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**Abstract**

We model a dynamic economy with strategic complementarity among investors and endogenous government interventions that mitigate coordination failures. We establish equilibrium existence and uniqueness, and show that one intervention can affect another through altering the public-information structure. A stronger initial intervention helps subsequent interventions through increasing the likelihood of positive news, but also leads to negative conditional updates. Our results suggest optimal policy should emphasize initial interventions when coordination outcomes tend to correlate. Neglecting informational externalities of initial interventions results in over- or under-interventions, depending on intervention costs. Moreover, saving smaller funds disproportionally more can generates greater informational benefits at smaller costs. Our paper is thus informative of the interaction of multiple intervention programs such as those enacted during the 2008 financial crisis.

**Keywords:** Coordination, Intervention, Dynamic Policy, Learning, Information Design, Global Games

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# 1 Introduction

Coordination failures are prevalent and socially costly. Effective interventions may ameliorate such damaging outcomes. For example, financial systems, especially short-term credit markets, are vulnerable to runs by investors. The 2008 financial crisis witnessed a series of runs on both financial and non-financial institutions. In response, governments and central banks around the globe employed an array of policy actions over time. Given the novelty, the scale, the cost, and the intertwined nature of such interventions, a study of how endogenous interventions relate to each other is natural.

More broadly, how should a government formulate intervention policy in a dynamic economy with strategic complementarity? How does intervention in one institution or market affect subsequent interventions in other institutions or markets? This paper tackles these questions by modeling the government as a large player in sequential global games and focusing on information transmission from one intervention to another. We have the following findings. First, an intervention not only improves welfare contemporaneously, but also affects agents' future coordination game and thus future interventions. Consequently, when intervention costs are comparable across coordination games, optimal policy often features an emphasis on the initial intervention. Second, decision makers for one intervention may not internalize the informational externality of the intervention outcome on other interventions, and thus may over- or under intervene, depending on the intervention costs. Third, an optimal policy may entail saving smaller funds disproportionally more. Such a policy generates an information structure with lower cost but greater benefits. The insights apply to situations with multiple interventions in which agents' actions exhibit strategic complementarity. Examples include interventions in currency attacks, bank runs, real estate programs, cross-sector industrialization, and technology subsidy programs.

We introduce the model in the context of runs on Money Market Mutual Funds (MMMFs) in September 2008 and subsequently on commercial papers, both triggered by investors' interpretation of Lehman's failure as a revelation of the credit risk and systemic illiquidity of commercial papers.[1] The initial successful intervention with insurance to all MMMF de-

[1]The run on commercial papers is primarily in financial commercial papers as opposed to ABCPs, according to Kacperczyk and Schnabl (2010).

positors and the Asset-Backed Commercial Paper Money Market Mutual Fund Liquidity Facilities (AMLF) arguably affected how investors reacted to later interventions in the commercial paper market, such as the Commercial Paper Funding Facility (CPFF) program.[2] Another context that motivates the study is the federal government's multiple attempts at stabilizing the housing market in a wide range of regions through the Neighborhood Stabilization Programs (NSPs) of 2008-2010.[3] These interventions provided funding for local housing authorities to purchase, renovate, and sell foreclosed properties in an effort to moderate the sizeable declines in home prices driven by the massive wave of foreclosures during the credit crisis.[4] Because the intervention outcomes were revealed gradually over time, and housing markets across neighborhoods share common components, people update their priors on the underlying health of housing markets from initial intervention outcomes, and behave differently in the local program.

Specifically, in a two-period economy, a group of atomistic investors in each period choose whether to run or remain invested in a fund.[5] Running guarantees a certain payoff, which is higher than that of staying if the fund fails, whereas staying pays more if the fund survives. The fund survives if and only if the total measure of investors who choose to stay is above a fundamental threshold $\theta$ – interpreted as an unhedgeable system-wide illiquidity shock or the persistent quality of the underlying investment, and is identical across the two periods. Following the global-games framework, $\theta$ in each period is unobservable and each investor receives a noisy signal. Prior literature has established that in static settings, a unique equilibrium exists in which the fund survives as long as the true $\theta$ is below a threshold $\theta^*$, and each investor stays if and only if his private signal is below a certain threshold $x^*$.

We then incorporate policy responses in a crisis and the formation of expectations by

[2]See Schmidt, Timmermann, and Wermers (2016) for more details on the run on MMMFs. As discussed in Bernanke (2015), page 283, the government was keenly aware that AIG's failure might affect market participants' beliefs and lead to runs in other markets, just like Lehman's commercial paper had triggered the run on money market funds.

[3]See Westrupp (2017) for a summary of the NSP and its impact on mitigating foreclosure externalities.

[4]One may view regional foreclosures as broadly analogous to "runs" due to the negative externality of a particular foreclosure on the values of neighboring properties. For example, Campbell, Giglio, and Pathak (2011) document a negative spillover effect of 1% per new foreclosure within a 0.10-mile radius. Guiso, Sapienza, and Zingales (2013) document the prevalence of strategic defaults during this period.

[5]In our context, running on the fund in the first period corresponds to the run on MMMFs, and running in the second period corresponds to the run in the financial commercial papers' market.

modeling interventions in our baseline setup as direct capital injections into funds experiencing run risks. $\theta$ is best interpreted as involving a solvency component or it concerns liquidity shock but intervention is not costless, and we capture in reduced-form the cost of intervention of scale $m$ by $k(m)$.[6] The equilibrium $\theta^*$ increases strictly with the size of the government's intervention: a greater liquidity injection makes the fund more likely to survive. Therefore, in a static economy, a benevolent government trades off this contemporaneous benefit and intervention costs.

In a dynamic setting, government intervention in the first period alters the informational environment in the second period. Indeed, agents' prior beliefs on $\theta$ are truncated, because whether the fund fails during the first period is public information. When the fund has survived in the first period, agents learn $\theta < \theta_1^*$, and their belief on $\theta$ shifts downward, making coordination easier. The opposite holds if the fund has failed in the first period. To the extent this public signal is useful, initial success increases the likelihood of subsequent success, and initial failure increases the likelihood of subsequent failure, endogenously giving rise to the greater tendency for the correlation of coordination outcomes across different periods (*endogenous correlation effect*). Initial intervention is thus more important because it increases the probability of survival in both periods.

However, initial intervention also has an informational cost, and thus its magnitude must be tempered. When a large intervention leads to a fund's survival, investors may infer the outcome is due to the intervention itself and not strong fundamentals. Conversely, if the fund fails despite a large initial intervention, investors become even more pessimistic about the market's fundamentals. This *conditional inference effect* harms investors' welfare and drives the government to intervene less for more favorable conditional updates. Therefore, the optimal policy has to consider the initial intervention's informational effect and trade off the two competing forces: intervening more to increase the likelihood of good news (truncating $\theta$ from above), and intervening less initially to encourage more favorable conditional updates

[6]Many crisis interventions indeed involves significant costs. For example, Duygan-Bump, Parkinson, Rosengren, Suarez, and Willen (2013) discuss how AMLF and CPFF were essentially capital injections that alleviated funds' pressure to meet redemption without suffering fire sales. Other examples include the Economic Stimulus Act of 2008 that reduced firms' tax obligations directly, or TARP, which intended to improve the liquidity of hard-to-value assets through secondary-market mechanisms. In Section 5.3, we discuss how our setup nests other forms of intervention. In almost all these cases, the intervention is not simply a costless promise, but entails actually putting aside or using the funding.

(lower $\theta^*$).

We establish results on the existence and uniqueness of equilibrium and study the implications for the optimal policy of a benevolent government. When the intervention costs are comparable in the sense that the endogenous intervention amounts are similar even absent public learning, the endogenous correlation effect dominates. Optimal policy then generally emphasizes initial intervention – the scale of intervention in the first period always exceeds that in the second period – to kill two birds with one stone (improving fund survival in both periods).

This result also implies that a decision maker who neglects the informational externality of one intervention on another would under-intervene initially. However, when the intervention costs across the two periods differ drastically, so much so that survival in the first period does not guarantee survival in the second period (when the second-period cost is too high relative to the first and private signals are relevant for the marginal investor), nor does failure lead to failure (when the second-period cost is so low that one can intervene more despite the negative update from the first period's failure), the conditional inference effect can dominate. The more the government considers the informational externality, the more it shades intervention. This result also applies to countries and regions sharing common fundamentals in which one country's investors learn from another country's intervention outcome. In that sense, a global social planner (e.g., the European Union) could have a role in mitigating inefficient interventions in member countries or states.

Finally, when the government endogenously chooses interventions in funds of different sizes and that choice also determines the order of the realizations of coordination outcomes, the larger fund is "too big to save first" if the government can decide on the order that these coordination outcomes get realized, because it costs less to intervene in the smaller fund first in order to induce the same updating on the fundamental $\theta$, and the larger fund benefits more from the reduced uncertainty. Even when the order of the realizations of coordination outcomes is exogenous, it can be beneficial to help the smaller fund disproportionally more relative to its size and likelihood of realizing the coordination outcome first. This result complements studies on institutions deemed to be "too big to fail" in that though bigger funds could be systemically important, helping smaller funds more or early could be more effective in terms of influencing the informational environment.

This paper contributes to our understanding of how interventions shape the informational environment during a crisis, and hence is useful for studying and assessing policies that aim to avoid inefficient outcomes. In particular, we highlight the role of government intervention on information structure: it not only affects the probability of good news versus bad news, but also the informativeness of news.[7] It thus complements existing work on government interventions in markets with strategic complementarity.[8] For example, Acharya and Thakor (2014) consider how liquidation decisions by informed creditors of one bank signal systematic shocks to other creditors and create contagions, and how selective bailout could be efficient when the regulator observes the systematic shock. Huang (2016) studies how the interaction between a policy maker's reputation building and speculators' learning of the policy maker's type determines speculative attacks and regime changes. Regarding the design of intervention policy, Bebchuk and Goldstein (2011) examine the effectiveness of various forms (rather than the extent) of exogenous government policies in avoiding self-fulfilling credit market freezes. Sakovics and Steiner (2012) analyze who matters in coordination failures and how to set intervention targets. Choi (2014) shows the importance of bolstering stronger financial institutions to prevent contagion. Like these studies that focus on one particular aspect of intervention, we demonstrate how information-structure design should play an important role in formulating intervention policies, and should be considered together with previously discussed factors. In addition, this paper concerns the dynamic interaction of multiple endogenous interventions under general cost functions.

This paper is also related to global games and equilibrium selection (Carlsson and Van Damme, 1993; Morris and Shin, 1998), especially in dynamic settings (Frankel and Pauzner, 2000; Angeletos, Hellwig, and Pavan, 2007), with the government as a large player

[7] Bernanke and Geithner spoke of the financial crisis as a bank run and emphasized the need to combat a financial crisis with the "use of overwhelming force to quell panics" (p. 397, Geithner (2014)), a tactic of "shock and awe" that often connotes managing market expectations by the government. The regulator's concern implies not only do the financial networks matter for intervention policies, information structure also plays a crucial role in coordination. Although isolating the informational aspect from systemic connectedness is challenging, it was an important element of both the Lehman Brothers episode and the Eurozone bank bailouts in 2010 and 2011.

[8] Strategic complementarity in financial markets is well-recognized in prior literature such as Diamond and Dybvig (1983), and more recently by empirical studies, including Chen, Goldstein, and Jiang (2010) and Hertzberg, Liberti, and Paravisini (2011).

(Corsetti, Dasgupta, Morris, and Shin, 2004; Angeletos, Hellwig, and Pavan, 2006). Our paper adds to earlier studies by explicitly modeling the government as a large player that endogenously selects coordination equilibrium through both static and dynamic channels. Thus, we provide theoretical insights on how endogenous interventions relate to one another. Different from Angeletos, Hellwig, and Pavan (2006) who demonstrate that endogenous intervention signals government type and leads to equilibrium multiplicity, our paper explores how endogenous intervention shapes information structure rather than signaling government's private information. This paper is also related to Angeletos, Hellwig, and Pavan (2007) who extend global games to a dynamic setup in which agents take actions over multiple periods and can learn about the fundamental over time. The authors point out that multiplicity resurfaces from the interaction between endogenous learning based on regime survivals and exogenous learning induced by private news arrivals.[9] We introduce endogenous interventions, which lead to endogenous equilibrium multiplicity and selection, and show that the policy and public learning feed back each other and have profound implications on the optimal policy design and coordination outcomes.

Finally, this papers adds to the emerging literature that apply information design and Bayesian persuasion (e.g., Gentzkow and Kamenica (2011); Bergemann and Morris (2017)) to financial institutions and markets (e.g., Azarmsa and Cong (2017) and Orlov, Zryumov, and Skrzypacz (2017)). In particular, it is related to Goldstein and Huang (2016), in which policy-makers costlessly and endogenously design information in a coordination game. The authors focus on a one-shot intervention in which the government commits to a regime-change policy to increase the probability of the survival of the status quo. The information transmission relies on the truncation of beliefs as in Angeletos, Hellwig, and Pavan (2007), but is endogenous, and does not concern multiple coordination games. Lenkey and Song (2016) also analyzes the tradeoffs in information design to study how a redemption fee affects runs on financial institutions when investors are asymmetrically informed about fundamentals. Our paper adds to both Goldstein and Huang (2016) and Lenkey and Song (2016) by introduc-

[9]The Bayesian learning from public signals without endogenous government actions has also been discussed in several other papers. For example, Manz (2010) studies information contagion; Ahnert and Bertsch (2015) study information choice and contagion after wake-up-calls; Taketa (2004), studies contagion via a common investor base; Li and Ma (2016) study contagion and fire sales after a bank run.

ing costly information design and underscoring its role in determining optimal information structure and policy (as well as characterizing the tradeoffs in information design more analytically). To our best knowledge, we are the first to derive implications of informational link between multiple endogenous interventions, the endogenous correlation effect of information design, and the role of relative intervention costs are entirely new.

The rest of the paper is organized as follows: Section 2 lays out the basic framework and establishes a static benchmark. Section 3 characterizes the equilibrium in dynamic settings. Section 4 solves for the optimal policy and presents its implications. Section 5 discusses the results and extends the model. Section 6 concludes. The Appendix contains all the proofs. We further relegate some interesting extensions to the online Appendix.

# 2 Model

This section introduces the model with a representative intervention form: government directly infusing liquidity into funds subject to runs in each period. We start by analyzing a static model as our benchmark in Section 2.1 and move to the dynamic setting in Section 2.2.

## 2.1 Static Benchmark

### 2.1.1 Model setup

A fund has a continuum of investors indexed by $i$ and normalized to unit measure. Each investor has 1 unit of capital invested in the fund, and simultaneously chooses between two actions: stay ($a_i = 1$) or withdraw ($a_i = 0$). For the remaining analysis, we interpret withdrawal as "run" on the fund, and staying can be interpreted as rolling over short-term debts. The net payoff from running on the fund and investing the proceeds in an alternative vehicle (e.g. a treasury bill) is always equal to $r$, whereas the payoff to each investor from staying is $R$ if the fund survives the run ($s = S$), and 0 if the fund fails ($s = F$). Let $R > r > 0$; then an investor finds it optimal to stay if and only if she expects the probability of survival to exceed the cost of illiquidity, defined as $c \equiv \frac{r}{R}$. Table 1 (left panel) shows the net payoff of each action under different states and actions. In the right panel of Table 1, we normalize the payoff matrix by subtracting $r$ and scaling by $\frac{1}{R}$. For notational convenience, we use the normalized net payoffs for the remainder of the paper, and extends to alternative payoff structures in Online Appendix Section O3.

Table 1: Net Payoffs and Normalized Net Payoffs

| | Stay | Run |
|---|---|---|
| Survive | $R$ | $r$ |
| Fail | 0 | $r$ |

| | Stay | Run |
|---|---|---|
| Survive | $1-c$ | 0 |
| Fail | $-c$ | 0 |

Agents' decisions are complements: the fund is more likely to survive as more agents choose to stay. Specifically, the fund survives if and only if

$$A + m \geq \theta, \tag{1}$$

where $A$ represents total measure of agents who choose to stay. $m \in [0, \bar{m}]$ is the size of the government's capital injection to the fund and is bounded above by a constant $\bar{m} > 0$. $\theta \in \mathbb{R}$ summarizes the underlying fundamental.[10] In the context of the run on MMMFs in 2008, $1 - A$ would represent the volume of net redemption of fund shares, and $m$ represents the magnitude of government intervention, such as insurance offerings and purchase facilities. $\theta$ represents the fundamental of the underlying assets, which is best interpreted as a shortfall in the fund's interim revenue that must be overcome to continue the investments. In that regard, the shock is similar to the so-called "liquidity shock" in Holmström and Tirole (1998), but in essence entails a solvency component. For the fund to survive, the total resource $A+m$ must dominate the fundamental shock $\theta$.

The government cares about social welfare comprising investors' total payoff less the intervention cost $k(m)$, which is weakly increasing and convex. $k(m)$ captures the political capital expended, tax distortion, moral hazard associated with the intervention policy, as well as the actual amount of $m$ used in the intervention. In our baseline we should think of $k(m)$ as non-trivial and it becomes so large that intervening more than $\bar{m}$ is infeasible. Oftentimes the intervention amount $m$ is actually used. For example, during the money market fund run in 2008, the Fed implemented the Asset-Backed Commercial Paper Money

[10] We assume the government publicly commits to the intervention.

Market Fund Liquidity Facility (AMLF), a lending program for troubled money market funds which actually lent out \$150 billion in the first 10 days of operation. Resources are actually spent and could be very costly for the Fed because the facility may lose money due to bad fundamentals of the funds' assets. Another example is the Troubled Asset Relief Program (TARP), which allows the government to purchase toxic assets and equity from financial institutions to strengthen its financial sector, in order to address the subprime mortgage crisis. Like many other intervention programs during the global recession, TARP operated primarily through secondary markets and entailed investing the resources. Moreover, intervention costs and political constraints are real, at least at the onset of the crisis (Swagel (2015)). Setting aside resources may require approvals from relevant committees. Moral hazard and tax distortions can also add to the cost. In the online appendix Section O2, we model fund managers' actions and microfound the cost using moral hazard induced by interventions. All these considerations rule out the simple solution of promising an unlimited amount of $m$ at no cost, which would have worked for a pure liquidity crisis.[11]

Apparently, coordination is needed when both $\theta$ and $m$ are commonly known by all agents. Indeed, if $\theta - m \in (0, 1)$, two equilibria coexist. In one equilibrium, all investors stay, and in the other one, all investors run. Global games resolve this issue of equilibrium multiplicity through introducing incomplete information. We apply the same technique to assume agents each observe a noisy private signal of $\theta$. In particular, agent $i$ observes

$$x_i = \theta + \varepsilon_i, \tag{2}$$

where the noise $\varepsilon_i \sim Unif[-\delta, \delta]$ is i.i.d. across investors. For simplicity, we assume the prior distribution of $\theta$ is uniform on $[-B, B]$, where $B \gg \max\{\delta, \bar{m}\}$.[12] We also assume the

[11] We thank an anonymous referee who kindly pointed this out and suggested another motivation for the fact that $m$ is used up. We follow his suggestion to allow $\theta$ to impact the eventual profitability of the firm's investment in the online appendix Section O3.

[12] We assume $B$ is sufficiently large relative to $\bar{m}$ so that the government cannot guarantee a successful intervention. Uninformative prior corresponds to $B \to \infty$. This is just an alternative way of saying $k(m)$ is large when $m$ is large. It allows us to apply our model even to situations where $\theta$ as a pure liquidity shock, because a weak government's unlimited promise of insurance is not always credible (the promised amount can break the government, or the insuring institution). One example is discussed in He and Manela (2016): half of running WaMu depositors were covered by FDIC insurance, and depositors may worry that DFIC could not afford to cover WaMu's insured deposits, which were several times the Deposit Insurance Fund at the time; Iyer and Puria (2012) also document that deposit insurance is only partially effective in preventing

government does not know the realization of the fundamental $\theta$ and does not have a private signal about it, which allows us to abstract away from signaling and focus on intervention as a form of information design.[13]

The timing in this single-period game is as follows: the government announces $m$, and then each investor $i$ receives a private signal $x_i$ and plays the game of choosing whether to stay, before their payoffs are realized. We restrict the equilibrium set to symmetric Perfect Bayesian Equilibria (PBE) in monotone strategies: all agents' strategies are symmetric and monotonic *w.r.t.* $x$ and $m$.[14] Specifically, agent $i$'s strategy $a_i(x_i, m)$ is non-increasing in $x_i$ and non-decreasing in $m$. We first examine the equilibrium given the government's intervention $m$. For the remainder of this paper, we will refer to this game as investors' stage game.

### 2.1.2 Investors' Stage Game Given Intervention

Because $B \gg \max\{\delta, \bar{m}\}$, it is *w.l.o.g.* to further restrict the equilibrium set to threshold equilibria denoted by $(\theta^*, x^*)$. The fund survives if and only if $\theta \leq \theta^*$, and each investor stays if and only if his signal $x \leq x^*$. Lemma 1 summarizes the equilibrium outcome in the static game.

**Lemma 1**

*In the stage game, $\forall m \in [0, \bar{m}]$, there exists a unique symmetric PBE in monotone strategies $(\theta^*, x^*)$, where*

$$\begin{cases} \theta^* &= 1 + m - c \\ x^* &= 1 + m - c + \delta(1 - 2c). \end{cases} \tag{3}$$

runs.

[13]Essentially we are assuming institutional investors are typically more informed than the government about the fundamental state of the market, which is consistent with Diamond and Kashyap (2015) (financial institutions know more about the fundamental illiquidity), Bond and Goldstein (2015) (government relies on market prices to learn fundamentals), Sakovics and Steiner (2012) (governments' inferior knowledge on utilizing or allocating resources leads to tax and subsidy distortions), and Bernanke (2015) (government is uncertain about how the market reacts to intervention).

[14]In fact, Van Zandt and Vives (2007) show that with regularity conditions ensuring the games are monotone supermodular, the restriction on monotone strategies is not needed. Although these conditions typically hold at least in many static games (Morris and Shin (1998)), it is not our main focus, and we simply concentrate on the natural case of monotone strategies.

*Each investor's strategy follows* $a_i = \mathbb{1}\{x_i \leq x^*\}$. *The fund's outcome* $s = S$ *if* $\theta \leq \theta^*$ *and* $s = F$ *otherwise.*

According to Lemma 1, the fund survives if and only if $\theta \leq \theta^*$. Each agent stays if and only if his private signal $x_i \leq x^*$. Note that $\theta^*$ increases in $m$ and so does $x^*$. In other words, the fund is more likely to survive and investors are more inclined to stay if the size of government intervention increases. This result shows the static effect of government intervention on coordination. In Section 3, we show government intervention has dynamic coordination effects.

#### 2.1.3 Welfare and Optimal Intervention

Let $V_i$ be investor $i$'s net payoff, and $W = E\left[\int_0^1 V_i di\right]$. Then investors' welfare is

$$W = \frac{1}{2B}\left[\underbrace{\int_{-B}^{\theta^*} (1-c)\, d\theta}_{\text{fundamental}} - \underbrace{\int_{x^*-\delta}^{\theta^*} (1-c)\left(1 - \frac{x^* - (\theta - \delta)}{2\delta}\right) d\theta}_{\text{overrun}} - \underbrace{\int_{\theta^*}^{x^*+\delta} c\frac{x^* - (\theta - \delta)}{2\delta} d\theta}_{\text{underrun}}\right].$$

Let us interpret the above payoff function. $\frac{1}{2B}$ is the probability density of the uniform distribution. The terms inside the square bracket split into three terms. The first term, *fundamental*, equals the net payoff if all agents stay when the fund survives. The second term, *overrun*, represents the net payoff loss due to the fact that some agents choose to run when the fund survives. The last term, *underrun*, is the net loss from agents who choose to stay when the fund fails.

Simple calculation suggests total welfare is

$$W - k(m) = \frac{(1-c)\left[1 + B - c\,(1+\delta) + m\right]}{2B} - k(m).$$

The marginal benefit of $m$ on $W$ is a constant, $\frac{(1-c)}{2B}$. This result comes from the fact that an increase in $m$ also raises $\theta^*$ linearly, making the fund more likely to survive. $\frac{(1-c)}{2B}$ is the net payoff from staying $1-c$, scaled by the probability density $\frac{1}{2B}$. Therefore, intervention improves coordination. Because $m$ lies in a compact set, an optimal intervention always exists:

$$m^* = \sup \left\{ m \in [0, \bar{m}] : \lim_{\epsilon \to 0} \frac{k(m+\epsilon) - k(m)}{\epsilon} \le \frac{1-c}{2B} \right\}.$$

For example, if $k(m) = \frac{1}{2} z m^2$, then $m^* = \min\left\{\frac{1-c}{2zB}, \bar{m}\right\}$.

## 2.2 Dynamic Economy

We now extend the static model to a two-period dynamic economy. In each period, a unit measure of agents choose whether to stay or run on a fund (not necessarily the same fund across the two periods).[15] The government intervenes in each period with $m_1$ and $m_2$. Agents in period 2 observe whether the first fund survives in period 1. To focus on Bayesian learning from public intervention outcomes, we assume the mass of agents in each period are non-overlapping, in that they do not observe the private signals in other periods. The government's cost of intervention now is $K(m_1, m_2)$, which is weakly increasing and convex in both arguments, and satisfies $K(0,0) = 0$, where $\{m_1, m_2\} \in I$, and $I \subset R^2$ indicates a convex set of feasible interventions. For ease of exposition, we assume for the remainder of the paper that the cost function lies in the space $\mathcal{C}[0, \bar{m}_1] \times \mathcal{C}[0, \bar{m}_2]$, where $\bar{m}_1$ and $\bar{m}_2$ are finite constants.

Importantly, the two periods are linked: (a) the fundamentals $\{\theta_t\}_{t=1,2}$ are identical across two periods. We omit the subscript of $\theta$ from now on and relax the assumption in Section 5.1 by only requiring positively correlated fundamentals; (b) agents in period 2 also observe the public outcome of whether investment has succeeded in the first period, indicated by $s_1 = S$ or $s_1 = F$; (c) the costs of intervention across these two periods may interact with each other.

The government chooses interventions to maximize investors' welfare, subtracting the intervention cost $K(m_1, m_2)$. In each period, agents simultaneously choose whether to stay with the fund ($a_t = 1$) or to run ($a_t = 0$).[16] The period-by-period normalized payoff structure is identical to the static game: running ($a_t = 0$) always guarantees 0 payoff, whereas staying ($a_t = 1$) pays off $1 - c$ in survival and $-c$ in failure. Agents' decisions within the same period

[15] An anonymous referee has kindly pointed out that if it is the same fund across two periods, the game ends conditional on failure in the first period. In that case, some of our results would be further strengthened.

[16] Because we focus on symmetric equilibrium, the subscript $i$ for agent $i$ is omitted without any confusion.

are complements: investment in period $t$ succeeds if and only if

$$A_t + m_t \geq \theta, \tag{4}$$

where $A_t$ is the total measure of investors who choose to invest, and $m_t$ denotes the size of liquidity injected by the government. Again, $\theta$ represents the fundamental. Similar to the interpretation of the static game, the runs could represent runs on MMMF and financial commercial papers, respectively, with $\theta$ representing the market-wide illiquidity or the credit quality of commercial paper issuers.

The timing within each period goes as follows. First, government announces $m_t$. Second, each investor $i$ in period $t$ receives a private signal $x_{it} = \theta + \varepsilon_{it}$ about the fundamental, where $\varepsilon_{it} \sim Unif\left[-\delta, \delta\right]$. Lastly, investors choose whether to stay, and their payoffs realize. The setup is dynamic in the sense that period 1's outcome is revealed before investors take actions in period 2.

In the baseline, we study a problem in which the government maximizes welfare by solving

$$\max_{m_1 \in [0, \bar{m}_1], m_2 \in [0, \bar{m}_2]} E\left[\int_0^1 V_{1i} di + \int_0^1 V_{2i} di\right] - K(m_1, m_2). \tag{5}$$

Given the set $[0, \bar{m}_1] \times [0, \bar{m}_2]$ is compact, an optimal policy exists in general, which exhibits interesting features. We solve this problem in two steps. The next section takes government interventions $(m_1, m_2)$ as given, and derives the stage-game equilibrium. Section 4 then examines a benevolent government's optimal policy design.

# 3 Coordination Equilibrium

We first examine the stage game of investors' coordination in each period, taking the intervention $\{m_1, m_2\}$ as given. Our equilibrium concept is symmetric Perfect Bayesian equilibria (PBE) in monotone strategies. Specifically, all agents' strategies are symmetric and monotonic $w.r.t.$ $x_t$ and $m_t$: agent $i$'s strategy in period $t$, $a_{it}\left(x_{it}\right)$, is non-increasing in $x_{it}$ and non-decreasing in $m_t$, $t = 1, 2$.

## 3.1 Equilibrium and Social Welfare in Period 1

The analysis in period 1 is identical to the static game. We relabel the unique threshold equilibrium with time subscripts $(\theta_1^*, x_1^*) = (1 + m_1 - c, 1 + m_1 - c + \delta(1 - 2c))$. The fate of the fund is $s_1 = S$ if $\theta \leq \theta_1^*$, and $s_1 = F$ otherwise. Agent $i$ adopts a threshold strategy $a_{i1} = \mathbb{1}\{x_{i1} \leq x_1^*\}$.

The social welfare in period 1 is also identical to the static economy,

$$W_1 - K(m_1, 0) = \frac{(1-c)\left[1 + B - c(1+\delta) + m_1\right]}{2B} - K(m_1, 0).$$

## 3.2 Equilibrium in Period 2

In period 2, the outcome of period-1 intervention (henceforth referred to as public news) is publicly known. As a result, beliefs on $\theta$ are truncated either from above or from below.

Despite that multiple equilibria easily emerge in dynamic global games (Angeletos, Hellwig, and Pavan (2007)), we show in our baseline model that when signals are imprecise but still reasonably informative, we obtain unique equilibrium that more clearly conveys the main economic intuition of our paper. To that end, we assume $2\delta > 1$ and $\frac{1}{2\delta+1} < c < \frac{2\delta}{1+2\delta}$ for the remainder of the paper. These assumptions correspond to the fact that during crisis, uncertainty is high and the cost of illiquidity is in an intermediate range in which agents do not overwhelmingly prefer to stay or to run. These assumptions ensure a unique threshold equilibrium in period 2 for both $s_1 = S$ and $s_1 = F$, and for all values that $m_1$ and $m_2$ take on. In online Appendix Section O1, we discuss the equilibrium outcomes for general $\delta$ and $c$, relate our findings to Angeletos, Hellwig, and Pavan (2007), and explain what factors determine equilibrium multiplicity in our model and in the literature.[17]

### 3.2.1 Survival News

If the fund in period 1 has survived ($s_1 = S$), the prior belief on $\theta$ is bounded above at $\theta_1^*$: $\theta \sim Unif\left[-B, \theta_1^*\right]$. In this case, investors might stay regardless of their signals. In fact, this equilibrium exists if and only if $m_2 > m_1 - c$. In this equilibrium, the (hypothetical) threshold

[17] We also demonstrate that either shutting down the interaction of public learning and private learning or reducing the private-signal precision would restore uniqueness in the dynamic setting.

$x_2^*$ satisfies $x_2^* \geq \theta_1^* + \delta$, which is always above all agents' realized signals. We call such equilibrium *Equilibrium with Dynamic Coordination* because the government's intervention in the first period has a dominant effect on improving coordination among investors in the second period.

**Lemma 2** (Stage Game Equilibrium with Dynamic Coordination)
*If $s_1 = S$, $(\theta_2^*, x_2^*) = (\theta_1^*, \theta_1^* + \delta)$ constitutes an equilibrium if and only if $m_2 > m_1 - c$.*[18]

Next, we turn to threshold equilibria with $\theta_2^* < \theta_1^*$ so that the fate of the fund in period 2 still has uncertainty. Likewise, any threshold equilibrium $(\theta_2^*, x_2^*)$ necessarily satisfies two conditions. First, when $\theta = \theta_2^*$, the fund is about to fail, that is, $A_2 + m_2 = \Pr\left(x_2 < x_2^* \middle| \theta = \theta_2^*\right) + m_2 = \theta_2^*$. Second, the marginal agent who receives the signal $x_2^*$ is indifferent between stay and run, $\Pr\left(\theta \leq \theta_2^* \middle| x_2 = x_2^*, \theta \in [-B, \theta_1^*]\right) = c$.

We analyze the equilibrium in two cases, depending on whether the marginal investor finds the public news "useful." Ignoring the public news, the marginal investor's posterior belief on $\theta$ is simply $\Pr\left(\theta \middle| x_2 = x_2^*\right) \sim Unif\left[x_2^* - \delta, x_2^* + \delta\right]$. If $x_2^* + \delta < \theta_1^*$, then the marginal investor finds the public news useless because it does not additionally help him learn about $\theta$, that is, $\Pr\left(\theta \leq \theta_2^* \middle| x_2 = x_2^*, \theta \in [-B, \theta_1^*]\right) = \Pr\left(\theta \leq \theta_2^* \middle| x_2 = x_2^*\right)$. We call such equilibrium *Equilibrium without Dynamic Coordination* because intervention in the first period has no effect on coordination in the second period.

**Lemma 3** (Stage Game Equilibrium without Dynamic Coordination)
*If $s_1 = S$ and $m_2 < m_1 - 2\delta(1-c)$, an equilibrium with thresholds $(\theta_2^*, x_2^*)$ exists, in which*

$$\begin{cases} \theta_2^* = 1 + m_2 - c \\ x_2^* = 1 + m_2 - c + \delta(1 - 2c). \end{cases} \tag{6}$$

Note that in this case, the dynamic game is simply a repeated version of the static game. This is not surprising because the public news is useless. However, if $x_2^* + \delta > \theta_1^*$, the marginal investor finds the public news useful, that is, $\Pr\left(\theta \leq \theta_2^* \middle| x_2 = x_2^*, \theta \in [-B, \theta_1^*]\right) \neq$

[18] Now that $\theta \leq \theta_1^*$ is common knowledge, any equilibrium with $(\theta_2^* > \theta_1^*, x_2^* > \theta_1^* + \delta)$ is equivalent to one with $(\theta_2^*, x_2^*) = (\theta_1^*, \theta_1^* + \delta)$.

$\Pr\left(\theta \leq \theta_2^* \middle| x_2 = x_2^*\right)$. We call this equilibrium *Equilibrium with Partial Dynamic Coordination* because government intervention in the first period partially influences the coordination among investors in the second period. Equilibrium without dynamic coordination is an artifact of bounded noise in the private signals. For unbounded noise, equilibrium always involves at least partial dynamic coordination.

**Lemma 4** (Stage Game Equilibrium with Partial Dynamic Coordination)
*If $s_1 = S$ and $m_1 - 2\delta(1-c) < m_2 < m_1 - c$, an equilibrium exists with thresholds*

$$\begin{cases} \theta_2^* = 1 + m_2 - c + \frac{c[m_2 - m_1 + 2\delta(1-c)]}{2\delta - c(1+2\delta)} \\ x_2^* = 1 + m_2 - c + \delta(1-2c) + \frac{c(1+2\delta)[m_2 - m_1 + 2\delta(1-c)]}{2\delta - c(1+2\delta)}. \end{cases} \tag{7}$$

Simple comparisons show that with partial dynamic coordination, both $\theta_2^*$ and $x_2^*$ are higher than their counterparts in the case without dynamic coordination. Therefore, the fund is more likely to survive, and investors are more likely to stay compared to the case without dynamic coordination. Intuitively, the public news that $\theta < \theta_1^*$ has eliminated the possibility of $\theta$ being too high so that in equilibrium, investors who take such elimination into account will behave more aggressively by choosing a higher threshold $x_2^*$. As a result, more investors tend to stay for a given $\theta$, and the fund is more likely to survive due to the increasing coordinated decisions to stay.

Combining Lemmas 2, 3, and 4, Proposition 1 describes the equilibrium outcome given any $(m_1, m_2)$ and $s_1 = S$.

**Proposition 1** (Equilibrium in period 2 when $s_1 = S$)

1. *If $m_2 < m_1 - 2\delta(1-c)$, the unique equilibrium is the Stage Game Equilibrium without Dynamic Coordination.*
2. *If $m_1 - 2\delta(1-c) < m_2 < m_1 - c$, the unique equilibrium is the Stage Game Equilibrium with Partial Dynamic Coordination.*
3. *If $m_1 - c < m_2$, the unique equilibrium is the Stage Game Equilibrium with Dynamic Coordination.*

The intuition for the results is as follows. First, let us ignore the public news from period 1 that $\theta \in [-B, \theta_1^*]$ so that the equilibrium in period 2 is without dynamic coordination.

Then the cutoffs $(\theta_2^*, x_2^*) = (1 + m_2 - c, 1 + m_2 - c + \delta(1 - 2c))$ are similar to those in the period-1 equilibrium. From the marginal investor's perspective, the true $\theta \in [x_2^* - \delta, x_2^* + \delta]$. If it further holds that $x_2^* + \delta < \theta_1^*$ (when $m_2 < m_1 - 2\delta(1 - c)$), the marginal investor's private signal dominates his inference on $\theta$ from the public news. In other words, the public information that $\theta < \theta_1^*$ does not further help him infer the true distribution of $\theta$ on top of this private signal.

Next, consider the case when the marginal investor finds the information at least partially useful, in which case both thresholds $(\theta_2^*, x_2^*)$ are augmented (See Lemma 4). If it further holds that $\theta_2^* > \theta_1^*$ (when $m_2 > m_1 - c$), the second fund survives for sure because $\theta$ is known to be lower than $\theta_1^*$. In this case, the public news dominates the private signal.

### 3.2.2 Failure News

If the fund in period 1 has failed ($s_1 = F$), the prior belief on $\theta$ is bounded below at $\theta_1^*$: $\theta \sim Unif[\theta_1^*, B]$. Proposition 2 summarizes the equilibrium outcome in this case. The detailed derivation can be found in Appendix B. Notice that in the Stage Game Equilibrium with Dynamic Coordination, investors choose to run regardless of their signals. The intuitions for different cutoffs are similar to those in Proposition 1.

**Proposition 2** (Equilibrium in period 2 when $s_1 = F$)

1. *If $m_2 < m_1 + 1 - c$, the unique equilibrium is the Stage Game Equilibrium with Dynamic Coordination.*

2. *If $m_1 + 1 - c < m_2 < m_1 + 2c\delta$, the unique equilibrium is the Stage Game Equilibrium with Partial Dynamic Coordination.*

3. *If $m_1 + 2c\delta < m_2$, the unique equilibrium is the Stage Game Equilibrium without Dynamic Coordination.*

### 3.2.3 Investors' Welfare and Dynamic Coordination

Let $W_{2S} = E\left[\int_0^1 V_{2i} di \middle| s_1 = S\right]$ be the total expected payoff in period 2 conditional on $s_1 = S$. Also, let $W_{2F} = E\left[\int_0^1 V_{2i} di \middle| s_1 = F\right]$ be the total expected payoff in period 2 when $s_1 = F$. Applying results from Proposition 1 and 2, we are able to obtain $W_{2S}$ and $W_{2F}$ for given values of $m_1$ and $m_2$. Corollary 1 below shows the results.

**Corollary 1** (Investors' Welfare in Period 2) *1. Conditional on* $s_1 = S$*,*

*(a) If* $m_2 < m_1 - 2\delta(1-c)$*,* $W_{2S}^{nc} = \frac{(1-c)[1+B-c(1+\delta)+m_2]}{B+\theta_1^*}$.

*(b) If* $m_1 - 2\delta(1-c) < m_2 < m_1 - c$*,*
$W_{2S}^{pc} = \frac{1-c}{\theta_1^*+B}\left[\theta_1^* + B + \frac{\delta c(c-m_1+m_2)^2+2\delta(c-m_1+m_2)[2\delta-c(1+2\delta)]}{[2\delta-c(1+2\delta)]^2}\right]$.

*(c)* $m_2 > m_1 - c$*,* $W_{2S}^{c} = (1-c)$.

*2. Conditional on* $s_1 = F$*,*

*(a) If* $m_2 < m_1 + 1 - c$*,* $W_{2F}^{c} = 0$.

*(b) If* $m_1 + 1 - c < m_2 < m_1 + 2c\delta$*,* $W_{2F}^{pc} = \frac{1-c}{B-\theta_1^*}\frac{c\delta(-1+c-m_1+m_2)^2}{(-1+c+2c\delta)^2}$.

*(c) If* $m_2 > m_1 + 2c\delta$*,* $W_{2F}^{nc} = \frac{1-c}{B-\theta_1^*}(m_2 - m_1 - c\delta)$.

The superscripts of $W_{2S}$ and $W_{2F}$ refer to equilibrium types. $nc$, $pc$, and $c$ respectively stand for equilibrium without dynamic coordination, with partial coordination, and with coordination. The expression of $W_{2S}^{nc}$ is isomorphic to $W_1$, except that the denominator is replaced by $\theta_1^* + B$ because the period-1 outcome is informative of the distribution of $\theta$. $W_{2S}^{pc}$ includes an additional positive term $\frac{\delta c(c-m_1+m_2)^2+2\delta(c-m_1+m_2)[2\delta-c(1+2\delta)]}{[2\delta-c(1+2\delta)]^2}$ which captures the investors' increasing coordinated decision to stay due to the public information. Thus, $W_{2S}^{pc} > W_{2S}^{nc}$. Finally, with coordination all investors stay irrespective of their signals and thus $W_{2S}^{c} = 1-c$. $W_{2F}$ can be interpreted similarly.

The left panel of Figure 1 plots $W_{2S}$ against $m_2$, including the welfare function in all three different types of equilibria. Given $m_1$, $W_{2S}$ is continuous, increasing in $m_2$, and convex in the region that involves partial dynamic coordination. Unlike in the first period, the marginal effect of $m_2$ on $W_{2S}$ is no longer a constant. Initially, $W_{2S}$ increases linearly in $m_2$ , in which case the intervention in the first period has no dynamic coordination effect. When $m_1 - 2\delta + 2c\delta < m_2 < m_1 - c$, the marginal effect of $m_2$ is increasing, due to the dynamic coordination effect of period 1 intervention. When $m_2 > m_1 - c$, the dynamic coordination effect is maximized and all agents' decisions are well coordinated towards an equilibrium without any run. In that case, further increasing $m_2$ has no effect.

Similarly, the right panel of Figure 1 plots $W_{2F}$ against $m_2$, including the welfare function in all three different types of equilibria. Given $m_1$, $W_{2F}$ is continuous, increasing in $m_2$, and convex when the equilibrium involves partial dynamic coordination. The effect of $m_2$ on $W_{2F}$ is not a constant either. When $m_2 < m_1 + 1 - c$, the failed intervention in period 1 makes all agents pessimistic. A slight increase in $m_2$ does not change people's belief, and therefore the marginal effect of $m_2$ on $W_{2F}$ is zero. When $m_1 + 1 - c < m_2 < m_1 + 2c\delta$, the marginal effect of $m_2$ on $W_{2F}$ is positive and increasing. Finally, when $m_2 > m_1 + 2c\delta$, the dynamic effect is zero and $W_{2F}$ increases linearly in $m_2$.

Clearly, $m_1$ affects both $W_{2S}$ and $W_{2F}$ by altering $\theta_1^*$, and thus the resulting informational structure. Because $W_{2S}$ and $W_{2F}$ are piecewise in $m_1$ and thus not everywhere differentiable, we define the left-hand derivative of $W_{2S}$ and $W_{2F}$ *w.r.t.* $m_1$ as the *conditional inference effect*, as Figure 2 illustrates.

**Proposition 3** (**Conditional Inference Effect**)
*Given $s_1$ and $m_2$, investors' welfare $W_{2S}$ and $W_{2F}$ decrease in $m_1$.*

The Conditional Inference Effect implies that given the outcome of the period-1 fund, and given the government's intervention into the period-2 fund, higher initial intervention always decreases investors' welfare. This effect is consistent with the fact that the government often faces uncertainties on intervention outcomes, and is aware of the informational detriments of using large interventions.[19] However, the overall effect of $m_1$ on the unconditional $E[W_2]$ is non-monotonic because besides the conditional inference effect, the probability of $s_1 = S$ also increases with $m_1$. Thus, to the extent that intervention outcomes are correlated across the two periods, increasing $m_1$ kills two birds with one stone. Figure 3 shows this non-monotonic property by plotting $E[W_2] = \Pr(s_1 = S) W_{2S} + (1 - \Pr(s_1 = S)) W_{2F}$ against $m_1$, taking $m_2$ as given. Clearly, the overall effect attains its highest level at $m_1 = m_2 + c$, and starts to decline afterward.[20]

That said, the intervention outcomes are perfectly correlated if $c - 1 < m_1 - m_2 < c$. In this region, the stage-game equilibrium features dynamic coordination no matter the fund survives in the first period or not. As we will show in the next section, if the intervention

[19] See, for example, Bernanke (2015), page 282, on the intervention in AIG: "A critical question was whether the proposed \$85 billion line of credit would in fact save the company. For us, the ultimate disaster would be to lend such an enormous amount and then see the company collapse."

[20] The decline is intuitive: conditional on $s_1$, a larger $m_1$ leads to a more negative update on $\theta_1^*$ because investors attribute the fund's survival more to the large intervention. However, they become pessimistic about the fundamental when the fund fails.

costs are similar across the two periods, the interventions $\{m_1, m_2\}$ also tend to be close to one another even without public learning. This force makes the public signal $s_1$ dominate over the private signal $x_{i2}$, leading to a greater tendency for highly correlated coordination outcomes, which we refer to as the *endogenous correlation effect*.

**Proposition 4** (**Endogenous Correlation Effect**)
*Investors' welfare $E[W_2]$ increases in $m_1$ when $c - 1 < m_1 - m_2 < c$.*

To get the general intuition for this effect, it is useful to compare equilibrium thresholds across different types of stage game equilibria. When $s_1 = S$ and $m_2 \in (m_1 - 2\delta(1-c), m_1 - c)$, both $x_2^*$ and $\theta_2^*$ in the Equilibrium with Partial Dynamic Coordination (Lemma 4) exceed their counterparts in the Equilibrium without Dynamic Coordination (Lemma 3), but are less than those in the Stage Game Equilibrium with Dynamic Coordination (Lemma 2). When the marginal agent finds the public news useful and realizes that the expected threshold level suggested by his signal alone is too stringent, he behaves more aggressively by choosing a higher threshold and running less often. As a result, $\theta_2^*$ is also higher and the fund is more likely to survive. Similarly, when $s_1 = F$, both $x_2^*$ and $\theta_2^*$ are the lowest (most stringent) in the Stage Game Equilibrium with Dynamic Coordination, followed by the Stage Game Equilibria with partial and no dynamic coordination. If we take coordination outcomes without dynamic learning as the benchmark, the dynamic coordination effect of initial intervention suggests that initial survival increases the likelihood of subsequent survival, and initial failure increases the likelihood of subsequent failure, which drives Proposition 4. Again, anecdotes during the recent financial crisis support the assertion that the policy makers place weights on this impact of one intervention on subsequent coordination outcomes.[21]

At the same time, recall that within the regions of partial dynamic coordination, conditional inference effect dominates and $\mathbb{E}[W_2]$ is decreasing in $m_1$. Endogenous correlation matters more when we consider the types of news (good, bad, irrelevant), whereas condi-

[21]For example, Geithner (2014), page 215, recounts how requiring haircuts in FDIC's involvement in Washington Mutual (WaMu) makes the intervention weaker, and the policy makers were concerned that the requirement might lead to a higher probability of failure, "more bank failures and much bigger FDIC losses down the road," and that "more failures would eventually require more aggressive government interventions." Bernanke (2015) also mentions on page 277 that "financial panics are a collective loss of the confidence essential for keeping the system functioning," and the FDIC's sale of WaMu would likely trigger downgrades and worsen market beliefs.

tional inference matters more when we consider the quality variation within the same type of news. What then determines which effect dominates? Note that the initial intervention outcome induces jumps in the belief in the second period's coordination outcome. Outside the region $c - 1 < m_1 - m_2 < c$, the jump is moderate and conditional inference weakly dominates; otherwise, the jump is sufficiently big (posterior on the outcome becomes either zero or one), and creates discontinuity in the impact of conditional inference, leading to endogenous correlation dominating in the absence of conditional inference. In other words, as long as $m_1$ relative to $m_2$ is such that the news is extremely informative, agents care more about whether it is good news or bad news because the conditional inferences for different values of $m_1$ in this case are all the same.[22]

In our model, the initial intervention essentially designs information for the subsequent intervention. Related is Goldstein and Huang (2016) which specializes to the case of costless information design for a single coordination game. In their setup, maintaining the regime too often reduces agents' positive updates, resembling our conditional inference effect. However, abandoning too often results in costly failures, and thus should be avoided. We generalize this desire for good news to multiple interventions and derive the novel endogenous correlation effect of intervention outcomes. Because information design is costless, the policymaker optimally commits to abandoning the regime with a high enough frequency so that a regime maintenance results in no attack, so maintenance leads to survival; and because the game ends if the regime is abandoned, abandonment leads to failure. We show that with costly information design and continuation game even upon initial failure, survival outcomes still tend to correlate. However, the correlation is in general imperfect and relative intervention costs matter. As discussed next, this phenomenon has profound implications when discussing endogenous intervention policy across countries or episodes of runs.

[22]We also note that the mechanism does not crucially rely on the distributional assumption of signals. For example, in the discussion of Normally distributed signals in Appendix C, we show that there are still an equilibrium outcome with full dynamic coordination and one with partial dynamic coordination. Therefore, either endogenous correlation or conditional reference can dominate.

# 4 Dynamic Intervention and Optimal Policy

The analysis so far has taken as given the government's interventions $\{m_1, m_2\}$ and studies investors' coordination for given interventions. In this section, we consider the government's problem. Specifically, given the costs and constraints of interventions, how should the government allocate resources across two periods, and how does the information-structure channel affect the scale and sequence of interventions? This section discusses three key implications for the optimal policy: emphasis on initial intervention, under- and over-intervention by myopic governments, and saving smaller funds disproportionally more.

Equation (5) states the government's objective, which is to maximize all investors' payoff net the intervention cost. The government's strategy space is to choose $m_1 \in [0, \bar{m}_1]$ and $m_2 \in [0, \bar{m}_2]$ subjecting to potential information set and implementation constraint. This section focuses on the case of *committed intervention*, which corresponds to choosing $m_2$ before $s_1$ is realized. In Section 5.2, we discuss how our main results and intuition carry through under some reasonable parameter ranges, for the case of *contingent intervention* that entails choosing $m_2$ after $s_1$ is realized. Committed intervention describes situations in which the government has to roll out policy programs or set up funding facilities before knowing the outcome of previous interventions, as was the case in the recent crisis.

## 4.1 Emphasis on Initial Intervention

Let us first consider a special case that the government has a total budget $M$ which can be used across the two periods, which admits explicit solutions.[23] A benevolent government solves the following problem:

$$\max_{m_1, m_2} W = E\left[\int_0^1 V_{1i} di + \int_0^1 V_{2i} di\right] \tag{8}$$

$$s.t.\ m_1 + m_2 = M. \tag{9}$$

We have shown earlier the information channel that arises from dynamic learning: while $W_1$ increases linearly with $m_1$, $W_2$ is non-monotonic in $m_1$ and increases with $m_2$ in a non-

[23] In other words, $K(m_1, m_2) = \frac{\mathbb{I}_{\{m_1+m_2>M\}}}{1-\mathbb{I}_{\{m_1+m_2>M\}}}$.

linear manner. Because the government also faces a hard budget constraint $m_1 + m_2 = M$, an increase in $m_1$ necessarily crowds out $m_2$ through the budget channel.[24] When the government optimally allocates resources in two periods, it needs to consider both.

Figure 4 plots a typical social welfare $W$ as $m_1$ varies. Both the intuitions and the pattern are similar to those in Figure 3. The intuitions again depend on the comparison between the endogenous correlation effect and the conditional inference effect. The patterns delivered by the figure hold for all parameters. (a) $W$ is always flat for either small or large $m_1$. (b) $W$ always attains its maximum at $m_1 = \frac{M+c}{2}$. Therefore, whenever $M$ is larger than $c$, the government should invest $m_1^* = \frac{M+c}{2}$. Lemma 5 in the Appendix summarizes the aggregate social welfare and the net benefit of initial intervention.

Proposition 5 characterizes the optimal intervention.

**Proposition 5** (**Optimal Intervention**)
*The optimal intervention under budget constraint $M$ is* $\min\left(\frac{c+M}{2}, M\right)$. *Optimal intervention always emphasizes initial intervention:* $m_1^* > m_2^*$.

The optimal intervention plan depends on $M$, the total resources available to the government. When $M$ is small ($M < \frac{M+c}{2}$), it is optimal to set $m_1 = M$. By contrast, when $M$ gets larger, setting $m_1 = M$ may be sub-optimal, and the optimal initial intervention is $m_1 = \frac{M+c}{2}$.

At the optimal intervention level, the fund in period 2 survives if and only if the fund in period 1 survives. The endogenous correlation effect completely dominates. The intuition for $m_1^* > m_2^*$ is then apparent: suppose the government equally splits the budget and invests $\frac{M}{2}$ in each period. Two periods' intervention outcomes are completely correlated. Knowing this, government always has incentives to kill two birds with one stone – increasing $m_1$ to increase the survival probability in both funds. The ratio $\frac{m_2^*}{m_1^*}$ is weakly increasing in $M$ and weakly decreasing in $c$. Thus, the tilt toward initial intervention is most significant when the government has a small budget or the illiquidity cost is high.

One may question whether the results are driven by the fact that imposing the budget constraint takes away the flexibility of $m_2$ after $m_1$ is chosen. By specifying a very general

[24]See Geithner (2014), pages 264-266, for an example of intervention cost and budget consideration.

$K(m_1, m_2)$, we show that emphasizing initial intervention is a robust phenomenon under committed interventions.

**Proposition 6** (**Emphasis on Early Intervention**)
*If the intervention cost satisfies* $K(m_1, m_2) > K\left(\frac{1}{2}[m_1 + m_2], \frac{1}{2}[m_1 + m_2 - 2c]\right)$*, the optimal policy strictly emphasizes initial intervention, that is,* $m_1^* > m_2^*$*.*

The condition in the proposition is satisfied by many plausible cost functions, such as one that is separable and symmetric in $m_1$ and $m_2$.[25] The $-2c$ in the condition derives from the endogenous correlation effect, if the first intervention optimally uses $m$, the second only needs $m - c$ because initial success makes later intervention easier to succeed due to positive updating and initial failure makes additional intervention later futile due to negative updating.

Note that this proposition is not about comparing the absolute sizes of the interventions. Given that we have normalized the total capital in the economy to one in both periods, we are really talking about a notion of intervention relative to the market size. Therefore, the conclusion could apply more broadly, especially when the coordination games are scale-invariant, that is, the normalized intervention, cost, and participation scale proportionally with the market size.[26]

## 4.2 Information Externality and Myopic Intervention

This section examines the situations in which the decision-maker for the initial intervention does not fully take into consideration the informational impact on subsequent interventions. This scenario happens when the incumbent government is not expecting to be re-elected and thus does not consider the impact of current intervention on coordination and interventions under the future government. This scenario could also happen when one

[25]Or one that emphasizes consistency in the sense that $K(m_1, m_2)$ only depends on $m_1 + m_2$ and $|m_1 - m_2|$ and is increasing in $|m_1 - m_2|$. One natural example is $K(m_1, m_2) = k(m_1) + k(m_2)$.

[26]Indeed, the eligible ABCPs for AMLF constitute less than half of the commercial paper markets. Thus, the scale of AMLF ($150 billion in the first 10 days relative to the magnitude of the run-$172 billion plummet from the $3.45-trillion MMF sector) is higher than CPFF ($144 billion usage in the first week, relative to a reduction of commercial paper outstanding, larger both in percentage (15%) and in level (330 billion)) that targets almost the entire commercial paper market. AMLF and its success also seem to have helped later interventions. For example, CPFF was also effective and even generated $5 billion in net income for the government.

European-Union country's intervention does not fully consider the informational externality on neighboring countries with correlated fundamentals.

To highlight the information externality from the initial intervention, we shut down the budget channel in our general intervention cost function by setting $K_{12}(m_1, m_2) = 0$ for the remainder of the paper.[27] For a given $m_1$, let us define

$$Y(m_1; \chi) \quad = \quad W_1 + \chi \max_{m_2} E\left[W_2\right] - K\left(m_1, m_2\right). \tag{10}$$

$Y(m_1; \chi)$ is the social welfare given the initial intervention $m_1$. Note that the choice of $m_2$ is already optimized, and the government chooses $m_1$ to maximize the social welfare. Here, $\chi \in [0, 1]$ measures how much the government cares about the fate of the fund in the second period. In particular, $\chi = 0$ corresponds to the static benchmark, and $\chi = 1$ corresponds to the case in which the second fund's fate is equally important. $\chi < 1$ corresponds to the short-termism of the government. Alternatively, in the context of the global economy in which countries' fundamentals are correlated, $\chi$ captures the extent to which one country considers the externality it imposes on others.

We are interested in $\frac{\partial m_1^*}{\partial \chi}$, the effect of government myopia on the initial intervention. By Theorem 2.1 in Athey, Milgrom, and Roberts (1998), $m_1^* \equiv \text{argmax}_{m_1} Y(m_1, \chi)$ is non-increasing in $\chi$ iff $Y$ has decreasing differences in $\chi$ and $m_1$, and is non-decreasing in $\chi$ iff $Y$ has increasing differences in $\chi$ and $m_1$.

**Proposition 7** (**Myopic Intervention**)

*A myopic government may under- or over- intervene initially. In particular, it*

1. *under-intervenes* ($\frac{\partial m_1^*}{\partial \chi} \geq 0$) *iff either* $m_1^* \geq c$ *and* $m_2^* = m_1^* - c$ *always or* $m_1^* \leq c$ *and* $m_2^* = 0$ *always;*

2. *over-intervenes* ($\frac{\partial m_1^*}{\partial \chi} \leq 0$) *iff either* $m_2^* > m_1^* + 1 - c$ *always or* $m_1^* > c$ *and* $m_2^* < m_1^* - c$ *always.*

Proposition 7 emphasizes $m_1$ relative to the case in which the intervention externality is absent. A myopic government under-intervenes initially when intervention outcomes are

[27] The case of a hard budget constraint trivially predicts that the more the government considers the welfare in the second period, the less it would intervene in the first period.

perfectly correlated. This scenario happens when the costs of intervention in the two periods are comparable. When they are both large ($m_1^* \leq c$ and $m_2^* = 0$) or both small ($m_1^* \geq c$ and $m_2^* = m_1^* - c$), the endogenous correlation effect dominates. Therefore, increasing the probability of survival by increasing $m_1$ also benefits investors in the second period, a fact that a myopic government neglects.

Interestingly, failure to consider dynamic coordination could also result in excessive intervention. This happens when the cost for the first intervention is sufficiently small such that the initial intervention is large scale, yet the second intervention is sufficiently costly that survival does not always lead to survival. Meanwhile, a high $m_1$ reduces the quality of good news, reducing the marginal benefit of $m_2$. When the costs of intervention in the two periods are rather disproportionate, outcomes are less correlated, and the conditional inference effect dominates. For a myopic government, shading $m_1$ makes intervention in the second period easier regardless of whether the fund survives or fails in the first period.

We illustrate the results in Figures 6 and 7. More generally, without global increasing or decreasing differences, $m_1^*$ can be non-monotonic in $\chi$, as seen in Figure 8. Furthermore, to link Proposition 7 to exogenous parameters, we provide some sufficient conditions in the appendix for both under-and over- intervention.

The above proposition calls for coordinated interventions across governments. For example, because economic fundamentals across EU countries are highly correlated, one member's isolated intervention imposes informational externality on other members. In the case of AMLF and CPFF, because the capacity to intervene using CPFF is comparable to that in AMLF, the later intervention was likely able to capture the benefit from investors' learning of earlier intervention. The above proposition thus provides additional justification for the overwhelming scale of AMLF.

## 4.3 Intervention with Heterogeneous Funds

In this section, we consider how the government intervenes in two funds of different sizes, given the dynamic coordination effect. Our main results are two-fold. First, the government tend to put more resources into the fund whose outcome is more likely to realize first, when cost function does not differ significantly across the interventions. This is essentially

Propositions 5 and 6 extended to funds of heterogeneous size and stochastic realization of ordering. Second, it is optimal to provide more resources to the smaller fund that are disproportional to its size and the likelihood that its outcome is realized first. In particular, when the two funds are equally likely to have their outcomes realized, the government puts disproportionally more resources into the smaller one.

Without loss of generality, we normalize the size of fund 1 to 1, and the size of fund 2 to $\lambda > 1$. Here, size simply refers to the total measure of investors. We continue to assume fund 1 survives if and only if

$$A_1 + m_1 \geq \theta,$$

where $A_1$, $m_1$, and $\theta$ have the same interpretations as before. In addition, fund 2 survives if and only if

$$\lambda A_2 + m_2 \geq \theta\lambda,$$

where $A_2 = \frac{\int_0^\lambda 1\{a_{2i}=1\}ds}{\lambda} \in [0,1]$ is the fraction of investors who choose to stay, and thus $\lambda A_2$ is the liquidity from remaining investors. Fund 2 survives if and only if the total liquidity is greater than $\theta\lambda$. The threshold is also augmented by $\lambda$ so that we are not distorting the funds' survival probability absent interventions.[28]

Our analysis so far has assumed without loss of generality that fund 1's outcome is always realized before fund 2's, because these two funds are homogeneous. When two funds differ in size, it matters which fund has its outcome realized first. To proceed, we assume that with probability $q \in (0,1)$, fund 1's outcome is realized first, and with probability $1-q$, fund 2's outcome is realized first. Given this probability, the government chooses $m_1$ and $m_2$ to continue to maximize the total social welfare.

In the case when the government has a budget constraint $m_1 + m_2 = M$ and allocates resources proportional to fund size, then $m_1 = \frac{M}{1+\lambda}$ and $m_2 = \frac{M\lambda}{1+\lambda}$. Proposition 8 shows that the government puts relatively more resources into the fund whose outcome is more likely

[28] $\theta$ in the baseline specification captures the systemic illiquidity for a market or fund of unit size. Therefore, we scale it up when the fund size scales up. This is a natural specification, because if we keep $\theta$ unscaled while changing the size of the fund, we are implicitly making larger fund more likely to survive, which clouds the informational effect on which we hope to focus. To see this, let us examine the static example. The survival threshold the larger fund becomes $\lambda(1-c)+m$, so with the same intervention, the larger fund survives with greater probability.

to realize first. When $q = \frac{1}{2}$ so that both funds' outcomes are equally likely, the government put more resources into the small fund.

**Proposition 8**

*A government facing hard budget constraint $m_1 + m_2 = M$ chooses the following optimal intervention policy*

1. *If $q > \frac{1}{\lambda+1}$, $m_1^* = \frac{M+c\lambda}{1+\lambda}$ and $m_2^* = \frac{(M-c)\lambda}{1+\lambda}$.*

2. *If $q < \frac{1}{\lambda+1}$, $m_1^* = \frac{M-c\lambda}{1+\lambda}$ and $m_2^* = \frac{(M+c)\lambda}{1+\lambda}$.*

3. *If $q = \frac{1}{\lambda+1}$, $m_1^* \in \left[\frac{M-c\lambda}{1+\lambda}, \frac{M+c\lambda}{1+\lambda}\right]$ and $m_2^* = M - m_1^*$.*

*When $q = \frac{1}{2}$, $m_1^* = \frac{M+c\lambda}{1+\lambda} > \frac{M}{1+\lambda}$.*

Appendix A.7 contains the proof. Note that $\frac{1}{\lambda+1} < \frac{1}{2}$, it means that even when the smaller fund is likely to realize its outcome later ($q \in (\frac{1}{\lambda+1}, \frac{1}{2})$ the government still favors disproportionally helping it more, for the following two reasons. First, given a probability to realize its outcome first, the smaller fund costs less intervention resources to generate the same informational environment. Second, the larger fund benefits more from the resolution of uncertainty due to the revelation of the initial intervention's outcome. These intuitions also imply that if the government can determine which fund realizes the outcome first, it would choose the smaller one first. The above result carries through with general intervention cost functions with $K\left(m_1, \frac{m_2}{\lambda}\right) > K\left(\frac{1}{2}\left[m_1 + \frac{m_2}{\lambda}\right], \frac{1}{2}\left[m_1 + \frac{m_2}{\lambda} - 2c\right]\right)$, a slight modification from the condition in Proposition 6.

Our result thus relates to the concept of "too big to fail." Rather than emphasizing financial networks and connectedness, we are adding an information-structure perspective to the debate on systemic fragility. Some institutions could be too big to fail, but the best way to save them may entail put more resources into the smaller ones to better boost market confidence.[29]

[29]Our results do not contradict 'too big to fail' in that we are not discussing which institutions to save, but which institutions to save first. One way to think about this is that among the too-big-to-fail institutions, if the government can commit to which institutions intervention outcome to be revealed first, it is informationally efficient to reveal the intervention outcome of the smaller fund first.

# 5 Discussions and Extensions

The economic intuition and main results in the paper are robust to a wide range of alternative specifications. In this section, we briefly discuss how imperfectly correlated fundamentals, contingent interventions, and alternative forms of interventions can be accommodated in our framework. In the online appendix we further extend the model to incorporate general ranges of $\delta$ and $c$, $\theta$-dependent payoff structures, moral hazards, Normally distributed signals, and general bounded noise distributions.

## 5.1 Imperfectly Correlated Fundamentals

So far we have assumed $\theta_1 = \theta_2$. What if the fundamentals across the two periods are positively correlated but non-identical? In this section, we introduce one tractable way to model this scenario.

Suppose at the beginning of period 2, everyone learns whether the fundamental in period 2 is identical to that in period 1, or just an independent one. In other words, whether period 2 is an extension of period 1's coordination game, or an independent one becomes public. Specifically, we assume that with probability $\pi$, $\theta_2 = \theta_1$, and with probability $1 - \pi$, $\theta_2$ is a random draw from $[-B, B]$ independent of $\theta_1$. Our baseline model corresponds to $\pi = 1$. In the case in which $\pi = 0$, the intervention problem is symmetric, which yields our benchmark policy $m_1^* = m_2^*$. With $\pi \in (0, 1)$, the intuition for all the implications continues to apply and the previous results are only affected qualitatively. We can show that similar to Proposition 7, the correlation positively increases $m_1^*$ when the cost functions are comparable (Corollary 2); the correlation reduces $m_1^*$ if the cost functions are asymmetric (Corollary 3); otherwise, the effect could be non-monotone, as illustrated in Figures 6 to 8 (replacing $\chi$ by $q$). Thus, how the correlation in the fundamental affects endogenous government intervention also depends on the intervention costs.

## 5.2 Contingent Interventions

In reality, government can sometimes choose the size of later intervention after the outcome of the initial intervention is realized. We analyze this case in this section. Let

$\{m_1, m_{2S}, m_{2F}\}$ be the government's intervention where $m_{2S}$ and $m_{2F}$ are the second-period's intervention upon $s_1 = S$ and $s_1 = F$. The intuition and key tradeoff in earlier discussions still apply. Note that any $m_{2F} \in \big(0, 1 + m_1^* - c\big]$ cannot be optimal since if $s_1 = F$, the fund in the second period still fails for sure despite the costly intervention. Given this, if $m_{2F}^* = 0$, the endogenous correlation effect is even reinforced; if $m_{2F}^* > 1 + m_1^* - c$, however, it is possible to have failed initial intervention but successful subsequent intervention, and the endogenous correlation effect is weaker. Nevertheless, the overall dynamic coordination still boils down to a tradeoff between the endogenous correlation effect and the conditional inference effect.

Intuitively, if the cost for the subsequent intervention (second period) increases with the amount of intervention $m_2$ fast enough relative to the first period, then $m_{2F}^* = 0$ because conditional on failure, it is not worthwhile to intervene more given the pessimistic posterior. Therefore, endogenous correlation dominates and initial intervention is emphasized, i.e., $m_1^* > m_{2S_1}^*$, due to the same reasoning as in Proposition 6.

Similarly, with contingent interventions, myopic government still under-or over- intervenes initially. If endogenous correlation effect dominates the conditional inference effect, the optimal initial intervention is weakly increasing in the extent it considers dynamic coordination, i.e., $\frac{\partial m_1^*}{\partial \chi} \geq 0$. Otherwise, it is weakly decreasing. Because when $m_{2F}^* = 0$, the endogenous correlation effect is the same as in the committed intervention case, and the same intuition carries through.

Finally, regarding the sequence of interventions in funds of different sizes, saving the smaller fund disproportionally more relative to its size and probability of realizing the coordination outcome first (and saving it first if the government can decides the order of the realizations) is still cheaper for creating the same informational environment, plus the larger fund still benefits more from the uncertainty reduction. A policy that induces perfectly correlated outcomes and saves the larger fund first or disproportionally more relative to its size cannot be optimal.

In Appendix C, we discuss how the intuition in Propositions 6, 7, and 8 extends to the case of contingent interventions.

## 5.3 Various Forms of Interventions

In the model, we have interpreted intervention as liquidity injection. We argue below that our model captures a broader array of interventions that are commonly used (Bebchuk and Goldstein, 2011; Diamond and Rajan, 2011). Here are a non-exhaustive list of examples.

**Direct lending and investing in borrower funds** This is exactly the interpretation in our model. During the financial crisis of 2008-2009, the US government directly participated in the commercial paper market through direct purchasing. Our general cost function to a large extent captures investment returns to the government and some inefficiencies discussed in Bebchuk and Goldstein (2011).

**Direct capital infusion to investors** Governments around the globe have injected capital to both retail and institutional investors. For instance, the U.S. Troubled Asset Relief Program (TARP) provided about US$250 billion to banks, and the UK injected about US$90 billion to its major banks. Tax breaks and related measures represent capital infusion to retail investors directly. To map these policies into our model, suppose the government injects a fraction $\alpha$ of investors' existing capital. This changes the capital of each investor from 1 unit to $1+\alpha$ without altering the investor's optimization problem. Consequently, the one period survival threshold becomes $\theta^* = (1-c)(1+\alpha)$. We can relabel $m = (1-c)\alpha$ and the model solutions are equivalent.

**Government guarantees** During the financial crisis, governments used guarantees that are similar to FDIC to limit the potential losses of the lenders. Specifically in our model, suppose that the government guarantees a proportion $\xi$ of a lender's or investors losses, then the lender who stays (rolls over) receives the return $R$ when the fund survives, and $-(1-\xi)c$ if it fails. Since our investors are risk neutral, the survival threshold now is $\theta^* = \frac{1-c}{1-c\xi}$. Again, we can relabel $m = \frac{c(1-c)\xi}{1-c\xi}$ and this is equivalent to an intervention that increases the probability of success.

**Interest Rate Reduction** During the financial crisis, the Fed Reserve Board cut the fed funds rate from 4.25% in Jan 2008 to 1% in Oct 2008. Many other countries took similar measures in the face of a global contraction in lending. In the model, this is equivalent to reducing $r$, the payoff for not investing. Under risk-neutrality, it is equivalent to increasing the survival probability through changing $c$, which is exactly the role of $m$ in our model.

# 6 Conclusion

How should a benevolent government choose policy for multiple interventions in a dynamic environment? Through the lens of sequential global games in which the government is a large player who mitigates coordination failures, we establish the existence of and characterize the equilibria, and show government interventions can affect coordination both contemporaneously and dynamically. A stronger initial intervention helps subsequent interventions through increasing the likelihood of positive news, but also leads to negative conditional updates. Our results suggest optimal intervention often emphasizes initial action, validating the conventional wisdom. However, depending on costs across interventions, an initial intervention could have either a positive or negative informational externality on subsequent coordination. Finally, some funds are "too big to save first," because they benefit more from resolution of uncertainty about the fundamentals, and first intervening in smaller funds leads to lower cost to generate this informational structure. Our paper thus has policy relevance to various intervention programs, such as the bailouts of money market mutual funds and of the financial commercial papers market during the 2008 financial crisis.

The dynamic learning mechanism and thus the information-structure effect also apply to broader contexts, such as interventions in currency attacks, credit market freezes, cross-sector industrialization, regulatory union, and green energy development. Our discussion therefore opens several avenues for future research. For example, how does the government simultaneously design information structure and signal private knowledge about economic fundamentals? Moreover, this paper only considers common forms of interventions. Understanding the optimal contingent intervention not only is of theoretical interest, but also provides new insights and guidance to policymakers.

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# Appendix

## A Derivations and Proofs

### A.1 Proof of Lemma 1

Suppose a threshold $x^* \in \mathbb{R}$ exists such that each agent invests if and only if $x \leq x^*$. The measure of agents who invest is thus

$$A(\theta) = \Pr\left(x \leq x^* \middle| \theta\right) = \begin{cases} 0 & \text{if } \theta > x^* + \delta \\ \frac{x^* - (\theta - \delta)}{2\delta} & \text{if } x^* - \delta \leq \theta \leq x^* + \delta \\ 1 & \text{if } \theta < x^* - \delta. \end{cases} \tag{11}$$

It follows that the investment succeeds if and only if $\theta \leq \theta^*$, where $\theta^*$ solves

$$A(\theta^*) + m = \theta^*. \tag{12}$$

By standard Bayesian updating, the posterior distribution about $\theta$ conditional on the private signal is also a uniform distribution with bandwidth $2\delta$. Therefore, the posterior probability of investment success is

$$\Pr\left(s = S \middle| x\right) = \Pr\left(\theta \leq \theta^* \middle| x\right) = \begin{cases} 0 & \text{if } x > \theta^* + \delta \\ \frac{\theta^* - (x - \delta)}{2\delta} & \text{if } \theta^* - \delta \leq x \leq \theta^* + \delta \\ 1 & \text{if } x < \theta^* - \delta. \end{cases} \tag{13}$$

For the marginal investor who is indifferent between investing or not, his signal $x^*$ satisfies

$$\Pr\left(s = S \middle| x^*\right) = c. \tag{14}$$

Jointly solving equations (12) and (14), we obtain the two thresholds:

$$\begin{cases} \theta^* = 1 + m - c \\ x^* = 1 - c + \delta - 2c\delta + m. \end{cases} \tag{15}$$

### A.2 Proof of Lemma 2

*Proof.* "if" $\Leftarrow$

If $m_2 > m_1 - c$, and if all agents know other agents will adopt a threshold strategy $x_2^* = \infty$,

$$A_2 + m_2 = 1 + m_2 > 1 + m_1 - c = \theta_1^* > \theta. \tag{16}$$

Therefore, the investment succeeds with probability 1. Therefore, it is individually rational for each agent to set $x_2^* = \infty$.

"only if" $\Rightarrow$

We prove by contradiction. Suppose an equilibrium exists in which all agents adopt a threshold $x_2^* = 1 + m_1 - c + \delta$ when $(m_1 - c) - m_2 = \Delta > 0$. Therefore, any agent with a signal $x_2 < \theta_1^* + \delta$ will invest. In other words,

$$\Pr\left(\theta < 1 + m_2 \middle| x_2, \theta < \theta_1^*\right) \geq c,$$

holds for any $x_2$.

Consider an agent who observes $\hat{x}_2 = m_1 + 1 - c + \delta - \frac{\Delta}{2}$. Such an agent exists when $\theta \in \left(m_1 + 1 - c + \delta - \frac{\Delta}{2}, m_1 + 1 - c + \delta\right)$. Apparently,

$$\Pr\left(\theta < 1 + m_2 \middle| x_2 = \hat{x}_2, \theta < \theta_1^*\right) \geq c = 0 < c$$

which violates the assumption that all agents invest irrespective of their signals.

□

## A.3 Proof of Lemmas 3, 4, Propositions 1, 2, and 12

Here we solve the equilibrium in period 2 under both $s_1 = S$ and $s_1 = F$, and under all parameter values. The solutions directly prove the lemmas and propositions.

Our solutions take two steps. First, we assume a solution pair $(\theta_2^*, x_2^*)$ exists and derive the equilibrium values. Second, we check the conditions these solutions must satisfy, and thus derive the parameter ranges such that they indeed constitute a solution.

**Case 1: The Period-1 fund survives:** $s_1 = S$

1. If $\theta_1^* - 2\delta < \theta_2^* < \theta_1^*$, then in equilibrium,

$$\theta_2^* - m_2 = A\left(\theta_2^*\right) = \begin{cases} 1 & \text{if } x_2^* - \theta_2^* > \delta \\ \frac{x_2^* - (\theta_2^* - \delta)}{2\delta} & \text{if -}\delta\text{¡}x_2^* - \theta_2^* < \delta \\ 0 & \text{if } x_2^* - \theta_2^* < -\delta \end{cases}$$

and

$$c = \begin{cases} 1 & \text{if } x_2^* < \theta_2^* - \delta < \theta_2^* < \theta_1^* \\ \frac{\theta_2^* - (x_2^* - \delta)}{2\delta} & \text{if } \theta_2^* - \delta < x_2^* < \theta_1^* - \delta \\ \frac{\theta_2^* - (x_2^* - \delta)}{\theta_1^* - \left(x_2^* - \delta\right)} & \text{if } \theta_1^* - \delta < x_2^* < \theta_2^* + \delta \\ 0 & \text{if } \theta_2^* + \delta < x_2^*. \end{cases}$$

Jointly solving the above equations, the solutions are as follows

(a) $\theta_2^* = 1 + m_2 - c$ and $x_2^* = 1 + m_2 - c + \delta\left(1 - 2c\right)$. The solution exists if $m_1 - 2\delta < m_2 < m_1 - 2\delta\left(1 - c\right)$.

(b) $\theta_2^* = 1 + m_2 - c + \frac{c[m_2 - m_1 + 2\delta(1-c)]}{2\delta - c(1+2\delta)}$ and $x_2^* = 1 + m_2 - c + \delta\left(1 - 2c\right) + \frac{c(1+2\delta)[m_2 - m_1 + 2\delta(1-c)]}{2\delta - c(1+2\delta)}$. The solution exists in two cases: 1) $m_1 - 2\delta\left(1 - c\right) < m_2 < m_1 - c$ if $0 < c < \frac{2\delta}{1+2\delta}$; 2)

$m_1 - c < m_2 < m_1 - 2\delta(1-c)$ if $\frac{2\delta}{1+2\delta} < c < 1$

2. If $\theta_2^* < \theta_1^* - 2\delta$, then in equilibrium,

$$\theta_2^* - m_2 = A(\theta_2^*) = \begin{cases} 1 & \text{if } x_2^* - \theta_2^* > \delta \\ \frac{x_2^* - (\theta_2^* - \delta)}{2\delta} & \text{if -}\delta\text{¡}x_2^* - \theta_2^* < \delta \\ 0 & \text{if } x_2^* - \theta_2^* < -\delta \end{cases}$$

and

$$c = \begin{cases} 1 & \text{if } x_2^* < \theta_2^* - \delta \\ \frac{\theta_2^* - (x_2^* - \delta)}{2\delta} & \text{if } \theta_2^* - \delta < x_2^* < \theta_2^* + \delta \\ 0 & \text{if } \theta_2^* + \delta < x_2^*. \end{cases}$$

Jointly solving the above equations, the solutions are as follows:

(a) $\theta_2^* = 1 + m_2 - c$ and $x_2^* = 1 + m_2 - c + \delta(1-2c)$. The solution exists if $m_2 < m_1 - 2\delta$.

Combining the above results, we prove Lemmas 3, 4, Proposition 1, and half of Proposition 12. The next case finishes the rest of the proof.

**Case 2: The Period-1 fund fails:** $s_1 = F$

The analysis is identical. We will list the results as below.

(a) $\theta_2^* = 1 + m_2 - c - \frac{(1-c)(m_1+2c\delta-m_2)}{c(1+2\delta)-1}$ and $x_2^* = 1 + m_2 - c + \delta(1-2c) - \frac{(1-c)(1+2\delta)(m_1+2c\delta-m_2)}{c(1+2\delta)-1}$.. The solution exists in two cases: 1) $m_1 + 2c\delta < m_2 < m_1 + (1-c)$ if $0 < c < \frac{1}{1+2\delta}$; 2) $m_1 + (1-c) < m_2 < m_1 + 2c\delta$ if $\frac{1}{1+2\delta} < c < 1$.

(b) $\theta_2^* = 1 + m_2 - c$ and $x_2^* = 1 + m_2 - c + \delta(1-2c)$. The solution exists if $m_1 + 2c\delta < m_2 < m_1 + 2\delta$.

(c) $\theta_2^* = 1 + m_2 - c$ and $x_2^* = 1 + m_2 - c + \delta(1-2c)$. The solution exists if $m_2 > m_1 + 2\delta$.

Combining this result, Proposition 2 and the other parts of 12 naturally follow.

## A.4 Proof of Proposition 5

Plugging in the government's budget constraint, we are able to obtain the aggregate social welfare as a function of $m_1$. As a by-product, we are also able to calculate the net benefit of initial intervention. Lemma 5 summarizes the results.

**Lemma 5**

*Aggregate social welfare $W$ and net benefit of initial intervention* $\left.\frac{\partial W}{\partial m_1}\right|_{m_1+m_2=M}$

1. *If* $m_1 > \frac{M+2\delta(1-c)}{2}$,

$$\begin{aligned}
W &= W_1 + \Pr(s_1 = S) W_{2S}^{nc} + \Pr(s_1 = F) W_{2F}^{c} \\
&= \frac{1-c}{2B}\left[2 + 2B - 2c(1+\delta) + M\right] \\
\frac{\partial W}{\partial m_1} &= 0.
\end{aligned}$$

*This case only exists for* $M > 2\delta(1-c)$.

2. *If* $\frac{M+c}{2} < m_1 < \frac{M+2\delta(1-c)}{2}$,

$$\begin{aligned}
W &= W_1 + \Pr(s_1 = S) W_{2S}^{pc} + \Pr(s_1 = F) W_{2F}^{c} \\
&= \frac{1-c}{2B}\Bigg[2 + 2B - c(2+\delta) + 2m_1 \\
&\quad + \frac{\delta c (c + M - 2m_1)^2 - 2\delta\left[c - 2(1-c)\delta\right](c + M - 2m_1)}{\left[c - 2(1-c)\delta\right]^2}\Bigg] \\
\frac{\partial W}{\partial m_1} &= \frac{(1-c)\left[2c(1+2\delta)\left[c - 2(1-c)\delta\right] - 4c\delta(c + M - 2m_1)\right]}{2B\left[c - 2(1-c)\delta\right]^2} < 0.
\end{aligned}$$

*This case only exists for* $M > c$.

3. *If* $\frac{M-(1-c)}{2} < m_1 < \frac{M+c}{2}$,

$$\begin{aligned}
W &= W_1 + \Pr(s_1 = S) W_{2S}^{c} + \Pr(s_1 = F) W_{2F}^{c} \\
&= \frac{1-c}{2B}\left[2 + 2B - c(2+\delta) + 2m_1\right] \\
\frac{\partial W}{\partial m_1} &= \frac{1-c}{B} > 0.
\end{aligned}$$

*This case always exists.*

4. *If* $\frac{M-2c\delta}{2} < m_1 < \frac{M-(1-c)}{2}$,

$$\begin{aligned}
W &= W_1 + \Pr(s_1 = S) W_{2S}^{c} + \Pr(s_1 = F) W_{2F}^{pc} \\
&= \frac{1-c}{2B}\left[2 + 2B - c(2+\delta) + 2m_1 + \frac{c\delta(-1 + c + M - 2m_1)^2}{(-1 + c + 2c\delta)^2}\right] \\
\frac{\partial W}{\partial m_1} &= \frac{1-c}{2B}\left[2 - \frac{4c\delta(c - 2m_1 + M - 1)}{(2c\delta + c - 1)^2}\right].
\end{aligned}$$

*This case only exists for* $M > 1 - c$. *We note the derivative changes sign from negative to positive exactly once in this region.*

5. *If* $m_1 < \frac{M-2c\delta}{2}$,

$$\begin{aligned} W &= W_1 + \Pr(s_1 = S) W_{2S}^{c} + \Pr(s_1 = F) W_{2F}^{nc} \\ &= \frac{1-c}{2B}\left[2 + 2B - 2c(1+\delta) + M\right] \\ \frac{\partial W}{\partial m_1} &= 0. \end{aligned}$$

*This case only exists for* $M > 2c\delta$.

Given that the welfare function is continuous, the maximum welfare in case 3 is higher than case 1 and 5, and how welfare varies with respect to $m_1$ in region 2 and 4, the result in the proposition follows.

## A.5 Proof of Proposition 6

*Proof.* Suppose the optimal $m_1 < m_2$. We show this leads to a contradiction. Notice welfare $W_1 + E[W_2] - K(m_1, m_2)$ is

$$L = -K(m_1, m_2) + \frac{1-c}{2B}\begin{cases} 2(m_1 + 1 - c + B) - c\delta & \text{if } m_2 < m_1 + (1-c) \\ 2(m_1 + 1 - c + B) - c\delta + \frac{c\delta(-1+c-m_1+m_2)^2}{(-1+c+2c\delta)^2} & \text{if } m_1 + (1-c) < m_2 < m_1 + 2c\delta \\ m_1 + m_2 + 2(1 - c + B - c\delta) & \text{if } m_2 > m_1 + 2c\delta. \end{cases} \tag{17}$$

We want to show the above is not optimal, because it is strictly dominated by the welfare at $\left(m_1^{'}, m_2^{'}\right) = \left(\frac{1}{2}[m_1 + m_2], \frac{1}{2}[m_1 + m_2 - 2c]\right)$, which equals

$$L^{'} = \frac{1-c}{2B}\left[m_1 + m_2 + 2(1-c+B) - c\delta\right] - K\left(\frac{1}{2}(m_1+m_2), \frac{1}{2}[m_1 + m_2 - 2c]\right). \tag{18}$$

The cases in which $m_2 < m_1 + (1-c)$ and $m_2 > m_1 + 2c\delta$ are straightforward. It remains to be shown that $W^{'} > W$ when $m_1 + (1-c) < m_2 < m_1 + 2c\delta$. Note that

$$\begin{aligned} sgn\left(L^{'} - L\right) &= sgn\left[(m_2 - m_1) - \frac{c\delta(-1+c-m_1+m_2)^2}{(-1+c+2c\delta)^2}\right] \\ &= sgn\left\{-c\delta\left[(-1+c)^2 + (m_2-m_1)^2 + 2(-1+c)(m_2-m_1)\right] + (m_2-m_1)(-1+c+2c\delta)^2\right\} \\ &= sgn\left\{-c\delta(m_2-m_1)^2 + \left[2c\delta(1-c) + (-1+c+2c\delta)^2\right](m_2-m_1) - c\delta(-1+c)^2\right\}. \end{aligned}$$

It suffices to show $sgn\left\{-c\delta(m_2-m_1)^2 + \left[2c\delta(1-c) + (-1+c+2c\delta)^2\right](m_2-m_1) - c\delta(-1+c)^2\right\} > 0$ at both $m_2 = m_1 + 2c\delta$ and $m_2 = m_1 + (1-c)$, which is straightforward algebra.

Notice that when the cost is separable and symmetric,

$$\begin{aligned} & K(m_1, m_2) - K\left(\frac{1}{2}[m_1+m_2], \frac{1}{2}[m_1+m_2-2c]\right) \\ = & \left[K(m_2) - K\left(\frac{1}{2}[m_1+m_2]\right)\right] - \left[K\left(\frac{1}{2}[m_1+m_2-2c]\right) - K(m_1)\right] > 0 \end{aligned} \tag{19}$$

due to $K$ being weakly convex and increasing. The above also holds when $\frac{1}{2}[m_1 + m_2 - 2c]$ is replaced by $\frac{1}{2}(m_1+m_2)$ and $K$ only depends on $m_1+m_2$ and $|m_1-m_2|$ and is increasing in $|m_1-m_2|$. Therefore initial intervention is strictly emphasized under those conditions. $\square$

## A.6 Proof of Proposition 7 and Conditions for Under- and Over-interventions

*Proof.* Let us first write the explicit expression for $Y(m_1;\chi)$:

$$\begin{aligned} Y(m_1;\chi) \quad &= \quad W_1 - K(m_1,0) + \chi \max_{m_2} \left[ \frac{B+m_1+1-c}{2B} [W_{2S} - (K(m_1,m_2) - K(m_1,0))] \right. \\ &\qquad \left. + \frac{B-m_1-1+c}{2B} [W_{2F} - (K(m_1,m_2)-K(m_1,0))] \right]. \end{aligned}$$

The following identity holds:

$$\begin{aligned} \frac{\partial}{\partial\chi} Y(m_1;\chi) \quad &= \quad \max_{m_2} \left[ \mathbb{E}[W_2(m_1,m_2) - (K(m_1,m_2) - K(m_1,0))] \right] \\ &= \quad \left[ \mathbb{I}_{\{m_2^* > m_1+1-c\}} + \mathbb{I}_{\{m_1>c\}} \mathbb{I}_{\{0<m_2^*<m_1-c\}} \right] (\mathbb{E}[W_2(m_1,m_2^*)] - (K(m_1,m_2^*) - K(m_1,0))) \\ &\qquad + \mathbb{I}_{\{m_1 \geq c\}} \mathbb{I}_{\{m_2^*=m_1-c\}} (\mathbb{E}[W_2(m_1,m_1-c)] - (K(m_1,m_1-c) - K(m_1,0))) \\ &\qquad + \mathbb{I}_{\{m_1\leq c\}} \mathbb{I}_{\{m_2^*=0\}} \mathbb{E}[W_2(m_1,0)]. \end{aligned} \tag{20}$$

Note that fixing $m_1$, increasing $m_2$ does not increase welfare $\mathbb{E}[W_2]$ in $[m_1-c, m_1+1-c]$; thus, the four indicator products sum to 1. This is seen in Figure 5, with the four indicators corresponding to $m_2 > m_1+1-c$, $m_2 < m_1-c$, $m_2 = m_1-c$, and $m_1 - c < m_2 \leq m_1+1-c$, respectively. The first term is non-increasing in $m_1$, whereas the last two terms are non-decreasing. In general, the overall expression is non-monotone in $m_1$ because its value could jump up or down when the indicators change values. However, if parameters are such that one indicator function is always 1, the expression is monotone in $m_1$ and we could draw robust comparative statics. If one of the first two indicators is always 1 as we vary $m_1$, $Y$ has decreasing differences in $m_1$ and $\chi$; if one of the last two indicators is always 1, $Y$ has increasing differences. The conclusions then follow from Theorem 2.1 in Athey, Milgrom, and Roberts (1998). $\square$

The next two corollaries illustrate sufficient conditions on exogenous parameters under which over- or under-interventions take place. These conditions are neither unique, nor restrictive. For simplicity in exposition, we assume for the remainder of the discussion that $K$ is twice-differentiable in a continuous feasible range of intervention $\mathcal{I} = [0,\bar{m}_1] \times [0,\bar{m}_2]$. This specification includes cases of budget constraint and separable quadratic intervention costs. Let $K_i$ denote the partial derivative w.r.t. $m_i$.

**Corollary 2**

*A myopic government under-intervenes initially if one of the two following conditions holds:*

1. $K_1(c,\cdot) > \frac{1-c}{B}$ *and* $K_2(\cdot,1-c) \geq \frac{1-c}{B}\frac{c\delta}{2c\delta+c-1}$.
2. *For some* $b > c$, *it holds that* $K_1(b,\cdot) > \frac{1-c}{B}$, $K_1(c,\cdot) < \frac{1-c}{B}\frac{\delta-c(1+2\delta)}{2\delta-c(1+2\delta)}$, $K_2(\cdot,b-c) \leq \frac{1-c}{2B}$, *and* $K_2(\cdot,1) \geq \frac{(1-c)c\delta}{B(2c\delta+c-1)}$.

**Corollary 3**

*A myopic government over-intervenes initially if one of the following conditions holds:*

1. *For some $b \geq 0$, it holds $K_1(c, \cdot) > \frac{1-c}{B}$ and $K_2(\cdot, b + 2c\delta) < \frac{1-c}{2B}\frac{\delta}{1+2\delta}$.*
2. *$K_1(b, \cdot) > \frac{1-c}{B}$, $K_1(c, \cdot) < \frac{1-c}{B}\frac{\delta - c(1+2\delta)}{2\delta - c(1+2\delta)}$, $K_2(\cdot, 0) > \frac{1-c}{2B}\frac{2\delta}{2\delta - c(1+2\delta)}$.*

We now prove them below, but note that other sufficient conditions exist, especially ones on cost parameters defined through levels rather than derivatives.

*Proof.* If $K_1(c, \cdot) > \frac{1-c}{B}$, we have $m_1^* < c$ because the maximum marginal benefit of $m_1$ on investors' welfare is $\frac{1-c}{B}$, $K_1(c, \cdot) \geq \frac{1-c}{B}$ implies $m_1^* \leq c$. Therefore, $m_2^* > m_1 - c$ for sure and $W_{2S} = (1-c)$. Given $K_2(\cdot, c(1+2\delta)) < \frac{1-c}{2B}\frac{\delta}{1+2\delta}$, we have $K(\cdot, c(1+2\delta)) < \frac{c(1-c)\delta}{2B}$, then $\frac{B-m_1-1+c}{2B}W_{2F} > K(m_1, m_1 + 2\delta c) - K(m_1, 0)$. Thus, the increase in $W_{2F}$ exceeds the intervention cost at $m_2 = m_1 + 2\delta c$, $m_2^* > m_1 + 1 - c$. When this happens, we know $\frac{\partial Y}{\partial \chi}$ equals the first term with the first indicator product being one, and is non-increasing in $m_1$. Thus, $Y$ has decreasing differences in $m_1$ and $\chi$.

Alternatively, if $K_1(b, \cdot) > \frac{1-c}{B}$, we have $m_1^* < b$ for $b$ potentially bigger than $c$. If the cost in the second intervention is sufficiently small such that $m_2^* > b + 2c\delta > 2c\delta + m_1^*$, we also have the first indicator product being 1, and $Y$ has decreasing differences in $m_1$ and $\chi$. One sufficient condition is $\left[W_2(m_1^*, m_1^* + 2c\delta) - W_2(m_1^*, m_1^* - c)\right]/c(1+2\delta) > K_2(b + 2c\delta)$, that is, $K_2(b + 2c\delta) < \frac{(1-c)\delta}{2B(1+2\delta)}$.

Next, for the last indicator to be 1 always, $K_1(c, \cdot) > \frac{1-c}{B}$. In addition, $K_2(\cdot, 1-c) \geq \frac{1-c}{B}\frac{c\delta}{2c\delta + c - 1}$ is a sufficient condition for $m_2^* = 0$ because this implies the cost exceeds the benefit at both $m_2 = 1 - c + m_1$ and $m_2 = m_1 + 2c\delta$, and the convexity of $K$ in $m_2$ excludes $m_2^* > 1 - c + m_1$. Then $\frac{\partial Y}{\partial \chi}$ equals the last term and is non-decreasing in $m_1$. We could alternatively use $K(\cdot, 1-c) \geq \frac{c(1-c)\delta}{2B}$ as a sufficient condition on cost, rather than on the derivative. The same goes for other sufficient conditions that we provide in this proposition.

Next, if $K_1(b, \cdot) > \frac{1-c}{B}$ and $K_1(c, \cdot) < \frac{1-c}{B}\frac{\delta - c(1+2\delta)}{2\delta - c(1+2\delta)} \leq \frac{\partial Y}{\partial m_1}$ for some $b > c$, we have $b > m_1^* > c$. On the one hand, if, in addition, the minimum marginal benefit in region $m_2 \leq m_1^* - c$ is bigger than the maximum marginal cost, that is, $\min\left\{\frac{1-c}{2B}, \frac{1-c}{2B}\frac{2\delta(1-c)}{2\delta - c(1+2\delta)}\right\} \geq \frac{1-c}{2B} \geq K_2(\cdot, b - c) > K_2(\cdot, m_1 - c)$, we have $\mathbb{E}[W_2] - (K(m_1, m_2) - K(m_1, 0))$ increasing in the entire region of $[0, m_1 - c]$. Moreover, if $K_2(m_1^*, m_1^* + 1 - c) > K_2(\cdot, 1) \geq \frac{(1-c)c\delta}{B(2c\delta + c - 1)}$, the lower bound on the marginal cost is weakly bigger than the maximum marginal benefit (RHS) in the region $m_2 \geq 1 - c + m_1^*$. Therefore, $m_2^* = m_1^* - c$ and the third indicator product is always 1. $Y$ has increasing differences in $m_1$ and $\chi$.

On the other hand, $K_2(\cdot, 0) > \frac{1-c}{2B}\frac{2\delta}{2\delta - c(1+2\delta)}$ implies $\frac{1-c}{2B}\frac{2\delta}{2\delta - c(1+2\delta)} < K_2(m_1, m_1 - c)$, which means the maximum marginal benefit in the region $m_2 \leq m_1^* - c$ taken at equality is less than the marginal cost. Thus, $m_2^* < m_1^* - c$ and the second indicator product is always 1. $Y$ has decreasing differences in $m_1$ and $\chi$.

These sufficient conditions are stated in the corollaries. We note that instead of directly computing the derivatives for the first term in $\frac{\partial Y}{\partial \chi}$, when $m_2^*$ is interior, we can apply the Envelope theorem to compute the partial derivative in $m_1$ of $\mathbb{E}[W_2(m_1, m_2)] - (K(m_1, m_2) - K(m_1, 0))$. Because $K_{12} = 0$, we know the partial derivative must be negative in the corresponding regions from Figure 3. $\square$

## A.7 Proof of Proposition 8

*Proof.* Define $n_1 = m_1$ and $n_2 = \frac{m_2}{\lambda}$. The two inequalities on funds' survival translate into

$$\begin{aligned} A_1 + n_1 &\geq \theta \\ A_2 + n_2 &\geq \theta. \end{aligned}$$

For the remaining analysis, we show the optimal intervention plan by a benevolent government that faces a hard budget constraint.

The proof is conducted in two steps. In the first step, we assume $q$, the probability of fund 1's outcome gets realized first is equal to 1 or 0 and study the optimal intervention plan $(n_1, n_2)$ when $\lambda$ varies. The budget constraint shows as $n_1 + n_2\lambda = M$. We will show that for both $\lambda > 1$ and $\lambda \in (0, 1)$, the government will intervene up to $n_2 = n_1 - c$. The case in which $\lambda \in (0, 1)$ is simply isomorphic to $q = 0$. Thus, we establish the result that the government will always induce perfectly correlated intervention outcomes. In step 2, we solve the optimal intervention with general $q$.

**Lemma 6**
*Suppose $q = 1$, $\forall\lambda > 0$; the optimal intervention plan is*

$$\begin{aligned} n_1^* &= \frac{M + c\lambda}{1 + \lambda} \\ n_2^* &= \frac{M - c}{1 + \lambda}. \end{aligned}$$

*Under $(n_1^*, n_2^*)$, intervention always leads to correlated outcomes: $s_1 = s_2$.*

*Proof.* With heterogeneous fund sizes, the aggregate social welfare naturally follows.

1. If $n_1 > \frac{M+2\delta\lambda(1-c)}{1+\lambda}$,

$$\begin{aligned} W &= \frac{1-c}{2B}\left\{\left[1 + B - c\left(1+\delta\right)\right]\left(1+\lambda\right) + M\right\} \\ \frac{\partial W}{\partial n_1} &= 0. \end{aligned}$$

2. If $\frac{M+c\lambda}{1+\lambda} < n_1 < \frac{M+2\delta\lambda(1-c)}{1+\lambda}$,

$$\begin{aligned} W &= \frac{1-c}{2B}\left[1 + B - c\left(1+\delta\right) + n_1\right] \\ &\quad + \lambda\frac{1-c}{2B}\left[1 + n_1 - c + B + \frac{\delta c\left(c - n_1 + n_2\right)^2 + 2\delta\left(c - n_1 + n_2\right)\left[2\delta - c\left(1+2\delta\right)\right]}{\left[2\delta - c\left(1+2\delta\right)\right]^2}\right] \\ \frac{\partial W}{\partial n_1} &< 0. \end{aligned}$$

3. If $\frac{M-(1-c)\lambda}{1+\lambda} < n_1 < \frac{M+c\lambda}{1+\lambda}$,

$$\begin{aligned} W &= \frac{1-c}{2B}\left[(1+B-c+n_1)(1+\lambda)-c\delta\right] \\ \frac{\partial W}{\partial n_1} &> 0. \end{aligned}$$

4. If $\frac{M-2c\delta\lambda}{1+\lambda} < n_1 < \frac{M-(1-c)\lambda}{1+\lambda}$,

$$\begin{aligned} W &= \frac{1-c}{2B}\left[1+B-c(1+\delta)+n_1\right]+\lambda\frac{1-c}{2B}\left[1+B-c+n_1\right] \\ &\quad +\lambda\frac{1-c}{2B}\frac{c\delta\left(-1+c-n_1+n_2\right)^2}{\left(-1+c+2c\delta\right)^2} \\ \frac{\partial W}{\partial n_1} &\quad \textit{changes from negative to positive exactly once.} \end{aligned}$$

5. If $n_1 < \frac{M-2c\delta\lambda}{1+\lambda}$,

$$\begin{aligned} W &= \frac{1-c}{2B}\left\{\left[1+B-c(1+\delta)\right](1+\lambda)+M\right\} \\ \frac{\partial W}{\partial n_1} &= 0. \end{aligned}$$

It easily establishes that $\frac{\partial W}{\partial n_1}=0$ in case 1 and 5, $\frac{\partial W}{\partial n_1}>0$ in case 3, and $\frac{\partial W}{\partial n_1}<0$ in case 2. Similar to the proof of Proposition 5, the aggregate welfare in case 1 equals that in case 5. The maximal welfare is attained at the right boundary of case 3, that is, when $n_1=\frac{M+c\lambda}{1+\lambda}$ $\square$

Next, we can compare the social welfare under general $q$.

When $n_1$ and $n_2$ are such that interventions are correlated, the overall welfare to be maximized is

$$q\frac{1-c}{2B}\left[(1+B-c+n_1)(1+\lambda)-c\delta\right]+(1-q)\frac{1-c}{2B}\left[(1+B-c+n_2)(1+\lambda)-c\lambda\delta\right].$$

Essentially, the government maximizes $qn_1+(1-q)n_2$ subject to the following constraints

$$\begin{aligned} n_1 &\geq n_2-c \\ n_2 &\geq n_1-c \\ n_1+n_2\lambda &= M \end{aligned}$$

The solution is

$$\begin{cases} n_1^*=\frac{M+c\lambda}{1+\lambda},\ n_2^*=\frac{M-c}{1+\lambda} & \text{if } q>\frac{1}{\lambda+1} \\ n_1^*=\frac{M-c\lambda}{1+\lambda},\ n_2^*=\frac{M+c}{1+\lambda} & \text{if } q<\frac{1}{\lambda+1}. \end{cases}$$

Therefore, if $q$ is very high, the government puts more resources into the small fund and vice versa if $q$ is very low. Since $\lambda>1$, the government puts more resources into the small fund when $q=\frac{1}{2}$. $\square$

# B Full Analysis of Section 3.2.2

Does any equilibrium exists that agents choose to run irrespective of their signals? In other words, the threshold $x_2^*$ that agents in period 2 adopt satisfies $x_2^* \leq \theta_1^* - \delta$. Such an equilibrium exists if and only if $m_2 < m_1 + 1 - c$. In this type of equilibrium, government intervention in the first period has a dominant effect on coordination among investors in the second period. Therefore, we name it the *Stage-Game Equilibrium with Dynamic Coordination*.

Lemma 7 describes this type of equilibrium. Because it is common knowledge that $\theta > \theta_1^*$, any equilibrium with $(\theta_2^* < \theta_1^*, x_2^* < \theta_1^* - \delta)$ is equivalent to $(\theta_2^*, x_2^*) = (-\infty, -\infty)$.

**Lemma 7** (Stage-Game Equilibrium with Dynamic Coordination)
*If $s_1 = F$, $(\theta_2^*, x_2^*) = (-\infty, -\infty)$ constitutes an equilibrium if and only if $m_2 < m_1 + 1 - c$.*

Next, we turn to threshold equilibria with $\theta_2^* > \theta_1^*$ so that the fate of the fund in period 2 still has uncertainty. Similar to the analysis when $s_1 = S$, we consider two types of equilibria, depending on whether the marginal investor finds the public news useful.

**Lemma 8** (Stage Game Equilibrium without Dynamic Coordination)
*If $s_1 = F$ and $m_2 > m_1 + 2c\delta$, an equilibrium exists with thresholds*

$$\begin{cases} \theta_2^* = 1 + m_2 - c \\ x_2^* = 1 + m_2 - c + \delta\,(1 - 2c)\,. \end{cases} \tag{21}$$

**Lemma 9** (Stage-Game Equilibrium with Partial Dynamic Coordination)
*If $s_1 = F$ and $\min\{m_1 + 2c\delta, m_1 + 1 - c\} < m_2 < \max\{m_1 + 2c\delta, m_1 + 1 - c\}$, there exists an equilibrium with thresholds*

$$\begin{cases} \theta_2^* = 1 + m_2 - c - \frac{(1-c)(m_1+2c\delta-m_2)}{c(1+2\delta)-1} \\ x_2^* = 1 + m_2 - c + \delta\,(1 - 2c) - \frac{(1-c)(1+2\delta)(m_1+2c\delta-m_2)}{c(1+2\delta)-1}. \end{cases} \tag{22}$$

Given any $(m_1, m_2)$ and $s_1 = F$, Proposition 2 clearly follows Lemmas 7, 8 and 9.

# C Contingent Interventions

**Proposition 9** (Emphasis on Initial Intervention (Contingent Case))
*When $K_2(0, 1-c) > \frac{(1-c)^2}{B-1}$, then contingent interventions strictly emphasizes initial intervention: $m_1^* > m_{2S_1}^*$.*

*Proof.* If $s_1 = S$, increasing $m_{2S}$ beyond $m_1^* - c$ incurs additional cost without increasing $\mathbb{E}[W_2]$, as is clear in Figure 1. Thus, $m_{2S}^* \leq m_1^* - c$. When $s_1 = F$, the condition on the parameter means the marginal cost of increasing $m_{2F}$ at $1-c$ exceeds the marginal benefit, which is bounded above by $\frac{(1-c)^2}{B-1}$. Thus, $m_{2F}^* < 1 - c$. Subsequently, $m_{2F}^* = 0$ because increasing $m_{2F}$ does not increase $\mathbb{E}[W_2]$, also clearly seen in Figure 1. □

This is just one example of the sufficient conditions under which the endogenous correlation effect dominates the conditional inference effect. Note this result applies to situations in which the initial intervention is more costly than the subsequent intervention.

**Proposition 10** (Myopic Intervention (Contingent Case))
*The government's initial intervention is weakly increasing in the extent it considers dynamic coordination, i.e.,* $\frac{\partial m_1^*}{\partial \chi} \geq 0$ *if one of the two following conditions hold:*

1. $K(\cdot, 1-c) - K(\cdot, 0) > \min\{\frac{(1-c)^2}{2c\delta - 1 + c}, \frac{c\delta(1-c)}{B-(1-c)}\}$ *and* $K_1(c, \cdot) \geq \frac{1-c}{B}$.

2. $K(\cdot, 1-c) - K(\cdot, 0) > \min\{\frac{(1-c)^2}{2c\delta - 1 + c}, \frac{c\delta(1-c)}{B-(1-c)}\}$ *and* $1 - c - K(\cdot, 1-c) + K(\cdot, 0) - (2 + B - c)K_2(\cdot, 1-c) > 0$.

*The government's contingent intervention is weakly decreasing in the extent it considers dynamic coordination, i.e.,* $\frac{\partial m_1^*}{\partial \chi} \leq 0$, *when* $K_2(\cdot, 0) > \frac{1-c}{B+c-2}\frac{2\delta}{2\delta - c(1+2\delta)}$ *and* $K_1(c, \cdot) < \frac{1-c}{B}\frac{\delta - c(1+2\delta)}{2\delta - c(1+2\delta)}$.

In general, the government chooses $\{m_1, m_{2S}, m_{2F}\}$ to maximize welfare. For a given $m_1$, define the objective as

$$\begin{aligned} Y(m_1; \chi) \quad = \quad & W_1 - K(m_1, 0) + \chi \left[ \frac{B + m_1 + 1 - c}{2B} \max_{m_{2S}} [W_{2S} - (K(m_1, m_{2S}) - K(m_1, 0))] \right. \\ & \left. + \frac{B - m_1 - 1 + c}{2B} \max_{m_{2F}} [W_{2F} - (K(m_1, m_{2F}) - K(m_1, 0))] \right]. \end{aligned} \tag{23}$$

Here, $\chi \in [0, 1]$ measures how much the government cares about the fate of the second period's fund.

*Proof.* The proof is similar to that in Proposition 7 and Corollaries 2 and 3, albeit algebraically more involved. We start with the first half of the proposition. Because the maximum marginal benefit of $m_1$ on the investors' total welfare is $\frac{1-c}{B}$, $K_1(c, \cdot) \geq \frac{1-c}{B}$ implies $m_1^* \leq c$. Figure 1 implies when $m_{2S} > m_1 - c$, welfare is weakly decreasing in $m_2$. Thus $m_{2S}^* = 0$.

$$\begin{aligned} & \frac{\partial}{\partial m_1} \frac{\partial}{\partial \chi} Y(m_1; \chi) \\ = \quad & \frac{d}{dm_1} \left[ \frac{B + m_1 + 1 - c}{2B} \max_{\{m_{2S}\}} [W_{2S} - (K(m_1, m_{2S}) - K(m_1, 0))] \right. \\ & \left. + \frac{B - m_1 - 1 + c}{2B} \max_{\{m_{2F}\}} [W_{2F} - (K(m_1, m_{2F}) - K(m_1, 0))] \right] \\ = \quad & \frac{1-c}{2B} + \frac{\partial}{\partial m_1} \left[ \frac{B - m_1 - 1 + c}{2B} \max_{\{m_{2F}\}} [W_{2F} - (K(m_1, m_{2F}) - K(m_1, 0))] \mathbb{I}_{\{m_{2F} > m_1 + 1 - c\}} \right] \\ = \quad & \frac{1-c}{2B} + \frac{1}{2B} [K(m_1, m_{2F}) - K(m_1, 0)] \mathbb{I}_{\{m_{2F}^* > m_1 + 1 - c\}} - \frac{1-c}{2B} \mathbb{I}_{\{m_{2F}^* > m_1 + 2c\delta\}} \\ & - \frac{1-c}{2B} \frac{2c\delta(-1 + c - m_1 + m_{2F}^*)}{(-1 + c + 2c\delta)^2} \mathbb{I}_{\{m_{2F}^* \in (m_1 + 1 - c, m_1 + 2c\delta]\}} \\ \geq \quad & 0. \end{aligned} \tag{24}$$

The second equality holds by the Envelope Theorem and by the fact that if $m_{2F} \leq m_1 + 1 - c$, taking $m_{2F} = 0$ dominates, as seen in Figure 1. When $K(\cdot, 1-c) - K(\cdot, 0) > \frac{c\delta(1-c)}{B-(1-c)}$, $W_{2F}(m_2 = m_1 + 2\delta c) < K(m_1, M_1 + 1 - c)$, and thus $m_{2F}^* = 0$; when $K(\cdot, 1-c) - K(\cdot, 0) > \frac{(1-c)^2}{2c\delta - 1 + c}$, the last two terms on the RHS of the third equality are dominated by the first two terms as $m_{2F}^* = 0$. In either case, we have the whole expression being non-negative.

From $1 - c - K(\cdot, 1-c) + K(\cdot, 0) - (2 + B - c)K_2(\cdot, 1-c) > 0$, we have $K_2(\cdot, 1-c) < \frac{1-c}{B+2-c}\frac{2\delta}{2\delta-(1+2\delta)c}$. The marginal benefit for increasing $m_{2S}$ in $W_{2S}$ exceeds the cost as long as $m_{2S} < m_1 - c$; therefore $m_{2S}^* = [m_1 - c]^+$. When $m_1 \leq c$, $m_{2S}^* = 0$, the local derivative is the same as above, and thus is positive. When $m_1 \geq c$, $m_{2S}^* = m_1 - c$, $m_{2F}^* = 0$, the local derivative is

$$\begin{aligned}
& \frac{\partial}{\partial m_1}\frac{\partial}{\partial \chi}Y(m_1;\chi) \\
= \; & \frac{1-c}{2B} - \frac{1}{2B}[K(m_1, m_{2S}) - K(m_1, 0)] - \frac{m_1 + B + 1 - c}{2B}\frac{\partial}{\partial m_1}[K(m_1, m_1 - c) - K(m_1, 0)] \\
= \; & \frac{1-c}{2B} - \frac{1}{2B}[K(m_1, m_{2S}) - K(m_1, 0)] - \frac{m_1 + B + 1 - c}{2B}[K_2(m_1, m_1 - c) \\
& + K_1(m_1, m_1 - c) - K_1(m_1, 0)], \quad \textit{note } K_1(m_1, m_1 - c) = K_1(m_1, 0) \\
= \; & \frac{1-c}{2B} - \frac{1}{2B}[K(m_1, m_{2S}) - K(m_1, 0)] - \frac{m_1 + B + 1 - c}{2B}[K_2(m_1, m_1 - c)] \\
\geq \; & \frac{1}{2B}[1 - c - K(m_1, 1-c) + K(m_1, 0) - (2 + B - c)K_2(m_1, 1-c)] \geq 0. \qquad (25)
\end{aligned}$$

The last two inequalities come from the fact that $m_1 \leq 1$ and the fact that $1 - c - K(\cdot, 1-c) + K(\cdot, 0) - (2 + B - c)K_2(\cdot, 1-c) > 0$. Therefore, we have that $Y$ has increasing differences in $(m_1, \chi)$.

To prove the second half of the theorem, note $K_2(\cdot, 0) > \frac{1-c}{B+c-2}\frac{2\delta}{2\delta - c(1+2\delta)}$, and thus $K(\cdot, 1-c) > \frac{1-c}{B+c-2}\frac{1-c}{1-c(1+\frac{1}{2\delta})} > \frac{1-c}{B-2+c}$, and $W_{2F}$ at $m_2 = m_1 + 2c\delta$ is still less than $K(m_1, m_1 + 1 - c) - K(m_1, 0)$. Consequently, $m_{2F}^* = 0$. $K_2(\cdot, 0) > \frac{1-c}{B+c-2}\frac{2\delta}{2\delta-c(1+2\delta)}$ also implies $\frac{1-c}{B+m_1+1-c}\frac{2\delta}{2\delta-c(1+2\delta)} < K_2(m_1, m_1 - c)$, which means $m_{2S}^* < m_1 - c$.

$$\begin{aligned}
& \frac{\partial}{\partial m_1}\frac{\partial}{\partial \chi}Y(m_1;\chi) \\
= \; & \frac{d}{dm_1}\left[\frac{B + m_1 + 1 - c}{2B}\max_{\{m_{2S}\}}[W_{2S} - (K(m_1, m_{2S}) - K(m_1, 0))]\right. \\
& \left. + \frac{B - m_1 - 1 + c}{2B}\max_{\{m_{2F}\}}[W_{2F} - (K(m_1, m_{2F}) - K(m_1, 0))]\right] \\
= \; & \frac{\partial}{\partial m_1}\left[\frac{B - m_1 - 1 + c}{2B}\max_{\{m_{2F}\}}[W_{2S} - (K(m_1, m_{2S}) - K(m_1, 0))]\right] \\
= \; & \frac{\partial}{\partial m_1}W_{2S}(m_{2S}^*) + \frac{B - m_1 - 1 + c}{2B}\frac{\partial}{\partial m_1}[K(m_1, m_{2S}^*) - K(m_1, 0)] \\
& - \frac{1}{2B}(K(m_1, m_{2S}) - K(m_1, 0)) < 0. \qquad (26)
\end{aligned}$$

The first term is negative because $m_{2S}^* < m_1 - c$. The second term is non-positive because $K$ is weakly increasing in second argument. Finally, the third term is zero because $K$ has zero cross-partials.

Finally, the above argument would not work if $m_1^* \leq c$. But this scenario can be ruled out in that the minimum $\frac{\partial Y}{\partial m_1} = \frac{1-c}{2B}\left[1 - \frac{c(1+2\delta)}{2\delta - c(1+2\delta)}\right] = \frac{1-c}{B}\frac{\delta - c(1+2\delta)}{2\delta - c(1+2\delta)}$. Notice we have used the fact that $m_{2F}^* = 0$. This is bigger than the marginal cost $K_1(c, \cdot)$, and thus $m_1^* > c$. Indeed, we have an interior $m_{2S}^*$.

□

These sufficient conditions for under- and over-interventions simply correspond to corollaries 2 and 3. Finally

**Proposition 11** (Too Big to Save First (Contingent Case))
*If interventions always lead to perfectly correlated outcomes ($s_1 = s_2$), it is socially efficient to save the small fund first when the government can determine the order of the realization of outcomes.*

*Proof.* We prove the case under separable cost functions to focus on the information channel: $K(m_1, m_2) = k(m_1) + k(m_2)$. Again, let $L$ ($L^{'}$) be the total welfare net the intervention cost if the smaller (larger) fund is saved first.

First, consider the case in which $m_1^* \le c$ and $m_1^{'*} \le c$. In this case,

$$\begin{aligned} L &= \max_{m_1} \frac{1-c}{2B}\left[(1+B-c+m_1)(1+\lambda) - c\delta\right] - k(m_1) \\ L^{'} &= \max_{m_1^{'}} \frac{1-c}{2B}\left[\left(1+B-c+\frac{m_1^{'}}{\lambda}\right)(1+\lambda) - c\lambda\delta\right] - k\left(m_1^{'}\right). \end{aligned}$$

Because $\lambda > 1$, obviously $L > L^{'}$.

Next, consider the case in which both $m_1^* > c$ and $m_1^{'*} > c$:

$$\begin{aligned} L &= \max_{m_1} \frac{1-c}{2B}\left[(1+B-c+m_1)(1+\lambda) - c\delta\right] - k(m_1) - \frac{B+m_1+1-c}{2B} k\left((m_1-c)\lambda\right) \\ L^{'} &= \max_{m_1^{'}} \frac{1-c}{2B}\left[\left(1+B-c+m_1^{'}\right)(1+\lambda) - c\lambda\delta\right] - k\left(\lambda m_1^{'}\right) - \frac{B+m_1^{'}+1-c}{2B} k\left(m_1\lambda - c\right). \end{aligned}$$

In this case, even if $m_1^* = \frac{m_1^{'*}}{\lambda}$, $L\big|_{m_1^*=\frac{m_1^{'*}}{\lambda}} - L^{'}_{m_1^{'}=m_1^{'*}}$ equals

$$L\big|_{m_1^*=\frac{m_1^{'*}}{\lambda}} - L^{'}_{m_1^{'}=m_1^{'*}} = c\delta(\lambda-1) + \left[k\left(m_1^{'*}\right) - k\left(\frac{m_1^{'*}}{\lambda}\right)\right] + \frac{B+\frac{m_1^{'*}}{\lambda}+1-c}{2B}\left[k\left(\frac{m_1^{'*}}{\lambda}-c\right) - k\left(\left(\frac{m_1^{'*}}{\lambda}-c\right)\lambda\right)\right].$$

Because $k(\cdot)$ is convex,

$$\begin{aligned} L\big|_{m_1^*=\frac{m_1^{'*}}{\lambda}} - L^{'}_{m_1^{'}=m_1^{'*}} &> \left[k\left(m_1^{'*}\right) - k\left(\frac{m_1^{'*}}{\lambda}\right)\right] - \left[k\left(\left(\frac{m_1^{'*}}{\lambda}-c\right)\lambda\right) - k\left(\frac{m_1^{'*}}{\lambda}-c\right)\right] \\ &> 0. \end{aligned}$$

□

When the order of outcome realizations are exogenous, we can similarly show that the smaller funds gets more interventions that disproportional to its size and probability of realizing the outcome first.

# Online Appendix

## O1. General $\delta$ and $c$, and Equilibrium Multiplicity

We now relate our paper to Angeletos, Hellwig, and Pavan (2007) and explain why our baseline model yields unique equilibrium. We show how equilibrium multiplicity is restored via a mechanism isomorphic to the one in their paper. More importantly, we highlight how equilibrium multiplicity could be endogenized by intervention policy.

Angeletos, Hellwig, and Pavan (2007) show that multiple equilibria emerge under the same conditions that guarantee uniqueness in static global games. The results rely on endogenous learning from regime survivals and exogenous learning from private news that arrives over time. Two elements are necessary for this multiplicity result. First, private information interacts with endogenous learning from earlier coordination outcomes. Second, the private information gets very precise as agents continuously receive private signals about the fundamental. Without the first element, the game is equivalent to one in which agents receive only one precise private signal.[30] Our paper shows the equilibrium results without the second element. We show that multiple equilibria may exist when private signals are very precise. That is, when $\delta$ gets very small. Likewise, Angeletos, Hellwig, and Pavan (2007) show that there always exists an equilibrium in which no attack occurs after the first period, and this would be the unique equilibrium if agents did not receive any private information after the first period.[31]

To see this, note that the set of parameters we have examined corresponds to imprecise signals ($2\delta > 1$ and $\frac{1}{1+2\delta} < c < \frac{2\delta}{1+2\delta}$. Moreover, the signal does not get more precise because agents are non-overlapping. If we relax the parameter assumptions, or allow agents' signals to become more precise over time, multiplicity follows. Proposition 12 complements Propositions 1 and 2.[32]

**Proposition 12** (Equilibria with general $\delta$ and $c$)

1. *If $s_1 = S$ and $\frac{2\delta}{2\delta+1} < c < 1$,*
   - (a) *If $m_2 < m_1 - c$, the unique equilibrium is the Subgame Equilibrium without Dynamic Coordination.*
   - (b) *If $m_1 - c < m_2 < m_1 - 2\delta(1-c)$, all three types of equilibria exist. However, in the Equilibrium with Partial Dynamic Coordination, the threshold $\theta_2^*$ decreases with $m_2$,*
   - (c) *If $m_1 - 2\delta(1-c) < m_2$, the unique equilibrium is the Subgame Equilibrium with Dynamic Coordination.*

[30] The variance of the signal is $Var\left(\frac{\sigma^2}{n}\right)$ with $n$ signals.

[31] One can easily write a two-period version of Angeletos, Hellwig, and Pavan (2007) and show this is the only equilibrium if the private signal is sufficiently imprecise.

[32] Technically, multiple equilibria resurface because we can apply the argument of iterated deletion of dominated regions only from one end of $\theta$ space. Despite this, with slight modifications on the intervention cost functions, the main intuitions for the results from earlier sections still apply as long as we are consistent with equilibrium selection.

*2. If $s_1 = F$ and $0 < c < \frac{1}{2\delta+1}$*

(a) *If $m_2 < m_1 + 2c\delta$, the unique equilibrium is the Subgame Equilibrium with Dynamic Coordination.*

(b) *If $m_1 + 2c\delta < m_2 < m_1 + 1 - c$, all three types of equilibria exist. However, in the Equilibrium with Partial Dynamic Coordination, the threshold $\theta_2^*$ decreases with $m_2$.*

(c) *If $m_2 > m_1 + 1 - c$, the unique equilibrium is the Subgame Equilibrium without Dynamic Coordination.*

Our model also differs from Angeletos, Hellwig, and Pavan (2007) in two additional ways: the government's action is endogenous and the private signal is bounded.[33] Government's action therefore affects equilibrium selection and learning. In particular, when the government's intervention induces equilibria with full or no dynamic coordination, it shuts down the interaction between private signal and public learning. Consequently, the equilibrium is unique even if the signal is infinitely precise. In this regard, the government's endogenous intervention can determine the equilibrium multiplicity.

# O2. Moral Hazard

Moral hazard is a big concern in government bailouts and interventions. Indeed, fund managers may divert the capital injected by the government, or gamble by investing in projects with risk profiles different from the pre-specification. We now demonstrate that our general cost specification already encompasses many forms of moral hazard. In particular, we show managerial stealing and risk shifting provide micro-foundations for the intervention cost. Moreover, by modeling moral hazard, we enrich the model with fund managers' utility function and endogenous actions, making the analysis more realistic and relevant.

## Cash Diversion

First consider the case in which the fund manager is able to divert a fraction $\eta \in [0,1]$ for any amount of liquidity $\mu$ injected by the government. Suppose the fund manager gets compensated a fraction of $\pi$ of the surplus she generates for investors, and among the diverted capital, she can consume $f(\eta)\mu \leq \eta\mu$, where $f : [0,1] \mapsto [0,1)$ satisfies $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$. The rest $\mu[\eta - f(\eta)]$ is inefficiently lost (iceberg costs), consistent with the standard assumption in the literature that cash diversion is increasingly inefficient in the amount diverted.[34] Because the fund manager only cares about her own fund, she does not internalize the intervention externality. Thus, to pin down the unique $\eta$, she equalizes the marginal benefit of keeping more injected liquidity in the fund, $\frac{\pi(1-c)\mu}{2B}$, to the marginal benefit of stealing more $f'(\eta)\mu$.

Under this setup, the optimal intervention problem is isomorphic to the problem solved earlier, where intervention incurs a cost $k(m)$ in the period. To see this, note the government is aware of the diverting technology. Therefore, to effectively inject $m$ into the fund, the government needs to spend $\mu$ such that $(1-\eta)\mu = m$. Equivalently, injecting $m$ into the fund costs the government $k(m) = k_o\left(\frac{m}{1-\eta}\right) - \frac{f(\eta)m}{1-\eta}$,

[33] (Uniform $[-\delta, \delta]$) in our model but unbounded support in their model ($N\left(z, \frac{1}{\alpha}\right)$).

[34] This specification captures the fact that the fraction of diversion matters for the efficiency loss. Alternatively, one could use $f(\eta\mu)$, where $f : [0,\mu] \mapsto [0,\mu)$ satisfies $f'(\cdot) > 0$ and $f''(\cdot) \leq 0$, which implies the diversion efficiency depends on the total amount. This alternative specification does not affect our conclusion.

where $k_o(\mu)$ is other social costs not associated with cash diversion. If $k_o$ is increasing and convex in $m$, adding the moral hazard cost preserves these properties, consistent with our cost specification.

### Risk Shifting

Now consider the case in which the fund manager could secretly choose projects with survival threshold $\theta + \Delta$ with a corresponding private payoff $\alpha\Delta$ conditional on success after paying the promised payoffs to investors (or some asymmetric split of the additional payoff). The project is thus more illiquid and risky (failure probability is higher), but the fund manager has an incentive to shift the risk, because she captures the upside (limited liability means she does not incur additional loss upon failure). Let the fixed cost of risk shifting be $c_o \geq 0$, and then given the liquidity injection $m$, the optimal risk shifting is

$$\Delta^* = argmax_\Delta \left[ \alpha\Delta \frac{1-c+m-\Delta}{2B} - c_o \mathbb{I}_{\{\Delta>0\}} \right] = \frac{1-c+m}{2} \mathbb{I}_{\left\{c_o < \frac{\alpha(1-c+m)^2}{8B}\right\}}. \tag{27}$$

The greater the intervention, the greater the distortion in investment by the manager. Compared to the case in which moral hazard is absent, the welfare is reduced by

$$\left[ c_o + \frac{\Delta^*(1-c)}{2B} - \frac{1-c+m-\Delta^*}{2B}\alpha\Delta^* \right] \mathbb{I}_{\left\{c_o < \frac{\alpha\Delta^*(1-c+m-\Delta^*)^2}{2B}\right\}}, \tag{28}$$

where the first term is the reduction in welfare due to a lower probability of fund survival, and the second term is the private benefit to the fund manager. For simplicity, we assume $c_o > \frac{\alpha(1-c)^2}{8B}$ and $\alpha$ is sufficiently small (e.g., $\alpha < \frac{1-c}{1-c+\bar{m}}$) so that moral hazard is only induced by the intervention. Then the moral hazard cost can be nested in $k(m) = k_o(m) + \left[ c_o + \frac{-\alpha m^2 + 2m(1-c)(1-\alpha) + (2-\alpha)(1-c)^2}{8B} \right] \mathbb{I}_{\left\{c_o < \frac{\alpha(1-c+m)^2}{8B}\right\}}$. Once again, $k(m)$ is weakly increasing and convex in $m$.

Because the sum of increasing and convex functions is still increasing and convex, our general cost function accommodates multiple types of moral hazard. In other words, moral hazard considerations constitute and motivate the general cost function we use. For example, when the moral hazard of risk shifting and stealing are both present, their costs can still be represented by the general cost function in our model, and such moral hazard costs motivate the cost specification.

## O3. $\theta$-dependent Payoff Structures

In this section, we extend the analysis in Section 3 to the case whereby the fundamental $\theta$ impacts not only the probability of the fund's survival but also investors' payoff structure. The government's intervention $\{m_1, m_2\}$ then impacts both the probability that the fund survives and the actual amount investors receive. We extend our model in three alternative specifications and show that most results and economic intuition remain. The Equilibrium without Dynamic Coordination may not exist under certain conditions in the extensions, but it is a sub-game equilibrium in which neither endogenous correlation nor conditional inference has effect, and do not matter for our results. Below we elaborate.

## Extensions 1 and 2

In this section, we introduce an alternative payoff structure in which $\theta$ directly impacts the eventual profitability of investment. We follow Goldstein and Pauzner (2005) by assuming that each fund's investment succeeds with probability $p(\theta)$ and fails with probability $1 - p(\theta)$, where $\theta \sim Unif[-B, B]$ is interpreted as the fundamental. To be consistent, we assume $p' \leq (\theta) < 0$ so that higher $\theta$ reduces the possibility of investment success. Conditional on fund's investment success, investors receive payoff $1 - c(\theta)$ if and only if the fund survives a run. Let $A$ continue to be the measure of investors who choose to stay. The condition for the fund to survive a run is $A \geq \theta$. If the fund fails, either due to fundamental investment failure which occurs with probability $1 - p(\theta)$ or due to more endogenous investor runs ($A < \theta$), each investor receives payoff $-d(\theta)$.

The government can intervene by increasing each investor's return by $m$ even if the fund fails. In other words, each investor's return increase to $m - d(\theta)$ with the intervention. Under this setup, the fundamental $\theta$ affects the NPV of investment through its effect on both the probability of investment success and the return conditional on success. The intervention therefore requires the government to allocate rather than just promise real resources. Following the baseline model, we continue to assume that $\theta$ is unobservable and each investor has a private signal about it.

Since investors in period 2 can observe the outcome of period 1—including the payoff $1-c(\theta)$ conditional on survival and $m_1 - d(\theta)$ conditional on failure, they could perfectly infer the realization of $\theta$ if there were a one-to-one mapping between either $c(\theta)$ and $\theta$, or $d(\theta)$ and $\theta$. In that case, the game in the second period no longer features incomplete information, and both the coordination issue and equilibrium multiplicity rise again. To avoid such perfect inference and to introduce fewer deviations from the baseline model, we assume $c(\theta)$ is flat and $d(\theta)$ is piecewise linear: $c(\theta) \equiv c$ and $d(\theta) \equiv \begin{cases} -c & \text{if } \theta \leq \bar{\theta} \\ -c-d & \text{if } \theta > \bar{\theta} \end{cases}$, where $d \geq 0$. $\bar{\theta}$ is a threshold which can be thought of as barriers in the fund's investment technology. Table 2 shows the payoff under this setup.

Table 2: Payoffs in the Extended Setup

| | Stay | Run |
|---|---|---|
| Survive | $1 - c$ | 0 |
| Fail | $\begin{cases} m - c & \text{if } \theta \leq \bar{\theta} \\ m - c - d & \text{if } \theta > \bar{\theta} \end{cases}$ | 0 |

For the remaining analysis, we study two special cases. In the first case, we assume $p(\theta) \equiv 1$ so that the fund can only fail due to investor runs. In this case, the fundamental $\theta$ only directly affects $c(\theta)$ and $d(\theta)$, the payoff conditional on survival/failure. We solve the model and show that the results from the baseline model go through. In the second case, we assume $d \equiv 0$ so that $\theta$ only directly affects $p(\theta)$, the probability of fund's fundamental investment success. We will see the results differ slightly in the sense that the sub-game equilibrium without coordination disappears in the latter case. In both cases, $\theta$ directly affects investors' payoff through its effect on the NPV of investment. Moreover, $\theta$ also indirectly affect payoff through coordination. Finally, to make the game non-trivial, we assume that both $m_1$ and $m_2$ are less than $\bar{m}$, which in turn is strictly less than $c$.

## Extension 1: $\theta$ affects conditional investment payoff

**Period 1** The equilibrium in period 1 is still characterized by two thresholds $(\theta_1^*, x_1^*)$: the fund survives a run if and only $\theta < \theta_1^*$; investor $i$ stays if and only if the observed signal $x_{1i} < x_1^*$. However, the equilibrium is no longer unique; investors' beliefs on the comparison between $\theta_1^*$ and $\bar{\theta}$ generate another source of self-fulfilling multiplicity. Intuitively, when investors expect $\theta_1^* < \bar{\theta}$, they behave more cautiously by choosing a lower threshold $x_1^*$. When investors are more cautious about the staying decision, fewer of them choose to stay for the same set of signals $\{x_{1i}\}_{i\in[0,1]}$, and $A_1$, the aggregate measure of investors who stay, indeed becomes smaller, leading to a lower survival threshold for the fund $\theta_1^*$ and thus fulfilling investors' expectation that $\theta_1^* < \bar{\theta}$. On the other hand, when investors expect $\theta_1^* > \bar{\theta}$, they behave less cautiously by choosing a higher threshold $x_1^*$. As a result, more of them choose to stay for the same set of signals $\{x_{1i}\}_{i\in[0,1]}$, and $A_1$ will indeed get higher, leading to a higher survival threshold for the fund $\theta_1^*$ and thus fulfilling investors' expectation that $\theta_1^* > \bar{\theta}$. More generally, if the payoff conditional on success or failure experiences jumps, the equilibrium multiplicity may come back. Proposition 13 summarizes the equilibrium in the first period.

**Proposition 13**
*The equilibrium in the first period is characterized by two thresholds $(\theta_1^*, x_1^*)$: the fund survives a run if and only if $\theta < \theta_1^*$; investor i stays if and only if the observed signal $x_{1i} < x_1^*$.*

1. *If $m_1 < \frac{\bar{\theta}-(1-c)}{\bar{\theta}}$, equilibrium is unique:*

$$\begin{cases} \theta_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1} \\ x_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1}(1+2\delta)-\delta. \end{cases}$$

   *Moreover, $\theta_1^* < \bar{\theta}$.*

2. *If $\frac{\bar{\theta}-(1-c)}{\bar{\theta}} < m_1 < \frac{(1+d)\bar{\theta}-(1-c)}{\bar{\theta}}$, there are two equilibria.*

$$\begin{cases} \theta_1^* = \frac{1-c}{1-m_1} \\ x_1^* = \frac{1-c}{1-m_1}(1+2\delta)-\delta \end{cases} \qquad \begin{cases} \theta_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1} \\ x_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1}(1+2\delta)-\delta. \end{cases}$$

   *In the first equilibrium, $\theta_1^* > \bar{\theta}$, where as in the second equilibrium, $\theta_1^* < \bar{\theta}$.*

3. *If $m_1 > \frac{(1+d)\bar{\theta}-(1-c)}{\bar{\theta}}$, equilibrium is unique*

$$\begin{cases} \theta_1^* = \frac{1-c}{1-m_1} \\ x_1^* = \frac{1-c}{1-m_1}(1+2\delta)-\delta. \end{cases}$$

   *Moreover, $\theta_1^* > \bar{\theta}$.*

*Proof.* When investors expect $\theta_1^* < \bar{\theta}$, the two equations determining equilibria are

$$\begin{aligned} A\left(\theta_1^*\right) = \theta_1^* \Rightarrow \frac{x_1^* - \left(\theta_1^* - \delta\right)}{2\delta} &= \theta_1^* \\ \Pr\left(\theta < \theta_1^* | x_1^*\right)(1-c) + \Pr\left(\theta_1^* < \theta < \bar{\theta}_1 | x_1^*\right)(m_1 - c) + \Pr\left(\theta > \bar{\theta}_1 | x_1^*\right)(m_1 - c - d) &= 0. \end{aligned}$$

Solving the two equations, we get

$$\begin{cases} \theta_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1} \\ x_1^* = \frac{1-c+\frac{\bar{\theta}}{2\delta}d}{1+\frac{1+2\delta}{2\delta}d-m_1}(1+2\delta) - \delta. \end{cases}$$

By imposing the requirement $\theta_1^* < \bar{\theta}$, we get $m_1 < \frac{(1+d)\bar{\theta}-(1-c)}{\bar{\theta}}$.

When investors expect $\theta_1^* > \bar{\theta}$, the two equations determining equilibria are

$$\begin{aligned} A\left(\theta_1^*\right) = \theta_1^* \Rightarrow \frac{x_1^* - \left(\theta_1^* - \delta\right)}{2\delta} &= \theta_1^* \\ \Pr\left(\theta < \theta_1^* | x_1^*\right)(1-c) + \Pr\left(\theta > \theta_1^* | x_1^*\right)(m_1 - c) &= 0. \end{aligned}$$

Solving the two equations, we get

$$\begin{cases} \theta_1^* = \frac{1-c}{1-m_1} \\ x_1^* = \frac{1-c}{1-m_1}(1+2\delta) - \delta. \end{cases}$$

By imposing the requirement $\theta_1^* > \bar{\theta}$, we get $m_1 > \frac{\bar{\theta}-(1-c)}{\bar{\theta}}$. □

**Period 2** We now turn to period 2. Obviously, the analysis differs in whether $\theta_1^*$, the equilibrium fund survival threshold in first period, is higher or lower than $\bar{\theta}$, the cutoff for more negative payment if the fund fails.

1. $\theta_1^* < \bar{\theta}$. In this case, the intervention in period 1 truncates the prior belief on $\theta$ into three regions: $[-B, \theta_1^*]$, $\left[\theta_1^*, \bar{\theta}\right]$, and $\left[\bar{\theta}, B\right]$, respectively corresponding to the case that the fund in the first period has succeeded, the fund in the first period has failed and investors who have stayed receive $m_1 - c$, the fund in the first period has failed and investors who have stayed receive $m_1 - c - d$.

2. $\theta_1^* \geq \bar{\theta}$. In this case, the intervention in period 1 again truncates the prior belief on $\theta$ into two regions: $[-B, \theta_1^*]$ and $[\theta_1^*, B]$.

Below we will study the first case $\theta_1^* < \bar{\theta}$ which is more general. The second case is a subcase of the first one.

If $s_1 = S$, then it becomes publicly known that $\theta \in [-B, \theta_1^*]$. As in the baseline model, there are also three types of equilibria: Equilibrium Without Dynamic Coordination, with Partial Coordination, and with Coordination.

**Proposition 14**
*Equilibrium in period 2 if $s_1 = S$ and $\theta_1^* < \bar{\theta}$*

1. *If* $m_2 < 1 - \frac{1+\frac{1+2\delta}{2\delta}d - m_1}{1-c+\frac{\bar{\theta}}{2\delta}d}(1-c)(1+2\delta)$*, the equilibrium in period 2 has no dynamic coordination:*

$$\begin{cases} \theta_2^* = \frac{1-c}{1-m_2} \\ x_2^* = \frac{1-c}{1-m_2}(1+2\delta) - \delta. \end{cases}$$

2. *If* $1 - \frac{1+\frac{1+2\delta}{2\delta}d - m_1}{1-c+\frac{\bar{\theta}}{2\delta}d}(1-c)(1+2\delta) < m_2 < c - 2(1-c)\delta$*,*[35] *the equilibrium in period 2 has partial dynamic coordination:*

$$\begin{cases} \theta_2^* = \frac{\theta_1^*(c-m_2)-2(1-c)\delta}{c-m_2-2(1-c)\delta} \\ x_2^* = \theta_2^*(1+2\delta) + \delta. \end{cases}$$

3. *If* $m_2 > c - 2(1-c)\delta$*, the equilibrium in period 1 has dynamic coordination.*

*Proof.* Consider first that there is no dynamic coordination. Since it is publicly known that $\theta \in [-B, \theta_1^*]$ and that $\theta_1^* < \bar{\theta}$, investors in the second period can never receive payoff $m_2 - c - d$. Therefore, ignoring the public news that $\theta < \theta_1^*$, the thresholds that determine equilibrium are similar to Case 3 of Proposition 13

$$\begin{cases} \theta_2^* = \frac{1-c}{1-m_2} \\ x_2^* = \frac{1-c}{1-m_2}(1+2\delta) - \delta. \end{cases}$$

The sufficient and necessary condition for the equilibrium to have no dynamic coordination is

$$x_2^* + \delta < \theta_1^*,$$

which leads to the condition $m_2 < 1 - \frac{1+\frac{1+2\delta}{2\delta}d - m_1}{1-c+\frac{\bar{\theta}}{2\delta}d}(1-c)(1+2\delta)$.

Consider next the Equilibrium with Dynamic Coordination, the thresholds are now determined by

$$\begin{aligned} A(\theta_2^*) = \frac{x_2^* - (\theta_2^* - \delta)}{2\delta} &= \theta_2^* \\ \Pr(\theta < \theta_2^* | x_2^*, \theta < \theta_1^*)(1-c) + \Pr(\theta > \theta_2^* | x_2^*, \theta < \theta_1^*)(m_2 - c) &= 0. \end{aligned}$$

The solutions are

$$\begin{cases} \theta_2^* = \frac{\theta_1^*(c-m_2)-2(1-c)\delta}{c-m_2-2(1-c)\delta} \\ x_2^* = \theta_2^*(1+2\delta) + \delta. \end{cases}$$

The sufficient and necessary condition for the equilibrium to have dynamic coordination is

$$\theta_2^* > \theta_1^*,$$

which lead to the condition $m_2 > c - 2(1-c)\delta$. □

The expressions for conditional payoffs $W_2^{nc}$, $W_2^{pc}$, and $W_2^{c}$ get lengthier and are omitted to avoid complication. However, it is clear that $\theta_1^*$ increases with $m_1$ and in the case with dynamic coordination,

[35] This condition either requires us to relax the assumption in the baseline model that $c < \frac{2\delta}{1+2\delta}$ or the assumption that $m_2 > 0$.

investors continue to receive $1-c$ if $s_1 = S$ and 0 if $s_1 = F$. Therefore, the *Endogenous Coordination Effect* and *Conditional Inference Effect* continue to exist.

**Corollary 4**
*The* Endogenous Coordination Effect *and* Conditional Inference Effect *continue to exist.*

The remaining cases $\theta \in \left[\theta_1^*, \bar{\theta}\right]$, $\theta \in \left[\bar{\theta}, B\right]$ under $s_1 = S$ and $\theta_1^* < \bar{\theta}$ can be analyzed in a similar way. So are the cases when $\theta_1^* > \bar{\theta}$ and $s_1 = F$. To avoid clunky and repeated expressions and statements, we omit the details. However, we would like to point out one exception when $\theta \in \left[\theta_1^*, \bar{\theta}\right]$. In this case, the length of the support of the updated belief on $\theta$ becomes $\bar{\theta} - \theta_1^*$, which can be smaller than $2\delta$. In this case, the Equilibrium without Dynamic Coordination may disappear. Indeed, this result depends on the prior belief on $\theta$ to be sufficiently wide. In the case $\theta \in \left[\theta_1^*, \bar{\theta}\right]$, the truncations $\theta > \theta_1^*$ and $\theta < \bar{\theta}$ at least partially helpsupdate the marginal investor's belief in additional to his private signal $x_2^*$. In Extension 2 and also Section C, we will show that the result of the Equilibrium without Dynamic Coordination further depends on the assumption that private noises follow uniform distribution, and the payoff conditional on success or failure is sufficiently flat and thus non-informative of the fundamental $\theta$.

However, since the other two types of sub-game equilibria continue to exist, we would like to emphasize that the two key channels in this paper, *Conditional Inference Effect* and *Endogenous Coordination Effect*, remain in the current extension. Therefore, the interactions with the intervention cost function lead to implications similar to those in Section 4.

## Extension 2: $\theta$ affects investment success probability

**Setup** In this section, we assume $p(\theta) \in (0,1)$ and $c(\theta) = d(\theta) \equiv c$. Table 3 shows the payoff under this extension. Note that survival requires two conditions. First, the fund's investment needs to succeed, which occurs with probability $p(\theta) < 1$. Second, enough investors must stay and the fund thus survives a run: $A > \theta$. We continue to assume that $m \in [0, \bar{m}]$ and $\bar{m} < c$. Otherwise, the government can guarantee higher payoff for investors who choose to stay.

Table 3: Payoffs in Case 2

| | Stay | Run |
|---|---|---|
| Survive | $1-c$ | 0 |
| Fail | $m-c$ | 0 |

**Period 1** The equilibrium in the first period is again captured by two thresholds $(\theta_1^*, x_1^*)$. If $\theta = \theta_1^*$, then $A(\theta_1^*) = \theta_1^*$ so that if the fund's investment succeeds, the extent of coordination among investors is just enough to guarantee the fund to survive. Moreover, each investor adopts a threshold strategy $a_i = \mathbb{1}\{x_i \leq x^*\}$: he stays if and only if the probability of receiving $1-c$ exceeds $\frac{c-m}{1-m}$. Therefore, the two

equations that characterize the two thresholds are

$$\begin{cases} A\left(\theta_1^*\right) = \frac{x_1^*-\left(\theta_1^*-\delta\right)}{2\delta} = \theta_1^* \\ \int_{x_1^*-\delta}^{\theta_1^*} \frac{p(\theta)}{2\delta} d\theta = \frac{c-m}{1-m}. \end{cases}$$

Note that if $p\left(\theta\right) \equiv 1$ and $m = 0$, the solutions are identical to that in standard regime shifting games: $\left(\theta_1^*, x_1^*\right) = \left(1-c, 1-c+\delta\left(1-2c\right)\right)$.

In general, $\theta_1^*$ is the solution to the following equation

$$\int_{\theta_1^*(1+2\delta)-2\delta}^{\theta_1^*} \frac{p\left(\theta\right)}{2\delta} = \frac{c-m}{1-m}.$$

To get closed-form solutions, one needs to specify a functional form for $p\left(\theta\right)$. Without narrowing us to any ad hoc functional form, we can still proceed with the qualitative analysis to show that the conditional inference effect remains: since $p\left(\theta\right)$ is a decreasing function of $\theta$ and the length of the interval $\left[\theta_1^*\left(1+2\delta\right)-2\delta, \theta_1^*\right]$ decreases with $\theta_1^*$, there exists a unique solution to $\theta_1^*$. Moreover, $\theta_1^*$ increases with $m_1$— the conditional inference effect. Finally, $x_1^* = \theta_1^*\left(1+2\delta\right)-2\delta$.

Next, we move on to the analysis in the second period. Upon observing that investors have received $1-c$ in the first period, investors again get more optimistic about the distribution of $\theta$. Their optimism can be decomposed into two effects.

1. Truncation. After observing a payoff $1-c$, investors can safely conclude that $\theta \in \left[-B, \theta_1^*\right]$. In other words, the regions where $\theta$ is very high gets excluded. Moreover, $\theta_1^*$ increases with $m_1$ so that the conditional inference effect carries over.

2. More optimistic on the untruncated region. While the support of $\theta$ is truncated from above: $\theta \in \left[-B, \theta_1^*\right]$, the belief on the distribution of $\theta$ on $\left[-B, \theta_1^*\right]$ is also updated. In particular, while the prior distribution of $\theta$ is uniform on $\left[-B, B\right]$, the updated distribution is no longer uniform. It involves updating on $p\left(\theta\right)$ as well.

   To see this, let's assume $p\left(\theta\right) = \frac{\theta+B}{2B} \sim Unif\left[0,1\right]$ for simplicity. Let us temporarily ignore the truncation effect so that upon observing investors have received $1-c$, the support of $\theta$ remains $\left[-B, B\right]$. Even so, observing $s_1 = S$ increases the probability density on high $p\left(\theta\right)$ (low $\theta$) and decreases the probability density on low $p\left(\theta\right)$ (high $\theta$). The updated belief distribution should entail this optimism. In particular, the updated belief becomes $p\left(\theta\right) \sim \beta\left(2,1\right)$ after $s_1 = S$.[36] In other words, the belief on $\theta$ gets skewed towards lower realizations. With the truncation effect, the updated distribution of $\theta$ becomes a conditional beta distribution on $\left[-B, \theta_1^*\right]$.

Combining both effects, the second-period stage equilibrium can never be one without dynamic coordination. In other words, the equilibrium in period 2 is either the *Stage Game Equilibrium with Partial Coordination* when the conditional inference effect more likely dominates; or *Stage Game Equilibrium with Coordination* when the endogenous correlation effect dominates. Consequently, since the updated belief of $\theta$ on $\left[-B, \theta_1^*\right]$ is no longer uniform, the second-period stage equilibrium may no longer be unique. However, the conditional

[36]The updated belief becomes $p\left(\theta\right) \sim \beta\left(1,2\right)$ after $s_1 = F$

inference effect also remains. The remaining analysis, which includes solving equilibrium cutoffs (numerically) for different equilibria, follows the analysis in Section O5.

## Extension 3

In this subsection, we introduce a different extension in which the fundamental $\theta$ and government intervention $m$ directly affect investors' payoff. Specifically, we keep the setup in Section 2.2 except for the following payoff structure for each investor:

$$V_{ti} = \begin{cases} 1-c & \text{if } A_t + m_t \geq \theta + \psi \\ -c + \frac{1}{2\psi}\left[A_t + m_t - (\theta - \psi)\right] & \text{if } \theta - \psi < A_t + m_t < \theta + \psi \\ -c & \text{if } A_t + m_t \leq \theta - \psi. \end{cases}$$

Each investor receives $1-c$ only if the fund fully survives: $A_t + m_t \geq \theta + \psi$. In contrast, each investor receives $-c$ if the fund fully fails: $A_t + m_t \leq \theta - \psi$. If the fund partially survives (or fails), however, i.e., if $\theta - \psi < A_t + m_t < \theta + \psi$, the investor's payoff is linear in the fundamental $\theta$. Note that the baseline model refers to one when $\psi \to 0$. We continue to assume that investors in period 2 can observe the outcome of period 1.

This payoff structure can be interpreted as debts or deposits, and generally applies to cases where an investor's payoff is capped above when the fund is very successful, but is zero if the fund fails. $A_t+m_t \geq \theta+\psi$ can be interpreted as that the project is successful enough that every staying investor gets the promised principal and interest (or promised return as in the case of wealth management products some funds offer); if the situation is really bad $A_t + m_t \leq \theta - \psi$, the fund has nothing left for the investors; if the fund does poorly but does not completely fail $\theta - \psi < A_t + m_t < \theta + \psi$, the investors get some payoff less than what is originally promised (think of them as debts who have senior claims on the limited revenue the fund generates). $\psi$ is then a parameter capturing the range of fundamental states that leads to the fund to survive but fails to deliver to the investors the full promised return.

Lemma 10 summarizes the equilibrium in period 1.

**Lemma 10**
*In the stage game, there exists a unique symmetric PBE in monotone strategies $\left(\theta_1^L, \theta_1^H, x_1^*\right)$ where*

$$\begin{cases} \theta_1^L & = 1 + m_1 - c - \frac{2\delta\psi}{1+2\delta\psi} \\ \theta_1^H & = 1 + m_1 - c + \frac{2\delta\psi}{1+2\delta\psi} \\ x_1^* & = 1 + m_1 - c + \delta\left(1 - 2c\right). \end{cases}$$

*Each investor's strategy follows $a_i = \mathbb{1}\left\{x_i \leq x_1^*\right\}$. The fund pays off each investor $1-c$ if $\theta \leq \theta_1^L$, $-c + \frac{1}{2\psi}\left[A_1 + m_1 - (\theta - \psi)\right]$ if $\theta \in \left(\theta_1^L, \theta_1^H\right)$ and $-c$ if $\theta \geq \theta_1^H$. Moreover, $A_1 = \frac{x_1^* - (\theta - \delta)}{2\delta}$.*

Lemma 10 is the extended version of Lemma 1. Note that if $\psi = 0$, both thresholds are identical to $\theta_1^*$: $\theta_1^L = \theta_1^H = \theta_1^*$, and the equilibrium is identical to one when $\theta$ only impacts the fund's survival. When $\theta$ also directly enters investor's payoff, the equilibrium is characterized by three thresholds. Specifically, $\theta_1^L$ is the threshold of $\theta$ under which the fund fully succeeds, and $\theta_1^H$ is the threshold above which the fund fully fails. If $\theta \in \left(\theta_1^L, \theta_1^H\right)$, the fund succeeds (or equivalently fails) only partially and repays between $-c$ and $1-c$.

*Proof.* The payoff structure satisfies A.1 to A.5 in Section 2.2.1 of Morris and Shin (2003). According to Proposition 2.1 there, each investor's strategy is a threshold strategy. We conjecture that the equilibrium is characterized by three thresholds that satisfy three equations

$$A\left(\theta_1^H\right) + m_1 - \left(\theta_1^H - \psi\right) = 0 \quad (29)$$

$$A\left(\theta_1^L\right) + m_1 - \left(\theta_1^L + \psi\right) = 0 \quad (30)$$

$$\Pr\left(\theta < \theta_1^L \middle| x_1^*\right)(1-c) + \int_{\theta_1^L}^{\theta_1^H} \left\{-c + \frac{1}{2\psi}\left[A(\theta) + m_1 - \theta + \psi\right]\right\} d\theta + \Pr\left(\theta > \theta_1^H \middle| x_2^*\right)(-c) = 0 \quad (31)$$

Solving the three equations, we get the solutions as below

$$\begin{cases} \theta_1^L & = 1 + m_1 - c - \frac{2\delta\psi}{1+2\delta\psi} \\ \theta_1^H & = 1 + m_1 - c + \frac{2\delta\psi}{1+2\delta\psi} \\ x_1^* & = 1 + m_1 - c + \delta(1-2c). \end{cases} \quad (32)$$

□

Lemma 11 shows the total welfare.

**Lemma 11**
*The total welfare is* $W_1 = \frac{(1-c)[1+B-c(1+\delta)+m_1]}{2B} - \frac{\delta\psi^2}{6B(1+2\delta)^2}$.

Note that when $\psi = 0$, the social welfare is identical to what we have in Section 2.1.3. As $\psi$ increases, the total social welfare decreases. Intuitively, larger $\psi$ increases the uncertainty on payoff, the difficulty in coordination, and thus the overall welfare.

Next, we turn to the equilibrium in period 2. After the outcome of period-1 intervention gets publicly known, the belief on $\theta$ is either partitioned or precisely known. Specifically, if investors receive $1-c$, it becomes public knowledge that $\theta \in \left[-B, \theta_1^L\right]$. In contrast, it becomes publicly known that $\theta \in \left[\theta_1^H, B\right]$ if investors receive $-c$. If investors receive $V_1 \in (1-c, c)$, they can perfectly infer the true realization $\theta$, which is the solution to the following equation:

$$V_1 = -c + \frac{1}{2\psi}\left[A_1(\theta) + m_1 - (\theta - \psi)\right],$$

where $A_1(\theta) = \frac{x_1^* - (\theta - \delta)}{2\delta}$. Since $m_1$ is publicly announced, and $A_1(\theta)$ decreases with $\theta$, the above equation admits a unique solution when $V_1 \in (-c, 1-c)$. In this case, $\theta$ becomes public information in period 2, and the equilibrium may not be unique. For the remainder of this section, we focus on the case that $\theta \in \left[-B, \theta_1^L\right]$ and $\theta \in \left[\theta_1^H, B\right]$ to avoid the issue of equilibrium selection.

If investors have received $1-c$ in period 1, the prior knowledge on $\theta$ is updated as $\left[-B, \theta_1^L\right]$. Similar to the baseline model, the equilibrium in period 2 can occur with, without or with partial dynamic coordination, summarized as follows. We assume $\psi$ is sufficiently small such that $\frac{\psi}{1+2\delta} < 1-c$.

**Proposition 15**

*Equilibrium in Period 2 when* $s_1 = S$

1. *If* $m_2 < m_1 - 2\delta(1-c) - \frac{2\delta\psi}{1+2\delta}$*, the unique equilibrium is the Stage Game Equilibrium without Dynamic Coordination. The total social welfare is* $W_{2S}^{nc} = \frac{(1-c)[1+B-c(1+\delta)+m_2]}{B+\theta_1^L} - \frac{\delta\psi^2}{3\left(B+\theta_1^L\right)(1+2\delta)^2}$.

2. *If* $m_1 - 2\delta(1-c) - \frac{2\delta\psi}{1+2\delta} < m_2 < m_1 - c - \frac{c\psi}{(1-c)(1+2\delta)}$*, the unique equilibrium is the Stage Game Equilibrium with Partial Dynamic Coordination.*

   (a) *If* $m_1 - 2\delta(1-c) - \frac{2\delta\psi}{1+2\delta} < m_2 < m_1 - c - \frac{\psi[4(1-c)\delta - c]}{(1-c)(1+2\delta)}$*, in the unique equilibrium, the fund may succeed, fail or partially fail. The total social welfare is*

$$\begin{aligned} W_{2S}^{pc} &= \frac{(1-c)}{B+\theta_1^L}\left[\theta_1^L + B + \frac{\delta c\left(c-m_1+m_2\right)^2 + 2\delta\left(c-m_1+m_2\right)\left[2\delta - c\left(1+2\delta\right)\right]}{\left[2\delta - c\left(1+2\delta\right)\right]^2}\right] \\ &- \frac{\delta\psi\left\{12\left(c-1\right)\delta\left(1+2\delta\right)\left[c\left(-1+c-m_1+m_2-2\delta\right)+2\delta\right] + \psi\left[c^2 - 4\left(1-c\right)\delta\left(c-\delta+4c\delta\right)\right]\right\}}{3\left(1+2\delta\right)^2\left[2\delta - c\left(1+2\delta\right)\right]^2\left(\theta_1^L + B\right)}. \end{aligned}$$

   (b) *If* $m_1 - c - \frac{\psi[4(1-c)\delta - c]}{(1-c)(1+2\delta)} < m_2 < m_1 - c - \frac{c\psi}{(1-c)(1+2\delta)}$*, in the unique equilibrium, the fund can never fail. The total social welfare is* $W_{2S}^{pc} = \frac{(1-c)}{B+\theta_1^L}\left[\theta_1^L + B + \frac{\delta c(c-m_1+m_2)^2 + 2\delta(c-m_1+m_2)[2\delta - c(1+2\delta)]}{[2\delta - c(1+2\delta)]^2}\right] - H_1(\psi)$ *where* $H_1(\psi) > 0$ *and converges to* $0$ *as* $\psi \to 0$.[37]

3. *If* $m_2 > m_1 - c - \frac{c\psi}{(1-c)(1+2\delta)}$*, the unique equilibrium is the Stage Game Equilibrium with Dynamic Coordination. In this equilibrium, the fund will always succeed and repay* $1-c$*. The social welfare is* $W_{2S}^c = 1-c$.

*Proof.* Again, we solve the model respectively in three cases: equilibrium with dynamic coordination, with partial coordination and without coordination.

1. Equilibrium without dynamic coordination. We solve the equilibrium assuming no dynamic coordination. The solutions are similar to those in period 1:

$$\begin{cases} \theta_2^L &= 1 + m_2 - c - \frac{2\delta\psi}{1+2\delta\psi} \\ \theta_2^H &= 1 + m_2 - c + \frac{2\delta\psi}{1+2\delta\psi} \\ x_2^* &= 1 + m_2 - c + \delta\left(1-2c\right). \end{cases} \tag{33}$$

   A necessary condition for the equilibrium to feature no dynamic coordination is the equilibrium $x_2^*$ satisfies $x_2^* + \delta < \theta_1^L$ so that the marginal investor finds the period-1 information useless. Moreover, $\theta_2^H < \theta_1^L$ so that it is indeed possible that the fund in period 2 still fails. Combining both inequalities and the assumption $\frac{\psi}{1+2\delta} < 1-c$, the condition for this case is derived as $m_2 < m_1 - 2\delta(1-c) - \frac{2\delta\psi}{1+2\delta}$.

2. Equilibrium with partial coordination. In this case, the marginal investor finds the information in period 1 being useful. As a result, the equation which shows the marginal investor is indifferent

[37] We leave out the complicated closed-form expression for $H_1(\psi)$ which is available upon request.

between two actions is

$$\begin{aligned}
\Pr\left(\theta < \theta_2^L \middle| x_2^*, \theta < \theta_1^L\right)(1-c) + \int_{\theta_2^L}^{\theta_2^H} \left\{-c + \frac{1}{2\psi}\left[A(\theta) + m_2 - \theta + \psi\right]\right\} d\theta + \Pr\left(\theta > \theta_2^H \middle| x_2^*, \theta < \theta_1^L\right)(-c) &= 0 \\
\Rightarrow \frac{\theta_2^L - (x_2^* - \delta)}{\theta_1^L - (x_2^* - \delta)} \times (1-c) + \frac{\theta_1^L - \theta_2^H}{\theta_1^L - (x_2^* - \delta)} \times (-c) + \frac{\theta_2^H - \theta_2^L}{\theta_1^L - (x_2^* - \delta)} \times \left(\frac{1}{2} - c\right) &= 0.
\end{aligned}$$

Note that the denominator is replaced by $\theta_1^L - (x_2^* - \delta)$ which is less than $2\delta$ since the information in period 1 truncates the range of $\theta$ coming from the marginal investor's private signal alone. The new thresholds are

$$\begin{cases}
\theta_2^L &= 1 + m_2 - c - \frac{c[m_2-(m_1-c)]}{c-2(1-c)\delta} - \frac{4\delta\psi}{1+2\delta}\frac{c-(1-c)\delta}{c-2(1-c)\delta} \\
\theta_2^H &= 1 + m_2 - c - \frac{c[m_2-(m_1-c)]}{c-2(1-c)\delta} - \frac{4\delta^2\psi}{1+2\delta}\frac{1-c}{c-2(1-c)\delta} \\
x_2^* &= 1 + m_2 - c + \delta(1-2c) - \frac{(1-c)(1+2\delta)(m_1+2c\delta-m_2)}{c(1+2\delta)-1} - \frac{2c\delta\psi}{c-2(1-c)\delta}.
\end{cases} \quad (34)$$

Note that if $\psi = 0$, the thresholds are exactly equal to Lemma 4. In this case, if $\theta_2^H > \theta_1^L$, the fund can never fully fail in the equilibrium with partial dynamic coordination. If it further holds that $\theta_2^L > \theta_1^L$, the fund always fully succeeds, and the equilibrium has dynamic coordination. The condition in 2(a) guarantees that $\theta_2^H < \theta_1^L$ so that the fund may still fail, whereas the condition in 2(b) enables $\theta_2^L < \theta_1^L < \theta_2^H$. Case 3 lists the condition for $\theta_2^L > \theta_1^L$.

□

Proposition 15 is the extended version of Proposition 1. Note that when $\psi \to 0$, the two propositions are identical. In case 1 when $m_2$ is small relative to $m_1$, the initial intervention is in vain. In case 3 when $m_2$ is large relative to $m_1$, the initial intervention guarantees success for the second fund. When $m_2$ is in between, the initial intervention has partial coordination effect. Case 2(a) describes a case similar to the equilibrium with partial coordination in the baseline setup. Case 2(b) describes a case when the initial intervention eliminates full failure but not partial failure (success) in the second period.

Proposition 16 summarizes the result when $s_1 = F$. We assume $\psi$ is sufficiently small such that $\psi < \frac{c}{1+2\delta c}$.

**Proposition 16**

*Equilibrium in Period 2 when $s_1 = F$*

1. *If $m_2 < m_1 + 1 - c + \frac{(1-c)\psi}{c(1+2\delta)}$, the unique equilibrium is the Stage Game Equilibrium with Dynamic Coordination. The total social welfare is $W_{2F}^c = 0$.*
2. *If $m_1 + 1 - c + \frac{(1-c)\psi}{c(1+2\delta)} < m_2 < m_1 + 2c\delta + \frac{2\delta\psi}{1+2\delta\psi}$, the unique equilibrium is the Stage Game Equilibrium with Partial Dynamic Coordination.*
   (a) *If $m_1 + 1 - c + \frac{(1-c)\psi}{c(1+2\delta)} < m_2 < m_1 + 1 - c + \frac{\psi[c(1+4\delta)-1]}{c(1+2\delta)}$, in the unique equilibrium, the fund can never fully succeed. The total social welfare is $W_{2F}^{pc} = \frac{1-c}{B-\theta_1^L}\frac{(-1+c-m_1+m_2)^2}{(-1+c+2c\delta)^2} + H_2(\psi)$ where $H_2(\psi) \to 0$ as $\psi \to 0$.*[38]

[38] The detailed expression for $H_2(\psi)$ is available upon request.

(b) *If* $m_1+1-c+\frac{\psi[c(1+4\delta)-1]}{c(1+2\delta)} < m_2 < m_1+2c\delta+\frac{2\delta\psi}{1+2\delta\psi}$*, in the unique equilibrium, the fund may succeed, fail or partially fail. The total social welfare is* $W_{2F}^{pc} = \frac{1-c}{B-\theta_1^L}\frac{(-1+c-m_1+m_2)^2}{(-1+c+2c\delta)^2} - \frac{\delta\psi\left\{12(-1+c)(-1+c-m_1+m_2)(1+2\delta)(-1+c+c\delta)+\left[-11-24\delta+c\left(34+4\delta(23+9\delta)+12c^2(1+2\delta)^2-c(35+4\delta(29+20\delta))\right)\right]\psi\right\}}{3(1+2\delta)^2(-1+c+2c\delta)^2\left(B-\theta_1^L\right)}$.

3. *If* $m_2 > m_1+2c\delta+\frac{2\delta\psi}{1+2\delta\psi}$*, the unique equilibrium is the Stage Game Equilibrium without Dynamic Coordination. The total social welfare is* $W_{2F}^{nc} = \frac{1-c}{B-\theta_1^L}(m_2-m_1-c\delta) - \frac{\psi\delta[\psi-6(1-c)(1+2\delta)]}{3(1+2\delta)^2\left(B-\theta_1^L\right)}$.

*Proof.* We solve the model respectively in three cases: equilibrium with dynamic coordination, with partial coordination and without coordination.

1. Equilibrium without dynamic coordination. Assuming no dynamic coordination. The solutions are:

$$\begin{cases}\theta_2^L &= 1+m_2-c-\frac{2\delta\psi}{1+2\delta\psi}\\ \theta_2^H &= 1+m_2-c+\frac{2\delta\psi}{1+2\delta\psi}\\ x_2^* &= 1+m_2-c+\delta(1-2c).\end{cases}$$

The conditions for no dynamic coordination are:

$$\begin{cases}x_2^*-\delta > \theta_1^H\\ \theta_2^L > \theta_2^H.\end{cases}$$

Then, as long as $\psi$ is sufficiently small such that $\psi < \frac{c}{1+2\delta c}$, the equilibrium without dynamic coordination exists if and only if $m_2 > m_1+2c\delta+\frac{2\delta\psi}{1+2\delta\psi}$. Note that if $\psi\to 0$, the condition is the same as Proposition 2.

2. Equilibrium with partial coordination. In this case, the equation which characterizes the marginal investor's indifference is:

$$\begin{aligned}\Pr\left(\theta<\theta_2^L\middle|x_2^*,\theta>\theta_2^H\right)(1-c)+\int_{\theta_2^L}^{\theta_2^H}\left\{-c+\frac{1}{2\psi}\left[A(\theta)+m_2-\theta+\psi\right]\right\}d\theta+\Pr\left(\theta>\theta_2^H\middle|x_2^*,\theta>\theta_2^H\right)(-c) &= 0\\ \Rightarrow \frac{\theta_2^L-\theta_1^H}{(x_2^*+\delta)-\theta_1^H}\times(1-c)+\frac{(x_2^*+\delta)-\theta_2^H}{(x_2^*+\delta)-\theta_1^H}\times(-c)+\frac{\theta_2^H-\theta_2^L}{(x_2^*+\delta)-\theta_1^H}\times\left(\frac{1}{2}-c\right) &= 0.\end{aligned}$$

The solutions are

$$\begin{cases}\theta_2^L &= 1+m_2-c-\frac{(1-c)(m_1+2c\delta-m_2)}{c(1+2\delta)-1}-\frac{4c\delta^2\psi}{(1+2\delta)[c(1+2\delta)-1]}\\ \theta_2^H &= 1+m_2-c-\frac{(1-c)(m_1+2c\delta-m_2)}{c(1+2\delta)-1}+\frac{4\delta(-1+c+c\delta)\psi}{(1+2\delta)[c(1+2\delta)-1]}\\ x_2^* &= 1+m_2-c+\delta(1-2c)-\frac{(1-c)(1+2\delta)(m_1+2c\delta-m_2)}{c(1+2\delta)-1}-\frac{2(1-c)\delta\psi}{c(1+2\delta)-1}.\end{cases}$$

In this case, if $\theta_2^L < \theta_1^H$, the fund can never fully repay $1-c$ in equilibrium. If it further holds that $\theta_2^H < \theta_1^H$, then the fund fails for sure and there is full dynamic coordination. The condition for $\theta_2^L < \theta_1^H$ to hold is $m_2 < m_1+1-c+\frac{\psi[c(1+4\delta)-1]}{c(1+2\delta)}$. The condition for $\theta_2^H < \theta_1^H$ is $m_2 < m_1+1-c+\frac{(1-c)\psi}{c(1+2\delta)}$

□

Given the results in Propositions 15 and 16, one can easily verify:

1. Conditional Inference Effect. Given $s_1$ and $m_2$, both $W_{2S}$ and $W_{2F}$ decrease in $m_1$.
2. Endogenous Correlation Effect. Investor's welfare $E[W_2]$ increases in $m_1$ when $m_1 - c - \frac{c\psi}{(1-c)(1+2\delta)} < m_2 < m_1 + 1 - c + \frac{(1-c)\psi}{c(1+2\delta)}$.

With this current payoff structure, one can also think about how much $m$ is actually used up ex post. For example, it could be the case that if the fund is very successful, $m$ is not used; if the fund survives and cannot deliver to the investors the full-promise of return, it has to use $m$ to cover that; if the fund fails, $m$ is just lost or taken away by the manager who runs away. This would result in an intervention cost $\tilde{k}(m, \theta)$ that is contingent on the true state. However, as the government makes the intervention decision ex ante, all it cares about is $\tilde{k}(m, \theta)$ integrated over the prior on $\theta$ — an expected cost that can be nested into our general cost specification.

Because the key economic channels remain, their interactions with the intervention cost function lead to implications similar to those in Section 4. What is integral to our model is that the investor payoff cannot be mapped one-to-one with the fundamental state, lest it is fully revealing and the private signal in the second period becomes completely useless, and the intervention loses the coordination effect.

## O4. Normally Distributed Signals

In this subsection, we discuss the results when the private signals follow Normal distribution, i.e., $\varepsilon_i \sim N(0, \delta)$.[39] We still assume that investors are non-overlapping to keep matters comparable with our baseline model. The case when investors perfectly overlap can be identically analyzed as one in which $\varepsilon_i \sim N\left(0, \frac{\delta}{2}\right)$. We characterize the equilibrium in each period and emphasize that government intervention in period 1 still has a dynamic coordination effect in period 2 through altering the informational environment.

Lemma 12 below summarizes equilibrium outcomes in two periods.

**Lemma 12**

*Equilibrium when signals follow Normal distribution*

1. *Given $m_1$, there exists unique equilibrium thresholds in period 1:*

$$\begin{aligned} \theta_1^* &= 1 + m_1 - c \\ x_1^* &= 1 + m_1 - c - \delta\Phi^{-1}(c). \end{aligned}$$

2. *Given $(m_1, m_2)$ and $s_1 = S$,*
   (a) *When $m_2 > m_1 - c$, $(\theta_2^* = \theta_1^*, x_2^* = \infty)$ consists a threshold equilibrium.*
   (b) *Equilibrium strategies $(\theta_2^*, x_2^*)$ which satisfy $\theta_2^* < \theta_1^*$ and $x_2^* < \infty$ may or may not exist. If they exist, they can be non-unique.*

[39] We use uninformative prior belief which is common in global games literature.

To see this, note that the equilibrium outcome in period 1 is characterized by two thresholds $(\theta_1^*, x_1^*)$ that satisfy

$$\begin{aligned} A_1(\theta_1^*) + m_1 &= \theta_1^* \\ \Pr\left(\theta < \theta_1^* \middle| x_1 = x_1^*\right) &= c. \end{aligned}$$

where $A_1(\theta_1^*) = \Pr\left(x_1 < x_1^* \middle| \theta = \theta_1^*\right)$ is the measure of investors who choose to roll over. Simple calculation shows that

$$\begin{aligned} \theta_1^* &= 1 + m_1 - c \\ x_1^* &= 1 + m_1 - c - \delta\Phi^{-1}(c). \end{aligned}$$

The equilibrium in period 2 is again state-independent. We discuss the outcomes when $s_1 = S$ here and the case of $s_1 = F$ is similar. When the intervention in the first period has succeeded, equilibrium in the second period will be either a stage-game equilibrium with full dynamic coordination (similar to Lemma 2), or one with partial dynamic coordination (similar to Lemma 4). The case without dynamic coordination vanishes as the support of the noise now spans between $(\infty, \infty)$. The first type of equilibrium is denoted as $(\theta_2^*, x_2^*) = (\infty, \infty)$ and any equilibrium with $(\theta_2^* > \theta_1^*, x_2^* = \infty)$ is equivalent. The necessary conditions that $(\theta_2^*, x_2^*) = (\infty, \infty)$ constitutes an equilibrium are

$$\begin{aligned} \Pr\left(1 + m_2 > \theta \middle| \theta < \theta_1^*\right) &= 1 \\ \Rightarrow m_2 &> m_1 - c. \end{aligned}$$

Likewise, the necessary conditions that an equilibrium with partial dynamic coordination exists is that the solution $(\theta_2^*, x_2^*)$ to the equation system

$$\begin{aligned} A_2(\theta_2^*) + m_2 &= \theta_2^* \\ \Pr\left(\theta < \theta_2^* \middle| x_2^*, \theta < \theta_1^*\right) &= c. \end{aligned}$$

exists and satisfies $\theta_2^* < \theta_1^*$. Equivalently, we are looking for $\theta_2^*$ that solves

$$1 - (\theta_2^* - m_2) = c\Phi\left(\frac{\theta_1^* - \theta_2^* - \delta\Phi^{-1}(\theta_2^* - m_2)}{\delta}\right) \tag{35}$$

We can numerically solve equation (35), and importantly, Table 4 presents the local comparative statics when there exists a unique equilibrium strategy. When $m_1$ increases from 0.7 to 0.9, both $\theta_2^*$ and $x_2^*$ decrease, validating the dynamic coordination effect.

## O5. General Distributions of Bounded Noise

In this section, we relax the assumption that each agent's private noise follows uniform distribution on $[-\delta, \delta]$. Instead, we assume that $\varepsilon_i$ follows a general distribution with CDF $G(\cdot)$ on $[-\delta, \delta]$. In addition, we

Table 4: $\theta_2^*$ as a function of $m_1$ ($s_1 = S$)

| $m_1$ | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 |
|---|---|---|---|---|---|
| $\theta_2^*$ | 0.7602 | 0.6981 | 0.6693 | 0.6511 | 0.6384 |
| $x_2^*$ | 0.9667 | 0.8222 | 0.7565 | 0.7152 | 0.6866 |

Other parameters are $c = 0.5, \delta = 0.5, m_2 = 0.1$.

continue to assume that $\varepsilon_i$ is i.i.d. across investors. The analysis is meant to show that the three-type of equilibria that we shown in Section 3 robust to more general noise distributions. Indeed, it is the assumption that noises have bounded support that drive the results. The assumption of uniform distribution helps us establish equilibrium uniqueness and derive the closed-form expressions for cutoffs and welfare.

The equilibrium in period 1 is still characterized by two equations

$$\begin{aligned} A_1(\theta_1^*) + m_1 &= \theta_1^* \\ \Pr\left(\theta < \theta_1^* \middle| x_1 = x_1^*\right) &= c. \end{aligned}$$

Given the noise distribution, $A_1(\theta_1^*) = G(x_1^* - \theta_1^*)$ and $\Pr\left(\theta < \theta_1^* \middle| x_1 = x_1^*\right) = 1 - G(x_1^* - \theta_1^*)$. Therefore, we can easily reach the solutions:

$$\begin{aligned} \theta_1^* &= 1 + m_1 - c \\ x_1^* &= 1 + m_1 - c + G^{-1}(1-c). \end{aligned}$$

Next, we turn to equilibrium in period 2. We will consider the case when $s_1 = S$, and the other case can be analyzed similarly.

If the equilibrium does not feature any dynamic coordination, the thresholds in the second period are

$$\begin{aligned} \theta_2^* &= 1 + m_2 - c \\ x_2^* &= 1 + m_2 - c + G^{-1}(1-c) \end{aligned}$$

In this case, if $x_2^* + \delta < \theta_1^* \Rightarrow m_2 < m_1 - \delta - G^{-1}(1-c)$ so that the marginal investor indeed finds the public information useless, then such an equilibrium without dynamic coordination exists.

Likewise, the equilibrium with partial coordination can also be characterized by two equations:

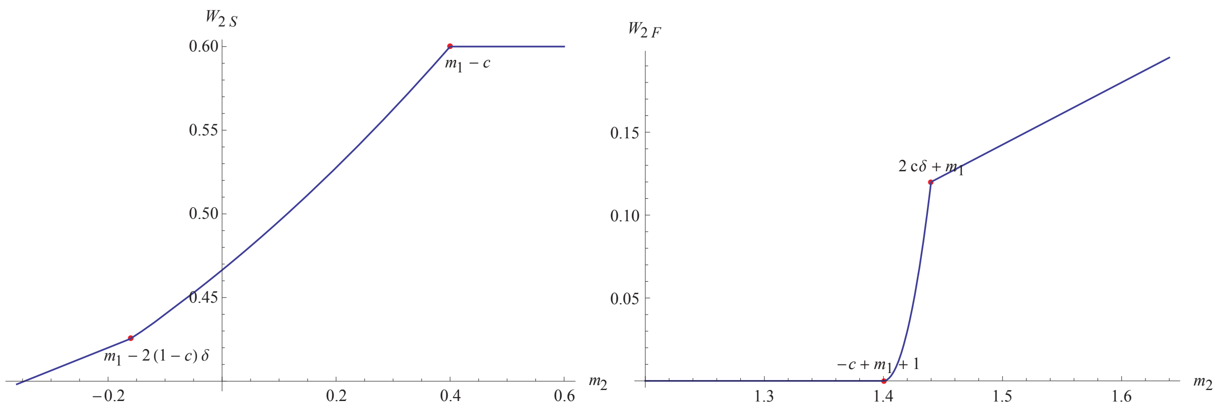
$$\begin{aligned} A_1(\theta_2^*) + m_2 &= \theta_2^* \Rightarrow G(x_2^* - \theta_2^*) + m_2 = \theta_2^* \\ \Pr\left(\theta < \theta_2^* \middle| x_2 = x_2^*, \theta < \theta_1^*\right) &= c \Rightarrow \frac{1 - (\theta_2^* - m_2)}{G(\theta_1^* - x_2^* + \delta)} = c. \end{aligned}$$

Any solution to the equation system comprises an equilibrium. Note that the second equation is non-monotonic in $x_2^*$ so that the solution in general is not unique. However, for each solution, as $m_1$ increases, $\theta_1^*$ increases and so is $\theta_2^*$. Therefore, the conditional inference effect continues to exist.

Finally, for any solution $(\theta_2^*, x_2^*)$ with partial coordination, the condition $\theta_2^* > \theta_1^*$ pins down the relation between $m_1$ and $m_2$ such that the equilibrium features full dynamic coordination.

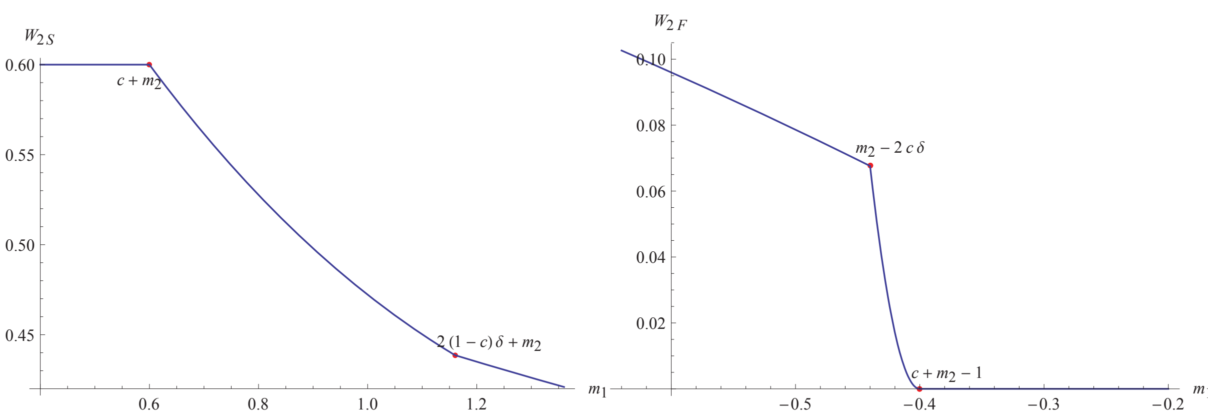
# Figures

Figure 1: $W_{2S}$ and $W_{2F}$ as a function of $m_2$



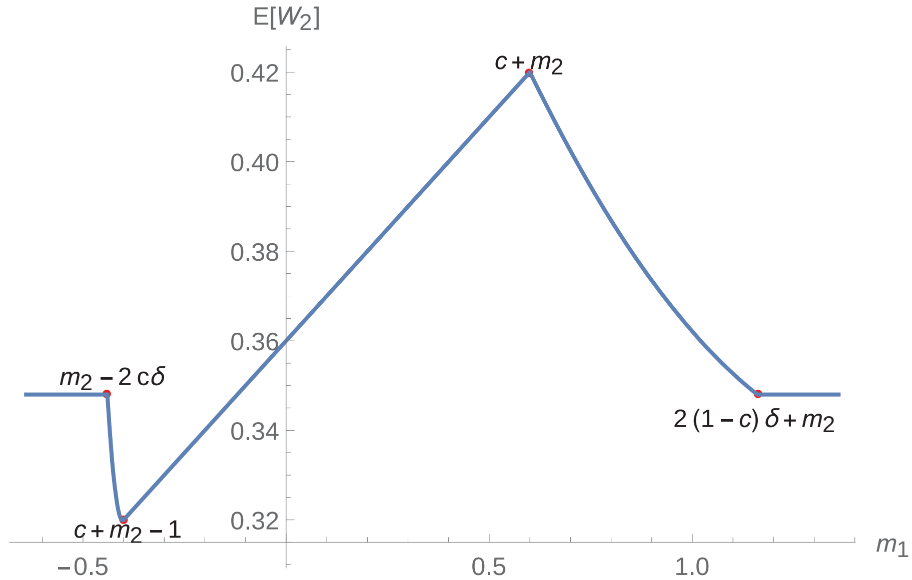

Parameters: $\delta = 0.8,\ c = 0.4,\ B = 3,\ m_1 = 0.8.$

Figure 2: $W_{2S}$ and $W_{2F}$ as a function of $m_1$



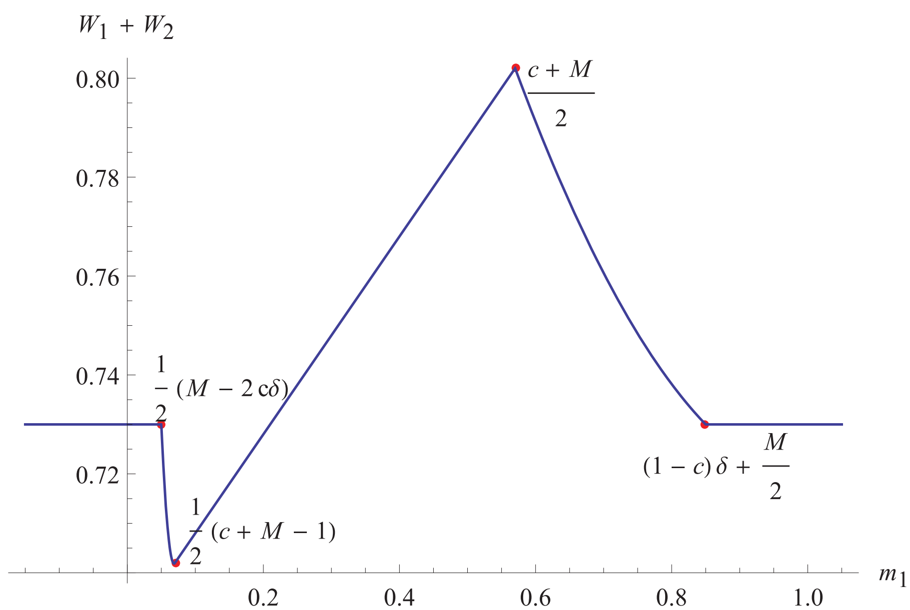

Parameters: $\delta = 0.8,\ c = 0.4,\ B = 3,\ m_2 = 0.2.$

Figure 3: $E[W_2]$ as a function of $m_1$



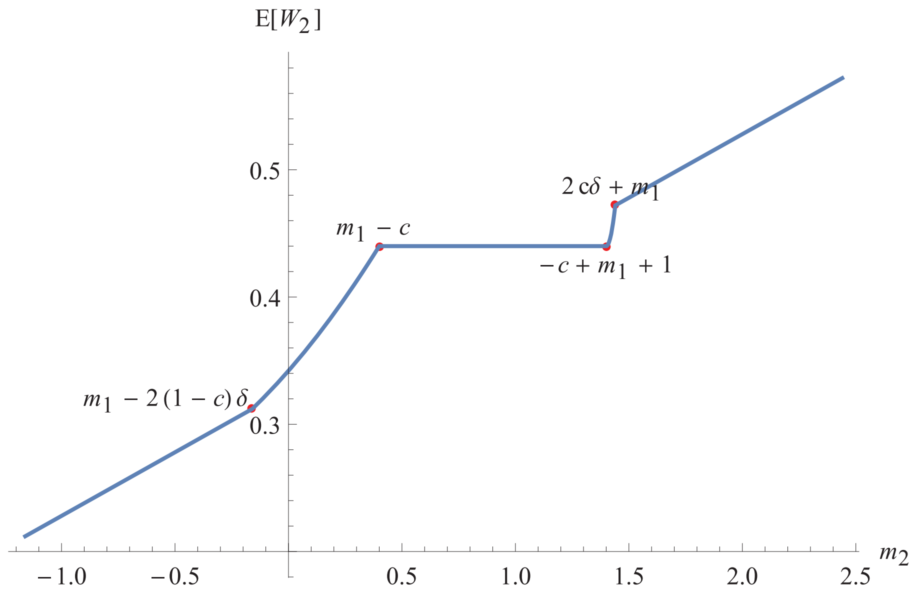

Parameters: $\delta = 0.8,\ c = 0.4,\ B = 3,\ m_2 = 0.2.$

Figure 4: $W_1 + W_2$ as a function of $m_1$ $(m_1 + m_2 = M > 2c\delta)$




Parameters: $\delta = 0.8,\ c = 0.4,\ B = 3,\ M = 0.74.$

Figure 5: $E[W_2]$ as a function of $m_2$




Parameters: $\delta = 0.8,\ c = 0.4,\ B = 3,\ m_1 = 0.8.$

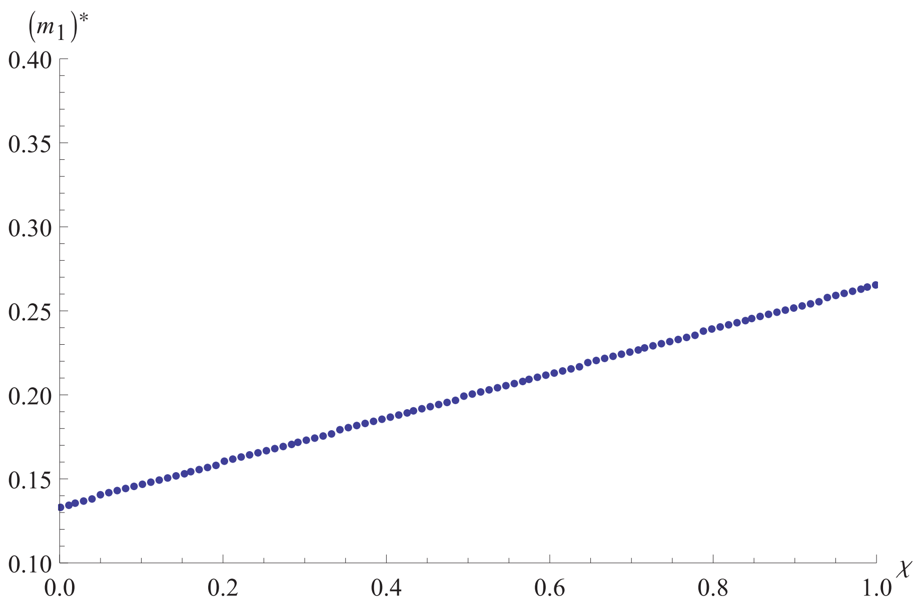



Figure 6: Endogenous initial intervention as a function of the extent to which the policy maker considers the subsequent intervention
Parameters: $\delta = 1.2,\ c = 0.6,\ B = 3,\ k_1 = 0.5,\ k_2 = 0.5,\ K(m_1, m_2) = \frac{1}{2}k_1 m_1^2 + \frac{1}{2}k_2 m_2^2.$

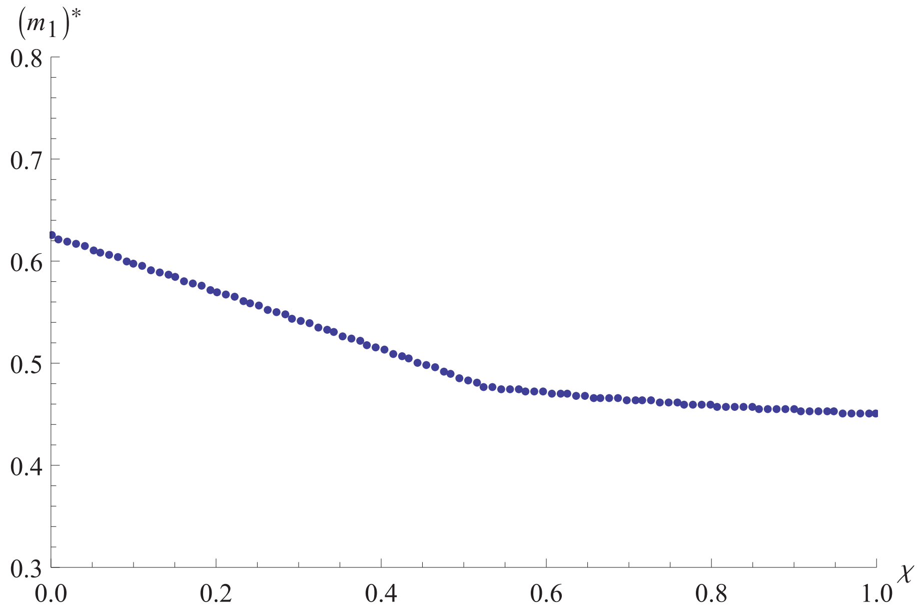



Figure 7: Endogenous initial intervention as a function of the extent to which the policy maker considers the subsequent intervention
Parameters: $\delta = 2,\ c = 0.25,\ B = 3,\ k_1 = 0.2,\ k_2 = 0.8,\ K(m_1, m_2) = \frac{1}{2}k_1 m_1^2 + \frac{1}{2}k_2 m_2^2.$

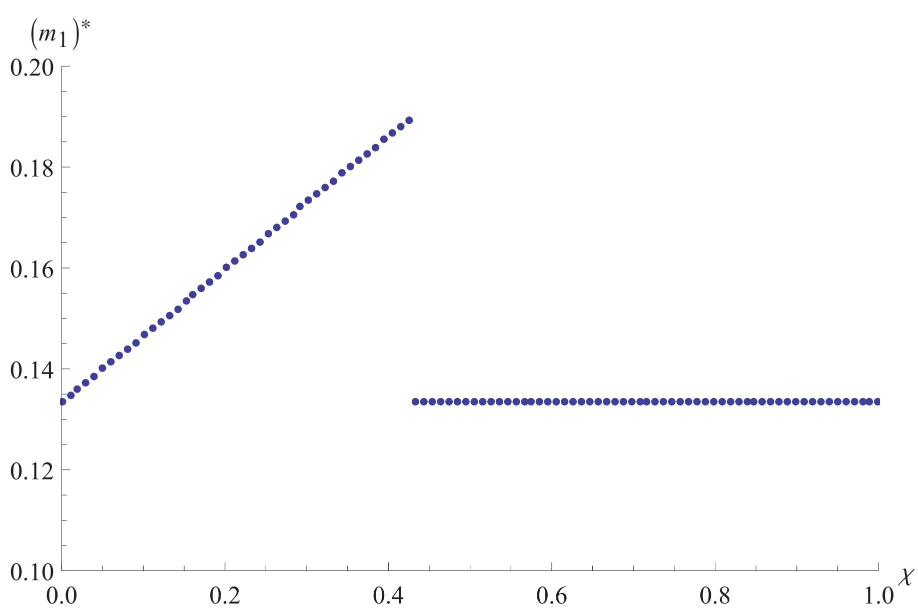



Figure 8: Endogenous initial intervention as a function of the extent to which the policy maker considers the subsequent intervention
Parameters: $\delta = 1.2,\ c = 0.6,\ B = 3,\ k_1 = 0.5,\ k_2 = 0.01,\ K(m_1, m_2) = \frac{1}{2}k_1 m_1^2 + \frac{1}{2}k_2 m_2^2.$