Contracting on Credit Ratings: Adding Value to Public Information*

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Abstract

We provide a novel interpretation of the role of credit ratings when contracts between investors and portfolio managers are incomplete. In our model, a credit rating on a bond provides a verifiable signal about an unverifiable state. We show that the rating will be contracted on only if it is sufficiently precise. Moderately precise ratings lead to wage contracts, and highly precise ones to contracts which directly restrict managers actions. In a market-wide equilibrium, surplus in the investor-manager transaction may decline when ratings become more precise. The widespread of use of credit ratings leads to excess volatility in bond returns.

1 Introduction

"For almost a century, credit rating agencies have been providing opinions on the creditworthiness of issuers of securities and their financial obligations."

Annette L. Nazareth; Director, U.S. SEC; Congressional testimony, April 2, 2003.

"Unlike other types of opinions, such as, for example, those provided by doctors or lawyers, credit rating opinions are not intended to be a prognosis or recommendation."

"What Credit Ratings Are & Are Not," Standard & Poor's web site.

Credit rating agencies and regulators routinely describe ratings as opinions, not sources of proprietary information. However, much of the academic literature characterizes ratings as an informative signal about the underlying security. Further, market participants use and react to credit ratings. The latter is especially surprising in cases such as sovereign bonds or insured municipal bonds, for which it is difficult to claim that the rating agency possesses information not already known to market participants. Yet, market prices on such bonds also react to rating changes.[1]

In this paper, we posit a novel explanation for the existence of this (seemingly) redundant information aggregation and reporting: When contracts are incomplete, the use of ratings allows market participants to write better contracts. We develop the implications of this idea in the context of delegated portfolio management. Our aim is threefold: first, to examine how credit ratings should be used in contracting between an investor and portfolio manager; second, to explore the equilibrium effects of the widespread use of ratings on bond returns; and

[1] For example, Brooks, et al. (2004) find that downgrades of sovereign debt adversely affect both the level of the domestic stock market and the exchange rate for the country's currency.

third, to understand the implications for market observables and policy of the "contracting view" as opposed to the "information view" of credit ratings.

Our model features a delegated portfolio management sector containing a continuum of investor-manager pairs. Each investor hires a manager to invest her portfolio. There are two states, high and low, and two feasible actions: hold a risky bond or a riskless asset. The investor prefers to hold the risky bond in the high state and the riskless bond in the low state. The action preferred by the manager depends both on the offered contract and on the realization of a stochastic private benefit. In contrast to many contracting frameworks, the size of the private perquisites the manager can extract are unrealized (and so unknown to both parties) at the time the contract is written. The potential inefficiency is that, due to these private benefits, the investor and manager may end up preferring different state-contingent actions. The state is not directly verifiable, so contracts are incomplete. In this setting, we interpret a credit rating as a verifiable, and therefore contractible, signal about the state. The precision of the signal captures the accuracy of the rating.

The first step in our analysis is to examine an optimal contract between one investor and one manager. An investor offers a contract that has two components. First, the manager is paid a compensation or wage based on both the portfolio return he delivers and the credit rating of the risky bond. Second, in contrast to a standard moral hazard model, the investor can also restrict the manager's action ex ante.

In our model, there is no conflict of interest between an investor and a manager in the high state. Correspondingly, if the rating on the risky bond is good, it is optimal to set wages to zero and to let the manager have access to an unrestricted action set.[2] Conversely,

[2]A zero wage is just a normalization in our model, and corresponds to the income from the manager's outside option.

if the rating is bad, the optimal contract depends on how precise the rating is as a signal of the state. If the rating is informative but relatively imprecise, the optimal contract again features zero wages and an unrestricted action set. In other words, the optimal contract in this range does not depend on whether the rating is good or bad. With a moderately precise rating, a bad rating leads to the manager being offered a strictly positive wage for delivering either a high return or the riskless return. Finally, with a bad rating that is relatively precise, the manager is prohibited from investing in the risky bond. That is, in choosing between an ex ante restriction on actions versus ex post compensation based on outcomes, the former is preferred when the rating is sufficiently precise, and the latter when the rating is somewhat (but not too) noisy.

It is common for mutual funds, pension funds, and insurance companies to have internal restrictions and investment policies that require minimum credit ratings on investments, and to use credit ratings to rule out potential counterparties in some transactions.[3] A prominent example is CalPERS, which manages about $300 billion in assets and is the largest public pension fund in the US. The CalPERS Total Investment Fund Policy establishes minimum credit ratings for different kinds of bonds in the various investment programs, and the Global Fixed Income Program policy document states that the portfolio formed under the Credit Enhancement Program will maintain an average rating of single A or higher.[4] There is also indirect evidence that such policies have bite: Chen, et al. (2014) examine a 2005 re-labeling of which split-rated bonds were eligible for index inclusion by Lehman Brothers. As they

[3]See "Report on the Role and Function of Credit Rating Agencies in the Operation of the Securities Markets," Securities and Exchange Commission, 2003. Kisgen and Strahan (2010) describe some of the ways in which credit ratings are used in the economy.

[4]Both documents are available at
`https://www.calpers.ca.gov/page/investments/about-investment-office/policies`.

mention, the change had no effect on the regulatory treatment of the bonds. Nevertheless, there was a significant change in price for the affected bonds, which the authors attribute to non-regulatory practices in the asset management sector, such as contractual investment mandates.

Our model implies that with moderately noisy ratings, the manager's compensation depends on the rating of the risky bond. This part of the model may be thought of either as prescriptive (a portfolio manager's compensation should depend not only on the portfolio return, but also on the risk of the portfolio, and lower-rated bonds are more likely to default) or as a reduced-form way to model the idea that in determining future fund flows, investors take into account both the return and risk of the portfolio.

The key intuition behind the form of the optimal contract is that restricting the manager's actions is costly if the rating is imprecise. Suppose that the rating on the risky bond is bad, sometimes the state with nevertheless be high; prohibiting investment in the risky bond requires the investor to forgo the return she can earn in this scenario. The other option is to use wages to induce the manager to hold the risky bond in this circumstance. When the rating is precise, the prohibitive contract is preferred, when the rating is less precise, a wage contract is preferred.

The second step in our analysis is to examine the market-wide equilibrium implications of credit ratings. This is a fixed-point problem because the contract (and hence a manager's action) depends on the portfolio return, whereas the equilibrium return of the risky bond in turn depends on the collective actions of the managers in the portfolio management sector. In the overall equilibrium, we find that there is an additional range of rating precision in which some proportion of investors rely only on wages in the optimal contract, whereas another proportion prevents investment in a bond with a bad rating.

We next turn to a comparison between the contracting and the information views of credit ratings. Observe that both views imply that bond prices react to rating changes. However, the contracting view implies that rating changes should not affect measures of adverse selection in the market. The two views also differ in their effect on demand for portfolio management services. Specifically, if ratings contain new information about the security or issuer, their existence makes it easier for individuals to construct their own portfolios, which should lead to a reduction in delegation. By contrast, under the contracting view, credit ratings reduce the cost of writing contracts with managers, which should increase the demand for portfolio management services. Finally, in the information view, increasing ratings precision leads to more efficient investment, and therefore typically increases welfare. In contrast, we show that, in our framework, increased precision of ratings can lead to a lower surplus in the transaction between the investor and the manager, because the manager's payoff is reduced when he is prohibited from investing in the risky asset. Further, we find that the widespread use of ratings can lead to bond returns being volatile even when fundamentals are fixed, as long as the rating is a noisy signal of the state.

Our focus on the use of non-informative credit ratings to mitigate contracting frictions is novel. Other work on non-informative ratings includes Boot, Milbourn, and Schmeits (2006), who present a framework in which a firm's funding costs depend on the market's beliefs about the type of project being chosen. The credit rating agency, by providing a rating, allows infinitesimal investors to coordinate on particular beliefs when multiple equilibria are possible. Further, the credit watch procedure provides a mechanism to monitor the firm if it can improve the payoff of its project. Manso (2014) considers how a credit rating might have real effects, in a model with multiple equilibria and self-fulfilling beliefs. In his framework, changes in a firm's credit rating affects its ability to raise capital, which then reinforce the original rating.

Much of the work in the literature considers credit ratings that communicate new information about the firm to the market, as well as frictions in the rating process that lead to noisy or inflated ratings. Several papers comment on the downside of regulators or investors relying on ratings. For example, Opp, Opp and Harris (2013) illustrate how the use of ratings by regulators leads to rating inflation, and so may have pernicious effects. Kartasheva and Yilmaz (2013) consider the optimal precision of ratings, and find in their model that efficiency is enhanced by reducing the reliance of regulation on credit ratings. Donaldson and Piacentino (2013) consider an environment in which the first-best outcome can be achieved by contracts that do not rely on credit ratings, and show that investment mandates based on ratings lead to inefficiency.

Our work provides a counter-perspective by focusing on the positive role of ratings in contracts. To make our point more starkly, we consider an extreme scenario in which credit ratings communicate no new information about the asset or issuer. We also abstract away from frictions in the process of producing and reporting ratings.

Many such frictions have been pointed out in the literature, building on the work of Lizzeri (1999) on certification intermediaries. Frictions in the rating process include rating inflation by the credit rating agency in a desire to capture high fees (Fulghieri, Strobl, and Xia, 2014), the breakdown of reputation as a disciplining device when flow income from new transactions is high (Mathis, McAndrews, and Rochet, 2009), and various inefficiencies stemming from rating shopping (Skreta and Veldkamp, 2009; Bolton, Freixas, and Shapiro, 2012; and Sangiorgi and Spatt, 2013). Goldstein and Huang (2015) consider the effect of such frictions on firm investment, and show that the existence of informative ratings sometimes reduces social welfare. In our model, introducing frictions into the rating process will necessarily reduce the precision of the rating. However, if these frictions are not too severe, the optimal contract remains contingent on the rating.

The core of our framework is inspired by some aspects of the model of Aghion and Bolton (1992), who present an incomplete contracting model with a principal and an agent in which states are observable, but not verifiable. In our framework, the credit rating is a verifiable signal, potentially improving efficiency in the contracting relationship.

We also build on the large literature on optimal contracts in delegated portfolio management. Bhattacharya and Pfleiderer (1985) consider such a problem with asymmetric information and Stoughton (1993) models the moral hazard version in which the manager chooses the proportion to invest in a risky asset (so the action set is continuous). We focus on the use of an outside signal in the contract, and simplify the action space to be binary. In other work, Admati and Pfleiderer (1997) and Das and Sundaram (1998) consider the use of benchmark evaluation measures. In our setting, we assume that other investors' performance is not verifiable, ruling out the possibility of relative performance evaluation.

Starting with Dasgupta and Prat (2006), some papers have considered the effects of career concerns on the part of portfolio managers on financial market equilibrium. Dasgupta and Prat (2008) introduce the notion of a reputational premium that a risky bond must earn to compensate for the risk that manager will be fired when a bond defaults. Guerrieri and Kondor (2012) construct a general equilibrium model that endogenizes reputational concerns, and show that the reputational premium amplifies price volatility.

We introduce our model in Section 2. In Section 3, we demonstrate the optimal contract for a single investor-manager pair, holding the price of the risky bond as fixed for each state and credit rating. We then step back to exhibit the equilibrium effects of the contract in Section 4. We provide some implications of our findings in Section 5. All proofs appear in the appendix.

2 Model

The delegated portfolio management sector of an economy comprises a continuum of investors and a continuum of portfolio managers, each with mass one. There are two assets, a risky bond and a risk-free one. Investors and managers are randomly matched in pairs, and contract exclusively with each other. The investor–manager relationship continues over four periods, $t = 1, \ldots, 4$. Contracts are signed at time $t = 1$, information is released at time $t = 2$, trading occurs at time $t = 3$, and payoffs are realized at time $t = 4$.

At time $t = 1$, an investor offers a manager a contract that specifies both a feasible action set at the trading date $t = 3$, and compensation or a wage at the final date $t = 4$. For simplicity, we assume that each manager may either purchase one unit of the risky asset or one unit of the risk-free asset. We denote the action of investing in the risky asset by a_h, and denote purchasing the risk-free bond, a_ℓ. The contract specifies a wage at time $t = 4$, conditional on the portfolio outcome and on the credit rating for the risky bond. In addition, the investor's contract can restrict the actions that the manager may take. This captures the idea that credit ratings are often used to restrict a manager's investment set.

At time $t = 2$, three pieces of information become available to market participants. First, a state, which affects the payoff to holding the risky bond, is realized and observed. There are two possible payoff states, h and ℓ, which correspond to the attractiveness of the bond to the investor. In particular, the probability that the bond will default is higher in state ℓ than in state h. State h has probability ϕ.

Critically, even though both parties know the state, it is not verifiable, and so not directly contractible. However, a contractible signal σ is available, in the form of a credit rating on the risky bond. We do not model the source of the credit rating. However, the rating is correlated with the state. Specifically, the rating takes on one of two values, g or b, and is potentially informative, with $\text{Prob}(\sigma = g \mid s = h) = \text{Prob}(\sigma = b \mid s = \ell) = \psi \geq \frac{1}{2}$. Thus, if $\psi = \frac{1}{2}$, the rating is completely uninformative, which is equivalent to the investor and manager being able to contract only on the final value of the investment, and if $\psi = 1$, the rating is perfectly informative, which is equivalent to the investor and manager being able to contract directly on the state. We refer to ψ as the precision of the rating.

There is a conflict of interest between the manager and investor in state ℓ. The investor suffers a private disutility $\delta > 0$ from holding the risky bond in state ℓ. The risky bond has a higher default probability in state ℓ, and the disutility δ may be interpreted as a reduced-form way to capture risk aversion on the part of the investor. Alternatively, relative to the external traders in the market, the investor is at a disadvantage in securing favorable terms in a bankruptcy negotiation. In contrast, the manager obtains a private benefit m from holding the risky bond in state ℓ. The private benefit corresponds to either synergies with his other funds ("soft money") or side transfers that he obtains from a sell-side firm if he places the risky bond in an investor's portfolio. The private benefit is random, and is drawn from a uniform distribution with support $[0, M]$. The size of the private benefit is independent across managers, and the size for each manager is realized at time $t = 2$. As is customary, the private benefit is not verifiable, so cannot be part of the contract.

At time $t = 3$, each portfolio manager chooses an action from his own feasible set. Collectively, their actions determine the demand for the risky bond, and hence the return on the bond between times $t = 3$ and $t = 4$. In state s, let q^s_σ denote the demand when the credit rating is σ. Clearly, $q^s_\sigma \in [0, 1]$. Further, let $r^s(q)$ denote the market return on the asset in state s when the aggregate demand for the asset from the delegated portfolio management (DPM) sector is q. We assume that r^s is decreasing in q. That is, a larger demand leads to a higher price and so a lower return.[5] In choosing the contract to offer a hired manager (at $t = 2$), an investor has rational expectations about the returns to the risky bond under different scenarios. That is, she correctly anticipates $r^s(q^s_\sigma)$ for each $s = h, \ell$ and $\sigma = g, b$. The return to holding the riskless asset is r^f, regardless of state or signal on the risky bond.

Let $\bar{r}^s = r^s(0)$ be the maximal return to the risky asset in state s. This return is realized if the price of the risky asset is low; that is, the demand for the asset from the DPM sector is zero. Correspondingly, let $\underline{r}^s = r^s(1)$ be the minimal return to the risky asset in state s, obtained when its price is high; specifically, when all investors wish to buy the risky asset, so that the demand from the DPM sector is one. We restrict attention to the case that $\underline{r}^h > r^f > \bar{r}^\ell - \delta$. Under these conditions, an investor purchasing bonds directly would prefer to buy the risky bond in state h (when the reward to bearing its risk is high) and the riskless bond in state ℓ (when the reward to bearing the risk on the risky bond is low). Given the agency conflict, managers may sometimes take an inefficient action. Potentially, there are gains from renegotiation between the investor and manager at that time. For now,

[5] Implicitly, we assume that there are traders external to the delegated portfolio management sector that, in the aggregate, produce an upward-sloping supply curve for the asset in each state.

we assume that renegotiation is costly enough to be infeasible, and return to a discussion of renegotiation in Section 3.1.

To summarize: There are four dates in the model, $t = 1$ through 4. Figure 1 shows the sequence of events in the model. It is important to note that the contract is written before the state and credit rating are realized. We have in mind a situation in which contracts are written on a periodic basis (say once a year), whereas the state (which could reflect other aspects of the investors' portfolio) can change frequently, indeed rapidly, in between. The credit rating need not be known as soon as the state is revealed, but it must be known before the manager takes an action. The private benefit of the manager reflects the effect of market events on other assets held by the manager or other payments he receives from his relationships, so is known only when the state itself is revealed.

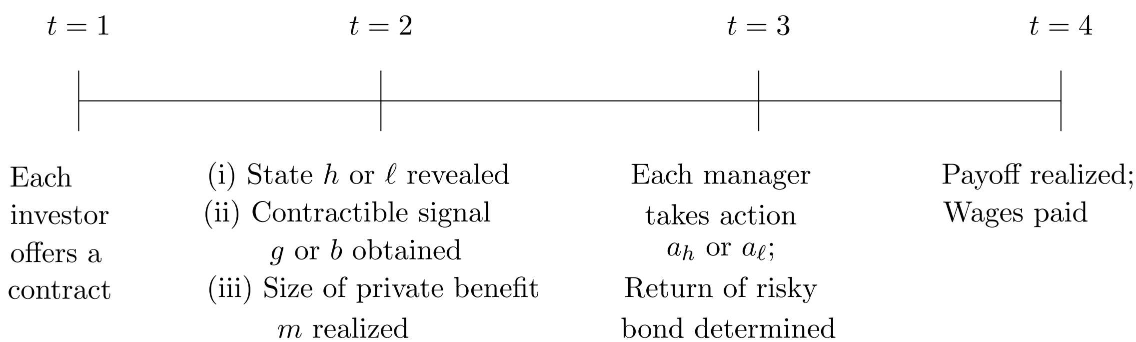


Figure 1: **Sequence of Events**

There is no discounting, and we model both parties as risk-neutral. The payoff to the investor from this relationship is the net return generated by the manager less the total compensation paid to the manager. The payoff to the manager is the sum of the wage and any private benefits he may garner. The manager enjoys limited liability that requires the wage in any state to be non-negative. His reservation utility is zero, so any contract that satisfies limited liability is also individually rational. When the outcome is realized at time 4, the manager is paid the wage specified by the contract signed at time 1, and the investor keeps any extra investment income.

We assume that the investor cannot directly invest in the risky bond on her own. Implicitly, the cost of direct investing is too high for her. This cost may be interpreted as either the opportunity cost of time for the investor or the direct cost of access to certain securities.[6] We also ignore an individual rationality constraint on the investor. That is, for now we assume that the payoff she obtains after contracting with the manager exceeds r^f, the payoff she could obtain if she invested in the riskless bond by herself. In Section 3, we show that the optimal contract satisfies this feature.

An equilibrium in this model has several components. First, each investor offers an optimal contract to the manager, anticipating the returns on the risky asset. The wage offered to the manager depends on both the rating and the return on the portfolio. In addition, we allow the investor to designate a specified action set for the manager, which depends on the rating. Second, each portfolio manager optimally decides whether to buy the risky bond or the riskless asset, given the state, credit rating, returns on the risky bond and his contract and his unique private benefit. Third, and finally, the market for the risky bond clears, which determines the return in each state and for each credit rating.

Formally, let $\mathbf{w} = \{w_\sigma^j\}_{\substack{\sigma=g,b \\ j=h,f,\ell}}$, $\mathbf{r} = \{r_\sigma^j\}_{\substack{\sigma=g,b, \\ j=h,f,\ell}}$, and $\mathbf{A} = \{A_g, A_b\}$ with $A_\sigma \subseteq \{a_h, a_\ell\}$ for each σ. A contract offered by investor i is denoted by $C_i = \{\mathbf{w}, \mathbf{A}\}_i$. Then,

Definition 1 *A market equilibrium in the model consists of:*

[6] For example, under SEC Rule 144A, only qualified institutional buyers may purchase certain private securities.

(a) An optimal contract $C_i = \{\mathbf{w}, \mathbf{A}\}_i$, offered by each investor i to her portfolio manager, where the wages $\mathbf{w}$ depend on the rating σ and the returns on the risky bond $\mathbf{r}$, and the feasible actions $\mathbf{A}$ depend on the rating σ.

(b) A payoff-maximizing action chosen by each portfolio manager i, given the returns on the risky bond $\mathbf{r}$, the state s, the credit rating σ, the contract C_i, and his private benefit m.

(c) Returns on the risky bond given state s and credit rating σ are determined by $r^s_\sigma = r^s(q^s_\sigma)$, where q^s_σ is the aggregate demand for the risky bond generated by portfolio managers in part (b).

An equilibrium is therefore a Nash equilibrium in contracts. Each investor offers an optimal contract given the returns on the risky bond, where the returns on the risky bond in turn depend on the contracts offered by all other investors. In that sense, each investor is offering an optimal contract given the contracts offered by all other investors.

We restrict the size of the maximal private benefit, M. M must be sufficiently large so that for some realizations of m, the agency conflict between investor and manager has bite. However, M is sufficiently small so that it is still effective to offer a wage contract to induce the manager to take the action preferred by the investor. Notice that $\underline{r}^h - r^f > M$ implies that $\underline{r}^h > r^f$ (as $M > 0$).

Assumption 1 $\frac{1-\phi}{\phi}(r^f - \underline{r}^\ell + \delta) \leq M < \underline{r}^h - r^f$.

3 Optimal Contract for a Single Investor-Manager Pair

As a first step, consider the optimal contract for a single investor-manager pair. After being randomly matched, the contract is entered into at time $t = 1$, before the state, credit rating and extent of the moral hazard problem (i.e., the size of the private benefit m) are known. The demand of each investor and each manager is infinitesimal, so they take as given the return on the risky asset for each possible state-rating pair.

Because all agents know the state when the manager takes the action, but cannot contract on it, the optimal contract depends on how precise the credit rating is. Define a threshold level of precision

$$\hat{\psi}(r_b^\ell) \quad = \quad \frac{1}{1 + \frac{(1-\phi)(r^f - r_b^\ell + \delta)}{\phi M}}. \tag{1}$$

Because each investor and manager treats r_b^ℓ as fixed, we suppress the dependence of $\hat{\psi}$ on r_b^ℓ in the notation for the rest of this section.

Our main result in this section is the structure of the optimal contract which is presented in Proposition 1. We state the result first, and build up the intuition in what follows.

Proposition 1 *The optimal contract for each investor is as follows.*

(i) If the rating is g, zero wages are offered and no action restriction is imposed.

(ii) If the rating is b, zero wages are offered when $\psi \leq \hat{\psi}$. Further, there exists a threshold rating precision $\psi_1 \in \left(\hat{\psi},\ 1\right)$ such that:

(a) If $\psi \in (\hat{\psi}, \psi_1)$, the contract relies only on wages, with no action restriction.

(b) If $\psi > \psi_1$, the contract prohibits investment in the risky asset, and offers zero wages.

To see the intuition behind Proposition 1, suppose first that the rating is good. If $\psi = \frac{1}{2}$, the rating is completely uninformative. It is immediate that the contract cannot depend on the rating in this case; that is, no wage is offered and no restriction is imposed on actions. When $\psi > \frac{1}{2}$, relative to prior beliefs, there is a greater likelihood that the state is high. As there is no conflict of interest between the investor and the manager in the high state, the investor has even less reason to pay the manager than when $\psi = \frac{1}{2}$. Therefore, a good rating leads to a contract with zero wages and no restriction on manager action.

Conversely, suppose the rating is bad. If ratings are relatively imprecise (less than $\hat{\psi}$), the contract remains one with zero wages and no restriction on manager action. In this region, investors do not use the rating in the contract. That is, neither the wages offered nor the permissible actions are contingent on the rating. However, if the rating is bad, as the rating becomes more precise (in particular, above $\hat{\psi}$, but below some threshold ψ_1), the investor chooses an optimal wage contract that does not restrict the manager's action. In this intermediate precision range, it is too costly to impose a restriction on action: When the state is high but the rating is bad (which can sometimes happen with imprecise ratings), forcing the manager to hold the risk-less asset entails giving up on the high return that can be obtained on the risky bond. Finally, as the rating becomes even more precise (above ψ_1), the investor prefers to restrict the manager's action when the rating is bad, rather than relying on wages to induce the right action. In particular, she bans the manager from investing in the risky bond and so chooses a prohibitive contract. The optimal contract conditional on a bad rating is illustrated in Figure 2 below.

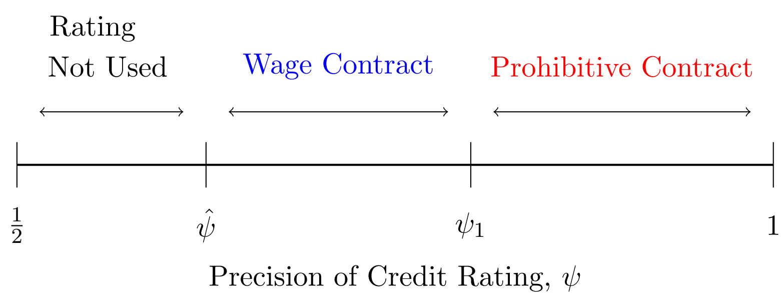


Figure 2: **Optimal Contract for a Single Investor, Given a Bad Rating**

In what follows, we first characterize the optimal prohibitive contract, followed by the optimal wage contract. Armed with these, we then compare an investor's payoff from each contract and determine the contract she will offer to the manager.

First, consider the prohibitive contract. Implicitly, we assume that the investor has a way to enforce a restriction on actions, either through a technological system, or perhaps due to a large reputational or legal penalty suffered by a manager who violates an imposed restriction. As any restriction on actions reduces the feasible action set to a singleton, it is immediate that no wages are offered.

Lemma 1 *Suppose an optimal contract restricts the manager's actions for some rating* σ*. Then, it must be that the wage offered is zero for that credit rating. Or,* $w_\sigma^j = 0$ *for each* $j = h, f, \ell$*.*

If the rating is not fully precise, the prohibitive contract may ban an action that is optimal. That is, sometimes the rating will be b even in state h, but the prohibitive contract prevents the manager from purchasing the risky asset. As an alternative, consider a contract in which there is no restriction on the manager's actions, so that $A_g = A_b = \{a_h, a_\ell\}$. In such a contract, the manager's action depends in part on the wages offered. We term this a wage contract.

In a wage contract, the investor writes a contract for the manager that depends on the return he delivers to the investor, in addition to the credit rating. A contract is therefore characterized by a payoff for each rating-state pair or $\mathbf{w} = \{w_g^h, w_g^\ell, w_g^f, w_b^h, w_b^\ell, w_b^f\}$, where w_σ^j denotes the compensation to the manager when the credit rating is $\sigma \in \{g, b\}$ and the portfolio return is r^j for $j \in \{h, \ell, f\}$.

At time 1, the investor chooses to offer the various wage levels, $\mathbf{w}$, to maximize her expected payoff, where π^h is her payoff when the state is h and π^ℓ is the payoff when the state is ℓ:

$$\Pi \quad = \quad \phi\pi^h + (1-\phi)\pi^\ell. \tag{2}$$

In the high state, h, there is no private benefit, and so the manager always takes the action that yields him the highest wage. Thus, he will take action a^h for a realization of a g credit rating if $w_g^h \geq w_g^f$, and for a realization of a b rating if $w_b^h \geq w_b^f$. Recall, in the h state, the credit rating is g with probability ψ and b with probability $1-\psi$. Further, if the investor induces the action a_h, her payoff is $r_\sigma^h - w_\sigma^h$; if she induces the action a_ℓ, her payoff is $r^f - w_\sigma^f$. Thus, the expected payoff of the investor in this state h is:

$$\begin{aligned} \pi^h \quad = \quad & \psi\left((r_g^h - w_g^h)1_{\{w_g^h \geq w_g^f\}} + (r^f - w_g^f)1_{\{w_g^h < w_g^f\}}\right) \\ & +(1-\psi)\left((r_b^h - w_b^h)1_{\{w_b^h \geq w_b^f\}} + (r^f - w_b^f)1_{\{w_b^h < w_b^f\}}\right), \end{aligned} \tag{3}$$

where $1_{\{x\}}$ is an indicator function that takes on the value of 1 if the event x occurs, and 0 otherwise.

Next, consider the low state, ℓ. The credit rating is g with probability $1-\psi$ and b with probability ψ. Given a signal σ, the manager invests in the riskless bond if $w_\sigma^f \geq w_\sigma^\ell + m$, or $m \leq w_\sigma^f - w_\sigma^\ell$. He buys the risky bond if $w_\sigma^f < w_\sigma^\ell + m$, or $m > w_\sigma^f - w_\sigma^\ell$. Of course, at the time the contract is established, neither party knows m, the size of the manager's private benefit. The investor therefore has to take expectations over the possible values it may take. Suppose for now that $w_\sigma^f - w_\sigma^\ell \in [0,1]$ (this is established in Lemmas 2 and 3 below). Then, recalling that m is uniform over $[0, M]$, the investor's expected payoff in the low state ℓ is

$$\begin{aligned}\pi^\ell &= (1-\psi)\left((r^f - w_g^f)\frac{w_g^f - w_g^\ell}{M} + (r_g^\ell - \delta - w_g^\ell)(1 - \frac{w_g^f - w_g^\ell}{M})\right) \\ &\quad +\psi\left((r^f - w_b^f)\frac{w_b^f - w_b^\ell}{M} + (r_b^\ell - \delta - w_b^\ell)(1 - \frac{w_b^f - w_b^\ell}{M})\right). \qquad (4)\end{aligned}$$

The manager's wage for investing in the risk-free asset (w_σ^f) affects incentive compatibility in both the high and low reward states, because the manager has the choice of investing in the risk-free asset in both states. To induce the manager to hold the risky asset when the state is h, the investor has to set the wage w_σ^h (earned when the portfolio return is r_σ^h) to at least w_σ^f. To minimize the cost of providing this incentive, the investor sets w_σ^h as low as possible; that is, equal to w_σ^f. In addition, we show that in an optimal wage contract, $w_\sigma^\ell = 0$. The investor does not want to hold the risky bond in state ℓ, so there is no reason to reward an manager who does so.

Lemma 2 *The optimal wage contract sets* $w_\sigma^h = w_\sigma^f$ *and* $w_\sigma^\ell = 0$ *for each credit rating*

$\sigma = g, b$.

Lemma 2 reduces the investor's problem of finding an optimal wage contract to two choice variables, w_g^f and w_b^f. Thus, the optimal contract is characterized by the compensation that the manager receives for investing in the risk-free asset, given the rating on the risky bond.

Broadly, the optimal wage contract involves no intervention when the rating is good, but rewards the manager for avoiding the risky bond in the low-return state ℓ when its credit rating is bad. If the signal embodied in the credit rating is sufficiently informative about the state (i.e., ψ is sufficiently high), the manager receives a positive wage w_b^f for buying the riskless asset when the risky bond has a low credit rating. He receives a zero wage for the same action when the risky bond has a good credit rating (i.e., $w_g^f = 0$). In other words, if the credit rating is sufficiently precise, the investor induces the manager to tilt toward the risky bond when it has a high credit rating and steer clear of the risky bond when it has a bad credit rating. Further, the wage w_b^f is capped at M, as it cannot be optimal to pay the manager more than his maximum private benefit.

Lemma 3 *In the optimal wage contract:*

(i) $w_g^f = 0$*, regardless of the rating precision* ψ*.*

(ii) w_b^f *depends on the rating precision* ψ*. Specifically,*

$$w_b^f = \begin{cases} \min\left\{\frac{1}{2}\left(r^f - r_b^\ell + \delta - \frac{\phi}{1-\phi}\,\frac{1-\psi}{\psi}\,M\right),\ M\right\} & \textit{if } \psi \geq \hat{\psi} \\ 0 & \textit{if } \psi < \hat{\psi}. \end{cases} \tag{5}$$

The optimal wage, when it is positive, trades off the investor's payoff across states. Specifically, increasing w^f_σ makes it more likely the manager takes the right action in the ℓ state. To see this, suppose the risky bond obtains a bad credit rating b. A higher wage w^f_b induces the manager to hold the riskless bond more often in the low state (i.e., for a larger set of private benefit realizations); this anti-shirking effect increases the investor's payoff. This is illustrated in Figure 3 below.

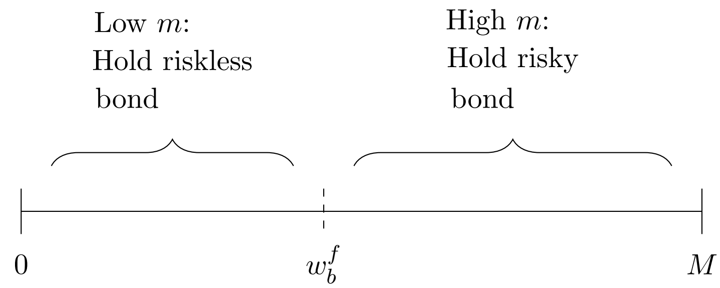


Figure 3: **Manager's action in low state when rating is bad**

There are two costs associated with increasing w^f_b. First, infra-marginal managers with low private benefits are paid more than they need to be. Second, in the high state, h, because of the incentive compatibility constraint ($w^h_b \geq w^f_b$), the investor has to pay the manager a higher amount to induce the manager to invest in the risky bond. The optimal wage w^f_b balances these two costs against the benefit of inducing more managers to take the right action in state ℓ.

The intuition for setting $w^f_g = 0$ is similar. On the one hand, in the low return state, ℓ, a positive w^f_g induces the manager to hold the riskless bond for a higher range of private benefit realizations. On the other, it requires the investor to increase w^h_g correspondingly, which lowers her payoff in the high return state, h. Under our assumptions, for a good rating, the incentive compatibility effect always dominates, so the investor sets w_g^f to zero.

It is clear from this discussion that the stochastic private benefit represents an important friction. If the highest value of the private benefit (i.e., M) is sufficiently high, even with a fully precise rating, the optimal contract does not always elicit the action preferred by the investor. Even if the investor could contract directly on the state when she offered a wage, she would prefer to let the manager sometimes deviate to the inefficient action in state ℓ (when the private benefit m is high enough), because by keeping w_b^f low, she sometimes obtains the efficient action at lower cost (when the private benefit m is low).

Finally, to complete the discussion of Proposition 1, we consider the range of signal precision over which the investor prefers the wage contract to the prohibitive contract, and vice versa. Let $\mu_b = \text{Prob}(s = h \mid \sigma = b) = \frac{\phi(1-\psi)}{\phi(1-\psi)+(1-\phi)\psi}$ be the probability the state is high given that the rating on the risky bond is b. From Lemma 2, the optimal wage contract satisfies $w_b^h = w_b^f$ and $w_b^\ell = 0$. The manager buys the risky bond in state h; in state ℓ she buys the risky bond if $m > w_b^f$ and the risk-less bond if $m \leq w_b^f$. Therefore, the payoff to the investor from using an optimal wage contract is

$$\Pi_{w,b} \quad = \quad \mu_b(r_b^h - w_b^f) + (1-\mu_b)\left[\frac{w_b^f}{M}(r^f - w_b^f) + (1 - \frac{w_b^f}{M})(r_b^\ell - \delta)\right], \tag{6}$$

where w_b^f is set as in Lemma 3, and $\frac{w_b^f}{M}$ represents the mass of managers with $m \leq w_b^f$.

If the investor bans the manager from investing in the risky asset, she offers zero wages (i.e., $w_b^h = w_b^f = w_b^\ell = 0$), her payoff is

$$\Pi_x \quad = \quad r^f, \tag{7}$$

since the wage is optimally set to zero. Equating these payoffs determines the ranges of rating precision defined in Proposition 1. The formal proof of the proposition, showing the optimality of the wage and prohibitive contracts in the respective ranges, is in the Appendix.

We note that by offering her manager this optimal contract, the investor is better off than if she invested her funds privately. A direct investor only has access to the risk-free asset, and earns r^f for sure. She can always earn this payoff by hiring a manager and offering the prohibitive contract that prevents the manager from buying the risky asset and pays a zero wage. When the rating is g, the optimal contract leaves the investor strictly better off, compared to buying the risk-free asset. The investor's individual rationality constraint is therefore satisfied.

3.1 Robustness: Renegotiation and Benchmarking

Renegotiation

Thus far, we have ignored the possibility of renegotiation between investor and manager, even though the manager is sometimes taking an inefficient action. In such cases, it is usual to consider renegotiation, which has the potential to increase the total surplus. Of course, renegotiation also affects how the surplus is split between the investor and the manager. In the delegated portfolio management problem, one suspects that renegotiation is infrequent. After all, an investor delegates her investment decisions because she does not want to monitor her portfolio closely. Nevertheless, in this section we argue that our results qualitatively survive when renegotiation is feasible, as long as either (i) renegotiation is sufficiently imperfect, or (ii) the manager has the bulk of the bargaining power at the renegotiation stage.

Suppose the state is ℓ and the credit rating is σ. Then, a manager with a private benefit in the range $(w^f_\sigma, r^f - r^\ell_\sigma + \delta)$ will take an inefficient action, by buying the risky bond when it would be efficient to hold the riskless asset. To incorporate the notion of renegotiation, consider the following amendment to the model. At time 3, given his contract and knowledge of the state and signal, the manager may renegotiate the contract. Suppose that renegotiation is costly, in the sense the opportunity to renegotiate is stochastic, and occurs with probability λ (so with probability $1-\lambda$, there is no renegotiation). We expect λ to be high, for example, if the manager is a private wealth manager, and negotiates separate contracts with each of his clients. Conversely, if the manager is a bond fund manager with dispersed investors all signing the same contract, λ will be zero.

Suppose further that, when renegotiation is feasible, the manager has all the bargaining power. The manager makes a take-it-or-leave-it offer to the investor that specifies both the action the manager will take and a new wage contract for the manager. If the investor accepts, the old contract is torn up and the new one holds. If the investor rejects, the old contract remains in force. Any gains to trade at the renegotiation stage are therefore captured by the manager.[7] After any possible renegotiation, the manager invests by taking action a_h or a_ℓ.

In such a set-up, the investor's payoff is not affected by the possibility of renegotiation, because the manager captures all gains from renegotiation. Thus, renegotiation has no effect on the optimal contract, on the investor's payoff, or on the decision to hire the manager. Of course, the payoff to the manager changes—the manager now earns not just what was promised in the contract at time 1, but also captures any extra surplus he can garner from renegotiation at time 3.

[7] Suppose, instead, we gave all bargaining power at the renegotiation stage to the investor. This would be equivalent to allowing the investor to write a contract *after* the state were known, going against the spirit of the idea that contracts are revised only at periodic intervals, whereas the state may change rapidly between contract revisions.

Now, instead of allocating all bargaining power to the manager, suppose that when renegotiation is feasible, with probability κ the manager has the right to make an offer, and with probability $1-\kappa$, the investor has the right to make an offer. In each case, the other party must take the offer or stick with the old contract. Then, with probability $\lambda(1-\kappa)$, the investor obtains an increased payoff at the renegotiation stage. If $\lambda(1-\kappa)=1$ (i.e., if renegotiation is perfect and the investor has all the bargaining power), the investor can effectively contract on state, rendering the credit rating irrelevant. Our base model assumes $\lambda=0$, and we argue above that when $\kappa=1$, the optimal contract remains the same as in Proposition 1. If $\lambda(1-\kappa)$ is strictly positive, but sufficiently low, the investor's payoff from both the wage contract and the prohibitive contract strictly increases. This changes the exact thresholds at which different contracts are optimal, but the same qualitative results are obtained.

Our results are therefore robust to the possibility of renegotiation, as long as either the manager has the bulk of the bargaining power, or renegotiation is sufficiently imperfect. Therefore, for simplicity, in the rest of the paper we assume there is no renegotiation.

Benchmarking

In the optimal wage contract, we do not allow for the possibility of benchmarking the contract to returns that may be earned by other investors in the market. Specifically, if the state is high and the manager delivers the riskfree return, the investor cannot penalize him because another investor in the market earned a high return. That is, we ignore the possibility of relative performance evaluation.

Our model represents a limiting case in which the return on the risky asset reveals the state. More generally, one can consider the scenario in which the return in each state is random, and the support of returns is the same in both states, with the risky bond being more likely to earn r^h in the good state and more likely to earn r^ℓ in the bad state. In the limit as the distribution over returns collapses to a single point, the return on the risky bond reveals the state. However, in the more general scenario, the return only results in a likelihood of the state being high or low, so that the state cannot directly be contracted on.

What effect would benchmarking have in our setting in the limit? We argue that our main result remains robust—the investor prefers a wage contract when the rating is less precise and the prohibitive contract when the rating is precise.

Observe that in a wage contract, allowing the investor to contract on the return on the risky bond (regardless of whether the manager actually held the risky bond) essentially allows the investor to contract directly on a state. Thus, the wage contract is no longer contingent on the credit rating. However, the prohibitive contract must continue to rely on the credit rating. Going back to the sequence of events in Figure 1, the prohibition has to be imposed before the action is taken, whereas benchmarking can only occur ex post (i.e., at time 4). Finally, notice that when the rating is sufficiently precise, the prohibitive contract must be optimal when the rating on the risky bond is bad, because the stochastic private benefit continues to represent a friction in the wage contract.

Going forward, we continue to assume there is no benchmarking. Effectively, we have in mind a scenario in which the manager constructs an individualized portfolio for the investor, and the investor does not know the feasible set of securities or the portfolios constructed for other investors.

4 Market Equilibrium

In a market equilibrium, the return on the risky asset in each state, and for each credit rating, is determined by the aggregate demand of all the portfolio managers. The aggregate demand is induced by the optimal contracts offered by each of the investors, and the realization of each manager's possible perquisites. This leads to a fixed point problem because, to characterize each individual contract, we made use of the fact that each investor is infinitesimal, and therefore takes the returns on the risky asset as given.

We begin with the following observation. Suppose that the proportion of principals who offer the wage contract is β, while a proportion $1-\beta$ offers the prohibitive contract. Fix β and let ψ, the rating precision vary. As ψ varies, the optimal wage in the wage contract will change, which in turn will affect r_b^ℓ. We show in Lemma 4 (stated and proved in the Appendix) that after taking into account all effects, the payoff to an investor from using the wage contract, $\Pi_{w,b}$, is strictly decreasing in ψ. This allows us to exhibit the overall market equilibrium in Proposition 2.

First, observe that $\hat{\psi}$, as defined in Equation (1), is increasing in r_b^ℓ, so is minimized when $r_b^\ell = \underline{r}^\ell$. Define $\underline{\psi} = \hat{\psi}(\underline{r}^\ell)$. Now, under Assumption 1, we have $M \geq \frac{1-\phi}{\phi}(r^f - \underline{r}^\ell + \delta)$, which implies that $\underline{\psi} \geq \frac{1}{2}$. If ψ remains below $\underline{\psi}$, the optimal contract offers zero wages and no restriction on actions. In other words, ratings do not play any role in the contract. As ψ increases beyond $\underline{\psi}$, all principals offer a wage contract over some range of ψ (so that $\beta = 1$). Over another range of ψ, the proportion β decreases continuously from 1 to 0, and when ratings become very precise, all principals offer the prohibitive contract (so that $\beta = 0$).

Proposition 2 *In a market equilibrium, for all values of ψ, the contracts offered by investors* *set $w_g^f = 0$ and have no restriction on actions if the rating is g. Further, there exist rating thresholds ψ_x and ψ_y, with $\underline{\psi} < \psi_x < \psi_y < 1$ such that, when the rating is b:*

(i) If $\psi \leq \underline{\psi}$, the contract offered by all investors has zero wages and no restriction on actions.

(ii) If $\psi \in (\underline{\psi}, \psi_x)$, the contract offered by all investors relies only on wages, and does not restrict the manager's action.

(iii) If $\psi \in (\psi_x, \psi_y)$, a mass of investors, $\beta(\psi) \in (0, 1)$, offer a contract that depends only on wages, with the remainder offering a contract that bans investment in a risky asset.

(iv) If $\psi > \psi_y$, the contract offered by all investors sets wages to zero and bans the manager from investing in the risky asset.

The market equilibrium, therefore, recovers some of the features of the single-investor problem. With a good rating, no wages are offered and no action restriction is imposed. With a bad rating, when the rating precision is low (below $\underline{\psi}$), all wages are also set to zero and there is no restriction on actions. In a low intermediate range (rating precision between $\underline{\psi}$ and ψ_x), all investors offer only a wage contract when the rating is b. Further, when the rating precision is very high (above ψ_y), the contract prohibits investment in a risky asset with a bad rating.

However, in contrast to Proposition 1, there is one additional region which features a mix of contracts. For precisions between ψ_x and ψ_y, a fraction $\beta(\psi)$ of investors offer a wage contract and a fraction $1 - \beta(\psi)$ offer a prohibitive contract. This region arises because if an investor offers a prohibitive contract, the demand for the risky asset is lower than with a wage contract. The return (r_b^ℓ) is therefore higher. The fraction $\beta(\psi)$ decreases as ψ increases, so that when the rating precision increases to ψ_y, in equilibrium all investors offer a prohibitive contract.

Next, consider the payoffs of the investor and the manager. Noting that over the region $[\psi_x, \psi_y]$ the investor is indifferent between offering a wage contract and a prohibitive contract, the investor's expected payoff from an optimal contract is

$$\Pi = \begin{cases} \phi\left[\psi r_g^h + (1-\psi)(r_b^h - w_b^f)\right] & \\ +(1-\phi)\left[(1-\psi)(r_g^\ell - \delta) + \psi\left(\frac{w_b^f}{M}(r^f - w_b^f) + (1-\frac{w_b^f}{M})(r_b^\ell - \delta)\right)\right] & \text{if } \psi \in [\underline{\psi}, \psi_x] \\ \phi(\psi r_g^h + (1-\psi)r^f) + (1-\phi)((1-\psi)(r_g^\ell - \delta) + \psi r^f) & \text{if } \psi \geq \psi_x. \end{cases}$$

The expected payoff to the manager in a wage contract is

$$\Gamma_w = \phi(1-\psi)w_b^f + (1-\phi)\left\{(1-\psi)\frac{M}{2} + \psi\left(\frac{(w_b^f)^2}{M} + \left(1 - \frac{w_b^f}{M}\right)\frac{w_b^f + M}{2}\right)\right\}, \quad (8)$$

resulting in an overall expected payoff to the manager of

$$\Gamma = \begin{cases} \Gamma_w & \text{if } \psi \in [\underline{\psi}, \psi_x] \\ \beta(\psi)\Gamma_w + (1-\beta(\psi))(1-\phi)(1-\psi)\frac{M}{2} & \text{if } \psi \in (\psi_x, \psi_y) \\ (1-\phi)(1-\psi)\frac{M}{2} & \text{if } \psi \geq \psi_y. \end{cases} \quad (9)$$

We show that, as the precision of the ratings increases, the payoff to an investor unambiguously increases. At low levels of precision (just above $\underline{\psi}$), the payoff to the manager also increases with ψ. However, as precision increases further and the prohibitive contract is used, the manager's payoff decreases as ψ goes up. In this range, the rating acts like a device to transfer utility from the manager to the investor. We also show that the total surplus increases in ψ both for low and for high levels of rating precision (but not necessarily for intermediate levels of precision; see Figure 4 (b) below). Note that we define the surplus to be the sum of the payoffs of the investor and manager, so that, in particular, the surplus includes the private benefit of the manager, m, whenever it is consumed.[8]

Proposition 3 *Suppose that $\psi \geq \underline{\psi}$. Then, an increase in the rating precision, ψ,*

(i) Strictly increases the payoff of the investor.

(ii) Strictly increases the payoff of the manager over some range $(\underline{\psi}, \psi')$, and strictly decreases the payoff of the manager over the range $(\psi_y, 1)$.

(iii) Strictly increases the surplus in the transaction between investor and manager over the ranges $(\underline{\psi}, \psi')$ and $[\psi_y, 1]$.

Suppose $\psi \in [\underline{\psi}, \psi_x]$, so that all investors offer a wage contract. An increase in the rating precision has the following effects on a manager's payoff. First, it reduces the probability of obtaining the bad rating in the high state, reducing the manager's payoff (recall that $w_b^f > 0$ and $w_g^f = 0$ when $\psi > \underline{\psi}$). Second, it increases the manager's payoff when the state the rating is bad—the manager earns w_b^f in the high state and $\max\{w_b^f, m\}$ in the low state, both of which increase as w_b^f increases. When $\psi = \underline{\psi}$, the optimal contract sets $w_b^f = 0$, so the first effect is not relevant. Thus, the manager's payoff strictly increases as the rating precision increases beyond $\underline{\psi}$. As ψ increases and correspondingly w_b^f increases, the first effect becomes

[8] To the extent that the private benefit m consists of a transfer by some other (unmodeled) agent in the economy to the manager, one could argue it should not be part of the surplus. However, in that case, a complete welfare calculation would take into account the surplus in the side transaction between the other agent and the manager.

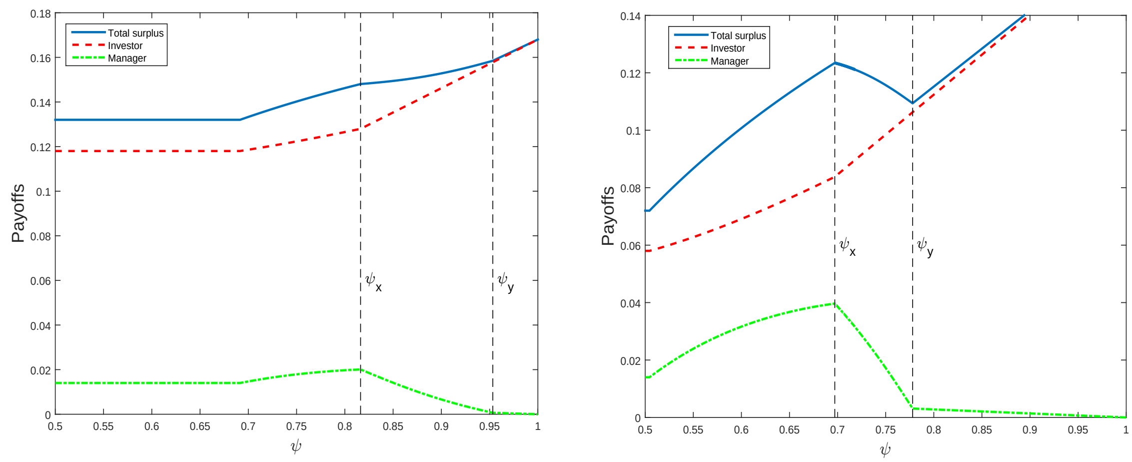
more important, so that there may be a rating precision beyond which the manager's payoff decreases as ψ increases.

Conversely, when the rating precision is greater than ψ_y, all investors offer a prohibitive contract. The manager only obtains a positive payoff if the state is low and the rating is good, so that he can invest in the risky asset and earn the private benefit m. The likelihood of attaining this payoff strictly decreases as ψ increases.

The surplus in the transaction between investor and manager is obtained by adding up their respective payoffs. This surplus of course increases over the range in which both investor's and manager's payoff is increasing. It also increases over the range in which only prohibitive contracts are offered, $[\psi_y, 1]$.

To gain some insight into the behavior of the surplus in the intermediate region between ψ' and ψ_y, we compute two numeric examples, exhibited in Figure 4 (the parameters are shown in the note to the figure). In Figure 4 (a), we find that surplus is monotonically increasing in ψ. However, in Figure 4 (b), surplus decreases for ψ in the range (ψ_x, ψ_y). The difference between the two examples is that δ is set to a high value (0.6) in Figure (b), and to a low value (0.3) in Figure (a). As a result, the wage in the optimal wage contract is significantly higher in Figure (b). When the signal precision ψ just exceeds ψ_x, a proportion of investors switch to the prohibitive contract. The resulting loss in payoff to the manager is sufficient to cause total surplus to decrease when δ is high (Figure (b)), but not when δ is low (Figure (a)).

Goldstein and Huang (2015) consider a model in which a credit rating provides information about a firm's fundamentals to creditors, the credit rating agency can engage in rating inflation, and the rating creates a feedback effect on whether a firm will engage in



(a) Surplus rises with ψ (b) Surplus falls in mixing region

This figure shows the payoffs to the investor, the manager and the total surplus. For each figure, common parameters are $\phi = 0.8$, $r^f = 0$, $\bar{r}^h = 0.25, \underline{r}^h = 0.21, \bar{r}^\ell = 0.25, \underline{r}^\ell = 0.05$, and $M = 0.14$. In each state, the return is linear in the demand from the delegated portfolio management sector. Figure (a) has $\delta = 0.3$, and figure (b) has $\delta = 0.6$.

Figure 4: **Equilibrium Payoffs and Surplus**

risk-shifting. They show that when firm fundamentals are in an intermediate range, the introduction of ratings reduces social welfare. In our model, an improvement in ratings quality reduces total surplus in the transaction between the investor and the manager, because the use of prohibitive contracts precludes the manager from ever obtaining his private benefit in the low state.

5 Discussion

As mentioned in the introduction, the view that credit ratings provide market participants with new information about the security or the issuer is pervasive in the academic literature. The policy implications of our contracting view and the information view are very different, making it important to differentiate between them.

5.1 Implications for Market Observables

We first note that the mere fact that bond prices react to rating changes cannot distinguish between the two views.

Observation 1 *An increase (decrease) in a bond price after a credit rating upgrade (downgrade) is consistent with both the contracting and the information views.*

In our model, whenever $\psi > \underline{\psi}$, an increase in the credit rating from b to g leads to more managers buying the risky bond in state ℓ regardless of the contract offered, and in state h as well if the contract is the prohibitive one. Thus, our model is consistent with the results of Tang (2009), who finds that when Moody's refined its rating system to include + and − levels, there was a response in bond prices, and Cornaggia, Cornaggia, and Israelsen (2014), who show that when Moody's revised its rating scale for municipal bonds, prices reacted accordingly. Therefore, such results cannot establish by themselves that ratings contain new information.

The logic of the information view suggests that bonds on which more information is available to the market should be less sensitive to changes in a rating. For example, the information content of credit ratings should be less important for bonds on which liquid CDS (credit default swap) contracts are traded. Therefore, the price responsiveness of a bond to a credit rating change should differ in the cross-section, depending on the existence and depth of the CDS market on the bond. However, in the contracting view, such cross-sectional differences should not exist.

Next, we turn to implications that differ across the two views. At any point, we expect some informed traders in the stock of a firm to possess superior information to the rest of the market, including the credit rating agency. To the extent a credit rating contains new information, some informed traders are likely to possess the information beforehand as well. When the credit rating is issued, the information is released publicly to the entire market. Therefore, measures of adverse selection in the stock (such as, e.g., microstructure or spread-based measures) should decrease. In the contracting view, the release of a credit rating should have no effect on adverse selection in the market for a stock.

Observation 2 *The release of a credit rating should decrease adverse selection in the market for the stock of a firm under the information view, and leave it unchanged under the contracting view.*

The two views also have different implications for the demand for portfolio management services. If credit ratings contain new information, releasing them makes it easier for investors to invest on their own rather than hire a portfolio manager. Conversely, if they facilitate contracting between investors and portfolio managers, widely available ratings lead to an increased demand for asset management services. Of course, this implication is difficult to test directly, as changes in demand for portfolio management services could manifest itself as either changes in the price of portfolio management services or in the flow of funds to such services.

Observation 3 *All else equal, the use of credit ratings leads to a reduced demand for portfolio management services in the information view, and an increased demand for such services in the contracting view.*

Next, we note that our model implies an absence of persistence in the performance of portfolio managers. If all managers are offered the prohibitive contract (i.e., when rating precision ψ is high), all take the same action, so there is no heterogeneity in performance. If some managers are offered a wage contract, ex post (based on the realization of their private benefits), actions are heterogeneous across these managers if the state is low and the rating is bad. The managers that deliver a low return in this scenario will be the ones with a high realized private benefit. Unless private benefits are correlated through time, there is no persistence of manager performance. However, if credit ratings communicate information to the market, managers that are skilled at generating that information on their own should exhibit persistent positive alphas.

Observation 4 *In the contracting view, if private benefits are uncorrelated through time, there is no persistence in portfolio manager performance.*

5.2 Policy Implications

We focus on three policy implications of our model: (1) Should contracts and regulations be contingent on credit ratings? (2) What is the optimal precision of credit ratings? (3) Who should pay for ratings?

Contracts in ratings

In our model, we focus on contracts between an investor and a fund manager, but the intuition applies just as straightforwardly when the principal is a regulator and the agent is a relevant participant in the financial market. That is, when states are unverifiable (and therefore regulation cannot be contingent on them), the use of credit ratings in regulation should reduce the cost to a regulator of inducing the right behavior from market participants. As Kisgen and Strahan (2010) point out, credit ratings have been used in regulation in the

US since 1931, to regulate institutions including banks, mutual funds, pension funds, and insurance companies. Our model offers a justification for this use of credit ratings.

Following the 2008-09 financial crisis, there has been much criticism of the use of credit ratings in regulation. For example, the Dodd-Frank Act of 2010, in Section 939A, requires each US federal agency to substitute an appropriate "standard of credit-worthiness" instead of credit ratings in regulations. In the models of Opp, Opp, and Harris (2013) and Kartasheva and Yilmaz (2014), the use of ratings in regulation has an adverse effect on the quality of the rating, and leads to rating inflation. In both these models, welfare is enhanced by reducing the use of ratings in regulation.

Our results on the benefits of ratings are qualitatively robust to frictions (such as rating inflation) in the rating process. Such frictions effectively lead to a lower precision of the credit rating. At the end of the day, if the eventual rating precision exceeds the threshold $\underline{\psi}$, making the contract contingent on the rating improves both the investor's payoff and the total surplus (compared to a situation in which ratings cannot be contracted on). In short, if credit ratings did not exist, investors would have to invent a substitute device. Only if the frictions are severe enough to reduce rating precision to close to one-half should the use of ratings be banned; however, in this case, rational investors would anyway reject the use of ratings. Therefore, moves such as those in the European Union in 2012 to ban the use of credit ratings are short-sighted at best. In designing regulation on ratings, it is critical to remember the positive role ratings play in contracting.

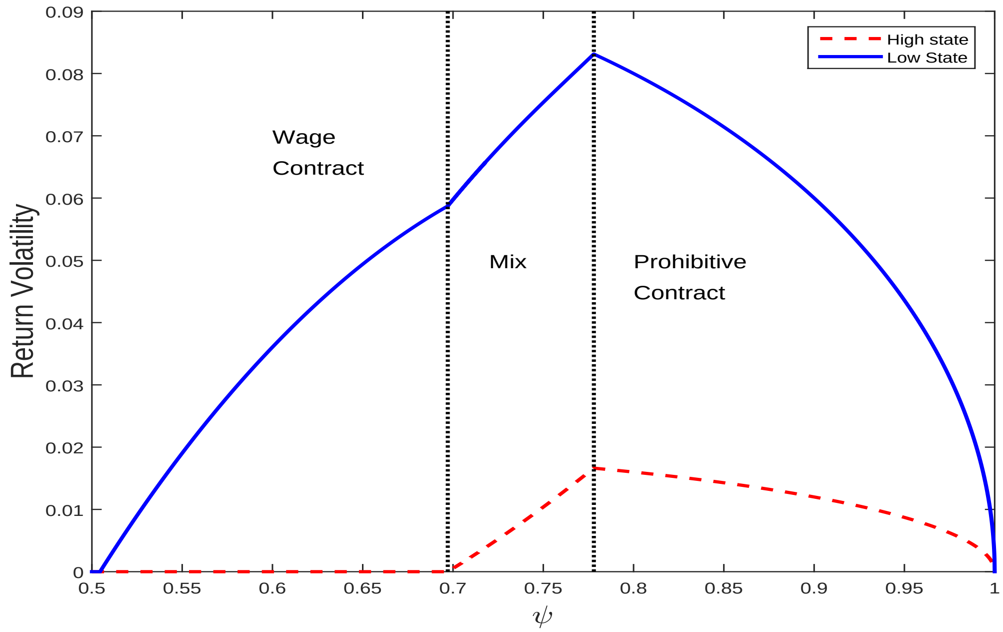
Optimal precision of ratings

Many of the ill-effects of ratings-based regulation are tied to rating inflation, which, under the information view can lead to over-investment in bad projects. Suppose that there is no technological cost to increasing the precision of credit ratings. In many information-based models, greater precision increases surplus by preventing inefficient investment.[9] In the contracting view, if the rating is fully precise, a first-best outcome is obtained. However, if there are technological or other barriers to achieving complete precision, it may not be desirable to seek greater precision.

There are two reasons for this. First, recall from Figure 4 that for some parameter values surplus actually declines as ratings become more precise. Second, a potential downside to increasing the precision of ratings is that the widespread use of credit ratings in the optimal contract induces a correlation in the actions of portfolio managers. In turn, fixing the state (and therefore the fundamentals on the bond), this induces a difference in the returns of a bond with a good rating and one with a low rating. That is, the bond return is volatile even when its fundamentals are held fixed, simply because of a noisy credit rating. We illustrate this phenomena in the context of a numeric example in Figure 5. The Y-axis plots the volatility of bond returns in each state in the example. This volatility is computed as the standard deviation of returns in each state, across a good and bad rating.

Given this effect, a welfare-maximizing regulator who can mandate (or incentivize) a minimum precision level for ratings must proceed with caution. If the parameters are set as in Figure 5, the surplus in the investor-manager transaction decreases in rating precision in the range $\psi \in [0.7, 0.77]$. Further, for all $\psi \leq 0.77$, an increase in rating precision also leads to greater return volatility. Depending on the current level of rating precision, both factors may make small increases in precision undesirable.

[9] Information production in such models is typically endogenous, as in (for example) Opp, Opp and Harris (2013). Increased precision can be obtained by suitably reducing the cost of investment.



This figure shows the standard deviation of returns across good and bad ratings in each state. The parameters are $\phi = 0.8$, $r^f = 0$, $\bar{r}^h = 0.25, \underline{r}^h = 0.21, \bar{r}^\ell = 0.25, \underline{r}^\ell = 0.05$, $\delta = 0.6$, and $M = 0.14$. In each state, the return is linear in the demand from the delegated portfolio management sector.

Figure 5: **Equilibrium Return Volatility in Each State**

Paying for ratings

The issuer-pay model for credit ratings has been much criticized, with some suggesting that the investor- or subscriber-pay model provides better incentives for a credit rating agency.[10] Kashyap and Kovrijnykh (2015) point out that, under the investor-pay model, investors may seek ratings too often relative to the social optimum. Our work suggests that, within an investor-pays model, the desired precision of ratings depends on whether the investor or the manager has the authority to request a rating.

As shown in Proposition 3 and as the examples in Figure 4 indicate, the investor's payoff strictly increases with the precision of the rating. However, the payoff of the manager

[10]See, for example the discussion on the respective models in "Report to Congress on Assigned Credit Ratings," prepared by the Staff of the Division of Trading and Markets of the U.S. Securities and Exchange Commission, December 2012.

decreases once the prohibitive contract is used. Thus, to the extent that ratings transfer surplus from manager to investors, the two parties will disagree on the desired precision in credit ratings, with managers preferring a lower precision. Therefore, if investors delegate to fund managers the authority to request and pay for credit ratings, even in an investor-pays model, low quality ratings may persist.

6 Conclusion

We have explored the optimal use of credit ratings in contracts between investors and managers when states are commonly known but unverifiable. In particular, we show that when ratings are precise, contracts should restrict managers' actions rather than rely on wages. The use of ratings improves overall efficiency in the relationship between a single investor and single manager. However, when credit ratings are used in contracts economy-wide, there is a feedback effect leading to increased volatility of risky bond returns.

In our investigation of the use of credit ratings, we took it as given that they were a contractible signal. However, if a state cannot be directly contracted on, what properties should a contractible signal have? This is an intriguing question, and one that we defer to future work. For now, a few observations are in order. A contractible signal should be forward-looking: For example, a firm that undertakes a new line of business may have payoffs that depend on states that were not obvious when the investor and manager agreed on the contract. Thus, macro-economic indices at the national level or reports by auditors at the firm level, which are inherently backward-looking, would be less useful. Second, the contracting signal cannot be too volatile—contracts have to be enforceable, and if signals change at too high a frequency relative to the actions of the manager, it is difficult to determine if he behaved appropriately given the contract. Thus, bond or CDS prices adjust at too high a frequency to be useful in contracts, and, in addition, can be affected by short-term changes in market liquidity. Given the way they are currently structured, credit ratings have a few characteristics that make them extremely useful in contracts—they are stable, change relatively infrequently, and are forward-looking.

Interestingly, the incomplete contracting approach suggests that credit ratings are not necessarily the most appropriate tool for investors and managers to use in contracts governing investments in structured finance vehicles. A credit rating can rely on a forward-looking business model to provide a useful summary of future states in which a government might change tax or monetary policies, or in which a company may sell assets to ensure its financial solvency. However, in the case of structured finance, if the vehicles are solvent, the interesting actions have been taken in the past, when the collateral was issued. Therefore, for these types of assets, whose quality is sunk at the time of origination, an auditor or a business entity that specializes in backward-looking analysis is most appropriate.

In conclusion, we note that for close to a century credit ratings have been produced and used in financial markets. In order to ensure that they provide the largest social benefit, it is important to understand how they add value. Fleshing out the implications of various possible uses is the first step to such an understanding.

Appendix: Proofs

Note: The sequence of proofs in the Appendix does not correspond to those presented in the text. The proof of Proposition 1 is presented after the proofs of Lemma 1 through Lemma 3. Expositionally, in the text, we think it is more helpful to first state Proposition 1 and then develop the supporting lemmata.

Proof of Lemma 1

The unrestricted action set is $\{a_h, a_\ell\}$. Suppose the rating on the risky bond is σ. Any restriction reduces the set to a single feasible action, so that the manager has no choice over actions. It is immediate that it is optimal to set all wages to zero; i.e., $w_\sigma^h = w_\sigma^f = w_\sigma^\ell = 0$.

■

Proof of Lemma 2

Suppose that the credit rating on the risky bond is g. In state h the manager takes action a_h if $w_g^h \geq w_g^f$ and action a_ℓ otherwise. In state ℓ, the manager takes action a_h if $w_g^f < w_g^\ell + m$, and action a_ℓ if $w_g^f \geq w_g^\ell + m$. It is immediate to see that it cannot be optimal to set $w_g^h > w_g^f$: Reducing w_g^h to w_g^f does not change the action in either state, and strictly reduces the amount paid to the manager in the h. Therefore, $w_g^h \leq w_g^f$.

Suppose $w_g^h < w_g^f$. Then, in state h, the manager takes action a_ℓ, so that the investor's payoff in this state is $r^f - w_g^f$. If the investor increases w_g^h to set it equal to w_g^f, the manager switches to action a_h. The investor's payoff in this state becomes $r_g^h - w_g^f$. Under Assumption 1, we have $r_g^h \geq \underline{r}^h > r^f + M > r^f$, so this strictly improves the investor's payoff. Therefore, it must be that $w_g^h = w_g^f$.

Next, consider the wage w_g^ℓ. Increasing w_g^ℓ above zero has two effects:

(i) The probability that the manager takes the inefficient action a_h in state ℓ is $\left[1 - \frac{w_g^f - w_g^\ell}{M}\right]$ when $w_g^f - w_g^\ell \in [0, M]$ (recall that m is uniform over $[0, M]$); this probability increases in w_g^ℓ. If $w_g^f < w_g^\ell$ or $w_g^f > M + w_g^\ell$, a small change in w_g^ℓ has no effect on this probability.

(ii) Conditional on action a_h being taken in state ℓ, the investor's payoff in that state is $r_g^\ell - \delta - w_g^\ell$, which strictly decreases in w_g^ℓ.

Therefore, it must be optimal to set $w_g^\ell = 0$ (i.e., for the limited liability constraint to bind when the portfolio return is r_g^ℓ).

A similar argument applies when the rating is b. ■

Proof of Lemma 3

Consider equations (3) and (4) in the text, which show the investor's payoff in states h and ℓ respectively. Use the fact that $w_\sigma^h = w_\sigma^f$ and $w_\sigma^\ell = 0$ for each σ. Further, observe that we must have $w_\sigma^f \leq M$, because when $w_\sigma^f = M$, all managers are purchasing the riskless bond in state ℓ. Therefore, we can re-write these equations as

$$\begin{aligned}
\pi_h &= \psi(r_g^h - w_g^f) + (1-\psi)(r_b^h - w_b^f) \\
\pi_\ell &= (1-\psi)\left((r^f - w_g^f)\frac{w_g^f}{M} + (r_g^\ell - \delta)(1 - \frac{w_g^f}{M})) + \psi((r^f - w_b^f)\frac{w_b^f}{M} + (r_b^\ell - \delta)(1 - \frac{w_b^f}{M})\right)
\end{aligned}$$

The investor's payoff is $\Pi = \phi\pi_h + (1-\phi)\pi_\ell$.

The first-order conditions for interior values of w_g^f and w_b^f are $\frac{\partial \Pi}{\partial w_g^f} = 0$ and $\frac{\partial \Pi}{\partial w_b^f} = 0$.

Taking these derivatives, we obtain

$$-\phi\psi + (1-\phi)(1-\psi)\left((r^f - w_g^f - r_g^\ell + \delta)\frac{1}{M} - \frac{w_g^f}{M}\right) = 0 \quad (10)$$

$$-\phi(1-\psi) + (1-\phi)\psi\left((r^f - w_b^f - r_b^\ell + \delta)\frac{1}{M} - \frac{w_b^f}{M}\right) = 0. \quad (11)$$

It is straightforward to see that the second-order conditions $\frac{\partial^2 \Pi}{\partial (w_g^f)^2} < 0$ and $\frac{\partial^2 \Pi}{\partial (w_b^f)^2} < 0$ are satisfied.

The solution to the first-order conditions is given by

$$w_g^f = \frac{1}{2}\left(r^f - r_g^\ell + \delta - \frac{\phi}{1-\phi}\frac{\psi}{1-\psi}M\right) \quad (12)$$

$$w_b^f = \frac{1}{2}\left(r^f - r_b^\ell + \delta - \frac{\phi}{1-\phi}\frac{1-\psi}{\psi}M\right). \quad (13)$$

Suppose that $\psi = \frac{1}{2}$, so that the rating is completely uninformative. The solution to the first-order condition given rating σ is $w_\sigma^f = \frac{1}{2}\left(r^f - r_\sigma^\ell + \delta - \frac{\phi}{1-\phi}M\right)$. From Assumption 1, $M \geq \frac{1-\phi}{\phi}(r^f - \underline{r}^\ell + \delta)$. By definition, r_g^ℓ and r_b^ℓ are each weakly greater than $\underline{r}^\ell$, so it follows that the wage that satisfies the first-order condition has the property that $w_\sigma^f \leq 0$ for $\psi = \frac{1}{2}$.

(i) Consider the wage offered when the rating is g. We have just shown that at $\psi = \frac{1}{2}$, the solution to the first-order condition is a weakly negative wage. Because the manager enjoys limited liability, the optimal wage at $\psi = \frac{1}{2}$ is $w_g^f = 0$. Further, the solution to the first-order condition, as shown in equation (12), decreases in ψ. Thus, for all $\psi > \frac{1}{2}$, the limited liability constraint binds and we have $w_g^f = 0$.

(ii) Now, consider the wage offered when the rating is b. As argued above, at $\psi = \frac{1}{2}$, we have

$w_b^f = 0$. However, the solution to the first-order condition (13) is increasing in ψ. Whenever that solution lies between 0 and M, it represents the optimal wage. From equation (13), $w_b^f \geq 0$ is equivalent to the condition

$$\begin{aligned} \frac{1-\psi}{\psi}\frac{\phi}{1-\phi}M &\leq r^f - r_b^\ell + \delta, \\ \text{Or,} \qquad \psi &\geq \frac{1}{1+\frac{1-\phi}{\phi}\frac{r^f - r_b^\ell + \delta}{M}} \quad \stackrel{d}{=} \quad \hat{\psi}(r_b^\ell). \end{aligned}$$

Finally, it is immediate that $w_b^f \leq M$. When $w_b^f = M$, the manager always chooses the riskless asset in state ℓ, so increasing the wage beyond M has no further effect on the manager's action. Thus, when $\psi \leq \hat{\psi}$, we have $w_b^f = 0$ in the optimal wage contract, and when $\psi > \hat{\psi}$, we have $w_b^f = \min\left\{\frac{1}{2}\left(r^f - r_b^\ell + \delta - \frac{\phi}{1-\phi}\frac{1-\psi}{\psi}M\right), M\right\}$. ■

Proof of Proposition 1

(i) Suppose the rating on the risky bond is g. As shown in Lemma 3, the optimal wage contract sets $w_g^f = 0$. From Lemma 2, we have in turn $w_g^h = w_g^\ell = 0$. Therefore, when the rating is g, it is optimal for the manager to purchase the risky bond. Restriction the action to insist that the manager purchases the risky bond has no further effect, and so the optimal contract has no restriction on action.

(ii) Suppose the rating on the risky bond is b. The investor can induce the manager to always purchase the risky bond by setting $w_b^f = 0$, so the only restriction on action that is meaningful to consider is banning investment in the risky bond; i.e., setting $A_b = \{a_\ell\}$. The payoff from the optimal wage contract is shown in equation (6) in the text, and the payoff from banning investment in the risky asset is r^f(as shown in equation (7)) . The wage contract is superior to banning iff

$$\mu_b(r_b^h - w_b^f) + (1-\mu_b)\left[\frac{w_b^f}{M}(r^f - w_b^f - r_b^\ell + \delta) + r_b^\ell - \delta\right] \;\; \geq \;\; r^f, \tag{14}$$

where w_b^f is given by the expression in equation (5) and $\mu_b = \frac{\phi(1-\psi)}{\phi(1-\psi)+(1-\phi)\psi}$.

Now, consider $\psi = \hat{\psi}$. At this value of ψ, we know from Lemma 3 that $w_b^f = 0$. When $w_b^f = 0$, the inequality in (14) reduces to

$$\mu_b r_b^h + (1-\mu_b)(r_b^\ell - \delta) \;\; \geq \;\; r^f, \tag{15}$$

$$\text{Or,} \quad r_b^h \;\; \geq \;\; \frac{r^f}{\mu_b} - \frac{1-\mu_b}{\mu_b}(r_b^\ell - \delta) \;\; = \;\; r^f + \frac{1-\mu_b}{\mu_b}(r^f - r_b^\ell + \delta). \tag{16}$$

When $\psi = \hat{\psi}$, straightforward algebraic calculation shows that $\frac{1-\mu_b}{\mu_b} = \frac{M}{r^f - r_b^\ell + \delta}$, so equation (16) reduces to

$$r_b^h \;\; \geq \;\; r^f + M, \tag{17}$$

an inequality that has been assumed to strictly hold in Assumption 1. Therefore, when $\psi = \hat{\psi}$, the investor strictly prefers an optimal wage contract to a prohibitive contract that bans investment in the risky asset.

Next, consider $\psi = 1$. In this case, it is clearly optimal for the investor to ban investment in the risky bond when the rating is b (because the state is ℓ for sure). Using a contract that relies only on wages has one of two effects:

(a) the optimal wage is M, and the manager always buys the riskless bond. Then, the prohibitive contract is strictly superior, because it achieves the same outcome at a lower cost (zero) for the investor, or

(b) the optimal wage is $w_b^f < M$, in which case the manager sometimes buys the risky bond (when $m > w_b^f$). The prohibitive contract achieves the investor's desired action for all m and at zero cost, so is again strictly superior.

Finally, going back to equation (6), take the derivative of $\Pi_{w,b}$, the investor's payoff from a wage contract, with respect to ψ. We have

$$\frac{\partial \Pi_{w,b}}{\partial \psi} = \frac{\partial \Pi_{w,b}}{\partial \mu_b}\frac{\partial \mu_b}{\partial \psi} + \frac{\partial \Pi_{w,b}}{\partial w_b^f}\frac{\partial w_b^f}{\partial \psi} \tag{18}$$

Whenever $w_b^f \in (0, M)$, the wage is chosen to satisfy $\frac{\partial \Pi_{w,b}}{\partial w_b^f} = 0$. Conversely, if $w_b^f = M$, we have $\frac{\partial w_b^f}{\partial \psi} = 0$. In either case, $\frac{\partial \Pi_{w,b}}{\partial \psi} = \frac{\partial \Pi_{w,b}}{\partial \mu_b}\frac{\partial \mu_b}{\partial \psi} < 0$, as $\frac{\partial \Pi_{w,b}}{\partial \mu_b} > 0$ and $\frac{\partial \mu_b}{\partial \psi} < 0$.

Therefore, $\Pi_{w,b}$, the investor's payoff from a wage-only contract when the rating is b, is strictly decreasing in the rating precision ψ, whereas the payoff from banning investment in the risky asset is independent of ψ, and remains r^f. Because $\Pi_{w,b}$ is continuous in ψ, it then follows that there exists some rating precision $\psi_1 \in (\hat{\psi}, 1)$ such that when $\psi \in (\hat{\psi}, \psi_1)$, the optimal contract conditional on a rating of b relies on wages with no restriction on actions, whereas when $\psi > \psi_1$, the optimal contract relies on banning the manager from investing in the risky asset when the rating is b, and offering zero wages. ■

Proof of Proposition 2

First, suppose the rating is g. From Proposition 1, for all $\psi \geq \frac{1}{2}$ and for all values of r_g^ℓ, it is optimal for each investor to offer a contract that sets $w_g^h = w_g^f = w_g^\ell = 0$, and imposes no restriction on the manager's action. Therefore, this contract remains the offered contract in a market equilibrium.

Next, consider the rating b. We consider each part of the Proposition in turn.

(i) Suppose that all investors offer a contract with $w_b^h = w_b^f = w_b^\ell = 0$ and no restriction on the manager's actions. Then, in equilibrium, the demand from the DPM sector is 1, and the return is $r^\ell(1) = \underline{r}^\ell$. Now, from Proposition 1 part (ii), if $\psi \leq \underline{\psi}$, it follows that it is a best response for each investor to offer a wage contract with zero wages. Therefore, this contract prevails in a market equilibrium. For completeness, note that under Assumption 1, it follows that $\underline{\psi} \geq \frac{1}{2}$.

(ii) For a generic $\psi > \underline{\psi}$, it is a best response for an investor to offer a wage contract if $\Pi_{w,b} \geq \Pi_x$, where $\Pi_{w,b}$ is shown in equation (6) and Π_x in equation (7). We first show that, fixing β (the proportion of investors who offer a wage contract), $\Pi_{w,b}$ is decreasing in ψ.

Lemma 4 *Fix β, the proportion of principals who offer the wage contract. Suppose $\psi \geq \underline{\psi}$. Then, $\Pi_{w,b}$ is strictly decreasing in ψ.*

Proof of Lemma 4

Recall that

$$\Pi_{w,b} = \mu_b(r_b^h - w_b^f) + (1-\mu_b)\left(\frac{w_b^f}{M}(r^f - w_b^f) + \left(1 - \frac{w_b^f}{M}\right)(r_b^\ell - \delta)\right). \qquad (19)$$

Here, μ_b, r_b^h, r_b^ℓ, and w_b^f are functions of ψ, and r_b^h, r_b^ℓ and w_b^f are functions of β. Fix β; going forward, in the notation we suppress the dependence on β.

Now,

$$\frac{\partial \Pi_{w,b}}{\partial \psi} = \frac{\partial \Pi_{w,b}}{\partial \mu_b}\frac{\partial \mu_b}{\partial \psi} + \frac{\partial \Pi_{w,b}}{\partial w_b^f}\frac{\partial w_b^f}{\partial \psi} + \frac{\partial \Pi_{w,b}}{\partial r_b^h}\frac{\partial r_b^h}{\partial \psi} + \frac{\partial \Pi_{w,b}}{\partial r_b^\ell}\frac{\partial r_b^\ell}{\partial \psi}. \qquad (20)$$

Consider the second term, $\frac{\partial \Pi_{w,b}}{\partial w_b^f} \frac{\partial w_b^f}{\partial \psi}$. For $\psi \geq \psi_x$, there are two possibilities: (i) w_b^f satisfies the first-order condition for optimal wages in the principal's problem, in which case $\frac{\partial \Pi_{w,b}}{\partial w_b^f} = 0$, or (ii) $w_b^f = M$, in which case $\frac{\partial w_b^f}{\partial \psi} = 0$. Therefore, the second term is equal to zero. Further, when β is fixed, the demand for the risky bond in the high state is $q_b^h = \beta$, which is invariant in ψ. Hence, $\frac{\partial r_b^h}{\partial \psi} = 0$, so that the third term is also zero. We therefore have

$$\frac{\partial \Pi_{w,b}}{\partial \psi} = \frac{\partial \Pi_{w,b}}{\partial \mu_b} \frac{\partial \mu_b}{\partial \psi} + \frac{\partial \Pi_{w,b}}{\partial r_b^\ell} \frac{\partial r_b^\ell}{\partial \psi} \tag{21}$$

Observe that $r_b^\ell = r^\ell(q_b^\ell)$, and in turn $q_b^\ell = \beta \left(1 - \frac{w_b^f}{M}\right)$. Therefore, we have $\frac{\partial r_b^\ell}{\partial \psi} = -\frac{\beta}{M} r^{\ell\prime}(q_b^\ell) \frac{\partial w_b^f}{\partial \psi}$, where $r^{\ell\prime}$ denotes the derivative of r^ℓ with respect to demand.

Further, if $w_b^f = M$, then $\frac{\partial w_b^f}{\partial \psi} = 0$. Instead, if w_b^f satisfies the first-order condition in the principal's problem, then $w_b^f = \frac{1}{2}\left(r^f - r_b^\ell + \delta - \frac{\phi}{1-\phi} \frac{1-\psi}{\psi} M\right)$. Therefore, we have

$$\begin{aligned} \frac{\partial w_b^f}{\partial \psi} &= \frac{1}{2}\left(\frac{\beta}{M} r^{\ell\prime} \frac{\partial w_b^f}{\partial \psi} + \frac{\phi}{1-\phi} \frac{M}{\psi^2}\right), \\ \text{or,} \quad \frac{\partial w_b^f}{\partial \psi} &= \frac{\phi M}{(1-\phi)\psi^2} \frac{1}{2 - \frac{\beta r^{\ell\prime}}{M}} = \frac{\phi M^2}{(1-\phi)\psi^2 \left(2M - \beta r^{\ell\prime}\right)}. \end{aligned} \tag{22}$$

Substituting the RHS of the last equation into the expression for $\frac{\partial r_b^\ell}{\partial \psi}$ we have

$$\frac{\partial r_b^\ell}{\partial \psi} = -\frac{\phi M}{(1-\phi)\psi^2} \frac{\beta r^{\ell\prime}}{2M - \beta r^{\ell\prime}} \tag{23}$$

Now, consider the various terms on the RHS of equation (21). It is straightforward to compute $\frac{\partial \Pi_{w,b}}{\partial \mu_b}$ and $\frac{\partial \Pi_{w,b}}{\partial r_b^\ell}$. Further, given $\mu_b = \frac{\phi(1-\psi)}{\phi(1-\psi)+(1-\phi)\psi}$, we can compute $\frac{\partial \mu_b}{\partial \psi} =$

$-\frac{\mu_b(1-\mu_b)}{\psi(1-\psi)}$. Putting all this together, we have

$$\frac{\partial \Pi_{w,b}}{\partial \psi} = -\left(r_b^h - w_b^f - \frac{w_b^f}{M}(r^f - w_b^f) - \left(1-\frac{w_b^f}{M}\right)(r_b^\ell - \delta)\right)\frac{\mu_b(1-\mu_b)}{\psi(1-\psi)} - (1-\mu_b)\left(1-\frac{w_b^f}{M}\right)\frac{\phi M}{(1-\phi)\psi^2}\frac{\beta r^{\ell'}}{2M-\beta r^{\ell'}} \tag{24}$$

$$= \frac{\phi(1-\mu_b)}{(1-\phi)\psi^2}\left[-(1-\mu_b)\left(r_b^h - w_b^f - r_b^\ell + \delta - \frac{w_b^f}{M}(r^f - w_b^f - r_b^\ell + \delta)\right) - M\left(1-\frac{w_b^f}{M}\right)\frac{\beta r^{\ell'}}{2M-\beta r^{\ell'}}\right], \tag{25}$$

where the second equation is obtained by substituting in for μ_b in the first equation and collecting terms.

Therefore, a sufficient condition for $\frac{\partial \Pi_{w,b}}{\partial \psi}$ to be strictly negative is

$$-M\left(1-\frac{w_b^f}{M}\right)\frac{\beta r^{\ell'}}{2M-\beta r^{\ell'}} < (1-\mu_b)\left(r_b^h - w_b^f - r_b^\ell + \delta - \frac{w_b^f}{M}(r^f - w_b^f - r_b^\ell + \delta)\right). \tag{26}$$

Observe that when $w_b^f = M$, the LHS of equation (26) is zero, and the RHS is $(1-\mu_b)(r_b^h - r^f) > 0$. Thus, the inequality is trivially satisfied when $w_b^f = M$.

Suppose, instead that $w_b^f < M$. Then, as $\psi \geq \psi_x$, the first-order condition for the optimal wage in the principal's problem is satisfied, so that $w_b^f = \frac{1}{2}\left(r^f - r_b^\ell + \delta - \frac{\mu_b}{1-\mu_b}M\right)$, or $r^f - 2w_b^f - r_b^\ell + \delta = \frac{\mu_b}{1-\mu_b}M$. Therefore, the RHS of (26) can be written as

$$(1-\mu_b)\left(r_b^h - w_b^f - r_b^\ell + \delta - \frac{(w_b^f)^2}{M} - \frac{\mu_b}{1-\mu_b}w_b^f\right) = (1-\mu_b)\left(r_b^h - \frac{w_b^f}{1-\mu_b} - r_b^\ell + \delta - \frac{(w_b^f)^2}{M}\right)$$

$$= -w_b^f + (1-\mu_b)\left(r_b^h - r_b^\ell + \delta - \frac{(w_b^f)^2}{M}\right). \tag{27}$$

Recall that $r^{\ell'}$ lies between $-\infty$ and 0. The LHS of equation (26) is strictly decreasing in $r^{\ell'}$, and so is maximized when $r^{\ell'} \to -\infty$. Its maximum value, in the limit, is $M - w_b^f$. Therefore, equation (26) holds if

$$(1-\mu_b)\left(r_b^h - r_b^\ell + \delta - \frac{(w_b^f)^2}{M}\right) \;\; > \;\; M. \tag{28}$$

In Claim 1 below, we show that this inequality holds for all $\psi \geq \psi_x$. Equation (28) in turn implies equation (26), which then implies that $\frac{\partial \Pi_{w,b}}{\partial \psi} < 0$, so that $\Pi_{w,b}$ is strictly decreasing in ψ.

Claim 1 For $\psi \geq \psi_x$, $(1-\mu_b)\left(r_b^h - r_b^\ell + \delta - \frac{(w_b^f)^2}{M}\right) > M$.

Proof of Claim

First, consider $\psi = \psi_x$. Recall that $\psi_x = \underline{\psi}(\underline{r}^\ell)$. At this value of ψ, the optimal wage w_b^f both satisfies the first-order condition for optimality from the principal's problem *and* is equal to zero. Therefore, the demand for the risky asset is 1 in both states, so that $r_b^h = \underline{r}^h$ and $r_b^\ell = \underline{r}^\ell$. Setting the optimal wage to zero, we have $\frac{1}{2}\left(r^f - \underline{r}^\ell + \delta - \frac{\mu_b}{1-\mu_b}M\right) = 0$, so that $\mu_b = \frac{r^f - \underline{r}^\ell + \delta}{M + r^f - \underline{r}^\ell + \delta}$, and $1 - \mu_b = \frac{M}{M + r^f - \underline{r}^\ell + \delta}$.

Now, substituting in the value of μ_b, $w_b^f = 0$, $r_b^h = \underline{r}^h$ and $r_b^\ell = \underline{r}^\ell$ into the LHS of equation (28), we obtain

$$\underline{r}^h - \underline{r}^\ell + \delta \; > \; M + r^f - \underline{r}^\ell + \delta, \quad \text{or,} \quad \underline{r}^h - r^f \; > \; M, \tag{29}$$

which has been assumed in Assumption 1, part (ii). Therefore, the claim holds for $\psi = \psi_x$.

Define the LHS of the claim to be $Z(\psi) = (1-\mu_b)\left(r_b^h - r_b^\ell + \delta - \frac{(w_b^f)^2}{M}\right)$. We have shown that $Z(\psi_x) > M$. If $\frac{\partial Z}{\partial \psi} \geq 0$, then it must be that $Z(\psi) > M$ for all $\psi \geq \psi_x$.

Now, $\frac{\partial Z}{\partial \psi} = -\left(r_b^h - r_b^\ell + \delta - \frac{(w_b^f)^2}{M}\right)\frac{\partial \mu_b}{\partial \psi} - (1-\mu_b)\left(\frac{\partial r_b^\ell}{\partial \psi} + \frac{2w_b^f}{M}\frac{\partial w_b^f}{\partial \psi}\right)$. Substitute in the values of $\frac{\partial w_b^f}{\partial \psi}$ from equation (22) and $\frac{\partial r_b^\ell}{\partial \psi}$ from equation (23), and note that $\frac{\partial \mu_b}{\partial \psi} = -\frac{\mu_b(1-\mu_b)}{\psi(1-\psi)}$.

Then, the condition $\frac{\partial Z}{\partial \psi} \geq 0$ is equivalent to

$$(1-\mu_b)\left(r_b^h - r_b^\ell + \delta - \frac{(w_b^f)^2}{M}\right) \quad \geq \quad M\left(\frac{2w_b^f - \beta r^{\ell'}}{2M - \beta r^{\ell'}}\right). \tag{30}$$

Now, at any given value of ψ, the wage w_b^f is fixed. Consider the term $\left(\frac{2w_b^f - \beta r^{\ell'}}{2M - \beta r^{\ell'}}\right)$ on the RHS of equation (30). This term is strictly decreasing in $r^{\ell'}$ if $w_b^f < M$, and invariant in $r^{\ell'}$ if $w_b^f = M$. In either case, a maximum value of $\frac{w_b^f}{M}$ is attained when $r^{\ell'} = 0$. Therefore, the RHS of equation (30) is less than or equal to $M \times \frac{w_b^f}{M} = w_b^f$.

That is, equation (30) holds whenever $(1-\mu_b)\left(r_b^h - r_b^\ell + \delta - \frac{(w_b^f)^2}{M}\right) \geq w_b^f$; i.e., whenever $Z(\psi) \geq w_b^f$.

Therefore, we have shown that (i) $Z(\psi_x) \geq M$, and (ii) whenever $Z(\psi) \geq w_b^f$, we have $\frac{\partial Z}{\partial \psi} \geq 0$. Consider ψ increasing just above ψ_x. As $Z(\psi_x) > M > w_b^f = 0$, we have $\frac{\partial Z}{\partial \psi} > 0$, so $Z(\psi)$ remains strictly above M. The same argument continues to hold as ψ increases, for any value of ψ. At some value of ψ, it is possible that $w_b^f = M$, in which case $\frac{\partial Z}{\partial \psi} = 0$. However, we never have $\frac{\partial Z}{\partial \psi} < 0$, so that $Z(\psi)$ remains strictly greater than M. That is, $Z(\psi) > M$ for all $\psi \geq \psi_x$, proving Claim 1. □

As argued in the paragraph before Claim 1, it now follows that, whenever $r^{\ell'} > -\infty$, $\Pi_{w,b}$ is strictly decreasing in ψ for all $\psi \geq \psi_x$, proving Lemma 4. □

We return to the proof of Proposition 2 (ii). From Lemma 4, for a fixed value of β, the payoff $\Pi_{w,b}$ overall falls as ψ increases. Conversely, Π_x remains unchanged at r^f. Further, for any value of β, when $\psi = 1$, we have $\Pi_{w,b} = \frac{w_b^f}{M}(r^f - w_b^f) + (1 - \frac{w_b^f}{M})(r_b^\ell - \delta)) < r^f$ (the last inequality holds because $w_b^f > 0$ and $r_b^\ell - \delta < r^f$). That is, an investor prefers a prohibitive contract that bans investment in the risky asset.

Fix $\beta = 1$. Then, by the mean value theorem, there must exist some ψ_x at which $\Pi_{w,b} = \Pi_x$ when w_b^f and r_b^f are allowed to adjust as ψ changes. That is, there exists some ψ_x such that if all other investors offer a wage contract with no restriction on actions, it is a best response for investor i to do the same. Then, for $\psi \in (\underline{\psi}, \psi_x)$, the contract offered in market equilibrium is a wage contract with no restriction on actions.

(iii) At $\psi = \psi_x$, all investors offer a contract with no restriction on actions. The total demand for the risky asset is 1 in state h (as $w_b^h = w_b^f$) and in state ℓ if the rating is g, and $1 - \frac{w_b^f}{M}$ in state ℓ when the rating is b (as only managers with $m > w_b^f$ buy the risky asset).

Suppose, instead, all investors except investor i switched to offering the prohibitive contract that banned investment in the risky asset. Consider the best response of investor i. As she is infinitesimal in the market, she takes the return on the risky asset as given in each state. The demand for the risky asset from the DPM sector is zero in both states, so $r_b^h = \bar{r}^h$ and $r_b^\ell = \bar{r}^\ell$. The return in both states increases, compared to the case in which all investors offer the wage contract. It is immediate that the payoff to the optimal wage contract, $\Pi_{w,b}$, also increases. Therefore, if all other investors switched to the prohibitive contract, investor i now strictly prefers the wage contract.

Now, consider ψ just greater than ψ_x. If all investors offer the wage contract, investor i strictly prefers the prohibitive contract. If all investors offer the prohibitive contract, investor

i strictly prefers the wage contract. It follows that there exists some fraction $\beta(\psi)$ such that if $\beta(\psi)$ investors offer the wage contract and $1-\beta(\psi)$ the prohibitive contract, investor i is indifferent between the two contracts. Therefore, in the market equilibrium, both contracts are offered by some proportion of investors. It follows that $\beta(\psi)$ must decrease in ψ; at a fixed $\beta(\psi)$, as ψ increases, the wage contract becomes strictly inferior to the prohibitive contract.

(iv) Suppose all investors are using a contract that bans investment in the risky asset when the rating is b, so that $\beta = 0$. Then, $r_b^\ell = \bar{r}^\ell$ and similarly $r_b^h = \bar{r}^h$. As argued above in part (ii), when $\psi = 1$, we have $\Pi_{w,b} < \Pi_x$. As argued in part (iii), if $\beta = 0$, at $\psi = \psi_x$ we have $\Pi_{w,b} > \Pi_x$. Because $\Pi_{w,b}$ decreases in ψ, there exists some $\psi_y < 1$ such that when $\psi \geq \psi_y$, all investors offer a contract with zero wages which prohibits investment in the risky asset when the rating is b.

Now, fix $\beta = 0$. At $\psi = \psi_x$, we have $\Pi_{w,b} > \Pi_x$, and at $\psi = \psi_y$, we have $\Pi_{w,b} = \Pi_x$. As $\Pi_{w,b}$ is decreasing in ψ, it follows that $\psi_y > \psi_x$. ■

Proof of Proposition 3

(i) First, suppose $\psi \in [\underline{\psi}, \psi_x]$. Over this range of ψ, the investor's payoff is

$$\begin{aligned}\Pi &= \phi\left[\psi r_g^h + (1-\psi)(r_b^h - w_b^f)\right] \\ &\quad +(1-\phi)\left[(1-\psi)(r_g^\ell - \delta) + \psi\left(\frac{w_b^f}{M}(r^f - w_b^f) + (1-\frac{w_b^f}{M})(r_b^\ell - \delta)\right)\right]. \end{aligned} \tag{31}$$

Using the definition of $\Pi_{w,b}$, the payoff from a wage contract when the rating is bad, shown in equation (6), we can write this payoff as

$$\Pi = \phi\psi r_g^h + (1-\phi)(1-\psi)(r_g^\ell - \delta) + (\phi(1-\psi) + (1-\phi)\psi)\Pi_{w,b}. \tag{32}$$

Then,

$$\frac{d\Pi}{d\psi} = \phi r_g^h - (1-\phi)(r_g^\ell - \delta) - (2\phi - 1)\Pi_{w,b} + [\phi(1-\psi) + (1-\phi)\psi]\frac{d\Pi_{w,b}}{d\psi}. \quad (33)$$

This expression is non-negative if

$$\phi r_g^h - (1-\phi)(r_g^\ell - \delta) \geq (2\phi - 1)\Pi_{w,b} - [\phi(1-\psi) - (1-\phi)\psi]\frac{d\Pi_{w,b}}{d\psi}. \quad (34)$$

We show in Lemma 4 that $\frac{d\Pi_{w,b}}{d\psi} < 0$. Further, in the proof of Proposition 2, we show that

$$\frac{d\Pi_{w,b}}{d\psi} = -\frac{\phi(1-\mu_b)}{(1-\phi)\psi^2}\left[(1-\mu_b)\left(r_b^h - w_b^f - r_b^\ell + \delta - \frac{w_b^f}{M}(r^f - w_b^f - r_b^\ell + \delta)\right) + M\left(1 - \frac{w_b^f}{M}\right)\frac{\beta r^{\ell'}}{2M - \beta r^{\ell'}}\right]. \quad (35)$$

Consider $\psi \in (\underline{\psi}, \psi_x)$. Then, $\beta = 1$, so that the last term can be written as $(M - w_b^f)\frac{r^{\ell'}}{M - r^{\ell'}}$. Further, $r^{\ell'} \leq 0$. Observe that $\frac{r^{\ell'}}{M - r^{\ell'}} = \frac{1}{\frac{M}{r^{\ell'}} - 1}$. This expression is equal to 0 when $r^{\ell'} = 0$ and goes to a limit of -1 as $r^{\ell'} \to -\infty$. Therefore, the minimum value of $\frac{d\Pi_{w,b}}{d\psi}$ (and hence the maximal value of the RHS of equation (34) is attained when $r^{\ell'} = 0$. Making this substitution into equation (34) and simplifying, the inequality in equation (34) holds if

$$\phi r_g^h - (1-\phi)(r_g^\ell - \delta) \geq (2\phi - 1)\Pi_{w,b} + \frac{\phi(1-\phi)}{\phi(1-\psi) + (1-\phi)\psi}\left(r_b^h - r^f + \left(1 - \frac{w_b^f}{M}\right)(r^f - w_b^f - r_b^\ell + \delta)\right). \quad (36)$$

Consider the RHS of equation (36). On the RHS, substitute in $\frac{\phi(1-\phi)}{\phi(1-\psi)+(1-\phi)\psi} = \mu_b\frac{1-\phi}{1-\psi}$

and $\Pi_{w,b} = \mu_b(r_b^h - w_b^f) + (1-\mu_b)\left(\frac{w_b^f}{M}(r^f - w_b^f) + (1 - \frac{w_b^f}{M})(r_b^\ell - \delta)\right)$, and simplify. The RHS can be shown to be equal to:

$$+\mu_b\left(2\phi - 1 + \frac{1-\phi}{1-\psi}\right)\left(1 - \frac{w_b^f}{M}\right)(r^f - w_b^f - r_b^\ell + \delta). \tag{37}$$

Now, $2\phi - 1 + \frac{1-\phi}{1-\psi} = \phi + (1-\phi)\frac{\psi}{1-\psi}$. Therefore, $\mu_b(2\phi - 1 + \frac{1-\phi}{1-\psi}) = \mu_b(\phi + (1-\phi)\frac{\psi}{1-\psi}) = \phi\mu_b + \phi(1-\mu_b) = \phi$. The expression in (37) simplifies to

$$\phi r_b^h - (1-\phi)(r_b^\ell - \delta) - \phi w^f - (1-\phi)\frac{w^f}{M}(r^f - w^f - r_b^\ell + \delta). \tag{38}$$

Then, equation (36) may be written as

$$\phi r_g^h - (1-\phi)(r_g^\ell - \delta) \geq \phi r_b^h - (1-\phi)(r_b^\ell - \delta) - \phi w^f - (1-\phi)\frac{w^f}{M}(r^f - w^f - r_b^\ell + \delta)$$
$$\text{or,} \quad \phi w^f + (1-\phi)\frac{w^f}{M}(r^f - w^f - r_b^\ell + \delta) \geq \phi(r_b^h - r_g^h) - (1-\phi)(r_b^\ell - r_g^\ell). \tag{39}$$

Now, for $\psi \in [\underline{\psi}, \psi_x]$, $r_b^h = r_g^h = \underline{r}^h$ (the manager always invests in the risky bond when the state is h) and $r_b^\ell \geq r_g^\ell = \underline{r}^\ell$, with strict inequality for all $\psi > \underline{\psi}$. Therefore, the RHS of equation (39) is zero at $\psi = \underline{\psi}$, and strictly negative for $\psi > \underline{\psi}$. The RHS is also zero at $\psi = \underline{\psi}$, and strictly negative for $\psi > \underline{\psi}$. That is, equation (39) holds as a strict inequality for $\psi \in (\underline{\psi}, \psi_x]$.

In turn, this implies that $\frac{d\Pi}{d\psi} > 0$ for $\psi \in (\underline{\psi}, \psi_x]$. Therefore, the investor's payoff Π is increasing in ψ for ψ in this range.

Now, consider $\psi \geq \psi_x$. For $\psi \in [\psi_x, \psi_y]$, each investor is indifferent between an optimal wage contract and a prohibitive contract. For $\psi > \psi_y$, the investor strictly prefers the prohibitive contract. Therefore, for all $\psi \geq \psi_x$, the investor's payoff is equal to that obtained under the prohibitive contract, and may be written as

$$\Pi = \phi(\psi r_g^h + (1-\psi)r^f) + (1-\phi)((1-\psi)(r_g^\ell - \delta) + \psi r^f). \tag{40}$$

As $r_g^h > r^f > r_g^\ell - \delta$, it follows that Π is increasing in ψ.

(ii) Consider the manager's payoff. First, suppose $\psi \in [\underline{\psi}, \psi_x]$. Recall that in this region, only a wage contract is offered, and that $w_g^f = 0$. Then, the agent's payoff is

$$\begin{aligned}\Gamma_w &= \phi(1-\psi)w_b^f + (1-\phi)\left\{(1-\psi)\frac{M}{2} + \psi\left(\frac{w_b^f}{M}w_b^f + \left(1 - \frac{w_b^f}{M}\right)E(m \mid m \geq w_b^f)\right)\right\} \\ &= \phi(1-\psi)w_b^f + (1-\phi)\frac{M}{2} + (1-\phi)\psi\frac{(w_b^f)^2}{2M}. \end{aligned} \tag{41}$$

Therefore,

$$\frac{d\Gamma_w}{d\psi} = -\phi w_b^f + (1-\phi)\frac{(w_b^f)^2}{2M} + \left(\phi(1-\psi) + (1-\phi)\psi\frac{w_b^f}{M}\right)\frac{dw_b^f}{d\psi}. \tag{42}$$

Recalling that $w_b^f = \frac{1}{2}\left(r^f - r_b^\ell + \delta - \frac{\phi}{1-\phi}\frac{1-\psi}{\psi}M\right)$, we have $\frac{dw_b^f}{d\psi} = \frac{\phi}{(1-\phi)\psi^2}\frac{M}{2}$. Therefore,

$$\frac{d\Gamma_w}{d\psi} = w_b^f\left((1-\phi)\frac{w_b^f}{2M} - \phi\left(1 - \frac{1}{2\psi}\right)\right) + \frac{\phi^2(1-\psi)}{(1-\phi)\psi^2}\frac{M}{2}. \tag{43}$$

Now, at $\psi = \underline{\psi}$, we have $w_b^f = 0$. It is clear that at this value of ψ, $\frac{d\Gamma_w}{d\psi} > 0$, so that the manager's payoff improves in ψ. Because the inequality is strict, it must be that $\frac{d\Gamma_w}{d\psi} > 0$ for ψ in some range $(\underline{\psi}, \psi')$.

Finally, consider $\psi \in [\psi_y, 1]$. In this range of ψ, all investors offer the prohibitive contract.

The wage is zero, and the manager earns a payoff only if the state is low and the credit rating is good, so that she can invest in the risky bond. Her payoff is therefore

$$\Gamma = (1-\phi)(1-\psi)\frac{M}{2}, \tag{44}$$

which is clearly decreasing in ψ.

(iii) The surplus in the transaction between the investor and the manager is equal to $\Lambda = \Pi + \Gamma$. It is immediate from above that the surplus is strictly increasing both for $\psi \in [\underline{\psi}, \psi')$, and in the region $\psi \in [\psi_y, 1]$. ■

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