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# Bank Capital and the Composition of Credit\*

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## Abstract

A growing empirical literature highlights the importance of compositional changes in credit for economic activity, including the buildup of leverage and risk. We develop a model of the composition of credit that transparently identifies features of an economy determining which types of borrowers are primarily affected by changes to bank capital and regulations governing it. Our theory echoes the complexity of compositional changes, revealing that even increases in capital ratio requirements can locally increase banks' riskiness. We derive new testable predictions, e.g., on the relations between prices and regulatory risk weights, and between interest rate pass-through and bank capital scarcity.

*Keywords:* Risk-taking, Credit-rationing, Composition of credit supply, Bank regulation, Bailouts, FDIC insurance.

*JEL Classification:* G21 (Banks, Depository Institutions, Mortgages), G28 (Government Policy and Regulation).

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# 1. Introduction

A central strand of research in economics aims to understand the channels through which banks and the financial system affect real activity. As highlighted by this literature, shocks to the banking sector can affect investment if some firms are “bank dependent” in the sense that they are unable to frictionlessly substitute to alternative funding sources when bank credit is unavailable.<sup>1</sup> Consistent with a bank lending channel, the empirical literature has indeed found that banks affect real activity by alleviating credit rationing, and thus, can help mitigate underinvestment.<sup>2</sup> On the other hand, evidence from the Great Recession and the European Sovereign Debt Crisis suggests that banks’ lending decisions may also be affected by risk-taking incentives, e.g., due to access to implicit or explicit public guarantees (see, e.g., Pennacchi, 2006, Allen et al., 2011a). Such risk taking may result in overinvestment, implying that a reduction in bank credit is not necessarily harmful.<sup>3</sup> Overall, these observations indicate that the real effects of changes in the credit supply crucially depend on which types of borrowers are affected. In other words, the *composition* of credit matters, rather than just its aggregate volume.<sup>4</sup>

Motivated by these facts, we develop an analytically tractable model of the composition of credit that transparently identifies the characteristics of those borrowers that are most affected by shocks to bank capital and regulations governing it. In particular, our framework sheds light on the forces determining the banking sector’s “marginal borrower type,” that is, the class of borrowers to which banks would extend credit if bank equity increased at the margin. Such an analysis is meaningful in the context of our model that captures multiple sources of relevant heterogeneity in the cross-section of borrowers, such as access to public markets, risk, profitability, and regulatory risk weight classification. We show that banks effectively rank the cross-section of borrowers based on a parsimonious “profitability index.” This summary statistic encodes all relevant dimensions of heterogeneity determining the *private* surplus a borrower can generate with bank funding.

Our theory echoes the economic relevance and complexity of compositional changes highlighted in the above-discussed literature. For example, increases in capital ratio requirements can paradoxically locally increase the riskiness of the average bank — this naturally happens when the marginal borrower type has relatively safe cash flows. Moreover, a firm is more likely not to obtain

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<sup>1</sup>See, e.g., Blum and Hellwig (1995).

<sup>2</sup>See, e.g., Kashyap et al. (1993, 1994), Gertler and Gilchrist (1994), Peek and Rosengren (2000), Khwaja and Mian (2008), Jiménez et al. (2012), Iyer et al. (2014), and Chodorow-Reich (2014).

<sup>3</sup>For example, recent studies by Acharya et al. (2014) and Jiménez et al. (2014) have identified the empirical importance of bank risk-taking in European data.

<sup>4</sup>See also Greenwood and Hanson (2013), who find that the composition of corporate debt financing, in particular issuer quality, is a more reliable signal of credit market overheating than rapid aggregate credit growth.

bank finance if it has low cash flow risk *relative* to the applied regulatory risk classification and has access to an outside option in the form of public market financing. Ultimately, the marginal borrower type is determined in equilibrium by the intersection of demand and supply of bank credit. Demand and supply, in turn, depend on economy-wide characteristic such as the aggregate level of bank capital, regulatory capital requirements (including risk weight classifications), and the borrower type distribution. As a result, the same policy intervention — such as a recapitalization of the banking sector — will have heterogeneous effects across economies that differ along these dimensions. Given this complexity, our theory can provide conceptual guidance which effects are likely to be relevant in a particular country or in a given state of the business cycle.

Despite featuring rich heterogeneity in bank portfolios and borrower characteristics, our model yields a parsimonious asset pricing condition that provides testable predictions regarding equilibrium loan rates, interest rate pass-through, and the expected returns on bonds trading in public markets. Our model suggests that empirical measures of interest rate pass-through should be informative about the current scarcity of bank capital and the marginal borrower type’s bank dependence. Expected returns increase with a security’s regulatory risk weight, reflecting a premium for the use of scarce bank capital. In addition, expected returns are reduced by a security-specific term reflecting the implicit funding subsidies banks obtain when investing in this type of security under a given regulatory regime.

Our analysis is deliberately static and qualitative, allowing us to cleanly characterize the interplay between bank capital and the composition of credit in the presence of multiple relevant sources of borrower heterogeneity. In contrast, computational challenges associated with dynamic analyses have motivated much of the existing literature to minimize cross-sectional heterogeneity and assume that banks have direct access to a simple investment technology. Homing in on the cross-sectional determinants of credit strikes us as important, given the above-discussed evidence that aggregate outcomes are strongly affected by the composition of credit, not just its aggregate volume. Ultimately, we view dynamic and cross-sectional analyses as complementary and essential for progress in this important field of research.

The following four features are central building blocks of our framework. First, the cross-sectional distribution of potential borrowers can be flexibly specified and captures borrowers’ value added, bank dependence, cash flow risk, and regulatory risk weight classification. Second, consistent with empirical evidence, banks can play both a socially valuable role by alleviating credit frictions of bank-dependent borrowers,<sup>5</sup> and a parasitic role by engaging in excessive risk-

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<sup>5</sup>See, e.g., Holmstrom and Tirole (1997).

taking that exploits public subsidies of banks’ debt financing cost. Access to deposit insurance, or equivalently, implicitly insured debt, allows banks to raise debt at rates that do not reflect banks’ true riskiness.<sup>6</sup> Third, since banks’ behavior in practice is also affected by the regulatory environment, we integrate central features of existing bank capital regulations into our framework — as in Basel I-III, equity ratio requirements limit the amount of debt that banks can raise as a function of equity capital and asset-specific regulatory risk-weights. Finally, we allow banks’ outside equity issuances to be costly,<sup>7</sup> which implies that shocks to bank capital can affect the credit supply, consistent with the above-mentioned literature on the bank lending channel. While we primarily refer to *banks* and *public markets* as potential funding sources in our economy, other financial institutions, such as systemically important insurance companies, may in practice also satisfy several (or all) of the characteristics we assign to banks. In that sense, our model may also shed light on these other types of financial institutions.<sup>8</sup>

In the following, we preview our model’s predictions and economic forces in more detail. Banks in our model endogenously segment into groups with different investment strategies with heterogeneous risks. When taking risks, banks optimally exploit the “put” value obtained from public guarantees of bank debt.<sup>9</sup> In equilibrium, where credit demand has to equal credit supply, this behavior implies that loans with different risk exposures are held and priced by different institutions, that is, marginal investors vary across securities. Nonetheless, we are able to recover the before-mentioned parsimonious asset pricing condition that holds for all bank-funded borrowers.

When the banking sector’s aggregate equity capital is sufficiently high — such that bank equity is not scarce — underinvestment does not occur. Yet access to subsidized debt causes banks to overvalue risky assets that command relatively low capital charges, such as, for example, mortgage backed securities (MBS) did in the pre-crisis period. This overvaluation causes banks to be willing to provide credit at interest rates that are lower than those at which uninsured investors in public markets are willing to provide funds. As a result, bank credit may not only cause overinvestment by borrowers with surplus-destroying projects, but also crowd out other investors.<sup>10</sup> That is, within our framework, banks may become marginal investors in publicly traded corporate (or

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<sup>6</sup>See Kareken and Wallace (1978) for a partial equilibrium analysis of bank risk-taking incentives due to FDIC deposit insurance. See also Hellmann et al. (2000) and Repullo and Suarez (2004, 2013).

<sup>7</sup>See, e.g., Hellmann et al. (2000), Decamps et al. (2011), and Bolton et al. (2013) for papers that also feature costly equity issuances.

<sup>8</sup>See Allen and Santomero (2001) for evidence on the activities of financial intermediaries, both over time and across countries.

<sup>9</sup>Consistent with these predictions, Acharya et al. (2014) find that Spanish and Italian banks primarily hold government debt of their respective home countries. In addition, Rappoport et al. (2014) find that banks indeed tend to hold specialized portfolios.

<sup>10</sup>See related empirical evidence by Abbassi et al. (2016).

sovereign) debt and bid up prices to the point where these securities earn negative expected excess returns, consistent with empirical evidence.<sup>11</sup>

When bank equity capital is scarce, shareholder value maximization dictates that banks shed borrowers according to the incremental *private* surplus that a borrower can generate when funded by a bank. This incremental surplus reflects not only the social value added of bank lending, but also loan-specific subsidies associated with distortions in banks' funding cost. Since the magnitude of these subsidies varies across borrower types (e.g., due to heterogeneous flow risks), the economy generally features simultaneously underinvestment, overinvestment, and crowding out of public market investors in different parts of the borrower distribution. Yet, the above-mentioned unidimensional profitability index summarizing all heterogeneity provides clear guidance regarding the relative likelihood of various borrowers being rationed by banks.

We evaluate three main types of policy interventions widely discussed in recent years: bank capital injections, changes to bank capital ratio requirements, and changes to interest rates. Injecting equity capital does not affect the relative profitability *ranking* of various borrower types from banks' perspective. Thus, an injection expands credit extension to the marginal borrower type, in case bank capital is scarce. As more and more capital is provided, bank equity becomes less scarce and bank loan yields decline. Larger (non-marginal) capital injections imply an expansion of bank financing to a whole set of marginal borrower types that have similar profitability rankings. Yet, as the profitability ranking is affected by several variables, this group may exhibit substantial heterogeneity. Moreover, since banks follow heterogeneous investment strategies, the theory predicts heterogeneous responses by banks, in line with empirical evidence from Japan by Giannetti and Simonov (2013).

Next, we consider a systemwide increase in capital *ratio* requirements, as proposed for example by Admati et al. (2011). Unlike an injection of capital, such a policy change does affect the banking sector's profitability ranking of loans to various borrower types. In particular, it lowers the *relative* profitability of loans to high-risk borrowers that benefit more from public guarantees of bank debt. As capital ratio requirements are increased, banks' equity holders have greater skin in the game, causing private loan profitability and allocative efficiency to become better aligned. When bank capital is not scarce, an increase in capital ratio requirements always weakly improves allocative efficiency. However, when an increase in capital ratio requirements either causes bank capital to

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<sup>11</sup>See Greenwood and Hanson (2013). In this context, it is worth noting that banks' securities holdings account for about 20% of their assets (see Laux and Leuz, 2010, Abbassi et al., 2016). In addition, insurance companies, which are also regulated financial institutions that may be subject to implicit too-big-to-fail guarantees, hold a large fraction of corporate debt.

become scarce, or scarcer than it previously was, aggregate bank credit also contracts. Paradoxically, in this case, there may be a non-monotonic relationship between capital ratio requirements and bank risk-taking, in particular when marginal borrower types have relatively safe cash flows. While risk-taking can always be eliminated with sufficiently large increases in capital requirements, such stringency may not be desirable, as it can cause credit rationing for bank-dependent borrowers.

Finally, we evaluate the effects of changes to interest rates (e.g., due to monetary policy shocks), in particular, how banks pass through interest rate changes to their borrowers. We show why pass-through can be incomplete or excessive, varies across borrower types in the cross-section, and depends on the scarcity of aggregate bank capital. Conversely, our analysis thus also reveals why empirical measures of interest rate pass-through should be informative about the current scarcity of bank capital and the marginal borrower type's bank dependence. When aggregate bank capital is scarce, pass-through may be severely limited: If the marginal borrower type is bank dependent, banks already extract all rents from this type, which prohibits banks from charging higher loan rates in response to a marginal interest rate increase. This, in turn, dampens pass-through for all inframarginal bank-funded borrowers.

**Relation to the literature.** As in Holmstrom and Tirole (1997) banks in our model can create social value by lending to borrowers that would otherwise be credit-rationed by public markets (see related empirical evidence mentioned above).<sup>12</sup> Following Diamond (1984), banks' advantage emanates from the ability to monitor borrowers and thus reduce moral hazard. Relative to these classic contributions, we allow borrowers to differ in terms of their ex-ante cash flow distributions (both in terms of riskiness and value creation), moral hazard problems, and regulatory treatment to be able to meaningfully analyze the implications of banks' risk-taking incentives for the composition of credit and, hence, allocative efficiency. While our paper thus focuses on the role of bankers as lenders, the existing literature has also highlighted that banks may also create important social value on their liability side through the provision of liquidity services.<sup>13</sup>

Concerns about the effects of shocks to intermediary capital on real activity is also at the core

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<sup>12</sup>In Chemmanur and Fulghieri (1994), borrowers can also choose between bank loans and publicly traded debt. Yet Chemmanur and Fulghieri's (1994) analysis highlights that banks, due to reputation concerns, have greater incentives to become informed about firms in distress, which improves their liquidation vs. renegotiation decisions.

<sup>13</sup>See Diamond and Dybvig (1983). DeAngelo and Stulz (2015) argue that tighter capital ratio requirements may be costly as they inhibit the production of liquid claims. Kashyap et al. (2014) also recognize the importance of liquidity creation in their theoretical analysis of macroprudential financial regulation.

of dynamic macroeconomic models with a financial sector.<sup>14</sup> Dynamic models have recently also been used to assess the effects of financial regulation, in particular capital requirements.<sup>15</sup> Our study has different objectives than these papers. In particular, instead of restricting our attention to bank investment technologies that are tractable in dynamic settings, our static framework can accommodate rich cross-sectional distributions of potential borrowers, as well as public markets. As highlighted above, empirical evidence suggests the importance of compositional changes in credit for allocative efficiency.

An important channel affecting the credit supply by banks in our model are risk-taking incentives. Risk-taking has implications for banks’ portfolio decisions, banking sector segmentation, and asset prices, connecting our paper to several strands of the literature.<sup>16</sup> In a partial equilibrium setting, Rochet (1992) shows theoretically that banks typically choose specialized, risky portfolios when their deposits are insured, even in the presence of capital ratio requirements.<sup>17</sup> In our general equilibrium framework with heterogeneous borrowers, risk-taking (“reaching for yield”) not only induces financial sector segmentation,<sup>18</sup> but also causes distortions in the cross-section of asset prices.<sup>19</sup> This feature relates our paper to a growing literature on the pricing of securities when intermediaries are marginal investors.<sup>20</sup> In the cross-section, asset prices are most distorted for securities that have high downside risk relative to the regulatory risk-weights they command. Since credit ratings played a significant role for regulatory risk-weights in the pre-crisis period (see, e.g., Opp et al., 2013), our theory predicts that credit ratings have a direct impact on security prices, holding security cash flows constant, which is consistent with empirical evidence by Kisgen and Strahan (2010).

Finally, our paper relates to the literature that explores the role of competition for financial stability and banks’ risk-taking incentives. A part of this literature highlights that competition *be-*

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<sup>14</sup>See, e.g., Bernanke and Gertler (1989) and Brunnermeier and Sannikov (2014).

<sup>15</sup>See, e.g., Gertler et al. (2012), Martinez-Miera and Suarez (2012), Nguyen (2014), Klimenko et al. (2015), and Begenau (2016).

<sup>16</sup>It is useful to contrast our prediction of “segmentation” with the “herding” result by Acharya and Yorulmazer (2007, 2008) and Farhi and Tirole (2012). In these models, herding by banks *induces* bailouts. In contrast, bailouts (or, equivalently deposit insurance) are a primitive of our environment. Yet, endogenous segmentation in our model still implies that subsets of banks choose the same portfolio strategies and default in the same states of the world. Allen et al. (2012) also develop a model that endogenizes asset commonality. In their setting, banks either hold common asset portfolios and default together, or an unclustered structure obtains where defaults are more dispersed.

<sup>17</sup>See also Repullo and Suarez (2004).

<sup>18</sup>Kahn and Winton (2004) show that “segmentation” may even obtain within a bank by creating subsidiaries without mutual recourse.

<sup>19</sup>Becker and Ivashina (2015) provide empirical evidence of reaching for yield behavior by life insurers, consistent with predictions of Pennacchi (2006).

<sup>20</sup>See, e.g., Garleanu and Pedersen (2011) and He and Krishnamurthy (2013).



*tween banks* reduces a bank's value of staying solvent and thus, encourages risk-taking.<sup>21</sup> Related implications of competition for regulation have also been studied in Hellmann et al. (2000) and Repullo (2004).<sup>22</sup> Allen and Gale (2004) theoretically analyze the complex relationship between competition and stability, finding that concentration may be socially preferable to perfect competition and perfect stability may be socially undesirable. In our static model, banks compete with each other and with investors in public markets. When the set of firms that can obtain funding from public markets becomes larger (e.g., due to improvements in the development of public markets), banks' profits from socially valuable lending decline, encouraging the funding of high-risk borrowers. Consistent with these predictions, Hoshi and Kashyap (1999, 2001) show empirically that deregulations leading up to the "Japanese Big Bang" allowed large corporations to switch from banks to public capital markets, which caused banks to take greater risks.

Our paper is organized as follows. We describe the economy in Section 2. In Section 3, we first characterize bank behavior in partial equilibrium and then analyze the composition of credit, prices, and investment in general equilibrium. Section 4 considers policy experiments, and Section 5 concludes. We relegate proofs to the Appendix.

## 2. Model Setup

We consider a discrete-state, incomplete-markets economy with two dates, 0 and 1. At date 1, the aggregate state of the world  $s \in S$  is realized. The ex-ante probability of state  $s$  is denoted by  $p_s > 0$ . The economy consists of entrepreneurs, investors, and bankers. All agents in the economy are risk-neutral and have a rate of time preference of zero.

### 2.1. Firms

There is a continuum of firms of total measure one, indexed by  $f \in \Omega_f$ .<sup>23</sup> Each firm  $f$  is owned by a cashless entrepreneur (borrower) who has access to a project that requires a fixed-scale

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<sup>21</sup>See Marcus (1984) and Keeley (1990). Boot et al. (1993) similarly argue that excessive risk-taking can be mitigated by reputation effects that in turn depend on imperfect competition. Parlour and Rajan (2001) analyze competition in a model of an unsecured loan market where a borrower may accept more than one contract. The authors show that despite the absence of asymmetric information, positive-profit equilibria are robust to increases in the number of lenders.

<sup>22</sup>Allen et al. (2011b) show that in perfectly competitive markets, banks can find it optimal to hold capital in excess of regulatory minimums, because doing so improves banks' ability to commit to monitoring.

<sup>23</sup>Formally,  $f = (f_1, f_2)$  with  $f_i \in [0, 1]$  for  $i \in \{1, 2\}$  and  $\Omega_f = [0, 1] \times [0, 1]$ . The double continuum assumption for firms will ensure that firms are atomistic relative to banks.

investment  $I$  at time 0, and produces state-contingent cash flows  $C_s$  at time 1. Firm cash flows  $C_s(q, a)$  are affected by the borrower's discrete quality type  $q_f \in \Omega_q$  and her *unobservable* binary action  $a_f \in \{0, 1\}$ . Going forward, we will at times omit firm subscripts when doing so does not create ambiguity.

Firms are subject to limited liability and have access to monitored financing from *banks* and unmonitored financing from *public markets*. In public markets, regular investors and banks compete for firms' securities. Both investors and bankers can observe the firm quality  $q$ , implying that there is no ex-ante asymmetric information between issuers and providers of capital. There is, however, a moral hazard problem following Holmstrom and Tirole (1997). Shirking,  $a = 0$ , allows the borrower to enjoy a private benefit of  $B(q)$  when unmonitored, and 0 when monitored.<sup>24</sup> As a result of this moral hazard problem, some firm types may be credit-rationed in public markets, providing a role for bank monitoring.

**Assumption 1** *Parameters satisfy the following relations:*

- 1)  $NPV(q) := \mathbb{E}[C_s(q, 1)] - I > 0$  for some  $q \in \Omega_q$ ,
- 2)  $B(q) + \mathbb{E}[C_s(q, 0)] - I < 0 \quad \forall q$ ,
- 3)  $C_s(q, 0) < I \quad \forall s, \forall q$ .

The first condition implies that there exists at least one quality type that generates positive social surplus under high effort, that is, a positive net present value ( $NPV$ ).<sup>25</sup> The second condition implies that no project generates positive social surplus (including the private benefit) under shirking. The third restriction is made for expositional reasons. It simplifies the firm's incentive problem when funding is provided by public markets and implies that debt is the optimal contract (see Lemma 1 below).

## 2.2. Investors

There is a continuum of competitive investors with sufficient wealth to finance all projects in the economy. At date 0, investors have access to the following investment opportunities: (1) securities issued by firms in public markets, (2) bank deposits and bank equity, and (3) a storage

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<sup>24</sup>The assumption that the private benefit under monitoring is zero simplifies the analysis, but it is not an essential ingredient for our main results. In principle, banks are effective monitors as long as they reduce the private benefit of shirking below  $B(q)$ .

<sup>25</sup>Otherwise, the optimal aggregate investment by firms is (trivially) zero.

technology with zero interest.<sup>26</sup> Competition, capital abundance, risk-neutrality, a zero rate of time preference, and access to the storage technology imply that investors' expected rate of return is zero on all investments in equilibrium.

Financing of firms via public markets first requires that the borrower's stake in her company provides her with sufficient incentives to exert effort ( $a = 1$ ), as Assumption 1.2 renders financing under shirking ( $a = 0$ ) infeasible. Second, securities purchased by investors must allow them to break even on their investment. Taken together, a firm  $f$  of quality  $q$  can obtain financing from regular investors in public markets if there exists a security with promised state- $s$  cash flows,  $CF_s \geq 0$ , that satisfies both the borrower's IC constraint and investors' IR constraint:

$$\mathbb{E}[\max\{C_s(q, 1) - CF_s, 0\}] \geq B(q) + \mathbb{E}[\max\{C_s(q, 0) - CF_s, 0\}], \quad (\text{IC})$$

$$\mathbb{E}[\min\{C_s(q, 1), CF_s\}] \geq I. \quad (\text{IR})$$

**Lemma 1** *A firm  $f$  of quality  $q_f$  can obtain financing from regular investors in public markets if and only if  $NPV(q_f) \geq B(q_f)$ . The optimal form of financing is debt, and the value of an entrepreneur's equity is  $NPV(q_f)$  if the entrepreneur receives funding from regular investors in public markets.*

Lemma 1 states that a firm can obtain funding from regular investors in public markets if it either has a sufficiently profitable investment opportunity (high  $NPV$ ) or if the required moral hazard rent is sufficiently small (low  $B$ ). While our model relates bank-dependence to moral hazard rents, one may also view  $B$  as a firm-specific parameter capturing other characteristics determining bank-dependence in reduced form. Empirically, large firms tend to be firms with low barriers to public markets (low  $B$ ) as compared to small- and medium sized firms (see e.g., Gertler and Gilchrist (1994) or Iyer et al. (2014)). The equity value obtained by entrepreneurs that obtain funding from regular investors in public markets is equal to the project  $NPV$  since investors act competitively and are not wealth constrained. Similar to Innes (1990), the optimal choice of outside financing is debt, as it provides entrepreneurs with the strongest incentives to exert effort.<sup>27</sup>

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<sup>26</sup>In Section 4.3, we consider the case where interest rates on cash holdings differ from zero and analyze the determinants of interest rate pass-through in the economy.

<sup>27</sup>Unlike in Innes (1990) the optimality of debt is implied by Assumption 1.3 rather than the joint assumption of the monotone likelihood ratio property (MLRP) and the monotonicity constraint of investors' payoff in firm cash flows. There are cash flow distributions that satisfy Assumption 1.3, but not MLRP, and vice versa. Hence, neither assumption is more general.

Although regular investors participate in public markets, debt issued by firms in these markets may also be purchased by banks, in particular, when banks are willing to invest at lower interest rates. However, firms with  $NPV(q) < B(q)$  can never obtain funding from regular investors in public markets and may resort only to monitored bank financing. We now turn to banks' financing and investment decisions.

### 2.3. Banks

There is a continuum of competitive bankers  $b \in \Omega_b$  of mass 1. While our baseline setup assumes that banks compete perfectly — a natural benchmark that yields analytical tractability — we highlight in Section 3.3 that many of our key results also apply when banks have market power. Each banker has initial wealth  $E_I > 0$  in the form of cash at time 0, so that aggregate bank capital is  $E_I$ . While both bankers and investors can observe a firm's type, only bankers have a costless monitoring technology that allows them to eliminate an entrepreneur's private benefit from shirking,  $B(q)$ .<sup>28</sup> This monitoring skill gives bankers a social role, as they can effectively raise the pledgeable income of firms and thus prevent credit rationing.

Banks may raise additional funds in the form of outside equity  $E_O$  and deposits  $D$ . Funds are used to finance investment in non-cash assets  $A$  or to hold cash (money)  $M$ . We obtain the standard balance sheet accounting identity:

$$A + M = E + D, \tag{1}$$

where  $E = E_I + E_O$  is the total book value of equity. To streamline the analysis we assume that firm projects requiring bank monitoring are funded by a loan that is fully held on the balance sheet of the monitoring bank.<sup>29</sup> In addition, banks can invest in senior debt issued in public markets by firms not requiring monitoring. Thus, non-cash assets  $A$  represent a bank's holdings of these two types of debt securities.<sup>30</sup> These assumptions ensure that we can abstract from security design

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<sup>28</sup>The assumption that bankers' monitoring technology is costless greatly increases the tractability of our analysis, but it is not essential for the main insights of our model.

<sup>29</sup>This assumption ensures that our model captures the "skin-in-the-game" requirement that is typical for models with moral hazard (see, e.g., Holmstrom and Tirole, 1997). Relatedly, Rajan and Winton (1995) show that covenants and collateral can be motivated as contractual devices that increase a lender's incentive to monitor. In addition, see Parlour and Winton (2013) for a model where a bank can transfer credit risk by either selling a loan or buying a credit default swap, where the latter has the feature that the originating bank retains the loan's control rights.

<sup>30</sup>In practice, U.S. banks are subject to a risk-weight of 300% for publicly traded stocks and 400% for non-publicly traded equity exposures under Basel III, which makes it very costly for banks to hold other securities, in particular, common stocks.

and the origination and trading of synthetic (derivative) securities.

**External financing frictions.** Banks are subject to limited liability and face external financing frictions, a necessary model ingredient to replicate the empirically relevant bank lending channel (see evidence in the motivating paragraph of the Introduction). Since we aim to develop a setup that remains tractable when considering rich cross-sectional distributions of firms — going beyond a homogeneous cash flow distribution as considered in Holmstrom and Tirole (1997) — we model bankers’ financing frictions in reduced form. Issuing outside equity is associated with a cost  $c(E_O)$ , where  $c$  is an increasing, convex function of outside equity  $E_O$ .<sup>31</sup> In contrast, issuing deposits is not associated with issuance costs, as in Klimenko et al. (2015). The differential cost associated with issuing deposits and equity captures the general insight of models where moral hazard impedes outside financing and debt provides better incentives than equity (see Innes, 1990, Tirole, 2005). It may also be interpreted as an adverse selection discount that investors demand for information-sensitive securities such as equity (see Gorton and Pennacchi, 1990).<sup>32</sup> We assume that banks’ costs of raising equity represent dead-weight losses, although the distinction between private and social costs is of less importance in the context of our *positive* analysis of the composition of credit.

**Bank capital regulation.** In reality, banks do not operate in a regulatory vacuum, and, one of our key premises is that banks’ financing and credit supply decisions are also affected by the regulatory and political environment. In particular, bank deposits are fully insured by FDIC insurance, and more generally, potentially via implicit bailout guarantees. The objective of our paper is to provide a flexible framework for positive analyses of the composition of credit, accounting for the effects of bank capital regulations in the presence of such bailout guarantees. As a result, both capital regulations and bailout guarantees are primitives of our environment.<sup>33</sup> Given our paper’s objective, we do not aim to analyze the potential distortions caused by the financing of bailouts and presume that the government finances bailouts via lump-sum taxes that are levied from investors. The implication of bailout guarantees is that competitive investors in our model are willing to provide debt financing to banks at a net promised interest rate of  $r_D = 0$ , regardless

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<sup>31</sup>See, e.g., Decamps et al. (2011) and Bolton et al. (2013) for papers using similar reduced-form specifications. It is irrelevant whether *firms* also face costs when raising outside equity since Lemma 1 already implies that equity is a suboptimal form of financing for firms even without these additional costs.

<sup>32</sup>See also Allen et al. (2015) who endogenize the cost of equity and deposit finance for banks in a model with bankruptcy costs and segmented deposit and equity markets.

<sup>33</sup>There is a substantial literature that provides rationales for deposit insurance (see, e.g., Diamond and Dybvig (1983)) or bailouts (see e.g., Bianchi (2016) or Chari and Kehoe (2016)).

of the asset positions of a bank.<sup>34</sup>

To mitigate risk-taking incentives, banks are subject to capital regulations that may be based on firm-specific signals of risk  $\rho_f \in \Omega_\rho$  but not directly on firm quality,  $q$ . We do not impose any restriction on the relationship between  $\rho$  and  $q$  other than that the signals  $\rho$  do not fully reveal different firm types  $q$ , which captures the idea that any risk signals used in practice, such as credit ratings, asset classifications, and accounting variables, are inherently noisy. In fact, existing regulations relying on credit ratings do not even use all the information provided by ratings: for example, capital regulations applying to U.S. insurance companies impose the same “risk based capital charges” no matter if a corporate bond is rated *AAA*, *AA*, or *A*.<sup>35</sup> Moreover, as argued by Iannotta and Pennacchi (2014), credit ratings are not even the right risk metric to use for capital regulation since they do not distinguish between idiosyncratic and systematic risk. That is, regulations in practice evidently disregard relevant public information, even if it is easily accessible.

We suppose that for any signal  $\rho$  and any quality  $q$ , the mass of firms with signal  $\rho$  and quality  $q$  is greater than 0. This technical condition ensures that an infinitesimal bank’s asset demand never exceeds the total supply of firms with quality  $q$  and signal  $\rho$ , for any combination of  $q$  and  $\rho$ . Let  $x(q, \rho)$  denote the weight of loans (or debt securities) of type  $(q, \rho)$  in a bank’s portfolio and let  $\mathbf{x}$  denote the associated vector containing all the portfolio weights. As in the regulatory frameworks of Basel I-III, bank capital regulation prescribes that the book equity ratio of every bank,  $e \equiv \frac{E}{A}$ , be above some minimum threshold  $e_{\min}(\mathbf{x})$  that depends on the asset portfolio of a bank,<sup>36</sup> that is,

$$e \geq e_{\min}(\mathbf{x}), \quad (2)$$

where the minimum capital requirement  $e_{\min}(\mathbf{x})$  is a weighted average of asset-specific capital

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<sup>34</sup>If guarantees were imperfect, the deposit rate would reflect a bank’s default risk, but less than justified by a bank’s asset risk. The qualitative results of our analysis would be unaffected in this case. See also Pennacchi (1987, 2006) and Iannotta and Pennacchi (2014) for analyses of deposit insurance pricing and implications for bank regulation and financial system risks.

<sup>35</sup>See the risk categories and risk based capital charges of the National Association of Insurance Commissioners documented in Table 1 by Becker and Opp (2013).

<sup>36</sup>We abstract from additional regulations directly targeting portfolio diversification. In practice, U.S. insurance companies’ capital requirements are multiplied by a “bond size factor” depending on the *number* of issuers in the bond portfolio. Holding RMBS issued by 400 different issuers implies the lowest possible bond size factor. Yet, if an insurance company intentionally chooses to hold RMBS from different issuers that tend to default in the same aggregate states of the world, such a rule does not effectively ensure a diversified portfolio. In our model, a bank typically invests in a continuum of borrowers (i.e., an infinite number), but if these borrowers exhibit correlated downside risks, doing so does not yield relevant diversification.

requirements  $\underline{e}(\rho)$ :

$$e_{\min}(\mathbf{x}) = \sum_{\forall q, \rho} x(q, \rho) \cdot \underline{e}(\rho). \quad (3)$$

For our subsequent comparative statics analysis, it is useful to express  $\underline{e}(\rho)$  as a product of a risk-weight,  $rw(\rho)$ , and an overall level of capital requirements,  $\underline{e}$ , that is,

$$\underline{e}(\rho) = rw(\rho) \cdot \underline{e}. \quad (4)$$

In this paper we focus on capital regulation as the most prominent regulatory tool in practice and make the simplifying assumption that banks are not additionally charged deposit insurance premia. This approach is also in line with our objective to capture the effects of implicit bailout guarantees, for which banks do not pay insurance premia.

In sum, our setup features two deviations from Modigliani-Miller that will play distinct roles for our analysis. First, deposit insurance generates risk-taking incentives for bankers even if depositors can observe bank risk-taking.<sup>37</sup> Second, the wedge between the cost of internal equity and outside equity implies balance sheet effects when the regulatory bank capital constraint binds.

**Bankers' Objective.** For banks, the typical asset is a loan (or debt purchase) of size  $I$  provided to a firm, where the loan has a promised yield  $y$ . Competitive banks take equilibrium yields  $y$  charged to firms of type  $(q, \rho)$  as given. Let  $y(q, \rho)$  denote this competitive equilibrium yield.<sup>38</sup> Since a bank receives a borrowing firm's total cash flow whenever the firm cannot fully repay its loan at date 1, the state-contingent return for a loan is given by:

$$r^s(q, \rho) = \min \left\{ y(q, \rho), \frac{C_s(q, 1)}{I} - 1 \right\}. \quad (5)$$

The overall rate of return on the bank's loan portfolio in state  $s$ ,  $r_A^s$ , satisfies:

$$r_A^s(\mathbf{x}) = \sum_{\forall q, \rho} x(q, \rho) \cdot r^s(q, \rho). \quad (6)$$

Thus, after issuing outside equity  $E_O$  and deposits  $D$ , the total market value of a bank's equity

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<sup>37</sup>Absent deposit insurance, an asset substitution problem may arise *after* a bank has issued debt. However, incentives for risk shifting would be reduced since depositors would require higher yields from banks that take risks (in particular, in the presence of covenants that address banks' asset choice).

<sup>38</sup>Note that at the equilibrium yield funding may be zero.

is

$$E_M = \mathbb{E} [\max \{ (1 + r_A^s(\mathbf{x})) A + M - D, 0 \}], \quad (7)$$

which accounts for a bank's limited liability. Before raising outside finance, a banker's objective is to maximize the value of her equity share, i.e., the market value of the inside equity,  $E_{M,I}$ , taking into account that the value outside equity holders obtain must be equal to the capital they provide,  $E_O$ , and that outside equity issuances are associated with cost  $c(E_O)$ . This value is given by

$$E_{M,I} = \max_{E_O, M, D, \mathbf{x}} E_M - E_O - c(E_O). \quad (8)$$

It is useful to express this objective function in terms of the equity ratio  $e = \frac{E_I + E_O}{A}$ . Using this definition and the balance sheet identity (1) we can eliminate the variables  $D$  and  $M$  and write the *expected* rate of return on bank book equity (ROE) before the cost of outside equity as follows:

$$r_E(\mathbf{x}, e) \equiv \mathbb{E} \left[ \max \left\{ \frac{r_A^s(\mathbf{x})}{e}, -1 \right\} \right]. \quad (9)$$

This representation allows us to rewrite a banker's maximization problem in the following tractable form:

$$E_{M,I} - E_I = \max_{E_O, e, \mathbf{x}} [(E_I + E_O) r_E(\mathbf{x}, e) - c(E_O)] \quad (10)$$

$$\text{s.t. } e \geq e_{\min}(\mathbf{x}). \quad (11)$$

That is, a banker maximizes her expected profit, which is the difference between the market value of her inside equity,  $E_{M,I}$ , and the book value of her inside equity,  $E_I$ .

### 3. Analysis

We now analyze optimal decisions by investors and firms in the economy.

**Definition 1** *A Competitive Equilibrium is a yield function,  $y(q, \rho)$ , specifying the promised yield on a loan to a firm of quality  $q$  with signal  $\rho$ , an investment and effort strategy for each entrepreneur, an outside equity, equity ratio, and portfolio strategy for each banker, and an investment strategy for each investor such that:*

- a) *Given its quality  $q$  and rating  $\rho$ , the entrepreneur of each firm  $f$  decides whether to raise  $I$*



units of capital at the equilibrium yield  $y(q, \rho)$  and whether to shirk or not.

- b) Each banker  $b$  chooses her outside equity  $E_O$ , her equity ratio  $e \geq \sum_{q,\rho} x(q, \rho) \cdot \underline{e}(\rho)$ , and her portfolio  $\mathbf{x} \geq 0$  to maximize (10).
- c) Investors decide on investments in cash, firm debt, deposits, and bank outside equity.
- d) Markets for loans, deposits, and bank outside equity clear.

Our analysis of the equilibrium proceeds as follows. We first study the optimal behavior of an individual bank in partial equilibrium. In the competitive equilibrium, an individual bank takes the promised loan yields  $y(q, \rho)$ , as exogenously given. In a second step, we will determine the prices of all assets in the economy as part of our general equilibrium analysis.

### 3.1. Bank Optimization in Partial Equilibrium

It is convenient to separate the maximization problem of an individual bank (10) into two steps — a problem of optimal outside equity issuance and the jointly optimal portfolio and leverage choice, that is,

$$E_{M,I} - E_I = \max_{E_O} \left[ (E_I + E_O) \max_{e, \mathbf{x}} r_E(\mathbf{x}, e) - c(E_O) \right],$$

First consider the inner (ROE) maximization problem given the exogenous yields on loans  $y(q, \rho)$ :

$$\max_{\mathbf{x}, e} r_E(\mathbf{x}, e) \text{ s.t. } e \geq e_{\min}(\mathbf{x}). \quad (12)$$

Given a solution  $(\mathbf{x}^*, e^*)$  to this maximization problem, define a bank's failure states and its survival states as  $\Sigma_F(\mathbf{x}^*, e^*)$  and  $\Sigma_S(\mathbf{x}^*, e^*)$ , respectively. In bank failure states,  $\Sigma_F(\mathbf{x}, e)$ , the loss on assets is greater than a bank's capital buffer, that is,  $-r_A^s(\mathbf{x}^*) > e$ .

**Lemma 2** *Jointly optimal bank leverage  $e^*$  and portfolio choices  $\mathbf{x}^*$*

- i) **Leverage:** The regulatory leverage constraint binds, that is,  $e^* = e_{\min}(\mathbf{x}^*)$ , if either
  - 1) there exists a portfolio  $\mathbf{x}$  that yields a strictly positive ROE,  $r_E(\mathbf{x}, e_{\min}(\mathbf{x})) > 0$ , or
  - 2) deposit insurance pays a positive amount in some state  $s$  if the bank takes maximum leverage,  $r_A^s(\mathbf{x}^*) < -e_{\min}(\mathbf{x}^*)$ .

ii) **Portfolio choice:** All loans of type  $(q, \rho)$  with a strictly positive weight in the portfolio of a particular bank  $(x^*(q, \rho) > 0)$

1) generate the same bank ROE conditional on bank survival, i.e.,

$$\frac{\mathbb{E}[r^s(q, \rho) | \Sigma_S(\mathbf{x}^*, e^*)]}{\underline{e}(\rho)} = k \quad \text{for some } k \geq 0,$$

2) and exhibit correlated downside risk, i.e.,

$$\begin{aligned} -r^s(q, \rho) > \underline{e}(\rho) &\Leftrightarrow -r_A^s(\mathbf{x}^*) > \underline{e}(\rho) \Leftrightarrow s \in \Sigma_F(\mathbf{x}^*, e^*), \\ -r^s(q, \rho) \leq \underline{e}(\rho) &\Leftrightarrow -r_A^s(\mathbf{x}^*) \leq \underline{e}(\rho) \Leftrightarrow s \in \Sigma_S(\mathbf{x}^*, e^*). \end{aligned}$$

**Leverage.** Part i.1 states that if the equilibrium loan yields allow banks to obtain a positive expected return on bank equity,  $r_E(\mathbf{x}, e_{\min}(\mathbf{x})) > 0$ , banks have a strict incentive to choose the maximum leverage allowed by the regulatory constraint. To understand condition 2, observe that upon bank default in some state  $s$ , government transfers to bank depositors,  $A \cdot (-r_A^s(\mathbf{x}) - e)$ , are strictly decreasing in  $e$ . Total payments to all security holders are thus increasing in leverage, a key departure from the Modigliani-Miller framework. While these transfers accrue *ex post* to depositors, competition among investors on the deposit rate ensures that the present value of these transfers,  $\mathbb{E}(A \max\{-r_A^s(\mathbf{x}) - e, 0\})$ , is passed on to bank equity holders *ex ante*. The present value of these transfers is the value of a put (see Merton, 1977).<sup>39</sup> Thus, shareholder value maximization requires the value of the put be maximized for any given optimal portfolio  $\mathbf{x}^*$ . This is achieved by taking on maximal leverage for a given portfolio  $\mathbf{x}^*$ . Consistent with this prediction, Kisin and Manela (2016) show empirically that capital requirements are indeed effectively binding for the largest banks in the US economy.

**Portfolio choice.** Lemma 2 highlights that optimally designed loan portfolios may consist of multiple, imperfectly correlated borrower types as long as the following properties are satisfied. First, each loan in the portfolio must generate the same expected asset return per unit of required equity capital  $\underline{e}(\rho)$  conditional on bank survival. This is because bank equity holders only care about asset returns in survival states  $s \in \Sigma_S(\mathbf{x}^*, e^*)$ . Second, a bank optimally aligns individual loan payoffs with overall bank survival states. That is, for each state  $s$ , the returns on each loan

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<sup>39</sup>Note, however, that once we endogenize loan yields for firms in general equilibrium, banks pass on (part of) the value of the put to firms.

type in a portfolio either wipe out each respective loan type’s capital cushion, or none of them. Forming portfolios with such correlated downside risk allows a bank to optimally respond to convexity in its payoff function implied by deposit guarantees. In our parameterized illustrations presented in the next section, a large set of borrower types will exhibit sufficiently large correlated downside risks — banks investing in any subset of these borrower types will default in the same aggregate state. Such banks are thus effectively taking gambles with respect to aggregate risk. Yet, at the same time, other banks optimally choose portfolios of securities that are sufficiently safe to ensure that the banks never default.

To illustrate the rationale underlying these portfolio choices, consider an example of a bank that can invest in US treasuries with zero default risk and in Greek sovereign bonds with positive default risk. Depending on the yields on the two types of bonds the bank might obtain a higher expected return on equity by investing either exclusively in US treasuries (and not default for sure) or exclusively in Greek bonds (and default in some states). Suppose that investing exclusively in Greek bonds yields the *weakly* higher ROE of the two options. Then the bank will receive a *strictly* lower ROE if, starting from a portfolio with 100% Greek bonds, it increases the portfolio weight of US treasuries. This is because the expected return on treasuries across the bank’s survival states must be *strictly* lower than for Greek bonds,<sup>40</sup> and the banker does not value the treasuries’ payoffs in bank default states. Conversely, if investing exclusively in treasuries *weakly* dominates investing exclusively in Greek bonds, the bank *strictly* lowers its ROE when marginally increasing the portfolio weight of Greek bonds, starting from a portfolio with 100% US treasuries. A bank that moves from a portfolio with 100% US treasuries to a portfolio with 99.99% US treasuries and 0.01% Greek sovereign bonds will still not default. Thus, it also cannot obtain the same marginal benefit from Greek bonds as when exclusively investing in Greek bonds and benefiting from the bailout put. In short, comparative advantages of financing different borrower types emerge endogenously in our model (see Rappoport et al. (2014) for related empirical evidence). More generally, the above mechanism would imply that banks have a natural incentive to acquire assets that exhibit downside risk that is correlated with their existing assets. For example, as Greek banks are generically more exposed to Greek risk factors, these banks have a comparative advantage specifically in holding Greek sovereign debt, rather than just any risky debt.

Given a solution  $e^*$  and  $\mathbf{x}^*$  yielding  $r_E^* = r_E(\mathbf{x}^*, e^*)$ , we can now characterize the incentives of an individual bank to issue additional equity in parsimonious way.

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<sup>40</sup>Recall that we started with the supposition that exclusively investing in Greek bonds (and defaulting in some states) yields a weakly higher ROE than exclusively investing in US treasuries (and defaulting in no states).

**Lemma 3** *Raising equity in partial equilibrium*

- 1) If  $r_E^* \leq c'(0)$ , a bank does not raise outside equity.
- 2) A bank has a strict incentive to increase outside equity as long as  $r_E^* > c'(E_O)$ .

### 3.2. Credit Supply and Loan Terms in General Equilibrium

In the previous section, we analyzed how individual banks optimally invest in loans taking the yields on each loan type,  $y(q, \rho)$ , as given. We now analyze how the pricing of all loans is determined in a competitive equilibrium where firms can potentially obtain funds from both banks and investors in public markets. We first study the capital supply decisions of regular investors in public markets.

Investors are willing to provide capital to borrower types with access to public markets (see Lemma 1) as long as loan yields are set such that

$$\mathbb{E}[r^s(q, \rho)] \geq 0, \quad (13)$$

where  $r^s(q, \rho)$  was defined in (5). This restriction pins down the minimum yield at which public market investors are willing to invest. Let  $y_P(q)$  denote the yield at which (13) holds with equality (which does not depend on  $\rho$ ). In equilibrium, the loan yield for a borrower with access to public markets and quality  $q$  cannot exceed  $y_P(q)$ , since investors in public markets have sufficient funds to finance all entrepreneurs.

Banks' investment decisions differ from those of regular investors for several reasons. First, bankers can fund borrower types that do not have access to public markets. Second, bankers have access to deposits at rate  $r_D = 0$  and have limited liability. Third, bankers' capital may be scarce.

Ex-ante identical banks typically choose heterogeneous portfolio and leverage strategies in equilibrium as doing so maximizes their private surplus from implicit government subsidies of banks' funding cost. In our setup with ex ante identical banks but heterogeneous strategies, there is naturally multiplicity with respect to the identities of banks that choose a particular strategy. Yet, the equilibrium is unique with respect to aggregate quantities and prices.<sup>41</sup> Definition 2 captures differences in portfolio and leverage strategies that will matter for loan supply decisions.

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<sup>41</sup>This type of multiplicity would generally not obtain if banks were not ex ante identical, for example, if their balance sheets were exposed to different types of legacy assets.

**Definition 2** We say that banks  $b$  and  $b'$  have the same “strategy type” if and only if they fail in the same states of the world, i.e.,

$$\Sigma_F(\mathbf{x}^*(b), e^*(b)) = \Sigma_F(\mathbf{x}^*(b'), e^*(b')). \quad (14)$$

We now turn to the aggregate implications of banks’ optimal leverage and portfolio choices (see Lemma 2).

**Proposition 1** *The competitive equilibrium satisfies the following two properties:*

1. *Banking sector segmentation: Suppose borrowers of two different types  $(q, \rho)$  and  $(q', \rho')$  obtain loans from banks in equilibrium and  $\mathbb{1}_{-r^s(q, \rho) > \underline{e}(\rho)} \neq \mathbb{1}_{-r^s(q', \rho') > \underline{e}(\rho')}$  for some state  $s$ , then the two borrowers must be financed by banks with different strategy types.*
2. *Loan yields: The cost of capital for all bank-financed borrowers satisfies*

$$\mathbb{E}[r^s(q, \rho)] = \underline{e}(\rho) r_E^* - \sigma(q, \rho), \quad (15)$$

where

$$\sigma(q, \rho) \equiv \mathbb{E} \left[ \max \left\{ -\frac{C_s(q, 1) - I}{I} - \underline{e}(\rho), 0 \right\} \right] \geq 0. \quad (16)$$

The first part of the Proposition follows immediately from Lemma 2.ii.2 (correlated downside risks). The second part follows from optimal bank loan supply decisions: borrower types that obtain loans from banks must have loan yields such that banks funding these borrower types obtain the same ROE — otherwise banks could improve their objective by providing loans only to the borrower types yielding the higher ROE. Given this condition, basic algebra yields the parsimonious asset pricing condition (15) for a firm’s cost of capital.

According to equation (15), the cost of capital can be decomposed into a premium related to the scarcity of bank capital  $r_E^*$  and a discount due to implicit government subsidies of banks’ funding cost.<sup>42</sup> While  $\sigma(q, \rho)$  is already defined as a function of *exogenous* parameters in equation (16), our analysis in this section will also yield analytical solutions for  $r_E^*$ . In particular,  $r_E^*$  will be determined by the rents banks can extract from the marginal borrower type they fund, which, in turn, also pins down the rents banks can extract from inframarginal borrowers. Subsidies arise

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<sup>42</sup>Section 4.3 shows how the cost of capital expression (15) can be extended in a straightforward way to accommodate a positive interest rate on cash.

as banks can finance their loan portfolio with deposits at a rate  $r_D = 0$  that is insensitive to banks' asset risk, implying a “put” as in Merton (1977). This put is captured by the term  $\sigma(q, \rho)$ , which represents the expected loan return shortfall relative to the regulatory capital cushion for a given loan, that is,  $\underline{e}(\rho)$ . Thus, the subsidy is high if a borrower type has high downside risk *relative* to the associated regulatory capital requirement  $\underline{e}(\rho)$ , a fitting description for structured securities in the period leading up to the Great Recession. Consistent with empirical evidence, our model predicts the prices of these structured securities to be too low relative to a frictionless benchmark. Our setup thus formalizes how risk signals used for regulation, such as credit ratings, will be reflected in prices, holding cash flow characteristics constant. The pricing implications of a ratings change, for example from  $\rho = AAA$  to  $\rho = B$ , thus depend on how capital requirements vary across ratings classes.

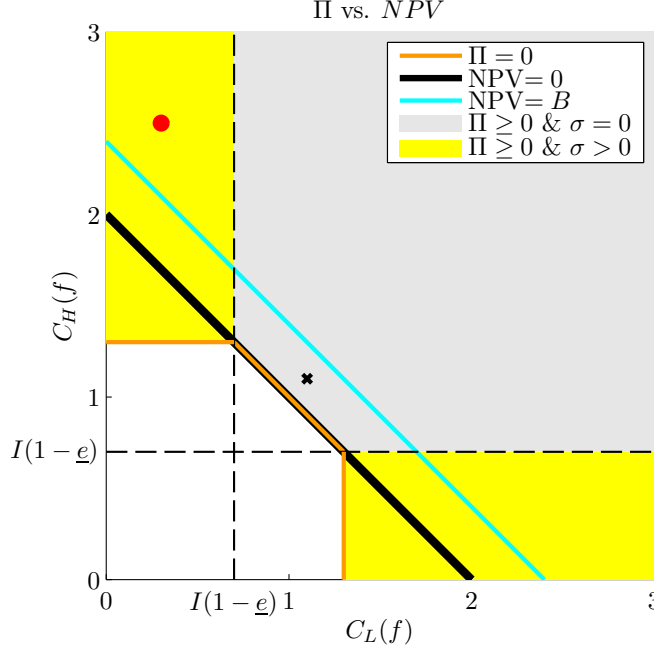
Equation (15) expresses a borrower's cost of capital in terms of exogenous variables, except for the equilibrium rate of return on bank equity,  $r_E^*$ . Although banks behave competitively in our model, bankers may earn a scarcity rent ( $r_E^* > 0$ ) if aggregate inside bank equity  $E_I$  is below some endogenous threshold. At this threshold level, a maximally levered banking sector has just enough funds to provide all weakly profitable loans. To determine this threshold and the equilibrium ROE  $r_E^*$ , we now characterize whether a borrower type can be profitably financed by the banking sector.

**Lemma 4** *The incremental private surplus that bank financing of a borrower of type  $(q, \rho)$  generates is given by*

$$\Pi(q, \rho) = NPV(q) \mathbb{1}_{B(q) > NPV(q)} + I\sigma(q, \rho). \quad (17)$$

$\Pi(q, \rho)$  reflects the two competitive advantages that banks possess relative to regular public market investors. First, banks can finance firms with  $B(q) > NPV(q)$  that are credit rationed by regular investors. Second, banks have access to implicit government subsidies. The value of this subsidy for a loan of size  $I$  is given by  $I\sigma(q, \rho)$ . The incremental private surplus  $\Pi(q, \rho)$  will be shared between banks and firms depending on the scarcity of bank capital (as reflected by  $r_E^*$ ).

Figure 1 illustrates how borrowers' state-contingent cash-flows relate to social surplus ( $NPV$ ), borrowers' access to public markets, and the incremental private surplus that bank financing generates ( $\Pi$ ). All firms to the “north” and “east” of the  $NPV = 0$  line have access to projects with positive NPV. To focus on the effects of cash flow characteristics, we illustrate a scenario where  $B$  and  $\underline{e}$  do not vary with cash flow characteristics. Thus, all firms to the north and east of the  $NPV = B$  line are firms with access to public markets. Firms in the corridor between  $NPV = 0$  and  $NPV = B$  — such as the borrower type marked with a black cross — are firms



**Figure 1.** Incremental private surplus from bank finance, social surplus, and access to public markets. The three solid lines (orange, black, blue) indicate cash flows in two equiprobable macro-states  $s \in \{L, H\}$  that give rise to zero incremental private surplus from bank finance ( $\Pi = 0$ ), zero social surplus ( $NPV = 0$ ), and  $NPV = B$ , respectively. The parameters are:  $I = 1$ ,  $B = 0.2$ , and  $\underline{\epsilon} = 0.3$ . The firm marked with a black cross has cash flows  $(C_L, C_H) = (1.1, 1.1)$ , such that  $NPV = \Pi = 0.1 < B$ .

with socially valuable investments that do not have access to public markets, and hence require bank finance.

Borrower types generally differ in terms of their exposure to downside risk. A borrower with sufficiently low cash flows in one state, that is,  $C_s < I(1 - \underline{\epsilon})$ , such as the borrower marked with the red circle, will receive finance from a bank that defaults in that state of the world (see Proposition 2). The orange line indicating  $\Pi = 0$  has a kink at  $C_L = I(1 - \underline{\epsilon})$ , as bank equity holders do not benefit from asset payoffs in such default states. Since further losses are borne by the government (e.g., deposit insurance), all borrowers in the yellow shaded region are associated with a strictly positive financing subsidy  $\sigma$ . This subsidy creates a wedge between the  $\Pi = 0$  line and the  $NPV = 0$  line. Borrowers in the area between the  $\Pi = 0$  line and the  $NPV = 0$  line have a negative  $NPV$  but generate incremental private surplus if financed by banks.

Finally, Figure 1 also illustrates banks' strategy types (see Proposition 1): If the “red-circle” borrower type is financed by a bank in equilibrium, then it must be financed by a bank that defaults in state  $L$  and invests in borrower types located in the yellow region in the “north-west.” In contrast, if the “black-cross” borrower type is financed by a bank, then this bank invests in

borrower types located in the grey region, implying that the bank is not exposed to default risk.

For the remaining analysis it is useful to define a bank profitability index, denoted by  $PI$ , which determines banks' private ranking of borrower types:

$$PI(q, \rho) = \frac{\Pi(q, \rho)}{I \cdot \underline{e}(\rho)}. \quad (18)$$

$PI(q, \rho)$  can be interpreted as the expected return on equity that a bank could obtain when funding only borrowers of type  $(q, \rho)$  if the bank did not face competition from other banks. Due to competition from other banks a bank may, however, not be able to extract this surplus in equilibrium. Since the profitability index is a function of social surplus and deposit insurance subsidies (see (17)), the private ranking based on  $PI(q, \rho)$  is generally not aligned with the social ranking based on  $NPV(q)$ . In particular, the private ranking is misaligned when a sufficient fraction of borrowers have high downside risk, low capital requirements, and a negative  $NPV$ . To streamline the analysis and avoid knife-edge cases, we presume that the finite number of borrower types  $(q, \rho)$  in the model have distinct profitability indices.

We also define  $E_R(\widetilde{PI})$  as the aggregate equity capital that a maximally levered banking sector requires to fund all borrower types with a profitability index greater or equal to  $\widetilde{PI}$ , that is:

$$E_R(\widetilde{PI}) = \sum_{(q, \rho): PI(q, \rho) \geq \widetilde{PI}} I \cdot \underline{e}(\rho) \cdot m(q, \rho), \quad (19)$$

where  $m(q, \rho)$  denotes the mass of firms of type  $(q, \rho)$ . We now characterize when bank inside equity is scarce in the economy.

**Definition 3** *Inside bank equity is scarce when maximally levered banks with aggregate equity  $E_I$  cannot fund all privately profitable borrowers in the economy, that is, when:*

$$E_I < E_R(0). \quad (20)$$

Note that scarcity of bank capital generally cannot be gauged based on data about banks' capital ratios  $e$  alone. Rather, scarcity depends on the relative magnitudes of banks' *aggregate* levered equity on the one hand and the aggregate quantity of privately profitable investment opportunities on the other hand. Put differently, apart from equity ratios, the total equity in the banking sector and the aggregate quantity of banks' profitable investment opportunities play an important role. Definition 3 accounts for these other factors determining the scarcity of bank



equity. In Section 4.3, we discuss how empirical measures of interest rate pass through can shed light on whether bank capital is scarce at a given point in time.

Going forward, our equilibrium analysis of investment decisions and prices in the economy will hence distinguish between the cases  $E_I > E_R(0)$  and  $E_I < E_R(0)$ . Before characterizing the equilibrium in full generality, it is useful to illustrate the basic concepts of the analysis with a simple example.

**Example 1** *The economy features a good and a bad borrower type  $q \in \{g, b\}$  and two equiprobable states  $s \in \{L, H\}$ . Signals,  $\rho$ , are uninformative, so that risk-weights  $rw(\rho)$  can be normalized to 1 without loss of generality. The borrower types' respective characteristics and payoffs in the two states are given by*

Type	$C_L(q, 1)$	$C_H(q, 1)$	$B(q)$	$NPV(q)$	$m(\rho, q)$
$g$	$1.05I$	$1.05I$	$0.3I$	$0.05I$	0.8
$b$	$0.4I$	$1.05I$	$0.2I$	$-0.1I$	0.2

Neither borrower type can access public markets, since  $NPV(q) < B(q)$ . In a first-best world, banks should finance all borrowers of the good type, but none of the bad type. However, for sufficiently lenient regulation,  $\underline{e} < 30\%$ , the bad type, which has risky cash flows, has a higher profitability index than the good type does, that is,  $PI(b, \rho) > PI(g, \rho) > 0$ . In this case, banks will only finance borrowers of the good type after all bad types have been financed. Aggregate bank capital  $E_I$  is scarce if it is below  $E_R(0) = \underline{e} \cdot I \cdot (0.2 + 0.8)$ , as specified in Definition 3.

When capital ratio requirements are more stringent,  $30\% < \underline{e} < 40\%$ , banks still rank investments in both borrower types above holding cash, but good types rank above bad types,  $PI(g, \rho) > PI(b, \rho) > 0$ . This happens, since increasing capital requirements reduces the effective subsidy for bad types,  $\sigma(b, \rho)$ , and hence reduces their relative attractiveness. Aggregate bank capital  $E_I$  is still scarce if it is below  $E_R(0) = \underline{e} \cdot I \cdot (0.2 + 0.8)$ .

Finally, when capital ratio requirements are sufficiently stringent,  $\underline{e} \geq 40\%$ , the required “skin in the game” makes financing bad projects unattractive relative to holding cash, so that the banking sector’s ranking of borrower types is aligned with the social ranking, that is,  $PI(g, \rho) > 0 > PI(b, \rho)$ . The threshold level for bank capital scarcity is now  $E_R(0) = \underline{e} \cdot I \cdot 0.8$ , where 0.8 reflects the mass of good types. Counter to conventional wisdom, an increase in capital ratio requirements from  $\underline{e} < 40\%$  to  $\underline{e} > 40\%$  may, thus, make bank capital less scarce by freeing up capital that was previously steered towards the financing of bad types. This highlights that bank capital scarcity is

*not simply a monotonic function of bank capital ratios. Instead, scarcity also crucially depends on the magnitude of banks' privately profitable investment opportunities, which themselves are affected by capital regulation.*

While this simple example illustrates some of the key concepts of our analysis (e.g., the central role of the profitability index  $PI$ ), the types of effects that can arise in equilibrium are limited by the specific parameterization. For example, more generally, good borrowers do not need to have safe cash flows (see Neuhaan and Saidi (forthcoming)) and may have access to public markets, whereas bad borrowers do not need to have risky cash flows. The analysis of our general framework accommodates any distribution of borrower types without losing tractability, and will thereby systematically uncover the various possible equilibrium effects of policy interventions and shocks affecting banks.

**Case:**  $E_I > E_R(0)$ . We first discuss the case when banks' inside equity is not scarce.

**Proposition 2** *When inside bank equity is not scarce, that is,  $E_I > E_R(0)$ ,*

1. *banks fund all borrowers with  $\Pi(q, \rho) > 0$ ,*
2. *of the remaining firms public market investors fund all firms with  $NPV(q) \geq B(q)$ ,*
3. *funded firms obtain all private surplus in equilibrium, that is,*

$$\Pi(q, \rho) + NPV(q) \mathbb{1}_{NPV(q) \geq B(q)}. \quad (21)$$

Since banks have excess inside equity, all borrower types with positive private surplus from bank finance are funded. In the economy illustrated in Figure 1, all borrowers to the north-east of the  $\Pi = 0$  line would be funded. This includes the set of all borrowers with positive-NPV projects, but also firms with negative-NPV projects, provided that their financing subsidy  $\sigma$  is sufficiently high.

**Case:**  $E_I < E_R(0)$ . In the case when banks' inside equity is scarce, banks cannot fund all privately profitable borrowers in the economy unless outside equity is raised. If the banking sector does not fund all such borrowers, it will rationally shed the borrower types with the lowest

profitability indices  $PI(q, \rho)$ . Let  $(\tilde{q}_{E_O}, \tilde{\rho}_{E_O})$  denote the type of the marginal firm funded by the banking sector given outside equity issuances  $E_O$ :

$$(\tilde{q}_{E_O}, \tilde{\rho}_{E_O}) = \arg \min_{(q, \rho)} PI(q, \rho) \text{ s.t. } E_R(PI(q, \rho)) \geq E_I + E_O \quad (22)$$

The borrower type  $(q, \rho)$  with the smallest profitability index in the set of bank-funded borrowers is the marginal borrower type. The profitability index of the marginal type absent outside equity issuances,  $PI(\tilde{q}_0, \tilde{\rho}_0)$ , plays a crucial role in determining whether the banking sector has incentives to raise outside equity. If  $c'(0) \geq PI(\tilde{q}_0, \tilde{\rho}_0)$  the banking sector does not raise outside equity. In this case, equilibrium yields are such that firms of the marginal type  $(\tilde{q}_0, \tilde{\rho}_0)$  pledge the entire incremental surplus from bank finance  $\Pi(\tilde{q}_0, \tilde{\rho}_0)$  to banks, while keeping the surplus from public market finance  $NPV(q) \mathbb{1}_{NPV(q_I) \geq B(q_I)}$ . The banking sector's return on equity from this loan is  $r_E^* = PI(\tilde{q}_0, \tilde{\rho}_0)$ . Equilibrium loan yields for all other borrower types funded by banks are such that all banks earn the same expected ROE.

In contrast, when  $c'(0) < PI(\tilde{q}_0, \tilde{\rho}_0)$ , the return on the marginal funded firm absent equity issuances exceeds the marginal cost of raising outside equity. Thus, an equilibrium without equity issuances cannot obtain. Banks raise outside equity up to the point where all borrower types with  $PI(q, \rho) > c'(E_O^*)$  are funded, where  $E_O^*$  refers to the equilibrium amount of outside equity that each bank raises.<sup>43</sup> Market clearing implies that the equilibrium rate of return of the banking sector is given by  $r_E^* = c'(E_O^*)$ . We summarize these insights in

**Proposition 3** *When inside bank equity capital is scarce, that is,  $E_I < E_R(0)$ ,*

1. *no bank raises equity if  $c'(0) \geq PI(\tilde{q}_0, \tilde{\rho}_0)$ . Otherwise,*

$$E_O^* = \sup\{E_O \geq 0 : PI(\tilde{q}_{E_O}, \tilde{\rho}_{E_O}) \geq c'(E_O)\}, \quad (23)$$

*where  $(\tilde{q}_{E_O}, \tilde{\rho}_{E_O})$  denotes the marginal borrower type after issuing outside equity  $E_O$ .*

2. *banks fund all firms of type  $(q, \rho)$  with  $PI(q, \rho) > r_E^* = \min\{PI(\tilde{q}_0, \tilde{\rho}_0), c'(E_O^*)\}$  and the*

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<sup>43</sup>If  $c(E_O)$  is strictly convex, then symmetric outside equity issuances are the unique equilibrium outcome. If  $c(E_O)$  is linear, individual banks may raise different amounts of equity, but the aggregate amount of issuance is still uniquely determined.

following fraction of firms of the marginal type  $(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})$ :

$$1 - \frac{E_R(PI(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})) - (E_I + E_O^*)}{I \cdot \underline{e}(\tilde{\rho}_{E_O^*}) \cdot m(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})}, \quad (24)$$

3. of the remaining firms public markets fund all firms with  $NPV(q) \geq B(q)$ ,
4. funded firms of type  $(q, \rho)$  obtain the following expected surplus

$$\Pi(q, \rho) \cdot \max \left\{ 1 - \frac{r_E^*}{PI(q, \rho)}, 0 \right\} + NPV(q) \cdot \mathbb{1}_{NPV(q) \geq B(q)}. \quad (25)$$

With scarce inside bank equity capital, it is possible that some firms with positive-NPV projects remain unfunded. Such capital rationing occurs for firm types that do not have access to public markets and are not sufficiently profitable for banks ( $PI(q, \rho) < r_E^*$ ). Scarcity not only affects the credit supply but also allows banks to earn equilibrium rents. Each firm that is funded by the banking sector now needs to pledge a fraction  $\frac{r_E^*}{PI(q, \rho)}$  of the private surplus  $\Pi(q, \rho)$  to bankers.

**Obtaining analytical solutions.** In summary, for any given distribution of borrower types, our model yields analytical solutions for aggregate quantities and prices. Solutions can be obtained as follows:

1. The profitability indices  $PI(q, \rho)$  defined in (18) and the function  $E_R(\widetilde{PI})$  specified in (19) are expressed explicitly in terms of exogenous parameters.
2. For  $E_I > E_R(0)$ , Proposition 2 states which borrower types are funded. Using  $r_E^* = 0$ , Equation (15) pins down the equilibrium yields for each borrower type that obtains funding.
3. For  $E_I < E_R(0)$ , the marginal borrower type  $(\tilde{q}_{E_O}, \tilde{\rho}_{E_O})$  is characterized as a step-function of  $E_0$  as specified in (22). Proposition 3 defines the optimal equity issuance  $E_O^*$  and the quantities and types of borrowers funded by banks and public markets. Using  $r_E^* = \min \{PI(\tilde{q}_0, \tilde{\rho}_0), c'(E_O^*)\}$ , (15) pins down the equilibrium yields for all borrower types funded by banks.

### 3.3. Discussion on the Role of Competition

While our baseline model assumes that banks are competing perfectly with each other, many of the insights of our analysis would also apply if banks had monopoly power. Consider for example

a simple variant of our model where each bank has monopolistic access to a set of borrowers in the economy, and where a each borrower type in that set has positive mass. As in our baseline model, borrower types with  $NPV > B$  still can access public markets, that is, they are not bank dependent.

In this economy with market power, the profitability index  $PI(q, \rho)$  introduced in equation (18) is still characterizing banks' private ranking of borrower types.<sup>44</sup> Banks' credit provision is thus still governed by the same types of incentives as in our baseline model. A key difference, however, is that monopolistic banks could charge higher equilibrium yields — loan rates would be set such that banks extract all rents from bank dependent borrowers that are funded.

## 4. Policy Experiments and Comparative Statics

Using the equilibrium characterization provided in the previous section we now discuss the comparative statics of our model to obtain its predictions and policy implications. The goal of our framework is to facilitate the analysis of the volume of credit and its composition. For the subsequent comparative statics analysis, it is useful to define a summary statistic of the efficiency of firm investment.

**Definition 4** *Total social surplus is the sum of the NPVs of all projects financed in the economy.*

It is important to emphasize that *total social surplus* as defined here should not be interpreted as a broad measure of welfare, as it does not account for the cost associated with banks' outside equity issuances. Moreover, our analysis does not aim to shed light on the magnitudes of distress cost and default externalities associated with bank failures, or tax distortions resulting from the financing of bailouts.

### 4.1. Changes to Banks' Inside Equity

First, we analyze the effects of changes to the aggregate amount of inside bank equity in the economy. In practice, various economic shocks can lead to declines or increases in aggregate bank capital. For example, a macroeconomic downturn might be associated with higher loan default

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<sup>44</sup>Recall that we also interpreted  $PI(q, \rho)$  previously as the expected return on equity that a bank could obtain when funding only borrowers of type  $(q, \rho)$  if the bank did not face competition from other banks.

rates, and thus, decreasing levels of aggregate bank equity. On the other hand, equity capital injections by governments during crises could increase aggregate bank equity (see, e.g., Giannetti and Simonov, 2013). The following Corollary to Propositions 2 and 3 summarizes general results regarding the effects of changes to aggregate bank capital.

**Corollary 1** *A decline in the aggregate amount of inside bank equity  $E_I$*

1. *weakly increases banks' ROE,  $r_E^*$ , and borrowers' loan yields  $y(q, \rho)$ ,*
2. *weakly decreases aggregate investment, but weakly increases funding by regular investors in public markets.*
3. *The local effect on total social surplus is*
  - (a) *negative, if the marginal borrower type  $(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})$  satisfies  $0 < NPV < B$ ,*
  - (b) *neutral, if the marginal borrower type  $(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})$  satisfies  $NPV > B$ ,*
  - (c) *positive, if the marginal borrower type  $(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})$  satisfies  $NPV < 0$ .*

The expected return on bank equity,  $r_E^*$ , is weakly decreasing in inside equity  $E_I$ , as higher levels of inside equity decrease the scarcity of bank equity.<sup>45</sup> As a result of this scarcity, all borrowers funded by banks after the negative shock have to pledge a larger fraction of private surplus to bankers to still obtain financing, i.e.,  $y(q, \rho)$  increases. The loan pricing condition (15) pins down by how much. The borrowers that are shed by banks can either access public markets at less favorable rates than before or lose funding altogether. Moreover, borrower types that previously did not receive bank finance will not start to receive bank finance either, since changes to  $E_I$  do not affect banks' ranking of borrower types (see definition of  $PI(q, \rho)$  in equation (18)). Taken together, aggregate investment must decrease, even though the reduction in bank lending is partially compensated by increased direct funding provided by regular investors in public markets (see empirical evidence for substitution by Becker and Ivashina (2014)).

The effect of this decline in aggregate bank investment on total social surplus depends on the  $NPV$  and the bank dependence of the marginal borrower type  $(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})$ . Since banks' private ranking of investment opportunities based on the profitability index  $PI$  is generally not aligned with the social ranking based on  $NPV$ , the marginal borrower type may have a positive or a

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<sup>45</sup>This holds even when banks are raising outside equity. A decline in  $E_I$  leads to a weakly higher amount of outside equity  $E_O^*$  being raised, and thus, a weakly higher expected ROE,  $r_E^* = c'_E(E_O^*)$  by convexity of  $c_E$ .

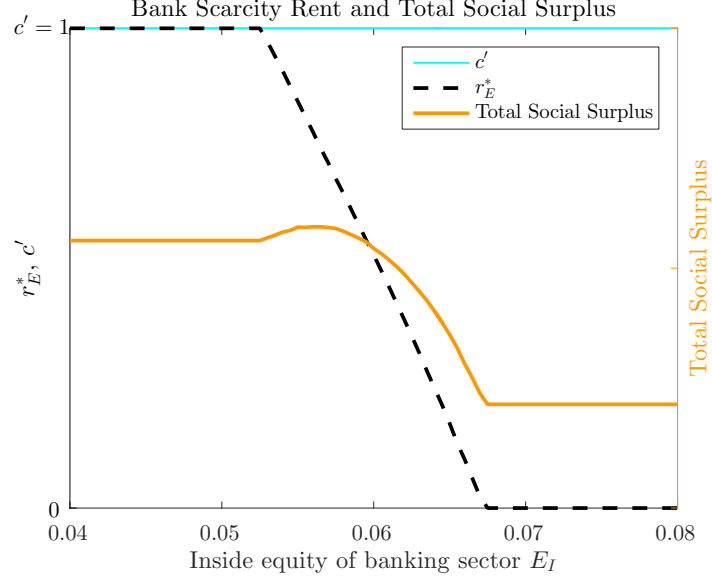
negative  $NPV$ . If the  $NPV$  is negative, a marginal reduction in bank funding has a positive impact on total social surplus. On the other hand, if the marginal borrower type has a positive  $NPV$  and is bank dependent ( $NPV < B$ ), social surplus falls. Finally, if the marginal borrower type is not bank dependent, social surplus is unaffected since firms of the marginal type can substitute to public market funding.

A *large* (non-local) decrease in bank equity capital will typically imply a reduction in bank financing of a whole *set* of marginal borrower types that have similar profitability rankings  $PI(q, \rho)$ , close to the cutoff value  $r_E^*$ . The overall effect on social surplus of such a discrete increase is then given by a corresponding weighted average of the above-discussed marginal effects. Overall, the set of marginal borrower types, which is crucially affected by the profitability indices  $PI(q, \rho)$ , is thus key for gauging the effects of shocks to bank capital.

If one interprets negative shocks to aggregate bank equity as typical for times of recessions then our model's predictions are also consistent with the main empirical findings of Santos and Winton (2008). Controlling for risk factors, firms with access to public debt markets pay lower spreads, and their spreads rise significantly less in recessions (when bank capital is particularly scarce).

**Parameterized example.** As in our previous example shown in Figure 1 of Section 3.2, we consider a stylized parameterization of the model with two aggregate states  $s \in \{L, H\}$ , which allows illustrating the qualitative effects of changes to aggregate bank capital with simple two-dimensional graphs. In the parameterization, borrower types only differ in terms of their state-contingent cash flows  $C_s(q, 1)$ . Thus, they share the same moral hazard parameters  $B(q)$  and risk-weights  $rw(\rho) = 1$ . In contrast to the previous simple example, we now consider a distribution of a large number of borrower types, which facilitates highlighting the various forces at play, and shows the tractability of our model. Rather than specifying the state-contingent cash flows of 100,000 borrower types by hand, we generate the distribution of borrower types from a bivariate log-normal distribution. This approach effectively produces a discrete approximation of a bivariate log-normal distribution of cash flows in the economy. Given such a discrete distribution, we then can simply use our analytical solutions to compute all equilibrium objects of interest (see the discussion at the end of Section 3.2).

Figures 2 and 3 illustrate how equilibrium outcomes vary with the level of inside equity  $E_I$ . Figure 2 plots the expected ROE  $r_E^*$  (black) and the total social surplus generated by firms funded in the economy (orange). It shows that the equilibrium ROE of the banking sector  $r_E^*$  is declining



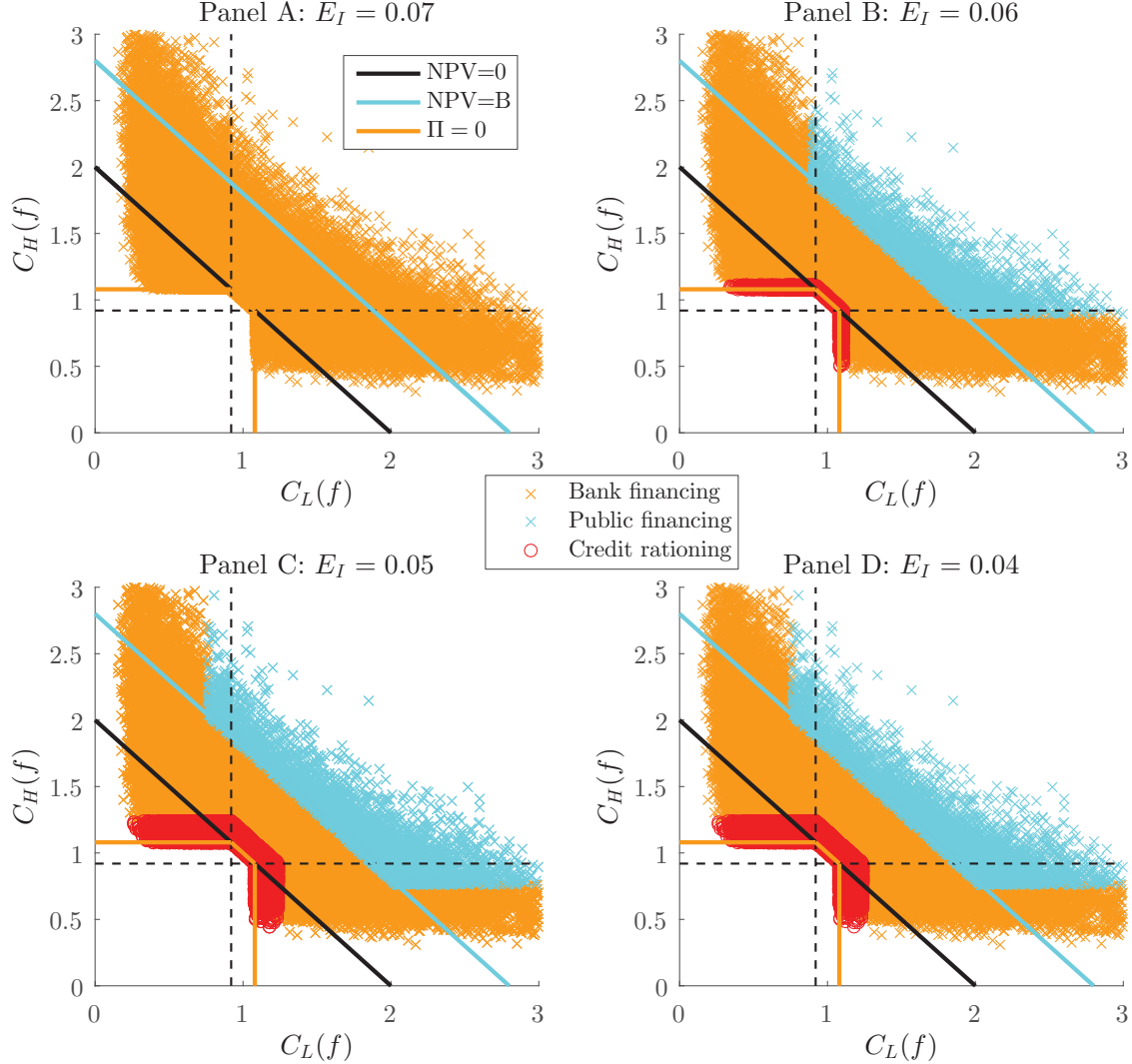
**Figure 2.** The figure plots equilibrium bank ROE and social surplus created by firms receiving credit as a function of aggregate bank inside equity  $E_I$ . Model parameters are chosen as follows: there are two aggregate states  $\{L, H\}$  with  $p_H = 0.5$  and 100,000 borrower types of equal mass with cash flows  $(C_L, C_H)$  drawn from a bivariate log-normal distribution with means  $(-0.1, 0.15)$ , variances  $(0.2, 0.1)$ , and a covariance of  $-0.1$ . All firms have a risk weight  $rw = 1$ ,  $B = 0.4$ , and  $I = 1$ . The capital requirement is  $\underline{e} = 0.08$ , and the marginal cost of outside equity is  $c'(E_O) \equiv 1$ .

in  $E_I$  (by Corollary 1), whereas the effect on total surplus is non-monotonic. The set of bank-funded firms is given by all borrower types  $(q, \rho)$  whose profitability index  $PI(q, \rho)$  is above  $r_E^*$ . Thus, in the two regions where  $r_E^*$  does not change with  $E_I$ , the composition of bank credit is also unaffected by changes in  $E_I$ .

While Figure 2 plots aggregate statistics as a function of  $E_I$ , Figure 3 illustrates the corresponding cross-sectional implications. In Panels A to D, we plot the equilibrium source of financing for the entire cross-section of borrowers for four distinct levels of bank capital  $E_I$ . A borrower type that receives funding in equilibrium is represented by either an orange cross (if funded by banks) or a blue cross (if funded by regular investors in public markets). Each cross indicates a borrower's state-contingent cash flows  $(C_L, C_H)$ . Borrower types that do not receive funding are represented by red circles. Figure 3 thus highlights which borrower types' funding depends on the capitalization of the banking sector. These affected borrower types must have profitability indices in the range between 0 and  $c'(E_O)$ . As previously discussed in reference to Figure 1, banks can follow three distinct strategy types in this type of economy — they either default in state  $L$ , they default in state  $H$ , or they default with zero probability. Risk-seeking banks thus effectively take



gamble with respect to aggregate risk, as they default in exactly one of the two aggregate states.<sup>46</sup>



**Figure 3.** The panels illustrate the cross-sectional distribution of borrower types and their access to finance for different values of aggregate bank inside equity  $E_I \in \{0.04, 0.05, 0.06, 0.07\}$ . Each borrower type is characterized by a cash flow tuple  $(C_L, C_H)$  corresponding to two aggregate states  $\{L, H\}$ . The graphs indicate only borrower types with  $\Pi(q, \rho) \geq 0$ . Model parameter values are detailed in the caption of Figure 2.

For levels of  $E_I$  below 0.053, the ROE is sufficiently high to incentivize banks to raise outside equity. Equity is issued up to the point where the ROE and the marginal cost of raising outside equity are equalized. Since the marginal cost of raising outside equity  $c'(E_O)$  is chosen to be

<sup>46</sup>The particular distribution of borrower types we consider implies that a significant fraction of banks default even in state  $H$ . The specification of the distribution of borrower types could be easily changed such that bank default rates in state  $H$  are very low. However, the goal of the example is to facilitate a discussion of the qualitative forces, not to obtain a realistic calibration.

constant in the parameterization, the equilibrium ROE is invariant to  $E_I$  for low levels of  $E_I$  (see Figure 2). In this region, the total social surplus created by funded borrowers (orange line) is also independent of the level of inside equity, since banks always end up with the same amount of total equity and the same aggregate assets.<sup>47</sup> Thus, the cross-sectional distribution of borrowers looks identical for  $E_I = 0.04$  and  $E_I = 0.05$ , as shown in Panels C and D of Figure 3. Banks use their scarce capital to focus on the funding of two broad groups of firms: (1) firms that do not have access to public markets (located below the  $NPV = B$  line) and (2) firms with large cash flow risks, that is, with highly asymmetric payoffs across aggregate states. These two groups correspond to banks' two competitive advantages — their monitoring ability and their access to cheap deposits irrespective of the riskiness of their asset holdings. The second advantage leads bank finance to crowd out regular public market investors for some borrower types that have access to public markets. Borrowers marked with red circles in Figure 3 are credit rationed, as they do not have access to public markets and rank low in terms of their profitability indices. Note that credit rationing obtains both for firms with negative NPV projects (below the solid black line) and for firms with positive NPV projects (above the solid black line).

For levels of inside equity  $E_I$  above 0.053, banks have sufficient capital to fund borrower types with profitability indices  $PI(q, \rho) < c'(0)$  such that banks do not raise costly outside equity. In this region, further increases in  $E_I$  lead to lower bank ROEs, since expansions in banks' aggregate loan book lead to declines in the profitability of the marginally funded borrower type (see Figure 2). The ROE bottoms out at zero for  $E_I \geq 0.067$ , as banks always prefer to hold cash or pay dividends instead of extending loans to firms with negative profitability. Increases in  $E_I$  beyond 0.067 thus have no effect on banks' ROE and the set of firms in the economy receiving credit.

However, before reaching  $E_I = 0.067$ , banks' loan book expansion first tends to increase total surplus (for  $0.053 < E_I < 0.057$ ) and then to decrease it (for  $0.057 < E_I < 0.067$ ).<sup>48</sup> For  $0.057 < E_I < 0.067$ , marginal borrower types have a negative NPV on average, such that increases in equity capital tend to be used to expand lending to surplus-destroying borrower types, reducing allocative efficiency. Figure 3 illustrates why the marginal borrower types tend to have a negative NPV in this region. At  $E_I = 0.06$  (Panel B), credit rationed firms indicated by red circles are mostly below the black solid line, and thus, have a negative NPV. Hence, an increase in equity from  $E_I = 0.06$  (Panel B) to  $E_I = 0.07$  (Panel A), which alleviates the credit rationing of these

<sup>47</sup>If the cost of raising equity was strictly convex, the aggregate assets would be affected, too.

<sup>48</sup>Note that the NPV of the marginal borrower type generally switches multiple times between positive and negative values when  $E_I$  is gradually increased. As the mass of each borrower type is small in the parameterization considered in Figure 2, these switches are not visible in the graph as positive and negative slopes for the orange line — the graph effectively smoothes out the NPVs of marginal borrower types within narrow regions of  $E_I$ .

borrower types, leads to a net-decline in total surplus.

## 4.2. Changes to Capital Ratio Requirements

**Channels of capital requirements.** Changes to capital ratio requirements affect equilibrium outcomes through two channels — a skin-in-the-game channel and a balance-sheet channel. Increases in ratio requirements have a positive effect on allocative efficiency through the skin-in-the-game channel: a system-wide increase in  $\underline{e}$  lowers the implicit deposit insurance subsidy for each borrower type  $\sigma(q, \rho)$  (see equation 16), and thus, weakly improves the alignment of banks' private ranking of borrowers based on  $PI(q, \rho)$  with the social ranking based on  $NPV(q)$ . On the other hand, increases in ratio requirements have an ambiguous effect on allocative efficiency through the balance sheet channel: imposing stricter limits on leverage causes bank balance sheets to contract (weakly), which can have either a positive, neutral, or negative effect on allocative efficiency. The balance sheet effect is positive if the marginal borrower types have negative NPV projects, since these borrower types will become credit-rationed. The effect is neutral when bank capital is not scarce, or when marginal borrower types that become credit rationed by banks have access to public markets (that is, when  $NPV(\tilde{q}) > B(\tilde{q})$ ). Finally, the effect is negative when marginal borrower types are bank dependent and have positive NPV projects, that is, for marginal borrower types  $\tilde{q}$  where  $0 < NPV(\tilde{q}) < B(\tilde{q})$ .

Overall, the allocative effects of changes to capital ratio requirements thus generally depend on the distribution of borrower types, and in particular, the marginal borrower types. Only when bank capital is not scarce do marginal increases in capital requirements always have a weakly positive effect on allocative efficiency, since only the skin-in-the-game channel is operational.

**Proposition 4** *The following comparative statics with respect to capital ratio requirements  $\underline{e}$  apply:*

1. *An increase in capital ratio requirements from  $\underline{e}$  to  $\underline{e} + \varepsilon$  (with  $\varepsilon > 0$ )*
  - (a) *weakly increases loan yields for all borrowers if bank capital is not scarce before the increase in capital ratio requirements,*
  - (b) *weakly decreases aggregate investment,*
  - (c) *weakly increases total social surplus if bank capital is not scarce after the increase.*

2. For  $PI(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*}) > c'(E_O^*)$ , a marginal increase in  $\underline{e}$  is compensated by additional equity issuances, leaving aggregate bank funding unchanged.

For  $PI(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*}) \leq c'(E_O^*)$ , a marginal increase in  $\underline{e}$  strictly reduces the fraction of firms of the marginal type  $(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})$  that receive bank funding.

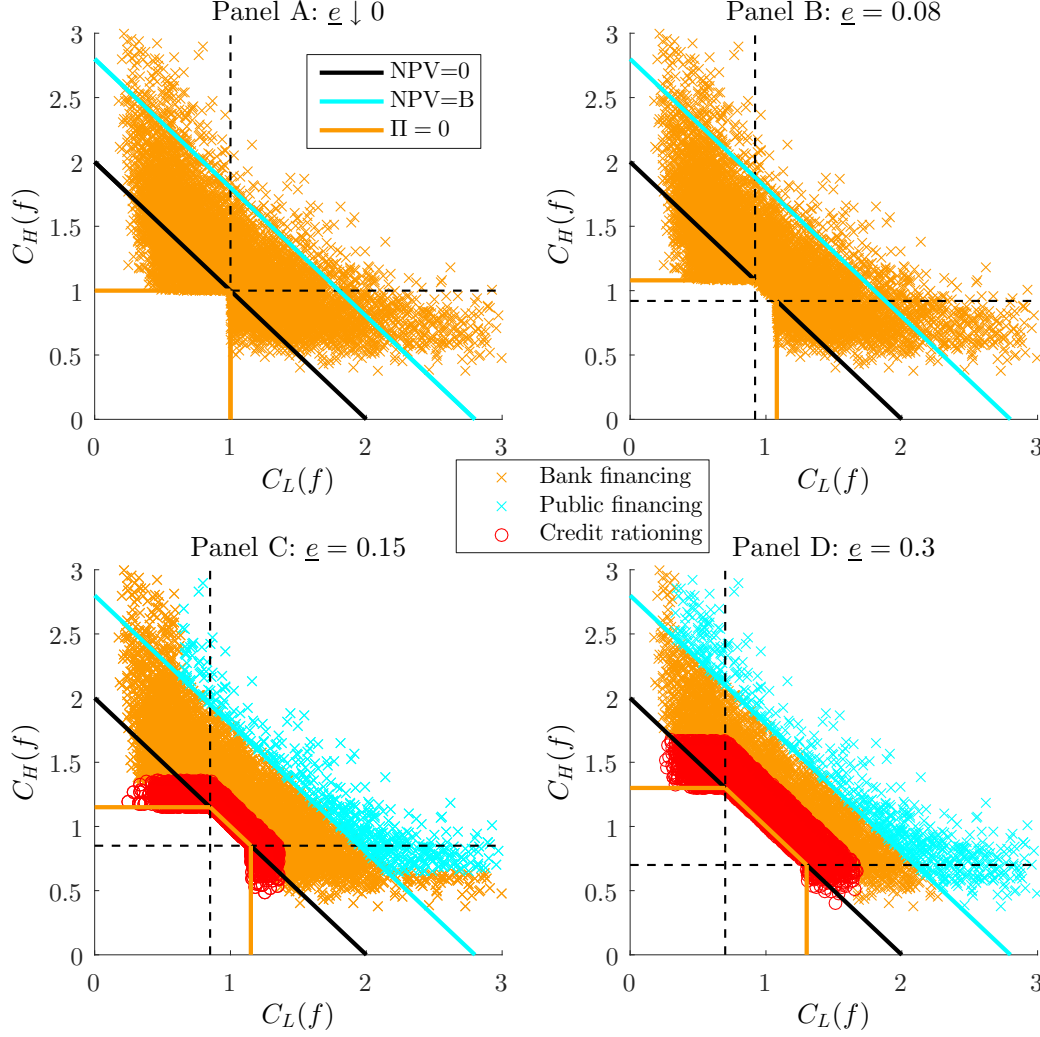
(a) If  $NPV(\tilde{q}_{E_O^*}) < B(\tilde{q}_{E_O^*})$  and  $NPV(\tilde{q}_{E_O^*})$  is negative (positive), this reduction in bank funding has a positive (negative) impact on total social surplus.

(b) If  $NPV(\tilde{q}_{E_O^*}) \geq B(\tilde{q}_{E_O^*})$  social surplus is unaffected.

If bank capital is not scarce *before* an increase in capital ratio requirements, loan rates to borrowers increase through two channels reflected in the loan pricing condition (15). First, the ROE of the banking sector must weakly increase (since  $\bar{r}_E = 0$  before the increase), which enables banks to capture a higher fraction of the private surplus  $\Pi$ . Second, the subsidy  $\sigma(q, \rho)$  is reduced (*strictly* so for all borrowers with  $\sigma(q, \rho) > 0$ ), such that the total private surplus  $\Pi$  is reduced. The second statement of Proposition 4 must hold in the aggregate. However, it masks compositional aspects. It is possible that some borrower types' credit rationing gets alleviated rather than worsened. (In Example 1, a change in capital requirements from  $\underline{e} < 30\%$  to  $\underline{e} > 30\%$  may increase credit extension to *good* borrowers as the relative profitability ranking of good and bad types is reversed). Third, if bank capital is not scarce after the increase in capital ratio requirements, all positive NPV projects in the economy must receive funding (either from banks or from investors in public markets). As a result, the firms that are shed by banks must have had negative NPV projects, such that total social surplus increases.

However, if bank capital is scarce after the increase in capital ratio requirements, the local effects of changes in  $\underline{e}$  on allocative efficiency depend on the profitability index, the  $NPV$ , and the bank dependence of the marginal borrower type  $(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})$ . The impact on social surplus is then analogous to the results discussed in the context of the effects of changes to bank equity (Section 4.1) — how potential reductions in the funding of the marginal borrower type affect social surplus again depends on the marginal borrower type's  $NPV$  and its bank dependence.

**Parameterized example.** We illustrate these general insights with a parameterization of the model, similar to the one considered in the previous subsection. For pedagogical reasons, we initially consider an economy where raising outside equity is prohibitively costly. Moreover, all bank loans are again subject to the same risk weight  $rw = 1$ . Figure 4 plots the cross-sectional distribution of borrower types and their access to finance for different capital requirement regimes



**Figure 4.** The panels illustrate the cross-sectional distribution of borrower types and their access to finance for different values of capital ratio requirements  $\underline{\epsilon} \in \{0, 0.1, 0.3, 0.7\}$ . Model parameters are chosen as follows: there are two aggregate states  $\{H, L\}$  with  $p_H = 0.5$  and 10,000 borrower-types of equal mass with cash flows  $(C_L, C_H)$  drawn from a bivariate log-normal distribution with means  $(0.15, -0.1)$ , variances  $(0.1, 0.2)$ , and a covariance of  $-0.1$ . All firms have a risk weight of 1,  $B = 0.4$ , and  $I = 1$ . Inside equity is  $E_I = 0.08$ , and the marginal cost of outside equity is  $c'_E(E_O) = 100$ . The graphs indicate only borrower types with  $\Pi(q, \rho) \geq 0$ .

in the economy (see the caption for additional details). Capital ratio requirements  $\underline{\epsilon}$  are increasing from Panel A to Panel D.

Panel A plots outcomes for an economy where capital ratio requirements are infinitely lax, that is, for  $\underline{\epsilon} \downarrow 0$ . In such an environment, banks providing credit to risky borrowers finance themselves with infinite leverage so that any potential losses on their loan book are absorbed by the government through bailouts of depositors. Since banks do not internalize credit risk, all borrowers with a cash flow greater than  $I = 1$  in at least one state of the world obtain bank

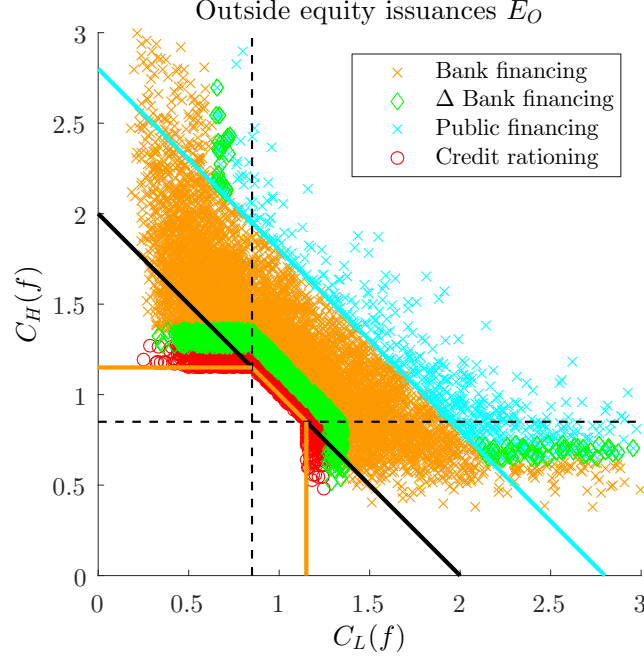
finance at a yield  $y = 0$ . Thus, there is substantial overinvestment in the economy and firms' credit spreads become completely uninformative about credit risk. Banks are the marginal investors for all loans and crowd out regular public market investors, who cannot compete at these distorted yields.

In Panel B of Figure 4 capital ratio requirements are increased to  $\underline{e} = 0.1$ . In the parameterization, this increase in  $\underline{e}$  does not cause bank capital to become scarce. Thus, all statements of Proposition 4 apply for the change from Panel A to Panel B. Only the skin-in-the-game effect operates, and allocative efficiency strictly improves — the  $\Pi = 0$  line (orange) shifts outwards, graphically illustrating the better “alignment” of private surplus with social surplus. Only negative-NPV borrower types are credit rationed, which are located below the solid black line (see part 3 of Proposition 4).

In Panel C, capital ratio requirements are further increased to  $\underline{e} = 0.3$ . Now, bank capital is scarce, such that banks cannot fund all borrower types that are privately profitable, which are located above the solid orange line. We indicate these rationed borrower types with red circles. Note that a significant fraction of these rationed borrowers lie above the black solid line and thus have a positive NPV. As a result, overall surplus created by firms in the economy declines in response to the increase in capital ratio requirements from  $\underline{e} = 0.1$  to  $\underline{e} = 0.3$ , even though there are also some negative-NPV borrowers that become credit rationed after the increase in  $\underline{e}$ .

Finally, in Panel D, capital ratio requirements are  $\underline{e} = 0.7$ . Relative to Panel C, two main changes occur: first, the set of credit-rationed borrower types becomes larger — banks fund almost no negative NPV borrowers, but many bank dependent borrowers with positive NPV projects are also rationed. Second, a greater fraction of the debt issued by risky borrowers with access to public markets (above the blue line) is held by public market investors rather than by banks. The latter shift does not affect social surplus, as these borrower types still receive funding after capital requirements are increased to  $\underline{e} = 0.7$ .

**Outside equity issuances.** While the parameterization considered in Figure 4 presumes that equity issuances are prohibitively costly, this assumption affects equilibrium outcomes only when bank capital is scarce. Thus, the possibility of equity issuances is only relevant in Panel C ( $\underline{e} = 0.3$ ), where banks are not funding all borrower types they consider to be profitable. In Figure 5 we therefore replicate the economy for  $\underline{e} = 0.3$  and sufficiently decrease equity issuance cost to create incentives for banks to raise outside equity. The newly funded borrower types (relative to Figure 4.C) are indicated by green diamonds in Figure 5. The graph illustrates why the effects of such



**Figure 5.** The graph illustrates the effects of equity issuances on the cross-sectional distribution of borrower types and their access to finance for the case of  $\underline{\epsilon} = 0.3$  (corresponding to Panel C of Figure 4). Each borrower type is characterized by a cash flow tuple  $(C_L, C_H)$  corresponding to two aggregate states  $\{L, H\}$ . Outside equity issuance cost are given by  $c(E_O) = 0.2 \cdot E_O$ . All other model parameter values are detailed in the caption of Figure 4.

equity issuances on allocative efficiency are generally ambiguous. First, banks use the newly available funds to provide loans to risky negative-NPV firms, thereby exacerbating overinvestment. Second, banks also extend more loans to bank-dependent, positive NPV firms that were previously rationed, absent equity issuances. Third, banks use their access to cheap deposit financing to crowd out regular public market investors. The net effect of equity issuances on total social surplus thus depends on the cross-sectional distribution of borrower types and the magnitude of equity issuance cost.

**Aggregate bank capital and capital ratio requirements.** While our paper does not aim to design optimal bank regulation, such analyses typically are also concerned with the effects of capital requirements on real activity.<sup>49</sup> In particular, our results suggest that procyclical capital requirements, as mandated by Basel III, are appropriate when the level of bank capital varies significantly over the business cycle (as argued by Kashyap and Stein, 2004).<sup>50</sup> In good times,

<sup>49</sup>Additionally, optimal bank regulation design would have to account for distortions caused by taxation required to fund government bailouts and bank default externalities. See Bhattacharya et al. (1998) and Thakor (2014) for survey articles on the literature analyzing optimal bank regulation.

<sup>50</sup>See also Malherbe (2015).

when bank capital is high relative to investment opportunities, underinvestment is less of a concern, and hence higher capital requirements have primarily a positive effect through the skin-in-the-game channel. In contrast, in bad times when capital is scarce, underinvestment is a first-order concern warranting lower capital requirements, even when such looser regulations incentivize an expansion in risk-taking.

### 4.3. Interest Rate Pass-through

One of the puzzles in the aftermath of the recent financial crises has been that bank lending has been sluggish despite the fact that interest rates have reached record lows. In this section, we analyze how changes in the interest rate agents earn on cash holdings (e.g., when depositing money with a central bank or investing in international safe assets) are passed through to various borrower types in the economy depending on the capitalization of the banking sector. In particular, we now consider the case where agents, including depositors, have access to a cash investment opportunity that yields a safe return of  $r_f \geq 0$  compared to the baseline case of  $r_f = 0$  that we considered so far. As a result, competitive depositors now demand a deposit rate of  $r_D = r_f$ ,<sup>51</sup> and a bank's expected return on equity (ROE) can be represented as follows:

$$r_E(\mathbf{x}, e) \equiv \mathbb{E} \left[ \max \left\{ r_f + \frac{r_A^s(\mathbf{x}) - r_f}{e}, -1 \right\} \right]. \quad (26)$$

In survival states, a bank's return on equity is thus equal to the sum of the return on cash  $r_f$  and the levered excess return on the asset portfolio.

If a borrower type is funded by regular investors in public markets, (marginal) pass-through,  $\frac{d\mathbb{E}[r^s(q, \rho)]}{dr_f}$ , is always equal to one, since equilibrium yields for such a borrower type are implied by the pricing condition

$$\mathbb{E}[r^s(q, \rho)] = r_f. \quad (27)$$

Yet, when a borrower type is funded by banks, its cost of capital reflects a premium for the use of scarce bank capital and a discount due to banks' implicit funding subsidies. In the presence of  $r_f \geq 0$ , the cost of capital expression in (15) generalizes to

$$\mathbb{E}[r^s(q, \rho)] = r_f + \underline{e}(\rho)(r_E^* - r_f) - \sigma(q, \rho), \quad (28)$$

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<sup>51</sup>Our analysis could be extended to account for a sluggish demand response of depositors to changes in deposit rates. See, e.g., Drechsler et al. (forthcoming).



where the private value of bailout guarantees is now given by:

$$\sigma(q, \rho) = \mathbb{E} \left[ \max \left\{ (1 + r_f)(1 - e(\rho)) - \frac{C_s(q, 1)}{I}, 0 \right\} \right] \geq 0. \quad (29)$$

To keep the analysis parsimonious we focus on marginal changes in interest rates, presuming that they leave banks' optimal equity issuances and ranking of borrowers unaffected. Using equation (28), we can measure the pass-through of interest rate changes for bank-funded borrowers by:

$$\frac{d\mathbb{E}[r^s(q, \rho)]}{dr_f} = \underline{e}(\rho) \cdot \frac{dr_E^*}{dr_f} + (1 - \underline{e}(\rho)) \cdot p_{sol}(q, \rho), \quad (30)$$

where we define

$$p_{sol}(q, \rho) = \Pr \left[ \frac{C_s(q, 1)}{I(1 - \underline{e}(\rho))} \geq (1 + r_f) \right] \quad (31)$$

as the probability with which a bank specializing in borrower type  $(q, \rho)$  remains solvent, that is, the probability with which the borrower type's gross-return on investment  $C_s/I$  times the ratio of bank assets to bank deposits,  $1/(1 - e) = A/D$ , exceeds the promised gross-return on deposits  $(1 + r_f)$ .

Equation (30) reveals that pass-through depends on loan-specific equity ratio requirements. To the extent that a bank has to finance a loan investment with equity (that is,  $\underline{e}(\rho)$ ), pass-through is determined by the response of banks' common expected return on equity to a change in  $r_f$ , which is further discussed below. To the extent that a bank can finance a loan investment with debt (that is,  $(1 - \underline{e}(\rho))$ ), pass-through is given by the probability with which a bank specializing in the borrower type remains solvent — pass-through is absent to the extent that marginal increases in the required payments to debt holders are covered by government bailouts.

Equation (30) reveals that pass-through for bank borrowers is always below one when pass-through for the expected return on bank equity,  $\frac{dr_E^*}{dr_f}$ , is weakly lower than one. The following proposition characterizes pass-through for banks' expected return on equity  $r_E^*$ .

**Proposition 5** *Pass-through of marginal changes in the interest rate  $r_f$  to banks' expected return on equity  $r_E^*$  depends on the scarcity of aggregate bank capital and the bank-dependence of the marginal borrower type.*

1.  $\frac{dr_E^*}{dr_f} = 1$  if bank capital is not scarce.
2.  $\frac{dr_E^*}{dr_f} < 0$  if bank capital is scarce and the marginal borrower type  $(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})$  is bank dependent,

that is,  $NPV(\tilde{q}_{E_O^*}) < B(\tilde{q}_{E_O^*})$ .

3.  $\frac{dr_E^*}{dr_f} > 1$  if bank capital is scarce and the marginal borrower type  $(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})$  is not bank dependent, that is,  $NPV(\tilde{q}_{E_O^*}) \geq B(\tilde{q}_{E_O^*})$ .

Proposition 5 provides sharp predictions on how pass-through  $\frac{dr_E^*}{dr_f}$  varies depending on whether bank capital is scarce and whether the banking sector's marginal borrower is bank dependent. Pass-through for banks' ROE is only greater than one when bank capital is scarce and the marginal borrower type is not bank dependent. Only in this case, pass-through for some borrower types' may be greater than one, in particular for those borrower types that command higher capital requirements than the marginal borrower type (see Proof of Proposition 5). Pass-through for banks' ROE is equal to one when bank capital is not scarce, as in that case banks' ROE is always equal to  $r_f$ . Finally pass-through for banks' ROE is below zero when bank capital is scarce and the marginal borrower type is bank dependent — as banks already extract all rents from marginal borrowers that are bank dependent, banks cannot pass on rate increases to these borrowers. As a result, banks' ROE falls in response to interest rate increases.

Overall, these predictions suggest that pass-through for bank borrowers in the years following the financial crisis may have been particularly low because bank capital has been scarce and bank-dependent borrowers have been the marginal borrowers of the banking sector.<sup>52</sup>

## 5. Conclusion

Following the recent crises in the United States and Europe, a substantive empirical literature has highlighted the complex behavior of the composition of credit and its importance for macroeconomic stability and efficiency. Nonetheless, the literature to date lacks tractable frameworks that can simultaneously explain the risk-taking behavior by banks in the years leading up to the financial crisis (see Jiménez et al. (2014)) and the credit crunch in its aftermath (see Ivashina and Scharfstein (2010)). To this end, we develop a general equilibrium framework in which banks can play both a socially valuable role by alleviating credit frictions of bank-dependent borrowers and a parasitic role by engaging in excessive risk-taking that exploits public subsidies of banks' debt financing cost. The key feature of our model is that it can flexibly capture cross-sectional

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<sup>52</sup>As noted before in the discussion of Definition 3 in Section 3.2, equity ratios by themselves generally do not reveal the extent to which bank capital is scarce. Instead, total bank equity in the economy, capital requirements, and the total supply of profitable bank investment opportunities jointly determine whether bank capital is scarce.

borrower heterogeneity along multiple dimensions known to be essential for credit decisions, such as risk characteristics, profitability, and bank dependence.

Our model yields a parsimonious intermediary asset pricing condition for a firms' cost of capital. Relative to a frictionless benchmark, a firm's cost of capital reflects a premium for a loan's use of scarce bank capital and a discount due to implicit government subsidies of banks' (debt) funding cost. As a result of these financing distortions, economies typically exhibit simultaneously over- and underinvestment in different parts of the borrower distribution. While relatively safe, bank-dependent borrower types are credit rationed, some surplus-destroying high-risk borrowers obtain funding from risk-seeking banks. We characterize how these cross-sectional misallocations are linked to the current capitalization of the banking sector and how they are affected by shocks to the financial system, such as capital injections, changes in capital ratio requirements, or changes in interest rates (e.g., via monetary policy). In particular, our model sheds light on heterogeneity in interest rate pass-through across borrower types and may explain why pass-through was relatively low in the aftermath of the Great Recession.

We believe that our framework suggests several fruitful avenues for future research. First, our framework could be applied to micro-level data of an entire economy, as studied in recent papers by e.g., Jiménez et al. (2012) and Iyer et al. (2014). By viewing the empirical results through the lens of our framework, it may be possible to address external validity concerns, that is, how empirical results obtained from a particular country during a specific episode extend to other countries and other time periods.

Second, to inform the debate on optimal financial regulation, it may be useful to analyze the impact of alternative financial regulations, going beyond the standard regulatory interventions that we consider for our positive analysis of the composition of credit. Further, the model could be extended to incorporate shadow banking activities, that is, the practice of moving assets and liabilities off the regulated balance-sheet in an effort to circumvent capital regulations.<sup>53</sup> A framework of the cross-section of credit like ours could help predict which types of assets are moved off balance-sheet in response to new regulations, and how the composition of credit is affected by regulatory arbitrage. Similarly, the framework could inform the financial sector's incentives to design securities to minimize regulatory burdens.

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<sup>53</sup> During the financial crisis, government support was provided not only to traditional banks, but also to investment banks, insurance companies, and other institutions in the shadow banking system, in particular money market mutual funds. See Gennaioli et al. (2013), Ordóñez (2014), Plantin (2014), and Moreira and Savov (forthcoming) for theories of shadow banking.

## A. Appendix

### A.1. Proof of Lemma 1

First, we show that if  $NPV(q) < B(q)$ , the borrower cannot raise financing under any contract. Assumption 1.2 implies that public financing requires high effort, i.e.,  $a = 1$ . If the borrower exerts effort, the maximum value of the borrower's stake is given by  $NPV(q)$ , since the IR constraint and investor competition imply that investors' expected payoff is equal to  $I$ , and  $NPV(q)$  is equal to the difference between the firm's total expected payoff  $\mathbb{E}[C_s(q, 1)]$  and  $I$ . Second, as reflected by the IC constraint, the borrower's payoff under shirking is bounded from below by  $B(q)$ , due to limited liability. Hence, if  $NPV(q) < B(q)$ , it is impossible to jointly satisfy IC and IR.

We next show that whenever  $NPV(q) \geq B(q)$ , the borrower can raise financing with a debt contract that gives all surplus to the borrower, which also proves the optimality of debt. Set  $CF_s = FV$  for all  $s$ . Then IR implies that  $FV \geq I$ . Moreover, using Assumption 1.3, we obtain that  $\mathbb{E}[\max\{C_s(q, 0) - CF_s, 0\}] = 0$  and the right hand side of IC achieves the lower bound  $B(q)$  under any debt contract that satisfies IR. Since investors are competitive, the face value of debt is set such that IR binds, so that the borrower's payoff is  $NPV(q)$ . We have thus proven that whenever  $NPV(q) \geq B(q)$ , there exists a debt contract that satisfies IR and allows the borrower to extract the entire NPV.

### A.2. Proof of Lemma 2

We consider a bank's individually optimal portfolio choice, taking as given state-dependent returns on securities  $r^s(q, \rho)$ . The analysis is in partial equilibrium and presumes that a bank faces a perfectly elastic supply of securities that yield these returns. In addition, banks can only make positive investments in loans  $x(q, \rho) \geq 0$ .

Recall a bank's inner (ROE) maximization problem given exogenous returns:

$$\max_{e, \mathbf{x}} r_E(\mathbf{x}, e) \quad \text{s.t. } e \geq e_{\min}(\mathbf{x}), \quad (32)$$

where we define

$$r_E(\mathbf{x}, e) = \frac{1}{e} \mathbb{E}[\max\{r_A^s(\mathbf{x}), -e\}].$$

**Leverage.** Taking the partial derivative of  $r_E(\mathbf{x}, e)$  w.r.t.  $e$  yields

$$\frac{\partial r_E(\mathbf{x}, e)}{\partial e} = -\frac{1}{e^2} \mathbb{E}[\max\{r_A^s(\mathbf{x}), -e\}] - \frac{1}{e} \Pr[r_A^s(\mathbf{x}) < -e].$$

Note that if  $r_E(e, \mathbf{x}) > 0$  for some  $(e, \mathbf{x})$  then it must be the case that

$$\mathbb{E} [\max \{r_A^s(\mathbf{x}), -e\}] > 0.$$

It follows that  $\frac{\partial r_E(\mathbf{x}, e)}{\partial e} < 0$  if  $r_E(\mathbf{x}, e) > 0$ . Further, if  $r_E(\mathbf{x}, e) = 0$  then  $\frac{\partial r_E(\mathbf{x}, e)}{\partial e} < 0$  as long as there is one state  $s$  with positive probability, where the bank defaults, that is,  $\Pr[r_A^s(\mathbf{x}) < -e] > 0$ .

Thus, for any choice  $(\mathbf{x}, e)$  that yields  $r_E(\mathbf{x}, e) > 0$  it is optimal to decrease  $e$  at the margin, unless the constraint  $e \geq e_{\min}$  is already binding. Since decreasing  $e$  increases  $r_E(\mathbf{x}, e)$ , the condition  $r_E(\mathbf{x}, e) > 0$  remains satisfied after any decrease in  $e$ . Thus, for any  $(\bar{\mathbf{x}}, \bar{e})$  such that  $r_E(\bar{\mathbf{x}}, \bar{e}) > 0$  it is the case that  $\arg \max_e r_E(\bar{\mathbf{x}}, \bar{e}) = e_{\min}$ .

Further, for any choice  $(\mathbf{x}, e)$  that yields  $r_E(\mathbf{x}, e) = 0$  and  $\Pr[r_A^s(\mathbf{x}) < -e] > 0$ , marginally decreasing  $e$  also increases  $r_E$  (provided such a decrease is feasible, that is, the constraint  $e \geq e_{\min}$  is not already binding). Since marginally decreasing  $e$  increases  $r_E(\mathbf{x}, e)$  (maintaining the condition that  $r_E(\mathbf{x}, e) \geq 0$ ) and weakly enlarges the set of default states (maintaining  $\Pr[r_A^s(\mathbf{x}) < -e] > 0$ ), it is optimal to decrease  $e$  until the constraint  $e \geq e_{\min}$  is binding. Formally, for any  $(\bar{\mathbf{x}}, \bar{e})$  such that  $r_E(\bar{\mathbf{x}}, \bar{e}) = 0$  it is the case that  $\arg \max_e r_E(\bar{\mathbf{x}}, \bar{e}) = e_{\min}$  if  $\Pr[r_A^s(\bar{\mathbf{x}}) < -e] > 0$ .

This concludes the proof of the two statements about optimum leverage.

**Portfolio choice.** The analysis in the previous paragraph implies that it is optimal for banks to choose  $e = e_{\min}$  as long as there exists a portfolio  $\mathbf{x}$  such that  $r_E(\mathbf{x}, e) > 0$ , or  $r_E(\mathbf{x}, e) = 0$  and  $\Pr[r_A^s(\mathbf{x}) < -e] > 0$ .

Presume that such a portfolio  $\mathbf{x}$  exists and that banks (optimally) choose  $e = e_{\min}$ . Then we can re-write the expected return on a bank's book equity as follows:

$$r_E(\mathbf{x}, e_{\min}) = \mathbb{E} \left[ \max \left\{ \frac{\sum_{q, \rho} x(q, \rho) r^s(q, \rho)}{\sum_{q, \rho} x(q, \rho) \underline{e}(\rho)}, -1 \right\} \right] \quad (33)$$

$$= \mathbb{E} \left[ \max \left\{ \sum_{q, \rho} \frac{r^s(q, \rho)}{\underline{e}(\rho)} \frac{x(q, \rho) \underline{e}(\rho)}{\sum_{\tilde{q}, \tilde{\rho}} x(\tilde{q}, \tilde{\rho}) \underline{e}(\tilde{\rho})}, -1 \right\} \right]. \quad (34)$$

Defining  $w(q, \rho) = \frac{x(q, \rho) \underline{e}(\rho)}{\sum_{\tilde{q}, \tilde{\rho}} x(\tilde{q}, \tilde{\rho}) \underline{e}(\tilde{\rho})} \in [0, 1]$  for all  $(q, \rho)$  as the new choice variables we obtain:

$$r_E(\mathbf{w}) = \mathbb{E} \left[ \max \left\{ \sum_{q, \rho} w(q, \rho) \frac{r^s(q, \rho)}{\underline{e}(\rho)}, -1 \right\} \right]$$

Maximizing subject to the constraint that  $\sum_{q, \rho} w(q, \rho) = 1$  and  $w(q, \rho) \geq 0$  (short-sales constraint), we obtain for all  $(q, \rho)$  with  $w^*(q, \rho) > 0$  the following condition at the optimum:

$$\frac{\partial r_E(\mathbf{w})}{\partial w(q, \rho)} = \nu,$$

where  $\nu$  is the Lagrange multiplier on the constraint  $\sum_{q,\rho} w(q,\rho) = 1$ . Further, we can write:

$$\frac{\partial r_E(\mathbf{w})}{\partial w(q,\rho)} = \mathbb{E} \left[ \frac{r^s(q,\rho)}{\underline{e}(\rho)} \mathbf{1}_{\{r_A^s > -e_{\min}\}} \right] = \mathbb{E} \left[ \frac{r^s(q,\rho)}{\underline{e}(\rho)} \middle| s : r_A^s > -e_{\min} \right] \cdot \Pr[s : r_A^s > -e_{\min}]. \quad (35)$$

Thus, we obtain at the optimum the following condition for all  $(q,\rho)$  with  $w^*(q,\rho) > 0$  (and thus  $x^*(q,\rho) > 0$ ):

$$\frac{\mathbb{E}[r^s(q,\rho) | s : r_A^s(\mathbf{x}^*) > -e_{\min}]}{\underline{e}(\rho)} = \frac{\nu}{\Pr[s : r_A^s > -e_{\min}]} = k. \quad (36)$$

**Correlated down-side risks.** First, note that we established above that for any optimal choice  $(\mathbf{x}^*, e^*)$  the expected asset return conditional on bank survival scaled by  $\underline{e}(\rho)$  is identical across firm types  $(q,\rho)$  with  $x^*(q,\rho) > 0$ . Suppose there is a type  $(\tilde{q}, \tilde{\rho})$  with  $x^*(\tilde{q}, \tilde{\rho}) > 0$  in the optimal portfolio that yields  $r^s(\tilde{q}, \tilde{\rho}) > -\underline{e}(\tilde{\rho})$  in some state  $s$  where the bank defaults, that is, where  $\sum_{q,\rho} x^*(q,\rho) r^s(q,\rho) < -e_{\min}$ . Then the bank could obtain a higher expected return on equity  $r_E > r_E(\mathbf{x}^*, e^*)$  by investing only in this asset  $(\tilde{q}, \tilde{\rho})$ , as it not only yields the same expected levered return across previous survival states (under the previous policy  $(\mathbf{x}^*, e^*)$ ) but also allows the bank to survive in at least one additional state  $s$ , yielding equity holders a levered return greater than  $-1$  in that state. Thus, any loan to a firm of type  $(q,\rho)$  that has a strictly positive weight in an optimal portfolio (that is,  $x^*(q,\rho) > 0$ ) can obtain returns  $r^s(q,\rho) > -\underline{e}(\rho)$  only in states  $s$  in which the bank survives.

Conversely, suppose  $\mathbf{x}^*$  is an optimal portfolio and there is an asset of type  $(\tilde{q}, \tilde{\rho})$  in the optimal portfolio with a strictly positive weight ( $x^*(\tilde{q}, \tilde{\rho}) > 0$ ) that yields  $r^{\tilde{s}}(\tilde{q}, \tilde{\rho}) \leq -\underline{e}(\tilde{\rho})$  in some state  $\tilde{s}$  where the bank survives and has strictly positive equity value, that is, where  $\frac{\sum_{q,\rho} x^*(q,\rho) r^{\tilde{s}}(q,\rho)}{e_{\min}} = \sum_{q,\rho} w^*(q,\rho) \frac{r^{\tilde{s}}(q,\rho)}{\underline{e}(\rho)} > -1$ . Then it must be the case that in this survival state  $\tilde{s}$  other assets in the portfolio yield  $\frac{r^{\tilde{s}}(q,\rho)}{\underline{e}(\rho)} > -1$ , otherwise the bank would default in that state. For notational simplicity define the set of states where the bank survives under policy  $(\mathbf{x}^*, e_{\min}(\mathbf{x}^*))$  as  $\Sigma_S(\mathbf{x}^*, e_{\min}(\mathbf{x}^*)) = \{s : r_A^s(\mathbf{x}^*) > -e_{\min}(\mathbf{x}^*)\}$ . We showed above that

$$\mathbb{E} \left[ \frac{r^s(q,\rho)}{\underline{e}(\rho)} \middle| \Sigma_S \right] = k \quad (37)$$

for all  $(q,\rho)$  with  $x^*(q,\rho) > 0$ . However, since asset  $(\tilde{q}, \tilde{\rho})$  performs worse than other assets in the portfolio in state  $\tilde{s}$ , that is,  $\frac{r^{\tilde{s}}(\tilde{q}, \tilde{\rho})}{\underline{e}(\tilde{\rho})} < -1 \leq \frac{r^{\tilde{s}}(q,\rho)}{\underline{e}(\rho)}$  it must outperform, relative to the other assets in the portfolio in expectation in the other survival states, to ensure that equation (36) can hold, that is:

$$\mathbb{E} \left[ \frac{r^s(\tilde{q}, \tilde{\rho})}{\underline{e}(\tilde{\rho})} \middle| \Sigma_S \setminus \tilde{s} \right] > \mathbb{E} \left[ \frac{r^s(q,\rho)}{\underline{e}(\rho)} \middle| \Sigma_S \setminus \tilde{s} \right] \text{ for all } (q,\rho) \neq (\tilde{q}, \tilde{\rho}) \text{ with } x^*(q,\rho) > 0. \quad (38)$$

If we set  $w(\tilde{q}, \tilde{\rho}) = 1$  and  $w(q,\rho) = 0$  for all  $(q,\rho) \neq (\tilde{q}, \tilde{\rho})$  we obtain the following expected return on

equity conditional on the states  $\Sigma_S$ :

$$\begin{aligned}
& (1 - \Pr[\tilde{s}|\Sigma_S]) \cdot \mathbb{E} \left[ \frac{r^s(\tilde{q}, \tilde{\rho})}{\underline{e}(\tilde{\rho})} \middle| \Sigma_S \setminus \tilde{s} \right] + \Pr[\tilde{s}|\Sigma_S] \cdot (-1) \\
& > (1 - \Pr[\tilde{s}|\Sigma_S]) \mathbb{E} \left[ \frac{r^s(\tilde{q}, \tilde{\rho})}{\underline{e}(\tilde{\rho})} \middle| \Sigma_S \setminus \tilde{s} \right] + \Pr[\tilde{s}|\Sigma_S] \frac{r^{\tilde{s}}(\tilde{q}, \tilde{\rho})}{\underline{e}(\tilde{\rho})} \\
& = k,
\end{aligned} \tag{39}$$

that is, we obtain a conditional expected return that is greater than the one obtained from portfolio  $\mathbf{x}^*$ . Further, in failure states  $\Sigma_F$ , this new portfolio cannot yield equity holders lower returns than the previous portfolio  $\mathbf{x}^*$ , since equity holders are protected by limited liability. This implies that setting  $x(\tilde{q}, \tilde{\rho}) = 1$  and  $x(q, \rho) = 0$  for all  $(q, \rho) \neq (\tilde{q}, \tilde{\rho})$  increases  $r_E$ , contradicting the supposition that  $\mathbf{x}^*$  was an optimal portfolio.

Thus, if  $\mathbf{x}^*$  is an optimal portfolio then any asset  $(q, \rho)$  in this optimal portfolio with a strictly positive weight ( $x^*(q, \rho) > 0$ ) must yield  $r^s(q, \rho) > -\underline{e}(\rho)$  in all states  $s$  where the bank survives and has strictly positive equity value.

### A.3. Proof of Lemma 3

Recall that the optimal bank inside equity value can be written as follows:

$$E_{M,I} = E_I + \max_{E_O} \left[ (E_I + E_O) \max_{e, \mathbf{x}} r_E(\mathbf{x}, e) - c(E_O) \right], \tag{40}$$

Let  $(\mathbf{x}^*, e^*)$  denote the optimal solution to the inner (ROI) maximization problem. It follows that if  $c'(0) \geq r_E(\mathbf{x}^*, e^*)$  the bank optimally sets  $E_O = 0$  (note that  $c$  is weakly convex). Further, at any  $E_O$  where  $c'(E_O) < r_E(\mathbf{x}^*, e^*)$  the bank can strictly increase its objective function at the margin by increasing  $E_O$ .

### A.4. Proof of Lemma 4

The firm's payoff under bank financing for a given  $\bar{r}_E^*$  is given by the expected cash flow for its project net of the total financing cost of the investment.

$$V_B(\rho, q) = \mathbb{E}[C_s(q, 1)] - I(1 + \mathbb{E}[r^s(q, \rho)]) \tag{41}$$

By setting  $\bar{r}_E^* = 0$  so that  $\mathbb{E}[r^s(q, \rho)] = -\sigma(q, \rho)$ , the banker is at his outside option. Thus, by using  $\mathbb{E}[r^s(q, \rho)] = -\sigma(q, \rho)$  in (41) we obtain the sum of the firm's payoff and the banker's payoff resulting

from the financing of this project

$$\Pi_{Gross}(q, \rho) = NPV(q) + \sigma(q, \rho) I. \quad (42)$$

By Lemma 1, the outside option for a borrower with quality  $q$  from public markets is given by:

$$V_P(q) = NPV(q) \mathbb{1}_{B(q) \leq NPV(q)} \quad (43)$$

Thus, the incremental surplus that bank financing delivers is given by the difference of (42) and (43).

## A.5. Proof of Proposition 1

The first part is proved in the main text. It remains to show how (15) is derived. Recall, that in equilibrium all bank-financed borrower types must satisfy:

$$\mathbb{E} \left[ \max \left\{ \frac{r^s(q, \rho)}{\underline{e}(\rho)}, -1 \right\} \right] = r_E^*. \quad (44)$$

Multiplying by  $\underline{e}(\rho)$  yields and rewriting yields

$$\mathbb{E}[r^s(q, \rho)] + \mathbb{E}[\max\{-\underline{e}(\rho) - r^s(q, \rho), 0\}] = \underline{e}(\rho) r_E^*. \quad (45)$$

Since  $y(q, \rho) \geq 0$ , we obtain that  $r^s(q, \rho) = \frac{C_s(q, 1) - I}{I}$  whenever  $r^s(q, \rho) < -\underline{e}(\rho)$ . Hence, we can rewrite (45) as (15).

## A.6. Proof of Proposition 2

**Part (1)** Since all banks are ex ante identical they have to obtain the same expected profits in equilibrium. Suppose there was an equilibrium where banks didn't obtain the same expected profits. By definition this equilibrium would have to satisfy market clearing. Yet, taking prices as given, banks with lower profits would prefer to deviate and mimic the strategies of banks that are more profitable. Suppose banks made identical, strictly positive profits in equilibrium and aggregate bank capital was not scarce. Then there would exist banks that are not at their leverage constraint (that have  $e > e_{\min}$ ) and who have  $r_E(\mathbf{x}, e) > 0$ . Yet, by Lemma 2 we know that these choices would not be optimal, since these banks could increase their expected profits by increasing leverage (that is, lower  $e$ ) and funding more borrowers of the same type. Only equilibrium yields that lead to zero expected profits ( $r_E(\mathbf{x}^*, e^*) = 0$ ) are thus consistent with an equilibrium where bank capital is not scarce and loan markets clear. Now suppose there is a borrower type  $(\tilde{q}, \tilde{\rho})$  with  $\Pi(\tilde{q}, \tilde{\rho}) > 0$  that does not obtain funding from any bank. Then a bank that makes zero profits would have a strict incentive to increase the portfolio weight of borrower type  $(\tilde{q}, \tilde{\rho})$ , violating optimality and contradicting the supposition that there was an equilibrium where



borrower type  $(\tilde{q}, \tilde{\rho})$  does not obtain bank finance.

**Part (2)** In the competitive equilibrium, yields on borrowers with  $NPV(q) \geq B(q)$  have to be set such that investors in public markets break even; otherwise there would be excess supply of capital, violating market clearing. For firms of type  $(q, \rho)$  with  $\Pi(q, \rho) \leq 0$  public market finance generates just as much private surplus as bank finance. Thus, for these borrowers, equilibrium yields are such that banks make at best zero profits on those borrowers as well. Thus, banks do not have an incentive deviate and expand the supply of capital to borrowers of type  $(q, \rho)$  with  $\Pi(q, \rho) \leq 0$  when those borrowers are funded by investors in public markets.

**Part (3)** Suppose that when bank capital is not scarce there existed an equilibrium where a borrower type  $(\tilde{q}, \tilde{\rho})$  did not extract all private rents in equilibrium. Then banks making zero expected profits would be strictly better off setting  $x(\tilde{q}, \tilde{\rho}) = 1$ , as doing so would yield them strictly positive expected profits. Yet, since banks have to make zero profits in equilibrium, it cannot be the case that there exists a borrower type that does not extract all private surplus in equilibrium.

## A.7. Proof of Proposition 3

*The profitability index  $PI(q, \rho)$ .* First, we derive the proposed profitability index  $PI(q, \rho)$ . The maximum surplus a bank can potentially extract from a borrower type  $(q, \rho)$  is given by  $\Pi(q, \rho)$ . For a given amount of outside equity  $E_O$ , the quantity of borrowers a bank can fund if it invests only in borrowers of same type  $(q, \rho)$  is:

$$n(q, \rho) = \frac{A}{I} = \frac{E_I + E_O}{\underline{e}(\rho) I}. \quad (46)$$

The total surplus a bank can extract by investing in these borrowers is at most:

$$\Pi(q, \rho) \cdot n(q, \rho) = \frac{\Pi(q, \rho)}{\underline{e}(\rho) I} (E_I + E_O) = PI(q, \rho) \cdot (E_I + E_O) \quad (47)$$

$PI(q, \rho)$  is thus a sufficient statistic for the maximum surplus a bank can extract from a borrower type given the bank has some equity  $(E_I + E_O)$ .  $PI(q, \rho)$  can be interpreted as the expected return on equity that a bank could obtain when funding only borrowers of type  $(q, \rho)$  if the bank did not face competition from other banks. Due to competition from other banks a bank may, however, not be able to extract this surplus in equilibrium.

**Part (1)** Lemma 3 implies that a bank will not raise equity when  $c'(0)$  weakly exceeds the maximum expected return on equity attainable.  $PI(\tilde{q}_0, \tilde{\rho}_0)$  is the maximum expected return on equity that is attainable if the bank extracts  $\Pi$  from the borrower type that is marginal absent equity issuances by the

banking sector. Thus, if  $c'(0) \geq PI(\tilde{q}_0, \tilde{\rho}_0)$  banks will not raise any outside equity. Lemma 3 further implies that banks optimally increase the amount of outside equity  $E_O$  up to the point where all firms of type  $(q, \rho)$  with  $PI(q, \rho) > c'(E_O)$  are funded, since otherwise, marginally increasing outside equity and funding additional borrowers with  $PI(q, \rho) > c'(E_O)$  strictly increases those banks' objectives that are invested in borrowers of type  $(q, \rho)$ .

**Part (2)** If there is no bank that raises outside equity in equilibrium then the marginal firm type that is funded has a profitability index  $PI(\tilde{q}_0, \tilde{\rho}_0)$  and all banks obtain an expected return on equity of  $r_E^* = PI(\tilde{q}_0, \tilde{\rho}_0) < c'(0)$ . If there was a portfolio and leverage strategy that yielded a higher expected return on equity, optimizing banks would deviate and choose that strategy.

For equilibria with equity issuances we focus on symmetric equilibria in equity issuances where all banks choose  $E_O^*$  in equilibrium. If  $E_O^* > 0$  in equilibrium then every bank's expected return on equity has to be equal to the marginal cost of outside equity,  $r_E^* = c'(E_O^*)$ . If there was some bank with a portfolio and leverage strategy that yielded  $r_E > c'(E_O^*)$  then this bank would want to raise more equity to increase its expected profits, contradicting the supposition that  $E_O^*$  is the equilibrium quantity of outside equity for all banks. If there was a bank that in equilibrium followed a strategy yielding an expected return  $r_E < c'(E_O^*)$  then this bank could improve its objective by choosing a smaller amount of outside equity than  $E_O^*$ , contradicting the supposition that  $E_O^*$  is the equilibrium quantity of outside equity for all banks. Finally, all banks have to obtain the same expected return on equity  $r_E = r_E^*$  in equilibrium as they are ex ante identical and can always mimic each others' portfolio and leverage strategies (which determine  $r_E$ ).

**Part (3)** By Lemma 1, Firms that do not obtain financing from banks obtain competitive financing from public markets as long as they are fundable, that is, as long as  $NPV(q) \geq B(q)$ .

**Part (4)** A borrower can always extract at least the component of private surplus that is attainable through public market financing, that is,  $NPV(q) \mathbb{1}_{NPV(q) \geq B(q)}$ . Further, borrowers may be able to extract a fraction of the surplus that is additionally generated through bank financing, which we denote by  $\Pi(q, \rho)$ . As noted above, for a given amount of outside equity  $E_O^*$ , the quantity of borrowers a bank can fund if it invests only in borrowers of type as  $(q, \rho)$  is:

$$n(q, \rho) = \frac{A}{I} = \frac{E_I + E_O^*}{\underline{e}(\rho) I} \quad (48)$$

Thus, after having raised outside equity  $E_O^*$ , a bank that specializes in lending to borrowers of type  $(q, \rho)$  makes the following expected surplus *per borrower*:

$$\frac{(E_I + E_O^*)r_E^*}{n(q, \rho)} = r_E^* \underline{e}(\rho) I. \quad (49)$$

A borrower receives the part of  $\Pi(q, \rho)$  that is not extracted by banks, that is:

$$\max\{\Pi(q, \rho) - r_E^* \varepsilon(\rho) I, 0\} = \Pi(q, \rho) \max\left\{1 - \frac{r_E^*}{PI(q, \rho)}, 0\right\}. \quad (50)$$

Overall, a borrower of type  $(q, \rho)$  thus obtains the following expected surplus:

$$\Pi(q, \rho) \max\left\{1 - \frac{r_E^*}{PI(q, \rho)}, 0\right\} + NPV(q) \mathbb{1}_{NPV(q) \geq B(q)}. \quad (51)$$

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## B. Online Appendix

### B.1. Proof of Proposition 4

**Part (1.a)** Let  $r_E^0$  and  $r_E^1$  denote the banking sector's ROE before and after the increase in capital ratio requirements, respectively. Likewise, let  $y^0(q, \rho)$  and  $y^1(q, \rho)$  denote the corresponding yields for a borrower of type  $(q, \rho)$ . If bank capital is not scarce before the increase, then Proposition 2 implies that  $r_E^0 = 0$ , so that  $r_E^1 \geq r_E^0$ . By (15), the respective loan pricing conditions before and after the increase are given by:

$$\begin{aligned}\mathbb{E}^0[r^s(q, \rho)] &= \underline{e}(\rho) r_E^0 - \sigma^0(q, \rho), \\ \mathbb{E}^1[r^s(q, \rho)] &= (\underline{e}(\rho) + \varepsilon) r_E^1 - \sigma^1(q, \rho),\end{aligned}$$

Since both  $r_E^0 \leq r_E^1$ ,  $\varepsilon > 0$ , and  $\sigma$  is decreasing in  $\underline{e}(\rho)$  it must be true that  $y^1(q, \rho) \geq y^0(q, \rho)$ . The result holds strictly if either  $r_E^1 > 0 = r_E^0$  or if the put value associated with borrower of type  $(q, \rho)$  before the increase was positive, i.e.,  $\sigma^0(q, \rho) > 0$ , which occurs if  $(1 - \underline{e}) - \frac{C_s(q, 1)}{I} > 0$  for at least one state  $s$ .

**Part (1.b)** A tighter capital ratio requirement weakly decreases the set of borrower types that the banking sector finds profitable to finance (since  $\Pi$  is decreasing in  $\underline{e}$ ) and decreases the size of the total loan portfolio for any given amount of equity  $E$ . As a result, aggregate investment by the banking sector must (weakly) decline. As long as some of the firms that are dropped by banks are bank dependent, i.e.,  $NPV(q) < B(q)$ , public markets will not fully compensate for the reduction in bank lending. In this case, aggregate investment strictly declines. If all dropped borrowers have access to public markets, then aggregate investment is unchanged.

**Part (1.c)** The proof shows that an increase in capital ratio requirements both leads to a weak reduction in underinvestment and a weak reduction in overinvestment. Taken together, these two results imply that social surplus must weakly increase.

*Weak reduction in underinvestment:* If bank capital is not scarce after the increase, the economy features no underinvestment, i.e., all socially valuable projects are financed. To see this, note that we can write the profitability index as follows:

$$PI(q, \rho) = \frac{NPV(q) \mathbb{1}_{B(q) > NPV(q)} + \mathbb{E}(\max\{I(1 - rw(\rho) \cdot \underline{e}) - C_s(q), 0\})}{rw(\rho) \cdot \underline{e} \cdot I} \quad (52)$$

All bank-dependent borrowers with positive-NPV projects have  $PI(q, \rho) > 0$ , so that all borrowers with positive-NPV projects are either funded by banks or public markets when bank capital is not scarce. Since there is no underinvestment after the increase, there has to be a weak reduction in underinvestment.

*Weak reduction in overinvestment:* For all borrower types  $(q, \rho)$  where the numerator of the prof-

itability index was negative before the increase in  $\underline{e}$  it will still be negative after the increase because:

$$\frac{\partial \{NPV(q) \mathbb{1}_{B(q) > NPV(q)} + \mathbb{E}(\max \{I(1 - rw(\rho) \cdot \underline{e}) - C_s(q), 0\})\}}{\partial \underline{e}} \leq 0. \quad (53)$$

Further, if the numerator is still negative after the increase, then  $PI(q, \rho) < 0$  and the negative-NPV borrower type will still not be funded after the increase in  $\underline{e}$  either. Thus, if any borrower type is dropped by the banking sector, it must have a negative NPV, which implies a decrease in overinvestment.

**Part (2)** If  $c'(E_O^*) > 0$  and  $PI(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*}) = c'(E_O^*)$ , a marginal increase in  $\underline{e}$  reduces the fraction of borrowers of the marginal type funded. If  $c'(E_O^*) = 0$  and  $PI(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*}) = c'(E_O^*)$  the marginal borrower type drops out completely. Finally, for  $PI(\tilde{q}_0, \tilde{\rho}_0) < c'(0)$ , that is,  $E_O^* = 0$ , the mass of firms of the marginal type that obtains bank funding is reduced.

## B.2. Proof of Proposition 5

We consider how banks' common return on equity  $r_E^*$  responds to changes in  $r_f$ . We distinguish between the case where bank capital is scarce and where it is not scarce. When bank capital is not scarce the expected return on bank equity equals the return on cash,  $r_E^* = r_f$ , implying that  $\frac{dr_E^*}{dr_f} = 1$ . Thus, pass-through is limited only because of the government absorbing marginal changes in bank default states. Formally, equation (30) simplifies to

$$\frac{d\mathbb{E}[r^s(q, \rho)]}{dr_f} = 1 - (1 - \underline{e}(\rho)) \cdot (1 - p_S(q, \rho)) \leq 1. \quad (54)$$

In contrast, when bank capital is scarce, banks' expected return on equity generally exceed  $r_f$ , and depends on the banking sector's marginal borrower type  $(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})$ . Since we are considering only marginal changes in  $r_f$  to evaluate pass-through, we suppose that the marginal change in  $r_f$  does not change the ranking of borrower types or the marginal borrower type. If the marginal borrower type is bank dependent ( $NPV(\tilde{q}_{E_O^*}) < B(\tilde{q}_{E_O^*})$ ) then banks extract the full surplus from this borrower type. As a result, a change in the interest rate on cash has no effect on yields and on the expected loan return of the marginal borrower type. By setting equation (30) to zero for the marginal borrower type, we obtain the implied change in the equilibrium rate of return of the banking sector:

$$\frac{dr_E^*}{dr_f} = (1 - 1/e(\rho))p_S(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*}) < 0.$$

When aggregate bank capital is scarce and the marginal borrower types are bank dependent, pass-through is thus lower for all borrower types when compared to the case where aggregate bank capital is abundant.

When the marginal borrower type  $(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})$  is not bank dependent, then this borrower type is exactly indifferent between receiving bank finance and public market finance, that is,  $\mathbb{E}[r^s(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})] = r_f$ . As

a result, pass through is equal to one for this marginal borrower type. By setting equation (30) to one for the marginal borrower type, we obtain the implied change in the equilibrium rate of return of the banking sector:

$$\begin{aligned}\frac{dr_E^*}{dr_f} &= 1/\underline{e}(\tilde{\rho}_{E_O^*}) - (1/\underline{e}(\tilde{\rho}_{E_O^*}) - 1) \cdot p_S(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*}) \\ &= 1/\underline{e}(\tilde{\rho}_{E_O^*}) (1 - p_S(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})) + p_S(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*}) \geq 1,\end{aligned}\tag{55}$$

that is, the return on bank equity responds more than one-for-one to an increase in the return on cash. Whereas the marginal borrower type experiences a pass-through of one other borrower types experience a pass-through that can be below or above one. Pass through for borrower types with equity requirements (risk classifications) that are marginally above the one of the marginal borrower type is greater than one since:

$$\frac{\partial \frac{d\mathbb{E}[r^s(q, \rho)]}{dr_f}}{\partial \underline{e}(\rho)} = \frac{dr_E^*}{dr_f} - p_S(q, \rho) > 0.$$

Conversely, borrower types that considered to be “safer” according to the regulatory framework will experience pass-through below one.

Borrower types that cause a marginally lower default risk for specialized banks also experience pass through greater than one since

$$\frac{\partial \frac{d\mathbb{E}[r^s(q, \rho)]}{dr_f}}{\partial p_S(q, \rho)} = 1 - \underline{e}(\rho) > 0.\tag{56}$$

**Lemma 5** *Pass-through of changes in the rate  $r_f$  to borrowers' cost of capital depends on the scarcity of aggregate bank capital and the bank-dependence of the marginal borrower type.*

1. *If bank capital is not scarce then pass through for all borrower types is weakly lower than one and given by:*

$$\frac{d\mathbb{E}[r^s(q, \rho)]}{dr_f} = \underline{e}(\rho) + (1 - \underline{e}(\rho))p_S(q, \rho) \leq 1.\tag{57}$$

2. *If bank capital is scarce and the marginal borrower type  $(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})$  is bank dependent, then pass through for all bank-funded borrowers is weakly lower than one and given by:*

$$\frac{d\mathbb{E}[r^s(q, \rho)]}{dr_f} = (1 - \underline{e}(\rho)) \cdot (p_S(q, \rho) - p_S(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})) \leq 1.\tag{58}$$

3. *If bank capital is scarce and the marginal borrower type  $(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})$  is not bank dependent, then pass*

through for bank-funded borrowers is given by:

$$\begin{aligned} \frac{d\mathbb{E}[r^s(q, \rho)]}{dr_f} = & (1 - \underline{e}(\rho)) \cdot (p_S(q, \rho) - p_S(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})) \\ & + \frac{\underline{e}(\rho)}{\underline{e}(\tilde{\rho}_{E_O^*})} (1 - p_S(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*})) + p_S(\tilde{q}_{E_O^*}, \tilde{\rho}_{E_O^*}), \end{aligned} \quad (59)$$

which can be greater or smaller than one.