



**FTG Working Paper Series**

Intermediation via Credit Chains

by

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Finance Theory Group

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# Intermediation via Credit Chains

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April 27, 2022

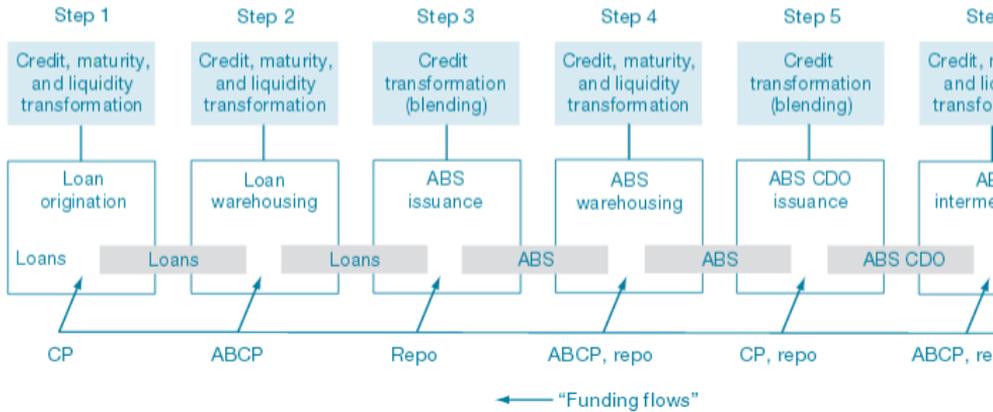
# Motivation: Long Credit Chains (1)

## Shadow banking system features long credit chains

### The Shadow Credit Intermediation Process

The shadow credit intermediation process consists of distinct steps. A credit intermediation chain, depending on the number of steps involved, may entail as few as three steps and as many as seven or more. The shadow banking system conducts these steps in sequential order. Each step is handled by specific types of financial entities, funded by specific types of liabilities.

"Asset flows" →

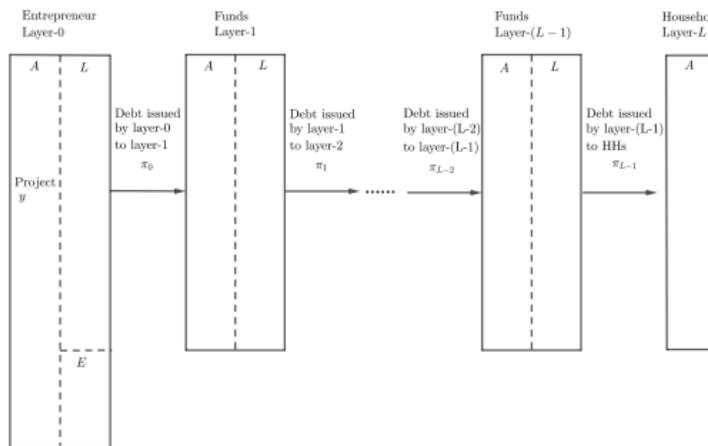


## Motivation: Long Credit Chains (2)

- Credit chain: **One institution's asset is another institution's liability**  $\Rightarrow$  Interconnected balance sheets
- During 2007-09 financial crisis, households invest in MBSs  
MMMFs hold ABCPs issued by special purpose vehicles (SPVs)  
 $\Rightarrow$  SPVs hold medium term bank notes  $\Rightarrow$  banks hold residential mortgages
- Nowadays, Mutual funds hold parts of collateralized debt obligations (CDOs)

# This Paper (1)

- Our model delivers intermediation via credit chain



- What is the benefit of intermediating via credit chain
- Is the equilibrium credit chain length socially efficient

# This Paper (2)

- Fundamental economic issue:
  - With market frictions, short-term debts facilitate **transformation** between households with short-term needs and long-term asset
  - But, short-term debt leads to excessive liquidation
- A fully dynamic model of endogenous credit chains following trade-off of layers
  - Benefit: insulate real project from liquidation given cash-flow
  - Cost: (exogenous) layer cost
- Today, mainly go through a three-period example
- Social planner can improve welfare by limiting credit

- **Credit chains:** Adrian and Shin (2010), Di Maggio and Lavezzi (2017), Donaldson and Micheler (2018), Glode and Opp (2016)
- **Debt runs:** Diamond and Dybvig (1983), Calomiris and Winton (2008), Goldstein and Puzner (2005), He and Xiong (2012), Gorton and Metrick (2012), Copeland, Martin and Walker (2014), Fahlenbrach, Nagel and Orlov (2014), He and Manela (2016), Schmitz and Wermers (2016)
- **Network and contagion:** Allen and Gale (2000), Allen and Carletti (2012), Elliott, Golub and Jackson (2014)
- **Asset trading chains:** Atkeson, Eisfeldt and Weill (2018), Opp (2016), Glode, Opp and Zhang (2019), Hugonnier and Weill (2019), Shen, Wei and Yan (2021)

# Outline

- 1 Example
- 2 Model
- 3 Equilibrium

# Example Setup

- $t = 0, 1, 2, 3$
- Households:
  - Two-period OLG born at  $t = 0, 1, 2$ , each endowed with a consumption good when born; no discount  $U_t =$
  - Can save in securities offered in the market

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- Entrepreneur owns a project which produces cash flow at the end of period 3
  - In  $t = 1$  or 2, **with prob.**  $p = 0.6$ , **good news** is revealed in period, then  $y = 1$ : two i.i.d. chances for “success”
  - Otherwise, fail,  $y = 0$

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  - Issue debt to maximize  $t = 0$  value  $P_0$  paid by OLG
- Zero-coupon debt securities, focus on different debt structures
  - (Optimal) principal payment  $F = 1$  upon mature

- When debt matures: refinance from new cohort to old cohort
  - If rollover fails, asset **liquidated** at discount  $\alpha$  (relative to the equilibrium market price) of the equilibrium market price
    - Fund's **asset** is entrepreneur's liability (debt); entrepreneur's liability is the real project itself
    - If long-term asset, kept being traded on the secondary market
  - If **rollover successful**  $\rightarrow$  no discount

## Setup Cont.

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- If debt does not mature: **sell at discount** in the secondary market
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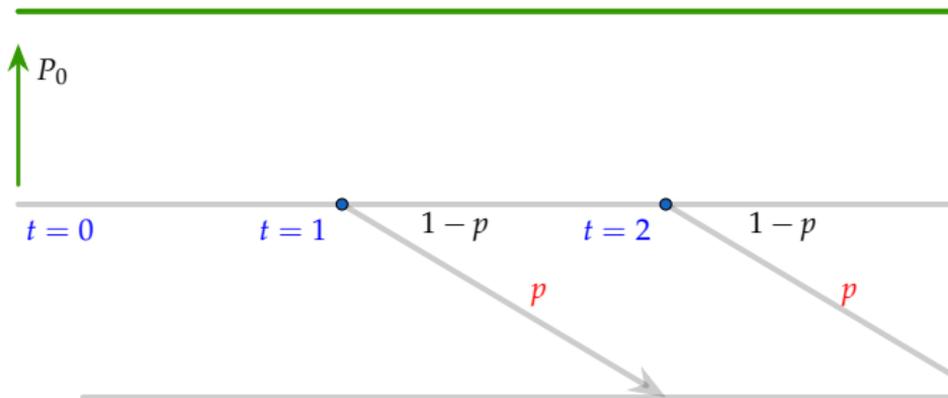
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  - Households trade debt via intermediaries in the secondary market at discount  $\alpha < 1$
- Both "discounts" apply to equilibrium market price
  - Micro-foundation: intermediaries buy the securities (liquidation or resell), put on their book for one month, and sell to households next morning

- In the example (as well as the model) we set  $\alpha =$ 
  - Liquidation  $\alpha_l$  could differ from secondary-market just need both to be less than 1
- **Rollover, if successful**, is the best way to transfer households (than **liquidation** or **secondary-market**)
  - Liquidation cost is prohibitive
  - Corporate bonds transaction cost 53 bps, but ABC underwriting/issuance fee 5-15 bps

# Case 0: Long-Term Three-Period Debt

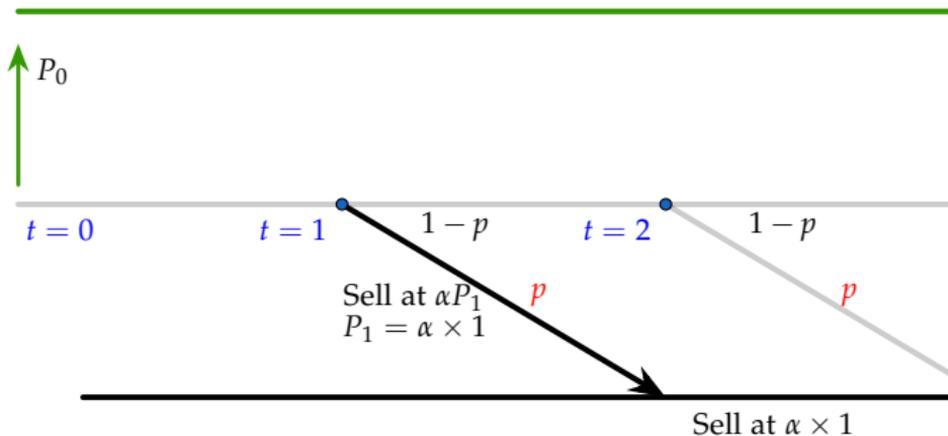
- Entrepreneur issues three-period debt to households



$$P_0 \text{ (three-period debt)} = 0.6 \times 0.5^2 + 0.4 \times 0.6 \times$$

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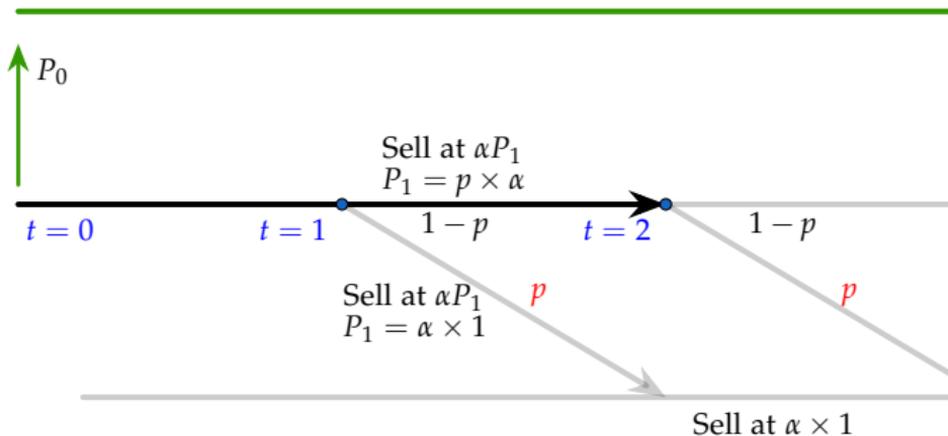
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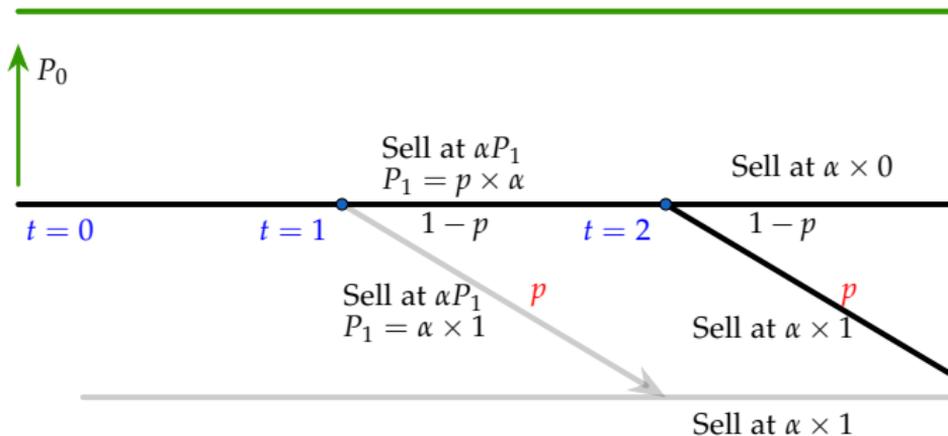
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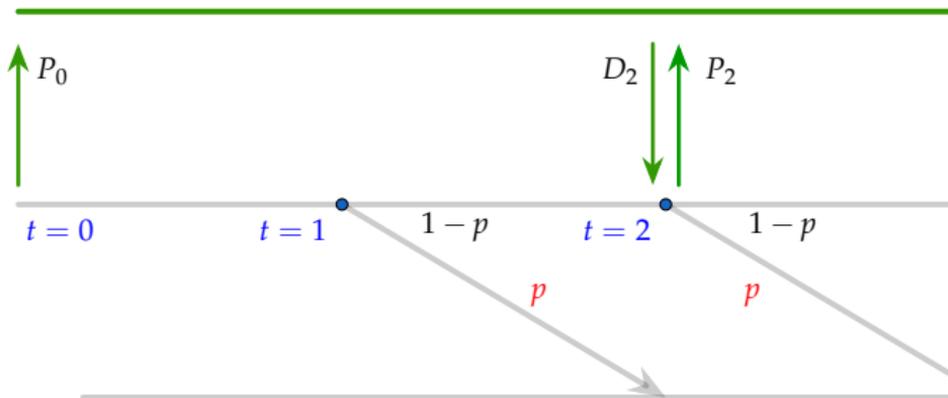
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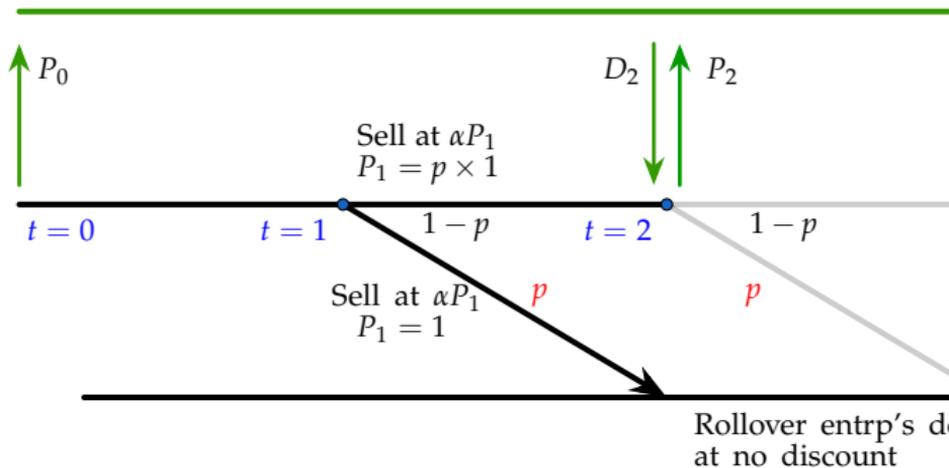
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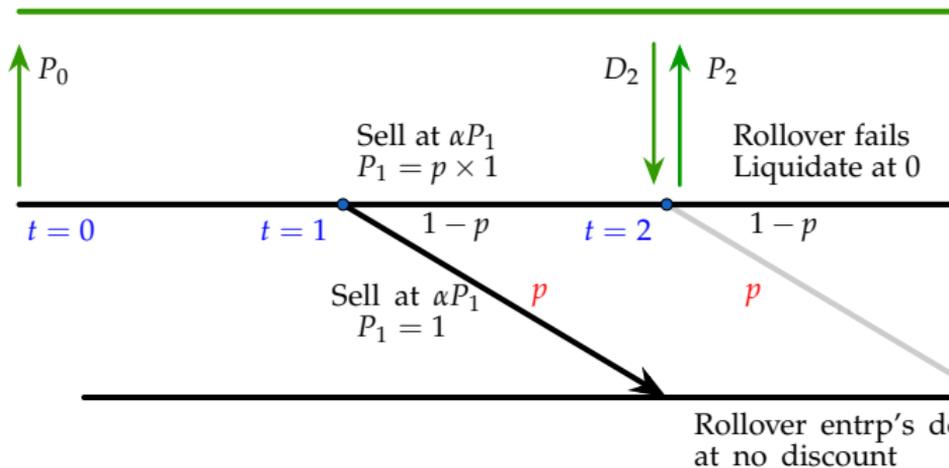
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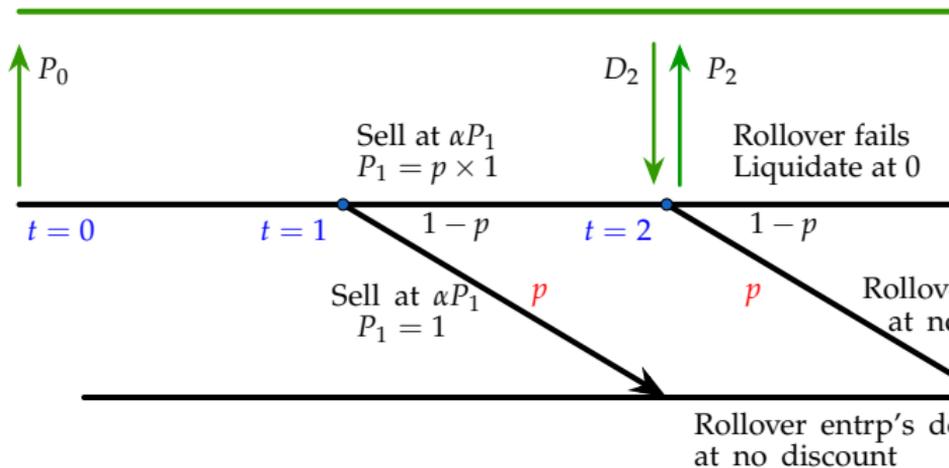
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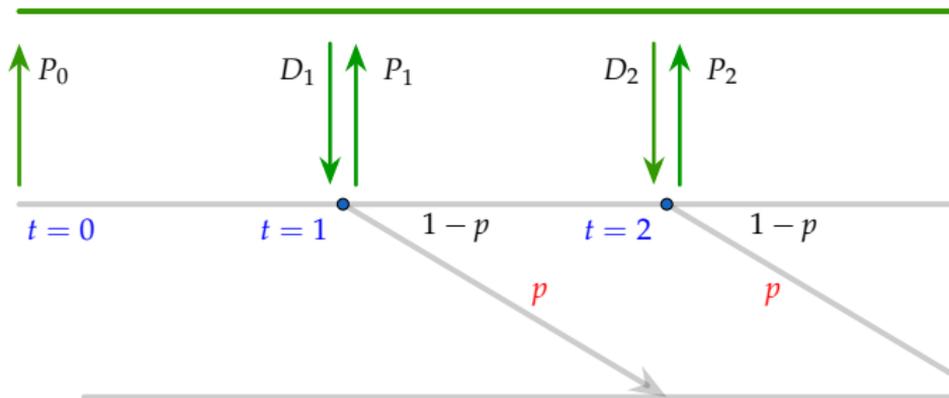
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## Case 2: Short-Term One-Period Debt

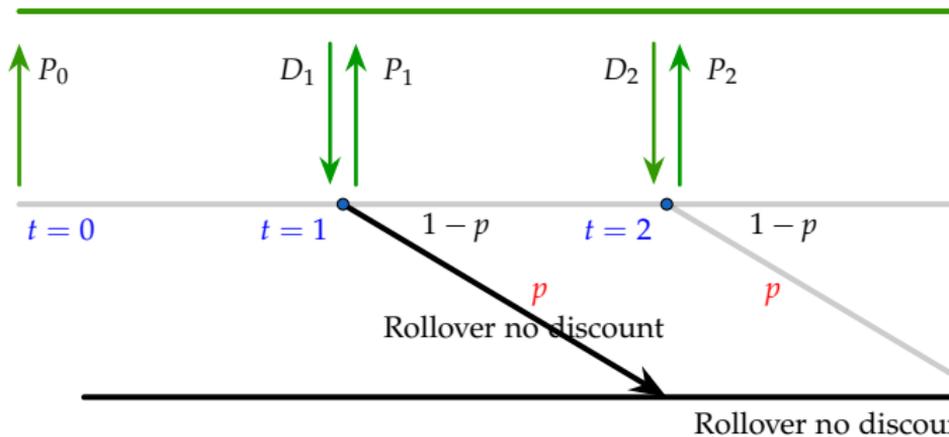
- One-period debt to households; the project liquid any rollover failure



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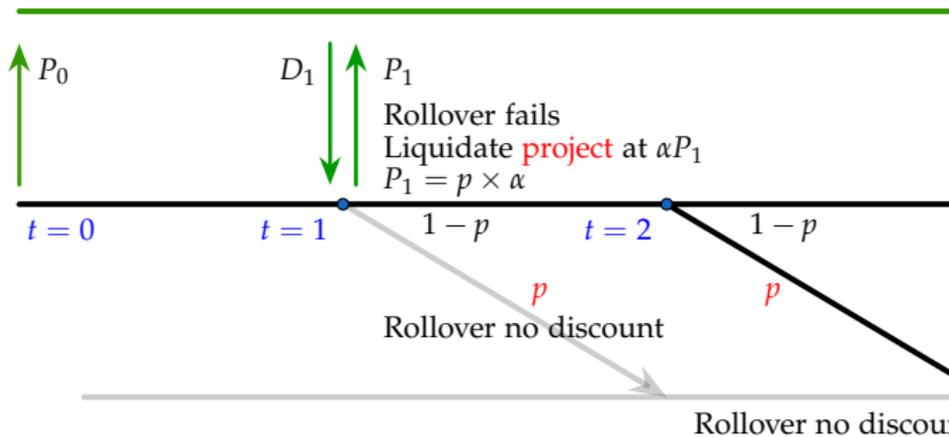
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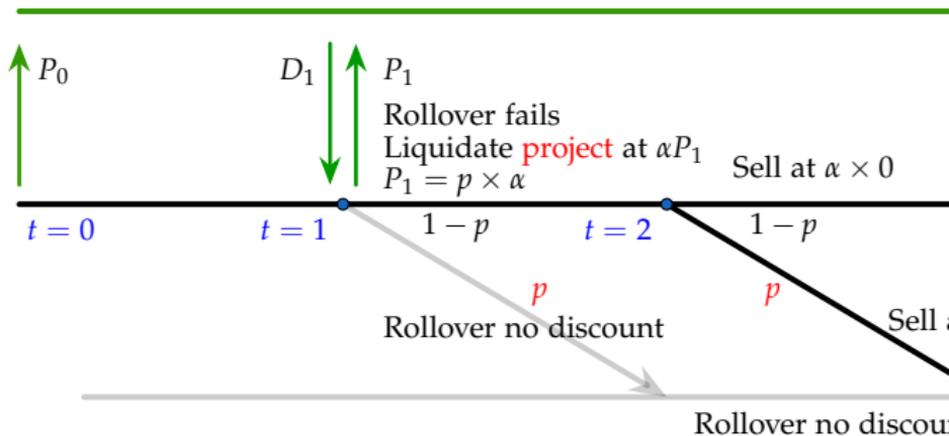
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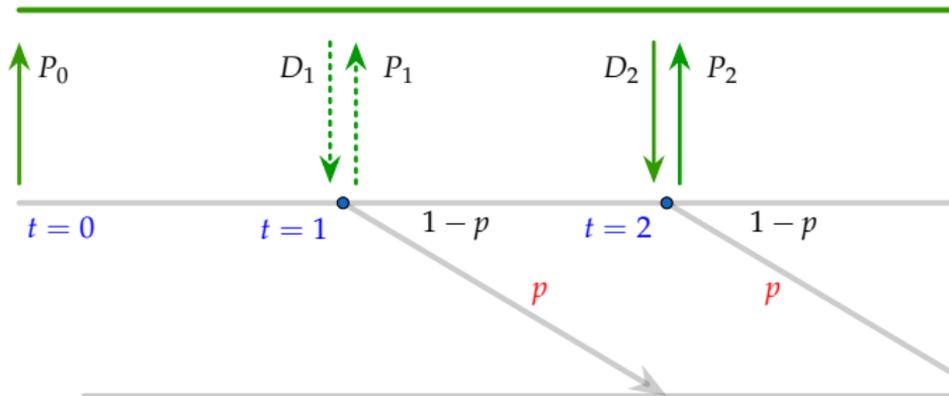
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# Special Case: State-Contingent Maturity

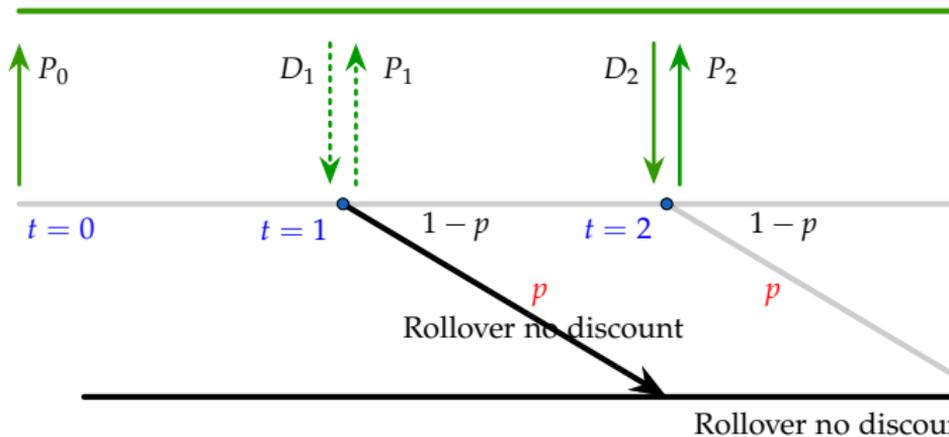
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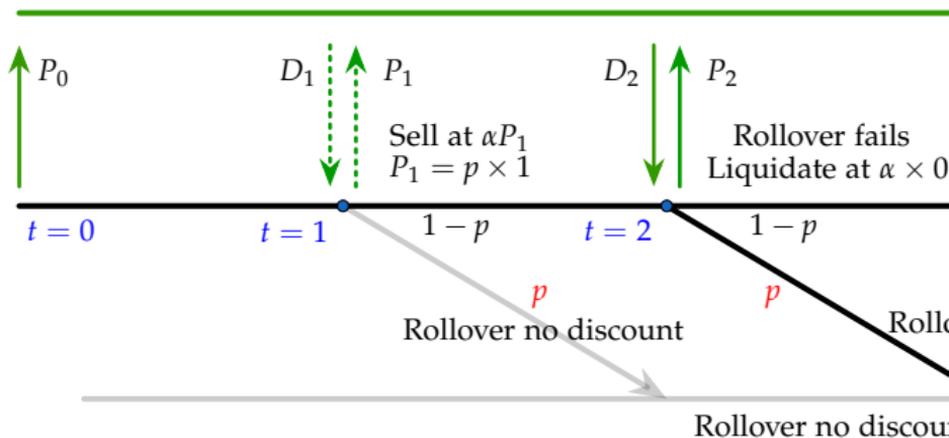
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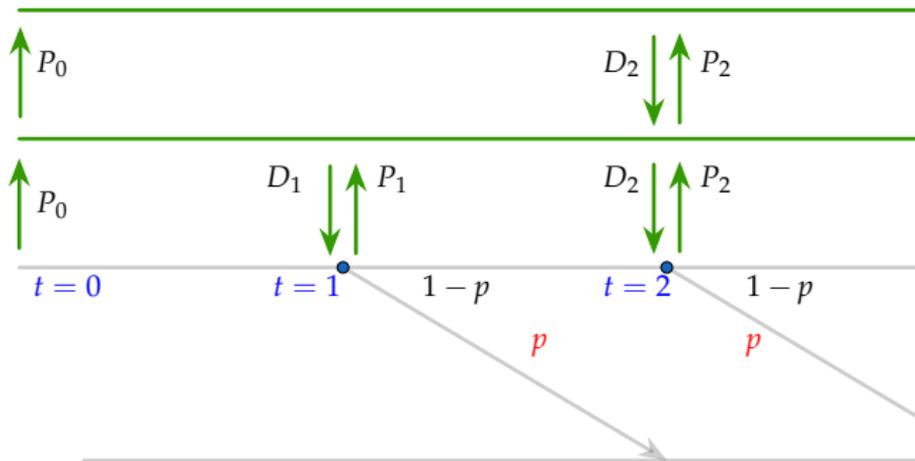
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## Case 3: Two-Layer Credit Chain

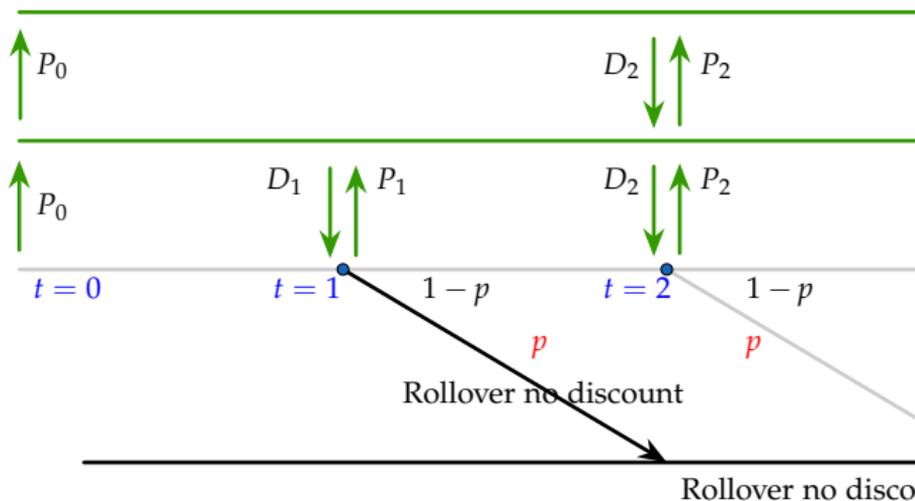
- Entrepreneur issues a two-period debt to fund, w one period debt to households
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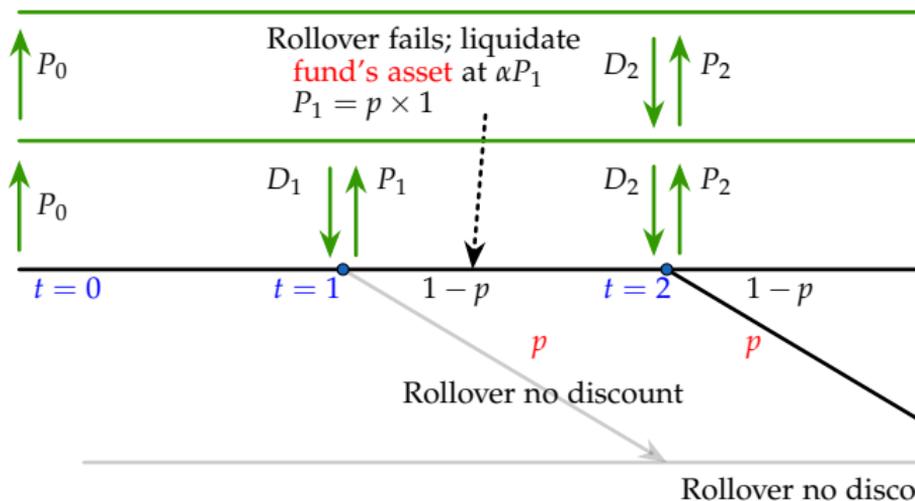
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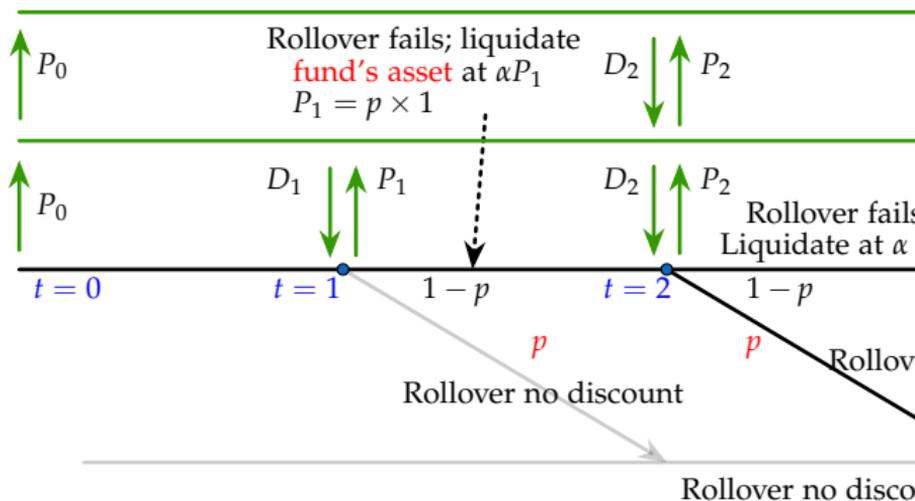
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- Short-term debt gets around secondary market trading but could trigger costly rollover failure
- Two-layer structure “replicates” the state-contingent maturity and dominates short-term debt structure
  - If period 1 rollover fails, the layer structure preserves subsequent short-term debt claims over cash-flow
  - In contrast, single-layer short-term debt: long-term over cash-flow gets traded repeatedly

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  - In contrast, single-layer short-term debt: long-term over cash-flow gets traded repeatedly
- Conceptual connection to Diamond and Rajan (2001)
  - There, intermediaries increase recovery value on the **continuation game** after asset liquidation (due to human capital)
  - Here, credit chain matters for **continuation game** project's future cash flows from heavy discount

# Extending to Multiple Layers

- The intuition extends to  $L$  layers
  - Suppose project matures in  $L$  periods
  - Entrepreneur (layer 0) issues debt with  $m = L - 1$
  - Fund in layer 1 holds debt with  $m = L - 1$  and issues debt with  $m = L - 2$
  - ...
  - Fund in layer  $l$  holds debt with  $m = L - l$  and issues debt with  $m = L - l - 1$ , for any  $l$
- In the model, for tractability we assume debt contracts have random maturity
  - Each layer's debt contract has random maturity  $\lambda_d$  and matures if above-layers' debts mature
  - Fund in layer  $l$  is holding debt with maturity rate  $\lambda_d$  and is issuing debt with maturity rate  $1 - (1 - \lambda_d)^{l+1}$

# Outline

1 Example

2 Model

3 Equilibrium

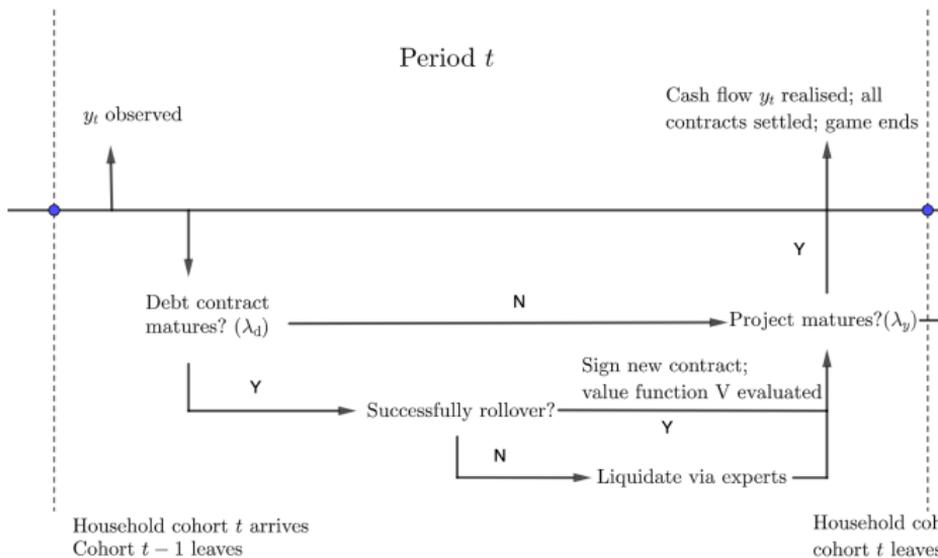
# Households and the Firm

- Time is discrete and infinite
- OLG Households: measure 1
  - Live for two periods: no discount

$$U_t = c_t^t + c_{t+1}^t$$

- Endowed with  $e$  units of consumption good when young
- Entrepreneur: infinitely lived with time discount
- Project: matures with probability  $\lambda_y$  each period
  - Produces  $y_t$  if matures in period  $t$ ; produces nothing otherwise
  - $y_t \sim H(\cdot)$  and is i.i.d. across periods
- Intermediaries: infinitely lived with time discount

# Timing



# Contracts

- Debt-like contract:  $\pi_t = \{\tilde{F}_{y,s}, F_{d,s+1}\}_{s=t}^T$  matures w.p.  $\lambda_d$  in each period,  $\mathcal{F}_s$ -measurable for any  $s \geq t$ 
  - $\tilde{F}_{y,s} \cdot \mathbf{1}_{\text{project matures at period } s}$ , w.p.  $\lambda_y$  +  $F_{d,s+1} \cdot \mathbf{1}_{\text{debt matures at period } s}$ 
    - $\tilde{F}_{y,s}(y_s) = \min(F_{y,s}, y_s)$
    - $y_s$  is known (and occurs if project matures) when contract at  $t = s$  which affects the rollover
- Creditors can renegotiate by “prepaying” the debt if it matures
  - Effectively, creditors can unilaterally trigger the debt
  - Focus on renegotiation-proof contracts; ensures s.p.
- Price  $P(\pi, y)$ . This is the endogenous proceeds from offering  $\pi$  given  $y$

# Credit Chain

- Consider credit chain of length  $L$ : entrepreneur is at layer 0, households are at layer  $L$
- A fund in layer  $l$  borrow from  $l + 1$  with contract
- Resource constraint:

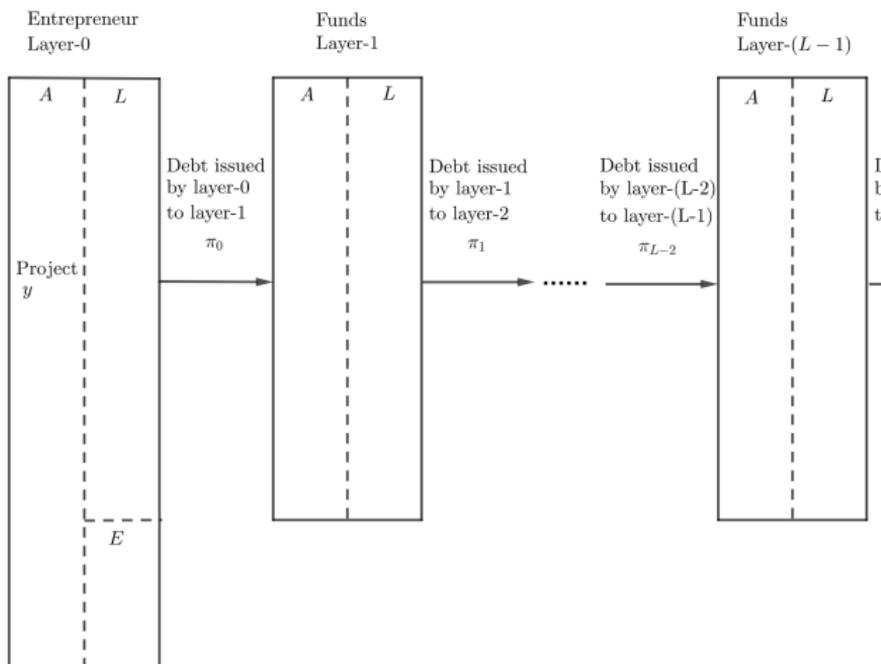
$$F_{y,l} \leq F_{y,l-1} \leq y \quad \text{for } 0 < l < L$$

- No-bubble constraint:

$$F_{d,l} \leq F_{d,l-1} \leq e \quad \text{for } 0 < l < L$$

- “Event-triggered prepayment” among contracts in layer  $l$ 
  - When the project matures,  $y$  trickles down
  - When  $l$ -layer debt matures and successful rollover occurs,  $e$  trickles down for  $l' > l$  (or just restore at  $l$ )

# Illustration of the Layer Structure



# Debt Rollover and Secondary Market

- When  $y_t \geq \hat{y}_l$ , layer  $l$  can rollover successfully
  - Nothing changes in the chain structure
- Otherwise, rollover fails
  - Layer  $l$ 's asset is worth  $B_l(y, L)$  to cohort  $t$  households  
 $B_l(y, L)$  endogenously determined
  - Liquidation will be intermediated by a distressed lender with discount  $\alpha \Rightarrow$  distressed lender pays only  $\alpha B_l(y, L)$
  - Restructuring/legal cost  $c \geq 0$  per layer
- Cohort  $t - 1$  households recover

$$\min(\alpha B_l(y, L), F_{d,l}) - c(L - l)$$

- To determine  $B_l(y, L)$ , we assume
  - With probability  $\beta \in [0, 1]$ , the chain is restored in
  - With probability  $1 - \beta$ ,
    - Cohort  $t$  households hold debt issued by layer  $l -$  period
    - The chain is restored with probability 1 in the fol absent another run
  - $\beta < 1$  makes the ex-ante chain formation matters
- The entrepreneur is always rehired after rollover project-specific human capital

- In the event of successful rollover  $y > \hat{y}$ ,  $l$ -fund takes  $y$  given, choose  $\pi_l$  to maximize proceeds and continue

$$\mathbf{1}_{\text{rollover}}^l (P_l(\pi_l, y; \pi_{l-1}, L) - F_{d,l} + V_l(y, \pi_l; \tau))$$

▶ Funds' Value Function

▶ Entrepreneur's Value Function

- Households' payoff

$$e - P_{L-1}(y) + V_L(y; \pi_{L-1}, L)$$

▶ Households' Value Function

# Outline

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# Equilibrium Definition

- Contracts  $\pi_{l,t}$  are chosen to maximize the respective payoff
- Equilibrium credit chain length  $L^*$  is such that firm managers ( $L^* - 1$ ) prefers to borrow directly from households
  - For all other funds  $0 < l < L^* - 1$ , they prefer to borrow from other funds than to borrow directly from households
- Intermediary funds are competitive to earn zero profit

# Optimal Contract

- Under Assumption 1 which says  $e$  is relatively small, contracts are **stationary** over time
  - $y_t$  is i.i.d. and  $F_{d,l,t} \leq e$  binds in all periods
- Contracts are **layer independent**:

$$F_{d,0} = F_{d,1} = \dots = F_{d,L-1} \quad F_{y,0} = F_{y,1} = \dots$$

- Lower layers (larger  $l$ ) are less concerned about rollover risk
  - Why? Any layer  $l$  only cares about the rollover risk of households ( $L$ )  $\Rightarrow$  larger  $l$  set higher  $F_{d,l}$
  - Hence  $F_{d,l} \leq F_{d,l-1}$  (no-Ponzi condition) binds for all  $l$
  - $F_{d,l} = F_{d,l-1} \Rightarrow F_{y,l} = F_{y,l-1}$  because of zero profit
- $F_y$  equals the run threshold (stationary, layer independent)
- $\pi^*$  is characterized in closed form

# Equilibrium Chain Length

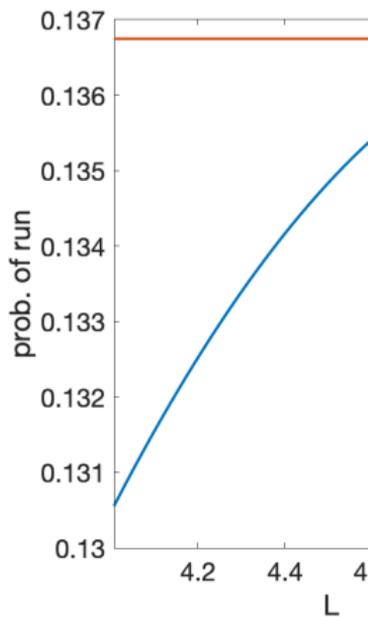
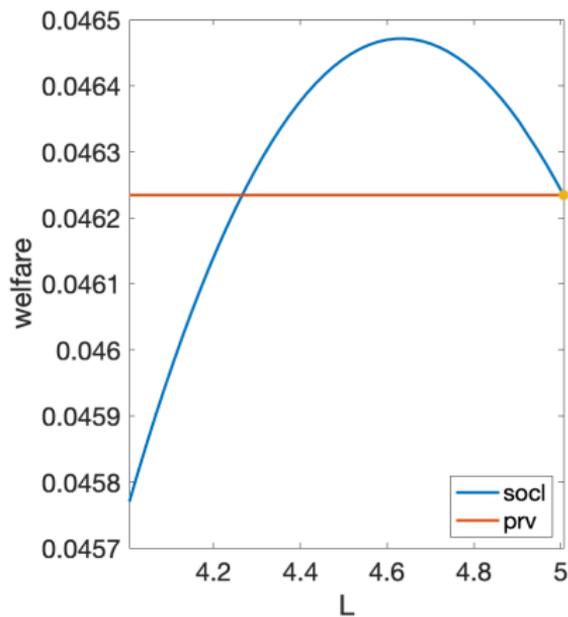
- The equilibrium chain length  $L^*$  minimizes the ro

$$L^* = \arg \min_L F_y(e, L) \quad \text{▶ Details}$$

- Social v.s. private: endogenize  $e$  and analyze whether credit chain length improves welfare ▶ Model Details
- Key friction: layer  $L - 1$  decides the chain length, takes as given when designing contract
  - Shorter chain  $\Rightarrow$  smaller maturity mismatch  $\Rightarrow$  smaller risks  $\Rightarrow$  larger borrowing amount  $F_d$
  - Decentralised  $F_d$  is too low; entrepreneur/funds cannot commit to future contract
  - Though  $F_d$  up, the overall stability increases

# Numerical Example

Social v.s. Private Welfare and Probability of



# Conclusion

- Provide a model of credit chains with endogenous chain length
- Highlight the asset insulation effect of the layer structure
- Despite the benefit of intermediating via layers, the equilibrium features too long chains

# Appendix: Value Functions—Funds

$$\begin{aligned}
 V_l(y, \pi_l; \pi_{l-1}, L) &= \lambda_y \underbrace{(\tilde{F}_{y,l-1} - \tilde{F}_{y,l})}_{\text{Project matures}} \\
 &+ (1 - \lambda_y) \alpha \left\{ (1 - \lambda_d)^{l+1} \mathbb{E} \left[ \underbrace{V_l(y', \pi_l; \pi_{l-1}, L)}_{\text{Neither debt issued by nor held by layer } l \text{ matures}} \right] \right. \\
 &+ \underbrace{\sum_{i=0}^{l-1} (1 - \lambda_d)^i \lambda_d \mathbb{E} \left[ \mathbf{1}_{\text{rollover}}^i (-F_{d,l} + F_{d,l-1} - P'_{l-1} + \max_{\pi'_l} (P'_l + V_l(y', \pi'_l; \pi_{l-1}, L))) \right]}_{\text{Debt held by layer } l \text{ matures}} \\
 &\left. + (1 - \lambda_d)^l \lambda_d \mathbb{E} \left[ \underbrace{\mathbf{1}_{\text{rollover}}^l (-F_{d,l} + \max_{\pi'_l} (P'_l + V_l(y', \pi'_l; \pi_{l-1}, L)))}_{\text{Debt held by layer } l \text{ does not mature but debt issued by layer } l \text{ matures}} \right] \right\}
 \end{aligned}$$

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$$\begin{aligned}
 V_0(y, \pi_0; L) = & \underbrace{\lambda_y(y - \tilde{F}_{y,l})}_{\text{Project matures}} + (1 - \lambda_y)\alpha \left\{ (1 - \lambda_d)\mathbb{E} \left[ \underbrace{V_0(y', \pi_0; L)}_{\text{Debt issued by layer-0 d}} \right] \right. \\
 & + \lambda_d \mathbb{E} \left[ \underbrace{\mathbf{1}_{\text{rollover}}^0 (-F_{d,0} + \max_{\pi'_0} (P'_0 + V_0(y', \pi'_0; L)))}_{\text{Debt issued by layer-0 matures and rollover succeeds}} \right] + \\
 & \left. \underbrace{(1 - \mathbf{1}_{\text{rollover}}^0) [(\beta + (1 - \beta)(1 - \lambda_y)\alpha)(-P'_{-1} + \max_{\pi'_0} (P'_0 + V_0(y', \pi'_0; L)))]}_{\text{Debt issued by layer-0 matures and rollover fails}} \right\}
 \end{aligned}$$

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$$\begin{aligned}
 V_L(y; \pi_{L-1}, L) = & \lambda_y \underbrace{\tilde{F}_{y,L-1}}_{\text{Project matures}} \\
 & + (1 - \lambda_y) \left\{ (1 - \lambda_d)^L \mathbb{E}[\underbrace{\alpha V_L(y'; \pi_{L-1}, L, L)}_{\text{Debt does not mature}}] \right. \\
 & \left. + \sum_{l=0}^{L-1} (1 - \lambda_d)^l \lambda_d \mathbb{E}[\underbrace{\mathbf{1}_{\text{rollover}}^l F_{d,l-1} + (1 - \mathbf{1}_{\text{rollover}}^l) (\alpha B_l(y))}_{\text{Debt matures}}] \right\}
 \end{aligned}$$

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# Appendix: Equilibrium Credit Chain Length

Rollover:  $F_y(e, L)$  is the solution to the following equation

$$\underbrace{V_L(F_y; \{e, F_y\}, L)}_{\text{Demand of debt}} = \underbrace{e}_{\text{Supply}}$$

## Proposition

*The equilibrium chain length  $L^*$  minimizes the rollover risk*

$$L^* = \arg \min_L F_y(e, L),$$

*which is characterized by the following equation uniquely:*

$$0 = \underbrace{\lambda_d(1 - \lambda_d)^{L^*}(1 - H(F_y))(1 - \alpha)e}_{\text{Marginal benefit of increasing } L} - \underbrace{cH(F_y)(1 - \alpha)}_{\text{Marginal cost}}$$

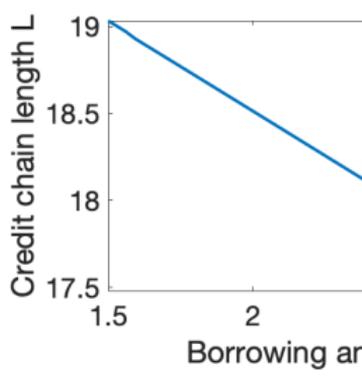
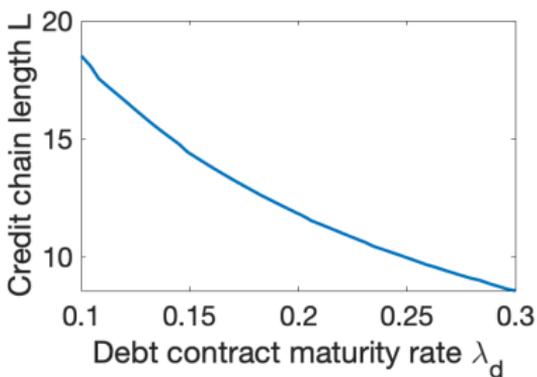
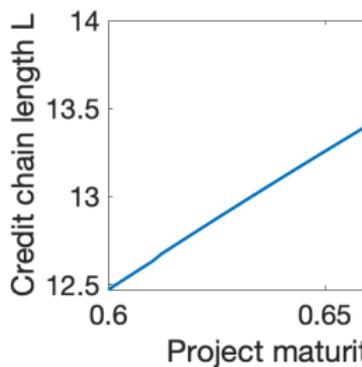
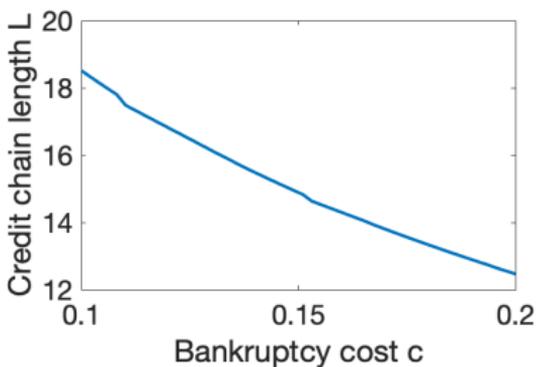
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## Special Case: $c = 0$

- When  $c = 0$ , equilibrium chain length is  $\infty$
- Hypothetical structure:  $L$  layers, but debt maturity households (layer  $L$ ) and layer  $L - 1$ ) is  $1 - (1 - \lambda_y)$ 
  - Effective debt maturity rate is the same as having
  - Denote its value by  $\tilde{V}_L(L)$
- Consider the change in value for households when

$$\begin{aligned} & V_{L+1}(L+1) - V_L(L) \\ = & \underbrace{\tilde{V}_L(L) - V_L(L)}_{\text{Benefit of shorter maturity}} + \underbrace{\frac{(1 - \lambda_y)\lambda_d m_L H(F_y)(B_L(L))}{1 - (1 - \lambda_y)\alpha(m_{L+1} + H)}}_{> 0 \text{ net benefit of more layers; illust}} \end{aligned}$$

# Comparative Statics of Credit Chain Length



## Appendix: Model Modification

- Richer setting to endogenize  $e$  while maintaining of the contracts
- Modified timing at period  $t$ :
  - Before  $y_t$  is realised, each household chooses  $c_t^D$
  - After  $y_t$  realises, households can choose to consume in addition, but only receive a utility of  $1 - \epsilon$  per unit
  - Minimum departure from baseline yet endogenize

$$e - c_t^D \rightarrow F_d, \quad c_t^N \begin{cases} \rightarrow 0 & \text{If debt matures and rollover} \\ > 0 & \text{Otherwise} \end{cases}$$

- In period  $t + 1$ , households collect money, consume in the economy
- No households observe the contracts, due to the credit chain

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## Appendix: Optimal Condition of $F_d$

- Consider one shot deviation in  $F_{d,t}$  and take as given  $F_{d,t+s} \leq e - c_{t+s}^D$
- There are a continuum of equilibria, we focus on the one that yields the highest welfare

$$(1 - \alpha)(1 - m_L)(1 - H(F_y)) - \frac{h(F_y)}{\lambda_y} \sum_{l=0}^{L-1} \lambda_d m_l (F_d - \alpha B_l(F_y))$$

- Higher  $F_d$ 
  - Benefit: larger value created from differences in information
  - Cost: higher probability of rollover failures

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## Appendix: Welfare Analysis

- Does limiting credit chain length improve welfare increasing borrowing amount  $F_d$

$$\underbrace{\frac{dW}{dF_d}}_{>0} \underbrace{\frac{dF_d}{dL}}_{<0} + \underbrace{\frac{dW}{dF_y}}_{=0} \underbrace{\frac{dF_y}{dL}}_{=0} + \underbrace{\frac{dW}{dL}}_{=0} < 0$$

- Key friction: layer  $L - 1$  decides the chain length, takes as given when designing contract;
  - Shorter chain  $\Rightarrow$  smaller maturity mismatch  $\Rightarrow$  smaller risks  $\Rightarrow$  larger borrowing amount  $F_d$
  - Decentralised  $F_d$  is too low; entrepreneur/funds cannot commit by period and cannot commit to future contract
  - Though  $F_d$  up, the overall stability increases
- Samuelson (1958)

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**Diamond, Douglas W. and Raghuram G. Rajan, “Liquidity Creation, and Financial Fragility: A Theory of the Money Market”**  
*Journal of Political Economy*, 2001, 109 (2), 287–327.