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Rational-expectations whiplash*

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Abstract

We present a financial market with investors who have nested private information. Small perturbations of price informativeness, originating from fat-finger errors or algorithmic glitches of well-informed investors, can trigger an oscillating shock throughout the economy that destabilizes the feedback loop between prices and expectations. Moreover, decreasing the volatility of liquidity trading makes the equilibrium less stable. We investigate what the distribution of informed investors implies for equilibrium stability and for the risk premium of the asset. We find that different investor distributions have different implications, depending on whether adverse-selection or risk-sharing effects dominate in the economy.

Keywords: Rational expectations equilibria, information hierarchy, market stability, risk premium.

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1 Introduction

The recent history of financial markets contains several episodes in which prices move violently in either direction within a few minutes. The press reports some of these episodes by alluding to two unusual trading patterns: fat-finger mistakes of human traders, or programming glitches in algorithmic trading. These phenomena are severe enough to have attracted the attention of several exchanges, who, through the Financial Industry Regulatory Authority (FINRA) and other self-regulatory organizations, have adopted increasingly sophisticated rules aimed at curbing what they call "extraordinary volatility."[1]

Although press reports offer little explanation of extraordinary volatility, they do contain three specific ingredients of a modern financial theory. First, an understanding that there is a usual pattern of economic behavior, that is, equilibrium. Second, a hypothesis that extraordinary volatility is due to out-of-equilibrium shocks coming from erroneous trading. Third, an intuition that the market may not able to recover from such shocks.

We present a model that formalizes the intuition that out-of-equilibrium shocks may indeed destabilize markets. Our financial market consists of different types of investors with nested information sets. The model is thus related to the rational-expectations equilibrium in Grossman and Stiglitz (1980) and several other papers with asymmetric information. Our information structure, however, is more general, and it can generate predictions about market stability that are not possible with simpler structures of asymmetric information. An important feature of our model is that the out-of-equilibrium shocks really come from mistakes

[1] On May 31, 2012, the Securities Exchange Commission (SEC) approved the Limit Up/Limit Down (LULD) plan. The plan allows the NYSE Group, Nasdaq, Bats Global Markets, and other FINRA exchanges to briefly pause trading following price movements outside a certain band, with the intent to protect investors and promote fair and orderly markets (U.S. Securities and Exchange Commission, 2012). The LULD plan differs from previous circuit-breaker rules in several regards. First, the plan imposes limits on bilateral price movements, not just declines. Second, the plan applies on trade of individual securities; in addition, price bands are calculated using the price history of the corresponding security, rather than using an index. Third, the exchanges do not suspend trade unless a security trades at the price-band limits for several seconds. Incidental hits of the price-band limits do not trigger suspension automatically. The plan has received a number of amendments, the latest of which—at the time of writing—took effect on November 20, 2017.

that investors make when they submit their trades—the shocks have a literal interpretation as either fat-finger errors or trading glitches.

Our motivating example is an equities market with three investor types: household investors, mutual funds, and hedge funds. We assume that households can only see public information about financial assets, that mutual funds have information about assets that households do not, and that hedge funds have even more information about assets than mutual funds do.[2]

In what follows, we take a general approach that does not rely on a specific number of investor types that may be present in the real world, or on whether the investors are humans or algorithms. We do, however, assume that the different investor types have strictly nested beliefs. We can therefore think of the model as a market where investors are split into different groups, which we call "information classes." All the groups together form a structure that we call the "information hierarchy." We also assume that all investors are rational utility maximizers given their information.

Because investors trade against each other in the market, their beliefs about fundamentals are interconnected through the price. An important question therefore arises. How does our system of interconnected beliefs respond to an out-of-equilibrium shock to the economy? Does the nested structure of information attenuate the effects of such a shock, or does it amplify it? Our model extends Grossman and Stiglitz (1980), which has a provably stable equilibrium. We might therefore be inclined to think that nested information stabilizes the economy, and that it attenuates external shocks. And yet, shocks can easily propagate throughout the economy, because investors' actions are interconnected through their beliefs. Thus, if we have an amplification mechanism anywhere in the market, then a small shock can potentially destabilize the entire economy.

As we explain below, we have, in fact, two distinct amplification mechanisms. Generally

[2]We give some examples of this information structure in Section 2, by appealing to geographical and temporal microfoundations.

speaking, there is a feedback loop between asset prices and expectations. The information content of prices determines the formation of expectations, and expectations determine aggregate demand and feed into the price through market clearing. The first mechanism, which we discuss next, makes shocks accumulate in investors' feedback loops. The second mechanism, which we discuss later, determines whether the investors' beliefs can withstand the accumulated shocks.

Let us consider a fat-finger error of a well-informed class. We assume that there are no changes whatsoever to any relevant state variables—the dividend information does not change, and the demand of liquidity traders remains the same. This type of error is equivalent to an out-of-equilibrium shock to the coefficients in the demand strategy of this class.

Because demands determine prices through market clearing, a fat-finger error provides a shock to price informativeness for all classes with less information. Moreover, because information is nested, the total effect of this shock for each less-informed class contains the accumulated reactions to that shock by all better-informed classes. Going back to our motivating example above, a hedge-fund trading glitch shocks the information content of prices for mutual funds and for households. Mutual funds respond by adjusting their own demand for the asset, which provides a further shock to the information content of prices for households. Herein, therefore, we have our first amplification channel—a small shock originating in a well-informed class grows as it progresses through the information hierarchy.

As we discuss next, the ramifications of this shock for sufficiently uninformed investors can be so large as to destabilize the feedback loop between prices and expectations. The end result is that the entire equilibrium becomes unstable. Given that our equilibrium is also unique, we interpret this phenomenon as complete market breakdown.

To explain how in more detail, we must first point out that when investors trade they use two different types of information: information they know directly, and information they learn from prices. We call the former type of information "first-hand information," because it is information that investors are endowed with. We call the latter type of information "second-hand information," because it is information that investors disentangle from prices by using their first-hand information.

From the perspective of individual investors, prices contain both first-hand and second-hand information of all better-informed investors. In our motivating example, prices partially reveal three things to households: what hedge funds know directly (first-hand information,) what mutual funds know directly (first-hand information,) and what mutual funds are learning about (second-hand information.) Of course, two of these things contain the same information; what mutual funds are learning about is what hedge funds know directly. Thus, when households use prices to learn about the first-hand information of hedge funds, they must be careful not to double-count the presence of information. One implication of this fact is that in equilibrium households downweigh how much information prices really contain.

A further implication, however, is that the feedback loop between expectations and prices for the less-informed investors exhibits negative feedback. For example, when households observe a downwards shock to the informativeness of prices, they adjust this shock in the opposite direction, to account for that prices contain second-hand information. This negative-feedback property creates oscillations in how the economy responds to out-of-equilibrium shocks in price informativeness.

The question now becomes whether the oscillations created by the shock self-attenuate or self-amplify. Herein we have a potentially second amplification mechanism. While it is intuitive that the oscillations should attenuate, it is not necessarily always so. It is well known that negative feedback can be too strong, and—as anyone who has ever held a microphone too close to a speaker can attest—that negative-feedback systems can be unstable. Whether oscillations attenuate or amplify depends on how much second-hand information is contained in prices. In our motivating example, the more mutual funds rely on prices to learn about what hedge funds know, the more households have to downweigh the true informativeness of prices, and the stronger the negative feedback is. We construct examples in which the economy is indeed unstable, and it cannot restore equilibrium in response to an external demand shock. We call this phenomenon the "rational-expectations whiplash," because the impulse-response function of investors' beliefs looks like a wave of ever-increasing fluctuations.

Our result on stability has an important policy implication. From the perspective of a policy maker, it may seem beneficial to discourage noise trading in financial markets, with the aim of improving information revelation and potentially aiding economic recovery. Our model suggests that such interventions may undermine market stability. Reducing the volatility of noise trading makes prices more informative, but it also forces investors to rely more on price information when they trade. Because shocks to price informativeness are transmitted from better-informed investors to less-informed investors through prices, making prices more informative amplifies the shock-transmission mechanism. Reducing price noise therefore makes markets more susceptible to shocks.

We also examine how the structure of different information hierarchies affects key features of the economy, such as the size of the risk premium and the stability of equilibrium. Using a partial-equilibrium analysis, we conduct two types of comparative statics. First, we increase the proportion of informed agents by changing the distribution of investors in the sense of first-order stochastic dominance (FOSD.) Second, we make the economy more concentrated around the average information class by changing the distribution of investors in the sense of second-order stochastic dominance (SOSD.) We are able to document the following main properties.

First, the risk premium is lower in hierarchies with higher proportions of well-informed investors. This finding reflects the casual intuition that more information makes the asset safer to hold.

Our second finding is perhaps more surprising. Under natural assumptions about investors' precisions, we find that the risk premium is higher in more concentrated hierarchies. This is essentially a converse risk-sharing effect. Concentrating the hierarchy makes the economy more homogeneous in terms of information. This erodes the informational advantage of the well-informed investors, because it reduces the number of investors they can exploit. Thus, concentrating the hierarchy makes the well-informed investors less able to bear risk, which reduces the overall risk-sharing capacity of the economy, and increases the risk premium.

In addition, we can show that increasing or decreasing the proportion of well-informed investors may not have a direct effect on stability. Nonetheless, we also find that concentrating the hierarchy around the average information class makes the economy less stable.

As in our discussion for the risk premium, decreasing the proportion of well-informed investors, either in a FOSD or a SOSD manner, reduces the risk-bearing capacity of an economy. This makes the economy more susceptible to shocks. Nevertheless, decreasing the proportion of well-informed investors also reduces adverse selection. In particular, it strengthens the ability of less-informed investors to absorb shocks because they face fewer adversarial trading partners, and it therefore makes the economy less susceptible to shocks. Exactly how these two effects combine depends on how we change the distribution of investors. For FOSD changes the two effects cancel each other out completely, whereas for SOSD changes the risk-sharing effect dominates.

1.1 Literature review

This paper complements a growing literature that documents various consequences of restricting market operations (Subrahmanyam, 1994; Chen et al., 2017).[3] This literature treats regulatory restrictions due to concerns about price swings as a point of departure. A less-well understood question, however, is where the price swings, the very cause of concern for regulators, actually come from. We propose an answer to this question in the form of market instabilities coming from fat-finger errors and trading glitches.

Our model can be thought of as a generalization of Grossman and Stiglitz (1980). In

[3]A related paper is Hong and Wang (2000), which examines the effect of periodic market closures on returns and trade.

Grossman and Stiglitz (1980) there is one class of informed agents and one class of uninformed agents, whereas our model accommodates an arbitrary number of different information classes with nested information sets. We can therefore think of the information structure in Grossman and Stiglitz (1980) as a special case, with a degenerate information hierarchy described by only two classes: one class with only public information, and another class with public and private information.

It is needless to say that there are numerous models with two classes of asymmetric information. Models with more than two levels of information asymmetry, however, are scarce. In a model in which a seller with market power inefficiently screens a privately informed buyer, Glode and Opp (2016) show that trade efficiency can improve by introducing moderately informed intermediaries. Their most general model features an arbitrary number of intermediaries with nested information sets. Chakraborty and Yilmaz (2008) use a model in which an informed trader bluffs to ambiguate his existence to the market maker. They show that the incentive for bluffing intensifies if the market involves rational traders with nested information. In both of these models, the traders are strategic and risk neutral. We study an economy with competitive, risk-averse investors.

Another related paper is Goldstein and Yang (2015), which discusses the trading and information-acquisition behavior of investors with two-dimensional information about the fundamentals of a security. Goldstein and Yang (2015) show that—in addition to the usual strategic substitutability in between differentially informed investors—there is also strategic complementarity, stemming from that aggressive trading in one dimension of information increases uncertainty about the other dimension of information. Our information structure is nested, rather than two-dimensional. Moreover, we study the stability of financial market equilibrium, rather than complementarities in trading and information acquisition. We show, in particular, that nested information can make the substitutability among investors' beliefs too strong, and that it can render the equilibrium unstable.

The implications of feedback loops on equilibrium stability are extensively studied by

Manzano and Vives (2011). They construct a general noisy rational expectations equilibrium model in which traders learn from up to three sources of information: private signals, the market price, and endowment shocks. As such, their model nests a spectrum of asymmetric and heterogeneous information structures employed by Grossman and Stiglitz (1980), Hellwig (1980), Diamond and Verrecchia (1981), and numerous subsequent extensions, but it does not nest ours. Manzano and Vives (2011) find that strong complementarity under correlated signals is necessary for the multiplicity of equilibria, and that the equilibrium with strong complementarity in information acquisition is unstable. Albagli (2015) constructs an overlapping generations model in which heterogeneously informed investors live for an arbitrary number of periods. He shows that the high volatility equilibrium is unstable, which can be proven for the full-information two-period case of Watanabe (2008). The unstable equilibria of Albagli (2015) and Watanabe (2008) are a strict subset of multiple equilibria, and are always unstable. In contrast, our equilibrium is unique, and whether it is stable or not depends on supply volatility.

All the models above confine attention to linear price functions. It is nevertheless possible to construct models with nonlinear price functions. Gennotte and Leland (1990) demonstrate that hedging strategies can turn the price into a nonlinear function of signals. This nonlinearity can bend the aggregate excess demand curve backwards, and it can create multiple equilibria. Moreover, one of the equilibria can slide down the demand curve upon negative news. The economy can then jump to another equilibrium, causing a discontinuous drop in price that resembles a crash. Likewise, non-normal distributions can have conditional moments that are nonlinear in conditioning signals. These distributions can therefore give rise to nonlinear price functions. Breon-Drish (2015) presents such an example for a binomially distributed payoff. We stay within the framework with normally-distributed state variables, constant absolute risk aversion (CARA) preferences, and continuous linear price functions.

2 The model

Our model for the financial market is a noisy rational expectations equilibrium. It is a standard model other than the structure of the information sets. The economy comprises a continuum of investors of total mass one. The utility function of each investor is

$$u(c) = -e^{-\alpha c}, \tag{1}$$

where α is the coefficient of constant absolute risk aversion. Every investor can invest in a safe bond with constant net interest rate normalized to zero, and in a risky, dividend-paying stock. We assume that the payoff D of the risky asset is

$$D = \sigma_\mu \sum_{i=1}^{N} q_{i-1}\, \mu_i + \varepsilon, \tag{2}$$

where μ_i, $i = 1, \ldots, N$, are independent identically distributed standard Normal random variables, and $\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$, independently of μ_i for all i. In addition, q_{i-1}, $i = 1, \ldots, N$ are constants, and N is a finite natural number. Each random variable μ_i, $i = 1, \ldots, N$, represents an incrementally acquirable piece of information about the dividend. The shock ε, represents residual risk, that is, it is not observable by anyone.

For notational convenience, we let

$$D_{1,i} = \sigma_\mu \sum_{j=1}^{i} q_{j-1}\, \mu_j \tag{3}$$

denote the sum of the first i components of dividend information, and we let

$$D_{i+1,N} = \sigma_\mu \sum_{j=i+1}^{N} q_{j-1}\, \mu_j \tag{4}$$

denote the sum of the last $N-i$ components of dividend information. (For regularity, we define $D_{1,0}=0$ and $D_{N+1,N}=0$.) We can think of the random variables $D_{1,i}$ and $D_{i+1,N}$ as partial payoffs of the asset. The total stock supply is

$$\bar{\theta}+\theta, \tag{5}$$

where $\theta \sim \mathcal{N}(0,\sigma_\theta^2)$, independently of ε and μ_i for all i, and $\bar{\theta}$ is a constant. The price of the asset is P, and it is publicly observable.

For a specific i between one and N, a proportion ϕ_i of investors observe the random variables $\mu_1,\ldots,\mu_{i-1},\mu_i$. We adopt the name *information class* i for the investors who observe this information. By convention, the investors in information class $i=0$ do not observe any dividend information—we have ϕ_0 of them. Moreover, for $j>i$, information class j has strictly more information than information class i. In sum, we have N classes with increasing amounts of private information and one class without any private information, for a total of $N+1$ classes. Our information structure implies that investors in information class i observe the dividend component $D_{1,i}$, and, as we explain below, they must look at the price in order to make inferences about $D_{i+1,N}$.

We must stress that although our information structure is ultimately a modeling assumption, it does arise naturally in different microeconomic settings. Examples involve securities for which information is nested due to geographical and temporal patterns.

We take securities of manufacturing companies as one general example, and the stock of the German car company BMW as a specific instance. German households trading the stock of BMW in the Frankfurt Stock Exchange may look at the stock price to infer information about BMW. Savvy investors, however, may have information about its sales prospects within Germany. In addition, mutual funds may have additional information about the sales prospects of BMW, by forecasting sales within the larger European Union (EU) market. What is more; hedge funds may have even more information than mutual funds, by forecasting sales globally.

We can model this setting by assuming that the information set of hedge funds contains that of the mutual funds', which contains that of the savvy investors', which in turn contains that of the households'. This example corresponds to our setting with $N = 3$, where μ_1 is information about German sales, μ_2 is information about sales in the EU excluding Germany, and μ_3 is information about global sales outside the EU.

Yet another context is that of forecasting the future dividends of a company. Here, we can represent the information environment with a nested structure if we have investors with increasing financial expertise—especially if better expertise enables investors to forecast deeper into the future. For example, households may not be able to forecast dividends, but savvy investors may be able to forecast next quarter's dividends. Mutual funds may able to forecast dividends for the next two quarters, while hedge funds may able to forecast for the next three quarters. In this example, $N = 3$, and μ_1, μ_2, and μ_3 are the dividend forecasts of one, two, and three quarters.

We note that ϕ is a probability mass function (pmf), so that

$$\sum_{i=0}^{N} \phi_i = 1. \tag{6}$$

We let

$$\Phi_i = \sum_{h=0}^{i} \phi_h \tag{7}$$

be the cumulative distribution function (cdf) corresponding to ϕ. In what follows we also use the cumulative sum of the cdf of ϕ, which we define as

$$\Psi_s = \sum_{k=0}^{s} \Phi_k, \tag{8}$$

for any s between zero and N. We adopt the name "super-cumulative" distribution function (scdf) for Ψ.

Definition 1 *For a given probability mass function of informed agents ϕ, an equilibrium is a price function P such that*

(i) the market clears, and

(ii) every agent sets their expectations of the dividend by learning rationally from their private information and the price.

2.1 Derivation of equilibrium

We conjecture that the price of the risky asset is

$$P = p_c + \sigma_\mu \sum_{i=1}^{N} p_\mu\,(i-1)\, q_{i-1}\, \mu_i + p_\theta\, \theta, \tag{9}$$

where p_c and p_θ are constants, and $p_\mu(\cdot)$ is a vector, to be determined in equilibrium. We let $\mathcal{F}_i$ denote the information set of class i, that is, the σ-algebra generated by P and $\mu_1, \ldots, \mu_i$. We define the ratio

$$p_{\mu\theta}\,(i) = \frac{p_\mu\,(i)}{p_\theta}, \tag{10}$$

which we call the price *informativeness* for classes $i = 0, \ldots, N-1$. We set

$$p_{\mu\theta}\,(N) = 0. \tag{11}$$

We also define the $N \times 1$ dimensional vector

$$p_{\mu\theta} = \begin{pmatrix} p_{\mu\theta}\,(0) & p_{\mu\theta}\,(1) & \ldots & p_{\mu\theta}\,(N-1) \end{pmatrix}^T, \tag{12}$$

which we call the *informativeness vector*.

We note that class i does not observe the partial payoff $D_{i+1,N}$ directly. Because, however, class i knows the shocks $\mu_1, \ldots, \mu_i$, the information content of the price for them is

$$K_i = \sigma_\mu \sum_{j=i+1}^{N} p_{\mu\theta}(j-1)\, q_{j-1}\, \mu_j + \theta. \tag{13}$$

The boundary condition (11) is equivalent to imposing that the information content of the price for class N is

$$K_N = \theta. \tag{14}$$

This says that the most-informed class observes the realization of supply. This is a standard feature in economies with asymmetric information.

Given that $D_{i+1,N}$ and K_i are normal for every i, the usual projection theorem gives the following result about the dividend inferences of class i (see Lemma A.1 of the Appendix for details.) The conditional expectation of class i about the unknown part of the dividend is

$$\mathbb{E}\left[D_{i+1,N} \middle| \mathcal{F}_i\right] = \beta_i K_i, \tag{15}$$

where β_i is a constant that depends on $p_{\mu\theta}(j)$ and q_j, $j = i, \ldots, N-1$. The conditional variance of the dividend for class i, $\mathrm{Var}\left(D \middle| \mathcal{F}_i\right)$, is also a constant that depends on $p_{\mu\theta}(j)$ and q_j, $j = i, \ldots, N-1$.

The market clears if and only if aggregate supply equals aggregate demand for every realization of stochastic supply. The market-clearing condition is therefore

$$\bar{\theta} + \theta = \sum_{i=0}^{N} \phi_i X_i, \tag{16}$$

where X_i is the optimal demand of information class i. It is straightforward to show that

$$X_i = \delta_{0i} + \sum_{j=1}^{i} \delta_{Di} \left(j-1\right) \mu_j - \delta_{Pi} P, \tag{17}$$

where the coefficients δ_{0i}, $\delta_{Di}(0), \ldots, \delta_{Di}(i-1)$, and δ_{Pi} are constants specific to class i. We show the demand coefficients in more detail in Lemma A.2 of the Appendix, but the important thing is that they depend on the equilibrium price coefficients, and more specifically, on the informativeness vector $p_{\mu\theta}$.

The expression in (17) gives the investors' demand as a linear function of their observables. The payoff components $\mu_1, \ldots, \mu_i$ are private state variables for investors in class i, because these investors observe them directly. What is more; the price P is publicly observable. Therefore—as is standard rational expectations equilibria—we can think of the demand in (17) as the best response function of investors to private and public signals.

We now derive the equilibrium. We substitute the demand function in (17) into the market-clearing condition (16). We rearrange the result into an equation that expresses P as a noisy aggregation of private signals and dividend expectations. We get

$$P = \left(\sum_{k=0}^{N} \phi_k \delta_{Pk}\right)^{-1} \cdot \left(\sum_{i=0}^{N} \phi_i \delta_{0i} - \bar{\theta} + \sum_{i=0}^{N} \phi_i \delta_{Di} \left(j-1\right) \mu_j - \theta\right). \tag{18}$$

Contained within (18) is a loop in which the vector of price coefficients $\left(p_\mu(0), \ldots, p_\mu(N-1), p_\theta, p_c\right)^T$ feeds back into itself. Investors know that the price has the form of (9), so they set their demand as in (17), wherein the demand coefficients depend on the price coefficients explicitly. Their demands clear the market as in (16). For the market to clear, the price must then be as in (18). This price, however, must also be consistent with the form of (9), which the investors use to set their demand. The fixed point of this feedback loop is our equilibrium.

For notational convenience, we define

$$\tau_i = \left[\mathrm{Var}\left(D \middle| \mathcal{F}_i\right)\right]^{-1} \tag{19}$$

to be the *precision* of class i. We obtain the equilibrium by adapting the usual method of matching coefficients to our model.

Theorem 2 *For a given distribution of informed agents ϕ, the informativeness vector $p_{\mu\theta}$ is the solution to the recursive equations*

$$p_{\mu\theta}(j) = -\frac{1}{\alpha}\sum_{i=j}^{N-1}\phi_{i+1}\tau_{i+1} + p_{\mu\theta}(j)\frac{1}{\alpha}\sum_{i=j}^{N-1}\phi_{i+1}\tau_{i+1}\beta_{i+1} \tag{20a}$$

for $j = N-1, \ldots, 0$, subject to the boundary condition

$$p_{\mu\theta}\left(N\right) = 0, \tag{20b}$$

where for $k > j$, the precision τ_k and the projection coefficient β_k depend on elements of $p_{\mu\theta}$ with indices strictly higher than j. Moreover, given the solution of the informativeness vector $p_{\mu\theta}$ as above, the supply coefficient is

$$p_\theta = \left(\sum_{i=0}^{N}\phi_i\tau_i\beta_i - \alpha\right)\left(\sum_{i=0}^{N}\phi_i\tau_i\right)^{-1}, \tag{20c}$$

the price offset is

$$p_c = -\alpha\bar{\theta}\left(\sum_{i=0}^{N}\phi_i\tau_i\right)^{-1}, \tag{20d}$$

and for $i = 0, \ldots, N$, the ith dividend coefficient is

$$p_\mu\left(i\right) = p_{\mu\theta}\left(i\right)p_\theta. \tag{20e}$$

The only price coefficient that appears in the fixed-point problem (20a) is the informativeness vector, $p_{\mu\theta}$. This is so because the vectors β and π are in terms of sums over the elements of $p_{\mu\theta}$ for a given distribution ϕ, and therefore the expression in (20a) pins down $p_{\mu\theta}$. Moreover, for every $i = 0, \ldots, N$, Equation (20e) gives $p_{\mu}(i)$ in terms of p_{θ}, where p_{θ} is itself in terms of the entire vector $p_{\mu\theta}$ as in (20c), given ϕ. Thus, for a given distribution of informed agents ϕ, Theorem 2 provides a complete characterization of the price coefficients.

What is more, we can solve for the equilibrium in a recursive manner. The right-hand side of Equation (20a) is linear in $p_{\mu\theta}(j)$. In addition, as we prove in Lemma A.1 of the Appendix, the precisions τ_k and the projection coefficients β_k for $k > j$ depend on elements of $p_{\mu\theta}$ with indices strictly higher than j. Thus, (20a) gives a unique analytical solution for $p_{\mu\theta}(j)$ recursively from $j = N-1$ to $j = 0$, expressed as

$$p_{\mu\theta}(j) = \frac{\sum_{i=j}^{N-1} \phi_{i+1}\tau_{i+1}}{\sum_{i=j}^{N-1} \phi_{i+1}\tau_{i+1}\beta_{i+1} - \alpha}, \tag{21}$$

which establishes the following important corollary.

Corollary 3 *For a given distribution of informed agents ϕ, the equilibrium of Theorem 2 is unique.*

2.2 Fat fingers and trading glitches

We now provide a formal representation of fat-finger errors and trading glitches originating from a particular class of investors. We consider class k to be the one committing the fat-finger error, and we show how to model the resulting out-of-equilibrium shock in our framework. In later sections of the paper we show how fat-finger errors propagate throughout the economy.

As we can see in (17), the demand of investors in information class k depends on the price, their direct private information, and a constant. We may also view (17) as an equilibrium demand strategy. Given the economic equilibrium, the investors need only consult what δ_{0k}, $\delta_{Dk}(0), \ldots, \delta_{Dk}(k-1)$, and δ_{Pk} are in order to form their demand.

Suppose now that investors in class k make a mistake when they submit their demand—that is, they do not submit X_k to the market, but rather, some other quantity X'_k. All other classes follow their equilibrium demand strategies correctly. We adopt the view that this error is not due to mistaken observations of state variables, but, rather literally, it is a fat-finger error. The investors observe their private information and the price correctly, but instead of typing the number X_k into their trading software, they type X'_k.[4]

This kind of glitch is equivalent to shocking at least one of the demand coefficients out of equilibrium. Our underlying mathematical representation of equilibrium, however, is the price informativeness vector $p_{\mu\theta}$. All other equilibrium objects, including the price coefficients and the demand coefficients, are functions of it. We must therefore provide a model of what a fat-finger error does to the vector $p_{\mu\theta}$.

We are agnostic about which demand coefficient is shocked by the fat-finger error, because, as we show shortly, it is not possible to shock the demand coefficients independently. We are similarly agnostic about the direction or the magnitude of the shock, because, as we see in later sections, such things do not matter explicitly for stability. The only thing that does matter is whether there is a shock to begin with.

Lemma 4 *An out-of-equilibrium shock to the demand of class k is equivalent to shocking the price informativeness for class $k-1$, leaving all other classes unchanged. This has the representation of perturbing the equilibrium vector $p_{\mu\theta}$ by an N-dimensional vector with one in the kth element and zeros everywhere else.*

As we can see in our Lemma above, fat-finger errors and trading glitches that come from class k have the representation of a unit vector pointing along the kth axis of an N-dimensional

[4]There are other potential sources of this kind of trading error. This error can arise as a programming bug in the investors' trading software. It can also arise in a situation where several investors have a common broker-dealer, who submits aggregated orders for all his clients together, and makes a mistake.

Cartesian coordinate system. We use the notation ι_k for this type of shock.

3 Properties of the equilibrium

In this section we discuss the properties of our fixed-point mapping. We begin with providing intuition for how price informativeness is determined in equilibrium. We then lay out some mathematical tools that can aid our analysis, and we apply these tools on our vector equilibrium. We conclude with comparative statics on the risk premium and on stability.

3.1 First-hand information, second-hand information, and stability of equilibrium as a sequence of scalars

If we look at fixed-point mapping of (20a) as an equation in the price informativeness $p_{\mu\theta}(j)$ of class j, we see two terms on the right-hand side. The first term,

$$-\frac{1}{\alpha}\sum_{i=j}^{N-1}\phi_{i+1}\tau_{i+1}, \tag{22}$$

is a constant that depends only on the precisions of higher information classes. The second term is the coefficient

$$\frac{1}{\alpha}\sum_{i=j}^{N-1}\phi_{i+1}\tau_{i+1}\beta_{i+1} \tag{23}$$

times the equilibrium object, $p_{\mu\theta}(j)$. Each term reflects a different type of information that investors use when they trade.

The first type of information on which investors trade is their own private information. As is standard in many rational-expectations models, the more intensely investors trade on their private information, the more of it they reveal to those who do not know it. We call this type of private information "first-hand" information. For information class j, the effect that first-hand information has on price informativeness is captured by the term in (22), which is nothing other than the aggregate trading intensity of all investors with strictly more information, adjusted by a negative sign to accommodate that $p_{\mu\theta}$ is a negative vector.

The second type of information on which investors trade is the dividend information that they disentangle from prices by utilizing their first-hand information. Prices, however, contain information that the higher classes infer from prices, in a recursive manner up to the highest class. Thus, what a particular investor learns from the price is what others learn about what others are learning. We call this type of private information "second-hand" information. For class j, the effect that second-hand information has on price informativeness is captured by the coefficient in Equation (23). We call this coefficient the "learning-adjusted trading intensity," because it is an aggregate version of trading intensities of all investors with strictly more information, where each trading intensity is adjusted for learning effects by the projection coefficient β.

It is important to note that the same piece of dividend information shows up in prices through two different channels. For example, a piece of information known only by the highest classes will be partially revealed to the lowest classes both through the trading of the highest classes and through the learning-and-trading of the middle classes. Therefore the amount of information that the price provides for any particular piece of dividend information must be discounted. The equilibrium accounts for this effect by adjusting the magnitude of price informativeness downwards. We can see this in Equation (20a) because the coefficient of $p_{\mu\theta}(j)$ on the right-hand side is negative (Equation (51) of Lemma A.1 shows that β is a negative vector.) Moreover, the magnitude of $p_{\mu\theta}(j)$ is adjusted downwards in a manner related to the amount of second-hand information. The higher the learning-adjusted trading intensity, the larger the adjustment for second-hand information.

This fact has important implications for the stability of the equilibrium. Let us consider a graphical interpretation of the fixed-point mapping of Equation (20a). The right-hand side is a linear function in the space of $p_{\mu\theta}(j)$, where the constant in (22) is the intercept, and the learning-adjusted trading intensity in (23) is the slope. The left-hand side of Equation (20a) is a 45° line in the space of $p_{\mu\theta}(j)$. The equilibrium price informativeness for class j is the point where these two lines cross. Of course, the intercept and slope of $p_{\mu\theta}(j)$ depend on the price informativeness of higher classes. Nevertheless, due to the recursive nature of the equilibrium solution, we can think of the vector equilibrium in Theorem 2 as a sequence of scalar fixed-point mappings. Setting aside, for the time being, that the equilibrium is a vector rather than a scalar, we can see that the economy responds to out-of-equilibrium shocks in the following manner.

Any perturbations away from the equilibrium point will produce oscillations around the equilibrium point. The intuition for why comes from iterating the fixed-point mapping of (20a) using an out-of-equilibrium value for $p_{\mu\theta}(j)$ as an initial condition. First, using an out-of-equilibrium value for $p_{\mu\theta}(j)$ is the same as shocking the value of $p_{\mu\theta}(j)$, and is therefore equivalent to a fat-finger shock of class $j+1$. Second, the slope of the mapping is negative due to second-hand-information effects. Thus, the reaction of the economy to a shock to $p_{\mu\theta}(j)$ is to overcorrect, and to move the shock to the opposite side of the equilibrium value.

This happens every time we iterate the fixed-point mapping, resulting in oscillations around the equilibrium value. Whether iterating the mapping attenuates these oscillations, in which case the equilibrium is eventually restored, or whether it amplifies these oscillations, in which case the equilibrium is destroyed, depends solely on the slope of the mapping. It is well-known that the mapping is unstable if its slope is smaller than negative one. We can see these effects clearly by drawing a traditional cobweb diagram for out-of-equilibrium analysis; we therefore turn to a brief graphical illustration.

In Figure 1 we show an example economy with $N = 4$ groups of nested private information. We can see that the slopes of the fixed-point mappings are negative, and that they are steeper for lower classes than for higher classes. This is intuitive, because the slope measures how much second-hand information the price contains for each class, and for lower classes the price conveys information about more unknown dividend components. We can also see that when the volatility of supply is low, the slope of the scalar mappings for the lowest classes can become quite steep. In fact, in our example with $\sigma_\theta = 0.1$ in panel (i), the fixed-point mapping of class zero has a slope of -1.56, and is therefore unstable. The fixed-point mapping of class one has a slope of -0.97, just above the threshold value of negative one, which implies that this mapping is stable.

[Figure 1 here]

In Figure 2 we show a demonstration of the oscillations around the equilibrium using a cobweb diagram. We draw examples of how the economy responds to an out-of-equlibrium shock, for two classes from the first panel of Figure 1: class zero, and class one. In either example we assume that each class responds in isolation from all other classes.[5]

As we can see on the left, the scalar mapping for $p_{\mu\theta}(0)$ is unstable. We move $p_{\mu\theta}(0)$ to the right of its equilibrium value, and the economy starts responding according to the rational-expectations feedback loop. Class zero adjusts the informativeness of prices downwards, which results in a subsequent shock to price informativeness, albeit on the opposite side. Because, however, the amount of second-hand information in prices is very large, class zero adjusts the informativeness of prices downwards so much that its reaction to the original shock is larger than the original shock itself. This happens every time the rational-expectations feedback loop operates, so that the price informativeness moves further and further away from its equilibrium value.

In contrast, as we can see on the right, the scalar mapping for $p_{\mu\theta}(1)$ is stable. The amount of second-hand information for class one is not as large as it is for class zero. Consequently, the reaction of class one to a shock in price informativeness is less pronounced—the reaction to a shock is smaller than the shock itself. Thus, in this case, successive iterations of the

[5]As we outline in the introduction, the shock in a particular class gets transmitted to all other less-informed classes. The scalar analysis we conduct here does not fully incorporate these shocks, but it is useful in terms of intuition for what follows. To account for shock transmission we need multidimensional tools, which we develop in Section 3.2 below.

rational-expectations feedback loop move the price informativeness closer and closer to the equilibrium value.

[Figure 2 here]

We show the solutions of some more examples in Figure 3. We use two different settings for how many information classes we have, $N = 4$ and $N = 2$, and for each N, we use two different distributions of informed investors.[6] The setting of $N = 2$ is the minimum number of information classes for which we have nested information effects; the information structure of Grossman and Stiglitz (1980) in our setting corresponds to $N = 1$. All of the effects that we describe below are feasible with $N = 2$, but we focus on our example with $N = 4$ to showcase how general our model is.

Two patterns stand out in Figure 3. First, the price informativeness for lower classes has a larger magnitude than that for higher classes. In addition, because information is nested, we have fewer investors with higher-class information than investors with lower-class information. Thus, one economic lesson from Figure 3 is that less-widely-known information matters less for prices.

Second, the price informativeness vector $p_{\mu\theta}$ has a lower magnitude for lower supply noise. This appears to be counterintuitive, because the elements of $p_{\mu\theta}$ are signal-to-noise ratios for the individual dividend components. We may thus have expected to see these ratios increase in magnitude as noise decreases. This line of argument, however, ignores an important source of noise for the lower information classes, which, when taken into account, explains the pattern in Figure 3. To convey the intuition, we appeal to our motivating example of hedge funds, mutual funds, and households. Nevertheless, we must clarify that our argument here is on the informativeness of prices as measured by a ratio of coefficients,

[6]For each N we show, we use a pair of distributions such that one is a mean-preserving spread of the other. The details of how we set each distribution are in the corresponding panels of Figure B.1 of the Internet Appendix. We return to the underlying distributions of informed investors in a later section of the paper, where we conduct comparative statics on the shapes of the distributions.

and not about conditional precision.

On the one hand, when supply noise decreases the demand strategy of hedge-fund investors does not change. These investors know every state variable, they are price-takers, and their proportion in the economy is fixed—there is thus no reason for them to modify how they trade collectively as a block. This implies that the informativeness of prices about what the hedge funds know, but the mutual funds do not, remains unaffected.[7]

On the other hand, when supply noise decreases the informativeness of prices about what the mutual funds know—but the households do not—actually increases. This happens because for the households the price contains two pieces of information. The first piece is what the mutual funds know, but the households do not; the second piece is what the mutual funds glean from prices. As supply noise decreases, the second piece of information becomes more accurate. This automatically implies that, in relative terms, the price becomes a more noisy version of the first piece of information.

The converse of this property is that as supply becomes more noisy the price becomes more informative about each incremental piece of dividend information. We therefore have a certain type of complementarity in information, similar to the effects discussed in different settings in Hellwig and Veldkamp (2009), Amador and Weill (2010), García and Strobl (2011), Goldstein and Yang (2015), Avdis (2016), and others.

[Figure 3 here]

It is important to point out that neither the instability of equilibrium nor the dependence of price informativeness on supply volatility is present in the seminal model of Grossman and Stiglitz (1980). In that model we have only one group of informed investors, which maps into our model by setting $N = 1$. Moreover, there is only one price-informativeness equation, for

[7]We can see this in the demand coefficients of Lemma A.2, where for the highest information class we obtain $\delta_{DN} = \delta_{PN} = \left(\alpha\sigma_\varepsilon^2\right)^{-1}$.

the investors with public information. From (21) and Lemma A.1 we obtain that

$$p_{\mu\theta}^{GS}(0) = -\frac{\phi_1}{\alpha\sigma_\varepsilon^2}. \tag{24}$$

This expression brings forth two points.[8] First, the equilibrium of Grossman and Stiglitz (1980) is always stable; there is no second-hand-information term on the right-hand side of (24), only a first-hand-information term. Second, to have $p_{\mu\theta}$ depend explicitly on supply noise we need more than one class of private information. We can think of the last two classes in isolation, classes $N-1$ and N, as a Grossman and Stiglitz (1980)-type economy embedded within a larger economy with $N \geq 2$. As we can see in Figure 3, the price informativeness does not react when we change supply noise for the last two classes in each graph. In this light, Figure 3 confirms that supply noise does not matter for $p_{\mu\theta}$ unless $N \geq 2$.

The larger economic takeaway here is that the equilibrium becomes more unstable as supply becomes less noisy. This fact is an important property of our model, and it has an interesting policy implication. From the perspective of a policy maker, it might, on occasion, seem beneficial to discourage noise trading in financial markets. The rationale would be that such a policy might promote information revelation, which might in turn help promote economic growth. Our model suggests that such interventions, however well-intended they might appear, would instead make markets more unstable.

Another important note is that the shock to price informativeness for class i does not necessarily have to come from a fat-finger error of class $i+1$. Because the price informativeness for a class depends recursively on the price informativeness for all higher classes, a shock to $p_{\mu\theta}(j)$, for $j > i$, will propagate down to $p_{\mu\theta}(i)$. Recalling that every element of the price informativeness vector reacts by oscillating around its equilibrium, we can see that a shock which starts in any class will cause a wave-like pattern of reactions in all lower

[8]We use the superscript "GS" in (24) to clarify that this computation is for class zero as it would apply in Grossman and Stiglitz (1980), but not in our setting for general N.

classes.

This pattern of reaction is the rational-expectations whiplash.[9] We must admit, however, that our analysis so far relies on scalars, and that we carry it out as if each mapping gets shocked independently of each other. To fully analyze the stability of our equilibrium we must be able to capture what happens in between classes. We must therefore turn to a vector-based analysis for stability, which is the subject of our next section.

3.2 A multidimensional analysis of stability

Our analysis would be standard if price informativeness was a scalar quantity. For example, it is well-known that a scalar fixed point x_0 of a scalar-valued mapping $f(x)$ is stable whenever

$$\left|f'(x_0)\right| < 1. \tag{25}$$

In our case, however, the equilibrium is a vector. We write our fixed-point mapping in vector notation as

$$p_{\mu\theta} = F\left(p_{\mu\theta}\right), \tag{26}$$

where the jth element of F is given by the right-hand side of (20a). We now discuss how the stability condition of (25) generalizes to multiple dimensions.

First, we note that the vector-space equivalent of the derivative is the Jacobian matrix. The Jacobian matrix is defined as the derivative of the mapping in (26) with respect to the vector $p_{\mu\theta}$, that is, a matrix where the $(i, j)th$ element is

$$\frac{\partial}{\partial p_{\mu\theta}\left(j\right)} F\left(\ldots, p_{\mu\theta}\left(i\right), \ldots\right). \tag{27}$$

[9]There is an analogous phenomenon in industrial organization, and more specifically in supply-chain management, called the "bullwhip effect." That phenomenon describes how an impulse in customer orders causes order fluctuations to build up in the supply chains of vertically integrated industries. See Forrester (1961) for seminal work and Forrester (1968) for an early survey of that literature.

We write this matrix in compact notation as

$$\mathcal{D}_{p_{\mu\theta}} F. \tag{28}$$

Second, the condition for equilibrium stability is that all eigenvalues of the Jacobian matrix, evaluated at the equilibrium vector $p_{\mu\theta}$, have magnitude less than one. In sum, the condition for stability of the equilibrium vector $p_{\mu\theta}$ is

$$\max\left\{|\chi| : \chi \text{ is an eigenvalue of } \mathcal{D}_{p_{\mu\theta}} F\right\} < 1 \tag{29}$$

Of course, each eigenvalue of the Jacobian matrix is associated with an eigenvector, which we denote as ν. At the equilibrium vector $p_{\mu\theta}$ we have

$$\left(\mathcal{D}_{p_{\mu\theta}} F\right) \nu = \chi \nu \tag{30}$$

for conformable eigenvectors ν. Our next theorem gives analytical expressions for the eigenvalue-eigenvector pairs of the fixed-point mapping.

Theorem 5 *For each $k = 0, \ldots, N-1$, the kth eigenvalue is*

$$\chi_k = \frac{1}{\alpha} \sum_{i=k}^{N-1} \phi_{i+1} \tau_{i+1} \beta_{i+1} \tag{31a}$$

with associated eigenvector

$$\nu_k = \begin{bmatrix} \xi_k \\ 1 \\ \mathbf{0}_{N-k-1} \end{bmatrix} \tag{31b}$$

of size N, where ξ_k is an auxiliary vector of size k which depends on the top left $(k+1)\times(k+1)$

entries of $\mathcal{D}_{p_{\mu\theta}}F$. Moreover, the eigenvectors $\nu_0, \ldots, \nu_{N-1}$ are linearly independent.[10]

The eigenvalue χ_k in Theorem 5 coincides with the slope of the scalar fixed point-mapping for class k in Theorem 2. This is intuitive, because the Jacobian matrix is the multidimensional slope of the vector fixed-point mapping. In addition, the eigenvector ν_k represents a direction in N-dimensional space along which we can push the equilibrium vector $p_{\mu\theta}$. If the associated eigenvalue χ_k has magnitude less than one, then the economy will return to equilibrium; otherwise it will not. In this sense, Theorem 5 says that the intuition we extract from the scalar analysis of section 3.1 extends to the multidimensional case. Of course, we must still address exactly how each information class may transmit shocks to others, and we do so below. Our point here is that the multidimensional condition for stability is identical to a "collection" of stability conditions we can derive from thinking in terms of a sequence of scalars.

According to (31a), the kth eigenvalue is the learning-adjusted trading intensity of class k. Since $\phi_{i+1}\tau_{i+1} > 0$ and $\beta_{i+1} \leq 0$, the magnitude of the kth eigenvalue decreases in k (χ_k is negative, and it decreases in k.) This is important, because if the magnitude of any eigenvalue exceeds one, our stability condition in (29) is violated, and the equilibrium is unstable. It is straightforward to see that the eigenvalue with the largest magnitude is that of class zero. Thus, a quick way to check whether the equilibrium is unstable is to check whether the eigenvalue for class zero falls below negative one. We therefore single out that particular eigenvalue,

$$\chi_0 = \frac{1}{\alpha}\sum_{i=0}^{N-1} \phi_{i+1}\tau_{i+1}\beta_{i+1}, \tag{32}$$

and we refer to it as the "public eigenvalue," because class zero consists of investors who have only public information in their hands.

As we discuss above, reducing the supply noise makes the equilibrium less stable because

[10]We note that the matrix $\mathcal{D}_{p_{\mu\theta}}F$ is upper triangular, so its eigenvalues are its diagonal elements. $\mathbf{0}_{N-k}$ is a column vector of $N-k$ zeros, with the convention that it is empty if $k = N$. The vector ξ_{k-1} is empty for $k = 1$. The eigenvalue χ_{N-1} is zero.

it makes the price more informative, and this in turn amplifies the second-hand information effect that is responsible for oscillations. Figure 4, in which we plot the eigenvalues, confirms this intuition. As we can see in panels (A) through (D), each eigenvalue becomes more negative as supply noise decreases. In each example we draw, the public eigenvalue is below the stability threshold for low enough noise. (It is also entirely possible for several classes to be unstable at the same time.) We can also see that how close each eigenvalue comes to the stability threshold depends on the distribution of informed investors. We return to this property in a later section, after we discuss how shocks propagate.

[Figure 4 here]

3.3 The propagation of fat-finger errors and the rational-expectations whiplash

We now discuss what happens to the economy when a particular class of investors commits a fat-finger error using the tools that we developed in Theorem 5. Suppose we perturb the economy away from the equilibrium vector $p_{\mu\theta}$ by an arbitrary vector ε of small magnitude. How will the economy respond? A first-order Taylor approximation of the resulting fixed-point mapping gives that

$$F\left(p_{\mu\theta}+\varepsilon\right)\approx F\left(p_{\mu\theta}\right)+\left(\mathcal{D}_{p_{\mu\theta}}F\right)\varepsilon=p_{\mu\theta}+\left(\mathcal{D}_{p_{\mu\theta}}F\right)\varepsilon, \tag{33}$$

where the last equality is due to that $F\left(p_{\mu\theta}\right)=p_{\mu\theta}$. Applying the fixed-point mapping and the Taylor approximation once more gives

$$F^{2}\left(p_{\mu\theta}+\varepsilon\right)=F\left(F\left(p_{\mu\theta}+\varepsilon\right)\right)\approx F\left(p_{\mu\theta}+\left(\mathcal{D}_{p_{\mu\theta}}F\right)\varepsilon\right)\approx p_{\mu\theta}+\left(\mathcal{D}_{p_{\mu\theta}}F\right)^{2}\varepsilon. \tag{34}$$

We may further iterate this argument an arbitrary number of times. The distance of the price-informativeness vector from its equilibrium after n iterations is

$$F^n\left(p_{\mu\theta}+\varepsilon\right)-p_{\mu\theta}\approx\left(\mathcal{D}_{p_{\mu\theta}}F\right)^n\varepsilon. \tag{35}$$

As we can see in (35), whether the economy comes closer to equilibrium or whether it moves farther away depends on the nth power of the Jacobian and the perturbation vector ε.

We are interested in what happens when a particular class commits a fat-finger error. According to Lemma 4, we can model a fat-finger error of class k as the perturbation vector $\varepsilon=\iota_k$. At this conjuncture we are able to use the linear-independence property of the eigenvectors to our advantage—the eigenvectors form a vector basis of $\mathbb{R}^N$. This implies that we can express the fat-finger shock ι_k as

$$\iota_k=\sum_{i=0}^{N-1}c_{i,k}\nu_i, \tag{36}$$

where the coefficients $c_{0,k},\ldots,c_{N-1,k}$ are the coordinates of ι_k in the eigenvector basis. We can now combine (35) and (36) with the definition of the eigenvalues in (30) to obtain[11]

$$F^n\left(p_{\mu\theta}+\iota_k\right)\approx p_{\mu\theta}+(-1)^n\sum_{i=0}^{k-1}c_{i,k}|\chi_i|^n\nu_i. \tag{37}$$

There are two economic lessons we can derive from (37). First, a shock that originates in a particular information class propagates to all other less-informed classes. In particular, as we can see from the right-hand side of (37), a shock to price informativeness for the kth class appears as a shock to price informativeness for the first $k-1$ classes within one iteration. By the same token, each of those shocks gets transmitted to all lower classes within one iteration, and so on. This keeps happening until all the shocks generated in this manner

[11] We note that the right-hand side of (37) does not depend on the last $N-k$ eigenvectors and eigenvalues, because the last $N-k$ coordinates of ι_k are zero. See Lemma A.3 of the Appendix for the proof.

either potentially die down, and the economy potentially converges to equilibrium, or until the economy diverges from equilibrium, and the market fails.

Second, the relation in (37) also shows that whether the economy converges or diverges depends on the largest eigenvalue associated with a non-zero coordinate in the eigenvector basis. It therefore suffices to check if the magnitude of the public eigenvalue is larger than one, and if it does, whether the coordinate $c_{0,k}$ is non-zero.

We can readily perform this check, because—as we prove in Lemma A.3 of the Appendix—we can obtain the coordinates of ι_k in the eigenvector basis by inverting a matrix composed of all the eigenvectors. As we know from Figure 4, the magnitude of the public eigenvalue is larger than one when supply noise is low enough. Furthermore, as we can see in Table 1, the coordinates of fat-finger errors associated with the public eigenvalue have non-zero values, no matter in which class they originate.[12]

[Table 1 here]

We now come to a demonstration of the rational expectations whiplash as a response to a fat-finger error. In Figure 5 we draw an example with four groups of nested private information, and one group with public information. This economy also appears in Figures 1, 2, and panel (A) of Figures 3 and 4; we note that class zero is unstable, and all other classes are stable.

We show how the economy responds to a one-time fat-finger error of class three in several frames. The leftmost frame of the first row shows the economy in equilibrium. The second frame of the first row shows the fat-finger error. (From Lemma 4 we know that a fat-finger error of class three shows up as an impulse to the price informativeness of class two.) The subsequent frames show successive iterations of the fixed-point mapping. We can see that

[12] In numerous equilibrium solutions we have always found that $c_{0,k} \neq 0$ for at least one k. Of course, it is possible to tweak the parameter values we use to make one particular $c_{0,k}$ equal zero (except for $k = 1$, as we discuss shortly) but that would be a knife-edge case. We have not been able to discover any constellations of parameter values that make all $c_{0,k}$ equal zero. Moreover, a result that is contained in the proof of Lemma A.3 is that $c_{0,1} = 1$ always.

the impact of the shock on class two dies down quickly. Moreover, the initial reactions of class zero and class one are quite small, almost imperceptible. In fact, the impact of the shock on class one also dies down quickly.

What the fat-finger error does to class zero, however, is a very different story. Even though the impact is initially small, the shock makes the price informativeness of class zero oscillate and grow with every iteration. These oscillations keep growing as long as the feedback loop between asset prices and expectations is operating. In addition, because the price coefficients are functions of price informativeness, fat-finger errors naturally translate to unstable price swings. Consequently, prices oscillate despite the complete lack of shocks to fundamentals or to liquidity trading.

One effective way to stop these oscillations is for the exchange authorities to halt trading momentarily, according to the provisions of the LULD plan. This brief pause can presumably give market participants the opportunity to recognize that the price swings are due to trading errors of no real economic significance, so that when trading restarts the economy resumes from an orderly equilibrium.

[Figure 5 here]

3.4 Comparative statics with respect to the information hierarchy

As we demonstrate above, certain distributions of investors can produce unstable equilibria when supply volatility is small enough. Can we avoid instability if we change the structure of the information hierarchy? For example, is stability better in an economy with relatively more highly-informed investors? Moreover, are economies where investors are spread out in the two extreme classes, 0 and N, more stable than economies where investors are concentrated around the middle information classes?

Furthermore, as we can see in Equation (20d), the risk premium of the asset depends explicitly on the aggregate precision of all investors in the economy. The precision of each investor depends on the price informativeness, which in turn depends on the distribution of investors in different information classes. It therefore stands to reason that different information hierarchies would be associated with different risk premiums. What effect does the shape of the distribution of investors have on the risk premium? Can we somehow "rank" different information hierarchies with respect to the size of their corresponding risk premium?

To answer these questions, we must first develop a good working definition of what we mean by "changing" the distribution of investors. We once again employ tools from vector calculus, and we derive two comparative statics. In the first comparative static, we shift the mass of the investors more towards the higher information classes, so that we can investigate what happens when the hierarchy contains more information overall. We ask what happens to a particular quantity of interest Q when we change the cdf of informed investors, Φ, in a direction υ in the space of all cumulative distribution functions (cdfs.) We thus calculate the quantity

$$(\mathcal{D}_\Phi Q)\,\upsilon \tag{38}$$

We note that a negative direction υ moves the distribution of informed agents towards first-order stochastically more dominant distributions—the effect is that Φ_i, the cumulative number of informed agents up to class i, decreases for every i. Of course, this is equivalent to increasing the number of informed agents above class i. In other words, a negative direction υ corresponds to increasing the amount of well-informed agents in the hierarchy.

In the second comparative static, we change the degree of homogeneity of information in the hierarchy. To simplify the exposition, we define υ^* to be a direction of differentiation with respect to the scdf Ψ, but one that leaves the average information class $\sum_{i=0}^{N} i\phi_i$ unchanged, that is,

$$\left(\mathcal{D}_\Psi \sum_{i=0}^{N} i\phi_i\right)\upsilon^* = 0. \tag{39}$$

We refer to such a υ^* as a "mean-preserving direction,"[13] and we calculate the quantity

$$(\mathcal{D}_\Psi Q)\,\upsilon^*. \tag{40}$$

Here we note that a negative direction υ^* moves the distribution of informed agents towards second-order stochastically more dominant distributions, because taking the derivative along $\upsilon^* < 0$ decreases Ψ. In addition, because υ^* does not affect the average information class, moving the distribution of informed investors in a negative direction υ^* makes the distribution more concentrated around the average information class.

We must note that our analysis below is admittedly partial, because we fix the informativeness of prices when we change the distribution of informed investors. We do so because it is not possible to obtain tractable closed-form results if we allow the vector $p_{\mu\theta}$ to incorporate changes in the vector ϕ. Nevertheless, we opt for cleaner results, with the understanding that our analysis below is meant to highlight the effect of certain economic forces, rather than to provide conclusive results. The effects we discuss below would, in any case, still play a major role in a more general analysis.

3.4.1 Hierarchical information and stability

Here we carry out comparative statics on the public eigenvalue, and we interpret changes in the information hierarchy that increase the public eigenvalue as making the economy more stable. We similarly interpret changes in the information hierarchy that decrease the public eigenvalue as making the economy less stable. To simplify our analysis, we assume that the direction of change is a vector with identical elements.

[13]Our terminology here echoes the "mean-preserving spread" of Rothschilds and Stiglitz (1970). We note, however, that condition (39) describes a direction of change in the space of distribution functions that leaves the mean unchanged, whereas the mean-preserving spread of Rothschilds and Stiglitz (1970) compares one distribution to another directly. In addition, condition (39) does not specify whether υ^* is a direction that moves distributions towards those that are more spread out or less spread out; it only says that the mean does not change.

Theorem 6 *For a given equilibrium informativeness vector $p_{\mu\theta}$,*

(i) the derivative of the public eigenvalue with respect to the cdf Φ in any direction υ with identical elements is zero.

(ii) The derivative of the public eigenvalue with respect to the scdf Ψ in a negative mean-preserving direction υ^* with identical elements is negative.

Let us consider a direction υ in the space of cdfs Φ. We recall that a negative direction moves the distribution of investors towards distributions with higher proportions of well-informed investors, and that a positive direction has the opposite effect. Let us pick a negative direction. As we can see in part (i) of Theorem 6, moving the distribution of informed investors along υ has no direct effect on the stability of the economy. This may seem puzzling, because increasing the proportion of well-informed investors definitely widens the information asymmetry in the economy. We have, however, two different economic factors associated with information asymmetry.

On the one hand, we have adverse selection—well-informed investors exploit poorly-informed investors. Widening the information asymmetry makes the economy less capable to withstand a shock, because the poorly-informed investors have to absorb part of the shock while simultaneously bearing the increased costs of adverse selection.

On the other hand, we also have risk sharing—investors with different asset information trade with each other to insure against uncertain payoffs. Here, widening the information asymmetry improves the amount of risk sharing in the economy. Having more investors with better information than others allows the better-informed investors to absorb more of the risk of the asset, precisely because these investors can handle risk better. Better risk sharing makes the asset safer overall, which increases the capacity of the economy to absorb shocks.

Bearing in mind that the differentiation along υ is partial—and that it therefore does not include indirect effects on the price coefficients—part (i) of Theorem 6 shows that adverse selection and risk sharing cancel each other out perfectly. In contrast, there is a clear winner in part (ii).

Let us consider a negative direction υ, which, as we discuss above, makes the economy more homogeneous in terms of information. Changing the hierarchy in this manner improves adverse selection, but it diminishes risk sharing. In this case, the risk-sharing properties of the economy deteriorate enough to overwhelm the improvement in risk sharing. The overall result is that the economy becomes less stable.

3.4.2 Hierarchical information and the risk premium

Equation (20d) implies that the risk premium of the asset is

$$\mathbb{E}[D-P] = -p_c = \alpha\bar{\theta}\left(\sum_{i=0}^{N}\phi_i\tau_i\right)^{-1}. \tag{42}$$

We use this expression to explore what happens to the risk premium if we change the information hierarchy, in a manner similar to the comparative statics above. Here we do not restrict the direction of change to have identical elements.

Theorem 7 *For a given equilibrium informativeness vector $p_{\mu\theta}$,*

(i) the derivative of the risk premium with respect to the cdf Φ in a negative direction υ is negative.

(ii) Suppose that the conditional precision τ_i is a convex function of the class i.[14] *Then, the derivative of the equity premium with respect to the scdf Ψ in a negative mean-preserving direction υ^* is positive.*

[14]In numerous computational solutions we have always found the conditional precision to be a convex function of the information class. See Figure B.3 of the Internet Appendix for examples with the economies that we have considered in Figures 1, 3, and 4 of the main text. It is, however, extremely algebraically elaborate to provide conditions under which τ_i is convex in i.

Part (i) of Theorem 7 has a straightforward interpretation. As we point out above, a negative υ is equivalent to shifting the distribution of informed investors towards first-order stochastically more dominant distributions. In other words, part (i) of Theorem 7 says that increasing the proportion of well-informed investors decreases the risk premium. This effect is consistent with the casual intuition that more information, in this case indicated by increased proportions of well-informed investors, makes the asset safer to hold.

To interpret part (ii) of Theorem 7, let us take the derivative of the risk premium with respect to Ψ in a negative mean-preserving direction υ^*. Because υ^* is negative, taking the derivative along υ^* decreases Ψ, which moves the distribution of informed agents towards second-order stochastically more dominant distributions. This has the effect of concentrating the distribution of informed agents, without affecting the mean. According to Theorem 7, concentrating the information hierarchy in this manner increases the risk premium.

It may appear a little paradoxical that a more concentrated information hierarchy is associated with a higher risk premium. After all, a more concentrated hierarchy is more homogeneous in terms of information, which implies that there is less adverse selection. As we discuss above, however, concentrating the hierarchy also affects the risk-sharing properties of the economy, albeit negatively. In this case the risk-sharing effect dominates the adverse-selection effect, so that a more homogeneous economy has a higher risk premium.

4 Conclusion

We construct an equilibrium where investors have increasing amounts of private information. Perturbing how much a particular information class contributes to overall price informativeness can trigger a whiplash that renders the rational-expectations mechanism unstable. Such perturbations may arise as out-of-equilibrium shocks to demand strategies of well-informed investors, and they have a natural interpretation as fat-finger errors or algorithmic-trading glitches. Our model suggests that reducing price noise can have a destabilizing effect on the economy, even though it makes prices more informative. An additional important implication of our model is that how investors are distributed in different information classes matters for the risk premium and for stability.

Our general message is that financial markets can be inherently fragile. This paper suggests that nested information is one possible avenue for subsequent work on market instability, so that, at some point in the future, we are able to go beyond what the financial press reports as "blaming the Flash Crash on a UK man who lives with his parents is like blaming lightning for starting a fire" (Traders Magazine, 2015).

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A Appendix

Lemma A.1 *For class* i*,* $0 \le i < N$*,*

$$\mathbb{E}\left[D_{i+1,N}\middle|\mathcal{F}_i\right] = \beta_i \left(\sigma_\mu \sum_{j=i+1}^{N} p_{\mu\theta}\left(j-1\right) q_{j-1}\, \mu_j + \theta\right), \tag{44}$$

where

$$\beta_i = \frac{\sum_{j=i+1}^{N} p_{\mu\theta}\left(j-1\right) q_{j-1}^2}{\sum_{j=i+1}^{N} p_{\mu\theta}^2\left(j-1\right) q_{j-1}^2 + \frac{\sigma_\theta^2}{\sigma_\mu^2}}, \tag{45}$$

and

$$\tau_i^{-1} = Var\left(D\middle|\mathcal{F}_i\right) = \sigma_\varepsilon^2 + \sigma_\mu^2 \left[\sum_{j=i+1}^{N} q_{j-1}^2 - \beta_i \sum_{j=i+1}^{N} p_{\mu\theta}\left(j-1\right) q_{j-1}^2\right]. \tag{46}$$

For class N*,*

$$\mathbb{E}\left[D_{N+1,N}\middle|\mathcal{F}_{s_N}\right] = 0, \tag{47}$$

$$\beta_N = 0, \tag{48}$$

and

$$\tau_N^{-1} = Var\left(D\middle|\mathcal{F}_N\right) = \sigma_\varepsilon^2. \tag{49}$$

Proof. For class i, $0 \le i < N$, by normality and the projection theorem we get

$$\mathbb{E}\left[D_{i+1,N}\middle|\mathcal{F}_i\right] = \beta_i K_i = \beta_i \left(\sigma_\mu \sum_{j=i+1}^{N} p_{\mu\theta}\left(j-1\right) q_{j-1} \mu_j + \theta\right), \tag{50}$$

where

$$\beta_i = \frac{Cov\left(D_{i+1,N}, K_i\right)}{\mathrm{Var}\left(K_i\right)} = \frac{\sum_{j=i+1}^{N} p_{\mu\theta}\left(j-1\right) q_{j-1}^2}{\sum_{j=i+1}^{N} p_{\mu\theta}^2\left(j-1\right) q_{j-1}^2 + \frac{\sigma_\theta^2}{\sigma_\mu^2}}, \tag{51}$$

Moreover, normality also implies

$$\operatorname{Var}\left(D\middle|\mathcal{F}_i\right) = \operatorname{Var}\left(D_{i+1,N}+\varepsilon\right) - \frac{\operatorname{Cov}^2\left(D_{i+1,N}+\varepsilon, K_i\right)}{\operatorname{Var}\left(K_i\right)}$$

$$= \sigma_\varepsilon^2 + \sigma_\mu^2\left[\sum_{j=i+1}^{N} q_{j-1}^2 - \beta_i \sum_{j=i+1}^{N} p_{\mu\theta}\left(j-1\right) q_{j-1}^2\right]. \quad (52)$$

We note that from (52), $\operatorname{Var}\left(D\middle|\mathcal{F}_i\right)$ depends only on $p_{\mu\theta}\left(j-1\right)$ and q_{j-1}, $j=1,\ldots,N$ and the parameters σ_ε^2, σ_μ^2 and σ_θ^2, because β_i depends only on $p_{\mu\theta}\left(j-1\right)$ and q_{j-1}, $j=1,\ldots,N$ and the parameters σ_μ^2 and σ_θ^2. Conditions (47), (48), and (49) are immediate for class N. ■

Lemma A.2 *The demand coefficients of information class i are*

$$\delta_{0i} = -\frac{p_c}{p_\theta}\frac{\tau_i}{\alpha}\beta_i \quad (53a)$$

$$\delta_{Di}\left(j-1\right) = \frac{\tau_i}{\alpha}\left[1-\beta_i p_{\mu\theta}\left(j-1\right)\right]\sigma_\mu q_{j-1} \quad (53b)$$

$$\delta_{Pi} = \frac{\tau_i}{\alpha}\left(1-\frac{1}{p_\theta}\beta_i\right) \quad (53c)$$

Proof.

It is straightforward to show that

$$X_i = \frac{\mathbb{E}\left[D-P\middle|\mathcal{F}_i\right]}{\alpha\operatorname{Var}\left(D\middle|\mathcal{F}_i\right)} = \frac{\tau_i}{\alpha}\left[-\frac{p_c}{p_\theta}\beta_i + \sigma_\mu\sum_{j=1}^{i}\left[1-\beta_i p_{\mu\theta}\left(j-1\right)\right] q_{j-1}\mu_j + \left(\frac{1}{p_\theta}\beta_i - 1\right)P\right], \quad (54)$$

which proves that the coefficients in the representation

$$X_i = \delta_{0i} + \sum_{j=1}^{i}\delta_{Di}\left(j-1\right)\mu_j - \delta_{Pi}P \quad (55)$$

are as in (53). ■

Proof of Theorem 2. Substituting (17), with the demand coefficients as in Lemma A.2 above, into the market clearing condition (16) and rearranging we get

$$P = \left[\sum_{i=0}^{N} \phi_i \tau_i \left(1 - \frac{\beta_i}{p_\theta}\right)\right]^{-1} \cdot \left\{-\alpha\theta - \frac{p_c}{p_\theta} \sum_{i=0}^{N} \phi_i \tau_i \beta_i - \alpha\bar{\theta} + \sigma_\mu \sum_{i=0}^{N} \phi_i \tau_i \sum_{j=1}^{i} \left[1 - \beta_i p_{\mu\theta}\left(j-1\right)\right] q_{j-1} \mu_j \right\}. \tag{56}$$

Matching the coefficient of supply on the right-hand side of (56) with the coefficient of supply in the price conjecture (9) gives

$$p_\theta = \left[\sum_{i=0}^{N} \phi_i \tau_i \left(\frac{\beta_i}{p_\theta} - 1\right)\right]^{-1} \alpha, \tag{57}$$

whereas matching constants gives

$$p_c = \left[\sum_{i=0}^{N} \phi_i \tau_i \left(\frac{\beta_i}{p_\theta} - 1\right)\right]^{-1} \left(\alpha\bar{\theta} + \frac{p_c}{p_\theta} \sum_{i=0}^{N} \phi_i \tau_i \beta_i\right). \tag{58}$$

Solving (57) for p_θ gives (20c), and solving (58) for p_c gives (20d). Condition (20b) obtains immediately by the regularity condition (11). What remains is to establish (20a).

After matching coefficients on supply and the constants, the remaining terms to match on the two sides of (56) are

$$\sigma_\mu \sum_{j=1}^{N} p_\mu \left(j-1\right) q_{j-1}\, \mu_j = \left[\sum_{i=0}^{N} \phi_i \tau_i \left(1 - \frac{\beta_i}{p_\theta}\right)\right]^{-1} \cdot \left\{\sigma_\mu \sum_{i=0}^{N} \phi_i \tau_i \sum_{j=1}^{i} \left[1 - \beta_i p_{\mu\theta}\left(j-1\right)\right] q_{j-1} \mu_j \right\}. \tag{59}$$

Diving through by $\sigma_\mu p_\theta$ we get, after using (57) on the right-hand side to express p_θ, that

$$\sum_{j=1}^{N} p_{\mu\theta} \left(j-1\right) q_{j-1}\, \mu_j =$$

$$-\frac{1}{\alpha}\left\{\sum_{i=0}^{N}\phi_i\tau_i\sum_{j=1}^{i}\left[1-\beta_i p_{\mu\theta}\left(j-1\right)\right]q_{j-1}\mu_j\right\} = -\frac{1}{\alpha}\left\{\sum_{i=1}^{N}\phi_i\tau_i\sum_{j=1}^{i}\left[1-\beta_i p_{\mu\theta}\left(j-1\right)\right]q_{j-1}\mu_j\right\}$$

$$= -\frac{1}{\alpha}\sum_{i=1}^{N}\phi_i\tau_i\sum_{j=1}^{i}q_{j-1}\mu_j + \frac{1}{\alpha}\sum_{i=1}^{N}\phi_i\tau_i\sum_{j=1}^{i}\beta_i p_{\mu\theta}\left(j-1\right)q_{j-1}\mu_j, \quad (60)$$

where the second equality follows because the $i=0$ term in the outer summation represents an empty sum in the inner summation (μ_0 is empty.) Exchanging the order of summation in the double summations on the right-hand side of (60) we obtain

$$\sum_{j=1}^{N}p_{\mu\theta}\left(j-1\right)q_{j-1}\,\mu_j = -\frac{1}{\alpha}\sum_{j=1}^{N}q_{j-1}\left(\sum_{i=j}^{N}\phi_i\tau_i\right)\mu_j + \frac{1}{\alpha}\sum_{j=1}^{N}p_{\mu\theta}\left(j-1\right)q_{j-1}\left(\sum_{i=j}^{N}\phi_i\tau_i\beta_i\right)\mu_j. \quad (61)$$

Matching the coefficient of each μ_j, $j=1,\ldots,N$ on the two sides of (61) and dividing by q_{j-1} shows that

$$p_{\mu\theta}\left(j-1\right) = -\frac{1}{\alpha}\sum_{i=j}^{N}\phi_i\tau_i + p_{\mu\theta}\left(j-1\right)\frac{1}{\alpha}\sum_{i=j}^{N}\phi_i\tau_i\beta_i. \quad (62)$$

Equation (20a) follows by shifting indices in (62). ■

Proof of Lemma 4. By inspection of system (53) of Lemma A.2 and Equation (46) of Lemma A.1, there is only one way to perturb the demand coefficients of class k without perturbing the demand coefficients of any other class, and this is to perturb β_k without perturbing any other quantity. From Equation (20a) of Theorem 2 we can see that perturbing β_k is mathematically equivalent to perturbing $p_{\mu\theta}\left(k-1\right)$, leaving $p_{\mu\theta}\left(j-1\right)$, $j\neq k$ unchanged. Note that $p_{\mu\theta}\left(k-1\right)$ appears in the kth entry of the vector $p_{\mu\theta}$. We may therefore represent a shock to β_k that leaves everything else in equilibrium as a unit vector pointing in the kth direction of the axes of an N-dimensional Cartesian coordinate system. ■

Proof of Theorem 5. By inspection of (20a) we obtain that

$$\mathcal{D}_{p_{\mu\theta}}F = \begin{bmatrix} \frac{\partial F\left(p_{\mu\theta}(0),\ldots\right)}{\partial p_{\mu\theta}(0)} & \frac{\partial F\left(p_{\mu\theta}(0),\ldots\right)}{\partial p_{\mu\theta}(1)} & \cdots & & \frac{\partial F\left(p_{\mu\theta}(0),\ldots\right)}{\partial p_{\mu\theta}(N-1)} \\ \frac{\partial F\left(\ldots,p_{\mu\theta}(1),\ldots\right)}{\partial p_{\mu\theta}(0)} & \frac{\partial F\left(\ldots,p_{\mu\theta}(1),\ldots\right)}{\partial p_{\mu\theta}(1)} & \ddots & & \vdots \\ \vdots & \ddots & \ddots & & \\ & & & \frac{\partial F\left(\ldots,p_{\mu\theta}(N-2),\ldots\right)}{\partial p_{\mu\theta}(N-2)} & \frac{\partial F\left(\ldots,p_{\mu\theta}(N-2),\ldots\right)}{\partial p_{\mu\theta}(N-1)} \\ \frac{\partial F\left(\ldots,p_{\mu\theta}(N-1)\right)}{\partial p_{\mu\theta}(0)} & & & \frac{\partial F\left(\ldots,p_{\mu\theta}(N-1)\right)}{\partial p_{\mu\theta}(N-2)} & \frac{\partial F\left(\ldots,p_{\mu\theta}(N-1)\right)}{\partial p_{\mu\theta}(N-1)} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\alpha}\sum_{i=0}^{N-1}\phi_{i+1}\tau_{i+1}\beta_{i+1} & \frac{\partial F\left(p_{\mu\theta}(0),\ldots\right)}{\partial p_{\mu\theta}(1)} & \cdots & & \frac{\partial F\left(p_{\mu\theta}(0),\ldots\right)}{\partial p_{\mu\theta}(N-1)} \\ 0 & \frac{1}{\alpha}\sum_{i=1}^{N-1}\phi_{i+1}\tau_{i+1}\beta_{i+1} & & & \vdots \\ \vdots & \ddots & \ddots & & \\ 0 & \cdots & 0 & \frac{1}{\alpha}\phi_{N-1}\tau_{N-1}\beta_{N-1} & \frac{\partial F\left(\ldots,p_{\mu\theta}(N-2),\ldots\right)}{\partial p_{\mu\theta}(N-1)} \\ 0 & \cdots & & 0 & 0 \end{bmatrix}.$$

Because $\mathcal{D}_{p_{\mu\theta}}F$ is upper triangular, it follows that the elements in its diagonal are its eigenvalues. This proves (31a).

We now derive the eigenvectors of $\mathcal{D}_{p_{\mu\theta}}F$. To simplify exposition, we use the following notational conventions. For a $N \times N$ matrix J, $J_{t:b,l:r}$ is the matrix block where the top-left corner has matrix coordinates (t,l) and the bottom right corner has matrix coordinates (b,r). I_n is the identity matrix of size $n \times n$. $\mathbf{0}_{n\times m}$ is an (n,m) matrix of zeros, with the convention that it is empty if $n = 0$ or $m = 0$.

We make the conjecture that eigenvector ν_k has the structure in (31b), with ξ_k to be determined. The eigenvalue-eigenvector equation for ν_k is

$$\left(\mathcal{D}_{p_{\mu\theta}}F - \chi_k I\right)\nu_k = 0, \tag{63}$$

which we can write in block form as

$$\begin{bmatrix} \left(\mathcal{D}_{p_{\mu\theta}}F\right)_{1:k,1:k} - \chi_k I_k & \left(\mathcal{D}_{p_{\mu\theta}}F\right)_{1:k,k+1:k+1} & \left(\mathcal{D}_{p_{\mu\theta}}F\right)_{1:k,k+2:N} \\ \mathbf{0}_{1\times k} & 0 & \left(\mathcal{D}_{p_{\mu\theta}}F\right)_{k+1:k+1,k+2:N} \\ \mathbf{0}_{(N-k-1)\times k} & \mathbf{0}_{(N-k-1)\times 1} & \left(\mathcal{D}_{p_{\mu\theta}}F\right)_{k+2:N,k+2:N} - \chi_k I_{N-k-1} \end{bmatrix} \begin{bmatrix} \xi_k \\ 1 \\ \mathbf{0}_{(N-k-1)\times 1} \end{bmatrix} = \mathbf{0}_{N\times 1}. \tag{64}$$

The bottom $N-k-1$ rows in (64) are satisfied due to the trailing zeros in ν_k. The $k+1$th row in (64) is satisfied because element $(k+1, k+1)$ of the matrix on the left-hand side is zero. Thus, ν_k has the structure of (31b), which verifies our conjecture. Solving the first k rows in (64) for ξ_k we get

$$\xi_k = \left[\chi_k I_k - \left(\mathcal{D}_{p_{\mu\theta}}F\right)_{1:k,1:k}\right]^{-1} \left(\mathcal{D}_{p_{\mu\theta}}F\right)_{1:k,k+1:k+1}. \tag{65}$$

This establishes (31b) with ξ_k as in (65). Finally, the eigenvectors $\nu_0, \ldots, \nu_{N-1}$ are linearly independent because the eigenvalues $\chi_0, \ldots, \chi_{N-1}$ are distinct. ■

Lemma A.3 *The coefficients of the fat-finger shock of class k in the equilibrium eigenvector basis is the kth column of the matrix*

$$\begin{bmatrix} \nu_0 & \ldots & \nu_{N-1} \end{bmatrix}^{-1}, \tag{66}$$

which is an upper triangular matrix.

Proof. The fat-finger shocks $\iota_1, \ldots, \iota_N$, are the vectors of the standard basis of $\mathbb{R}^N$. Collecting them into a matrix we obtain the identity matrix I_N. Let $\mathcal{V}$ denote the eigenvectors basis in the form of a matrix,

$$\mathcal{V} = \begin{bmatrix} \nu_0 & \ldots & \nu_{N-1} \end{bmatrix}, \tag{67}$$

and let G denote the matrix of coordinates of the standard basis of $\mathbb{R}^N$ in the eigenvector basis. We note that the matrix $\mathcal{V}$ is invertible because by Theorem 5 its columns are linearly indepedent. Changing bases from $\mathcal{V}$ to the standard basis of $\mathbb{R}^N$ implies that G satisfies

$$I_N = \mathcal{V}G, \tag{68}$$

which is equivalent to

$$G = \mathcal{V}^{-1}. \tag{69}$$

By Theorem 5 the matrix $\mathcal{V}$ is upper triangular, and hence so is its inverse G. ■

Lemma A.4 *Any change in the vector Ψ such that the underlying vector ϕ remains a probability mass function must satisfy the constraint $\Psi_N = \Psi_{N-1} + 1$. Consequently, any direction of change υ in the space of Ψ such that ϕ remains a probability mass function must satisfy $\upsilon_{N-1} = \upsilon_N$.*

Proof. Notice that by definition of Ψ, we can write

$$\phi_i = (\Psi_i - \Psi_{i-1}) - (\Psi_{i-1} - \Psi_{i-2}). \tag{70}$$

We define $\Psi_{-1} = \Psi_{-2} = 0$. This is for notational convenience and does not affect our proof, but it does make the exposition easier. A necessary condition for the vector ϕ to be a probability mass function is that

$$\sum_{i=0}^{N} \phi_i = 1. \tag{71}$$

Substituting (70) into (71) and rearranging gives

$$\sum_{i=0}^{N} (\Psi_i - \Psi_{i-1}) = \sum_{i=0}^{N} (\Psi_{i-1} - \Psi_{i-2}) + 1 \tag{72}$$

Telescoping the sums on both sides gives, after using $\Psi_{-1} = \Psi_{-2} = 0$, that

$$\Psi_N = \Psi_{N-1} + 1. \tag{73}$$

It now follows that any change in Ψ_N must be reflected by changing Ψ_{N-1} in the same amount, and therefore any direction υ in the space of Ψ such that ϕ remains a probability mass function must satisfy $\upsilon_{N-1} = \upsilon_N$. ■

Lemma A.5 *Let*

$$\bar{\mathbb{E}}\left[k\right] = \sum_{k=0}^{N} k\phi_k \tag{74}$$

denote the average information class. Any change in any element of the vector Ψ affect only Ψ_N through the average information class $\bar{\mathbb{E}}\left[k\right]$, and thus, if the average information class remains the same, Ψ_N is not allowed to vary. Consequently, any direction of change υ in the space of Ψ such that $\bar{\mathbb{E}}\left[k\right]$ remains the same must satisfy $\upsilon_N = 0$.

Proof. Notice that by definition of Ψ, we can express Ψ_N as

$$\Psi_N = \sum_{k=0}^{N}(N+1-k)\phi_k = (N+1)\sum_{k=0}^{N}\phi_k - \sum_{k=0}^{N}k\phi_k = (N+1) - \bar{\mathbb{E}}\left[k\right] \tag{75}$$

where the second equality follows by summation by parts. Equation (75) proves that any changes in Ψ only affect Ψ_N through the average information class $\bar{\mathbb{E}}\left[k\right]$. Thus, holding $\bar{\mathbb{E}}\left[k\right]$ fixed we can allow Ψ_i to vary for any i strictly less than N. It immediately follows that any direction of change υ in the space of Ψ such that $\bar{\mathbb{E}}\left[k\right]$ remains fixed must satisfy $\upsilon_N = 0$. ■

Proposition A.6 *For a given equilibrium informativeness vector $p_{\mu\theta}$,*

(i) the derivative of the public eigenvalue with respect to the cdf Φ in the direction υ is

$$\left(\mathcal{D}_{\Phi}\chi_0\right)\upsilon = -\upsilon_0\tau_1\beta_1 + \sum_{i=1}^{N-1}\upsilon_i\left(\tau_i\beta_i - \tau_{i+1}\beta_{i+1}\right). \tag{76a}$$

(ii) The derivative of the public eigenvalue with respect to the scdf Ψ in a mean-preserving

direction υ^* *is*

$$\alpha\left(\mathcal{D}_\Psi\chi_0\right)\upsilon^* = \upsilon_0^\dagger\left[(\tau_2\beta_2-\tau_1\beta_1)-\tau_1\beta_1\right]+\sum_{i=1}^{N-2}\upsilon_i^*\left[(\tau_{i+2}\beta_{i+2}-\tau_{i+1}\beta_{i+1})-(\tau_{i+1}\beta_{i+1}-\tau_i\beta_i)\right]. \tag{76b}$$

Proof. We have

$$\chi_0 = \frac{1}{\alpha}\sum_{i=0}^{N-1}\phi_{i+1}\tau_{i+1}\beta_{i+1}. \tag{77}$$

Because $\phi_i = \Phi_i - \Phi_{i-1}$, we can write

$$\alpha\chi_0 = \left(\Phi_1-\Phi_0\right)\tau_1\beta_1 + \ldots + \left(\Phi_{N-1}-\Phi_{N-2}\right)\tau_{N-1}\beta_{N-1} + \left(\Phi_N-\Phi_{N-1}\right)\tau_N\beta_N. \tag{78}$$

Note that because $\Phi_N = 1$ by definition, we are not allowed to vary Φ_N, and thus we can only vary Φ_i, $i = 0,\ldots,N-1$. Differentiating (78) with respect to Φ_i, $i = 1,\ldots,N-1$ we obtain that

$$\alpha\frac{\partial}{\partial\Phi_i}\chi_0 = \tau_i\beta_i - \tau_{i+1}\beta_{i+1}, \tag{79}$$

and differentiating (78) with respect to Φ_0 we obtain that

$$\alpha\frac{\partial}{\partial\Phi_0}\chi_0 = -\tau_1\beta_1, \tag{80}$$

and therefore

$$\alpha\left(\mathcal{D}_\Phi\chi_0\right)\upsilon = -\upsilon_0\tau_1\beta_1 + \sum_{i=1}^{N-1}\upsilon_i\left(\tau_i\beta_i - \tau_{i+1}\beta_{i+1}\right). \tag{81}$$

This establishes part (i).

Next, because $\phi_i = (\Psi_i - \Psi_{i-1}) - (\Psi_{i-1} - \Psi_{i-2})$, we can write

$$\alpha\chi_0 = \left[(\Psi_1-\Psi_0)-(\Psi_0-\Psi_{-1})\right]\tau_1\beta_1 + \left[(\Psi_2-\Psi_1)-(\Psi_1-\Psi_0)\right]\tau_2\beta_2 + \ldots$$

$$+\left[(\Psi_{N-1}-\Psi_{N-2})-(\Psi_{N-2}-\Psi_{N-3})\right]\tau_{N-1}\beta_{N-1}+\left[(\Psi_{N}-\Psi_{N-1})-(\Psi_{N-1}-\Psi_{N-2})\right]\tau_{N}\beta_{N}. \tag{82}$$

Differentiating (82) with respect to Ψ_j, $j=1,\ldots,N-2$ we obtain that

$$\alpha\frac{\partial}{\partial\Psi_j}\chi_0=(\tau_j\beta_j-\tau_{j+1}\beta_{j+1})-(\tau_{j+1}\beta_{j+1}-\tau_{j+2}\beta_{j+2}), \tag{83}$$

where $j=N-1$ is a special case, for which

$$\alpha\frac{\partial}{\partial\Psi_{N-1}}\chi_0=(\tau_{N-1}\beta_{N-1}-\tau_N\beta_N)-\tau_N\beta_N, \tag{84}$$

$j=N$ is another special case, for which

$$\alpha\frac{\partial}{\partial\Psi_N}\chi_0=\tau_N\beta_N, \tag{85}$$

and $j=0$ is yet another special case, for which

$$\alpha\frac{\partial}{\partial\Psi_0}\chi_0=-\tau_1\beta_1-(\tau_1\beta_1-\tau_2\beta_2). \tag{86}$$

The derivative of the public eigenvalue with respect to the vector $(\Psi_0,\ldots,\Psi_N)$ in an arbitrary direction $\upsilon^\dagger$ is

$$\alpha\left(\mathcal{D}_\Psi\chi_0\right)\upsilon^\dagger=\upsilon_0^\dagger\left[-\tau_1\beta_1-(\tau_1\beta_1-\tau_2\beta_2)\right]+\sum_{i=1}^{N-2}\upsilon_i^\dagger\left[(\tau_i\beta_i-\tau_{i+1}\beta_{i+1})-(\tau_{i+1}\beta_{i+1}-\tau_{i+2}\beta_{i+2})\right]$$
$$+\upsilon_{N-1}^\dagger\left[(\tau_{N-1}\beta_{N-1}-\tau_N\beta_N)-\tau_N\beta_N\right]+\upsilon_N^\dagger\tau_N\beta_N. \tag{87}$$

By Lemma A.4 above, any direction υ^* in the space of well-defined distributions must be such that $\upsilon_{N-1}^*=\upsilon_N^*$. In addition, by Lemma A.5 above, we must also have $\upsilon_N^*=0$ for υ^* to be mean-preserving, which implies that $\upsilon_{N-1}^*=\upsilon_N^*=0$. Therefore, the derivative of the public eigenvalue with respect to the vector $(\Psi_0, \dots, \Psi_{N-1})$ in the mean-preserving direction υ^* is

$$\alpha\left(\mathcal{D}_\Psi \chi_0\right)\upsilon^* = \upsilon_0^*\left[-\tau_1\beta_1 - \left(\tau_1\beta_1 - \tau_2\beta_2\right)\right] + \sum_{i=1}^{N-2} \upsilon_i^*\left[\left(\tau_{i+2}\beta_{i+2} - \tau_{i+1}\beta_{i+1}\right) - \left(\tau_{i+1}\beta_{i+1} - \tau_i\beta_i\right)\right]. \tag{88}$$

This establishes part (ii). ■

Proof of Theorem 6. To prove (i), consider Equation (76a) of Proposition A.6 above. Set every element of the vector υ that is allowed to be nonzero to be the same, that is, $\upsilon_i = L$, $i = 0, \dots, N-1$, where L is a non-zero scalar. The sum in (76a) telescopes, which gives

$$\alpha\left(\mathcal{D}_\Phi \chi_0\right)\upsilon = -L\tau_1\beta_1 + L\left(\tau_1\beta_1 - \tau_N\beta_N\right) = -L\tau_N\beta_N = 0, \tag{89}$$

where the second equality follows because $\beta_N = 0$ by Lemma A.1.

To prove (ii), consider Equation (76b) of Proposition A.6 above. Set every element of the vector υ^* that is allowed to be nonzero to be the same. In particular, set $\upsilon_i^* = L^*$, $i = 0, \dots, N-2$, where L^* is a negative scalar. The sum in (76b) telescopes, and it gives

$$\begin{aligned}\alpha\left(\mathcal{D}_\Psi \chi_0\right)\upsilon^* &= L^*\left[\left(\tau_2\beta_2 - \tau_1\beta_1\right) - \tau_1\beta_1\right] + L^*\left[-\left(\tau_2\beta_2 - \tau_1\beta_1\right) + \left(\tau_N\beta_N - \tau_{N-1}\beta_{N-1}\right)\right] \\ &= -L^*\left(\tau_1\beta_1 + \tau_{N-1}\beta_{N-1}\right) < 0, \end{aligned} \tag{90}$$

where the second equality follows because $\beta_N = 0$ by Lemma A.1, and the inequality follows because $L^* < 0$ and $\beta_1, \beta_{N-1} < 0$ by inspection. ■

Proposition A.7 *For a given equilibrium informativeness vector $p_{\mu\theta}$,*

(i) the derivative of the equity premium with respect to the cdf Φ in the direction υ is

$$\left(\mathcal{D}_\Phi \mathbb{E}\left[D - P\right]\right)\upsilon = \alpha\bar{\theta}\left(\sum_{i=0}^{N}\phi_i\tau_i\right)^{-2}\sum_{i=0}^{N-1}\upsilon_i\left(\tau_{i+1} - \tau_i\right), \tag{91a}$$

(ii) the derivative of the equity premium with respect to Ψ, the cumulative sum of the cdf, in a mean-preserving direction υ^* is

$$\left(\mathcal{D}_\Psi \mathbb{E}\left[D-P\right]\right)\upsilon^* = -\alpha\bar{\theta}\left(\sum_{i=0}^{N}\phi_i\tau_i\right)^{-2}\left\{\sum_{i=0}^{N-2}\upsilon_i^*\left[\left(\tau_{i+2}-\tau_{i+1}\right)-\left(\tau_{i+1}-\tau_i\right)\right]\right\}, \quad (91b)$$

Proof. The proof follows along the lines of the proof of Proposition A.6 above. Defining $\Phi_{-1}=0$, and because $\phi_i = \Phi_i - \Phi_{i-1}$, we can write

$$\sum_{i=0}^{N}\phi_i\tau_i = \left(\Phi_0-\Phi_{-1}\right)\tau_0+\left(\Phi_1-\Phi_0\right)\tau_1+\ldots+\left(\Phi_{N-1}-\Phi_{N-2}\right)\tau_{N-1}+\left(\Phi_N-\Phi_{N-1}\right)\tau_N. \quad (92)$$

As in the proof of Proposition A.6 above, we are not allowed to vary Φ_N because $\Phi_N = 1$ by definition, and thus we can only vary Φ_i, $i=0,\ldots,N-1$ only. Differentiating (92) with respect to Φ_i, $i=0,\ldots,N-1$ we obtain that

$$\frac{\partial}{\partial\Phi_i}\sum_{i=0}^{N}\phi_i\tau_i = \tau_i - \tau_{i+1} \quad (93)$$

and therefore

$$\left(\mathcal{D}_\Phi \mathbb{E}\left[D-P\right]\right)\upsilon = \alpha\bar{\theta}\left(\mathcal{D}_\Phi\left(\sum_{i=0}^{N}\phi_i\tau_i\right)^{-1}\right)\upsilon = \alpha\bar{\theta}\left(\sum_{i=0}^{N}\phi_i\tau_i\right)^{-2}\sum_{i=0}^{N-1}\upsilon_i\left(\tau_{i+1}-\tau_i\right), \quad (94)$$

where the second equality follows by (92) and the chain rule. This proves part (i).

Next, because $\phi_i = \left(\Psi_i-\Psi_{i-1}\right)-\left(\Psi_{i-1}-\Psi_{i-2}\right)$, we can write

$$\begin{aligned}\sum_{i=0}^{N}\phi_i\tau_i &= \left[\left(\Psi_0-\Psi_{-1}\right)-\left(\Psi_{-1}-\Psi_{-2}\right)\right]\tau_0+\left[\left(\Psi_1-\Psi_0\right)-\left(\Psi_0-\Psi_{-1}\right)\right]\tau_1+\ldots\\ &+\left[\left(\Psi_{N-1}-\Psi_{N-2}\right)-\left(\Psi_{N-2}-\Psi_{N-3}\right)\right]\tau_{N-1}+\left[\left(\Psi_N-\Psi_{N-1}\right)-\left(\Psi_{N-1}-\Psi_{N-2}\right)\right]\tau_N. \quad (95)\end{aligned}$$

Differentiating (95) with respect to Ψ_j, $j = 0, \ldots, N-2$ we obtain that

$$\frac{\partial}{\partial \Psi_j} \sum_{i=0}^{N} \phi_i \tau_i = (\tau_j - \tau_{j+1}) - (\tau_{j+1} - \tau_{j+2}), \tag{96}$$

where $j = N-1$ is a special case, for which

$$\frac{\partial}{\partial \Psi_{N-1}} \sum_{i=0}^{N} \phi_i \tau_i = (\tau_{N-1} - \tau_N) - \tau_N, \tag{97}$$

and $j = N$ is another special case, for which

$$\frac{\partial}{\partial \Psi_N} \sum_{i=0}^{N} \phi_i \tau_i = \tau_N. \tag{98}$$

The derivative of the risk premium with respect to the vector $(\Psi_0, \ldots, \Psi_{N-1})$ in an arbitrary direction $\upsilon^\dagger$ is

$$\begin{aligned} \left(\mathcal{D}_\Psi \mathbb{E}\left[D - P\right]\right) \upsilon^\dagger &= \alpha \bar{\theta} \left(\mathcal{D}_\Psi \left(\sum_{i=0}^{N} \phi_i \tau_i \right)^{-1} \right) \upsilon^\dagger \\ &= -\alpha \bar{\theta} \left(\sum_{i=0}^{N} \phi_i \tau_i \right)^{-2} \left\{ \sum_{i=0}^{N-2} \upsilon_i^\dagger \left[(\tau_{i+2} - \tau_{i+1}) - (\tau_{i+1} - \tau_i) \right] \right. \\ &\qquad \left. + \upsilon_{N-1}^\dagger \left[(\tau_{N-1} - \tau_N) - \tau_N \right] + \upsilon_N^\dagger \tau_N \right\}, \end{aligned} \tag{99}$$

where the second equality follows by (95) and the chain rule.

As in the proof of Proposition A.6 above, for υ^* to be a mean-preserving direction in the space of well-defined distributions we must constrain $\upsilon_{N-1}^* = \upsilon_N^* = 0$. Thus, the derivative of the risk premium with respect to the mean-preserving direction υ^* is

$$\left(\mathcal{D}_\Psi \mathbb{E}\left[D-P\right]\right)\upsilon^* = -\alpha\bar{\theta}\left(\sum_{i=0}^{N}\phi_i\tau_i\right)^{-2}\left\{\sum_{i=0}^{N-2}\upsilon_i^*\left[\left(\tau_{i+2}-\tau_{i+1}\right)-\left(\tau_{i+1}-\tau_i\right)\right]\right\}, \tag{100}$$

which establishes part (ii). ■

Proof of Theorem 7. Consider Equation (91a) of Proposition A.7 above. The difference term inside the summation of Equation is positive, because the conditional precision of class i is increasing in i. Therefore, if the direction υ is negative, the derivative of the risk premium is negative. This proves part (i).

To prove part (ii), consider Equation (91b) of Proposition A.7. If the conditional precision is a convex function of the class i, the sign of the second difference of precisions in (91b) is positive, and because the direction υ^* is negative, the sign of (91b) is positive. ■

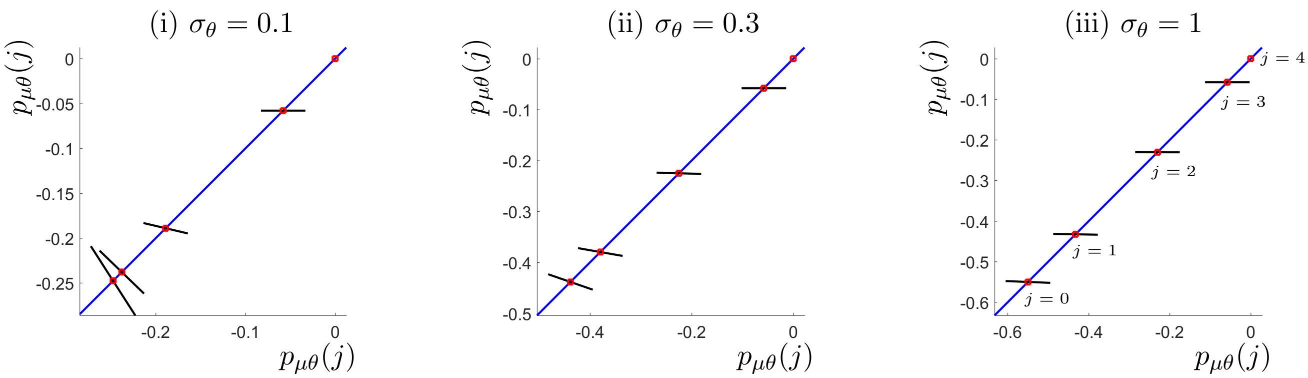

Figure 1. Fixed-point-mapping plot for an economy with $N = 4$ groups with nested private information, and one class with publicly available information (class zero), for three different supply volatilities. The blue line is the 45° line. The four black lines are the slopes of the fixed-point mapping on the right-hand side of (20a). The four red points are the solutions of the price informativeness $p_{\mu\theta}(j)$ for each information class j. The order of the different information classes for all panels is as in panel (iii): lower classes are in lower parts of the 45° line. Class N is the most-informed class (we have $p_{\mu\theta}(N) = 0$ as a boundary condition.) The fixed-point mapping of class $N - 1$ always has a slope of zero. Lower information classes have fixed-point mappings with steeper slopes, an effect which comes from accumulated second-hand information. Panel (i) shows an unstable equilibrium, for which the slope of the mapping becomes steeper than -1 for class zero in the bottom left region. In contrast, Panels (ii) and (iii) show stable equilibria because the slopes of the linear fixed-point mappings are within the threshold, -1, for every information class. Our parameter values are $\alpha = 1$, $\sigma_\mu = 0.5$, $\sigma_\varepsilon = 1$, $\bar{\theta} = 1$, and $q_j = 1$ for $j = 0, \ldots, N$. The distribution ϕ of informed investors is a discretized version of a Beta distribution with shape parameters a and b, set to $a = b = 3$. We show this discretized distribution in more detail in Figure B.1, panel (A), of the Internet Appendix.

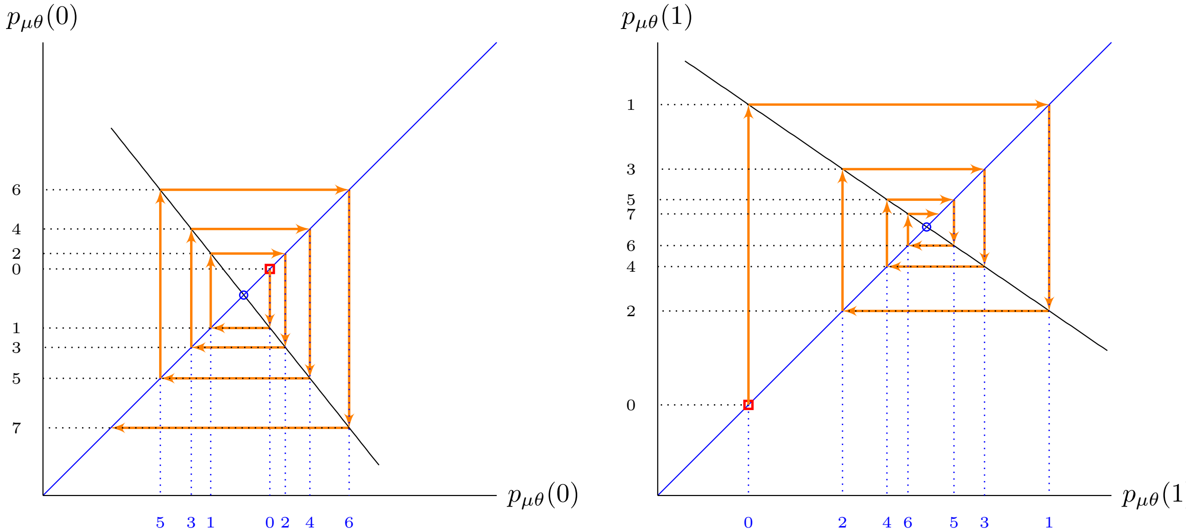

Figure 2. Cobweb diagrams for the fixed-point mappings of class zero and class one of Figure 1, assuming that the equilibrium for other classes does not change. The economy has $N = 4$ groups with nested private information, and one class with publicly available information (class zero). The blue line is the 45° line. The black lines are the slopes of the fixed-point mapping on the right-hand side of (20a). The blue circles are the equilibrium price informativeness $p_{\mu\theta}(j)$ for information classes $j = 0, 1$. The red squares represent the shocked price informativeness $p_{\mu\theta}(j)$ for information classes $j = 0, 1$. The numbers on the x and y axes are the number of iterations of the fixed-point mapping for each class. The orange arrows represent how $p_{\mu\theta}(j)$ changes when we apply the fixed-point mapping to a previous value (vertical arrows) and how it changes when we feed the new value back into the price (horizontal arrows). Panel (i) shows an unstable scalar mapping, for which the slope is steeper than -1. Successive iterations of the fixed-point mapping move the price informativeness farther away from equilibrium. In contrast, Panel (ii) shows a stable scalar mapping because the slope is within the threshold, -1. Here, successive iterations of the fixed-point mapping bring the price-informativeness closer to equilibrium. Our parameter values are $\alpha = 1$, $\sigma_\mu = 0.5$, $\sigma_\varepsilon = 1$, $\sigma_\theta = 0.1$, $\bar{\theta} = 1$, and $q_j = 1$ for $j = 0, \ldots, N$. The distribution ϕ of informed investors is a discretized version of a Beta distribution with shape parameters a and b, set to $a = b = 3$. We show this discretized distribution in more detail in Figure B.1, panel (A).

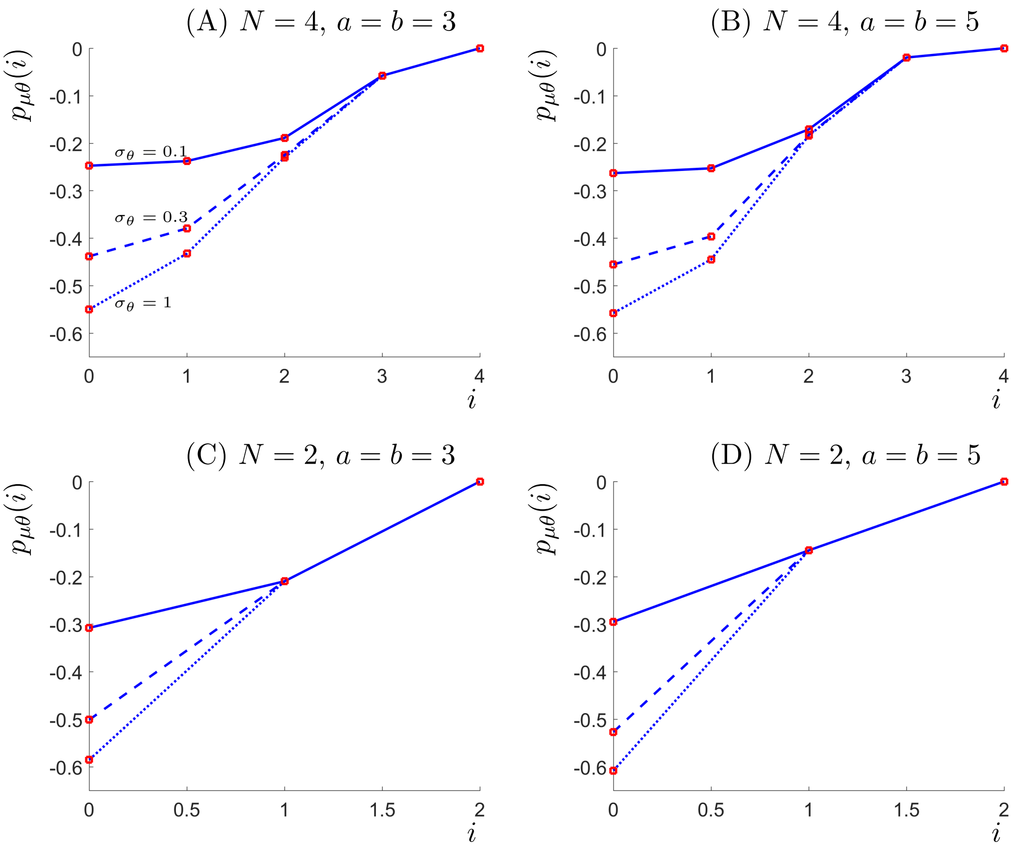

Figure 3. Equilibrium price informativeness vector for four economies with N groups of nested private information, for three different supply volatilities for each economy: $\sigma_\theta = 1$ (dotted lines,) $\sigma_\theta = 0.3$ (dashed lines,) and $\sigma_\theta = 0.1$ (solid lines.) Our parameter values are $\alpha = 1$, $\sigma_\mu = 0.5$, $\sigma_\varepsilon = 1$, $\bar{\theta} = 1$, and $q_j = 1$ for $j = 0, \ldots, N$. In panels (A) through (D) the distribution ϕ of informed investors is a discretized version of a Beta distribution with shape parameters a and b. The economy of Figure 1 is in panel (A). For each N we consider, the distributions with $a = b = 3$ are mean-preserving spreads of the distributions with $a = b = 5$. We show the discretized distributions in more detail in the corresponding panels of Figure B.1 of the Internet Appendix.

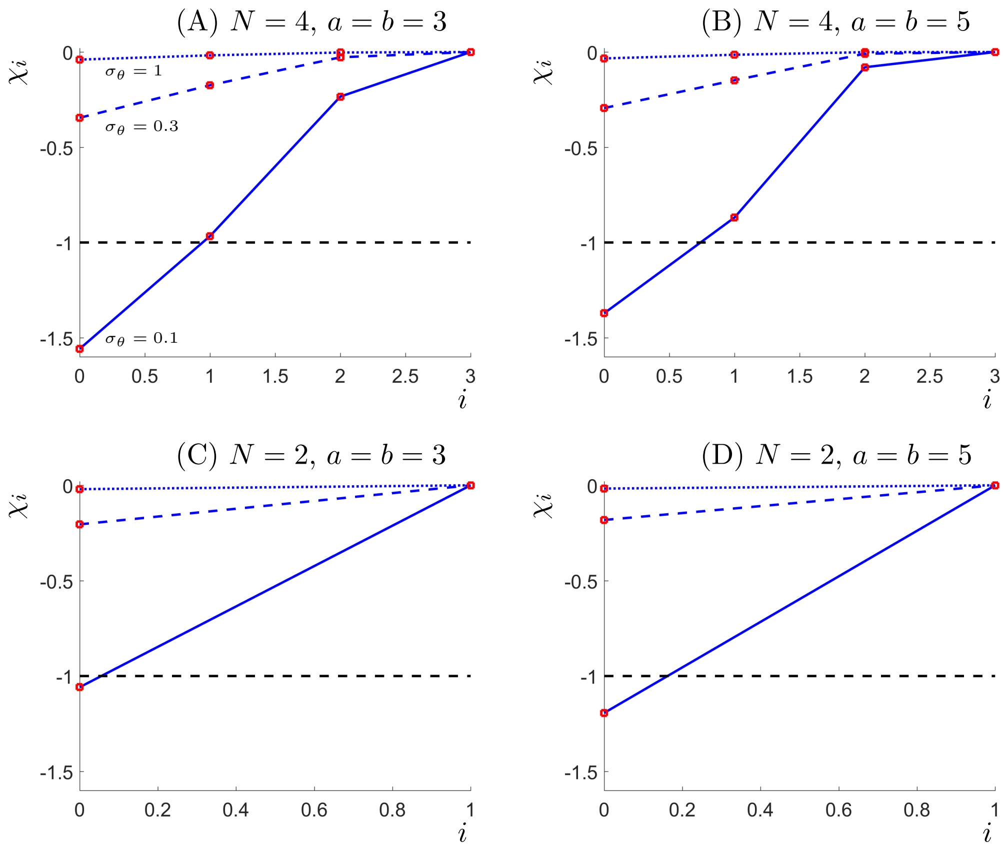

Figure 4. The eigenvalues χ_i of the Jacobian matrix of the fixed-point mapping for four economies with N groups of nested private information, for three different supply volatilities for each economy: $\sigma_\theta = 1$ (dotted lines,) $\sigma_\theta = 0.3$ (dashed lines,) and $\sigma_\theta = 0.1$ (solid lines.) For each N we consider, the distributions with $a = b = 3$ are mean-preserving spreads of the distributions with $a = b = 5$. The values of the remaining parameters are $\alpha = 1$, $\sigma_\mu = 0.5$, $\sigma_\varepsilon = 1$, $\bar{\theta} = 1$, and $q_j = 1$ for $j = 0, \ldots, N$. In panels (A) through (D) the distribution ϕ of informed investors is a discretized version of a Beta distribution with shape parameters a and b. We show these discretized distributions in more detail in the corresponding panels of Figure B.1 of the Internet Appendix.

Table 1. The loadings $c_{0,k}$ of Equation (37), which show how strongly the public eigenvalue χ_0 matters for reactions to fat-finger shocks originating in class $k = 1, \ldots, N$. In panels (A) through (D) we show the values of the loadings $c_{0,k}$ for the different N and the different distributions ϕ of the economies in the corresponding panels in Figures 3 and 4. In this table we confine ourselves to the unstable case, in which supply volatility is $\sigma_\theta = 0.1$. Our remaining parameters are $\alpha = 1$, $\sigma_\mu = 0.5$, $\sigma_\varepsilon = 1$, $\bar{\theta} = 1$, and $q_j = 1$ for $j = 0, \ldots, N$. We show graphs of the coefficients $c_{0,k}$ for all the values of σ_θ that we consider in Figure B.4 of the Internet Appendix.

(A) $N = 4$, $a = b = 3$

	k			
	1	2	3	4
$c_{0,k}$	1.000	−0.604	2.937	118.590

(B) $N = 4$, $a = b = 5$

	k			
	1	2	3	4
$c_{0,k}$	1.000	−0.043	9.181	828.014

(C) $N = 2$, $a = b = 3$

	k	
	1	2
$c_{0,k}$	1.000	9.716

(D) $N = 2$, $a = b = 5$

	k	
	1	2
$c_{0,k}$	1.000	8.945

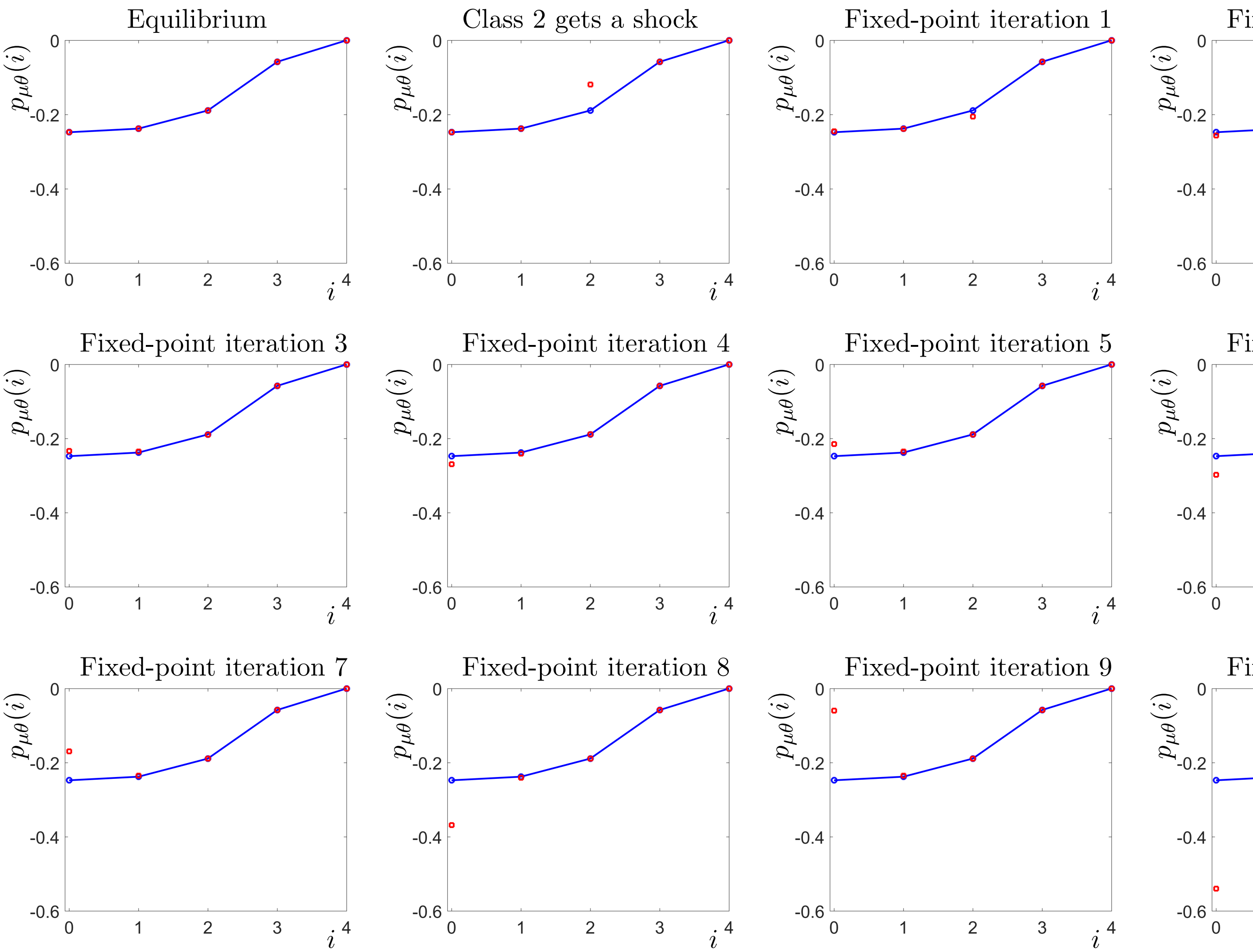

Figure 5. Example of the rational-expectations whiplash with $N = 4$ classes with nested private information; cla observe prices. We show the evolution of a fat-finger error committed by class three (this corresponds to an out-of-e received by class two.) Classes one, two, and three are stable, but class zero is unstable. The economy we show he that in panel (i) of Figure 1 and panel (A) of Figures 3 and 4 (drawn as a solid line.) Our parameters are $\alpha = 1$, $\sigma_\theta = 0.1$, $\bar{\theta} = 1$, $N = 4$, and $q_j = 1$ for $j = 0, \ldots, N$. The distribution ϕ of informed investors is a discretized distribution with shape parameters a and b, set to $a = b = 3$. We show this distribution in more detail in panel (A) the Internet Appendix.

B Internet Appendix: Supplementary Figures

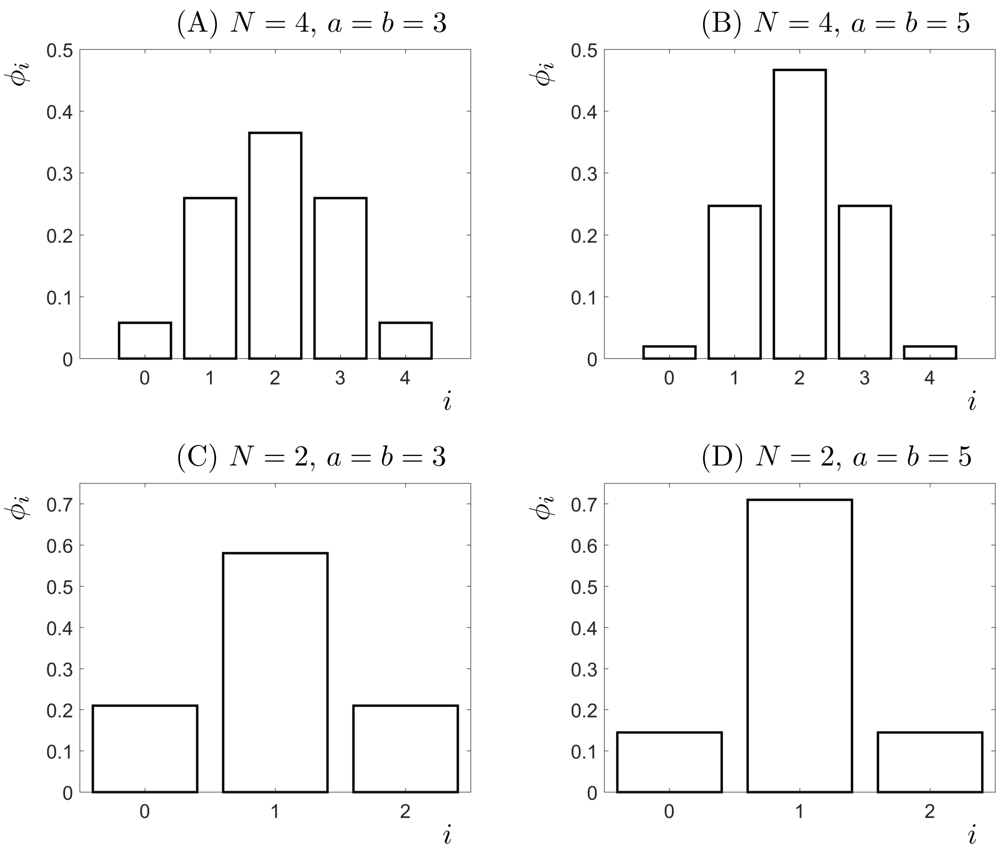

Figure B.1. The distribution ϕ of informed investors is based on a continuous symmetric Beta distribution $\mathcal{B}(a, b)$ with shape parameters a and b. We discretize the probability distribution function of the $\mathcal{B}(a, b)$ distribution by chopping the unit interval $[0, 1]$ into $N+1$ pieces, and assigning the total mass that corresponds to each piece of $[0, 1]$ to the integers $0, \ldots, N$.

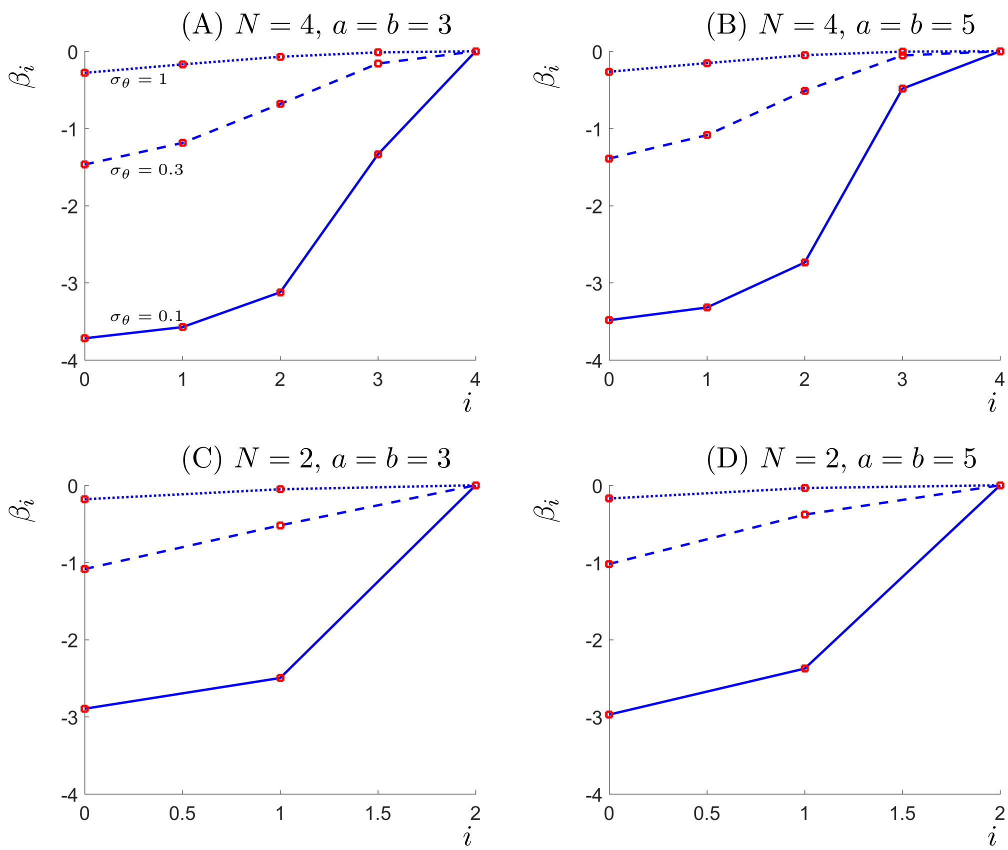

Figure B.2. The projection coefficient β_i of each information class $i = 0, \ldots, N$, for different N and different distributions, with three different supply volatilities: $\sigma_\theta = 1$ (dotted lines,) $\sigma_\theta = 0.3$ (dashed lines,) and $\sigma_\theta = 0.1$ (solid lines.) In panels (A) through (D) the distribution ϕ is that of the corresponding panel in figure B.1. Our parameters $\alpha = 1$, $\sigma_\mu = 0.5$, $\sigma_\varepsilon = 1$, $\bar{\theta} = 1$, and $q_j = 1$ for $j = 0, \ldots, N$.

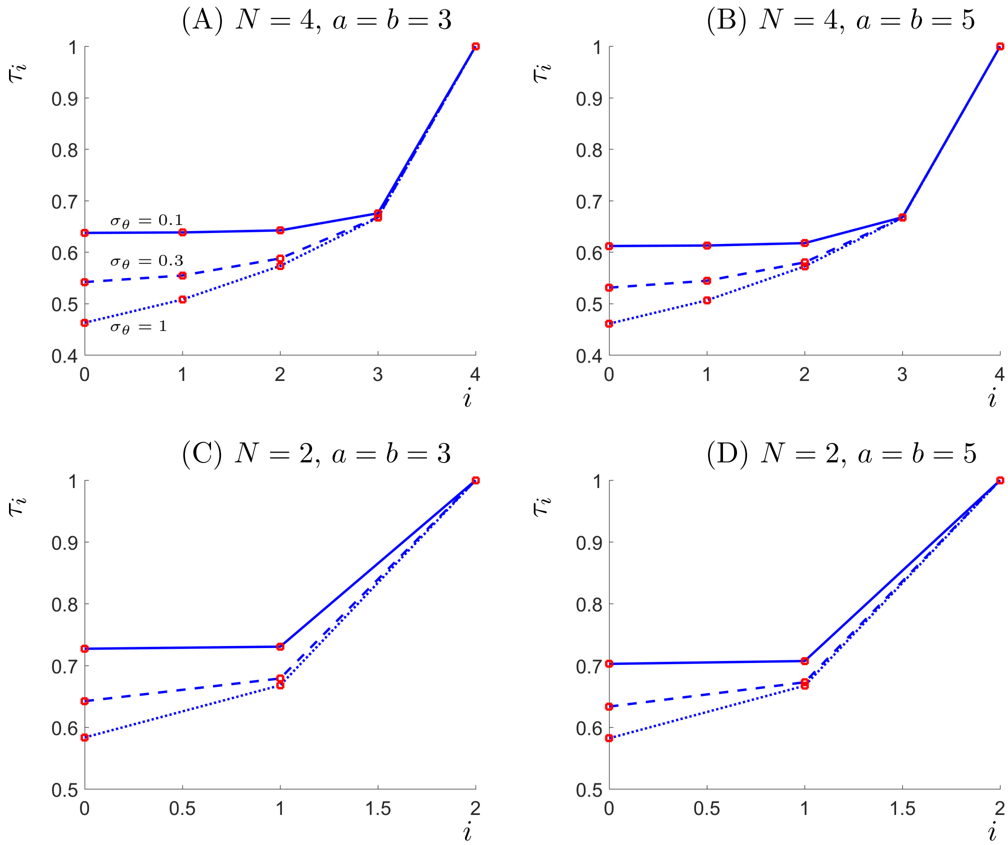

Figure B.3. The precisions τ_i of each information class $i = 0, \ldots, N$, for different N and different distributions, with three different supply volatilities: $\sigma_\theta = 1$ (dotted lines,) $\sigma_\theta = 0.3$ (dashed lines,) and $\sigma_\theta = 0.1$ (solid lines.) In panels (A) through (D) the distribution ϕ is that of the corresponding panel in figure B.1. Our parameters $\alpha = 1$, $\sigma_\mu = 0.5$, $\sigma_\varepsilon = 1$, $\bar{\theta} = 1$, and $q_j = 1$ for $j = 0, \ldots, N$.

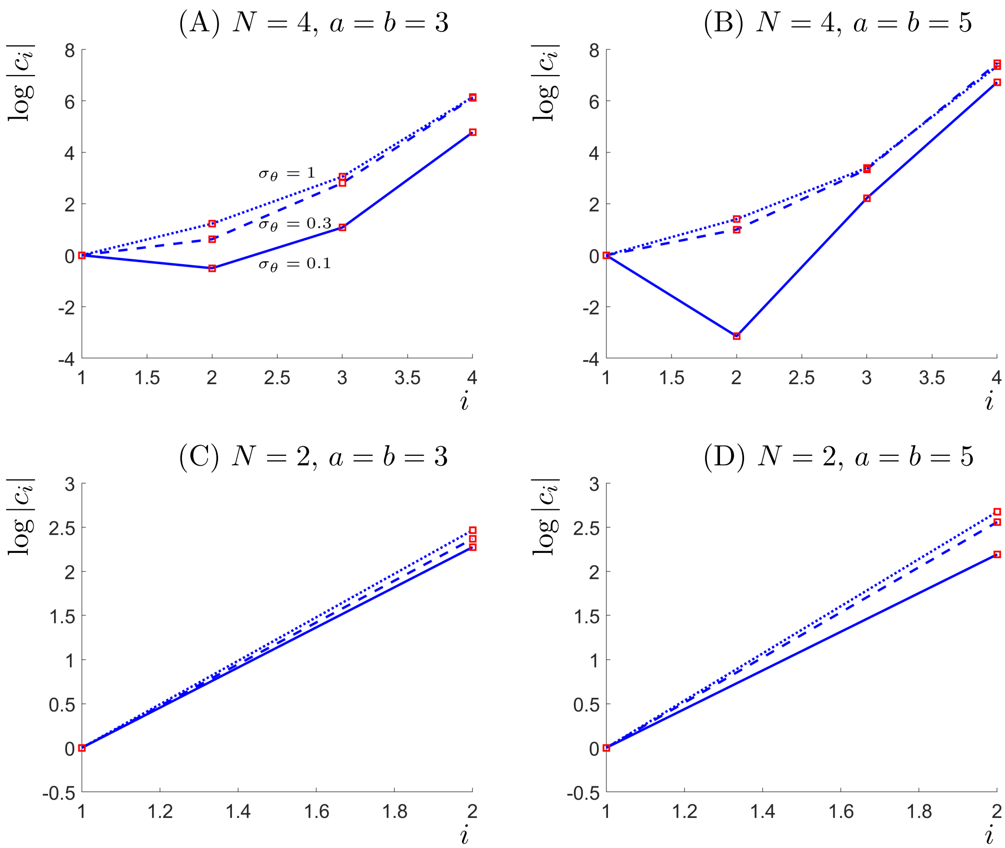

Figure B.4. The absolute values of the loadings $c_{0,i}$ of Equation (37) in logarithmic scales. We show the loading attached to the public eigenvalue χ_0, for fat-finger shocks originating in each class $i = 1, \ldots, N$, for different N and different distributions, with three different supply volatilities: $\sigma_\theta = 1$ (dotted lines,) $\sigma_\theta = 0.3$ (dashed lines,) and $\sigma_\theta = 0.1$ (solid lines.) In panels (A) through (D) we plot the loadings for the distribution ϕ of the corresponding panel in figure B.1. The loading $c_{0,1}$ attached to the public eigenvalue for the eigenvector of class one is always one. Our parameters $\alpha = 1$, $\sigma_\mu = 0.5$, $\sigma_\varepsilon = 1$, $\bar{\theta} = 1$, and $q_j = 1$ for $j = 0, \ldots, N$.