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by

Yaron Leitner
Bilge Yilmaz
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Yaron Leitner[†] Bilge Yilmaz[‡]

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Abstract

We study a situation in which a regulator relies on risk models that banks produce in order to regulate them. A bank can generate more than one model and choose which models to reveal to the regulator. The regulator can find out the other models by monitoring the bank, but in equilibrium, monitoring induces the bank to produce less information. We show that a high level of monitoring is desirable when the bank's private gain from producing more information is either sufficiently high or sufficiently low (e.g., when the bank has a very little or a very large amount of debt). When public models are more precise, banks produce more information, but the regulator may end up monitoring more.

Keywords: bank regulation, Bayesian persuasion, information design, internal-risk models, model-based regulation

JEL Classifications: D82, D83, G21, G28

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[†]Corresponding author: Yaron Leitner, Research Department, Federal Reserve Bank of Philadelphia, Ten Independence Mall, Philadelphia, PA 19106; email: YaronLeitner@gmail.com, Tel: 215-574-3963.

[‡]Finance Department, Wharton School, University of Pennsylvania; email: yilmaz@wharton.upenn.edu.

1 Introduction

Regulators rely on information that banks produce in order to regulate them. One example is the advanced internal ratings-based approach to calculating capital requirements for credit risk. Under this approach, banks use their internal risk models to estimate the borrower's probability of default, loss given default, and exposure at default. These estimates are then used to calculate risk weights and capital requirements. Another example is stress tests. Banks' internal credit ratings are used to project losses on corporate loans to companies that are not publicly traded. Banks' internal models are also used to estimate losses on trading positions.

The idea behind this approach is that banks may know more than regulators about their own risk. After all, banks have strong incentives to develop good models for their own portfolio decisions. Yet, a concern exists that, while banks will use their best models for trading purposes, for the purpose of regulation they will select models that underestimate risk. This concern is supported by empirical evidence.[1]

The first question we address in this paper relates to this concern: Should regulators attempt to find out all relevant information from banks, even if regulators can do so without incurring any cost? In our model, a bank can create more than one model and choose which models to reveal to the regulator. The regulator uses the information from the models he observes to decide whether to allow the bank to invest in some risky asset. The regulator also decides how much to monitor the bank, which leads to an endogenous probability q that the regulator will find out the other models that the bank produces.

As we explain here, there are two forces that push the optimal q in different directions. A higher q allows the regulator to learn more from the information that the bank produces. This can lead to better investment decisions from the regulator's point of view. However, a higher q may also induce the bank to produce less information overall (in the sense of Blackwell, 1951). This is because, if the regulator finds out the information, he can use it to restrict investment when the bank wants to invest

[1] See, for example, Behn, Haselmann, and Vig (2014); and Plosser and Santos (2014).

but the regulator does not.

The conflict of interest between the bank and the regulator could arise for various reasons. Banks do not internalize externalities, while regulators are concerned with systemic risk and take into account total social costs. Similarly, banks have limited liability, and, as such, shareholders do not necessarily maximize total firm value. Our main insights do not depend on the specific source of conflict.

For concreteness, we focus on one source of conflict. In our model, the regulator wants to invest only if a project has a positive net present value (NPV), but the bank, facing limited liability, wants to invest also in some projects that have negative NPV. This conflict of interest is also impacted by the fact that the bank has other assets, which could be used to pay debt holders when the project fails. Since both the value of these assets and the value of the project depend on the state of the world, the bank's ideal investment rule may not be just a simple cutoff rule. For example, for some parameter values, it is optimal for the bank to invest in good states (when the project has positive NPV) and bad states (when the project has negative NPV and other assets are worthless) but not in intermediate states (when the value of the other assets is too high to lose). More generally, the bank's ideal investment rule includes intervals of the state space in which the bank wants to invest and intervals in which the bank does not want to invest.

The bank can generate information to guide its investment decisions. Specifically, the bank generates *models*, which are information partitions of the state space. We restrict attention to information partitions, such that each element in the partition is a convex set (i.e., an interval or a singleton). Then, the bank faces a tradeoff. When the bank generates more information, the bank can make better investment decisions for its equity holders. However, if the regulator finds out the information, the regulator can use the information against the bank to ban investment when the bank wants to invest but the regulator does not. The outcome of this tradeoff is that, when the regulator sets a higher level of monitoring, the bank produces less information. Consequently, the optimal level of monitoring could have an interior solution.

We characterize the optimal level of monitoring. In particular, we provide necessary and sufficient conditions under which it is optimal to set $q = 1$. When these conditions hold, it is optimal that the regulator observes all the models that the bank creates. When these conditions do not hold, it is optimal that the bank creates two sets of models. The bank reveals to the regulator the first set of models but does not reveal the second set of models. The regulator allows the bank to invest as long as he is convinced that the state of the world is sufficiently high for the project to have a nonnegative NPV, in expectation. The bank maintains discretion whether to invest when the regulator allows it to invest.

Interestingly, it is optimal to set a high level of monitoring $q = 1$ when the bank's private gain from producing information is either sufficiently high or sufficiently low. When the private gain is high, the bank produces a lot of information even if it is highly monitored. In this case, it is optimal to set $q = 1$ because the regulator can learn everything the bank knows without impacting the amount of information that the bank produces. If instead, the bank's gain is intermediate, the regulator must set a lower level of monitoring $q < 1$ to induce the bank to produce information. In this range, the optimal level of monitoring decreases when the bank's gain from producing information falls because it becomes harder to induce the bank to produce information. Finally, when the bank's private gain from producing information is very low, the regulator can induce the bank to produce information only if the level of monitoring is very low. But then the regulator does not learn much from the information that the bank produces, and it is again optimal to set $q = 1$, even though this reduces the overall amount of information that the bank produces.

Using this insight, we derive comparative statics as to how the optimal q changes with respect to model parameters, such as the amount of debt that the bank owes, the value of its existing assets, or the quality of its new project. We also discuss the case in which the bank faces some exogenous cost of producing information and the case in which the regulator can impose penalties on the bank. In general, the relationship is nonmonotone. For example, for some parameter values, it is optimal to set a high level of monitoring for banks that have either a low cost or a high cost of producing information and a lower level of monitoring for banks that have an intermediate cost. Similarly, banks that have either high levels of debt or low levels of debt could face a high level of monitoring, while banks with an intermediate level of debt could face a lower level of monitoring. As for the amount of information produced, our model predicts that for a given (positive) level of monitoring, banks will produce less information when they have more debt, when the value of their existing assets falls, and, perhaps surprisingly, when they have higher quality projects. All these changes increase the bank's gain from investing, and so the fact that the regulator could use the bank's information to ban investment has a more significant effect on the bank.

We also analyze the role of public information. We show that when public models become more informative, the bank generates more information. However, there is a nonmonotone relationship between the informativeness of the public models and the optimal level of monitoring q. Intuitively, there are two forces that push the optimal q in different directions. On the one hand, with more public information, the regulator can monitor less because he already has some information. On the other hand, public information can affect the bank's private gain from producing information. In particular, under some circumstances it can increase the bank's incentive to produce more information. This makes it easier for the regulator to induce the bank to produce information and allows the regulator to increase the probability of monitoring.

The paper proceeds as follows. In Section 2, we review the literature, and in Section 3, we provide an example. The formal model is in Section 4. In Section 5, we analyze the benchmark case of an unregulated bank, and in Section 6, we provide an equilibrium analysis of a regulated bank (and most of our results). Comparative statics are in Section 7. Section 8 contains an analysis of the case of public models, and in Section 9, we discuss applications, such as corporate governance, new drug approval, bank stress tests, and capital requirements. We conclude in Section 10, which also contains a discussion of model assumptions. Proofs are in the Appendix.

2 Literature review

Our paper is closely related to the Bayesian persuasion literature (Kamenica and Gentzkow, 2011), in which one agent (the bank) generates a signal (a model) to persuade another agent (the regulator) to allow some action. A key difference between our paper and existing literature is that, in our setting, the bank can generate a second signal, which it does not reveal to the regulator. The bank can then use information from both signals to decide whether to take the action. Our paper allows us to answer questions such as whether and under what conditions the regulator can gain by committing not to find out what the second signal is. More generally, our paper is an example of an information design problem (e.g., Bergemann and Morris, 2016, 2017), in which the bank (the information designer) designs the information structure for itself and for the regulator, and in which payoffs in the induced game depend on the (endogenous) monitoring intensity that the regulator sets.[2]

Our paper also relates to the literature on delegation of authority within organizations, in particular, the literature that focuses on the tradeoff between incorporating more information into decision making and controlling decision-making authority when the agent has relevant information. In a strategic communication setting, Dessein (2002), Harris and Raviv (2005, 2008, and 2010), Chakraborty and Yilmaz (forthcoming), and Grenadier, Malenko, and Malenko (2016) study conditions under which the principal should allocate decision-making authority to the agent. In this framework, keeping decision-making authority may hurt the principal because the gains from doing so are outweighed by the losses arising from imperfect information transmission between the agent and the principal. In a related work, Aghion and Tirole (1997) also analyze the optimal allocation of authority but without strategic communication. They emphasize a distinction between formal and real authority, showing that often the party with formal authority will delegate authority to another party with information. In our framework, the regulator can gain by delegating au-

[2] Other papers that study information design (or Bayesian persuasion, in particular) in the context of bank regulation include Goldstein and Leitner (2015); Gick and Pausch (2014); Williams (2017); and Orlov, Zryumov, and Skrzypacz (2017).

thority to the bank to decide whether to invest. A key difference between our paper and the existing literature is that, in our setting, the regulator allocates authority based on the realization of a signal that the bank produces (endogenously). The bank obtains authority only if the signal realization is above some threshold (and the regulator does not observe the other signal(s) the bank produces).[3]

Our paper also relates to the literature on bank regulation in which the regulator uses information that banks provide to set capital requirements. Prescott (2004) studies a framework in which the bank has private information about its own risk, and the regulator sets capital requirements based on the risk that the bank reports.[4] The source of inefficiency is that the bank can misrepresent its true risk. The regulator can mitigate this problem by monitoring the bank (e.g., auditing and imposing penalties).[5] In our framework, the bank does not know its own risk but can generate information. In contrast to earlier literature on the benefits of monitoring, our model shows that too much monitoring could have perverse effects. Monitoring diminishes the banks' incentive to produce valuable information.

There is also a growing empirical literature on the effect of regulation that relies on banks' internal models. Plosser and Santos (2014) found systematic differences in risk estimates that large U.S. banks provided for the same (syndicated) loan. In particular, banks with less regulatory capital reported lower probability of default, and their risk estimates had less explanatory power with regard to the loan prices. Behn, Haselmann, and Vig (2014) compare risk estimates that German banks provided on loan portfolios that shifted to the internal rating-based approach and loan portfolios that were waiting for approval for the new approach and for which capital was calculated based on the traditional risk-weights method. They show that, for the first group of loans, banks provided lower probability of default estimates. Yet, the

[3] Aghion and Bolton (1992) also analyze allocation of control rights based on a signal realization, but in their setting, the signal is exogenous.

[4] See also Marshall and Prescott (2006).

[5] Blum (2008) shows that, when the regulator has limited ability to audit or impose penalties, minimum leverage ratios in addition to capital requirements can help. However, Colliard (2014) shows that imposing minimum leverage ratio could lead to unintended consequences with respect to equilibrium loan supply. In his setting, the bank has private information regarding the *distribution* of true risks.

interest rates they charged on these loans and actual default rates were higher. The findings of this literature are consistent with the idea that banks don't reveal all the information they have to the regulator.[6]

In a different context, Bond and Goldstein (2015) study a situation in which a government relies on a firm's stock price to decide how to intervene in the firm. They show that it might be optimal for the government to commit to limit its reliance on the firm's stock price because relying on the price could harm the aggregation of information from market participants into the price. Instead, in our setting, the regulator relies on information that is produced by the regulated party itself.

Finally, the idea that a principal can benefit by committing not to monitor too much also appears in Crémer (1995). In his model, a principal has a monitoring technology, which allows him to obtain the reason behind a low output. The principal may benefit from inefficient monitoring because it allows him to precommit to fire a high-quality agent who produced low output. Cohn, Rajan, and Strobl (2016) show that, when credit rating agencies screen issuers more heavily, issuers could have stronger incentive to manipulate the information they provide to the credit rating agency. In the context of regulation of disclosure in the product market, Polinsky and Shavell (2012) show that forcing firms to disclose information about product risk may lead firms to gather less information.[7] In the context of corporate governance, Burkart, Gromb, and Panunzi (1997) show that excessive monitoring can reduce managerial effort to learn about new investments and that dispersed ownership could act as a commitment device not to exercise excessive control.

3 An example

There is a bank and a regulator. The bank has a debt liability with a face value of \$1. The bank also has \$1 in cash, other existing assets, and a new investment opportunity (project). The project requires an investment of \$1. It can either succeed and yield

[6] Other related work includes Begley, Purnanandam, and Zheng (forthcoming); Firestone and Rezende (2016); Mariathasan and Merrouche (2014); and Rajan, Seru, and Vig (2015).

[7] See also Shavell (1994).

\$2 or fail and yield nothing. The project's success probability and the value of the bank's existing assets (other than cash) depend on the unobservable state of nature, as described in Table 1. Table 1 also shows the project's NPV in each state, namely, the project's expected payoff minus the initial investment.

Table 1

State	s_1	s_2	s_3	s_4
Probability of state	0.25	0.25	0.25	0.25
Project's success probability	0.1	0.4	0.4	0.8
Value of existing assets	0.3	0.3	0.8	1
Project's NPV	-0.8	-0.2	-0.2	0.6

There is a conflict of interest between the bank and the regulator. The regulator wants to maximize total surplus, which is the sum of payoffs to debt holders and equity holders. Hence, the regulator wants to invest only if the project has positive NPV. The bank has limited liability and acts to maximize the expected payoff to its equity holders. Hence, as we illustrate below, the bank wants to invest not only when the project has positive NPV but also in some states in which the project has negative NPV.

To see that, consider state s_2. If the bank does not invest, debt holders are fully paid. However, in the case of investment, if the project fails, which occurs with probability 0.6, the bank cannot pay off its debt. In this case, the bank's debt holders obtain the bank's existing assets, which are worth only 0.3. The expected loss for debt holders due to investment in state s_2 is then $0.6 \times (1 - 0.3) = 0.42$. From the perspective of the bank's equity holders, this is beneficial because this is a transfer of wealth from debt holders. Since the sum of this gain (0.42) and the project's NPV in state s_2 (-0.2) is positive, the bank wants to invest in state s_2. Table 2 repeats these calculations for the other three states. It follows that the bank wants to invest in states s_2 and s_4 but not in states s_1 and s_3.

Table 2

State	s_1	s_2	s_3	s_4
Gain from default	0.63	0.42	0.12	0
Project's NPV + Gain from default	-0.17	0.22	-0.08	0.6
Bank's ideal investment rule	Don't invest	Invest	Don't invest	Invest

If the regulator knew the state, he would allow the bank to invest only in state s_4. However, the regulator is not an expert in producing information. The only way for the regulator to learn more about the state is to rely on information that the bank produces.

The bank can generate two types of signals. The first signal is very informative. It fully reveals the state. The second signal is less informative. It tells only whether the success probability is 0.1 or above 0.1. In other words, the second signal tells whether the true state is in the set $\{s_1\}$ or in the set $\{s_2, s_3, s_4\}$. The regulator cannot dictate to the bank which signal (or signals) to generate, but he can force the bank to disclose the signal realization. Putting it differently, the regulator has a monitoring technology that allows him to find out the signals that the bank generates. Note that if the bank does not generate any signal, the regulator would ban investment because the project's expected NPV, averaged across all states, is negative.

Suppose first that the regulator monitors the bank. Which signal will the bank generate? If the bank generates the very informative signal, the regulator would allow it to invest only in state s_4. The expected gain for the bank's equity holders is then 0.25×0.6. If the bank generates the less informative signal, the regulator will ban investment when he learns that the state is s_1 but will allow investment when the state is in $\{s_2, s_3, s_4\}$. The last part follows because conditional on being in $\{s_2, s_3, s_4\}$, the project has positive NPV. The bank will then invest in the three states s_2, s_3, s_4, yielding an expected gain of $0.25 \times (0.22 - 0.08 + 0.6)$ to its equity holders. Hence, the bank will generate the less informative signal, as it leads to a higher gain to its equity holders.

Now suppose the bank can generate both signals and reveal to the regulator only the less informative one. In other words, the regulator does not monitor the bank. As before, the regulator will allow investment only in states $\{s_2, s_3, s_4\}$. However, now the bank can use the information from the more informative signal to decide whether to invest. So, the bank will invest only in states s_2 and s_4, in which the gain to its equity holders is positive. From the regulator's point of view (and also from the bank's), this outcome is preferred to the outcome when the bank is monitored because the bank does not invest in state s_3. Hence, the regulator will not monitor the bank.

The result above changes when the bank has less debt. Suppose the face value of debt is only \$0.8. Now the bank's equity holders gain from defaulting on the bank's debt only in states s_1 and s_2, in which the value of existing assets is less than the face value of debt. As we show in Table 3, the bank will still want to invest only in states s_2 and s_4. However, the bank's incentives to produce information change. Now the bank will produce the more informative signal even if it is being monitored. To see why, note that if the bank generates the more informative signal, it obtains 0.25×0.6, as in the case of debt with face value of 1. If the bank generates the less informative signal, it obtains only $0.25 \times (0.1 - 0.2 + 0.6)$. Hence, the bank will generate the more informative signal, although it knows that it will be forced to reveal the information. From the regulator's perspective, this is the best possible outcome. Hence, the regulator will monitor the bank.

Table 3

State	s_1	s_2	s_3	s_4
Gain from default	0.45	0.3	0	0
Project's NPV + Gain from default	−0.35	0.1	−0.2	0.6
Bank's ideal investment rule	Don't invest	Invest	Don't invest	Invest

We can interpret the signals in this example as internal risk models that the bank generates for regulatory purpose. The example illustrates three points. First, the regulator could benefit from relying on internal risk models that the bank generates. Second, the regulator could gain by allowing the bank to produce two sets of models. The first model is used to persuade the regulator to allow the bank to invest, while the second model — which is not shared with the regulator — is used by the bank to decide whether to invest when the regulator allows it to do so. Third, whether the regulator could gain by allowing the bank to produce two sets of models depends on the bank's private gain from producing information, which, in turn, depends on bank characteristics, such as how much debt the bank has.

4 The model

The formal model generalizes the example in Section 3. In particular, we assume a continuous state space and allow the bank to choose from a larger set of signals. We refer to these signals as models. In addition, we allow for partial monitoring, which induces a probability $q \in [0, 1]$ that the regulator will find out all the models that the bank has generated.

4.1 Economic environment

As in the example, the bank's assets consist of cash, which is normalized to 1; a risky asset; and a new investment opportunity (project). The value of the risky asset depends on the unobservable state ω, according to some continuous function $v(\omega)$. The state ω is drawn from the set $\Omega = [0, 1]$, according to a continuous cumulative distribution function F (everything is common knowledge). The value of the new project also depends on the state. The new project requires an investment of 1. It generates $x > 1$ with probability ω and 0 with probability $1 - \omega$.[8] The bank also has a debt liability with face value $D \leq 1$. The bank has limited liability.

The bank acts to maximize the expected payoff to its equity holders. The regulator maximizes total surplus, which is the sum of payoffs to debt holders and equity holders.

4.2 Information production

The bank can generate information about ω by creating *models*. A model is an information partition of Ω, with the added requirement that each set in the information partition is convex (i.e., a singleton or an interval). Formally:

Definition 1 *A model is defined by a set of indexes $\mathcal{I} \subset R$ and a collection of sets $\mathbf{P} = \{P_i\}_{i \in \mathcal{I}}$, such that the following hold:*

1. $\cup_{i \in \mathcal{I}} P_i = \Omega$.

[8] The nature of the results remains if the project's probability of success depends on the state according to some arbitrary function that is (weakly) increasing in the state.

2. For every $i \neq j$, $P_i \cap P_j = \emptyset$.

3. For every $P_i \in \mathbf{P}$ and $\lambda \in (0,1)$, if $\omega_1, \omega_2 \in P_i$, then $\lambda\omega_1 + (1-\lambda)\omega_2 \in P_i$.

We let $P(\omega)$ stand for the set in $\mathbf{P}$ to which ω belongs. So, when the realized state is $\omega \in \Omega$, the model $\mathbf{P}$ tells that the event $P(\omega)$ has occurred. For example, a model that consists of only singletons fully reveals ω. A model that consists of two intervals tells whether ω is above or below some threshold. We can think of a model as a collection of experiments, where each experiment tells whether the state is above or below some threshold. Note that $\{P(\omega)\}_{\omega \in \Omega}$ fully defines a model.[9]

The bank can create more than one model. With probability q, the regulator observes all the models that the bank creates. With probability $1-q$, the regulator observes only the models that the bank chooses to reveal. The probability q is endogenous and determined by the regulator before the bank creates models.

We refer to q as the probability of monitoring and assume that the regulator can precommit to acting according to it. In practice, monitoring could take various forms. One example is on-site audits. In this case, a higher q could capture the idea that the regulator devotes more resources into monitoring the bank (e.g., by having more staff on site); commitment to act according to q arises naturally in this case. As we explain later (Section 7.3), a higher q could also capture the outcome of penalties that the regulator imposes on the bank if the project fails and the regulator finds out that the bank knew that the project had negative NPV. Here, commitment could arise from the fact that the regulator must follow some pre-specified rules that determine what the regulator can or cannot do.

For simplicity, we assume that all choices of q entail the same cost.[10] The regulator can allow or ban investment based on the information it has about ω. However, the regulator cannot precommit to investment rules that are suboptimal ex post.

[9]In practice, the models that the bank produce could be based on hard information (e.g., statistical analysis of past returns to project future returns) and/ or soft information (e.g., loan officer opinions).

[10]This assumption helps us focus on the main tradeoff in our paper. It is easy to relax this assumption, but relaxing this assumption does not provide any interesting insights.

Without loss of generality, we can assume that the bank generates only two models, $\mathbf{P}^B$ and $\mathbf{P}^R$, such that model $\mathbf{P}^B$ contains all the information that the bank produces and model $\mathbf{P}^R$ contains only the information that the bank chooses to reveal to the regulator. In particular, if the bank creates m models $\mathbf{P}^1, ..., \mathbf{P}^m$ and reveals to the regulator only the first $l \leq m$ models, we can define for every $\omega \in \Omega$, $P^B(\omega) = \cap_{j=1}^m P^j(\omega)$ and $P^R(\omega) = \cap_{j=1}^l P^j(\omega)$. We refer to models $\mathbf{P}^B$ and $\mathbf{P}^R$ as the bank model and regulator model, respectively. Note that $\mathbf{P}^B$ is at least as informative as $\mathbf{P}^R$.

4.3 Sequence of events

The sequence of events is as follows:

1. The regulator chooses $q \in [0, 1]$ and publicly announces it.
2. The bank chooses models $\mathbf{P}^B$ and $\mathbf{P}^R$.
3. Nature draws the state ω. The bank observes $P^B(\omega)$. As for the regulator, with probability q, he observes $P^B(\omega)$, and with probability $1 - q$, he observes $P^R(\omega)$.
4. The regulator allows or bans investment.
5. If investment is allowed, the bank chooses whether to invest.
6. The project either succeeds or fails. Debt holders and equity holders get paid.

We focus on perfect Bayesian equilibria of the game above. Assume that if the bank is indifferent between investing and not investing, the bank invests. If the regulator is indifferent between allowing and banning investment, the regulator allows investment.

5 Unregulated bank (benchmark)

We start with the benchmark case in which the bank is unregulated; that is, the regulator cannot ban investment. In this case, we can assume, without loss of generality, that the bank generates only one model, which fully reveals the state. That is, for every $\omega \in \Omega$, $P^R(\omega) = P^B(\omega) = \omega$.

We derive the bank's ideal investment rule as follows. If the bank does not invest, debt is riskless, and the bank's equity holders obtain

$$v(\omega) + 1 - D. \tag{1}$$

If the bank invests, debt holders obtain D when the project succeeds and $\min\{v(\omega), D\}$ when the project fails. So, if the bank invests, the expected payoff to the bank's equity holders is

$$\omega[x + v(\omega) - D] + (1 - \omega)\max\{v(\omega) - D, 0\}. \tag{2}$$

Denote the project's NPV in state ω by

$$N(\omega) \equiv \omega x - 1. \tag{3}$$

The expected gain to the bank's equity holders from investing in state ω [i.e., (2) minus (1)] is then

$$G(\omega) \equiv N(\omega) + (1 - \omega)\max\{D - v(\omega), 0\}. \tag{4}$$

The second term in (4) is the expected gain to the bank's equity holders from defaulting on the bank's debt when the project fails. This gain arises when the value of the risky asset is less than the face value of debt. In this case, equity holders benefit at the expense of debt holders, who get paid less than the promised amount. The bank invests if and only if $G(\omega) \geq 0$.

Lemma 1 *$G(\omega) \geq 0$ if and only if either (i) $\omega \geq \frac{1}{x}$; or (ii) $\omega < \frac{1}{x}$ and $v(\omega) \leq D + \frac{N(\omega)}{1-\omega}$.*

Lemma 1 says that an unregulated bank invests if either (i) the project has positive NPV; or (ii) the project has negative NPV, and the value of the bank's existing asset, $v(\omega)$, is less than $D + \frac{N(\omega)}{1-\omega}$.

Figure 1 illustrates the function $D + \frac{N(\omega)}{1-\omega}$, which is convex and increasing in ω, and the function $v(\omega)$. The bank's ideal investment rule depends on how the two functions intersect. In general, it is composed of intervals in which the banks invests and intervals in which the bank does not invest. To simplify the exposition, we assume that the functions $D + \frac{N(\omega)}{1-\omega}$ and $v(\omega)$ intersect at a finite number of points and that there exists a finite set of numbers $b_1 > a_1 > ... > b_l > a_l$, such that $v(\omega) \leq D + \frac{N(\omega)}{1-\omega}$ if and only if $\omega \in \cup_{i=1}^{l}[a_i, b_i]$. Then, $G(\omega) \geq 0$ if and only if $\omega \in \cup_{i=1}^{l}[a_i, b_i]$.

[Insert Figure 1 here.]

6 Regulated bank

Consider now a regulated bank. If $v(\omega) > D + \frac{N(\omega)}{1-\omega}$ for every $\omega < \frac{1}{x}$, it follows from Lemma 1 that the bank wants to invest if and only if the project has positive NPV. In this case, there is no conflict of interest between the bank and regulator, and so, regulation is unnecessary.

The rest of this paper focuses on the case in which $v(\omega) \leq D + \frac{N(\omega)}{1-\omega}$ for some $\omega < \frac{1}{x}$. In this case, the bank and the regulator do not agree on the investment rule. Both want to invest when the project has a positive NPV ($\omega \geq \frac{1}{x}$), but the bank wants to invest also in some states in which the project has a negative NPV [Part (ii) in Lemma 1]. We explore optimal regulation in this case. We first characterize equilibrium outcomes, taking q as given. Then, we solve for an optimal q (i.e., a q that the regulator chooses in an equilibrium).

Remark 1 The analysis below does not depend on the specific microfoundations assumed above. It applies to any continuous bank's and regulator's payoff functions $G(\omega)$ and $N(\omega)$, such that $N(\omega)$ is increasing in ω and $G(\omega) \geq N(\omega)$.

6.1 Equilibrium outcomes for a given q

We solve the game backward. Suppose the bank chooses models $\mathbf{P}^B$ and $\mathbf{P}^R$. If the regulator allows investment, the bank invests if and only if the expected gain to its equity holders is positive. Anticipating the bank's behavior, the regulator allows investment if and only if he expects the project to have positive NPV, conditional on the bank investing.

The next lemma simplifies the analysis.

Lemma 2 *For any equilibrium outcome, there exist $\omega_B, \omega_R \in \Omega$, such that $\omega_R \leq \omega_B$ and:*

(i) When the regulator observes model $\mathbf{P}^B$, investment takes place if $\omega > \omega_B$ but not if $\omega < \omega_B$.

(ii) When the regulator observes model $\mathbf{P}^R$, (a) investment takes place if $\omega > \omega_B$; (b) investment does not take place if $\omega < \omega_R$; and (c) if $\omega \in (\omega_R, \omega_B)$, investment takes place when $G(\omega) > 0$ but not when $G(\omega) < 0$.

The first part in Lemma 2 follows from two observations. First, if the regulator allows investment in state ω (and the bank invests), the regulator allows investment also in higher states $\omega' > \omega$. This follows because $N(\omega)$ is increasing in ω and each set in the model partition is convex. Second, the bank invests whenever the regulator allows it to invest. This follows because in the first part, the bank and regulator share the same information (they both observe $\mathbf{P}^B$) and because $G(\omega) \geq N(\omega)$.

The second part also uses the first observation above. However, now the regulator has less information than the bank. So, whenever the regulator allows investment, the bank decides whether to invest based on information from the more informative model $\mathbf{P}^B$, which is not shared with the regulator. In equilibrium, the bank chooses $\mathbf{P}^B$, so that, if $\omega \in (\omega_R, \omega_B)$, the bank invests according to its ideal investment rule: namely, if $G(\omega) > 0$ but not if $G(\omega) < 0$.

In fact, we can assume, without loss of generality, that $\mathbf{P}^B$ fully reveals the state when $\omega < \omega_B$. To see that, note that, if the regulator observes $\mathbf{P}^B$, revealing the exact state below ω_B does not hurt the bank because the regulator does not allow investment anyway. If, instead, the regulator does not observe $\mathbf{P}^B$, learning the exact state below ω_B helps the bank because it can invest according to its ideal investment rule.

Hence, we can assume, without loss of generality, that model $\mathbf{P}^B$ takes the simple form:

$$P^B(\omega) = \begin{cases} \omega & \text{if } \omega < \omega_B \\ [\omega_B, 1] & \text{otherwise.} \end{cases} \tag{5}$$

That is, model $\mathbf{P}^B$ fully reveals the state below ω_B but does not generate any information above ω_B. Intuitively, pooling the states above ω_B together makes it easier for the bank to persuade the regulator to allow investment in some projects that have negative NPV.

Similarly, we can assume, without loss of generality, that model $\mathbf{P}^R$ takes the simple form:

$$P^R(\omega) = \begin{cases} \omega & \text{if } \omega < \omega_R \\ [\omega_R, 1] & \text{otherwise.} \end{cases} \tag{6}$$

Formally:

Lemma 3 *For any equilibrium outcome, there exists an equilibrium that achieves that outcome and in which the bank chooses models $\mathbf{P}^B$ and $\mathbf{P}^R$ that take simple form as in Equations (5) and (6).*

The problem of finding models $\mathbf{P}^B$ and $\mathbf{P}^R$ reduces to finding the thresholds ω_B and ω_R. The two thresholds must satisfy the following:

$$E[N(\tilde{\omega})|\tilde{\omega} \geq \omega_B] \geq 0 \tag{7}$$

$$E[N(\tilde{\omega})|\ \tilde{\omega} \geq \omega_B \text{ or } \tilde{\omega} \in \{\omega \in [\omega_R, \omega_B) : G(\tilde{\omega}) \geq 0\} \geq 0. \tag{8}$$

Equation (7) ensures that when the regulator observes $\mathbf{P}^B$, he allows the bank to invest when $\omega \geq \omega_B$. Equation (8) ensures that when the regulator observes only $\mathbf{P}^R$, he allows the bank to invest when $\omega \geq \omega_R$. Note that in the second case, the regulator understands that the bank will use the second model $\mathbf{P}^B$ to decide whether to invest.

The bank's expected payoff is

$$V(\omega_B, \omega_R) \equiv (1-q) \int_{\omega_R}^{\omega_B} 1_{\{\omega: G(\omega)>0\}} G(\omega) dF(\omega) + \int_{\omega_B}^{1} G(\omega) dF(\omega). \tag{9}$$

In particular, the bank always invests when $\omega \geq \omega_B$. But if $\omega \in (\omega_R, \omega_B)$, the bank invests only if the regulator does not observe $\mathbf{P}^B$, which happens with probability $1-q$, and $G(\omega) \geq 0$.

Let $\bar{\omega}_R(\omega_B)$ be the lowest $\omega_R \in \Omega$ that satisfies Equation (8). Since the term inside the first integral in (9) is positive, we can assume, without loss of generality, that

$$\omega_R = \bar{\omega}_R(\omega_B). \tag{10}$$

That is, the bank sets ω_R as low as possible subject to Equation (8).[11]

As for ω_B, we proceed in two steps. First, we derive a necessary and sufficient condition for ω_B to be an equilibrium threshold. Then, we derive a closed-form solution.

Denote the lowest $\omega_B \in \Omega$ that satisfies Equation (7) by $\bar{\omega}_B$. Clearly, we must have

$$\omega_B \geq \bar{\omega}_B. \tag{11}$$

In addition, for every $\omega'_B \geq \bar{\omega}_B$, the following equilibrium condition must hold:

$$V(\omega_B, \bar{\omega}_R(\omega_B)) \geq V(\omega'_B, \bar{\omega}_R(\omega_B)). \tag{12}$$

Equation (12) rules out a deviation in which the bank chooses model $\mathbf{P}^B$ with threshold ω'_B instead of ω_B, while keeping model $\mathbf{P}^R$ unchanged with threshold $\bar{\omega}_R(\omega_B)$.

It turns out that ruling out the deviation above is not only a necessary equilibrium condition but is also a sufficient condition. Formally,

Lemma 4 $\omega_B \in \Omega$ *is an equilibrium threshold if and only if* $\omega_B \geq \bar{\omega}_B$ *and Equation (12) holds for every* $\omega'_B \in \Omega$, *such that* $\omega'_B \geq \bar{\omega}_B$.

For use below, recall that $G(\omega) \geq 0$ if and only if $\omega \in \cup_{i=1}^{l}[a_i, b_i]$. Let $K = \{a_1, a_2, ..., a_l\}$, and let

$$\Omega_0 = \begin{cases} \{\omega \in K : \omega \geq \bar{\omega}_B\} \cup \{\bar{\omega}_B\}, & \text{if } \bar{\omega}_B \in (a_i, b_i) \text{ for some } i \in \{1, .., l\} \\ \{\omega \in K : \omega \geq \bar{\omega}_B\}, & \text{otherwise.} \end{cases} \tag{13}$$

That is, K includes the left corners of the intervals in which the bank wants to invest. Ω_0 includes the points in K that are above $\bar{\omega}_B$, and if $\bar{\omega}_B$ belongs to an interval in

[11] Any $\omega_R \in [\bar{\omega}_R(\omega_B), \hat{\omega}_R(\omega_B)]$ will give the same outcome, where $\hat{\omega}_R(\omega_B)$ denotes the lowest $\omega_R \in \Omega$ that satisfies both Equation (8) and $G(\omega_R) \geq 0$. Any other ω_R will give a worse outcome for the bank.

which the bank wants to invest, Ω_0 also includes $\bar{\omega}_B$. Lemma 5 below shows that, when looking for the equilibrium threshold ω_B, we can restrict attention to the set Ω_0.

Before we present Lemma 5, we illustrate the sets K and Ω_0 in special cases.

Example 1 Suppose $D = 0.9$, $x = 1.2$, and $\bar{\omega}_B = 0.5$.

1. If $v(\omega) = 0.5$, it follows from Lemma 1 and Figure 2 that the bank wants to invest if and only if $\omega \geq 0.75$. Hence, $K = \{0.75\}$. Hence, $\Omega_0 = \{0.75\}$.

2. If $v(\omega) = \max\{0, 4.4\omega - 2.64\}$, it follows from Lemma 1 and Figure 3 that the bank wants to invest if and only if $\omega \in [0.33, 0.67] \cup [0.83, 1]$. Hence, $K = \{0.33, 0.83\}$. Since $\bar{\omega}_B \in (0.33, 0.67)$, it follows that $\Omega_0 = \{0.5, 0.83\}$.

3. If $v(\omega)$ is as in Figure 1, then $K = \{a_1, a_2, a_3\}$, where a_1, a_2, and a_3 are the left corners of the red intervals. Since $a_1 < 0.5 < a_2 < a_3$ and $\bar{\omega}_B$ does not lie inside an interval in which the bank wants to invest (it lies at the corner), $\Omega_0 = \{a_2, a_3\}$.

[Insert Figures 2 and 3 here.]

Lemma 5 *$\omega_B \in \Omega$ is an equilibrium threshold if and only if $\omega_B \in \Omega_0$ and Equation (12) holds for every $\omega_B' \in \Omega_0$.*

The idea behind Lemma 5 is simple. As noted earlier, $\omega_B \geq \bar{\omega}_B$. Moreover, unless $\omega_B = \bar{\omega}_B$, the equilibrium threshold ω_B cannot lie inside an interval in which the bank wants to invest because the bank could increase its payoff by reducing ω_B. The equilibrium ω_B can also not lie inside an interval in which the bank does not want to invest because the bank could increase its payoff by increasing ω_B. Hence, the equilibrium ω_B must be in the set Ω_0. In the proof, we show that we can also restrict attention to Ω_0 when we consider deviations.

Using Lemma 5, we can derive a closed-form solution for ω_B as a function of q. An easy case is when Ω_0 contains only one threshold ω_1 (e.g., part 1 in Example 1). In this case, $\omega_B = \omega_1$, independently of q.

The more interesting case is when Ω_0 contains more than one threshold.[12] To

[12] A necessary condition for this is that $G(\omega)$ is nonmonotone.

obtain intuition, we start with the case in which Ω_0 contains only two thresholds, ω_1 and ω_2, where $\omega_1 > \omega_2$ (e.g., parts 2 and 3 in Example 1). From Lemma 5, ω_1 is an equilibrium threshold if and only if

$$V(\omega_1, \bar{\omega}_R(\omega_1)) \geq V(\omega_2, \bar{\omega}_R(\omega_1)). \tag{14}$$

Similarly, ω_2 is an equilibrium threshold if and only if

$$V(\omega_2, \bar{\omega}_R(\omega_2)) \geq V(\omega_1, \bar{\omega}_R(\omega_2)). \tag{15}$$

Let

$$\rho(\omega_1, \omega_2) \equiv \frac{|\int_{\omega_2}^{\omega_1} 1_{\{\omega:G(\omega)<0\}} G(\omega) dF(\omega)|}{\int_{\omega_2}^{\omega_1} 1_{\{\omega:G(\omega)>0\}} G(\omega) dF(\omega)}. \tag{16}$$

Equation (14) reduces to $q \leq \rho(\omega_1, \omega_2)$, and Equation (15) reduces to $q \geq \rho(\omega_1, \omega_2)$. (The appendix contains more details.) Hence, if $q < \rho(\omega_1, \omega_2)$, the unique equilibrium threshold is $\omega_B = \omega_1$. If $q > \rho(\omega_1, \omega_2)$, the unique equilibrium threshold is $\omega_B = \omega_2$. If $q = \rho(\omega_1, \omega_2)$, both ω_1 and ω_2 are equilibrium thresholds.

In what follows, whenever there is more than one equilibrium threshold ω_B, we focus on the equilibrium that is most preferred by the regulator: namely, the equilibrium with the highest threshold. This equilibrium is also weakly preferred by the bank.[13] We let $\omega_B(q)$ stand for the equilibrium threshold ω_B for a given q. We obtain the following.

Proposition 1 *If Ω_0 contains only two thresholds $\omega_1 > \omega_2$, then:*

1. *If $\rho(\omega_1, \omega_2) \geq 1$, then $\omega_B(q) = \omega_1$ for every $q \in [0, 1]$.*
2. *If $\rho(\omega_1, \omega_2) < 1$, then*

$$\omega_B(q) = \begin{cases} \omega_1 & \text{if } q \leq \rho(\omega_1, \omega_2) \\ \omega_2 & \text{if } q > \rho(\omega_1, \omega_2) \end{cases}. \tag{17}$$

Choosing a lower ω_B corresponds to producing less information. Hence, Proposition 1 captures the intuition that when the regulator monitors more, the bank produces less information.[14]

[13] In particular, when ω_B is higher, the bank can satisfy Equation (8) by setting a weakly lower ω_R.

[14] Consistent with Blackwell (1951), less information here means less information that is relevant to investment decisions.

Intuitively, the bank faces a tradeoff. Producing more information is beneficial for the bank because the bank can make better investment decisions for its equity holders. However, producing more information can also be costly for the bank because if the regulator finds out the information, he can use it to ban investment when the bank wants to invest but the regulator does not want to invest. This tradeoff is captured by the ratio ρ in Equation (16). Specifically, the numerator reflects the gain from choosing the higher threshold $\omega_B = \omega_1$, namely the bank avoids investment when $\omega \in (\omega_2, \omega_1)$ and $G(\omega) < 0$. The denominator (times q) captures the cost; namely if the regulator observes model $\mathbf{P}^B$, he bans investment when $\omega \in (\omega_2, \omega_1)$ and $G(\omega) > 0$. Since the expected cost is increasing in q, the bank prefers the higher threshold only if q is sufficiently low.

We refer to the ratio ρ in Equation (16) as the bank's private gain (relative to cost) from producing information.

Example 2 Consider Part 2 in Example 1. So, $\omega_1 = 0.83$ and $\omega_2 = 0.5$. The bank's private gain from producing information is

$$\rho(\omega_1, \omega_2) = \frac{|\int_{0.67}^{0.83} G(\omega) dF(\omega)|}{\int_{0.5}^{0.67} G(\omega) dF(\omega)}. \tag{18}$$

Figure 4 illustrates $\rho(\omega_1, \omega_2)$ graphically as the ratio of two "areas" under the function $G(\omega)$.

[Insert Figure 4 here.]

The next theorem extends Proposition 1 to the general case in which Ω_0 contains n thresholds $\omega_1 > \omega_2 > ... > \omega_n$. The theorem shows that the equilibrium threshold ω_B can be described by a step function, which is decreasing in q.[15]

Theorem 1 *There exist $\delta_1, \delta_2, ..., \delta_m \in \Omega_0$, such that*

$$\omega_B(q) = \begin{cases} \delta_1 & \text{if } q \in [0, \bar{q}_1] \\ \delta_2 & \text{if } q \in (\bar{q}_1, \bar{q}_2] \\ \vdots & \\ \delta_m & \text{if } q \in (\bar{q}_{m-1}, 1], \end{cases} \tag{19}$$

[15] At the corners of the steps there could be more than one equilibrium threshold ω_B, and as noted earlier, we focus on the equilibrium with the highest threshold.

where $\bar{q}_i = \rho(\delta_i, \delta_{i+1})$ *for* $i \in \{1, 2, ..., m-1\}$*. Moreover,* $\delta_1 = \omega_1 > \delta_2 > ... > \delta_m$*.*

The proof (in the Appendix) fully defines the number of steps m and the values for $\delta_1, \delta_2, ..., \delta_m$. Note that each $\bar{q}_i$ represents the bank's private gain from producing information by moving from threshold δ_{i+1} to the next (higher) threshold δ_i.

As an aside, observe that, while ω_B is (weakly) decreasing in q, ω_R is (weakly) increasing. In other words, when the regulator monitors more heavily, the bank reveals more information to the regulator but produces less information overall. This follows because $\omega_R = \bar{\omega}_R(\omega_B)$, which is weakly decreasing in ω_B. Intuitively, when the regulator monitors the bank more heavily, the bank produces less information for itself, and so, the bank is more likely to invest in negative NPV projects even though the bank's equity holders do not gain from such investment. Since the regulator is aware of this, it becomes harder to persuade the regulator to allow investment, and so, the model that the bank reveals to the regulator becomes more informative.

6.2 Optimal q

The regulator's payoff from choosing a probability of monitoring q is

$$u(q) \equiv (1-q) \int_{\bar{\omega}_R(\omega_B(q))}^{\omega_B(q)} 1_{\{\omega: G(\omega) \geq 0\}} N(\omega) dF(\omega) + \int_{\omega_B(q)}^{1} N(\omega) dF(\omega). \qquad (20)$$

The regulator's payoff is similar to the bank's [Equation (9)], but instead of $G(\omega)$, we have $N(\omega)$. In equilibrium, the regulator chooses $q \in [0, 1]$ to maximize (20).

The next lemma shows that the regulator's problem has a solution that lies at the (right) corners of the intervals that define the step function in Theorem 1. That is, there is a solution $q \in \{\bar{q}_1, ..., \bar{q}_{m-1}, 1\}$. This follows because $\omega_B(q)$ is left-continuous and the first integral in (20) is at most zero.[16] In general, there are no solutions that lie inside an interval $(\bar{q}_{i-1}, \bar{q}_i)$.

Lemma 6 *If* K *contains only one state or the highest state in* K *is at most* $\bar{\omega}_B$ *(i.e.,* $a_1 \leq \bar{\omega}_B$*), then every* $q \in [0, 1]$ *is optimal. Otherwise, any solution* q *to the regulator's problem must be in the set* $\{\bar{q}_1, ..., \bar{q}_{m-1}, 1\}$*.*

[16] In particular, since $\omega_B(q) \leq \frac{1}{x}$, the project has negative NPV when $\omega < \omega_B(q)$.

A special case is when $\Omega_0 = \{\omega_1\}$. In this case, the step function in Theorem 1 has only one step: $\omega_B(q) = \omega_1$ for every $q \in [0,1]$. Hence, $q = 1$ is optimal. From Lemma 6, $q = 1$ is uniquely optimal if $|K| \geq 2$ and $a_1 > \bar{\omega}_B$; otherwise, every $q \in [0,1]$ is optimal. Examples when every q is optimal are the first case in Example 1 or the second case in Example 1 but with $\bar{\omega}_B > 0.83$.

The rest of this section focuses on the more interesting case in which Ω_0 contains more than one state. In this case, $|K| \geq 2$ and $a_1 > \bar{\omega}_B$. So, by Lemma 6, any solution to the regulator's problem satisfies $q \in \{\bar{q}_1, ..., \bar{q}_{m-1}, 1\}$.

We illustrate our main result for the case $\Omega_0 = \{\omega_1, \omega_2\}$, where $\omega_1 > \omega_2$. If $\rho(\omega_1, \omega_2) \geq 1$, we know from Proposition 1 that $\omega_B(q) = \omega_1$ for every $q \in [0,1]$. Hence, $q = 1$ is uniquely optimal. If, instead, $\rho(\omega_1, \omega_2) < 1$, then $\omega_B(q)$ is given by the step function in (17), and any solution to the regulator's problem satisfies $q \in \{\rho(\omega_1, \omega_2), 1\}$. Consequently, if $u(1) > u(\rho(\omega_1, \omega_2))$, it is uniquely optimal to set $q = 1$, and if $u(1) < u(\rho(\omega_1, \omega_2))$, it is uniquely optimal to set $q = \rho(\omega_1, \omega_2)$. In the first case, the bank responds by choosing the lower level of information production ($\omega_B = \omega_2$). In the second case, the bank responds by choosing the higher level of information production ($\omega_B = \omega_1$).

Let

$$\hat{q} \equiv \frac{|\int_{\bar{\omega}_R(\omega_1)}^{\omega_2} 1_{\{\omega:G(\omega)\geq 0\}} N(\omega) dF(\omega)| - |\int_{\omega_2}^{\omega_1} 1_{\{\omega:G(\omega)<0\}} N(\omega) dF(\omega)|}{|\int_{\bar{\omega}_R(\omega_1)}^{\omega_2} 1_{\{\omega:G(\omega)\geq 0\}} N(\omega) dF(\omega)| + |\int_{\omega_2}^{\omega_1} 1_{\{\omega:G(\omega)\geq 0\}} N(\omega) dF(\omega)|}. \tag{21}$$

Observe that $\hat{q} < 1$. The condition $u(1) < u(\rho(\omega_1, \omega_2))$ reduces to $\rho(\omega_1, \omega_2) > \hat{q}$. Hence, we have established the following:

Proposition 2 *If Ω_0 contains only two thresholds $\omega_1 > \omega_2$, then*

1. If $\rho(\omega_1, \omega_2) > \hat{q}$, the regulator sets $q = \min\{\rho(\omega_1, \omega_2), 1\}$, and the bank responds by choosing $\omega_B = \omega_1$.

2. If $\rho(\omega_1, \omega_2) < \hat{q}$, the regulator sets $q = 1$, and the bank responds by choosing $\omega_B = \omega_2$.

3. If $\rho(\omega_1, \omega_2) = \hat{q}$, both $q = \rho(\omega_1, \omega_2)$ and $q = 1$ are optimal.

Part 1 in Proposition 2 captures the intuition that, if the bank's private gain from producing information is relatively high, it is optimal to induce the bank to produce information. If the private gain is very high, it is possible to do so even if $q = 1$, and so $q = 1$ is optimal. Otherwise, a lower $q < 1$ is necessary. Part 2 captures the intuition that, if the bank's private gain from producing information is low, it is possible to induce the bank to produce information only if the regulator precommits to a very low level of monitoring. But then, the regulator cannot make much use of the information that the bank produces and is better off setting $q = 1$, even though this induces the bank to produce less information overall.

Example 3 Consider Example 2. Suppose $\bar{\omega}_R(0.83) = 0.4$.[17] Then

$$\hat{q} \equiv \frac{|\int_{0.4}^{0.5} N(\omega)dF(\omega)| - |\int_{0.67}^{0.83} N(\omega)dF(\omega)|}{|\int_{0.4}^{0.5} N(\omega)dF(\omega)| + |\int_{0.5}^{0.67} N(\omega)dF(\omega)|}. \tag{22}$$

If $\hat{q} \leq 0$, it is optimal to set $q = \min\{\rho(\omega_1, \omega_2), 1\}$. In this case, the optimal q falls as $\rho(\omega_1, \omega_2)$ falls. If instead $\hat{q} \in (0, 1)$, we obtain a nonmonotone relationship, as follows. If $\rho(\omega_1, \omega_2) < \hat{q}$, it is optimal to set $q = 1$; if $\rho(\omega_1, \omega_2) \in (\hat{q}, 1)$, it is optimal to set $q = \rho(\omega_1, \omega_2)$; and if $\rho(\omega_1, \omega_2) > 1$, it is again optimal to set $q = 1$.

The next theorem extends the previous intuition to the general case. The theorem shows that it is optimal to set a high level of monitoring $q = 1$ when the bank's private gain from producing more information is either sufficiently high or sufficiently low. Formally, let

$$\hat{q}_i \equiv \frac{|\int_{\bar{\omega}_R(\delta_i)}^{\delta_m} 1_{\{\omega:G(\omega)\geq 0\}} N(\omega)dF(\omega)| - |\int_{\delta_m}^{\delta_i} 1_{\{\omega:G(\omega)<0\}} N(\omega)dF(\omega)|}{|\int_{\bar{\omega}_R(\delta_i)}^{\delta_m} 1_{\{\omega:G(\omega)\geq 0\}} N(\omega)dF(\omega)| + |\int_{\delta_m}^{\delta_i} 1_{\{\omega:G(\omega)\geq 0\}} N(\omega)dF(\omega)|} < 1. \tag{23}$$

Then:

Theorem 2 *If* $|\Omega_0| \geq 2$*, then:*

1. $q = 1$ *is optimal if and only if either* $\min_{\omega\in\Omega_0:\omega<\omega_1} \rho(\omega_1, \omega) \geq 1$ *or* $\rho(\delta_i, \delta_{i+1}) \leq \hat{q}_i$ *for every* $i \in \{1, 2, ..., m-1\}$*.*

2. $q = 1$ *is uniquely optimal if and only if either* $\min_{\omega\in\Omega_0:\omega<\omega_1} \rho(\omega_1, \omega) \geq 1$ *or* $\rho(\delta_i, \delta_{i+1}) < \hat{q}_i$ *for every* $i \in \{1, 2, ..., m-1\}$*.*

[17] Note that we must have $\bar{\omega}_R(0.83) < \bar{\omega}_B = 0.5$.

($m, \delta_1, \delta_2, ..., \delta_m$ are from Theorem 1.)

The condition $\min_{\omega \in \Omega_0 : \omega < \omega_1} \rho(\omega_1, \omega) \geq 1$ in Theorem 2 says that the bank gains a lot from producing information by moving from threshold $\omega < \omega_1$ to the (highest) threshold ω_1. In the proof, we show that when this condition holds, the step function in Theorem 1 has only one step, and so, $q = 1$ is uniquely optimal.

The second condition, $\rho(\delta_i, \delta_{i+1}) \leq \hat{q}_i$ for every $i \in \{1, 2, ..., m-1\}$, applies to the case in which the step function in Theorem 1 has $m > 1$ steps. In this case, any solution to the regulator's problem satisfies $q \in \{\bar{q}_1, ..., \bar{q}_{m-1}, 1\}$, and so, $q = 1$ is optimal if and only if $u(1) \geq u(\bar{q}_i)$ for every $i \in \{1, 2, ..., m-1\}$. In the proof, we show that the condition $u(1) \geq u(\bar{q}_i)$ reduces to $\rho(\delta_i, \delta_{i+1}) \leq \hat{q}_i$. The last condition says that the bank's private gain from producing information by moving from threshold δ_{i+1} to the next (higher) threshold δ_i is sufficiently low.

7 Comparative statics

We use the insights above to derive comparative statics on how the optimal q and the amount of information that the bank produces change with respect to model parameters. We also discuss the case in which information production is costly and the case in which the regulator can impose penalties. We illustrate for the case in which changing the bank's gain from producing information has a nonmonotone effect on the optimal q.

7.1 Cost of producing information

Suppose the bank incurs a cost $z > 0$ if it produces information on states above ω_2. That is, the bank incurs the cost if it chooses a model that includes a proper subset of $(\omega_2, 1]$. As before, we can focus, without loss of generality, on models that take the simple form as in Equations (5) and (6). So, the bank's expected payoff is $V(\omega_B, \omega_R) - z1_{\{\omega_B > \omega_2\}}$. If $\omega_B > \omega_2$, the cost z also reduces the regulator's payoff.

Proposition 1 continues to hold, but we need to replace the gain from producing information $\rho(\omega_1, \omega_2)$ with the following:

$$\zeta(z) = \frac{|\int_{\omega_2}^{\omega_1} 1_{\{\omega:G(\omega)<0\}} G(\omega) dF(\omega)| - z}{\int_{\omega_2}^{\omega_1} 1_{\{\omega:G(\omega)\geq 0\}} G(\omega) dF(\omega).} \tag{24}$$

Intuitively, the cost z reduces the bank's gain from producing information, and this is reflected in the numerator of Equation (24).[18]

Using similar logic as in Proposition 2, we obtain the following:

Proposition 3 *There exists $\bar{z} \in \mathbb{R}$, such that:*

1. If $z < \bar{z}$, the regulator sets $q = \min\{1, \zeta(z)\}$ and the bank responds by choosing $\omega_B = \omega_1$. In this range, q is decreasing in z.

2. If $z > \bar{z}$, the regulator sets $q = 1$, and the bank responds by choosing $\omega_B = \omega_2$.

3. If $z = \bar{z}$, the regulator is indifferent between setting $q = 1$ and setting $q = \zeta(z)$.

Part 1 captures the intuition that, if the cost of producing information is sufficiently low, the regulator can induce the bank to produce information while still maintaining a relatively high level of monitoring. In this range, as the cost of producing information increases, the regulator monitors less because it becomes harder to induce the bank to produce information. Part 2 captures the intuition that if the cost of producing information is very high, it becomes too costly, or even impossible, for the regulator to induce the bank to produce information. In this case, it is optimal to set a high level of monitoring, even though the bank will produce less information.

7.2 Changes in model parameters

We explore the effect of changes in project cash flow x, the face value of debt D, and the value of existing assets $v(\omega)$. As we show below, these model parameters affect the bank's gain from investing $G(\omega)$, which, in turn, affect the bank's gain from producing information $\rho(\omega_1, \omega_2)$ and the optimal monitoring intensity q.

[18] Given our assumption on the cost of information production, the set Ω_0 from which the bank chooses ω_B does not change. However, a different specification for the cost function (e.g., assuming that the cost is $z\omega_B$) could potentially affect Ω_0. Although the main force and intuition we identify here will still be valid, some of the derivations and arguments may be more convoluted (see also Footnote 20).

Specifically, from Equation (4), an increase in x, an increase in D, or a reduction in $v(\omega)$ increase $G(\omega)$. And from Equation (16), an increase in $G(\omega)$ leads to a lower $\rho(\omega_1, \omega_2)$, noting that the numerator decreases and the denominator increases.

Hence, for a given $q > 0$, an increase in x, an increase in D, or a reduction in $v(\omega)$ lead the bank to produce less information.[19] Intuitively, when the bank's gain from investment increases, the bank has less incentive to produce information because the cost that the regulator will use the information to ban investment becomes more significant, while at the same time, the gain from producing information to avoid investment in negative NPV projects becomes less significant.

As for the optimal q, under some regularity conditions,[20] the fact that model parameters affect $\rho(\omega_1, \omega_2)$ leads to similar implications as in Proposition 2. Specifically:

With respect to the bank debt level, when D is very low, the bank has strong incentives to produce information, and, therefore, the regulator chooses a high level of monitoring without much perverse effect. As D increases, the regulator needs to lower the level of monitoring to induce information production. Finally, when D is sufficiently high, the regulator moves back to full monitoring as information production is very difficult to induce.

As for the asset value $v(\omega)$, the predictions are opposite to those related to D. When asset values are high, the regulator monitors extensively, as the bank has strong incentives to produce information. When asset values are moderate, the regulator monitors less to induce the bank to produce more information. When asset values are low, the regulator monitors extensively, but the bank produces less information.

Finally, as for the project cash flow x, when x is low, the regulator monitors extensively, and the bank produces a lot of information. When x is medium, the regulator monitors less to induce the higher level of information production. When

[19] The parameter changes above could also affect the set $\{\omega : G(\omega) = 0\}$, and hence, the set Ω_0 from which the bank chooses ω_B. For example, when D increases, the thresholds ω_1 and ω_2 become lower. This effect also works to reduce the amount of information that the bank produces.

[20] Such conditions are needed because, as noted earlier, the set Ω_0 could change. This could affect the regulator's gain from implementing a higher level of information production versus a lower level of information production. Moreover, with respect to x, we should also take into account the direct effect of x on the regulator's payoff.

x is sufficiently high, the regulator monitors extensively, but the bank produces less information.

7.3 Penalties

Although we do not conduct a formal analysis of the effect of penalties, our model provides some insights. We discuss two types of penalties. The first type of penalty is imposed on the bank whenever its project fails. The second type of penalty is also imposed upon project failure but only if it is found that the bank knowingly invested in a negative NPV project (without the regulator's approval). In our model, the bank has limited liability, so it is natural to think of nonpecuniary penalties.

The first type of penalty reduces the bank's gains from investing $G(\omega)$. For example, a fixed penalty upon failure c will reduce $G(\omega)$ by $(1-\omega)c$. As we showed above, this leads the bank to produce more information. However, if $G(\omega)$ is close to $N(\omega)$, this penalty could also lead to a negative $G(\omega)$ even when $N(\omega)$ is positive. So, the bank could forego investment in some positive NPV projects.

The second type of penalty solves the underinvestment problem above. Moreover, taking the information that the bank produces as given, this penalty reduces the conflict of interest between the bank and the regulator because the bank's gain from investing in negative NPV projects is reduced. However, a higher penalty could also induce the bank to produce less information overall because the regulator could use the information against the bank to show that the bank knowingly invested in negative NPV projects. This effect is similar to that of increasing q. Our model suggests that imposing high penalties of the second type could be beneficial if the bank's private gain from producing information is either sufficiently high or sufficiently low. Otherwise, such penalties could be suboptimal.

8 Public information

In this section, we analyze how public information affects the optimal q as well as the bank's incentives to produce information.

Suppose it is common knowledge that everyone is endowed with some model $\hat{\mathbf{P}}$. In other words, $\hat{\mathbf{P}}$ is public information. For example, $\hat{\mathbf{P}}$ could represent existing rules that are used to determine risk weights without relying on bank internal models. $\hat{\mathbf{P}}$ could also represent publicly available credit ratings or models that the regulator produces and shares with the bank.[21] So, in Step 3 in the sequence of events, the bank observes $P^B(\omega)$ and $\hat{P}(\omega)$. As for the regulator, with probability q, he observes $P^B(\omega)$ and $\hat{P}(\omega)$, and with probability $1-q$, he observes $P^R(\omega)$ and $\hat{P}(\omega)$.

Let ϕ_1 and ϕ_2 be the corners of the information set in $\hat{\mathbf{P}}$ that contains the state $\frac{1}{x}$. Formally, $\phi_1 = \inf \hat{P}(\frac{1}{x})$ and $\phi_2 = \sup \hat{P}(\frac{1}{x})$. For any $\mathbf{P}^B$ and $\mathbf{P}^R$, if $\omega < \phi_1$, investment will not take place because the regulator knows that the project has negative NPV. Similarly, if $\omega > \phi_2$, investment will take place because both the bank and the regulator know that the project has positive NPV. Hence, the models that the bank chooses affect the outcome only when $\omega \in (\phi_1, \phi_2)$ (and potentially at the corners ϕ_1 and ϕ_2).

The problem reduces to finding the thresholds ω_B and ω_R, as in the previous section, but instead of $\bar{\omega}_B$ and $\bar{\omega}_R(\omega_B)$, we now have $\breve{\omega}_B$ and $\breve{\omega}_R(\omega_B)$, which are defined as follows: $\breve{\omega}_B$ is the lowest $\omega_B \in [\phi_1, \phi_2]$ that satisfies

$$E[N(\tilde{\omega})|\tilde{\omega} \in [\omega_B, \phi_2)] \geq 0, \tag{25}$$

and $\breve{\omega}_R(\omega_B)$ is the lowest $\omega_R \in [\phi_1, \phi_2]$ that satisfies

$$E[N(\tilde{\omega})|\tilde{\omega} \in [\omega_B, \phi_2) \text{ or } \tilde{\omega} \in [\omega_R, \omega_B) \text{ and } G(\omega) \geq 0] \geq 0. \tag{26}$$

An interesting question is how the optimal q changes when ϕ_1 or ϕ_2 change. We illustrate for the case in which $\Omega_0 = \{\omega_1, \omega_2\}$, where $\omega_1 > \omega_2 > \bar{\omega}_B$, and focus on the more interesting case in which $\rho(\omega_1, \omega_2) < \hat{q}$. So, without any information, the regulator sets $q = 1$ and the bank responds by choosing $\omega_B = \omega_2$ (Proposition 2).[22]

Start with the case $\phi_2 = 1$.

[21] We abstract from an important question of whether the regulator should share its models with the bank.

[22] If $\rho(\omega_1, \omega_2) > \hat{q}$, the first case in Proposition 4 is absent and the second case holds for every $\phi_1 < \omega_2$ with the regulator setting $q = \min\{1, \rho(\omega_1, \omega_2)\}$.

Proposition 4 *If $\phi_2 = 1$ and Ω_0 contains only two thresholds $\omega_1 > \omega_2$, such that $\rho(\omega_1, \omega_2) < \hat{q}$, then there exists $\hat{\phi} \in (\bar{\omega}_R(\omega_1), \omega_2)$, such that:*

1. If $\phi_1 < \hat{\phi}$, the regulator sets $q = 1$, and the bank responds by choosing $\omega_B = \omega_2$.

2. If $\phi_1 \in (\hat{\phi}, \omega_2]$, the regulator sets $q = \rho(\omega_1, \omega_2)$, and the bank responds by choosing $\omega_B = \omega_1$.

3. If $\phi_1 \in (\omega_2, \omega_1]$, the regulator sets $q = \min\{1, \rho(\omega_1, \phi_1)\}$, and the bank responds by choosing $\omega_B = \omega_1$. In this range, the optimal q is increasing in ϕ_1.

4. If $\phi_1 \in (\omega_1, \frac{1}{x})$, any q is optimal, and the bank choice of models is irrelevant.

(When $\phi_1 = \hat{\phi}$, there are two /

Proposition 5 *equilibria: the equilibrium from Part 1 and the equilibrium from Part 2.)*

Proposition 4 illustrates two forces that push q in different directions. When the public model is more informative (ϕ_1 increases), the benefit from monitoring the bank is reduced because the regulator can use the public model to ban investment. This could lead to a lower q. However, it could also become easier to induce the bank to produce information because if the regulator uses the public model to ban investment, the bank gains less from not producing information. This could lead to a higher q. Part 2 in Proposition 4 illustrates the first force, while part 3 illustrates the second force. As an aside, note that the Proposition also shows that, when ϕ_1 increases, the bank produces more information (i.e., ω_B increases).

Figure 5 illustrates. If $\phi_1 < \omega_2$, an increase in ϕ_1 affects q because it increases $\breve{\omega}_R(\omega_1)$, which, in turn, increases the regulator's payoff $u(\rho(\omega_1, \omega_2))$ from implementing ω_1. Since there is no effect on $u(1)$, the regulator switches from $q = 1$ to $q = (\omega_1, \omega_2)$. If $\phi_1 > \omega_2$, then $\breve{\omega}_B = \phi_1$, and the set Ω_0 becomes $\{\phi_1, \omega_1\}$. In this case, the regulator sets $q = \rho(\phi_1, \omega_1)$, which is increasing in ϕ_1.

[Insert Figure 5 here.]

Finally, reducing ϕ_2 has a similar effect to increasing ϕ_1. If ϕ_2 is sufficiently large so that $\breve{\omega}_B < \omega_2$, reducing ϕ_2 increases $\breve{\omega}_R(\omega_1)$, which leads to a lower q. If ϕ_2 is lower, so that $\breve{\omega}_B > \omega_2$, reducing ϕ_2 increases $\breve{\omega}_B$, which leads to a higher q.

9 Applications

The insights from our model can be applied in other settings. One example is when the regulator uses bank internal risk models to set minimum capital requirements. Suppose capital can be either high or low and the state ω represents how safe the bank portfolio is. Suppose the social gain and bank's private gain from having low capital rather than high capital are given, respectively, by some (exogenous) functions $N(\omega)$ and $G(\omega)$, which satisfy the conditions in Remark 1. That is, the bank's gain from having a lower amount of capital is larger than the regulator's gain, and the regulator's gain is increasing in the state (it could take both positive and negative values). To apply the insights from our model, relabel "investment" to mean having low capital. So, allowing investment means that the regulator allows the bank to have low capital (but the bank can still choose to maintain high capital), while banning investment means that the regulator requires high capital.

Our theory suggests that when existing models (e.g., Basel I risk weights) measure risk imperfectly, it might be socially optimal to rely on bank internal risk models. Moreover, under some conditions, it is optimal to allow the bank to produce two sets of models: one for regulation (to determine capital requirements) and one for its own portfolio decisions. Our theory also suggests that when existing models measure risk more precisely, the bank will produce more information, but it may be optimal to monitor the bank more.

In practice, the bank's actions (e.g., the interest rate it charges on its loans) may reveal information to the regulator beyond the information that the bank revealed initially for the purpose of regulation. The regulator may then be tempted to set capital requirements based on this additional information. Our theory suggests that such practice could be suboptimal because it could induce the bank to produce less information. An extension of our theory could provide conditions under which such practice is optimal and under which such practice is suboptimal.[23]. Another interest-

[23] The idea that incentives to produce information should be taken into account also appears in Rajan, Seru, and Vig (2015). They provide evidence suggesting that securitization reduces lenders' incentives to collect soft information, which leads to a breakdown of statistical models used to predict

ing extension would be a setting in which multiple banks hold correlated information. In this case, the regulator could use information from one bank to regulate another bank.

We obtain similar implications in the context of bank stress tests. Here the regulator decides whether the bank can pay dividends (pass the test) or not (fail the test), and the functions $N(\omega)$ and $G(\omega)$ represent the social gain and the bank's private gain from paying dividends.

There are also applications outside the banking industry. For example, in the context of regulation in the pharmaceutical industry, we can think of $N(\omega)$ and $G(\omega)$ as the benefits to society and to the drug company from introducing a new drug. Here the state ω could represent, for example, potential side effects (which are decreasing in the state).

Our framework can also be applied in corporate governance. The optimality of delegation of authority to the management has been studied in the context of information transmission and incorporation of private information into decision making. Our model provides new insights on the benefits of curbing boards' and shareholders' power to create shareholder value. In particular, when the agency costs are intermediate, it may be optimal for shareholders to commit to a less stringent monitoring mechanism.

10 Concluding remarks

We analyze a situation in which a regulator relies on information that a bank produces to regulate the bank. We show that monitoring induces a tradeoff. By monitoring the bank, the regulator obtains information, which could be used to the regulator's advantage. However, a higher level of monitoring could induce the bank to produce less information overall. Solving for the optimal level of monitoring, we show that, in general, the optimal level of monitoring should be high when the bank's private gain from producing information is either sufficiently high or sufficiently low. We use this

default.

result to derive comparative statics as to how the optimal level of monitoring varies with respect to model parameter, such as the bank's level of debt. We also analyze the case in which some public information already exists. We show that when public models are more precise, banks produce more information, but the regulator may end up monitoring more.

One can think of our framework as a persuasion game in which the bank produces two signals. The first signal is used to persuade the regulator to delegate authority to the bank, and the second signal is used to make better investment decisions, once authority is delegated. As is standard in the Bayesian persuasion literature, we assumed that the bank has full control in choosing the information technology, and as such, the regulator cannot (or chooses not to) force the bank to choose a specific information technology. This could be due to the fact that the bank's ability to produce information is private information.

Two assumptions are crucial for our results. First, the regulator can commit to a prespecified monitoring intensity, but he cannot commit to contracts that bind him to allow investment when it is inefficient to do so given his information ex post.[24] Second, we need a restriction on the information technology. In particular, the bank cannot produce information partitions that pool together high and low states while excluding the states in the middle. If we maintained only the first assumption but not the second, the bank would not need to generate a second signal, and the optimal signal could be obtained along the lines of Kamenica and Gentzkow (2011).

In practice, the regulator's commitment to a monitoring technology often arises via various mechanisms and rules that dictate what the regulator and banks can or cannot do. Our findings lend support in favor of simple policy rules, as opposed to complicated recipes that try to get all available information from banks to fine tune regulation. Our main tradeoff continues to hold in dynamic settings or in settings where full commitment is not feasible, as long as the regulator cannot completely adjust the monitoring technology perfectly ex post.

[24]If the regulator could commit to allowing investment in negative NPV projects, he could induce the bank to produce more information even under monitoring.

Our framework can be extended in several directions. One possible path is to allow the regulator to choose whether to rely on banks' internal models. The main tradeoff we identified will continue to hold in such an extension. However, a richer strategy space will enable us to address other issues in bank regulation, such as the impact of using banks' models on banks' risk-taking behavior. Another path is to allow the regulator to use banks' internal models to impose capital requirements coupled with restrictions on investment decisions or endogenous investment decisions by the bank. Such an extension will enable us to make more complete policy prescriptions.

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Appendix

Proof of Lemma 1. We first show that, if $G(\omega) \geq 0$, then either (i) or (ii) holds. To see that, note that, if (i) does not hold, then $N(\omega) < 0$. So, to satisfy $G(\omega) \geq 0$, we must have $D > v(\omega)$. Hence, $N(\omega) + (1-\omega)[D - v(\omega)] \geq 0$, which reduces to $v(\omega) \leq D + \frac{N(\omega)}{1-\omega}$. Hence, (ii) holds. Next, we show that, if (i) or (ii) holds, then $G(\omega) \geq 0$. Clearly, if (i) holds, then $N(\omega) \geq 0$, and so, $G(\omega) \geq 0$. If (ii) holds, then $N(\omega) + (1-\omega)[D - v(\omega)] \geq 0$, and so $N(\omega) + (1-\omega)\max[D - v(\omega), 0] \geq 0$. Hence, $G(\omega) \geq 0$.

Proof of Lemma 2. Consider an equilibrium in which the bank chooses models $\mathbf{P}^B$ and $\mathbf{P}^R$. The equilibrium outcome can be described by a pair of functions $I_B, I_R : \Omega \to \{0,1\}$, where for $\gamma \in \{B, R\}$, $I_\gamma(\omega) = 1$ if and only if investment takes place when the state is ω and the regulator observes $P^\gamma(\omega)$. Clearly, $I_\gamma(\omega) = 1$ if and only if the regulator allows investment and the bank chooses to invest. Let

$$\omega_\gamma = \begin{cases} \inf\{\omega \in \Omega : I_\gamma(\omega) = 1\} & \text{if } I_\gamma(\omega) = 1 \text{ for some } \omega \in \Omega \\ 1 & \text{otherwise.} \end{cases} \tag{A-1}$$

We show that ω_B, ω_R satisfy parts (i) and (ii) in the lemma, and that $\omega_R \leq \omega_B$.

Part (i) follows from the observation that, if $I_B(\omega) = 1$ for some $\omega \in \Omega$, then $I_B(\omega') = 1$ for every $\omega' > \omega$. To see that, suppose $I_B(\omega) = 1$. Then, $E[N(\tilde{\omega})|\tilde{\omega} \in P^B(\omega)] \geq 0$ because otherwise the regulator would ban investment upon observing $P^B(\omega)$. Now consider $\omega' > \omega$. Because $N(\omega)$ is increasing in ω and each set in the model partition is convex, it follows that $E[N(\tilde{\omega})|\tilde{\omega} \in P^B(\omega')] > 0$. Moreover, since $G(\omega) \geq N(\omega)$, it follows that $E[G(\tilde{\omega})|\tilde{\omega} \in P^B(\omega')] > 0$. Hence, upon observing $P^B(\omega')$, the regulator allows investment and the bank invests. That is, $I_B(\omega') = 1$. The result $\omega_R \leq \omega_B$ follows because, if this were not true, the bank could strictly increase its expected payoff by choosing model $\hat{\mathbf{P}}^R = \mathbf{P}^B$ instead of the original model $\mathbf{P}^R$.

Part (ii) is satisfied as follows.

If $\omega < \omega_R$, then $I_R(\omega) = 0$, by the definition of ω_R.

If $\omega > \omega_B$, then $I_R(\omega) = 1$, as follows. Suppose to the contrary that $I_R(\omega) = 0$. From part (i), the bank wants to invest at state ω, given its information $P^B(\omega)$. Hence, $I_R(\omega) = 0$ would imply that $\inf P^R(\omega) < \frac{1}{x}$ because otherwise the regulator would allow investment and the bank would invest. Now consider $\omega' \leq \inf P^R(\omega)$, such that $\omega' \notin P^R(\omega)$. Because $N(\omega)$ is increasing in ω and each set in the model partition is convex, it follows that upon observing $P^R(\omega')$, the regulator knows that the project has negative NPV, and so, investment does not take place. That is, $I_R(\omega') = 0$. But then it follows from the definition of ω_R that $\omega_R > \omega_B$, which contradicts what we showed earlier.

Finally, consider $\omega \in (\omega_R, \omega_B)$. We first show that if $G(\omega) > 0$, then $I_R(\omega) = 1$. Suppose to the contrary that $I_R(\omega) = 0$. Then either $E[G(\tilde{\omega})|\tilde{\omega} \in P^B(\omega)] < 0$, so the bank does not want to invest, or $E[G(\tilde{\omega})|\tilde{\omega} \in P^B(\omega)] \geq 0$ and the regulator does not allow investment. Following similar logic as above, the second case cannot happen because this would imply that $I_R(\omega') = 0$ for every $\omega' < \omega$, which leads to a contradiction $\inf\{\omega \in \Omega : I_\gamma(\omega) = 1\} > \omega_R$. Hence, $E[G(\tilde{\omega})|\tilde{\omega} \in P^B(\omega)] < 0$. Hence, $P^B(\omega)$ is an interval. Since G is continuous, $P^B(\omega)$ contains a positive measure of states with $G(\omega) > 0$ in which the bank does not invest according to its ideal investment rule. But then the bank can increase its payoff by choosing model $\hat{\mathbf{P}}^B$ instead of $\mathbf{P}^B$, where

$$\hat{P}^B(\omega) = \begin{cases} P^B(\omega) & \text{if } \omega > \omega_B \\ \omega & \text{otherwise.} \end{cases} \quad \text{(A-2)}$$

In particular, if the regulator observes $\hat{P}^B(\omega)$, investment will continue to take place when $\omega > \omega_B$, and revealing the state when $\omega < \omega_B$ does not make the bank worse off. However, if the regulator does not observe $\hat{P}^B(\omega)$, the bank is better off learning the exact state and investing according to its ideal investment rule. Using similar arguments, we can show that if $G(\omega) < 0$, we must have $I_R(\omega) = 0$. This completes the proof.

Proof of Lemma 3. Consider an equilibrium outcome I_B, I_R, as described in the proof of Lemma 2. Let ω_B, ω_R be the corresponding thresholds.

We first show that Equations (7) and (8) must hold. Let $\gamma \in \{B, R\}$. For every

ω, such that $I_\gamma(\omega) = 1$, we must have $E[N(\tilde{\omega})|\tilde{\omega} \in P^\gamma(\omega), I_\gamma(\tilde{\omega}) = 1] \geq 0$. That is, the project must have positive NPV conditional on the regulator's information $P^\gamma(\omega)$ and investment taking place. It then follows that $E[N(\tilde{\omega})|\tilde{\omega} \in \cup_{\omega>\omega_\gamma} P^\gamma(\omega), I_\gamma(\tilde{\omega}) = 1] \geq 0$. Hence, $E[N(\tilde{\omega})|\omega > \omega_\gamma, I_\gamma(\tilde{\omega}) = 1] \geq 0$. Hence, Equations (7) and (8) hold.

Now suppose the bank chooses simple models as in Equations (5) and (6). Denote the equilibrium outcome in this case by $\hat{I}_B, \hat{I}_R$. We show that for every $\omega \in \Omega$, $\hat{I}_B(\omega) = I_B(\omega)$ and $\hat{I}_R(\omega) = I_R(\omega)$. Since Equation (7) holds, we know that $\hat{I}_B(\omega) = 1$ if $\omega > \omega_B$, and since Equation (8) holds, we know that $\hat{I}_R(\omega) = 1$ if $\omega > \omega_R$ and $I_R(\omega) = 1$. Moreover, $\hat{I}_B(\omega) = 0$ if $\omega < \omega_B$ because if this were not true, if follows from the continuity of G (similar to the proof of Lemma 2) that the bank could increase its payoff by choosing the simple models instead of the original models. Similarly, $\hat{I}_R(\omega) = 0$ if $\omega < \omega_R$. Hence, the simple models lead to the same outcome as the original models.

Proof of Lemma 4. Consider an equilibrium in which the bank chooses models $\mathbf{P}^B$ and $\mathbf{P}^R$ with thresholds ω_B and ω_R, as in Equations (5) and (6), and assume, without loss of generality, that $\omega_R = \bar{\omega}_R(\omega_B)$. To ensure that the regulator allows the bank to invest when $\omega > \omega_B$, Equation (7) must be satisfied. Hence, $\omega_B \geq \bar{\omega}_B$. Next, consider a deviation in which instead of model $\mathbf{P}^B$, the bank chooses (a simple) model $\hat{\mathbf{P}}^B$ with a threshold $\omega'_B \geq \bar{\omega}_B$, while keeping model $\mathbf{P}^R$ unchanged. The bank's payoff from this deviation is at least $V(\omega'_B, \bar{\omega}_R(\omega_B))$ because if the regulator observes $\hat{\mathbf{P}}^B$, he allows investment if $\omega > \omega'_B$, and if the regulator observes $\mathbf{P}^R$, he continues to believe that the other model is $\mathbf{P}^B$, and so he continues to allow investment when $\omega \geq \bar{\omega}_R(\omega_B)$. Hence, to rule out this deviation, Equation (12) must hold for every $\omega'_B \in \Omega$, such that $\omega'_B \geq \bar{\omega}_B$.

It remains to show the other direction. Suppose $\omega_B \geq \bar{\omega}_B$ and Equation (12) holds for every $\omega'_B \in \Omega$, such that $\omega'_B \geq \bar{\omega}_B$. We show that there is an equilibrium in which the bank chooses models $\mathbf{P}^B$ and $\mathbf{P}^R$ with thresholds ω_B and $\omega_R = \bar{\omega}_R(\omega_B)$. In such an equilibrium, the bank's payoff is $V(\omega_B, \bar{\omega}_R(\omega_B))$. We need to show that the bank cannot increase its payoff by choosing different models.

Consider a deviation in which the bank chooses models $\hat{\mathbf{P}}^B$ and $\hat{\mathbf{P}}^R$, which do not necessarily take the simple form in Equations (5) and (6). Without loss of generality, we focus on payoffs from the best possible (or limit of best possible) deviation for the bank. Assign out-of-equilibrium beliefs, such that when the regulator observes $\hat{\mathbf{P}}^R \neq \mathbf{P}^R$, he believes that the other model is $\mathbf{P}^B$. Then, upon observing $\hat{\mathbf{P}}^R(\omega)$, the regulator anticipates that if he allows investment, the bank will invest if and only if $E[G(\tilde{\omega})|\tilde{\omega} \in \mathbf{P}^B(\omega)] \geq 0$.

Following the logic of Lemma 2, we can show that $\hat{\omega}_B \in \Omega$ exists, such that the outcome of the deviation is as follows. When the regulator observes model $\hat{\mathbf{P}}^B(\omega)$, the bank invests if $\omega > \hat{\omega}_B$ but not if $\omega < \hat{\omega}_B$; and when the regulator observes models $\hat{\mathbf{P}}^R(\omega)$, he allows the bank to invest when $\omega > \hat{\omega}_R$, but investment does not take place when $\omega < \hat{\omega}_R$. Since the regulator allows investment only if given his information, the project has nonnegative NPV, we must have $\hat{\omega}_B \geq \bar{\omega}_B$ and $E[N(\tilde{\omega})|\tilde{\omega} > \omega_B$ or $\tilde{\omega} \in [\hat{\omega}_R, \omega_B)$ and $G(\tilde{\omega}) \geq 0] \geq 0$. Hence, $\hat{\omega}_R \geq \bar{\omega}_R(\omega_B)$ (from the definition of $\bar{\omega}_R$). Hence, the bank's payoff from the deviation is at most $V(\hat{\omega}_B, \bar{\omega}_R(\omega_B))$. But since Equation (12) holds for every $\omega'_B \in \Omega$, such that $\omega'_B \geq \bar{\omega}_B$, this payoff is at most $V(\omega_B, \bar{\omega}_R(\omega_B))$. This completes the proof.

Proof of Lemma 5. As a preliminary, we show that for every $\omega \notin \Omega_0$ and $\omega_R \in \Omega$, such that $\omega \geq \bar{\omega}_B \geq \omega_R$, there exists $\omega' \in \Omega_0$, such that $V(\omega, \omega_R) < V(\omega', \omega_R)$. Consider $\omega \notin \Omega_0$ and $\omega_R \in \Omega$, such that $\omega \geq \bar{\omega}_B \geq \omega_R$. There exists $i \in \{1, ..., l\}$, such that either $\omega \in (a_i, b_i)$ or $\omega \in [b_{i+1}, a_i)$. ($b_{l+1} \equiv 0$.) If $\omega \in (a_i, b_i)$, let $\omega' = \max\{a_i, \bar{\omega}_B\}$. If $\omega \in [b_{i+1}, a_i)$, let $\omega' = a_i$. Then $\omega' \in \Omega_0$ and $V(\omega, \omega_R) < V(\omega', \omega_R)$.

We use the observation above to prove the lemma. Suppose $\omega_B \in \Omega$ is an equilibrium threshold. From Lemma 4, $\omega_B \geq \bar{\omega}_B$ and Equation (12) holds for every $\omega'_B \in \Omega$ such that $\omega'_B \geq \bar{\omega}_B$. Hence, Equation (12) holds under the weaker condition $\omega'_B \in \Omega_0$. Moreover, we must have $\omega_B \in \Omega_0$ because otherwise the observation above would imply that there exists $\omega' \in \Omega_0$, such that $V(\omega_B, \bar{\omega}_R(\omega_B)) < V(\omega', \bar{\omega}_R(\omega_B))$, which contradicts Lemma 4.

Now suppose $\omega_B \in \Omega_0$ and Equation (12) holds for every $\omega'_B \in \Omega_0$. We show that

Equation (12) also holds for every $\omega'_B \notin \Omega_0$, such that $\omega'_B \geq \bar{\omega}_B$, and hence, by Lemma 4, ω_B is an equilibrium threshold. Suppose to the contrary that $\omega'_B \notin \Omega_0$ exists such that $\omega'_B \geq \bar{\omega}_B$ and $V(\omega_B, \bar{\omega}_R(\omega_B)) < V(\omega'_B, \bar{\omega}_R(\omega_B))$. From the observation above, $\omega' \in \Omega_0$ exists, such that $V(\omega'_B, \bar{\omega}_R(\omega_B)) < V(\omega', \bar{\omega}_R(\omega_B))$. Hence, $V(\omega_B, \bar{\omega}_R(\omega_B)) < V(\omega', \bar{\omega}_R(\omega_B))$, which contradicts the starting assumption that Equation (12) holds for every $\omega'_B \in \Omega_0$.

Lemma A-1 *For any $\omega_R \in \Omega$ and $\omega, \omega' \in \Omega_0$, such that $\omega > \omega'$, the following holds:*

1. *If $q < \rho(\omega, \omega')$, then $V(\omega, \omega_R) > V(\omega', \omega_R)$.*
2. *If $q = \rho(\omega, \omega')$, then $V(\omega, \omega_R) = V(\omega', \omega_R)$.*
3. *If $q > \rho(\omega, \omega')$, then $V(\omega, \omega_R) < V(\omega', \omega_R)$.*

Proof of Lemma A-1. The proof applies to a more general case in which there is a cost $z \geq 0$, as in Section 7. Consider $\omega_R \in \Omega$ and $\omega, \omega' \in \Omega_0$, such that $\omega > \omega'$. Observe that $V(\omega, \omega_R) - z > V(\omega', \omega_R)$ is equivalent to

$$\begin{aligned} & (1-q)\int_{\omega_R}^{\omega} 1_{\{\omega:G(\omega)>0\}} G(\omega)dF(\omega) + \int_{\omega}^{1} G(\omega)dF(\omega) - z \\ > \ & (1-q)\int_{\omega_R}^{\omega'} 1_{\{\omega:G(\omega)>0\}} G(\omega)dF(\omega) + \int_{\omega'}^{1} G(\omega)dF(\omega). \end{aligned} \tag{A-3}$$

After rearranging terms, (A-3) reduces to

$$(1-q)\int_{\omega'}^{\omega} 1_{\{\omega:G(\omega)>0\}} G(\omega)dF(\omega) > \int_{\omega'}^{\omega} G(\omega)dF(\omega) + z, \tag{A-4}$$

which reduces to

$$\int_{\omega'}^{\omega} 1_{\{\omega:G(\omega)\geq 0\}} G(\omega)dF(\omega) - \int_{\omega'}^{\omega} G(\omega)dF(\omega) - z > q\int_{\omega'}^{\omega} 1_{\{\omega:G(\omega)>0\}} G(\omega)dF(\omega). \tag{A-5}$$

Since $\omega, \omega' \in \Omega_0$ and $\omega > \omega'$, the integral on the right-hand side of (A-5) is positive. Hence, (A-5) reduces to

$$q < \frac{-\int_{\omega'}^{\omega} 1_{\{\omega:G(\omega)<0\}} G(\omega)dF(\omega) - z}{\int_{\omega'}^{\omega} 1_{\{\omega:G(\omega)>0\}} G(\omega)dF(\omega)} = \frac{|\int_{\omega'}^{\omega} 1_{\{\omega:G(\omega)<0\}} G(\omega)dF(\omega)| - z}{\int_{\omega'}^{\omega} 1_{\{\omega:G(\omega)>0\}} G(\omega)dF(\omega)}. \tag{A-6}$$

Hence, we proved part 1. Parts 2 and 3 follow in a similar fashion.

Proof of Proposition 1. From Lemma 5, the equilibrium threshold ω_B belongs to $\{\omega_1, \omega_2\}$. Moreover, ω_1 is an equilibrium threshold if and only if Equation (14) holds, and ω_2 is an equilibrium threshold if and only if Equation (15) holds. From Lemma A-1, Equation (14) holds if and only if $q \leq \rho(\omega_1, \omega_2)$, and Equation (15) holds if and only if $q \geq \rho(\omega_1, \omega_2)$. The results then follow easily.

Proof of Theorem 1. We construct the step function $\omega_B(q)$ as follows. Let $\bar{q}_0 = 0$, $\delta_1 = \omega_1$, and for integers $i \geq 1$, define recursively:

$$\bar{q}_i = \begin{cases} \min\{1, \min_{\omega \in \Omega_0 \cap [0,\delta_i)} \rho(\delta_i, \omega)\} & \text{if } \delta_i > \omega_n \\ 1 & \text{otherwise.} \end{cases} \tag{A-7}$$

$$\delta_{i+1} = \begin{cases} \min\{\omega \in \Omega_0 \cap [0, \delta_i) : \rho(\delta_i, \omega) = \bar{q}_i\} & \text{if } \bar{q}_i < 1 \\ \delta_i & \text{otherwise.} \end{cases} \tag{A-8}$$

Let $m = \min\{i : \bar{q}_i = 1\}$.

From the definition of m, $\bar{q}_i < 1$ for every $i < m$. Hence, from Equation (A-8), $\delta_1 > \delta_2 > ... > \delta_m$ and

$$\bar{q}_i = \rho(\delta_i, \delta_{i+1}) \text{ for every } i < m. \tag{A-9}$$

In addition, from Equation (A-7), $\delta_i > \omega_n$ for every $i < m$.

We show that $\omega_B(0) = \delta_1$, and that for $i \in \{1, ..., m\}$, $\omega_B(q) = \delta_i$ if $q \in (\bar{q}_{i-1}, \bar{q}_i]$. The proof is by induction. For $i = 1$, we know from the definition of $\bar{q}_1$ that $\bar{q}_1 \leq \min_{\omega \in \Omega_0 \cap [0,\omega_1)} \rho(\omega_1, \omega)$. Hence, when $q \leq \bar{q}_1$, we know from Lemma A-1 that $V(\omega_1, \bar{\omega}_R(\omega_1)) \geq V(\omega, \bar{\omega}_R(\omega_1))$ for every $\omega \in \Omega_0 \cap [0, \omega_1)$, and so by Lemma 5, ω_1 is an equilibrium threshold. Since in the case of multiple equilibria, we focus on the one with the highest threshold, it follows that $\omega_B(q) = \omega_1$ when $q \in [0, \bar{q}_1]$.

Now suppose $i < m$ and $\omega_B(q) = \delta_i$ if $q \in (\bar{q}_{i-1}, \bar{q}_i]$. We show that $\omega_B(q) = \delta_{i+1}$ if $q \in (\bar{q}_i, \bar{q}_{i+1}]$. There are a few steps:

1. From Lemma A-1, it follows that, if $V(\omega_B, \bar{\omega}_R(\omega_B)) \geq V(\omega'_B, \bar{\omega}_R(\omega_B))$, then $V(\omega_B, \omega_R) \geq V(\omega'_B, \omega_R)$ for every $\omega_R \in \Omega$. Hence, we can rewrite Lemma 5 as follows: $\omega_B \in \Omega$ is an equilibrium threshold if and only if $\omega_B \in \Omega_0$ and $V(\omega_B, \omega_R) \geq V(\omega', \omega_R)$ for every $\omega' \in \Omega_0$ and $\omega_R \in \Omega$.

2. We show that, if $q = \bar{q}_i$, then $V(\delta_{i+1}, \omega_R) \geq V(\omega', \omega_R)$ for every $\omega' \in \Omega_0$ and $\omega_R \in \Omega$. In other words, if $q = \bar{q}_i$, then δ_{i+1} weakly dominates any other threshold candidate. This follows from the following two observations. First, if $q = \bar{q}_i$, we know from the induction assumption that δ_i is an equilibrium threshold, and so, from Step 1, $V(\delta_i, \omega_R) \geq V(\omega', \omega_R)$ for every $\omega' \in \Omega_0$ and $\omega_R \in \Omega$. Second, since $\bar{q}_i = \rho(\delta_i, \delta_{i+1})$, it follows from Lemma A-1 that $V(\delta_{i+1}, \omega_R) = V(\delta_i, \omega_R)$ for every $\omega_R \in \Omega$.

3. Now we show that, if $q > \bar{q}_i$, then δ_{i+1} strictly dominates any other threshold candidate that is greater than δ_{i+1}. It follows from Step 2 and Lemma A-1 that $\bar{q}_i \geq \rho(\omega', \delta_{i+1})$ for every $\omega' \in \Omega_0 \cap (\delta_{i+1}, 1]$. It then follows from Lemma A-1 that, if $q > \bar{q}_i$, then $V(\delta_{i+1}, \omega_R) > V(\omega', \omega_R)$ for every $\omega' \in \Omega_0 \cap (\delta_{i+1}, 1]$ and $\omega_R \in \Omega$.

4. It also follows from Step 2 and Lemma A-1 that $\bar{q}_i \leq \rho(\delta_{i+1}, \omega)$ for every $\omega \in \Omega_0 \cap [0, \delta_{i+i})$.

5. If $\delta_{i+1} = \omega_n$, then $\bar{q}_{i+1} = 1$, and from Steps 1 and 3, δ_{i+1} is a unique equilibrium when $q \in (\bar{q}_i, \bar{q}_{i+1}]$. So, the proof is complete.

6. If $\delta_{i+1} > \omega_n$, then $\bar{q}_{i+1} \leq \min_{\omega \in \Omega_0 \cap [0, \delta_{j+1})} \rho(\delta_{i+1}, \omega)$. Hence, from Lemma A-1, if $q \leq \bar{q}_{i+1}$, then $V(\delta_{i+1}, \omega_R) \geq V(\omega', \omega_R)$ for every $\omega' \in \Omega_0 \cap [0, \delta_{i+1})$ and $\omega_R \in \Omega$. In other words, δ_{i+1} weakly dominates any other threshold candidate that is smaller than δ_{i+1}. Note that, if $\bar{q}_{i+1} \neq 1$, then $\bar{q}_{i+1} = \rho(\delta_{i+1}, \delta_{i+2}) \geq \bar{q}_i$, where the last inequality follows from Step 4.

7. It follows from Steps 1, 3, and 6 that, if $q \in (\bar{q}_i, \bar{q}_{i+1}]$, δ_{i+1} is an equilibrium and any other $\omega_B \in \Omega_0 \cap (\delta_{i+1}, 1]$ is not an equilibrium. Since in the case of multiple equilibria we focus on the one with the highest threshold, it follows that $\omega_B(q) = \delta_{i+1}$ if $q \in (\bar{q}_i, \bar{q}_{i+1}]$.

Proof of Lemma 6 As a preliminary, observe that for every $q \in [0, 1]$, $\omega_B(q) \geq \bar{\omega}_B \geq \bar{\omega}_R(\omega_B(q))$. Moreover, $\omega_1 \leq \frac{1}{x}$. Hence, for every q and $\omega < \omega_B(q)$, $N(\omega) < 0$. Hence, the first integral in (20) is either zero or negative. Let $\bar{q}_m \equiv 1$.

If $|K| = 1$ and $a_1 > \bar{\omega}_B$, then $\Omega_0 = \{a_1\}$ and $\omega_B(q) = a_1$. Since $G(\omega) = 0$ for every $\omega < a_1$, the first integral in (20) equals zero, and so, every q is optimal. Similarly, if $a_1 \leq \bar{\omega}_B$, then $\Omega_0 = \{\bar{\omega}_B\}$ and $\omega_B(q) = \bar{\omega}_B$. Since $\bar{\omega}_R(\bar{\omega}_B) = \bar{\omega}_B$, the first integral in

(20) equals zero, and again, every q is optimal.

Now suppose $|K| > 1$ and $a_1 > \bar{\omega}_B$. We show that any solution q to the regulator's problem satisfies $q \in \{\bar{q}_1, ..., \bar{q}_{m-1}, 1\}$. Suppose to the contrary that there exists a solution $q \notin \{\bar{q}_1, ..., \bar{q}_{m-1}, 1\}$. There are two cases:

Case 1: $\omega_B(q) > a_l$ (recall a_l is the lowest element in K). In this case, $\bar{\omega}_R(\omega_B(q)) < \omega_B(q)$, and the first integral in (20) is negative. In particular, either $\omega_B(q) > \bar{\omega}_B \geq \bar{\omega}_R(\omega_B(q))$ or $\omega_B(q) = \bar{\omega}_B > \bar{\omega}_R(\omega_B(q))$, where the last inequality follows since $a_1 > \bar{\omega}_B$ and $\omega_B(q) > a_l$. Consequently, if $q \in [0, \bar{q}_1)$, we know from Theorem 1 that $\omega_B(q) = \omega_B(\bar{q}_1)$, and so, $u(\bar{q}_1) > u(q)$, which contradicts the optimality of q. Similarly, if $q \in (\bar{q}_{i-1}, \bar{q}_i)$ for some $i \in \{2, ..., m\}$, then $\omega_B(q) = \omega_B(\bar{q}_i)$, and so, $u(\bar{q}_i) > u(q)$, which is, again, a contradiction.

Case 2: $\omega_B(q) = a_l$. In this case, the first integral in (20) is zero, and the regulator's payoff reduces to $\int_{a_l}^{1} N(\omega) dF(\omega)$. We obtain a contradiction because the regulator can increase his payoff by setting $q = \bar{q}_1$. Then the bank responds by choosing $\omega_B(q) = a_1 > a_l$, and the regulator's payoff increases because, with probability $\bar{q}_1$, the bank does not invest when $\omega < a_1$.

Proof of Proposition 2. Suppose $\Omega_0 = \{\omega_1, \omega_2\}$, where $\omega_1 > \omega_2$. Then in Theorem 1, $\delta_1 = \omega_1$ and $\bar{q}_1 = \min\{1, \rho(\omega_1, \omega_2)\}$.

If $\rho(\omega_1, \omega_2) \geq 1$, then $m = 1$, and by Lemma 6, $q = 1$ is uniquely optimal (because $|\Omega_0| \geq 2$ implies that $|K| \geq 2$ and $a_1 > \bar{\omega}_B$). The bank responds by choosing $\omega_B = \delta_1 = \omega_1$.

If $\rho(\omega_1, \omega_2) < 1$, then $\bar{q}_1 = \rho(\omega_1, \omega_2)$, $\delta_2 = \omega_2$, $\bar{q}_2 = 1$, and $m = 2$. Hence, from Lemma 6, $q \in \{\bar{q}_1, 1\}$. There are three subcases. If $\bar{q}_1 \in (\hat{q}, 1)$, then $u(1) < u(\bar{q}_1)$. (The proof of Theorem 2 contains more details.) So, $q = \bar{q}_1$ is uniquely optimal, and the bank responds by choosing $\omega_B = \delta_1 = \omega_1$. If $\bar{q}_1 < \hat{q}$, then $u(1) > u(\bar{q}_1)$. So, $q = 1$ is uniquely optimal, and the bank responds by choosing $\omega_B = \delta_2 = \omega_2$. Finally, if $\bar{q}_1 = \hat{q}$, then $u(1) = u(\bar{q}_1)$. Hence, both $q = \bar{q}_1$ and $q = 1$ are optimal.

Proof of Theorem 2. Suppose Ω_0 contains $n \geq 2$ thresholds $\omega_1 > \omega_2 > ... > \omega_n$. Then, in Theorem 1, $\delta_1 = \omega_1$ and $\bar{q}_1 = \min\{1, \min_{\omega \in \Omega_0 \cap [0, \omega_1)} \rho(\omega_1, \omega)\}$.

If $\min_{\omega\in\Omega_0:\omega<\omega_1}\rho(\omega_1,\omega)\geq 1$, then $m=1$, and by Lemma 6, $q=1$ is uniquely optimal. If instead, $\min_{\omega\in\Omega_0:\omega<\omega_1}\rho(\omega_1,\omega)<1$, then $m>1$, and any solution to the regulator's problem satisfies $q\in\{\bar{q}_1,...,\bar{q}_{m-1},1\}$. Hence, $q=1$ is optimal if and only if $u(1)\geq u(\bar{q}_i)$ for every $i\in\{1,...,m-1\}$, and it is uniquely optimal if and only if $u(1)>u(\bar{q}_i)$ for every $i\in\{1,...,m-1\}$. From Theorem 1, $\bar{q}_i=\rho(\delta_i,\delta_{i+1})$ for every $i\in\{1,...,m-1\}$. Hence, to complete the proof, we need to show that, for every $i\in\{1,...,m-1\}$, $u(1)\geq u(\bar{q}_i)$ reduces $\bar{q}_i\leq\hat{q}_i$. (Similarly, $u(1)>u(\bar{q}_i)$ reduces $\bar{q}_i<\hat{q}_i$.) The details are as follows.

Observe that $u(1)=\int_{\delta_m}^{1}N(\omega)dF(\omega)$, and for $i\in\{1,...,m-1\}$, $u(\bar{q}_i)=(1-\bar{q}_i)\int_{\bar{\omega}_R(\delta_i)}^{\delta_i}1_{\{\omega:G(\omega)\geq 0\}}N(\omega)dF(\omega)+\int_{\delta_i}^{1}N(\omega)dF(\omega)$. Hence, after rearranging terms, $u(1)\geq u(\bar{q}_i)$ reduces to

$$\int_{\delta_m}^{\delta_i}N(\omega)dF(\omega)\geq(1-\bar{q}_i)\int_{\bar{\omega}_R(\delta_i)}^{\delta_i}1_{\{\omega:G(\omega)\geq 0\}}N(\omega)dF(\omega). \tag{A-10}$$

Since $\delta_m<\delta_{m-1}<...<\delta_1\leq\frac{1}{x}$, it follows that $N(\omega)<0$ when $\omega<\delta_i$. Hence, the integrals on both sides of Equation (A-10) are negative. Hence, Equation (A-10) reduces to

$$1-\bar{q}_i\geq\frac{\int_{\delta_m}^{\delta_i}N(\omega)dF(\omega)}{\int_{\bar{\omega}_R(\delta_i)}^{\delta_i}1_{\{\omega:G(\omega)\geq 0\}}N(\omega)dF(\omega)}=\frac{|\int_{\delta_m}^{\delta_i}N(\omega)dF(\omega)|}{|\int_{\bar{\omega}_R(\delta_i)}^{\delta_i}1_{\{\omega:G(\omega)\geq 0\}}N(\omega)dF(\omega)|}, \tag{A-11}$$

or equivalently,

$$\begin{aligned}\bar{q}_i &\leq \frac{|\int_{\bar{\omega}_R(\delta_i)}^{\delta_i}1_{\{\omega:G(\omega)\geq 0\}}N(\omega)dF(\omega)|-|\int_{\delta_m}^{\delta_i}N(\omega)dF(\omega)|}{|\int_{\bar{\omega}_R(\delta_i)}^{\delta_i}1_{\{\omega:G(\omega)\geq 0\}}N(\omega)dF(\omega)|} \qquad \text{(A-12)}\\ &= \frac{|\int_{\bar{\omega}_R(\delta_i)}^{\delta_m}1_{\{\omega:G(\omega)\geq 0\}}N(\omega)dF(\omega)|-|\int_{\delta_m}^{\delta_i}1_{\{\omega:G(\omega)<0\}}N(\omega)dF(\omega)|}{|\int_{\bar{\omega}_R(\delta_i)}^{\delta_m}1_{\{\omega:G(\omega)\geq 0\}}N(\omega)dF(\omega)|+|\int_{\delta_m}^{\delta_i}1_{\{\omega:G(\omega)\geq 0\}}N(\omega)dF(\omega)|}=\hat{q}_i.\end{aligned}$$

Proof of Proposition 3. From the proof of Lemma A-1, Proposition 1 continues to hold if we replace $\rho(\omega_1,\omega_2)$ with $\zeta(z)$. Observe that $\zeta(z)$ is decreasing in z. Let z_1 be the unique z that satisfies $\zeta(z)=1$, and let z_2 be the unique z that satisfies

$$\int_{\omega_2}^{1}N(\omega)dF(\omega)=(1-\zeta(z))\int_{\bar{\omega}_R(\omega_1)}^{\omega_1}1_{\{\omega:G(\omega)\geq 0\}}N(\omega)dF(\omega)+\int_{\omega_1}^{1}N(\omega)dF(\omega)-z. \tag{A-13}$$

A unique z_2 exists because $\int_{\bar{\omega}_R(\omega_1)}^{\omega_1} 1_{\{\omega:G(\omega)\geq 0\}} N(\omega)dF(\omega) < 0$, and so the right-hand side in Equation (A-13) is decreasing in z. Moreover, $z_2 > z_1$. To see that, it is sufficient to show that, when $\zeta(z) = 1$, the right-hand side in Equation (A-13) is greater than the left-hand side. This follows because, from Equation (24), $\zeta(z) = 1$ implies that

$$\begin{aligned} -z &> -\left|\int_{\omega_2}^{\omega_1} 1_{\{\omega:G(\omega)<0\}} G(\omega)dF(\omega)\right| = \int_{\omega_2}^{\omega_1} 1_{\{\omega:G(\omega)<0\}} G(\omega)dF(\omega) \\ &\geq \int_{\omega_2}^{\omega_1} 1_{\{\omega:G(\omega)<0\}} N(\omega)dF(\omega) \geq \int_{\omega_2}^{\omega_1} N(\omega)dF(\omega). \end{aligned} \tag{A-14}$$

If $z \leq z_1$, then $\zeta(z) \geq 1$, and so $\omega_B(q) = \omega_1$ for every $q \in [0, 1]$. In this case, the regulator's payoff is $u(q) - z$, which is increasing in q. Hence, $q = 1$ is uniquely optimal.

If $z > z_1$, then $\zeta(z) < 1$, and so,

$$\omega_B(q) = \begin{cases} \omega_1 & \text{if } q \leq \zeta(z) \\ \omega_2 & \text{if } q > \zeta(z). \end{cases} \tag{A-15}$$

In this case, the regulator's payoff is $u(q) - z1_{\{\omega_B(q)=\omega_1\}}$. Hence, the left-hand side in Equation (A-13) is the regulator's payoff if $q = 1$, and the right-hand side is the regulator's payoff if $q = \zeta(z)$. Hence, when $z = z_2$, the regulator is indifferent between choosing $q = 1$ and choosing $q = \zeta(z)$. If $z \in (z_1, z_2)$, choosing $q = \zeta(z)$ is preferred to choosing $q = 1$. Similarly, if $z > z_2$, choosing $q = 1$ is preferred to choosing $q = \zeta(z)$. To complete the proof, set $\bar{z} = z_2$, and note that from Lemma 6, it follows that any $q \notin \{\zeta(z), 1\}$ is suboptimal.

Proof of Proposition 4.

Parts 1 and 2. If $\phi_1 \leq \omega_2$, then

$$\omega_B(q) = \begin{cases} \omega_1 & \text{if } q \leq \rho(\omega_1, \omega_2) \\ \omega_2 & \text{otherwise} \end{cases} \tag{A-16}$$

and

$$u(q) = (1-q)\int_{\max\{\phi_1, \bar{\omega}_R(\omega_B(q))\}}^{\omega_B(q)} 1_{\{\omega:G(\omega)\geq 0\}} N(\omega)dF(\omega) + \int_{\omega_B(q)}^{1} N(\omega)dF(\omega). \tag{A-17}$$

From Proposition 2, we know that, when $\phi_1 = 0$, the regulator sets $q = 1$. Since $\hat{q} > \rho(\omega_1, \omega_2) > 0$ and $\omega_2 > \bar{\omega}_B$, we know that $G(\omega) > 0$ for some $\omega < \bar{\omega}_B$. In addition, $G(\bar{\omega}_B) < 0$. Let $\omega_2' = \max\{\omega < \bar{\omega}_B : G(\omega) \geq 0\}$. When $\phi_1 = \omega_2'$, it is optimal to set $q = \rho(\omega_1, \omega_2)$ because

$$\begin{aligned} u(\rho(\omega_1, \omega_2)) &= (1-q)\int_{\omega_2}^{\omega_1} 1_{\{\omega : G(\omega) \geq 0\}} N(\omega) dF(\omega) + \int_{\omega_1}^{1} N(\omega) dF(\omega) \\ &> \int_{\omega_2}^{1} N(\omega) dF(\omega) = u(1). \end{aligned} \tag{A-18}$$

Since $u(q)$ is continuous and increasing in ϕ, and $u(1)$ does not depend on ϕ_1, there exists $\hat{\phi} \in (\bar{\omega}_R(\omega_1), \omega_2')$, such that, when $\phi_1 < \hat{\phi}$, the regulator sets $q = 1$, and when $\phi_1 \in (\hat{\phi}, \omega_2')$, the regulator sets $q = \rho(\omega_1, \omega_2)$.

Part 3. Let $\omega_1' = \max\{\omega < \omega_1 : G(\omega) \geq 0\}$. If $\phi_1 \in (\omega_2, \omega_1')$, the set Ω_0 changes to $\{\phi_1, \omega_1\}$. Hence,

$$\omega_B(q) = \begin{cases} \omega_1 & \text{if } q \leq \rho(\omega_1, \phi_1) \\ \phi_1 & \text{otherwise} \end{cases} \tag{A-19}$$

and

$$u(q) = (1-q)\int_{\phi_1}^{\omega_B(q)} 1_{\{\omega : G(\omega) \geq 0\}} N(\omega) dF(\omega) + \int_{\omega_B(q)}^{1} N(\omega) dF(\omega). \tag{A-20}$$

Since $u(\rho(\omega_1, \phi_1)) > u(1)$, it is (uniquely) optimal to choose $q = \rho(\omega_1, \phi_1)$. The bank responds by choosing $\omega_B = \omega_1$ and $\omega_R = \phi_1$.

Part 4. If $\phi_1 > \omega_1'$, the set Ω_0 includes only one state: $\max\{\omega_1, \phi_1\}$. Hence, $\omega_B(q) = \max\{\omega_1, \phi_1\}$ for every $q \in [0, 1]$. In this case, the first integral in Equation (A-20) equals zero, and any q is optimal.

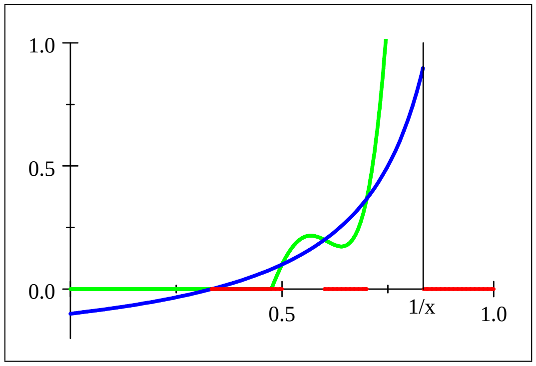

Figure 1. The figure illustrates the function $v(\omega)$ (in light green) and the function $D + \frac{N(\omega)}{1-\omega}$ (in blue). The bank's ideal investment rule is to invest when the state ω is in the red intervals. This happens when either (i) $\omega \geq \frac{1}{x}$, so the project has positive NPV; or (ii) $\omega < \frac{1}{x}$, and the green line is below the blue line.

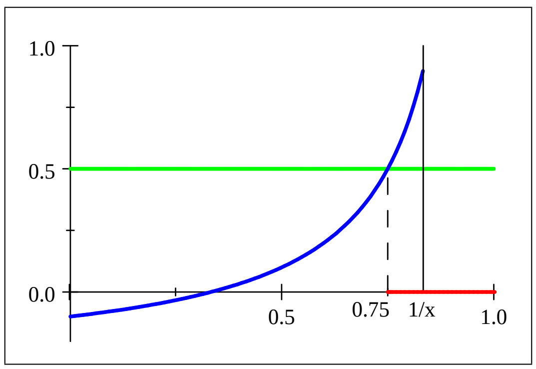

Figure 2. The figure illustrates the function $v(\omega)$ (in light green) and the function $D + \frac{N(\omega)}{1-\omega}$ (in blue) when $D = 0.9$, $x = 1.2$, and $v(\omega) = 0.5$. The bank's ideal investment rule is to invest when $\omega \geq 0.75$.

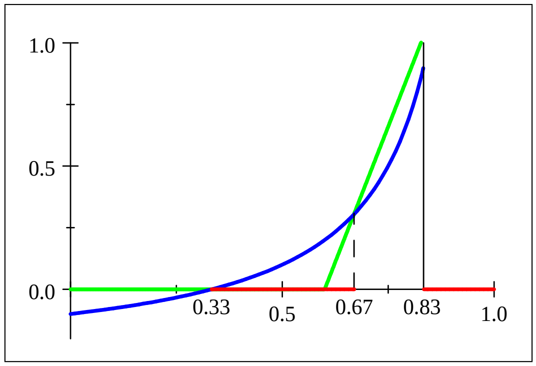

Figure 3. The figure illustrates the function $v(\omega)$ (in light green) and the function $D + \frac{N(\omega)}{1-\omega}$ (in blue) when $D = 0.9$, $x = 1.2$, and $v(\omega) = \max\{0, 4.4\omega - 2.64\}$. The bank's ideal investment rule is to invest when ω is in the red intervals.

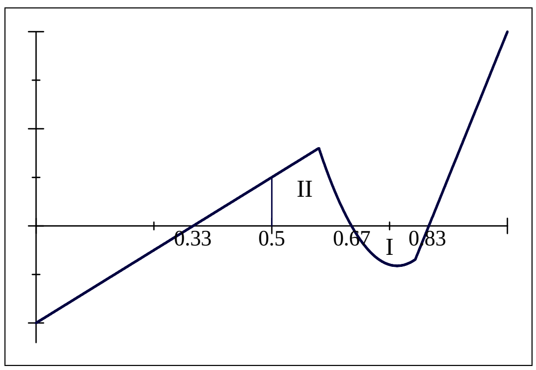

Figure 4. The figure plots the function $G(\omega)$ when $D = 0.9$, $x = 1.2$, and $v(\omega) = \max\{0, 4.4\omega - 2.64\}$. The bank's private gain from producing information is represented by the ratio of "area I" to "area II," where both areas are calculated according to the probability distribution function F.

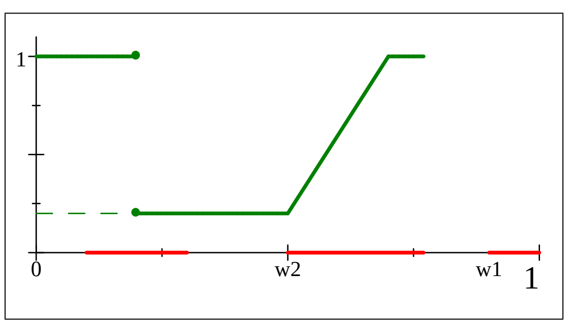

Figure 5. The figure illustrates the optimal q as a function of ϕ_1 (green line). The red intervals represent the states in which an unregulated bank wants to invest.