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Central Bank Digital Currency and Financial Stability

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Abstract

We develop a model of financial intermediation with remunerated Central Bank Digital Currency (CBDC) as consumers' alternative to bank deposits and an endogenous risk of bank runs. Echoing widespread concerns, higher CBDC remuneration raises bank fragility by increasing consumers' withdrawal incentives. On the other hand, it also induces banks to offer more attractive deposit contracts in order to retain funding, thereby reducing fragility. This results in a U-shaped relationship between bank fragility and CBDC remuneration. We evaluate policy proposals aimed at mitigating the financial-stability risks of CBDC, such as holding limits and contingent CBDC remuneration.

Keywords: Central Bank Digital Currency, Bank Fragility, Financial Stability, CBDC Remuneration, Global Games.

JEL Codes: D82, G01, G21.

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1 Introduction

Central banks around the globe are researching the costs and benefits of central bank digital currency, or CBDC (Kosse and Mattei, 2022). These efforts are a response to the declining importance of cash as means of payment and the challenges associated with the proliferation of new forms of private digital money such as stablecoins. While CBDC aims to preserve the role of public money and fend off threats to monetary sovereignty, some policy makers are concerned about its potentially adverse effects on the financial system (Ahnert et al., 2022) and on the business model of traditional banks (Vives, 2019).

One issue that has received particular attention is the effect of CBDC on financial stability (Bank for International Settlements, 2020). Its status as safe asset with potentially positive remuneration—a key difference to physical cash—could render it an attractive store of value and thus increase the risk of bank runs during crisis times. The 2023 U.S. regional banking crisis highlights that bank runs continue to be an important real-world phenomenon.

This paper aims to inform this debate by incorporating remunerated CBDC in a global-games bank-run model (Goldstein and Pauzner, 2005; Vives, 2005; Carletti et al., 2023). At the initial date, a profit-maximizing bank with access to profitable but risky long-term investment opportunities raises uninsured deposits. At the interim date, consumers receive a noisy private signal about the investment's profitability ("economic fundamentals") and decide whether to withdraw their balances or roll them over. Funds that are not kept in the monopolist bank can be held either in cash or CBDC. In line with the existing literature, we assume that CBDC holdings are possibly remunerated, which improves consumers' outside option and thus curtails the bank's market power in the deposit market (see e.g. Chiu et al. 2023; Andolfatto 2021; Whited et al. 2023).

When making withdrawal decisions, consumers trade off the returns from keeping their funds in the bank and storing them in CBDC. Accordingly, our model allows us to study how the terms of the deposit contract and CBDC remuneration affect the probability of a bank run, our measure of bank fragility.

In this economy, an increase in CBDC remuneration has two effects. First, it makes withdrawals at the interim date more attractive by offering a higher payoff from storing funds with the central bank for consumption at the final date. This *direct effect* makes the bank more fragile, consistent with the line of argument underlying the ongoing policy debate (see also Williamson, 2022). Second, a higher CBDC remuneration improves consumers' outside option at the funding stage. This induces the bank to offer more attractive deposit contracts, which is achieved via the long-term interest rate promised to depositors who keep their funds in the bank until the final date. As a consequence, consumers have lower incentives to withdraw their funds at the interim date. This *indirect effect* renders the bank more stable.

In equilibrium, the total effect of CBDC remuneration on bank fragility depends on the relative strengths of these two countervailing forces. The indirect effect dominates if and only if the elasticity of the run probability with respect to the bank deposit rate exceeds one. A sufficient condition for this to obtain is that the profitability of the bank's investment project is high relative to the level of remuneration on CBDC. In this case, bank fragility is minimized (corresponding to maximized utilitarian welfare) for a strictly positive level of CBDC remuneration.

Importantly, CBDC remuneration has redistributive effects. A higher CBDC rate moves rents from banks (whose expected profits shrink) towards depositors (who earn higher deposit rates). As long as the CBDC rate is not too high, this redistribution is socially desirable because it helps to make banks more stable.

Next, we examine the potential effects of two CBDC design features that have received attention in the policy debate. Various central banks (including the European Central Bank and the Bank of England) have proposed the introduction of individual holding limits with the aim of reducing financial stability concerns associated with CBDC (Bindseil et al., 2021; Bank of England, 2023). In our model, such a policy reduces the effective remuneration that consumers earn on withdrawn funds because they can store at most a fraction of their wealth in CBDC and must hold the remainder in cash. We show that holding limits do not improve welfare when the interest on CBDC can be set freely. However, they can be a relevant tool if the CBDC rate is exogenous from a financial stability perspective, for example because it mainly aims at monetary policy objectives outside of the model. Building on our previous insights, such limits can improve social welfare for high levels of CBDC remuneration because they reduce the attractiveness of consumers' outside option. By contrast, it is optimal not to impose any holding limits if the CBDC rate is low. This result serves as a cautionary note to policymakers, suggesting that holding limits are not a panacea from a financial stability perspective.

Alternatively, we study the possibility that CBDC remuneration is contingent on the state of the financial system, i.e. that CBDC holdings earn a lower rate in crisis times. We distinguish between two dimensions of such a policy, namely i) the threshold in terms of interim withdrawals at which it enters into effect, and ii) the reduction in remuneration relative to tranquil times. We show that a more restrictive design along the first dimension always reduces fragility (akin to a partial suspension of convertibility), and provide conditions for this to be the case along the second dimension. Intuitively, this policy tool dampens the direct effect by reducing the return from withdrawing deposits. On the other hand, it has little impact on the ex-ante return from holding CBDC, so its impact on the indirect effect is weak. We conclude that contingent remuneration can be an effective tool to reduce unwanted financial stability implications of CBDC.

We extend the model in three directions. First, we limit the bank's market power in the deposit market by assuming that the deposit contract is determined by Nash bargaining with depositors. In line with the intuition from the baseline model, a decline in bank market power weakens the indirect effect by reducing the bank's incentives to adjust the deposit rate in response to changes in CBDC remuneration. We show analytically that a higher CBDC rate always increases bank fragility with perfect competition, because the deposit rate is already quite high and thus not very responsive to changes in CBDC remuneration. We also provide numerical examples which show that our baseline result still holds when the bank has sufficient market power in the deposit market. In this case, the bank's profit margin is large enough to afford the higher deposit rate required to attract deposit funding.

Second, we explore the existence of the indirect effect and its beneficial effect on financial fragility in a traditional bank-run model where banks provide liquidity to risk-averse depositors (Diamond and Dybvig, 1983). To this end, we introduce a remunerated CBDC into the model of Goldstein and Pauzner (2005) where, unlike our baseline framework, the bank adjusts the interim repayment to make deposits more attractive. This extension serves two purposes. First, it allows us to fully micro-found the deposit contract and the existence of panic runs. Second, it allows us to show that the bank always adjusts the deposit contract in a way that, ceteris paribus, reduces financial fragility, irrespective of whether this occurs through the interim or final repayment. We show that when consumers' risk aversion is sufficiently low, the interim deposit rate decreases in the level of CBDC remuneration, consistent with the result in Keister and Monnet (2022).

Third, we extend our model to allow for bank risk-taking on the asset side, which broadens the analysis to a more complete notion of financial stability. Specifically, we assume that the bank can exert costly monitoring effort to increase the success probability of the risky investment. In this context, we define financial stability as the probability that the bank survives, i.e. there is no run *and* investment succeeds. We show analytically that higher CBDC remuneration affects these two separate components in different ways. It induces the bank to increase its monitoring intensity, but at the same time increases fragility due to higher withdrawal incentives. Notably, the indirect effect from our baseline model is absent as we assume an exogenous deposit contract for tractability. Accordingly, the overall effect on financial stability is ambiguous. Consistent with the result in our main analysis, we provide a numerical example that shows that an increase in CBDC remuneration can lead to higher financial stability.

Literature. Our paper is part of a fast-growing literature on CBDC. An overview of recent work is found in Ahnert et al. (2022). A key feature of our model is that the bank is not passive, but instead adjusts its behaviour (here its deposit rates) in response to the introduction to CBDC. This channel is also present in recent papers that study the effects of CBDC on credit supply and liquidity services (Brunnermeier and Niepelt, 2019; Andolfatto, 2021; Chiu et al., 2023; Keister and Sanches, 2022; Whited et al., 2023; Niepelt, 2023; Paul et al., 2024). In contrast, our focus is on financial stability.

Several other papers connect CBDC to financial stability. Using a Diamond and Dybvig (1983) model, Fernández-Villaverde et al. (2021) and Schilling et al. (2024) study the implications for bank runs. They show that the introduction of CBDC completely removes the risk of bank runs by fostering a flow of deposits from the banking system into the central bank (also featured in Skeie, 2020), but create a trade-off between efficiency and price stability. Keister and Monnet (2022) also consider the implications of CBDC for bank runs, but focus on the efficacy of government interventions. In their framework, CBDC allows the central bank to have more accurate information about the health of the banking sector and thus to intervene promptly to mitigate the risk of a run. In Williamson (2022), the fragility of banks induced by the introduction of CBDC is ex-ante efficient.

A key difference relative to these papers is our use of global-games methods to uniquely pin down the probability of a bank run. This approach allows us to study how CBDC design affects bank fragility, both directly via withdrawal incentives and indirectly via the bank's response in deposit rates. Global games were introduced by Carlsson and van Damme (1993), and have been widely applied to study run-like behaviour (e.g. Rochet and Vives, 2004; Goldstein and Pauzner, 2005; Vives, 2014; Bouvard et al., 2015; Liu, 2016; Ahnert, 2016; Eisenbach, 2017; Ahnert et al., 2019; Liu, 2023; Carletti et al., 2023; Schilling, 2023).[1] Our contribution to the standard global-games bank-run model is to allow for a portfolio choice of investors. In particular, they can choose between CBDC and bank deposits at both the funding and the withdrawal stage.

[1] Morris and Shin (2003) and Vives (2005) survey the theory and applications of global games.

2 Model

The model builds on Goldstein and Pauzner (2005) and Carletti et al. (2023). The economy extends over three dates $t = 0, 1, 2$ and is populated by a bank and a unit continuum of consumers indexed by $i \in [0, 1]$. There is a single divisible good for consumption and investment. All agents are risk neutral, and there is no discounting. Consumers are endowed with one unit of funds at $t = 0$ only.

At $t = 0$, the bank has access to a profitable but risky investment technology. A unit investment returns $L \in (0, 1)$ if liquidated at $t = 1$ (the liquidation value) and $R\theta$ upon maturity at $t = 2$, where $\theta \sim U[0, 1]$ represents the economic fundamental and $R > 2$ is a constant that reflects the return from lending.

To finance investment, the bank raises funds from consumers in exchange for demandable deposit contracts.[2] The bank chooses the deposit contract that maximizes expected profits. The contract specifies a repayment $r_1 \geq 1$ at $t = 1$ and r_2 at $t = 2$. As will become clear later, the lower bound on r_1 has two implications. First, it implies that debt is demandable (at least at par). Second, it supports the occurrence of panic runs in equilibrium. In Section 5.2, we relax this assumption and fully endogenize the interim deposit rate via a classical liquidity insurance motive based on Diamond and Dybvig (1983).[3]

Depositors choose whether to withdraw their funds before the bank's investment matures based on an imperfect signal about the fundamental. At $t = 1$, each depositor receives a noisy private signal

$$s_i = \theta + \varepsilon_i, \tag{1}$$

with $\varepsilon_i \sim U[-\varepsilon, +\varepsilon]$. In addition to being informative about the profitability of

[2] Uninsured deposits refer to any short-term or demandable debt instrument, including uninsured retail deposits and insured deposits when deposit insurance is not credible (Bonfim and Santos, 2023). Three quarters of U.S. commercial bank funding are deposits, half of which are uninsured (Egan et al., 2017).

[3] In the banking literature, the demandability of debt has also emerged as a commitment device to overcome agency conflicts, as in e.g. Calomiris and Kahn (1991) and Diamond and Rajan (2001)—see also Rochet and Vives (2004) for such an approach in the context of a global-games bank run model.

the bank's investment project, it also provides information about the signals (and withdrawal actions) of other depositors. As is standard in much of the global-games literature, we assume vanishing noise, $\varepsilon \to 0$, to simplify the analysis.[4]

The bank satisfies interim withdrawals by liquidating investment. Let $n \in [0,1]$ be the fraction of consumers who withdraw at $t = 1$. When the liquidation proceeds at $t = 1$ are insufficient to meet withdrawals, $n > \overline{n} \equiv \frac{L}{r_1}$, the bank is bankrupt due to illiquidity. Otherwise, it continues to operate until $t = 2$. If the bank cannot meet the remaining withdrawals, $n > \widehat{n} \equiv \frac{R\theta - r_2}{R\theta \frac{r_1}{L} - r_2}$, it is bankrupt due to insolvency, where $\widehat{n}$ solves the insolvency condition

$$R\theta \left(1 - \frac{\widehat{n} r_1}{L}\right) = (1 - \widehat{n})\, r_2. \tag{2}$$

The left-hand side is the return on the part of the project that was not liquidated at $t = 1$, and the right-hand side represents the remaining withdrawals at $t = 2$. Bankruptcy is costly and we assume zero recovery for simplicity.[5] We relax the assumption on bankruptcy costs in Appendix I and show that our main results are qualitatively unchanged.

As alternative to bank deposits, consumers can store their wealth in CBDC or cash. A deep-pocketed central bank offers consumers deposits with a per-period gross return $\omega \geq 1$, while cash is unremunerated.[6] Accordingly, consumers strictly prefer CBDC over cash as long as $\omega > 1$. They are indifferent for $\omega = 1$, so that this case is equivalent to a model without CBDC.

Our main interpretation is that ω represents the remuneration of CBDC.

[4]The assumption of vanishing private noise ensures a unique equilibrium and simplifies the analysis of the bank's choice of deposit rate at the initial date. In a similar framework, Vives (2014) studies the properties of the multiple equilibria that arise when this assumption is relaxed.

[5]Bankruptcy costs are large in practice. For example, James (1991) measures the losses associated with bank failure as the difference between the book value of assets and the recovery value net of direct expenses associated with failure. These losses amount to about 30% of failed banks' assets.

[6]We abstract from both raising funds (e.g. via taxation) and an investment choice of the central bank at $t = 0$. This is without loss of generality in our model because the central bank disburses no funds on the equilibrium path. At $t = 0$, the bank sets deposit rates high enough such that consumers prefer bank deposits over CBDC. A run leads to costly liquidation of assets, so no funds are re-deposited with the central bank at $t = 1$. Nonetheless, the option of remunerated CBDC affects both withdrawal incentives at $t = 1$ and the deposit rate at $t = 0$.

However, it could also capture other benefits relative to cash such as the reduced risk of theft or the additional convenience derived from digital payment means, such as the ability to settle e-commerce transactions.[7] Our focus on CBDC remuneration (and the convenience of CBDC more broadly) reflects the main argument in the academic and policy debate on the impact of CBDC on financial stability.

Relative to an economy with only deposits and cash, the introduction of CBDC has two effects. First, it improves the outside option of consumers deciding at $t = 0$ whether to deposit funds with the bank from 1 to ω^2 (the compound return on CBDC over two periods). Second, it pays interest ω on funds withdrawn from the bank at $t = 1$. Table 1 summarizes the timeline of the economy.

$t = 0$	$t = 1$	$t = 2$
1. CBDC design **2.** Bank sets rates **3.** Consumers deposit	**1.** Fundamental shock **2.** Private signals **3.** Withdrawal choice	**1.** Investment matures **2.** Consumption

Table 1: Timeline

3 Equilibrium

To solve for the equilibrium, we work backwards. First, for a given deposit contract and CBDC remuneration, we characterize a bank failure threshold $\theta^*(r_1, r_2, \omega)$. Next, we solve for the bank's choice of deposit contract $(r_1^*(\omega), r_2^*(\omega))$ for a given remuneration. Finally, we study how remuneration affects overall bank fragility.

3.1 Bank fragility

We use global-games methods to solve for the unique equilibrium at the withdrawal stage, building on Goldstein and Pauzner (2005) and Carletti et al. (2023).

[7]In principle, ω could also reflect an improved outside option arising from the existence of alternative stores of value such as treasury bonds or money market funds. However, these do not have the means-of-payment function of cash, bank deposits, and CBDC.

Some preliminary steps are useful for the analysis of the individual withdrawal decisions. We can limit attention to deposit contracts that satisfy $r_2 > \omega r_1$. To see this, suppose that $r_2 \leq \omega r_1$. Then, a depositor who is certain that the bank will not fail would still prefer to withdraw at $t = 1$ in order to re-deposit in CBDC, which gives a higher return than rolling over funds at $t = 1$ and only withdrawing at $t = 2$. Thus, there would be bank runs with probability 1 and the bank would make zero expected profits, which cannot be optimal for the bank. Hence, the bank offers a deposit contract that satisfies $r_2 > \omega r_1$.

Then, we establish the usual dominance bounds that yield ranges of the fundamental θ for which consumers have a dominant strategy. First, withdrawing is a dominant strategy for $\theta < \underline{\theta}$. This lower dominance bound solves

$$R\underline{\theta} - r_2 = 0, \tag{3}$$

so that $\underline{\theta} = \frac{r_2}{R}$. Since the bank always chooses a deposit contract $r_2^* < R$ (as deposit-taking would otherwise be unprofitable), we have that $\underline{\theta} \in (0, 1)$. The intuition for the lower dominance bound is as follows. When no other depositor withdraws ($n = 0$), the bank is always liquid at $t = 1$ and insolvent at $t = 2$ for $R\theta < r_2$. Therefore, withdrawing yields a payoff of r_1, while not withdrawing returns zero due to bankruptcy. So withdrawing is a dominant strategy for $\theta < \underline{\theta}$.

Second, we establish the upper dominance region. As in Goldstein and Pauzner (2005), we assume that there exists a threshold $\overline{\theta}$ such that the liquidation value improves to R for $\theta > \overline{\theta}$.[8] In this range, it is optimal for a depositor not withdraw irrespective of the withdrawal decision of all other depositors. In the upper dominance region, a depositor receives r_2 at date 2 and ωr_1 from withdrawing at date 1 and re-depositing in CBDC. Thus, not withdrawing at date 1 is a dominant action. The range $[\overline{\theta}, 1]$ identifies the upper dominance region, where the bound $\overline{\theta}$ can be arbitrarily close to (but strictly below) 1.[9] For simplicity, we assume $\overline{\theta} \to 1$.

[8]This assumption can be viewed as a liquidation value of the bank's project that depends on the fundamental θ. Specifically, it is equal to $L < 1$ for $\theta \in [0, \overline{\theta})$ and to R for $\theta \in [\overline{\theta}, 1]$.

[9]An endogenous upper dominance region arises in Kashyap et al. (2024), where depositors receive signals about the asset's interim liquidation value instead.

In the intermediate range $(\underline{\theta}, \overline{\theta})$, a consumer's decision to withdraw depends on what she expects the other consumers to do. Using global-games techniques, we can solve for the bank failure threshold, characterized in the next proposition.

Proposition 1. ***Failure threshold.*** *There exists a unique failure threshold $\theta^* \in \left(\underline{\theta}, \overline{\theta}\right)$. Each consumer withdraws their deposits and the bank fails if and only if $\theta < \theta^*$, where*

$$\theta^* \equiv \underline{\theta} \frac{r_2 - \omega L}{r_2 - \omega r_1} > \underline{\theta}. \tag{4}$$

The threshold θ^* decreases in L and R, increases in ω and r_1, and is non-monotonic in r_2: $\frac{\partial \theta^*}{\partial L} < 0$, $\frac{\partial \theta^*}{\partial R} < 0$, $\frac{\partial \theta^*}{\partial \omega} > 0$, $\frac{\partial \theta^*}{\partial r_1} > 0$, $\frac{\partial \theta^*}{\partial r_2} < 0$ if and only if $r_2 < r_2^{max}$.

Proof. See Appendix A, which also defines the threshold r_2^{max}. □

Under vanishing noise, the bank failure threshold θ^* corresponds to the probability of a bank run, which we thus use as our measure of bank fragility. A higher liquidation value L or higher profitability R reduce depositors' incentives to run.

The terms of the deposit contract (r_1, r_2) also affect the failure threshold. As in Diamond and Dybvig (1983) and Goldstein and Pauzner (2005), a higher short-term deposit rate increases fragility. Liquidity provision by the bank $(r_1 > L)$ gives rise to strategic complementarity in consumer withdrawal decisions, so that both panic runs and fundamental runs exist, $\theta^* > \underline{\theta}$.

Moreover, the relationship between the long-term deposit rate r_2 and bank fragility is non-monotonic: when the deposit rate is low, higher rates reduce fragility while the opposite holds for high deposit rates. Two opposing factors are at play. On the one hand, a higher long-term deposit rate implies that depositors receive a higher payoff when they wait and the bank is solvent. On the other hand, a higher long-term rate makes it more likely for the bank to be insolvent.

All else equal, the probability of a bank run increases with CBDC remuneration, since it increases the payoff from storing wealth outside the bank between $t = 1$ and $t = 2$, and thus makes withdrawing more attractive. However, this direct effect, $\frac{\partial \theta^*}{\partial \omega}$, fails to capture the overall impact because r_1 and r_2 are held fixed.

As we show below, changes in CBDC remuneration induce the bank to adjust the terms of the deposit contract, which in turn affects θ^*. To see this formally, we can use total differentiation:

$$\frac{d\theta^*}{d\omega} = \frac{\partial\theta^*}{\partial\omega} + \frac{\partial\theta^*}{\partial r_1}\frac{dr_1^*}{d\omega} + \frac{\partial\theta^*}{\partial r_2}\frac{dr_2^*}{d\omega}. \tag{5}$$

We next study these indirect effects of CBDC remuneration on bank fragility via the equilibrium deposit rates r_1^* and r_2^*.

3.2 Deposit rates

Since a bank run leads to zero profits, the bank internalizes the effects of the deposit contract on fragility, $\theta^* = \theta^*(r_1, r_2)$. With vanishing noise, $\epsilon \to 0$, consumer behaviour is fully symmetric. For $\theta > \theta^*$, there are no interim withdrawals and the investment matures at $t = 2$ with return $R\theta$. The banker pays the promised return r_2 to consumers and pockets the difference, $R\theta - r_2$. For $\theta < \theta^*$, all consumers withdraw at $t = 1$, there is bankruptcy, and the bank makes zero profits. Using $\overline{\theta} \to 1$, the banker's problem at $t = 0$ is therefore[10]

$$\max_{r_1 \geq 1, r_2} \Pi \equiv \int_{\theta^*}^{1} (R\theta - r_2)\, d\theta \tag{6}$$

$$\text{s.t. } V \equiv \int_{\theta^*}^{1} r_2\, d\theta - \omega^2 \geq 0. \tag{7}$$

Equation (7) is consumers' participation constraint. The first term is the expected payoff from keeping funds in the bank until $t = 2$, which is the long-term deposit rate in case there is no bank run. Because of our assumption on costly bankruptcy, a depositor's gross return is zero upon a bank run. The second term reflects the outside option, which is to store wealth in remunerated CBDC for a per-period return ω.

The following proposition characterizes the bank deposit rates in equilibrium.

[10]Expected bank profits can be written as $\Pi = (1-\theta^*)\left[\frac{R}{2}(1+\theta^*) - r_2\right]$, which is naturally interpreted as the probability of no run times the expected bank profits conditional on no run.

Proposition 2. ***Deposit rates.*** *Let* $\omega < \widetilde{\omega}$ *and* $R > \underset{\sim}{R}$. *The equilibrium deposit rates are* $r_1^* = 1$ *and* $r_2^* < r_2^{max}$. *The long-term deposit rate solves* $V(r_2^*) \equiv 0$ *(the participation constraint binds), increases in CBDC remuneration, and decreases in the liquidation value and investment profitability:* $\frac{dr_2^*}{d\omega} > 0$, $\frac{dr_2^*}{dL} < 0$, *and* $\frac{dr_2^*}{dR} < 0$.

Proof. See Appendix B, which also defines the bounds $\widetilde{\omega}$ and $\underset{\sim}{R}$. □

A higher short-term deposit rate r_1 reduces expected bank profits because the bank is more fragile (see Proposition 1). This also tightens consumers' participation constraint, since they are repaid less often. Accordingly, the bank chooses the lowest possible value for r_1, which is independent of CBDC remuneration. In other words, in equilibrium the bank responds to changes in CBDC remuneration ω by exclusively changing the long-term deposit rate r_2^*. We consider an alternative and complementary version of the model in Section 5.2 in which the bank alters the attractiveness of the deposit contract by varying both deposit rates.

In general, the long-term deposit rate r_2^* is pinned down by either the bank's first-order condition or the consumer participation constraint. The bounds on R and ω are sufficient conditions for the participation constraint to bind. Intuitively, they ensure that the bank has a large enough margin to adjust the deposit contract.[11] We henceforth assume that these conditions are met.

An increase in CBDC remuneration improves consumers' outside option, both at the initial and interim dates. Accordingly, to remain attractive and guarantee consumer participation, the bank needs to offer a more attractive long-term deposit rate r_2^*.[12] A higher liquidation value or investment profitability has the opposite effect. Because they reduce bank fragility, consumer participation can be satisfied with a lower long-term deposit rate.

Since it can exert market power, the bank offers a deposit rate r_2 strictly

[11]These assumptions ensure that the bank can always raise its deposit rate in response to higher CBDC remuneration, so financial intermediation continues to be feasible in our model. See Fernández-Villaverde et al. (2021) for a model in which CBDC crowds out all bank deposits.

[12]Garratt et al. (2022) also find that higher CBDC remuneration leads to higher deposit rates offered by banks. In a model with heterogeneous banks, they show that large banks increase their deposit rates more.

below the fragility minimizing level r_2^{max}. While such a lower rate renders the bank more fragile, it increases profitability in the states of the world where the bank survives.

Combining Propositions 1 and 2, a change in CBDC remuneration ω has two opposing effects on bank fragility θ^* in equilibrium. On the one hand, a higher remuneration leads to a higher incentive to withdraw at $t = 1$ and thus a larger threshold θ^*. On the other hand, the bank responds to higher remuneration by increasing the long-term deposit rate r_2^* offered at $t = 0$. This reduces bank fragility ceteris paribus. The overall effect of a change in ω on θ^* depends on which of these two effects dominates. The next result offers some insight into their relative strength.

Lemma 1. ***Elasticity of the failure threshold.*** *Let* $\eta \equiv -\frac{r_2}{\theta^*}\frac{\partial \theta^*}{\partial r_2}$ *be the elasticity of the failure threshold with respect to the deposit rate. Higher CBDC remuneration reduces bank fragility,* $\frac{d\theta^*}{d\omega} < 0$*, if and only if* $\eta > 1$.

Proof. See Appendix C. □

Lemma 1 states that the indirect effect of higher CBDC remuneration dominates the direct effect whenever the failure threshold θ^* is very elastic to changes in the bank deposit rate r_2. That is, higher CBDC remuneration needs to induce a sufficiently strong increase in deposit rates for overall fragility to fall. The elasticity η depends on equilibrium deposit rates and, thus, ultimately on parameters.

We now state our main positive result on CBDC remuneration and bank fragility. It is also shown in Figure 1.

Proposition 3. ***CBDC remuneration and bank fragility.*** *Bank fragility is U-shaped in CBDC remuneration with a unique minimum* $\omega_{min} > 1$.

Proof. See Appendix C. □

This proposition shows that the introduction of a positively remunerated CBDC can lead to a lower probability of runs (relative to an economy in which only cash is available to depositors as alternative to bank deposits). Furthermore, as illustrated in Figure 1, bank fragility is a convex function of CBDC remuneration: It first decreases with ω, and then increases. The introduction of CBDC induces the bank to increase the long-term deposit rate in an effort to maintain its deposit base. How much the deposit rate can be increased depends on investment profitability R and the bank's market power, as we illustrate in Section 5.1.

The non-monotonicity of the effect of CBDC remuneration on the run threshold arises because the direct and indirect effects vary with the level of ω. When the CBDC rate is low, the indirect effect is stronger, improving financial stability overall. For a high level of CBDC remuneration, the increase in the long-term deposit rate required for the indirect effect to dominate would reduce the bank's profit margin by too much, so the bank prefers to accept a higher level fragility instead. On the contrary, when CBDC remuneration is low, the positive direct effect of higher CBDC remuneration on bank fragility is limited and so is the increase in the deposit rate required for the indirect effect to dominate.

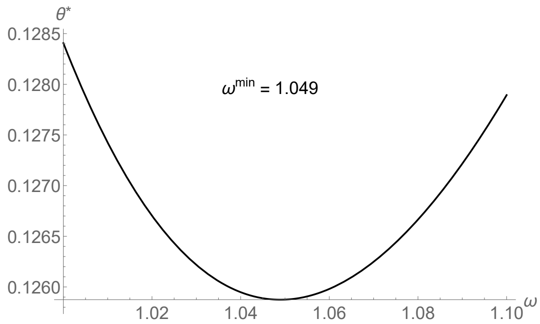

Figure 1: Bank failure threshold θ^* and CBDC remuneration ω. Parameters: $L = 0.9$, $R = 15$, so $\widetilde{\omega} \approx 1.32$.

4 CBDC design

Financial stability concerns feature prominently in the policy debate on CBDC (Bank for International Settlements, 2020). Accordingly, several central banks have advanced concrete proposals on specific CBDC design features that aim to mitigate potentially adverse effects. While our results in the previous section have shown that such concerns may already be mitigated by an appropriate level of CBDC remuneration, this section aims to link our model explicitly to this debate.

In order to do so, we consider a central bank who operates as a constrained planner and takes the informational friction and the privately optimal behaviour of consumers and the bank as given. Throughout we assume that the central bank can commit to a CBDC design and maximizes utilitarian welfare W, which is given by the sum of expected bank profits and expected consumer payoffs

$$W \equiv \int_{\theta^*}^{1} (R\theta - r_2) d\theta + \int_{\theta^*}^{1} r_2 d\theta = \frac{R}{2} \left[1 - (\theta^*)^2\right]. \tag{8}$$

Accordingly, maximizing welfare is equivalent to minimizing fragility in our economy. Following Proposition 3, a central bank that can freely set CBDC remuneration will choose $\omega^* = \omega_{min}$.

While the case in which the central bank can freely set CBDC remuneration represents a useful benchmark, it is rather unrealistic because most central banks do not consider CBDC to be a policy tool (European Central Bank, 2020; Bank of England, 2023). Even if they did, monetary policy considerations would likely be the overriding determinant of the level of CBDC remuneration (see, e.g., Lilley and Rogoff, 2020; Jiang and Zhu, 2021). Therefore, in what follows we assume that the remuneration of CBDC is fixed and study two design features that have emerged in the current policy debate as potential remedies to tackle the financial stability concerns associated with the introduction of CBDC: holding limits and contingent remuneration.

4.1 Holding limits

The introduction of individual holding limits has been proposed by various policy makers (e.g. Bindseil et al., 2021; Bank of England, 2023) as potential tool to limit the attractiveness of CBDC as store of value and thus for reducing run incentives.[13]

[13]The European Central Bank stated that it is exploring individual-specific holding limits in the context of its digital euro project. An amount of 3,000 EUR has been forwarded (see "Digital

In order to provide a formal analysis of this tool in the context of our model, we modify the baseline framework and assume that consumers can only hold up to a proportion γ of their wealth in a CBDC. Since the remuneration of CBDC exceeds that of cash, $\omega \geq 1$, depositors will always choose the maximum level of CBDC consistent with the holding limit. While policy makers are considering nominal limits in practice (Bindseil et al., 2021), all consumers are identical in our model at both $t = 0$ and $t = 1$ in equilibrium, so a proportional limit is equivalent to a nominal limit.[14]

Holding limits imply that not all withdrawn funds earn the same return, so the depositors' withdrawal decision at $t = 1$ may become more complicated. They can either withdraw only some of their funds at $t = 1$ and hold them in CBDC, or withdraw all of their funds and hold a portfolio of CBDC and cash. However, the following Lemma shows that partial withdrawals are never optimal. Intuitively, depositors withdraw in equilibrium only when the bank fails, so it is optimal for each consumer to fully withdraw in this case.

Lemma 2. ***No partial withdrawals.*** *Under holding limits, depositors never withdraw only a fraction of their funds.*

Proof. See Appendix D. □

Since depositors always withdraw their entire balance when running on the bank, the effective per-period remuneration on wealth held outside of the bank changes from ω (without holding limits) to

$$\omega^{HL} \equiv \gamma\omega + 1 - \gamma, \tag{9}$$

because the remaining $1 - \gamma$ must be held in cash. Proposition 4 provides the

euro will protect consumer privacy, ECB executive pledges", Financial Times, 20 June 2021). Similarly, the Bank of England has recently proposed a holding limit of 10,000-20,000 GBP in a public consultation. Other proposals include tiered remuneration, as suggested in Bindseil (2020). If the second tier is remunerated at zero or below, this is equivalent to holding limits in our model because consumers would prefer to hold amounts exceeding the first tier in cash.

[14] Nominal and proportional holding limits may have different implications when consumers are heterogeneous, e.g. in ex-ante endowments.

results of both a positive and normative analysis of holdings limits on CBDC.

Proposition 4. ***Holding limits.*** *Holding limits, $\gamma < 1$, increase (reduce) bank fragility for low (high) levels of CBDC remuneration ω. Hence, the central banks optimally sets holding limits as*

$$\gamma^* = \begin{cases} \frac{\omega_{min}-1}{\omega-1} & \text{if } \omega > \omega_{min} \\ 1 & \text{if } \omega \leq \omega_{min} \end{cases}$$

Proof. See Appendix D. □

From a positive perspective, the introduction of holding limits reduces the pass-through of CBDC remuneration to consumers' outside option at both $t = 0$ and $t = 1$. In line with our previous analysis, this leads to two opposite effects on bank fragility. At the interim date, holding limits reduce the return that consumers earn on withdrawn funds. Since only part of their wealth held outside the bank can be stored in remunerated CBDC, the remainder must be held as cash and earns a return of 1. This corresponds to the direct effect and makes the bank less fragile. However, holding limits soften the competition that the bank faces at the funding stage, and thus imply a lower equilibrium deposit rate r_2^*. This increases consumers' withdrawal incentives at $t = 1$ and thus makes the bank more fragile (the "indirect" effect).

The overall effect on bank fragility depends on which of these two effects dominates, which is determined by the (exogenous) level of CBDC remuneration. As shown in Figure 2, holding limits only have a beneficial effect on bank fragility when the level of CBDC remuneration is sufficiently high. Since holding limits reduce the responsiveness of the deposit rate to changes in CBDC remuneration, they also limit the beneficial effects of higher CBDC remuneration that are attained when ω is sufficiently low.

Building on this insight, the socially optimal holding limit depends on the level of CBDC remuneration. Whenever ω exceeds the social optimum ω_{min}, the central bank uses holding limits to implement this optimum by setting $\gamma = \frac{\omega_{min}-1}{\omega-1}$.

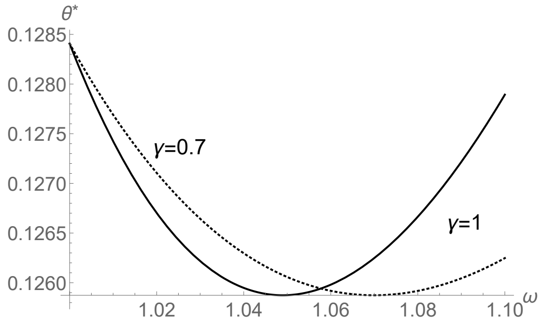

Figure 2: Bank failure threshold θ^*, CBDC remuneration ω, and holding limits γ. The solid line captures an economy without holding limits ($\gamma = 1$), while the dotted line captures an economy in which consumers can hold up to 70% of their funds in CBDC. Parameters: $L = 0.9$, $R = 15$.

Otherwise, it is socially optimal to not impose any holding limit. Overall, Proposition 4 contains a note of caution for policymakers: the calibration of CBDC remuneration and holding limits should avoid inefficient outcomes via higher bank fragility. Whenever CBDC remuneration is high, holding limits can be used as a tool to achieve the constrained-efficient outcome. However, this is not true for low levels of CBDC remuneration.

Finally, holding limits also have a distributional impact. Since they decrease the effective remuneration of consumers' outside option, they reduce the competitive pressure of CBDC remuneration on bank deposit rates, and thus lead to higher bank profits.

4.2 Contingent remuneration

Next, we consider the possibility that CBDC remuneration can be contingent on the state of the financial system. The underlying idea is that a reduction of CBDC remuneration in crisis times can serve as a tool to reduce depositors' withdrawal incentives whenever they become acute (see Bindseil, 2020; Bindseil et al., 2021).

To study the effectiveness of this policy, we modify the baseline model to specify CBDC remuneration as follows. In the first period, CBDC balances earn $\omega_1 = \omega$, as before. By contrast, the second period remuneration depends on consumers' withdrawals,

$$\omega_2(n) = \begin{cases} \omega & \text{if } n \leq \widetilde{n} \\ \underline{\omega} & \text{if } n > \widetilde{n}, \end{cases} \tag{10}$$

where $\underline{\omega} \in [1, \omega]$ is the reduced CBDC remuneration in crisis times, which is characterized by withdrawals of at least $\widetilde{n}$ depositors. The lower bound on $\underline{\omega}$ arises because we continue to assume that cash is available as alternative storage at each date.[15] Moreover, we focus on $\widetilde{n} < \overline{n}$ because a reduced remuneration cannot have a beneficial effect when the bank is illiquid at the interim date.

A stricter intervention is captured by lower values of the policy parameters $\widetilde{n}$ or $\underline{\omega}$.[16] While their impact on depositors' withdrawal incentives for a given deposit rate r_2 is similar, they affect depositors' participation constraint (and thus the bank's choice r_2^*) in different ways.

To see this difference formally, it is useful to first consider the marginal depositor's payoff upon withdrawal at $t = 1$, which can be written as

$$\pi_{1,CR} = \omega \int_0^{\widetilde{n}} r_1 dn + \underline{\omega} \int_{\widetilde{n}}^{\overline{n}} r_1 dn \leq \omega \int_0^{\overline{n}} r_1 dn = \pi_1, \tag{11}$$

where the subscript CR refers to contingent remuneration. As before, $\overline{n}$ is the bank's illiquidity threshold, so π_1 is the equivalent in the baseline model without contingent remuneration (see also Appendix A). For a given deposit rate r_2, a decrease in both $\widetilde{n}$ and $\underline{\omega}$ yields a lower expected payoff from withdrawing, and thus a lower run threshold relative to the baseline model, $\theta_{CR}^*(r_1, r_2) < \theta^*(r_1, r_2)$.

Recall that the deposit rate is pinned down by depositors participation con-

[15] This is consistent with most central banks already having committed to continue supplying cash after a potential CBDC introduction. We abstract from the inconvenience associated with the handling and storage of physical cash, which may in practice enable central banks to set a slightly negative CBDC rate (the "effective lower bound") without triggering a complete substitution by consumers into cash.

[16] As the policy is implemented based on observed withdrawals at the final date (i.e. after consumers' interim-date choices), contingent remuneration does not require superior information by the central bank at the interim date. Moreover, this specification rules out any potential complications associated with an endogenous public signal. See also Angeletos et al. (2006).

straint. With contingent remuneration, it reads[17]

$$V_{CR} = \int_0^{\theta^*_{CR}} r_2 d\theta - \omega \left(\int_0^{\theta^*_{CR}} \underline{\omega} d\theta + \int_{\theta^*_{CR}}^1 \omega d\theta \right) \geq 0. \tag{12}$$

Unlike $\widetilde{n}$, the reduced remuneration rate $\underline{\omega}$ does not only affect depositors' participation constraint indirectly via a change in the run threshold, but also directly. Hence, its impact on the deposit rate r_2 is more complex, as detailed below.

Proposition 5. ***Contingent remuneration.*** *A more restrictive design of contingent remuneration affects bank fragility as follows.*
(1) $\frac{d\theta^*_{CR}}{d\widetilde{n}} > 0$*;*
(2) $\frac{d\theta^*_{CR}}{d\underline{\omega}} > 0$ *if*

$$(1-\theta^*_{CR}) \frac{r_2^2(1-L)}{RL(r_2 - \frac{\pi_{1,CR}}{L})}(\overline{n} - \widetilde{n}) + \omega\theta^*_{CR}\frac{\partial\theta^*_{CR}}{\partial r_2} > 0.$$

Moreover, $\frac{d\theta^*_{CR}}{d\underline{\omega}} < 0$ *for* $\widetilde{n} \to \overline{n}$*.*

Proof. See Appendix E. □

Similar to the effect of CBDC remuneration in the main model, there is both a direct and an indirect effect of the policy parameters $\widetilde{n}$ and $\underline{\omega}$ on bank fragility. An earlier reduction of CBDC remuneration upon withdrawals (a lower $\widetilde{n}$) directly benefits financial stability. Moreover, the cutoff $\widetilde{n}$ does not directly enter the participation constraint of investors (as it only enters via the failure threshold), resulting in a weak indirect effect. Thus, the direct effect always dominates and fragility is unambiguously reduced. This result is reminiscent of a partial suspension of convertibility in run models, but also considers the effects on ex-ante deposit rates. However, this ex-ante effect is always weak and dominated by the effect on withdrawal incentives, yielding the first result of the proposition.

The effect of a reduced remuneration (a lower $\underline{\omega}$) is generally more complex. As intended, the lower return on withdrawn funds reduces withdrawal incentives.

[17] We use the fact that with vanishing noise, either all or no depositors runs, so that the reduced (full) CBDC rate is earned in the second period in case of failure (survival) for any $\widetilde{n} \in [0, 1]$.

However, $\underline{\omega}$ enters the participation constraint directly (see Equation 12), which strengthens the indirect effect that operates via equilibrium deposit rates.

Unfortunately, the total effect of CBDC remuneration is difficult to sign in general. For the special case where $\widetilde{n} \to \overline{n}$, however, the beneficial direct effect of contingent remuneration on withdrawal incentives vanishes because the bank is always illiquid and fails. We are thus left with an indirect effect of contingent remuneration, which lowers equilibrium deposit rates and therefore raises bank fragility. Figure 3 offers a numerical example that shows that a lower remuneration during financial turmoil can lower bank fragility.

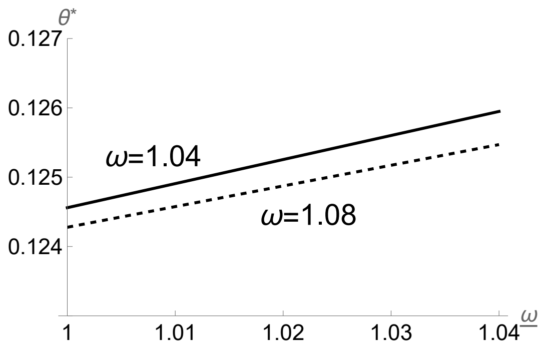

Figure 3: Bank failure threshold θ^*, CBDC remuneration ω, and reduced remuneration $\underline{\omega}$. The solid line and dashed lines are drawn for different values of CBDC remuneration, respectively $\omega = 1.04$ and $\omega = 1.08$. Parameters: $L = 0.9$, $R = 15$.

Interestingly, introducing contingent remuneration does not affect the impact of CBDC remuneration ω on financial fragility. This is shown in Figure 4. In line with the above results, a decrease in $\underline{\omega}$ simply leads to a downward shift in the curve illustrating the impact of CBDC remuneration on fragility, thus stressing the beneficial effect of this design feature.

5 Extensions

In this section, we consider three extensions of the baseline model. We first consider the case when the bank has limited market power in the deposit market. Next, we embed CBDC in a model of liquidity provision following Goldstein and

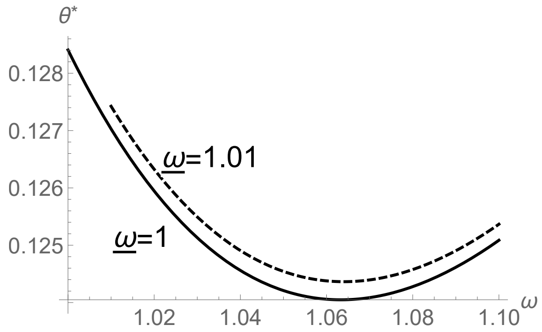

Figure 4: Bank failure threshold θ^*, CBDC remuneration ω, and reduced remuneration $\underline{\omega}$. The solid line captures an economy in which consumers receive a zero remuneration on CBDC in the event of a run, while the dotted line captures an economy in which they still receive a positive remuneration but below that they receive when there is no run. Parameters: $L = 0.9$, $R = 15$.

Pauzner (2005). Finally, we examine the bank's risk-taking incentives on the asset side of its balance sheet.

5.1 Limited market power in the deposit market

So far, we have considered a bank that acts as a monopolist in the deposit market. In this subsection, we relax this assumption.[18] This approach is partly motivated by theoretical work on the effects of CBDC on bank credit supply, which reaches different conclusions depending on the level of competition in deposit markets (Keister and Sanches, 2022; Andolfatto, 2021; Chiu et al., 2023).

More specifically, we assume that the deposit contract is determined by Nash bargaining between the bank and depositors. This approach is attractive because it allows us to model the degree of deposit market competition by varying the bank's bargaining power $\beta \in (0, 1)$. Formally, the deposit contract is the solution to

$$\max_{r_1, r_2} \left(\int_{\theta^*}^{1} R\theta - r_2 d\theta \right)^{\beta} \left(\int_{\theta^*}^{1} r_2 d\theta - \omega^2 \right)^{1-\beta}. \tag{13}$$

where the first (second) bracket represents the bank's (depositors') surplus in excess of their outside option. As in the main text, $r_1^* = 1$ follows immediately.

[18] A large literature documents imperfect competition in retail deposit markets, including Neumark and Sharpe (1992), Hannan and Berger (1997), and Drechsler et al. (2017).

For $\beta \to 1$, this collapses to the baseline model where the bank maximizes expected profits subject to depositors' participation constraint. By contrast, $\beta \to 0$ corresponds to a model with perfect competition where the deposit rate r_2 maximizes the expected value of the deposit claim subject to non-negative bank profits. The following proposition summarizes the resulting implications of this polar case for the effect of CBDC remuneration on bank fragility.

Proposition 6. ***Perfect competition in the deposit market.*** *For $\beta \to 0$, an increase in CBDC remuneration increases bank fragility, $\frac{d\theta^*}{d\omega} > 0$.*

Proof. See Appendix F. □

Depositors' gross surplus from the deposit contract is $(1 - \theta^*)r_2$, stating that the deposit rate r_2 is earned whenever the bank survives (this happens with probability $1-\theta^*$). Under perfect competition, the value of the deposit contract is maximized. The resulting equilibrium deposit rate is higher than under monopoly. In fact, it exceeds the level r_2^{max} beyond which an increase in the deposit rate raises the risk of bank failure (see also Proposition 1).[19]

As in the baseline model, higher CBDC remuneration affects bank fragility both directly (via the failure threshold) and indirectly (via the deposit contract). As before, the direct effect is positive. However, the indirect effect is no longer unambiguously negative, but its sign varies with parameters. In any case, deposit rates are so high that the indirect effect is of second order, so that the overall effect of higher CBDC remuneration is to unambiguously increase bank fragility.

We now turn to the intermediate case of limited market power, $0 < \beta < 1$. The following result states how the equilibrium deposit rates are pinned down.

Proposition 7. ***Deposit rates with limited bank market power.*** *The deposit*

[19] Bank profits evaluated at the competitive deposit rate are positive. That is, the value of the deposit claim is maximized for a deposit rate below the rate at which bank profits are zero.

rate r_2^* *solves*

$$\frac{\beta}{\int_{\theta^*}^{1}\left(R\theta - r_2^*\right)d\theta}\left(1-\theta^* + (R\theta^* - r_2^*)\frac{\partial\theta^*}{\partial r_2}\right) = \frac{1-\beta}{\int_{\theta^*}^{1} r_2 d\theta - \omega^2}\left(1-\theta^* - r_2^*\frac{\partial\theta^*}{\partial r_2}\right), \tag{14}$$

where $\theta^* = \theta^*(r_2^*)$. *For sufficiently high bank market power,* $\beta > \underline{\beta}$, *the deposit rate increases in CBDC remuneration, i.e.,* $\frac{dr_2^*}{d\omega} > 0$.

Proof. See Appendix F. □

Intuitively, a decline in the bank's bargaining power dampens the impact of CBDC remuneration on deposit rates because the bank is no longer a monopolist. Unfortunately, a full analytical characterization of the general case is difficult. To provide some insight, Figure 5 provides numerical examples showing that the U-shaped relationship between bank fragility and CBDC remuneration is preserved when the bank's bargaining power in the deposit market is sufficiently high, whereas a monotonically increasing relationship is obtained for more competitive deposit markets. This result clarifies that sufficient bank market power in the deposit is a necessary ingredient for our main result of a U-shape relationship between CBDC remuneration and bank fragility.

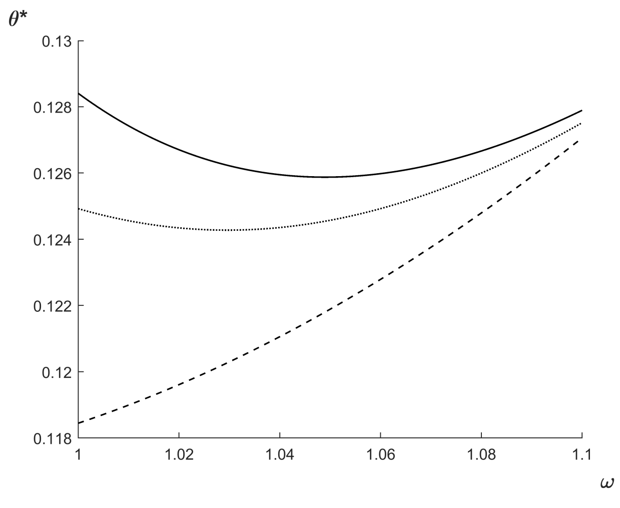

Figure 5: Bank failure threshold θ^*, CBDC remuneration ω, and bargaining power in the deposit market β. The solid line is for monopoly, the dotted line for high market power of the bank, and the dashed line are for low market power of the bank in the deposit market. Parameters: $L = 0.9$, $R = 15$, $\beta \in \{1, 0.998, 0.9\}$.

5.2 Liquidity provision to risk-averse depositors

So far, adjustments in the deposit rates upon the introduction of a remunerated CBDC have only translated into a change in the final promised repayment r_2 because the bank acts as monopolist in the deposit market, while the interim repayment r_1 has remained at the lower bound. This differs from the traditional bank-run literature (Diamond and Dybvig, 1983), where the interim deposit rate is at the center of attention because depositors are risk-averse and subject to a liquidity shock. In that context, banks provide liquidity insurance by offering an interim repayment r_1 in excess of the liquidation value despite the fact that this exposes banks to panic runs.[20]

In order to acknowledge the key role of the interim repayment in the bank-run literature, we show in this extension that the beneficial effect of an adjustment in the deposit contract on financial stability following a CBDC introduction is also present when the bank varies the interim deposit rate in equilibrium. To do so, we incorporate a remunerated CBDC into the seminal global-games model of bank runs with liquidity provision of Goldstein and Pauzner (2005).[21] Since the bank is perfectly competitive in this setup, we do not expect to recover a U-shaped relationship between CBDC remuneration and bank fragility—consistent with the result in the previous section.[22] However, we show that the bank now adjusts both deposit rates in response to a change of CBDC remuneration, and these adjustments are consistent with the main model in the sense that the resulting indirect effects reduce bank fragility.

Model. As before, there is a unit mass continuum of consumers endowed with 1 unit of funds who can choose to deposit their wealth into a bank at date 0 or

[20] In this class of models, this feature of the deposit contract emerges endogenously in equilibrium and captures the liquidity insurance service that banks offer to their depositors. By setting $r_1 > L$, banks transfer resources from the final to the interim date and so allows depositors to obtain a higher expected utility that would obtain if they were to invest directly in the asset.

[21] Importantly, these departures from our baseline model allow us to fully endogenize the deposit contract and so provide a micro-foundation for the lower bound on the interim deposit rate imposed in the baseline model.

[22] Additionally, large adjustments in r_1 are not optimal in this setting as they distort liquidity provision by too much. This limits the strength of the indirect effect as we will show below.

into CBDC at both date 0 and date 1, which earns $\omega \geq 1$ per period. Unlike in the main text, and closely following Goldstein and Pauzner (2005), we assume that consumers are risk-averse with utility function $u(c)$ such that $u(0) = 0$, $u'(c) > 0$, $u''(c) < 0$, and the relative risk aversion $RRA > 1$ for any $c \geq 1$. While consumers are identical ex-ante, a fraction $\lambda \in (0, 1)$ is hit by a preference shock at date 1 and must consume immediately ("early" or "impatient" depositors). The remaining $1 - \lambda$ depositors are indifferent between consuming at dates 1 and 2, and are called "late" or "patient".

At date 0, the bank raises funds in exchange for a deposit contract $\{r_1, r_2\}$, where r_1 is the promised repayment upon withdrawal at the interim date and r_2 is the promised repayment upon withdrawal at the final date. The bank uses the resources raised initially to invest in the risky asset. The project can be liquidated at par at date 1 (i.e. $L = 1$), and yields a return R with probability $p(\theta) = \theta$ at date 2 (and zero otherwise). Unlike in the main text, the bank operates in a perfectly competitive environment and maximizes depositors' expected utility at $t = 0$. As a result, late depositors waiting until the final date are just residual claimants of the bank and receive a pro-rata share of the bank's available resources. If the bank has insufficient resources to repay r_1 in the case of a run at the interim date, the early withdrawing depositors also receive a pro-rata share of the available resources. This small technical deviation from the sequential service constraint assumed in Goldstein and Pauzner (2005) improves tractability.

Equilibrium. We again work backwards, starting with depositors' withdrawal choices. A first result with bank liquidity provision (LP) follows.

Proposition 8. ***Failure threshold.*** *There exists a unique threshold equilibrium. When $\varepsilon \to 0$, all depositors withdraw and the bank fails if and only if $\theta < \theta^*_{LP}$, where*

$$\theta^*_{LP} = \frac{\int_{\lambda}^{\overline{n}} u(\omega r_1)\, dn + \int_{\overline{n}}^{1} u\left(\frac{\omega}{n}\right) dn}{\int_{\lambda}^{\overline{n}} u\left(\frac{1 - n r_1}{1 - n} R\right) dn}. \tag{15}$$

The failure threshold θ^*_{LP} increases in both r_1 and ω, i.e. $\frac{\partial \theta^*_{LP}}{\partial r_1} > 0$ and $\frac{\partial \theta^*_{LP}}{\partial \omega} > 0$.

Proof. See Appendix G. □

A higher CBDC remuneration increases a patient depositor's expected payoff from withdrawing at the interim date, which raises bank fragility. This result corroborates the direct effect from our main analysis. As usual, a higher short-term deposit rate also raises bank fragility due to greater strategic complementarity in withdrawal decisions.

Having characterized the run probability θ^*_{LP}, we turn to the choice of deposit contract. A competitive bank chooses r_1 to maximize depositors' expected utility[23]

$$\max_{r_1} \int_0^{\theta^*_{LP}} \left[\lambda u\left(1\right) + \left(1-\lambda\right) u\left(\omega\right)\right] d\theta + \int_{\theta^*_{LP}}^1 \left[\lambda u\left(r_1\right) + \left(1-\lambda\right)\theta u\left(\frac{1-\lambda r_1}{1-\lambda} R\right)\right] d\theta. \tag{16}$$

The first term in Equation (16) represents depositors' expected utility upon a run. All depositors withdraw from the bank at date 1 and receive the pro-rata share of 1. Impatient consumers must consume immediately and therefore obtain $u\left(1\right)$, while impatient depositors keep their funds in their CBDC account for one period and receive $u\left(\omega\right)$ at date 2. The second term in (16) captures depositors' expected utility in the absence of a run. Impatient depositors withdraw at date 1 and receive $u(r_1)$, while patient depositors wait until the final date and receive the pro-rata share $\frac{1-\lambda r_1}{1-\lambda} R$ when the project is successful (with probability θ). This expression shows the negative relationship between short-term and long-term deposit rates. A higher interim deposit rate r_1 reduces the repayment obtained at the final date and transfers resources from the final to the interim date (Diamond and Dybvig, 1983; Goldstein and Pauzner, 2005).

Let σ denote a depositor's absolute risk aversion. We have the following result about the equilibrium interim deposit rate.

Proposition 9. ***Deposit rates.*** *The equilibrium deposit rate $r_1^* > 1$ solves the*

[23] We are implicitly focusing on values of ω that are not too large. Very large values of ω imply that runs occur all the time (so $\theta^*_{LP} = 1$) and render the choice of the deposit contract irrelevant.

following first-order condition

$$
\begin{aligned}
FOC \equiv & -\frac{\partial \theta_{LP}^{*}}{\partial r_1}\left[\lambda u\left(r_1\right)+(1-\lambda)\, \theta_{LP}^{*}\, u\left(\frac{1-\lambda r_1}{1-\lambda} R\right)-\lambda u(1)-(1-\lambda)\, u(\omega)\right]+ \\
& +\lambda \int_{\theta_{LP}^{*}}^{1}\left[u^{\prime}\left(r_1\right)-\theta R u^{\prime}\left(\frac{1-\lambda r_1}{1-\lambda} R\right)\right] d \theta=0. \qquad (17)
\end{aligned}
$$

If depositors' risk aversion is sufficiently low, it decreases in CBDC remuneration, i.e. $\frac{dr_1^*}{d\omega} < 0$ *for* $\sigma < \widehat{\sigma}$.

Proof. See Appendix G. □

As in Goldstein and Pauzner (2005), the equilibrium deposit rate r_1 trades off the marginal cost associated with a higher run probability (the first term in (17)) with the marginal benefit due to better risk-sharing (the second term in (17)). Moreover, $r_1^* > L = 1$ implies that the bank provides liquidity in equilibrium: the deposit rate exceeds the liquidation value of its investments. This gives rise to strategic complementarities in depositors' withdrawal decisions, and thus panic runs.

The proposition also states that the equilibrium deposit rate r_1^* decreases in the level of CBDC remuneration ω when the level of risk aversion is sufficiently small. Because of the above mentioned negative relationship between interim and final deposit rates, the final deposit rate therefore increases in ω, just as in the baseline model. Overall, we can rewrite the total effect of CBDC remuneration from equation (5) as follows

$$
\frac{d\theta_{LP}^*}{d\omega} = \frac{\partial \theta_{LP}^*}{\partial \omega} + \left[\frac{\partial \theta_{LP}^*}{\partial r_1} - \frac{\partial \theta_{LP}^*}{\partial r_2}\frac{R\lambda}{1-\lambda}\right]\frac{dr_1^*}{d\omega},
$$

where we have used the fact that $\frac{dr_2^*}{dr_1^*} = -\frac{R\lambda}{1-\lambda}$. This decomposition highlights how the adjustment in r_1^* following a change in ω counteracts the direct effect represented by the first term $\frac{\partial \theta_{LP}^*}{\partial \omega}$, thus reducing financial fragility.

Unfortunately, a full analytical characterization of the general case is difficult, so we rely on a numerical analysis. It shows that the result in Proposition 9 on the inverse relationship between the interim deposit rate r_1 and CBDC remuneration ω holds more broadly than the sufficient condition stated in the proposition. Furthermore, the numerical analysis also highlights the beneficial impact of the interim deposit rate adjustment on stability. Specifically, Figure 6 shows that the interim deposit rate can decrease in CBDC remuneration (and, accordingly, the final date deposit rate increases in CBDC remuneration). Figure 7 shows that this indirect effect—equal to the difference between the dashed and the solid lines—reduces bank fragility, just as in the main model.

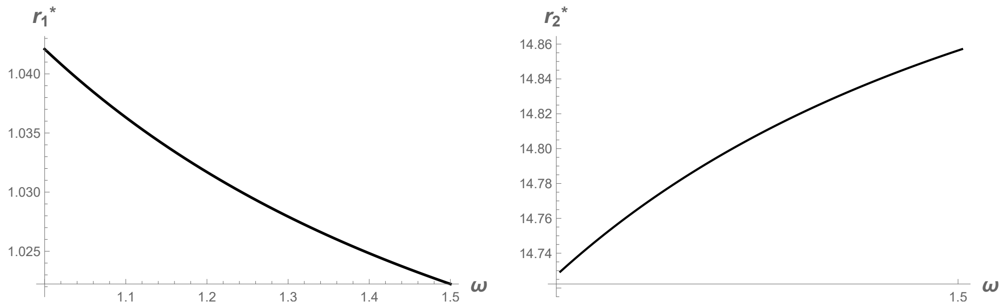

Figure 6: Bank deposit rates r_1^* (left panel) and r_2^* (right panel) as a function of CBDC remuneration ω. Parameters: $R = 15$, $\lambda = 0.3$, $u(c) = \frac{(c+f)^{1-\sigma}}{1-\sigma} - \frac{f^{1-\sigma}}{1-\sigma}$, $\sigma = 3$, $f = 4$. For these parameter values, the cutoff $\widehat{\sigma} \simeq 1.3$.

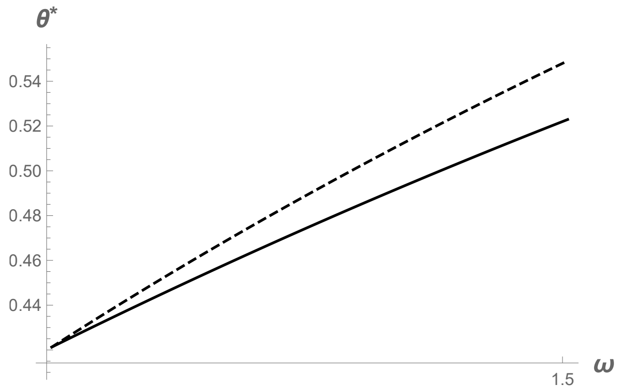

Figure 7: Bank fragility θ^* and CBDC remuneration ω. The dashed line shows the effect on bank fragility if deposit rates were kept constant (at the optimal level for $\omega = 1$), while the solid line account for the endogenous response in deposit rates. Parameters: $R = 15$, $\lambda = 0.3$, $u(c) = \frac{(c+f)^{1-\sigma}}{1-\sigma} - \frac{f^{1-\sigma}}{1-\sigma}$, $\sigma = 3$, $f = 4$.

5.3 Bank risk-taking on the asset side

So far we have focused on fragility on the liability side of the bank balance sheet (uninsured deposits) as a source of financial instability. However, financial instability can also result from banks' risk-taking decisions on the asset side (e.g., risk choices and asset substitution). To explore this issue, this section builds on Carletti et al. (2023) and extends the model by allowing the bank to choose its monitoring effort at $t = 0$, which gives rise to interactions between asset and liability sides of bank balance sheet.[24]

Consistent with the literature (e.g., Holmstrom and Tirole (1997), Hellmann et al. (2000), Morrison and White (2005), Dell'Ariccia and Marquez (2006), Allen et al. (2011), DellAriccia et al. (2014)), the effort q fully determines the probability of success of bank investment, whose return changes to

$$P = \begin{cases} R\theta & \text{w.p. } q \\ 0 & \text{w.p. } 1-q \end{cases},$$

where bank effort is publicly observable. Higher monitoring leads to a higher success probability, but it entails a non-pecuniary cost $\frac{c}{2}q^2$. To keep the analysis tractable, we consider an exogenous deposit contract (r_1, r_2) that satisfies the participation constraint of investors. This assumption shuts down the channel along which higher CBDC remuneration improves financial stability in the main text (the "indirect effect"), which allows us to focus on the effects of CBDC remuneration ω on bank risk choice q^* instead.

To solve the model, we proceed as in the main text. We continue to assume that $\overline{\theta} \to 1$ and $\epsilon \to 0$. Moreover, we require that $qr_2 > \omega r_1$ to rule out bank runs taking place with certainty. We begin by deriving the endogenous run threshold θ_q^*, so that all depositors run on the bank at $t = 1$ if and only if $\theta < \theta_q^*$. Following

[24] Monnet et al. (2023) also study the effect of CBDC on bank risk-taking. They find that banks respond to the introduction of CBDC by becoming safer, even excessively safe.

the same steps as in Section 3.1, we obtain

$$\theta_q^* = \frac{r_2}{R}\frac{qr_2 - \omega L}{qr_2 - \omega r_1}. \tag{18}$$

Better monitoring increases the probability that the bank is able to repay depositors at $t = 2$, and therefore reduces incentives to run ($\frac{\partial \theta_q^*}{\partial q} < 0$). This means that lower risk on the asset side leads to lower risk on the liability side.

Taking the run threshold θ_q^* into account, the bank's optimal choice of monitoring effort q at $t = 0$ solves

$$\max_q \Pi_q \equiv q \int_{\theta_q^*}^{1} (R\theta - r_2)\, d\theta - \frac{cq^2}{2}. \tag{19}$$

Provided that c is sufficiently high, there exists a unique and interior solution q^*, which is given by the solution to the following first-order condition

$$FOC_q \equiv \int_{\theta_q^*}^{1} (R\theta - r_2)\, d\theta - q\frac{\partial \theta_q^*}{\partial q}\left(R\theta_q^* - r_2\right) - cq = 0. \tag{20}$$

The bank's risk choice at $t = 0$ reflects a trade-off. The last term in equation (20) reflects the marginal cost of monitoring effort. The other two terms represent the marginal benefit of higher monitoring. First, more monitoring increases the probability that the project is successful, so that the bank reaps the residual claim $R\theta - r_2$ more often (provided there is no bank run, $\theta > \theta_q^*$). Second, the bank benefits from the interaction between the bank's asset and liability sides. An increase in monitoring reduces depositors' incentives to run, so that costly bankruptcy can be avoided.

Since we allow for risk-taking on the asset side of the balance sheet, it is important to note that there are now two potential sources of bank failure: bank runs and an unsuccessful investment project. When the project fails, then there is always a run. When the project succeeds, then there is a run on the bank for $\theta < \theta_q^*$. Taken together, financial stability can be measured by the overall probability that the bank survives, which we define as

$$\Phi^* \equiv q^* \left(1 - \theta_q^*\right). \tag{21}$$

The following proposition shows that an increase in CBDC remuneration affects its two separate components in opposite ways.

Proposition 10. ***Risk taking on the asset side.*** *Higher CBDC remuneration improves monitoring,* $\frac{dq^*}{d\omega} > 0$*, but also increases fragility,* $\frac{d\theta_q^*}{d\omega} > 0$.

Proof. See Appendix H. □

Changes in CBDC remuneration affect the marginal benefit of bank monitoring, which follows directly from the first-order condition (20). The direction of this effect depends both on the direct effect of CBDC remuneration on the run threshold ($\frac{\partial \theta^*}{\partial \omega}$) as well as the threshold's sensitivity to changes in monitoring ($\frac{\partial^2 \theta^*}{\partial q \partial \omega}$). Proposition 10 states that the overall effect is always positive, that is an increase in CBDC remuneration always leads to higher bank monitoring ($\frac{dq^*}{d\omega} > 0$) and thus renders the bank's asset side more stable.

The total effect of CBDC remuneration of the probability of a bank run $\frac{d\theta_q^*}{d\omega}$ takes into account the (indirect) effect that CBDC remuneration has on bank monitoring q and can be written as

$$\frac{d\theta_q^*}{d\omega} = \frac{\partial \theta_q^*}{\partial \omega} \left[1 - \frac{\omega}{q} \frac{dq^*}{d\omega}\right], \tag{22}$$

where $\frac{\partial \theta_q^*}{\partial \omega}$ is the direct effect of CBDC remuneration ω on the failure threshold θ_q^*. Just as in the main text, an increase in CBDC remuneration affects the run threshold θ_q^* both directly and indirectly. However, in this case, the indirect operates through bank monitoring, $\frac{dq^*}{d\omega}$, and not through the deposit contract (which is assumed to be exogenous). While these two effects go in opposite directions, Proposition 10 states that the direct effect always dominates, so that a higher CBDC remuneration always increases the risk of bank runs.

Since CBDC affects both aspects of financial stability in opposite ways, its overall effect on financial stability is ambiguous. While it is difficult to derive sufficient conditions analytically, Figure 8 provides a numerical example for which the beneficial effect of higher bank monitoring dominates. Accordingly, higher CBDC remuneration can increase financial stability, as in the baseline model.

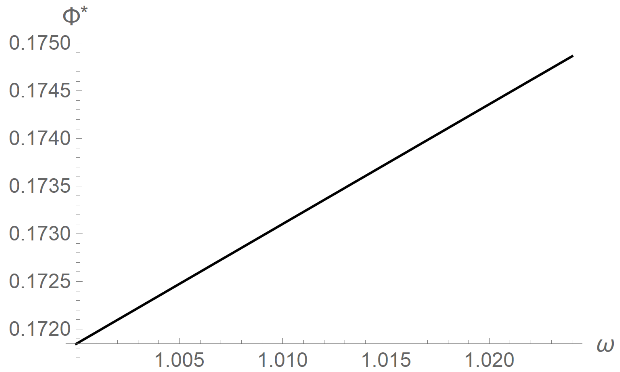

Figure 8: Financial stability Φ^* and CBDC remuneration ω. Parameters: $L = 0.9$, $R = 20$, $c = 0.1$, $r_1 = 1$, and $r_2 = 6$.

To isolate the impact on asset-side risk, we have assumed an exogenous deposit contract for tractability. If the deposit contract were endogenous, two additional effects would arise. First, as shown in Section 3, higher remuneration of CBDC would increase the deposit rate, which directly reduces the bank's margin and, in turn, its incentive to exert monitoring effort. Second, the reduced fragility due to the higher deposit rate would translate into higher expected profits, and thus induce a higher risk management effort. These additional effects, which come from the interaction of the asset and liability sides of the bank balance sheet, go in opposite directions, thus making the characterization of the overall effect of CBDC on financial stability even more involved.

6 Conclusion

This paper aims to examine the impact of CBDC on financial stability. To do so, we develop a global-games model of financial intermediation and bank runs in which a remunerated CBDC provides consumers with an alternative to bank deposits (and cash). Consistent with concerns among policymakers, a higher CBDC remuneration raises bank fragility by increasing consumers' withdrawal incentives. However, it also induces the bank to offer more attractive deposit contracts in an effort to retain funding, which reduces fragility. Accordingly, the overall relationship between bank fragility and CBDC remuneration is U-shaped.

Within this framework, we evaluate several policy proposals aimed at allaying financial stability concerns connected to an introduction of CBDC. We find that a positive remuneration of CBDC can be socially desirable because it lowers bank fragility. When CBDC remuneration is exogenous from a financial stability perspective (e.g. when it is determined by monetary objectives or bound by previous commitments), holding limits can be socially beneficial for a high CBDC rate. However, they are ineffective for low levels of CBDC remuneration. Contingent remuneration, which lowers rates in times of financial distress, can be effective in reducing withdrawal incentives without having a large effect on deposit rates.

We extend the model to allow for imperfect competition in the deposit market, liquidity provision, and bank risk-taking on its asset side. These analyses support the robustness of our baseline results. Further extensions may generate additional insights. For example, one could consider the role of bank equity and liquid reserves on the interaction between CBDC remuneration and financial stability. One could also examine how CBDC affects the bank's funding mix, as cheaper but more unstable wholesale deposits may become attractive, or the supply of deposits may change. Exploring general equilibrium effects via the response of asset prices and exchange rates after the introduction of CBDC may be interesting. Finally, the interaction of CBDC design features (remuneration, holding limits, and contingent remuneration) with traditional tools for mitigating run risk, such as lender of last resort policies, is an interesting avenue for further research.

A Proof of Proposition 1

The proof builds on Carletti et al. (2023). The only difference is that our model exhibits global strategic complementarity (depositor's incentive to withdraw at $t = 1$ monotonically increases in the number of depositors withdrawing). The arguments in their proofs establish that, in the limit of $\epsilon \to 0$, there is a unique threshold value of the fundamental, denoted as θ^*, below which all consumers choose to withdraw from the bank. We first prove the existence of a unique equilibrium and then study its comparative statics.

Existence and uniqueness. For $\theta \in (\underline{\theta}, \overline{\theta})$, a depositor's decision to withdraw depends on the withdrawal choices of others. Suppose that all depositors use a threshold strategy s^*. Then, the fraction of depositors withdrawing at $t = 1$, $n(\theta, s^*)$, equals the probability of receiving a signal below s^*:

$$n(\theta, s^*) = \begin{cases} 1 & \text{if } \theta \leq s^* - \epsilon, \\ \frac{s^* - \theta + \epsilon}{2\epsilon} & \text{if } s^* - \epsilon < \theta \leq s^* + \epsilon, \\ 0 & \text{if } \theta > s^* + \epsilon. \end{cases} \tag{23}$$

Thus, a depositor's withdrawal decision is characterized by the pair of thresholds $\{s^*, \theta^*\}$, which solves the following system of equations:

$$R\theta^* \left(1 - \frac{n(\theta^*, s^*) r_1}{L}\right) - (1 - n(\theta^*, s^*)) r_2 = 0, \tag{24}$$

$$r_2 Pr(\theta > \theta^* | s^*) = \omega r_1 Pr(\theta > \theta_n | s^*), \tag{25}$$

where $\theta_n = s^* + \epsilon - 2\epsilon \frac{L}{r_1}$ solves $n(\theta, s^*)\, r_1 = L$. Condition (24) identifies the level of fundamentals θ at which the bank is just able to repay the promised repayment to non-withdrawing depositors, pinning down the cutoff θ^*. Condition (25) states that at the signal threshold s^* a depositor is indifferent between withdrawing at $t = 1$ and waiting until $t = 2$, since the expected payoff at $t = 2$, as captured by the LHS in (25), is equal to the expected $t = 1$ payoff, which is captured by the RHS in (25). Hence, given θ^* from (24), it pins down the threshold signal s^*. The payoff at $t = 1$ is received whenever the bank is liquid, while the payoff at $t = 2$ is received whenever the bank is solvent.

Differentiating the LHS of (24) with respect to θ, we obtain $R\left(1 - \frac{n(\theta, s^*) r_1}{L}\right) - \frac{\partial n(\theta, s^*)}{\partial \theta}\left[R\theta \frac{r_1}{L} - r_2\right] > 0$ for any $\theta > \underline{\theta}$ since $r_1 > L$ and $\frac{\partial n(\theta, s^*)}{\partial \theta} \leq 0$. Taking the derivative of (24) with respect to $n(.)$, we obtain $-R\theta\frac{r_1}{L} + r_2 < 0$ for any $\theta > \underline{\theta}$ because $r_1 > L$. Overall, this implies that the LHS in (24) monotonically increases in θ and the signal s_i and so does the LHS in (25). Furthermore, rearranging (24) as $R\theta^* - r_2 - n(\theta^*, s^*)\left[R\theta^*\frac{r_1}{L} - r_2\right] = 0$, it follows that (24) is negative at $\theta = \underline{\theta}$ and positive at $\theta = \overline{\theta}$. Using (25), this means that at $\theta = \underline{\theta}$, a depositor expects to receive 0 when waiting and thus strictly prefers to withdraw. At $\theta = \overline{\theta}$ such that the LHS in (24) is strictly positive, a depositor expects to receive $r_2 > \omega r_1$ when waiting. Since ωr_1 exceeds the RHS in (25), it follows that, at $\theta = \overline{\theta}$, a depositor strictly prefers not to withdraw. Overall, the analysis above implies that $\underline{\theta} < \theta^* < \overline{\theta}$ and analogously that the threshold signal s^* falls within the range $\left(\underline{\theta} + \epsilon, \overline{\theta} - \epsilon\right)$. Given that $\underline{\theta} > 0$ and $\overline{\theta} \to 1$, it follows that the equilibrium pair $\{\theta^*, s^*\}$ falls in the range $(0, 1)$. To obtain a closed-form expression, we perform a change of variable using (23) from which we obtain $\theta(n) = s^* + \epsilon(1 - 2n)$. For $\epsilon \to 0$, $\theta(n) = s^*$ and yields the run threshold, which solves:

$$\int_0^{\widehat{n}(\theta^*)} r_2 dn = \int_0^{\overline{n}} \omega r_1 \, dn \equiv \pi_1 \Rightarrow \widehat{n}\left(\theta^*\right) r_2 = \omega L, \tag{26}$$

where π_1 is the expected payoff from withdrawing at $t = 1$. Solving for θ^* yields the closed-form expression stated. And $\theta^* > \underline{\theta}$ directly follows from $L < 1 \leq r_1$.

Comparative statics. To complete the proof, we study how bank fragility θ^* changes with deposit rates r_1 and r_2, as well as CBDC remuneration ω, liquidation value L, and the investment profitability R. We have the following:

$$\frac{\partial \theta^*}{\partial r_1} = \frac{\omega \theta^*}{(r_2 - \omega r_1)} > 0, \tag{27}$$

$$\frac{\partial \theta^*}{\partial r_2} = \frac{1}{R}\frac{r_2 - \omega L}{r_2 - \omega r_1} - \frac{\underline{\theta}\omega(r_1 - L)}{(r_2 - \omega r_1)^2} = \frac{r_2^2 - 2\omega r_1 r_2 + \omega^2 L r_1}{R(r_2 - \omega r_1)^2}, \tag{28}$$

$$\frac{\partial \theta^*}{\partial \omega} = \underline{\theta}\frac{r_2(r_1 - L)}{(r_2 - \omega r_1)^2} > 0, \quad \frac{\partial \theta^*}{\partial L} = -\frac{\omega \underline{\theta}}{r_2 - \omega r_1} < 0, \quad \frac{\partial \theta^*}{\partial R} = -\frac{\theta^*}{R} < 0, \tag{29}$$

where we used $r_1 > L$ and $r_2 > \omega r_1$. To establish the sign of $\frac{\partial \theta^*}{\partial r_2}$, we need to determine the sign of the numerator since the denominator is positive. The numerator is negative whenever $r_2^A < r_2 < r_2^B$, where r_2^A and r_2^B denote the roots of the associated quadratic equation $r_2^2 - 2\omega r_1 r_2 + \omega^2 L r_1 = 0$ since $\Delta = 4\omega^2 r_1^2 - 4\omega^2 L > 0$. The two roots are:

$$r_2^{A/B} = \omega r_1 \left(1 \pm \sqrt{1 - \frac{L}{r_1}}\right). \tag{30}$$

The smaller root r_2^A is inadmissible as it implies $r_2 < \omega r_1$, a contradiction. Thus, only the bigger root $r_2^B > \omega r_1$ is admissible. Since this value is the maximum of the relevant deposit rates considered by the bank, as we will show shortly, we label it $r_2^{max} \equiv r_2^B$. To summarize, $\frac{\partial \theta^*}{\partial r_2} < 0$ if $r_2 < r_2^{max}$ and $\frac{\partial \theta^*}{\partial r_2} > 0$ if $r_2 > r_2^{max}$.

B Proof of Proposition 2

A higher short-term deposit rate r_1 increases fragility (Proposition 1), so it reduces expected bank profits because the bank is solvent less often, $\frac{\partial \Pi}{\partial r_1} = (R\theta^* - r_2)\frac{\partial \theta^*}{\partial r_1} < 0$. A higher short-term deposit rate also tightens the participation constraint of consumers because they are repaid less often, $\frac{\partial V}{\partial r_1} = -r_2 \frac{\partial \theta^*}{\partial r_1} < 0$. Thus, $r_1^* = 1$.

The proof of the remaining claims is in several steps. We first derive sufficient conditions for the participation constraint of consumers to bind in equilibrium. Then, we derive comparative statics of the equilibrium deposit rate. As they would be useful later, we state some partial derivatives (evaluated at r_1^*):

$$\frac{\partial \theta^*}{\partial r_2} = \frac{(r_2 - \omega)^2 - \omega^2(1-L)}{R(r_2-\omega)^2} = \frac{1}{R} - \frac{\omega^2(1-L)}{R(r_2-\omega)^2}, \tag{31}$$

$$\frac{\partial^2 \theta^*}{\partial \omega \partial r_2} = -\frac{2(1-L)\omega r_2}{R(r_2-\omega)^3} < 0, \tag{32}$$

$$\frac{\partial^2 \theta^*}{\partial r_2^2} = \frac{2(1-L)\omega^2}{R(r_2-\omega)^3} > 0. \tag{33}$$

B.1 Binding participation constraint of consumers

Step 1: We derive bounds on the deposit rate chosen by the bank. A profit-maximizing bank never chooses a rate that entails $\theta^* = 1$. If a run is certain, the bank is certain to make zero (expected) profits. Hence, the bank chooses $r_2 > r_2^{min}$ where r_2^{min} solves $\theta^*(r_2^{min}) \equiv 1$, yielding a lower bound on the deposit rate:

$$r_2^{min} = \frac{R+\omega L}{2} - \sqrt{\left(\frac{R+\omega L}{2}\right)^2 - R\omega}. \tag{34}$$

Proposition 1 states that bank fragility decreases in the long-term deposit rate as long as $r_2 < r_2^{max}$. We now impose constraints on parameters to ensure that the participation constraint of consumers is slack at $r_2 = r_2^{max}$, that is $V(r_2^{max}) > 0$. Note that $\theta^*(r_2^{max}) = \frac{\omega}{R}\left(1+\sqrt{1-L}\right)^2$ and $V(r_2^{max}) = \omega\left(1+\sqrt{1-L}\right) - \frac{\omega^2}{R}\left(1+\sqrt{1-L}\right)^3 - \omega^2$, resulting in a lower bound on investment profitability:

$$R > \underline{R}_1 \equiv \frac{\omega\left(1+\sqrt{1-L}\right)^3}{1+\sqrt{1-L}-\omega}. \tag{35}$$

An upper bound on CBDC remuneration ensures that the denominator of $\underline{R}_1$ is always positive:

$$\omega < \widetilde{\omega} \equiv 1 + \sqrt{1-L}. \tag{36}$$

Note that $r_2^{min} < r_2^{max}$, which justifies our labels, and ensures that the bank does not always fail, $\theta^*(r_2^{max}) < 1$, which is the economically interesting case.

Step 2: We can write marginal bank profits as

$$\frac{d\Pi}{dr_2} \equiv -\frac{\partial\theta^*}{\partial r_2}\left(R\theta^* - r_2\right) - \int_{\theta^*}^{1} d\theta. \tag{37}$$

Since $(R\theta^* - r_2) = r_2\omega\frac{(r_1-L)}{r_2-\omega r_1} > 0$ and $1-\theta^* > 0$ (given the bounds on r_2) as well as the parameter constraints ensuring that higher long-term deposit rates reduce bank fragility, there is a non-trivial trade-off for the bank: higher long-term deposit rates make the bank more stable but also reduce its profit margin.

Evaluating marginal profits at $r_2 = r_2^{\max}$ (where, by definition, $\frac{\partial\theta^*}{\partial r_2} = 0$), gives $\frac{d\Pi}{dr_2} < 0$. Moreover, $\frac{\partial\Pi}{\partial r_2} < 0$ for all $r_2 > r_2^{max}$. Thus, the bank chooses a deposit rate $r_2 < r_2^{max}$ if feasible (i.e. if the participation constraint of consumers holds). Given the parameter constraints on investment profitability and CBDC remuneration (see step 1), the participation constraint is indeed slack, so the bank chooses a rate $r_2^* < r_2^{max}$ (establishing an upper bound on the deposit rate).

Similarly, evaluating at $r_2 = r_2^{\min}$ (where, by definition, $\theta^* = 1$) gives $\frac{d\Pi}{dr_2} > 0$. Furthermore, at $r_2 = r_2^{\min}$, we also have $V < 0$ (i.e. the participation constraint is violated), so the bank always chooses a higher deposit rate, $r_2^* > r_2^{\min}$ (establishing a lower bound on the deposit rate).

Step 3: Next, we show that expected bank profits Π are globally concave. As a result, the unconstrained choice of deposit rate that ignores the participation constraint of consumers, denoted by r_2^{Π} and solving $\frac{d\Pi}{dr_2} \equiv 0$, is unique. To establish global concavity, we show that the second-derivative is always negative:

$$\frac{d^2\Pi}{dr_2^2} \equiv -\frac{\partial^2\theta^*}{\partial r_2^2}(R\theta^* - r_2) - \left(\frac{\partial\theta^*}{\partial r_2}\right)^2 R + 2\frac{\partial\theta^*}{\partial r_2} < 0,$$

because $\frac{\partial\theta^*}{\partial r_2} < 0$ and $\frac{\partial^2\theta^*}{\partial r_2^2} > 0$. Consider $\underset{\sim}{r}_2 = \omega^2$, which solves the participation constraint (PC) of investors with no bank failure. Since the bank sometimes fails, $\theta^* > 0$, $\underset{\sim}{r}_2$ is clearly a lower bound on the value that solves the binding PC, $r_2^{PC} > \underset{\sim}{r}_2$. This bound is helpful in establishing sufficient conditions for the relevant equilibrium condition to be the binding PC. By global concavity of Π, a sufficient condition for $r_2^{\Pi} < \underset{\sim}{r}_2$ is $\frac{d\Pi(\underset{\sim}{r}_2)}{dr_2} < 0$. Intermediate results are $\theta^*(\underset{\sim}{r}_2) = \frac{\omega^2(\omega-L)}{R(\omega-1)}$, $R\theta^*(\underset{\sim}{r}_2) - \underset{\sim}{r}_2 = \frac{\omega^2(1-L)}{\omega-1}$, and $\frac{\partial\theta^*(\underset{\sim}{r}_2)}{\partial r_2} = \frac{\omega^2-2\omega+L}{R(\omega-1)^2}$. Thus, we can express $\frac{d\Pi(\underset{\sim}{r}_2)}{dr_2} < 0$ as a lower bound:

$$R > \underline{R}_2 \equiv \frac{\omega^2}{\omega-1}\left[-\frac{1-L}{(\omega-1)^2}\left(\omega^2 - 2\omega + L\right) + \omega - L\right] = \omega^2\left(1 + \frac{(1-L)^2}{(\omega-1)^3}\right). \quad (38)$$

As a result, we have shown that $r_2^{PC} > r_2^{\Pi}$. Finally, we verify that $\underset{\sim}{r}_2 \geq r_2^{min}$. Rewriting $\theta^*(\underset{\sim}{r}_2) < 1$ yields another lower bound on profitability:

$$R > \underline{R}_3 \equiv \frac{\omega^2(\omega-L)}{\omega-1}. \quad (39)$$

Since $\omega < \widetilde{\omega}$, which implies $\omega^2 - 2\omega + L < 0$, we can rank these bounds $\underline{R}_2 > \underline{R}_3$. Thus, we can drop the bound $\underline{R}_3$. Taking stock, we define $\underset{\sim}{R}$ as the largest of all lower bounds on the investment returns (see below for the definition).

B.2 Existence of a unique deposit rate, comparative statics

Having established that the deposit rate r_2^* corresponds to the solution to the binding participation constraint, we next prove its existence and uniqueness. Recall that the net value of the deposit claim is $V = \int_{\theta^*}^{1} r_2\, d\theta - \omega^2$. So, $V(r_2^*) \equiv 0$. Note that $V(r_2^{min}) = -\omega^2 < 0$ and $V(r_2^{max}) > 0$ given the parameter constraints on R and ω. Differentiating V with respect to r_2:

$$\frac{dV}{dr_2} = -\frac{\partial\theta^*}{\partial r_2} r_2 + (1-\theta^*) > 0, \quad (40)$$

so a higher (long-term) deposit rate increases the value of the deposit claim for two reasons: consumers receive a high payment in the absence of a bank run and the bank is less fragile (Proposition 1). Given the monotonicity of V in r_2 and its change of signs from the bound r_2^{min} to r_2^{max}, a solution for r_2^* exists and is unique.

Next, we study the comparative statics of r_2^*. First, consider CBDC remuneration ω, using the implicit function theorem (IFT), $\frac{dr_2}{d\omega} = -\frac{\frac{\partial V}{\partial \omega}}{\frac{\partial V}{\partial r_2}}$. The denominator is positive, as shown in (40). Hence, the sign of $\frac{dr_2}{d\omega}$ is the opposite of the sign of the numerator:

$$\frac{\partial V}{\partial \omega} = -2\omega - \frac{\partial \theta^*}{\partial \omega} r_2 < 0. \tag{41}$$

It follows that r_2 monotonically increases in CBDC remuneration ω:

$$\frac{dr_2^*}{d\omega} = \frac{2\omega + \frac{\partial \theta^*}{\partial \omega} r_2}{1 - \theta^* - \frac{\partial \theta^*}{\partial r_2} r_2} > 0. \tag{42}$$

Finally, we derive the comparative statics of the equilibrium deposit rate with respect to investment characteristics. Using the implicit function theorem again, the results $\frac{dr_2^*}{dL} < 0$ and $\frac{dr_2^*}{dR} < 0$ follow from $\frac{\partial V}{\partial L} = -r_2 \frac{\partial \theta^*}{\partial L} > 0$ and $\frac{\partial V}{\partial R} = -r_2 \frac{\partial \theta^*}{\partial R} > 0$.

C Proof of Lemma 1 and Proposition 3

We first prove the lemma and then the proposition. Using the expression for $\frac{dr_2}{d\omega}$ in Equation (42), we expand the expression for $\frac{d\theta^*}{d\omega}$:

$$\frac{d\theta^*}{d\omega} = \frac{\partial \theta^*}{\partial \omega} + \frac{\partial \theta^*}{\partial r_2}\frac{dr_2^*}{d\omega} = \frac{\partial \theta^*}{\partial \omega} + \frac{\partial \theta^*}{\partial r_2}\frac{2\omega + \frac{\partial \theta^*}{\partial \omega} r_2^*}{1 - \theta^* - r_2^* \frac{\partial \theta^*}{\partial r_2}}. \tag{43}$$

Since the denominator of the second term is positive, we get $\frac{d\theta^*}{d\omega} < 0$ whenever $\frac{\partial \theta^*}{\partial \omega}\left(1 - \theta^* - r_2^*\frac{\partial \theta^*}{\partial r_2}\right) + \frac{\partial \theta^*}{\partial r_2}\left(2\omega + \frac{\partial \theta^*}{\partial \omega} r_2^*\right) < 0$. This inequality simplifies to

$$\frac{\partial \theta^*}{\partial \omega}(1 - \theta^*) + 2\omega \frac{\partial \theta^*}{\partial r_2} < 0. \tag{44}$$

Using the equilibrium deposit rate (so $1 - \theta^* = \frac{\omega^2}{r_2^*}$) and $\frac{\partial \theta^*}{\partial r_2} = \frac{1}{r_2}\left[\theta^* - \omega \frac{\partial \theta^*}{\partial \omega}\right]$, we can re-express this condition as:

$$\theta^* + r_2^* \frac{\partial \theta^*}{\partial r_2} < 0, \tag{45}$$

which has the intuitive interpretation of an elasticity. In particular, the elasticity of the failure threshold with respect to deposit rate, $\eta = -\frac{r_2^*}{\theta^*}\frac{\partial \theta^*}{\partial r_2}$, has to exceed 1 for the indirect effect to dominate and thus $\frac{d\theta^*}{d\omega} < 0$, where r_2^* solves $V(r_2^*) = 0$.

Using $1-\theta^* = \frac{\omega^2}{r_2^*}$ to rewrite Condition (44) yields $\omega\frac{\partial\theta^*}{\partial\omega}+2r_2\frac{\partial\theta^*}{\partial r_2} < 0$. Inserting the expressions for the partial derivatives dividing by the positive common term $\frac{r_2^*}{R(r_2^*-\omega)^2}$, we obtain $\eta > 1$ if and only if $\omega r_2^*(1-L)+2\left[(r_2^*)^2 - 2\omega r_2^* + \omega^2 L\right] < 0$. Rewriting yields the following condition with a quadratic term:

$$h(r_2^*,\omega) \equiv (r_2^*)^2 - \frac{3+L}{2}\omega r_2^* + \omega^2 L < 0. \tag{46}$$

We turn to the proof of the proposition. First, we determine whether $\frac{d\theta^*}{d\omega} < 0$ when evaluated at $\omega = 1$ is possible. Using condition (46), this boils down to $(r_2^*)^2 - \frac{3+L}{2}r_2^* + L < 0$. Thus, we can find the roots $r_2^C \equiv \frac{\omega}{4}\left(3+L-\sqrt{L^2-10L+9}\right)$ and $r_2^D \equiv \frac{\omega}{4}\left(3+L+\sqrt{L^2-10L+9}\right)$ such that $h < 0$ if and only if $r_2^C < r_2^* < r_2^D$. Since $r_2^C < \omega$ is inadmissible, r_2^D is the relevant root, which is independent of R.

Second, we impose parameter constraints to ensure $r_2^D \in \left(r_2^{min}, r_2^{max}\right)$. Using the expression for r_2^{max} as given in (30) and evaluating it at $r_1 = 1$ and $\omega = 1$, $r_2^D < r_2^{max}$ can be expressed as $\frac{1-L}{4} + \sqrt{1-L} > \frac{1}{4}\sqrt{L^2-10L+9}$. Squaring and rewriting yields $8(1-L)(1+\sqrt{1-L}) > 0$, which always holds for $L < 1$. Moreover, for $r_2^D > r_2^{min}$ to hold at $\omega = 1$, it suffices to show that $\theta^*(\omega = 1, r_2 = r_2^D) < 1$. This yields another lower bound on profitability:

$$R > \underline{R}_4 \equiv \frac{r_2^D(r_2^D - L)}{r_2^D - 1}. \tag{47}$$

Third, r_2^* decreases in R, while r_2^D is independent of it. Thus, there exists a critical value, $\underline{R}_5$, such that $r_2^* < r_2^D$ for all $R > \underline{R}_5$. Importantly, $\underline{R}_5 < \infty$. One can easily show that $r_2^D > 1 > L$ because $\sqrt{L^2-10L+9} > 1-L$ can be rearranged by squaring to $8(1-L) > 0$. By contrast, $r_2^* \to 1$ for $R \to \infty$ since $\theta^* \to 0$ and thus $r_2^* \to 1$ for a given $L < 1$ and $\omega = 1$.

The reader may notice that the bound $\underline{R}_2$ characterized in the proof of Proposition 2 converges to ∞ as $\omega \to 1$, thus becoming the binding bound on profitability. However, it is important to stress that this simple sufficient condition is quite restrictive. In fact, the numerical example in the main text shows that our results also hold for much lower levels of the investment profitability R.

Fourth, we show that $\frac{d\theta^*}{d\omega} > 0$ for large ω. Recall that $\frac{dr_2^*}{d\omega} > 0$ and $r_2^* < r_2^{max}$. Then, we can denote ω^{max} such that $r_2^* \to r_2^{max}$ when $\omega \to \omega^{max}$. In this limit, Condition (45) is violated because $\frac{\partial\theta^*}{\partial r_2} \to 0$ when $r_2 \to r_2^{max}$. Thus, $\frac{d\theta^*}{d\omega} > 0$.

Note that $\omega^{max} < \widetilde{\omega}$. To see this, recall that (i) $\underline{R}_1 = +\infty$ at $\omega = \tilde{\omega}$ and (ii) $\underline{R}_1 < \infty$ for any $\omega < \widetilde{\omega}$. That is, for any $\omega < \widetilde{\omega}$, there exists a finite $\underline{R}_1$ such that the participation constraint binds exactly at $r_2 = r_2^{max}$. Hence, $\frac{\partial \underline{R}_1}{\partial \omega} > 0$ implies that there exists an $\omega < \widetilde{\omega}$ and $R > \underline{R}_1$ for which $r_2^* = r_2^{max}$, denoted as ω^{max}.

Taken these steps together, we have $\frac{d\theta^*}{d\omega} > 0\big|_{\omega=1} < 0$ and $\frac{d\theta^*}{d\omega}\big|_{\omega^{max}} > 0$. Hence, there is at least a value of ω, denoted as ω^{min}, at which θ^* is minimized.

Fifth, we show that ω^{min} is unique. The value ω^{min} solves $h(r_2^*, \omega_{min}) = 0$, where $h(r_2, \omega)$ is given in (46). Since r_2^* is a function of ω, $h(r_2(\omega), \omega)$ is a polynomial where ω is the main variable. The degree of the polynomial determines the number of possible values ω^{min}. Since $\frac{d\theta^*}{d\omega}\big|_{\omega=1} < 0$ and $\frac{d\theta^*}{d\omega}\big|_{\tilde{\omega}} > 0$, the number of solutions ω^{min} must be odd. To determine the degree of the polynomial $h(r_2(\omega), \omega)$, it is useful to characterize a closed-form solution for r_2^*. Since r_2^* solves $V(r_2^*, \omega) = 0$ given in (7). Substituting the expression for θ^* from (4), we obtain:

$$r_2^3 - r_2^2(R + \omega L) + r_2 R\omega(\omega + 1) - R\omega^3 = 0. \tag{48}$$

Equation (48) has three roots, which solve the corresponding depressed cubic equation

$$y^3 + Py + Q = 0, \tag{49}$$

where $y = r_2 - \frac{R+\omega L}{3}$, $P = \frac{3R\omega(1+\omega)-(R+\omega L)^2}{3}$ and $Q = \frac{-2(R+\omega L)^3+9(R+\omega L)R\omega(\omega+1)-27R\omega^3}{27}$. We focus on parameters such that $4P^3 + 27Q^2 > 0$, so there is only one real root: $y = \sqrt[3]{-\frac{Q}{2} + \sqrt{\frac{Q^2}{4} + \frac{P^3}{27}}} + \sqrt[3]{-\frac{Q}{2} - \sqrt{\frac{Q^2}{4} + \frac{P^3}{27}}}$. The expression pinning down y and, in turn, r_2^* is a function of ω. One can show that ω only appears at a power of 1. This implies that $h(r_2(\omega), \omega)$ has at most two roots, of which only one can be in the range $1 < \omega < \tilde{\omega}$. Since the derivative is initially negative and eventually positive, there must be an odd number of crossings with zero within $[1, \tilde{\omega}]$. Hence, ω_{min} is unique.

D Proof of Lemma 2 and Proposition 4

Proof of Lemma. To simplify the exposition, the proof is done under the assumption of $r_1^* = 1$. This is without loss of generality because $r_1^* = 1$ emerges in equilibrium, as in the main analysis, given that the run threshold increases in r_1.

Under partial withdrawals, the depositor wishes to redeposit the maximum amount γ with the central bank at $t = 1$, so it withdraws deposits worth γ from the bank at $t = 1$. The remaining $1 - \gamma$ are kept in the bank until $t = 2$. Thus, the bank requires resources worth γ at $t = 1$ to meet withdrawals. The proof considers separately the cases of $\gamma > L$ and $\gamma \leq L$ because this ranking of aggregate resources determines the presence of strategic complementarity in withdrawal decisions. For each case, we compare a depositor's expected payoffs from partial and full withdrawals. Recall that full withdrawals imply that γ is held in CBDC and $1 - \gamma$ in cash.

Case 1: When $\gamma > L$, the bank does not have enough resources to meet total withdrawals when all depositors partially withdraw. This gives rise to strategic complementarity in depositors' withdrawal decisions. Hence, we follow the same steps as in the main text to compute the run threshold, denoted as θ^*_{PW} for partial withdrawals (PW). The relevant equations are the insolvency condition, $R\theta\left(1 - \frac{n\gamma}{L}\right) - (1 - n\gamma)\, r_2 = 0$, which pins down $\widehat{n}_{PW} = \widehat{n}/\gamma$, and the indifference condition:

$$\gamma \int_0^{\widehat{n}_{PW}} r_2 dn = \gamma\omega \int_0^{\overline{n}_{PW}} dn, \tag{50}$$

where $\overline{n}_{PW} = \overline{n}/\gamma$ scales similarly as $\widehat{n}_{PW}$. As a result, we obtain the same indifference condition as before, $\widehat{n} r_2 = \omega\overline{n}$.

Consider next the case of full withdrawals (FW), assuming that a depositor withdraws the entire amount when choosing to run on the bank. Thus, the thresholds for illiquidity and insolvency are unchanged relative to the baseline model, $\overline{n}_{FW} = \overline{n}$ and $\widehat{n}_{FW} = \widehat{n}$, but the indifference condition changes to:

$$\int_0^{\widehat{n}_{FW}} r_2 dn = [\gamma\omega + (1 - \gamma)] \int_0^{\overline{n}_{FW}} dn. \tag{51}$$

We can now compare partial and full withdrawals. The LHS of (50) is the same as the LHS of (51). For $\omega > 1$ and $\gamma < 1$, the RHS of (51) is smaller than the RHS of (50). As a result, bank fragility is lower under full withdrawals, $\theta^*_{FW} < \theta^*_{PW}$. The second benefit of full withdrawal is that depositors get more of their funds out in a run: $\gamma\omega$ with partial withdrawals and $\gamma\omega + (1 - \gamma)$ with full withdrawals. As a result, the expected payoff for depositors is higher under full withdrawals. Hence, partial withdrawals are never privately optimal. Given the linearity of depositors' expected payoffs in γ and ω, it follows that any partial withdrawal $\gamma + x$ with $x > 0$ is dominated by an even larger partial equal to $\gamma + y$, with $x < y \leq 1 - \gamma$. Therefore, full withdrawals are optimal.

Case 2: Consider now $\gamma \leq L$, a situation in which partial withdrawals does not give rise to any coordination failure since the bank has enough asset liquidity. Thus, only fundamental-driven runs occur when $\theta < \underline{\theta}$, which is the same under the partial and full withdrawal assumption and equal to $\underline{\theta} = \frac{r_2}{R}$, as before. For any $\theta < \underline{\theta}$, any resource kept at the bank is lost due to costly bankruptcy. Hence, it immediately follows that it is never optimal to only withdraw a fraction γ of deposits. This concludes the proof.

Proof of Proposition. Introducing holding limits affects consumers' decisions at $t = 0$ and $t = 1$. At $t = 0$, holding limits changes the consumers' participation constraint to:

$$\int_{\theta^*}^{1} r_2 \, d\theta \geq (\omega^{HL})^2 = [1 + \gamma(\omega - 1)]^2 \,. \tag{52}$$

The left-hand side is the value of the deposit claim to the consumer (unchanged relative to the main text). The right-hand side is the expected return of holding CBDC, which differs from the main text because only a fraction γ of funds can be held in CBDC. At $t = 0$ an consumer invests γ in CBDC and $1 - \gamma$ in storage/cash. At $t = 1$, the initial investment returns ω on the γ units, whose a fraction γ is held in the CBDC account while the remainder is held in storage/cash. Thus, the analysis in the main text is a special case for no holding limits, $\omega^{HL}(\gamma = 1) = \omega$.

At $t = 1$, holding limits only affects a depositor's expected payoff from withdrawing, $r_1\omega^{HL}$, so they have the intended effect of directly reducing withdrawal incentives by lowering the remuneration of the withdrawn funds stored until $t = 2$. Thus, the effective CBDC remuneration with holding limits is $\omega^{HL} \equiv 1 + \gamma(\omega - 1)$. Once this transformation is made, the economy is identical to the one without holding limits with the only difference that ω is replaced by ω^{HL}.

The bank run threshold is $\theta^*_\gamma = \frac{r_2}{R}\frac{r_2 - L\omega^{HL}}{r_2 - r_1\omega^{HL}}$, where θ^*_γ increases in γ because $\frac{\partial \theta^*_\gamma}{\partial \gamma} = \frac{r_2}{R}\frac{r_2(\omega-1)(r_1-L)}{(r_2 - r_1(1+\gamma(\omega-1)))^2} > 0$ whenever $\omega > 1$. This result captures the "common wisdom" about holding limits: introducing them (i.e., setting $\gamma < 1$) reduces bank fragility, effectively mitigating the direct effect of CBDC remuneration on fragility.

However, introducing holding limits also affects the sensitivity of the run threshold to changes in r_2, thus leading to a potential ambiguous effect on fragility when banks respond to the introduction of CBDC. The derivative of the threshold θ^*_γ with respect to r_2 is now a function of γ and equal to $\frac{\partial \theta^*_\gamma}{\partial r_2} = \frac{\theta^*_\gamma}{r_2} - \frac{r_2}{R}\frac{(r_1 - L)\omega^{HL}}{(r_2 - r_1\omega^{HL})^2}$. Hence, the total effect of holding limits on fragility is thus not obvious and again depends on both a direct effect (via lower withdrawal incentives) and an indirect effect (via equilibrium deposit rates).

E Proof of Proposition 5

We derive the total effect of changes in policy parameters on fragility. There is the usual direct effect and an indirect effect via deposit rates. As in the main model, we start with the direct effect on withdrawal incentives at $t = 1$. In the main model, $\pi_1 = \omega r_1 \overline{n} = \omega L$, while with contingent remuneration (CR) we have $\pi_{1,CR}$, as given in the main text. As a result, the failure threshold in the withdrawal subgame is lower with contingent remuneration, $\theta^*_{CR} < \theta^*$, which is the intended objective of the policy. To see this, we write the failure threshold as a function of the expected payoff from withdrawing:

$$\theta^* = \frac{r_2}{R}\frac{r_2 - \pi_1}{r_2 - \pi_1 \frac{r_1}{L}}, \tag{53}$$

so $\frac{d\theta^*}{d\pi_1} = \frac{r_2^2(r_1 - L)}{RL(r_2 - \pi_1 r_1/L)^2} > 0$ and $\frac{\partial \pi_{1,CR}}{\partial \underline{\omega}} = (\overline{n} - \widetilde{n})r_1 > 0$ and $\frac{\partial \pi_{1,CR}}{\partial \widetilde{n}} = (\omega - \underline{\omega})r_1 > 0$. Thus, contingent remuneration achieves its direct objective of reducing withdrawal incentives, the more so, the more restrictive the policy is (lower $\underline{\omega}$ and lower $\widetilde{n}$).

Since $\frac{d\pi_{1,CR}}{dr_1} = \widetilde{n}(\omega - \underline{\omega}) > 0$ and $\frac{\partial \widehat{n}}{\partial r_1} < 0$, higher short-term deposit rates again increase fragility, $\frac{d\theta^*_{CR}}{dr_1} > 0$. Moreover, when evaluated at $r_1 = 1$, we have

$$\begin{aligned} \frac{\partial \theta^*_{CR}}{\partial r_2} &= \frac{(r_2 - \frac{\pi_{1,CR}}{L})^2 - \left(\frac{\pi_{1,CR}}{L}\right)^2 (1 - L)}{R(r_2 - \frac{\pi_{1,CR}}{L})^2} = \frac{1}{R} - \frac{\pi_{1,CR}^2(1 - L)}{RL^2(r_2 - \frac{\pi_{1,CR}}{L})^2} && (54) \\ \pi_{1,CR} &= \underline{\omega}L + \widetilde{n}(\omega - \underline{\omega}), && (55) \end{aligned}$$

so $\pi_{1,CR}$ is a sufficient statistic for the effect of intervention parameters on fragility at $t = 1$ and comprises the effects of both $\widetilde{n}$ and $\underline{\omega}$.

As in the main model, there is also an effect of CR on the ex-ante deposit rate and, thus, an indirect effect on fragility. For vanishing private noise, the remuneration of CBDC is ω for $\theta > \theta^*$ and $\underline{\omega}$ for $\theta < \theta^*$, resulting in the changed participation constraint of consumers given in the main text. Expected bank profits change to

$$\Pi_{CR} = \int_{\theta^*_{CR}}^{1} (R\theta - r_2) d\theta. \tag{56}$$

The incentives of the bank and consumers are again aligned in setting the lowest feasible short-term deposit rate, $r^*_{1,CR} = 1$, because $\frac{d\Pi_{CR}}{dr_1} < 0$ and $\frac{dV_{CR}}{dr_1} = \frac{d\theta^*_{CR}}{dr_1}[-r_2 + \omega(\omega - \underline{\omega})] < 0$ because $r^*_2 > \omega^2$ in any equilibrium. Using the same steps as in the main text, we find that $r^*_{2,CR}$ solves a binding participation constraint.

There is an asymmetry in policy parameters on the ex-ante choices. Lowered CBDC remuneration $\underline{\omega}$ affects the value of the deposit claim directly, while the intervention threshold $\widetilde{n}$ only affects the ex-ante choice via its effect on fragility. Formally, changes in $\widetilde{n}$ translate into changes in $\pi_{1,CR}$, so we study how the latter affects fragility. To derive the effect of the intervention on the deposit rate, we use the IFT and the following partial derivatives:

$$\frac{\partial V_{CR}}{\partial r_2} = 1 - \theta^*_{CR} - \big[r_2 - \omega(\omega - \underline{\omega})\big] \frac{\partial \theta^*_{CR}}{\partial r_2} > 0, \tag{57}$$

$$\frac{\partial V_{CR}}{\partial \pi_{1,CR}} = -\big[r_2 - \omega(\omega - \underline{\omega})\big] \frac{\partial \theta^*_{CR}}{\partial \pi_{1,CR}} < 0, \tag{58}$$

$$\frac{\partial V_{CR}}{\partial \underline{\omega}} = -\big[r_2 - \omega(\omega - \underline{\omega})\big] \frac{\partial \theta^*_{CR}}{\partial \pi_{1,CR}} \frac{\partial \pi_{1,CR}}{\partial \underline{\omega}} - \omega\theta^*_{CR} < 0. \tag{59}$$

Thus, the effects on the deposit rate are:

$$\frac{dr^*_{2,CR}}{d\pi_{1,CR}} = \frac{\big[r_2 - \omega(\omega - \underline{\omega})\big] \frac{\partial \theta^*_{CR}}{\partial \pi_{1,CR}}}{1 - \theta^*_{CR} - \big[r_2 - \omega(\omega - \underline{\omega})\big] \frac{\partial \theta^*_{CR}}{\partial r_2}} > 0, \tag{60}$$

$$\frac{dr^*_{2,CR}}{d\underline{\omega}} = \frac{\big[r_2 - \omega(\omega - \underline{\omega})\big] \frac{\partial \theta^*_{CR}}{\partial \pi_{1,CR}} \frac{\partial \pi_{1,CR}}{\partial \underline{\omega}} + \omega\theta^*_{CR}}{1 - \theta^*_{CR} - \big[r_2 - \omega(\omega - \underline{\omega})\big] \frac{\partial \theta^*_{CR}}{\partial r_2}} > 0. \tag{61}$$

Totally differentiating the failure threshold with respect to each policy parameter (taking into account the indirect effect via deposit rates) yields the following:

$$\frac{d\theta^*_{CR}}{d\pi_{1,CR}} = \frac{(1 - \theta^*_{CR}) \frac{\partial \theta^*_{CR}}{\partial \pi_{1,CR}}}{1 - \theta^*_{CR} - \big[r_2 - \omega(\omega - \underline{\omega})\big] \frac{\partial \theta^*_{CR}}{\partial r_2}} > 0, \tag{62}$$

$$\frac{d\theta^*_{CR}}{d\underline{\omega}} = \frac{(1 - \theta^*_{CR}) \frac{\partial \theta^*_{CR}}{\partial \pi_{1,CR}} \frac{\partial \pi_{1,CR}}{\partial \underline{\omega}} + \omega\theta^*_{CR} \frac{\partial \theta^*_{CR}}{\partial r_2}}{1 - \theta^*_{CR} - \big[r_2 - \omega(\omega - \underline{\omega})\big] \frac{\partial \theta^*_{CR}}{\partial r_2}}, \tag{63}$$

whose denominator is positive. To obtain the inequality in the proposition, we simply substitute the expressions for $\frac{\partial\theta^*_{CR}}{\partial\pi_{1,CR}}$ and $\frac{\partial\pi_{1,CR}}{\partial\underline{\omega}}$. For $\widetilde{n} \to \overline{n}$, the beneficial direct effect vanishes because $\frac{\partial\pi_{1,CR}}{\partial\underline{\omega}} \to 0$. As the detrimental indirect effect via lower deposit rates remains bounded away from zero, the overall effect on fragility is detrimental.

F Proof of Propositions 6 and 7

Perfect competition. Consider perfect competition, $\beta \to 0$. It implies that the bank maximizes the expected return of its deposit claim (which is equivalent to maximizing V) subject to non-negative profits, $\Pi \geq 0$. Our approach will be to consider the unconstrained problem and then check whether bank profits are indeed non-negative. The first-order condition pins down the equilibrium deposit rate r_2^*:

$$H(r_2^*) \equiv \left.\frac{dV}{dr_2}\right|_{r_2=r_2^*} = 0. \tag{64}$$

Since $H(r_2^{max}) > 0$, we deduce that $r_2^{max} < r_2^*$ and, as a result, fragility increases in the deposit rate around the equilibrium, $\left.\frac{\partial\theta^*}{\partial r_2}\right|_{r_2=r_2^*} > 0$. Since

$$\frac{\partial H}{\partial r_2} \equiv -2\frac{\partial\theta^*}{\partial r_2} - r_2\frac{\partial^2\theta^*}{\partial r_2^2} < 0, \tag{65}$$

a unique global maximum exists. Using the IFT and

$$\frac{\partial H}{\partial\omega} \equiv -\frac{\partial\theta^*}{\partial\omega} - r_2\frac{\partial^2\theta^*}{\partial r_2\partial\omega}, \tag{66}$$

we obtain

$$\frac{dr_2^*}{d\omega} = \frac{\frac{\partial\theta^*}{\partial\omega} + r_2\frac{\partial^2\theta^*}{\partial r_2\partial\omega}}{-2\frac{\partial\theta^*}{\partial r_2} - r_2\frac{\partial^2\theta^*}{\partial r_2^2}}, \tag{67}$$

so $\frac{dr_2^*}{d\omega} > 0$ whenever $r_2^* < 3\omega$, which arises from inserting the partial derivatives into (66) and re-arranging. To obtain a sufficient condition for $r_2^* < 3\omega$, note that $H(3\omega) < 0$ suffices. Using $\theta^*(r_2 = 3\omega) = 3\omega\frac{3-L}{2R}$ and $\left.\frac{\partial\theta^*}{\partial\omega}\right|_{r_2=3\omega} = \frac{3+L}{4R}$, we obtain a sufficient condition for $\frac{dr_2^*}{d\omega} > 0$ under perfect competition, which is $R < 3\omega\frac{9-L}{4}$.

Next, we turn from the effect of CBDC remuneration on deposit rates to its effect on bank fragility. Using the total derivative of fragility, $\frac{d\theta^*}{d\omega} = \frac{\partial\theta^*}{\partial\omega} + \frac{\partial\theta^*}{\partial r_2}\frac{dr_2^*}{d\omega}$, we obtain

$\frac{d\theta^*}{d\omega} > 0$ (after some rearrangement) whenever $-\frac{\partial\theta^*}{\partial\omega}\frac{\partial\theta^*}{\partial r_2} - r_2\frac{\partial\theta^*}{\partial\omega}\frac{\partial^2\theta^*}{\partial r_2^2} + r_2\frac{\partial\theta^*}{\partial r_2}\frac{\partial^2\theta^*}{\partial r_2\partial\omega} < 0$, which always holds given the signs of the various partial derivatives already established. Hence, higher CBDC remuneration increases fragility.

Finally, we need to establish that $\Pi(r_2^*) \geq 0$. Note that expected profits can be written as $\Pi = (1-\theta^*)\left[\frac{R}{2}(1+\theta^*(r_2)) - r_2\right]$, where the first factor is strictly positive because of $\theta^* < 1$ from the upper-dominance region. The second factor is also positive because $r_2^* \in (\omega, R)$ and the following result:

$$\frac{R}{2}(1+\theta^*) - r_2 = \frac{R}{2} + \frac{r_2(r_2-\omega L)}{2(r_2-\omega)} - r_2 = \frac{R-r_2}{2} + \frac{r_2}{2}\frac{\omega(1-L)}{r_2-\omega} > 0, \tag{68}$$

so expected profits at the equilibrium deposit rate are strictly positive.

Imperfect competition. For $\beta \in (0,1)$, the equilibrium deposit rate r_2^* arises from taking logs and differentiating, yielding the expression in Equation (14). That is, the first-order condition that pins down r_2^* is written as $\Lambda(r_2^*) = 0$, where

$$\begin{aligned}\Lambda(r_2) \quad \equiv \quad & -\beta\left(1-\theta^* + (R\theta^* - r_2)\frac{\partial\theta^*}{\partial r_2}\right)\left[\int_{\theta^*}^1 r_2 d\theta - \omega^2\right] + \cdots \\ & \cdots + (1-\beta)\left(1-\theta^* - r_2\frac{\partial\theta^*}{\partial r_2}\right)\int_{\theta^*}^1 (R\theta - r_2)\, d\theta.\end{aligned} \tag{69}$$

Thus, the partial derivative $\frac{\partial\Lambda}{\partial\omega}$ contains both positive and negative terms as well as an ambiguous one. All non-positive terms are multiplied by $(1-\beta)$, so a high enough value of β suffices for $\frac{\partial\Lambda}{\partial\omega} > 0$ and, thus, $\frac{dr_2^*}{d\omega} > 0$ from the IFT. To see this, note that

$$\begin{aligned}\frac{\partial\Lambda}{\partial\omega} \quad = \quad & -\beta\left(-\theta_\omega + (R\theta^* - r_2)\theta_{r\omega} + R\theta_\omega\theta_r\right)\left[\int_{\theta^*}^1 r_2 d\theta - \omega^2\right] + \cdots \\ & \cdots - \beta\left(1-\theta^* + (R\theta^* - r_2)\theta_r\right)\left[-r\theta_\omega - 2\omega\right] + \cdots \\ & \cdots + (1-\beta)\int_{\theta^*}^1 (R\theta - r_2)\, d\theta\left[-\theta_\omega - r\theta_{r\omega}\right] - (1-\beta)\theta_\omega(R\theta^* - r_2)\left(1-\theta^* - r_2\theta_r\right)\end{aligned} \tag{70}$$

where the subscripts denote the partial derivatives of θ^*. Recall that $\theta_\omega > 0$, and $\theta_{r\omega} < 0$.

G Proof of Propositions 8-9

The proof builds on Goldstein and Pauzner (2005) because introducing ω does not affect the properties of the relevant functions. We consider bank fragility at $t = 1$ first and then look at equilibrium deposit rates at $t = 0$.

Bank fragility. All impatient depositors withdraw, while patient depositors decide between withdrawing and re-depositing with the central bank to earn ω and keeping their funds in the bank. The lower dominance bound is the value of θ that makes a patient depositor indifferent when only impatient depositors withdraw ($n = \lambda$):

$$u(\omega r_1) \equiv \underline{\theta} u\left(c_{2\lambda}\right), \tag{71}$$

where the consumption level is $c_{2\lambda} \equiv R\frac{1-\lambda r_1}{1-\lambda}$. Similarly, $c_{2n} \equiv R\frac{1-nr_1}{1-n}$. The depositor's utility differential is $v(\theta, n) = \theta u(c_{2n}) - u(\omega r_1)$ if the bank is liquid at $t = 1$, $\lambda \leq n \leq \overline{n}$. Otherwise, $\overline{n} < n \leq 1$, $v(\theta, n) = 0 - u\left(\frac{\omega}{n}\right)$ because all withdrawing depositors receive an equal share of the liquidation proceeds and patient depositors will redeposit these funds, earn a return ω, and then consume at $t = 2$. One-sided strategic complementarity applies, so we apply the results in Goldstein and Pauzner (2005). The equilibrium failure threshold arises from the indifference condition of the marginal depositor for $\epsilon \to 0$:

$$\theta^* \int_{\lambda}^{\overline{n}} u(c_{2n})dn = \int_{\lambda}^{\overline{n}} u(\omega r_1)dn + \int_{\overline{n}}^{1} u\left(\frac{\omega}{n}\right) dn, \tag{72}$$

so rewriting yields Condition (15). We turn to the comparative statics:

$$\frac{\partial \theta^*}{\partial r_1} = \frac{\int_{\lambda}^{\overline{n}} u'\left(r_1\omega\right)\omega dn + \theta^* \int_{\lambda}^{\overline{n}} u'\left(c_{2n}\right) R\frac{n}{1-n}dn}{\int_{\lambda}^{\overline{n}} u\left(c_{2n}\right)dn} > 0 \tag{73}$$

$$\frac{\partial \theta^*}{\partial \omega} = \frac{\int_{\lambda}^{\overline{n}} u'\left(r_1\omega\right) r_1 dn + \int_{\overline{n}}^{1} u'\left(\frac{\omega}{n}\right)\frac{1}{n}dn}{\int_{\lambda}^{\overline{n}} u\left(c_{2n}\right)dn} > 0 \tag{74}$$

$$\frac{\partial^2 \theta^*}{\partial r_1 \partial \omega} = \frac{\int_{\lambda}^{\overline{n}} \left[u''\left(r_1\omega\right)\omega r_1 + u'\left(r_1\omega\right)\right]dn}{\int_{\lambda}^{\overline{n}} u\left(c_{2n}\right)dn} + \frac{\partial \theta^*}{\partial \omega}\frac{\int_{\lambda}^{\overline{n}} u'\left(c_{2n}\right) R\frac{n}{1-n}dn}{\int_{\lambda}^{\overline{n}} u\left(c_{2n}\right)dn} \tag{75}$$

Equilibrium deposit rates. We first prove that $r_1^* > 1$ and then we show that $\frac{dr_1^*}{d\omega} < 0$. Consider the FOC in the proposition. Evaluating (17) at $r_1 = 1$, we obtain

$$\frac{\partial \theta^*}{\partial r_1}(1-\lambda)[\underline{\theta}u(R) - u(\omega)] + \lambda \int_{\underline{\theta}}^{1} \left[u'(1) - \theta R u'(R)\right] d\theta \quad = \quad \lambda \int_{\underline{\theta}}^{1} \left[u'(1) - \theta R u'(R)\right] d\theta > 0,$$

as $\theta^* = \underline{\theta}$ when $r_1 = 1$, $\underline{\theta}u(R) - u(\omega) = 0$ from the definition of $\underline{\theta}$ at $r_1 = 1$, and $u'(1) - \theta R u'(R) > 0$ given that $RRA > 1$. Hence, this cannot be optimal. So $r_1^* > 1$.

In order to complete the proof, we need to show that r_1^* decreases in ω. To do so, we can use the FOC for r_1, the IFT, and $SOC < 0$ (as r_1^* is a local maximum). We derive this partial derivative next; it has three parts:

$$\begin{aligned}\frac{\partial FOC}{\partial \omega} &= -\frac{\partial^2 \theta_{LP}^*}{\partial r_1 \partial \omega}\left[\lambda u(r_1) + (1-\lambda)\theta_{LP}^* u(c_{2\lambda}) - \lambda u(1) - (1-\lambda) u(\omega)\right] \\ &\quad + \frac{\partial \theta_{LP}^*}{\partial r_1}(1-\lambda)\left[u'(\omega) - \frac{\partial \theta_{LP}^*}{\partial \omega} u(c_{2\lambda})\right] - \lambda\left[u'(r_1) - \theta_{LP}^* R u'(c_{2\lambda})\right]\frac{\partial \theta_{LP}^*}{\partial \omega}.\end{aligned} \tag{76}$$

The third term is negative because of the risk-sharing benefit of liquidity provision.

The first term is negative whenever the cross-partial $\frac{\partial^2 \theta_{LP}^*}{\partial r_1 \partial \omega}$ is positive. Using (75), one can see that the second term of this cross-partial is positive, while its first one can be rearranged as follows:

$$\frac{1 - RRA(\omega r_1^*)}{\int_\lambda^{\overline{n}} u(c_{2n})\, dn \int_\lambda^{\overline{n}} u'(r_1 \omega) dn},$$

where RRA is the relative risk aversion. Note that RRA monotonically increases in the level of absolute risk aversion σ and in the level of consumption. Hence, we denote as $\widehat{\sigma}$ the level of risk aversion such that, for $\sigma < \widehat{\sigma}$, the RRA when evaluated at $\omega r_1 \to R$, which is the upper bounds of ωr_1^*, approaches 1. Thus, for any $\sigma < \widehat{\sigma}$, the first term in (75) is dominated by the second one. As a result of the positive cross-partial derivative, the first term in (76) is also negative.

Thus, it only remains to be shown that the second term in (76) is also negative. This is the case when $\frac{\partial \theta_{LP}^*}{\partial \omega} u(c_{2\lambda}) - u'(\omega) \leq 0$. Using the expression for $\frac{\partial \theta_{LP}^*}{\partial \omega}$ and noting that $\left(\frac{1-nr_1}{1-n} R\right) \leq \left(\frac{1-\lambda r_1}{1-\lambda} R\right)$ for all $n \geq \lambda$, a sufficient condition for this inequality to hold is $u'(\omega) \leq \frac{1}{\overline{n}-\lambda}\left[\int_\lambda^{\overline{n}} u'(\omega r_1) r_1 dn + \int_{\overline{n}}^1 u'\left(\frac{\omega}{n}\right)\frac{1}{n} dn\right]$. Summarizing the first term on the right hand side and multiplying by ω yields

$$\omega u'(\omega) \leq \omega r_1 u'(\omega r_1) + \frac{1}{\overline{n}-\lambda}\int_{\overline{n}}^1 u'\left(\frac{\omega}{n}\right)\frac{\omega}{n} dn. \tag{77}$$

Note that the term on LHS is bigger than the first term on the RHS. A lower bound for the second term on the RHS is $\frac{1-\overline{n}}{\overline{n}-\lambda}\omega r_1 u'(\omega r_1)$. Thus, the RHS of Condition (77) is at least $\frac{1-\lambda}{\overline{n}-\lambda}\omega r_1 u'(\omega r_1)$, so a simple sufficient condition is

$$\omega u'(\omega) \leq \frac{1-\lambda}{\overline{n}-\lambda}\omega r_1 u'(\omega r_1). \tag{78}$$

One can show that this condition is always satisfied. Substituting $\overline{n} = \frac{1}{r_1}$ and dividing by ω each side, the inequality above can be rewritten as follows:

$$u'(\omega) \leq \frac{(1-\lambda)}{1-\lambda r_1} r_1^2 u'(\omega r_1).$$

It is immediate that the inequality is satisfied for $r_1 = 1$. It is also satisfied for $r_1 = \frac{1}{\lambda}$, which is the upper bound of r_1. Hence, the inequality must hold true for all r_1 in the admissible range, unless the RHS is not monotonically changing with r_1. To exclude this case, we take the derivative:

$$\frac{\partial RHS}{\partial r_1} = \frac{(1-\lambda)r_1}{(1-\lambda r_1)^2}\left[2u'(\omega r_1) + r_1 u''(\omega r_1)\omega + \lambda r_1 u'(\omega r_1)\right],$$

which, using the definition of RRA, can be rearranged as follows:

$$\frac{\partial RHS}{\partial r_1} = \frac{(1-\lambda)r_1}{(1-\lambda r_1)^2 u'(\omega r_1)}\left[2 - RRA(\omega r_1^*) + \lambda r_1\right].$$

For any $\sigma < \widehat{\sigma}$, the RHS increases monotonically in r_1 and so the fact that the inequality holds when $r_1 = 1$ establishes that it is always satisfied. This completes the proof.

H Proof of Proposition 10

This proof has three parts. First, we derive the run threshold. This part uses the same argument as the proof of Proposition 1. The threshold θ_q^* corresponds to the solution to $\int_0^{\widehat{n}(\theta)} q r_2 dn = \int_0^{\overline{n}} \omega r_1 dn$, because the bank repays depositors r_2 at $t = 2$ only when the project succeeds, where both $\widehat{n}(\theta)$ and $\overline{n}$ are independent of q and identical to the corresponding cutoffs in the main text. Some algebra yields the threshold θ_q^* stated in the proposition. Differentiating this threshold with respect to q and ω, we obtain:

$$\frac{\partial \theta_q^*}{\partial q} = \frac{r_2}{R}\frac{r_2 q r_2 - r_2 \omega r_1 - q r_2 r_2 + \omega L r_2}{(q r_2 - \omega r_1)^2} = -\frac{r_2^2}{R}\frac{\omega(r_1 - L)}{(q r_2 - \omega r_1)^2} < 0, \tag{79}$$

$$\frac{\partial \theta_q^*}{\partial \omega} = \frac{r_2}{R}\frac{-L q r_2 + L \omega r_1 + q r_2 r_1 - \omega L r_1}{(q r_2 - \omega r_1)^2} = \frac{r_2}{R}\frac{q r_2 (r_1 - L)}{(q r_2 - \omega r_1)^2} > 0. \tag{80}$$

Second, we solve for the bank's choice at $t = 0$. Differentiating the expected profits (19) with respect to q, we obtain Equation (20). A high enough c ensures that the solution q^* is interior and unique (because $SOC_q < 0$ for high c).

Third, and finally, we study how an increase in CBDC remuneration affects financial stability. Note that the monitoring effort q^* directly depends on CBDC remuneration. Formally, the overall effect of a change in CBDC remuneration on bank monitoring effort can be expressed as follows (because of the IFT):

$$\frac{\partial q^*}{\partial \omega} = -\frac{\frac{\partial FOC_q}{\partial \omega}}{SOC_q}. \tag{81}$$

Since $SOC_q < 0$, the sign of $\frac{dq^*}{d\omega}$ is equal to the sign of $\frac{\partial FOC_q}{\partial \omega}$, which is equal to

$$\begin{aligned}
\frac{\partial FOC_q}{\partial \omega} &= -\frac{\partial \theta_s^*}{\partial \omega}\left(R\theta_q^* - r_2\right) - q\frac{\partial \theta_q^*}{\partial q}\frac{\partial \theta_s^*}{\partial \omega}R - q\frac{\partial^2 \theta_q^*}{\partial q \partial \omega}\left(R\theta_q^* - r_2\right) \\
&= -\left[\frac{\partial \theta_q^*}{\partial \omega} + q\frac{\partial^2 \theta_q^*}{\partial q \partial \omega}\right]\left(R\theta_q^* - r_2\right) - q\frac{\partial \theta_q^*}{\partial q}\frac{\partial \theta_q^*}{\partial \omega}R \\
&= -\left[\frac{\partial \theta_q^*}{\partial \omega} + q\frac{\partial^2 \theta_q^*}{\partial q \partial \omega}\right]\left(R\theta_q^* - r_2\right) + \omega\left(\frac{\partial \theta_s^*}{\partial \omega}\right)^2 R,
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial^2 \theta_q^*}{\partial q \partial \omega} &= -\frac{r_2^2}{R}(1-L)\frac{(qr_2-\omega)^2 + 2\omega\,(qr_2-\omega)}{(qr_2-\omega)^4} = -\frac{r_2^2}{R}(1-L)\frac{qr_2+\omega}{(qr_2-\omega)^3} < 0 \\
&= -\frac{\partial \theta_q^*}{\partial \omega} - 2\omega\frac{qr_2^2}{R}\frac{(1-L)}{(qr_2-\omega)^3}.
\end{aligned}$$

Substituting the expressions for θ_q^*, $\frac{\partial \theta_s^*}{\partial \omega}$, $\frac{\partial^2 \theta_q^*}{\partial q \partial \omega}$, and $\frac{\partial \theta_q^*}{\partial q}$, we obtain

$$\frac{\partial FOC_q}{\partial \omega} = 2\omega\frac{qr_2^2}{R}\frac{(1-L)}{(qr_2-\omega)^3}\left(R\theta_s^* - r_2\right) + \omega\left(\frac{\partial \theta_q^*}{\partial \omega}\right)^2 R = \frac{1}{R}q\omega r_2^3(L-1)^2\frac{qr_2+2\omega}{(qr_2-\omega)^4} > 0,$$

which implies, in turn, that $\frac{dq^*}{d\omega} > 0$.

Finally, we move on to the total effect of CBDC remuneration on bank fragility:

$$\frac{d\theta^*}{d\omega} = \frac{\partial \theta^*}{\partial \omega}\left[1 - \frac{\omega}{q}\frac{dq^*}{d\omega}\right] > 0, \tag{82}$$

where the sign arises because $\frac{\partial \theta^*}{\partial \omega} > 0$ and one can show that

$$\left[1 - \frac{\omega}{q}\frac{dq^*}{d\omega}\right] = 1 + \frac{\omega}{q}\frac{\partial^2\left(q\int_{\frac{r_2}{R}\frac{qr_2-\omega L}{qr_2-\omega}}^{1}\left(R\theta - r_2\right)d\theta - \frac{cq^2}{2}\right)}{\partial q \partial \omega}\left[\frac{\partial^2\left(q\int_{\frac{r_2}{R}\frac{qr_2-\omega L}{qr_2-\omega}}^{1}\left(R\theta - r_2\right)d\theta - \frac{cq^2}{2}\right)}{\partial q^2}\right]^{-1} > 0.$$

I Relaxing the bankruptcy cost assumption

In this section, we check the robustness of our main result, namely the non-monotonicity of the run threshold θ^* as a function of CBDC remuneration ω, to a different assumption concerning bankruptcy costs. Unlike the baseline model, we now assume that there are no bankruptcy costs in case the bank fails at date 1. This implies that in the event of a run, depositors receive a pro-rata share of bank's available resources. These funds can then be placed into a CBDC account to yield the return ω between date 1 and 2.[25]

As in the main text, we start from the characterization of the run threshold. The bounds for the lower and upper dominance regions are unchanged and so are still given by $\underline{\theta} = \frac{r_2}{R}$ and $\overline{\theta} \to 1$. The absence of bankruptcy costs at date 1 changes depositors' expected payoff at date 1 as follows:

$$\pi_1^{BC} = \int_0^{\overline{n}} \omega r_1 \, dn + \int_{\overline{n}}^1 \omega \frac{L}{n} \, dn,$$

where the lack of bankruptcy costs results in the second term, which is new relative to the main text. This term captures the pro rata payment to depositors that withdraw in case of a bank run. As a result, the indifference condition in (26) now becomes:

$$\int_0^{\widehat{n}(\theta^*)} r_2 dn = \pi_1^{BC} \Rightarrow \widehat{n}\left(\theta^*\right) r_2 = \omega L \left(1 - \log\left(\frac{L}{r_1}\right)\right), \tag{83}$$

which yields

$$\theta^*_{BC} = \underline{\theta} \frac{r_2 - \omega L \left(1 - \log\left(\frac{L}{r_1}\right)\right)}{r_2 - \omega r_1 \left(1 - \log\left(\frac{L}{r_1}\right)\right)}.$$

Comparing (26) and (83) immediately yields $\theta^*_{BC} > \theta^*$, since the expected payoff from running in the absence of interim bankruptcy costs is larger, while the expected payoff from not running is unchanged. This also implies that the direct effect of ω on θ^*_{BC} is positive as in the baseline model, but stronger because of the extra payoff (relative to the baseline model) that depositors accrue in the event of a run.

[25] As in the baseline model, we still consider full bankruptcy costs in case the bank fails at date 2. Carletti et al. (2023) show that the asymmetric treatment of bankruptcy costs simplifies the analysis without affecting the basic properties of the equilibrium for the depositors' withdrawal game. Furthermore, in this framework, removing full bankruptcy costs only at date 1 should reinforce the direct effect of CBDC remuneration, making the emergence of a non-monotone relationship between fragility and CBDC remuneration—our main result—less likely.

As in the main text, the run threshold also depends *indirectly* on CBDC remuneration via the terms of the deposit contract r_1 and r_2. Those rates are chosen by the bank to solve the following problem:

$$\max_{r_1 \geq 1, r_2} \Pi \equiv \int_{\theta^*_{BC}}^{1} (R\theta - r_2)\, d\theta \tag{84}$$

$$\text{s.t. } V \equiv \int_0^{\theta^*_{BC}} L\omega d\theta + \int_{\theta^*_{BC}}^{1} r_2\, d\theta - \omega^2 \geq 0. \tag{85}$$

This problem is similar to the one characterized by (6) and (7) with two differences: First, the relevant run threshold is now θ^*_{BC}; second, the participation constraint has the extra term that captures the depositors' payoff in the event of a run.

As in the main text, both the bank and the depositors benefit from a low r_1, so $r_1^* = 1$ emerges in equilibrium. Hence, the indirect effect of ω on the run threshold again works through the change in the equilibrium deposit rate r_2^*. The two figures below show that the bank responds to the higher CBDC remuneration by increasing r_2 in equilibrium and that such an increase in r_2 reduces the run threshold θ^*_{BC}.

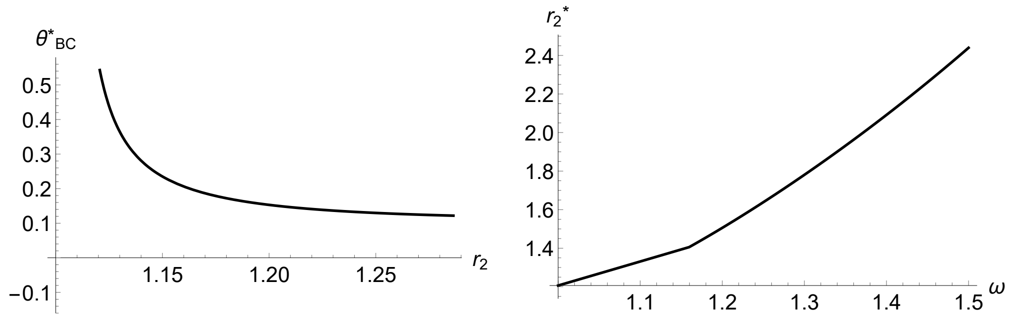

Figure 9: The indirect effect of CBDC remuneration on bank fragility via equilibrium deposit rates. The left panel shows that an increase in r_2 leads to a lower run threshold θ^*_{BC}. The right panel shows how the equilibrium deposit rate r_2^* changes with CBDC remuneration ω. The parameters are: $L = 0.9$ and $R = 17$. CBDC remuneration is set to be $\omega = 1$ in the left panel.

The equilibrium deposit rate r_2^* is either pinned down by the binding depositors' participation constraint or corresponds to the solution to the first order condition, i.e., $r_2^*(\omega) = max\{r_2^{FOC}, r_2^{PC}\}$, where r_2^{FOC} is the solution to the FOC with respect to r_2, while r_2^{PC} corresponds to the level of r_2 that solves the binding participation constraint. This explains the different slope of the function in the right panel of Figure 9.

Hence, the overall effect of ω on θ^*_{BC} is determined by the interplay between the direct and indirect effect. Figure 10 shows that we still obtain a non-monotonic relationship between bank fragility and CBDC remuneration.

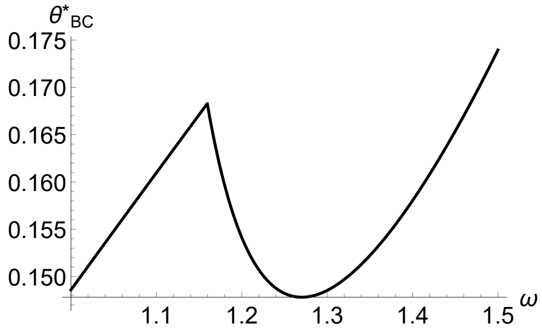

Figure 10: Bank failure threshold θ^*_{BC} and CBDC remuneration ω. Parameters: $L = 0.9$, $R = 17$.

Compared to the baseline model with full bankruptcy costs, a U-shape only emerges for intermediate to large values of ω. Yet, and as in the baseline model, a positively remunerated CBDC has a beneficial effect on the probability of a run, since bank fragility is minimized for $\omega > 1$. For smaller values of ω, an increase in CBDC remuneration leads to a higher run threshold θ^*_{BC}. This range corresponds to the parameter space in which depositors' participation constraint is slack and the equilibrium deposit rate is pinned down by the FOC. While in that range the indirect effect is still present, i.e., r^*_2 still increases with ω, thus leading to a reduction in θ^*_{BC} ceteris paribus, its strength is not enough to offset the direct effect in this particular parameter space. This can be seen from the right panel in Figure 10, where it can be seen that the slope of the function is much more negative in the first part of the graph (corresponding to the range where r^*_2 solves the FOC) than in the second, where r^*_2 solves the binding participation constraint.

References

Ahnert, T. (2016). Rollover risk, liquidity and macroprudential regulation. *Journal of Money, Credit and Banking 48*(8), 1753–85.

Ahnert, T., K. Anand, J. Chapman, and P. Gai (2019). Asset encumbrance, bank funding, and fragility. *Review of Financial Studies 32*(6), 2422–55.

Ahnert, T., K. Assenmacher, P. Hoffmann, A. Leonello, C. Monet, and D. Porcellacchia (2022). The economics of central bank digital currency. *ECB Working Paper 2713.*

Allen, F., E. Carletti, and R. Marquez (2011). Credit market competition and capital regulation. *Review of Financial Studies 24*(4), 983–1018.

Andolfatto, D. (2021). Assessing the impact of central bank digital currency on private banks. *The Economic Journal 131*(634), 525–540.

Angeletos, G., C. Hellwig, and A. Pavan (2006). Signaling in a global game: Coordination and policy traps. *Journal of Political Economy 114*(3), 452–84.

Bank for International Settlements (2020). Central bank digital currencies: financial stability implications. *Report no 4*.

Bank of England (2023). The digital pound: a new form of money for households and businesses? Consultation paper, Bank of England.

Bindseil, U. (2020). Tiered cbdc and the financial system. *ECB Working Paper 2351.*

Bindseil, U., F. Panetta, and I. Terol (2021). Central bank digital currency: functional scope, pricing and controls. *ECB Occasional Paper 286.*

Bonfim, D. and J. A. Santos (2023). The importance of deposit insurance credibility. *Journal of Banking & Finance 154*, 106916.

Bouvard, M., P. Chaigneau, and A. DeMotta (2015). Transparency in the financial system: Rollover risk and crises. *Journal of Finance 70*(4), 1805–37.

Brunnermeier, M. K. and D. Niepelt (2019). On the equivalence of private and public money. *Journal of Monetary Economics 106*, 27–41.

Calomiris, C. and C. Kahn (1991). The role of demandable debt in structuring optimal banking arrangements. *American Economic Review 81*(3), 497–513.

Carletti, E., A. Leonello, and R. Marquez (2023). Loan guarantees, bank underwriting policies and financial fragility. *Journal of Financial Economics 149*(2), 260–95.

Carlsson, H. and E. van Damme (1993). Global games and equilibrium selection. *Econometrica 61*(5), 989–1018.

Chiu, J., S. M. Davoodalhosseini, J. Jiang, and Y. Zhu (2023). Bank market power and central bank digital currency: Theory and quantitative assessment. *Journal of Political Economy 131*(5), 1213–48.

DellAriccia, G., L. Laeven, and R. Marquez (2014). Real interest rates, leverage, and bank risk-taking. *Journal of Economic Theory 149*(C), 65–99.

Dell'Ariccia, G. and R. Marquez (2006). Lending booms and lending standards. *The Journal of Finance 61*(5), 2511–46.

Diamond, D. and P. Dybvig (1983). Bank runs, deposit insurance and liquidity. *Journal of Political Economy 91*, 401–19.

Diamond, D. and R. Rajan (2001). Liquidity risk, liquidity creation, and financial fragility: A theory of banking. *Journal of Political Economy 109*(2), 287–327.

Drechsler, I., A. Savov, and P. Schnabl (2017). The deposits channel of monetary policy. *The Quarterly Journal of Economics 132*(4), 1819–1876.

Egan, M., A. Hortacsu, and G. Matvos (2017). Deposit competition and financial fragility: Evidence from the us banking sector. *American Economic Review 107*(1), 169–216.

Eisenbach, T. M. (2017). Rollover risk as market discipline: A two-sided inefficiency. *Journal of Financial Economics 126*(2), 252–69.

European Central Bank (2020). Report on a digital euro.

Fernández-Villaverde, J., D. Sanches, L. Schilling, and H. Uhlig (2021). Central bank digital currency: Central banking for all? *Review of Economic Dynamics 41*, 225–42.

Garratt, R., J. Yu, and H. Zhu (2022). How central bank digital currency design choices impact monetary policy pass-through and market composition. *Mimeo*.

Goldstein, I. and A. Pauzner (2005). Demand deposit contracts and the probability of bank runs. *Journal of Finance 60*(3), 1293–1327.

Hannan, T. H. and A. N. Berger (1997). The rigidity of prices: Evidence from the banking industry. *J. Reprints Antitrust L. & Econ. 27*, 245.

Hellmann, T. F., K. C. Murdock, and J. E. Stiglitz (2000). Liberalization, moral hazard in banking, and prudential regulation: Are capital requirements enough? *American Economic Review 90*(1), 147–65.

Holmstrom, B. and J. Tirole (1997). Financial intermediation, loanable funds, and the real sector. *The Quarterly Journal of Economics 112*(3), 663–91.

James, C. (1991). The losses realized in bank failures. *The Journal of Finance 46*(4), 1223–1242.

Jiang, J. H. and Y. Zhu (2021). Monetary policy pass-through with central bank digital currency. Technical report, Bank of Canada Staff Working Paper.

Kashyap, A., D. Tsomocos, and A. Vardoulakis (2024). Optimal bank regulation in the presence of credit and run risk. *Journal of Political Economy 132*(3), 772–823.

Keister, T. and C. Monnet (2022). Central bank digital currency: Stability and information. *Journal of Economic Dynamics and Control 142*, 104501.

Keister, T. and D. Sanches (2022). Should Central Banks Issue Digital Currency? *The Review of Economic Studies 90*(1), 404–31.

Kosse, A. and I. Mattei (2022). Gaining momentum–Results of the 2021 BIS survey on central bank digital currencies. *BIS Papers No. 125*.

Lilley, A. and K. Rogoff (2020). The case for implementing effective negative interest rate policies. In J. Cochrane and J. Taylor (Eds.), *Strategies for Monetary Policy*, Volume 2, pp. 27–90. Hoover Institution Press.

Liu, X. (2016). Interbank market freezes and creditor runs. *Review of Financial Studies 29*, 1860–1910.

Liu, X. (2023). A model of systemic bank runs. *Journal of Finance 78*(2), 731–93.

Monnet, C., A. Petursdottir, and M. Rojas-Breu (2023). Central bank account for all: Efficiency and risk taking. *Mimeo*.

Morris, S. and H. Shin (2003). Global games: Theory and applications. In M. Dewatripont, L. Hansen, and S. Turnovsky (Eds.), *Advances in Economics and Econometrics*, Volume 1, pp. 57–114. Cambridge University Press.

Morrison, A. D. and L. White (2005, December). Crises and capital requirements in banking. *American Economic Review 95*(5), 1548–72.

Neumark, D. and S. A. Sharpe (1992). Market structure and the nature of price rigidity: evidence from the market for consumer deposits. *The Quarterly Journal of Economics 107*(2), 657–680.

Niepelt, D. (2023). Money and banking with reserves and cbdc. *The Journal of Finance*.

Paul, P., M. Ulate, and J. C. Wu (2024). A macroeconomic model of central bank digital currency. *Federal Reserve Bank of San Francisco Working Paper 2024-11*.

Rochet, J.-C. and X. Vives (2004). Coordination failures and the lender of last resort: Was bagehot right after all? *Journal of the European Economic Association 2*(6), 1116–47.

Schilling, L. (2023). Optimal forbearance of bank resolution. *The Journal of Finance 78*(6), 3621–75.

Schilling, L., J. Fernández-Villaverde, and H. Uhlig (2024). Central bank digital currency: When price and bank stability collide. *Journal of Monetary Economics*, 103554.

Skeie, D. R. (2020). Digital currency runs. *WBS Finance Group Research Paper*.

Vives, X. (2005). Complementarities and games: New developments. *Journal of Economic Literature 43*, 437–479.

Vives, X. (2014). Strategic complementarity, fragility, and regulation. *Review of Financial Studies 27*(12), 3547–92.

Vives, X. (2019). Digital disruption in banking. *Annual Review of Financial Economics 11*(1), 243–72.

Whited, T., Y. Wu, and K. Xiao (2023). Will central bank digital currency disintermediate banks? *Working Paper*.

Williamson, S. (2022). Central bank digital currency: Welfare and policy implications. *Journal of Political Economy 130*(11), 2829–61.