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An Equilibrium Model of Housing and Mortgage Markets with State-Contingent Lending Contracts*

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Abstract

We develop a tractable general equilibrium framework of housing and mortgage markets with aggregate and idiosyncratic risks, costly liquidity and strategic defaults, empirically relevant informational asymmetries, and endogenous mortgage design. We show that adverse selection plays an important role in shaping the form of an equilibrium contract. If borrowers' homeownership values are known, the equilibrium state-contingent contract depends on both aggregate wages and house prices. However, when lenders cannot observe borrowers' homeownership values, the equilibrium contract only depends on house prices and takes the form of a home equity insurance mortgage (HEIM) that eliminates the strategic default option and insures the borrower's equity position. Interestingly, we show that widespread adoption of such loans has ambiguous effects on the homeownership rate and household welfare. In economies in which recessions are expected to be severe, the HEIM equilibrium Pareto dominates the equilibrium with fixed-rate mortgages. However, if economic downturns are not severe, HEIMs can lower the homeownership rate and make some marginal home buyers worse-off. We also note that adjustable-rate mortgages (ARMs) may share some benefits with HEIMs. Finally, we find that unrestricted competition in contract design among lenders may lead to a non-existence of equilibrium. This suggests that government-sponsored enterprises may stabilize mortgage markets by subsidizing certain lending contracts.

1 Introduction

Residential mortgage contracts are of first-order importance for households, financial institutions, and for the broader economy. The Great Depression of 1929-1939 showed that forms of mortgage lending are extremely important to how the economy responds to shocks. At that time, mortgage contracts were predominantly short-term loans. The inability to roll them over was a major contributor to the collapse of the financial and housing markets. In response to the lessons learned from the Great Depression, federal regulators developed long-term fully amortizing fixed rate mortgage contracts, also known as FRMs, which have become the most popular form of mortgage lending in the United States.¹

The recent Great Recession associated with millions of costly foreclosures revived the debate regarding the appropriate structure of residential lending contracts. A key lesson from the Great Recession is that the rigidity of mortgage contract terms coupled with a variety of frictions preventing effective renegotiation or refinancing of loans of vulnerable borrowers may have exacerbated the foreclosure crisis and the severity of the economic downturn (e.g., Piskorski et al. 2010; Keys et al. 2013; Mayer et al. 2014; Agarwal et al. 2015, 2017; Scharfstein and Sunderam 2016).² At the core of this debate are a variety of proposals concerning the redesign of mortgage contracts so that they allow for more efficient sharing of aggregate risk between borrowers and lenders to lower the incidence of costly foreclosures and the severity of future housing downturns (e.g., Shiller 2008; Caplin et al. 2008; Piskorski and Tchisty 2011; Campbell 2013; Keys et al. 2013; Mian and Sufi 2014; Eberly and Krishnamurty 2014).

Despite the fundamental importance of this question, there is little theoretical analysis investigating the effects of a widespread adoption of state-contingent lending contracts in a general equilibrium setting with aggregate uncertainty. The life-cycle models of mortgage contract choice (e.g., Campbell and Cocco 2003, 2015) and mortgage design studies of state-contingent contracts employing dynamic contracting tools (e.g., Piskorski and Tchisty 2010, 2011) commonly adopt a partial equilibrium perspective that takes some key variables such as house prices as given. How-

¹See Green and Wachter (2005) for more discussion of the historical evolution of mortgage contracts in the U.S.

²These frictions include barriers to loan renegotiation due to securitization (Piskorski et al. 2010), lenders' concerns regarding borrowers' strategic behavior that can limit ex-post loan work-outs (Mayer et al. 2014), and the limited organizational ability of servicers to provide debt relief to a large number of borrowers in a crisis (Agarwal et al. 2017). In addition, borrowers with FRMs left with little housing equity can be ineligible for loan refinancing, and limited competition in the refinancing market may adversely affect the effectiveness of policies aimed at facilitating loan refinancing activity during economic downturns (Agarwal et al. 2015, Scharfstein and Sunderam 2016).

ever, due to its large size, developments in the mortgage lending market can have pronounced effects on house prices, construction, home ownership rates, and the allocation of credit in the economy, feeding back into mortgage market outcomes.

To study such effects, we develop a tractable general equilibrium framework of housing and mortgage markets in a setting with aggregate and idiosyncratic risks, costly liquidity and strategic defaults, empirically relevant informational asymmetries, endogenous housing supply and home prices, and endogenous mortgage design. Our framework captures feedback effects between features of mortgage contracts and housing construction, homeownership rate, prices, and default rates, which are important in assessing the welfare implications of state-contingent contracts.

Our work is complementary with recent quantitative dynamic equilibrium models of housing markets with heterogenous agents and aggregate risk (e.g., Favilukis, Ludvigson, Van Nieuwerburgh 2016; Kaplan, Mitman, and Violante 2016) and contemporaneous research that studies the role of mortgage contracts in such settings (e.g., Greenwald, Landvoigt, and Van Nieuwerburgh 2017; Guren, Krishnamurty, and McQuade 2017).³ Such models can provide many valuable insights including the quantitative assessment of various effects. However, their complex settings usually require the use of numerical methods to analyze equilibria. This makes it hard to derive results analytically and to study the impact of factors such as informational asymmetries and competition among lenders on endogenous mortgage design and other equilibrium outcomes.⁴ In this regard, our objective is to develop a framework that is rich enough to capture the complex interplay of various factors in a market equilibrium while still being tractable enough to develop key qualitative insights transparently including a set of closed form solutions. We hope that this framework could be used in future research on housing and mortgage markets.

In our setting the economy is subject to aggregate productivity shocks that determine the capital returns and aggregate wages. Households need financing from owners of capital to buy homes. We incorporate two sources of informational asymmetries between borrowers and lenders that were documented to be quantitatively important by empirical literature.⁵ First, we assume

³In this regard, our paper is also related to Kung (2015) who explores a number of counterfactuals related to credit availability and mortgage contract forms in a quantitative equilibrium model of the housing market.

⁴In dynamic quantitative equilibrium models with aggregate uncertainty and individual heterogeneity the fully rational equilibrium is usually not computable because a large number of agents typically face an infinite dimensional state space (e.g., see Favilukis et al. 2016).

⁵The empirical literature has documented that informational asymmetries are a pervasive feature of the housing finance market. For recent evidence see, among others, Keys et al. (2010), Jiang et al. (2014), Garmaise (2015), Piskorski et al. (2015), Griffin and Maturana (2016), Stroebel (2016).

that households differ in the values they attach to homeownership, which are not observable by the lenders. Second, after obtaining their loans, the households are subject to individual private productivity shocks leading to hard-to-verify variation in their disposable income. To provide borrowers with incentives to repay their loans the lenders have to repossess properties of delinquent borrowers, which results in deadweight losses. Consequently, in equilibrium borrowers may default due to both liquidity (inability to pay) and strategic (unwillingness to pay) reasons⁶.

The model timing is as follows. At time 0, before the aggregate and idiosyncratic shocks are realized, the households obtain loans from competitive lenders. Next, the aggregate and idiosyncratic shocks are realized, affecting the equilibrium returns to capital, wages, and households repayment decisions. The lenders repossess homes of borrowers who defaulted on their loans. At time 1, the secondary market for homes is open, where homeowners and lenders with repossessed inventory can sell their homes. The potential buyers include renters and households who lost their homes to foreclosures.

We first consider a market equilibrium when the lenders are restricted to only offer fixed-rate mortgage contracts. As expected, mortgage defaults are lower and equilibrium house prices are higher in good economic times. Interestingly, an equilibrium with FRM contracts features strategic defaults in the bad economic state by borrowers with relatively low homeownership values. Because homeownership value is private, the FRM equilibrium has borrowers with high homeownership values effectively cross-subsidizing those with lower values.

As an intermediate step towards the equilibrium with state-contingent mortgages, we derive an equilibrium mortgage contract assuming the borrower's homeownership utility is known to be sufficiently high, so that the borrower will not default strategically. We find that a FRM contract is inefficient relative to the equilibrium contract that depends explicitly on realization of aggregate wages and home prices. Intuitively, when wages are higher, the borrowers can afford higher mortgage payments. Moreover, higher house prices in the good state imply that liquidity default is less costly to the lender due to the higher collateral value. A higher required payment in the good state allows a reduction of payments in the bad state, which reduces the overall chances of liquidity default and the associated losses.⁷

⁶The empirical literature suggests that both reasons played an important role in accounting for mortgage default patterns during the Great Recession. While liquidity considerations coupled with the collapse of house prices are believed to play a dominant role, strategic motives may have accounted for more than 20% of mortgage defaults (see Keys et al. 2013).

⁷The features of the equilibrium contract in this case are broadly consistent with an optimal mortgage derived by

Next, we characterize an equilibrium with no restrictions on mortgage forms assuming lenders cannot fully observe homeownership values of potential borrowers. The state-contingent contract discussed above cannot be sustained in an equilibrium, since this contract would be attractive to borrowers with relatively low homeownership value who would strategically default with positive probability. More broadly, we show that the adverse selection associated with strategic defaulters implies that there cannot be strategic defaults in equilibrium with no restrictions on mortgage contract forms. As a result, the equilibrium contract takes the form of a *home equity insurance mortgage* (HEIM) that *only* depends on the realization of house prices. This contract fully insures borrower's equity position against the movement in house prices and completely eliminates the strategic default option and associated default inefficiencies.

We find, however, that in some cases unrestricted competition in contract design among lenders may lead to a non-existence of equilibrium. In particular, the equilibrium is less likely to exist when there are fewer households with homeownership values close to that of the marginal homebuyer. This result underscores the importance of distribution of homeownership values among households.

We finish our main analysis with a relative comparison of the equilibrium outcomes in an economy restricted to only FRM contracts to the one with no restrictions on mortgage contract form. This comparison importantly builds on the general equilibrium setting of our model. Interestingly, we find that the homeownership rate is not necessarily higher and that some households can be worse off in the equilibrium with HEIM contracts. This is because FRMs come with the embedded strategic default option, which is valuable to marginal homebuyers. We further show that the impact of HEIMs on the welfare of borrowers is importantly tied to the severity of economic downturns. In the economies in which recessions are expected to be sufficiently severe (e.g., household incomes are sufficiently low in the bad state) HEIMs lead to an increase in the homeownership rate and higher welfare for all households compared to the FRM equilibrium. However, if economic downturns are not severe, a widespread adoption of HEIMs can lower the homeownership rate and make some marginal potential homeowners worse-off.

Our paper is broadly related to a large literature on financing models with informational asymmetries (e.g., Stiglitz and Weiss 1981; Diamond 1984; Bolton and Scharfstein 1990; DeMarzo and Sannikov 2006; DeMarzo and Fishman 2007; Biais et al. 2007; Farhi, Golosov, and Tsyvinski 2009; DeMarzo et al. 2012; Malenko 2016) and to the literature studying the role of collateral, leverage,

Piskorski and Tchisty (2011) in a dynamic partial equilibrium setting with a known homeownership value.

and default in various equilibrium settings (e.g., Kiyotaki and Moore 1997; Dubey et al. 2005; Board 2007; Rampini and Viswanathan 2010 and 2016; Makarov and Plantin 2013; Brunnermeier and Sannikov 2014; He and Milbradt 2014; Bolton et al. 2016). Our model contributes to these literatures by recognizing the importance of the consumption value of housing assets and by focusing on the equilibrium state-contingent mortgage contracts in a general equilibrium setting with informational asymmetries and costly default. Our setting also relates to earlier partial equilibrium real estate finance literature that recognized the importance of informational asymmetries for mortgage contract terms (e.g., Dunn and Spatt 1985, 1988; Chari and Jagannathan 1989; Stanton and Wallace 1998).

The rest of this paper is organized as follows. Section 2 presents the setup of our model. Section 3 characterizes the equilibrium with FRM contracts. Section 4 characterizes an equilibrium state-contingent contract in a setting where borrowers' homeownership values are known. Section 5 characterizes the equilibrium with state-contingent mortgage contracts in our main setting where lenders are uncertain about homeownership values. Section 6 studies the welfare implications of a widespread adoption of state-contingent contracts. Section 7 discusses the robustness of our findings and a number of possible extensions of our setting. Among other points, in Section 7, we note that adjustable-rate mortgage (ARM) contracts may share some benefits with HEIMs. We also note that in some cases imposing a restricted contract choice may help facilitate an existence of a stable market. Section 8 concludes.

2 Model Setup

For simplicity, we assume that there is only one generation of workers who live for two periods - production and retirement. Production happens only in the first period, however the agents enjoy consumption and housing utility in both periods. Since workers have no assets in the first period they need to borrow from owners of capital to finance their housing purchases. They repay their loans during the first period. This simple setup allows us to analyze housing and mortgagee market equilibrium while maintaining the analytical tractability of the model.

In particular, we assume that there is a unit continuum of risk-neutral workers, living and

consuming during two periods. Worker i per period utility is given by

$$u_{i,t}(c_t, h_{i,t}, \theta_i) = c_t + \theta_i 1_{h_{i,t}=1},$$

where c_t is consumption in period t , $h_{i,t} = 1$ if worker i is a homeowner in period t , and $h_{i,t} = 0$ otherwise. For simplicity we assume that there is no time discounting and there is no disutility of labour.⁸ We normalize the cost and utility of rental to zero.⁹ In other words, homeownership means living in a more expensive housing unit and enjoying additional utility $\theta_i \in [0, \bar{\theta}]$ because of it. Worker i 's utility θ_i of owning a house for one period is his or her private information and remains constant for two periods. The utility of homeownership among workers is distributed according to the cumulative and probability density functions F and f , with $f(\theta_i) > 0$ for $\theta_i \in [0, \bar{\theta}]$.

Our assumption that homeownership value θ is private can be justified by the fact that it reflects the utility assigned by the household to specific features of the house and the neighborhood. This includes, among others, factors such as specific features of the floor plan, view, backyard, perceived quality of local schools, expected family size, neighborhood amenities, and proximity to relatives and friends. Households differ in their preferences across such factors, making the homeownership value of the specific borrower hard to observe by the lenders.

Each worker supplies one unit of labor in the first period and retires in the second period. Workers's idiosyncratic labor productivity $l(i) \in [0, 1]$ is i.i.d. across workers, with cdf $G(l)$, and pdf $g(l) > 0$ for $l \in [0, 1]$. The average idiosyncratic worker's labor productivity is denoted by

$$L = \int_0^1 l(i)g(i)di.$$

Since there is a unit mass continuum of workers, L is also the aggregate labor supply. For simplicity, assume that workers have no capital endowment.

In addition to the workers, the economy is also populated by risk-neutral capitalists who are endowed with capital \bar{K} and do not work. The risk-neutral capitalists already have houses, which they will not sell. The capitalists can consume their capital, use it to build new houses, or invest it into production of the consumption good. The capitalists can also give mortgage loans to the

⁸Allowing for time discounting and disutility of labour have no qualitative impact on the results. We also note that risk neutrality assumption is not essential for our key findings (see Section 7.4).

⁹See Section 7.5 for a discussion of this assumption.

workers.

Building one home requires q units of capital. For simplicity, we assume that home production happens instantaneously at the beginning on the first period and does not require any labor.

Aggregate production of the consumption good in the first period is given by the linear production function with constant return to scale

$$Y(K, L, s) = A^K(s)K + A^L(s)L, \tag{1}$$

where $A^K(s)$, $A^L(s)$ are the total factor productivity of capital and labour in state s , respectively and K is the capital allocated to the production. There are two states of the economy, "good" s_g with probability π_g and "bad" s_b with probability $\pi_b = 1 - \pi_g$, and $A^K(s_g) > A^K(s_b)$ and $A^L(s_g) > A^L(s_b)$. As we discuss in Section 7.1 while the production function form (1) simplifies some of our arguments, our key findings should hold in a large class of production functions.

The timing of the events is as follows. At $t = 0$, the capitalists allocate capital between production and housing. The workers decide whether to buy homes knowing their personal utility θ_i of homeownership, but before knowing the future state of the economy and their idiosyncratic labor productivity shocks. Homebuyers take on mortgages to finance the home purchases. Borrowers for whom buying a home given the financing terms yields them strictly higher utility relative to not buying, become homeowners at $t = 0$.

At time $t = 0+$, both macro and idiosyncratic labor productivity shocks are realized. The workers learn their income and decide whether to pay the mortgage or default and vacate their homes, which will remain unoccupied until the end of the first period. Defaults result in deadweight losses because foreclosed homes remain unoccupied for one period and their housing utility is lost.

At time $t = 1$, the secondary market for homes clears. The supply includes all foreclosed homes in $t = 0+$ and homes that the existing homeowners decide to put on the market. Renters including those who defaulted at time $t = 0+$ decide whether to buy homes. There are no costs associated with real estate transactions.

One can interpret $t = 0+$ as the immediate aftermath of the productivity shock, and $t = 1$ is the long term equilibrium with the markets fully adjusted to the shock.

We assume perfect competition at the production, home building and financial sectors of the

economy. The production function (1) implies that

$$\begin{aligned} R(s) &= A^K(s), \\ w(s) &= A^L(s), \end{aligned}$$

are the gross return on capital and the wage per unit of labor productivity in the state s , respectively. Both wage $w(s)$ and interest rate $R(s)$ are higher in the good state of the economy. The income of worker i is equal to $w(s)l_i$.

Let

$$\bar{R} \equiv \pi_b R(s_b) + \pi_g R(s_g)$$

denote the expected return on the capital invested in the production sector. In equilibrium, the capitalists should earn the same expected return on mortgages and the capital invested in the production sector. The price of a home at time zero is equal to the construction cost

$$P_0 = q.$$

3 Equilibrium with Fixed Rate Mortgages

In this section we characterize an equilibrium in mortgage and housing markets when the only allowed mortgage contract is the fixed-rate mortgage (FRM). Under the FRM contract, the borrower must pay a fixed amount \bar{m} in every state of the economy in period one. If the borrower does not repay the loan the foreclosure happens and the lender repossesses the home. We will use superscript F to emphasize the key variables related to the FRM equilibrium. Below we define an equilibrium with FRMs .

Definition 1 *An FRM equilibrium consists of the allocation of capital K^H and K into housing and production sectors, mortgage payment \bar{m} and prices $\{w(s), R(s), P_0, P_1^F(s)\}$, such that taking the equilibrium prices and mortgage payment as given the following is true.*

At time $t = 0$, i.e., before aggregate and idiosyncratic shocks s and l are known:

- (i) Capitalists expect the same return on their investments in housing and production sectors,*
- (ii) Households buy homes if and only if homeownership results in strictly higher expected utility compared to renting in period one,*

(iii) The housing market clears at time $t = 0$, i.e., all homes built by capitalists are bought by households;

At time $t = 0+$, i.e., when aggregate and idiosyncratic shocks s and l are revealed:

(iv) Homeowners with income less than \bar{m} default for liquidity reason,

(v) Homeowners with income greater than \bar{m} default strategically if and only if default increases their utility;

At time $t = 1$, i.e., after economic shocks are fully absorbed by the economy:

(vi) Homeowners with housing utility less than $P_1^F(s)$ sell their homes in state s ,

(vii) Renters including those who defaulted at time $t = 0+$ buy homes in state s if their housing utility and their income are greater than $P_1^F(s)$,

(viii) The housing market clears at time $t = 1$, i.e., no homes are left unoccupied in the second period.

We start our analysis of the equilibrium by characterizing the borrowers' default decisions.

3.1 Borrowers' default decisions

Borrowers obtain mortgages at $t = 0$ and decide whether to default at $t = 0+$ knowing all shocks, as well as the equilibrium home price $P_1^F(s)$ at time $t = 1$. If $w(s)l_i < \bar{m}$, borrower i does not have money to pay the mortgage and *liquidity default* happens.¹⁰ In this case the borrower keeps his labor income, but loses housing utility θ_i in the first period. If $P_1^F(s) \leq w(s)l_i$, the defaulted borrower can buy a home at $t = 1$ and enjoy housing utility θ_i in the second period. Thus, the utility of workers who experience liquidity default in state s is given by:

$$w(s)l_i + 1_{w(s)l_i \geq P_1^F} \max(0, \theta_i - P_1^F(s)). \quad (2)$$

If $w(s)l_i \geq \bar{m}$, the borrower can repay the loan but may decide to default strategically, in which case his utility is the same as above¹¹. If he pays the mortgage, his utility will be

$$w(s)l_i - \bar{m} + \theta_i + \max(\theta_i, P_1^F(s)), \quad (3)$$

¹⁰In principle borrowers who cannot repay their mortgages could also try to sell their homes in the secondary market to repay their loans. However, in equilibrium the required debt repayment is larger than the secondary market price ($\bar{m} > P_1(s)$ for any s) so this would not be possible.

¹¹We assume that mortgages are non-recourse loans. We discuss this assumption in Section 7.3.

where the last term reflects the fact that the borrower would sell the home at time $t = 1$ when $P_1^F(s) > \theta_i$.

When the borrower obtains a lower utility from paying the mortgage, the borrower chooses *strategic default*. The following proposition summarizes simple rules for liquidity and strategic defaults.

Proposition 1 *Borrowers default for liquidity reasons in state s when*

$$\bar{m} > w(s)l_i.$$

Borrowers default strategically in state s when the following two conditions hold:

$$\begin{aligned} \bar{m} &\leq w(s)l_i, \\ \bar{m} &> \theta_i + P_1^F(s). \end{aligned} \tag{4}$$

Proof is in Appendix. ■

This condition (4) is very intuitive. If $P_1^F(s) < \theta_i$, the borrower will buy back home at $t = 1$ if he defaults at $t = 0+$. In this case the benefit of default is the money he saves on defaulting less cost of buying back the home: $\bar{m} - P_1^F(s)$. The cost is the lost housing utility θ_i due to not living in the house in the first period.

If $P_1^F(s) > \theta_i$, the borrower will not buy back the house. Instead, he will sell it at time 1, provided he does not default earlier. In this case, $\bar{m} > \theta_i + P_1^F(s)$ means that the benefit of default \bar{m} has to be larger than housing utility θ_i in the first period plus the market value $P_1^F(s)$ of the house at $t = 1$.

In an equilibrium, a borrower does not strategically default in both states at $t = 0+$ since then he would not be better off if he were a renter at $t = 0$. Hence strategic default can only occur in one of the states. Moreover, this state cannot be the one with higher home prices at $t = 1$ as according to Proposition 1 this borrower would also default in the state with lower home prices. We summarize this observation in the proposition below.

Proposition 2 *In an FRM equilibrium, strategic defaults can only occur in the state with lower home prices.*

3.2 Housing market

At $t = 0$, given home prices $P_0 = q$ and mortgage repayment \bar{m} worker i decides whether to become a home owner or a renter. Since the state of the economy and the labor income are not known at that time, the worker's housing utility θ_i is the only relevant variable for this decision. As a result, there is a marginal homebuyer with housing utility $\underline{\theta}^F$ who is indifferent between buying and renting at $t = 0$. Workers with $\theta > \underline{\theta}^F$ will get a mortgage \bar{m} and buy homes at $t = 0$. Workers with $\theta \leq \underline{\theta}^F$ will initially rent, but may decide to buy at $t = 1$.

We note that home prices $P_1^F(s)$ in each state s at time $t = 1$ cannot be greater than the housing utility of the marginal home buyer $\underline{\theta}^F$. If $P_1^F(s) \geq \underline{\theta}^F$, then workers with $\theta \in (\underline{\theta}^F, P_1^F(s))$ will put their houses on the market in state s . However, workers with $\theta < \underline{\theta}^F$ will not buy them because their housing utility is lower than the price, and workers with $\theta > P_1^F(s)$ will not buy them because they are already homeowner. Thus, it must be the case that $P_1^F(s) < \underline{\theta}^F$ for the market to clear in an equilibrium.

Since $P_1^F(s) < \underline{\theta}^F$, only foreclosed homes will be offered for sale at $t = 1$. In addition, $P_1^F(s) < \underline{\theta}^F$ means that if homeowners default at $t = 0+$, they will buy houses again at time $t = 1$ if they have enough income.

The marginal homebuyer will strategically default effectively becoming a renter in the state with lower prices. Thus, in order for the marginal homebuyer to be indifferent between renting and buying at $t = 0$, the marginal homebuyer has to be indifferent between renting and buying in the state with higher prices, which we denote by s^* . Hence, it must be the case that

$$\underline{\theta}^F = \bar{m} - P_1^F(s^*). \quad (5)$$

Indeed, $\underline{\theta}^F$ is the housing utility, and $\bar{m} - P_1^F(s^*)$ is the savings from being a renter in the first period. Equation (5) also implies that $P_1^F(s) < \bar{m}$ for both s .

The defaulted homes will be offered for sale at $t = 1$. We remember that strategic default can happen only in the state with lower prices. As a result, the aggregate supply of homes in the state s^* associated with with higher prices will be

$$(1 - F(\underline{\theta}^F))G\left(\frac{\bar{m}}{w(s^*)}\right),$$

where $(1 - F(\underline{\theta}^F))$ is the number of home built and sold at $t = 0$, and $G\left(\frac{\bar{m}}{w(s^*)}\right)$ is the fraction of

the homeowners who defaulted due to liquidity reasons at $t = 0+$.

The homes will be bought by the first time homebuyers with housing utility $\theta \in (P_1^F(s^*), \underline{\theta}^F)$ and idiosyncratic income shocks $l \geq \frac{P_1^F(s^*)}{w(s^*)}$. In addition, borrowers with $\theta > \underline{\theta}^F$ who defaulted at $t = 0+$ will buy homes again at $t = 1$ if $l \in (\frac{P_1^F(s^*)}{w(s^*)}, \frac{\bar{m}}{w(s^*)})$. As a result, the aggregate demand will be

$$(F(\underline{\theta}^F) - F(P_1^F(s^*)))(1 - G(\frac{P_1^F(s^*)}{w(s^*)})) + (1 - F(\underline{\theta}^F))[G(\frac{\bar{m}}{w(s^*)}) - G(\frac{P_1^F(s^*)}{w(s^*)})].$$

In the equilibrium, the supply equals the demand. Thus, in the state with high home prices the market clearing condition is given by

$$(F(\underline{\theta}^F) - F(P_1^F(s^*)))(1 - G(\frac{P_1^F(s^*)}{w(s^*)})) = (1 - F(\underline{\theta}^F))G(\frac{P_1^F(s^*)}{w(s^*)}). \quad (6)$$

In the state with low home prices, denoted by s' , borrowers with $\theta \in [\underline{\theta}^F, \hat{\theta}^F)$ will default strategically even though they can afford to pay the mortgage, where

$$\hat{\theta}^F = \bar{m} - P_1^F(s'),$$

and $P_1^F(s') \leq P_1^F(s^*)$. In addition, some borrowers with $\theta \geq \hat{\theta}^F$ will default for liquidity reasons, when $l_i < \frac{\bar{m}}{w(s')}$. As a result, the aggregate supply of homes in the bad state will be

$$(F(\hat{\theta}^F) - F(\underline{\theta}^F)) + (1 - F(\hat{\theta}^F))G(\frac{\bar{m}}{w(s')}).$$

Workers with $\theta > P_1^F(s')$ who are not homeowners will buy homes at $t = 1$ if they have enough income, i.e., $l \geq \frac{P_1^F(s')}{w(s')}$. Thus, the aggregate demand in the state with low home prices will be

$$(F(\underline{\theta}^F) - F(P_1^F(s')))(1 - G(\frac{P_1^F(s')}{w(s')})) + (F(\hat{\theta}^F) - F(\underline{\theta}^F))(1 - G(\frac{P_1^F(s')}{w(s')})) + (1 - F(\hat{\theta}^F))[G(\frac{\bar{m}}{w(s')}) - G(\frac{P_1^F(s')}{w(s')})],$$

where the first term represents the demand from the first time homebuyers with housing utility $\theta \in (P_1^F(s'), \underline{\theta}^F)$, the second term represents the demand from the homeowners with $\theta \in (\underline{\theta}^F, \hat{\theta}^F)$ who (strategically) defaulted at $t = 0+$, and the last term represents the demand from the homeowners with $\theta > \hat{\theta}^F$ who defaulted for liquidity reasons at $t = 0+$.

Equating the supply and demand and cancelling terms on both sides gives the following equation

for $P_1^F(s')$:

$$(F(\underline{\theta}^F) - F(P_1^F(s')))(1 - G(\frac{P_1^F(s')}{w(s')})) = (1 - F(\underline{\theta}^F))G(\frac{P_1^F(s')}{w(s')}). \quad (7)$$

The market clearing equations (6) and (7) for the two states are similar because workers who default for liquidity or strategic reason at time $t = 0+$ reenter the housing market at time $t = 1$. The main difference between these equations is that wages are higher in the good state. This means that home prices are higher in the good state, i.e., $s^* = s_g$ (see the proof of Proposition 3 in Appendix).

We summarize these findings in Proposition 3.

Proposition 3 *In an FRM equilibrium, $1 - F(\underline{\theta}^F)$ homes are built at time zero and the price of one home is $P_0 = q$. Workers with $\theta \leq \underline{\theta}^F$ become renters, while workers with $\theta > \underline{\theta}^F$ take FRMs and buy homes, where*

$$\underline{\theta}^F = \bar{m} - P_1^F(s_g).$$

At time $t = 0+$ workers with $\underline{\theta}^F < \theta < \hat{\theta}^F$ strategically default, where

$$\hat{\theta}^F = \bar{m} - P_1^F(s_b).$$

At time $t = 1$ only foreclosed homes are offered for sale. Home prices are higher in the good state:

$$P_1^F(s_g) > P_1^F(s_b), \quad (8a)$$

and satisfy

$$(F(\underline{\theta}^F) - F(P_1^F(s_g)))(1 - G(\frac{P_1^F(s_g)}{w(s_g)})) = (1 - F(\underline{\theta}^F))G(\frac{P_1^F(s_g)}{w(s_g)}), \quad (9)$$

$$(F(\underline{\theta}^F) - F(P_1^F(s_b)))(1 - G(\frac{P_1^F(s_b)}{w(s_b)})) = (1 - F(\underline{\theta}^F))G(\frac{P_1^F(s_b)}{w(s_b)}). \quad (10)$$

Proof of equation (8a) is in Appendix. ■

Proposition 3 shows that FRM contracts embed a strategic default option. While this option creates inefficiencies due to strategic defaults in the bad state, it benefits the borrowers with lower homeownership values $\theta \in (\underline{\theta}^F, \hat{\theta}^F]$, which contributes positively to the homeownership rate $\underline{\theta}^F$. As a result, borrowers with higher homeownership values ($\theta \geq \hat{\theta}^F$) end up cross-subsidizing the low θ borrowers.

Threshold $\underline{\theta}^F$ determines the homeownership rate $1 - F(\underline{\theta}^F)$ in the economy, i.e., the fraction of households who are homeowners at time $t = 0$ and $t = 1$. The higher $\underline{\theta}^F$, the lower the homeownership rate.

Appendix B provides a closed form solution for home prices in the FRM equilibrium where both θ and l are uniformly distributed.

3.3 Equilibrium FRM contract

The capitalists issue mortgages before knowing the future state of the economy. They lend P_0 to borrowers, who promise to pay back \bar{m} . However, if a borrower defaults, the capitalist's payoff in state s is going to be $\bar{P}(s) < \bar{m}$.¹² In an equilibrium, the expected return on a mortgage is equal to the expected return \bar{R} on capital invested in the production sector.

The expected mortgage payment in the good state, in which there are no strategic defaults, is:

$$\Pi(s_g, \bar{m}) = \left(1 - G\left(\frac{\bar{m}}{w(s_g)}\right)\right) \bar{m} + G\left(\frac{\bar{m}}{w(s_g)}\right) P_1^F(s_g). \quad (11)$$

In the bad state, all borrowers with $\theta \in (\underline{\theta}^F, \hat{\theta}^F)$ default. As a result, the expected mortgage payment is:

$$\hat{\Pi}(s_b, \bar{m}) = \left(\frac{F(\hat{\theta}^F) - F(\underline{\theta}^F)}{1 - F(\underline{\theta}^F)}\right) P_1^F(s_b) + \left(\frac{1 - F(\hat{\theta}^F)}{1 - F(\underline{\theta}^F)}\right) \left(\left(1 - G\left(\frac{\bar{m}}{w(s_b)}\right)\right) \bar{m} + G\left(\frac{\bar{m}}{w(s_b)}\right) P_1^F(s_b)\right). \quad (12)$$

The equilibrium FRM is the smallest \bar{m} that satisfies the lender's break even condition:

$$P_0 \bar{R} = \pi_g \Pi(s_g, \bar{m}) + \pi_b \hat{\Pi}(s_b, \bar{m}).$$

Throughout the paper we will use functions $\Pi(s, m)$ and $\hat{\Pi}(s, m)$ to denote lenders' payoffs in the state without strategic defaults and in the state with strategic defaults correspondingly.¹³ One can see that strategic defaults reduce the expected payoff to the lender. Indeed, $\hat{\Pi}(s_b, \bar{m})$ can be

¹²We assume no additional foreclosure costs for lenders.

¹³Unlike the FRM equilibrium, in which strategic defaults can occur only in the bad state, in an equilibrium with state contingent mortgage contracts strategic defaults can occur in the good state.

rewritten as

$$\hat{\Pi}(s_b, \bar{m}) = \Pi(s_b, \bar{m}) - \frac{(F(\hat{\theta}^F) - F(\underline{\theta}^F))}{(1 - F(\underline{\theta}^F))} \left(1 - G\left(\frac{\bar{m}}{w(s_b)}\right)\right) (\bar{m} - P_1^F(s_b)) < \Pi(s_b, \bar{m}), \quad (13)$$

where $\Pi(s_b, \bar{m})$ is the lender's expected payoff in the bad state in the absence of strategic defaults.

3.4 FRM modification

While FRM payments are contractually fixed, the actual payments on a loan could be modified on state-contingent basis through voluntary ex-post loan renegotiation. In this section, we consider the possibility of reducing the FRM payment in the bad state of the economy, which was not anticipated by the borrowers and lenders at time zero. We only consider modifications that make both borrowers and lenders better-off.

Specifically, at time $t = 0+$ after the bad state s_b is realized, but before borrowers default, a lender can unilaterally reduce mortgage payments. This mortgage modification will affect all his borrowers, because the lender cannot identify defaulters without knowing borrowers' wage shocks l and housing utility θ . We assume that mortgage modification is happening on a small scale and thus has no effect on the equilibrium home price $P_1^F(s_b)$.

We need the following condition for Theorem 1.

Condition 1 *Distribution functions G and F are such that $\frac{1-G(x)}{g(x)}$ and $\frac{1-F(x)}{f(x)}$ are decreasing in x .*

We note that uniform distribution functions satisfy Condition 1. We also note that this is a rather strong condition that can be potentially relaxed. For example, for the proof of Theorem 1, we need G and F to satisfy Condition 1 only in some neighborhoods of $\frac{\bar{m}}{w(s_b)}$ and $\hat{\theta}^F$ respectively.

Theorem 1 *Assume Condition 1 holds. When wages w_b in the bad state are low compared to mortgage payments \bar{m} , then FRM modification is Pareto improving, i.e., the lender can increase the revenue by lowering mortgage payment in the bad state. When wages w_b in the bad state are high compared to mortgage payments \bar{m} , then lowering mortgage payment in the bad state reduces the lender's payoff, and the equilibrium mortgage is renegotiation-proof. [Change to Proposition]*

Proof is in Appendix. ■

Theorem 1 is very intuitive. A reduction in the mortgage payment in the bad state will reduce both strategic and liquidity defaults, which can benefit the lender. However, when wages in the bad state $w(s_b)$ are high compared to mortgage payments \bar{m} , most borrowers are solvent. As a result, a reduction in the default rate will not be enough to offset the loss of revenue from the solvent borrowers. In this case, the mortgage modification does not benefit the lender. On the other hand, when wages in the bad state $w(s_b)$ are low compared to mortgage payments \bar{m} , many borrowers are insolvent. In this case, the mortgage modification leads to a significant reduction in the number of defaults, which increases the lender's payoff.

Since the mortgage payment \bar{m} is directly related to the home price P_0 at time zero, one can interpret Theorem 1 as follows. When wages in the bad state w_b are low compared to the home price P_0 at time zero, then FRM modification is Pareto improving in the bad state. On the other hand, when wages in the bad state w_b are high compared to the home price P_0 at time zero, then the FRM mortgage is renegotiation-proof.

Theorem 1 assumes the ex-post contract changes that are unanticipated by the borrowers and lenders at time zero. In the next section we consider a general space of contracts that allows for state-contingent mortgage repayments and study their impact on the market equilibrium.

4 Equilibrium Mortgage when Homeownership Value Is Known to Be Sufficiently High

In this section, as an intermediate step towards the equilibrium with state-contingent mortgage contracts, we consider a setting in which the lender offers the borrower a state-contingent mortgage contract knowing the borrower's homeownership utility θ_i is sufficiently high, so that the borrower will not default strategically in any state. The borrower has to pay an amount $m(s)$ in state s . If the borrower fails to pay this amount the home is foreclosed by the lender. For the purpose of this section, we assume that home prices $P_1(s)$ are exogenous, and $P_1(s_g) > P_1(s_b)$.¹⁴

Since the borrower does not default strategically, his expected utility in state s is given by

$$v(s, \theta) = w(s)L + \left[1 - G\left(\frac{m(s)}{w(s)}\right) \right] (2\theta - m(s)) + \left[G\left(\frac{m(s)}{w(s)}\right) - G\left(\frac{P_1(s)}{w(s)}\right) \right] (\theta - P_1(s)), \quad (14)$$

¹⁴According to Proposition 4 in the next section, equilibrium home prices are always higher in the good state.

where the first term is the borrower's expected income, the second term represents the homeownership utility if he pays the mortgage, and the last term represents the housing utility in the second period when the borrower defaults at time $t = 0+$ and buys a house at time $t = 1$.

The optimal mortgage contract maximizes the borrower's expected utility

$$\max_{m(s)} \pi_g v(s_g, \theta) + \pi_b v(s_b, \theta), \quad (15)$$

subject to the lender's break even condition:

$$P_0 \bar{R} = \pi_g \Pi(s_g, m(s_g)) + \pi_b \Pi(s_b, m(s_b)). \quad (16a)$$

Combining equations (15) with (16a) (see the proof of Theorem 2 in Appendix for details) allows us to rewrite the optimal contracting problem as

$$\min_{m(s)} \pi_g G\left(\frac{m(s_g)}{w(s_g)}\right) + \pi_b G\left(\frac{m(s_b)}{w(s_b)}\right)$$

subject to (16a). Thus, when the borrower does not default strategically, an optimal contract is the contract that minimizes the overall probability of liquidity default subject to the lender's break even condition.

Let

$$Q(s, m(s)) = \frac{w(s)}{g\left(\frac{m(s)}{w(s)}\right)} \left(1 - G\left(\frac{m(s)}{w(s)}\right)\right) - (m(s) - P_1(s)) \quad (17)$$

The optimal state-contingent mortgage contract for borrowers with high homeownership values is given by Theorem 2.

Theorem 2 *Let $\{\tilde{m}(s_g), \tilde{m}(s_b)\}$ satisfy*

$$Q(s_g, \tilde{m}(s_g)) = Q(s_b, \tilde{m}(s_b)), \quad (18)$$

and the lender's break even condition (16a). Then $\{\tilde{m}(s_g), \tilde{m}(s_b)\}$ is the optimal contract for all borrowers with known $\theta \geq \tilde{\theta}$, where $\tilde{\theta} = \max_s [\tilde{m}(s) - P_1(s)]$.

Moreover, if the distribution function G satisfies Condition 1, then

$$\frac{\tilde{m}(s_g)}{w(s_g)} > \frac{\tilde{m}(s_b)}{w(s_b)},$$

which implies that the required mortgage repayment is higher in the good state.

In the uniform example, equation (18) becomes

$$\tilde{m}(s_g) - \tilde{m}(s_b) = \frac{1}{2}[w(s_g) - w(s_b)] + \frac{1}{2}[P_1(s_g) - P_1(s_b)]. \quad (19)$$

Proof is in Appendix. ■

Theorem 2 says that the optimal contract for borrowers with high homeownership values balances probabilities of liquidity default, which are determined by the wages $w(s)$ and the recovery values $P_1(s)$. Equation (19) is very intuitive. It is optimal to set $\tilde{m}(s_g) > \tilde{m}(s_b)$ when $w(s_g) > w(s_b)$ and $P_1(s_g) > P_1(s_b)$. Indeed, when $w(s_g) > w(s_b)$, a higher payment in the good state reduces overall chances of default. In addition, when $P_1(s_g) > P_1(s_b)$, default is less costly to the lender in the good state due to the higher collateral value, which benefits the borrower in the form of lower mortgage payments.

The optimal state-contingent mortgage contract will be offered in an equilibrium to borrowers whose homeownership values are known to be sufficiently high, so that they do not default strategically.

5 General Equilibrium with State-Contingent Mortgage Contracts

In this section, we assume that lenders can offer any state-contingent mortgage contracts $m(s)$. A borrower has to pay $m(s)$ in state s , or he loses his home to the lender who would sell it at time $t = 1$. Lenders do not know borrowers' homeownership values θ , and the home prices are endogenous. FRM is a special case of a state-contingent mortgage contract.

5.1 Equilibrium with a single state-contingent mortgage contract

We start our equilibrium analysis assuming that there is only one type of state-contingent mortgage contract, i.e., all borrowers have to pay $m(s_g)$ and $m(s_b)$ in the good and bad states correspondingly. We are not making any assumptions about what payments $m(s_g)$ and $m(s_b)$ should be, other than they should satisfy the lender's break even condition. In this subsection, lenders are not allowed to offer other mortgage contracts.

We will use superscript S to denote key variables in an equilibrium with a state-contingent mortgage. The definition of equilibrium with a single state-contingent mortgage is similar to that

in the case with an FRM contract with the only change being that mortgage payments can be state-contingent.

Definition 2 *An equilibrium with a single state-contingent mortgage contract consists of allocation of capital K^H and K into housing and production sectors, mortgage payments $m(s)$ and prices $\{w(s), R(s), P_0^S, P_1^S(s)\}$, such that taking the equilibrium prices and mortgage payments as given, the following is true.*

At time $t = 0$, i.e., before aggregate and idiosyncratic shocks s and l are known:

- (i) Capitalists expect the same return on their investments in housing and production sectors,*
- (ii) Households buy homes if and only if homeownership results in strictly higher expected utility compared to renting in period one,*
- (iii) The housing market clears at time $t = 0$, i.e., all homes built by capitalists are bought by households.*

At time $t = 0+$, i.e., when aggregate and idiosyncratic shocks s and l are revealed:

- (iv) Homeowners with income less than $m(s)$ default for liquidity reason,*
- (v) Homeowners with income greater than $m(s)$ default strategically if and only if default increases their utility.*

At time $t = 1$, i.e., after economic shocks are fully absorbed by the economy:

- (vi) Homeowners with housing utility less than $P_1^S(s)$ sell their homes in state s ,*
- (vii) Renters including those who defaulted at time $t = 0+$ buy homes in state s if their housing utility and their income are greater than $P_1^S(s)$,*
- (viii) Housing market clears at time $t = 1$, i.e., no homes are left unoccupied in the second period.*

It is straightforward to adjust the argument of Section 3 to the setting with a state-contingent mortgage. Let $\underline{\theta}^S$ denote the housing utility of the marginal homebuyer who is indifferent between buying and renting at $t = 0$ under the state-contingent mortgage contract $m(s)$. As in the FRM equilibrium, home prices $P_1^S(s)$ at time $t = 1$ must be such that $P_1^S(s) < \underline{\theta}^S$, otherwise the housing market would not clear due to the lack of home buyers. In addition, $P_1^S(s) < \underline{\theta}^S$ means that households who defaults at $t = 0$ would buy homes again at time $t = 1$ if they can afford them.

Similar to the FRM setting, households with housing utility $\theta \in (\underline{\theta}^S, m(s) - P_1^S(s))$ strategically default in state s at $t = 0+$. However, in the setting with a state-contingent mortgage strategic default may occur in either state depending on the mortgage payments $m(s_g)$ and $m(s_b)$. Let's define \hat{s} as the state in which strategic default occurs, i.e.,

$$\hat{s} = \arg \max_s \{m(s) - P_1^S(s)\},$$

and \hat{s}^- as the other state, i.e., the state in which strategic default does not happen.

We note that strategic default cannot happen in both states since it would mean that homebuyers who default strategically in both states are not better off than renters. The marginal homebuyer's housing utility is given by

$$\underline{\theta}^S = m(\hat{s}^-) - P_1^S(\hat{s}^-).$$

Indeed, a worker with $\underline{\theta}^S$ defaults strategically in state \hat{s} , and is indifferent between defaulting and paying the mortgage in state \hat{s}^- .

Workers with housing utility $\theta \in (\underline{\theta}^S, \hat{\theta}^S)$ strategically default in state \hat{s} at $t = 0+$, where

$$\hat{\theta}^S = m(\hat{s}) - P_1^S(\hat{s}).$$

Finally, one can verify that the housing market clearing conditions will have the same functional form for a given $\underline{\theta}^S$ and $m(s)$ as in the FRM equilibrium. The proposition below summarizes this discussion.

Proposition 4 *An equilibrium with a single state-contingent mortgage contract $m(s)$ is characterized by the state*

$$\hat{s} = \arg \max_s \{m(s) - P_1^S(s)\},$$

in which strategic default occurs at $t = 0+$, and two thresholds of θ :

$$\begin{aligned} \hat{\theta}^S &= m(\hat{s}) - P_1^S(\hat{s}), \\ \underline{\theta}^S &= m(\hat{s}^-) - P_1^S(\hat{s}^-). \end{aligned}$$

At time $t = 0$, $(1 - F(\underline{\theta}^S))$ homes are built, and the price of one home is $P_0 = q$. Workers

with $\theta \leq \underline{\theta}^S$ become renters, while workers with $\theta > \underline{\theta}^S$ take the state-contingent mortgages and buy homes.

At time $t = 0+$ workers with $\underline{\theta}^S < \theta < \hat{\theta}^S$ strategically default.

At time $t = 1$ only foreclosed homes are offered for sale and home prices $P_1^S(s)$ satisfy

$$\begin{aligned} (F(\underline{\theta}^S) - F(P_1^S(s_g)))(1 - G(\frac{P_1^S(s_g)}{w(s_g)})) &= (1 - F(\underline{\theta}^S))G(\frac{P_1^S(s_g)}{w(s_g)}), \\ (F(\underline{\theta}^S) - F(P_1^S(s_b)))(1 - G(\frac{P_1^S(s_b)}{w(s_b)})) &= (1 - F(\underline{\theta}^S))G(\frac{P_1^S(s_b)}{w(s_b)}). \end{aligned}$$

The home prices are always higher in the good state:

$$P_1^S(s_g) > P_1^S(s_b). \quad (20)$$

Proof of equation (20) is analogous to the proof of equation (8a). ■

It may be surprising that home prices are always higher in the good state no matter what mortgage payments are. Indeed, mortgage payments determine default rates at time $t = 0+$. However, people who defaulted at $t = 0+$ reenter the housing market at time $t = 1$. As a result, home prices at time $t = 1$ are determined by the homeownership rate $\underline{\theta}^S$, which is the same in both state, and the purchasing power of the population, which is higher in the good state since $w(s_g) > w(s_b)$. Thus, $P_1^S(s_g) > P_1^S(s_b)$.

Proposition 4 says that a state contingent mortgage contract $m(s)$ comes with a strategic default option, which is valuable for borrowers with $\theta \in (\underline{\theta}^S, \hat{\theta}^S)$. The only contract that eliminates the strategic default option is the one for which $\underline{\theta}^S = \hat{\theta}^S$, i.e., the difference in mortgage payments is equal to the difference in the home prices:

$$m^*(s_g) - m^*(s_b) = P_1(s_g) - P_1(s_b). \quad (21)$$

This mortgage insures borrowers against uncertainty in home values. We call a mortgage m^* that satisfies (21) a *home equity insurance mortgage*, or *HEIM*.

We note that the HEIM contract is similar to the continuous workout mortgages advocated by Shiller (2008).

Applying Proposition 4 to the home equity insurance mortgage yields the following corollary.

Corollary 1 *The housing equilibrium with the home equity insurance mortgage contracts m^* is*

characterized by the single threshold

$$\theta^* = m^*(s_g) - P_1^*(s_g) = m^*(s_b) - P_1^*(s_b).$$

At time $t = 0$, $1 - F(\theta^*)$ homes are built and the price of one home is $P_0 = q$. Workers with $\theta < \theta^*$ become renters, while workers with $\theta \geq \theta^*$ take the home equity insurance mortgages and buy homes. No strategic defaults occur in either state of the economy.

At time $t = 1$ only homes foreclosed due to liquidity defaults are offered for sale and the home price $P_1^*(s)$ in state $s \in \{s_g, s_b\}$ satisfies

$$(F(\theta^*) - F(P_1^*(s)))(1 - G(\frac{P_1^*(s)}{w(s)})) = (1 - F(\theta^*))G(\frac{P_1^*(s)}{w(s)}). \quad (22)$$

When θ and l are uniformly distributed on $[0, 1]$, the home prices at $t = 1$ are given by

$$P_1^*(s) = \frac{1}{2}[1 + 2w(s) - \sqrt{(1 + 2w(s))^2 - 4m^*(s)w(s)}].$$

5.2 A home equity insurance mortgage as the equilibrium mortgage contract

In this subsection, we analyze an equilibrium in which competitive lenders can offer any mortgage contract to borrowers. Borrowers choose mortgage contracts that maximize their expected utility. In an equilibrium, lenders break even on the equilibrium mortgages, and cannot make a positive profit on any off-equilibrium mortgage contract. Below is the formal definition of the equilibrium.

Definition 3 *An equilibrium in the housing and mortgage markets with no restrictions on mortgage design consists of allocation of capital K^H and K into housing and production sectors, a set of mortgage contract $\{m^j\}$ and prices $\{w(s), R(s), P_0^*, P_1^*(s)\}$, such that taking the equilibrium prices and mortgage contracts as given, the following is true.*

At time $t = 0$, i.e., before aggregate and idiosyncratic shocks s and l are known:

- (i) *The expected return on every equilibrium mortgage contract is equal to \bar{R} , and there exists no mortgage contract resulting in a strictly expected return for a capitalist,*
- (ii) *Workers buy homes if and only if homeownership results in strictly higher expected utility compared to renting in period one,*
- (iii) *Each homebuyer chooses a mortgage contract that maximizes his expected utility,*

(iv) *The housing market clears at time $t = 0$, i.e., all homes built by capitalists are bought by households.*

At time $t = 0+$, i.e., when aggregate and idiosyncratic shocks s and l are revealed:

(v) *Homeowners who have mortgage m^j default for liquidity reason if their income is less than $m^j(s)$,*

(vi) *Homeowners who have mortgage m^j and income greater than $m^j(s)$ default strategically if and only if default increases their utility.*

At time $t = 1$, i.e., after economic shocks are fully absorbed by the economy:

(vii) *Homeowners with housing utility less than $P_1^*(s)$ sell their homes in state s ,*

(viii) *Renters including those who defaulted at time $t = 0+$ buy homes in state s if their housing utility and their income are greater than $P_1^*(s)$,*

(ix) *Housing market clears at time $t = 1$, i.e., no homes are left unoccupied in the second period.*

A key insight of this section is to show that no mortgage contract that comes with a strategic default option can survive in an equilibrium. It follows from the observation that a strategic defaulter cares only about the mortgage payment in the state in which he does not default strategically. Indeed, if a borrower defaults strategically in state \hat{s} , his payoff does not depend on the mortgage payment in this state. As a result, a lender can lose strategic defaulters by slightly increasing the mortgage payment in state \hat{s}^- and lowering the mortgage payment in state \hat{s} . Since strategic defaults are costly for the lender, such a modification of a mortgage contract will increase the lender's expected payoff compared to the original mortgage.

More formally, suppose lenders break even on a mortgage contract m' , and a positive mass of borrowers strategically defaults under this contract in state \hat{s} . This means that a lender loses money on a borrower with low θ , who strategically defaults, and makes a strictly positive profit on a borrower with high θ , who does not default strategically. A lender can offer a new contract m'' :

$$\begin{aligned} m''(\hat{s}^-) &= m'(\hat{s}^-) + \varepsilon^-, \\ m''(\hat{s}) &= m'(\hat{s}) - \varepsilon, \end{aligned}$$

with $\varepsilon > 0$ and $\varepsilon^- > 0$, such that borrowers who do not strategically default under contract m' are weakly better off under contract m'' . Since ε and ε^- can be made arbitrarily small, the lender

will keep making a strictly positive profit on these borrowers. On the other hand, the strategic defaulters would strictly prefer mortgage m' over m'' , since only the mortgage payment in the state \hat{s}^- , in which they do not default strategically, matters to them. As a result, the lender will stop losing money on them under contract m'' . Thus, the lender will make a strictly positive profit with contract m'' .

The above argument shows that a mortgage with a strategic default option cannot be an equilibrium contract. Consequently, only a home equity insurance mortgage can be an equilibrium mortgage contract.

Theorem 3 *If there is an equilibrium with state contingent mortgage contracts, then an equilibrium contract m^* takes the form of a home equity insurance mortgage:*

$$m^*(s_g) - m^*(s_b) = P_1^*(s_g) - P_1^*(s_b),$$

and the housing equilibrium is characterized by Corollary 1.

Proof is in Appendix. ■

We will refer to an equilibrium with a HEIM contract as the HEIM equilibrium.

5.3 Existence of an equilibrium with state-contingent mortgage contracts

Theorem 3 establishes that HEIM is the only mortgage contract that can exist in equilibrium, but does not guarantee the existence of the equilibrium. If a HEIM equilibrium exists, then it must be impossible for a lender to make a positive profit by offering a non-HEIM mortgage contract.

Let's consider an arbitrary mortgage m . We assume that the mortgage is offered on a small scale, and it does not change the HEIM equilibrium prices $P_1^*(s_g)$ and $P_1^*(s_b)$. If a borrower with $\theta \in (\underline{\theta}, \hat{\theta})$ takes this mortgage he will strategically default in state \hat{s} , where

$$\begin{aligned} \hat{s} &= \arg \max_s \{m(s) - P_1^*(s)\}, \\ \hat{s}^- &= \arg \min_s \{m(s) - P_1^*(s)\}, \\ \hat{\theta} &= m(\hat{s}) - P_1^*(\hat{s}), \\ \underline{\theta} &= m(\hat{s}^-) - P_1^*(\hat{s}^-). \end{aligned}$$

Let z and z^* denote the probabilities¹⁵ that a borrower does not default for liquidity reasons under mortgages m and m^* :

$$\begin{aligned} z &= \pi_g(1 - G(\frac{m(s_g)}{w(s_g)})) + \pi_b(1 - G(\frac{m(s_b)}{w(s_b)})), \\ z^* &= \pi_g(1 - G(\frac{m^*(s_g)}{w(s_g)})) + \pi_b(1 - G(\frac{m^*(s_b)}{w(s_b)})). \end{aligned}$$

To make a positive profit on mortgage m , the lender must transfer costs associated with strategic defaults on borrowers with $\theta \geq \hat{\theta}$. Those borrowers could still choose contract m over m^* but only if they have a better chance of remaining homeowners under mortgage m , i.e., when $z > z^*$. If $z \leq z^*$, only strategic defaulters would choose m , and the lender would lose money on it.

The following Lemma shows that borrowers' preferences over the mortgage contracts are determined by their homeownership values.

Lemma 1 *When $z > z^*$, borrowers with $\theta \in (\theta^1, \theta^2)$ choose mortgage m^* over mortgage m , while borrowers with $\theta \in (\underline{\theta}, \theta^1) \cup (\theta^2, \bar{\theta})$ choose m over m^* , where*

$$\theta^1 = \min \left\{ \hat{\theta}, \frac{z^* \theta^* - \pi_{\hat{s}^-} (1 - G(\frac{m(\hat{s}^-)}{w(\hat{s}^-)})) \underline{\theta}}{z^* - \pi_{\hat{s}^-} (1 - G(\frac{m(\hat{s}^-)}{w(\hat{s}^-)}))} \right\}, \quad (23)$$

$$\theta^2 = \max \left\{ \hat{\theta}, \frac{z \underline{\theta} - z^* \theta^* + \pi_{\hat{s}} (1 - G(\frac{m(\hat{s})}{w(\hat{s})})) (\hat{\theta} - \underline{\theta})}{z - z^*} \right\}. \quad (24)$$

Borrowers with $\theta \in (\underline{\theta}, \theta^1)$ strategically default in state \hat{s} .

Proof is in Appendix. ■

Intuitively, borrowers with high homeownership values $(\theta^2, \bar{\theta})$ prefer m , because it improves their chance to enjoy homeownership utility when $z > z^*$. Borrowers with intermediate homeownership values (θ^1, θ^2) do not like m , because it is more expensive than m^* due to costs imposed by strategic defaults. Borrowers with low homeownership values $(\underline{\theta}, \theta^1)$ prefer m , because they take advantage of the strategic default option.

We note that

$$\underline{\theta} < \theta^* < \theta^1 \leq \hat{\theta} \leq \theta^2 \leq \bar{\theta}$$

¹⁵ z and z^* are functions of the mortgage payments. We omit the arguments to simplify the notation.

whenever $m \neq m^*$.

Lemma 1 implies that the probability of a strategic default in state \hat{s} under mortgage m is equal to

$$\xi(m(s_g), m(s_b)) = \frac{F(\theta^1) - F(\underline{\theta})}{1 - F(\underline{\theta}) - (F(\theta^2) - F(\theta^1))}.$$

One can see that $\xi(m^*(s_g), m^*(s_b)) = 0$ and $\xi(m(s_g), m(s_b)) > 0$ whenever $m \neq m^*$.

The following Theorem characterizes the existence of an equilibrium with state-contingent mortgage contracts.

Theorem 4 *If a HEIM contract is a solution to the following problem:*

$$\max_{m(s_g), m(s_b)} \pi_{\hat{s}^-} (1 - G(\frac{m(\hat{s}^-)}{w(\hat{s}^-)})) \underline{\theta} + \pi_{\hat{s}} (1 - G(\frac{m(\hat{s})}{w(\hat{s})})) (1 - \xi(m(s_g), m(s_b))) \hat{\theta}, \quad (25)$$

then the HEIM equilibrium is the unique equilibrium. Otherwise, there is no equilibrium with state-contingent mortgage contracts.

Proof is in Appendix. ■

The objective function (25) represents the lender's payoff under mortgage m without terms that do not depend on mortgage payments. If a HEIM contract maximizes this function, then the lender cannot make a positive profit on a different mortgage contract. In this case, the HEIM is the equilibrium contract and the equilibrium is unique.

If a non-HEIM mortgage maximizes (25), then there is a profitable deviation and the HEIM equilibrium unravels. According to the argument of Section 5.2, a non-HEIM mortgage cannot be sustained in an equilibrium either. Thus, there is no equilibrium with state-contingent mortgage contracts in this case.

The distribution of the homeownership values $F(\theta)$ is important for the existence of the equilibrium, as it determines the losses associated with the strategic defaults. When $f(\theta)$ is high in a neighborhood of θ^* , then a deviation from the HEIM contract becomes too costly due to the large mass $(F(\theta^1) - F(\underline{\theta}))$ of strategic defaulters, and the HEIM equilibrium exists. On the other hand, when $f(\theta)$ is low in a neighborhood of θ^* , the HEIM equilibrium may unravel, since the low mass $(F(\theta^1) - F(\underline{\theta}))$ of strategic defaulters may allow a profitable non-HEIM contract.

Wage levels in the two states are also important for equilibrium existence. When wages are much lower in the bad state than in the good state, the HEIM contract will result in much higher

liquidity default rates in the bad state even though HEIM payments are already lower in the bad state. As a result, further lowering payments in the bad state and increasing them in the good state may lead to a profitable deviation from the HEIM contract, unraveling the HEIM equilibrium.

To sum up, the economic intuition suggests that the equilibrium is less likely to exist when there are fewer households with homeownership values close to that of the marginal homebuyer θ^* , and when wages in the bad state are much lower than those in the good state.

Appendix B provides examples illustrating the existence and non-existence of equilibrium.

6 Welfare Implications of Home Equity Insurance Mortgage Contracts

The previous section has established that in the absence of restrictions on mortgage design the home equity insurance mortgage emerges as the equilibrium contract. In this section, we compare the equilibrium with the HEIM contract m^* of Section 5 versus the equilibrium with the FRM contract \bar{m} of Section 3. We start our analysis with the effect of mortgage design on home prices and homeownership rate.

Proposition 5 *If in the good state the HEIM payment is higher (equal, lower) than the FRM payment the home prices in the HEIM equilibrium in both states are higher (equal, lower) than the corresponding prices in the FRM equilibrium. Moreover, the homeownership is lower (equal, higher) in the HEIM equilibrium compared to the FRM equilibrium:*

$$\text{if } m^*(s_g) > \bar{m}, \text{ then } \theta^* > \underline{\theta}^F \text{ \& } P_1^*(s) > P_1^F(s) \text{ for } s \in \{s_g, s_b\},$$

$$\text{if } m^*(s_g) = \bar{m}, \text{ then } \theta^* = \underline{\theta}^F \text{ \& } P_1^*(s) = P_1^F(s) \text{ for } s \in \{s_g, s_b\},$$

$$\text{if } m^*(s_g) < \bar{m}, \text{ then } \theta^* < \underline{\theta}^F \text{ \& } P_1^*(s) < P_1^F(s) \text{ for } s \in \{s_g, s_b\},$$

where the homeownership thresholds are given by

$$\underline{\theta}^F = m^F(s_g) - P_1^F(s_g),$$

$$\theta^* = m^*(s_g) - P_1^*(s_g) = m^*(s_b) - P_1^*(s_b).$$

Proof is in Appendix. ■

Proposition 5 shows that mortgage payments in the good state are the key determinant of the homeownership rate and the long-term home prices. When $m^*(s_g) > \bar{m}$, more homes are built in the FRM equilibrium. This is because the FRM contract attracts strategic defaulters with low homeownership values, who do not care about mortgage payments in the bad state. Higher housing inventories translate into lower home prices on the secondary market at time $t = 1$.

On the other hand, when $m^*(s_g) < \bar{m}$, all homebuyers with the FRM mortgage would also buy homes with the HEIM mortgage. A higher homeownership rate associated with HEIM leads to lower home prices at time $t = 1$.

It may seem counterintuitive that if $m^*(s_g) = \bar{m}$ the prices are the same in the bad state, even though there are more strategic and liquidity defaults under FRM contract. However, those who default at $t = 0+$ buy their homes back at $t = 1$ if their income is greater than the home price at $t = 1$. Thus, the net effect of defaults at time $t = 0+$ on housing demand and supply at $t = 1$ is zero.

Theorem 5 *If wages in the bad state are sufficiently low, then the equilibrium with home equity insurance mortgages Pareto dominates the equilibrium with fixed-rate mortgages. Specifically, the homeownership rate is higher, while the mortgage payments, default rates, and home prices are lower in both states in the HEIM equilibrium.*

If wages in the bad state are not too low, then the homeownership rate is lower and prices are higher in both states with HEIMs compared to the equilibrium with FRMs. Households with low homeownership values $\theta \in (\underline{\theta}^F, \theta^]$ are strictly worse off in the HEIM equilibrium, while households with higher homeownership values $\theta \geq \hat{\theta}^F$ are better off in the HEIM equilibrium.*

Proof is in Appendix. ■

To understand Theorem 5, we recall that FRM contracts embed a strategic default option, which contributes positively to the homeownership rate. The home equity insurance mortgage eliminates the strategic default option with associated default inefficiencies. However, without this option, homeownership may become less attractive to some borrowers with relatively low θ . As a result, it is possible that homeownership can be lower and these marginal borrowers are worse off in the HEIM equilibrium.

When wages are not too low in the bad state, the default rate is not that sensitive to a reduction in the mortgage payment associated with the HEIM contract. In order to compensate for the lower

revenue in the bad state, the HEIM contract requires a higher than the FRM payment in the good state. This leads to a lower homeownership rate in the HEIM equilibrium. Consequently, workers with $\theta \in (\underline{\theta}^F, \theta^*]$ are better off in the FRM equilibrium, where they buy homes at time $t = 0$ and enjoy homeownership utility in the first period in the good state, and strategically default in the bad state. On the other hand, borrowers with higher homeownership values ($\theta \geq \hat{\theta}^F$) are worse off in the FRM equilibrium, since they end up cross-subsidizing the strategic defaulters.

When wages are sufficiently low in the bad state, the default rate is highly sensitive to a reduction in mortgage payments, and the FRM contract is very inefficient. As a result, it is possible to have the HEIM contract with lower than FRM payments in both states and break even. In this case, the HEIM equilibrium Pareto dominates the FRM equilibrium, since all homeowner are strictly better off with the HEIM contract and there are more of them.

7 Robustness and Extensions

In order to keep our analysis tractable and transparent, we have made a number of simplifying assumptions. This has allowed us to highlight several novel relationships among mortgage design, default decisions, home prices and the homeownership rate. We hope that our tractable framework will be helpful in future research on housing and mortgage markets. Below we discuss several potential extensions of our setting.

7.1 Alternative production technologies

For tractability we assumed a simple linear production function form (1). However, our key findings would be preserved in environments with other production function forms. In particular, Theorem 3, which establishes HEIM as the equilibrium contract, does not depend on the shape of the production function.

We also note that other production functions can induce additional general equilibrium effects. For example with the Cobb-Douglas form, $Y(K, L, s) = A(s)K^\alpha L^{1-\alpha}$, with $\alpha \in (0, 1)$ if HEIMs increase the housing supply, the shift of capital towards the housing sector will result in higher equilibrium returns to capital and lower wages compared to the FRM equilibrium. While this would mute to some extent the effects of home equity insurance loans on the equilibrium homeownership rate and welfare of households, we expect our key insights (including Theorem 5) to hold for more

general production forms.

We also assumed that the construction of homes can only happen at time 0. However, if home construction is possible in the intermediate period, home prices will still be lower in the bad state relative to the good state. Intuitively, there will be no new home construction in the bad state as there is already excessive supply of homes built at time zero given the reduced purchasing power of the population. While new home construction can occur in the good state of the economy, it would not fully eliminate the relative difference in prices across states that drive our key results.

7.2 Multiple credit categories and down payments

For simplicity, we assumed that all borrowers have the same expected income and no endowment for down payment. In a setting with multiple observable credit characteristics, mortgage contracts would be conditional on these characteristics. State-contingent contracts would be particularly beneficial to borrowers with risky income profiles and low down payments as these contracts can eliminate strategic defaults and reduce liquidity defaults. HEIM would still arise as the equilibrium contract in the segment of borrowers with no down payment.

A positive down payment would relax the strategic default condition, and as a result, an equilibrium contract could depend on both home prices and aggregate wages. On the other hand, the benefits of state-contingent contracts can be small for borrowers with high down payments and income, as these borrowers are unlikely to default under FRM contracts.

Overall, one could see our analysis of state-contingent contracts as being particularly relevant for the least creditworthy segment of borrowers who can only afford to buy homes with no or little down payment and who display a nontrivial default risk.¹⁶ Among more creditworthy borrowers who can afford substantial downpayments there may be little efficiency gain from moving away from traditional contracts such as FRMs that dominate this market segment.

7.3 Recourse Loans

Throughout the paper, we assumed that the set of feasible mortgage contracts is limited to non-recourse loans. In practice, the value of recourse clauses in the US residential mortgage market is open to serious question. In many states, mortgages are non-recourse, in others they are effectively

¹⁶We note that borrowers with no or very little down payment have accounted for a substantial share of residential borrowers in the US (see Keys et al. 2013).

so, given costly and limited recovery. In recourse states, deficiency judgments are rarely observed, as they can be avoided or defeated¹⁷. For example, debtors can hide or transfer their assets to other people, or challenge lenders in court. On the other hand, lenders pursuing deficiency judgments may end up paying not only legal fees, but also face substantial political and reputation costs.

7.4 Risk aversion

For tractability, we have assumed a linear utility in consumption goods and focused on the effects of mortgage contracts on foreclosure inefficiencies. In a setting with borrower risk aversion, state-contingent lending contracts will provide additional benefits to households by partially insuring their labor income risk and hence allowing them to better smooth their consumption profiles. This additional benefit should increase the value of state-contingent contracts relative to fixed-rate mortgages.

We also assumed that capitalists are risk neutral. However, the ability of the financial sector to bear aggregate risk during economic downturns may be limited.¹⁸ As HEIMs require lower mortgage repayments in the bad state, this could make them costly to financial intermediaries. On the other hand, HEIMs may have a positive effect on risk averse lenders in some cases, since HEIMs reduce liquidity defaults and eliminate strategic defaults in the bad economic state. This suggests that the HEIM effect on risk averse lenders can be ambiguous.

We also note that there are mechanisms such as securitization that could partly transfer aggregate risk away from the banking sector by allowing more wealthy households (capitalists) to hold the aggregate portfolio of mortgage debt and effectively insure the less wealthy (workers in our model). We expect that in such settings, the equilibrium state-contingent mortgage contracts would still require lower repayments in states with lower home prices.

7.5 Rental market

For simplicity we have normalized the cost and utility of rental to zero. This implies that the rental and the owner-occupied housing markets are fully segmented in our setting. We note that while there is some conversion between owner-occupied homes and rental homes, the rental and owner-

¹⁷See Brueggeman and Fisher (2011).

¹⁸See recent work by Greenwald, Landvoigt, and Van Nieuwerburgh (2017), who explore the effect of shared-appreciation mortgages on financial intermediaries with limited risk-bearing capacity in the quantitative general equilibrium model.

occupied markets are quite segmented in reality. In addition rental units are more standardized, smaller in size, and commonly placed in multifamily units which limits agency issues related to maintenance and customizing of rental units.

In principle we could relax these assumptions and formally model the rental market with rents being determined by the equilibrium market clearing condition. However, as long as homeownership is sufficiently valuable relative to the rental option, our key insights would remain unchanged.

7.6 Cost of credit and house prices

Our general equilibrium framework allows us to study the relationship between the cost of mortgage credit and house prices. For that purpose, one could introduce a loan origination cost faced by the lenders that would affect the equilibrium mortgage rate. This cost could depend on lending technology, securitization, regulations, and government subsidies. For example, cheaper mortgage credit could be a result of government interest rate policies and implicit subsidies to too-big-too-fail financial institutions and government sponsored enterprises, such as Fannie Mae and Freddie Mac. It could be also caused by financial innovations, such as securitization, and by technological progress, such as computers and the Internet.

For example, we could investigate the impact of cost of credit on house prices in the FRM equilibrium. Appendix B.3 shows that a *decrease* in the mortgage payment \bar{m} leads to an *increase* in the homeownership rate and a *decrease* of home prices at time $t = 1$. While it may seem surprising that home prices at time $t = 1$ decrease with the cheaper mortgage credit, the explanation is straightforward. Lower mortgage payments make homeownership more attractive, which increases the homeownership rate. A higher housing supply in the secondary market at time $t = 1$ translates into lower home prices. In addition, with convex construction costs, cheaper credit would also increase home prices at time $t = 0$, exacerbating a boom and bust cycle in the housing market.

7.7 Foreclosure externalities

In our setting foreclosures generate deadweight losses because houses remain unoccupied for one period. The number of foreclosures, however, does not directly affect house prices in the secondary market as these are determined by the initial housing supply and the state of the economy with

associated labor market conditions.¹⁹ There is no direct impact of foreclosures on house prices as borrowers who default can buy back their homes at time 1 and the foreclosures do not affect the quality of housing. In that sense, we could think about the equilibrium at time 1 as the longer term equilibrium when the markets fully absorb the shock.

We could capture the more immediate impact of a foreclosure crisis on equilibrium outcomes by introducing some additional costs of foreclosures. For example, we could assume that some borrowers who default on their mortgages are excluded from the housing market at time 1.²⁰ This modification implies that foreclosures will have an adverse impact on house prices by limiting the number of potential buyers at time 1, resulting in foreclosure externality. In an FRM equilibrium, this will contribute to the larger spread between house prices across states, which will reinforce the value of state-contingent contracts.

7.8 Local housing markets

For simplicity, we focused on one housing market. In reality, there is some heterogeneity in house price movements across locations. One could consider a model with multiple regions. In a setting with no mobility of population across regions, the HEIM will still be the equilibrium mortgage since it remains the only contract that eliminates the strategic default option. In a more general model with mobility, the borrower's default decisions may also depend on house price movements in other regions. We expect that if mobility costs are substantial for most of the borrowers or if mobility benefits are not known ex-ante, the equilibrium contract will still be primarily indexed to the local house prices.

7.9 Refinancing

In our analysis we did not consider an option to refinance a mortgage. This additional option effectively turns FRMs into state-contingent mortgages as some borrowers could refinance their loans when market interest rates are low. However, it would not in general lead to similar outcomes to equilibrium with HEIMs. This is because the HEIM contract implies an *automatic* reduction in mortgage payments during the bad economic times while refinancing is *conditional* on borrower qualifying for better mortgage terms. As lower rates during the economic downturns are also

¹⁹However, the expected foreclosures in our setting indirectly affect prices at time 1 through their impact on mortgage rates, homeownership rate, and the housing supply.

²⁰Alternatively, some borrowers who defaulted may lose interest in homeownership due to their adverse experience.

associated with higher default premia due to declines in home values and borrower creditworthiness, some borrowers may not be able to reduce their mortgage payments through refinancing. This observation is consistent with the empirical evidence showing that during the Great Recession many borrowers were not able to refinance their mortgages despite historically low risk-free rates (see Agarwal et al. 2015).

7.10 HEIMs versus other existing state-contingent contracts

In this section we compare the features of HEIM with state-contingent contracts currently used in the marketplace. We also discuss issues related to implementation of HEIM contracts in practice.

7.10.1 HEIMs vs ARMs

The fundamental feature of the HEIM contract is its dependence on house price level, which eliminates a strategic default option by fully insuring borrower's equity position against the movement in house prices. It happens that in our setting we can also implement this contract as a special form of an adjustable rate-mortgage. In particular consider an ARM contract with required payments equal to $m^{ARM}(r(s)) = P_0[1 + \gamma(r(s))]$ where $r(s) = R(s) - 1$ and γ is a non-decreasing function of $r(s)$ such that $m^*(s) = m^{ARM}(r(s))$ for all s . Such a function γ exists in our setting as market interest rate and house prices co-move together with bad economic states being associated with a decline in house prices and market interest rates and vice versa.

However, we note that such a function may be different from typical indexation forms used in the marketplace on ARM contracts. Nonetheless, if home prices, wages, and interest rates are sufficiently positively correlated our analysis suggests that even standard ARM contracts may be more efficient than FRMs because lower payments in the bad state reduce expected liquidity and strategic defaults.²¹ This may also help justify a documented high concentration of ARMs among riskier borrowers (e.g, see Mayer et al. 2009 and Keys et al. 2013).

Recent empirical evidence is consistent with this observation. In particular, Di Maggio et al. (2017) show that mortgage rate declines during the Great Recession due to ARM contracts resetting to a low rate had a direct positive impact on borrowers who experienced these reductions by reducing their default rate and increasing their consumption. Moreover, regions more exposed to

²¹Moreover, Piskorski and Tchisty (2010) show in a dynamic partial equilibrium setting that an option ARM that combines a flexible repayment feature with adjustable rate can further enhance efficiency.

mortgage rate declines due to ARM resets saw a relatively faster recovery in house prices, increased consumption, and decline in foreclosure rate. Overall, this evidence highlights the potential role of ARM contracts in insuring households against recessions if interest rate indices to which these loans are indexed are expected to be lower during economic downturns and higher during expansions.

This being said, there are a number of factors that may limit the ability of contracts indexed to national interest rate indices to provide effective insurance to households. First, historical correlation between interest rates and aggregate employment and output has been far from perfect.²² Second, a significant regional heterogeneity in economic conditions, including the local housing markets, may limit the effectiveness of "one-size fits all" adjustments of mortgage terms based on the national indices. In this regard, the HEIM type-contracts indexed to local house price indices could be more efficient in insuring households against housing downturns.

7.10.2 HEIMs vs SAMs and other participating mortgages

There are contracts whose repayments are explicitly tied to the real estate values. A shared appreciation mortgage (SAM) is a participating loan in which the borrower agrees to pay the lender a fixed percentage of the gain in the individual property value in exchange for reduction in mortgage rate.²³ However, there are important differences between HEIMs and SAMs. First, the payments of SAMs do not go down when home prices decline, which may limit their ability to reduce defaults during severe housing downturns. Second, due to their dependence on the individual home price, SAMs may lead to the moral hazard problem resulting in under investment in house maintenance by the borrower. The HEIMs do not create this moral hazard problem since they depend on local house price indices rather than the individual home value.

Participating mortgages are also used in commercial real estate market. A shared income mortgage (SIM) allows the lender to share in part of the property income. A shared equity mortgage (SEM) allows the lender to share in both part of the property income and resale proceeds. Such contracts are not feasible in a residential real estate market since the borrower's housing utility cannot be simply shared with the lender, unlike the rent revenue from commercial properties.

²²The US economy experienced periods when national interest rate indices substantially increased during economic downturns such as the stagflation episode of the 1980s. In an economy with high share of ARMs the monetary policy could take into account ARM's potential role as an automatic stabilizer in setting interest rate policy. However, a system with a larger share of ARMs could also complicate the central bank price stability objective as increases in interest rates can be highly unpopular with homeowners creating political pressure to keep interest rates low for an extended period of time.

²³Shared appreciation mortgages were offered by Barclays and the Bank of Scotland in the UK.

7.11 Implementation of HEIMs in practice

While state-contingent mortgage contracts are not uncommon, HEIMs have not yet been seen in practice. Below we highlight a number of challenges facing implementation of such contracts.

First, substantial government involvement in the mortgage market through a system of subsidies and regulations favours traditional contracts like FRMs or ARMs.²⁴ This potentially suppresses adoption of new mortgage designs.

Second, implementation of new designs may require a significant amount of time due to private market inertia, learning, or low perceived value of such innovations from the ex-ante perspective. For example, prior to the Great Depression, mortgage contracts were predominantly short-term loans. The inability to roll these loans over was a major contributor to the collapse of the financial and housing markets. As a result, the government helped private market develop and standardize the fully-amortizing long-term mortgage contracts such as FRMs that currently dominate the US residential market.

Third, the widespread adoption of HEIMs would require timely and accurate regional house price indices. Such indices were unavailable in the past. Zip-code level house price indices have only recently been developed and offered by data providers (e.g, Zillow.com).²⁵

7.12 Non-existence of equilibrium

Finally, our findings in Section 5.3 indicate that in some cases unrestricted competition in mortgage design may lead to a non-existence of mortgage market equilibrium with state-contingent lending contracts.²⁶ This finding is connected to the prior literature pointing that in insurance markets private information can lead to non-existence of equilibrium (Rothschild and Stiglitz 1976).

Non-existence of equilibrium may seem hard to interpret, as taken on its face value it implies that the model cannot predict a market outcome. This finding, however, suggests that in a richer dynamic setting in which new mortgage contracts cannot be introduced immediately, there might

²⁴Currently about 90 percent of residential mortgages originated in the US carry explicit or implicit government guarantees (based on the Corelogic data). See Hurst et al. (2016) and Gete and Zecchetto (2017) for a recent analysis of distributional implications of government guarantees in mortgage markets.

²⁵In addition, Hartman-Glaser and Hebert (2016) point out that if there are informational asymmetries between borrowers and lenders about the ability of such indices to measure underlying states, the risk-sharing ability of state-contingent contracts based on such indices can be limited.

²⁶In this regard we note that the period preceding the recent crisis and the collapse of the private residential lending market was characterized by a significant growth in nontraditional mortgage products and a fierce competition among lenders (Keys et al. 2013).

be no stationary equilibrium leading to some market instability. This may require creating a set of laws or rules to foster market stability. This observation highlights the potential understudied role of government sponsored enterprises (GSEs).²⁷ By subsidizing a restricted contract choice through its guarantee system the GSEs may help facilitate the existence of a stable mortgage market by limiting private competition in mortgage design. In this regard, the government may also play a potentially important role in practical implementation of HEIM contracts.

8 Conclusion

In this paper, we develop a tractable general equilibrium framework of housing and mortgage markets with aggregate and idiosyncratic risks, costly liquidity and strategic defaults, empirically relevant informational asymmetries, and endogenous mortgage design. We show that the equilibrium contract takes the form of the home equity insurance mortgage that solves the adverse selection problem imposed by the strategic defaulters with low homeownership values. We also find that, while beneficial for households with high homeownership values, a widespread adoption of HEIM loans may have ambiguous effects on the homeownership rate and welfare of marginal homebuyers.

We note that while the HEIM contract is similar to some of the recent proposals (e.g., Shiller 2008), such mortgages have not yet been widely adopted in the marketplace. The widespread adoption of these loans requires timely and accurate regional house price indices, which were not widely available in the past. Alternatively, appropriately structured ARM contracts may share some of the benefits of the state-contingent mortgage contracts as long as the relevant interest rate indices to which these loans are indexed closely co-move with home prices and borrowers' income. In such settings rather than exposing borrowers to interest rate risk, ARMs can effectively provide households with valuable insurance against recessions by lowering mortgage payments in the states when wages and house prices are low. This observation suggests that in economies with high share of ARMs the monetary policy should take into account ARM's potential role as an automatic stabilizer (e.g., by introducing a house price component to Taylor-type rules). This being said, as we discussed in Section 7.10, there are a number of factors that may limit the ability of contracts indexed to national interest rate indices to provide effective insurance to households.

²⁷The common rationale for the GSEs role is the facilitation of the secondary market liquidity through the system of repayment guarantees that limit the asymmetric information problem in the secondary market.

In this regard, the HEIM type-contracts indexed to local house price indices can be more efficient in insuring households against housing downturns.

Our tractable framework has the potential to be used to address a number of policy questions related to the design of mortgage contracts, mortgage market regulations, and the welfare implications of policy interventions during the housing crisis. In addition, one can incorporate nominal prices and interest rates in our setting to investigate the interplay between the central bank policies and mortgage and housing markets.

There are several reasons why it may be optimal for the government to intervene in our setting. First, our results show that unrestricted competition between lenders may lead in some cases to a non-existence of equilibrium. In such cases, public policy may help facilitate the existence of a stable mortgage market by subsidizing a restricted set of contracts. In this regard, GSEs could foster the adoption of HEIMs by prominently including these contracts in the menu of mortgages eligible for financing with government guarantees. Second, policy interventions could also be justified by the inability of market participants to fully internalize the impact of their actions on equilibrium outcomes. For example, in the setting with foreclosure externalities that we discussed in Section 7.7, it may be socially optimal to provide households with more insurance against housing downturns than the level implied by the private market equilibrium. We are currently exploring these questions in our ongoing research.

Appendix A: Proofs

Proof of Proposition 1

Strategic default occurs if and only if a borrower obtains a higher utility from defaulting than paying the mortgage. According to (2) and (3), this happens when

$$w(s)l_i + 1_{w(s)l_i \geq P_1^F(s)} \max(0, \theta_i - P_1^F(s)) > w(s)l_i - \bar{m} + \theta_i + \max(\theta_i, P_1^F(s)),$$

implying

$$1_{w(s)l_i \geq P_1^F(s)} \max(0, \theta_i - P_1^F(s)) > \theta_i - \bar{m} + \max(\theta_i, P_1^F(s)).$$

There are two cases to consider.

Case 1: $\bar{m} \leq w(s)l_i < P_1^F(s)$, i.e., the borrower can pay the mortgage, but cannot afford to buy a home at time 1. The above strategic default condition becomes

$$\bar{m} \geq \theta_i + \max(\theta_i, P_1^F(s)).$$

This inequality contradicts the assumption that $\bar{m} < P_1^F(s)$ used to define Case 1. As a result, Case 1 is not relevant to our analysis.

Intuitively, the borrower would never default strategically when the house is worth more than the mortgage payment.

Case 2: $w(s)l_i \geq P_1^F(s)$, i.e., the borrower can buy a home at time 1. The strategic default condition can be rewritten as follows

$$\max(0, \theta_i - P_1^F(s)) > \theta_i - \bar{m} + \max(\theta_i - P_1^F(s), 0) + P_1^F(s),$$

which simplifies to

$$\bar{m} > \theta + P_1^F(s).$$

Proof of Proposition 3

The following argument demonstrates that it is impossible to have $P_1^F(s^*) \geq P_1^F(s')$, and $w(s^*) < w(s')$ at the same time. To see it suppose that by contradiction $w(s^*) < w(s')$ and $P_1^F(s^*) \geq P_1^F(s')$.

From (6) and the fact that $\frac{P_1^F(s^*)}{w(s^*)} > \frac{P_1^F(s')}{w(s')}$ we get that

$$\begin{aligned} & (F(\underline{\theta}^F) - F(P_1^F(s')))(1 - G(\frac{P_1^F(s')}{w(s')})) > \\ & (F(\underline{\theta}^F) - F(P_1^F(s^*)))(1 - G(\frac{P_1^F(s^*)}{w(s^*)})) = (1 - F(\underline{\theta}^F))G(\frac{P_1^F(s^*)}{w(s^*)}) \\ & > (1 - F(\underline{\theta}^F))G(\frac{P_1^F(s')}{w(s')}), \end{aligned}$$

which contradicts the market clearing condition (7). Thus, the home price must be higher in the state with higher wages, which proves (8a).

Proof of Theorem 1

According to (12), in the bad state the expected payment from the mortgage is:

$$\hat{\Pi}(\bar{m}, w(s_b), P_1^F(s_b)) = \left(\frac{F(\hat{\theta}^F) - F(\underline{\theta}^F)}{1 - F(\underline{\theta}^F)} \right) P_1^F(s_b) + \left(\frac{1 - F(\hat{\theta}^F)}{1 - F(\underline{\theta}^F)} \right) \left(G\left(\frac{\bar{m}}{w(s_b)}\right) P_1^F(s_b) + \left(1 - G\left(\frac{\bar{m}}{w(s_b)}\right)\right) \bar{m} \right),$$

where $\hat{\theta}^F = \bar{m} - P_1^F(s_b)$. The homeownership rate $\underline{\theta}^F$ is not affected by the loan modification in the bad state. We will investigate how $\hat{\Pi}(\bar{m}, w(s_b), P_1^F(s_b))$ responds to a change in \bar{m} . Taking into account the fact that $\frac{d\hat{\theta}^F}{d\bar{m}} = 1$, we can write

$$\begin{aligned} \frac{d\hat{\Pi}(\bar{m}, w(s_b), P_1^F(s_b))}{d\bar{m}} &= \frac{f(\hat{\theta}^F)}{1 - F(\underline{\theta}^F)} P_1^F(s_b) - \frac{f(\hat{\theta}^F)}{1 - F(\underline{\theta}^F)} \left(G\left(\frac{\bar{m}}{w(s_b)}\right) P_1^F(s_b) + \left(1 - G\left(\frac{\bar{m}}{w(s_b)}\right)\right) \bar{m} \right) + \\ &+ \left(\frac{1 - F(\hat{\theta}^F)}{1 - F(\underline{\theta}^F)} \right) \left(g\left(\frac{\bar{m}}{w(s_b)}\right) \frac{P_1^F(s_b) - \bar{m}}{w(s_b)} + \left(1 - G\left(\frac{\bar{m}}{w(s_b)}\right)\right) \right) \\ &= \frac{1}{1 - F(\underline{\theta}^F)} \left[\begin{aligned} & -f(\hat{\theta}^F) \left(1 - G\left(\frac{\bar{m}}{w(s_b)}\right)\right) (\bar{m} - P_1^F(s_b)) + \\ & \left(1 - F(\hat{\theta}^F)\right) \left(g\left(\frac{\bar{m}}{w(s_b)}\right) \frac{P_1^F(s_b) - \bar{m}}{w(s_b)} + \left(1 - G\left(\frac{\bar{m}}{w(s_b)}\right)\right) \right) \end{aligned} \right] \\ &= \frac{1}{1 - F(\underline{\theta}^F)} \left[\begin{aligned} & -f(\hat{\theta}^F) \left(1 - G\left(\frac{\bar{m}}{w(s_b)}\right)\right) \hat{\theta}^F + \\ & \left(1 - F(\hat{\theta}^F)\right) \left(1 - G\left(\frac{\bar{m}}{w(s_b)}\right) - g\left(\frac{\bar{m}}{w(s_b)}\right) \frac{\hat{\theta}^F}{w(s_b)} \right) \end{aligned} \right] \\ &= \frac{1}{(1 - F(\underline{\theta}^F)) w(s_b)} \left[w(s_b) \left(1 - G\left(\frac{\bar{m}}{w(s_b)}\right)\right) \left(1 - F(\hat{\theta}^F) - f(\hat{\theta}^F) \hat{\theta}^F\right) - \hat{\theta}^F \left(1 - F(\hat{\theta}^F)\right) g\left(\frac{\bar{m}}{w(s_b)}\right) \right]. \end{aligned}$$

Thus $\frac{d\hat{\Pi}(\bar{m}, w(s_b), P_1^F(s_b))}{d\bar{m}} > 0$ if and only if

$$w(s_b) \left(1 - G\left(\frac{\bar{m}}{w(s_b)}\right)\right) \left(1 - F(\hat{\theta}^F) - f(\hat{\theta}^F) \hat{\theta}^F\right) - \hat{\theta}^F \left(1 - F(\hat{\theta}^F)\right) g\left(\frac{\bar{m}}{w(s_b)}\right) > 0,$$

which can be rewritten as

$$w(s_b) \frac{1 - G\left(\frac{\bar{m}}{w(s_b)}\right)}{g\left(\frac{\bar{m}}{w(s_b)}\right)} > \frac{\hat{\theta}^F}{1 - \frac{f(\hat{\theta}^F)\hat{\theta}^F}{1-F(\hat{\theta}^F)}}.$$

We note that when wages in the bad state $w(s_b)$ are high and mortgage payments \bar{m} are low and Condition 1 is satisfied, the left hand side is high. At the same time, the right hand side is low, since $\hat{\theta}^F = \bar{m} - P_1^F(s_b)$ is bounded by \bar{m} . Thus, when $w(s_b)$ is high and \bar{m} is low, $\frac{d\hat{\Pi}(\bar{m}, w(s_b), P_1^F(s_b))}{d\bar{m}} > 0$, which means that a reduction in \bar{m} will reduce the lender's payoff.

When when $w(s_b)$ is low, \bar{m} is high and Condition 1 is satisfied, the opposite is true, i.e., $\frac{d\hat{\Pi}(\bar{m}, w(s_b), P_1^F(s_b))}{d\bar{m}} < 0$, which means that a reduction in \bar{m} will increase the lender's payoff.

Proof of Theorem 2

Adding (16a) to the above objective function (15) and simplifying allows us to rewrite this optimization problem as

$$\max_{m(s)} \left(\begin{array}{c} \bar{w}L + 2\theta - P_0\bar{R} \\ -\pi_g[\theta G\left(\frac{m(s_g)}{w(s_g)}\right) + G\left(\frac{P_1(s_g)}{w(s_g)}\right)(\theta - P_1(s_g))] \\ -\pi_b[\theta G\left(\frac{m(s_b)}{w(s_b)}\right) + G\left(\frac{P_1(s_b)}{w(s_b)}\right)(\theta - P_1(s_b))] \end{array} \right) \quad (26)$$

subject to (16a), where

$$\bar{w} \equiv \pi_b w(s_b) + \pi_g w(s_g).$$

Equation (26) follows from the fact that

$$v(s, \theta) + \Pi(s, m(s)) = w(s)L + 2\theta - \theta G\left(\frac{m(s)}{w(s)}\right) - G\left(\frac{P_1(s)}{w(s)}\right)(\theta - P_1(s)).$$

Deleting terms that do not depend on $m(s_g)$ or $m(s_b)$ from (26) simplifies the optimal contracting problem to

$$\min_{m(s)} \pi_g G\left(\frac{m(s_g)}{w(s_g)}\right) + \pi_b G\left(\frac{m(s_b)}{w(s_b)}\right)$$

subject to (16a). Thus, when the borrower does not default strategically, an optimal contract is the contract that minimizes the probability of liquidity default subject to the lender's break even condition.

The first order conditions with respect to $m(s_g)$ and $m(s_b)$ are given by:

$$\begin{aligned} m(s_g) &: \frac{1}{w(s_g)}g\left(\frac{m(s_g)}{w(s_g)}\right) + \lambda\left(1 - \frac{1}{w(s_g)}g\left(\frac{m(s_g)}{w(s_g)}\right)(m(s_g) - P_1(s_g)) - G\left(\frac{m(s_g)}{w(s_g)}\right)\right) = 0 \\ m(s_b) &: \frac{1}{w(s_b)}g\left(\frac{m(s_b)}{w(s_b)}\right) + \lambda\left(1 - \frac{1}{w(s_b)}g\left(\frac{m(s_b)}{w(s_b)}\right)(m(s_b) - P_1(s_b)) - G\left(\frac{m(s_b)}{w(s_b)}\right)\right) = 0, \end{aligned}$$

where λ is the Lagrangian multiplier associated with constraint (16a). These conditions yield equation (18), since

$$-\frac{1}{\lambda} = Q(s_g, m(s_g)) = Q(s_b, m(s_b)),$$

where $Q(s, m(s))$ is given by (17).

Equation (18) can be rewritten as

$$w(s_g)\left(\frac{1 - G\left(\frac{m(s_g)}{w(s_g)}\right)}{g\left(\frac{m(s_g)}{w(s_g)}\right)} - \frac{m(s_g)}{w(s_g)}\right) + P_1(s_g) = w(s_b)\left(\frac{1 - G\left(\frac{m(s_b)}{w(s_b)}\right)}{g\left(\frac{m(s_b)}{w(s_b)}\right)} - \frac{m(s_b)}{w(s_b)}\right) + P_1(s_b).$$

Since $P_1(s_g) > P_1(s_b)$ and $w(s_g) > w(s_b)$, we have

$$\frac{1 - G\left(\frac{m(s_g)}{w(s_g)}\right)}{g\left(\frac{m(s_g)}{w(s_g)}\right)} - \frac{m(s_g)}{w(s_g)} < \frac{1 - G\left(\frac{m(s_b)}{w(s_b)}\right)}{g\left(\frac{m(s_b)}{w(s_b)}\right)} - \frac{m(s_b)}{w(s_b)}. \quad (27)$$

If G satisfies Condition 1, then $\frac{1-G(x)}{g(x)} - x$ is decreasing in x . Hence, equation (27) implies that

$$\frac{m(s_g)}{w(s_g)} > \frac{m(s_b)}{w(s_b)},$$

meaning the default rate is going to be higher in the good state:

$$G\left(\frac{m(s_g)}{w(s_g)}\right) > G\left(\frac{m(s_b)}{w(s_b)}\right).$$

In the uniform example, $Q(s, m(s))$ becomes

$$Q(s, m(s)) = w(s) + P_1(s) - 2m(s),$$

which yields (19).

Finally, Section 5.1 shows that the borrower will not default strategically if and only if $\theta \geq$

$$\max_s [\tilde{m}(s) - P_1(s)].$$

Proof of Theorem 3

Consider a contract m' such that $m'(s_g) - m'(s_b) \neq P_1'(s_g) - P_1'(s_b)$, where $P_1'(s)$ are home prices at time $t = 1$. We will show that this contract cannot be an equilibrium contract.

Let's assume, for the sake of the argument, that

$$\underline{\theta}' \equiv m'(s_g) - P_1'(s_g) < m'(s_b) - P_1'(s_b) \equiv \hat{\theta}'.$$

Then, borrowers with $\theta \in (\underline{\theta}', \hat{\theta}')$ will buy homes at time $t = 0$, and default strategically in the bad state at time $t = 0+$. A borrowers with $\theta \geq \hat{\theta}'$ will not default strategically, and will enjoy expected utility

$$\pi_g v(s_g, \theta) + \pi_b v(s_b, \theta),$$

where

$$v(s, \theta) = w(s)L + \left[1 - G\left(\frac{m'(s)}{w(s)}\right)\right] (2\theta - m'(s)) + \left[G\left(\frac{m'(s)}{w(s)}\right) - G\left(\frac{P_1'(s)}{w(s)}\right)\right] (\theta - P_1'(s)).$$

is the borrower's expected utility in state s according to (14). Taking the derivative of v with respect to $m'(s)$ yields

$$\frac{\partial v(s, \theta)}{\partial m'(s)} = - \left[1 - G\left(\frac{m'(s)}{w(s)}\right)\right] - \frac{g\left(\frac{m'(s)}{w(s)}\right)}{w(s)} (\theta - (m'(s) - P_1'(s))). \quad (28)$$

Now, let's consider mortgage m'' such that

$$\begin{aligned} m''(s_g) &= m'(s_g) + \varepsilon_g, \\ m''(s_b) &= m'(s_b) + \varepsilon_b, \end{aligned}$$

where $\varepsilon_g > 0$ and $\varepsilon_b < 0$ are small. A borrowers with $\theta \geq \hat{\theta}'$ prefers contract m'' over m' if

$$\pi_g \frac{\partial v(s_g, \theta)}{\partial m'(s_g)} \varepsilon_g + \pi_b \frac{\partial v(s_b, \theta)}{\partial m'(s_b)} \varepsilon_b \geq 0. \quad (29)$$

Plugging (28) into (29) yields

$$\varepsilon_b \leq - \frac{\pi_g [1 - G\left(\frac{m'(s_g)}{w(s_g)}\right) + \frac{g\left(\frac{m'(s_g)}{w(s_g)}\right)}{w(s_g)}(\theta - \underline{\theta}')] }{\pi_b [1 - G\left(\frac{m'(s_b)}{w(s_b)}\right) + \frac{g\left(\frac{m'(s_b)}{w(s_b)}\right)}{w(s_b)}(\theta - \hat{\theta}')] } \varepsilon_g.$$

Let

$$\varepsilon_b = - \max_{\theta \in [\hat{\theta}', \bar{\theta}]} \frac{\pi_g [1 - G\left(\frac{m'(s_g)}{w(s_g)}\right) + \frac{g\left(\frac{m'(s_g)}{w(s_g)}\right)}{w(s_g)}(\theta - \underline{\theta}')] }{\pi_b [1 - G\left(\frac{m'(s_b)}{w(s_b)}\right) + \frac{g\left(\frac{m'(s_b)}{w(s_b)}\right)}{w(s_b)}(\theta - \hat{\theta}')] } \varepsilon_g.$$

Then, all borrowers who do not default strategically, i.e., with $\theta \in [\hat{\theta}', \bar{\theta}]$, prefer contract m'' over m' . However, borrowers who default strategically in the bad state, i.e., with $\theta \in (\underline{\theta}', \hat{\theta}')$, prefer contract m' over m'' , since they do not pay mortgage in the bad state and contract m' has lower payment in the good state.

If m' is an equilibrium mortgage contract, then lenders must make zero profit:

$$\pi_g \Pi(s_g, m'(s_g)) + \pi_b \hat{\Pi}(s_b, m'(s_b)) - P_0 \bar{R} = 0, \quad (30)$$

where

$$\begin{aligned} \Pi(s_g, m'(s_g)) &= \left(1 - G\left(\frac{m'(s_g)}{w(s_g)}\right)\right) m'(s_g) + G\left(\frac{m'(s_g)}{w(s_g)}\right) P_1'(s_g), \\ \hat{\Pi}(s_b, m'(s_b)) &= \Pi(s_b, m'(s_b)) - \frac{(F(\hat{\theta}') - F(\underline{\theta}'))}{(1 - F(\underline{\theta}'))} \left(1 - G\left(\frac{m'(s_b)}{w(s_b)}\right)\right) (m'(s_b) - P_1'(s_b)) \end{aligned} \quad (31)$$

are the lender's expected payoffs in the good and the bad states according to (11) and (13).

$$\frac{\partial \Pi(s, m'(s))}{\partial m'(s)} = \left(1 - G\left(\frac{m'(s)}{w(s)}\right)\right) - \frac{g\left(\frac{m'(s)}{w(s)}\right)}{w(s)} (m'(s) - P_1'(s)).$$

Since there are no strategic defaults under contract m'' , the lender's payoff under contract m'' is

$$\begin{aligned}
& \pi_g \Pi(s_g, m''(s_g)) + \pi_b \Pi(s_b, m''(s_b)) - P_0 \bar{R} \\
= & \pi_g \left(\Pi(s_g, m'(s_g)) + \frac{\partial \Pi(s_g, m'(s_g))}{\partial m'(s_g)} \varepsilon_g \right) + \pi_b \left(\Pi(s_b, m'(s_b)) + \frac{\partial \Pi(s_b, m'(s_b))}{\partial m'(s_b)} \varepsilon_b \right) - P_0 \bar{R} \\
= & \pi_b \frac{(F(\hat{\theta}') - F(\underline{\theta}'))}{(1 - F(\underline{\theta}'))} \left(1 - G\left(\frac{m'(s_b)}{w(s_b)}\right) \right) (m'(s_b) - P_1'(s_b)) + \pi_g \frac{\partial \Pi(s_g, m'(s_g))}{\partial m'(s_g)} \varepsilon_g + \pi_b \frac{\partial \Pi(s_b, m'(s_b))}{\partial m'(s_b)} \varepsilon_b,
\end{aligned} \tag{32}$$

where equation (32) follows from (30) and (31). The first term in (32) is strictly positive since $m'(s_b) - P_1'(s_b) = \hat{\theta}'$, and the last two terms can be made arbitrarily small compared to the first term, since they are proportional to ε_g and ε_b . Thus, the lender makes a strictly positive profit with contract m'' . As a result, mortgage m' cannot be an equilibrium contract.

If borrowers default strategically in the good state under contract m'' , i.e.,

$$m'(s_g) - P_1'(s_g) > m'(s_b) - P_1'(s_b),$$

then we can use the above argument to show that a lender can make a positive profit under m'' , if he slightly increases the mortgage payment in the bad state ($\varepsilon_g < 0$ and $\varepsilon_b > 0$). Thus, an equilibrium contract m^* must satisfy

$$m^*(s_g) - m^*(s_b) = P_1^*(s_g) - P_1^*(s_b).$$

Proof of Lemma 1

A borrowers with $\theta \geq \hat{\theta}$ does not default strategically, and his payoffs in state s under contracts m^* and m are given by

$$\begin{aligned}
v(s, \theta, m^*(s)) &= w(s)L + (1 - G(\frac{m^*(s)}{w(s)}))(2\theta - m^*(s)) + (G(\frac{m^*(s)}{w(s)}) - G(\frac{P_1(s)}{w(s)}))(\theta - P_1^*(s)) \\
&= w(s)L + (1 - G(\frac{P_1(s)}{w(s)}))(\theta - P_1^*(s)) + (1 - G(\frac{m^*(s)}{w(s)}))(\theta - (m^*(s) - P_1^*(s))) \tag{33} \\
v(s, \theta, m(s)) &= w(s)L + (1 - G(\frac{m(s)}{w(s)}))(2\theta - m(s)) + (G(\frac{m(s)}{w(s)}) - G(\frac{P_1(s)}{w(s)}))(\theta - P_1^*(s)) \\
&= w(s)L + (1 - G(\frac{P_1(s)}{w(s)}))(\theta - P_1^*(s)) + (1 - G(\frac{m(s)}{w(s)}))(\theta - (m(s) - P_1^*(s))). \tag{34}
\end{aligned}$$

We note that the first two terms in (33) and (34) do not depend on mortgage payments.

The borrower prefers mortgage m over m^* , if

$$\pi_g v(s_g, \theta, m(s_g)) + \pi_b v(s_b, \theta, m(s_b)) \geq \pi_g v(s_g, \theta, m^*(s_g)) + \pi_b v(s_b, \theta, m^*(s_b)).$$

Both sides of the inequality can be further simplified as follows.

$$\begin{aligned} & \pi_g (1 - G(\frac{m^*(s_g)}{w(s_g)})) (\theta - (m^*(s_g) - P_1^*(s_g))) + \pi_b (1 - G(\frac{m^*(s_b)}{w(s_b)})) (\theta - (m^*(s_b) - P_1^*(s_b))) \\ = & \pi_g (1 - G(\frac{m^*(s_g)}{w(s_g)})) (\theta - \theta^*) + \pi_b (1 - G(\frac{m^*(s_b)}{w(s_b)})) (\theta - \theta^*) \\ = & z^* (\theta - \theta^*). \end{aligned}$$

Similarly,

$$\begin{aligned} & \pi_g (1 - G(\frac{m(s_g)}{w(s_g)})) (\theta - (m(s_g) - P_1^*(s_g))) + \pi_b (1 - G(\frac{m(s_b)}{w(s_b)})) (\theta - (m(s_b) - P_1^*(s_b))) \\ = & z (\theta - \underline{\theta}) - \pi_{\hat{s}} (1 - G(\frac{m(\hat{s})}{w(\hat{s})})) (\hat{\theta} - \underline{\theta}). \end{aligned}$$

Thus, the borrower prefers mortgage m over m^* , if

$$z (\theta - \underline{\theta}) - \pi_{\hat{s}} (1 - G(\frac{m(\hat{s})}{w(\hat{s})})) (\hat{\theta} - \underline{\theta}) \geq z^* (\theta - \theta^*),$$

or

$$\theta \geq \frac{\underline{\theta} z - \theta^* z^* + \pi_{\hat{s}} (1 - G(\frac{m(\hat{s})}{w(\hat{s})})) (\hat{\theta} - \underline{\theta})}{z - z^*},$$

which yields (24).

A borrower with $\theta < \hat{\theta}$ defaults strategically in state \hat{s} , and his payoff in this state is given by

$$\hat{v}(\hat{s}, \theta, m(\hat{s})) = w(\hat{s})L + (1 - G(\frac{P_1(\hat{s})}{w(\hat{s})})) (\theta - P_1^*(\hat{s})).$$

The borrower prefers mortgage m over m^* , if

$$\begin{aligned} \pi_{\hat{s}} \hat{v}(\hat{s}, \theta, m(\hat{s})) + \pi_{\hat{s}^-} v(s_{\hat{s}^-}, \theta, m(s_{\hat{s}^-})) & \geq \pi_g v(s_g, \theta, m^*(s_g)) + \pi_b v(s_b, \theta, m^*(s_b)), \\ \pi_{\hat{s}^-} (1 - G(\frac{m(s_{\hat{s}^-})}{w(s_{\hat{s}^-})})) (\theta - (m(s_{\hat{s}^-}) - P_1^*(s_{\hat{s}^-}))) & \geq z^* (\theta - \theta^*), \\ \theta & \leq \frac{z^* \theta^* - \pi_{\hat{s}^-} (1 - G(\frac{m(s_{\hat{s}^-})}{w(s_{\hat{s}^-})})) \theta}{z^* - \pi_{\hat{s}^-} (1 - G(\frac{m(s_{\hat{s}^-})}{w(s_{\hat{s}^-})}))}, \end{aligned}$$

which yields (23).

Proof of Theorem 4

The lender's payoff in state \hat{s}^- under mortgage m is equal to

$$\begin{aligned}\Pi(\hat{s}^-, m(\hat{s}^-)) &= (1 - G(\frac{m(\hat{s}^-)}{w(\hat{s}^-)}))m(\hat{s}^-) + G(\frac{m(\hat{s}^-)}{w(\hat{s}^-)})P_1^*(\hat{s}^-) \\ &= (1 - G(\frac{m(\hat{s}^-)}{w(\hat{s}^-)}))(m(\hat{s}^-) - P_1^*(\hat{s}^-)) + P_1^*(\hat{s}^-) \\ &= (1 - G(\frac{m(\hat{s}^-)}{w(\hat{s}^-)}))\underline{\theta} + P_1^*(\hat{s}^-).\end{aligned}$$

The payoff in state \hat{s} is given by

$$\begin{aligned}\hat{\Pi}(\hat{s}, m(\hat{s})) &= (1 - \xi(m(s_g), m(s_b))) \left((1 - G(\frac{m(\hat{s})}{w(\hat{s})}))m(\hat{s}) + G(\frac{m(\hat{s})}{w(\hat{s})})P_1^*(\hat{s}) \right) + \xi(m(s_g), m(s_b))P_1^*(\hat{s}) \\ &= (1 - G(\frac{m(\hat{s})}{w(\hat{s})}))(1 - \xi(m(s_g), m(s_b)))(m(\hat{s}) - P_1^*(\hat{s})) + P_1^*(\hat{s}) \\ &= (1 - G(\frac{m(\hat{s})}{w(\hat{s})}))(1 - \xi(m(s_g), m(s_b)))\hat{\theta} + P_1^*(\hat{s}).\end{aligned}$$

The expected payoff under mortgage m is

$$\begin{aligned}&\pi_{\hat{s}^-}\Pi(\hat{s}^-, m(\hat{s}^-)) + \pi_{\hat{s}}(1 - G(\frac{m(\hat{s})}{w(\hat{s})}))\hat{\Pi}(\hat{s}, m(\hat{s})) \\ &= \pi_{\hat{s}^-}(1 - G(\frac{m(\hat{s}^-)}{w(\hat{s}^-)}))\underline{\theta} + \pi_{\hat{s}}(1 - G(\frac{m(\hat{s})}{w(\hat{s})}))(1 - \xi(m(s_g), m(s_b)))\hat{\theta} + \pi_{\hat{s}^-}P_1^*(\hat{s}^-) + \pi_{\hat{s}}P_1^*(\hat{s}).\end{aligned}$$

In an equilibrium, this payoff must be maximized by a HEIM mortgage. Since the last two terms do not depend on the mortgage payments, an equilibrium HEIM contract must solve the problem given by equation (25).

Proof of Proposition 5

Equations (9), (10) and (22) that determine equilibrium prices $P_1^F(s_g)$, $P_1^F(s_b)$, $P_1^*(s_g)$ and $P_1^*(s_b)$ can be rewritten as

$$F(\bar{m} - P_1^F(s_g)) - F(P_1^F(s_g)) - G\left(\frac{P_1^F(s_g)}{w(s_g)}\right)(1 - F(P_1^F(s_g))) = 0 \quad (35)$$

$$F(\underline{\theta}^F) - F(P_1^F(s_b)) - G\left(\frac{P_1^F(s_b)}{w(s_b)}\right)(1 - F(P_1^F(s_b))) = 0 \quad (36)$$

$$F(m^*(s_g) - P_1^*(s_g)) - F(P_1^*(s_g)) - G\left(\frac{P_1^*(s_g)}{w(s_g)}\right)(1 - F(P_1^*(s_g))) = 0 \quad (37)$$

$$F(\theta^*) - F(P_1^*(s_b)) - G\left(\frac{P_1^*(s_b)}{w(s_b)}\right)(1 - F(P_1^*(s_b))) = 0 \quad (38)$$

Applying the implicit function theorem to equation (35), we can write

$$\begin{aligned} \frac{dP_1^F(s_g)}{d\bar{m}} &= - \frac{f(\bar{m} - P_1^F(s_g))}{-f(\bar{m} - P_1^F(s_g)) - f(P_1^F(s_g)) + f(P_1^F(s_g))G\left(\frac{P_1^F(s_g)}{w(s_g)}\right) - (1 - F(P_1^F(s_g)))g\left(\frac{P_1^F(s_g)}{w(s_g)}\right)\frac{1}{w(s_g)}} \\ &= \frac{f(\bar{m} - P_1^F(s_g))}{f(\bar{m} - P_1^F(s_g)) + f(P_1^F(s_g))\left(1 - G\left(\frac{P_1^F(s_g)}{w(s_g)}\right)\right) + (1 - F(P_1^F(s_g)))g\left(\frac{P_1^F(s_g)}{w(s_g)}\right)\frac{1}{w(s_g)}} \end{aligned}$$

We note that $0 < \frac{dP_1^F(s_g)}{d\bar{m}} < 1$.

Equations (35) and (37) are identical, except for \bar{m} and $m^*(s_g)$. Since $\frac{dP_1^F(s_g)}{d\bar{m}} > 0$, then $P_1^*(s_g) > P_1^F(s_g)$ if and only if $m^*(s_g) > \bar{m}$.

Since $\frac{dP_1^F(s_g)}{d\bar{m}} < 1$, then $\theta^* > \underline{\theta}^F$ if and only if $m^*(s_g) > \bar{m}$.

Finally, applying the implicit function theorem to equation (36) gives

$$\frac{dP_1^F(s_b)}{d\underline{\theta}^F} = \frac{f(\underline{\theta}^F)}{f(P_1^F(s_b))\left(1 - G\left(\frac{P_1^F(s_b)}{w(s_b)}\right)\right) + (1 - F(P_1^F(s_b)))g\left(\frac{P_1^F(s_b)}{w(s_b)}\right)\frac{1}{w(s_b)}} > 0$$

Since equations (36) and (38) are identical, except for $\underline{\theta}^F$ and θ^* , the fact that $\frac{dP_1^F(s_b)}{d\underline{\theta}^F} > 0$ implies that $P_1^*(s_b) > P_1^F(s_b)$ if and only if $m^*(s_g) > \bar{m}$.

Proof of Theorem 5

Let \bar{m} be the equilibrium FRM. Consider the following modification of the FRM mortgage: $\ddot{m}(s_g) = \bar{m}$, and $\ddot{m}(s_b) = \bar{m} - (P_1^F(s_g) - P_1^F(s_b))$, where $P_1^F(s_g)$ and $P_1^F(s_b)$ are the home prices in the FRM equilibrium. This modification does not change the lender's payoff in the good state. According to

Theorem 1, if wages in the bad state $w(s_g)$ are high compared to the FRM payments \bar{m} , then this modification will reduce the lender's payoff in the bad state. On the other hand, if $w(s_g)$ is low compared \bar{m} , then this modification will increase the lender's payoff in the bad state.

Consider the special case, in which the FRM modification does not change the lender's payoff in the bad state. In this case, $P_1^F(s_g)$ and $P_1^F(s_b)$ are also the home prices in the HEIM equilibrium, and $\ddot{m}(s_g)$ and $\ddot{m}(s_b)$ is the equilibrium HEIM contract. Indeed, according to Proposition 5, when $\ddot{m}(s_g) = \bar{m}$, home prices at time $t = 1$ are the same in the FRM and HEIM equilibria, and the lender breaks even under the HEIM contract.

In the case when $w(s_g)$ is sufficiently low compared to \bar{m} , the modification leads to a higher expected payoff for the lender. Thus, an equilibrium HEIM contract will charge less than \bar{m} in both states. According to Proposition 5, the HEIM equilibrium will have lower home prices in both states and higher homeownership than those in the equilibrium with FRMs. In this case, the HEIM equilibrium Pareto dominates the FRM equilibrium, since all homeowner are strictly better off with the HEIM contract and there are more of them at $t = 0$ and $t = 1$. In addition, foreclosure costs are lower in the HEIM equilibrium.

In the case when $w(s_g)$ is sufficiently high compared to \bar{m} , the modification leads to a lower expected payoff for the lender. Thus, the equilibrium HEIM contract must charge more than \bar{m} in the good state to break even. As a result, according to Proposition 5, the HEIM equilibrium will have higher home prices in both states and lower homeownership compared to the FRM equilibrium. Consequently, workers with $\theta \in (\underline{\theta}^F, \theta^*]$ are better off in the FRM equilibrium, where they buy homes at time $t = 0$ and enjoy homeownership utility in the first period in the good state. In the HEIM equilibrium, these workers enjoy no homeownership utility in the first period, and have to pay higher prices at time $t = 1$ to enjoy it in the second period.

Appendix B: Examples and Extensions

Appendix B1: A Uniform Example of the FRM Equilibrium

Here we assume that both θ and l are uniformly distributed on $[0, 1]$, i.e., $F(\theta) = \theta$, and $G(l) = l$. With the uniform distribution functions, equation (9) becomes²⁸.

$$(\underline{\theta}^F - P_1^F(s_g))(1 - \frac{P_1^F(s_g)}{w(s_g)}) = (1 - \underline{\theta}^F) \frac{P_1^F(s_g)}{w(s_g)}.$$

It can be further simplified to

$$P_1^F(s_g) - P_1^F(s_g)(1 + w(s_g)) + \underline{\theta}^F w(s_g) = 0.$$

Since $\underline{\theta}^F = \bar{m} - P_1^F(s_g)$, $P_1^F(s_g)$ solves the following quadratic equation

$$P_1^F(s_g) - P_1^F(s_g)(1 + 2w(s_g)) + \bar{m}w(s_g) = 0. \quad (39)$$

Equation (10) becomes

$$(\underline{\theta}^F - P_1^F(s_b))(1 - \frac{P_1^F(s_b)}{w(s_b)}) = (1 - \underline{\theta}^F) \frac{P_1^F(s_b)}{w(s_b)}.$$

Thus, $P_1^F(s_b)$ solves

$$P_1^F(s_b) - P_1^F(s_b)(1 + w(s_b)) + \underline{\theta}^F w(s_b) = 0. \quad (40)$$

Solving the system of the two quadratic equations (39)-(40) yields the following equilibrium home prices:

$$\begin{aligned} P_1^F(s_g) &= \frac{1}{2}[1 + 2w(s_g) - \sqrt{(1 + 2w(s_g))^2 - 4\bar{m}w(s_g)}], \\ P_1^F(s_b) &= \frac{1}{2}[1 + w(s_b) - \sqrt{(1 + w(s_b))^2 - 4(\bar{m} - P_1^F(s_g))w(s_b)}]. \end{aligned}$$

²⁸In the uniform example, one has to check that $P_1(s)$, $\frac{P_1(s)}{w(s)}$, $\underline{\theta} \in [0, 1]$.

Appendix B2: Numerical Examples Illustrating the Equilibrium Existence

To investigate the existence of equilibrium with state-contingent mortgage contracts, we solve the optimal deviation problem (25) numerically. The following table shows parameter values that we keep unchanged in all our examples below.²⁹

Table 1: Parameter Values

Parameter	Value
Price of home at time zero, P_0	0.45
Probability of good state, π_g	0.5
Expected return on capital, \bar{R}	1.2
Idiosyncratic labor productivity pdf, $g(l)$	1, for $l \in [0, 1]$
Wage level in good state, $w(s_g)$	3

We consider two examples of probability density functions of homeownership value:

$$\begin{aligned}
 f_1(\theta) &= 1, \text{ for } \theta \in [0, 1], \\
 f_2(\theta) &= \begin{cases} 1.0421, & \text{for } \theta \in [0, 0.35) \\ 0.2, & \text{for } \theta \in [0.35, 0.4] \\ 1.0421, & \text{for } \theta \in (0.4, 1] \end{cases} .
 \end{aligned}$$

The main difference between these two examples is that the density of households with θ close to θ^* is five times higher in the first example, which corresponds to the uniform example, than in the second example.

We vary the wage level $w(s_b)$ from 1 to 3, and find that the HEIM equilibrium always exists in the uniform example. However, in the second example, the HEIM equilibrium exists only for $w(s_b) \geq 1.5$. These findings are consistent with our earlier observation that the HEIM equilibrium may unravel when the density of strategic defaulters is low and when wages in the bad state are low.

Table 2 below shows parameter values in the HEIM equilibrium and solutions $(\dot{m}(s_g), \dot{m}(s_b))$ to the optimal deviation problem (25) for the two pdf functions when $w(s_b) = 1.2$. It shows that

²⁹Since idiosyncratic labor productivity l is distributed on the interval $[0, 1]$, wage levels $w(s_g)$ and $w(s_b)$ are the highest possible labor incomes in the good and bad states respectively.

the HEIM equilibrium exists in the uniform example, since the solution to the optimal deviation problem (25) is equal to the HEIM payments. However, the optimal deviation mortgage in Example 2 has a higher (lower) payment in the good (bad) state than the corresponding HEIM payment. As a result, Example 2 has no equilibrium with state-contingent mortgage contracts.

Table 2: Numerical Examples

Parameter	Example 1	Example 2
Homeownership value pdf, $f(\theta)$	f_1	f_2
Wage level in bad state, $w(s_b)$	1.2	1.2
HEIM marginal homebuyer, θ^*	0.359	0.361
HEIM payment in good state, $m^*(s_g)$	0.650	0.649
HEIM payment in bad state, $m^*(s_b)$	0.577	0.579
Optimal deviation in good state, $\hat{m}(s_g)$	0.650	0.658
Optimal deviation in bad state, $\hat{m}(s_b)$	0.577	0.568
Equilibrium exists	Yes	No

Appendix B3: Cost of Credit and House Prices in the FRM Equilibrium

In this appendix we prove the following lemma.

Lemma 2 *The homeownership rate $1 - F(\underline{\theta}^F)$ is decreasing and home prices $P_1^F(s_g)$ and $P_1^F(s_b)$ at time $t = 1$ are increasing in the mortgage payment \bar{m} .*

Proof: Equations (9), (10) that determine equilibrium prices $P_1^F(s_g)$, $P_1^F(s_b)$ can be rewritten as

$$F(\bar{m} - P_1^F(s_g)) - F(P_1^F(s_g)) - G\left(\frac{P_1^F(s_g)}{w(s_g)}\right)(1 - F(P_1^F(s_g))) = 0 \quad (41)$$

$$F(\underline{\theta}^F) - F(P_1^F(s_b)) - G\left(\frac{P_1^F(s_b)}{w(s_b)}\right)(1 - F(P_1^F(s_b))) = 0 \quad (42)$$

Applying the implicit function theorem to equation (41) yields

$$\begin{aligned} \frac{dP_1^F(s_g)}{d\bar{m}} &= - \frac{f(\bar{m} - P_1^F(s_g))}{-f(\bar{m} - P_1^F(s_g)) - f(P_1^F(s_g)) + f(P_1^F(s_g))G\left(\frac{P_1^F(s_g)}{w(s_g)}\right) - (1 - F(P_1^F(s_g)))g\left(\frac{P_1^F(s_g)}{w(s_g)}\right)\frac{1}{w(s_g)}} \\ &= \frac{f(\bar{m} - P_1^F(s_g))}{f(\bar{m} - P_1^F(s_g)) + f(P_1^F(s_g))\left(1 - G\left(\frac{P_1^F(s_g)}{w(s_g)}\right)\right) + (1 - F(P_1^F(s_g)))g\left(\frac{P_1^F(s_g)}{w(s_g)}\right)\frac{1}{w(s_g)}} \end{aligned}$$

Thus, $0 < \frac{dP_1^F(s_g)}{d\bar{m}} < 1$. As a result, $P_1^F(s_g)$ is increasing in \bar{m} . In addition, since $\underline{\theta}^F = \bar{m} - P_1^F(s_g)$, we have

$$\frac{d\underline{\theta}^F}{d\bar{m}} = 1 - \frac{dP_1^F(s_g)}{d\bar{m}} > 0,$$

which means that the homeownership rate $1 - F(\underline{\theta}^F)$ is declining in \bar{m} .

Applying the implicit function theorem to equation (42) gives

$$\frac{d\underline{\theta}^F}{d\bar{m}} = \frac{f(\underline{\theta}^F)}{f(P_1^F(s_b))\left(1 - G\left(\frac{P_1^F(s_b)}{w(s_b)}\right)\right) + (1 - F(P_1^F(s_b)))g\left(\frac{P_1^F(s_b)}{w(s_b)}\right)\frac{1}{w(s_b)}} > 0.$$

Moreover,

$$\frac{dP_1^F(s_b)}{d\bar{m}} = \frac{dP_1^F(s_b)}{d\underline{\theta}^F} \cdot \frac{d\underline{\theta}^F}{d\bar{m}} > 0.$$

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