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The Cash-Cap Model: A Two-State Model of Firm Dynamics

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Abstract

How does a firm's optimal cash, issuance, default, payout, and investment policies vary over its life cycle? To answer this question, we relax the assumption of homogeneity in size in models of firm dynamics. Our Cash-Cap model has two state variables: cash and capital. We find support in the data for new predictions: (1) issuance-to-capital ratios decrease convexly in capital, (2) payout-to-capital ratios are concave in capital, (3) investment-to-cash holdings sensitivities decrease in capital, and (4) cash holdings-to-volatility sensitivities increase in capital. We prove the uniqueness of the model solution and convergence of the numeric algorithm.

JEL classification: E22, G12, G32, G33, G35

Keywords: Cash holdings, payouts, investment, issuance, default, liquidity management, risk management, viscosity solutions

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1. Introduction

Understanding a firm's demand for cash has captured the attention of economists for decades (Baumol, 1952; Miller and Orr, 1966; Whalen, 1966). Mulligan (1997) documents significant cross-sectional variation in cash holdings across small and large firms, and Bates et al. (2009) document significant increases in cash balances over time. Existing models of the firm have difficulty explaining the cross-sectional distribution of cash holdings, which is important for thinking about such questions as the cross-section of financial constraints (Kaplan and Zingales, 1997) and monetary policy transmission (Cole and Ohanian, 2002; Adão and Silva, 2020). Part of the challenge is that a firm's policy for cash holdings is closely intertwined with its policies for investment, issuance, default, and payout in the presence of costly financing and bankruptcy; for instance, investment rates depend on the availability of internal funds, the costs of external funds, a firm's solvency, and the marginal returns to investing. While existing models of firm dynamics have begun to examine many of these firm decisions together (e.g., Bolton et al., 2011), these models generally employ dimensionality reduction techniques, assuming homogeneity in size, so that a firm's decision problem depends on the cash-to-capital ratio. However, over a firm's life cycle, a firm's optimal cash and capital positions are unlikely to evolve in tandem for several reasons. As firms mature, the marginal returns to investing may diminish (e.g., Caballero, 1991), and the intensity of financing frictions may decrease (e.g., Altınkılıç and Hansen, 2000).

In this paper, we relax the typical homogeneity assumption by solving a dynamic model of the firm with two state variables—cash and capital. Our new "Cash-Cap" model highlights that a firm's optimal policies are a rich function of both a firm's physical capital and cash positions. By allowing for differences in firm size, our model can incorporate leverage and costly default, external costs of financing that decrease relative to firm size, and diminishing returns to scale. We provide empirical support for four novel predictions about payouts, issuances, investments, and cash holdings.

The model firm generates cash flows from production using its stock of capital. Cash flows are stochastic, and production exhibits diminishing returns to scale. Capital depreciates, and investment is irreversible and subject to convex adjustment costs. Cash held in the firm carries a liquidity premium and hence earns a low rate of return. The firm is financed by equity and long-term debt. We model debt as a consol bond with a constant coupon rate and principal. We model a fixed component of issuance costs (that does not scale with capital) and a proportional component (which scales with issuance size). Thus, issuance costs are relatively higher when firms are smaller, and issuance costs scaled by proceeds decline in proceeds. The firm weighs the costs of holding cash against the flexibility that cash offers in funding investment and meeting liabilities without having to pay the cost of raising new equity. At default, equityholders liquidate the capital in a fire sale and apply the proceeds to the debt claim, with any remaining proceeds paid out as a dividend.

We characterize the firm's optimal value via a dynamic programming equation with two state variables: capital k and cash c. We prove that the value function of the firm is the unique solution to this equation. We numerically solve this equation with a reasonable set of parameters and prove that the numerical algorithm converges to the unique solution of this dynamic programming equation (the numeric solution converges to the firm's value function) by proving, in the appendix, a comparison theorem for (discontinuous) viscosity solutions to the Hamilton–Jacobi–Bellman (HJB) equation. We present the solution in a series of simple yet insightful figures.

Our solution shows that the state space (k, c) is partitioned into several regions with different strategies: a dividend region, an investment region, a continuation region, and an issuance or default region. When the cash reserve is sufficiently high, the firm pays out a dividend. The firm continually compares the best value of the firm with issuance and without issuance and chooses to issue the optimal amount if the former is greater. Issuances are discrete and lumpy because of the fixed component of issuance costs. We prove that the firm only issues equity when the cash reserve is depleted, which may occur because of negative cash flow shocks or investment. The firm liquidates the first time it runs out of cash and chooses not to raise equity. Between the payout, issuance, and default regions, the firm invests or retains its earnings.

In the data, we find support for new predictions about issuance size, payout amounts, investment rates, and cash holdings.

First, the amount a firm optimally issues scaled by capital is declining and convex in capital. Intuitively, when capital is low, the cash flows are low. Thus, a firm relies more on the cash reserve to invest in the capital (which has high marginal returns when k is low) and pay the bond coupon. This higher reliance on the cash reserve incentivizes a larger issuance size. Also, a larger issue size when capital is low economizes over the fixed component of issue costs, which constitutes a larger proportion of firm value when capital is low. Characterizing a firm's optimal issuance amount complements prior work on trade-off theory (Hovakimian et al., 2001), market timing (Chang et al., 2006), and issuance costs (Smith, 1977; Lee et al., 1996; Chen and Ritter, 2000; Altınkılıç and Hansen, 2000; Butler et al., 2005; Gomes, 2001; Hennessy and Whited, 2007). The extant papers generally omit controls for a firm's relative capital and cash positions prior to issuance and the convexity of the issuance-to-capital ratio. In the Internet Appendix, we examine the results in Chang et al. (2006) and Hovakimian et al. (2001) and show that the predicted convexity holds when accounting for their covariates and improves their explanatory power (adjusted R^2) by 50%-80%.[1]

Second, the optimal payout amount scaled by capital is concave in capital. The marginal value of cash is high when a firm is small, which discourages payouts, because cash flows are low, default risk is high, investment incentives are strong, and issuance costs are high relative to firm value. As capital increases, the larger expected cash flows substitute for the role of the cash reserve in funding investments and meeting liabilities, which reduces the marginal value of cash and thus frees up the cash reserve for payouts. Additionally, the incentives to invest decline in capital because of diminishing returns to scale. Contributing to the concavity is diminishing returns to scale, because incremental cash flows scaled by capital are declining in capital. Also, the nominal size of the volatility shock continues to grow with capital, which increases the risk of paying the issuance costs. This prediction is important for work examining how firms adjust their payout policies to manage cash in response to the costs and benefits of internal and external financing sources. Recent empirical studies examining payouts do not account for the concavity of the payout-to-capital ratio (Becker et al., 2011, 2013; Desai and Jin, 2011; Hail et al., 2014; Hanlon and Hoopes, 2014; Bliss et al., 2015; Kumar and Vergara-Alert, 2020). In Internet Appendix Table D.12, we show for the main specification in Bliss et al. (2015) that our predicted concavity holds and is statistically significant despite a large battery of controls. Additionally, controlling for a firm's cash and capital positions increases the specification's adjusted-R^2 by 15% and the magnitude of interest by 46%.

Third, the sensitivity of investment to cash holdings is declining in capital. In our model, when a firm is larger, a firm's investment opportunities are relatively less attractive and the marginal value of cash is lower because issuance costs relative to firm size are declining in capital and default risk is declining in capital. Consequently, investment is predicted to be less sensitive to the cash-to-capital ratio when a firm is larger. Interest in how financial constraints distort investment goes back to Myers and Majluf (1984). Bolton

[1]Internet Appendix Tables D.8, D.9, and D.10 include a discussion of the results in Chang et al. (2006) and Hovakimian et al. (2001).

et al. (2011) predicts that investment is increasing with cash holdings in the presence of costly financing. The Cash-Cap model contributes by characterizing how the sensitivity of investment to cash holdings varies with a firm's size. Prior empirical work finds that investment varies with cash holdings, comparing level differences in investment and cash across firms in the same industry (Opler et al., 1999; Denis and Sibilkov, 2009).[2] Our empirical contribution is to show that year-to-year fluctuations in a firm's cash reserves are associated with that same firm's investment rates and that the sensitivity of investment to cash is higher when a firm is smaller.

Fourth, when a firm is larger, the firm's cash holdings are more sensitive to volatility. While Bolton et al. (2011) predicts that increases in volatility lead to larger cash reserves, the Cash-Cap model predicts that the sensitivity of cash holdings to volatility is larger when a firm is larger. Intuitively, when a firm is smaller, the marginal value of cash is already high and thus increasing volatility changes the firm's marginal value of cash less. The extant empirical literature provides cross-sectional evidence that firms facing more volatility hold more cash (e.g., Opler et al., 1999; Almeida et al., 2004; Palazzo, 2012). Few papers examine changes in cash holdings and changes in volatility. Duchin (2010) finds that more diversifying acquisitions lead to lower cash holdings. Bates et al. (2009) finds that cash holdings increase more over time for industries experiencing larger increases in idiosyncratic volatility. There is scant work on how the sensitivity of cash holdings to volatility varies with firm size. Pinkowitz and Williamson (2006) and Song and Lee (2012) provide some evidence that large firms increase their cash reserves more after a financial crisis; however, firms may exit the sample during a crisis and government intervention may differentially affect small and large firms. Our empirical contribution is to show that the sensitivity of a firm's cash holdings to changes in both firm-level volatility (measured using return volatility, cash flow volatility, and implied volatility from options) and the VIX is larger when the firm is larger.

The Cash-Cap model extends the dynamic liquidity management literature in several ways. Our paper extends the seminal Bolton et al. (2011) (BCW) by relaxing their homogeneity assumption, which assumes that all decisions depend only on a firm's cash-to-capital ratio. By relaxing this assumption, we allow a firm's policies to vary with

[2] While there is a large literature on investment and cash flows (Fazzari et al., 1987; Kaplan and Zingales, 1997), cash holdings are functionally different from cash flows. Cash holdings help firms hedge against cash flow volatility, relaxing the impact of low cash flow realizations on investment and other firm outcomes. Also, while there is a literature on investment and financing costs (Minton and Schrand, 1999; Harford et al., 2014), these papers do not characterize the investment-to-cash holdings sensitivity.

its size. Our framework allows for additional realistic assumptions such as leverage, declining returns to scale, and issuance costs that decline in intensity with size. We generate several novel predictions about how a firm's policies vary with its size, which we find support for in the data. Beyond BCW, most other models examining a firm's cash policy assume that cash-flow shocks are independent and identically distributed through time, which implies that a firm's solvency — or economic health — is constant through time.[3] In our model, firm cash flows depend on the dynamic capital stock. Gomes (2001) and Hennessy and Whited (2005, 2007) numerically solve discrete-time dynamic capital structure models with investment for financially constrained firms. They allow for stochastic investment opportunities; however, they do not include adjustment costs for investment that would spread out investment over time and do not model cash accumulation. Gamba and Triantis (2008) model cash but ignore adjustments costs and costly issuance. Anderson and Carverhill (2012) model cash and debt but do not consider issuance and investment. In contrast, for our model firm, cash and capital continually change, and the firm may raise equity or default depending on the level of capital and cash in the firm. The capital state variable also gives rise to a rich cash policy. Dai et al. (2021) extend BCW to a two-divisional firm with the relative size of the divisions and the cash-to-capital ratio as state variables. Our paper differs in that a firm's policies vary with the firm's size.

We contribute to the broader theoretical corporate finance literature in a few ways. First, most models of risky debt assume that a firm defaults when the market value of its assets falls below the face value of debt or when the option value of equity is not high enough to keep the firm alive (e.g., Black and Cox (1976), Leland (1994), Longstaff and Schwartz (1995)). In most such models, there is no cash balance, and if the firm's cash flow is insufficient for debt service while the asset value is still high enough, equityholders can contribute additional funds at no cost. As a result, these models cannot speak to how financial distress interacts with issuance costs to define a firm's liquidity management. This interaction may be important as approximately 10% of firms in default are solvent (market value of assets exceeding the face value of liabilities), and these firms face high external financing costs (Davydenko, 2013). In our model, as external financing costs increase, firms with larger capital stocks (more solvent firms) begin to find defaulting more optimal than paying the issuance costs to keep the firm running. Second, models

[3] See Girgis (1968), Riddick and Whited (2009), Bolton et al. (2011, 2014), Décamps et al. (2011), He and Milbradt (2014), and Hugonnier et al. (2015).

in the contingent claims literature generally assume that default occurs due to severe negative shocks to the market value of a firm's assets. There are no frictions to issuing equity and so no role for cash in these models.[4] More work on default and cash holdings is warranted since about 13% of defaulting firms are insolvent but have more than enough cash to cover their current liabilities (Davydenko, 2013). In our model, low capital firms may choose to default even with positive cash if costly financing reduces the continuation value of the firm enough relative to liquidation.

Lastly, our model exhibits behavior of particular theoretical and mathematical interest. First, costly financing and default lead to a multi-band strategy for payouts, for which an optimal policy cycles on and off two times. Specifically, for certain levels of capital, the firm initially does not pay dividends for small cash levels c, and then, as c increases, it crosses into the dividend-payout region. As c increases further, the dividend payout region is again exited and once again entered for higher c, creating the second band. Second, we prove that equity issuance is put off until cash equals zero (the very last moment), which occurs because of negative cash flow shocks or investment. Finally, we provide a comparison theorem for (discontinuous) viscosity solutions to the HJB equation. Such results are powerful and also rare, with the impulse control structure coming from equity issuance. The comparison theorem ensures that the HJB equation has a unique solution and that the numerical method indeed converges to the value function.

2. Model

The firm's cash flows are stochastic and are a function of its capital stock and a shock. We assume that the shock evolves according to

$$dZ_t = \mu dt + \sigma dW_t, \tag{1}$$

where W is a one-dimensional Brownian motion under the risk-neutral measure and μ and σ are positive constants. Thus, shocks dZ_t are assumed to be i.i.d. with mean μdt and variance $\sigma^2 dt$. The firm's cumulative cash flows Y follow the dynamics

$$dY_t = k_t^\alpha dZ_t, \tag{2}$$

[4] See Merton (1974), Leland (1994), Leland and Toft (1996), Longstaff and Schwartz (1995), Goldstein et al. (2001), and Collin-Dufresne and Goldstein (2001).

where k is the size of the firm's capital stock and $\alpha \in (0,1)$ is a scale parameter following Bertola and Caballero (1994).[5] Therefore, production exhibits decreasing returns to scale, which is the first difference between our model and that of Bolton et al. (2011) (BCW), who consider a constant returns-to-scale technology.

The firm can invest in the capital stock. As is standard in capital accumulation models, for an investment process i, the dynamics of the capital stock follows

$$dk_t = \Big(i_t - \delta k_t\Big)dt, \tag{3}$$

where $\delta \geq 0$ is the depreciation rate. We assume that investment is irreversible, i.e., $i \geq 0$. Similar to BCW, investment is subject to a convex adjustment cost

$$g(k,i) = \frac{\theta}{2}\left(\frac{i}{k}\right)^2 k, \tag{4}$$

for a positive constant θ that measures the degree of the adjustment cost.

The firm is financed by equity and long-term debt. As is common in the literature, we model debt as a fixed consol bond (e.g., Hennessy, 2004; Manso, 2008). The firm pays a coupon at a constant rate b. Modeling debt as a fixed consol bond is a simple way to vary a firm's solvency. Setting $b \geq 0$ allows us to study the impact of debt on a firm's payout, issuance, investment, and cash holdings decisions. Extending the model to allow a firm to retire or raise debt would likely amplify the four novel predictions.[6] The modeling of debt is the second difference with BCW, who assume $b = 0$ and thus do not consider default and leverage.

The firm determines its investment and cash management strategies, which include when to pay a dividend and when to raise equity. The value of the cash reserve follows

[5]Our model can accommodate $\alpha = 1$ or $\alpha > 1$; however, we find strong support in the data for the model's predictions with regard to investment when assuming $\alpha < 1$. Diminishing returns to scale is quite common in the literature. See Caballero (1991), Basu and Fernald (1997), Gomes (2001), and Grullon and Ikenberry (2021). Also, empirically, the investment-to-depreciation ratio is declining and convex in a firm's capital, consistent with diminishing returns to scale (See Internet Appendix D.5).

[6]Issuance scaled by capital may exhibit more convexity because cash raised can also be used to pay down debt, which is particularly valuable when a firm is smaller. The payout boundary may become more concave because the marginal value of cash is higher when a firm is small and that cash can also be used to pay down debt. The sensitivity of investment to liquidity is also likely to be greater when a firm is smaller because investment competes with alternative uses of cash, which would include paying down debt. Lastly, when firms are smaller, their cash holdings likely respond even less to changes in volatility because the marginal value of cash is even higher when the cash can retire debt.

the dynamics

$$dc_t = (r - \lambda_c)c_t dt + dY_t - bdt - i_t dt - g(k_t, i_t)dt - dD_t + dI_t. \tag{5}$$

Here, r is the interest rate, λ_c is the cash holding cost (liquidity premium), D is the cumulative dividend payout, and I is the cumulative equity issuance. Both D and I are non-decreasing processes. Cash earns a return equal to the risk free rate (r) net of a carry cost of holding cash (λ_c).[7] Even though cash earns a lower rate of return, the firm holds cash for precautionary reasons to lower the expected issuance or default costs if it runs out of liquid funds. The firm manages an optimal cash policy to trade off the risk management benefits of maintaining a cash reserve against the delay in dividend payouts.

Equity issuance is costly. For a lump sum issuance of size I, the cost is

$$\lambda(I) = \lambda_f + \lambda_p I, \tag{6}$$

where λ_f and λ_p are constants representing the constant component and the proportional components of issuance costs, respectively. The fixed component of issuance costs is the third modelling difference with BCW. In BCW, the "fixed" issuance cost λ_f is proportional to the capital size, resulting in a constant ratio between the fixed issuance cost and capital (λ_c/k). In our model, this ratio is decreasing in capital because the fixed costs associated with issuance do not scale with capital.[8]

Even if a firm neither pays out cash nor invests, its cash reserve can run out due to negative productivity shocks. When this happens, the firm compares the benefit of equity issuance and continuing (continuation value) with the residual value for equityholders after liquidation and applying proceeds first to paying off the debt (liquidation value). If

[7]This assumption is standard in models with cash. For example, see Bolton et al. (2011, 2019), Kim et al. (1998) and Riddick and Whited (2009). If $\lambda_c = 0$, then the firm finds it optimal to hold as much cash as it can (indefinitely postponing the dividend) to prevent costly equity issuance. The equity is still valuable because equityholders could always choose to extract the cash via a dividend. The more realistic case is when $\lambda_c > 0$. Cash may earn low returns because interest earned on a firm's cash holdings is taxed at the corporate tax rate, which generally exceeds the personal tax rate (Graham, 2000; Faulkender and Wang, 2006). Agency problems may lower cash returns (Jensen, 1986; Harford, 1999; Dittmar and Shivdasani, 2003; Pinkowitz et al., 2006; Dittmar and Mahrt-Smith, 2007; Harford et al., 2008; Caprio et al., 2011; Gao et al., 2013). For agency with private information see for example Demarzo et al. (2012) and Ward and Ying (2022).

[8]BCW scale fixed issuance costs by capital for their variable reduction method for tractability. Nevertheless, they acknowledge, "In practice, external costs of financing scaled by firm size are likely to decrease with firm size." Prior work finds declining costs relative to proceeds and size (Lee et al., 1996; Altınkılıç and Hansen, 2000).

the latter outweighs the former, the firm defaults. Therefore, the default time of the firm is

$$\tau = \inf\{t \geq 0 : c_t < 0\}.$$

When the firm defaults, its capital stock k_τ is fire sold. The recovery rate ℓ is assumed to be constant. The liquidation value ℓk_τ is used to pay off the long-term debt with the face value b/r_{debt}, where r_{debt} is the cost of financing for long-term debt. If there is any value after paying the long-term debtholder, the remaining value, $\left(\ell k_\tau - b/r_{\text{debt}}\right)_+$, is distributed to the equityholders.[9]

2.1. The firm's problem

The firm chooses investment, dividend payout, equity issuance, and default to maximize the present value of dividend payouts net of equity issuance costs:

$$\sup_{i\geq 0, D, \{\sigma_j, I_j\}} \mathbb{E}\left[\int_0^\tau e^{-rs} dD_s - \sum_j e^{-r\sigma_j}\left(I_j + \lambda(I_j)\right) + 1_{\{\tau<\infty\}} e^{-r\tau}\left(\ell k_\tau - b/r_{\text{debt}}\right)_+\right], \quad (7)$$

where $\{\sigma_j\}$ is a sequence of stopping times when the lump sum of equity of size I_j is issued at each σ_j.[10]

The capital stock size and the cash reserve value, k and c, are the two state variables for the firm's problem. The firm's value function is

$$\begin{aligned} V(k_t, c_t) = \sup_{i\geq 0, D, \{\sigma_j, I_j\}} \mathbb{E}_t \Bigg[\int_t^\tau e^{-r(s-t)} dD_s - \sum_{\sigma_j \geq t} e^{-r(\sigma_j - t)}\left(I_j + \lambda(I_j)\right) \\ + 1_{\{\tau<\infty\}} e^{-r(\tau-t)} (\ell k_\tau - b/r_{\text{debt}})_+ \Bigg]. \end{aligned} \quad (8)$$

It follows from the dynamic programming that the value function V satisfies the HJB equation:

$$\begin{aligned} 0 = \min \Bigg\{ rV - \sup_{i \geq 0} \left\{ \left[i - \delta k\right] \partial_k V + \left[(r - \lambda_c)c + k^\alpha \mu - b - i - g(k,i)\right] \partial_c V + \frac{1}{2} k^{2\alpha} \sigma^2 \partial^2_{cc} V \right\}, \\ \partial_c V - 1, V(k,c) - \sup_{I\geq 0}\left[V(k, c+I) - I - \lambda(I)\right] \Bigg\}. \end{aligned} \quad (9)$$

In the equation above, the firm chooses among three alternatives: investment (the group

[9] $a_+ = \max\{a, 0\}$.

[10] Note that because there is no information asymmetry between existing and new investors, one can simply think of the problem through the lens of one representative investor.

of terms on the first line of the right-hand side of the equation), dividend payout (the first group of terms on the second line), and equity issuance (the second group on the second line).[11]

Regarding the investment alternative, rV represents the required rate of return on equity, which equals the risk free rate demanded by risk-neutral investors. The term $\partial_k V$ is a firm's marginal benefit of capital; hence, $[i - \delta k]\partial_k V$ captures the marginal effect of net investment on equity value. The term $\partial_c V$ is firm's marginal cost of cash; hence, $[(r - \lambda_c)c + k^\alpha \mu - b - i - g(k,i)]\partial_c V$ is the effect of a firm's expected savings on equity value. The term $\frac{1}{2}k^{2\alpha}\sigma^2\partial^2_{cc}V$ captures the effect of the volatility of cash holdings due to volatility in production on equity value.

To determine the optimal investment strategy, note that the first order condition of i in Equation (9) and the constraint $i \geq 0$ yields

$$\partial_k V - \left[1 + \theta \frac{i^*}{k}\right]\partial_c V \leq 0 \tag{10}$$

for the optimal investment i^*. This inequality is an equality when $i^* > 0$. Equation (10) shows that $i^* > 0$ if and only if $\partial_c V < \partial_k V$, i.e., when the marginal benefit of capital is larger than the marginal cost of cash. Moreover, $i^* > \delta k$ if and only if $1 + \theta\delta < \frac{\partial_k V}{\partial_c V}$. That is, when the ratio between the marginal benefit of capital and the marginal cost of cash is sufficiently high, the firm invests more than depreciation.

Regarding the dividend payout term, the firm postpones dividend payout until the marginal cost of reducing the cash reserve matches the marginal benefit of dividend payout, i.e., $\partial_c V = 1$.

Regarding the issuance term, at each point (k,c) in the state space, the equityholders compare the value of the firm without issuance $V(k,c)$ to the best value for issuance $\sup_{I\geq 0}\left[V(k,c+I) - I - \lambda(I)\right]$, where $V(k,c+I) - I - \lambda(I)$ is the firm value post issuance net of issuance costs. The firm only issues equity when the latter value is strictly larger.

The dynamic programming principal implies that all three groups are nonnegative, that only one group equals zero at each point (k,c) in the state space, and that the corresponding action is optimal for the firm.

The HJB Equation (9) is coupled with several boundary conditions. The boundary

[11] The HJB equation also implicitly incorporates the option of default at $c > 0$, in which case firm's value is the sum of c and firm's liquidation value after debt payment. We call this strategic default. This case is incorporated in the second group of terms $\partial_c V - 1$ in (9).

condition at $c = 0$ is determined by comparing the default and issuance values:

$$0 = \min \left\{ V(k, 0) - \Big(\ell k - b/r_{\text{debt}}\Big)_+, V(k, 0) - \sup_{I \geq 0} \Big[V(k, I) - I - \lambda(I) \Big] \right\}. \quad (11)$$

In the equation above, the boundary value $V(k, 0)$ dominates the default value $(\ell k - b/r_{\text{debt}})_+$ and the best issuance value $\sup_{I \geq 0}[V(k, I) - I - \lambda(I)]$, and is only equal to one of the terms for each value of k. If $V(k, 0)$ equals the former value, it is optimal for the firm to default; otherwise, issuance is optimal with an optimal size.[12] The HJB equation (9) is numerically solved in a sufficiently large domain, the payout and issuance boundaries are determined endogenously in this domain.

3. The Model Solution

In this section, we present and discuss the results of our model for a baseline set of parameters. Then, we vary the different parameters to examine their impact on the model firm's predicted behavior.

3.1. Numeric results

The baseline values for the parameters are presented in Table 1 along with rationale.

[Insert Table 1 Here]

Figure 1 depicts a firm's choices for the baseline set of parameters. There are several grey regions. The light grey region is the dividend payout region. The medium and dark grey regions are the investment regions. The white region is the continuation region, where the firm retains its earning and neither pays dividends nor invests in capital. We now discuss these regions in more detail.

Dividend payout. Let us first focus on the dividend payout boundary ABE. The firm only pays out a dividend when the marginal cost of reducing the cash reserve matches the marginal benefit of dividend payout. If c is initially higher than the boundary ABE, the marginal value of cash equals one, which is the marginal benefit of dividend payout. As a result, when c is higher than the boundary ABE, a lump sum dividend is paid out

[12] When the cash reserve is sufficiently high, we expect that the firm optimally pays out excess cash. Therefore, we impose the boundary condition $\partial_c V = 1$ at $c = c_{max}$ with a sufficiently high c_{max}. When $k = 0$, the adjustment cost g is infinitely large for $i > 0$; When $k = k_{max}$ with a sufficiently high k_{max}, the benefit of investment vanishes in comparison to the adjustment cost, due to diminishing returns to scale. In both ends, we impose the value of no-investment as boundary conditions. See the appendix for more technical discussions on the boundary conditions and convergence of the numeric scheme.

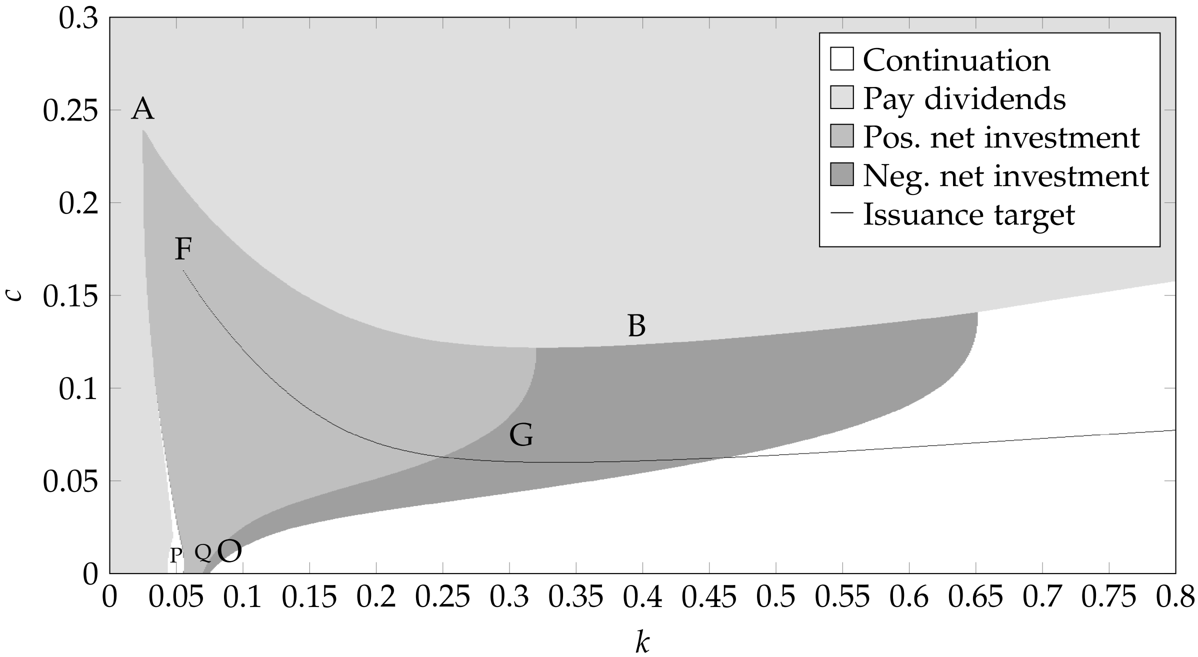


Figure 1: **Optimal regions**

The ■ colored region indicates dividend payouts, delimited by the lines connecting PABE; the ■ colored region below AB indicates positive net investment; the ■ colored region below BE indicates negative net investment; the white region is the continuation region where the firm neither pays dividends nor invests in capital; and the solid black line FGH indicates the state after equity issuance from the points on the line QR. Parameters used are summarized in Table 1.

so that the state process (k, c) lands on ABE exactly after dividend payout. When the state process (k, c) reaches the boundary ABE from below, a minimal dividend is paid out to reflect the state process below so that the state process remains lower than ABE.

The firm's dividend payout boundary ABE varies with a firm's capital position k. The payout boundary first decreases and then increases with k. The intuition is that when k is low, the firm generates low cash flows. In order to invest in the capital and pay the bond coupon, the firm needs a large cash buffer to avoid costly default and equity issuance. Therefore, the firm chooses a high dividend payout boundary. As k increases, cash flows from production partially substitute for the cash reserve's role in supporting the coupon and funding investment; hence, the need for the cash reserve decreases in capital, resulting in a lower dividend payout boundary. As k further increases, even though the expected cash flow also increases and further substitutes for the cash reserve, the cash flow volatility rises as well (see Equation (2)), resulting in an increasing likelihood of costly issuance. When k is sufficiently high, the volatility effect dominates the substitution effect. Therefore, a deeper cash buffer is needed to hedge against the larger cash flow variations.

Investment. Due to diminishing returns to scale and the risk of issuance costs, the optimal investment depends on the stock of capital and cash. In the medium-grey region, investment i is larger than the depreciation δk; thus, the capital is built up and the firm grows. In the dark-grey region, the firm still invests, i.e., $i > 0$, but investment is less than the depreciation δk. In the white region, the firm neither invests nor pays out a dividend. Hence, in the dark-grey and white regions, capital declines and the firm shrinks. Holding capital fixed, the white continuation region emerges when cash decreases because the marginal value of cash balloons as the cash reserve approaches zero. This ballooning occurs because the likelihood of paying the cost of issuance or default is increasing. Therefore, preserving cash to mitigate costly issuance or liquidation becomes more important than investment.

The interface between the medium- and dark-grey regions, the boundary BGO in Figure 1, is where investment exactly offsets depreciation — a stationary region for capital size, because capital grows to the left of the boundary BGO and shrinks to the right. This continuum of stationary points, where investment equals depreciation, arises because the optimal exposure to the productivity shock (amount of capital) is a function of the cash reserve.

Figure 2 provides a heat map of the net investment $i - \delta k$. Evidently, as capital increases, net investment decreases because the marginal returns to investing are lower. For a given level of capital, as cash decreases, investment decreases because the likelihood of costly issuance or default increases, raising the marginal value of cash.

Interestingly, for some levels of capital, when cash is low, investment is still desirable even though the investment immediately results in cash hitting zero, the firm paying the issuance costs, and the firm issuing up to the issuance target. The rationale is that when capital is low, the cash flows available to build the cash reserve are small. Consequently, it would take a long time to build the cash reserve. Additionally, when capital is low, the investment opportunities are strong. Thus, when capital is low, the firm may find it optimal to take advantage of the strong investment opportunities immediately by raising costly equity instead of waiting to fund the investment internally. In other words, the firm considers the amount of time it would have to wait to build the cash position as a cost that is compares with the cost of issuing equity immediately.

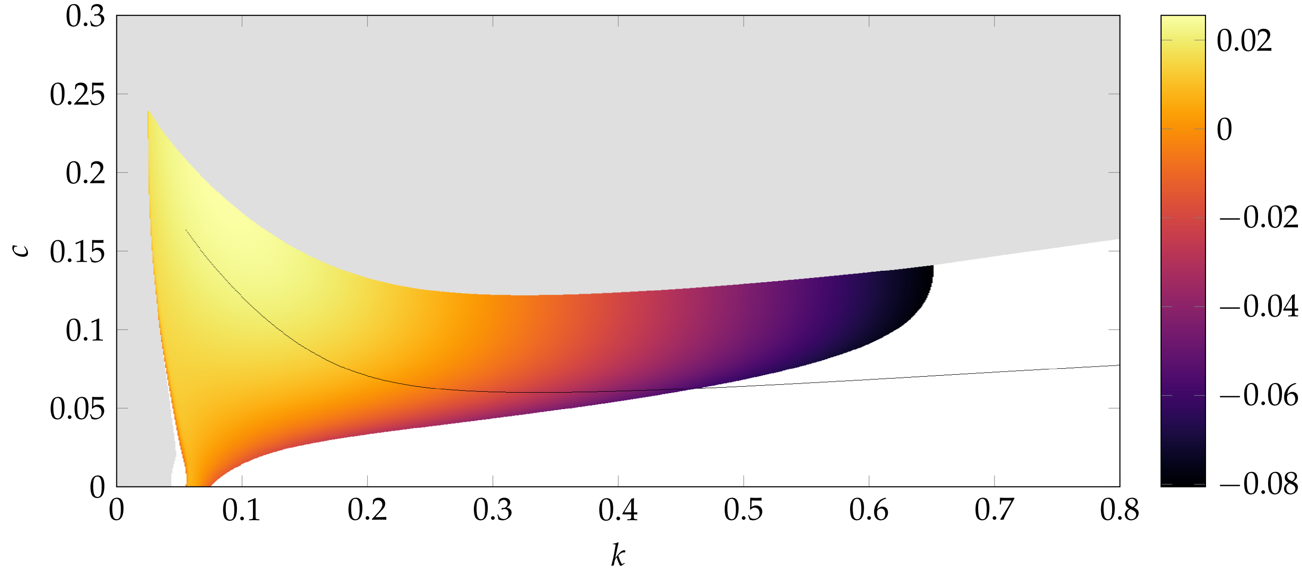


Figure 2: **Net investment heat map**

The ■ colored region indicates dividend payouts and the solid black line indicates the state after equity issuance. The white region is the continuation region where the firm neither pays dividends nor invests in capital. The investment region is replaced by a color gradient representing the net investment rate ($i - \delta k$). For example, when the net investment scale on the right indicates -0.04, that means that capital drops 0.04 (i.e. from 0.5 to 0.46). Parameters used are summarized in Table 1.

To better understand the determinant of investment, let us examine the tradeoff between the marginal cost of cash $\partial_c V$ and the marginal benefit of capital $\partial_k V$ for the baseline result in Figure 1. For a fixed capital level, Figure 3(a) shows that $\partial_c V$ is decreasing in c, implying the concavity of V in c and the firm's effective risk aversion in cash. For different capital levels, Figure 3(a) shows that $\partial_c V$ does not scale with k linearly. As we have discussed for Equation (10), investment is optimal only when $\partial_c V < \partial_k V$. Comparing $\partial_c V$ and $\partial_k V$ in Figure 3 (a), we see that the minimal level of cash reserve needed for positive investment is lower when $k = 0.1$ than when $k = 0.4$. This observation is consistent with Figure 2. For fixed cash levels, Figure 3(b) plots $\partial_k V$ and $\partial_c V$ for different k. The marginal benefit of capital $\partial_k V$ is non-monotone in k in general. But when k is higher than a threshold, 3(b) shows that $\partial_k V$ decreases with k, reflecting the firm's decreasing returns to scale. We will explain the non-monotone pattern of $\partial_k V$ when we discuss default next.

Equity issuance and default. Even though Equation (9) indicates that for each point (k, c) in the state space, the firm compares the value of the firm without issuance to the best value for issuance, the next result shows that the firm optimally issues equity only

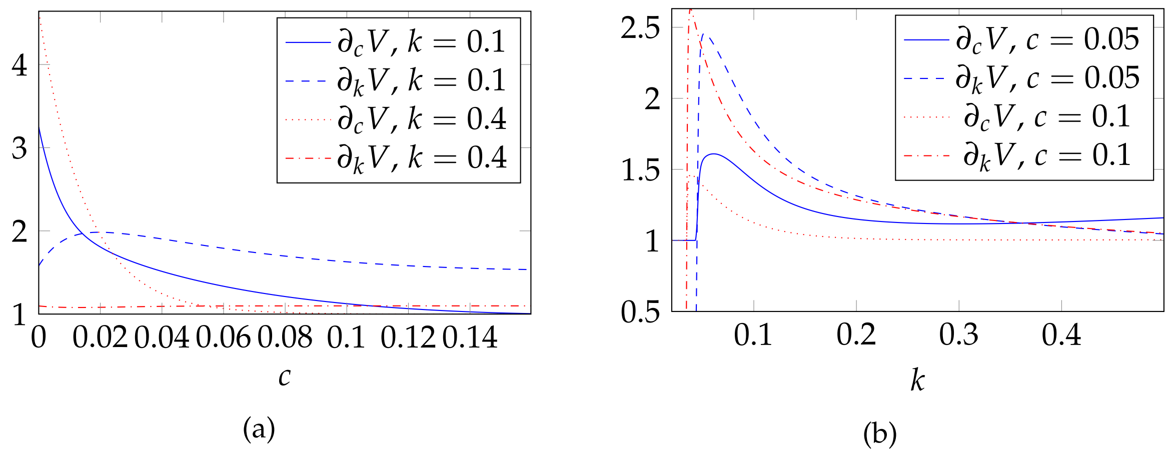


Figure 3: **Marginal cost of cash and marginal benefit of capital**

Parameters used are summarized in Table 1.

when the cash reserve runs out.

Proposition 1. *Firms only consider issuance when the cash reserve reaches zero.*

Since the firm faces a fixed cost of financing, it chooses to raise equity infrequently. It will optimally raise cash only when it has to. First, cash within the firm earns a below-market return $r - \lambda_c$. Second, there is time value for external financing costs. Therefore, without any benefit for early issuance, it is always better to defer external financing when $c > 0$. This argument highlights the pecking order between cash and external financing in the model.[13]

When c reaches zero, due to either negative productivity shocks or investment, the firm compares the best issuance value with the liquidation value to determine issuance or default. In Figure 1(a), the issuance boundary is the interval QR on the horizontal axis, and the default boundary is the interval PQ. When c reaches zero, if k lands on the interval QR, the firm issues a lump sum amount of equity, the size of which is specified by the curve FGH. We call this curve the issuance target. The lumpiness of issuance is to economize fixed issuance costs. After issuance, the state process jumps upward and lands on FGH before continuing its dynamics. If k lands on the interval PQ when c reaches zero, then the firm chooses not to issue and instead defaults.

A related prediction of our model is the existence of a strategic default region, which is the light grey region to the left of the boundary AP in Figure 1(a). When the state process

[13]Relatedly, because of costly issuance and no information asymmetry, the firm never finds it optimal to issue equity and immediately pay a dividend.

(k, c) reaches the boundary AP, rather than continuing operations, equityholders pay out the remaining cash reserve as a lump sum dividend and default afterward. Intuitively, when the debt coupon is high but capital is low, even though the marginal productivity of capital is high, it takes substantial investment and time to build up capital and increase cash flows to service the debt. When comparing this option with the continuation choice, equityholders may find it more attractive to pay out the remaining cash immediately and then liquidate the firm.[14] In the Internet Appendix, we show that banning such strategic defaulting when the firm is insolvent does not affect the model's main predictions. We allow for strategic defaulting to keep the model similar to that of BCW and to avoid assuming a particular default rule. The strategic default also induces the non-monotone pattern of $\partial_k V$ in Figure 3(b): when k is low, strategic default is imminent, rather than using cash to invest, equityholders prefers to hold back investment and wait for payout and strategic default. This strategic consideration yields low marginal benefit of capital $\partial_k V$ when k is low in Figure 3(b).

In practice, the incentive to strategically default may explain the sharp increase in dividend covenants as credit quality deteriorates (Sufi, 2007) and the prevalence of laws banning fraudulent conveyance by equityholders in the zone of bankruptcy. Also, consistent with strategic defaulting, financially distressed firms are reluctant to cut dividends even though dividends exceed cash flows (Deangelo and Deangelo, 1990), perhaps to reduce cash reserves. In addition, equityholders may pay large dividends ahead of predicted insolvency. For example, creditors of SemGroup sued equityholders to recoup $56 million in dividends paid shortly before filing for bankruptcy in 2008.

A firm's positioning over time. Figure 4 shows a heat map of the firm's position in (k, c) space over time. We simulate paths until $t = 10$ starting in a 1000×1000 grid in the figure domain. Evidently, the firm spends more time in the area of the stationary point B in Figure 1.

3.2. Comparative statics

This section examines how a firm's choices vary when we adjust the assumed values of specific parameters. We specifically vary the coupon rate, expected productivity, the volatility of the productivity shock, the issuance costs, and the cash liquidity premium.

[14]The existence of the strategic default region is ensured by Proposition 2 in Appendix A.

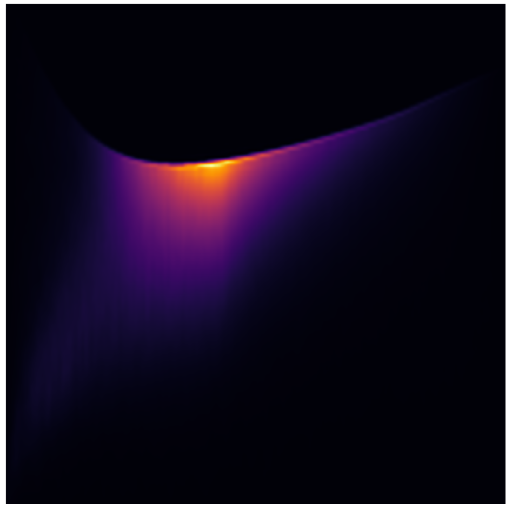


Figure 4: **A firm's time series location in capital-cash (k, c) space**

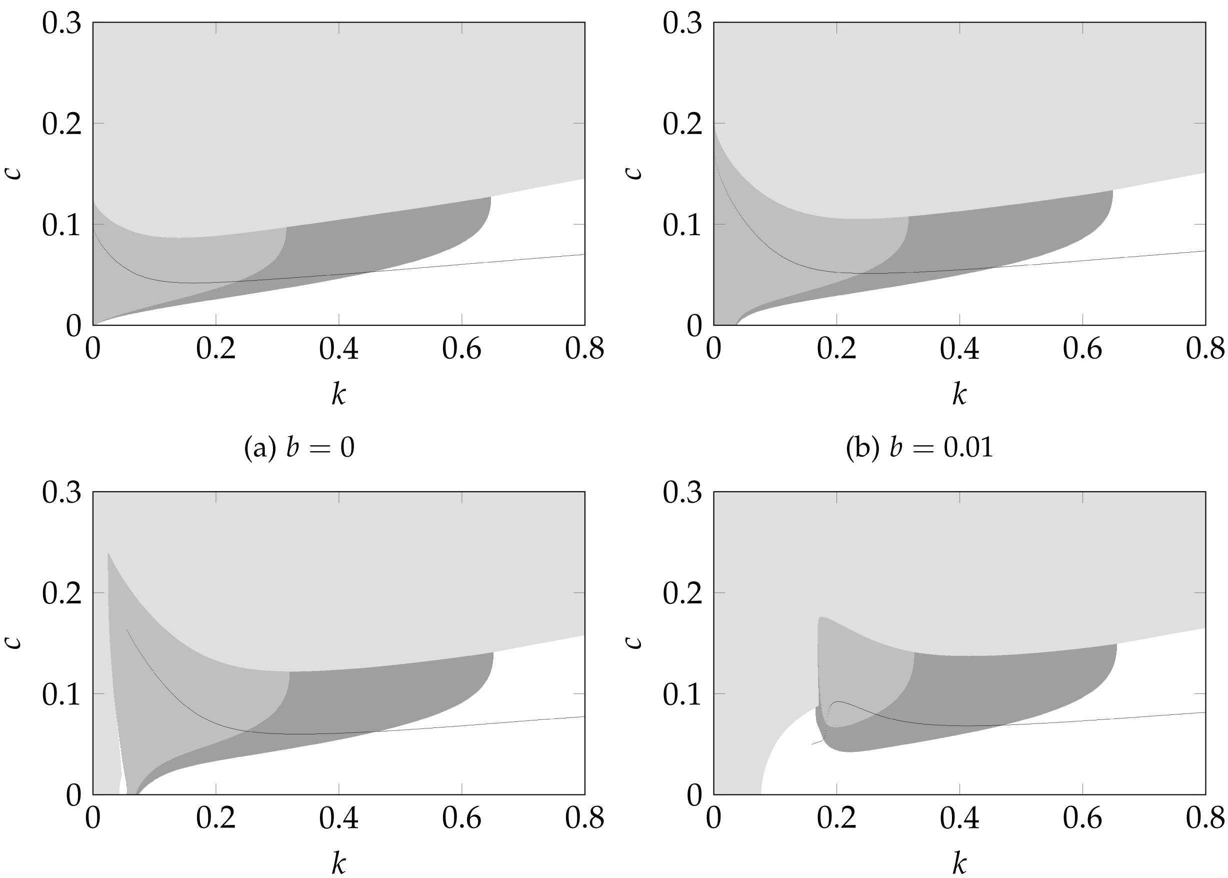
The heatmap is generated by simulating paths until $t = 10$ starting from each grid point in a 1000×1000 grid in the figure domain. Parameters used are summarized in Table 1.

3.2.1. Impact of leverage

As the cash coupon, b, increases, the burden from debt becomes increasingly heavier. Figure 5 shows how the debt burden alters the firm's choices in a few ways.

First, when b increases, the default region expands. Specifically, increasing the debt burden shifts the issuance target to the right. For example, in Figure 5(c), the issuance target only begins to appear around $k = 0.1$. For the region $0 \leq k < 0.1$, if the firm runs out of cash, the equityholders optimally choose to default rather than to issue new equity. Despite this intuitive prediction, the existing literature mostly examines equity offerings and bankruptcy separately and does not consider the impact of leverage on the decision of whether to issue or default. In Internet Appendix D.4, we find empirical support for the prediction that low-capital firms with more leverage are more likely to default than issue new equity.

Second, as b increases, debt overhang emerges as equityholders of distressed firms are less willing to invest than those holding equity in firms with similar characteristics but with less debt. Specifically, as Figure 5(d) shows, as capital decreases, investment decreases because the dark-grey region begins to envelop the mid-grey region from below. Moreover, a continuation (white) region emerges between the strategic default region and investment regions. This continuation region first emerges as a white stripe in panel (c) and expands dramatically in panel (d). This underinvestment leads to lower capital levels in the short term, which decreases firm cash flows, but underinvestment also builds the



(a) $b = 0$ (b) $b = 0.01$

(c) $b = 0.02$ (d) $b = 0.03$

Figure 5: **Impact from** b

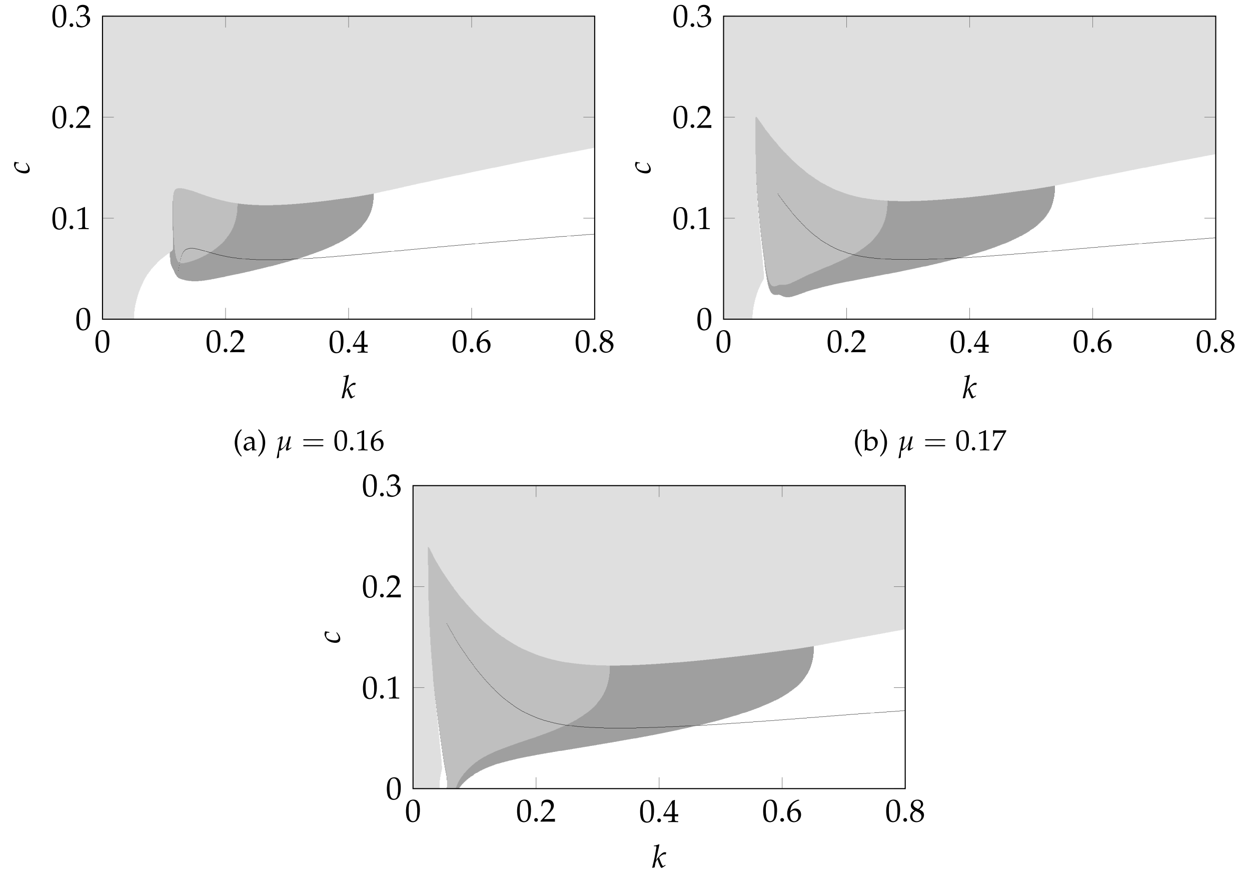
The ■ colored region indicates dividend payouts; the ■ colored region indicates positive net investment; the ■ colored region indicates negative net investment; the white region is the continuation region where the firm neither pays dividends nor invests in capital; and the solid black line indicates the state after equity issuance. Other parameters used are summarized in Table 1.

cash reserve, which equityholders can use to delay costly liquidation or issuance.

Third, when capital k is low, Figure 5 panels (a) to (c) show that increases in leverage sharply raise the dividend payout boundary and the equity issuance target. By contrast, when k is high, the dividend payout boundary and issuance target are less sensitive to the coupon b. The intuition is that when k is low, there is less cash flow generated, so the firm needs a larger cash buffer to fund the higher coupon. When k is high, the cash flows generated by production can better support the debt coupon payment, reducing the need for a cash buffer.

3.2.2. Impact of expected productivity

As μ decreases, the expected cash flows of the firm decrease. Figure 6 shows how lower expected cash flows change the firm's choices (moving from 6(c) to 6(a)).



(a) $\mu = 0.16$

(b) $\mu = 0.17$

(c) $\mu = 0.18$

Figure 6: **Impact from** μ

The ■ colored region indicates dividend payouts; the ■ colored region indicates positive net investment; the ■ colored region indicates negative net investment; the white region is the continuation region where the firm neither pays dividends nor invests in capital; and the solid black line indicates the state after equity issuance. Other parameters used are summarized in Table 1.

First, when the expected returns to capital are lower, the investment region (combination of the medium- and dark-grey regions) shrinks in size.

Second, the default region grows as the issuance boundary shifts to the right. Intuitively, when capital and profitability are low, the benefits of issuing equity to continue investing declines. Consequently, equityholders find it more attractive to default when cash reaches zero. Relatedly, when capital is low, increases in expected profitability increase the optimal cash balance due to the drop in endogenous default risk.

Third, lower expected cash flows lower the dividend payout boundary. When a firm is less profitable and is less willing to invest, then equityholders find it optimal to extract more cash from the firm, resulting in lower cash balances and earlier dividend payouts. Lower cash balances increase the risk of default because it is harder for the firm to recover from bad productivity shocks, but the lost capital at default is also less valuable.

Fourth, the white continuation region wraps underneath the investment region. Evidently, when profitability is low, firms find it optimal to start investing only when cash balances are sufficiently high. Lower profitability leads the marginal value of cash to balloon more because it takes longer for the firm's cash reserve to recover from negative productivity shocks. Also, default risk is higher and equityholders prefer to have more cash than capital when default or issuance is imminent.

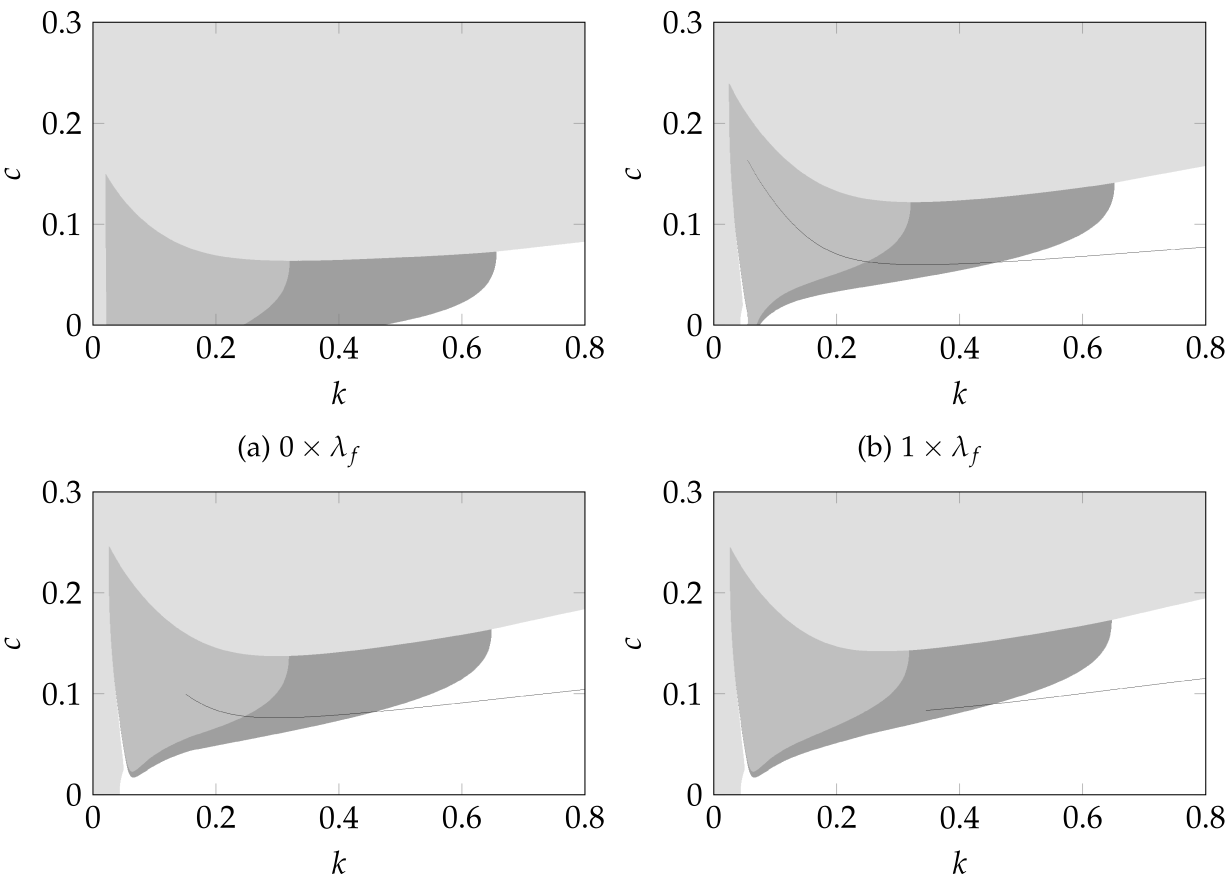
3.2.3. Impact of the fixed component of issuance costs

As λ_f increases, the fixed costs of issuing equity increase. Figure 7 (moving from (a) to (d)) shows how higher fixed issuance costs change the firm's choices.

First, Figure 7 shows that the fixed component of issuance costs drives the issuance boundary of the firm. Specifically, Figure 7(a) shows that when there are no fixed issuance costs, the issuance boundary collapses to the horizon axis. Intuitively, firms still value a cash reserve to hedge against costly issuance (there is still a proportional cost), but when the cash reserve reaches zero, firms raise just enough cash, leading the cash reserve back into the positive domain. In the presence of fixed issuance costs, the firm optimally issues equity in lumps to economize over the fixed issuance costs. As the fixed issuance costs increase, the issuance boundary rises because firms have a larger incentive to avoid paying the fixed costs in future issuances.

Second, higher fixed issuance costs incentivize a higher issuance target and a higher dividend payout boundary. Intuitively, if fixed issuance costs are higher, then the firm optimally raises more cash in any given issuance. Similarly, Figure 8 shows that without fixed issuance costs, increases in the proportional issuance costs (λ_p) increase the dividend boundary because hedging against costly issuance becomes more important.

Third, higher fixed issuance costs move the equity issuance boundary to the right, expanding the default region. In the default region, when the firm runs out of cash, there is no optimal issuance amount (no issuance target boundary). Instead, the firm liquidates. The default region expands because higher issuance costs increase the expected costs of operating the firm, which reduces the incentive for equityholders to invest and build up the capital.



(a) $0 \times \lambda_f$ (b) $1 \times \lambda_f$ (c) $4 \times \lambda_f$ (d) $7 \times \lambda_f$

Figure 7: **Impacts of fixed issuance costs**

The ■ colored region indicates dividend payouts; the ■ colored region indicates positive net investment; the ■ colored region indicates negative net investment; the white region is the continuation region where the firm neither pays dividends nor invests in capital; and the solid black line indicates the state after equity issuance. Other parameters used are summarized in Table 1.

3.2.4. Impact of production volatility

Figure 9 shows how a change in the volatility of the cash flow shock, σ, alters a firm's choices.

When cash flow shocks are more volatile, the dividend payout boundary and the equity issuance boundaries increase. Equityholders facing more volatility demand higher cash reserves to hedge against the possibility of larger negative cash flow shocks that could lead to costly issuance. Relatedly, conditional on issuance, equityholders raise more capital. This effect is more pronounced when capital k is high because the nominal size of the shock is larger for higher capital levels. Also, when a firm is small, the marginal value of cash is already high because of high default risk, strong investment opportunities, and

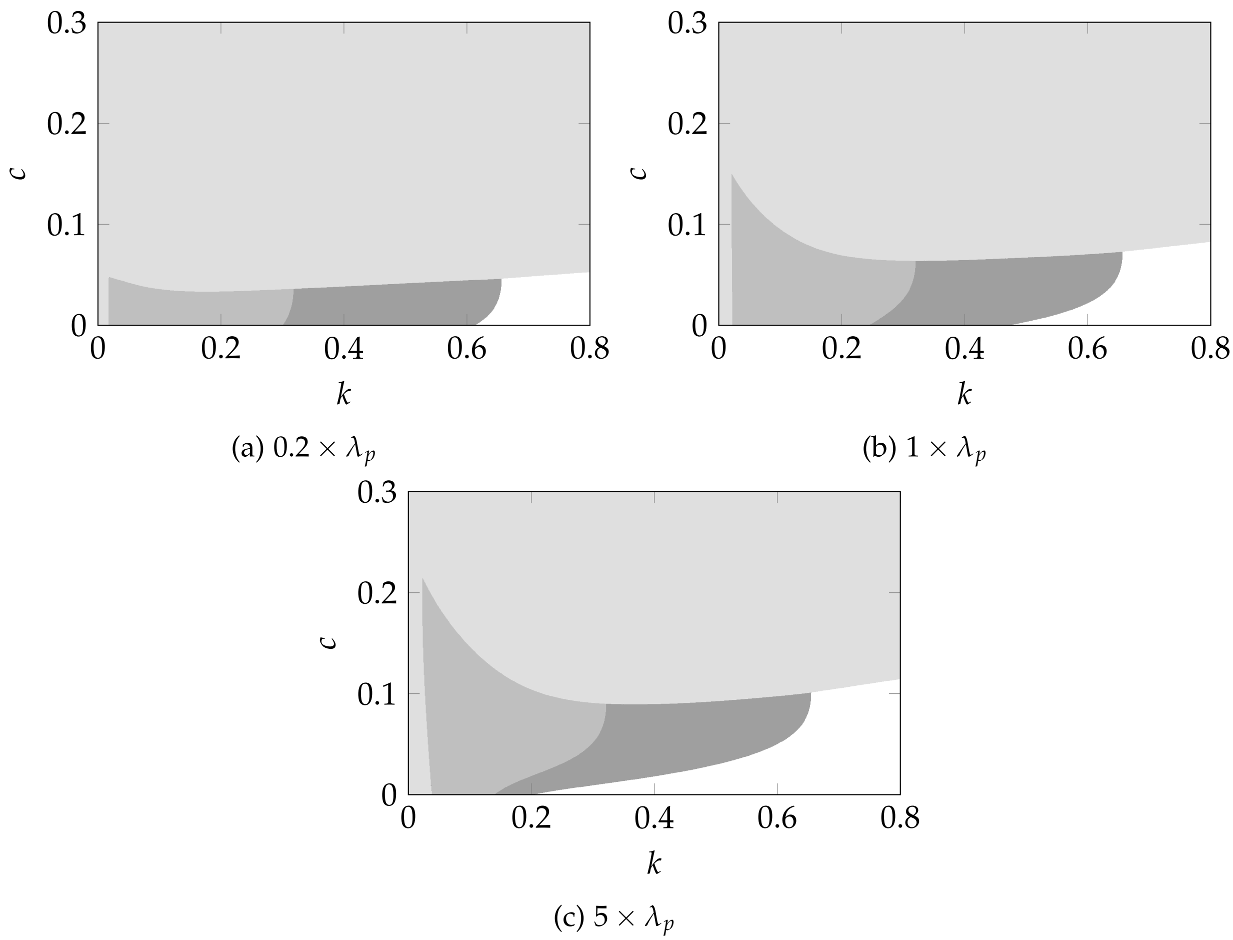


Figure 8: **No fixed issuance cost,** $\lambda_f = 0$

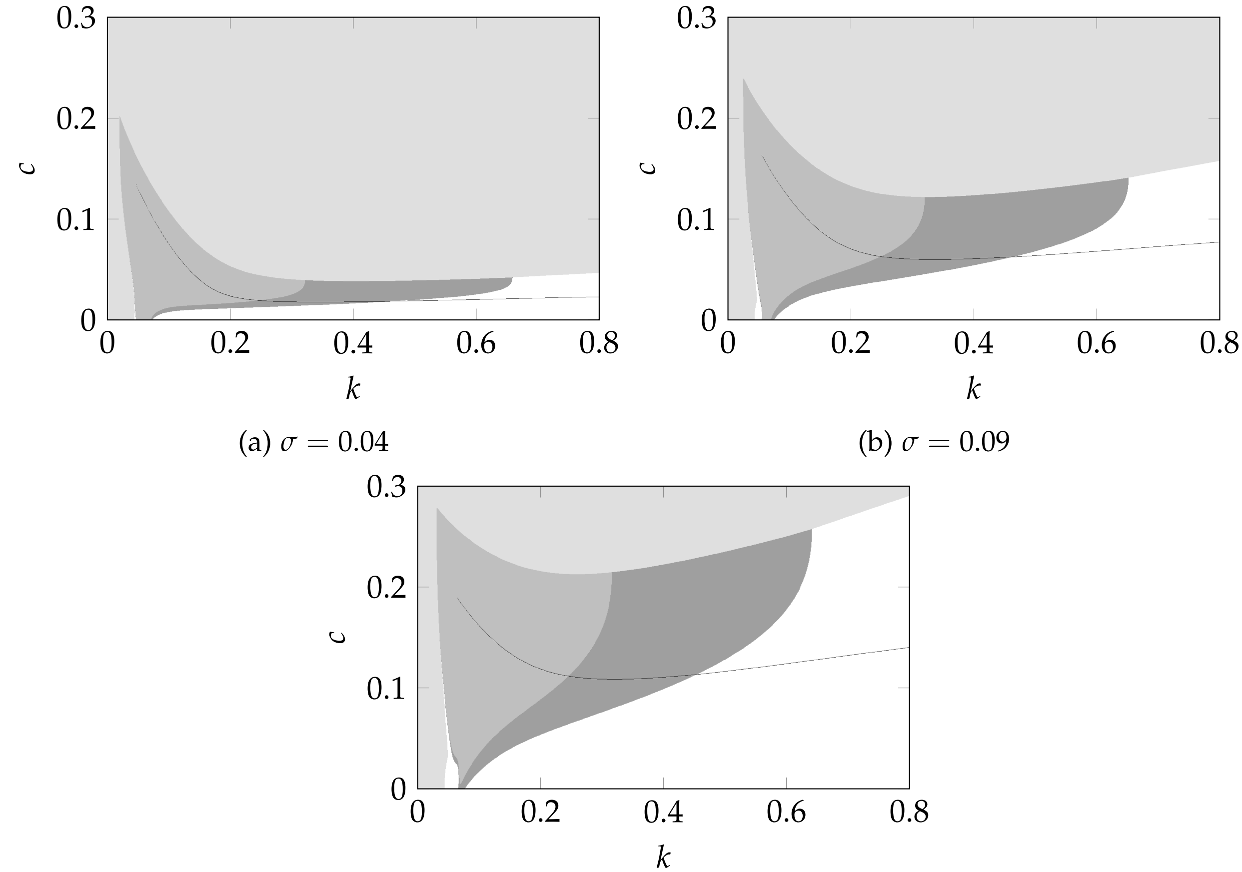
The colored region indicates dividend payouts; the colored region indicates positive net investment; the colored region indicates negative net investment; the white region is the continuation region where the firm neither pays dividends nor invests in capital; and the solid black line indicates the state after equity issuance. Other parameters used are summarized in Table 1.

low cash flows. Thus, raising the volatility of the cash flow shock has a relatively smaller impact on the marginal value of cash when a firm is small.

3.2.5. *Impact of the liquidity premium* λ_c

Figure 10 shows how a firm's choices vary with the liquidity premium λ_c.

When the liquidity premium is lower (the top panel of Figure 10), the dividend boundary is higher. Intuitively, lower liquidity premiums reduce the difference between the net rate of return for cash and the equityholders' discounting rate. Hence, early dividend payout becomes less attractive, and the dividend payout boundary increases.



(a) $\sigma = 0.04$

(b) $\sigma = 0.09$

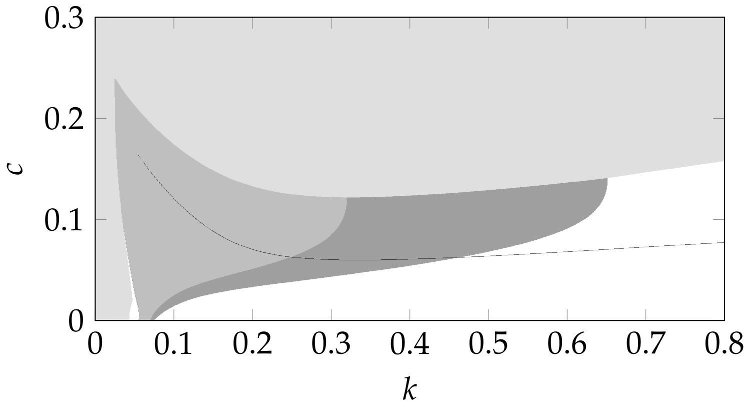
(c) $\sigma = 0.14$

Figure 9: **Different** σ

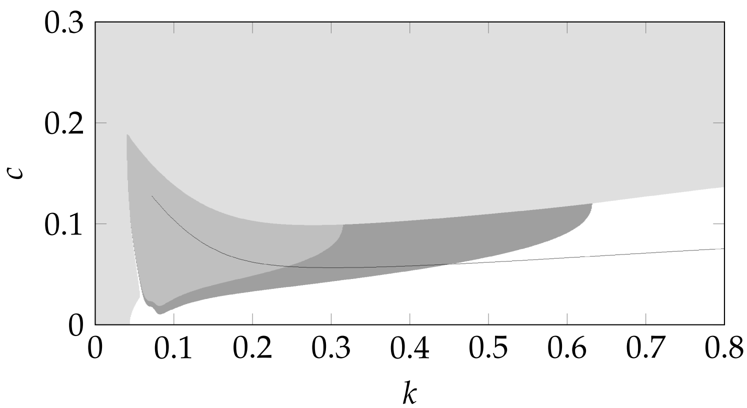
The ■ colored region indicates dividend payouts; the ■ colored region indicates positive net investment; the ■ colored region indicates negative net investment; the white region is the continuation region where the firm neither pays dividends nor invests in capital; and the solid black line indicates the state after equity issuance. Other parameters used are summarized in Table 1.

4. Empirical Support

The analyses thus far has been mostly theoretical. In the preceding sections, we build a dynamic model of a firm with two states — cash and capital — and connect the predicted firm dynamics to the existing empirical literature. This section provides empirical support for several novel predictions about a firm's issuance, payout, investment and cash holdings policies.



(a) $\lambda_c = r/6$



(b) $\lambda_c = r/2$

Figure 10: **Different λ_c (Cash liquidity premium)**

The ■ colored region indicates dividend payouts; the ■ colored region indicates positive net investment; the ■ colored region indicates negative net investment; the white region is the continuation region where the firm neither pays dividends nor invests in capital; and the solid black line indicates the state after equity issuance. Other parameters used are summarized in Table 1.

4.1. Data

The primary data source is the annual Compustat data file, which provides detailed financial statement information on public firms from 1962 through 2017.[15] After filtering the data, we have 10,027 firms and 123,793 firm-years. (See Internet Appendix Table D.1 for details on the filtering.) For certain tests, the data series is more limited in time frame due to the availability of certain key variables.

The two primary state variables in the model are a firm's cash and capital positions. To proxy for the cash state variable, we use a firm's cash and cash equivalents (*che*) from

[15] The original data file is from 1950 to 2021. However, a primary variable — total capital — is only available for 1960 to 2017 (Peters and Taylor, 2017).

the fiscal year–end balance sheet. The capital position includes both the tangible and intangible capital on the balance sheet and the intangible capital not on the balance sheet, as estimated by Peters and Taylor (2017).

The main outcome variables capture a firm's issuance, payout, and investment activity.

To proxy for a firm's issuance activity, we use the firm's total sales of common stock listed on the cash flow statement (*sstk*, available starting in 1971). In the Internet Appendix, we repeat the related analyses using common stock offering data from Securities Data Company (SDC) Platinum.

To proxy for a firm's payout activity, we primarily use the firm's annual amount of dividends (*dvt*) and annual amount spent on repurchasing shares (*prstkc*).

To proxy for a firm's net investment activity, we incorporate spending on physical and intangible capital following Peters and Taylor (2017). For investment in physical capital, we use a firm's capital expenditures on property, plants, and equipment (*capx*). For investment in intangible capital, we assume that 30% of SG&A spending, net of R&D spending, is investment in intangible capital.[16] We add to that 100% of R&D spending. We compare spending on investments to depreciation of the existing capital stock to determine net investment. To find the total depreciation rate, we add the depreciation of physical capital (*dp*) and assume a depreciation rate of 20% for the intangible capital.

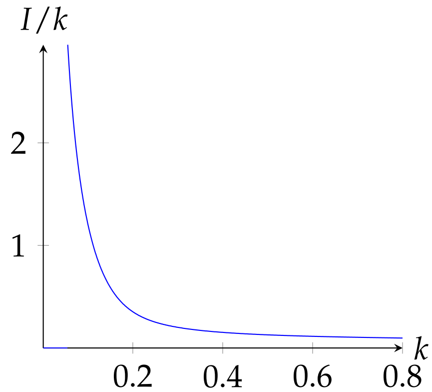
4.2. The amount issued as a proportion of capital is declining and convex in capital

While the empirical literature on issuances is predominately concerned with when firms choose to issue, few studies examine how much firms choose to issue conditional on issuance. Characterizing issuance amounts is important because issuance amounts are relevant for work on trade-off theory (Hovakimian et al., 2001), market timing (Chang et al., 2006), and issuance costs (Smith, 1977; Lee et al., 1996; Chen and Ritter, 2000; Altınkılıç and Hansen, 2000; Butler et al., 2005; Gomes, 2001; Hennessy and Whited, 2007).

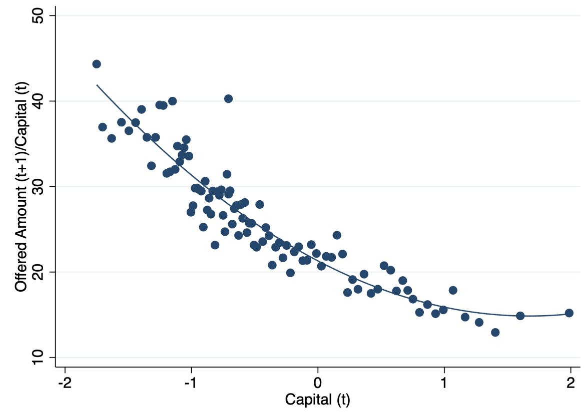
Figure 11(a) shows that the target issuance amount in Figure 1 as a proportion of capital is declining and convex in capital. Driving this convexity is variation in a firm's marginal value of cash with capital, costly financing, and costly bankruptcy.

First, the fixed component of issuance costs incentivizes lumpy issuance and thus drives the issuance target. The issuance target scaled by capital is declining, in part because the fixed component of issuance costs becomes relatively less important as capital

[16] SG&A is equal to total SG&A (*xsga*) minus R&D expenses (*xrd*) minus in-process R&D (*rdip*) if *xsga* is not missing and *xrd* is less than *xsga* but greater than the cost of goods sold (*cogs*); otherwise, SG&A is equal to *xsga*.



(a) Figure 11a is the issuance target in Figure 1 scaled by capital. The vertical axis shows the amount raised scaled by capital; the horizontal axis plots capital. The parameters used are summarized in Table 1.



(b) The offered amount is the value of stock sold in year $t+1$ scaled by capital at the end of year t. The horizontal axis shows a firm's total capital (tangible and intangible) standardized within firm. The sample is all offerings in year $t+1$ with proceeds greater than or equal to 5% of capital stock at the end of year t.

Figure 11: Predicted Issuance from Figure 1 vs. Actual

increases. In other words, for low levels of capital, the fixed component of issuance costs represents a larger proportion of firm value. Thus, conditional on issuing, firms with low capital have a stronger precautionary motive to issue more equity to reduce the possibility of paying the fixed issuance costs again in the future.

Second, because of diminishing returns to scale, firms with low capital have stronger incentivizes to invest because the marginal investment returns are higher. Additionally, when capital is low, firms rely more on the cash reserve to fund investment, which puts downward pressure on the cash reserve. As capital increases, cash flows fund a larger proportion of investment.

Third, when capital is low, the marginal value of cash is also higher because the firm is closer to default. A larger cash reserve helps the firm make the required coupon payments for longer to avoid default.

Overall, when capital is low, issuing more equity enables the firm to capitalize on the higher investment returns while managing the risks of future fixed issuance costs and default.

This reasoning leads to the following hypothesis:

H1: The amount issued as a proportion of capital is declining and convex in capital.

Figure 11(b) provides visual empirical evidence consistent with **H1**, which is based on the model prediction illustrated in Figure 11(a). The sample is the subset of firm-years for which the amount of stock sold in year $t+1$ is at least 5% of a firm's total capital at the end of year t.[17] The vertical axis plots the amount offered in year $t+1$ scaled by total capital at the end of year t. The horizontal axis is a firm's total capital standardized within firm. The figure clearly shows that the amount issued as a proportion of capital is declining and convex in capital.[18]

To provide further empirical support of this hypothesis, we estimate the following empirical specification:

$$\frac{\text{Amt Raised}_{i,t+1}}{\text{Capital}_{i,t}} = \beta_1 \text{Capital}_{i,t} + \beta_2 \text{Capital}^2_{i,t} + \beta_3 \text{Cash}_{i,t} + \mu_i + \delta_{j,t} + \epsilon_{i,t}. \tag{12}$$

The outcome is the proceeds from common stock sales by firm i in year $t+1$ scaled by firm i's total capital (tangible plus intangible) at the end of year t. The main explanatory variable is a firm's total capital at the end of year t, standardized within firm and winsorized at the 1% level.[19] Thus, capital is positive (negative) when above (below) a firm's mean capital. We calculate within-firm capital so that the quadratic form of capital captures the extent to which a firm is different from its average capital rather than the average across firms. β_2 multiplies the quadratic form of capital, standardized within firm. We control for a firm's cash and equivalents (standardized within firm). Because the outcome is not within-firm, we include firm fixed effects, μ_i.[20] To account for industry trends, we include SIC-2–by–year industry trends, ($\delta_{j,t}$). $\epsilon_{i,t}$ is the unexplained variation. Standard errors are double-clustered by firm and year (not firm-year). The sample begins in 1971 because of the availability of issuance proceeds data. Additionally, the sample is restricted to firm-years in which more than 5% of capital is issued (See Footnote 17).

Table 2 presents results consistent with **H1**. Column (1) shows the full sample

[17] In Internet Appendix Figure D.1, we find similar patterns using offering size cutoffs of 1%, 10%, and 20%. The 5% threshold reduces the prevalence of smaller issuances from employee options exercises or at-the-market offerings. A 5% threshold is used in other studies examining issuance (Hovakimian et al., 2001; Chang et al., 2006).

[18] Internet Appendix Figure D.2 shows a similar convexity using offering data from SDC Platinum.

[19] To standardize capital within firm, for each firm, we determine the mean and standard deviations of total capital. In Internet Appendix Table D.7, we find similar results using the natural log of capital at the end of year t divided by the firm's average capital in the sample.

[20] Firm fixed effects limit the sample to serial issuers. In the Internet Appendix, we repeat the related analyses without firm fixed effects and get similar results.

regression. β_1, which multiplies a firm's capital position, is -8.36 and is highly significant. β_2, which multiplies the quadratic form of a firm's capital, is +3.86 and is highly significant. Together, these results provide strong evidence that the amount raised scaled by capital is declining in capital in a convex manner. Columns (2) and (3) show that the relation holds similarly when we split the sample by the median year for the issuance sample of 1997. Column (4) shows somewhat larger coefficients when the sample is restricted to offerings exceeding 10% of capital, rather than 5% of capital. Columns (5) to (11) show that the results hold similarly across SIC-1 industry classifications.[21]

[Table 2 Here]

4.3. The amount paid out to equityholders is concave in capital

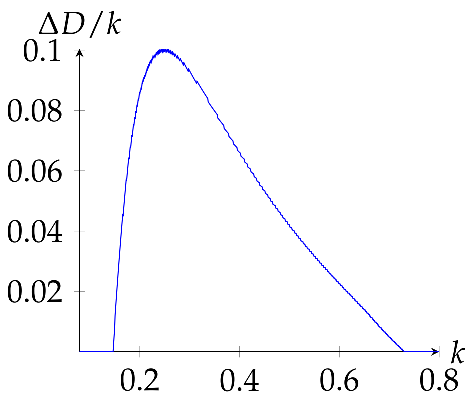
Whereas a substantial empirical literature examines payout policy, we know less about how cash and capital jointly affect payout policy in the presence of costly external financing. We help characterize how firms adjust payout policies to manage cash in response to the costs and benefits of internal and external financing sources.

Figure 12(a) shows that for certain levels of cash, the optimal payout amount for the firm in Figure 1 scaled by capital is concave in capital. Intuitively, in the region of low capital, as capital increases, the larger cash flows substitute for the role of the cash reserve in funding expected investments and meeting liabilities. Also, as capital grows, the incentives to invest decline and the risk of default shrinks. These factors reduce the marginal value of cash, allowing for larger payouts. Additionally, as capital grows, the nominal size of the cash flow shocks continues to increase, raising the likelihood of running out of cash and incurring issuance costs (incentivizing a larger cash reserve). Also, the marginal increase in expected cash flows falls relative to capital because of diminishing returns to scale. Together, these dynamics predict a concave relation between scaled payouts and capital.

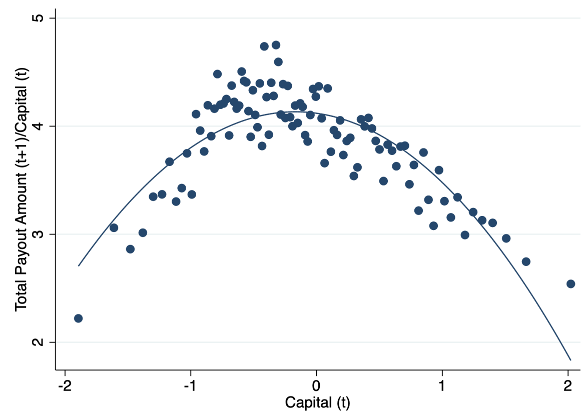
H2: Holding cash fixed, the total payout to equityholders scaled by capital is concave in capital.

[21] Internet Appendix Table D.3 finds similar results without including firm fixed effects and thus allowing for non-serial issuers. Internet Appendix Table D.4 finds similar results when using offering data from SDC Platinum. Internet Appendix Table D.5 finds similar results when we drop IPOs. Internet Appendix Table D.6 finds similar results when controlling for the size of the prior offering.

Figure 12(b) provides visual empirical evidence consistent with **H2**, which is based on the model prediction illustrated in Figure 12(a). The vertical axis plots the amount distributed via dividends and share repurchases in year $t+1$ scaled by total capital at the end of year t. The horizontal axis represents a firm's total capital standardized within firm. The figure clearly shows that payout amounts scaled by capital are concave in capital.



(a) Figure 12a shows the amount distributed via dividends in Figure 1 scaled by capital at $c = 0.15$. Δ denotes distance to the dividend boundary for a certain (k,c) position. The parameters used are summarized in Table 1.



(b) The vertical axis shows the total amount distributed to shareholders through dividends and share repurchases in year $t+1$ scaled by capital at the end of year t. The horizontal axis plots a firm's total capital (tangible and intangible) standardized within firm. Accounts for industry trends.

Figure 12: Predicted Payout from Figure 1 vs. Actual

To evaluate this hypothesis, we estimate the following empirical specification:

$$\frac{\text{Payout}_{i,t+1}}{\text{Capital}_{i,t}} = \beta_1 \text{Capital}_{i,t} + \beta_2 \text{Capital}^2_{i,t} + \beta_3 \text{Cash}_{i,t} + \mu_i + \delta_{j,t} + \epsilon_{i,t}. \tag{13}$$

The outcome is the total amount distributed via dividends and share repurchases by firm i in year $t+1$ scaled by firm i's total capital (tangible plus intangible) at the end of year t. The main explanatory variable is a firm's total capital at the end of year t standardized within firm and winsorized at the 1% level.[22] β_2 captures the relation between payouts and the quadratic form of capital (standardized). We control for a firm's cash and equivalents, firm fixed effects μ_i, and SIC-2–by–year industry trends ($\delta_{j,t}$). $\epsilon_{i,t}$ is

[22]Internet Appendix Table D.11 finds similar results without standardizing within firm. Instead, we use the natural logarithm of a firm's total payouts in a given year divided by that firm's sample average of total payouts.

the unexplained variation. Standard errors are double-clustered by firm and year.

Table 3 presents the results. Column (1) uses the full sample of firms.[23] β_1, which multiplies a firm's capital position, is -0.48 and is highly significant. β_2, which multiplies the quadratic form of a firm's capital, is -0.24 and is also highly significant. The quadratic term, β_2, indicates that a firm further from its average size (either smaller or larger) will have lower payouts because *Capital* is standardized within firm. However, as capital initially declines below the average, the linear term dominates, leading to an increase in payouts (i.e., $0.5 > (0.5)^2$). Together, these findings provide strong evidence that payout scaled by capital is concave in capital. Columns (2) and (3) show that the concave relation may be stronger in the more recent half of the sample. Columns (4) to (10) show the concavity exists across most SIC-1 industry classifications, with the exception of SIC=1 (Mining and Construction).

[Table 3 Here]

4.4. The sensitivity of investment to the cash-to-capital ratio is declining in capital

A primary point in Bolton et al. (2011) (BCW) is that costly financing distorts investment from standard q-theory. In their model, as the cash-to-capital ratio increases, a firm's marginal value of cash decreases because the firm is further from costly issuance and thus less constrained. Accordingly, investment is increasing in the cash-to-capital ratio. By relaxing the homogeneity assumption in BCW, the Cash-Cap model sheds light on how financial constraints matter for investment when a firm is smaller versus larger.

In our model, the sensitivity of investment to the cash-to-capital ratio is predicted to be lower for when a firm is larger for several reasons. First, when a firm is smaller, it faces relatively higher issuance costs. Second, a firm's overall incentives to invest decline with size because of diminishing returns to scale. As Figure 1b illustrates, investment rates decline in capital. In the white continuation region, large firms stop investing completely. One can also see that the investment region is shrinking as capital increases.[24]

Figure 13a shows the model predicted relation between investment and the cash-to-capital ratio for two firm sizes. Specifically, we choose $k = 0.1$ and $k = 0.5$ to examine how a firm's investment changes with the cash-to-capital ratio when it is small and large,

[23]Note that the sample has fewer observations than derived in Table D.1 because we are regressing payouts at $t + 1$ on characteristics at time t.

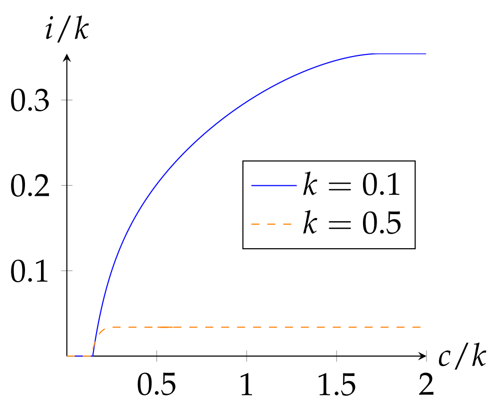
[24]As Figure 10 shows, even when issuance costs are similar across firms of different sizes (i.e. no fixed component of issuance costs), diminishing returns to scale leads to a lower sensitivity of investment to the cash-to-capital ratio.

respectively. One can see that when capital is high, investment does not vary much with the cash-to-capital ratio. In fact, when capital is high, the firm quickly hits the payout boundary as the cash-to-capital ratio increases, resulting in the "red" line plateauing earlier. By contrast, when capital is low, investment varies much more dramatically with the cash-to-capital ratio as the payout boundary is at a higher cash-to-capital ratio and the returns to investing are higher when capital is low. Overall, the figure makes clear that the investment rates of small firms are more sensitive to the cash-to-capital ratio.

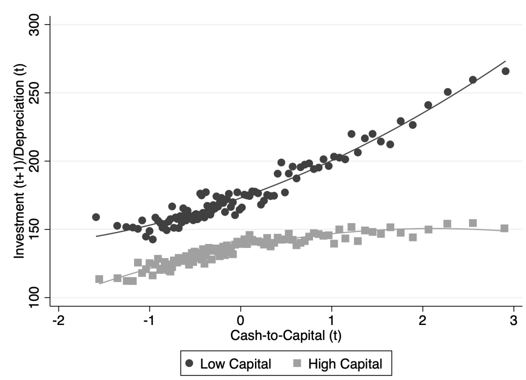
This reasoning leads to the following hypothesis:

H3: The sensitivity of investment to the cash-to-capital ratio is declining in capital.

Figure 13b provides visual empirical evidence consistent with **H3**. The vertical axis plots the amount spent on physical and intangible capital in year $t+1$ scaled by the amount of depreciation of physical and intangible capital in year t. This investment-to-depreciation ratio captures the investment rate. The horizontal axis is a firm's cash-to-capital ratio at the end of year t. One can see a generally positive trend between investment and a firm's cash-to-capital ratio consistent with BCW's prediction. Our contribution is to show that the sensitivity of investment to the cash-to-capital ratio is declining in capital. Evidently, when firms are below their average level of capital, investment is increasing more steeply in the cash-to-capital ratio.



(a) The vertical axis is the model predicted investment rate (investment i scaled by δk) versus the cash-to-capital ratio on the horizontal axis. The blue line is when capital is low ($k = 0.10$), and the red line is when capital is high ($k = 0.50$). The red line stops plateaus earlier along the cash-to-capital axis because when capital is high, the firm hits the payout boundary at lower cash-to-capital levels. Overall, investment by firms with high capital is less sensitive to the cash-to-capital ratio.



(b) The vertical axis shows the total investment in physical and intangible capital in year $t+1$ relative to the total depreciation of physical and intangible capital in year t. The horizontal axis is a firm's cash-to-capital ratio at the end of year t, standardized within firm. "Low Capital" firms have a capital stock (standardized within firm) below the sample mean. Accounts for industry trends.

Figure 13: Investment, Cash-to-Capital, and Firm Size

To evaluate this hypothesis more rigorously, we estimate the following empirical specification:

$$\frac{\text{Investment}_{i,t+1}}{\text{Depreciation}_{i,t}} = \beta_1 \frac{\text{Cash}}{\text{Capital}}_{i,t} + \beta_2 \text{Capital}_{i,t} + \beta_3 \frac{\text{Cash}}{\text{Capital}}_{i,t} \times \text{Capital}_{i,t} + \mu_i + \delta_{j,t} + \epsilon_{i,t}. \tag{14}$$

The outcome is total investments in tangible and intangible capital in year $t+1$ scaled by the total depreciation of tangible and intangible capital in year t. This investment-to-depreciation ratio captures the extent to which the firm is growing the capital stock. β_1 captures the relation between investment and a firm's cash-to-capital ratio, calculated at the end of year t, standardized within firm, and winsorized at the 1% level. β_2 captures the relation between investment and the size of a firm's capital stock at the end of year t, standardized within firm, and winsorized at the 1% level. We control for firm fixed effects μ_i (because the outcome is not within firm), and SIC-2–by–year industry trends ($\delta_{j,t}$). $\epsilon_{i,t}$ is the unexplained variation. Standard errors are double-clustered by firm and year.

Table 4 presents the results. Column (1) finds evidence consistent with BCW's basic point that investment rates are increasing in a firm's cash-to-capital ratio. Column (1) also shows that investment rates are generally decreasing in a firm's capital stock, which is consistent with diminishing returns to scale. Lastly, column (1) shows that a firm's sensitivity of investment to the cash-to-capital ratio is declining in a firm's capital stock. Columns (2) and (3) show that the result is robust to splitting the sample around the median year. Columns (4) to (10) show similar results across SIC-1 industry classifications.

4.5. Volatility has a larger affect on cash balances for larger firms

Figure 9 illustrates how volatility affects the behavior of our model firm. Evidently, as volatility increases, the marginal value of cash increases because the payout boundary shifts upwards. This prediction is consistent with that in BCW. Our contribution is to relax BCW's homogeneity assumption to characterize how the sensitivity in cash holdings to volatility varies in the cross section of firm size.

One of the more striking results in Figure 9 is that the cash reserve responds more to an increase in cash flow volatility when a firm is larger than when it is smaller. For example, examining Figures 9a and 9c, an increase in cash flow volatility from 0.04 to 0.14 increases the payout boundary from about 0.05 (0.20) to 0.25 (0.28) when a firm is large (small). Intuitively, in Figure 9a, when a firm is small, the marginal value of cash is high even when volatility is low for several reasons. When a firm is small, it (1) is close to default; (2) faces high issuance costs because of the fixed component of issuance costs; and (3) has strong investment incentives because of diminishing returns to scale. By contrast, when a firm is large and cash flow volatility is low, it holds much less cash because default risk is low, issuance costs are low, and the marginal returns to investing are low. Thus, when a firm is large and volatility increases, it experiences a larger change in the marginal value of cash because it starts at a low marginal value of cash. In other words, when a firm is small, it is already so incentivized to hold cash that increasing volatility matters less for its optimal cash reserve.

Figure 14a presents the marginal value of cash by size of the firm when volatility is low ($\sigma = 0.04$) and when volatility is high ($\sigma = 0.14$). Evidently, the difference between the marginal value of cash when volatility is low and high is generally increasing in capital.

This reasoning leads to the following hypothesis:

H4: Cash flow volatility has a larger affect on cash balances when a firm is larger.

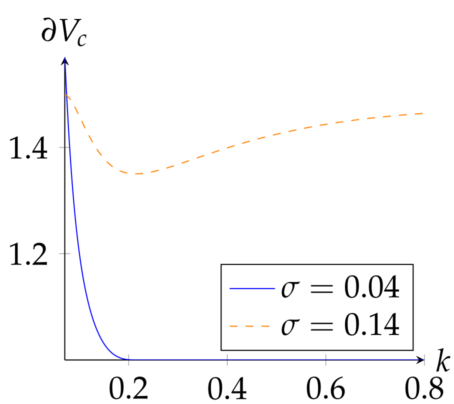
Figure 14b provides visual empirical evidence consistent with **H4**. On the vertical axis, we plot the change in the cash reserve from the end of year t to the end of year $t+1$ (standardized within firm). On the horizontal axis, we plot capital standardized within firm. To proxy for the volatility of a firm's future cash flows, we calculate changes in the volatility of a firm's stock returns. Specifically, to measure firm i's volatility in year t, we follow Ang et al. (2006, 2009), Duffee (1995), and Grullon et al. (2012) among others, and calculate the standard deviation of the firm's daily returns during year t:

$$VOL_{i,t} = \sqrt{\frac{\sum_{\tau \in t} (r_{i,\tau} - \overline{r_{i,t}})^2}{n_t - 1}}. \tag{15}$$

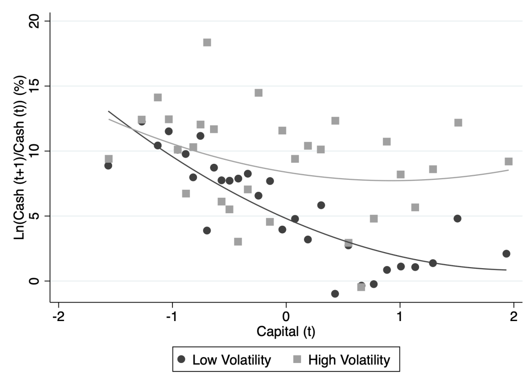
$r_{i,\tau}$ is the natural logarithm of day $\tau \in t$ gross excess return on firm i's stock, $\overline{r_{i,t}}$ is the mean of the logarithms of gross daily returns on firm i's stock during year t, and n_t is the number of nonmissing return observables during year t. We use logarithmic returns to mitigate the potential mechanical effect of return skewness (see Duffee (1995)) on the relation between returns and contemporaneous return volatilities. The change in volatility in year t, $\Delta VOL_{i,t}$, is computed as the difference between the estimated volatility in year t and the estimated volatility in year $t-1$:

$$\Delta VOL_{i,t} = VOL_{i,t} - VOL_{i,t-1} \quad (16)$$

In Figure 14b, a firm-year falls in the "High Volatility" group if the change in a firm's volatility ($\Delta VOL_{i,t}$) from year $t-1$ to year t is one standard deviation above the average year-to-year change in daily price volatility for that firm in the sample. Consistent with the prior literature, Figure 14b shows that when volatility increases, firms increase cash holdings as the "High Volatility" group generally increases the cash reserve more than the "Low Volatility" group. Our contribution is to show that the cash behavior of large firms responds more to increases in volatility. This cross-sectional difference in the sensitivity of cash holdings to volatility is evidenced by the widening of the changes in cash between low and high volatility firm-years as the capital stock increases.



(a) The vertical axis is the model predicted marginal value of cash (∂V_c), and the horizontal axis is capital k. The blue line is when volatility is low ($\sigma = 0.04$), and the red line is when volatility is high ($\sigma = 0.14$). The difference in the red and blue lines is generally increasing, indicating that the marginal value of cash for larger firms varies more with volatility.



(b) The vertical axis shows the change in cash from year t to $t+1$. The horizontal axis is a firm's capital standardized within firm. "High Volatility" firms experience a change in their average daily price volatility of returns in year t that is one standard deviation above the average year-to-year change in daily price volatility for that firm in the sample.

Figure 14: Change in Cash and Volatility

To evaluate this hypothesis more rigorously, we estimate the following empirical specification:

$$\text{Ln}\left(\frac{\text{Cash}_{i,t+1}}{\text{Cash}_{i,t}}\right) = \beta_1 \text{Capital}_{i,t} + \beta_2 \Delta\text{VOL}_{i,t} + \beta_3 \text{Capital}_{i,t} \times \Delta\text{VOL}_{i,t} + \mu_i + \delta_{j,t} + \epsilon_{i,t}. \quad (17)$$

The outcome is the change in the cash reserve from the end of year t to the end of year

$t+1$. β_1 captures the relation between the change in the cash reserve and the size of a firm's total capital (physical capital plus intangible capital). Capital is standardized within firm. Our expectation is that $\beta_1 < 0$ because larger firms have a lower marginal value of cash. β_2 captures the relation between changes in the cash reserve and changes in the firm's average daily volatility of excess returns between years $t-1$ and t. Changes in the volatility of stock returns proxies for changes in the uncertainty in a firm's future cash flows. We standardize changes in volatility within firm. Our expectation is that $\beta_2 > 0$ because increases in volatility increase the marginal value of cash, which incentivize larger cash reserves. β_3 is the interaction of interest for **H4**. We predict that $\beta_3 > 0$, which indicates that cash holdings are more sensitive to volatility when a firm is larger. We control for firm fixed effects μ_i (because the outcome is not within firm). $\epsilon_{i,t}$ is the unexplained variation. Standard errors are double-clustered by firm and year.

Table 5 presents results consistent with **H4**. Column (1) shows that a firm holds less cash on average when the firm is larger, consistent with the marginal value of cash being lower when a firm is larger. Column (1) also shows that when the volatility of the firm's stock increases, the firm hoards more cash in the following year. Lastly, Column (1) shows the sensitivity of cash holdings to volatility is larger when the firm is larger. Columns (2) and (3) examine the result before and after the median year in this sample of 1994. While the sensitivity of cash holdings to volatility is similar, the interaction with size is more significant in the earlier part of the sample. Column (4) to (10) find a positive coefficient on the interaction term of size and volatility for all industries. The coefficient is statistically significant for three of the seven industries. In Internet Appendix Table D.13, we find similar results using a firm's average daily volatility (rather than changes in volatility), the average VIX, the year-over-year change in the VIX, and the implied volatility derived from near-to-maturity call options. In Internet Appendix Table D.14, we also find similar results in a quarterly panel using changes in a firm's historical cash flow volatility as a proxy for changes in a firm's expected cash flow volatility. In Internet Appendix Table D.15, we also find similar results controlling for changes in the average return from year $t-1$ to year t since changes in volatility may coincide with changes in expected performance.

5. Conclusion

Our two-state Cash-Cap model of the firm provides new insights into how cash and capital jointly determine a firm's optimal dynamics in the presence of costly financing and bankruptcy. We present the solution to the firm's two-state decision problem and provide proofs of convergence in Appendix A and Appendix B. The model's predictions connect several existing empirical studies and also provide novel predictions for future work. We examine and find support for several novel predictions about a firm's dynamic policies for issuance size, payout amounts, investment rates, and cash holdings.

Appendix A. Omitted proofs

The following result provides a sufficient condition for a strategic default region to exist. This result shows that a strategic default region exists when the debt coupon rate b is higher than a threshold, which is an increasing function of the expected productivity μ. Therefore, firms with lower expected productivity are more likely to have a strategic default region. This result also provides for the existence of a strong type of strategic default region, for which strategic default is optimal regardless of initial cash reserves.

Proposition 2. *For $b > \mu(1-\alpha)(\mu\alpha/(r+\delta))^{\alpha/(1-\alpha)}$, there is a region $[0,\infty) \times [0,\underline{k}]$ where strategic default is optimal for all c, i.e, $V(c,k) = c$ for $k \in [0,\underline{k}]$.*

Proof. Consider the following optimization problem:

$$\begin{aligned} V_R(k,c) &= c + V_R(k) \\ &= c + \sup_{\tau, i \geq 0} \mathbb{E}\left[\int_0^\tau e^{-rt}\Big(k^\alpha \mu - b - i_t - g(k_t,i_t)\Big)dt + 1_{\{\tau<\infty\}} e^{-r(\tau-t)}(\ell k_\tau - b/r_{\text{debt}})_+\right]. \end{aligned}$$

This is the optimization problem for a firm that is subject to neither external financing costs nor a cash liquidity premium, and that can invest and choose its default time optimally, i.e., a real option. This value function V_R dominates the value function V in Equation (8), i.e., $V(k,c) \leq V_R(k,c)$ for any (k,c). The equation for $V_R(k)$ is given by dynamic programming:

$$0 = \min\{rV_R - \sup_{i \geq 0}((i-\delta k)V_R' + k^\alpha \mu - b - i - g(k,i)), V_R - (\ell k - b/r_{\text{debt}})\}. \tag{A.1}$$

We assume that this equation satisfies the comparison principle. This can be proven as in Theorem 3, which proves comparison for a more complicated but related case.

Let

$$v(k) = (k - \underline{k})_+$$

for some $\underline{k} > 0$. We show that this is a viscosity supersolution to Equation (A.1) for some sufficiently small $\underline{k}$. First, we restrict $\underline{k}$ so that $\underline{k} \leq b/r_{\text{debt}}$; then, clearly $v - (\ell k - b/r_{\text{debt}})_+ \geq 0$. Let ϕ be any C^1 function such that $v - \phi$ attains a local minimum at k. Then,

$$\phi'(k) \in \begin{cases} \{0\}, & k < \underline{k} \\ [0,1], & k = \underline{k} \\ \{1\}, & k > \underline{k} \end{cases}.$$

For all these cases, because $\phi' \geq 1$, we obtain

$$r(k-\underline{k})_+ - k^\alpha \mu + b - \sup_{i \geq 0}((i-\delta k)\phi' - i - g(k,i)) = r(k-\underline{k})_+ + \delta k \phi' - k^\alpha \mu + b. \quad \text{(A.2)}$$

When $k < \underline{k}$, the previous expression is

$$b - k^\alpha \mu,$$

which is nonnegative for $\underline{k} \leq (b/\mu)^{1/\alpha}$.

When $k > \underline{k}$, the right-hand side of Equation (A.2) is $r(k-\underline{k}) + \delta k - k^\alpha \mu + b$. This expression is minimized at $((r+\delta)/\mu\alpha)^{1/(\alpha-1)}$, with the minimum value

$$(r+\delta)(1-1/\alpha)(\mu\alpha/(r+\delta))^{1/(1-\alpha)} - r\underline{k} + b.$$

If

$$b > (r+\delta)(1/\alpha - 1)(\mu\alpha/(r+\delta))^{1/(1-\alpha)},$$

we can choose a sufficiently small $\underline{k} > 0$ such that the previous minimum value is still nonnegative.

Finally, when $k = \underline{k}$,

$$\delta \underline{k} \phi' - (\underline{k})^\alpha \mu + b \geq -(\underline{k})^\alpha \mu + b \geq 0$$

is satisfied if we choose $\underline{k} \leq (b/\mu)^{1/\alpha}$. Combining the previous three cases, we confirm that v is a viscosity supersolution to Equation (A.1).

By the comparison principle, $V_R \leq v$, so we have

$$c \leq V(c,k) \leq c + V_R(k) \leq c + v(k) = c$$

for $k \leq \underline{k}$. The first inequality above holds because for Equation (8) in the paper, the firm can always pay out the remaining cash and default. Hence, $V(c,k) = c$ for all $k \leq \underline{k}$, which is attained by the immediate payout of all cash as dividends. □

Next, we prove Proposition 1.

Proof of Proposition 1. Because of the term $(r - \lambda_c)c$ appearing in the dynamics for c, the proof of Akyildirim et al. (2014) does not extend to this case. Instead, we define an equivalent control problem by letting

$$\tilde{c}_t = e^{-(r-\lambda_c)t} c_t, \quad \tilde{D}_t = \int_0^t e^{-(r-\lambda_c)s} dD_s, \quad \tilde{I}_t = \int_0^t e^{-(r-\lambda_c)s} dI_s.$$

Then,

$$d\tilde{c}_t = e^{-(r-\lambda_c)t} dY_t - e^{-(r-\lambda_c)t}(b + i_t + g(k_t, i_t))dt - d\tilde{D}_t + d\tilde{I}_t,$$

and the optimization problem is

$$V(k,c) = \sup_{i \geq 0, \tilde{D}, \{\sigma_j, \tilde{I}_j\}} \mathbb{E}\Big[\int_0^\tau e^{-\lambda_c t} d\tilde{D}_t - \sum_{\sigma_j \geq 0} e^{-\lambda_c \sigma_j} \Big(\tilde{I}_j + \lambda_p \tilde{I}_j + e^{-(r-\lambda_c)\sigma_j} \lambda_f \Big) \Big) \\ + 1_{\{\tau < \infty\}} e^{-r(\tau - t)} (\ell k_\tau - b/r_{\text{debt}})_+ \Big],$$

where we note that τ can be defined equivalently for c or $\tilde{c}$.

For this alternative formulation of the control problem, we note that issuance can be delayed at a discount. To make things clear, we fix an investment strategy i and a dividend strategy D, index $\tilde{c}^\nu$ by an issuance strategy $\nu = \{\sigma_j, \tilde{I}_j\}_{j \in \mathbb{N}}$ with at least one issuance time say σ_i at which $\tilde{c}^\nu_{\sigma_i} > 0$. Let $\tilde{I}_i$ be the corresponding issuance amount. Consider another issuance strategy ν^- that omits this issuance, but keep the rest of the issuance strategy as ν. Finally, construct a third strategy ν' like ν^- but with an additional issuance of size $\tilde{I}_i$ at a time $\sigma' = \inf\{t > \sigma_i : \tilde{c}^{\nu^-}_t < 0\}$.[25] Note that for the same (i, D), the increment of $\tilde{c}^\nu$ and $\tilde{c}^{\nu'}$ are the same as between σ_i and σ'. With the same issuance size, we have $\tilde{c}^\nu_{\sigma'} = \tilde{c}^{\nu'}_{\sigma'}$, so the continuation values must coincide, because the strategies are identical after σ'_i. Moreover, dividends and issuance until σ' have been identical, with one exception, for which ν' has resulted in a larger discounting factor and a smaller discounted fixed cost.

We therefore conclude that the original strategy is dominated by the one issuing equity only at $c = 0$. As this is true for any strategy, we may consider only strategies that issue equity when $c = 0$. □

Appendix B. Comparison result and numeric algorithm

Proposition 1 lets us simplify the HJB equation where $c \neq 0$. We further restrict ourselves to the case where there is a maximal permitted investment rate $i_{\max} < \infty$. The

[25] If $\tilde{c}^{\nu^-}_t$ fell below zero due to a lump sum dividend payout, we can balance out the dividend payout and the issuance to obtain the same result in the next step. If there are multiple issuances in ν between σ_i and σ', we omit all of them in ν^- and issue the sum of all missed size at σ' in ν'.

proof here only relies on the boundedness of $i_{\max}$, but not on its size. We are now ready to state the resulting HJB equation. Let $\mathcal{O} = (0, \infty) \times (0, k_{\max})$.

$$
\begin{aligned}
0 = \min \Bigg\{ rV - \sup_{i \in [0, i_{\max}]} \bigg(& \Big[i - \delta k\Big] \partial_k V + \Big[(r - \lambda_c)c + k^\alpha \mu - b - i - g(k,i)\Big] \partial_c V \\
& + \frac{1}{2} k^{2\alpha} \sigma^2 \partial^2_{cc} V \bigg), \\
\partial_c V - 1 \Bigg\} \quad & \text{in } \mathcal{O}.
\end{aligned}
\tag{B.1}
$$

At $k = 0$, $g(k,i) = \infty$ for any $i > 0$, i.e., investment is infinitely costly, so k will remain zero forever. Hence, at the boundary, the value function satisfies

$$
0 = \min\{rV - (r - \lambda_c)c + b, \partial_c V - 1\},
$$

which has the solution $V = c$. Consider, therefore, the following boundary condition (with precedence to the first one in the corner):

$$
\begin{aligned}
c &= V && \text{at } k = 0, \\
0 &= \min\{V - (\ell k - b/r_{\text{debt}}), V - \mathcal{I} V\} && \text{at } c = 0.
\end{aligned}
\tag{B.2}
$$

Here, $\mathcal{I}(V) = V(0,k) - \sup_{I \geq 0} \Big[V(I,k) - I - \lambda(I)\Big]$.

Theorem 3. *Let u and v be, respectively, possibly discontinuous viscosity sub- and supersolutions to (B.1) with the above boundary conditions. Assume further that u and v are both of linear growth in c and polynomial growth in k, i.e., they take values in $[c, c + M + p(k)]$ for some constant $M > 0$ and polynomial p. Then, $u \leq v$ everywhere in $\mathcal{O}$.*

Proof. Suppose there exists a point at which $u > v$. Fix some $\eta > 0$ and consider a maximizing sequence $(c_n, k_n)_{n \geq 1}$ to $\sup_{\mathcal{O}} e^{-\eta k}(u - v) > 0$. By the growth condition, k_n is bounded by some k^*, where k^* depends only on η. Now, for any $\zeta > 0$ small enough, there exists a point $(\bar{c}, \bar{k})$ such that $e^{-\eta \bar{k}}(u - v)(\bar{c}, \bar{k}) = \delta_\zeta \geq \sup_{\mathcal{O}} e^{-\eta k}(u - v) - \zeta > 0$. We emphasize that $\bar{k}$ remains bounded, irrespective of ζ. In particular, for any η, $\delta_\zeta / (\zeta + \sqrt{\zeta})$ can be chosen arbitrarily large.

We begin by showing that if such a point lies on the boundary $c = 0$, then there is another with the same property on the interior. Consider points $(\bar{c}, \bar{k})$ such that $\bar{c} = 0$. Then, depending on whether $u(0, \bar{k}) \leq \bar{k} - \ell / r_{\text{debt}}$ or $u(0, \bar{k}) \leq \mathcal{I} u$, we have

$$
(u - v)(0, \bar{k}) \leq \ell \bar{k} - b/r_{\text{debt}} - \max\{\ell \bar{k} - b/r_{\text{debt}}, \mathcal{I} v\} \leq 0
$$

or

$$(u-v)(0,\bar{k}) \le \mathcal{I}u - \max\{\ell\bar{k} - b/r_{\text{debt}}, \mathcal{I}v\} \le \sup_{I>0}\Big[u(I,\bar{k}) - v(I,\bar{k})\Big].$$

The first case contradicts $\delta_\zeta > 0$, and the second shows that another a point with the same properties exists in the interior. Similarly, for $\bar{k} = 0$, we also get $(u-v)(\bar{c},0) \le \bar{c} - \bar{c} = 0$. Hence, without loss of generality, we may assume $(\bar{c},\bar{k})$ lies away from $c = 0$ and $k = 0$.

Define

$$\begin{aligned}\Phi^{\epsilon,\gamma}(c,k,d,\ell) = (1-\gamma)e^{-\eta k}u(c,k) - e^{-\eta\ell}v(d,\ell)\\ - \beta(c-\bar{c})^4 - \frac{1}{2\epsilon}\Big((c-d)^2 + (k-\ell)^2\Big) \quad \text{in } \mathcal{O}\times\mathcal{O}.\end{aligned}$$

Clearly,

$$\sup_{\mathcal{O}\times\mathcal{O}} \Phi^{\epsilon,\gamma} \ge \Phi^{\epsilon,\gamma}(\bar{c},\bar{k},\bar{c},\bar{k}) = e^{-\eta\bar{k}}\Big((1-\gamma)u(\bar{c},\bar{k}) - v(\bar{c},\bar{k})\Big) > \delta_\zeta,$$

for $\gamma > 0$ small enough. In particular, for any $\gamma > 0$ and $\eta > 0$, $\Phi^{\epsilon,\gamma}$ has a maximizer $(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}, d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma})$, because of the growth conditions on u and v. Moreover, the growth conditions give an upper bound for this maximizer, depending only on γ and η. Therefore, $(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}, d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma})$ converges along a subsequence as $\epsilon \to 0$. From here on, let us only consider ϵ along this subsequence. Because the lower bound at the maximum above is independent of ϵ,

$$0 < \delta_\zeta < \liminf_{\epsilon\to 0} \Phi^{\epsilon,\gamma}(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}, d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma}),$$

which implies

$$\limsup_{\epsilon\to 0} \frac{1}{2\epsilon}\Big((c_{\epsilon,\gamma} - d_{\epsilon,\gamma})^2 + (k_{\epsilon,\gamma} - \ell_{\epsilon,\gamma})^2\Big) < \infty,$$

so $(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}), (d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma}) \to (c_\gamma, k_\gamma)$. Note that $k_\gamma \le k^*$, again because of the growth condition.

Rearranging terms and letting $\epsilon \to 0$,

$$\begin{aligned}\lim_{\epsilon\to 0} \beta(c_{\epsilon,\gamma} - \bar{c})^4 + \lim_{\epsilon\to 0} \frac{1}{2\epsilon}\Big((c_{\epsilon,\gamma} - d_{\epsilon,\gamma})^2 + (k_{\epsilon,\gamma} - \ell_{\epsilon,\gamma})^2\Big)\\ &\le \limsup_{\epsilon\to 0} e^{-\eta k_{\epsilon,\gamma}}(1-\gamma)u(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}) - e^{-\eta\ell_{\epsilon,\gamma}}v(d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma}) - \delta_\zeta\\ &\le e^{-\eta k_\gamma}\Big((1-\gamma)u(c_\gamma, k_\gamma) - v(c_\gamma, k_\gamma)\Big) - \delta_\zeta\\ &\le \zeta.\end{aligned}$$

That is,

$$\lim_{\epsilon\to 0}\beta(c_{\epsilon,\gamma}-\bar{c})^4+\lim_{\epsilon\to 0}\frac{1}{2\epsilon}\Big((c_{\epsilon,\gamma}-d_{\epsilon,\gamma})^2+(k_{\epsilon,\gamma}-\ell_{\epsilon,\gamma})^2\Big)\leq\zeta. \tag{B.3}$$

As β may be taken arbitrarily large, we ensure that $\zeta<\beta\bar{c}^4$, so that $c_\gamma>0$.

If $k_\gamma=0$, we directly obtain $u(c_\gamma,0)\leq c_\gamma\leq v(c_\gamma,0)$, which is a contradiction. Hence, (c_γ,k_γ) must lie in the interior, and so will $(c_{\epsilon,\gamma},k_{\epsilon,\gamma})$ and $(d_{\epsilon,\gamma},\ell_{\epsilon,\gamma})$ for sufficiently small ϵ.

Because the maxima are attained in interior points, we proceed to use Ishii's lemma, from which we obtain $(p^u_\gamma,X)\in\bar{J}^{2,+}(e^{-\eta k_{\epsilon,\gamma}}(1-\gamma)u(c_{\epsilon,\gamma},k_{\epsilon,\gamma}))$ and $(p^v_\gamma,Y)\in\bar{J}^{2,-}(e^{-\eta\ell_{\epsilon,\gamma}}v(d_{\epsilon,\gamma},\ell_{\epsilon,\gamma}))$ (Crandall et al., 1992, Theorem 3.2), satisfying

$$p^u_\gamma=(p^u_c,p^u_k)=(p^v_c+4\beta(c_{\epsilon,\gamma}-\bar{c})^3,p^v_k),\quad p^v_\gamma=(p^v_c,p^v_k)=\left(\frac{c_{\epsilon,\gamma}-d_{\epsilon,\gamma}}{\epsilon},\frac{k_{\epsilon,\gamma}-\ell_{\epsilon,\gamma}}{\epsilon}\right)$$

and

$$k^{2\alpha}_{\epsilon,\gamma}X-\ell^{2\alpha}_{\epsilon,\gamma}Y\leq k^{2\alpha}_{\epsilon,\gamma}12\beta(c_{\epsilon,\gamma}-\bar{c})^2+\frac{(k^\alpha_{\epsilon,\gamma}-\ell^\alpha_{\epsilon,\gamma})^2}{\epsilon}+o(1),$$

where $o(1)$ denotes a term that converges to 0 as $\epsilon\to 0$.

Because u is a subsolution, $\tilde{u}=(1-\gamma)e^{-\eta k}u$ satisfies

$$\begin{aligned}0\geq\min\Bigg\{r\tilde{u}-\sup_{i\in[0,i_{\max}]}&\Bigg(\Big[i-\delta_\zeta k_{\epsilon,\gamma}\Big](\eta\tilde{u}+\partial_k\tilde{u})\\&+\Big[(r-\lambda_c)c_{\epsilon,\gamma}+k^\alpha_{\epsilon,\gamma}\mu-b-i-g(k_{\epsilon,\gamma},i)\Big]\partial_c\tilde{u}\\&+\frac{1}{2}k^{2\alpha}_{\epsilon,\gamma}\sigma^2\partial^2_{cc}\tilde{u}\Bigg),\\&\partial_c\tilde{u}-(1-\gamma)e^{-\eta k_{\epsilon,\gamma}}\Bigg\}.\end{aligned}\tag{B.4}$$

Similarly, $\tilde{v}=e^{-\eta k}v$ satisfies

$$\begin{aligned}0\leq\min\Bigg\{r\tilde{v}-\sup_{i\in[0,i_{\max}]}&\Bigg(\Big[i-\delta_\zeta\ell_{\epsilon,\gamma}\Big](\eta\tilde{v}+\partial_k\tilde{v})\\&+\Big[(r-\lambda_c)d_{\epsilon,\gamma}+\ell^\alpha_{\epsilon,\gamma}\mu-b-i-g(\ell_{\epsilon,\gamma},i)\Big]\partial_c\tilde{v}\\&+\frac{1}{2}\ell^{2\alpha}_{\epsilon,\gamma}\sigma^2\partial^2_{cc}\tilde{v}\Bigg),\\&\partial_c\tilde{v}-e^{-\eta\ell_{\epsilon,\gamma}}\Bigg\}.\end{aligned}\tag{B.5}$$

We split into two cases, depending on which expression is smallest in Equation (B.4).

We begin with the simple case of

$$p_c^u \leq (1-\gamma)e^{-\eta k_{\epsilon,\gamma}}.$$

Subtracting the two equations (B.4) and (B.5) thus gives

$$4\beta(c_{\epsilon,\gamma}-\bar{c})^3 = p_c^u - p_c^v \leq (e^{-\eta k_{\epsilon,\gamma}} - e^{-\eta \ell_{\epsilon,\gamma}}) - \gamma e^{-\eta k_{\epsilon,\gamma}}.$$

Letting $\epsilon \to 0$ in the last inequality,

$$4\beta(c_\gamma - \bar{c})^3 \leq -\gamma e^{-\eta k_\gamma},$$

which contradicts with Equation (B.3) because ζ can be chosen arbitrarily small, independently of k^*.

In the other case, we subtract the equations and get

$$\begin{aligned}
r(\tilde{u}-\tilde{v}) \leq \sup_{i\in[0,i_{\max}]} & \Big\{ \Big[i - \delta_\zeta k_{\epsilon,\gamma}\Big](\eta \tilde{u}(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}) + p_k^u) \\
& + \Big[(r-\lambda_c)c_{\epsilon,\gamma} + k_{\epsilon,\gamma}^\alpha \mu - b - i - g(k_{\epsilon,\gamma}, i)\Big](p_c^v + 4\beta(c_{\epsilon,\gamma}-\bar{c})^3) + \frac{1}{2}k_{\epsilon,\gamma}^{2\alpha}\sigma^2 X \\
& - \Big[i - \delta_\zeta \ell_{\epsilon,\gamma}\Big](\eta \tilde{v}(d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma}) + p_k^v) \\
& - \Big[(r-\lambda_c)d_{\epsilon,\gamma} + \ell_{\epsilon,\gamma}^\alpha \mu - b - i - g(\ell_{\epsilon,\gamma}, i)\Big]p_c^v - \frac{1}{2}\ell_{\epsilon,\gamma}^{2\alpha}\sigma^2 Y \Big\} \\
\leq \sup_{i\in[0,i_{\max}]} & \Big\{ i\eta(\tilde{u}(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}) - \tilde{v}(d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma})) \\
& + \Big[(r-\lambda_c)c_{\epsilon,\gamma} + k_{\epsilon,\gamma}^\alpha \mu - b - i - g(k_{\epsilon,\gamma}, i)\Big]4\beta(c_{\epsilon,\gamma}-\bar{c})^3 \\
& - \delta_\zeta(\ell_{\epsilon,\gamma} - k_{\epsilon,\gamma})p_k^u + \Big[(k_{\epsilon,\gamma}^\alpha - \ell_{\epsilon,\gamma}^\alpha)\mu - (g(k_{\epsilon,\gamma}, i) - g(\ell_{\epsilon,\gamma}, i))\Big]p_c^v \\
& + 6k_{\epsilon,\gamma}^{2\alpha}\sigma^2\beta(c_{\epsilon,\gamma}-\bar{c})^2 + \frac{(k_{\epsilon,\gamma}^\alpha - \ell_{\epsilon,\gamma}^\alpha)^2}{\epsilon} \Big\} + o(1).
\end{aligned}$$

Let $\eta < (r-\Delta)/i_{\max}$ for $\Delta \in (0,r)$. Then, taking $\limsup$ as $\epsilon \to 0$, and using that $g(\cdot, i)$ and $k \mapsto k^\alpha$ are Lipschitz in the neighborhood of (c_γ, k_γ), i.e.,

$$|g(k_{\epsilon,\gamma}, i) - g(\ell_{\epsilon,\gamma}, i)| + \mu|k_{\epsilon,\gamma}^\alpha - \ell_{\epsilon,\gamma}^\alpha| \leq R|k_{\epsilon,\gamma} - \ell_{\epsilon,\gamma}|,$$

we get

$$\limsup_{\epsilon\to 0} \Delta(\tilde{u}(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}) - \tilde{v}(d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma}))$$
$$\leq \lim_{\epsilon\to 0} \Bigg[(\delta_\zeta + R^2)\frac{(k_{\epsilon,\gamma} - \ell_{\epsilon,\gamma})^2}{\epsilon} + R\frac{(c_{\epsilon,\gamma} - d_{\epsilon,\gamma})}{\sqrt{\epsilon}}\frac{(k_{\epsilon,\gamma} - \ell_{\epsilon,\gamma})}{\sqrt{\epsilon}}$$
$$+ R'(|c_{\epsilon,\gamma} - \bar{c}|^2 + |c_{\epsilon,\gamma} - \bar{c}|^3) + o(1)\Bigg]$$

for some constant R', depending on k^* (i.e., η), $i_{\max}$, β, and the model parameters. In other words, R' is independent of ζ. By Equation (B.3), the right-hand side is bounded by $R''(\zeta + \sqrt{\zeta})$, for some constant $R'' > 0$ that is also independent of ζ. Finally, because $\Delta > 0$,

$$\delta_\zeta \leq e^{-\eta k_\gamma}((1-\gamma)u - v)(c_\gamma, k_\gamma) \leq \limsup_{\epsilon\to 0}(\tilde{u}(c_{\epsilon,\gamma}, k_{\epsilon,\gamma}) - \tilde{v}(d_{\epsilon,\gamma}, \ell_{\epsilon,\gamma})) \leq \frac{R''}{\Delta}(\zeta + \sqrt{\zeta}),$$

which is a contradiction because $\delta_\zeta/(\zeta + \sqrt{\zeta})$ can be chosen arbitrarily large. Hence, there cannot exist a point (c, k) such that $(u - v)(c, k) > 0$.

□

Following the steps of the proof of Proposition 2, it can be easily proven that $V(c, k) \leq M + c + k$ for some M. As a consequence, V satisfies the assumptions of Theorem 3. The following results are standard consequences of comparison of viscosity solutions.

Corollary 4. *The value function V is the (continuous) unique solution to Equation* (B.1) *with its boundary conditions.*

We now present our numerical algorithm to solve the model. Equation (B.1) is solved in a square domain $[0, c_{\max}] \times [0, k_{\max}]$ via policy iteration, which produces the value function $V(k, c)$ and investment policy function $i(k, c)$ in addition to the regions of dividend and equity issuance. The singular structure of the dividends is approximated as in (Reppen et al., 2020, Section 4), which also describes the policy iteration algorithm, and the impulse control issuance as in (Reppen et al., 2020, Section 6.1.2).

In addition to (B.2), the boundary conditions where $c = c_{\max}$ and $k_{\max}$ are given by

$$0 = \partial_c V - 1 \qquad \text{at } c = c_{\max}$$
$$0 = \min\left\{ rV + \delta k \partial_k V - \Big[rc + k^\alpha \mu - b \Big] \partial_c V - \frac{1}{2}k^{2\alpha}\sigma^2 \partial^2_{cc} V, \partial_c V - 1, \mathcal{I}V \right\} \quad \text{at } k = k_{\max}$$

At the corners, the c-conditions are used.

Another consequence of the comparison result in Theorem 3 is the convergence of the numerical scheme (see Barles and Souganidis (1991)).

Corollary 5. *Numerical solutions converge to the value function as the discretization gets finer.*

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Table 1: Model Parameters

Parameter	Name	Values	Comments
r	Interest rate	6%	In line with long term average yield to maturity on 30 year U.S. Treasuries.
λ_c	Cash holding cost, liquidity premium	1%	In line with Bolton et al. (2011, 2019).
μ	Expected productivity shock	0.18	In line with the estimates of Eberly et al. (2009) for large U.S. firms.
σ	Volatility of productivity shock	0.09	In line with the estimates of Eberly et al. (2009) for large U.S. firms.
θ	Degree of adjustment cost	1.5	See Whited (1992).
δ	Depreciation rate	10.07%	In line with the estimates of Eberly et al. (2009) for large U.S. firms.
b	Long-term debt coupon rate	0.02	Together with r_{debt} below yields a long-term debt face value 0.22.
λ_p	Variable issuance cost	6.4%	In line with the estimates of Altınkılıç and Hansen (2000).
λ_f	Fixed issuance cost	0.05	λ_f is chosen so that the fixed issuance cost for firms of average size in our model is at the same magnitude as the fixed cost rate in Bolton et al. (2011) multiplied by our average size.
α	Curvature of production function. When $\alpha < 1$, then diminishing returns to scale	0.7	$\alpha = 0.75$ in Riddick and Whited (2009) and $\alpha = 0.627$ with std 0.219 for the full sample of firms in Hennessy and Whited (2007).
ℓ	Recovery rate in liquidation of capital	90%	The choice of ℓ is consistent with Hennessy and Whited (2007), where the recovery rate is estimated to be 0.896 for the full sample of firms.
r_{debt}	Cost of financing for long-term debt	9%	r_{debt} is chosen as $1.5r$. Results are insensitive to this parameter, because firms are left with no proceeds after the bondholder is repaid after liquidation in most of numeric experiments.

Table 2: H1: Amount issued as a proportion of capital is declining and convex in capital

The outcome variable is the amount of common stock sold in year $t+1$ divided by the total capital of the firm (tangible plus intangible) at the end of year t. The sample is all offerings in year $t+1$ that exceed 5% of capital as of the end of year t. The main explanatory variable is the total capital of the firm (standardized within firm) and its square (standardized). We control for the cash reserve (standardized within firm), industry trends, and firm fixed effects. Columns (2) and (3) split the sample after and on or before 1997, respectively. Column (4) restricts the sample to larger offerings—those exceeding 10% of capital. Columns (5) to (11) restrict the sample to industries based on the SIC-1 identifier. See Internet Appendix Table D.2 for SIC-1 category descriptions. Standard errors are double-clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Amt Raised (t+1) / Capital (t) $\times$ 100										
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Capital (t)	-8.36***	-8.28***	-8.00***	-9.98***	-12.26***	-9.59***	-6.81***	-8.20***	-9.96***	-6.28***	-7.76***
	(0.45)	(0.72)	(0.85)	(0.64)	(1.14)	(1.52)	(0.89)	(1.77)	(1.95)	(1.26)	(1.95)
Capital (t)^2	3.86***	2.99***	3.93***	4.37***	5.52***	3.47***	3.81***	2.52*	4.54***	4.23***	2.87*
	(0.41)	(0.56)	(0.82)	(0.61)	(0.75)	(1.01)	(0.61)	(1.28)	(1.29)	(0.78)	(1.68)
Cash (t)	-3.30***	-3.48***	-4.53***	-2.79***	-0.05	-3.76***	-4.50***	-3.55*	-1.58	-4.64***	-0.66
	(0.41)	(0.66)	(0.82)	(0.55)	(1.15)	(1.02)	(0.92)	(1.88)	(1.28)	(1.03)	(1.28)
Constant	22.29***	22.42***	21.56***	31.48***	26.30***	23.40***	21.50***	25.85***	21.60***	19.28***	23.31***
	(0.20)	(0.25)	(0.52)	(0.32)	(0.24)	(0.57)	(0.33)	(0.69)	(0.87)	(0.62)	(0.73)
Specification	All	Yr>'97	Yr≤'97	Off.>10%	SIC1=1	SIC1=2	SIC1=3	SIC1=4	SIC1=5	SIC1=7	SIC1=8
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
SIC-2$\times$Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
% Adjusted R^2	31.28	35.20	29.67	32.35	33.68	22.85	27.29	38.54	35.15	36.85	30.96
% Within R^2	8.42	7.35	7.79	10.04	14.92	8.12	7.75	7.54	10.58	9.03	5.44
Observations	10,107	4,544	4,765	5,526	1,184	1,525	3,431	708	932	1,729	581

Table 3: H2: The total payouts to equityholders scaled by capital is declining and concave in capital

The outcome variable is the total dividends paid in year $t + 1$ plus the total value of the shares repurchased divided by the total capital of the firm (tangible plus intangible) at the end of year t. The main explanatory variable is the total capital of the firm (standardized within firm) and its square (standardized). We control for the cash reserve (standardized within firm), industry trends, and firm fixed effects. Columns (2) and (3) split the sample after and on or before 1994, respectively. Columns (4) to (10) restrict the sample to industries based on the SIC-1 identifier. See Internet Appendix Table D.2 for SIC-1 category descriptions. Standard errors are double-clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Dividends (t+1) / Capital (t) × 100									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Capital (t)	-0.48***	-0.44***	-0.61***	-0.65***	-0.51***	-0.52***	-0.81***	-0.14	-0.53***	-0.54***
	(0.05)	(0.09)	(0.06)	(0.15)	(0.10)	(0.09)	(0.18)	(0.11)	(0.15)	(0.19)
Capital (t)^2	-0.24***	-0.35***	0.05	0.05	-0.22***	-0.19***	-0.11	-0.25***	-0.57***	-0.51***
	(0.05)	(0.06)	(0.04)	(0.09)	(0.06)	(0.06)	(0.10)	(0.08)	(0.13)	(0.14)
Cash (t)	0.98***	1.14***	0.66***	0.54***	0.97***	1.15***	0.69***	0.89***	1.10***	0.95***
	(0.05)	(0.07)	(0.04)	(0.12)	(0.08)	(0.07)	(0.13)	(0.11)	(0.15)	(0.19)
Constant	3.84***	4.30***	3.28***	2.54***	4.24***	3.88***	3.43***	3.23***	4.87***	3.97***
	(0.00)	(0.02)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
Specification	All	Yr>'94	Yr≤'94	SIC1=1	SIC1=2	SIC1=3	SIC1=4	SIC1=5	SIC1=7	SIC1=8
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
SIC-2×Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
% Adjusted R^2	36.15	39.76	39.74	27.16	39.35	30.70	33.79	37.18	41.03	40.71
% Within R^2	2.68	3.05	1.99	1.81	3.31	3.20	2.26	2.57	2.55	2.44
Observations	96520	48041	47863	7391	19721	33012	7224	14122	10574	4075

Table 4: H3: The sensitivity of investment to the cash-to-capital ratio is declining in capital

The outcome variable is investment in physical and intangible capital in year $t+1$ divided by the amount of depreciation of tangible and intangible capital in year t. The main explanatory variable is the cash-to-capital ratio at the end of year t (standardized within firm). We interact the cash-to-capital ratio with the firm's total capital at the end of year t (standardized within firm). Columns (2) and (3) split the sample after and on or before 1993, respectively. Columns (4) to (10) restrict the sample to industries based on the SIC-1 identifier. See Internet Appendix Table D.2 for SIC-1 category descriptions. Standard errors are double-clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Investment (t+1) / Depreciation (t) × 100									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Cash-to-Capital (t)	13.28***	17.52***	10.18***	18.45***	8.66***	13.39***	16.67***	12.68***	15.39***	13.11***
	(0.79)	(1.01)	(0.87)	(3.48)	(0.88)	(0.99)	(2.53)	(1.35)	(2.51)	(2.31)
Capital (t)	-34.23***	-30.63***	-36.06***	-45.42***	-25.19***	-29.84***	-39.07***	-41.87***	-42.80***	-30.44***
	(1.71)	(3.14)	(1.59)	(5.83)	(1.70)	(1.82)	(4.29)	(3.17)	(3.73)	(4.31)
Cash-to-Capital (t) × Capital (t)	-12.01***	-13.61***	-9.09***	-10.01***	-8.26***	-10.13***	-16.34***	-15.77***	-18.42***	-15.46***
	(1.13)	(1.54)	(1.14)	(3.56)	(1.19)	(1.44)	(3.16)	(1.73)	(2.73)	(3.36)
Constant	154.11***	140.00***	167.35***	245.29***	132.86***	142.18***	169.78***	165.38***	147.91***	141.43***
	(0.30)	(0.42)	(0.54)	(0.71)	(0.35)	(0.30)	(0.76)	(0.56)	(1.01)	(0.98)
Specification	All	≤'93	>'93	SIC1=1	SIC1=2	SIC1=3	SIC1=4	SIC1=5	SIC1=7	SIC1=8
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
SIC-2×Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
% Adjusted R^2	51.70	57.49	49.69	46.87	44.06	46.27	49.52	53.92	52.43	59.73
% Within R^2	9.25	11.14	6.54	5.31	7.02	11.44	8.80	13.56	13.46	8.69
Observations	109678	55507	53638	8188	23112	37984	8214	15529	11720	4494

Table 5: H4: Volatility has a larger affect on the cash balances of larger firms

The outcome variable is the change in cash from the end of year t to the end of year $t+1$ (standardized within firm). Capital is the firm's total capital at the end of year t (standardized within firm). ΔVOL is the change in a firm's average daily volatility from year $t-1$ to t (standardized within firm). Columns (2) and (3) split the sample after and on or before 1994, respectively. Columns (4) to (10) restrict the sample to industries based on the SIC-1 identifier. See Internet Appendix Table D.2 for SIC-1 category descriptions. Standard errors are clustered by firm. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Ln(Cash (t+1)/Cash(t))× 100									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Capital (t)	-3.01***	-3.53***	-4.71***	-0.01	-0.02***	-0.03***	-0.03***	-0.04***	-0.04***	-0.03***
	(0.23)	(0.39)	(0.40)	(0.01)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
Ch. Volatility (t)	0.85***	1.35***	1.02**	-0.00	0.02***	0.02***	0.01	-0.01	-0.01	-0.01
	(0.31)	(0.47)	(0.45)	(0.01)	(0.01)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
Capital (t) × Ch. Volatility (t)	1.25***	1.92***	0.85	0.04**	0.00	0.01**	0.01	0.02**	0.00	0.01
	(0.36)	(0.56)	(0.52)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
Constant	6.26***	4.72***	8.24***	0.06***	0.06***	0.07***	0.07***	0.06***	0.06***	0.06***
	(0.03)	(0.13)	(0.10)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Specification	All	≤'94	>'94	SIC1=1	SIC1=2	SIC1=3	SIC1=4	SIC1=5	SIC1=7	SIC1=8
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
% Adjusted R^2	-3.65	-4.08	-4.57	-3.89	-3.36	-3.39	-3.96	-3.71	-3.66	-4.71
% Within R^2	0.13	0.14	0.22	0.12	0.09	0.18	0.14	0.19	0.24	0.13
Observations	101275	51092	49592	7212	21526	35651	7401	14318	10733	4051

Internet Appendix to
The Cash-Cap Model:
A Two-State Model of Firm Dynamics

This Internet Appendix contains supplementary analyses. These include the following:

1. Appendix C provides additional model outputs
   (a) Figure C.1 repeats Figure 1 without the ability to default strategically and seize the cash.
2. Appendix D provides additional empirical work.
   (a) Table D.1 shows the sample selection criteria.
   (b) Table D.2 describes the SIC-1 categories used for robustness in the main tables.
   (c) Section Appendix D.1 provides robustness for the empirical support of **H1**.
      - Figure D.1 replicates Figure 11b using different cutoffs for offering size.
      - Figure D.2 replicates Figure 11b using offering data from SDC Platinum and different cutoffs for offering size.
      - Table D.3 repeats Table 2 without firm fixed effects.
      - Table D.4 repeats Table 2 using offering data from SDC.
      - Table D.5 repeats Table 2 dropping IPOs.
      - Table D.6 repeats Table 2 controlling for the size of the previous offering.
      - Table D.7 repeats Table 2 without standardizing capital and cash within firm.
      - Table D.8 shows that a firm's cash and capital improve the explanatory power of the specification in Chang et al. (2006).
      - Tables D.9 and D.10 show that a firm's cash and capital matter for the tests in Hovakimian et al. (2001).
   (d) Section Appendix D.2 provides robustness for the empirical support of **H2**.
      - Table D.11 repeats Table 3 without standardizing capital and cash within firm.
      - Table D.12 shows that a firm's cash and capital improve the explanatory power and significance of the tests in Bliss et al. (2015).
   (e) Section Appendix D.3 provides robustness for the empirical support of **H4**.
      - Table D.13 repeats Table 5 using the total volatility, VIX, change in VIX, and option implied volatility.
      - Table D.14 repeats Table 5 at the quarterly frequency using changes in a firm's cash flow volatility over the prior 16 quarters as a proxy for changes in a firm's expected cash flow volatility.
      - Table D.15 repeats Table 5 controlling for changes in average returns from years $t+1$ to t.

(f) Section Appendix D.4 provides evidence that the default region expands with leverage so that more levered firms running out of cash are more likely to choose to default rather than issue equity.

- Figure D.3a shows a reduction in offerings when capital is low and leverage is high.
- Figure D.3b shows an increase in default when capital is low and leverage is high.
- Table D.16 shows a decrease in the probability of issuing equity and an increase in the probability of defaulting when capital is low and leverage is high.
- Figure D.4a repeats Figure D.3a using offering data from SDC.
- Table D.17 repeats Table D.16 without firm fixed effects.
- Table D.18 replicates Table D.16 Panel A using data from SDC.

(g) Section Appendix D.5 provides support in the data for the model's predicted relation between the investment-to-depreciation ratio and capital.

- Figure D.5(a) shows that the model predicts a convex relation between the investment-to-depreciation ratio and capital. Figure D.5(b) provides empirical support.
- Table D.19 provides evidence of the convex relation between the investment-to-depreciation ratio and capital.
- Table D.20 shows that firms in the data generally exhibit diminishing returns to scale.

Appendix C.

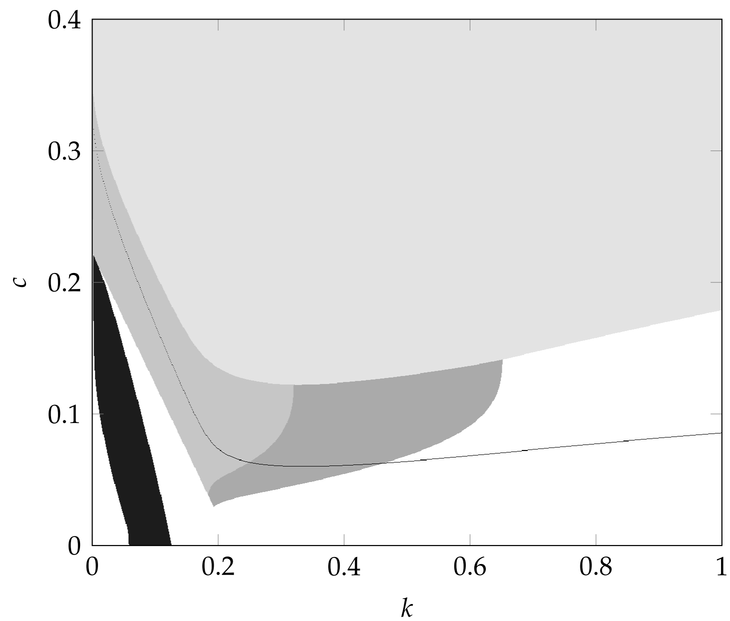


Figure C.1: **Repeats Figure 1 after eliminating the opportunity to seize the cash in default (i.e., no strategic defaulting)**

Discussion: In this figure, we forbid the firm from paying a dividend when capital is below the face value of debt, i.e., the firm is insolvent. In this setup, the important takeaway is that the dynamics for much of the (k, c) state space look similar to those in Figure 1. For example, the issuance target and dividend payout boundaries are convex.

There are some differences in the left-hand corner: (1) The new black region is an issuance region, at which the firm issues a lump sum of equity that places the firm on the issuance target line. This new issuance region emerges because removing the option of seizing the cash in a strategic default raises the stakes for equityholders because default results in losing the cash and capital. Instead, the equityholders determine whether it is better to wait and avoid the issuance cost or to issue, moving away from default and invest. When the combination of cash and capital is large enough (evidenced by the negative slope in the black region), the equityholders prefer to issue and avoid default. Issuing leads to investment and dividends more quickly.

(2) In between the black issuance region and the grey investment regions is a new continuation region. In this continuation region, the equityholders prefer to wait to avoid the issuance costs. While the drift in capital is negative without investment, the expected productivity shocks are positive, leading to an increase in cash. The expected northwest movement means that it is possible that the firm can avoid issuing altogether and reenter the investment region within a reasonable amount of time.

(3) In the bottom left corner is another continuation region. This region is an example of debt overhang. Equityholders are not willing to issue new equity because the benefits of issuing primarily accrue to the debtholders in the form of safer debt and higher recoveries. If the firm runs out of cash in this region, the firm chooses default.

Parameters used are summarized in Table 1.

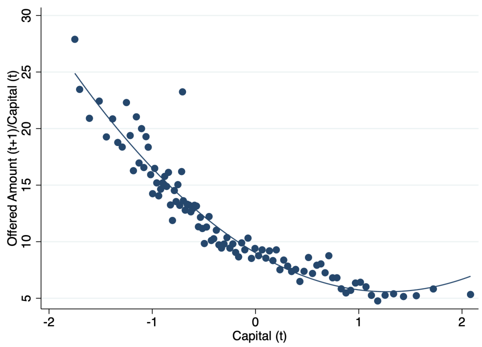
Appendix D. Empirical Appendix

Table D.1: Sample Selection

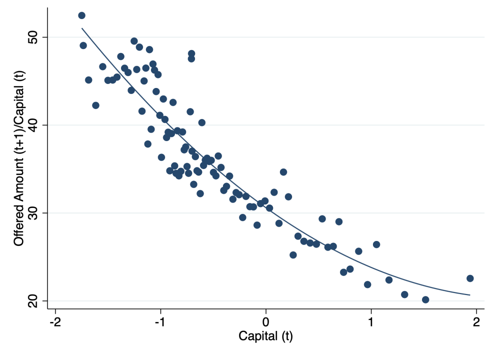
Criteria	Obs. Lost	Obs. Remaining
COMPUSTAT, 01/01/1950 – 06/30/2021		578,958
Less:		
Firms missing Intangible Capital Data (1960-2017)	(94,440)	484,518
Pre-IPO Data	(28,778)	455,740
Firms headquartered outside of USA	(77,440)	378,300
Firms incorporated outside of USA	(4,117)	374,183
Financials (SIC-1==6)	(94,916)	277,267
Utilities (SIC-2==49)	(18,997)	258,270
Public Administration (SIC-1==9)	(4,943)	253,327
Firms with missing or zero assets	(8,009)	245,318
Firms with missing common stock price at close of fiscal year	(39,158)	206,160
Firms with missing shares outstanding	(577)	205,583
Firms with missing or zero book value of equity	(1,547)	204,036
Firms with cash and cash equivalents	(24)	204,012
Firms with missing or zero total liabilities	(363)	203,649
Firms with missing net income	(440)	203,209
Firms with missing operating income	(7)	203,202
Firms with missing retained earnings	(2,124)	201,078
Firms with missing working capital	(4,493)	196,585
Firms with PP&E less than $5M or missing PP&E	(67,706)	128,879
Firms with negative cash and cash equivalents	(9)	128,870
Firms with less than $1M in sales	(1,056)	127,814
Firms with zero market equity	(35)	127,779
Singleton Firms	(1,066)	126,713
SIC-4 industries with one firm	(2,920)	123,793
Final sample (10,027 firms)		123,793

Table D.2: SIC Code Reference

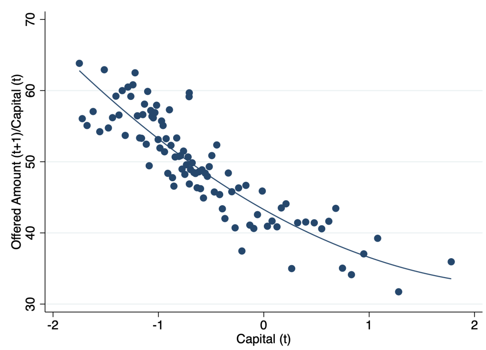
SIC First Digit	Category Description
1	Mining & Construction
2	Manufacturing (Food, Apparel, Furniture, Chemicals, Petroleum)
3	Manufacturing (Rubber, Leather, Stone, Metals, Transportation Equipment)
4	Transportation and Public Utilities
5	Wholesale and Retail Trade
7	Services (Hotel, Personal, Business, Auto Repair)
8	Services (Health, Legal, Educational, Social, Museums)



(a) The sample is the set of firms issuing shares worth at least 1% of their capital stock in year $t+1$. The y-axis represents the amount offered scaled by a firm's total capital. The x-axis shows a firm's total capital (tangible and intangible) standardized within firm.

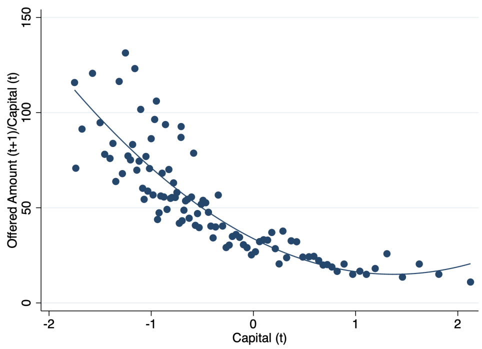


(b) The sample is the set of firms issuing shares worth at least 10% of their capital stock in year $t+1$. The y-axis shows the amount offered scaled by a firm's total capital. The x-axis represents a firm's total capital (tangible and intangible) standardized within firm.

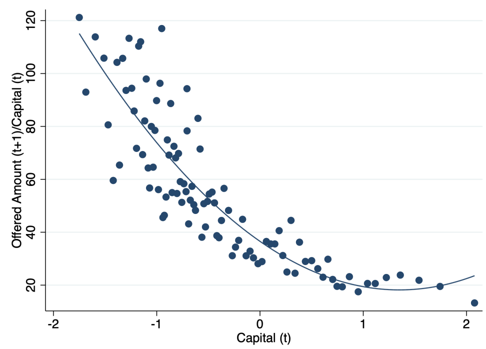


(c) The sample is the set of firms issuing shares worth at least 20% of their capital stock in year $t+1$. The y-axis represents the amount offered scaled by a firm's total capital. The x-axis plots a firm's total capital (tangible and intangible) standardized within firm.

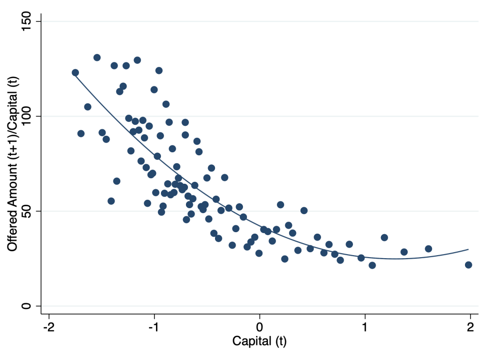
Figure D.1: **Evaluating H1 using different cutoffs for offering size**



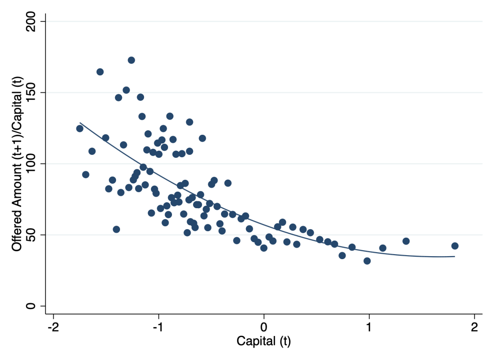
(a) The sample is the set of firms issuing shares worth at least 1% of their capital stock in year $t+1$. The y-axis represents the amount offered scaled by a firm's total capital. The x-axis shows a firm's total capital (tangible and intangible) standardized within firm.



(b) The sample is the set of firms issuing shares worth at least 5% of their capital stock in year $t+1$. The y-axis represents the amount offered scaled by a firm's total capital. The x-axis shows a firm's total capital (tangible and intangible) standardized within firm.



(c) The sample is the set of firms issuing shares worth at least 10% of their capital stock in year $t+1$. The y-axis represents the amount offered scaled by a firm's total capital. The x-axis shows a firm's total capital (tangible and intangible) standardized within firm.



(d) The sample is the set of firms issuing shares worth at least 20% of their capital stock in year $t+1$. The y-axis represents the amount offered scaled by a firm's total capital. The x-axis shows a firm's total capital (tangible and intangible) standardized within firm.

Figure D.2: **Evaluating H1 using Offering Data from SDC Platinum and Different Cutoffs for Offering Size**

Table D.3: Repeats Table 2 without firm fixed effects

The outcome variable is the amount of common stock sold in year $t+1$ divided by the total capital of the firm (tangible plus intangible) at the end of year t. The sample is all offerings in year $t+1$ that exceed 5% of capital as of the end of year t. The main explanatory variable is the total capital of the firm (standardized within firm) and its square. We control for the cash reserve (standardized within firm) and industry trends. Columns (2) and (3) split the sample after and on or before 1997, respectively. Column (4) restricts the sample to larger offerings—those exceeding 10% of capital. Columns (5) to (11) restrict the sample to industries based on the SIC-1 identifier. Standard errors are double-clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Amt Raised (t+1) / Capital (t) × 100										
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Capital (t)	-7.54***	-7.13***	-7.88***	-8.44***	-10.69***	-9.32***	-5.61***	-7.31***	-9.29***	-6.24***	-8.62***
	(0.40)	(0.49)	(0.62)	(0.48)	(0.88)	(0.72)	(0.60)	(1.25)	(1.00)	(0.83)	(1.32)
Capital $(t)^2$	2.22***	1.94***	2.44***	1.83***	3.34***	2.56***	2.26***	0.46	2.87***	2.17***	2.20*
	(0.34)	(0.51)	(0.43)	(0.42)	(0.72)	(0.78)	(0.47)	(0.78)	(0.94)	(0.72)	(1.28)
Cash (t)	-3.17***	-3.64***	-2.72***	-2.91***	-0.51	-3.75***	-4.28***	-2.57**	-1.31	-4.29***	-2.53**
	(0.34)	(0.32)	(0.62)	(0.41)	(0.95)	(0.80)	(0.64)	(1.18)	(1.01)	(0.94)	(1.06)
Constant	22.89***	23.12***	22.63***	31.84***	26.04***	23.15***	22.19***	25.89***	21.89***	21.26***	23.35***
	(0.22)	(0.32)	(0.34)	(0.31)	(0.51)	(0.42)	(0.37)	(0.75)	(0.61)	(0.50)	(0.82)
Specification	All	Yr>'97	Yr≤'97	Off.>10%	SIC1=1	SIC1=2	SIC1=3	SIC1=4	SIC1=5	SIC1=7	SIC1=8
Firm FE	No	No	No	No	No	No	No	No	No	No	No
SIC-2×Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
% Adjusted R^2	14.08	13.72	13.65	13.90	15.27	14.96	11.23	20.92	20.92	11.92	12.07
% Within R^2	9.17	9.07	9.37	9.30	11.96	12.65	8.20	7.67	11.97	8.09	10.06
Observations	12708	5982	6719	8159	1415	1962	4185	908	1311	2163	740

Table D.4: Evaluating **H1** using offering data from SDC

The outcome variable is the amount of common stock sold in year $t+1$ (from SDC Platinum's offering database) divided by the total capital of the firm (tangible plus intangible) at the end of year t. The sample is all offerings in year $t+1$ that exceed 5% of capital as of the end of year t. The main explanatory variable is the total capital of the firm (standardized within firm) and its square. We control for the cash reserve (standardized within firm), industry trends, and firm fixed effects. Columns (2) and (3) split the sample after and on or before 2002, respectively. Column (4) restricts the sample to larger offerings—those exceeding 10% of capital. Standard errors are double-clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Amt Raised (t+1) / Capital (t) $\times$ 100			
	(1)	(2)	(3)	(4)
Capital (t)	-22.61***	-17.88***	-30.54**	-27.92***
	(4.32)	(3.09)	(13.72)	(6.71)
Capital (t)^2	15.00***	10.47***	16.66**	15.73***
	(2.47)	(2.58)	(6.04)	(3.11)
Cash (t)	-1.82	-0.89	0.26	0.78
	(2.61)	(3.06)	(13.40)	(4.17)
Constant	44.31***	37.27***	48.94***	50.81***
	(1.68)	(0.55)	(11.31)	(3.31)
Specification	All	Yr>'02	Yr$\leq$'02	Off.>10%
Firm FE	Yes	Yes	Yes	Yes
SIC-2$\times$Year FE	Yes	Yes	Yes	Yes
% Adjusted R^2	53.76	48.94	55.08	48.61
% Within R^2	12.85	16.63	8.08	11.77
Observations	1,356	653	495	1,006

Table D.5: Repeats Table 2 dropping IPOs

The outcome variable is the amount of common stock sold in year $t+1$ divided by the total capital of the firm (tangible plus intangible) at the end of year t. The sample is all offerings in year $t+1$ that exceed 5% of capital as of the end of year t. The main explanatory variable is the total capital of the firm (standardized within firm) and its square. We control for the cash reserve (standardized within firm) and industry trends. Columns (2) and (3) split the sample after and on or before 1997, respectively. Column (4) restricts the sample to larger offerings—those exceeding 10% of capital. Standard errors are double-clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Amt Raised (t+1) / Capital (t) × 100			
	(1)	(2)	(3)	(4)
Capital (t)	-5.77***	-5.64***	-5.60**	-6.94***
	(0.57)	(0.48)	(1.99)	(0.56)
Capital (t)^2	1.37**	0.71	2.60*	1.03
	(0.65)	(0.56)	(1.30)	(0.97)
Cash (t)	-4.30***	-4.06***	-4.92***	-4.41***
	(0.40)	(0.42)	(0.91)	(0.59)
Constant	23.77***	22.03***	27.64***	33.27***
	(0.35)	(0.38)	(1.47)	(0.43)
Specification	All	Yr>'97	Yr≤'97	Off.>10%
Firm FE	No	No	No	No
SIC-2×Year FE	Yes	Yes	Yes	Yes
% Adjusted R^2	12.83	11.32	6.78	12.70
% Within R^2	6.03	6.21	6.00	6.72
Observations	5,351	3,812	1,529	3,352

Table D.6: Repeats Table 2 controlling for the size of the last offering

The outcome variable is the amount of common stock sold in year $t+1$ divided by the total capital of the firm (tangible plus intangible) at the end of year t. The sample is all offerings in year $t+1$ that exceed 5% of capital as of the end of year t with data on the size of a previous offering. The main explanatory variable is the total capital of the firm (standardized within firm) and its square. We control for the amount raised in the prior offering, the cash reserve (standardized within firm), and year fixed effects. Columns (2) and (3) split the sample after and on or before 1997, respectively. Standard errors are double-clustered by firm. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Amt Raised (t+1) / Capital (t) $\times$ 100		
	(1)	(2)	(3)
Capital (t)	-3.67***	-4.40***	-3.81***
	(0.34)	(0.59)	(0.47)
Capital (t)^2	0.76***	1.09***	0.60
	(0.28)	(0.34)	(0.52)
Cash (t)	-1.48***	-2.38***	-0.39
	(0.24)	(0.32)	(0.38)
Last Offering Amt Raised/Capital	0.15***	0.13***	0.14***
	(0.01)	(0.01)	(0.02)
Constant	12.19***	12.71***	12.84***
	(0.32)	(0.33)	(0.51)
Specification	All	Yr>'97	Yr≤'97
Firm FE	No	No	No
Year FE	Yes	Yes	Yes
% Adjusted R^2	9.24	14.59	9.80
% Within R^2	7.44	8.53	6.36
Observations	9,134	5,183	3,426

Table D.7: Repeats Table 2 without standardizing cash and capital within firm

The outcome variable is the amount of common stock sold in year $t+1$ divided by the total capital of the firm (tangible plus intangible) at the end of year t. The sample is all offerings in year $t+1$ that exceed 5% of capital as of the end of year t. The main explanatory variable is the natural logarithm of total capital of the firm divided by its sample mean for the firm (standardized) and its square. We control for the natural logarithm of the cash reserve divided by its sample mean for the firm (standardized). These capital and cash measures are standardized but not within firm. We control for industry trends. Columns (2) and (3) split the sample after and on or before 1997, respectively. Column (4) restricts the sample to larger offerings—those exceeding 10% of capital. Columns (5) to (11) restrict the sample to industries based on the SIC-1 identifier. See Internet Appendix Table D.2 for SIC-1 category descriptions. Standard errors are double-clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Amt Raised (t+1) / Capital (t) × 100										
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Capital (t)	-6.74***	-7.25***	-6.79***	-7.42***	-6.43***	-8.54***	-5.29***	-6.98***	-8.60***	-7.14***	-7.85***
	(0.33)	(0.54)	(0.51)	(0.40)	(0.74)	(0.94)	(0.54)	(1.10)	(1.20)	(0.84)	(1.34)
Capital^2 (t)	0.59***	1.44***	0.29	0.49**	0.82**	0.76	0.69**	-0.05	0.30	0.37	0.40
	(0.18)	(0.41)	(0.22)	(0.19)	(0.37)	(0.50)	(0.28)	(0.44)	(0.48)	(0.40)	(0.68)
Cash (t)	-0.78**	-1.31***	-0.54	0.12	0.55	-0.05	-2.44***	-0.75	0.80	-1.76	-0.08
	(0.30)	(0.39)	(0.44)	(0.35)	(0.68)	(0.68)	(0.66)	(0.99)	(0.88)	(1.27)	(1.22)
Constant	22.71***	22.87***	22.42***	32.04***	25.57***	23.19***	21.83***	26.34***	21.71***	20.75***	23.13***
	(0.29)	(0.52)	(0.37)	(0.36)	(0.78)	(0.52)	(0.45)	(0.92)	(0.75)	(0.64)	(0.92)
Specification	All	Yr>'97	Yr≤'97	Off.>10%	SIC1=1	SIC1=2	SIC1=3	SIC1=4	SIC1=5	SIC1=7	SIC1=8
Firm FE	No	No	No	No	No	No	No	No	No	No	No
SIC-2×Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
% Adjusted R^2	15.05	14.74	14.99	15.60	14.83	13.61	11.93	22.59	23.57	13.97	13.84
% Within R^2	10.25	10.22	10.80	11.12	11.49	11.26	8.93	10.00	14.91	10.29	11.88
Observations	12730	5990	6733	8173	1423	1962	4189	911	1312	2168	741

Table D.8: Re-Examines the results on issue size in Table VI of Chang et al. (2006), accounting for our predicted determinants of issue size

Discussion: This table shows that accounting for the novel non-linearities in issue size predicted by our model improves the explanatory power of the regressions in Chang et al. (2006) and does not overturn their results. Specifically, the coefficient on *PSTKRTN (t)×NbrAnal (t-3)* remains negative and significant, suggesting that issuances by firms with more analyst coverage are less sensitive to past stock price performance. One takeaway is that the adjusted within R^2 from their baseline specification in column (1) increases from 5.73% to 10.48%, or 83%, when we add controls for a firm's relative cash and capital positions. A second takeaway is that the non-linearity in issuance size and capital is significant as in Table 2 after accounting for all of the controls in Chang et al. (2006).

As in Chang et al. (2006), data are collected from Compustat, CRSP, and I/B/E/S for the years 1985 to 2000. Note, the details in Chang et al. (2006) are not precise enough for a perfect replication. The dependent variable is the amount of net equity issued, scaled by the firm's prior-year total capital, which is the sum of a firm's physical and intangible capital per Peters and Taylor (2017). Regressions include issue years in which the net equity issued exceeds 1% of capital. Net equity issued is the sale of common stock minus the purchase of common stock. *NbrAnal* is the maximum number of analysts that make annual earnings forecasts in any month over a 12-month period that ends 3 years prior to the issue decision. *Return on assets* is the earnings before depreciation and amortization divided by total assets. *Market-to-book ratio* is defined as (market value of equity plus book value of debt)/book value of assets. *PSTKRTN* is the compounded monthly stock return over the past 24-month period. *Capital* is the amount of physical and intangible capital per Peters and Taylor (2017) (standardized within firm) in the prior year and its square. *Cash* is the firm's cash reserve (standardized within firm). Standard errors clustered by firm are reported, and *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Amt. Raised (t)/Capital (t-1) × 100	
	(1)	(2)
PSTKRTN (t)	6.05***	5.89***
	(1.45)	(1.41)
NbrAnal (t-3)	-1.13***	-0.92***
	(0.17)	(0.16)
PSTKRTN (t) × NbrAnal (t-3)	-0.56***	-0.59***
	(0.11)	(0.11)
PSTKRTN^2 (t)	1.73**	1.88**
	(0.76)	(0.73)
PSTKRTN^3 (t)	-0.01	-0.04
	(0.75)	(0.73)
Return on assets (t)	-4.96	-16.87***
	(5.18)	(5.13)
Market-to-book ratio (t)	1.80***	1.45***
	(0.57)	(0.55)
Ln(Assets) (t)	14.85***	21.04***
	(1.58)	(1.91)
Cash (t-1)		-8.42***
		(0.60)
Capital (t-1)		-9.84***
		(1.04)
Capital^2 (t-1)		1.59**
		(0.65)
Constant	-64.89***	-95.53***
	(8.12)	(9.70)
Firm FE	Yes	Yes
Year FE	Yes	Yes
% Adjusted Within R^2	5.73	10.48
Observations	9,016	9,016

Table D.9: Predicting a firm's target debt-to-asset ratio per Hovakimian et al. (2001), which is used later in Internet Appendix Table D.10

Discussion: The purpose of this table is to determine the target capital structure for firms used in the tests in Internet Appendix Table D.10. We replicate as closely as possible the Tobit model in Hovakimian et al. (2001) used to predict a firm's target debt-to-asset ratio (1979–1997). The details in Hovakimian et al. (2001) are not precise enough for a perfect replication. Nevertheless, we achieve a fairly similar sample size, and the coefficients are generally similar. Column (1) shows the coefficients for our sample, and column (2) shows the results in Hovakimian et al. (2001) table 3, panel C, which is the main specification used in the subsequent tables in that paper.

All variables are defined as differences from three-digit SIC industry means for a given year. The dependent variable, debt/assets, equals the (book value of debt)/(book value of debt + market value of equity). The *tangible assets ratio* is the ratio of property, plant, and equipment to the book value of assets. *Firm size* is the natural log of total assets. Robust standard errors are reported, and *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Our Sample (1)	Hovakimian et al. (2001) Sample (2)
R&D expenditures/sales	-0.26***	-0.27***
	(0.02)	
Selling expenses/sales	-0.08***	-0.06***
	(0.01)	
Tangible assets ratio	0.10***	0.20***
	(0.01)	
Firm size	-0.02***	0.00**
	(0.00)	
Constant	-0.05***	-0.01***
	(0.00)	
Observations	36,254	39,387

Table D.10: Re-examining column (1) of table 7 of Hovakimian et al. (2001) relating distance to target capital structure and equity issuance amount

Discussion: In Hovakimian et al. (2001), table 7, column (1), there is a positive but insignificant relation between distance of a firm to its target capital structure and equity issuance. That paper measures distance to the target capital structure by comparing the estimated target debt-to-assets ratio (D/A) derived as in Internet Appendix Table D.9 to the industry average D/A and separately by comparing the industry average D/A to the firm's actual D/A. Column (1) in our table shows their estimated coefficients. We are not able to match their sample exactly given the information provided. However, we construct all the same controls and then update the sample through 2017. Many coefficients are similar. Surprisingly, in column (2) for our sample, the positive coefficients on the two measures of distance to the target capital structure become statistically significant. These positive coefficients are the wrong direction because tradeoff theory would predict that firms that are further below their target D/A should issue less equity. However, in column (3), when we control for a firm's relative cash and capital positions and the non-linearity in capital, we find that the positive relations weaken. Also, the adjusted R^2 increases from 6.9% to 10.26% or by 49%. Also, note that the relative cash and capital positions remain statistically significant when controlling for the covariates in that paper. In Column (4), because our model shows that financing costs are related to issuance size, we add controls for the number of analysts and the standard deviation of analyst estimates as proxies of a firm's financing costs. The coefficients related to target capital structure are insignificant, and the coefficients on a firm's relative cash and capital positions remain statistically significant. Also, the adjusted R^2 now increases from 6.88% to 11.85%, or by 72%. Below we define the key variables in the specification.

The details in Hovakimian et al. (2001) are not precise enough for a perfect replication. As in that paper, the sample is restricted to firms defined as issuing a security, which is when the net amount issued divided by the book value of assets exceeded 5%. Cases where firms issued both debt and equity in a given fiscal year are omitted. D/A is the debt/assets measured with equity at market in the year prior to the issuance period. Target D/A is estimated as the fitted value from the regression in their table 3, panel C. Our Table D.9 shows a similar specification. ROA is earnings before interest, taxes, depreciation, and amortization divided by the book value of assets. NOLC is the net operating loss carryforwards scaled by the book value of assets. The two-year stock return is defined as the split- and dividend-adjusted percentage return from the beginning of the pre-issue year until close of the issue year. The market-to-book ratio is defined as (market value of equity + book value of debt)/total assets. The dummy for whether an equity issue could dilute earnings is set to zero except when one minus the assumed tax rate times the yield on Moody's Baa-rated debt is less than a firm's after-tax earnings-price ratio. The tax rate is assumed to be 50% before 1987 and 34% afterward. Robust standard errors are reported, and *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Common Stock Issue (Net Amount Issued (t) / Capital (t-1))			
	Hovakimian et al. (2001) Table 7 Column (1) (1)	Our Sample (2)	Adding Cash (3)	Adding Analyst (4)
Target D/A-industry mean D/A	0.24	0.13**	0.10*	0.07
		(0.06)	(0.06)	(0.06)
Industry mean D/A-actual D/A	0.06	0.04*	0.02	0.04
		(0.02)	(0.02)	(0.02)
Three-year mean ROA	-0.34***	-0.34***	-0.34***	-0.27***
		(0.04)	(0.04)	(0.04)
NOLC	0.03**	-0.00	-0.01	-0.01
		(0.01)	(0.01)	(0.01)
Two-year stock return	0.06***	0.04***	0.03***	0.03***
		(0.01)	(0.01)	(0.01)
Market-to-book ratio	0.05***	0.01*	0.00	0.01*
		(0.00)	(0.00)	(0.00)
Dummy for M/B > 1	0.04	0.04***	0.04***	0.04***
		(0.01)	(0.01)	(0.01)
Dilution dummy	-0.05***	0.01	0.01	0.00
		(0.01)	(0.01)	(0.01)
FD3	0.04*	0.04**	0.03**	0.02
		(0.02)	(0.02)	(0.02)
Loss dummy × FD3	-0.03	0.01	0.01	0.01
		(0.02)	(0.02)	(0.02)
Cash (t)			-0.02***	-0.02***
			(0.00)	(0.00)
Capital (t)			-0.05***	-0.04***
			(0.01)	(0.01)
Capital^2 (t)			0.01*	0.01**
			(0.01)	(0.01)
Number of Analyst Estimates				-0.00***
				(0.00)
Standard Deviation of Analyst Estimates				0.68**
				(0.32)
Constant	0.10***	0.22***	0.21***	0.21***
		(0.02)	(0.02)	(0.02)
Sample Period	'79-'97	'79-'17	'79-'17	'79-'17
Year FE	Yes	No	No	No
SIC-2–by–Year FE	No	Yes	Yes	Yes
% Adjusted Within R^2		6.88	10.26	11.85
Observations	2,231	3,452	3,452	3,452

Appendix D.2. Robustness for the empirical support for ***H2***

Table D.11: Repeats Table 3 without standardizing capital and cash within firm

The outcome variable is the total dividends paid in year $t+1$ plus the total value of the shares repurchased divided by the total capital of the firm (tangible plus intangible) at the end of year t. The main explanatory variable is the natural logarithm of total capital divided by its sample mean for a firm (standardized) and its square. We control for the natural log of the cash reserve divided by its sample mean for a firm (standardized), industry trends, and firm fixed effects. Columns (2) and (3) split the sample after and on or before 1994, respectively. Columns (4) to (10) restrict the sample to industries based on the SIC-1 identifier. See Internet Appendix Table D.2 for SIC-1 category descriptions. Standard errors are double-clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Dividends (t+1) / Capital (t) × 100									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Capital (t)	-0.70***	-0.71***	-0.89***	-0.72***	-0.75***	-0.58***	-1.19***	-0.23	-0.96***	-1.07***
	(0.08)	(0.13)	(0.09)	(0.14)	(0.14)	(0.12)	(0.26)	(0.17)	(0.22)	(0.31)
Capital^2 (t)	-0.24***	-0.25***	-0.14***	-0.01	-0.22***	-0.28***	-0.12	-0.28***	-0.39***	-0.38***
	(0.03)	(0.05)	(0.03)	(0.04)	(0.06)	(0.04)	(0.09)	(0.06)	(0.09)	(0.11)
Cash (t)	0.69***	0.70***	0.60***	0.38***	0.77***	0.81***	0.55***	0.46***	0.78***	0.76***
	(0.03)	(0.05)	(0.04)	(0.09)	(0.06)	(0.05)	(0.12)	(0.08)	(0.13)	(0.14)
Constant	4.07***	4.70***	3.27***	2.56***	4.44***	4.15***	3.54***	3.50***	5.28***	4.38***
	(0.03)	(0.06)	(0.04)	(0.05)	(0.05)	(0.04)	(0.08)	(0.05)	(0.09)	(0.09)
Specification	All	Yr>'94	Yr≤'94	SIC1=1	SIC1=2	SIC1=3	SIC1=4	SIC1=5	SIC1=7	SIC1=8
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
SIC-2×Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
% Adjusted R^2	35.62	38.73	39.89	27.48	38.94	30.03	33.90	36.36	40.50	40.70
% Within R^2	1.86	1.40	2.19	1.98	2.61	2.26	2.52	1.29	1.62	2.44
Observations	96700	48142	47942	7412	19746	33031	7266	14127	10636	4079

Table D.12: Examining column (1) of table 3 of Bliss et al. (2015), looking at dividend reductions during the 2008-2009 financial crisis

Discussion: The results in this table show a similar increase in the probability that a firm reduces payouts during a crisis as in Bliss et al. (2015). One takeaway is that when we go from column (2) to column (3) and control for the relation between payouts and a firm's cash and capital stock, the coefficient on *Crisis* increases in size by 46% (0.73/0.50) in our sample. A second takeaway is that the pseudo R^2 increases by 14.5% (10.77/9.41) when accounting for a firm's cash and capital stock. A third takeaway is that the relations between cash and capital and the non-linearity in capital hold after controlling for the other variables in Bliss et al. (2015). It is important to note that this test looks at reductions in payouts, so the signs on *Cash (t)* and *Capital (t)*^{2} are flipped.

The details in Bliss et al. (2015) are not precise enough for a perfect replication. We focus on their table 3, column (1). As in that paper, the sample is restricted to firms with positive average payouts over the prior two years. The sample period is 2005 to 2009. The outcome is a binary variable that equals to one if the firm reduces the total payout (dividends plus repurchases) by at least 5%. The main explanatory variable is *Crisis*, which is a binary variable equal to one if fiscal year is 2008 or 2009, and zero otherwise. The next 10 control variables are constructed as in the appendix to Bliss et al. (2015). We add the following based on H2 of our paper: *Capital* is the total capital of the firm (standardized within firm) and its square. *Cash* is the cash and cash equivalents of the firm (standardized within firm). The independent variables are lagged except contemporaneous *Cash flow/Lag TA* and *Tobin's Q*. Industry fixed effects are based on the Fama-French 48-industry definitions. Robust standard errors are reported, and *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	$\mathbb{1}$(Reduction in Total Payout>5%)		
	Bliss et al. (2015) Table 3 Column (1) (1)	Our Sample (2)	Adding Cash&Capital (3)
Crisis	0.11**	0.50***	0.73***
	(0.05)	(0.19)	(0.22)
Crisis × Market leverage	0.14**	-0.04	-0.06
	(0.07)	(0.06)	(0.06)
Crisis × Cash/TA	-0.34***	-0.30	-0.24
	(0.08)	(0.43)	(0.47)
Crisis × Tobin Q	0.08***	0.04	-0.06
	(0.03)	(0.13)	(0.13)
Crisis × Cash flow/TA	-0.00	-1.63**	-1.59**
	(0.13)	(0.76)	(0.77)
Crisis × (R&D+CapEx)/TA	-0.54***	0.61	0.26
	(0.15)	(0.80)	(0.81)
Crisis × Capital (t)			0.01
			(0.14)
Crisis × Capital (t)^2			-0.16
			(0.10)
Crisis × Cash (t)			-0.16*
			(0.08)
Ln(Assets)	-0.01***	-0.14***	-0.09***
	(0.00)	(0.02)	(0.03)
Losses	0.02**	0.02	0.01
	(0.01)	(0.03)	(0.03)
(R&D+CapEx)/TA	0.88***	2.05***	1.84***
	(0.15)	(0.51)	(0.52)
Market leverage	0.31***	0.15***	0.16***
	(0.07)	(0.06)	(0.06)
Cash flow/TA	-0.88***	-2.19***	-2.13***
	(0.12)	(0.51)	(0.51)
Cash/TA	0.00	-0.17	0.78**
	(0.07)	(0.31)	(0.37)
Tobin Q	-0.15***	-0.46***	-0.51***
	(0.02)	(0.09)	(0.09)
Cash flow volatility	0.29**	1.96**	1.70*
	(0.13)	(0.91)	(0.90)
Total Payout/TA	3.27***	13.55***	13.73***
	(0.17)	(1.41)	(1.44)
Payout Reduction	0.32***	0.35***	0.33***
	(0.01)	(0.07)	(0.08)
Capital (t)			-0.13
			(0.10)
Capital (t)^2			0.17***
			(0.06)
Cash (t)			-0.27***
			(0.06)
Industry FE	Yes	Yes	Yes
% Psuedo R^2	21.9	9.41	10.77
Observations	6,886	5,354	5,354

*Appendix D.3. Robustness for the empirical support of **H4***

Table D.13: Repeats Table 5 using alternative measures of expected cash flow volatility

The outcome variable is the change in cash from the end of year t to the end of year $t+1$ (standardized within firm). Capital is the firm's total capital at the end of year t (standardized within firm). Volatility is a firm's average daily volatility in year t (standardized within firm). Ch. Volatility is the year-over-year change in the average daily volatility. VIX is the average daily VIX in year (t). Ch. VIX is the year-over-year change in the average daily VIX. Option Implied Volatility is the implied volatility of the stock derived from near to maturity call options. Standard errors are clustered by firm. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Ln(Cash (t+1)/Cash(t)) × 100				
	(1)	(2)	(3)	(4)	(5)
Capital (t)	-2.93***	-2.42***	-3.90***	-3.37***	-2.32***
	(0.23)	(0.23)	(0.37)	(0.35)	(0.56)
Ch. Volatility (t)	0.83***				
	(0.30)				
Capital (t) × Ch. Volatility (t)	1.22***				
	(0.35)				
Volatility (t)		3.68***			
		(0.27)			
Capital (t) × Volatility (t)		0.85***			
		(0.32)			
Ch. VIX (t)			0.60		
			(0.37)		
Capital (t) × Ch. VIX (t)			1.36***		
			(0.44)		
VIX (t)				3.21***	
				(0.33)	
Capital (t) × VIX (t)				0.77*	
				(0.40)	
Option Implied Volatility (t)					6.48***
					(0.52)
Capital (t) × Option Implied Volatility (t)					0.97*
					(0.57)
Constant	6.27***	6.10***	7.68***	6.61***	8.14***
	(0.01)	(0.03)	(0.09)	(0.05)	(0.23)
Specification	All	All	All	All	≤'94
Firm FE	Yes	Yes	Yes	Yes	Yes
% Adjusted R^2	-3.91	-3.63	-4.89	-4.56	-4.47
% Within R^2	0.12	0.25	0.16	0.25	0.90
Observations	101212	105983	56158	63991	30127

Table D.14: Repeats Table 5 at the quarterly frequency using changes in historical cash flow volatility as a proxy for changes in a firm's expected cash flow uncertainty

The outcome variable is the a firm's cash normalized by assets in quarter $t+1$. The primary explanatory variable of interest is cash flow volatility, which is the coefficient of variation of quarterly cash flows over the prior 16 quarters, with at least 12 non-missing cash flows. Cash flow is the operating income before extraordinary items plus depreciation scaled by beginning of period assets. Capital is the firms total physical and intangible capital. Tobin q is the assets plus market value of equity less book value of equity scaled by the book value of assets. Leverage is a firm's long term debt scaled by total assets. Specifications include firm and year-quarter fixed effects. Standard errors are double-clustered by firm and quarter. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Cash / Assets (t+1) × 100		
	(1)	(2)	(3)
Cash Flow Volatility (t)	0.01***	0.01***	0.01***
	(0.00)	(0.00)	(0.00)
Ln(Assets) (t)	-0.69***	-0.70***	
	(0.07)	(0.07)	
Cash Flow Volatility (t) × Ln(Assets) (t)		0.01**	
		(0.00)	
Capital (t)			-1.16***
			(0.08)
Cash Flow Volatility (t) × Capital (t)			0.00**
			(0.00)
Cash Flow (t+1)	4.65***	4.66***	4.57***
	(1.20)	(1.20)	(1.18)
Tobin Q (t)	0.27***	0.27***	0.25***
	(0.03)	(0.03)	(0.03)
Leverage (t)	-0.99***	-1.00***	-0.89***
	(0.13)	(0.13)	(0.13)
Cash/Assets (t)	0.74***	0.74***	0.74***
	(0.01)	(0.01)	(0.01)
Cash/Assets (t-1)	0.05***	0.05***	0.05***
	(0.01)	(0.01)	(0.01)
Cash/Assets (t-2)	0.05***	0.05***	0.05***
	(0.01)	(0.01)	(0.01)
Constant	1.70***	1.70***	1.89***
	(0.07)	(0.07)	(0.08)
Firm FE	Yes	Yes	Yes
Qtr FE	Yes	Yes	Yes
% Adjusted R^2	92.12	92.12	92.14
% Within R^2	70.27	70.28	70.35
Observations	235061	235061	235061

Table D.15: Repeats Table 5 controlling for changes in average returns

The outcome variable is the change in cash from the end of year t to the end of year $t+1$ (standardized within firm). Capital is the firm's total capital at the end of year t (standardized within firm). ΔVOL is the change in a firm's average daily volatility from year $t-1$ to t (standardized within firm). ΔRET is the change in a firm's average excess return from year $t-1$ to t (standardized within firm). Columns (2) and (3) split the sample after and on or before 1994, respectively. Columns (4) to (10) restrict the sample to industries based on the SIC-1 identifier. See Internet Appendix Table D.2 for SIC-1 category descriptions. Standard errors are clustered by firm. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Ln(Cash (t+1)/Cash(t))× 100									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Capital (t)	-3.01***	-3.30***	-4.77***	-0.02	-0.02***	-0.04***	-0.03***	-0.03***	-0.04***	-0.02**
	(0.24)	(0.42)	(0.41)	(0.01)	(0.00)	(0.00)	(0.01)	(0.01)	(0.01)	(0.01)
Δ VOL (t)	1.74***	1.65***	2.49***	0.01	0.03***	0.03***	0.01	-0.00	0.00	0.00
	(0.32)	(0.48)	(0.48)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)
Capital (t) × Δ VOL (t)	1.18***	1.89***	0.25	0.04**	0.00	0.01	0.01	0.03**	-0.00	0.00
	(0.38)	(0.57)	(0.55)	(0.02)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
Δ Return (t)	4.54***	4.23***	4.68***	0.03**	0.05***	0.06***	0.01	0.06***	0.03***	0.03*
	(0.35)	(0.59)	(0.46)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
Capital (t) × Δ Return (t)	-1.28***	-1.46**	-1.66***	-0.02	-0.03***	-0.01**	0.00	-0.00	-0.01	-0.01
	(0.40)	(0.68)	(0.52)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.01)	(0.02)
Constant	6.54***	4.88***	8.69***	0.06***	0.06***	0.07***	0.07***	0.06***	0.07***	0.06***
	(0.03)	(0.12)	(0.12)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)	(0.00)
Specification	All	≤'94	>'94	SIC1=1	SIC1=2	SIC1=3	SIC1=4	SIC1=5	SIC1=7	SIC1=8
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
% Adjusted R^2	-3.39	-4.12	-4.16	-3.75	-3.04	-2.96	-4.04	-3.39	-3.75	-3.99
% Within R^2	0.44	0.40	0.58	0.29	0.48	0.66	0.14	0.70	0.38	0.27
Observations	95343	47361	47460	6846	20480	33608	7121	13304	9898	3733

Appendix D.4. The default region and leverage

The literatures on offerings and defaults are generally distinct and concerned with the timing of offerings and defaults, respectively. Few studies examine how firms facing costly financing and distress choose whether to issue more equity or to default and how that decision interacts with leverage.

To examine default behavior, we compile bankruptcy information from various sources. First, we retrieve bankruptcy years from SDC Platinum. We drop a firm's observations following the bankruptcy year. Because firms may delist before declaring bankruptcy, we designate the final year a firm is in the sample as a bankruptcy year if SDC says the bankruptcy year is within two years of the end of a firm's time in the sample. Second, we use the status identifier in Compustat (*stalt*), which equals "TL" for companies in bankruptcy or liquidation. We drop observations after the first year the status changes to "TL". Third, we examine the deletion code (*dlrsn*), which equals two for bankruptcy and three for liquidation. Fourth, we retrieve delisting information from CRSP (codes between 450 and 490 and between 550 and 587). Collectively, these bankruptcy data are most complete starting in 1984. In total, there are 2,080 bankruptcies in our data.

Figure 5 shows that, in the presence of costly financing, more levered firms facing distress are less likely to issue equity to continue and are more likely to default. Specifically, the target issuance boundary shifts to the right, expanding the default region. Intuitively, when capital is low, cash flows are low. Thus, equityholders have to decide whether to default or pay the issuance costs (which are larger for small firms because of the fixed component) to raise cash to fund the coupon payments and invest over time to rebuild the capital (because of the convex adjustment costs). For more leveraged firms, the coupon rates are higher, which increases the coupon burden on the firms, especially when capital stock is low.

This reasoning leads to the following hypothesis:

H: When capital is low, levered firms are less likely to conduct offerings and more likely to default.

Figure D.3(a) provides visual evidence consistent with **H3**. The y-axis plots the propensity to conduct an offering for both firms with high leverage (above the 90th percentile of the sample distribution of debt to market equity) and low leverage. The x-axis represents a firm's total capital standardized within firm. The figure shows that the propensity to conduct an offering is increasing as capital decreases. More importantly, the figure shows that the propensity to conduct an offering is lower for highly leveraged firms with low capital than for low-leveraged firms with low capital. By contrast, for firms with high capital, there is no difference between offering propensity and leverage.

Figure D.3(b) also provides visual evidence consistent with this hypothesis. The y-axis plots the propensity to declare bankruptcy in year $t+1$. The x-axis shows a firm's total capital standardized within firm. The figure shows that the propensity to declare bankruptcy is higher for high-leverage firms with low capital than for low-leverage firms with low capital. By contrast, when capital is high, there is less of a difference in the bankruptcy propensity for high- and low-leverage firms.

To evaluate this hypothesis more rigorously, we estimate the following two empirical specifications:

$$\begin{aligned}\text{Offering}_{i,t+1} = {} & \beta_1 \text{Capital}_{i,t} + \beta_2 \text{Capital}_{i,t} \times \text{High Lev.}_{i,t} \\ & + \beta_3 \text{Cash}_{i,t} + \mu_i \times \text{High Lev.}_{i,t} + \delta_{j,t} \times \text{High Lev.}_{i,t} + \epsilon_{i,t}\end{aligned} \tag{D.1}$$

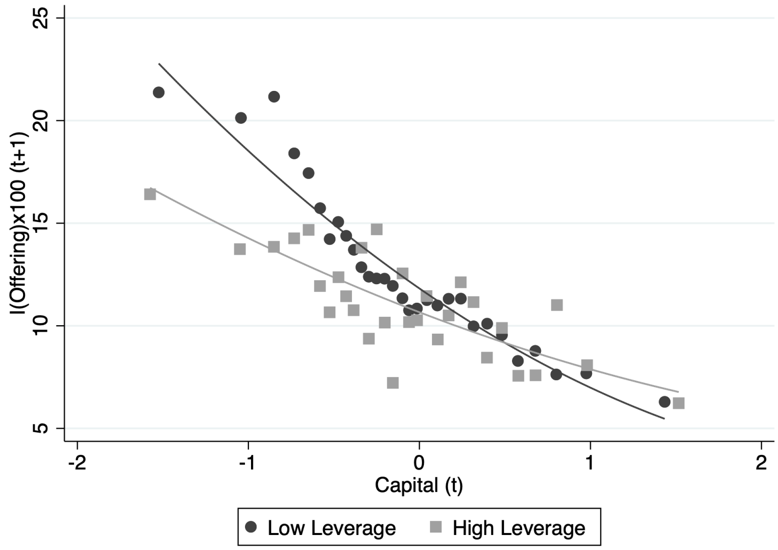
$$\begin{aligned}\text{Bankruptcy}_{i,t+1} = {} & \beta_1 \text{Capital}_{i,t} + \beta_2 \text{Capital}_{i,t} \times \text{High Lev.}_{i,t} \\ & + \beta_3 \text{Cash}_{i,t} + \mu_i \times \text{High Lev.}_{i,t} + \delta_{j,t} \times \text{High Lev.}_{i,t} + \epsilon_{i,t}\end{aligned} \tag{D.2}$$

The outcome in Equation D.1 is an indicator that equals one if firm i conducts an offering in year $t+1$ amounting to more than 5% of firm i's capital at the end of year t. The outcome in Equation D.2 is an indicator that equals one if firm i declares bankruptcy in year $t+1$. The main explanatory variable is a firm's total capital at the end of year t standardized within firm and winsorized at the 1% level. β_2 multiplies the interaction of capital and an indicator *High Lev.*$_{i,t}$, which equals one if the firm's leverage in year t is above the 90th percentile in the panel. We control for a firm's cash and equivalents, firm fixed effects μ_i, and SIC-2–by–year industry trends ($\delta_{j,t}$). We interact the firm and time trends with *High Lev.*$_{i,t}$ to allow a firm with high leverage to make different decisions on average than the same firm without high leverage (e.g., a higher propensity to default) and to allow high-leverage firms to have different industry trends than low-leverage firms (e.g., leveraged firms may have greater market sensitivity). $\epsilon_{i,t}$ is the unexplained variation. Standard errors are double-clustered by firm and year.

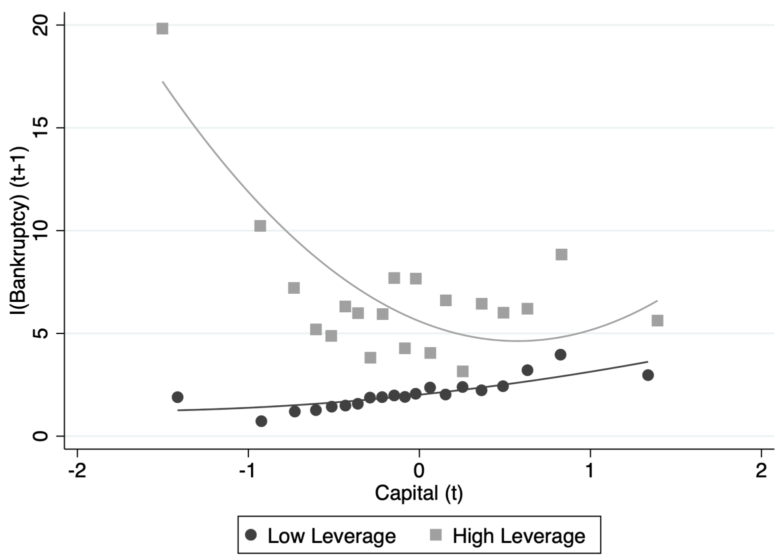
Table D.16 presents the results. Panel A, column (1), shows that firms with high leverage and low capital are less likely to conduct an offering. Columns (2) and (3) show that the result holds before and after 1995. Column (4) shows that the result holds when the outcome is whether the firm conducts an offering in year $t+1$ that is greater than

10% of capital as of the end of year t. Columns (5)–(11) show that the relation is fairly common across industries.[26]

[26] Internet Appendix Table D.18 finds similar results using SDC Platinum offering data.



(a) The y-axis plots the percentage of firms issuing shares worth at least 5% of their capital stock in year $t+1$. The x-axis shows a firm's total capital (tangible and intangible) standardized within firm. Controls for year and firm fixed effects are included. *High leverage* indicates a debt-to-market equity ratio above the 90th percentile in the firm-year sample.



(b) The y-axis plots the percentage of firms declaring bankruptcy in year $t+1$. The x-axis shows a firm's total capital (tangible and intangible) standardized within firm. Controls for year and firm fixed effects are included. *High leverage* indicates a debt-to-market equity ratio above the 90th percentile in the firm-year sample.

Figure D.3: **Leverage, Offerings, and Bankruptcy**

Table D.16: Firms with high leverage and low capital are less likely to issue stock and more likely to default

In Panel A, the outcome variable is an indicator that equals 100 if the a firm receives proceeds from stock sales of at least 5% of capital in year $t+1$. In Panel B, the outcome variable is an indicator that equals 100 if the firm declares bankruptcy in year $t+1$. The main explanatory variable in both panels is the total capital of the firm (standardized within firm). We interact capital with an indicator, *High Lev.*, that equals one if a firm's debt–to–market-equity ratio exceeds the cross-section's 90th percentile. We control for the cash reserve (standardized within firm), industry trends, and firm fixed effects. Because *High Lev.* is time varying, even within firm, we interact the fixed effects with the indicator for high leverage, allowing industry trends to differ for high-leverage firms and allowing for different fixed-firm characteristics when a firm has high leverage. These interacted fixed effects help isolate the role of capital in offering and bankruptcy decisions. In Panel A, columns (2) and (3) split the sample after and on or before 1995, respectively. Column (4) restricts the outcome to offerings of at least 10% of capital. Columns (5) to (11) restrict the sample to industries based on the SIC-1 identifier. In Panel B, Columns (2) and (3) split the sample after and on or before 1999 (the median year in the sample with bankruptcy data), respectively. Columns (4) to (10) restrict the sample to industries based on the SIC-1 identifier. Standard errors are double-clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

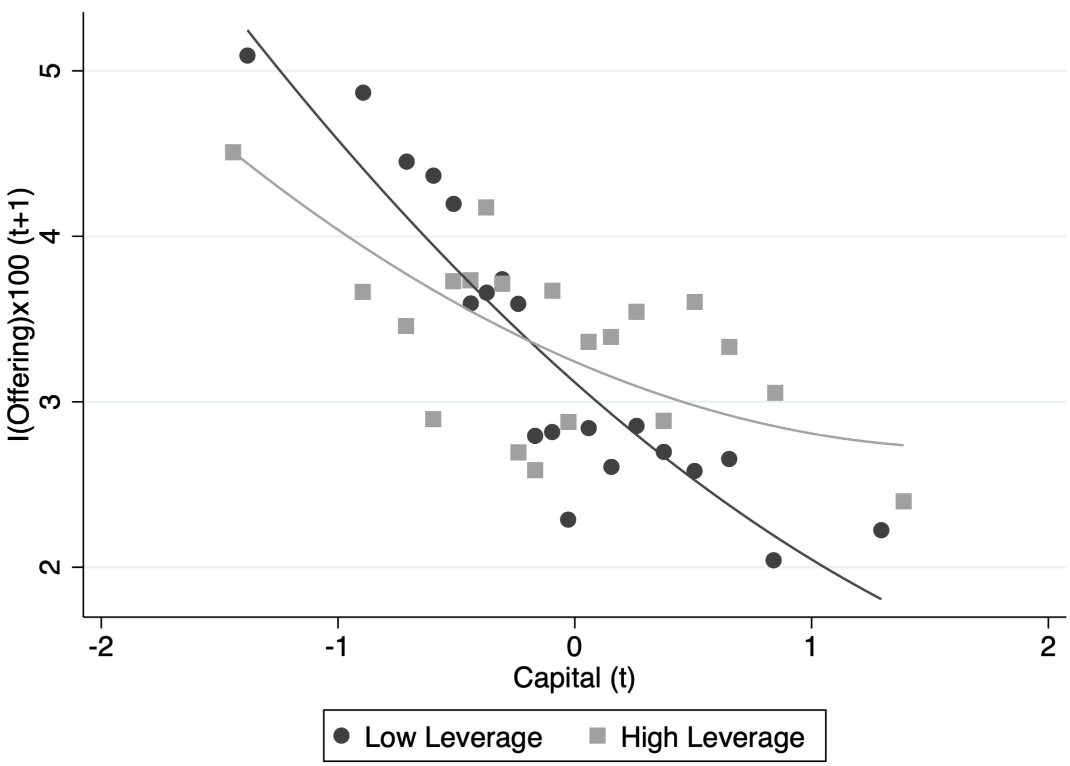
Panel A: Levered firms are less likely to issue when capital is low

	I(Offering (t+1)) × 100										
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Capital (t)	-5.36***	-6.16***	-4.31***	-4.58***	-5.72***	-3.19***	-4.74***	-5.97***	-5.61***	-9.13***	-7.73***
	(0.34)	(0.59)	(0.48)	(0.28)	(0.85)	(0.36)	(0.41)	(0.82)	(0.69)	(0.85)	(1.19)
Capital (t)×High Lev. (t)	2.82***	3.92***	2.00*	2.46***	1.35	0.53	1.68	2.73*	4.24**	8.16***	6.67**
	(0.61)	(1.01)	(0.97)	(0.59)	(2.84)	(1.09)	(1.18)	(1.45)	(1.72)	(2.31)	(2.61)
Cash (t)	-1.90***	-2.33***	-1.56***	-1.91***	-2.85***	-2.01***	-1.95***	-1.02*	-1.43***	-2.13***	-1.73**
	(0.18)	(0.21)	(0.30)	(0.16)	(0.58)	(0.32)	(0.24)	(0.54)	(0.33)	(0.53)	(0.75)
Constant	11.94***	15.64***	8.89***	7.73***	18.08***	9.15***	11.29***	11.82***	8.70***	17.57***	16.04***
	(0.03)	(0.12)	(0.19)	(0.03)	(0.07)	(0.04)	(0.05)	(0.09)	(0.08)	(0.08)	(0.15)
Specification	All	>'95	≤'95	10%	SIC1=1	SIC1=2	SIC1=3	SIC1=4	SIC1=5	SIC1=7	SIC1=8
Firm FE×High Lev.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
SIC-2×Year FE×High Lev.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
% Adjusted R^2	22.68	27.70	19.27	19.25	19.18	24.59	20.56	21.65	15.86	28.15	19.12
% Within R^2	1.64	2.05	0.88	1.79	1.67	1.03	1.48	1.62	1.62	3.47	2.40
Observations	102,552	49,722	52,026	102,552	7,706	20,963	35,402	7,632	14,694	11,382	4,398

Panel B: Levered firms are more likely to default when capital is low

	I(Bankruptcy (t+1)) × 100									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Capital (t)	0.77***	-0.09	1.23***	-0.36	1.25***	0.58***	0.57*	1.46***	0.67***	1.06**
	(0.10)	(0.13)	(0.17)	(0.39)	(0.23)	(0.15)	(0.30)	(0.37)	(0.23)	(0.49)
Capital (t)× High Lev. (t)	-5.80***	-5.51***	-5.13***	-5.20**	-7.36***	-8.20***	-2.95*	-3.93**	-6.17***	-5.69***
	(0.71)	(1.24)	(0.96)	(2.43)	(1.79)	(1.57)	(1.64)	(1.92)	(1.78)	(2.09)
Cash (t)	-0.63***	-0.65***	-0.48***	-0.73**	-0.38***	-0.54***	-0.66**	-0.85***	-0.75***	-0.92***
	(0.06)	(0.08)	(0.10)	(0.29)	(0.14)	(0.10)	(0.26)	(0.19)	(0.16)	(0.30)
Constant	1.89***	1.97***	1.98***	2.53***	1.74***	1.51***	2.26***	2.63***	1.65***	2.22***
	(0.01)	(0.04)	(0.08)	(0.04)	(0.03)	(0.02)	(0.06)	(0.05)	(0.03)	(0.07)
Specification	All	Yr>'99	Yr≤'99	SIC1=1	SIC1=2	SIC1=3	SIC1=4	SIC1=5	SIC1=7	SIC1=8
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
SIC-2×Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
% Adjusted R^2	25.25	30.04	24.61	27.08	28.54	23.01	27.71	22.73	25.16	22.73
% Within R^2	0.79	0.65	0.81	0.76	1.00	1.13	0.49	0.71	1.14	1.22
Observations	75,898	39,431	35,605	5502	14,281	25,765	5,645	10,966	9,696	3,804

By contrast, Table D.16, Panel B, column (1), shows that leveraged firms with low capital are more likely to declare bankruptcy. Columns (2) and (3) show that the relation holds before and after the median year of 1999 for the bankruptcy sample. Columns (4) to (10) show directionally similar results across industries.



(a) This figure replicates Figure D.3(a) using offering data from SDC. The y-axis shows the percentage of firms issuing shares worth at least 5% of their capital stock in year $t + 1$. The x-axis represents a firm's total capital (tangible and intangible) standardized within firm.

Figure D.4: **Leverage and offerings, using SDC data**

Table D.17: This table repeats Table D.16(a) without firm fixed effects
The outcome variable is an indicator that equals 100 if the a firm receives proceeds from stock sales of at least 5% of capital in year $t + 1$. The main explanatory variable in both panels is the total capital of the firm (standardized within firm). We interact capital with an indicator, *High Lev.*, that equals one if a firm's leverage exceeds the cross-section's 90th percentile. We control for the cash reserve (standardized within firm) and industry trends. Columns (2) and (3) split the sample after and on or before 1995, respectively. Column (4) restricts the outcome to offerings of at least 10% of capital. Columns (5) to (11) restrict the sample to industries based on the SIC-1 identifier. Standard errors are double-clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

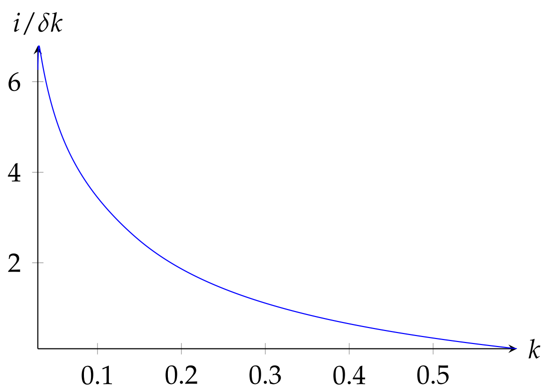
	I(Offering (t+1)) × 100										
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Capital (t)	-6.35***	-7.48***	-4.96***	-5.14***	-7.23***	-5.12***	-5.77***	-5.83***	-5.77***	-10.10***	-7.91***
	(0.38)	(0.46)	(0.49)	(0.28)	(0.84)	(0.50)	(0.45)	(0.81)	(0.58)	(0.83)	(1.00)
Capital (t)×High Lev. (t)	4.20***	4.85***	3.09***	3.26***	2.71	3.80***	3.60***	2.32*	4.26***	8.62***	6.59***
	(0.60)	(0.76)	(0.90)	(0.49)	(2.28)	(1.00)	(1.13)	(1.37)	(1.25)	(1.50)	(2.39)
Cash (t)	-1.92***	-2.34***	-1.41***	-1.82***	-2.83***	-2.10***	-2.02***	-0.66	-1.06***	-2.59***	-2.02***
	(0.19)	(0.24)	(0.27)	(0.15)	(0.59)	(0.34)	(0.26)	(0.50)	(0.29)	(0.56)	(0.73)
Constant	11.95***	16.04***	8.88***	7.77***	18.22***	9.09***	11.25***	11.94***	8.69***	17.66***	15.90***
	(0.15)	(0.25)	(0.24)	(0.11)	(0.57)	(0.29)	(0.25)	(0.45)	(0.27)	(0.53)	(0.60)
Specification	All	>'95	≤'95	10%	SIC1=1	SIC1=2	SIC1=3	SIC1=4	SIC1=5	SIC1=7	SIC1=8
Firm FE×High Lev.	No	No	No	No	No	No	No	No	No	No	No
SIC-2×Year FE×High Lev.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
% Adjusted R^2	8.18	9.65	6.26	6.80	6.32	7.20	6.12	8.68	5.76	11.06	7.60
% Within R^2	2.88	4.07	1.71	2.84	2.85	2.59	2.67	2.03	2.60	5.24	3.32
Observations	104,705	51,250	53,435	104,705	7,910	21,335	35,946	7,815	15,024	11,768	4,518

Table D.18: This table replicates Table D.16(a) using offering data from SDC
The outcome variable is an indicator that equals 100 if the a firm receives proceeds from stock sales of at least 5% of capital in year $t + 1$. The main explanatory variable is the total capital of the firm (standardized within firm). We interact capital with an indicator, *High Lev.*, that equals one if a firm's leverage exceeds the cross-section's 90th percentile. We control for the cash reserve (standardized within firm), industry trends, and firm fixed effects. Columns (2) and (3) split the sample after and on or before 1995, respectively. Column (4) restricts the outcome to offerings of at least 10% of capital. Columns (5) to (11) restrict the sample to industries based on the SIC-1 identifier. Standard errors are double-clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

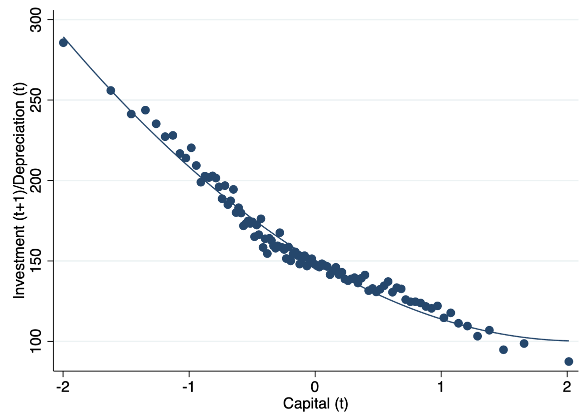
	I(Offering (t+1)) $\times$ 100										
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
Capital (t)	-1.32***	-1.71***	-0.66***	-1.32***	-1.68***	-1.12***	-1.03***	-1.72***	-1.63***	-1.60***	-1.68***
	(0.14)	(0.19)	(0.18)	(0.14)	(0.43)	(0.25)	(0.16)	(0.35)	(0.30)	(0.27)	(0.45)
Capital (t)$\times$ High Lev (t)	0.84***	0.74*	0.27	0.84***	1.65	0.41	0.65**	0.80	1.25***	1.26**	1.68***
	(0.21)	(0.41)	(0.21)	(0.21)	(1.55)	(0.48)	(0.29)	(0.62)	(0.44)	(0.49)	(0.46)
Cash (t)	-0.79***	-1.30***	-0.24***	-0.79***	-0.91***	-0.77***	-0.87***	-0.92***	-0.27**	-1.40***	-0.36
	(0.09)	(0.11)	(0.07)	(0.09)	(0.30)	(0.17)	(0.13)	(0.25)	(0.13)	(0.20)	(0.32)
Constant	2.41***	4.08***	1.29***	2.41***	3.19***	2.20***	2.39***	2.44***	1.99***	2.70***	2.75***
	(0.01)	(0.04)	(0.07)	(0.01)	(0.06)	(0.03)	(0.02)	(0.06)	(0.03)	(0.05)	(0.06)
Specification	All	>'95	$\leq$'95	10%	SIC1=1	SIC1=2	SIC1=3	SIC1=4	SIC1=5	SIC1=7	SIC1=8
Firm FE$\times$High Lev.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
SIC-2$\times$Year FE$\times$High Lev.	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
% Adjusted R^2	11.32	13.14	11.89	11.32	18.19	17.16	9.03	5.03	7.46	10.80	8.54
% Within R^2	0.53	0.81	0.12	0.53	0.69	0.48	0.46	0.71	0.46	1.06	0.49
Observations	102,552	49,722	52,026	102,552	7,706	20963	35,402	7,632	14,694	11,382	4,398

Appendix D.5. Investment-to-depreciation is declining and convex in capital

In our model, the assumption that capital exhibits diminishing returns creates convexity in the investment-to-depreciation ratio. Figure D.5(a) shows that the model implied investment-to-depreciation ratio is declining and convex in capital. Intuitively, as capital increases, the marginal returns to investing decrease, discouraging investment. As capital increases, depreciation increases linearly in capital at the rate δ. Together, the ratio of investment to depreciation should be declining and convex in capital.



(a) Figure D.5a plots the amount a firm invests scaled by depreciation at $c = 0.15$ from Figure 1. Here, $b = 0.02$ and other parameters used are summarized in Table 1.



(b) The y-axis plots a firm's investment in physical and intangible capital in year $t + 1$. Depreciation is the depreciation of tangible and intangible capital in year t. The x-axis represents a firm's total capital (tangible and intangible) standardized within firm.

Figure D.5: Predicted investment from Figure 1 vs. actual

This reasoning leads to the following hypothesis:

H: The ratio of investment to depreciation is declining and convex in capital.

Examining the data, Figure D.5(b) provides initial support for this hypothesis. The bin-scatter plot shows a similarly convex pattern in the investment-to-depreciation ratio and a firm's capital stock, standardized within firm.

To evaluate this hypothesis more formally, we estimate the following empirical specification:

$$\frac{\text{Investment}_{i,t+1}}{\text{Depreciation}_{i,t+1}} = \beta_1 \text{Capital}_{i,t} + \beta_2 \text{Capital}^2_{i,t} + \beta_3 \text{Cash}_{i,t} + \mu_i + \delta_{j,t} + \epsilon_{i,t} \tag{D.3}$$

The outcome is firm i's investment in physical and intangible capital in year $t+1$ scaled by the depreciation of physical and intangible capital in year t.[27] The main explanatory variable is a firm's total capital at the end of year t standardized within firm and winsorized at the 1% level. β_2 captures the sensitivity of the investment-to-depreciation ratio to the quadratic form of capital (standardized). We control for a firm's cash and equivalents, firm fixed effects μ_i, and SIC-2–by–year industry trends ($\delta_{j,t}$). $\epsilon_{i,t}$ is the unexplained variation. Standard errors are double-clustered by firm and year.

Table D.19 presents the results. Column (1) uses the full sample of firms. β_1 multiplying a firm's capital position is -42.50 and is highly significant. β_2, which captures the relation between the investment-to-depreciation ratio and the quadratic form of a firm's capital, is +17.39 and is also highly significant. Together, these results provide strong evidence that payouts are declining in capital in a convex pattern. Columns (2) and (3) show the relation is similar when splitting the sample for the investment analyses at the median year of 1995. Columns (4) to (10) show the convexity exists across SIC-1 industry classifications.

[27] To proxy for a firm's net investment activity, we use proxies for a firm's investment in physical and intangible capital. The measure of expenditures on physical capital is a firm's capital expenditures on property, plant, and equipment from the cash flow statement (*capx*). The measure of expenditures on intangible capital follows from Hulten and Hao (2008), Eisfeldt and Papanikolaou (2014), and Peters and Taylor (2017). Specifically, a firm's expenditures on intangible capital is the firm's R&D (*xrd* plus *rdip*) plus 30% of a firm's SG&A (*xsga*) minus R&D expenses (*xrd*) minus in-process R&D (*rdip*). When *xrd* exceeds *xsga* but is less than the cost of goods sold (*cogs*), or when *xsga* is missing, then we measure SG&A as *xsga* with no further adjustments, or zero if missing. R&D is substracted from SG&A because Compustat adds R&D expenses to SG&A. Also, the literature interprets the remaining 70% of SG&A as operating costs that support the current period's profits. Both R&D and SG&A are set to zero if the corresponding data elements on Compustat are missing. We compare the investment amount to the depreciation on the firm's physical capital (*dp*) and intangible capital. Like the literature, we assume that the depreciation on intangible capital is 20% of the stock of intangible capital.

Table D.19: The ratio of investment to depreciation is convex and declining in capital
The outcome variable is the ratio of investment to depreciation. Investments include spending on physical capital as well as intangible capital on and off the balance sheet. We also determine the depreciation of physical and intangible capital. The main explanatory variable is the total capital of the firm (standardized within firm) and its square. We control for the cash reserve (standardized within firm), industry trends, and firm fixed effects. Columns (2) and (3) split the sample after and on or before 1995, respectively. Columns (4) to (10) restrict the sample to industries based on the SIC-1 identifier. Standard errors are double-clustered by firm and year. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Investment (t+1) / Depreciation (t+1) $\times$ 100									
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Capital (t)	-45.56***	-43.43***	-44.09***	-64.70***	-32.79***	-41.26***	-52.70***	-50.66***	-56.78***	-39.36***
	(1.70)	(3.41)	(1.91)	(5.38)	(1.89)	(1.89)	(5.02)	(3.13)	(3.87)	(4.44)
Capital (t)^2	18.33***	15.81***	17.07***	23.76***	11.89***	18.09***	17.40***	21.84***	25.73***	15.26***
	(0.99)	(1.86)	(1.17)	(2.82)	(1.10)	(1.18)	(3.21)	(1.67)	(2.31)	(2.90)
Cash (t)	10.35***	9.87***	11.95***	20.47***	8.25***	9.64***	16.24***	9.29***	7.53***	8.99***
	(0.63)	(0.85)	(0.68)	(3.32)	(0.95)	(0.88)	(2.38)	(1.67)	(2.42)	(2.49)
Constant	157.40***	140.29***	171.57***	248.28***	135.16***	144.45***	173.55***	169.66***	155.89***	146.87***
	(0.05)	(0.78)	(0.53)	(0.25)	(0.12)	(0.09)	(0.31)	(0.20)	(0.20)	(0.26)
Specification	All	Yr>'95	Yr≤'95	SIC1=1	SIC1=2	SIC1=3	SIC1=4	SIC1=5	SIC1=7	SIC1=8
Firm FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
SIC-2×Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
% Adjusted R^2	51.78	58.05	50.51	47.49	44.20	46.59	49.23	53.86	52.32	58.95
% Within R^2	9.36	9.93	6.88	6.42	7.26	11.96	8.22	13.44	13.06	6.93
Observations	109711	50292	58810	8188	23114	37986	8229	15529	11726	4502

We provide evidence of diminishing returns to scale in our sample by examining how profitability varies with capital stock. Table D.20 shows that a firm's return on assets and return on equity decline with capital. Note that the specification includes firm and industry-by-year fixed effects.

Table D.20: Decreasing returns to scale

This table provides evidence of decreasing returns to scale. The outcome variable in column (1) is the return on assets (EBITDA/book value of assets), measured in percentage points. The outcome variable in column (2) is the return on equity (net income/book value of equity). All explanatory variables are standardized to facilitate comparisons. Each specification includes firm fixed effects and industry trends. Standard errors are double-clustered by stock and quarter. *, **, and *** indicate significance at the 10%, 5%, and 1% level, respectively.

	Return on Assets (%)	Return on Equity (%)
	(1)	(2)
Capital (t)	-6.46***	-1.93***
	(0.66)	(0.25)
Cash (t)	5.32***	1.33***
	(0.38)	(0.14)
Constant	6.04***	13.16***
	(0.01)	(0.00)
Firm FE	Yes	Yes
SIC-4 x Year FE	Yes	Yes
% Adjusted R^2	15.25	61.46
% Within R^2	0.48	0.88
Observations	93,916	93,840