Information Tradeoffs in Dynamic Financial Markets*

by

Efstathios (Stathi) Avdis
Masahiro Watanabe
Masahiro Watanabe

Working Paper No. 00013-00

Finance Theory Group

www.financetheory.com

Information tradeoffs in dynamic financial markets *

Efstathios Avdis[†]

University of Alberta, Canada

Abstract

In dynamic financial markets the stochastic supply of risky assets has a significant informational role. Contrary to static models, where it acts as "noise," in dynamic markets stochastic supply contains information about risk premiums. Acquiring private dividend information helps investors disentangle dividend information from discount-rate information contained in prices. For uninformed investors, however, as more informed investors enter the economy prices become more informative about dividends but less informative about discount rates. This tradeoff creates complementarities in information acquisition and multiple equilibria in the information market.

Keywords: Information acquisition, Dynamic financial markets, Rational expectations, Market efficiency, Complementarities.

JEL codes: D53, D82, D84, G14.

---

*I would like to thank Bradyn Breon-Drish (the referee), and Philipp Illeditsch, Ivan Shaliastovich, Itay Goldstein, and especially my advisor, Richard Kihlstrom, for their input. I would also like to thank Mark Huson, Vincent Glode, Christian Opp, Doron Levit, Liyan Yang, and Giovanni Cespa for helpful discussions, and seminar participants at the Wharton School, Rice University, University of Alberta, Sveriges Riksbank (Central Bank of Sweden), the Northern Finance Association 2012 Conference, and the 2014 Barcelona GSE Summer Forum for their valuable feedback.

†University of Alberta, 3-30B Business Building, Edmonton, Alberta, Canada T6G 2R6, phone: 780-492-8876, fax: 780-492-9924, email: avdis@ualberta.ca

1. Introduction

I develop a theory of information acquisition in dynamic settings by extending Grossman and Stiglitz (1980) to the case where the asset market is dynamic. How information gets incorporated into asset prices is a central question in finance, yet our understanding of endogenous information acquisition comes from static descriptions of financial markets. For example, in the model of Grossman and Stiglitz (1980), an investor's decision to become informed is completely characterized by the informativeness of prices about dividends. This description of information acquisition rests crucially on the assumption that financial markets are static. In dynamic markets, however, investors care not only about how much an asset will pay as dividends, but also about how the price of the asset will change over time. As I show below, introducing dynamic trading into an otherwise standard model of information acquisition can lead to significantly different economic conclusions.

My first main result is that in dynamic financial markets the value of private information can be increasing in the number of informed agents. This complementarity is very different from the well-known negative relation between the value of information and the number of informed agents in the static version of the economy. The distinction arises because in a dynamic setting the presence of informed agents affects the informativeness of prices about future prices in a different way than it affects the informativeness of prices about dividends. As I explain below, investors learn information about future capital gains through the stochastic supply of the asset, rather than through dividend information.

In fact, existing theories of dynamic financial markets recognize explicitly that the stochastic supply of the asset is a proxy for the conditional risk premium. For example, in Campbell and Kyle (1993) and Wang (1993) the quantity of risk borne by investors is proportional to stochastic supply. This interpretation delivers an information-based version of the Gordon growth formula. Prices contain both dividend information and information about discount rates, where shocks to discount rates are modeled as shocks to supply. Thus, the dynamics of conditional risk premiums are fully represented by the dynamics of supply, and therefore stochastic supply may predict future price changes.

To keep the analysis tractable, I assume that investors decide whether to purchase information about dividends once and for all time at the beginning of the economy. Moreover, prices are publicly observable and they depend only on two variables, dividend information and supply information. This may sound trivial, but its consequence is that investors who acquire information about dividends are better able to infer the level of supply by looking at prices. Therefore, purchasing dividend information provides the extra benefit of revealing non-dividend information about conditional expected returns. In contrast, agents who do not purchase dividend information must rely on noisy prices to learn not only about dividends, but also about supply.

In equilibrium, the information content of prices is a weighted average of dividend information and supply information. The weight of each type of information is a function of the number of informed agents. Similarly to the static model, as more informed agents enter the economy prices become less noisy signals of dividends because there is more dividend information available. This implies that the weight of dividend information in the total information content of prices increases. Correspondingly, the weight of supply decreases, making prices noisier signals of supply.

This tradeoff affects the ex ante willingness of agents to pay for dividend information as the number of informed agents increases. On the one hand, that prices become more informative about dividends makes agents less willing to pay for dividend information. On the other hand, when prices become noisier signals about supply they also become less informative about the conditional risk premium. This makes agents more willing to pay for dividend information, because agents can infer supply levels from prices only if they know the dividend information. The latter effect dominates in the overall value of information when prices contain little dividend information, so that others becoming informed is a complement for acquiring information directly.

My second main result is that such complementarities in information acquisition are prominent when the mean-reversion of stochastic supply is high. This is because the mean-reversion of supply determines whether the level of supply contains information about the capital gain of the asset.

Note that if a stochastic process is mean-reverting, then today's level predicts tomorrow's change: for example, if supply is high today it will most likely decrease tomorrow. In the model, having a mean-reverting supply is formally equivalent to today's discount rate level containing information about changes in future discount rates. I show that as the time-series of supply becomes more persistent, and thus discount rate changes become less predictable, the complementarities in the information market become less pronounced. In the extreme case of when supply follows a random walk the aggregate demand for information is decreasing in the number of informed agents. In this limiting case information acquisition in dynamic markets works in the same manner as in static markets.

My model features a single risky asset, but its conclusions extend to a multi-asset economy, as long as one can argue that there is asymmetric information about risk premiums. There is empirical support for the existence of private information about market-wide economic forces, which ultimately determine risk premiums. For example, Albuquerque, De Francisco, and Marques (2008) generalize the structural model of Easley, Kiefer, O'Hara, and Paperman (1996) to allow for trading in multiple stocks and for private information in two levels, firm-specific and market-wide. Their method strongly rejects a null hypothesis of no market-wide private information for the NYSE. Moreover, across eight developed international equity markets, Albuquerque, Bauer, and Schneider (2009) exhibit that a common global factor explains about 50% of the variation of trades due to private information. In addition, some theoretical studies show that even firm-specific information may affect expected returns. Lambert, Leuz, and Verrecchia (2007) use a model similar to the Capital Asset Pricing Model (CAPM) to show that higher quality of firm-specific accounting information lowers the cost of capital of firms. Finally, Armstrong, Banerjee, and Corona (2013) establish that firm-specific information can affect expected returns if it pertains to the firm's systematic factor loadings.[1]

The model applies to markets of professionals who produce information, such as analysts who

[1] In this light we can think of learning about supply as a reduced-form model of effects similar to those in Armstrong, Banerjee, and Corona (2013).

cover a particular stock. It proposes that initiations of analyst coverage might be positively related to existing analyst coverage. Extant empirical work contains evidence that can be interpreted as both in favor (Rao et al., 2001; Das et al., 2006; Chen and Ritter, 2000; Corwin and Schultz, 2005) and against (Bradley et al., 2008) complementarity, while there is also evidence against substitutability (Frankel et al., 2006). This study suggests that looking at periods with mean-reverting conditional risk premiums might help identify complementarities in analyst initiations.

The next section relates this paper to the existing literature. Section 3 describes the setup of the model and the equilibrium. Section 4 discusses and analyzes the information tradeoff, complementarities, and further results. Section 5 summarizes the paper. The supplementary internet Appendix shows how the information market works in a continuous-time extension of the model.

2. Literature review

In this paper I explain how the information market works when the financial market is intertemporal. My model builds on Grossman and Stiglitz (1980), which considers information acquisition in a single-period financial market. It also builds on Wang (1993), which describes a dynamic financial market with an exogenous number of informed agents. In environments where agents can trade before they acquire information, after which they trade again, Holden and Subrahmanyam (2002) generate momentum in asset returns and Chordia and Subrahmanyam (2004) establish predictability of returns by trade imbalances. Manela (2014) calibrates a two-trading-period Grossman-Stiglitz economy and studies how exogenous rates of information dissemination through media affect returns in the market for pharmaceutical drugs. These papers build on the concept of noisy rational expectations introduced by Lucas (1972) and further developed by Green (1973), Grossman (1976, 1978), Kihlstrom and Mirman (1975), and Grossman, Kihlstrom, and Mirman (1977).

The existing theory literature features a variety of assumptions about the mean-reversion of price noise. My model shows that these assumptions can have stark implications about the qualitative nature of information acquisition. In Allen et al. (2006) the noisy supply is independent and identically distributed (i.i.d.) over time. Other assumptions include a general AR(1) structure for noisy supply, as in Campbell and Kyle (1993), Wang (1993, 1994), and He and Wang (1995), and more recently in Cespa and Vives (2015) and Albagli (2015), among others. The random-walk assumption is used in Kyle (1985), Grundy and McNichols (1989), Brown and Jennings (1989), and Vives (1995).

This paper also relates to a growing literature about complementarities in information acquisition. My contribution here is to show that complementarities obtain when financial markets are dynamic without any further enrichments of the economy. In contrast, all of the currently known mechanisms for information complementarities are static. They include fixed costs in the information production sector as in Veldkamp (2006), relative-wealth concerns as in García and Strobl (2011), supply signals (Ganguli and Yang, 2009; Manzano and Vives, 2011), non-normal returns as in Breon-Drish (2015b), ambiguity aversion as in Mele and Sangiorgi (2015), and information acquisition in segmented markets as in Goldstein, Li, and Yang (2014). My discount-rate mechanism has empirical implications that are distinct from those of the above models: demand for information can increase whenever (i) the conditional risk premium becomes more volatile or (ii) the autocorrelation coefficient of stochastic supply, or any other kind of price noise, is low.

Another part of the literature discusses the aggregation of diverse information, such as in Hellwig (1980), Diamond and Verrecchia (1981), and Verrecchia (1982). The most related papers are Grundy and McNichols (1989), who study the volume of trade in a multiperiod economy, and Cespa and Vives (2015), who study the effect of persistent noisy supply on equilibria in the financial market, rather than in the information market, as here.

Lastly, in this study the dependence of prices on supply plays a dual role. It conveys non-dividend information about future prices and it makes prices noisy signals of dividend information. Using economic variables other than supply to replicate this role will not change the information tradeoff and the main result. Examples of such interpretations can be found in Diamond and

Verrecchia (1981), Wang (1994), and Vives (2008, Section 4.4.1).

3. The model

The economy is made up of two markets: an information market and a financial market. So as to simplify the analysis, I assume that investors make information decisions once and for all before trading begins. They decide whether to obtain information about the dividend at a fixed cost or remain uninformed for free, knowing that they will be observing prices in the financial market. All agents consume at the liquidation date of the asset, when dividends are paid out and private information is revealed. I present the economy with two trading periods, but note that as the supplementary internet Appendix shows, all the results extend to a continuous-time infinite-horizon economy with intermediate consumption and interim dividend payouts that follow a general autoregressive process. The advantage of the model I present here is that it is possible to derive a closed-form expression for the value of information.

There is a continuum of ex ante identical agents of total mass one. Each agent has constant absolute risk aversion (CARA) preferences with coefficient δ. Everyone has initial wealth W_0 which he can invest in a safe storage technology with constant net return normalized to zero and in a risky stock. There are two trading periods, $t = 1$ and $t = 2$, during which agents trade the stock but do not consume. This is followed by a consumption-only period, $t = 3$. The stock pays off a risky dividend D_3 only in the consumption period. It is known that the dividend is made up of two parts,

$$D_3 = \tilde{\mu} + \tilde{\zeta}, \tag{1}$$

where $\tilde{\mu} \sim \mathcal{N}(0, \sigma_\mu^2)$ and $\tilde{\zeta} \sim \mathcal{N}(0, \sigma_\zeta^2)$ with $\tilde{\mu}$ independent of $\tilde{\zeta}$. The value of $\tilde{\mu}$ does not change between trading periods.[2] Before the two trading periods there is an information-acquisition period,

[2]This models that dividend information is persistent. It can be modified by adding a shock to μ at $t = 1$, or by modeling the dividend information as a mean-reverting process, but only at the expense of more complicated calculations. I invite the interested reader to look at the extension of the model in the supplementary internet Appendix, where the dividend information process is generally mean-reverting.

[Fig. 1 here]

$t = 0$, during which every agent can pay κ_0 so as to observe the value of $\tilde{\mu}$ right before trading starts at $t = 1$. An agent might also decide not to pay κ_0 at the information-acquisition stage and thereby remain uninformed for the duration of trading. Thus, for uninformed agents D_3 is a random variable over which they have the prior $D_3 \sim \mathcal{N}(0, \sigma_\mu^2 + \sigma_\zeta^2)$.

At $t = 1$ the price is P_1 and at $t = 2$ the price is P_2. Because prices depend on the value of $\tilde{\mu}$, the uninformed agents use prices to update their beliefs about $\tilde{\mu}$. Let $\hat{\mu}_1$ and $\hat{\mu}_2$ be the uninformed agents' estimate of $\tilde{\mu}$ at $t = 1$ and $t = 2$.

If prices contained no more unknowns than just the mean dividend $\tilde{\mu}$, the uninformed would be able to infer the mean dividend perfectly. Thus, the presence of some kind of noise is necessary in prices. As is standard in the literature, I fill this modeling necessity by making the supply of stock at time t, $\tilde{\theta}_t$, stochastic. As I show in more detail below, stochastic supply plays a dual role: it makes prices noisy signals of dividend information, but it also conveys non-dividend information about future prices. I assume that

$$\tilde{\theta}_2 = \rho\, \tilde{\theta}_1 + \tilde{\eta}, \tag{2}$$

where I take $0 \leq \rho \leq 1$. Here, $\tilde{\theta}_1$ and $\tilde{\eta}$ are independent of each other and of $\tilde{\mu}$ and $\tilde{\zeta}$. The priors over supply are $\tilde{\theta}_1 \sim \mathcal{N}(0, \sigma_\theta^2)$ and $\tilde{\eta} \sim \mathcal{N}(0, \sigma_\eta^2)$. If an agent decides to become informed he will not only see $\tilde{\mu}$, but also the price at $t = 1$ and $t = 2$. As explained shortly, the informed agents will be able to deduce perfectly the level of supply at each point in time. At the same time, the uninformed agents will also be able to estimate this piece of information by observing prices. Let $\hat{\theta}_1$ be the uninformed agents' estimate of $\tilde{\theta}_1$ at $t = 1$ and let $\hat{\theta}_2$ be his estimate of $\tilde{\theta}_2$ at $t = 2$.

Each agent anticipates rationally the trade-and-update process at the two trading periods. In the information-acquisition period he compares the benefits of being informed versus the cost of giving up κ_0. The benefit of being informed, however, depends on the number of agents that will have decided to be informed, because the presence of informed agents influences the informativeness of prices. Let λ denote the fraction of informed agents. The information market will equilibrate at the λ at which every agent is indifferent between paying to become informed and remaining uninformed for free. It is also possible that information is too cheap, in which case the economy will equilibrate at $\lambda = 1$, or that information is too expensive, in which case the economy will equilibrate at $\lambda = 0$. See Fig. 1 for a graphical depiction of the sequence of events in the model.

3.1. Financial market equilibrium

I construct the equilibrium in the financial market in a conjecture-and-verify approach. In particular, I use a discrete-time finite-horizon version of the continuous-time steady-state equilibrium of Wang (1993). Let i denote the informed agents and u denote the uninformed agents.

Definition 3.1. A *Financial Market Equilibrium at a fraction λ of informed agents* is a pair of price functions $\left(P_1^\lambda, P_2^\lambda\right)$ such that

(a) Agents in group j, $j = i, u$, select their demand in period t based on their information set $\mathcal{F}_t^j$ so as to maximize expected utility.

(b) The price functions $\left(P_1^\lambda, P_2^\lambda\right)$ are such that in each period total demand for the stock equals total supply of the stock.

(c) Agents in group j, $j = i, u$, extract their information sets $\mathcal{F}_t^j$ rationally from the history of observed prices and any other information available to them in period t, for $t = 1, 2$.

At $t = 1$ the value function of the uninformed is $J^u(W_0, P_1^\lambda; \lambda)$ and the value function of the informed is $J^i(W_0, P_1^\lambda, \tilde{\mu}; \lambda)$. For the rest of this section I drop dependence of the value functions and prices on λ. For expositional convenience I present a summary of the construction of the equilibrium, whereas I give the details in Appendix B.

Fix the fraction $\lambda \in [0, 1]$ of informed agents. I conjecture that prices are linear in state variables:

$$P_1 = p_\mu \tilde{\mu} + p_\theta \tilde{\theta}_1 + p_{\hat{\mu}} \hat{\mu}_1 + p_{\hat{\theta}} \hat{\theta}_1, \tag{3a}$$

$$P_2 = q_\mu \tilde{\mu} + q_\theta \tilde{\theta}_2 + q_{\hat{\mu}} \hat{\mu}_2 + q_{\hat{\theta}} \hat{\theta}_2. \tag{3b}$$

The only objects in prices that do not depend on λ are $\tilde{\mu}$, $\tilde{\theta}_1$, and $\tilde{\theta}_2$. At $t = 1$ an uninformed agent observes P_1 only and at $t = 2$ he observes P_2 only, but remembers P_1. Thus, the information sets of the uninformed are the σ-algebras

$$\mathcal{F}_1^u = \sigma(P_1), \tag{4a}$$

$$\mathcal{F}_2^u = \sigma(P_1, P_2). \tag{4b}$$

The uninformed investors estimate mean dividends and supply level given their information at each point in time,

$$t = 1: \;\; \hat{\mu}_1 = \mathbb{E}\left[\mu|\mathcal{F}_1^u\right], \;\; \hat{\theta}_1 = \mathbb{E}\left[\theta_1|\mathcal{F}_1^u\right], \tag{5a}$$

$$t = 2: \;\; \hat{\mu}_2 = \mathbb{E}\left[\mu|\mathcal{F}_2^u\right], \;\; \hat{\theta}_2 = \mathbb{E}\left[\theta_2|\mathcal{F}_2^u\right]. \tag{5b}$$

The inferences $\hat{\mu}_1$ and $\hat{\theta}_1$ belong to $\mathcal{F}_1^u$ so the uninformed treat them as known. Therefore, from Eq. (3a) what the uninformed observe when they see P_1 is the price signal

$$y_1 = p_\mu \tilde{\mu} + p_\theta \tilde{\theta}_1. \tag{6}$$

The inferences $\hat{\mu}_1$ and $\hat{\theta}_1$ are linear transformations of this price signal. Similarly, at $t = 2$ what the uninformed observe when they see P_2 is the price signal

$$y_2 = q_\mu \tilde{\mu} + q_\theta \tilde{\theta}_2, \tag{7}$$

so the inferences $\hat{\mu}_2$ and $\hat{\theta}_2$ are linear combinations of this price signal and y_1.

Because every constant is known in equilibrium, observing the price signals y_1 and y_2 is equivalent to observing

$$\frac{1}{p_\theta} y_1 = p_{\mu\theta} \tilde{\mu} + \tilde{\theta}_1 \tag{8}$$

and

$$\frac{1}{q_\theta} y_2 = q_{\mu\theta} \tilde{\mu} + \tilde{\theta}_2, \tag{9}$$

where $p_{\mu\theta}$ denotes the ratio p_μ / p_θ and $q_{\mu\theta}$ denotes the ratio q_μ / q_θ. The quantities $p_{\mu\theta}$ and $q_{\mu\theta}$ measure the sensitivity of price information to dividend information.

The informed agents also observe the price signals y_1 and y_2. But because they know $\tilde{\mu}$, they effectively observe $\tilde{\theta}_1$ and $\tilde{\theta}_2$. Thus, the information sets of the informed are the σ-algebras

$$\mathcal{F}_1^i = \sigma(\tilde{\mu}, P_1) = \sigma(\tilde{\mu}, \tilde{\theta}_1), \tag{10a}$$

$$\mathcal{F}_2^i = \sigma(\tilde{\mu}, P_1, P_2) = \sigma(\tilde{\mu}, \tilde{\theta}_1, \tilde{\theta}_2). \tag{10b}$$

The information market is open only at time zero, at which time all agents are identically uninformed. Everyone's initial information set $\mathcal{F}_0^u$ is the trivial sigma algebra.

Each agent in group j, $j = i, u$, selects first-period demand x_1^j and second-period demand x_2^j to maximize the first-period expectation of third-period utility by solving a dynamic programming problem. Doing so he obtains value $J^j(W_0, S^j)$ at time one,

$$J^j(W_0, S^j) = \max_{x_1^j} \mathbb{E}\left[\max_{x_2^j} \mathbb{E}\left[-e^{-\delta c_3^j} \middle| \mathcal{F}_2^j \right] \middle| \mathcal{F}_1^j \right], \tag{11}$$

subject to

$$c_3^j = W_0 + x_2^j (D_3 - P_2) + x_1^j (P_2 - P_1), \tag{12}$$

where $S^u = P_1$ is the state vector of the uninformed and $S^i = [P_1, \tilde{\mu}]^T$ is the state vector of the informed. A discount factor does not appear above because there is only one period of consumption.

The market clears at each point in time, that is, the total agent demand equals total supply,

$$\lambda x_t^{i*} + (1 - \lambda) x_t^{u*} = \tilde{\theta}_t, \tag{13}$$

for $t = 1, 2$. Solving for the optimal agent demands, substituting into market clearing, and rearranging for prices gives an expression that I compare to the price conjecture. By matching coefficients

I obtain expressions for the information sensitivities $q_{\mu\theta}$ and $p_{\mu\theta}$.

Proposition 3.2. *In equilibrium, for a fixed fraction of informed investors λ,*

(i) The second-period information sensitivity $q_{\mu\theta}$ is

$$q_{\mu\theta} = -\frac{\lambda}{\delta\sigma_\zeta^2}. \tag{14}$$

(ii) The first-period information sensitivity $p_{\mu\theta}$ is the solution to the equation

$$\gamma_3 p_{\mu\theta}^3 + \gamma_2 p_{\mu\theta}^2 + \gamma_1 p_{\mu\theta} + \gamma_0 = 0, \tag{15}$$

where the coefficient γ_i, $i = 0, \dots 3$, is a polynomial of order $4 - i$ in $q_{\mu\theta}$.

I give γ_3, γ_2, γ_1, and γ_0 explicitly in terms of the model parameters and $q_{\mu\theta}$ in Appendix B. The next proposition summarizes results on existence and uniqueness.

Proposition 3.3. *For a fixed fraction of informed investors λ,*

(i) A financial market equilibrium at λ always exists.

(ii) Among the financial market equilibria, there always exists one such that

(a) if $\lambda = 0$, then $p_{\mu\theta} = 0$,

(b) if $\lambda > 0$, then $p_{\mu\theta} < 0$.

(iii) Within the class of linear equilibria, the financial market equilibrium at λ is unique if the discriminant of Eq. (15) *is non-positive.*

For every solution, example, and graph that I provide in this paper I have checked through the discriminant non-positivity condition that each equilibrium is unique.[3] Next I characterize the rate at which the equilibrium $p_{\mu\theta}$ changes for small numbers of informed agents.

[3] At the time of this writing, I have not found counterexamples to existence and uniqueness. Future research could attempt proving uniqueness of this linear equilibrium within the class of continuous equilibria by using the recent global uniqueness result of Breon-Drish (2015a) in backward induction.

Lemma 3.4. *Consider the first-period information sensitivity $p_{\mu\theta}$ and the second-period information sensitivity $q_{\mu\theta}$ as functions of the informed agents λ. Their derivatives at the origin are equal,*

$$\frac{d}{d\lambda}p_{\mu\theta}\Big|_{\lambda=0} = \frac{d}{d\lambda}q_{\mu\theta}\Big|_{\lambda=0}. \tag{16}$$

In fact, this lemma delivers an approximation of $p_{\mu\theta}$ as a function of λ for small λ. Whereas the solution of $q_{\mu\theta}$ is provably affine in λ, Eq. (15) shows that $p_{\mu\theta}$ does not necessarily share this property. However, from above we can see that, as functions of λ, the quantities $p_{\mu\theta}$ and $q_{\mu\theta}$ have the same intercept and the same slope at the origin. Therefore, we can informally argue that for small numbers of informed agents that $p_{\mu\theta}$ and $q_{\mu\theta}$ are roughly equal. Finally, in the special case of when supply follows a random walk I can solve for $p_{\mu\theta}$ explicitly.

Proposition 3.5. *When $\rho = 1$, $p_{\mu\theta} = q_{\mu\theta}$ for every λ.*

I finish the computation of the equilibrium by solving for the price coefficients, which are completely determined by the ratios $q_{\mu\theta}$ and $p_{\mu\theta}$. Please refer to Proposition B.4 and Lemma B.6 of Appendix B for more details.

3.2. Information market equilibrium

Having obtained the equilibrium in the financial market for each fixed fraction λ, I step one period back to $t = 0$ to endogenize λ. For an exogenous cost of information κ_0, a candidate equilibrium in the information market is a tuple of a fraction of informed investors $\lambda \in [0, 1]$ and a Financial Market Equilibrium $\left(P_1^\lambda, P_2^\lambda\right)$.

Definition 3.6. For a given cost of information κ_0,

(a) If $\lambda^* \in [0, 1]$ and $\mathbb{E}\left[J^u(W_0, P_1^{\lambda^*}; \lambda^*)\middle|\mathcal{F}_0^u\right] = \mathbb{E}\left[J^i(W_0 - \kappa_0, P_1^{\lambda^*}, \tilde{\mu}; \lambda^*)\middle|\mathcal{F}_0^u\right]$, then $\left(\lambda^*, \left(P_1^{\lambda^*}, P_2^{\lambda^*}\right)\right)$ is an *Information Market Equilibrium.*

(b) If $\mathbb{E}\left[J^u(W_0, P_1^0; 0)\middle|\mathcal{F}_0^u\right] > \mathbb{E}\left[J^i(W_0 - \kappa_0, P_1^0, \tilde{\mu}; 0)\middle|\mathcal{F}_0^u\right]$, then $\left(0, \left(P_1^0, P_2^0\right)\right)$ is an Information Market Equilibrium.

(c) If $\mathbb{E}\left[J^u(W_0, P_1^1; 1)\middle|\mathcal{F}_0^u\right] < \mathbb{E}\left[J^i(W_0 - \kappa_0, P_1^1, \tilde{\mu}; 1)\middle|\mathcal{F}_0^u\right]$, then $\left(1, \left(P_1^1, P_2^1\right)\right)$ is an Information Market Equilibrium.

At $t = 0$, before the agents decide on their information status, everyone is uninformed with wealth W_0. If an agent decides to remain uninformed he will enjoy value given by the value function J^u. If an agent decides to become informed he will pay κ_0 and he will switch value functions to J^i. But $J^u(W_0, P_1^\lambda; \lambda)$ and $J^i(W_0 - \kappa_0, P_1^\lambda, \tilde{\mu}; \lambda)$ are random variables; before an agent sees P_1^λ or $\tilde{\mu}$ he does not know what their realization will be. Thus, the comparison of values at $t = 0$ is done conditional on $\mathcal{F}_0^u$.

Each type of agent anticipates rationally the workings of the financial market and compares the benefit of being informed versus the cost of giving up κ_0. The benefit of being informed depends on the fraction λ of agents that are informed, because how many informed agents exist influences the informativeness of prices about $\tilde{\mu}$, $\tilde{\theta}_1$, and $\tilde{\theta}_2$. To facilitate the derivation of this equilibrium, I define the value of information at λ as the relative value of being informed against being uninformed.

Definition 3.7. The *value of information* $\psi_0(\lambda)$ is the relative certainty-equivalent value of being informed,

$$\mathbb{E}\left[J^u\left(W_0, P_1^\lambda; \lambda\right)\middle|\mathcal{F}_0^u\right] = \mathbb{E}\left[J^i\left(W_0 - \psi_0(\lambda), P_1^\lambda, \tilde{\mu}; \lambda\right)\middle|\mathcal{F}_0^u\right]. \tag{17}$$

Due to CARA utility I can write

$$e^{\delta\psi_0(\lambda)} = \frac{\mathbb{E}\left[J^u(W_0, P_1^\lambda; \lambda)\middle|\mathcal{F}_0^u\right]}{\mathbb{E}\left[J^i(W_0, P_1^\lambda, \tilde{\mu}; \lambda)\middle|\mathcal{F}_0^u\right]}, \tag{18}$$

and to calculate $\psi_0(\lambda)$ I need to calculate the conditional expectations of each value function conditional on prior information. The CARA-normal environment allows this calculation in closed form in moments of returns. I provide the details in Appendix C.

The cost of information acquisition, κ_0, is an exogenous parameter. The equilibrium fraction of informed agents λ^* is such that every agent finds that the value of information is the same as its cost,

$$\kappa_0 = \psi_0(\lambda^*). \tag{19}$$

If $\kappa_0 > \psi_0(0)$, then $\lambda^* = 0$ is an equilibrium and if $\kappa_0 < \psi_0(1)$, then $\lambda^* = 1$ is an equilibrium.

Thus, to determine λ^* I need to derive the entire value-of-information curve as a function of λ. I give this curve in the main theorem of the paper, where I drop dependence on λ for expositional clarity.

Theorem 3.8. *The value of information is*

$$\psi_0 = \frac{1}{2\delta}\log\frac{Var(D_3 - P_2|\mathcal{F}_2^u)}{Var\left(D_3 - P_2|\mathcal{F}_2^i\right)} + \frac{1}{2\delta}\log\frac{Var(P_2 - P_1|\mathcal{F}_1^u)}{Var\left(P_2 - P_1|\mathcal{F}_1^i\right)} + \frac{1}{2\delta}\log\frac{1 - Corr^2\left(D_3 - P_2, P_2 - P_1|\mathcal{F}_1^u\right)}{1 - Corr^2\left(D_3 - P_2, P_2 - P_1|\mathcal{F}_1^i\right)}, \quad (20)$$

where

$$Corr\left(D_3 - P_2, P_2 - P_1|\mathcal{F}_1^j\right) = \frac{Cov\left(D_3 - P_2, P_2 - P_1|\mathcal{F}_1^j\right)}{\sqrt{Var\left(D_3 - P_2|\mathcal{F}_1^j\right)\, Var\left(P_2 - P_1|\mathcal{F}_1^j\right)}}. \quad (21)$$

I provide a detailed discussion of the economics behind the value of information in Section 4.2 below. Finally, I note that if the cost of acquiring information is arbitrarily small, it is always better to be informed.

Proposition 3.9. *For* $\lambda \in [0, 1]$, $\psi_0(\lambda) > 0$.

4. Results

In this section I lay out the main results of the model. First I establish that there is a tradeoff between dividend information and supply information. This information tradeoff happens regardless of whether the market is static or dynamic, but as I explain below it only creates complementarities in information acquisition when markets are dynamic.

4.1. The information tradeoff

Proposition 4.1. *Holding* σ_μ^2 *and* σ_θ^2 *fixed, the derivatives of* $Var\left(\tilde{\theta}_1|\mathcal{F}_1^u\right)$ *and* $Var(\tilde{\mu}|\mathcal{F}_1^u)$ *with respect to any other parameter have opposite signs.*

I provide a formal proof of the information tradeoff in Appendix D whereas here I give an intuitive explanation. The total amount of information contained in prices is a weighted average of dividend information and supply information. Consider the price signal at $t = 1$, $y_1 = p_\mu \tilde{\mu} + p_\theta \tilde{\theta}_1$. It is a noisy signal of $\tilde{\mu}$ because of the presence of $\tilde{\theta}_1$, but also a noisy signal of $\tilde{\theta}_1$ because of the presence of $\tilde{\mu}$. In equilibrium, every quantity that is not a random variable is known by every investor. Therefore, when the uninformed see y_1 they can divide it by its standard deviation and thus glean the quantity

$$\frac{y_1}{\sqrt{\mathrm{Var}(y_1)}} = \frac{p_\mu \tilde{\mu} + p_\theta \tilde{\theta}_1}{\sqrt{p_\mu^2 \sigma_\mu^2 + p_\theta^2 \sigma_\theta^2}} = \frac{p_{\mu\theta} \sigma_\mu}{\sqrt{p_{\mu\theta}^2 \sigma_\mu^2 + \sigma_\theta^2}} \frac{\tilde{\mu}}{\sigma_\mu} + \frac{\sigma_\theta}{\sqrt{p_{\mu\theta}^2 \sigma_\mu^2 + \sigma_\theta^2}} \frac{\tilde{\theta}_1}{\sigma_\theta}. \tag{22}$$

This standardized information content of prices is a linear combination of two independent standard normal random variables. Notice that the squares of the coefficients of each standard normal add up to one. In this manner each coefficient measures how much weight the amount of each type of information carries in the overall amount of information contained in prices. Thus, changes in the economy that increase the amount of dividend information contained in prices decrease the amount of supply information contained in prices and vice versa.

I plot the inference qualities of mean dividends and supply in Fig. 2. As the number of informed investors in the market increases, prices become more informative about dividends and $\mathrm{Var}\left(\tilde{\mu}|\mathcal{F}_1^u\right)$ decreases. This is the classic effect of Grossman and Stiglitz (1980), whereby a higher amount of traders with access to dividend information decreases the dividend uncertainty for uninformed investors. At the same time, $\mathrm{Var}\left(\tilde{\theta}_1|\mathcal{F}_1^u\right)$ increases, which means that prices become less informative about supply. In other words, the uninformed investors' estimation of dividend information improves but their estimation of supply information worsens.

[Fig. 2 here]

4.2. Complementarities

Complementarities in information acquisition obtain when the slope of the value of information $\psi_0(\lambda)$ with respect to the number of informed traders λ is positive. That Eq. (15) is a cubic polynomial in λ presents a bottleneck of tractability in $\partial\psi_0(\lambda)/\partial\lambda$ for arbitrary λ. Nevertheless, the value of information is in closed form in moments of cash flows and prices, even though these moments are not themselves a closed form of λ. As Theorem 3.8 shows, the value that agents attach to information has three components: the relative informativeness of price histories about liquidating cash flows, the relative informativeness of first-period prices about capital gains in the second period, and a conditional correlation component. This says that information is valuable because it allows informed investors to respectively (i) make better decisions at the terminal trading date, (ii) manage their myopic demand better at the intermediate trading date, and (iii) hedge better against future changes in the investment opportunity set. I show graphs of each component of the value of information in Fig. 3.

[Fig. 3 here]

The value of information coming from information about liquidating cash flows is

$$\frac{1}{2\delta}\log\frac{\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_2^u\right)}{\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_2^i\right)}. \tag{23}$$

This component is always decreasing in the number of informed investors, following the intuition in Grossman and Stiglitz (1980) that as more informed agents enter the market, prices become more informative about dividends. Therefore, any remaining uninformed agents are less willing to pay for information. That is, other agents becoming informed is a substitute for uninformed agents acquiring information directly. But what prices can reveal about liquidating cash flows is not the only informational concern of uninformed investors. Investors also worry about how the price of the asset is going to move around over time. In other words, investors care about being able to predict the capital gain of the asset. This part of informational value is measured by

$$\frac{1}{2\delta}\log\frac{\mathrm{Var}\left(P_2 - P_1|\mathcal{F}_1^u\right)}{\mathrm{Var}\left(P_2 - P_1|\mathcal{F}_1^i\right)}. \tag{24}$$

The next proposition establishes that the relative value of information about capital gains is one-to-one with the uninformed investors' conditional variance of supply.

Proposition 4.2. *The relative informativess of first-period prices about second-period returns is*

$$\frac{Var\left(P_2 - P_1|\mathcal{F}_1^u\right)}{Var\left(P_2 - P_1|\mathcal{F}_1^i\right)} = 1 + \left(\frac{q_{\mu\theta}}{p_{\mu\theta}} - \rho\right)^2 \frac{Var\left(\tilde{\theta}_1|\mathcal{F}_1^u\right)}{\sigma_\eta^2}. \tag{25}$$

The coefficient of $\mathrm{Var}\left(\tilde{\theta}_1|\mathcal{F}_1^u\right)$ depends on the intertemporal ratio of informativeness and the value of the persistence of supply.[4] In particular, the smaller the persistence ρ is, the more supply information matters for the relative value of information about capital gains. To see why, notice that we can write the capital gain from trade as

$$P_2 - P_1 = g_\eta\ \tilde{\eta} + g_\mu\ \hat{\mu}_1 + g_\eta \left(\frac{q_{\mu\theta}}{p_{\mu\theta}} - \rho\right)\tilde{\theta}_1, \tag{26}$$

where the return coefficients g_η and g_μ are deterministic functions of λ, given in Appendix D. The contribution of $P_2 - P_1$ to the value of information comes from the relative posterior variance of each state variable conditional on first-period information. The first term on the right-hand side of Eq. (26) depends on $\tilde{\eta}$, but this is an unpredictable shock. Neither the informed nor the uninformed have any information about it after the first period. There is also dependence on $\hat{\mu}_1$, but this is common information. Hence, $\tilde{\eta}$ and $\hat{\mu}_1$ do not contribute to the relative value of information. The only part of the capital gain that matters for the value of information is supply.

In Fig. 4 I show the breakdown of the coefficient of first-period supply into two components. The first component, shown on the left, is the same as the coefficient of the shock $\tilde{\eta}$. The second component, shown on the right, is a measure of how well the supply level predicts the capital gain. The loading of the capital gain on first-period supply is the product of these two terms. Here, we can see that if supply is mean-reverting, the capital gain loads heavily on the level of first-period supply for all levels of fractions of informed agents λ.

[Fig. 4 here]

The intuition for why supply persistence affects the value of information comes by thinking in

[4] For small λ this coefficient is roughly equal to $(1-\rho)^2/\sigma_\eta^2$ by Lemma 3.4.

terms of first differences over time. Prices are driven by state variables and therefore capital gains are driven by changes in state variables. When supply is mean-reverting, the level of supply in the first period predicts the forthcoming change in supply. For example, suppose that today the level of stochastic supply is higher than its mean, so that the price level is lower than its mean. Because stochastic supply reverts to its mean, it will most likely decrease tomorrow and prices will correspondingly increase. In this case, having access to dividend information has a valuable ancillary benefit. Because agents with dividend information are able to infer the level of supply by looking at prices, they can form sharper expectations of the capital gain. In other words, having dividend information is equivalent to also having better information about the conditional expected return.[5]

In contrast, the uninformed agents have to rely on what prices convey about supply to predict the capital gain. Here, the information tradeoff becomes important. As informed agents enter the economy they make prices more informative about dividends. But at the same time they also make prices less informative about supply, and thus less informative about capital gains. As a result, the marginal effect of additional informed agents is to make the remaining uninformed agents value information about capital gains more.

The third component of the value of information is

$$\frac{1}{2\delta} \log \frac{1 - \mathrm{Corr}^2 \left(D_3 - P_2, P_2 - P_1 | \mathcal{F}_1^u\right)}{1 - \mathrm{Corr}^2 \left(D_3 - P_2, P_2 - P_1 | \mathcal{F}_1^i\right)}. \tag{27}$$

This term comes out of the hedging component of the agents' demand. We can interpret the quantity $1 - \mathrm{Corr}^2(D_3 - P_2, P_2 - P_1 | \mathcal{F}_1^j)$ for $j = i, u$, as one minus the R^2 of a regression of the liquidating return $D_3 - P_2$ on the myopic return $P_2 - P_1$, but conditional on first-period information. Whenever this R^2 increases the agents know, as of $t = 1$, that when they observe the realized myopic return at

[5]Information asymmetry about supply is formally equivalent to information asymmetry about discount rates. From Eq. (26), the conditional risk premium for the informed is $\mathbb{E}\left[P_2 - P_1 \middle| \mathcal{F}_1^i\right] = g_\mu \; \hat{\mu}_1 + g_\eta \left(\frac{q_{\mu\theta}}{p_{\mu\theta}} - \rho\right) \tilde{\theta}_1$. The uninformed know $\hat{\mu}_1$, so what stops them from having the same conditional expected returns as the informed is that they do not know $\tilde{\theta}_1$.

$t = 2$ they will have a more precise signal of the future liquidating return. This improves the agents' ability to plan out their intertemporal investment as of $t = 1$, which further allows the agents to substitute away from obtaining information. Thus, whenever the ability to hedge improves for the uninformed in relation to the informed, the value of acquiring information decreases.

As we can see in Fig. 3(c), for large numbers of informed agents and for low mean-reversion of supply, the hedging component of the value of information is decreasing in the number of informed agents λ. We can also see, however, that the opposite can be true, but only if there are few informed agents and the mean-reversion of supply is high. When the information-tradeoff is dominant, increasing the number of informed agents makes supply information less precise, which makes the myopic return a less precise signal of future liquidating returns. The uninformed thus perceive the asset as a poorer hedge, and they become more willing to purchase information. To summarize, what triggers complementarities in the hedging component is the same economic force that triggers the complementarities in the myopic component: that the supply process is mean-reverting.

In fact, when supply is highly mean-reverting, as in the case of when it is i.i.d., it is possible to prove analytically that complementarities in information acquisition are always present. To make the calculation a little easier, I calculate the slope of $e^{2\delta\psi_0(\lambda)}$ instead, which has the same monotonicity in λ.

Theorem 4.3.

$$\left.\frac{de^{2\delta\psi_0(\lambda)}}{d\lambda}\right|_{\lambda=0} = \left(\frac{\sigma_\zeta^2 + \sigma_\mu^2}{\sigma_\zeta^2}\right)^2 \frac{2\frac{\sigma_\mu^2}{\sigma_\zeta^2}}{1 + \delta^2\sigma_\eta^2(\sigma_\zeta^2 + \sigma_\mu^2)} \left[\frac{1 + \delta^2\sigma_\eta^2\sigma_\zeta^2}{1 + \delta^2\sigma_\eta^2(\sigma_\zeta^2 + \sigma_\mu^2)} - \rho\right]. \tag{28}$$

Here, the effect of changing ρ on the presence of complementarities is clear. As supply becomes less persistent, ρ decreases and the complementarity in the value of information for small numbers of informed agents becomes more pronounced. In particular, when supply is independent of its past, the value of information is increasing for small numbers of informed agents.

In contrast, in economies where supply is more persistent, the level of supply predicts the changes in supply to a lesser degree. Those who acquire dividend information are still able to infer the supply level by looking at prices. But in this case the ancillary benefit of better predicting the capital gain is not as large, and thus the complementarity in information acquisition is less pronounced. In the extreme case of when supply follows a random walk the change in supply is completely unpredictable, so there are no benefits whatsoever to knowing what the supply level is. Here, the complementarity dissipates and consequently information acquisition is all about the dividends, similarly to the static market.

I exhibit this pattern in Fig. 5, where I plot the value of information for two extreme values of ρ and one intermediate case: $\rho = 0$, $\rho = 0.5$, and $\rho = 1$. The remaining parameters of the model are $\delta = 1$, $\sigma_\zeta = 1$, $\sigma_\mu = 1$, $\sigma_\theta = 2$, and $\sigma_\eta = 1$. The value of information here is the sum of the three terms shown individually in Fig. 3. For low ρ, when few informed investors exist in the market the supply information effect dominates. As more informed investors enter the economy, they start making prices less informative about supply, pushing the value of information upwards but more informative about dividends, pushing the value of information downwards. Eventually when λ is large enough the dividend information effect dominates. As ρ increases to 0.5 the supply information effect starts to become less pronounced. For example, when $\rho = 0$ the value of information is increasing for $\lambda \leq 0.33$, but when $\rho = 0.5$, $\psi_0(\lambda)$ is increasing for $\lambda \leq 0.15$. Finally, when $\rho = 1$ the value of information is decreasing in λ because there is no supply information effect. I give more details about this special case in the next section.

[Fig. 5 here]

4.3. Persistent supply

Thus far I have argued that adding another trading period introduces complementarities in information acquisition when supply is mean-reverting. But what would happen if one of the two trading periods was informationally redundant, in the sense that I could remove all information effects from it? The economy would then be informationally equivalent to a one-period economy. We know that under the assumptions of Grossman and Stiglitz (1980) there are no information complementarities in static economies. Therefore, in dynamic economies that are informationally equivalent to a static economy there should not be any information complementarities, either. As Fig. 5 shows, this is exactly what happens when supply follows a random walk. In fact, the reason why there are no complementarities when $\rho = 1$ is precisely that there are no informational effects coming from the second period. Notice that the first-period price contains the information

$$\frac{1}{p_\theta} y_1 = p_{\mu\theta} \tilde{\mu} + \tilde{\theta}_1, \tag{29}$$

and the second-period price contains the information

$$\frac{1}{q_\theta} y_2 = q_{\mu\theta} \tilde{\mu} + \rho\, \tilde{\theta}_1 + \tilde{\eta}. \tag{30}$$

In addition, because agents learn over time, the contribution of the second period to the value of information is due to information not already seen in the first period. When agents see the information content y_2 in the second period they remember the information content y_1 in the first period, so that they can calculate any function of y_1 and y_2. In particular, they effectively observe

$$\frac{1}{q_\theta} y_2 - \frac{1}{p_\theta} y_1 = (q_{\mu\theta} - p_{\mu\theta})\, \tilde{\mu} + (\rho - 1)\, \tilde{\theta}_1 + \tilde{\eta}, \tag{31}$$

which is nothing other than the innovation to the information content of prices.

When $\rho = 1$, in which case $q_{\mu\theta} = p_{\mu\theta}$ also holds, the innovation to the information content of prices reveals the shock $\tilde{\eta}$ to the uninformed agents at $t = 2$. Moreover, the new price signal at $t = 2$ is $\frac{1}{q_\theta} y_2 = \frac{1}{p_\theta} y_1 + \tilde{\eta}$. But because the uninformed know $\tilde{\eta}$ at $t = 2$, what they glean from y_2 is the first-period information content of prices $\frac{1}{p_\theta} y_1$. In other words, the uninformed agents do not see anything about the dividend at $t = 2$ that they did not already know at $t = 1$.

There are two implications of this fact. First, the only role of any trading between the informed and the uninformed in the second period is to accommodate the incremental liquidity $\tilde{\eta}$, without any asymmetric-information effects. Second, conditional on first-period information, the capital gain $P_2 - P_1$ depends only on $\tilde{\eta}$. But $\tilde{\eta}$ is completely unpredictable by both the informed and the uninformed in the first period. Therefore, any trading considerations about the capital gain in the first period have nothing to do with information asymmetry either. In particular, the capital gain is not predictable by the level of supply in the first period, and thus the information tradeoff between $\tilde{\mu}$ and $\tilde{\theta}_1$ in the first period does not matter. Consequently, and as the next proposition establishes formally, when supply is a random walk the complementarities disappear.

Proposition 4.4. *When supply is a random walk, the value of information is always decreasing in the number of informed agents.*

5. Conclusion

To conclude, supply information is valuable in dynamic financial markets because it captures information about future prices, which are determined in equilibrium, in a way that is distinct from information about dividends, which are fixed exogenously. This implies that uninformed investors use the time-series of prices to learn information about two quantities, dividend information and supply. In this joint estimation dividends and supply act as noise with respect to each other. As a result, changes in the information market make the estimation qualities of dividends and supply move in opposite directions.

This tradeoff in estimation creates a similar tradeoff in informativeness of prices about dividends and capital gains. As more informed agents enter the economy they make prices more informative about future cash flows, but less informative about capital gains. The combination of the two effects gives a value of information that is not monotonic in the number of informed agents. This phenomenon creates multiple equilibria in the information market and makes prices fragile in perturbations of information costs.

This price fragility is often interpreted as a source of "structural breaks," or "regime switching" in prices. The intuition is that if the information decision was to be repeated over time, then the multiplicity of equilibria would translate to time-varying moments of prices. To argue this point further the iteration of the information decision must be part of the model. Moreover, in reality, information markets are dynamic environments. Because asset markets and information markets interact, if we want to understand either market we must understand intertemporal information acquisition. The dynamic model of this paper provides a foundation for such studies.

Appendix A. Some auxiliary results

Lemma A.1. *For two σ-algebrae $\mathcal{H}_1$ and $\mathcal{H}_2$ where $\mathcal{H}_1$ is contained in $\mathcal{H}_2$ and the jointly normal random variables Z and W,*

$$Cov\left(\mathbb{E}\left[Z|\mathcal{H}_2\right], \mathbb{E}\left[W|\mathcal{H}_2\right]|\mathcal{H}_1\right) = Cov\left(Z, W|\mathcal{H}_1\right) - Cov\left(Z, W|\mathcal{H}_2\right). \tag{32}$$

Proof. The law of total covariance states that for the random variables X and Y, conditional on the σ-algebra $\mathcal{G}$,

$$\mathrm{Cov}(X, Y) = \mathbb{E}\left[\mathrm{Cov}(X, Y|\mathcal{G})\right] + \mathrm{Cov}(\mathbb{E}[X|\mathcal{G}], \mathbb{E}[Y|\mathcal{G}]). \tag{33}$$

When X and Y are jointly normal the conditional covariance is constant. Thus, with $X = \mathbb{E}\left[Z|\mathcal{H}_2\right]$, $Y = \mathbb{E}\left[W|\mathcal{H}_2\right]$, and the σ-algebra $\mathcal{H}_1$, I get

$$\begin{aligned}
\mathrm{Cov}\left(\mathbb{E}\left[Z|\mathcal{H}_2\right], \mathbb{E}\left[W|\mathcal{H}_2\right]|\mathcal{H}_1\right) &= \mathrm{Cov}(\mathbb{E}\left[Z|\mathcal{H}_2\right], \mathbb{E}\left[W|\mathcal{H}_2\right]) - \mathrm{Cov}(\mathbb{E}\left[Z|\mathcal{H}_1\right], \mathbb{E}\left[W|\mathcal{H}_1\right]) \\
&= \mathrm{Cov}\left(Z, W|\mathcal{H}_1\right) - \mathrm{Cov}\left(Z, W|\mathcal{H}_2\right) \qquad (34)
\end{aligned}$$

by the law of iterated expectations and the law of total covariance. □

Lemma A.2. *For two jointly normal random variables X_1 and X_2, where $X_1 \sim \mathcal{N}(\mu_1, \sigma_1^2)$, $X_2 \sim \mathcal{N}(\mu_2, \sigma_2^2)$, and $Cov(X_1, X_2) = \sigma_{12}$,*

$$\begin{aligned}
&\mathbb{E}\big[\exp\{b_1 X_1 + b_2 X_2 + a_{11} X_1^2 + 2a_{12} X_1 X_2 + a_{22} X_2^2\}\big] = \\
&\qquad \frac{1}{S^{\frac{1}{2}}} \exp\Bigg\{\frac{1}{S}\Bigg\{\frac{1}{2}\left[b_1^2\left(\sigma_1^2 - 2a_{22}|\Sigma|\right) + 2b_1 b_2\left(\sigma_{12} + 2a_{12}|\Sigma|\right) + b_2^2\left(\sigma_2^2 - 2a_{11}|\Sigma|\right)\right] \\
&+ \mu_1\left[b_1 + 2(a_{11}b_2 - a_{12}b_1)\sigma_{12} + 2(a_{12}b_2 - a_{22}b_1)\sigma_2^2\right] + \mu_2\left[b_2 + 2(a_{12}b_1 - a_{11}b_2)\sigma_1^2 + 2(a_{22}b_1 - a_{12}b_2)\sigma_{12}\right] \\
&\qquad + \mu_1^2 a_{11}\left(1 - 2a_{22}\sigma_2^2\right) + 2\mu_1\mu_2\left(a_{12} + 2|A|\sigma_{12}\right) + \mu_2^2 a_{22}\left(1 - 2a_{11}\sigma_1^2\right)\Bigg\}\Bigg\}, \qquad (35)
\end{aligned}$$

where

$$S = |I - 2\Sigma A| = 1 - 2(a_{11}\sigma_1^2 + 2a_{12}\sigma_{12} + a_{22}\sigma_2^2) + 4|A||\Sigma|, \tag{36a}$$

$$|A| = a_{11}a_{22} - a_{12}^2, \tag{36b}$$

$$|\Sigma| = \sigma_1^2\sigma_2^2 - \sigma_{12}^2. \tag{36c}$$

Proof. This is a special case of a standard property of the multivariate normal distribution for two random variables, see, for example, Vives (2008, Section 10.2.4) and references therein. □

Lemma A.3. *The first-period inferences of the uninformed are*

$$\hat{\mu}_1 = \frac{p_{\mu\theta}\sigma_\mu^2}{p_{\mu\theta}^2\sigma_\mu^2 + \sigma_\theta^2}\left(p_{\mu\theta}\tilde{\mu} + \tilde{\theta}_1\right), \tag{37a}$$

$$\hat{\theta}_1 = \frac{\sigma_\theta^2}{p_{\mu\theta}^2\sigma_\mu^2 + \sigma_\theta^2}\left(p_{\mu\theta}\tilde{\mu} + \tilde{\theta}_1\right). \tag{37b}$$

The precisions of the first-period inferences are

$$Var\left(\tilde{\mu}|\mathcal{F}_1^u\right) = \frac{\sigma_\mu^2\sigma_\theta^2}{p_{\mu\theta}^2\sigma_\mu^2 + \sigma_\theta^2}, \tag{38a}$$

$$Cov\left(\tilde{\mu}, \tilde{\theta}_1|\mathcal{F}_1^u\right) = -p_{\mu\theta}\, Var\left(\tilde{\mu}|\mathcal{F}_1^u\right), \tag{38b}$$

$$Var\left(\tilde{\theta}_1|\mathcal{F}_1^u\right) = p_{\mu\theta}^2\, Var\left(\tilde{\mu}|\mathcal{F}_1^u\right). \tag{38c}$$

The forecast is

$$\mathbb{E}\left[\tilde{\mu}|\mathcal{F}_1^u\right] = \hat{\mu}_1, \quad (\tilde{\mu} \textit{ does not change}) \tag{39a}$$

$$\mathbb{E}\left[\tilde{\theta}_2|\mathcal{F}_1^u\right] = \rho\hat{\theta}_1. \tag{39b}$$

The second-period inferences of the uninformed are

$$\hat{\mu}_2 = \hat{\mu}_1 + (q_{\mu\theta} - \rho p_{\mu\theta})\frac{Var\left(\tilde{\mu}|\mathcal{F}_2^u\right)}{\sigma_\eta^2}\left[q_{\mu\theta}(\tilde{\mu} - \hat{\mu}_1) + \tilde{\theta}_2 - \rho\hat{\theta}_1\right], \tag{40a}$$

$$\hat{\theta}_2 = \rho\hat{\theta}_1 + \left[\frac{Var\left(\tilde{\mu}|\mathcal{F}_2^u\right)}{Var\left(\tilde{\mu}|\mathcal{F}_1^u\right)} - \rho p_{\mu\theta}(q_{\mu\theta} - \rho p_{\mu\theta})\frac{Var\left(\tilde{\mu}|\mathcal{F}_2^u\right)}{\sigma_\eta^2}\right]\left[q_{\mu\theta}(\tilde{\mu} - \hat{\mu}_1) + \tilde{\theta}_2 - \rho\hat{\theta}_1\right]. \tag{40b}$$

The precisions of the second-period inferences are

$$Var\left(\tilde{\mu}|\mathcal{F}_2^u\right) = \frac{Var\left(\tilde{\mu}|\mathcal{F}_1^u\right)\sigma_\eta^2}{Var\left(\tilde{\mu}|\mathcal{F}_1^u\right)\left(q_{\mu\theta} - \rho p_{\mu\theta}\right)^2 + \sigma_\eta^2}, \tag{41a}$$

$$Cov\left(\tilde{\mu}, \tilde{\theta}_2|\mathcal{F}_2^u\right) = -q_{\mu\theta}\, Var\left(\tilde{\mu}|\mathcal{F}_2^u\right), \tag{41b}$$

$$Var\left(\tilde{\theta}_2|\mathcal{F}_2^u\right) = q_{\mu\theta}^2\, Var\left(\tilde{\mu}|\mathcal{F}_2^u\right). \tag{41c}$$

The increase in precision over time is characterized by

$$\frac{1}{Var\left(\tilde{\mu}|\mathcal{F}_2^u\right)} = \frac{1}{Var\left(\tilde{\mu}|\mathcal{F}_1^u\right)} + \left(\frac{q_{\mu\theta} - \rho p_{\mu\theta}}{\sigma_\eta}\right)^2. \tag{42}$$

Proof. The results follow directly from the solution of the Kalman filter of the uninformed agents. □

Appendix B. Financial market

Proposition B.1. *For $j = i, u$,*

(i) The agents' second-period demand is

$$x_2^{j*} = \frac{\mathbb{E}\left[D_3 - P_2 | \mathcal{F}_2^j\right]}{\delta\, Var\left(D_3 - P_2 | \mathcal{F}_2^j\right)}. \tag{43}$$

(ii) The agents' first-period demand is

$$x_1^{j*} = \frac{\mathbb{E}\left[P_2 - P_1 | \mathcal{F}_1^j\right]}{\delta\, Var\left(P_2 - P_1 | \mathcal{F}_1^j\right)} - \beta^j \frac{\mathbb{E}\left[D_3 - P_2 - \beta^j (P_2 - P_1) | \mathcal{F}_1^j\right]}{\delta\, Var\left(D_3 - P_2 - \beta^j (P_2 - P_1) | \mathcal{F}_1^j\right)}, \tag{44}$$

where

$$\beta^j = \frac{Cov\left(D_3 - P_2, P_2 - P_1 \middle| \mathcal{F}_1^j\right)}{Var(P_2 - P_1 | \mathcal{F}_1^j)}. \tag{45}$$

Proof. The value function of each agent group $j = i, u$ is

$$J^j(W_0, S^j) = -e^{-\delta W_0} \min_{x_1^j} \left\{ e^{\delta x_1^j P_1} \mathbb{E}\left[e^{-\delta x_1^j P_2} \min_{x_2^j} \left\{ e^{\delta x_2^j P_2} \mathbb{E}\left[e^{-\delta x_2^j D_3} \middle| \mathcal{F}_2^j \right] \right\} \middle| \mathcal{F}_1^j \right] \right\}. \tag{46}$$

Let

$$E_2\left(x_2; \mathcal{F}_2^j\right) = e^{\delta x_2 P_2} \mathbb{E}\left[e^{-\delta x_2 D_3} \middle| \mathcal{F}_2^j \right], \tag{47a}$$

$$E_1\left(x_1; \mathcal{F}_1^j\right) = e^{\delta x_1 P_1} \mathbb{E}\left[e^{-\delta x_1 P_2} \min_{x_2^j} \left\{ E_2^j\left(x_2^j; \mathcal{F}_2^j\right) \right\} \middle| \mathcal{F}_1^j \right]. \tag{47b}$$

The objective function of the innermost optimization is

$$E_2\left(x_2; \mathcal{F}_2^j\right) = e^{-\delta x_2 \mathbb{E}\left[D_3 - P_2 | \mathcal{F}_2^j\right] + \frac{1}{2}\delta^2 x_2^2 \mathrm{Var}\left(D_3 - P_2 | \mathcal{F}_2^j\right)}, \tag{48}$$

so the first-order condition gives

$$x_2^{j*} = \frac{\mathbb{E}\left[D_3 - P_2 | \mathcal{F}_2^j\right]}{\delta \mathrm{Var}\left(D_3 - P_2 | \mathcal{F}_2^j\right)}, \tag{49}$$

which establishes (i). Plugging the first-order condition back into the value function shows that the objective function of the outermost optimization is

$$E_1\left(x_1;\mathcal{F}_1^j\right)=\mathbb{E}\left[e^{-\delta x_1^j(P_2-P_1)-\frac{\mathbb{E}^2\left[D_3-P_2|\mathcal{F}_2^j\right]}{2\mathrm{Var}\left(D_3-P_2|\mathcal{F}_2^j\right)}}\middle|\mathcal{F}_1^j\right]. \tag{50}$$

To carry out the calculation of this expectation, apply Lemma A.2 with $X_1=P_2-P_1$ and $X_2=\mathbb{E}\left[D_3-P_2|\mathcal{F}_2^j\right]$ conditionally on $\mathcal{F}_1^j$, i.e., with

$$\mu_1=\mathbb{E}\left[P_2-P_1|\mathcal{F}_1^j\right], \tag{51a}$$

$$\mu_2=\mathbb{E}\left[D_3-P_2|\mathcal{F}_1^j\right], \tag{51b}$$

$$\sigma_1^2=\mathrm{Var}\left(P_2-P_1|\mathcal{F}_1^j\right), \tag{51c}$$

$$\sigma_2^2=\mathrm{Var}\left(\mathbb{E}[D_3-P_2|\mathcal{F}_2^j]|\mathcal{F}_1^j\right), \tag{51d}$$

$$\sigma_{12}=\mathrm{Cov}\left(P_2-P_1,\mathbb{E}[D_3-P_2|\mathcal{F}_2^j]|\mathcal{F}_1^j\right), \tag{51e}$$

and $b_1=-\delta x_1^j$, $b_2=0$, $a_{11}=a_{12}=0$, $a_{22}=-\frac{1}{2\mathrm{Var}\left(D_3-P_2|\mathcal{F}_2^j\right)}$. Then $|A|=0$ and $S=1-2a_{22}\sigma_2^2$ so that the expectation is

$$E_1\left(x_1;\mathcal{F}_1^j\right)=\frac{1}{S^{\frac{1}{2}}}\exp\left\{\frac{1}{S}\left\{\frac{1}{2}\delta^2{x_1}^2\left(\sigma_1^2-2a_{22}|\Sigma|\right)-\delta x_1\left[(1-2a_{22}\sigma_2^2)\mu_1+2a_{22}\sigma_{12}\mu_2\right]+\mu_2^2a_{22}\right\}\right\}. \tag{52}$$

The first-order condition at $t=1$ gives

$${x_1^j}^*=\frac{(1-2a_{22}\sigma_2^2)\mu_1+2a_{22}\sigma_{12}\mu_2}{\delta\left(\sigma_1^2-2a_{22}|\Sigma|\right)}=\frac{\left(\sigma_2^2-\frac{1}{2a_{22}}\right)\mu_1-\sigma_{12}\mu_2}{\delta\left(|\Sigma|-\frac{1}{2a_{22}}\sigma_1^2\right)}. \tag{53}$$

Using Lemma A.1 I obtain

$$\sigma_{12}=\mathrm{Cov}\left(P_2-P_1,D_3-P_2|\mathcal{F}_1^j\right), \tag{54a}$$

$$\sigma_2^2-\frac{1}{2a_{22}}=\mathrm{Var}\left(\mathbb{E}[D_3-P_2|\mathcal{F}_2^j]|\mathcal{F}_1^j\right)+\mathrm{Var}\left(D_3-P_2|\mathcal{F}_2^j\right)=\mathrm{Var}\left(D_3-P_2|\mathcal{F}_1^j\right), \tag{54b}$$

$$|\Sigma|-\frac{1}{2a_{22}}\sigma_1^2=\sigma_1^2\left(\sigma_2^2-\frac{1}{2a_{22}}\right)-\sigma_{12}^2. \tag{54c}$$

Now define

$$h^j=\frac{\mathrm{Cov}\left(D_3-P_2,P_2-P_1\middle|\mathcal{F}_1^j\right)}{\mathrm{Var}(D_3-P_2|\mathcal{F}_1^j)}, \tag{55}$$

so that the first-order condition at $t = 1$ becomes

$$x_1^{j\,*} = \frac{\mu_1 - h^j \mu_2}{\delta \left(\sigma_1^2 - \frac{\sigma_{12}^2}{\mathrm{Var}(D_3 - P_2 | \mathcal{F}_1^j)} \right)}. \tag{56}$$

Moreover,

$$\begin{aligned} \sigma_1^2 - \frac{\sigma_{12}^2}{\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_1^j\right)} &= \mathrm{Var}\left(P_2 - P_1|\mathcal{F}_1^j\right) - h^j \mathrm{Cov}\left(D_3 - P_2, P_2 - P_1 \middle| \mathcal{F}_1^j\right) \\ &= \mathrm{Var}\left(P_2 - P_1 - h^j(D_3 - P_2)|\mathcal{F}_1^j\right), \end{aligned} \tag{57}$$

which establishes that

$$x_1^{j\,*} = \frac{\mathbb{E}\left[P_2 - P_1 - h^j(D_3 - P_2)|\mathcal{F}_1^j\right]}{\delta \mathrm{Var}\left(P_2 - P_1 - h^j(D_3 - P_2)|\mathcal{F}_1^j\right)}. \tag{58}$$

Result (ii) now follows by plugging the definition of β^j into the first-period demand as stated in the proposition and carrying out the algebra to show that it is equal to $x_1^{j\,*}$ as written just above.

Finally, define K_1^j to be the inverse of S. Then Lemma A.1 implies that

$$K_1^j = \frac{\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_2^j\right)}{\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_1^j\right)}. \tag{59}$$

□

Remark B.2.

$$\mathrm{Var}\left(P_2 - P_1 - h^j(D_3 - P_2)|\mathcal{F}_1^j\right) = \mathrm{Var}\left(P_2 - P_1|\mathcal{F}_1^j\right) - \left(h^j\right)^2 \mathrm{Var}\left(D_3 - P_2|\mathcal{F}_1^j\right). \tag{60}$$

Definition B.3. For $j = i, u$, let

$$\Pi_2^j = \left[\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_2^j\right)\right]^{-1}, \tag{61a}$$

$$\Pi_1^j = \left[\mathrm{Var}\left(P_2 - P_1 - h^j(D_3 - P_2)|\mathcal{F}_1^j\right)\right]^{-1}, \tag{61b}$$

be the *precisions* for each information group and let

$$\Pi_2 = \lambda \Pi_2^i + (1 - \lambda)\Pi_2^u, \tag{62a}$$

$$\Pi_1 = \lambda \Pi_1^i + (1 - \lambda)\Pi_1^u. \tag{62b}$$

be the *aggregate precisions*.

Proposition B.4. *In equilibrium, for a fixed fraction of informed investors* λ*,*

(i) The second-period price coefficients are

$$q_\mu = \lambda \frac{\Pi_2^i}{\Pi_2}, \tag{63a}$$

$$q_{\hat{\mu}} = 1 - q_\mu, \tag{63b}$$

$$q_\theta = -\frac{\delta}{\Pi_2}, \tag{63c}$$

$$q_{\hat{\theta}} = 0, \tag{63d}$$

where

$$\left(\Pi_2^i\right)^{-1} = \sigma_\zeta^2, \tag{64a}$$

$$\left(\Pi_2^u\right)^{-1} = \sigma_\zeta^2 + Var\left(\tilde{\mu}|\mathcal{F}_2^u\right). \tag{64b}$$

(ii) The first-period price coefficients are given by

$$p_\mu = \lambda \frac{\Pi_1^i}{\Pi_1}\left[1 - \left(1 + h^i\right) q_{\hat{\mu}} \frac{Var\left(\tilde{\mu}|\mathcal{F}_2^u\right)}{Var\left(\tilde{\mu}|\mathcal{F}_1^u\right)}\right], \tag{65a}$$

$$p_\theta = \lambda \frac{\Pi_1^i}{\Pi_1}\left(1 + h^i\right)\rho q_\theta - \frac{\delta}{\Pi_1}, \tag{65b}$$

$$p_{\hat{\mu}} = 1 - p_\mu, \tag{65c}$$

$$p_{\hat{\theta}} = (1-\lambda)\frac{\Pi_1^u}{\Pi_1}\left(1 + h^u\right)\rho q_\theta, \tag{65d}$$

where, in addition, $p_\theta < 0$*, and, if the discriminant of Eq.* (15) *is non-positive, then* $p_\mu \geq 0$*.*

Proof. Establishing the expressions for the second-period precisions Π_2^i and Π_2^u is straightforward. Second-period optimal demands, Eq. (3b), and coefficient-matching in second-period market clearing establish part (i). Next, notice that $\mathbb{E}\left[D_3|\mathcal{F}_1^i\right] = \tilde{\mu}$, $\mathbb{E}\left[D_3|\mathcal{F}_1^u\right] = \hat{\mu}_1$, and that from Eq. (42)

$$\mathbb{E}\left[P_2|\mathcal{F}_1^i\right] = \bar{q}_\mu\tilde{\mu} + \bar{q}_{\hat{\mu}}\hat{\mu}_1 + \rho q_\theta\tilde{\theta}_1, \tag{66}$$

where

$$\bar{q}_\mu = q_\mu + q_{\hat{\mu}}\frac{\sigma_\mu^2\sigma_\theta^2(q_{\mu\theta} - \rho p_{\mu\theta})^2}{\sigma_\mu^2\sigma_\theta^2(q_{\mu\theta} - \rho p_{\mu\theta})^2 + \sigma_\eta^2(p_{\mu\theta}^2\sigma_\mu^2 + \sigma_\theta^2)} = q_\mu + q_{\hat{\mu}}(q_{\mu\theta} - \rho p_{\mu\theta})^2\frac{\mathrm{Var}\left(\tilde{\mu}|\mathcal{F}_2^u\right)}{\sigma_\eta^2}, \tag{67a}$$

$$\bar{q}_{\hat{\mu}} = q_{\hat{\mu}}\frac{\sigma_\eta^2(p_{\mu\theta}^2\sigma_\mu^2 + \sigma_\theta^2)}{\sigma_\mu^2\sigma_\theta^2(q_{\mu\theta} - \rho p_{\mu\theta})^2 + \sigma_\eta^2(p_{\mu\theta}^2\sigma_\mu^2 + \sigma_\theta^2)} = 1 - \bar{q}_\mu = q_{\hat{\mu}}\frac{\mathrm{Var}\left(\tilde{\mu}|\mathcal{F}_2^u\right)}{\mathrm{Var}\left(\tilde{\mu}|\mathcal{F}_1^u\right)}. \tag{67b}$$

Using iterated conditional expectations I get

$$\mathbb{E}\left[P_2|\mathcal{F}_1^u\right] = \hat{\mu}_1 + \rho q_\theta \hat{\theta}_1. \tag{68}$$

The above conditional expectations, first-period optimal demands, Eq. (3a), and coefficient-matching in first-period market clearing give

$$\Pi_1 p_\mu = \lambda \Pi_1^i \left[\left(1+h^i\right)\bar{q}_\mu - h^i\right], \tag{69a}$$

$$\Pi_1 p_\theta = \lambda \Pi_1^i \left(1+h^i\right)\rho q_\theta - \delta, \tag{69b}$$

$$\Pi_1 p_{\hat{\mu}} = \lambda \Pi_1^i \left(1+h^i\right)\bar{q}_{\hat{\mu}} + (1-\lambda)\Pi_1^u, \tag{69c}$$

$$\Pi_1 p_{\hat{\theta}} = (1-\lambda)\Pi_1^u \left(1+h^u\right)\rho q_\theta, \tag{69d}$$

where I note that

$$p_\mu + p_{\hat{\mu}} = 1. \tag{70}$$

The last five equations together with Eq. (42) establish part (ii). Finally, I establish that $p_\theta < 0$ and $p_\mu \geq 0$. First, solving Eq. (63c) for q_θ and plugging into Eq. (69b) gives

$$p_\theta = -\frac{\delta}{\Pi_1}\left[\frac{\lambda \Pi_1^i}{\Pi_2}\left(1+h^i\right)\rho + 1\right], \tag{71}$$

where $\Pi_2 > 0$, $\Pi_1^i > 0$, and $\Pi_1 > 0$ because they are either precisions or convex combinations of precisions, and $1 + h^i > 0$ from Lemma B.6 below. Thus, $p_\theta < 0$. In addition, whenever an equilibrium exists and the discriminant of Eq. (15) is non-positive, Proposition 3.3 implies that $p_\mu/p_\theta \leq 0$, which implies that $p_\mu \geq 0$. □

Remark B.5. The above computes the values of the price coefficient ratios $p_{\mu\theta}$ and $q_{\mu\theta}$, which are sufficient to construct the equilibrium. The above is also sufficient for computation of the price coefficients in period $t = 2$. To complete the computation of the price coefficients for the period $t = 1$, it is very helpful to use an alternative representation of prices. Since $\hat{\mu}_2$ and $\hat{\theta}_2$ are linear combinations of y_1 and y_2, $\hat{\mu}_2$ and $\hat{\theta}_2$ are linear combinations of $\tilde{\mu}$, $\tilde{\theta}_1$, and $\tilde{\theta}_2$. As a result the price representation Eq. (3) is equivalent to

$$P_1 = \varphi_\mu \tilde{\mu} + \varphi_\theta \tilde{\theta}_1, \tag{72a}$$

$$P_2 = \chi_\mu \tilde{\mu} + \chi_{\theta_2}\tilde{\theta}_2 + \chi_{\theta_1}\tilde{\theta}_1, \tag{72b}$$

where φ_μ and φ_θ are linear combinations of the price coefficients in Eq. (3a) and χ_μ, χ_{θ_2}, and χ_{θ_1} are linear combinations of the price coefficients in Eq. (3b).

Lemma B.6. *The first-period precisions are given by*

$$\left(\Pi_1^i\right)^{-1} = \left(1+h^i\right)^2 {\chi_{\theta 2}}^2 \sigma_\eta^2 + {h^i}^2 \sigma_\zeta^2, \tag{73a}$$

$$
\begin{aligned}
\left(\Pi_1^u\right)^{-1} &= \left(1+h^u\right)^2 {\chi_{\theta 2}}^2 \sigma_\eta^2 + {h^u}^2 \sigma_\zeta^2 && (73b)\\
&+ \left[\left(1+h^u\right)\chi_\mu - h^u\right]^2 Var\left(\tilde{\mu}|\mathcal{F}_1^u\right) + \left(1+h^u\right)^2 \left(\rho\chi_{\theta 2}+\chi_{\theta 1}\right)^2 Var\left(\tilde{\theta}_1|\mathcal{F}_1^u\right) && (73c)\\
&+ 2\left[\left(1+h^u\right)\chi_\mu - h^u\right]\left(1+h^u\right)\left(\rho\chi_{\theta 2}+\chi_{\theta 1}\right) Cov\left(\tilde{\mu},\tilde{\theta}_1|\mathcal{F}_1^u\right). && (73d)
\end{aligned}
$$

The first-period conditional moments are

$$
\begin{aligned}
Cov\left(D_3-P_2, P_2-P_1\middle|\mathcal{F}_1^i\right) &= -\chi_{\theta 2}^2\sigma_\eta^2, && (74a)\\
Var\left(P_2-P_1\middle|\mathcal{F}_1^i\right) &= \chi_{\theta 2}^2\sigma_\eta^2, && (74b)\\
Var\left(D_3-P_2\middle|\mathcal{F}_1^i\right) &= \sigma_\zeta^2 + \chi_{\theta 2}^2\sigma_\eta^2, && (74c)
\end{aligned}
$$

and

$$
\begin{aligned}
Cov\left(D_3-P_2, P_2-P_1\middle|\mathcal{F}_1^u\right) &= -\chi_{\theta 2}^2\sigma_\eta^2\\
&+ \left[\chi_\mu - p_{\mu\theta}\left(\rho\chi_{\theta 2}+\chi_{\theta 1}\right)\right]\left[1-\chi_\mu + p_{\mu\theta}\left(\rho\chi_{\theta 2}+\chi_{\theta 1}\right)\right] Var\left(\tilde{\mu}|\mathcal{F}_1^u\right), && (74d)\\
Var\left(P_2-P_1|\mathcal{F}_1^u\right) &= \chi_{\theta 2}^2\sigma_\eta^2 + \left[\chi_\mu - p_{\mu\theta}\left(\rho\chi_{\theta 2}+\chi_{\theta 1}\right)\right]^2 Var\left(\tilde{\mu}|\mathcal{F}_1^u\right), && (74e)\\
Var\left(D_3-P_2\middle|\mathcal{F}_1^u\right) &= \sigma_\zeta^2 + \chi_{\theta 2}^2\sigma_\eta^2 + \left[1-\chi_\mu + p_{\mu\theta}\left(\rho\chi_{\theta 2}+\chi_{\theta 1}\right)\right]^2 Var\left(\tilde{\mu}|\mathcal{F}_1^u\right), && (74f)
\end{aligned}
$$

where

$$
\begin{aligned}
\chi_\mu &= 1 - q_{\hat{\mu}}\frac{Var\left(\tilde{\mu}|\mathcal{F}_2^u\right)}{\sigma_\mu^2}, && (75a)\\
\rho\chi_{\theta 2}+\chi_{\theta 1} &= \rho q_\theta + q_{\hat{\mu}} p_{\mu\theta}\frac{Var\left(\tilde{\mu}|\mathcal{F}_2^u\right)}{\sigma_\theta^2}, && (75b)\\
\chi_{\theta 2} &= q_\theta + q_{\hat{\mu}}\left(q_{\mu\theta} - \rho p_{\mu\theta}\right)\frac{Var\left(\tilde{\mu}|\mathcal{F}_2^u\right)}{\sigma_\eta^2}. && (75c)
\end{aligned}
$$

Proof. These expressions follow by Eq. (3), Eq. (72), the solution to the Kalman filter, and omitted algebraic manipulations. The hedging coefficients h^i and h^u are given by the above and definition Eq. (55). □

Proof of Proposition 3.2. Divide Eq. (63a) by Eq. (63c) to get

$$
q_{\mu\theta} = -\frac{\lambda\Pi_2^i}{\delta} = -\frac{\lambda}{\delta\sigma_\zeta^2}, \tag{76}
$$

which establishes part (i). Next, divide Eq. (69a) by Eq. (69b) to get

$$
p_{\mu\theta} = \frac{\lambda\Pi_1^i\left[\left(1+h^i\right)\bar{q}_\mu - h^i\right]}{\lambda\Pi_1^i\left(1+h^i\right)\rho q_\theta - \delta}. \tag{77}
$$

Rearranging this and carrying out the algebra gives

$$\begin{aligned}&\left[\delta q_{\mu\theta}^2\left(q_{\mu\theta}-\rho p_{\mu\theta}\right)+\delta\frac{\sigma_\eta^2}{\sigma_\zeta^2}(q_{\mu\theta}-p_{\mu\theta})-\left(\frac{1}{\sigma_\zeta^2}+\delta q_{\mu\theta}\right)\left(q_{\mu\theta}-\rho p_{\mu\theta}\right)\left(q_{\mu\theta}-p_{\mu\theta}\right)\right]\\&=\left[q_{\mu\theta}\left(q_{\mu\theta}-\rho p_{\mu\theta}\right)-\delta\sigma_\eta^2\left(q_{\mu\theta}-p_{\mu\theta}\right)\right]\left[\frac{\left(q_{\mu\theta}-\rho p_{\mu\theta}\right)^2}{\sigma_\eta^2}+\frac{p_{\mu\theta}^2\sigma_\mu^2+\sigma_\theta^2}{\sigma_\mu^2\sigma_\theta^2}\right].\end{aligned} \tag{78}$$

Further algebraic manipulations show that Eq. (78) can be written as

$$\gamma_3 p_{\mu\theta}^3+\gamma_2 p_{\mu\theta}^2+\gamma_1 p_{\mu\theta}+\gamma_0=0, \tag{79}$$

where

$$\gamma_3=\sigma_\mu^2\left(\rho q_{\mu\theta}-\delta\sigma_\eta^2\right)\left(\rho^2\sigma_\theta^2+\sigma_\eta^2\right), \tag{80a}$$

$$\gamma_2=\frac{\sigma_\mu^2}{\sigma_\zeta^2}\left\{-\sigma_\zeta^2(\sigma_\eta^2+3\rho^2\sigma_\theta^2)q_{\mu\theta}^2+\delta\sigma_\zeta^2\sigma_\eta^2\left(\sigma_\eta^2+\rho\left(1+\rho\right)\sigma_\theta^2\right)q_{\mu\theta}-\rho\sigma_\theta^2\sigma_\eta^2\right\}, \tag{80b}$$

$$\gamma_1=\frac{\sigma_\theta^2}{\sigma_\zeta^2}\left\{3\rho\sigma_\zeta^2\sigma_\mu^2q_{\mu\theta}^3-2\delta\rho\sigma_\eta^2\sigma_\zeta^2\sigma_\mu^2q_{\mu\theta}^2+\sigma_\eta^2\left(\sigma_\mu^2+\rho\left(\sigma_\zeta^2+\sigma_\mu^2\right)\right)q_{\mu\theta}-\delta\sigma_\eta^4\left(\sigma_\zeta^2+\sigma_\mu^2\right)\right\}, \tag{80c}$$

$$\gamma_0=-\frac{\sigma_\theta^2}{\sigma_\zeta^2}q_{\mu\theta}\left(q_{\mu\theta}-\delta\sigma_\eta^2\right)\left(\sigma_\eta^2\sigma_\mu^2+\sigma_\zeta^2(\sigma_\eta^2+\sigma_\mu^2q_{\mu\theta}^2)\right), \tag{80d}$$

which proves part (ii). □

Proof of Proposition 3.3.

(i) A financial market equilibrium exists when Eq. (79) has a real root. Because Eq. (79) is a cubic polynomial in $p_{\mu\theta}$, it always has at least one real root.

(ii) (a) When $\lambda = 0$, $q_{\mu\theta} = 0$ so $\gamma_0 = 0$ and therefore one of the solutions of Eq. (79) is $p_{\mu\theta} = 0$.

(b) When $\lambda > 0$, $q_{\mu\theta} < 0$ and $\gamma_0, \gamma_1, \gamma_2, \gamma_3 < 0$. By Descartes' rule of signs the polynomial in Eq. (79) has no positive roots, but from above it has at least one real root. Therefore, that root must be negative.

(iii) The financial market equilibrium is unique when Eq. (79) has a unique real root. This is true when the discriminant of the polynomial in Eq. (79), $\Delta^{p_{\mu\theta}}$, is non-positive. The discriminant is

$$\Delta^{p_{\mu\theta}}=18\gamma_3\gamma_2\gamma_1\gamma_0-4\gamma_2^3\gamma_0+\gamma_2^2\gamma_1^2-4\gamma_3\gamma_1^3-27\gamma_3^2\gamma_0^2. \tag{81}$$

□

Proof of Lemma 3.4. By the Implicit Function Theorem and Eq. (15) I get that

$$\frac{d}{dq_{\mu\theta}}p_{\mu\theta} = -\left(p_{\mu\theta}^3\frac{\partial}{\partial q_{\mu\theta}}\gamma_3 + p_{\mu\theta}^2\frac{\partial}{\partial q_{\mu\theta}}\gamma_2 + p_{\mu\theta}\frac{\partial}{\partial q_{\mu\theta}}\gamma_1 + \frac{\partial}{\partial q_{\mu\theta}}\gamma_0\right)\left(3p_{\mu\theta}^2\gamma_3 + 2p_{\mu\theta}\gamma_2 + \gamma_1\right)^{-1}. \tag{82}$$

Recall from Propositions 3.2 and 3.3 that $q_{\mu\theta}\big|_{\lambda=0} = 0$ and $p_{\mu\theta}\big|_{\lambda=0} = 0$. Thus,

$$\frac{d}{dq_{\mu\theta}}p_{\mu\theta}\Big|_{\lambda=0} = -\left(\frac{\partial}{\partial q_{\mu\theta}}\gamma_0\big|_{\lambda=0}\right)\left(\gamma_1\big|_{\lambda=0}\right)^{-1} = -\left(\delta\sigma_\eta^4\sigma_\theta^2\frac{\sigma_\zeta^2+\sigma_\mu^2}{\sigma_\zeta^2}\right)\left(-\delta\sigma_\eta^4\sigma_\theta^2\frac{\sigma_\zeta^2+\sigma_\mu^2}{\sigma_\zeta^2}\right)^{-1} = 1. \tag{83}$$

The result now follows by the chain rule of differentiation with respect to λ. □

Proof of Proposition 3.5. The result follows by inspection of Eq. (78). □

Appendix C. Information market

Proof of Theorem 3.8. The value of information is

$$e^{\delta\psi_0(\lambda)} = \frac{\mathbb{E}\left[E_1\left(x_1^{u*};\mathcal{F}_1^u\right)\Big|\mathcal{F}_0^u\right]}{\mathbb{E}\left[\mathbb{E}\left[E_1\left(x_1^{i*};\mathcal{F}_1^i\right)\Big|\mathcal{F}_0^i\right]\Big|\mathcal{F}_0^u\right]} = \frac{\mathbb{E}\left[E_1\left(x_1^{u*};\mathcal{F}_1^u\right)\Big|\mathcal{F}_0^u\right]}{\mathbb{E}\left[E_1\left(x_1^{i*};\mathcal{F}_1^i\right)\Big|\mathcal{F}_0^u\right]}, \tag{84}$$

where the last equality follows from the law of iterated expectations. I need to calculate two conditional expectations, which are very similar. For $j = i, u$, plugging x_1^{j*} back into the value function gives

$$\begin{aligned}E_1\left(x_1^{j*};\mathcal{F}_1^j\right) = \sqrt{K_1^j}\exp\Bigg\{&-\frac{1}{2\mathrm{Var}\left(P_2-P_1-h^j(D_3-P_2)|\mathcal{F}_1^j\right)}\\ &\Big\{\mathbb{E}^2\left[P_2-P_1|\mathcal{F}_1^j\right] - 2h^j\mathbb{E}\left[P_2-P_1|\mathcal{F}_1^j\right]\mathbb{E}\left[D_3-P_2|\mathcal{F}_1^j\right]\\ &+\frac{\mathrm{Var}\left(P_2-P_1|\mathcal{F}_1^j\right)}{\mathrm{Var}\left(D_3-P_2|\mathcal{F}_1^j\right)}\mathbb{E}^2\left[D_3-P_2|\mathcal{F}_1^j\right]\Big\}\Bigg\}.\end{aligned} \tag{85}$$

To calculate the conditional expectation of $E_1\left(x_1^{j*};\mathcal{F}_1^j\right)$, apply Lemma A.2 for $j = i, u$ with $X_1 = \mathbb{E}\left[P_2-P_1|\mathcal{F}_1^j\right]$ and $X_2 = \mathbb{E}\left[D_3-P_2|\mathcal{F}_1^j\right]$ conditionally on $\mathcal{F}_0^u$. Because ex ante all random variables have zero means

$$\mu_1 = \mathbb{E}\left[\mathbb{E}\left[P_2-P_1|\mathcal{F}_1^j\right]\Big|\mathcal{F}_0^u\right] = 0, \tag{86a}$$

$$\mu_2 = \mathbb{E}\left[\mathbb{E}\left[D_3-P_2|\mathcal{F}_1^j\right]\Big|\mathcal{F}_0^u\right] = 0. \tag{86b}$$

Moreover, take

$$\sigma_1^2 = \mathrm{Var}\left(\mathbb{E}\left[P_2 - P_1|\mathcal{F}_1^j\right]\Big|\mathcal{F}_0^u\right), \tag{87a}$$

$$\sigma_2^2 = \mathrm{Var}\left(\mathbb{E}\left[D_3 - P_2|\mathcal{F}_1^j\right]\Big|\mathcal{F}_0^u\right), \tag{87b}$$

$$\sigma_{12} = \mathrm{Cov}\left(\mathbb{E}\left[P_2 - P_1|\mathcal{F}_1^j\right], \mathbb{E}\left[D_3 - P_2|\mathcal{F}_1^j\right]\Big|\mathcal{F}_0^u\right). \tag{87c}$$

and

$$b_1 = b_2 = 0, \tag{88a}$$

$$a_{11} = -\frac{1}{2\mathrm{Var}\left(P_2 - P_1 - h^j(D_3 - P_2)|\mathcal{F}_1^j\right)}, \tag{88b}$$

$$a_{12} = -h^j a_{11}, \tag{88c}$$

$$a_{22} = \frac{\mathrm{Var}\left(P_2 - P_1|\mathcal{F}_1^j\right)}{\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_1^j\right)} a_{11}. \tag{88d}$$

Then

$$|A| = a_{11}^2\left[\frac{\mathrm{Var}\left(P_2 - P_1|\mathcal{F}_1^j\right)}{\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_1^j\right)} - {h^j}^2\right] = -\frac{a_{11}}{2\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_1^j\right)}, \tag{89}$$

where the last equality follows from the variance decomposition in Remark B.2 and as a result

$$\begin{aligned}
S &= 1 - 2a_{11}\left[\sigma_1^2 - 2h^j\sigma_{12} + \frac{\mathrm{Var}\left(P_2 - P_1|\mathcal{F}_1^j\right)}{\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_1^j\right)}\sigma_2^2\right] - 2a_{11}\frac{\sigma_1^2\sigma_2^2 - \sigma_{12}^2}{\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_1^j\right)} \\
&= 1 + \frac{\sigma_1^2 - 2h^j\sigma_{12} + {h^j}^2\sigma_2^2}{\mathrm{Var}\left(P_2 - P_1 - h^j(D_3 - P_2)|\mathcal{F}_1^j\right)} + \frac{\left(\frac{\mathrm{Var}\left(P_2 - P_1|\mathcal{F}_1^j\right)}{\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_1^j\right)} - {h^j}^2\right)\sigma_2^2}{\mathrm{Var}\left(P_2 - P_1 - h^j(D_3 - P_2)|\mathcal{F}_1^j\right)} \\
&\quad + \frac{\sigma_1^2\sigma_2^2 - \sigma_{12}^2}{\mathrm{Var}\left(P_2 - P_1 - h^j(D_3 - P_2)|\mathcal{F}_1^j\right)\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_1^j\right)} \\
&= \frac{\mathrm{Var}\left(P_2 - P_1|\mathcal{F}_0^u\right)\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_0^u\right)}{\mathrm{Var}\left(P_2 - P_1 - h^j(D_3 - P_2)|\mathcal{F}_1^j\right)\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_1^j\right)}.
\end{aligned} \tag{90}$$

The last equality follows from Remark B.2, the definition of h^j, and Lemma A.1. Now define K_0^{ju} to be the inverse of S for $j = i, u$ as in the last equation. Applying Lemma A.2 gives that the value of information is

$$\begin{aligned} e^{\delta\psi_0(\lambda)} &= \frac{\mathbb{E}\left[E_1\left(x_1^{u*};\mathcal{F}_1^u\right)\Big|\mathcal{F}_0^u\right]}{\mathbb{E}\left[E_1\left(x_1^{i*};\mathcal{F}_1^i\right)\Big|\mathcal{F}_0^u\right]} = \sqrt{\frac{K_1^u}{K_1^i}}\sqrt{\frac{K_0^{uu}}{K_0^{iu}}} \\ &= \frac{\mathrm{Var}(D_3 - P_2|\mathcal{F}_2^u)}{\mathrm{Var}(D_3 - P_2|\mathcal{F}_2^i)}\frac{\mathrm{Var}\left(P_2 - P_1 - h^u(D_3 - P_2)|\mathcal{F}_1^u\right)}{\mathrm{Var}\left(P_2 - P_1 - h^i(D_3 - P_2)|\mathcal{F}_1^i\right)}. \end{aligned} \tag{91}$$

From this expression, different writings of the value of information can be obtained, by either using Remark B.2 or by writing h^j in terms of β^j for $j = i, u$ and carrying out the algebra. □

Proof of Proposition 3.9. Lemma B.6 yields that $h^i < h^u < 0$, thus $-\left(h^i\right)^2 < -\left(h^u\right)^2 < 0$. By Lemma A.1, $\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_2^u\right) > \mathrm{Var}\left(D_3 - P_2|\mathcal{F}_2^i\right)$, $\mathrm{Var}\left(D_3 - P_2|\mathcal{F}_1^u\right) > \mathrm{Var}\left(D_3 - P_2|\mathcal{F}_1^i\right)$, and $\mathrm{Var}\left(P_2 - P_1|\mathcal{F}_1^u\right) > \mathrm{Var}\left(P_2 - P_1|\mathcal{F}_1^i\right)$. Remark B.2 and Theorem 3.8 now imply that $\psi_0(\lambda) > 0$ for any $\lambda \in [0, 1]$. □

Appendix D. Information tradeoffs and complementarities

Lemma D.1. *The coefficients of the capital gain $P_2 - P_1$ are*

$$g_\eta = q_\theta + q_{\hat{\mu}}(q_{\mu\theta} - \rho p_{\mu\theta})\frac{Var(\tilde{\mu}|\mathcal{F}_2^u)}{\sigma_\eta^2}, \tag{92a}$$

$$g_\mu = \left[(q_{\mu\theta} - \rho p_{\mu\theta})g_\eta + p_{\mu\theta}\left(\rho q_\theta - (p_\theta + p_{\hat{\theta}})\right)\right]\frac{\sigma_\theta^2}{p_{\mu\theta}^2\sigma_\mu^2}. \tag{92b}$$

Proof. This follows from Proposition B.4, the solution to the Kalman filter in Lemma A.3, and omitted algebra. □

Proof of Proposition 4.1. Combining the expressions for $\mathrm{Var}\left(\tilde{\mu}|\mathcal{F}_1^u\right)$ and $\mathrm{Var}\left(\tilde{\theta}_1|\mathcal{F}_1^u\right)$ from the Kalman filter I get

$$\mathrm{Var}\left(\tilde{\theta}_1|\mathcal{F}_1^u\right) = \sigma_\theta^2\left[1 - \frac{\mathrm{Var}\left(\tilde{\mu}|\mathcal{F}_1^u\right)}{\sigma_\mu^2}\right], \tag{93}$$

which establishes the result. □

Proof of Proposition 4.2. Eq. (26) and Lemma D.1 give

$$\frac{\mathrm{Var}\left(P_2 - P_1|\mathcal{F}_1^u\right)}{\mathrm{Var}\left(P_2 - P_1|\mathcal{F}_1^i\right)} = \frac{g_\eta^2\sigma_\eta^2 + g_\eta^2\left(\frac{q_{\mu\theta}}{p_{\mu\theta}} - \rho\right)^2\mathrm{Var}\left(\tilde{\theta}_1|\mathcal{F}_1^u\right)}{g_\eta^2\sigma_\eta^2} = 1 + \left(\frac{q_{\mu\theta}}{p_{\mu\theta}} - \rho\right)^2\frac{\mathrm{Var}\left(\tilde{\theta}_1|\mathcal{F}_1^u\right)}{\sigma_\eta^2}. \tag{94}$$

The state variable $\hat{\mu}_1$ is common information in the first period, so it does not contribute anything to the conditional variance of capital gains. □

Proof of Theorem 4.3. Total differentiation of $e^{2\delta\psi_0(\lambda)}$ gives

$$\frac{de^{2\delta\psi_0(\lambda)}}{d\lambda} = \frac{\partial e^{2\delta\psi_0(\lambda)}}{\partial\lambda} + \frac{\partial e^{2\delta\psi_0(\lambda)}}{\partial q_{\mu\theta}}\frac{dq_{\mu\theta}}{d\lambda} + \frac{\partial e^{2\delta\psi_0(\lambda)}}{\partial p_{\mu\theta}}\frac{dp_{\mu\theta}}{d\lambda}. \tag{95}$$

Evaluating each partial derivative of the right-hand side above at $\lambda = 0$ yields

$$\left.\frac{\partial e^{2\delta\psi_0(\lambda)}}{\partial\lambda}\right|_{\lambda=0} = 2\frac{\sigma_\mu^2(\sigma_\zeta^2+\sigma_\mu^2)}{\sigma_\zeta^4\left[1+\delta^2\sigma_\eta^2(\sigma_\zeta^2+\sigma_\mu^2)\right]}, \tag{96}$$

$$\left.\frac{\partial e^{2\delta\psi_0(\lambda)}}{\partial q_{\mu\theta}}\right|_{\lambda=0} = -2\delta\frac{\sigma_\mu^4(\sigma_\zeta^2+\sigma_\mu^2)}{\sigma_\zeta^4\left[1+\delta^2\sigma_\eta^2(\sigma_\zeta^2+\sigma_\mu^2)\right]^2}, \tag{97}$$

$$\left.\frac{\partial e^{2\delta\psi_0(\lambda)}}{\partial p_{\mu\theta}}\right|_{\lambda=0} = 2\rho\delta\frac{\sigma_\mu^2(\sigma_\zeta^2+\sigma_\mu^2)^2}{\sigma_\zeta^4\left[1+\delta^2\sigma_\eta^2(\sigma_\zeta^2+\sigma_\mu^2)\right]}. \tag{98}$$

The intermediate algebraic manipulations are trivial but very long and are thus omitted. Combining the above with Lemma 3.4 and carrying out the algebra establishes the result. □

Proof of Proposition 4.4. When $\rho = 1$, Propositions 3.5 and 4.2 imply that

$$q_{\mu\theta} - \rho p_{\mu\theta} = 0, \tag{99}$$

$$\mathrm{Var}\left(\tilde{\mu}|\mathcal{F}_2^u\right) = \mathrm{Var}\left(\tilde{\mu}|\mathcal{F}_1^u\right), \tag{100}$$

$$\frac{\mathrm{Var}\left(P_2 - P_1|\mathcal{F}_1^u\right)}{\mathrm{Var}\left(P_2 - P_1|\mathcal{F}_1^i\right)} = 1. \tag{101}$$

These relations, together with Lemma B.6, imply that the value of information in Theorem 3.8 is

$$e^{2\delta\psi_0} = \left(\frac{\sigma_\zeta^2 + \mathrm{Var}\left(\tilde{\mu}|\mathcal{F}_1^u\right)}{\sigma_\zeta^2}\right)^2 \frac{\sigma_\zeta^2 + q_\theta^2\sigma_\eta^2}{\sigma_\zeta^2 + \mathrm{Var}\left(\tilde{\mu}|\mathcal{F}_1^u\right) + q_\theta^2\sigma_\eta^2}. \tag{102}$$

Here, p_θ and $\mathrm{Var}\left(\tilde{\mu}|\mathcal{F}_1^u\right)$ are functions of λ. Taking the derivative of $e^{2\delta\psi_0}$ with respect to λ and carrying out the algebra shows that the value of information is decreasing in λ. □

References

Albagli, E., 2015. Investment horizons and asset prices under asymmetric information. Journal of Economic Theory 158, Part B, 787–837.

Albuquerque, R., Bauer, G. H., Schneider, M., 2009. Global private information in international equity markets. Journal of Financial Economics 94, 18–46.

Albuquerque, R., De Francisco, E., Marques, L. B., 2008. Marketwide private information in stocks: Forecasting currency returns. The Journal of Finance 63, 2297–2343.

Allen, F., Morris, S., Shin, H. S., 2006. Beauty contests and iterated expectations in asset markets. Review of Financial Studies 19, 719–752.

Armstrong, C. S., Banerjee, S., Corona, C., 2013. Factor-loading uncertainty and expected returns. Review of Financial Studies 26, 158–207.

Bradley, D. J., Jordan, B. D., Ritter, J. R., 2008. Analyst behavior following IPOs: The "bubble period" evidence. Review of Financial Studies 21, 101–133.

Breon-Drish, B., 2015a. On existence and uniqueness of equilibrium in a class of noisy rational expectations models. The Review of Economic Studies 82, 868–921.

Breon-Drish, B. M., 2015b. The value of information in financial markets with non-Gaussian shocks. Unpublished working paper. Stanford University, California.

Brown, D., Jennings, R., 1989. On technical analysis. Review of Financial Studies 2, 527–551.

Campbell, J. Y., Kyle, A. S., 1993. Smart money, noise trading and stock price behaviour. The Review of Economic Studies 60, 1–34.

Cespa, G., Vives, X., 2015. The beauty contest and short-term trading. The Journal of Finance 70, 2099–2154.

Chen, H.-C., Ritter, J. R., 2000. The seven percent solution. The Journal of Finance 55, 1105–1131.

Chordia, T., Subrahmanyam, A., 2004. Order imbalance and individual stock returns: Theory and evidence. Journal of Financial Economics 72, 485–518.

Corwin, S. A., Schultz, P., 2005. The role of IPO underwriting syndicates: Pricing, information production, and underwriter competition. The Journal of Finance 60, 443–486.

Das, S., Guo, R.-J., Zhang, H., 2006. Analysts' selective coverage and subsequent performance of newly public firms. The Journal of Finance 61, 1159–1185.

Diamond, D. W., Verrecchia, R. E., 1981. Information aggregation in a noisy rational expectations economy. Journal of Financial Economics 9, 221–235.

Easley, D., Kiefer, N. M., O'Hara, M., Paperman, J. B., 1996. Liquidity, information, and infrequently traded stocks. The Journal of Finance 51, 1405–1436.

Frankel, R., Kothari, S., Weber, J., 2006. Determinants of the informativeness of analyst research. Journal of Accounting and Economics 41, 29–54.

Ganguli, J. V., Yang, L., 2009. Complementarities, multiplicity, and supply information. Journal of the European Economic Association 7, 90–115.

García, D., Strobl, G., 2011. Relative wealth concerns and complementarities in information acquisition. Review of Financial Studies 24, 169–207.

Goldstein, I., Li, Y., Yang, L., 2014. Speculation and hedging in segmented markets. Review of Financial Studies 27, 881–922.

Green, J. R., 1973. Information, efficiency and equilibrium. Discussion Paper 284. Harvard Institute of Economic Research, Cambridge, Massachusetts.

Grossman, S., 1976. On the efficiency of competitive stock markets where trades have diverse information. The Journal of Finance 31, 573–585.

Grossman, S., 1978. Further results on the informational efficiency of competitive stock markets. Journal of Economic Theory 18, 81–101.

Grossman, S. J., Kihlstrom, R. E., Mirman, L. J., 1977. A Bayesian approach to the production of information and learning by doing. The Review of Economic Studies 44, 533–547.

Grossman, S. J., Stiglitz, J. E., 1980. On the impossibility of informationally efficient markets. The American Economic Review 70, 393–408.

Grundy, B. D., McNichols, M., 1989. Trade and the revelation of information through prices and direct disclosure. Review of Financial Studies 2, 495–526.

He, H., Wang, J., 1995. Differential informational and dynamic behavior of stock trading volume. Review of Financial Studies 8, 919–972.

Hellwig, M. F., 1980. On the aggregation of information in competitive markets. Journal of Economic Theory 22, 477–498.

Holden, C. W., Subrahmanyam, A., 2002. News events, information acquisition, and serial correlation. The Journal of Business 75, 1–32.

Kihlstrom, R. E., Mirman, L. J., 1975. Information and market equilibrium. Bell Journal of Economics 6, 357–376.

Kyle, A. S., 1985. Continuous auctions and insider trading. Econometrica 53, 1315–1335.

Lambert, R., Leuz, C., Verrecchia, R. E., 2007. Accounting information, disclosure, and the cost of capital. Journal of Accounting Research 45, 385–420.

Lucas, R. E., 1972. Expectations and the neutrality of money. Journal of Economic Theory 4, 103–124.

Manela, A., 2014. The value of diffusing information. Journal of Financial Economics 111, 181–199.

Manzano, C., Vives, X., 2011. Public and private learning from prices, strategic substitutability and complementarity, and equilibrium multiplicity. Journal of Mathematical Economics 47, 346–369.

Mele, A., Sangiorgi, F., 2015. Uncertainty, information acquisition, and price swings in asset markets. The Review of Economic Studies 82, 1533–1567.

Rao, H., Greve, H. R., Davis, G. F., 2001. Fool's gold: Social proof in the initiation and abandonment of coverage by Wall Street analysts. Administrative Science Quarterly 46, 502–526.

Veldkamp, L., 2006. Media frenzies in markets for financial information. The American Economic Review 96, 577–601.

Verrecchia, R. E., 1982. Information acquisition in a noisy rational expectations economy. Econometrica 50, 1415–1430.

Vives, X., 1995. Short-term investment and the informational efficiency of the market. The Review of Financial Studies 8, 125–160.

Vives, X., 2008. Information and Learning in Markets: The Impact of Market Microstructure. Princeton University Press, Princeton, NJ.

Wang, J., 1993. A model of intertemporal asset prices under asymmetric information. The Review of Economic Studies 60, 249–282.

Wang, J., 1994. A model of competitive stock trading volume. Journal of Political Economy 102, 127–168.

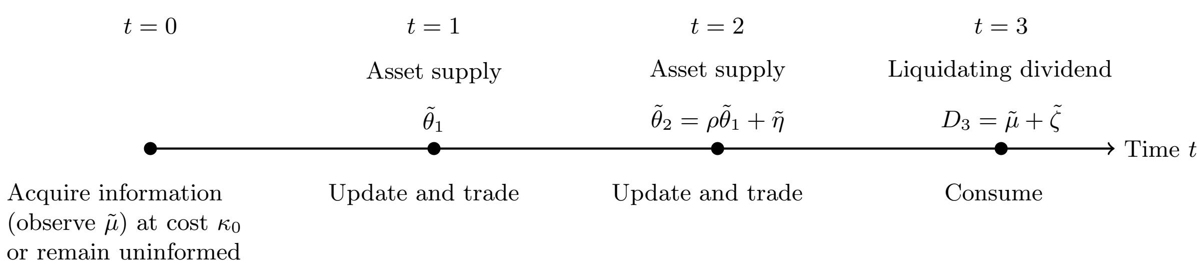

Fig. 1. The sequence of events in the model. At time $t = 0$ all agents choose whether to pay κ_0 to acquire information about the liquidating dividend D_3 of a risky asset. The dividend is made up of two random variables, $\tilde{\mu} \sim \mathcal{N}(0, \sigma_\mu^2)$ and $\tilde{\zeta} \sim \mathcal{N}(0, \sigma_\zeta^2)$ with $\tilde{\mu}$ independent of $\tilde{\zeta}$. Agents that choose to acquire information observe the value of $\tilde{\mu}$ right before time $t = 1$. Agents that choose not to acquire information must rely on prices in later periods to update their beliefs about the value of $\tilde{\mu}$. At times $t = 1$ and $t = 2$ all agents are free to trade the risky asset. The value of the dividend information $\tilde{\mu}$ does not change over time. The supply of the asset at time $t = 1$ is $\tilde{\theta}_1 \sim \mathcal{N}(0, \sigma_\theta^2)$, independent of $\tilde{\mu}$ and $\tilde{\zeta}$. The supply of the asset at time $t = 2$ is $\tilde{\theta}_2 = \rho\tilde{\theta}_1 + \tilde{\eta}$, where ρ is the persistence of supply and $\tilde{\eta} \sim \mathcal{N}(0, \sigma_\eta^2)$, independent of $\tilde{\theta}_1$, $\tilde{\mu}$, and $\tilde{\zeta}$. All agents consume out of their dividend holdings at time $t = 3$ and the economy ends.

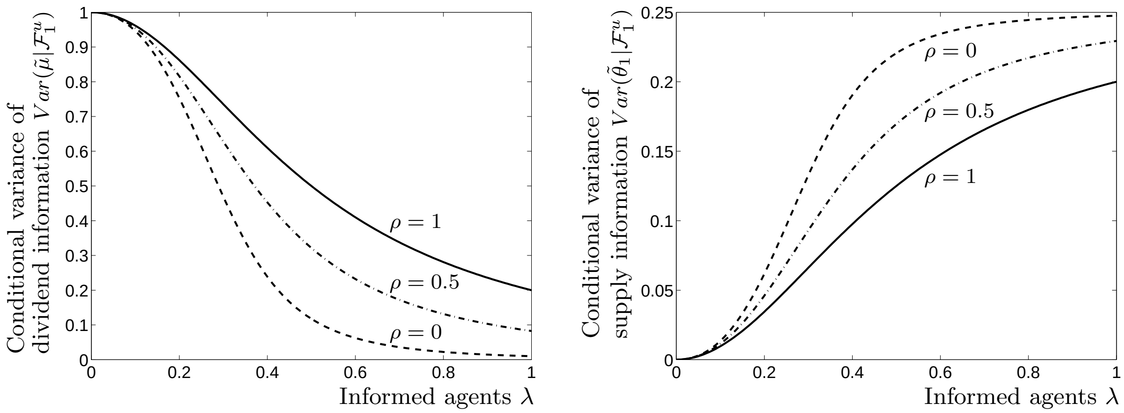

Fig. 2. Conditional variance of dividend information (left) and conditional variance of supply information (right) for the uninformed agents at time $t = 1$, for different values of supply persistence ρ. Solid curves for $\rho = 1$, dash-dotted curves for $\rho = 0.5$, and dashed curves for $\rho = 0$. The information set $\mathcal{F}_1^u$ of the uninformed agents at time $t = 1$ contains only the first-period price. The risk-aversion coefficient is $\delta = 1$, and the standard deviations of the random variables as in the timeline of Fig. 1 are $\sigma_\zeta = 1$, $\sigma_\mu = 1$, $\sigma_\theta = 0.5$, and $\sigma_\eta = 0.5$.

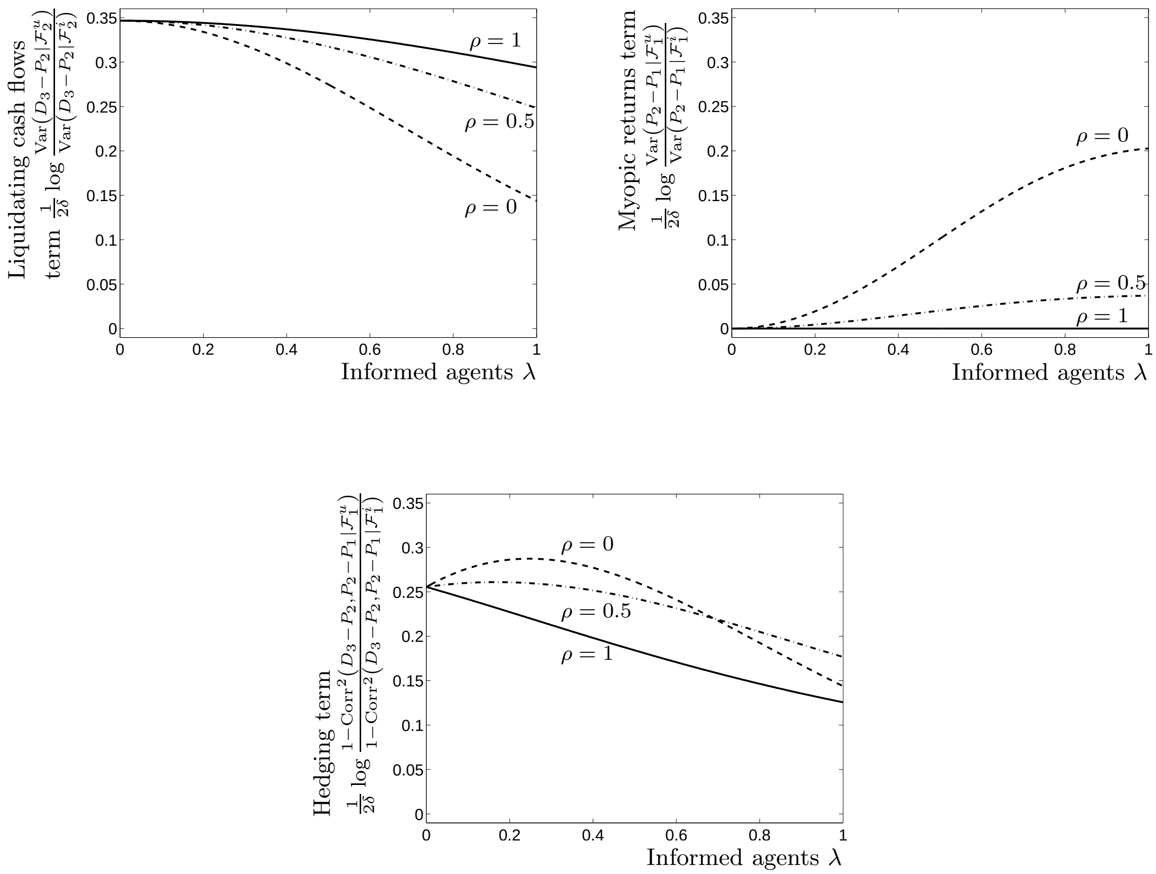

Fig. 3. The components of the total value of information as a function of the fraction of informed agents λ for a two-trading-period market with first-period price P_1, second-period price P_2, and liquidating dividends D_3. For each trading period $t = 1, 2$, the informed agents have information set $\mathcal{F}_t^i$ and the uninformed agents have information set $\mathcal{F}_t^u$. All agents have CARA preferences with risk-aversion coefficient δ. The value of information has three components: one capturing the relative informativeness of price histories about liquidating cash flows (top left), one capturing the relative informativeness of first-period prices about myopic returns (top right), and one capturing the relative value of using information to hedge better (bottom.) Each plot shows the value of information for different levels of supply persistence ρ, with solid curves for $\rho = 1$, dash-dotted curves for $\rho = 0.5$, and dashed curves for $\rho = 0$. The risk-aversion coefficient is $\delta = 1$, and the standard deviations of the random variables as in the timeline of Fig. 1 are $\sigma_\zeta = 1$, $\sigma_\mu = 1$, $\sigma_\theta = 2$, and $\sigma_\eta = 1$.

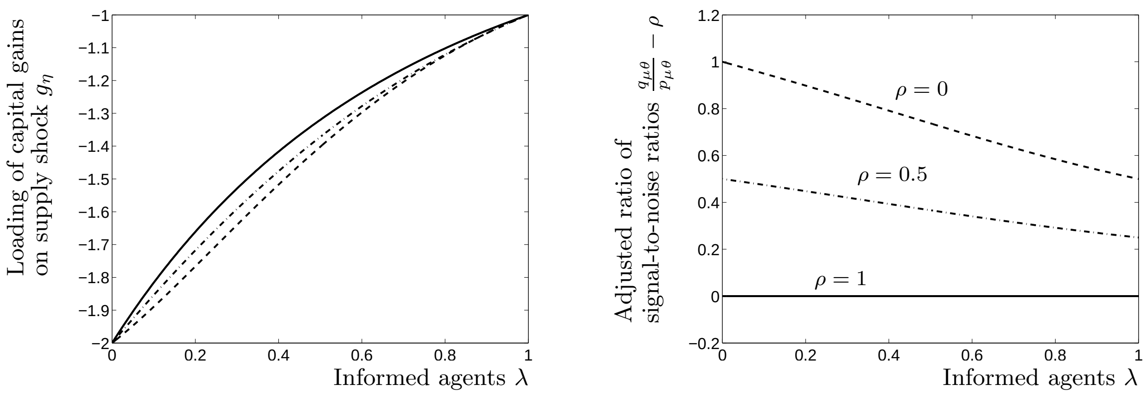

Fig. 4. The breakdown of the coefficient of first-period supply in capital gains as a function of the fraction of informed agents λ, for different values of supply persistence ρ. This coefficient is the product $g_\eta \left(\frac{q_{\mu\theta}}{p_{\mu\theta}} - \rho \right)$, where g_η is the loading of capital gains on the second-period supply shock (shown on the left) and $q_{\mu\theta}/p_{\mu\theta}$ is the ratio of the price-to-noise ratios $q_{\mu\theta}$ and $p_{\mu\theta}$ (shown on the right, adjusted by subtracting ρ.) Solid curves for $\rho = 1$, dash-dotted curves for $\rho = 0.5$, and dashed curves for $\rho = 0$. The risk-aversion coefficient is $\delta = 1$, and the standard deviations of the random variables as in the timeline of Fig. 1 are $\sigma_\zeta = 1$, $\sigma_\mu = 1$, $\sigma_\theta = 2$, and $\sigma_\eta = 1$.

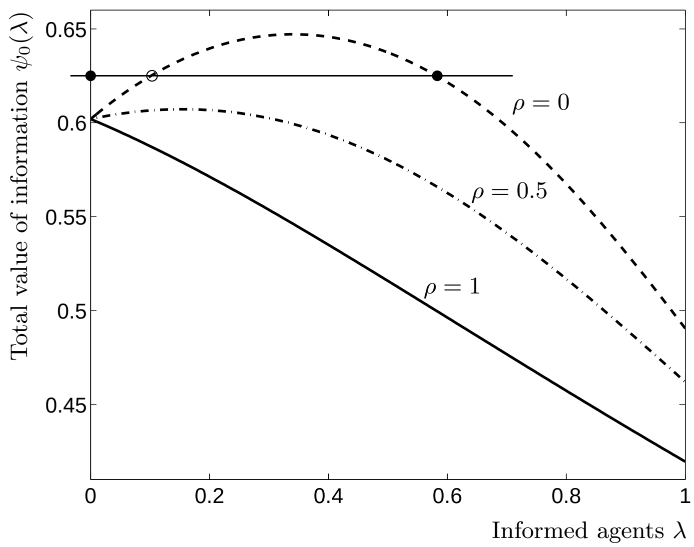

Fig. 5. The total value of information $\psi_0(\lambda)$ as a function of the fraction of informed agents λ for different values of supply persistence ρ. Solid curves for $\rho = 1$, dash-dotted curves for $\rho = 0.5$, and dashed curves for $\rho = 0$. For $\rho = 0$, when the cost of information is $\kappa_0 = 0.625$ there is one corner equilibrium at $\lambda_0 = 0$ (left filled circle), one interior equilibrium at $\lambda_* = 0.10$ (empty circle), and another interior equilibrium at $\lambda^* = 0.58$ (right filled circle). The risk-aversion coefficient is $\delta = 1$, and the standard deviations of the random variables as in the timeline of Fig. 1 are $\sigma_\zeta = 1$, $\sigma_\mu = 1$, $\sigma_\theta = 2$, and $\sigma_\eta = 1$.