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Compensation in High Finance: A Theory of Periodic Labor Markets and Guaranteed Bonuses\*

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# Compensation in high finance: A theory of periodic labor markets and guaranteed bonuses\*

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## **Abstract**

We present a general equilibrium model of labor market flows that features a periodic equilibrium in which turnover is high in some periods and low in others. If a firm finds itself in a periodic equilibrium, it is optimal to time compensation in the form of low wages and a large bonus delivered just prior to periods with deep labor markets. This ensures that employees depart when replacements are available. The theory generates large, coordinated bonuses, high overall pay, and seasonal turnover, each of which is consistent with evidence from the labor market in high finance.

JEL: D47, D83, G24, J31, J63

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# 1 Introduction

Bankers' bonuses are unusual in at least three ways. First, they are large as a share of total pay. Table 1 presents data on salary and bonus levels for analysts at major investment banks in calendar year 2016.<sup>1</sup> Analysts are recent graduates from college and, as is the case in law, academia, and elsewhere, firms coordinate on compensation packages. As is clear in Table 1, analysts expected to receive 35% or more of total compensation in the form of a bonus.

Second, bonuses are largely guaranteed, implicitly if not explicitly. Table 1 shows that bonuses typically have a floor that employees can be reasonably confident their bonus will exceed. First year analysts, for example, expected to receive at least \$45,000 as a bonus, and an additional \$20,000 if they performed exceptionally well.<sup>2</sup> Guaranteed bonuses are common for higher-level bankers as well, though compensation levels are not standardized and so cannot be presented in a simple table.

Are these bonuses really guaranteed? In 2008, during the depths of the financial crisis in which investment banks were provided government assistance, many of those banks still paid out large bonuses to workers. According to a report by the office of Andrew Cuomo, the Attorney General of New York at the time, nine large banks that received government assistance paid out bonuses of over \$1 million apiece to approximately 5,000 bankers and traders.<sup>3</sup> In March 2009, AIG famously announced nearly \$200 million in bonus payments to its financial services division that accounted for a record loss of over \$60 billion in 2008.

Figure 1 displays monthly bonus payments in the United Kingdom (UK) Finance and Business Services industry. Bonuses fell substantially in 2008, during the depths of the financial crisis, but still amounted to over 50% of those paid in 2007. Bonuses quickly recovered in 2009, even though the financial services industry was still undergoing substantial stress.

Third, banks coordinate the timing of their bonuses. Table 2 displays the timing of when bonuses were announced and paid for a selection of banks worldwide, in calendar year 2017.<sup>4</sup> United States (US) based banks typically set bonuses in January, presumably for work done in January to December of the prior year. European banks determine bonuses in late February and

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<sup>1</sup>The data were collected by the online financial employment newspaper Mergers and Inquisitions, and are described here: <https://www.mergersandinquisitions.com/2016-investment-banking-bonuses/>. We present data for analysts because offers tend to be standardized. Bonuses for more senior bankers are even larger, but not standard and therefore not presentable in a simple table.

<sup>2</sup>The relatively low level of performance pay does not mean that there are no incentives for high performance. Banks often feature an up-or-out culture, in which even mediocre performers may not expect to remain at the firm for long.

<sup>3</sup><http://www.nytimes.com/2009/07/31/business/31pay.html>

<sup>4</sup>These data were generously provided by the online financial newspaper HITC and represent the practice of each bank in 2017. The data include all banks for which HITC collects bonus information.

early March (a fact that is clearly displayed in Figure 1). Asian and Australian banks determine bonuses between mid-April and early May, and Canadian banks determine bonuses in December. These facts suggest regional coordination among banks in the timing of their bonuses.

The variation in the timing of bonuses across geographies suggests that this timing is not related to the seasons or religious holidays. Rather, it appears that the precise date of coordination is not relevant, so long as there is coordination on some date. Indeed, while Table 2 shows that bonus payments are not coordinated *within an industry but across geographies*, Figure 2 shows that bonus payments are not coordinated *within a geography but across industries*: in New York, lawyers are paid bonuses in the fourth quarter while financiers are paid bonuses in the first.

We note that there are four exceptions in the data. The first two are Rothschild and Fortis, which are asset managers focused on private wealth management, rather than investment banks engaged in high finance like the others. These exceptions suggest that the coordination of bonuses is specific to a labor market. Bonuses are not coordinated among all firms in finance – they are only coordinated within high finance. The second two are Credit Suisse and Barclay’s which, while based in Europe, pay bonuses at the same time as the US-based banks. Credit Suisse and Barclay’s have large presences in New York, so these exceptions suggest that the coordination date depends on where the workers are located, not where the bank is headquartered. A further exception, not shown in the table but previously referenced, is the financial services division of AIG. While AIG is an American insurer, it paid bonuses to its London-based traders in March, again suggesting that firms choose the timing of bonus pay to coordinate with other firms operating within the same local labor market.

These three facts about bankers’ bonuses demand an explanation. There does not exist a theory of bonuses in the economics and finance literatures that is consistent with large, guaranteed bonuses. Many contracting problems, including those of moral hazard, screening, and signaling, can be solved with large, performance-based bonuses. Periodic, guaranteed bonuses have at least one theoretical justification as a way to deliver money to financially constrained, present-biased workers just when their marginal utility of consumption is highest (Parsons and Van Wesep, 2013).<sup>5</sup> But the settings that call for large bonuses and those that call for guaranteed bonuses are quite different. Further, the theoretical literature is silent concerning coordination of bonuses across firms, as nearly all models of optimal compensation are solved in partial-equilibrium. Given the apparent empirical facts of bonuses in the finance labor market, a theory is called for.

Suppose that a firm finds that when an employee quits, it can be easier or harder to find a replacement depending on the time of year. It may be easy, for example, to find a replacement in

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<sup>5</sup>Van Wesep (2010) also provides a rationale for signing bonuses, which are guaranteed, as signals to potential hires that they will be a good match with the firm.

January, and increasingly difficult during the year. If the cost of vacancies is large, the firm will want to provide incentives for employees who are considering quitting to delay until January, when filling the vacancy is easier. How can it do this? The simplest way is a large bonus paid if the employee is with the firm at the end of December. If she would like to quit in September, then she may delay quitting until she receives the bonus. Importantly, the bonus will only be effective for retention if it is large relative to pay, and the incentive for the firm to provide the bonus is only large if labor markets are sufficiently seasonal. This means that highly seasonal labor markets can induce firms to offer large, periodic bonuses.

Now suppose that many firms offer large bonuses. Some may pay the bonus in January and others in July, but there is variation in the fraction of firms offering bonuses in different months. Since employees delay quitting until they receive their bonuses, this implies variation in the number of employees available to be hired in each month. Large bonuses generate seasonal variation in labor market depth.

Finally, consider the firm's problem in choosing a time for its bonus. It notes the variation in labor market depth and decides when to offer the bonus. The most profitable time to offer a bonus coincides with the time of greatest labor market depth which, as noted above, also coincides with the time that the most other firms are offering bonuses. There is an incentive to coordinate. That is, if a firm offers a bonus to time departures of its employees, the most profitable time to do so is the time in which most other firms are also offering bonuses.

Large bonuses beget highly seasonal labor markets which beget large bonuses, and firms each have an individual incentive to pay bonuses when other firms pay theirs. There is therefore an equilibrium in which firms all pay large bonuses at the same times. In fact, there are equilibria in which firms coordinate on any periodicity, whether it be paying bonuses daily, monthly, annually, etc. Given a periodicity, the model is silent as to the particular time that the bonus is paid. For example, bonuses can be annual, with the bonus paid in January, February, March, or any other time within the year. While any coordination periodicity is possible in equilibrium, however, *there do not exist robust equilibria in which firms pay periodic bonuses but do not fully coordinate their bonuses.*

It should be clear from Table 2 that coordination is a key stylized fact that should be addressed in a model of banker pay, but that the model need not speak to the specific coordination date. This leaves the coordination dates themselves to be explained by another story. For example, firms may coordinate to pay bonuses at the beginning or end of the industry's tax year, which would be largely consistent with the data presented thus far.<sup>6</sup>

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<sup>6</sup>We note that many stories that one might think of to explain the precise coordination dates in Table 2 are hard to justify. Clearly, calendar years do not explain the data. Fiscal years do not either, as all of the major US and

The model provides three additional implications. The first is that turnover should be seasonal. It should be high in the months after bonuses are paid and low just prior to the payment of next-year’s bonuses.

This prediction holds in US data. We do not have access to turnover data specific to high finance, but the Bureau of Labor Statistics provides turnover data by industry. The bottom panel in Figure 3 presents a back-of-the-envelope estimate of turnover in high finance. Turnover is clearly highest in January, relative to the rest of the US labor market, and is lowest at the end of the year before bonuses are paid.<sup>78</sup>

The second additional implication of the model is that workers can earn rents. That is, they can, as a group, earn more in financial sector jobs than they could earn working outside finance. This is because firms need to provide bonuses large enough to retain employees until the labor market is deep, but these employees might or might not like working for their employers. The employee’s desire to stay is unobservable to the firm, allowing the employee to earn an information rent. If labor markets were not periodic and workers were paid fixed wages, then they would leave when they become dissatisfied with their employers and would not earn information rents. A substantial literature has arisen discussing apparent rents in the financial sector, so we do not provide new stylized facts concerning high pay and instead point to existing research. Pay in high finance has increased dramatically since the 1980s (Kaplan and Rauh, 2010; Philippon and Reshef, 2012; Bell and Van Reenen, 2013a,b; and Boustanifar, Grant, and Reshef, 2016). Much of this is because finance is attracting more skilled workers (Goldin and Katz, 2008; Oyer, 2008; and Gupta and Hacamo, 2017), but some of the increase appears to be rents, in that the same worker doing the same amount of work is paid more in high finance than she would be paid elsewhere (Böhm,

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European banks have fiscal years ending in December. Tax years are more promising, as the tax year in the US ends in December and the tax year in the UK ends in March. The argument is somewhat weaker because both UK banks and US law firms pay in the last month of the tax year, while US banks pay in the first month of the tax year, but this difference could be considered marginally important. Regardless, the model we present does not take a stand on the specific coordination date – only that there is perfect coordination within an industry-geography pair.

<sup>7</sup>The top panel of Figure 3 shows the average turnover (average of hirings and separations) in the Finance and Insurance industry from December 2000 to August 2017, relative to the average across all months. January turnover is 123% of the average and December turnover is only 81% of the average. The middle panel of Figure 3 shows the same calculation for all nonfarm industries in the United States (US). While January turnover is 6% higher than in the average month for all nonfarm industries, the difference is considerably lower than in finance and insurance.

Most workers in Finance and Insurance are not associated with high finance. Some sell insurance for State Farm and others build risk models at Capital One. Even among those employed at JP Morgan Chase, some work in back offices and others are tellers at Chase branches. Presumably, most of the turnover in Finance and Insurance follows similar seasonal patterns to other industries. To separate out the turnover in high finance alone, we subtract abnormal nonfarm turnover from turnover in Finance and Insurance and present the difference in the bottom panel of Figure 3. We see that January is associated with 17 percentage points more turnover in Finance than in the US as a whole, while the second half of the year features considerably less turnover in Finance than in the US as a whole.

<sup>8</sup>Israelson and Yonker (2017) document that CEO turnover is also seasonal: resignation rates are more than 50% higher in January than in other months. They speculate that this is due to the timing of CEO bonuses.

Metzger, and Strömberg, 2015; Célérier and Vallée, 2018). The emerging empirical evidence has driven theorists to study the issue as well (Axelson and Bond, 2015; Glode, Green and Lowrey, 2012; and Glode and Lowrey, 2016). Our model provides an additional explanation for rents in high finance.

The third additional implication of the model is that it is best to grant performance bonuses at the same time as guaranteed bonuses, even if performance can be measured more frequently. To those accustomed to the bonus structure in high human capital professions, it may not seem odd that performance pay is annual, but this is not the practice in other professions. Auto salespeople typically receive bonuses on a bi-weekly or monthly basis. Realtors and stock brokers receive performance pay when they close on a sale or execute a trade, respectively. These examples suggest that it would be natural for a banker working on an IPO to receive a bonus when the IPO occurs, and that it would be natural to offer traders performance pay monthly.

While our model is not one of performance pay, which has been extensively studied elsewhere, we discuss an extension in which workers need to be incented to exert effort every period. Even if the task and its outcome take place every period, we show that it is weakly optimal (and sometimes strictly optimal) to time the performance pay at the same time as the guaranteed bonus. For example, if the guaranteed bonus is paid in January, then it is optimal to pay performance bonuses in January as well, even if performance can be measured at the monthly level. This implication of the model is consistent with the data in Tables 1 and 2, that show that guaranteed and performance bonuses are paid at the same time.

The existence of a periodic equilibrium requires some assumptions on parameters of the model. Most importantly, matching must be fairly efficient. If it takes a long time for workers to find new employers after they quit, then labor markets will not be particularly seasonal, regardless of firms' compensation policies. If labor markets are not particularly seasonal, then firms will not offer large, guaranteed bonuses and periodic labor markets will not arise. Is matching efficient in practice? Theory suggests that it is: all of the workers in high finance live in just a few cities worldwide, most importantly New York, London, and Hong Kong, and they often interact with each other personally. Empirical facts also suggest efficient matching: bankers do not suffer from high levels of unemployment.

In sum, we observe three unusual aspects of bankers' bonuses – they are large, partly guaranteed, and coordinated across firms within an industry-geography pair, but not across industries or geographies – that cannot be reconciled with existing theory in finance or personnel economics. We provide a theory reconciling these facts, which produces additional predictions. Workers in finance will earn rents, in that they earn more in finance than in other professions in which they are equally productive. Firms will time their guaranteed and periodic bonuses to coincide. Turnover

in high finance will be seasonal, peaking immediately after bonuses are determined and declining thereafter. These three predictions also appear to hold in the data.

In principle, this model can apply to any labor market. Periodic labor markets can arise any time there is excess demand for labor and matching between workers and firms is relatively efficient. We believe that high finance is a clear example of an industry that meets these criteria, but other markets likely provide additional examples. A notable counter-example is the market for programmers in Silicon Valley, which is not periodic. This may be because aperiodic equilibria exist as well. It may also be because firms exit frequently in Silicon Valley, whether through an acquisition, a funding squeeze, or a shutdown. We show that markets with substantial exogenous firm exits cannot sustain periodic equilibria.

## 2 Literature

This paper speaks to literatures in many fields including finance, labor economics, personnel economics, economic geography, and market design.

### 2.1 Compensation in high finance

A small literature has provided theories concerning pay in the financial industry. Efficiency wages have long been seen as a basis for “overpaying” workers in many industries. Axelson and Bond (2015) show that the argument can account for high pay in finance even when contracts can be more sophisticated than the flat wages found in the efficiency wage literature. The model accounts for a variety of facts concerning the labor market in high finance, including rapid promotion, an up-or-out culture, drastically different pay for apparently similar workers, and seemingly unnecessarily long hours for young workers. As in our model, employee rents are but one of a relatively disparate set of implications from the model. Glode, Green and Lowrey (2012) and Glode and Lowrey (2016) consider the effects of arms-race style competition in finance, where one trader’s expertise is costly for the party with whom it trades. As in an actual arms race, this leads to overinvestment in financial expertise and results in high pay. Similarly to Axelson and Bond (2015), the model in Glode and Lowrey (2016) shows that similar workers - traders and bankers - can earn substantially different wages. Traders are paid not just to produce value for their employers, but also to prevent them from producing value for competitors. Bond and Glode (2014) consider career concerns of individuals who can work as financiers or regulators. Boom periods draw the best regulators into banking, increasing misbehavior, and individuals looking to enter the financial sector may start their careers in regulation in order to acquire human capital.



## 2.2 Coordinating with market design

Our paper is concerned with firms coordinating their personnel policies so that the industry tends to hire when the labor market is deep. In a sense, this is a form of coordination akin to more formal methods of matching firms to workers covering an entire industry. Classic examples are the National Residency Matching Program, established in 1952, and the ASSA market for young economists. Each of these institutions aims to produce more efficient matches between workers and firms by coordinating meetings and offers. Matching mechanisms have been analyzed formally since the seminal work of Gale and Shapley (1962), who studied a matching theory in the context of college admissions and marriage markets, and Roth (1984), who brought the ideas into the economic mainstream (and later used his models to update the National Residency Matching Program to make it stable and able to handle dual-career problems). Just as with these institutions and this literature, our model centers on coordination among firms to make the labor market more efficient, and just like these models, we do not require players to commit to any set of rules that they would later prefer to break. Unlike existing institutions and work, we investigate labor market coordination through the use of compensation rather than through a designated coordinator.

## 2.3 Coordinating by agglomerating

Urban agglomeration economies have been extensively studied since Adam Smith (1776), and much of the work concerns the benefits of having deep pools of skilled labor in the same geographic area as the firms that employ that labor. For example, Wheaton and Lewis (2002) find that workers in urban areas with many other workers in their industry earn substantially more than those in areas with fewer similarly skilled workers, suggesting an agglomeration economy in labor.<sup>9</sup> The relation to our paper is that this only makes economic sense if matching is important in the labor market. An economy with many employers and many equivalent workers should not see a different overall wage level than an economy with fewer firms and fewer equivalent workers. If match quality is important, however, then the deeper the pool, the higher the average match quality, which generates a higher average productivity and thus a higher average wage. In our model, match quality is also important, though the impact on wages follows a different logic.

## 2.4 Seasonality

A substantial literature concerns predictable seasonal variation in employment, and the most frequently cited examples of seasonality are the bulge in the labor force during the summer months,

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<sup>9</sup>See also Andersson, Burgess, and Lane (2007) who find greater assortative matching between workers and firms in thicker labor markets.

the bulge in retail employee demand during the holiday season, and the bulge in agricultural labor demand during the harvest. These literatures have been concerned with documenting general patterns (e.g., Fisher, 1951, Hansen, 1955); with the effect on government policies such as unemployment insurance (e.g., Myers, 1931, O’Conner, 1962); and with the relationship of seasonality to business cyclicity (e.g., Myron and Beaulieu, 1996, Barsky and Miron, 1989).

More importantly for our paper, two more recent strands of this literature have focused on how firms respond to seasonal demands. First, micro-founded macroeconomic models that produce business cycles often rely on firm-level frictions. One such friction is adjustment costs, which have been extensively studied (e.g., Anderson, 1993, LaFontaine and Sivadasan, 2009). Second, researchers have asked how firms’ compensation responds to seasonal demands (e.g., Del Bono and Weber, 2008, Rosenzweig and Udry, 2014). Our paper is related to this literature in that both are concerned with predictable variation in the labor market and the optimal firm response. The two diverge in that we focus on endogenous periodicity and an optimal contracting response.<sup>10</sup>

## 2.5 Timing of pay

A substantial literature has developed concerning the timing of pay (see Edmans, Gabaix, and Jenter, 2017, for a review). On the one hand, many actions have consequences that develop over time, so maximal incentives are provided by paying later rather than sooner. On the other hand, workers are often assumed to have higher discount rates than shareholders (perhaps because it can be hard to borrow against future earnings), so paying earlier can be less expensive (e.g., Lambert, 1983, Rogerson, 1985, Edmans, Gabaix, Sadzik, and Sannikov, 2012, Opp and Zhu, 2015, Hoffmann, Inderst, and Opp, 2018). Most of this literature has focused on timing because time is associated with information (what actions did the worker take?) and discounting (workers prefer money sooner rather than later). Parsons and Van Wesep (2013) consider the timing of payments for an altogether separate reason: when workers are present-biased, the timing of their pay affects their consumption patterns and therefore their welfare. They show that pay should be more frequent as workers are more present-biased, in order to help them smooth consumption.

## 2.6 Structuring bonuses

There is a long-standing literature on the structure of bonuses. Holmström (1979) provides the classic treatment of optimal bonus policy, and Lazear (2000) its classic empirical investigation. The literature spans hundreds of papers that we cannot cite here. See Edmans, Gabaix, and Jenter

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<sup>10</sup>In addition, Oyer (1998) shows that, in response to seasonal bonus payments, firms’ sales are higher at the end of year and lower at the start of year as salespeople manipulate the timing of customer purchases in order to meet year-end quotas.

(2017) for a review. Our paper provides a rationale for guaranteed bonuses – that is, bonuses whose size is fixed. The bonus does not provide incentives for effort, and does not screen workers. It simply coordinates workers’ decisions to quit.

We are aware of only two other theories that features guaranteed bonuses. The first is Parsons and Van Wesep (2013). As with our paper, the bonuses in their paper do not provide screening or incentive effects. Instead, bonuses are timed to match periods of a high value of consumption for present-biased workers, who have difficulty saving for those needs. This generates bonuses around holidays and vacations, both of which are periods of high consumption needs. As in our paper, these are periodic guaranteed bonuses, though the rationale is different. The second is Van Wesep (2010), who provides a rationale for the signing bonus. Here, the bonus is a signal to workers of the quality of their match with the firm. The size of the bonus is fixed and it is guaranteed, but it is not periodic.

Our model is also related to that in Oyer (2004) which, like ours, studies bonus payments used to retain employees rather than induce effort. While the bonuses in our model are guaranteed, Oyer shows that bonuses tied to firm performance (such as in stock option plans) can be optimal when firm performance is correlated with workers’ outside opportunities.

### 3 The model

Let there be a continuum of firms of mass  $M$ , each employing 0 or 1 worker at any time, and let there be a continuum of workers of mass  $N < M$ . Worker  $i$  has utility  $u_{i,t} = C_{i,t} - x_t^{i,m}$  from working at firm  $m$  where  $x_t^{i,m} \in \{x^L, x^H\}$  is the worker’s firm-specific disutility from work, and where  $0 < x^L < x^H < 1$ .  $C_{i,t}$  is the pay that the worker receives in time  $t$ . Firm revenue is 1 if a worker is employed and 0 otherwise. Firm profit is revenue net of the worker’s wage.<sup>11</sup>

The game takes place over an infinite number of periods indexed by  $t$  with workers and firms discounting future utility according to the common discount factor  $\beta \in (0, 1)$ . Within each period, there are a five sub-periods. First, unmatched firms and unmatched workers search for new matches, and some firms and workers become matched. For simplicity, we assume that newly matched workers have disutility from working at their new employers of  $x_t^{i,m} = x^L$ . Second, newly matched firms make take-it-or-leave-it (TIOLI) offers to their matched workers, who can accept or reject these offers.<sup>12</sup> If rejected, the worker and the offering firm earn no revenue or pay in the period and

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<sup>11</sup>The assumption that the worker-firm match affects worker utility rather than productivity or revenue may appear peculiar. Most theories of turnover are concerned with the firm’s decision to fire, in which case match quality must affect firm profitability. We are concerned with the employee’s decision to quit, so match quality must affect worker utility. In reality, both forces are likely present. Like the existing literature, we focus on a single force to keep the model as simple as possible.

<sup>12</sup>The assumption of take-it-or-leave-it offers means that firms will earn the surplus from employment. We are

re-enter the matching market at the beginning of the next period. Third, if the offer is accepted, or if the worker and firm are already under contract from a prior period, then the firm earns revenue of 1 and pays the agreed-upon wage. Fourth, with probability  $p \in (0, 1)$ , each employed worker who was satisfied with her employer at the start of the period becomes dissatisfied, and her disutility of working at that employer becomes  $x_t^{i,m} = x^H$ . Once dissatisfied, employees do not change types again while at their same employers. Fifth, workers tell firms if they are leaving the firm to enter the matching market in the next period. The series of events is shown in Figure 4.

The game generates a collection of state variables. Let the masses of unmatched workers and firms at the start of period  $t$  be denoted  $N_t^0$  and  $M_t^0$ , respectively. The mass that are matched well, so that the employee's disutility of work is  $x^L$  is denoted  $N_t^L = M_t^L$ . The mass that are matched poorly, because the employee has in some prior period become dissatisfied but has not quit, is denoted  $N_t^H = M_t^H$ . Finally, note, as well, that market clearing requires that  $N_t^L + N_t^H + N_t^0 = N$  and  $M_t^L + M_t^H + M_t^0 = M$  and for all  $t$ .

**Assumption A1:** If  $M_t^0$  firms are searching for a worker and  $N_t^0$  workers are searching for a firm at the start of period  $t$ , then the mass that will match is  $\mu \min\{M_t^0, N_t^0\}$ , where  $\mu \in (0, 1]$ .<sup>13</sup>

Because  $N < M$  and each matched firm has a single employee, the mass of unmatched workers,  $N_t^0$ , in any period is less than the mass of unmatched firms,  $M_t^0$ . The mass of workers and firms that will match is therefore  $\mu N_t^0$ .

Note that  $\min$  is a constant-returns-to-scale matching function. If the number of firms and workers searching both double, then the number of matches doubles as well. The fact that deeper labor markets are better for employers is therefore not due to an increase in the efficiency of matching.

**Assumption A2:**  $\mu = 1$ .

In our baseline model, we assume both A1 and A2. We then relax A2 in Section 5.1. If  $\mu = 1$ , all unmatched workers will match but  $M - N > 0$  unmatched firms will be left unmatched. The fact that it is easier for workers to find new employers than vice versa is critical for the model: firms must fear the loss of their employees.

**Assumption A3:** Workers can be offered contracts featuring a flat wage  $w$  in every period, a bonus  $b$  every  $T$  periods, and a signing bonus/fee  $s$  in the first period of employment. The firm is free to choose any positive or negative value of  $w$ ,  $b$ , and  $s$ , and  $T$  must be a natural number.

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concerned with the employee's decision to quit, so the firm must suffer a consequence if the worker quits. Most theories of turnover are concerned with the firm's decision to fire, in which case the authors assume that workers earn some benefit from employment and fear being fired. That benefit may be exogenous (e.g., stemming from bargaining power) or endogenous (e.g. stemming from efficiency wages).

<sup>13</sup>In Appendix C, we show that our results are not qualitatively dependent on using the minimum function. Any constant-returns-to-scale matching function suffices, but the minimum is convenient.

**Assumption A4:** Workers are free to quit at any time. Firms can commit to a series of wages and can commit not to fire workers.

Assumption A4 is consistent with current labor law in the United States in which labor cannot be compelled but payments to workers can be.

### 3.1 The equilibrium with periodicity $T = 2$

The game defined above is a game of incomplete information, in that, in periods following the match of a worker to a firm, only the worker knows whether she is well matched with the firm. In the period of matching, workers are always well matched with a new employer, and therefore both parties know that  $x_t^{i,m} = x^L$ . In every subsequent period for which  $i$  and  $m$  are matched, only the worker knows  $x_t^{i,m}$ . The firm commits to a long-term contract when it first matches with the employee (and at this time, it knows the employee's type). It takes no action again unless it becomes separated from that employee, in which case the employee's type is irrelevant.

**Definition D1:** *An equilibrium of this game requires the following:*

1. In each period, all unmatched workers search for work and all unmatched firms search for workers.
2. When matched, firms choose  $w$ ,  $b$ , and  $s$  to maximize expected discounted profit. Firms' beliefs are consistent with workers' actions.
3. Workers accept offers if (given  $w$ ,  $b$ , and  $s$ ) the expected present value of employment is weakly higher than the expected present value of re-entering the matching market next period.
4. In each period, matched workers leave their existing firms if and only if the expected present value of re-entering the matching market is strictly greater than the expected present value of staying with their current employers.
5. The labor market state variables  $\{N_t^0, M_t^0, N_t^L, M_t^L, N_t^H, M_t^H\}$  are consistent with equilibrium actions.

**Definition D2:** *An equilibrium is periodic of length  $T$  if there exists  $\tau \geq 1$  such that, for all*

$t$ , the state variables  $\{N_t^0, M_t^0, N_t^L, M_t^L, N_t^H, M_t^H\}$  satisfy

$$\begin{aligned} N_t^0 &= N_{t+\tau}^0 \\ N_t^L &= N_{t+\tau}^L \\ N_t^H &= N_{t+\tau}^H \\ M_t^0 &= M_{t+\tau}^0 \\ M_t^L &= M_{t+\tau}^L \\ M_t^H &= M_{t+\tau}^H. \end{aligned}$$

We denote the period length  $T$  by the smallest  $\tau$  satisfying this set of equations.

In our base model, we construct an equilibrium in which  $T = 2$ .<sup>14</sup> That is, the mass of well matched, poorly matched, and unmatched firms and workers depend only on whether the period is odd or even. For clarity, we denote all periods for which  $t$  is odd as  $t = 1$  and all periods for which  $t$  is even as  $t = 2$ . Without loss of generality, we assume that the labor market is deep in odd periods and shallow in even periods ( $N_1^0 \geq N_2^0$ ).

To make the exposition easier to follow, we briefly outline the construction of the equilibrium. All workers will be well matched with a firm at the beginning of each odd period. As a result of assumptions A1 and A2, any unmatched worker will successfully match with a new firm at the beginning of each period. In equilibrium, all firms that match will make offers to their workers that workers will accept. Because the offers are take-it-or-leave-it, firms will make offers such that the workers' individual rationality constraints bind. At the end of the first period of a worker's time with the firm, she is free to quit. The firm will offer contracts to induce the worker to stay if she is poorly matched at the end of an odd period, and it does so via a bonus  $b > 0$  that is paid only if the worker is employed by the firm in the following even period. If the worker is poorly matched at the end of an even period, she will choose to quit and re-enter the labor market, becoming well matched with a new firm at the start of the following odd period (since  $\mu = 1$ ).

In this equilibrium, there are no unmatched workers searching for firms in even periods, yet both workers and firms need beliefs about what would happen if a worker happened to be available for hire in even periods. In Section 5.1, we relax assumption A2 so that some unmatched workers in odd periods remain unmatched in even periods, and we assume in the base model that agents' off-equilibrium beliefs are identical to the actions that take place in the limit as  $\mu \rightarrow 1$  in the more general model.

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<sup>14</sup>In Section 4 we construct an equilibrium for any  $T$ . Of particular interest is the case in which  $T = 1$ , and the state variables do not change over time.

### 3.1.1 Relations defining the set of state variables

This equilibrium features 12 state variables. Eight are pinned down with market clearing conditions, and the remaining four with flow equations.

At all times, the mass of firms is  $M$  and the mass of employees is  $N$ . This yields four market clearing conditions.

$$M = M_1^L + M_1^H + M_1^0 \quad (1)$$

$$M = M_2^L + M_2^H + M_2^0 \quad (2)$$

$$N = N_1^L + N_1^H + N_1^0 \quad (3)$$

$$N = N_2^L + N_2^H + N_2^0, \quad (4)$$

In each period, because each firm employs a single worker, the mass of well matched firms must equal the mass of well matched workers. The same follows for poorly matched firms and workers. This yields four additional market clearing conditions.

$$M_1^L = N_1^L \quad (5)$$

$$M_2^L = N_2^L \quad (6)$$

$$M_1^H = N_1^H \quad (7)$$

$$M_2^H = N_2^H \quad (8)$$

The four flow equations require some additional discussion. The set of workers that are well matched with firms at the start of an odd period comes from two sources. First are workers that were well matched at the start of the preceding period and who did not become dissatisfied during that period. This group has mass  $N_2^L(1-p)$ . Second are workers hired at the start of the preceding period who did not become dissatisfied during that period. By assumptions A1 and A2, this group has mass  $N_2^0(1-p)$ . Therefore, we have

$$N_1^L = (N_2^L + N_2^0)(1-p). \quad (9)$$

A similar equation holds for the set of workers that are well matched at the start of an even period,

$$N_2^L = (N_1^L + N_1^0)(1-p). \quad (10)$$

The set of workers that is unmatched at the start of an odd period equals the set of workers that quit their employers at the end of the prior, even, period. This set comes from three groups.

First are workers who were dissatisfied with their prior employers at the start of the prior period but were induced not to quit,  $N_2^H$ . Second are workers who were not dissatisfied at the start of the prior period and became dissatisfied,  $N_2^L p$ . Third are workers hired in the prior period who promptly became dissatisfied at the end of that period,  $N_2^0 p$ . Therefore, we have

$$N_1^0 = N_2^H + (N_2^L + N_2^0)p. \quad (11)$$

Finally, because firms will offer contracts that induce workers who are poorly matched at the end of odd periods to nonetheless remain with their employers, because assumptions A1 and A2 ensure that all workers who search for jobs find them, and because firms will always make contract offers that workers will accept, no workers will be unmatched at the start of even periods. Therefore, we have

$$N_2^0 = 0. \quad (12)$$

**Lemma 1** *In a periodic equilibrium with  $T = 2$ , the set of state variables is given by*

$$N_1^L = M_1^L = N(1-p)^2 \quad (13)$$

$$N_1^H = M_1^H = 0 \quad (14)$$

$$N_1^0 = N(1 - (1-p)^2) \quad (15)$$

$$M_1^0 = N(1 - (1-p)^2) + M - N \quad (16)$$

$$N_2^L = M_2^L = N(1-p) \quad (17)$$

$$N_2^H = M_2^H = Np \quad (18)$$

$$N_2^0 = 0 \quad (19)$$

$$M_2^0 = M - N \quad (20)$$

**Proof.** The Lemma follows directly from equations (1) to (12). ■

### 3.1.2 The likelihood of matching for a firm searching for a worker

Let  $f_t$  denote the probability that an unmatched firm finds a new worker at the beginning of period  $t$ .  $M_t^0$  firms are searching for a worker and  $N_t^0 < M_t^0$  workers are available to match. By assumptions A1 and A2, the mass of matches is therefore  $N_t^0$ . For a given firm, the probability of a match is therefore given by

$$f_t = \frac{N_t^0}{M_t^0}.$$



Substituting from the solutions to equations (1) to (12) yields the following result on a firm's likelihood of matching in a periodic equilibrium with  $T = 2$ .

**Lemma 2** *In a periodic equilibrium with  $T = 2$ , a firm's likelihood of matching in odd and even periods is given by*

$$f_1 = \frac{N_1^0}{M_1^0} = \frac{N(1 - (1 - p)^2)}{N(1 - (1 - p)^2) + M - N} \quad (21)$$

$$f_2 = \frac{N_2^0}{M_2^0} = 0. \quad (22)$$

**Proof.** The Lemma follows directly from the expressions in Lemma 1. ■

These matching likelihoods make clear why firms are concerned about losing workers at the end of odd periods: they will be unable to hire in the following even period.

### 3.1.3 Optimal contracting

Given the likelihoods of finding a new employee in odd and even periods, we can determine the optimal contract for a firm to offer if it matches with a new employee in odd and even periods. In equilibrium, no employees are available in even periods, so firms need not concern themselves with that case. We therefore focus on the case in which firms find a new employee in an odd period.

By assumption A3, we restrict attention to “wage-plus-bonus” contracts in which a wage  $w$  is paid in every period, and a bonus  $b > 0$  is paid every  $T$  periods. In our base model, in which  $T = 2$ , this is not restrictive. Any periodic contract can be written as a wage-plus-bonus contract. We allow for the payment in the hiring period to differ from what is specified by  $w$  and  $b$  via a signing fee  $s$ . The signing fee is important when workers may be hired “off-cycle.” Since workers are only hired in odd periods in the base model, the signing fee in odd periods is  $s = 0$ .

Since firm revenue is equal to 1 whenever a worker is employed, the optimal wage-plus-bonus contract offered to a worker who matches in an odd period minimizes the worker's total discounted compensation subject to the constraints that the worker remains with her firm at the end of even periods if she is well matched (constraint 24) and the worker remains with her firm at the end of odd periods even if she is poorly matched (constraint 25). The firm's problem is to choose  $w$  and  $b$  in order to

$$\min_{\{w,b\}} w + \beta(w + b) \quad (23)$$

$$\text{s.t. } 0 \leq w - x^L + \beta(w + b - (1 - p)x^L - (1 - (1 - p))x^H) \quad (24)$$

$$0 \leq w + b - x^H \quad (25)$$

If a worker is well matched after receiving her bonus, the first constraint ensures that her expected utility is higher if she stays with the firm than if she quits and rematches. Since firms make TIOLI offers, workers' outside options are zero, which is why the left-hand-side of constraint (24) is zero. In addition, if a worker is poorly matched at the end of the period in which she does not receive a bonus, then the second constraint ensures that her expected utility is higher if she stays for another period and receives the bonus than if she quits.

Define  $\underline{b} \equiv (1 + \beta(1 - p))(x^H - x^L) > 0$ . Then we have

**Lemma 3** *In a periodic equilibrium with  $T = 2$ , there is a continuum of optimal pairs of wage and bonus. Any optimal pair satisfies*

$$b \geq \underline{b} \tag{26}$$

$$w = x^L - \beta(1 - p)(x^H - x^L) - \frac{\beta}{1 + \beta}(b - \underline{b}) \tag{27}$$

**Proof.** The firm's problem is linear in  $w$  and  $b$  with two linear constraints. Constraint (24) must be satisfied with equality. If constraint (24) is slack, the firm can reduce the wage  $w$  (and if necessary increase the bonus  $b$ ) in order to reduce the total compensation paid to the worker while still satisfying the two participation constraints. When constraint (25) is also satisfied with equality,  $b = \underline{b}$ , which is the smallest bonus that induces the employee to remain with the firm even if poorly matched at the end of odd periods. Because the problem is linear, the firm could offer a higher bonus and a correspondingly lower wage, yielding the same employee behavior at the same total cost. ■

### 3.1.4 Firm values in equilibrium

The firm's value from retaining (or hiring) a well matched worker in period 1 is given by

$$J_1 = 1 - w + \beta(1 - w - b) + \beta^2((1 - p)^2 J_1 + (1 - (1 - p)^2) V_1), \tag{28}$$

where  $V_1$  is the value of a vacant (unmatched) firm in period 1.  $V_1$  is given by

$$V_1 = f_1 J_1 + (1 - f_1) \beta^2 V_1. \tag{29}$$

This expression for  $V_1$  is intuitive. With probability  $f_1$ , the firm can hire a worker, and is therefore worth  $J_1$ . With probability  $1 - f_1$ , it is unable to hire a worker and earns zero. In period 2, it will certainly be unable to hire a worker as no workers quit in even periods. Therefore, the firm must suffer two periods with zero profit, and try to hire again in two periods.

The expression for  $J_1$  is also intuitive. Once the firm has hired a worker in an odd period, it earns revenue of 1 and pays a wage of  $w$  in that period. In the following even period, it earns revenue of 1 and pays  $w + b$  to the worker. With probability  $(1 - p)^2$ , the worker is still matched well with the firm in two periods and stays with the firm, which is once again worth  $J_1$ . With the complementary probability, the worker quits and the firm must find a new worker, in which case it is worth  $V_1$ .

**Lemma 4** *In a periodic equilibrium with  $T = 2$ , the value of the firm that has just successfully hired a worker in an odd period is  $J_1$  and the value of the firm that is attempting to hire a worker in an odd period is  $V_1$ , as defined by*

$$\begin{aligned}
 J_1 &= \left( \frac{(1 - x^H)(1 + \beta) + (x^H - x^L)(1 + \beta(1 - p))}{1 - \beta^2} \right) \\
 &\quad \times \left( \frac{1 - (1 - f_1)\beta^2}{1 - (1 - f_1)\beta^2(1 - p)^2} \right) \\
 V_1 &= \frac{f_1 J_1}{1 - (1 - f_1)\beta^2},
 \end{aligned}$$

where  $f_1$  is given in equation (21).

**Proof.** These functions follow immediately from equations (26) to (29). ■

Thus far we have posited the possibility of an equilibrium in which firms contract with workers so that workers stay with the firm at the end of odd periods in all cases, and quit at the end of even periods if and only if poorly matched. In this equilibrium, workers will be available for hire only at the start of odd periods, which provides an incentive for firms to retain their workers at the end of odd periods. We have shown that the contract that implements this equilibrium features periodic bonuses.

What we have not done is establish that firms would prefer to offer this contract versus a contract that induces different employee behavior. We have not yet, therefore, established that our candidate equilibrium is actually an equilibrium.

### 3.2 Existence of equilibrium

Lemma 3 establishes a continuum of optimal contracts. The firm is indifferent between any contract in this set at the time that it makes the contract offer, but the specific contract chosen from this set affects the value of the firm at subsequent points. For example, if it offers a higher bonus and a lower wage leaving the value  $J_1$  unchanged, the firm will earn a higher profit in odd periods

and a lower profit in even periods. The firm making the offer is indifferent to the trade-off (as is the employee receiving the offer), but one period later, the firm is worth less – the higher cash flow in odd periods has now been earned, but the higher bonus in even periods must still be paid. For the purposes of valuing the firm at different points in time, we must choose one particular contract. Note that the conditions for equilibrium existence are not dependent on which  $\{w, b\}$  pair we choose. For simplicity, we choose the lowest possible bonus,  $\underline{b}$ , and the associated wage  $w = x^L - \beta(1 - p)(x^H - x^L)$ .

Let  $V_2$  be the value of an unmatched firm in even periods. As unmatched firms in even periods are never able to hire, we have

$$V_2 = \beta V_1.$$

In addition, let  $J_2^H$  be the value of a firm with a poorly matched worker in even periods. These firms earn revenue of 1 and pay a wage and bonus that sum to  $x^H$  in even periods, and always lose their workers in the following period. Therefore,

$$J_2^H = 1 - x^H + \beta V_1.$$

For the proposed periodic equilibrium with  $T = 2$  to exist, the contract pair  $\{w, b\}$  defined in Lemma 3 must maximize firm profit given the labor market state variables in equations (13) to (20). In particular, two conditions must be met. The first condition requires that the firm prefer to be matched poorly with a worker in an even period rather than not matched at all,

$$J_2^H \geq V_2.$$

Substituting from above, this condition can be rewritten as

$$1 - x^H + \beta V_1 \geq \beta V_1,$$

which is satisfied for all  $x^H \leq 1$ . Since the firm never matches with a new employee in even periods ( $f_2 = 0$ ), the firm earns higher profit from employing a poorly matched worker in an even period than it does from searching for a new worker.

The second condition requires that the firm prefer to induce employees to quit when poorly matched at the end of an even period rather than stay for one more two-period cycle. Keeping an employee for one more two-period cycle requires paying the worker at least  $x^H$  per period, as the firm cannot observe whether or not the employee is well matched. Therefore, this condition can be written

$$V_1 \geq (1 + \beta)(1 - x^H) + \beta^2 V_1,$$

which is satisfied if and only if

$$\frac{x^H - x^L}{1 - x^H} \geq \left( \frac{1 - f_1}{f_1} \right) (1 + \beta)(1 - \beta(1 - p)). \quad (30)$$

The left-hand side of equation (30) is the percentage difference in surplus generated from employing a well matched worker relative to a poorly matched worker and therefore captures the importance of match quality.<sup>15</sup> When matching is sufficiently important (and/or the probability of matching with a new worker  $f_1$  is large enough), the firm prefers to separate from a poorly matched worker at the end of an even period in order to search for a new well matched worker at the beginning of the following odd period.

**Proposition 1** *A periodic equilibrium with  $T = 2$  exists if and only if*

$$\frac{x^H - x^L}{1 - x^H} \geq \left( \frac{M - N}{N} \right) (1 + \beta) \left( \frac{1 - \beta(1 - p)}{1 - (1 - p)^2} \right).$$

*In this equilibrium, all firms offer wage-bonus contracts defined by*

$$\begin{aligned} b &\geq \underline{b} \\ w &= x^L - \beta(1 - p)(x^H - x^L) - \frac{\beta}{1 + \beta} (b - \underline{b}). \end{aligned}$$

*Turnover is periodic, positive in the period after bonuses are paid, and zero in the period before bonuses are paid.*

**Proof.** Substituting for  $f_1$  from equation (21) into equation (30) yields

$$\frac{x^H - x^L}{1 - x^H} \geq \left( \frac{M - N}{N} \right) (1 + \beta) \left( \frac{1 - \beta(1 - p)}{1 - (1 - p)^2} \right). \quad (31)$$

■

### 3.3 Non-existence of equilibria without full coordination

We have found one particular equilibrium of this game, but there are many others. In Section 4 for example, we establish the existence of other periodic equilibria, in which firms coordinate by paying bonuses every  $T$  periods for values of  $T$  other than two. It is worth emphasizing that there is a class of equilibria that cannot exist. There do not exist equilibria (that are robust to small

<sup>15</sup>Note that the left-hand side of equation (30) is given by  $\frac{(1-x^L)-(1-x^H)}{1-x^H} = \frac{x^H-x^L}{1-x^H}$ .

changes in parameter values) in which firms pay periodic bonuses but do not fully coordinate their bonuses.<sup>16</sup> For example, there are no equilibria in which some fraction  $\varphi$  of firms pay bonuses in even periods and  $1 - \varphi$  pay bonuses in odd periods. The intuition is straightforward. If  $\varphi > 1/2$ , then all firms would strictly prefer to pay bonuses in even periods, and if  $\varphi < 1/2$  then all firms would strictly prefer to pay bonuses in odd periods. If some firms are paying bonuses in even periods and some in odd periods, then they must be indifferent between the two, which means that  $\varphi = 1/2$ . In this case, however, the labor market would not be periodic, and the entire motivation for paying periodic bonuses fails. All firms would therefore strictly prefer either (i) to pay  $w = x^L$ , retaining workers only when they are well matched, or (ii) pay  $w = x^H$ , always retaining workers.

The empirical implication is clear: if most banks in the United States coordinate on January bonuses, then all must coordinate. It will not occur that many coordinate on January and the remainder coordinate on July. This implication is consistent with the data.

This completes the base model. We have established that there can exist an equilibrium in which labor markets are periodic, alternating between being deep and being non-existent. In this equilibrium, the rewards to firms for keeping employees in periods with shallow labor markets is high enough to outweigh the cost of retaining potentially dissatisfied employees. Firms induce optimal separation behavior on the part of employees with periodic bonuses. The bonus must be large enough to prevent separation when the bonus is imminent, but not so high that workers never leave.

For this equilibrium to exist, matching must be sufficiently important. If matching is unimportant, then the cost of retaining employees permanently is low relative to the benefit of always having a worker, so firms will offer contracts that keep employees permanently.

In this equilibrium, bonuses are large as a share of pay (potentially comprising 100% or more of total compensation). Only with large bonuses can the firm generate periodic separations.

In this equilibrium, bonuses are also guaranteed. This could be weakened – as the model is written, a bonus of, for example,  $2b$  paid with probability  $1/2$  would be equivalent, in the eyes of both the firm and the worker, to a bonus of  $b$  paid always. For guaranteed bonuses to be strictly optimal, adding risk-aversion on the part of the worker would suffice.

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<sup>16</sup>For example, suppose that  $m = 1$ ,  $n = 0.9$ ,  $p = 0.5$ ,  $x^L = 0.32$ , and  $x^H = 0.5$ , and consider an equilibrium in which half of firms pay bonuses in even periods and half pay bonuses in odd periods. Since an equal number of firms pay bonuses in even and odd periods, the labor market depth in both periods is the same,  $f_1 = f_2 = 0.75$ . Furthermore, under the given parameter values, inequality (30) is satisfied with equality. Since firms are therefore indifferent between employing poorly matched workers or searching for new workers in either period, an equilibrium exists in which half of firms offer bonuses in odd periods and half offer bonuses in even periods. However, this equilibrium is not robust to a small change in parameters. For example, consider an increase in  $x^H$  by some  $\epsilon > 0$ . In this case, all firms now strictly prefer to search for new employees in either period, and bonuses are no longer optimal. In addition, consider, as well, a small decrease in  $x^H$ . Now, all firms prefer to retain poorly match employees in both periods, and bonuses which incent workers to quit after receiving their bonus cannot be optimal.

In this equilibrium, bonuses are coordinated across firms. All pay bonuses at the end of even periods and none pay bonuses at the end of odd periods.

Finally, in this equilibrium, the coordination date is arbitrary. Coordinating bonuses at the end of odd periods would be equivalent for firms and workers. The precise timing is not important. The only thing that matters is that there is full coordination.

We now consider cycles with  $T \neq 2$ . If  $T = 1$ , then the equilibrium features constant pay and labor markets. This is a useful baseline that captures most labor markets. We show that there are also equilibria for any  $T > 2$ , and the intuition from the base model continues to hold. The objective of this extension is twofold. First, we establish that  $T = 2$  is special only in that it is easy to analyze. The principles from this work apply to any  $T$ . Second, we analyze how welfare depends on  $T$ . Is there an optimal length of period for firms, for employees, and for society? It turns out that under assumptions A1 and A2 both firms and society are best off with  $T = 1$ .

## 4 Equilibria with $T = 1$ or $T > 2$

In the base model, we focus upon the case of  $T = 2$  because it is the shortest possible cycle for which periodicity is present. The most obvious other value of  $T$  that is most clearly interesting is  $T = 1$ , because the labor market does not periodically fluctuate. Without periodic fluctuations, firms will not have an incentive to offer bonuses, let alone coordinate them. Given that most real-world labor markets do not appear to be periodic, analyzing the equilibrium with  $T = 1$  is an important exercise.

Equilibria with  $T > 2$  may also be interesting. Periodicity trades off a cost and a benefit. On the one hand, a longer time between periods of high turnover is costly in that workers choose to stay with employers with which they are dissatisfied. On the other hand, a longer time between periods of high turnover means deeper labor markets when they do come about. One might imagine that there is an optimal value of  $T$  that trades off the cost and benefit. In this section we construct an equilibrium for any arbitrary  $T$  and establish that the same basic facts characterize the equilibrium as in the  $T = 2$  case for all  $T > 1$ .

The state variables are similar to those derived in Lemma 1. Workers leave their employers if they are dissatisfied at the end of period  $T$ , so  $N_1^0 = N(1-p)^T > 0$ . For all other periods, workers never quit, so  $N_\tau^0 = 0$  for  $\tau \in \{2, 3, \dots, T\}$ . Similarly, no workers are poorly matched in the first period of a cycle, so  $N_1^H = 0$  and by market clearing,  $N_1^L = N(1 - (1-p)^T)$ . Over the course of a cycle, the mass of well matched workers declines at rate  $p$  per period (after all matching successfully during period 1), so  $N_\tau^L = N(1-p)^{\tau-1}$  and  $N_\tau^H = N(1 - (1-p)^{\tau-1})$  for  $\tau \in \{2, \dots, T\}$ .

Because  $T > 2$ , it will be convenient to restrict the contracting space. In the base model, the

contract takes a “wage-plus-bonus” form, but this was without loss of generality. Since the contract specifies one payment in even periods and a (potentially) different payment in odd periods, it can always be written as wage-plus-bonus. When  $T > 2$ , one could have a different payment to the worker in each of the  $T$  periods. We restrict attention to wage-plus-bonus contracts, but note that there is a continuum of equivalent contracts that cost the firm the same amount and induce the same employee behavior.

The optimal wage-plus-bonus contract solves

$$\min_{\{w,b\}} w + \beta w + \dots + \beta^{T-2}w + \beta^{T-1}(w + b) \quad (32)$$

$$\begin{aligned} \text{s.t. } 0 \leq & w - x^L + \beta(w - (1-p)x^L - (1 - (1-p))x^H) + \dots \\ & + \beta^{T-1}(w + b - (1-p)^{T-1}x^L - (1 - (1-p)^{T-1})x^H) \end{aligned} \quad (33)$$

$$0 \leq w - x^H + \beta(w - x^H) + \dots + \beta^{T-2}(w + b - x^H) \quad (34)$$

Inequality (33) ensures that a well matched worker remains with the firm at the start of the cycle. Inequality (34) ensures that even a poorly-match worker remains with the firm off-cycle. Note that if a poorly matched worker is willing to remain with the firm in period 2, then the worker must strictly prefer to remain with the firm in periods 3 through  $T$ , so a single constraint suffices.<sup>17</sup>

Now let  $\underline{b} = \left(\frac{1}{\beta^{T-2}}\right) \left(\frac{1-\beta^{T-1}}{1-\beta}\right) \left(\frac{1-\beta^T(1-p)^T}{1-\beta(1-p)}\right) (x^H - x^L)$ .

**Lemma 5** *In a periodic equilibrium with  $T \geq 2$ , there is a continuum of optimal pairs of wage and bonus pairs defined by*

$$\begin{aligned} b & \geq \underline{b} \\ w & = x^L - \frac{\beta(1-p)(1-\beta^{T-1}(1-p)^{T-1})}{1-\beta(1-p)}(x^H - x^L) - \frac{\beta^{T-1}(1-\beta)}{1-\beta^T}(b - \underline{b}). \end{aligned}$$

**Proof.** As in the base model, the firm’s optimization problem is linear in  $w$  and  $b$  with two linear constraints. Furthermore, constraint (33) must be satisfied with equality. When constraint (34) is also satisfied with equality,  $b = \underline{b}$ , which is the smallest bonus that induces the employee to remain with the firm even if poorly matched at the end of the first period in a cycle. Because the problem is linear, the firm could offer a higher bonus and a correspondingly lower wage, yielding the same employee behavior at the same total cost. ■

It should be clear that the wage-bonus pairs follow a similar format to those found in the base

<sup>17</sup>As in the base model, workers are hired only in the first period of a cycle, and the signing fee at this time is  $s_1 = 0$ . While no workers are hired off-cycle, all agents believe the signing fee  $s_\tau$  in period  $\tau$  is given by  $0 = w - s_\tau - x^L + \beta(w - (1-p)x^L - (1 - (1-p))x^H) + \dots + \beta^{T-\tau}(w + b - (1-p)^{T-\tau}x^L - (1 - (1-p)^{T-\tau})x^H)$ .



model with  $T = 2$ . The bonus must be large enough to retain a dissatisfied worker for  $T - 1$  periods. A natural result that follows immediately is that, the larger is the period  $T$ , the larger must be the bonus.

**Corollary 1** *The minimum bonus  $\underline{b}$  that can be used in a periodic equilibrium is increasing in the period length  $T$ .*

We leave a full characterization of the equilibrium for an arbitrary  $T$  to Appendix A. As the derivation largely follows the lines of the base model, repeating it in the body of the paper is not helpful. We simply note here that parameters exist that support an equilibrium for any  $T > 0$ .

We move directly to the analysis of welfare, wages, and profit as a function of the equilibrium periodicity. In practice, banks have coordinated on a 12 month bonus cycle. We would like to understand why that particular periodicity arose. Is it optimal for society? Is it profit maximizing? We now show that, within our model, the answer to both questions is “no”.

Suppose that

$$\frac{x^H - x^L}{1 - x^H} \geq \max \left\{ \left( \frac{M - N}{N} \right) \left( \frac{1 - \beta(1 - p)}{p} \right), \left( \frac{M - N}{N} \right) \left( \frac{1 - \beta(1 - p)}{1 - \beta} \right) \right\},$$

so that an equilibrium exists for all  $T \geq 1$ .

**Proposition 2** *Social welfare and aggregate firm profits are decreasing in  $T$ .*

**Proof.** Since  $\mu = 1$ , workers are never unmatched in equilibrium. Social welfare is therefore inversely related to workers’ aggregate dissatisfaction with their employers. In the first period of a  $T$ -period cycle, no workers are poorly matched. In the second period, a fraction  $p$  become poorly matched. In the third period,  $p + (1 - p)p$  are poorly matched, etc. In a  $T$ -period cycle, therefore, the average surplus per period is given by

$$\begin{aligned} \text{Welfare} &= N \frac{1}{T} \left[ 1 - x^L + (1 - p)(1 - x^L) + (1 - (1 - p))(1 - x^H) + \right. \\ &\quad \left. \dots + (1 - p)^{T-1}(1 - x^L) + (1 - (1 - p)^{T-1})(1 - x^H) \right] \\ &= N \left[ 1 - x^H + (x^H - x^L) \left( \frac{1}{T} \sum_{t=1}^T (1 - p)^{t-1} \right) \right]. \end{aligned}$$

$\frac{1}{T} \sum_{t=1}^T (1 - p)^{t-1}$  is decreasing in  $T$  and  $(x^H - x^L) > 0$ , so welfare is decreasing in  $T$ . Regardless of  $T$ , worker utility is zero. Therefore, aggregate firm profitability is equal to social welfare and is decreasing in  $T$ . ■

As the length of a cycle increases, workers wait longer to separate from their employers after becoming poorly matched. This increases workers' aggregate dissatisfaction and therefore decreases social welfare. Since firms extract all surplus via TIOLI offers, aggregate firm profit is also decreasing in  $T$ . If this is the case, then why might firms find themselves in an equilibrium with  $T > 1$ ? One answer is that firms do not collectively choose the equilibrium in which the industry finds itself. An alternative answer is that bonuses may be useful for reasons that are outside the model, as evidenced by the fact that some bonuses are, in practice, performance-based. Once many firms offer annual bonuses, an incentive to coordinate immediately arises and firms may find themselves stuck in a sub-optimal equilibrium. In Section 3.3, we showed that an equilibrium in which a variety of firms choose different bonus dates cannot exist: if periodic bonuses are offered, then they will be coordinated.

In addition, parameters may be such that an equilibrium with  $T = 1$  does not exist. Corollary 2 and Proposition 2 (found in Appendix A) combine to form an interesting and counter-intuitive result. If  $\beta < 1 - p$ , then cycles can be sustained for a larger set of parameters as  $T$  increases. Regardless of  $\beta$  and  $p$ , welfare is decreasing in  $T$ , so longer cycles are both easier to sustain and less efficient. If employers could coordinate to pay wages without bonuses, both profit and welfare would rise, but it may not be possible to sustain this coordination if any individual firm would like to deviate, pay more, and retain scarce employees.

A final answer may be that there are costs to turnover that are not modeled. The equilibrium with  $T = 1$  features substantial turnover. If turnover is costly, it may be beneficial for firms to incent employees to stay even if poorly matched. In this case, it would be sensible for firms to coordinate on an equilibrium with  $T > 1$  which features lower turnover.

We should note as well that one assumption is critical for the negative relationship between welfare and cycle length. Throughout the analysis, we assume that the matching market exhibits constant returns to scale. If, instead, the matching market featured increasing returns to scale, there would be an additional benefit of increasing  $T$ . A higher value of  $T$  causes the average time that workers spend at firms with which they are poorly matched to increase, which is costly. A higher value of  $T$  also increases the depth of the matching market every  $T$  periods. In a constant returns to scale model, this depth does not increase welfare. Employers care about depth because they are more likely to find workers in a deeper market, but welfare overall does not increase. If we were to hard-wire a sufficient benefit of depth, we could presumably find that the optimal  $T$  is greater than one.

We now provide five extensions that generate additional implications of the model. First, we allow for matching to be less perfect by relaxing assumption A2. We will show that the basic intuition of the model holds, but that matching must be sufficiently efficient for periodic equilibria

to exist. This may help explain why the model pertains to finance: given that most workers live and work in just a few cities world-wide, finance probably has relatively efficient matching relative to more dispersed labor markets.

Second, we impose a new assumption of limited liability for workers:  $w \geq 0$ . We will show that this can generate rents for workers, in that all workers in the industry earn more than their outside options require. This result is consistent with recent empirical research on high pay in high finance. If limited liability binds, then we can reconsider the optimal value of  $T$ . We show that firms and society continue to be best off if  $T = 1$ , but workers are better off for higher values of  $T$ .

Third, we consider an extension in which a fraction  $\delta$  of firms exogenously exit the labor market every period and are immediately replaced. Just as when matching is imperfect, this process means that some workers search for work in every period. If the exit rate is higher, then it is harder to sustain a periodic equilibrium. This fact may explain why the market for computer programmers in Silicon Valley, which shares many other characteristics with the markets for high finance, does not feature periodic contracting.

Fourth, we discuss a repeated moral hazard problem that justifies performance bonuses. If there is no limited liability, then this addition does not affect the model in any interesting way. If there is limited liability, however, there is a new result. Even though the action and payout are realized every period, it is efficient to time variable bonuses to coincide with guaranteed bonuses. For example, if the labor market is periodic at the annual level, justifying annual guaranteed bonuses, then variable bonuses should also come at the annual level, even if output can be measured at higher frequencies. This matches the empirical result outlined in Table 1 that shows that the non-guaranteed component of banker bonuses matches the timing of the guaranteed component.

Fifth, we discuss an extension in which there is variation in employee ability. In the model above, the bonus is guaranteed. This was not important: a bonus of  $2b$  paid with probability  $1/2$  would serve as well as a bonus  $b$  paid for sure. What matters is that the expected bonus is sufficient to retain employees until bonus season. A more realistic (though complicated) model would allow for employees to vary in their abilities. One could imagine some employees who are so low ability that the firm would like to fire them. In order to address this possibility, we can adapt the earlier statement: the expected bonus must be sufficient to retain the employees *that the firm wants to retain* until bonus season. In this extension, the expected bonus  $b$  would vary as the employee's type is learned and could fall enough for the employee to quit. It is rare in practice for a banker not to receive any bonus at all, but it does happen and it is associated with the firm firing the worker.

## 5 Extensions

### 5.1 Extension 1: Why is coordination local?

In this extension, we consider matching functions that are less efficient than in the base model. Specifically, we eliminate assumption A2, and allow for the matching function to be  $\mu \min\{M_t^0, N_t^0\}$  where  $0 < \mu \leq 1$ .

The derivation of results that correspond to the base model is left to Appendix B, as it closely follows the base model and does not provide additional insight on its own. Instead, we move directly to the key result on equilibrium existence for  $\mu < 1$ .

Define  $\underline{b}$  as in the base model with  $T = 2$ , and define  $\underline{\alpha}$  and  $\bar{\alpha}$  such that

$$\underline{\alpha} \equiv \frac{(1 - f_1)(1 + (1 - f_2)\beta)(1 - \beta(1 - p))}{1 - (1 - f_1)(1 - f_2)\beta} \quad (35)$$

$$\bar{\alpha} \equiv \frac{1 - f_2(1 + (1 - f_1)\beta(1 - p)) - (1 - f_1)(1 - f_2)\beta^2(1 - p)^2}{f_2(1 + (1 - f_1)\beta(1 - p))}. \quad (36)$$

**Proposition 3** *A periodic equilibrium with  $T = 2$  and  $0 < \mu \leq 1$  exists if and only if  $\underline{\alpha} \leq \frac{x^H - x^L}{1 - x^H} \leq \bar{\alpha}$ . In this equilibrium, all firms offer compensation contracts defined by*

$$\begin{aligned} b &\geq \underline{b} \\ w &= x^L - \beta(1 - p)(x^H - x^L) - \frac{\beta}{1 + \beta}(b - \underline{b}) \\ s_1 &= 0 \\ s_2 &= x^H - x^L + \frac{1}{1 + \beta}(b - \underline{b}). \end{aligned}$$

*Separations are periodic, positive in the period after bonuses are paid, and zero in the period before bonuses are paid.*

**Proof.** See Appendix B. ■

As in the base model, for the equilibrium to exist two conditions must be met: (i) it must be optimal for the firm to retain a poorly matched worker at the end of odd periods, and (ii) it must be optimal for the firm to release a poorly matched worker at the end of even periods. The first condition requires

$$1 - x^H + \beta V_1 \geq V_2,$$

which is satisfied if and only if match quality is not too important:

$$\frac{x^H - x^L}{1 - x^H} \leq \bar{\alpha}.$$

When match quality is not too important, the relative cost of employing a poorly matched worker in even periods is not too high. Thus, it is optimal for the firm to retain a poorly matched worker at the end of an odd period rather than re-enter the matching market in the following even period when the labor market is shallow.

The second condition requires

$$V_1 \geq (1 - x^H)(1 + \beta) + \beta^2 V_1,$$

which is satisfied if and only if match-quality is sufficiently important:

$$\frac{x^H - x^L}{1 - x^H} \geq \underline{\alpha}.$$

When this condition is satisfied, the relative cost of keeping a poorly matched worker in odd periods is high enough that the firm prefers to re-enter the matching market when the labor market is deep rather than employ a poorly matched worker in odd periods. Thus, cycles can be sustained in equilibrium if and only if

$$\underline{\alpha} \leq \frac{x^H - x^L}{1 - x^H} \leq \bar{\alpha}.$$

**Lemma 6** *When matching is more efficient, the labor market is more periodic. That is,  $\Delta f = f_1 - f_2$  is larger when matching is more efficient.*

**Proof.** See Appendix E. ■

Thus, when  $\mu$  is larger and matching is more efficient, a periodic equilibrium can be sustained over a larger range of parameters, as shown in Figure 6. The reason that more parameters are consistent with periodic equilibrium is that higher values of  $\mu$  generate stronger periodicity in labor market depth. In the extreme when matching is perfect, then there are no workers available off-cycle. If matching is less perfect, more workers are available off-cycle, and this reduces the incentive for any individual firm to stay on-cycle. This may explain why coordination is local. Since matching between workers and firms is likely to be more efficient within New York or London than between New York and London, firms have a strong incentive to coordinate the timing of bonus payments with firms in the same region and little incentive to coordinate with firms in other geographies.

### 5.1.1 The stub bonus

Before continuing, we must clarify one aspect of the optimal contract in this section that may seem odd: the signing fee. In practice, signing fees are not common. Mathematically, the signing fee in this model is identical to a lower bonus offered to workers who join the firm mid-cycle. That is, a firm would not describe the offered contract as featuring a wage, bonus, and signing fee. Instead, a firm would offer simply a wage and periodic bonus, but the first bonus would be pro-rated to reflect that the employee was not working for the entire bonus cycle. A banker hired in September, for example, would receive a wage and a bonus, but the bonus will be smaller than the one earned in subsequent years. This implication of the model is borne out in practice: in the banking industry, these small first-year bonuses are called “stub” bonuses.

## 5.2 Extension 2: Why do bankers earn rents?

One feature of pay in high finance that has attracted substantial empirical and theoretical attention is the level of pay. This manuscript is primarily devoted to pay structure and labor market dynamics, but the model does speak to the level of pay as well. We need one additional assumption in order to account for the level of compensation.

**Assumption A5:** Pay in a period cannot be less than 0.

Assumption A5 is a standard limited-liability constraint. Workers cannot be required to pay the firm money. Whether this constraint is plausible in high finance is debatable, but it is interesting to consider for two reasons. First, in practice, workers do not often pay their employers money; pay is almost always positive, so perhaps limited liability is an improvement regarding the realism of the model. Second, it provides some additional results that speak to the literature on the level of pay in high finance.

Specifically, we show that limited liability can deliver rents to workers: they earn more employed in high finance than they would earn in other professions, not because their skills are better suited to finance or because they work harder, but because periodic labor markets deliver rents.

**Proposition 4** *If  $x^L - \beta(1 - p)(x^H - x^L) < 0$ , then the optimal contract is  $w = 0$  and  $b = x^H$ , and workers earn expected rents worth  $\beta(1 - p)(x^H - x^L) - x^L$  every two periods.*

**Proof.** With limited liability, the optimal wage-plus-bonus contract solves

$$\min_{\{w,b\}} w + \beta(w + b) \quad (37)$$

$$s.t. 0 \leq w - x^L + \beta(w - (1 - p)x^L - (1 - (1 - p))x^H) \quad (38)$$

$$0 \leq w + b - x^H \quad (39)$$

$$0 \leq w. \quad (40)$$

The first constraint ensures that a well matched worker remains with the firm at the start of the cycle. The second constraint ensures that even a poorly-match worker remains with the firm off-cycle. The third constraint is limited liability. Assuming that limited liability does not bind, an optimal contract (not unique) is one in which both constraints (38) and (40) are satisfied with equality:

$$\begin{aligned} w &= x^L - \beta(1 - p)(x^H - x^L) \\ b &= (1 + \beta(1 - p))(x^H - x^L) \end{aligned}$$

If  $x^L - \beta(1 - p)(x^H - x^L) < 0$ , the above contract implies a negative wage payment, which violates limited liability. If limited liability binds, then  $w = 0$  and the bonus is determined by the second constraint,  $b \geq x^H$ . In this case, the first constraint is slack and the worker earns rents. The worker is paid  $x^H$  for two periods of work. She is unpaid in the first period, so her utility in that period is  $-x^L$ . In the second period, with probability  $p$  she is a bad match and receives no utility. With probability  $(1 - p)$ , she is a good match and receives  $(x^H - x^L)$  in utility, which is discounted by  $\beta$  to period 1. Thus, in expectation workers earn positive rents equal to

$$rent = \beta(1 - p)(x^H - x^L) - x^L > 0$$

over two periods of work even though firms make take-it-or-leave-it offers at the time of hiring. ■

For the sake of brevity, we do not fully characterize equilibrium. We simply note that it is no more difficult to sustain an equilibrium with rents than one without. The reason is that, in equilibrium, other firms are paying rents so any given firm must deliver at least as much regardless of whether it offers a periodic contract. Rents essentially raise each employee's outside option – they do not fundamentally change the calculus of whether to offer a periodic contract.

### 5.3 Extension 3: Why are employees in Silicon Valley not paid coordinated bonuses?

We have thus far assumed that firms live in perpetuity. In this section, we allow for exogenous births and deaths of firms and show that if the churn rate is high enough, then periodic equilibria cannot exist. Much of the intuition and equilibrium construction follows similarly to Section 5.1. For this reason, we present here the main result and provide the full equilibrium construction in Appendix C. Specifically, suppose that in each period, with probability  $\delta \in [0, 1)$ , a firm is shut down and the match between the firm and its worker is broken (if the firm is matched at that time). This occurs after revenue and wages are paid and concurrent with the worker learning whether she becomes dissatisfied. The firm is then replaced with a new identical firm and both the new firm and the previously matched worker return to the matching market in the next period. For simplicity, we do not impose limited liability.

**Lemma 7** *When the likelihood of a firm death is smaller, the labor market is more periodic. That is,  $\Delta f = f_1 - f_2$  is larger when the probability of a firm dying is smaller.*

**Proof.** See Appendix E. ■

When firms die, their workers need to find new employers. This means that when firms die at a faster rate, there will be more workers searching for firms off-cycle. If the mass of workers available for hire off-cycle is large enough, then the incentive for a firm to remain on cycle is low and our periodic equilibrium breaks down. Just as a poor matching technology destroys periodic equilibria, so too does exogenous firm death. Panels A and B of Figure 7 mimic Panels A and B of Figure 6, but instead of varying the match intensity  $\mu$ , we vary the likelihood of a firm dying in a period,  $\delta$ . In Panel A, it is clear that as  $\delta$  increases, the likelihood that a firm finds a worker in both odd ( $f_1$ ) and even ( $f_2$ ) periods increases. This is because a higher death rate of firms means a higher mass of unmatched workers looking for a new employer. Exogenous firm deaths have an especially large effect on  $f_2$ , because without exogenous deaths, no workers would be looking for work in even periods (since  $\mu = 1$ ). This provides the intuition for Panel B. As the exogenous firm death rate  $\delta$  increases, more workers are available both on- and off-cycle. Just as in the case in which  $\mu$  is low, plentiful workers off-cycle make it difficult to sustain a periodic equilibrium.

Lemma 7 may explain why the market for programmers in Silicon Valley is not periodic even though it shares many features with the market for financiers. Since firms exit frequently in Silicon Valley, whether through an acquisition, a funding squeeze, or a shutdown, periodic equilibria are likely to be difficult to sustain.



#### 5.4 Extension 4: Why are performance bonuses paid at the same time as guaranteed/retention bonuses?

We have deliberately left aside any justification for bonuses that is not related to coordinating labor markets, but it is clear from Table 1 that bonuses have a variable component as well as a floor. The reasons for this are outside the model, but it is not hard to imagine explanations. For our purposes, the notable feature of the variable bonuses is that they are timed to match the timing of the guaranteed bonuses. That is, even if the effort that variable pay is trying to incent can be measured at, say, a monthly or quarterly frequency, the actual bonus is annual and timed to match the guaranteed bonus. There is no obvious reason that this should be true: if trading profits, for example, are measurable at the monthly frequency, why not offer monthly bonuses? Our theory offers an answer.

Consider a simple extension to the theory in which employees generate revenue by exerting costly effort in each period. In order to get employees to exert that effort, employers would pay based on some performance measure. The only question is *when* they would pay. In the model with no limited liability, the timing of that pay would be irrelevant: offering incentive pay every period or every other period would be equally profitable for the firm. If limited liability binds, however, then incentive pay should be paid at the same time as the guaranteed bonus. The reason is that employees earn information rents in even periods when their employers do not know their types. Firms would like to offset these with negative rents in odd periods by paying a low wage. Limited liability places a lower bound on their ability to do this. Paying even more than zero in odd periods only exacerbates the problem.

We therefore have the prediction that, if limited liability binds, then variable pay will be timed to coincide with guaranteed bonuses, as is the case empirically.

#### 5.5 Extension 5: Must the bonuses actually be guaranteed?

In the model, the bonus is assumed to be guaranteed. This was not necessary: all that matters is that the expected value of the bonus be sufficiently high to retain workers. In extending this model, we could consider a game in which the employee's type is initially high or low, and unknown to all players. If her type is low, then expected revenue is less than  $x^L$  and it is efficient for her to exit the labor market. If it is high then the game works as above. Learning about each employee's type occurs over time and that information is public.

In this setting, it seems likely that the optimal contract would specify that if the posterior falls low enough, the employee should quit and let the firm find a new worker. How would it do this? One possibility is that, if the posterior is denoted  $\rho$ , then the bonus is  $b(\rho)$ . There is a threshold

posterior  $\bar{\rho}$  such that for  $\rho < \bar{\rho}$  it is efficient for the worker to leave the business. The efficient contract would set  $b(\rho)$  such that the worker would quit if  $\rho < \bar{\rho}$ . One such contract would specify  $b = 0$  if  $\rho < \bar{\rho}$  at any time prior to bonus season.

The precise design of this extension is left to future work. It would be a substantial undertaking to combine the problems above with a learning game, and it does not seem worth the space in this manuscript. The goal of this section is simply to note that, if bonuses are not guaranteed, that fact does not invalidate the model. The model allows for guaranteed bonuses, but does not require them, and we can easily imagine a realistic scenario in which an employee fares especially poorly and receives no bonus. Given that it is relatively easy to see the intuition behind such an extension, we leave a more detailed analysis to future work.

## 6 Concluding remarks

We began our investigation in this paper by identifying three unusual aspects of bonuses in high finance. They are large relative to pay and largely guaranteed. Firms within a geography coordinate on the timing of these bonuses, but that timing differs across geographies. We build a model of labor markets in which workers periodically become dissatisfied with their employers and have the option to quit and find a new employer with which they are better matched. We show that when match quality is important (and the matching technology is efficient) there exist equilibria in which labor markets are periodic: workers only quit every  $T$  periods, even if they become dissatisfied in the interim. This means that labor markets are deep every  $T$  periods, making it easy to hire in these periods, and shallow in the remaining periods, making it hard to hire.

This periodicity in labor markets provides an incentive for firms to design contracts that generate periodic quitting by employees. The contracts that induce this behavior feature periodic bonuses: even if dissatisfied, as employees approach their bonus dates, they prefer to stay with their employers rather than to quit. These contracts must feature large bonuses so that employees have a sufficient incentive to stay even if dissatisfied. The bonuses are guaranteed, as uncertainty serves no purpose. They are also coordinated across firms: the purpose of the bonus is to retain employees until the labor market is deep and the firm can easily replace them. Only if bonuses are coordinated does this logic apply. Finally, the precise dates of coordination in the model are arbitrary. The model therefore matches the motivating facts regarding bankers' bonuses.

The model makes additional predictions as well. First, turnover should be seasonal, high immediately after bonuses are paid and low prior to the next bonus date. This prediction appears to match the data well: turnover in US high finance is highest in January, when bonuses are set, and low in the fall. Second, if limited liability binds, then the model also predicts that pay will be

“high”, in the sense that workers earn more in high finance than they would earn in other industries that do not feature large periodic bonuses. Recent empirical work suggests that this prediction holds. Third, if some performance pay is necessary and limited liability binds, then the timing of the performance pay should match the timing of the guaranteed bonus. Even if performance can be measured at higher frequency than the frequency of guaranteed bonuses, there is an incentive for firms to delay performance grants to coincide with the guaranteed bonus. This appears to be borne out in practice.

Why finance? This model applies to any labor market, and it may not be clear why it is specific to finance. For a periodic equilibrium to exist, workers have to be scarce, worker-firm matching must be at least somewhat important, the matching technology has to be fairly efficient, and employers cannot exit or fire employees too often. These conditions are satisfied for finance, but probably other industries as well. The labor markets in corporate law, management consulting, and economic consulting, for example, probably satisfy these conditions. Casual observation suggests that bonuses are common and coordinated in these markets as well (New York law firms, for example, typically pay bonuses in December). We believe that the model appears to apply especially well to finance, but do not argue that it does not apply elsewhere.

An interesting and important example in which we do not see periodicity is the market for programmers in Silicon Valley. Good workers are scarce, the local labor market is liquid, and there is probably a moderate match importance. Periodicity may not arise in Silicon Valley because of the high rate of firm exits (deaths or acquisitions) and funding losses. As we show in Section 5.3, if workers are forced from their employers throughout the year, then labor markets will never be thin enough to support a periodic equilibrium.

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## 8 Appendix A – The full model with an arbitrary $T > 0$

In this Appendix, we replicate the results from the base model for an arbitrary  $T > 0$ .

In an equilibrium with periodicity  $T$ , the probability that a firm finds a new worker in the first period of the cycle is given by

$$f_1 = \frac{N(1 - (1 - p)^T)}{N(1 - (1 - p)^T) + M - N}. \quad (41)$$

The probability that a firm finds a new worker off-cycle is  $f_2 = f_3 = \dots = f_T = 0$ . The firm's value from retaining (or hiring) a well matched worker in the first period of a cycle is therefore given by

$$\begin{aligned} J_1 = & 1 - w + \beta(1 - w) + \dots + \beta^{T-2}(1 - w) + \beta^{T-1}(1 - w - b) \\ & + \beta^T((1 - p)^T J_1 + (1 - (1 - p)^T)V_1), \end{aligned}$$

where  $V_1$  is the value of a vacant firm in the first period of a cycle, and where  $V_1$  is given by

$$V_1 = f_1 J_1 + (1 - f_1)\beta^T V_1.$$

Note that the last term in the expression for  $V_1$  follows from

$$f_2 = f_3 = \dots = f_T = 0.$$

Solving for  $V_1$  and  $J_1$  yields

$$\begin{aligned} V_1 &= \frac{f_1 J_1}{1 - (1 - f_1)\beta^T} \\ J_1 &= \left( \frac{(1 - x^H) \left( \frac{1 - \beta^T}{1 - \beta} \right) + (x^H - x^L) \left( \frac{1 - \beta^T(1 - p)^T}{1 - \beta(1 - p)} \right)}{1 - \beta^T} \right) \\ &\quad \times \left( \frac{1 - (1 - f_1)\beta^T}{1 - (1 - f_1)\beta^T(1 - p)^T} \right), \end{aligned}$$

where  $f_1$  is given in equation (41).

**Proposition 5** *A periodic equilibrium with  $T \geq 2$  exists if and only if*

*$\frac{x^H - x^L}{1 - x^H} \geq \left( \frac{1 - n}{n} \right) \left( \frac{1 - \beta^T}{1 - \beta} \right) \left( \frac{1 - \beta(1 - p)}{1 - (1 - p)^T} \right)$ . In this equilibrium, all firms offer wage-bonus contracts de-*

fined by

$$\begin{aligned} b &\geq \underline{b} \\ w &= x^L - \frac{\beta(1-p)(1-\beta^{T-1}(1-p)^{T-1})}{1-\beta(1-p)}(x^H - x^L) - \frac{\beta^{T-1}(1-\beta)}{1-\beta^T}(b - \underline{b}). \end{aligned}$$

Turnover is periodic, positive in the period after bonuses are paid, and zero in all other periods.

**Proof.** For the equilibrium to exist two conditions must be met: (i) first, it must be optimal for the firm to retain a poorly-match worker off-cycle, and (ii) second, it must be optimal for the firm to release a poorly matched worker on-cycle. The first condition requires

$$J_\tau^H \geq V_\tau \text{ for } \tau \in \{2, 3, \dots, T\}, \quad (42)$$

where

$$J_\tau^H = (1 - x^H) \left( \frac{1 - \beta^{T+1-\tau}}{1 - \beta} \right) + \beta^{T+1-\tau} V_1,$$

is the value of retaining a poorly matched worker off-cycle (and allowing her to leave on-cycle). Substituting for  $V_\tau$  and  $J_\tau^H$  in inequality (42) yields

$$(1 - x^H) \left( \frac{1 - \beta^{T+1-\tau}}{1 - \beta} \right) + \beta^{T+1-\tau} V_1 \geq \beta^{T+1-\tau} V_1,$$

which is satisfied for all  $x^H \leq 1$ . Thus, the firm will never deviate to a contract which incents workers to leave off-cycle. The second condition requires

$$V_1 \geq (1 - x^H) \left( \frac{1 - \beta^T}{1 - \beta} \right) + \beta^T V_1,$$

which is satisfied if and only if match-quality is sufficiently important:

$$\frac{x^H - x^L}{1 - x^H} \geq \left( \frac{1 - f_1}{f_1} \right) \left( \frac{1 - \beta^T}{1 - \beta} \right) (1 - \beta(1 - p)).$$

And, substituting for  $f_1$  from above yields

$$\frac{x^H - x^L}{1 - x^H} \geq \left( \frac{1 - n}{n} \right) \left( \frac{1 - \beta^T}{1 - \beta} \right) \left( \frac{1 - \beta(1 - p)}{1 - (1 - p)^T} \right).$$

■



**Corollary 2** *If  $\beta < 1 - p$ , cycles can be sustained over a larger parameter space as  $T$  increases. If  $\beta > 1 - p$ , cycles can be sustained over a larger parameter space as  $T$  decreases.*

Corollary 2 shows that, when  $\beta < 1 - p$ , a periodic equilibrium may be easier to sustain when the length of the cycle is longer. When  $1 - p$  is large, the rate at which workers become dissatisfied with their current employers is small, and as a result, the mass of workers separating from employers at the end of a cycle and available for hire at the beginning of a cycle is also small. If the labor market at the start of a cycle is not sufficiently deep, firms prefer to retain poorly matched workers, and periodic equilibria do not exist. In this case, increasing the length of the cycle increases labor market depth, thus making periodic equilibria easier to sustain.

Perhaps the most important special case other than cycles with  $T = 2$  features  $T = 1$ , since most real-world labor markets do not appear to be periodic. These equilibria feature firms offering fixed-wage contracts and workers quitting as soon as they become poorly matched. In such an equilibrium, all workers are matched well.

**Corollary 3** *In an equilibrium with  $T = 1$ , workers are offered a wage  $w = x^L$  and no bonus, and workers quit when they become dissatisfied with their current employer.*

**Proof.** Plugging  $T = 1$  into the expressions from Lemma 5 yields  $\underline{b} = 0$  and

$$\begin{aligned} b &\geq \underline{b} \\ w &= x^L - (b - \underline{b}). \end{aligned}$$

Since the bonus is paid every  $T$  periods and  $T = 1$ , the bonus and wage are both paid every period. The sum of the two is  $x^L$ . At this wage, workers quit if  $x_t^{i,m} = x^H$  and immediately rematch. ■

## 9 Appendix B – The base model with $\mu < 1$

In this appendix, we extend the base model to allow for imperfect matching, in which  $\mu < 1$ .

Following the motivation in the base model, in a cycle of length  $T = 2$ , labor market dynamics are given by eight market-clearing conditions and four flow equations. The eight market clearing conditions are given by equations (1) to (8). The flow equations, however, differ from the base

model:

$$N_1^L = (N_2^L + \mu N_2^0)(1 - p) \quad (43)$$

$$N_1^0 = N_2^0(1 - \mu) + N_2^H + (N_2^L + \mu N_2^0)p \quad (44)$$

$$N_2^L = (N_1^L + \mu N_1^0)(1 - p) \quad (45)$$

$$N_2^0 = N_1^0(1 - \mu). \quad (46)$$

**Lemma 8** *In a periodic equilibrium with  $T = 2$  and  $0 < \mu \leq 1$ , the set of state variables is given by*

$$\begin{aligned} N_1^L &= M_1^L = \phi \mu N(1 - p)(1 - p + 1 - \mu) \\ N_1^H &= M_1^H = 0 \\ N_1^0 &= \phi N(1 - (1 - p)^2) \\ M_1^0 &= \phi N(1 - (1 - p)^2) + M - N \\ N_2^L &= M_2^L = \phi \mu N(1 - p)[1 + (1 - p)(1 - \mu)] \\ N_2^H &= M_2^H = \phi \mu N p[1 + (1 - p)(1 - \mu)] \\ N_2^0 &= \phi N(1 - (1 - p)^2)(1 - \mu) \\ M_2^0 &= \phi N(1 - (1 - p)^2)(1 - \mu) + M - N \end{aligned}$$

where

$$\phi = \frac{1}{1 - (1 - \mu)(1 - p)(1 - p - \mu)}.$$

**Proof.** The Lemma follows directly from equations (1) to (8) and (43) to (46). ■

Continuing to follow the procedure from the base model, we can calculate the likelihood of a firm matching in a particular period. Recall that  $f_t$  is the likelihood that a firm searching in period  $t$  matches with a worker. Because the mass of unmatched firms is always greater than the mass of unmatched workers, the mass of new matches is  $\mu N_t^0$ , and the probability that a firm matches is given by

$$f_t = \mu \frac{N_t^0}{M_t^0}.$$

Substituting from the solutions to the above equations gives

**Lemma 9** *In a periodic equilibrium with  $T = 2$  and  $0 < \mu \leq 1$ , a firm's likelihood of matching in*

odd and even periods is given by

$$f_1 = \mu \left( \frac{N_1^0}{M_1^0} \right) = \mu \left( \frac{N\phi(1 - (1-p)^2)}{N\phi(1 - (1-p)^2) + M - N} \right) \quad (47)$$

$$f_2 = \mu \left( \frac{N_2^0}{M_2^0} \right) = \left( \frac{f_1}{1 - f_1} \right) (1 - \mu). \quad (48)$$

Labor market depth is periodic: higher in odd periods and lower in even periods,  $f_1 > f_2$  for all  $\mu \in (0, 1]$ .

**Proof.** The Lemma follows directly from the expressions in Lemma 8. ■

Since constraints (24) and (25) do not depend on the labor market state variables, the wage and bonus given in the base model

$$\begin{aligned} b &\geq \underline{b} & (49) \\ w &= x^L - \beta(1-p)(x^H - x^L) - \frac{\beta}{1+\beta}(b - \underline{b}). & (50) \end{aligned}$$

are optimal for all  $\mu \in (0, 1]$ .

Given the preceding wage and bonus, a worker hired in an odd period earns zero rents in expectation, and thus the signing fee in odd periods is equal to zero,  $s_1 = 0$ . When a worker is hired in an even period, the firm knows the worker is well matched and that her disutility from work is  $x^L$ . The firm therefore charges the worker a signing fee  $s_2$  such that  $0 = w + b - s_2 - x^L$  which gives<sup>18</sup>

$$s_2 = x^H - x^L + \frac{1}{1+\beta}(b - \underline{b}). \quad (51)$$

As in the base model, the firm is indifferent between contracts that satisfy equations (49) to (51), but the specific contract chosen from this set affects the value of the firm in even periods. For simplicity, we again focus on the lowest possible bonus,  $\underline{b}$ , and the associated wage  $w = x^L - \beta(1-p)(x^H - x^L)$  and signing fee  $s_2 = x^H - x^L$ . Under this contract, the equations defining

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<sup>18</sup>Thus in the base model, we assume that agents believe workers hired in even periods will face a signing fee given by  $s_2 = x^H - x^L + \frac{1}{1+\beta}(b - \underline{b})$ .

firm values are

$$J_1 = 1 - x^L + \beta \left[ \begin{array}{c} (1 - x^L)(1 - p) \\ + (1 - x^H)(1 - (1 - p)) \end{array} \right] + \beta^2 \left[ \begin{array}{c} J_1(1 - p)^2 \\ + V_1(1 - (1 - p)^2) \end{array} \right] \quad (52)$$

$$J_2 = 1 - x^L + \beta [J_1(1 - p) + V_1(1 - (1 - p))] \quad (53)$$

$$V_1 = f_1 J_1 + (1 - f_1) \beta V_2 \quad (54)$$

$$V_2 = f_2 J_2 + (1 - f_2) \beta V_1, \quad (55)$$

where  $J_1$  is again the value of hiring or retaining a well match worker in odds periods,  $J_2$  is the value of hiring a well matched worker in even periods, and  $V_1$  and  $V_2$  are the values of a vacant firm in odd and even periods, respectively.

These continuation values form the basis of proposition 3 in the text, whose proof is found in Appendix E.

## 10 Appendix C – The base model with a constant returns to scale matching function

In the base model, we assume for ease of exposition in assumption A1 that the mass of matches between unmatched firms and unmatched workers is equal to  $\mu \min\{M_t^0, N_t^0\}$ , where  $\mu \in (0, 1]$ . In this short appendix, we show that the intuition from the base model does not depend on this assumption. We replace A1 with the following:

**Assumption B1:** If  $M_t^0$  firms are searching for a worker and  $N_t^0$  workers are searching for a firm at the start of period  $t$ , then the mass that will match is  $m_t = m(M_t^0, N_t^0)$  where the matching function  $m$  is continuous, strictly increasing in both arguments, and homogeneous of degree one.

Define  $n_t^0 = \frac{N_t^0}{M_t^0}$  to be the labor market depth (from a firm's perspective) at time  $t$ .

**Lemma 10** *Equilibrium labor market depth is increasing in the mass of unmatched workers  $N_t^0$ .*

**Proof.** *The following market clearing conditions*

$$\begin{aligned} N_t^L &= M_t^L \\ N_t^H &= M_t^H \\ M_t^0 + M_t^L + M_t^H &= M \\ N_t^0 + N_t^L + N_t^H &= N \end{aligned}$$

imply that for all  $t$

$$M_t^0 = N_t^0 + M - N,$$

and this implies that the labor market depth for all  $t$  is given by

$$n_t^0 = \frac{N_t^0}{N_t^0 + M - N}.$$

Finally,

$$\frac{dn_t^0}{dN_t^0} = \frac{M - N}{(N_t^0 + M - N)^2} > 0,$$

since workers are scarce ( $M > N$ ). ■

By Assumption B1, the probability that an unmatched firm finds a match at time  $t$  is

$$f_t = \frac{m(M_t^0, N_t^0)}{M_t^0} = m(1, n_t^0),$$

where the last equality follows because the matching function  $m$  is homogeneous of degree one.

**Lemma 11** *In a periodic equilibrium with  $T=2$ , an unmatched firm matches with a new worker with a higher probability in odd periods than in even periods,  $f_1 > f_2$ .*

**Proof.** *Since no workers quit at the end of odd periods, the mass of unmatched workers at the beginning of an even period is given by*

$$N_2^0 = N_1^0 - m(M_1^0, N_1^0) < N_1^0.$$

*The last inequality follows because  $m(M_1^0, N_1^0) > 0$  for  $N_1^0 > 0$  (and  $N_1^0 > 0$ , since all high-disutility workers quit at the end of even periods). Then from Lemma 10, equilibrium labor market depth satisfies  $n_1^0 > n_2^0$ . Finally,*

$$f_1 = m(1, n_1^0) > m(1, n_2^0) = f_2$$

since  $m$  is strictly increasing and  $n_1^0 > n_2^0$ . ■

**Proposition 6** *For any matching function satisfying Assumption B1, there exists a nonempty set  $\mathbf{X}^H \subset (x^L, 1)$  such that a periodic equilibrium with  $T=2$  exists for  $x^H \in \mathbf{X}^H$ .*

**Proof.** *Since firm values depend on the labor market state variables only through the matching probabilities  $f_1$  and  $f_2$ , the equations defining firm values under assumption B1 are the same as those defined in equations (79)-(82) in the proof of Proposition 3. Following the proof of Proposition*

3, an equilibrium exists if and only if the following two conditions are satisfied:

$$\begin{aligned} 1 - x^H + \beta V_1 &\geq V_2 \\ V_1 &\geq (1 - x^H)(1 + \beta) + \beta^2 V_1, \end{aligned}$$

or equivalently if and only if

$$V_1 - \beta V_1 \geq 1 - x^H \geq V_2 - \beta V_1.$$

Now, let  $\Delta V = V_1 - V_2$ . From equations (81) and (82),

$$\Delta V = \rho(1 - \beta) \left[ \begin{array}{c} (1 - x^H)\beta p[f_1 - f_2(1 - f_1)\beta(1 - p)] \\ -(1 - x^L)[f_1 f_2 \beta p - (f_1 - f_2)(1 + \beta(1 - p))] \end{array} \right]$$

which is linear and decreasing in  $x^H$ , since  $f_1 > f_2$ . Furthermore, it is straight forward to show that  $\Delta V(x^H) > 0$  for  $x^H = x^L$ . Now, define  $\hat{x}^H$  (potentially greater than one) such that  $\Delta V(\hat{x}^H) = 0$ , then

$$\hat{x}^H = 1 - \frac{(1 - x^L)[f_1 f_2 \beta p - (f_1 - f_2)(1 + \beta(1 - p))]}{\beta p[f_1 - f_2(1 - f_1)\beta(1 - p)]} > x^L.$$

Thus,  $\Delta V(x^H) > 0$  for all  $x^H \in [x^L, \hat{x}^H]$ .

We now consider two cases: (i)  $\hat{x}^H > 1$  and (ii)  $\hat{x}^H \leq 1$ . When  $\hat{x}^H > 1$ ,  $\Delta V(x^H) > 0$  for all  $x^H \in [x^L, \hat{x}^H]$ . We now show that  $V_1 - \beta V_1 \geq 1 - x^L$  for all  $x^H \in [x^L, \hat{x}^H]$ . To see this, note that  $V_1 < \frac{1 - x^L}{1 - \beta}$  when  $x^H = x^L$  and  $f_1 = 1$ . Since workers are scarce,  $f_1 < 1$  implying that  $V_1 < \frac{1 - x^L}{1 - \beta}$  for  $x^H = x^L$  (since  $V_1$  is decreasing in  $f_1$ ). Thus, in the limit as  $x^H \rightarrow x^L$ , we have  $V_2 - \beta V_1 < V_1 - \beta V_1 < 1 - x^H$ .

Because  $f_1 > f_2 \geq 0$ , it is clear that  $V_2 > 0$  for all  $x^H \in [x^L, 1]$ . Then in the limit as  $x^H \rightarrow 1$ , we have  $V_1 - \beta V_1 > V_2 - \beta V_1 > 1 - x^H = 0$ . Finally, the value functions  $V_1$  and  $V_2$  are continuous in  $x^H$ , and thus by the intermediate value theorem, there exists  $\tilde{x}^H \in (x^L, 1)$  such that

$$V_1(\tilde{x}^H) - \beta V_1(\tilde{x}^H) > 1 - \tilde{x}^H > V_2(\tilde{x}^H) - \beta V_1(\tilde{x}^H).$$

When  $\hat{x}^H \leq 1$ , then  $\Delta V(x^H) > 0$  for all  $x^H \in [x^L, \hat{x}^H]$ . As in case (i), we have  $V_2 - \beta V_1 < V_1 - \beta V_1 < 1 - x^H$  in the limit as  $x^H \rightarrow x^L$ .

We now show that  $V_2(\hat{x}^H) - \beta V_1(\hat{x}^H) > 1 - \hat{x}^H$ . Substituting  $x^H = \hat{x}^H$  into equations (81) and (82) gives

$$V_2(\hat{x}^H) - \beta V_1(\hat{x}^H) = f_2(1 - x^L),$$

and

$$V_2(\hat{x}^H) - \beta V_1(\hat{x}^H) - (1 - \hat{x}^H) = \frac{(1 - x^L)(1 - f_2)(1 + \beta(1 - p))}{\beta p} > 0.$$

By the intermediate value theorem, there exists  $\tilde{x}^H \in (x^L, \hat{x}^H)$  such that

$$V_1(\tilde{x}^H) - \beta V_1(\tilde{x}^H) > 1 - \tilde{x}^H > V_2(\tilde{x}^H) - \beta V_1(\tilde{x}^H).$$

■

We have established that equilibria with  $T = 2$  exist for an arbitrary constant returns to scale matching function. These equilibria are qualitatively equivalent to the equilibria evaluated in the base model.

## 11 Appendix D – Equilibrium construction with firm exits

When firms exit and are replaced with probability  $\delta \in [0, 1)$  each period, the eight market clearing conditions are given by equations (1) to (8). The four flow equations differ from the base model:

$$N_1^L = (N_2^L + N_2^0)(1 - p)(1 - \delta) \quad (56)$$

$$N_1^0 = N_2^H + (N_2^L + N_2^0)(1 - (1 - p)(1 - \delta)) \quad (57)$$

$$N_2^L = (N_1^L + N_1^0)(1 - p)(1 - \delta) \quad (58)$$

$$N_2^0 = N\delta. \quad (59)$$

**Lemma 12** *In a periodic equilibrium with  $T = 2$  and  $0 \leq \delta < 1$ , the set of state variables is given by*

$$N_1^L = M_1^L = N(1 - p)(1 - \delta)(1 - p(1 - \delta))$$

$$N_1^H = M_1^H = 0$$

$$N_1^0 = N[1 - (1 - p)(1 - \delta)(1 - p(1 - \delta))]$$

$$M_1^0 = N[1 - (1 - p)(1 - \delta)(1 - p(1 - \delta))] + M - N$$

$$N_2^L = M_2^L = N(1 - p)(1 - \delta)$$

$$N_2^H = M_2^H = Np(1 - \delta)$$

$$N_2^0 = N\delta$$

$$M_2^0 = N\delta + M - N.$$

**Proof.** The Lemma follows directly from equations (1) to (8) and (56) to (59). ■

Continuing to follow the procedure from the base model, we can calculate the likelihood of a firm matching in a particular period. Recall that  $f_t$  is the likelihood that a firm searching in period  $t$  matches with a worker. Because the mass of unmatched firms is always greater than the mass of unmatched workers, the mass of new matches is  $N_t^0$ , and the probability that a firm matches is given by

$$f_t = \frac{N_t^0}{M_t^0}.$$

Substituting from the solutions to the above equations gives

**Lemma 13** *In a periodic equilibrium with  $T = 2$  and  $0 \leq \delta < 1$ , a firm's likelihood of matching in odd and even periods is given by*

$$f_1 = \frac{N_1^0}{M_1^0} = \frac{N[1 - (1-p)(1-\delta)(1-p(1-\delta))]}{N[1 - (1-p)(1-\delta)(1-p(1-\delta))] + M - N} \quad (60)$$

$$f_2 = \frac{N_2^0}{M_2^0} = \frac{N\delta}{N\delta + M - N}. \quad (61)$$

*Labor market depth is periodic: higher in odd periods and lower in even periods,  $f_1 > f_2$  for all  $\delta \in [0, 1)$ .*

**Proof.** The Lemma follows directly from the expressions in Lemma 12. ■

We now turn to the firm's problem and again follow the procedure in the base model. When a firm may be shutdown with probability  $\delta$ , the firm's problem is to choose  $w$  and  $b$  in order to solve

$$\min_{\{w,b\}} w + (1-\delta)\beta(w+b) \quad (62)$$

$$\text{s.t. } 0 \leq w - x^L + (1-\delta)\beta(w+b - (1-p)x^L - (1-(1-p))x^H) \quad (63)$$

$$0 \leq w + b - x^H \quad (64)$$

If a worker is well matched after receiving her bonus, the first constraint ensures that her expected utility is higher if she stays with the firm than if she quits and rematches. Since firms make TIOLI offers, workers' outside options are zero, which is why the left-hand-side of constraint (63) is zero. In addition, if a worker is poorly matched at the end of the period in which she does not receive a bonus, then the second constraint ensures that her expected utility is higher if she stays for another period and receives the bonus than if she quits.



Define  $\underline{b}^\delta \equiv (1 + (1 - \delta)\beta(1 - p))(x^H - x^L) > 0$ . Also, define  $\widehat{\beta} = \beta(1 - \delta)$ .  $\widehat{\beta}$  is the product of the discount factor  $\beta$  and the likelihood that a firm does not die before the next period, and is therefore the relevant discount factor for a firm.

**Lemma 14** *In a periodic equilibrium with  $T = 2$ , there is a continuum of optimal pairs of wage and bonus. Any optimal pair satisfies*

$$b \geq \underline{b}^\delta \tag{65}$$

$$w = x^L - \widehat{\beta}(1 - p)(x^H - x^L) - \frac{\widehat{\beta}}{1 + \widehat{\beta}}(b - \underline{b}) \tag{66}$$

**Proof.** The firm's problem is linear in  $w$  and  $b$  with two linear constraints. As in the base model, constraint (63) must be satisfied with equality. If constraint (63) is slack, the firm can reduce the wage  $w$  (and if necessary increase the bonus  $b$ ) in order to reduce the total compensation paid to the worker while still satisfying the two participation constraints. When constraint (64) is also satisfied with equality,  $b = \underline{b}$ , which is the smallest bonus that induces the employee to remain with the firm even if poorly matched at the end of odd periods. Because the problem is linear, the firm could offer a higher bonus and a correspondingly lower wage, yielding the same employee behavior at the same total cost. Given the preceding wage and bonus, a worker hired in an odd period earns zero rents in expectation, and thus the signing fee in odd periods is equal to zero,  $s_1 = 0$ . When a worker is hired in an even period, the firm knows the worker is well matched and that her disutility from work is  $x^L$ . The firm therefore charges the worker a signing fee  $s_2$  such that  $0 = w + b - s_2 - x^L$  which gives

$$s_2 = x^H - x^L + \frac{1}{1 + \widehat{\beta}}(b - \underline{b}). \tag{67}$$

■

The firm is indifferent between any contract which satisfies equations (65) to (67), but the specific contract chosen from this set affects the value of the firm in even periods. For simplicity, we again focus on the lowest possible bonus,  $\underline{b}^\delta$ , and the associated wage  $w = x^L - \widehat{\beta}(1 - p)(x^H - x^L)$  and signing fee  $s_2 = x^H - x^L$ . Under this contract, the equations defining firm values are

$$J_1 = 1 - x^L + \widehat{\beta} \left[ \begin{array}{c} (1 - x^L)(1 - p) \\ + (1 - x^H)(1 - (1 - p)) \end{array} \right] + \widehat{\beta}^2 \left[ \begin{array}{c} J_1(1 - p)^2 \\ + V_1(1 - (1 - p)^2) \end{array} \right] \quad (68)$$

$$J_2 = 1 - x^L + \widehat{\beta} [J_1(1 - p) + V_1(1 - (1 - p))] \quad (69)$$

$$V_1 = f_1 J_1 + (1 - f_1) \widehat{\beta} V_2 \quad (70)$$

$$V_2 = f_2 J_2 + (1 - f_2) \widehat{\beta} V_1. \quad (71)$$

Now, define  $\underline{\alpha}^\delta$  and  $\bar{\alpha}^\delta$  such that

$$\underline{\alpha}^\delta \equiv \frac{(1 - f_1) \left(1 + (1 - f_2) \widehat{\beta}\right) \left(1 - \widehat{\beta}(1 - p)\right)}{1 - (1 - f_1)(1 - f_2 \widehat{\beta})} \quad (72)$$

$$\bar{\alpha}^\delta \equiv \frac{1 - f_2(1 + (1 - f_1) \widehat{\beta}(1 - p)) - (1 - f_1)(1 - f_2) \widehat{\beta}^2(1 - p)^2}{f_2(1 + (1 - f_1) \widehat{\beta}(1 - p))}, \quad (73)$$

we have

**Proposition 7** *A periodic equilibrium with  $T = 2$  and  $0 \leq \delta < 1$  exists if and only if  $\underline{\alpha}^\delta \leq \frac{x^H - x^L}{1 - x^H} \leq \bar{\alpha}^\delta$ . In this equilibrium, all firms offer compensation contracts defined by*

$$\begin{aligned} b &\geq \underline{b}^\delta \\ w &= x^L - \widehat{\beta}(1 - p)(x^H - x^L) - \frac{\widehat{\beta}}{1 + \widehat{\beta}}(b - \underline{b}) \\ s_1 &= 0 \\ s_2 &= x^H - x^L + \frac{1}{1 + \widehat{\beta}}(b - \underline{b}). \end{aligned}$$

*Voluntary separations are periodic, positive in the period after bonuses are paid, and zero in the period before bonuses are paid.*

**Proof.** The proof follows the preceding proof of Proposition 3. The firm value functions follow

directly from equations (68) to (71).

$$J_1 = \rho \left[ \begin{aligned} &(1 - x^H) \left[ 1 + \widehat{\beta} \left( 1 - \widehat{\beta} (1 - f_1) \left( 1 + \widehat{\beta} - f_2 \left( 1 - p + \widehat{\beta} (1 + p(1 - p)) \right) \right) \right) \right] \\ &+ (x^H - x^L) \left[ 1 + \widehat{\beta} \left( -\widehat{\beta}^2 (1 - f_1) (1 - f_2 - p) \right) \right] \end{aligned} \right] \quad (74)$$

$$J_2 = \rho \left[ \begin{aligned} &(1 - x^H) \left[ 1 + \widehat{\beta} \begin{pmatrix} 1 - \widehat{\beta} (1 + \widehat{\beta}) (1 - f_1) (1 - f_2) \\ -p (1 - f_1) (1 - \widehat{\beta}) (1 - \widehat{\beta}^2 (1 - f_2)) \\ -(1 - f_1) \widehat{\beta} p^2 (1 - \widehat{\beta}^2 (1 - f_2)) \end{pmatrix} \right] \\ &+ (x^H - x^L) \left[ 1 + \widehat{\beta} \begin{pmatrix} 1 - \widehat{\beta} (1 + \widehat{\beta}) (1 - f_1) (1 - f_2) \\ -p (1 - f_1) (1 - \widehat{\beta}) - \widehat{\beta}^2 (1 - f_1) (1 - f_2) \end{pmatrix} \right] \end{aligned} \right] \quad (75)$$

$$V_1 = \rho \left[ \begin{aligned} &(1 - x^H) \left[ \begin{aligned} &f_2 \widehat{\beta} (1 + \widehat{\beta} (1 - p) (1 + \widehat{\beta} p)) \\ &+ f_1 (1 + \widehat{\beta} (1 - f_2 (1 + \widehat{\beta} (1 - p) (1 + \widehat{\beta} p)))) \end{aligned} \right] \\ &+ (x^H - x^L) \left[ (f_1 + f_2 \widehat{\beta} (1 - f_1)) (1 + \widehat{\beta} (1 - p)) \right] \end{aligned} \right] \quad (76)$$

$$V_2 = \rho \left[ \begin{aligned} &(1 - x^H) \left[ \begin{aligned} &f_2 + \widehat{\beta} f_2 (1 - f_1) (1 - p) + f_1 \widehat{\beta} (1 + \widehat{\beta}) \\ &- f_2^2 \widehat{\beta} (f_1 - p (1 - f_1) + p^2 (1 - f_1)) \end{aligned} \right] \\ &+ (x^H - x^L) \left[ f_2 + f_1 \widehat{\beta} + f_2 \widehat{\beta} (1 - f_1) (1 - p) + f_1^2 \widehat{\beta} (1 - f_2 - p) \right] \end{aligned} \right] \quad (77)$$

For the equilibrium to exist two conditions must be met: (i) it must be optimal for the firm to retain a poorly matched worker off-cycle, and (ii) it must be optimal for the firm to release a poorly matched worker on-cycle. The first condition requires

$$1 - x^H + \widehat{\beta} V_1 \geq V_2.$$

Substituting from the solutions above gives

$$\frac{x^H - x^L}{1 - x^H} \leq \bar{\alpha}^\delta \equiv \frac{1 - (1 - f_1)(1 - f_2)(1 - p)^2 \widehat{\beta}^2}{f_2(1 + (1 - f_1)(1 - p)\widehat{\beta})} - 1.$$

If match quality is not too important, it is optimal for the firm to retain a poorly matched worker at the end of period 1. The second condition requires

$$V_1 \geq (1 - x^H)(1 + \widehat{\beta}) + \widehat{\beta}^2 V_1,$$

which is satisfied if and only if match-quality is sufficiently important:

$$\frac{x^H - x^L}{1 - x^H} \geq \underline{\alpha}^\delta \equiv \left( \frac{1 - f_1}{f_1 + (1 - f_1) \widehat{\beta} f_2} \right) \left( 1 + \widehat{\beta}(1 - f_2) \right) \left( 1 - \widehat{\beta}(1 - p) \right).$$

Thus, cycles can be sustained in equilibrium if and only if

$$\underline{\alpha}^\delta \leq \frac{x^H - x^L}{1 - x^H} \leq \bar{\alpha}^\delta.$$

■

As in the base model, for the equilibrium to exist two conditions must be met: (i) it must be optimal for the firm to retain a poorly matched worker at the end of odd periods, and (ii) it must be optimal for the firm to release a poorly matched worker at the end of even periods. The first condition requires

$$1 - x^H + (1 - \delta)\beta V_1 \geq V_2,$$

which is satisfied if and only if match quality is not too important:

$$\frac{x^H - x^L}{1 - x^H} \leq \bar{\alpha}^\delta.$$

When match quality is not too important, the relative cost of employing a poorly matched worker in even periods is not too high. Thus, it is optimal for the firm to retain a poorly matched worker at the end of an odd period rather than re-enter the matching market in the following even period when the labor market is shallow.

The second condition requires

$$V_1 \geq (1 - x^H) (1 + (1 - \delta)\beta) + \widehat{\beta}^2 V_1,$$

which is satisfied if and only if match-quality is sufficiently important:

$$\frac{x^H - x^L}{1 - x^H} \geq \underline{\alpha}^\delta.$$

When this condition is satisfied, the relative cost of keeping a poorly matched worker in odd periods is high enough that the firm prefers to re-enter the matching market when the labor market is deep rather than employ a poorly matched worker in odd periods. Thus, cycles can be sustained in equilibrium if and only if

$$\underline{\alpha}^\delta \leq \frac{x^H - x^L}{1 - x^H} \leq \bar{\alpha}^\delta.$$

## 12 Appendix E – Proofs omitted in the body of the manuscript

**Proof of Proposition 3.** First, let

$$\rho = \frac{1}{(1 - \beta^2)(1 - \beta^2(1 - f_1)(1 - p)(1 - f_2 - p))}. \quad (78)$$

The firm value functions follow directly from equations (52) to (55).

$$J_1 = \rho \left[ \begin{aligned} &(1 - x^H) [1 + \beta (1 - \beta (1 - f_1) (1 + \beta - f_2 (1 - p + \beta (1 + p(1 - p)))))] \\ &+ (x^H - x^L) [1 + \beta (-\beta^2 (1 - f_1) (1 - f_2 - p))] \end{aligned} \right] \quad (79)$$

$$J_2 = \rho \left[ \begin{aligned} &(1 - x^H) \left[ 1 + \beta \begin{pmatrix} 1 - \beta (1 + \beta) (1 - f_1) (1 - f_2) \\ -p (1 - f_1) (1 - \beta) (1 - \beta^2 (1 - f_2)) \\ -(1 - f_1) \beta p^2 (1 - \beta^2 (1 - f_2)) \end{pmatrix} \right] \\ &+ (x^H - x^L) \left[ 1 + \beta \begin{pmatrix} 1 - \beta (1 + \beta) (1 - f_1) (1 - f_2) \\ -p (1 - f_1 (1 - \beta) - \beta^2 (1 - f_1) (1 - f_2)) \end{pmatrix} \right] \end{aligned} \right] \quad (80)$$

$$V_1 = \rho \left[ \begin{aligned} &(1 - x^H) \left[ \begin{aligned} &f_2 \beta (1 + \beta (1 - p) (1 + \beta p)) \\ &+ f_1 (1 + \beta (1 - f_2 (1 + \beta (1 - p) (1 + \beta p)))) \end{aligned} \right] \\ &+ (x^H - x^L) [(f_1 + f_2 \beta (1 - f_1)) (1 + \beta (1 - p))] \end{aligned} \right] \quad (81)$$

$$V_2 = \rho \left[ \begin{aligned} &(1 - x^H) \left[ \begin{aligned} &f_2 + \beta f_2 (1 - f_1) (1 - p) + f_1 \beta (1 + \beta) \\ &- f_2 \beta^2 (f_1 - p (1 - f_1) + p^2 (1 - f_1)) \end{aligned} \right] \\ &+ (x^H - x^L) [f_2 + f_1 \beta + f_2 \beta (1 - f_1) (1 - p) + f_1 \beta^2 (1 - f_2 - p)] \end{aligned} \right] \quad (82)$$

For the equilibrium to exist two conditions must be met: (i) it must be optimal for the firm to retain a poorly matched worker off-cycle, and (ii) it must be optimal for the firm to release a poorly matched worker on-cycle. The first condition requires

$$1 - x^H + \beta V_1 \geq V_2.$$

Substituting from the solutions above gives

$$\frac{x^H - x^L}{1 - x^H} \leq \bar{\alpha} \equiv \frac{1 - (1 - f_1)(1 - f_2)(1 - p)^2 \beta^2}{f_2(1 + (1 - f_1)(1 - p)\beta)} - 1.$$

If match quality is not too important, it is optimal for the firm to retain a poorly matched worker at the end of period 1. The second condition requires

$$V_1 \geq (1 - x^H) (1 + \beta) + \beta^2 V_1,$$

which is satisfied if and only if match-quality is sufficiently important:

$$\frac{x^H - x^L}{1 - x^H} \geq \alpha \equiv \left( \frac{1 - f_1}{f_1 + (1 - f_1)\beta f_2} \right) (1 + \beta(1 - f_2))(1 - \beta(1 - p)).$$

Thus, cycles can be sustained in equilibrium if and only if

$$\alpha \leq \frac{x^H - x^L}{1 - x^H} \leq \bar{\alpha}.$$

■

**Proof of Lemma 6.** Using equations (47) and (48), the difference  $\Delta f = f_1 - f_2$  is given by

$$\Delta f = \frac{\mu^2(M - N)N\phi(1 - (1 - p)^2)}{[M - N(1 - \phi(1 - (1 - p)^2))][M - N(1 - \phi(1 - \mu)(1 - (1 - p)^2))]} > 0.$$

Differentiating with respect to  $\mu$  gives

$$\begin{aligned} \frac{d\Delta f}{d\mu} &= (M - N)N(2 - p)p\mu \\ &\times \left[ \frac{2(M - N)^2\phi + N^2(2 - p)^2p^2(2 - \mu)\phi^3 + (M - N)^2\mu\phi' + N(2 - p)p\phi^2((M - N)(4 - \mu) - N(2 - p)p(1 - \mu)\mu\phi')}{[M - N(1 - \phi(1 - (1 - p)^2))]^2[M - N(1 - \phi(1 - \mu)(1 - (1 - p)^2))]^2} \right] > 0, \end{aligned}$$

where

$$\phi' = \frac{-(2(1 - \mu) - p)(1 - p)}{(1 - (1 - \mu)(1 - p)(1 - p - \mu))^2}.$$

Thus,  $\Delta f$  is increasing in  $\mu$  for  $\mu \in (0, 1]$ . ■

**Proof of Lemma 7.** Using equations (60) and (61), the difference  $\Delta f = f_1 - f_2$  is given by

$$\Delta f = \frac{(M - N)N(1 - \delta)[1 - (1 - p)(1 - p(1 - \delta))]}{[N(1 - (1 - p)(1 - \delta)(1 - p(1 - \delta))) + M - N][N\delta + M - N]} > 0.$$

Differentiating with respect to  $\delta$  gives

$$\begin{aligned} \frac{d\Delta f}{d\delta} &= -(M - N)Np \\ &\times \left[ \frac{M^2(3 - 2p(1 - \delta) - 2\delta) - 2MN(1 - p)(1 - \delta)^2 - N^2(1 - p)(1 - \delta)^2(1 + p(3 - p(1 - \delta) - \delta))}{[N(1 - (1 - p)(1 - \delta)(1 - p(1 - \delta))) + M - N]^2[N\delta + M - N]^2} \right] < 0. \end{aligned}$$

Thus,  $\Delta f$  is decreasing in  $\delta$  for  $\delta \in [0, 1)$ . ■

## 13 Tables and figures

Employee Rank	Tier	Bonus	Total Compensation	Bonus Ratio
First-year analyst	1	\$65,000	\$150,000	43%
	2	\$55,000	\$140,000	39%
	3	\$45,000	\$130,000	35%
Second-year analyst	1	\$75,000	\$165,000	45%
	2	\$65,000	\$155,000	42%
	3	\$55,000	\$145,000	38%
Third-year analyst	1	\$95,000	\$190,000	50%
	2	\$85,000	\$180,000	47%
	3	\$75,000	\$170,000	44%

Table 1: Banker pay varies by rank and performance. In 2016, modal base pay was \$85,000, \$90,000, and \$95,000 for first, second, and third year analysts, respectively, while bonuses were as high as \$65,000, \$75,000, and \$95,000. A majority of each level's bonus is implicitly guaranteed, as even relatively poor performers still expect bonuses at least 70% as large as the best performers.

Bank	Headquarters	Bonuses Announced	Bonuses Paid
Bank of America Merrill Lynch	USA	End January	End February
Citigroup	USA	Mid January	End January
Goldman Sachs	USA	Early January	January/February payday
Jeffries	USA	Early January	Early February
JP Morgan Chase	USA	Mid January	End January
Morgan Stanley	USA	End January	End February
CIBC World Markets	Canada	Early December	Late December
Fortis	Canada	End February	April
Royal Bank of Canada	Canada	Early December	Mid December
Barclays	Europe	End January	End February
BNP Paribas	Europe	Early March	End March
Commerzbank	Europe	Early March	Mid March
Credit Agricole CIB	Europe	End February	End March
Credit Suisse	Europe	End January	Mid February
Deutsche Bank	Europe	Early March	End March
HSBC Investment Bank	Europe	Mid March	End March
ING Wholesale Banking	Europe	Early March	Mid March
Rabobank	Europe	End February	Mid March
Rothschild	Europe	May	June
Royal Bank of Scotland	Europe	End February	March-June
SG Corporate and Investment Banking	Europe	End January	End March
Standard Chartered	Europe	Early March	End March
UBS Investment Bank	Europe	Mid February	End February
Daiwa Securities	Australasia	Mid April	Early May
Nomura International	Australasia	Early May	End May
Macquarie	Australasia	Early May	End May

Table 2: This table summarizes the timing of bonus determination and bonus payment for major global investment banks, as compiled by the financial newspaper HITEC in 2017. All US banks determine bonuses in January. Canadian banks determine bonuses in early December, with the exception of Fortis. European banks determine bonuses in late February and early March, with the exceptions of Rothschild, Barclay's, and Credit Suisse. Australian and Asian banks determine bonuses from mid-April to early May.



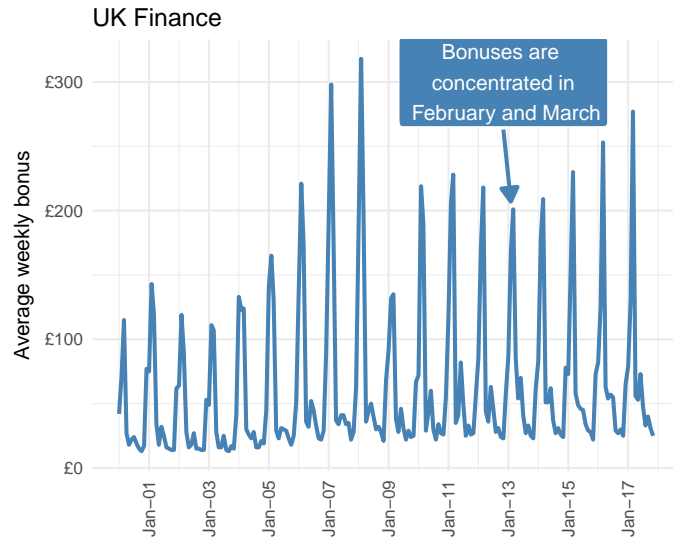


Figure 1: This figure plots average weekly bonuses by month from January 2000 through November 2017 in the United Kingdom for the Finance and Business Services industry. Bonus payments peak in February and March. Note that this plot is an average over all workers in these industries. The typical bonuses for the subset in high finance would be much larger.

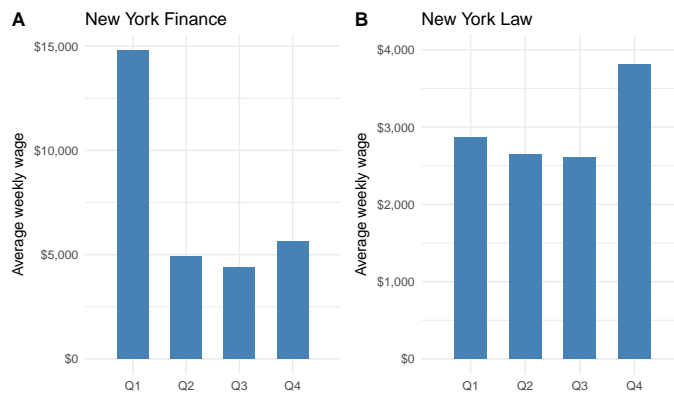


Figure 2: This figure plots average weekly wages by quarter in New York, since 2013. Panel A plots the data for the high finance and Panel B plots the data for law. Total compensation is substantially higher in the first quarter in finance, whereas it is considerably higher in the fourth quarter in law. Assuming that much of the variation is due to bonuses, this suggests differential timing of bonus payments for different industries in the same geography.

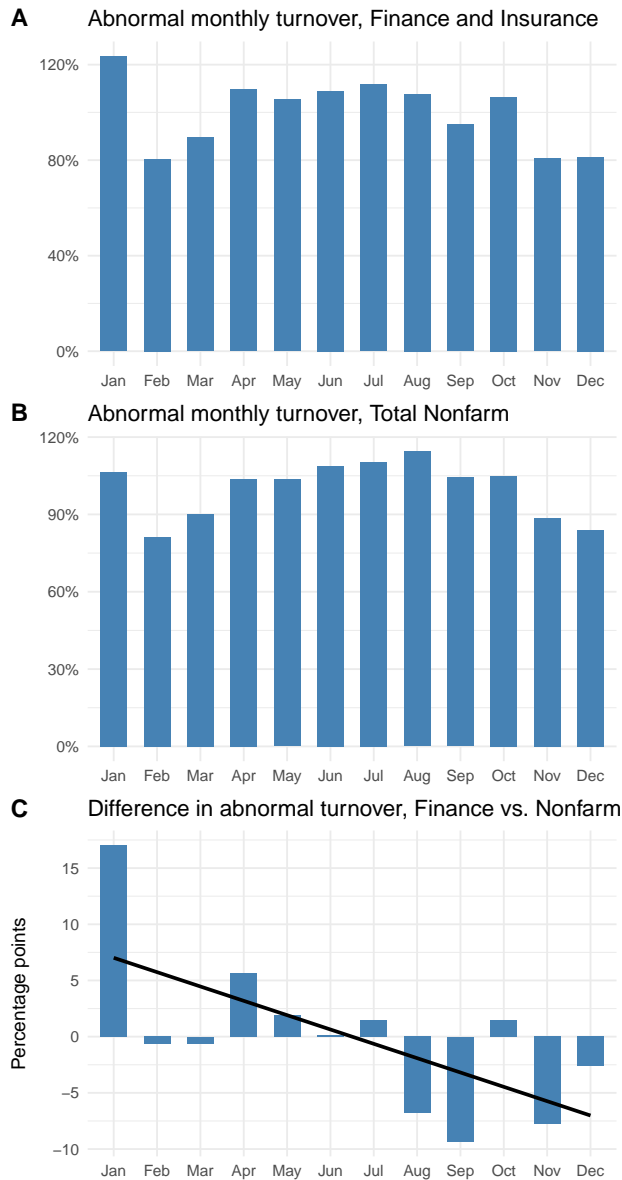


Figure 3: Panels A and B plot abnormal turnover by month from December 2000 to August 2017. In Finance and Insurance, turnover (average of separations and hiring) is 23% higher in January than in an average month. For all nonfarm industries, turnover is higher in January by only 6%. Turnover for all industries peaks in August, as summer workers return to school. Panel C provides the difference in abnormal turnover in finance relative to all nonfarm industries. Turnover in January is 17 percentage points higher in Finance than in the US as a whole, while over the last five months of the year turnover in finance is, on average, 5 percentage points lower than in all nonfarm industries.

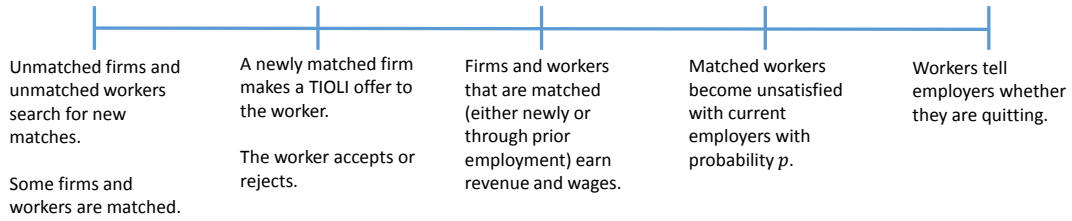


Figure 4: Timeline within a period  $t$ .  $M_t^0$  firms and  $N_t^0$  workers begin the period unmatched and search for new matches.  $\mu \min\{N_t^0, M_t^0\}$  matches are made. A newly matched firm makes a take-it-or-leave-it offer to the worker, whose outside option is to receive 0 at time  $t$  and re-enter the matching market at time  $t + 1$ . Matched firms earn revenue 1 and pay workers based on their previously agreed-upon contract. After revenue and pay are earned, matched workers become dissatisfied with their current employers with probability  $p$ . Whether satisfied or not, workers may quit. If a firm's worker quits, both the firm and the worker are unmatched to start period  $t + 1$ . If a firm's worker does not quit, then both skip the matching and contracting market in period  $t + 1$ .

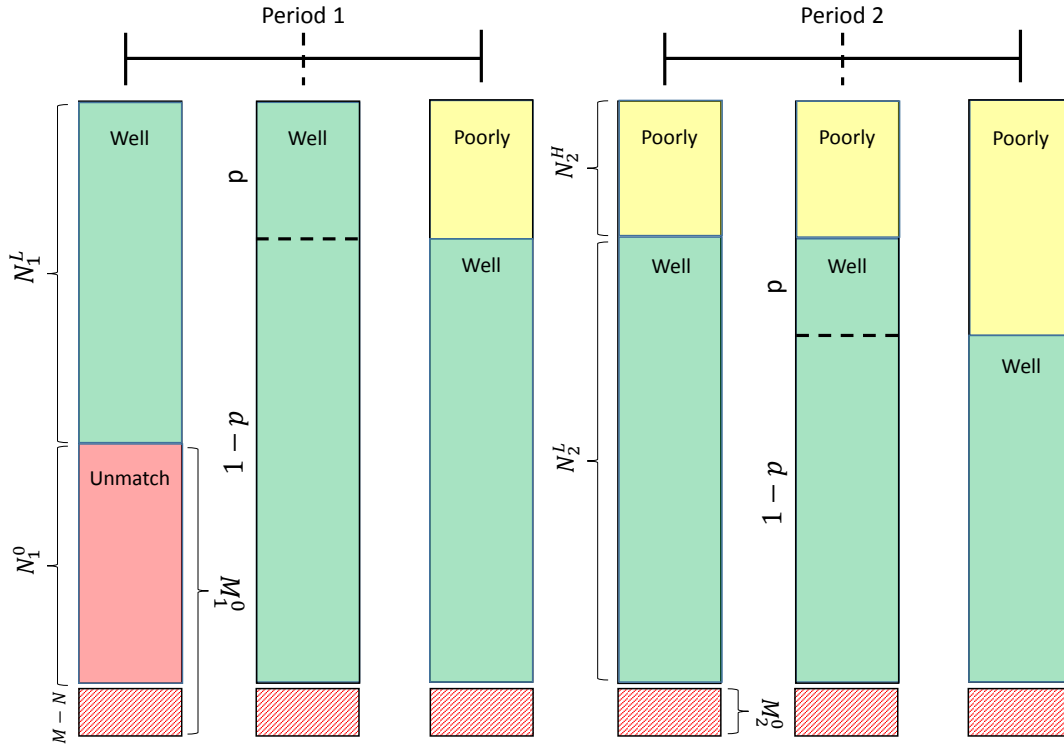


Figure 5: Worker flows in the  $T = 2$ ,  $\mu = 1$  base model. At the start of odd periods, workers are either well matched or searching for work, as those who were poorly matched at the end of the preceding period quit. The mass of unmatched firms exceeds the mass of unmatched workers by  $M - N$ . Because  $\mu = 1$ , all unmatched workers match well and  $M - N$  firms remain unmatched. Then, a share  $p$  of workers become dissatisfied and are poorly matched at the end of the period. Because, in equilibrium, these workers do not quit, they are poorly matched at the start of the following even period. Of the workers who are well matched at the start of the even period, a share  $p$  become poorly matched. All workers who are poorly matched at the end of even periods become unmatched for the start of the following odd period.

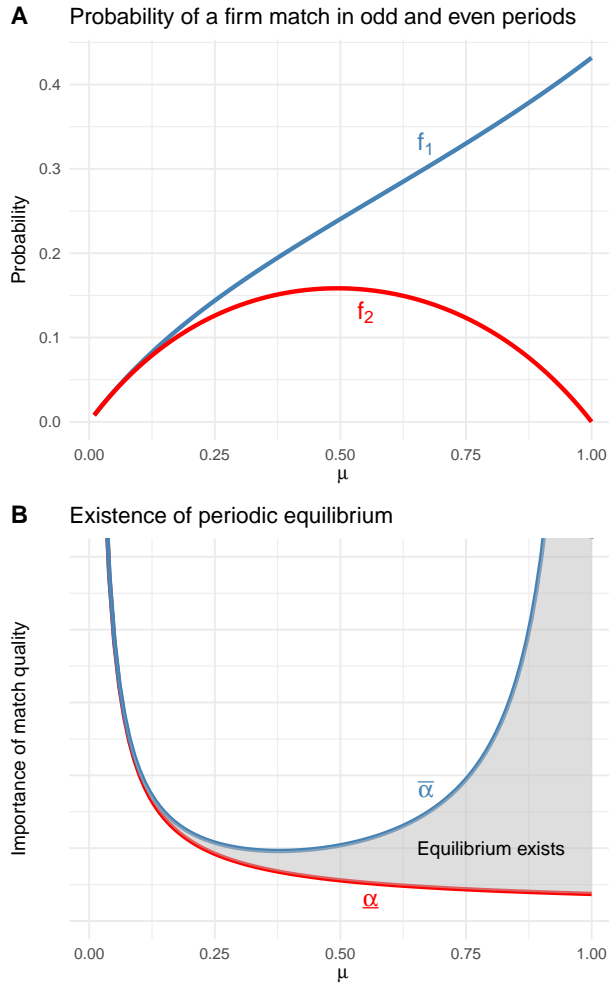


Figure 6: Panel A plots the probability that an unmatched firm matches with an unmatched worker in odd ( $f_1$ ) and even ( $f_2$ ) periods as a function of the efficiency of matching,  $\mu$ . When matching is more efficient, the difference between  $f_1$  and  $f_2$  is larger, implying that the labor market experiences stronger cyclical variation. Panel B plots the thresholds  $\bar{\alpha}$  and  $\underline{\alpha}$  also as a function of  $\mu$ . Since the labor market is more cyclical when matching is more efficient, a periodic equilibrium exists for a larger range of parameters when  $\mu$  is larger. Parameters used to generate these plots are  $M = 1, N = 0.8, p = 0.1, \beta = 0.95$ .

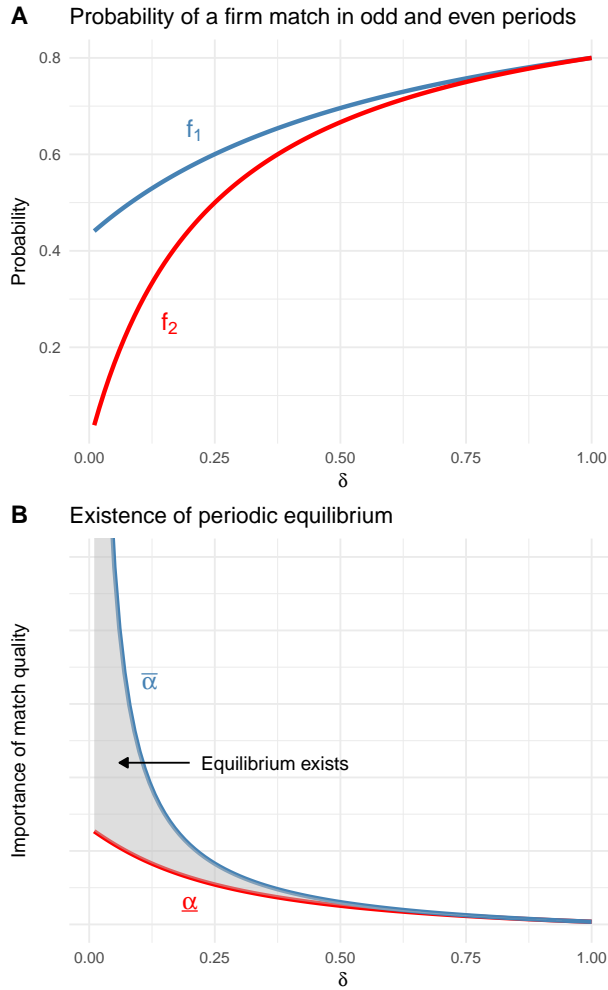


Figure 7: Panel A plots the probability that an unmatched firm matches with an unmatched worker in odd ( $f_1$ ) and even ( $f_2$ ) periods as a function of the likelihood of a firm death,  $\delta$ . When the likelihood of a firm dying in a period decreases, the difference between  $f_1$  and  $f_2$  is larger, implying that the labor market experiences stronger cyclical variation. Panel B plots the thresholds  $\bar{\alpha}$  and  $\underline{\alpha}$  also as a function of  $\delta$ . Since the labor market is more cyclical when the likelihood of a firm death is smaller, a periodic equilibrium exists for a larger range of parameters when  $\delta$  is smaller. Parameters used to generate these plots are  $M = 1, N = 0.8, p = 0.1, \beta = 0.95$ .