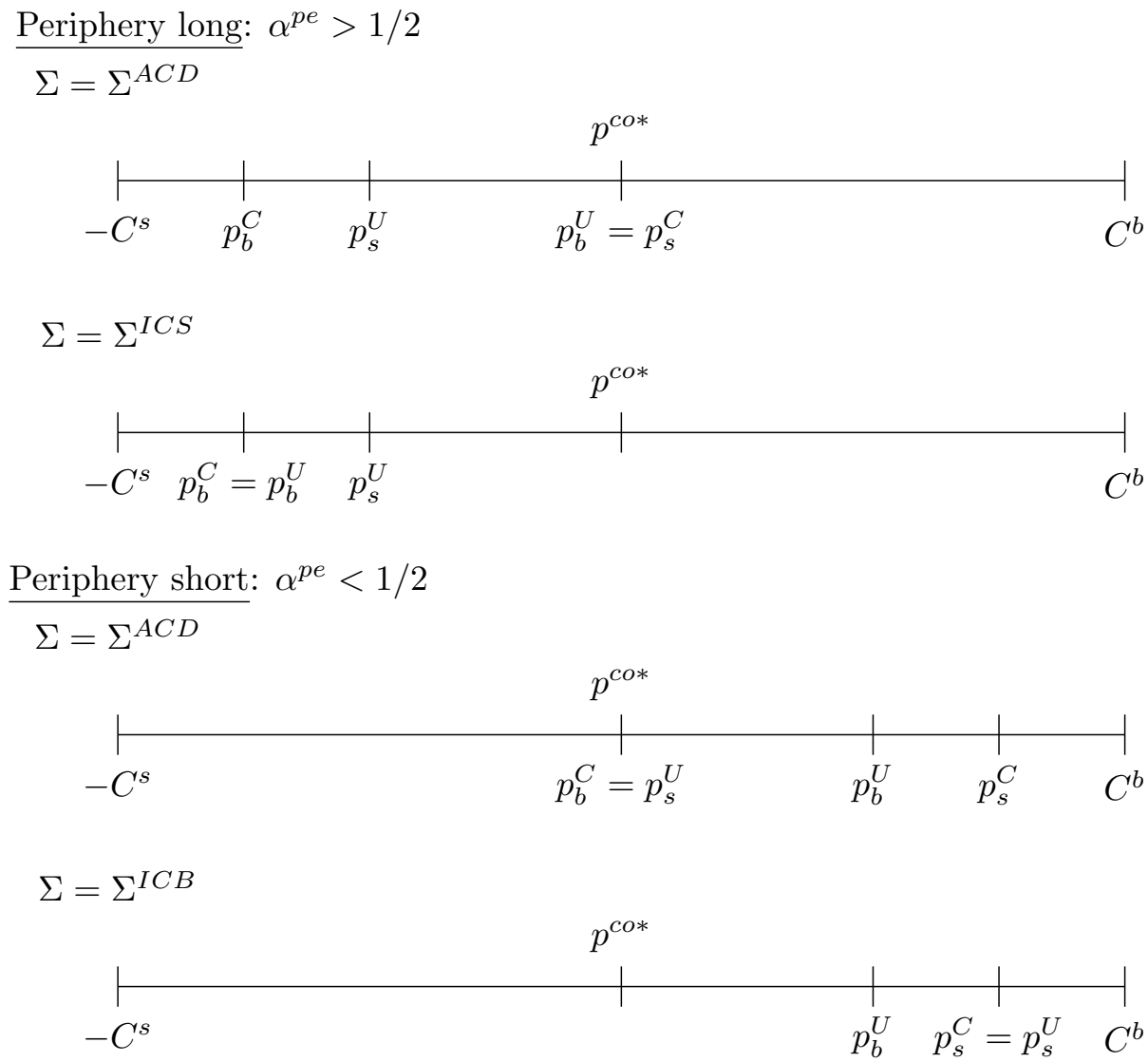


Figure 4: Equilibrium Prices. This figure shows the position of equilibrium prices in the peripheral market relative to the equilibrium core market price in each possible equilibrium regime when $\alpha^{pe} > 0.5$ and $\alpha^{pe} < 0.5$;

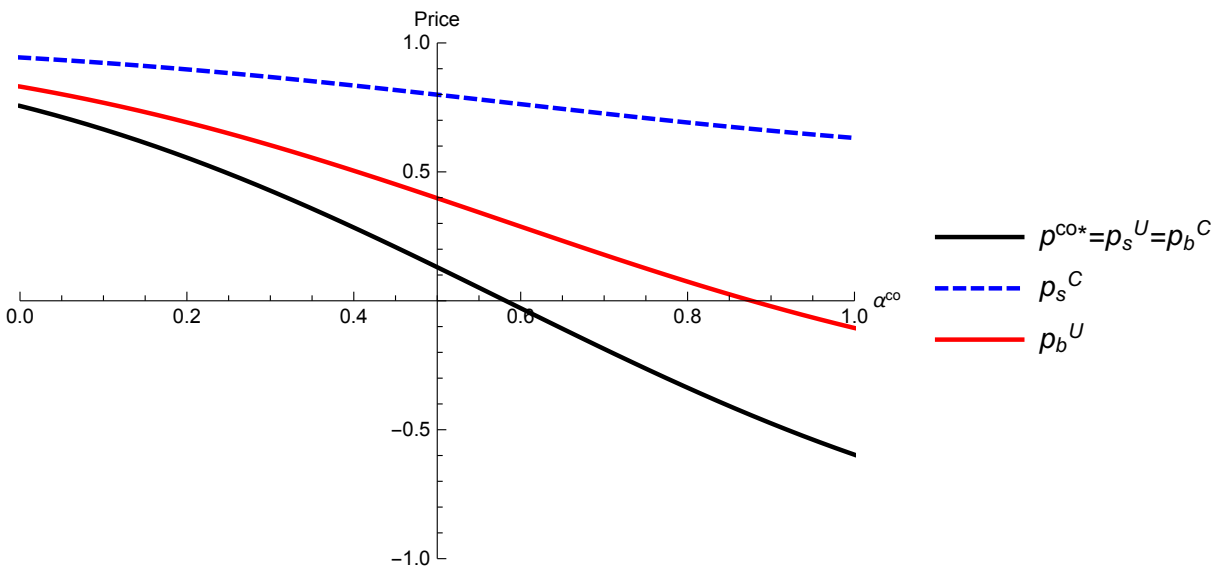


Figure 5: Equilibrium prices as a function the core dealers' aggregate inventory. The figure illustrates Corollary 3. It shows the core market price (black line) and the different prices at which trades take place among peripheral dealers as a function of the mass of sellers among core dealers (α^{co}). The mass of sellers in the peripheral market is $\alpha^{pe} = 0.25$, i.e., in aggregate, peripheral dealers have a short position in the asset ($z_0^{pe} < 0$). When core dealers' initial aggregate inventory increases, prices in the core and peripheral markets go down (Corollary 2). However, the core market price drops faster than the highest price in the peripheral market. Thus, price dispersion in the peripheral market increases (Corollary 3).

	$\bar{\alpha}^{pe} = \bar{\alpha}^{co} = 0.5$		$\bar{\alpha}^{pe} = 0.2, \bar{\alpha}^{co} = 0.8$	
	Avg.	St.Dev.	Avg.	St.Dev.
Core price	0.00	0.10	-0.54	0.08
Periphery prices				
$\lambda = 0.05$	0.00	0.10	-0.53	0.13
$\lambda = 0.10$	0.00	0.11	-0.52	0.17
$\lambda = 0.15$	0.00	0.11	-0.51	0.20
$\lambda = 0.20$	0.00	0.11	-0.49	0.23
$\lambda = 0.25$	0.00	0.12	-0.48	0.26
$\lambda = 0.30$	0.00	0.12	-0.45	0.30
$\lambda = 0.35$	0.00	0.13	-0.42	0.34
$\lambda = 0.40$	0.00	0.13	-0.39	0.37
$\lambda = 0.45$	0.00	0.14	-0.36	0.40
$\lambda = 0.50$	0.00	0.15	-0.32	0.43

Table 1: This table reports the mean and the standard deviation of transaction prices in (a) the core market and (b) the peripheral market for different parameter values for (i) λ and (ii) $\bar{\alpha}^{pe}$ and $\bar{\alpha}^{co}$.

Figure 6: Distribution of equilibrium prices in the peripheral market. We randomly draw $1,000,000$ realizations of $(\alpha^{co}, \alpha^{pe})$. For each draw, we compute equilibrium prices of trades among peripheral dealers and the *likelihood* that each price is observed in equilibrium based on the stationary probability distributions derived in Appendix B.3. We then use this likelihood as the number of times a given price is observed in a given draw (we normalize the number of trades per draw to one). We then divide the interval between $-C^s$ and C^b in 100 bins and plot the respective number of times a price in each bin is observed. We then normalize so that the sum of all frequencies across bins is equal to 1. All simulations use $\sigma_\eta^2 = 0.05, \sigma_\epsilon^2 = 0.8, C^b = C^s = 1, \kappa \to \infty$. We set $\bar{\alpha}^{pe} = \bar{\alpha}^{co} = 0.5$ in Panel A, and $\bar{\alpha}^{pe} = 0.2, \bar{\alpha}^{co} = 0.8$ in Panel B.

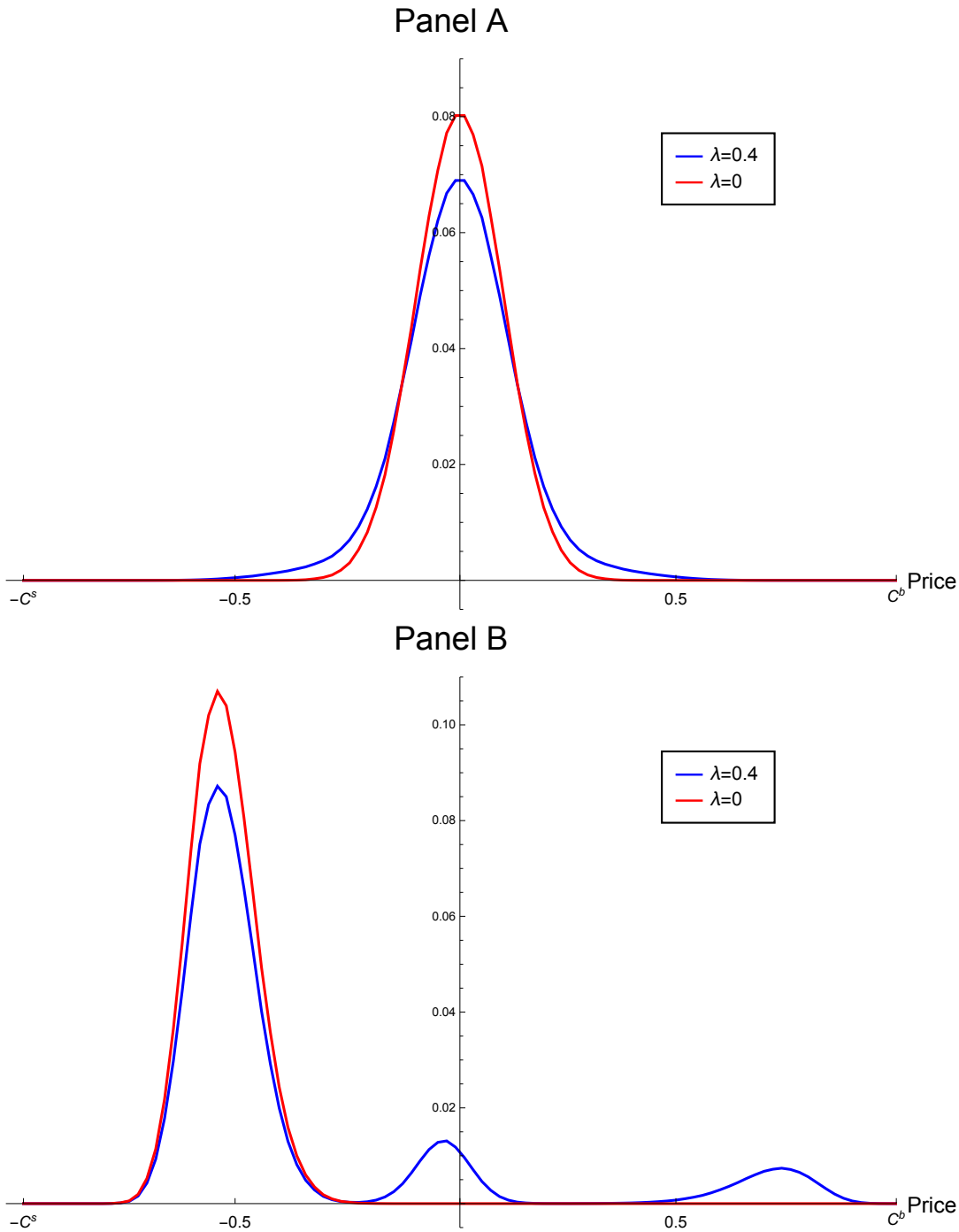
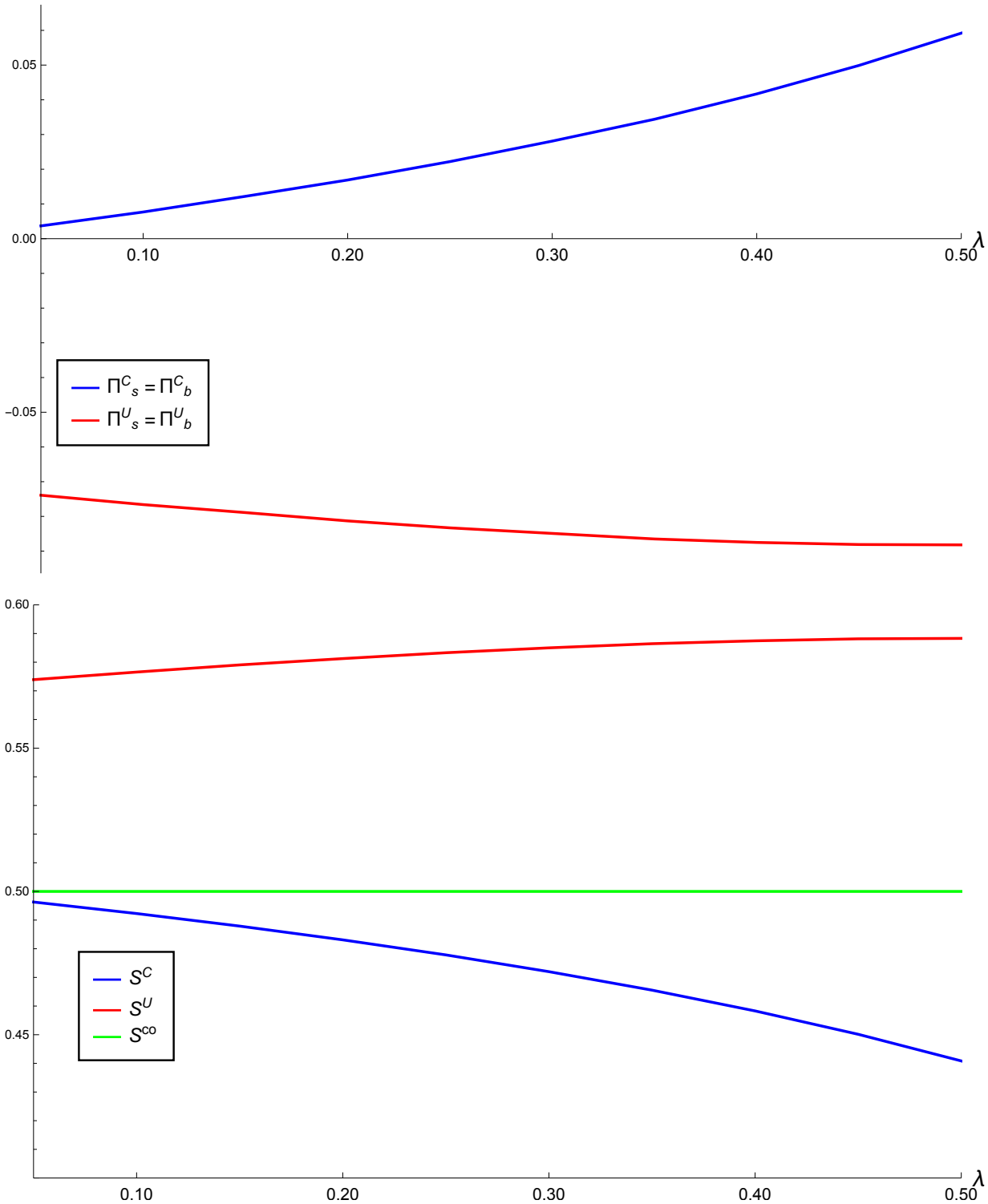


Figure 7: Dealers' equilibrium payoffs and bid-ask spreads charged to clients. Using derivations in Section A.3.15 of the Appendix, the top panel plots the average payoff of the four types of peripheral dealers: connected sellers and buyers (blue line); unconnected sellers and buyers (red line); over one million simulations (performed as described in Section 5.4) as a function of λ. The bottom panel plots the half bid-ask spreads charged to their clients by connected peripheral dealers (blue line), unconnected peripheral dealer (red line), and core dealers (green line). The parameters are $\sigma_\eta^2 = 0.05, \sigma_\epsilon^2 = 0.8, C^b = C^s = 1, \bar{\alpha}^{pe} = \bar{\alpha}^{co} = 0.5$.



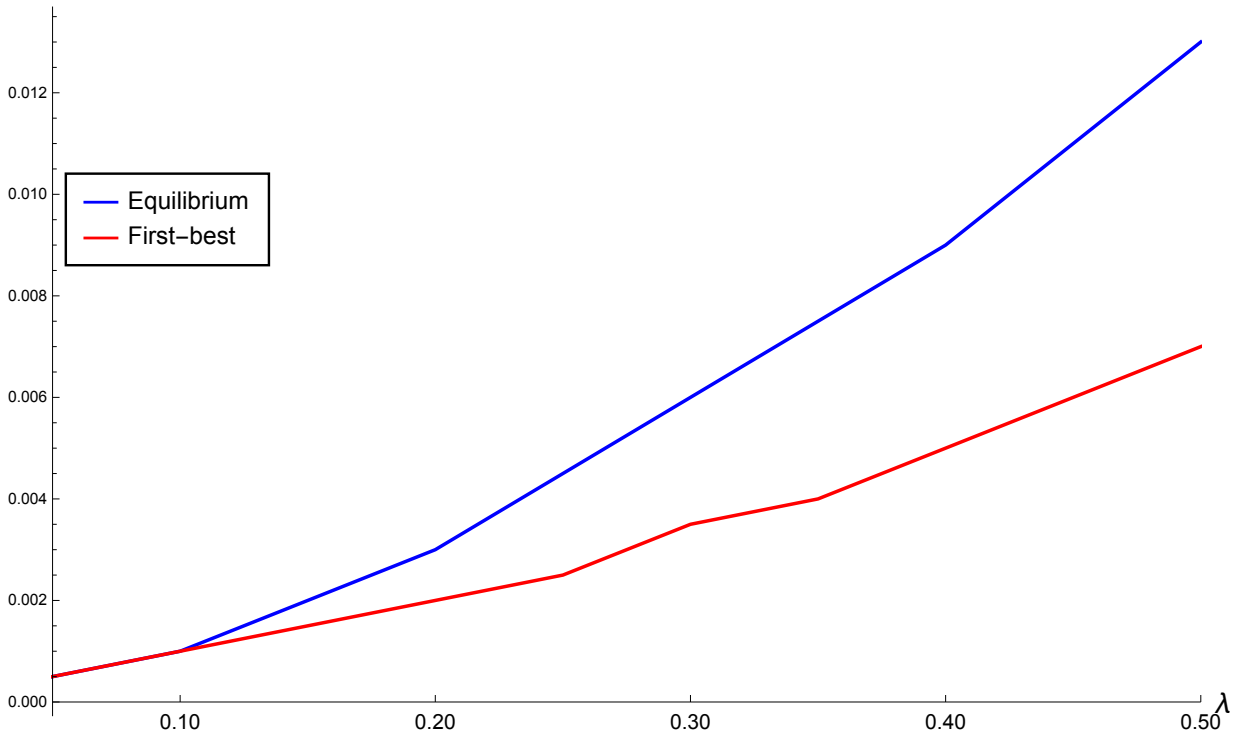


Figure 8: Average clients' transaction costs. This figure plots the average total transaction costs of clients (as defined in Appendix A.3.15) over one million numerical simulations of the model (performed as described in Section 5.4) as a function of λ, when dealers behave according to the equilibrium strategy profile (blue line) and when they behave according to the efficient strategy profile (red line). The parameters are $\sigma_\eta^2 = 0.05, \sigma_\epsilon^2 = 0.8, C^b = C^s = 1, \bar{\alpha}^{pe} = \bar{\alpha}^{co} = 0.5$.

A.3 Proofs

A.3.1 Proof of Lemma 1

Follows from the discussion preceding the proposition.

A.3.2 Proof of Lemma 2

Follows from the observations that precede the lemma.

A.3.3 Proof of Lemma 3

ACD Regime. We start by deriving the conditions under which the active connected dealers (ACD) regime obtains. In this equilibrium, a connected seller's offer is accepted if and only if the seller is matched with an unconnected buyer. This event has probability $\lambda\pi_b$, so that:

$$V_s^{C*} = \lambda\pi_b p_s^{C*} + (1 - \lambda\pi_b)p^{co*}. \tag{32}$$

Symmetrically, a connected buyer's offer is accepted with probability $\lambda\pi_s$. Therefore:

$$V_b^{C*} = -\lambda\pi_s p_b^{C*} - (1 - \lambda\pi_s)p^{co*}. \tag{33}$$

Moreover, an unconnected seller's offer is accepted if and only if he is matched with a buyer, of any type, which gives:

$$V_s^{U*} = \pi_b p_s^{U*} - (1 - \pi_b)C^s. \tag{34}$$

And, symmetrically:

$$V_b^{U*} = -\pi_s p_b^{U*} - (1 - \pi_s)C^b. \tag{35}$$

Furthermore, from Lemma 2, we know that $p_s^{C*} = -V_b^{U*}$, $p_b^{C*} = V_s^{U*}$, $p_s^{U*} = -V_b^{C*}$, and $p_b^{U*} = V_s^{C*}$.

Combining these conditions with (32)-(35), we obtain a system of 4 equations with 4

unknowns $(p_s^{C*}, p_s^{U*}, p_b^{C*}, p_b^{U*})$. Solving this system, we obtain:

$$p_b^{U*} = V_s^{C*} = \frac{(1-\pi_b\lambda)p^{co*} + \pi_b\lambda(1-\pi_s)C^b}{1-\pi_s\pi_b\lambda}, p_b^{C*} = V_s^{U*} = \frac{\pi_b(1-\pi_s\lambda)p^{co*} - (1-\pi_b)C^s}{1-\pi_s\pi_b\lambda} \tag{36}$$

$$p_s^{U*} = -V_b^{C*} = \frac{(1-\pi_s\lambda)p^{co*} - \pi_s\lambda(1-\pi_b)C^s}{1-\pi_s\pi_b\lambda}, p_s^{C*} = -V_b^{U*} = \frac{\pi_s(1-\pi_b\lambda)p^{co*} + (1-\pi_s)C^b}{1-\pi_s\pi_b\lambda}. \tag{37}$$

Equations (36) and (37) yield equilibrium prices in the ACD regime.

It remains to derive the conditions under which dealers have no incentive to deviate from their equilibrium behavior in the ACD regime. First, observe that we have $p_b^{C*} \leq p^{co*}$ and $p_s^{C*} \geq p^{co*}$. Hence, $\gamma_s^* = \gamma_b^* = 1$ is optimal for connected dealers.[28] Second, we have $p_b^{U*} \leq C^b$ and $p_s^{U*} \geq -C^s$ so that unconnected dealers are better off trading with another peripheral dealer rather than bearing their inventory holding cost.

Last, we need to check that $\theta_s^* = \theta_b^* = 1$ is optimal for unconnected dealers. Consider an unconnected seller. In equilibrium, he obtains an expected payoff of V_s^{U*} with an offer at $p_s^{U*} = -V_b^{C*}$. As explained in the text, his most profitable deviation is to offer the same price as that of a connected seller, i.e., $p_s^{C*} = -V_b^{U*}$. This offer is accepted only by unconnected buyers, i.e., with probability $\pi_b\lambda$. Thus, an unconnected seller is better off not deviating if and only if:

$$V_s^{U*} > -\pi_b\lambda V_b^{U*} - (1-\pi_b\lambda)C^s. \tag{38}$$

Symmetrically, an unconnected borrower is better off not deviating if and only if:

$$V_b^{U*} > -\pi_s\lambda V_s^{U*} - (1-\pi_s\lambda)C^b. \tag{39}$$

[28] When $\alpha^{pe} \geq 1/2$, we have $\pi_s = 1$ and thus $p_s^{C*} = p^{co}$. Thus, in this case, connected sellers are indifferent between making an offer and directly trading in the core market. Thus, they could play a mixed strategy between making an offer or, instead, trading in the core market. However, this indifference breaks down as soon as the likelihood that the successor of a buyer is not a seller with probability 1, as is always the case when $\zeta < 1$ (see (4)). Indeed in this case, $p_s^{C*} > p^{co}$ for all values of α^{pe} in the ACD regime and a connected seller is therefore strictly better off making an offer.

Using (36) and (37), Conditions (38) and (39) can be rewritten as:

$$\frac{p^{co*} + C^s}{C^b + C^s} > \frac{\lambda(1-\pi_s)}{1-\pi_s\lambda(2-\pi_b\lambda)}, \tag{40}$$

$$\frac{C^b - p^{co*}}{C^b + C^s} > \frac{\lambda(1-\pi_b)}{1-\pi_b\lambda(2-\pi_s\lambda)}. \tag{41}$$

After straightforward manipulations, these two conditions can be shown to be equivalent to Condition (19) in Lemma 3.

We now derive conditions under which the other regimes are obtained in equilibrium.

Inactive connected sellers (ICS) regime. In this equilibrium, connected buyers make offers that are accepted by unconnected sellers only, i.e., $p_b^{C*} = V_s^{U*}$. Moreover, connected sellers reject offers from buyers (whether unconnected or connected) and trade with a core seller with probability 1 ($\gamma_s^* = 0$). Hence, in this equilibrium, we have:

$$V_s^{C*} = p^{co*} \tag{42}$$

$$V_b^{C*} = -\lambda\pi_s V_s^{U*} - (1-\lambda\pi_s)p^{co*}. \tag{43}$$

In addition, in this equilibrium, unconnected sellers make offers that are accepted by all buyers, i.e., $p_s^{U*} = -V_b^{C*}$, while unconnected buyers make offers that are accepted by unconnected sellers only, i.e., $p_b^{U*} = V_s^{U*}$. Thus, we have:

$$V_s^{U*} = -\pi_b V_b^{C*} - (1-\pi_b)C^s \tag{44}$$

$$V_b^{U*} = -\pi_s\lambda V_s^{U*} - (1-\pi_s\lambda)C^b. \tag{45}$$

Solving the previous system of equations for V_s^{U*}, V_b^{U*}, and V_b^{C*}, we obtain that in an ICS regime:

$$p_b^{U*} = p_b^{C*} = V_s^{U*} = \frac{(1-\pi_s\lambda)\pi_b p^{co*} - (1-\pi_b)C^s}{1-\lambda\pi_s\pi_b}, \tag{46}$$

$$V_b^{U*} = -\left(\frac{\lambda\pi_s\pi_b(1-\pi_s\lambda)p^{co*} - \pi_s\lambda(1-\pi_b)C^s}{1-\lambda\pi_s\pi_b} + (1-\pi_s\lambda)C^b\right), \tag{47}$$

$$p_s^{U*} = -V_b^{C*} = \frac{(1-\pi_s\lambda)p^{co*} - \pi_s\lambda(1-\pi_b)C^s}{1-\lambda\pi_s\pi_b}, \tag{48}$$

$$V_s^{C*} = p^{co*}. \tag{49}$$

We now establish show that no peripheral dealer has an incentive to deviate from the ICS equilibrium if and only if $p^{co} > (1-\omega_b)C^b - \omega_b C^s$. We first check that this is the case for buyers.

From (48), we deduce that $V_b^{C*} > -p^{co}$, so that connected buyers are better off making an offer at rate p_b^{C*} to another peripheral dealer when they reject an offer rather than directly contacting a core dealer. This implies $\gamma_b^* = 1$ is optimal for a connected buyer, as it should in the ICS regime.

In the ICS regime, unconnected buyers offer prices that are accepted by unconnected sellers only. As explained in the text, their best deviation is to offer a price V_s^{C*} that is accepted by all types of sellers. This deviation is not optimal when other dealers behave as in the ICS regime iff:

$$V_b^{U*} > -\pi_s V_s^{C*} - (1-\pi_s)C^b. \tag{50}$$

When $\alpha^{pe} < \frac{1}{2}$, $\pi_b = 1$ and $\pi_s < 1$. Thus, substituting into Condition (50) V_b^{U*} and V_s^{C*} by their expressions in (47) and (49), we observe that Condition (50) requires $p^{co} > C_b$, which is impossible. Thus, the ICS regime cannot obtain when $\alpha^{pe} < \frac{1}{2}$. When $\alpha^{pe} > \frac{1}{2}$, $\pi_s = 1$ and $\pi_b < 1$. Thus, substituting into Condition (50) V_b^{U*} and V_s^{C*} by their expressions in (47) and (49), we can rewrite Condition (50) as:

$$\frac{C^b - p^{co}}{C^b + C^s} < \frac{\lambda(1-\pi_b)}{1-\lambda\pi_b(2-\lambda)}, \tag{51}$$

which is equivalent to $p^{co} > (1-\omega_b)C^b - \omega_b C^s$. Thus, $p^{co} > (1-\omega_b)C^b - \omega_b C^s$ and $\alpha^{pe} > \frac{1}{2}$ are necessary conditions to obtain the ICS equilibrium.

Now, we show that peripheral sellers have no incentive to deviate from their strategy profile in an ICS equilibrium if $p^{co} > (1-\omega_b)C^b - \omega_b C^s$. In the ICS regime, a connected seller directly contacts a core dealer if he rejects an offer ($\gamma_s^* = 0$). His best deviation (see the discussion preceding Lemma 2) is to make an offer at the highest price that unconnected buyers are willing to accept, i.e., $-V_b^{U*}$. This deviation cannot be optimal if $p^{co*} > -V_b^{U*}$. Substituting V_b^{U*} by its expression in ((47)), we find that when $\alpha^{pe} > \frac{1}{2}$, this condition is satisfied if (51), or equivalently $p^{co} > (1-\omega_b)C^b - \omega_b C^s$, is satisfied. Thus, when $\alpha^{pe} > \frac{1}{2}$ (which, as just explained, is a necessary condition for the ICS regime) and

$p^{co} > (1-\omega_b)C^b - \omega_b C^s$, $\gamma_s^* = 0$ is a best response for connected sellers.

Finally, unconnected sellers offer a price that is accepted by all types of buyers, i.e., a price equal to $p_s^{U*} = -V_b^{C*}$. Their best deviation is to offer a price $-V_b^{U*}$ that is accepted by unconnected buyers only (see the discussion preceding Lemma 2). This deviation is not optimal iff:

$$V_s^{U*} > -\pi_b \lambda V_b^{U*} - (1-\pi_b\lambda)C^s. \tag{52}$$

Substituting V_s^{U*} and V_b^{U*} by their expressions, we deduce that, when $\alpha^{pe} > \frac{1}{2}$, (52) is equivalent to:

$$\frac{C^b - p^{co}}{C^b + C^s} < \frac{1-\lambda}{1-\lambda^2\pi_b}. \tag{53}$$

When $\lambda < 1/2$, (53) is satisfied if (51) (or equivalently $p^{co} > (1-\omega_b)C^b - \omega_b C^s$) is satisfied.

We deduce from this analysis that $\alpha^{pe} > \frac{1}{2}$ and $p^{co} > (1-\omega_b)C^b - \omega_b C^s$ are necessary and sufficient conditions for obtaining the ICS regime. As $\alpha^{pe} > \frac{1}{2}$ is a necessary condition for $p^{co} > (1-\omega_b)C^b - \omega_b C^s$, we deduce that $p^{co} > (1-\omega_b)C^b - \omega_b C^s$ is a necessary and sufficient condition an ICS equilibrium.

Inactive connected buyers (ICB) regime. We can show that this equilibrium obtains if and only if $\alpha^{pe} < \frac{1}{2}$ and:

$$\frac{p^{co} + C^s}{C^b + C^s} < \frac{\lambda(1-\pi_s)}{1-\lambda\pi_s(2-\lambda)}, \tag{54}$$

by proceeding exactly as we did for ICS regime. This condition is equivalent to the condition on the third line of (19).

A.3.4 Proof of Lemma 4

See the Online Appendix B.4.

A.3.5 Proof of Proposition 1

The threshold values are defined by $\alpha_{ACD}^+ = \alpha^+(\Sigma^{ACD})$, $\alpha_{ICB}^+ = \alpha^+(\Sigma^{ICB})$, $\alpha_{ACD}^- = \alpha^-(\Sigma^{ACD})$, $\alpha_{ICS}^- = \alpha^-(\Sigma^{ICS})$, with:

$$\alpha^{+}(\Sigma) = \frac{1}{2}\left(1 - \left[\Phi^{-1}(\omega_s) + \frac{\Delta(\alpha^{pe}, \lambda, \Sigma)}{\kappa}\right]\right), \tag{55}$$

$$\alpha^{-}(\Sigma) = \frac{1}{2}\left(1 - \left[\Phi^{-1}(1-\omega_b) + \frac{\Delta(\alpha^{pe}, \lambda, \Sigma)}{\kappa}\right]\right), \tag{56}$$

where, for brevity, we omit the fact that $\alpha^{+}(\Sigma)$ and $\alpha^{-}(\Sigma)$ are ultimately functions of the exogenous parameters λ and α^{pe}. Suppose that $\alpha^{pe} < \frac{1}{2}$. In this case, we know from Lemma 3 that the only possible equilibrium strategy profiles are $\Sigma^* = \Sigma^{ACD}$ or $\Sigma^* = \Sigma^{ICB}$. A full equilibrium in which $\Sigma^* = \Sigma^{ICB}$ obtains if and only if the condition on the third line of (19) is satisfied for $p^{co} = p^{co*}(z^{co}, \Delta^*(\alpha^{pe}, \lambda, \Sigma^{ICB}))$. As $p^{co*}(z^{co}, \Delta) = \Phi(-\kappa^{-1}(z^{co} + \Delta))C^b - C^s(1 - \Phi_{co}(-(z^{co} + \Delta)))$ and $z^{co} = \kappa(2\alpha^{co} - 1)$, this is equivalent to:

$$\alpha^{co} > \alpha^{+}(\Sigma^{ICB}). \tag{57}$$

A full equilibrium in which $\Sigma^* = \Sigma^{ACD}$ obtains if and only if the condition on the second line of (19) is satisfied for $p^{co} = p^{co*}(z^{co}, \Delta^*(\alpha^{pe}, \lambda, \Sigma^{ACD}))$. As $p^{co*}(z^{co}, \Delta) = \Phi(-\kappa^{-1}(z^{co} + \Delta))C^b - C^s(1 - \Phi(-\kappa^{-1}(z^{co} + \Delta))$ and $z^{co} = \kappa(2\alpha^{co} - 1)$, this is equivalent to:

$$\alpha^{co} < \alpha^{+}(\Sigma^{ACD}). \tag{58}$$

For $\alpha^{pe} < \frac{1}{2}$, $\Delta(\alpha^{pe}, \lambda, \Sigma^*) \leq 0$, since $\Sigma^* = \Sigma^{ACD}$ or $\Sigma^* = \Sigma^{ICB}$ (see Lemma 4). Moreover, $\omega_s < 1/2$ and therefore $\Phi^{-1}(\omega_s) < 0$ (since $\Phi(\frac{1}{2}) = 0$).[29] Lastly, as shown in Lemma 4, $\Delta(\alpha^{pe}, \lambda, \Sigma^{ICB}) < \Delta(\alpha^{pe}, \lambda, \Sigma^{ACD}) < 0$. It follows that: $\frac{1}{2} < \alpha^{+}(\Sigma^{ACD}) < \alpha^{+}(\Sigma^{ICB})$. The proof for the other cases ($\alpha^{pe} > \frac{1}{2}$ and $\alpha^{pe} = \frac{1}{2}$) is similar and is therefore skipped for brevity.

A.3.6 Proof of Proposition 2

The proof of the proposition follows directly from the expressions for equilibrium prices in the peripheral market obtained in the proof of Lemma 3.

[29] We have $\omega_s < 1/2$ because $\lambda \leq \frac{1}{2}$.

A.3.7 Proof of Proposition 3

Suppose $\alpha^{pe} \geq \frac{1}{2}$. From Proposition 1, we know that in this case $\Sigma = \Sigma^{ACD}$ if $\alpha^{co} > \alpha^{-}_{ACD}$. We look at the limit of α^{-}_{ACD} when $\lambda \to 0$, using (56). The term $\Delta(\alpha^{pe}, \lambda, \Sigma^{ACD})$ is positive and bounded above by 1. As λ goes to zero, ω_b goes to zero and thus $\Phi^{-1}(1-\omega_b)$ goes to $+\infty$. As a result $\lim_{\lambda\to 0} \alpha^{-}_{ACD} = -\infty$, so that the only possible equilibrium in the peripheral market is ACD.

In this case, we deduce from Appendix B.3 that $\lim_{\lambda\to 0} \mu_1^* = (1+\pi_b)^{-1}$ and $\lim_{\lambda\to 0} \mu_5^* = \pi_b(1+\pi_b)^{-1}$. As $\alpha^{pe} \geq \frac{1}{2}$, we have $\pi_b < 1$ and, using (B.7), we deduce that:

$$\lim_{\lambda\to 0} \Delta(\alpha^{pe}, \lambda, \Sigma^{ACD}) = (2\alpha^{pe} - 1) = z_0^{pe}. \tag{59}$$

Thus, using (12), we deduce that

$$\lim_{\lambda\to 0} p^{co*}(\alpha^{pe}, \Delta^*(\alpha^{pe}, \lambda, \Sigma^{ACD})) = p^{co*}(\alpha^{pe}, z_0^{pe}) = p^{bench*}. \tag{60}$$

Finally, using the expressions for equilibrium rates in Proposition 2, we also obtain that

$$\lim_{\lambda\to 0} p_b^{U*} = \lim_{\lambda\to 0} p_s^{U*} = p^{bench*}. \tag{61}$$

A symmetric reasoning shows that the same findings obtain when $\alpha^{pe} < \frac{1}{2}$.

A.3.8 Proof of Corollary 1

Part 1: Suppose $\alpha^{pe} > \frac{1}{2}$ or equivalently $z_0^{pe} > 0$. Thus, $\pi_s = 1$ and $\pi_b < 1$. In this case either $\Sigma^* = \Sigma^{ACD}$ or $\Sigma^* = \Sigma^{ICS}$. Consider first the case in which $\Sigma^* = \Sigma^{ACD}$. Using Proposition 2, we obtain:

$$p_s^{C*} = p_b^{U*} = p^{co*}, \tag{62}$$

$$p_s^{U*} = p^{co*} - \frac{\lambda(1-\pi_b)(p^{co*} + C^s)}{1-\pi_b\lambda}, \tag{63}$$

$$p_b^{C*} = p^{co*} - \frac{(1-\pi_b)(p^{co*} + C^s)}{1-\pi_b\lambda}. \tag{64}$$

When a transaction between two unconnected dealers happens, either the seller made the offer and this offer is accepted by the buyer, or the buyer made the offer and this offer is accepted by the seller. In the first case, the transaction price is p_s^{U*} while in the second case the transaction price is p_b^{U*}. Conditional on observing a trade between two unconnected dealers, the likelihood of the first event is $\frac{\mu_3^* \times \pi_b \lambda}{\mu_3^* \times \pi_b \lambda + \mu_7^* \times \lambda}$, where μ_3^* and μ_7^* are the stationary probabilities derived in Appendix B.3 when $\Sigma^* = \Sigma^{ACD} = (1, 1, 1, 1)$. Thus

$$\overline{p}^{UU} = \frac{\mu_3^* \times \pi_b \lambda}{\mu_3^* \times \pi_b \lambda + \mu_7^* \times \lambda} p_s^{U*} + \frac{\mu_7^* \times \lambda}{\mu_3^* \times \pi_b \lambda + \mu_7^* \times \lambda} p_b^{U*}. \tag{65}$$

We deduce from (62) and (63) that

$$M^{UU} = -\frac{\mu_3^* \times \pi_b \lambda}{\mu_3^* \times \pi_b \lambda + \mu_7^* \times \lambda} \frac{\lambda(1 - \pi_b)(p^{co*} + C^s)}{(1 - \pi_b \lambda)} < 0. \tag{66}$$

A similar reasoning implies that:

$$\overline{p}^{CU} = \frac{\mu_1^* \times \pi_b \lambda}{\mu_1^* \times \pi_b \lambda + \mu_7^* \times (1 - \lambda)} p_s^{C*} + \frac{\mu_7^* \times (1 - \lambda)}{\mu_1^* \times \pi_b \lambda + \mu_7^* \times (1 - \lambda)} p_b^{U*} = p^{co*}, \tag{67}$$

and therefore $M^{CU} = 0$. Finally, trades where an unconnected dealer sells to a connected dealer occur if either the offer from an unconnected seller is accepted by a connected buyer, or vice versa. Hence we have

$$\overline{p}^{UC} = \frac{\mu_3 \times \pi_b (1 - \lambda)}{\mu_3 \times \pi_b (1 - \lambda) + \mu_5 \times \lambda} p_s^{U*} + \frac{\mu_5 \times \lambda}{\mu_3 \times \pi_b (1 - \lambda) + \mu_5 \times \lambda} p_b^{C*} \in [p_b^{C*}, p_s^{U*}]. \tag{68}$$

As $p_b^{C*} < p_s^{U*} < p_b^{U*}$, we have $\overline{p}^{UC} < \overline{p}^{UU}$ and thus $M^{UC} < M^{UU} < M^{CU} = 0$.

Now consider the case in which $\Sigma^* = \Sigma^{ICS}$. Using Proposition 2, we obtain:

$$p_s^{U*} = p^{co*} - \frac{\lambda(1 - \pi_b)(p^{co*} + C^s)}{1 - \pi_b \lambda}, \tag{69}$$

$$p_b^{C*} = p_b^{U*} = p^{co*} - \frac{(1 - \pi_b)(p^{co*} + C^s)}{1 - \pi_b \lambda}. \tag{70}$$

As $\Sigma^* = \Sigma^{ICS}$, the likelihood of observing an offer from a connected seller to a unconnected buyer is zero. Thus, $\bar{p}^{CU}$ is not observed. For other possible matches, using the same logic

as before, we obtain:

$$\overline{p}^{UU} = \frac{\mu_3 \times \pi_b \lambda}{\mu_3 \times \pi_b \lambda + \mu_7 \times \lambda} p_s^{U*} + \frac{\mu_7 \times \lambda}{\mu_3 \times \pi_b \lambda + \mu_7 \times \lambda} p_b^{U*} \tag{71}$$

$$\overline{p}^{UC} = \frac{\mu_3 \times \pi_b (1-\lambda)}{\mu_3 \times \pi_b (1-\lambda) + \mu_5 \times \lambda} p_s^{U*} + \frac{\mu_5 \times \lambda}{\mu_3 \times \pi_b (1-\lambda) + \mu_5 \times \lambda} p_b^{C*}, \tag{72}$$

where μ_3^*, μ_5^*, and μ_7^* are the stationary probabilities derived in Appendix B.3 when $\Sigma^* = \Sigma^{ICS} = (1,0,0,1)$, with $\mu_3^* = \frac{\lambda(1-\pi_b)}{(1+\pi_b)(1-\pi_b\lambda)}$, $\mu_5^* = \frac{\pi_b(1-\lambda)^2}{(1+\pi_b)(1-\pi_b\lambda)}$, and $\mu_7^* = \frac{\pi_b\lambda(1-\lambda)}{(1+\pi_b)(1-\pi_b\lambda)}$. Using these expressions and the fact that $p_b^{C*} = p_b^{U*}$ in an ICS equilibrium, we find that $\overline{p}^{UU} = \overline{p}^{UC}$. Moreover, $\overline{p}^{UU} < p_s^{U*} < p^{co*}$. Thus, $M^{UU} = M^{UC} < 0$.

Part 2: The proof for the case $z_0^{pe} < 0$ (i.e., $\alpha^{pe} < \frac{1}{2}$) is symmetric and omitted for brevity.

A.3.9 Proof of Corollary 2

Follows directly from Lemma 1 (core market price) and Proposition 2 (peripheral market prices).

A.3.10 Proof of Corollary 3

Differentiating (28) with respect to z_0^{co} and using the fact that $z^* = \Delta^* + z_0^{co}$, we have:

$$\frac{\partial Spread}{\partial z_0^{co}} = \begin{cases} -\frac{\Phi'(-\kappa^{-1}z^*)}{\kappa}\frac{1-\pi_b}{1-\pi_b\lambda}(C^b + C^s) < 0 & \text{if } z_0^{pe} > 0, \\ \frac{\Phi'(-\kappa^{-1}z^*)}{\kappa}\frac{1-\pi_s}{1-\pi_s\lambda}(C^b + C^s) > 0 & \text{if } z_0^{pe} < 0. \end{cases} \tag{73}$$

A.3.11 Proof of Corollary 4

The expression for $\Delta(\alpha^{pe}, \lambda, \Sigma^{ACD})$ is given in Lemma 4. Using this expression, tedious calculations (see Section B.5 in the Online Appendix for details) show that the absolute value of $\Delta(\alpha^{pe}, \lambda, \Sigma^{ACD})$ decreases in λ. As $\Delta(\alpha^{pe}, \lambda, \Sigma^{ACD}) > 0$ when $\alpha^{pe} > 1/2$ and $\Delta(\alpha^{pe}, \lambda, \Sigma^{ACD}) \leq 0$ when $\alpha^{pe} \leq 1/2$, we deduce that $\Delta(\alpha^{pe}, \lambda, \Sigma^{ACD})$ decreases with λ when $\alpha^{pe} > 1/2$ and increases λ when $\alpha^{pe} \leq 1/2$. Then, Corollary 4 follows directly from (29).

A.3.12 Proof of Corollary 5

Differentiating (28) with respect to λ, we have:

$$\frac{\partial Spread}{\partial \lambda} = \begin{cases} \left[\Phi(-\kappa^{-1}z^*)\frac{\pi_b}{1-\pi_b\lambda} - \Phi'(-\kappa^{-1}z^*)\frac{\partial \Delta^*}{\partial \lambda}\frac{1}{\kappa}\right]\frac{1-\pi_b}{1-\pi_b\lambda}(C^b + C^s) & \text{if } z_0^{pe} > 0, \\ \left[(1-\Phi(-\kappa^{-1}z^*))\frac{\pi_s}{1-\pi_s\lambda} + \Phi'(-\kappa^{-1}z^*)\frac{\partial \Delta^*}{\partial \lambda}\frac{1}{\kappa}\right]\frac{1-\pi_s}{1-\pi_s\lambda}(C^b + C^s) & \text{if } z_0^{pe} < 0. \end{cases} \tag{74}$$

Remember that the sign of Δ^* is the same as the sign of z^{pe} in absolute value (see Lemma 4) and that $|\Delta^*|$ decreases with λ in the ACD equilibrium. We deduce from (74) that the dispersion of prices increases when more dealers become unconnected to core dealers.

A.3.13 Proof of Proposition 4

See the Online Appendix B.6.

A.3.14 Proof of Corollary 6

In the ACD regime, unconnected peripheral dealers on the uncrowded side of the market find a counterparty with probability 1. Indeed, either they receive an offer from a dealer with an opposite trading need that they accept with probability 1 or they make offers which are accepted with probability 1. Thus, they do not hold inventories after trading. In contrast, in the ICB or ICS regimes, unconnected peripheral dealers on the uncrowded side of the market make offers that are only accepted by unconnected dealers on the crowded side. Hence, the likelihood that unconnected peripheral dealers on the uncrowded side of the market hold inventories after the trading session is higher in the ICS or ICB regimes than in the ACD regime. Now, the ICS or ICB obtain when core dealers' aggregate position is large with a sign opposite to that of peripheral dealers' aggregate inventory (Proposition 1). This proves Corollary 6.

A.3.15 Extension

To analyze how inefficiencies in the interdealer market affect trading costs for end-investors (dealers' clients), we add to the baseline model a prior stage at date 0, in which dealers

trade with their clients. Prior to this stage, dealers have a zero position. Hence, dealers' initial positions (z_{i0}) stem from their trades with clients.

We assume that, at date 0, there is a mass α^{pe} of investors who want to sell the asset to peripheral dealers, and a mass $(1-\alpha^{pe})$ who want to buy the asset from peripheral dealers. Similarly, there are masses $\kappa\alpha^{co}$ and $\kappa(1-\alpha^{co})$ of investors who want to sell the asset to or buy the asset from core dealers, respectively. Each client is randomly matched to only one dealer. Buyers have a private valuation $+L$ for the asset, while sellers have a private valuation $-L$, with $L > \max(C_s, C_b)$. Thus, as dealers start with a zero position in the asset, gains from trade always exist between dealers and clients. Each client knows whether the dealer with whom he is matched is connected or not, and the price charged to the client is determined by Nash bargaining, with $\beta \in (0,1)$ being the client's bargaining power.[30]

Dealers' Bid-ask Spreads. We first derive the ask and bid prices at which peripheral dealers trade with their clients. Let $\mathbb{E}(\Pi_k^i)$ be the expected payoff (as defined in (3)) of a peripheral dealer with type $(i,k) \in \{C,U\} \times \{b,s\}$, and denote ask^i, bid^i the ask and bid prices charged by a dealer of type i. Consider a client who wants to buy the asset. The client's expected payoff is nil if does not trade and $(L - ask^i)$ if he does (remember that the expected payoff of the asset is normalized to zero). If the trade takes place, the dealer sells the asset and therefore becomes a buyer in the interdealer market. Thus, the dealer's expected payoff is $(ask^i + \mathbb{E}(\Pi_b^i))$ if he sells the asset to his client and is nil otherwise. Thus, Nash bargaining between the dealer and his client yields the following ask price:

$$ask^i = L - \beta[\mathbb{E}(\Pi_b^i) + L]. \tag{75}$$

If the dealer's client wants to sell the asset, a similar reasoning implies that the dealer's bid price is:

$$bid^i = -L + \beta[\mathbb{E}(\Pi_s^i) + L]. \tag{76}$$

[30]Search-and-matching models of trades in OTC markets have focused on the trading process between dealers and their clients. In these models, prices between dealers and clients are set by Nash bargaining, as we assume here, but they usually allow for a more complex matching process between dealers and clients. We deliberately keep this process as simple as possible to directly analyze how inefficiencies in the interdealer market affect trading costs for clients.

It is easily checked that the dealer and his client are always better off trading at these prices than not trading because $L > \max(C_s, C_b)$. Thus, the proportion of sellers among peripheral dealers in the interdealer market is α^{pe}, as assumed in the baseline model.

To obtain values of dealers' bid and ask quotes, we compute the expected payoff ($\mathbb{E}(\Pi_k^i)$) of all types of dealers in the interdealer market. We do so by computing the *total* expected payoff of each category of dealer, and then divide by the number of dealers in each category to obtain the expected payoff per capita.

$$\mathbb{E}(\Pi_s^C) = \frac{\mu_1[\varphi_s(p_s^C)p_s^C + (1-\varphi_s(p_s^C))p^{co}] + \mu_7\theta_b(1-\lambda)\pi_s p_b^U}{\alpha^{pe}(1-\lambda)}, \tag{77}$$

$$\mathbb{E}(\Pi_s^U) = \frac{\mu_3[\varphi_s(p_s^U)p_s^U - (1-\varphi_s(p_s^U))C^s] + \mu_5\gamma_b\lambda\pi_s p_b^C + \mu_7\lambda\pi_s p_b^U}{\alpha^{pe}\lambda}, \tag{78}$$

$$\mathbb{E}(\Pi_b^C) = -\frac{\mu_5[\varphi_b(p_b^C)p_b^C + (1-\varphi_b(p_b^C))p^{co}] + \mu_3\theta_s(1-\lambda)\pi_b p_s^U}{(1-\alpha^{pe})(1-\lambda)}, \tag{79}$$

$$\mathbb{E}(\Pi_b^U) = -\frac{\mu_7[\varphi_b(p_b^U)p_b^U + (1-\varphi_b(p_b^U))C^b] + \mu_1\gamma_s\lambda\pi_b p_s^C + \mu_3\lambda\pi_b p_s^U}{(1-\alpha^{pe})\lambda}, \tag{80}$$

where the μs' are the stationary probabilities derived in the Online Appendix (Section B.3).

Now we derive the ask and bid prices at which core dealers trade with their clients. The analysis is slightly different than that for peripheral dealers because core dealers' expected payoff in the interdealer market is different from zero even if they do not trade with their clients (since they can be hit by an inventory shock between $t = 2$ and $t = 3$). Let $\Pi^{co}(z_{i0})$ be a core dealer's equilibrium expected payoff if he has a position z_{i0} at date 0. After trading, each core dealer has the same position equilibrium, equal to $z_{i0} + q_{i2}^{co*}(p^{co*}, z_{i0}) = z_0^{co} = 2\alpha^{co} - 1$, where the second equality comes from the expressions for q_{i2}^{co*} and p^{co*} derived in Section 4.1. Thus, core dealers final position at date 3 is $z_{i3} = (2\alpha^{co} - 1) + \epsilon_i$. Replacing z_{i3} by this expression in the core dealers' expected payoff (5), we obtain:

$$\begin{aligned}\Pi^{co}(z_{i0}) &= z_{i0}p^{co*} + (2\alpha^{co}-1)[p^{co*} + \Pr(\epsilon_i > 2\alpha^{co}-1)C^s - \Pr(\epsilon_i < 2\alpha^{co}-1)C^b] \\ &+ \Pr(\epsilon_i < 2\alpha^{co}-1)C^b\mathbb{E}[\epsilon_i|\epsilon_i < 2\alpha^{co}-1]C^b - \Pr(\epsilon_i > 2\alpha^{co}-1)\mathbb{E}[\epsilon_i|\epsilon_i > 2\alpha^{co}-1]C^s. \end{aligned} \tag{81}$$

For brevity, we denote by (i) $\Pi_s^{co} = \Pi^{co}(-1), (ii)\Pi_b^{co} = \Pi^{co}(+1), (iii)\Pi_0^{co} = \Pi^{co}(0)$, a core dealer's expected payoff in equilibrium when he respectively, (i) buys the asset from a client, (ii) sells the asset to a client, (iii) does not trade with a client. Thus, the surplus of a core dealer when he trades with a client is (i) $(\Pi_s^{co} - \Pi_0^{co})$ if he buys the asset from his

client and (ii) $(\Pi_b^{co} - \Pi_0^{co})$ if he sells the asset to a client. Thus, Nash bargaining between a core dealer and his client yields the following ask and bid prices:

$$ask^{co} = L - \beta[\Pi_b^{co} - \Pi_0^{co} + L] \tag{82}$$

$$bid^{co} = -L + \beta[\Pi_s^{co} - \Pi_0^{co} + L]. \tag{83}$$

We can now use the expressions of ask^C, ask^U, ask^{co}, bid^C, bid^U, and bid^{co} to compute the equilibrium half bid-ask spread, denoted S^i, charged by each type of dealer to his clients. We obtain:

$$S^C = \frac{ask^C - bid^C}{2} = L(1-\beta) - \beta\frac{\mathbb{E}(\Pi_b^C) + \mathbb{E}(\Pi_s^C)}{2} \tag{84}$$

$$S^U = \frac{ask^U - bid^{CU}}{2} = L(1-\beta) - \beta\frac{\mathbb{E}(\Pi_b^U) + \mathbb{E}(\Pi_s^U)}{2} \tag{85}$$

$$S^{co} = \frac{ask^{co} - bid^{co}}{2} = L(1-\beta) - \beta\frac{\Pi_b^{co} + \Pi_s^{co} - 2\Pi_0^{co}}{2} = L(1-\beta), \tag{86}$$

where the last equality follows from the fact that $\Pi_b^{co} + \Pi_s^{co} = 2\Pi_0^{co}$ (see (81)). We use these expressions for computing numerically the average bid-ask spread charged by various types of dealers across various realizations of α^{co} and α^{pe}. Results from these simulations are reported in Figure 7.

Now we prove that, as mentioned in the text, we have $S^C \leq S^{co} \leq S^U$ which, using (84)-(86), is equivalent to $\mathbb{E}(\Pi_s^C) + \mathbb{E}(\Pi_b^C) \geq 0 \geq \mathbb{E}(\Pi_s^U) + \mathbb{E}(\Pi_b^U)$. For the first inequality, using (77) and (79) it is easy to show that $\mathbb{E}(\Pi_s^C) \geq p^{co}$ and $\mathbb{E}(\Pi_b^C) \geq -p^{co}$, so that indeed $\mathbb{E}(\Pi_s^C) + \mathbb{E}(\Pi_b^C) \geq 0$.

For the second inequality, suppose that $\alpha^{pe} \geq 1/2$ (the analysis for $\alpha^{pe} < 1/2$ is similar and omitted for brevity). In this case, the equilibrium regime is either ACD or ICS. If the equilibrium is ACD, we obtain using (78) and (80) (and the equilibrium values of the stationary probabilities and φ_k appearing in these equations) that $\mathbb{E}(\Pi_s^U) + \mathbb{E}(\Pi_b^U) = -(C^s + p^{co}) \times (a_1/b_1)$, with:

$$a_1 = (1-\pi_b)[1 - \lambda\pi_b[3 - \pi_b(2 + \lambda - \pi_b(1 + \lambda(1+\lambda)(1-\lambda)^2) + \lambda\pi_b^2(1-\lambda)^2)]] \tag{87}$$

$$b_1 = (1-\lambda\pi_b)[1 - \lambda\pi_b(2 - \lambda - \lambda\pi_b(1-\lambda)^2)]. \tag{88}$$

Simple manipulations, omitted for brevity, show that a_1 and b_1 are both positive, so that indeed $\mathbb{E}(\Pi_s^U) + \mathbb{E}(\Pi_b^U) \leq 0$.

If the equilibrium is ICS, (78) and (80) yield:

$$\mathbb{E}(\Pi_s^C) + \mathbb{E}(\Pi_b^C) = -\frac{a_2[C^b - p^{co}] + b_2[C^s + p^{co}]}{(1-\lambda\pi_b)^2} \tag{89}$$

$$\text{with} \qquad a_2 = (1-\lambda)[1 - \lambda(2 - \pi_b(1 + \lambda - \pi_b))] \tag{90}$$

$$b_2 = (1-\pi_b)[1 - \lambda\pi_b(3 + \pi_b^2 - (2+\lambda)\pi_b)]. \tag{91}$$

Again, it is easily shown that a_2 and b_2 are both positive, so that $\mathbb{E}(\Pi_s^U) + \mathbb{E}(\Pi_b^U) \leq 0$.

Clients average trading costs. We also consider the difference between the average ask price and the average bid price charged to the clients of peripheral dealers, denoted ATC^{pe}, as a measure of effective trading costs for these clients (where the average is taken by weighting each quote by the fraction of clients trading at this quote). We obtain:

$$ATC^{pe} = (1-\lambda)(1-\alpha^{pe})ask^C - (1-\lambda)\alpha^{pe}bid^C + \lambda(1-\alpha^{pe})ask^U - \lambda\alpha^{pe}bid^U \tag{92}$$

$$= L(1-\beta) - \beta[(1-\lambda)(1-\alpha^{pe})\mathbb{E}(\Pi_b^C) + (1-\lambda)\alpha^{pe}\mathbb{E}(\Pi_s^C) + (1-\alpha^{pe})\lambda\mathbb{E}(\Pi_b^U) + \alpha^{pe}\lambda\mathbb{E}(\Pi_s^U)] \tag{93}$$

$$= L(1-\beta) - \beta W(\Sigma^*). \tag{94}$$

where the last equality follows from the fact that $(1-\lambda)(1-\alpha^{pe})\mathbb{E}(\Pi_b^C) + (1-\lambda)\alpha^{pe}\mathbb{E}(\Pi_s^C) + (1-\alpha^{pe})\lambda\mathbb{E}(\Pi_b^U) + \alpha^{pe}\lambda\mathbb{E}(\Pi_s^U)$ is the expected payoff of a peripheral dealer before observing his type. Thus, it must be equal to gains from trade among dealers, i.e., $W(\Sigma^*)$.

For core dealers, using (81)-(83) we have:

$$ATC^{co} = (1-\alpha^{co})ask^{co} - \alpha^{co}bid^{co} = L(1-\beta) - \beta p^{co}[2\alpha^{co} - 1]. \tag{95}$$

We use these expressions for computing numerically the average trading costs paid by clients across various realizations of α^{co} and α^{pe} when dealers use equilibrium strategy profiles (blue line of Fig. 8). The expressions for ATC^{pe} and ATC^{co} when peripheral dealers follow the efficient strategy profile are similar, except that $W(\Sigma^*)$ is replaced by $W(\Sigma^{FB-})$ if $\alpha^{pe} < 1/2$ or $W(\Sigma^{FB+})$ if $\alpha^{pe} > 1/2$. Expressions for $W(\Sigma^{FB-})$ and $W(\Sigma^{FB+})$ are given in Section B.6 of the Online Appendix and are used to derive the average trading costs when peripheral dealers follow the efficient strategy profile (red line in Figure 8).

References

Adrian, T., N. Boyarchenko, and O. Shachar (2017): "Dealer balance sheets and bond liquidity provision," *Journal of Monetary Economics*, 89, 92 – 109.

Afonso, G., and R. Lagos (2015): "Trade Dynamics in the Market for Federal Funds," *Econometrica*, 83(1), 263–313.

Atkeson, A., A. Eisfeldt, and P.-O. Weill (2015): "Entry and Exit in OTC Derivatives Markets," *Econometrica*, 83, 2231–2292.

Babus, A., and C. Parlatore (2018): "Strategic Fragmented Markets," Working paper.

Bessembinder, H., W. Maxwell, S. Jacobsen, and K. Venkataraman (2018): "Capital Commitment and Illiquidity in Corporate Bonds," *Journal of Finance*, Forthcoming.

Biais, B. (1993): "Price Formation and Equilibrium Liquidity in Fragmented and Centralized Markets," *The Journal of Finance*, 48, 157–185.

Caballero, R. J., and A. Simsek (2013): "Fire sales in a model of complexity," *The Journal of Finance*, 68(6), 2549–2587.

Chang, B., and S. Zhang (2016): "Endogenous Market Making and Network Formation," Working paper.

Chordia, T., R. Roll, and A. Subrahmanyam (2002): "Order imbalance, liquidity, and market returns," *Journal of Financial Economics*, 65(1), 111–130.

Chordia, T., and A. Subrahmanyam (2004): "Order imbalance and individual stock returns: Theory and evidence," *Journal of Financial Economics*, 72(3), 485–518.

Colliard, J.-E., and G. Demange (2017): "Cash Providers: Asset Dissemination over Intermediation Chains," Working paper.

Collin-Dufresne, P., B. Junge, and A. Trolle (2016): "Market Structure and Transaction Costs of Index CDSs," Working paper.

Comerton-Forde, C., T. Hendershott, C. M. Jones, P. C. Moulton, and M. S. Seasholes (2010): "Time Variation in Liquidity: The Role of Market-Maker Inventories and Revenues," *The Journal of Finance*, 65(1), 295–331.

Di Maggio, M., A. Kermani, and Z. Song (2017): "The value of trading relations in turbulent times," *Journal of Financial Economics*, 124(2), 266 – 284.

Duffie, D. (2012a): *Dark Markets: Asset Pricing and Information Transmission in Over-the-Counter Markets*. Princeton University Press, Princeton.

——— (2012b): "Market Making Under the Proposed Volcker Rule," Working paper.

Duffie, D., N. Garleanu, and L. H. Pedersen (2005): "Over-the-Counter Markets," *Econometrica*, 73(6), 1815–1847.

Dunne, P. G., H. Hau, and M. J. Moore (2015): "Dealer Intermediation Between Markets," *Journal of the European Economic Association*, 13(5), 770–804.

Farboodi, M., G. Jarosh, and R. Shimer (2017): "The emergence of market structure," Working paper.

Fleming, M. (2007): "The round the clock market for U.S. treasury securities," *Federal Reserve Bank of New York, Economic Policy Review*, 3, 9–32.

Fleming, M., and J. Rosenberg (2008): "How do treasury dealers manage their positions," Working paper.

Friewald, N., and F. Nagler (2017): "Over-the-Counter Market Frictions and Yield Spread Changes," Working paper.

Grossman, S., and M. Miller (1988): "Liquidity and Market Structure," *The Journal of Finance*, 43(3), 617–633.

Hendershott, T., and A. J. Menkveld (2014): "Price pressures," *Journal of Financial Economics*, 114(3), 405 – 423.

Ho, T. S. Y., and H. R. Stoll (1983): "The Dynamics of Dealer Markets under Competition," *Journal of Finance*, 38(4), 1053–1074.

Hugonnier, J., B. Lester, and P.-O. Weill (2016): "Heterogeneity in decentralized markets," Working paper.

Lagos, R., and G. Rocheteau (2009): "Liquidity in Asset Markets With Search Frictions," *Econometrica*, 77(2), 403–426.

Lester, B., G. Rocheteau, and P.-O. Weill (2015): "Competing for order flow in OTC markets," *Journal of Money, Credit and Banking*, 47(S2), 77–126.

Li, D., and N. Schuerhoff (2017): "Dealer Networks," Working paper.

Madhavan, A., and S. Smidt (1993): "An Analysis of Changes in Specialist Inventories and Quotations," *The Journal of Finance*, 48(5), 1595–1628.

Neklyudov, A. (2014): "Bid-Ask Spreads and the Over-the-Counter Interdealer Markets: Core and Peripheral Dealers," Working paper.

Neklyudov, A., and B. Sambalaibat (2017): "Endogenous specialization and dealer networks," Working paper.

Randall, O. (2015a): "How Inventory Costs Affect Dealer Behavior in the US Corporate Bond Market," Working paper.

——— (2015b): "Pricing and Liquidity in Over-the-Counter Markets," Working paper.

Reiss, P., and I. Werner (1998): "Does Risk Sharing Motivate Interdealer Trading?," *The Journal of Finance*, 53(5), 1657–1703.

Schultz, P. (2012): "The market for new issues of municipal bonds: the roles of transparency and limited access to retail investors," *Journal of Financial Economics*, 106, 492–512.

——— (2017): "Inventory management by corporate bond dealers," *Working paper*.

Stoll, H. R. (1978): "The Supply of Dealer Services in Securities Markets," *The Journal of Finance*, 33(4), 1133–1151.

Wang, C. (2017): "Core-periphery trading networks," Working paper.