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Abstract

We develop a model to study the efficiency of socially responsible investments (SRI) as a market-based mechanism to control firms' externalities. When responsible and profit-motivated investors interact, the former tend to concentrate on a subset of firms in the economy, while excluding others. This concentration of responsible capital can mitigate free-riding and coordination issues in the adoption of green technologies, but it can also create product market power and crowd out the green investments of excluded firms. If the crowding-out dominates, firms' aggregate green investments and welfare are higher without SRI. In equilibrium, responsible capital concentrates the most when concentration is the least desirable.

Keywords: Socially responsible investment, engagement, externalities, governance.

JEL Classification: D62, G34, M14.

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1 Introduction

Companies face increasing pressure from investors to include environmental and social factors in their policies. By the end of 2021, 4,375 investors managing $121 trillion have signed the United Nations Principles for Responsible Investment (UN PRI), pledging to incorporate corporate social responsibility (CSR) issues into their investment analysis and ownership policies.[1] In 2020, $1 in every $3 under professional management in the United States was allocated to sustainable investments (Edmans, 2023). The primary rationale for this socially responsible investing (SRI) is to change or divest from firms that exert negative externalities, to reduce their harm to society.

In a world where political institutions fail to control externalities, responsible investors can have a positive impact by serving as substitutes for regulation and other forms of government intervention. While there is a large literature on the limits to government intervention (e.g., Acemoglu, 2003; Besley and Persson, 2022), the inefficiencies that may arise when capital markets try to control firms' externalities are less understood. The objective of this article is to investigate the impact of SRI in a framework where its potential inefficiencies, and the externalities that such inefficiencies may impose on other stakeholders, are carefully articulated and analyzed.

We consider a model where firms' CSR policies reflect both their interaction with other firms (in input or output markets) and the social attitudes of their shareholders (through shareholders' engagement in shaping CSR policies or management catering to their preferences). Firm ownership is endogenous: responsible and non-responsible investors trade in a decentralized stock market. In practice, sustainable funds are typically evaluated based on the average ESG scores of the stocks in their portfolios (see, e.g., Morningstar's "globe" ratings of funds). Consistent with this observation, we assume that responsible investors suffer disutility from holding firms that generate negative externalities. Non-responsible investors are instead purely profit-motivated. Since firms internalize their shareholders' preferences, a firm invests more in CSR and, as a consequence, produces fewer externalities when responsible investors hold a larger fraction of its shares.

Our main insight is that the interaction between SRI and CSR policies can be such that SRI either

[1] In 2006, only 63 investors managing a total of $6.5 trillion had signed the UN PRI.

prevents a transition to a greener economy from materializing or pushes one where firms have more market power and consumers are worse off. The basic idea is easy to explain. Responsible investors are more selective in their investment choices and have more impact on a firm's CSR policy as a group. In equilibrium, this naturally leads responsible capital to concentrate in a subset of firms in the economy. This feature of the equilibrium is broadly consistent with the concentration of green capital we observe in the data (Figure 2). The abundance of responsible capital spurs the CSR investments of the targeted firms but also has an *indirect* effect on those excluded.

Suppose that firms' CSR investments are strategic complements so that the indirect effect is positive – that is, the excluded firms follow the lead of the targeted firms and invest more in CSR. Complementarity is most likely to arise when the firms' products are complements in consumers' preferences (e.g., electric vehicles and charging stations) or there are positive spillovers in developing green technologies (as documented by Aghion, Dechezleprêtre, Hemous, Martin, and Van Reenen (2016) for the auto industry). In this case, the aggregate greenness of the economy always increases with SRI. Moreover, concentrated responsible capital is sometimes the most efficient way to overcome free-riding and coordination issues in the adoption of green technologies.

Consider now a setting where firms' CSR investments are strategic substitutes so that the indirect effect is negative – that is, the investments of the targeted firms crowd out the CSR investments of the excluded ones. Substitutability is most likely to arise when firms compete for consumers in product markets (e.g., when the traditional "Schumpeterian effect" of competition is at play, as documented by Aghion, Bloom, Blundell, Griffith, and Howitt 2005) or for resources in input markets (e.g., lithium batteries for electric vehicles (Speirs, Contestabile, Houari, and Gross 2014)). We show that the crowding-out effect can outweigh the direct effect, so the impact of SRI on aggregate greenness can be negative. Even when SRI increases aggregate CSR, it may still decrease welfare: the concentration of responsible capital helps the targeted firms gain product market power. This allows them to charge higher prices so that consumption and social welfare fall.

Perhaps surprisingly, responsible capital tends to concentrate the most in our model when concentration is least desirable – that is, when the crowding out effect described above is strongest.

The reason is that crowding out makes excluded firms less green and less attractive to responsible investors, reinforcing their tendency to concentrate. It is worth emphasizing that, past a certain threshold, SRI's concentration reduces as more and more investors become responsible. Therefore, a sufficient mix of responsible and purely profit-motivated investors is crucial for the unintended consequences of SRI to arise in equilibrium. Thus, our model suggests that SRI is likely to help the transition to a greener economy if it becomes the norm in financial markets, but it may generate inefficiencies in the interim or if it remains a somewhat limited phenomenon.

The equilibrium strategies of responsible investors resemble some of the typical ESG strategies (e.g., best-in-class investing and ESG exclusion) and the type of coordinated engagements documented in the literature (e.g., Dimson, Karakaş, and Li, 2021). A central insight of our analysis is that these investment strategies may arise in equilibrium even when they (a) are ineffective at promoting *systematic* change in the economy and (b) reduce welfare by introducing too much differentiation across firms. There are a number of implications stemming from this insight.

First, a more uniform pressure on CSR issues deriving, for example, from a broader shift in consumption norms, may be both more effective and less distortionary. In practice, most information about firms' CSR policies is catered to investors and is often unavailable to consumers. There may be significant welfare gains from making this information more accessible to consumers, so they can incorporate it more easily into their shopping habits. Interestingly, SRI may hinder the transition to more responsible consumption norms: when the concentration of green capital leads to product market power and higher prices for green products, some consumers steer away from green products, reducing the level of responsible consumption in the economy.

Second, SRI is most efficient when targeted at firms whose CSR investments stimulate those of other firms. Conventional measures of firms' greenness that responsible investors typically target in their portfolio decisions (e.g., firms' ESG scores) do not account for these spillovers. Augmenting firms' ESG scores with proxies for positive spillovers, like their positions in the supply network and complementarity with other clean products, may help direct responsible investors to such "catalyst firms," leading to a more efficient allocation of green capital.

Third, our model has implications for how the distribution of investors' social attitudes affects aggregate greenness. In equilibrium, aggregate CSR investments may be non-monotonic in the fraction of responsible investors in the economy. Under strategic substitutability, responsible investors exclude all firms when they are only a small fraction of the population of investors, and they target an increasing number of firms as this fraction goes up. However, aggregate CSR investments may be the lowest when the fraction of responsible investors is at some intermediate value so that they participate in the financial market but are also highly concentrated. Recent empirical work (e.g., Hartzmark and Shue, 2023) finds evidence of distributional inefficiencies of SRI that are broadly consistent with our results. Our model suggests that these inefficiencies may become worse over time, but are also unlikely to persist if SRI becomes sufficiently popular.

Contributions. We make three main contributions to the literature. First, we provide, to the best of our knowledge, the first study of SRI in a general equilibrium model with interdependence in firms' CSR policies and imperfect competition. The interdependence allows us to study the indirect effects of SRI concentration for excluded firms and aggregate CSR. Because of imperfect competition, we can then explore how these spillovers affect product market competition.

While there is extensive literature on the direct effects of SRI on firms' CSR policies (Heinkel, Kraus, and Zechner, 2001; Oehmke and Opp, 2020; Gupta, Kopytov, and Starmans, 2022; Broccardo, Hart, and Zingales, 2022; Edmans, Levit, and Schneemeier, 2022), the spillovers to other firms and markets are less understood. The papers closest to ours are those that explore SRI in general equilibrium models with perfect competition (Landier and Lovo, 2020; Hakenes and Schliephake, 2021). Besides exploring imperfect competition, we consider a general model of interdependence in CSR policies, which allows for both complementarity and substitutability.[2] The analysis of the interaction between SRI and other forms of CSR incentives (e.g., the pricing of CSR issues in product markets) also connects our paper to those exploring the interplay between SRI and regulation (Biais and Landier, 2022; Piatti, Shapiro, and Wang, 2022; Huang and Kopytov, 2023).[3]

[2]Landier and Lovo (2020) consider a setting with two production sectors where supply-chain interactions generate a form of complementarity in firms' toxic emissions. Hakenes and Schliephake (2021) abstract from interdependence in CSR policies.

[3]Other work on SRI focuses on its implications for financial market outcomes, like expected returns (Pedersen, Fitzgibbons, and Pomorski, 2021; Pástor, Stambaugh, and Taylor, 2021) and price informativeness (Goldstein, Kopytov, Shen, and Xiang, 2022).

Second, we contribute to the vast literature on the objectives of the firm. The traditional view that firms should primarily maximize profits (Friedman, 1970) has been challenged by several recent papers (Elhauge, 2005; Bénabou and Tirole, 2010; Hart and Zingales, 2017). They argue that when political institutions fail to control externalities (due to commitment problems and political failures (e.g., Acemoglu, 2003; Besley and Persson, 2022)), firms should internalize shareholders' social preferences, potentially pursuing social goals at the expense of profits. We explore the efficiency of this market-based approach to controlling firm externalities. Our paper fits into a broader line of research that explores how individual pro-social preferences influence public goods provision (e.g., Bénabou and Tirole, 2006; Besley and Ghatak, 2007; Kaufmann and Koszegi, 2023).

Last, we provide a parsimonious model for studying the general equilibrium implications of heterogeneity in investor preferences. While we focus on investors' social attitudes, other dimensions of heterogeneity may represent interesting avenues for other applications. There is a large body of evidence that investors differ along multiple dimensions, like their investment horizons (Bushee, 1998; Gaspar, Massa, and Matos, 2005), governance attitudes (Bubb and Catan, 2022), and social and political ideologies (Bolton, Li, Ravina, and Rosenthal, 2020). Several recent articles study how such heterogeneity jointly affects shareholders' trading and governance decisions (e.g., Levit, Malenko, and Maug, 2023; Kakhbod, Loginova, Malenko, and Malenko, 2023) in single-firm settings. We derive a tractable characterization of (asymmetric) equilibria where ex-ante identical firms differ in their ownership and policies, which allows us to study how investors' heterogeneity affects the distribution of ownership and policies in the economy.

2 The model

The model consists of two dates, $t \in \{1, 2\}$, and a discrete number of publicly traded firms, $j \in \mathcal{J} \equiv \{1, \ldots, N\}$. At time $t = 1$, investors trade claims to the firms' terminal values in a financial market, which shapes their ownership structures. At time $t = 2$, firms first choose their CSR policies; subsequently, their terminal values realize and are distributed to shareholders. All agents in the model are rational and, for simplicity, we assume that there is no discounting. Next, we describe our baseline model, which will be the focus of our analysis. Section 5 discusses the robustness of our main results to alternative modeling assumptions.

2.1 CSR policies and firm values

We consider $N \geq 2$ ex-ante identical firms. Each firm generates a negative externality λ at a rate $1-\sigma_j$. One can interpret λ as the pollution generated by the production process or the societal cost of a negative corporate culture. Firms can invest resources to reduce the externality they generate. Each firm j chooses a CSR policy $\sigma_j \in [0,1]$ at a cost $C(\sigma_j) \equiv \frac{c}{2}\sigma_j^2$, where σ_j equals the percentage reduction in the externality. Alternatively, σ_j can also be interpreted as the probability with which the firm adopts a clean technology. We denote the collection of CSR policies by $\vec{\sigma} \equiv (\sigma_1, \ldots, \sigma_N)$. Firm j's expected terminal value Π_j is given by $\Pi\left(\sigma_j, \vec{\sigma}_{-j}\right) \equiv \pi_j - C_j$ where $\pi_j \equiv \pi\left(\sigma_j, \vec{\sigma}_{-j}\right)$ is the expected profit gross of CSR investment costs $C_j = C(\sigma_j)$. π_j depends on firm j's CSR policy σ_j as well as the CSR policies of other firms, collected in the vector $\vec{\sigma}_{-j}$. We assume that π_j is weakly positive and increasing and concave in σ_j: $\pi_j \geq 0$, $\frac{\partial \pi_j}{\partial \sigma_j} \geq 0$, and $\frac{\partial^2 \pi_j}{\partial \sigma_j^2} \leq 0$. The assumption that $\frac{\partial \pi_j}{\partial \sigma_j}$ can be positive captures firms' *private* incentives to invest in CSR, that is, independent of SRI. These incentives could reflect pressure from consumers, workers, or government regulation.

An important aspect of our model is that we allow for strategic interactions between firms. First, we capture these interactions in a general way through the properties of π_j. Later, we micro-found these interactions by explicitly modeling a product market that is populated by responsible and non-responsible consumers. If $\frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_{-j}} > 0$, firms' CSR policies are strategic complements, and more CSR investments by another firm strengthen firm j's incentives to increase σ_j. Examples of strategic complementarities include technological spillovers in green technologies or firms selling complementary goods. Vice versa, if $\frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_{-j}} < 0$, CSR investments are strategic substitutes. This scenario arises, for instance, when firms compete for responsible consumers in product markets or for shared resources in input markets.

2.2 Ownership market and firm objective

At time $t = 1$, the firms' ownership structure is determined. Each firm has a fixed supply of shares, normalized to one, traded in a financial market. A unit mass of atomistic investors, indexed by $i \in [0, 1]$, simultaneously submit their demand schedules for the shares of each firm. The market-clearing price equates demand and supply.

Investors have heterogeneous social attitudes. A fraction $\chi \in [0, 1)$ of investors is socially responsible ($i \in \mathcal{R}$). They internalize the externality λ generated by firms in their portfolio net of any mitigating effects through the firm's CSR investments (see, e.g., Heinkel et al., 2001; Pástor et al., 2021; Goldstein et al., 2022). Non-responsible investors ($i \in \mathcal{N}$) are purely profit-motivated. They maximize the expected monetary return from their holdings, i.e., the expected value of the claim to the firm's profits net of its share price p_j and the trading cost K_j.

Formally, for a given vector of CSR policies $\vec{\sigma}$, investor i solves:

$$\max_{s_{ij} \geq 0} \sum_j s_{ij} \left(\Pi_j - p_j - \mathbb{1}_{i,\mathcal{R}} \lambda (1 - \sigma_j) \right) - K_j, \tag{1}$$

where s_{ij} is the number of shares investor i holds in firm j, and $\mathbb{1}_{i,\mathcal{R}} = 1$ if i is responsible ($i \in \mathcal{R}$) and 0 otherwise ($i \in \mathcal{N}$). We assume that the trading cost of purchasing $\sum_j s_{ij}$ shares is $K_j = K\left(\sum_j s_{ij}\right)$ and, in order to obtain closed-form solutions for the trading strategies, we set $K(x) \equiv \frac{\kappa}{2} x^2$ (similar to Banerjee, Davis, and Gondhi (2018) and Dávila and Parlatore (2021)). In Online Appendix D.1, we show that our main insights continue to hold if we assume $K_j = \sum_j K\left(s_{ij}\right)$.[4]

At the beginning of $t = 2$, firms simultaneously choose their CSR policies, given their ownership structure. They incorporate shareholders' social preferences into their objective function so that ownership influences the choice of CSR policies. Formally, given its ownership $\{s_{ij}\}$ for $i \in [0, 1]$, firm j chooses σ_j to solve:

$$\max_{\sigma_j \in [0,1]} \Pi_j - \lambda (1 - \sigma_j) \eta s_j^{\mathcal{R}}, \tag{2}$$

where $s_j^{\mathcal{R}} \equiv \int_0^1 s_{ij} \mathbb{1}_{i,\mathcal{R}} di$ denotes the shares of firm j held by responsible investors and $\vec{s}^{\mathcal{R}} \equiv$

[4]The assumption that $K(\cdot)$ depends on the total size of the investor's portfolio, not on the allocation of shares across firms, is consistent with this cost reflecting direct transaction costs as well as indirect costs, such as the borrowing or opportunity costs of raising funds for the investor. Under certain conditions on the distribution of Π_j, the specification in Online Appendix D.1 is instead equivalent to a portfolio-choice problem with CARA utility.

$\left(s_1^{\mathcal{R}}, \ldots, s_N^{\mathcal{R}}\right)$ describes the distribution of SRI in the industry.

The objective in Program (2) is a weighted average of the expected payoff per share to investors, where the weights are the shares held by each shareholder.[5] This specification is commonly referred to as *proportional control* assumption (see, e.g., O'Brien and Salop, 1999; López and Vives, 2019), and captures different channels through which shareholders can influence managerial decisions in proportion to their stake in the firm.[6]

2.3 Sequence of events

The timing of the model is summarized in what follows.

Time $t = 1$:

(i) Investors trade and form their portfolios $\{s_{ij}\}$ for $i \in [0, 1]$ and $j \in \mathcal{J}$.

Time $t = 2$:

(ii) Having observed the distribution of SRI in the industry ($\vec{s}^{\mathcal{R}}$), firms choose CSR policies $\vec{\sigma}$.

(iii) Firms' terminal values realize and are distributed to shareholders.

We use *subgame perfect equilibrium* as the solution concept and restrict our attention to pure-strategy equilibria. An equilibrium of the game is a collection $\{\{s_{ij}\}, \vec{\sigma}\}$, where $i \in [0, 1]$ and $j \in \mathcal{J}$. The equilibrium collection solves Programs (1) and (2), and satisfies sequential rationality.

It is worth noticing that at $t = 1$ investors choose their portfolios based on their conjectures about firms' CSR policies, which are set at $t = 2$. Moreover, investors' portfolios shape firms' CSR policies, as they determine the firms' ownership and objective functions in Program (2). This two-way interaction between investors' portfolios and firms' CSR policies is the focus of the equilibrium

[5] At the beginning of time $t = 2$, investors have already paid the share price and transaction cost. Therefore, an $\mathcal{N}$-type investor receives an expected payoff Π_j from holding a share of firm j. An $\mathcal{R}$-type receives an expected payoff $\Pi_j - \lambda(1 - \sigma_j)$, since the investor incurs the disutility λ with probability (or at a rate) $1 - \sigma_j$.

[6] Examples of these channels include voting (e.g., Levit and Malenko, 2011; Levit et al., 2023), exit, and voice (e.g., Edmans and Manso, 2011; Brav, Dasgupta, and Mathews, 2022). Our qualitative results hold under a more general formulation of Program (2), that is, one where j maximizes $\Pi_j - \lambda(1 - \sigma_j) f(s_j^{\mathcal{R}})$, with $f : [0, 1] \to [0, 1]$ increasing in $s_j^{\mathcal{R}}$. We assume that f is linear to simplify the exposition. It is worth noticing that, if we assume $f = 1$ if $s_j^{\mathcal{R}} > \tau \in [0.5, 1)$ and 0 otherwise (consistent with a majority voting system), $\mathcal{R}$ investors' tendency to concentrate is stronger, as they need a majority of shares to influence the firm's policies.

analysis. In Online Appendix D.2, we show that our main insights continue to hold in a setting where firms choose their CSR policies first, and investors trade afterward.

As we will see, depending on the model parameters, two types of equilibria may arise: *Symmetric equilibria*, in which firms have identical ownership structures and CSR policies; *Asymmetric equilibria*, in which firms have different ownership structures and CSR policies.

3 Equilibrium analysis

We work our way backward by first deriving firms' CSR policies (Section 3.1), taking their ownership structures as given. We then determine the investors' optimal choices given the correctly anticipated CSR policies (Section 3.2). Finally, we characterize the symmetric (Section 3.3) and asymmetric (Section 3.4) equilibria of the game.

3.1 CSR policies

First, we impose two technical assumptions that help us to simplify the exposition.

Assumption 1 *We make the following two parametric assumptions: (i)* $c \geq \underline{c} > 0$, *and (ii)* $\frac{\partial^3 \pi_j^3}{\partial \sigma_j^3} < \frac{\partial^3 \pi_j}{\partial \sigma_j \partial \sigma_{-j}^2}$.

The first assumption states that the CSR investment cost is sufficiently convex (the threshold $\underline{c}$ is defined in Appendix B.1). This assumption guarantees that the second-order condition holds and makes the symmetric equilibrium unique. The second assumption ensures symmetry of the benchmark equilibrium without responsible investors.

Next, we characterize the firms' optimal choice of CSR policies. Taking the first-order condition of Problem (2), the optimal CSR policy for firm j solves:

$$\sigma_j = \frac{1}{c}\left(\eta \lambda s_j^{\mathcal{R}} + \frac{\partial \pi_j}{\partial \sigma_j}(\sigma_j, \vec{\sigma}_{-j})\right). \tag{3}$$

Equation (3) shows that firm j's willingness to implement a greener CSR policy depends on two forces: the internalization of shareholder preferences and the impact on expected profits. A greater share of responsible investors reduces the effective cost of greener CSR policies and, thus, encourages the firm to invest in them. The strength of this channel scales with the severity of the externality, λ. Moreover, the marginal impact of σ_j on π_j depends on the CSR investments of other firms. When other firms invest more in CSR, firm j's willingness to invest in CSR decreases if CSR policies are strategic substitutes, and increases if they are strategic complements.

The following lemma describes the firms' equilibrium CSR policies for a given ownership structure. All proofs are in Appendix B.

Lemma 1 (Equilibrium CSR Policies) *For a given distribution of SRI ($\vec{s}^{\mathcal{R}}$), an equilibrium $\sigma_j^*(\vec{s}^{\mathcal{R}}) \in [0, 1]$ of the subgame in which firms choose their CSR policies always exists.*

1. ***Symmetric equilibrium:*** *If $s_j^{\mathcal{R}} \equiv s^{\mathcal{R}}$ for all $j \in \mathcal{J}$, then $\sigma_j^*(\vec{s}^{\mathcal{R}})$ is unique, and all firms choose the same CSR policy $\sigma^*(s^{\mathcal{R}})$, which is increasing in $\eta s^{\mathcal{R}}$.*

2. ***Asymmetric equilibria:*** *If $s_1^{\mathcal{R}} \geq s_2^{\mathcal{R}} \geq \ldots \geq s_N^{\mathcal{R}}$, with at least one strict inequality, then $\sigma_1^* \geq \sigma_2^* \geq \ldots \geq \sigma_N^*$, with $\sigma_j^* > \sigma_{j'}^*$, if and only if $s_j^{\mathcal{R}} > s_{j'}^{\mathcal{R}}$; moreover:*

   (a) Under strategic substitutability, $\sigma_1^* > \sigma^*(\vec{0}) \equiv \sigma_0 > \sigma_j^*$ whenever $s_j^{\mathcal{R}} = 0$.

   (b) Under strategic complementarity, $\sigma_j^* > \sigma_0 \, \forall j \in \mathcal{J}$.

If responsible investors hold the same positions in all firms, there is a unique equilibrium, which is symmetric. In this case, an increase in responsible capital or a stronger internalization of shareholder preferences (i.e., a higher η) unambiguously increases CSR investments by all firms.

However, if the ownership structure is differentiated across firms, this subgame only admits asymmetric equilibria. If $\mathcal{R}$ investors hold different shares in different firms, then firms implement different CSR policies in equilibrium, with firms with higher $s_j^{\mathcal{R}}$ choosing higher σ_j^*. Hence, the internalization of shareholder preferences may generate differentiation in firms' CSR policies.

To better understand this result, suppose that, initially, no responsible investors exist in any firm. Then, an increase in $s_j^{\mathcal{R}}$ for firm j has two effects. First, there is a direct effect because management starts internalizing the negative externality. As the perceived cost of CSR decreases, the firm has stronger incentives to invest. Second, there is an indirect effect because j's rivals will respond. In anticipation of j's investments, they will increase (decrease) their CSR investments under strategic complementarity (substitutability).

If CSR policies are strategic substitutes, then a firm owned by $\mathcal{R}$ investors increases its CSR investment beyond the level in the symmetric equilibrium without SRI. At the same time, firm j's rivals invest less, increasing j's incentive to invest. Hence, the direct and indirect effects reinforce each other, so asymmetries in SRI may generate substantial heterogeneity in CSR investments.

By contrast, if CSR policies are strategic complements, concentrated SRI encourages all firms to invest more. In this case, anticipating that a firm owned by $\mathcal{R}$ investors has stronger incentives to invest, its rivals optimally expand their CSR investments. This increase, in turn, incentivizes a firm targeted by responsible investors to invest even more, and so on. As a result, strategic complementarities in CSR policies imply that the increased presence of responsible capital always spurs CSR investments of all firms in the market, no matter whether $\mathcal{R}$ investors increase their positions in all firms symmetrically or instead target only a subset of firms.

3.2 Equilibrium ownership

Having described how firms' equilibrium strategies depend on their ownership, we can now close the model by solving for the investors' portfolio choices and the distribution of SRI across firms. Since investors are atomistic, they take the vector of stock prices $\vec{p} \equiv (p_1, \ldots, p_N)$ as given when deciding on their asset holdings. Moreover, they rationally anticipate firms' CSR choices and expected profits and how these depend on the distribution of SRI in the economy.

Taking the first-order condition of Program (1) for investor i with respect to s_{ij} yields:

$$\Pi_j - p_j - \mathbb{1}_{i,\mathcal{R}}\lambda(1-\sigma_j) \leq \kappa \sum_{j=1}^{N} s_{ij} \tag{4}$$

where the inequality is strict if and only if the investor does not invest in firm j, i.e., if $s_{ij} = 0$.

Note that the marginal cost, i.e., the right-hand side of Equation (4), is constant across firms. To break ties, we assume that investors incur a small cost of acquiring shares in multiple firms, so each individual investor prefers to hold shares in one firm only.[7]

[7]Without this tie-breaking assumption, each of the equilibria we characterize coexists with observationally equivalent ones (i.e., featuring the same distribution of SRI) where investors hold diversified portfolios, i.e., divide their optimal demands across firms.

We can then write investor i's demand for firm j's shares as:

$$s_{ij} = \begin{cases} \max\left\{\frac{1}{\kappa}\left[\Pi_{j^*} - p_{j^*} - \mathbb{1}_{i,\mathcal{R}}\lambda(1-\sigma_{j^*})\right], 0\right\} & \text{for } j^* \in \text{argmax}_j\{\Pi_j - p_j - \mathbb{1}_{i,\mathcal{R}}\lambda(1-\sigma_j)\} \\ 0 & \text{for } j \neq j^*. \end{cases} \tag{5}$$

Hence, each investor i is willing to invest if the preference-adjusted return, $\Pi_j - p_j - \mathbb{1}_{i,\mathcal{R}}\lambda(1-\sigma_j)$, is positive for at least one firm. Otherwise, she does not invest in any firm.

Let α_j^θ denote the fraction of $\theta \in \{\mathcal{R}, \mathcal{N}\}$ investors buying positive shares of firm j, where the quantity purchased is given in Equation (5) above. $\mathcal{R}$ investors suffer disutility from holding brown firms. Hence, in any interior equilibrium (i.e., for all $\sigma_j < 1$), these investors have lower valuations than $\mathcal{N}$ investors. Thus, all else equal, responsible investors have lower demand than non-responsible ones. As a result, $\mathcal{R}$ investors may choose not to invest in any firm. $\mathcal{N}$ investors, however, must always buy some shares to ensure market clearing. That is, in equilibrium we may have $\alpha_j^\mathcal{R} = 0 \; \forall j \in \mathcal{J}$, but we always have $\alpha_j^\mathcal{N} > 0$ for at least some $j \in \mathcal{J}$.

The following market clearing condition determines the equilibrium share price p_j for firm j:

$$\chi\alpha_j^\mathcal{R}\frac{1}{\kappa}\left[\Pi_j - p_j - \lambda(1-\sigma_j)\right] + (1-\chi)\alpha_j^\mathcal{N}\frac{1}{\kappa}\left(\Pi_j - p_j\right) = 1. \tag{6}$$

Solving this expression for p_j leads to:

$$p_j = \Pi_j - \frac{\kappa}{\chi\alpha_j^\mathcal{R} + (1-\chi)\alpha_j^\mathcal{N}} - \frac{\lambda(1-\sigma_j)\chi\alpha_j^\mathcal{R}}{\chi\alpha_j^\mathcal{R} + (1-\chi)\alpha_j^\mathcal{N}}. \tag{7}$$

Equation (7) shows that p_j equals j's expected profits net of two distinct discounts. The first term captures a standard liquidity discount, which is necessary to incentivize investors to trade in the asset. This term vanishes as the trading cost κ goes to zero. The second discount arises due to the presence of responsible investors and is thus increasing in their mass $\chi\alpha_j^\mathcal{R}$ and the externality λ. In the limit of $\sigma_j \to 1$, firm j does not generate an externality, so this term disappears.

As we will show, depending on the parameters of the model, there are three possible types of ownership structures in equilibrium.

Definition 1 (Ownership structure) *We define the following types of ownership structures:*

1. ***No SRI*** *($s_j^{\mathcal{R}} = 0\ \forall j \in \mathcal{J}$): no firm is held by $\mathcal{R}$ investors.*

2. ***SRI without concentration*** *($s_j^{\mathcal{R}} = s^{\mathcal{R}} \in (0,1)\ \forall j \in \mathcal{J}$): each firm has the same positive share of $\mathcal{R}$ investors.*

3. ***SRI with concentration*** *($\exists j, j' \in \mathcal{J}$ such that $s_j^{\mathcal{R}} \neq s_{j'}^{\mathcal{R}}$): firms differ in the share of $\mathcal{R}$ investors.*

Before we formally characterize the different equilibrium outcomes, it is helpful to introduce the function $\widehat{\chi}$, which will be used as a threshold for the fraction χ of $\mathcal{R}$ investors:

$$\widehat{\chi}(\sigma, n) \equiv \frac{1}{2} + \frac{\sqrt{4\kappa\lambda n(1-\sigma) + (\lambda(1-\sigma) - N\kappa)^2} - N\kappa}{2\lambda(1-\sigma)} \in \left[\frac{n}{N}, 1\right], \tag{8}$$

for $\sigma \in [0,1)$ and $\widehat{\chi}(1,n) \equiv \frac{n}{N}$, where $n \in \{0, \ldots, N\}$ and $\widehat{\chi}(\cdot)$ is decreasing in σ and increasing in n.

3.3 Symmetric equilibria (no SRI and SRI without concentration)

As a first step, we consider equilibria in which all firms choose the same CSR policy $\sigma^* \in (0,1)$. In this scenario, all firms must have the same share price p^* in equilibrium because they generate identical expected profits Π^*. Indeed, if two firms had different prices, responsible and non-responsible investors would be strictly better off buying shares from the one with the lower price. Lemma 1 implies that equilibria, where firms have identical CSR policies, can exist only when all firms have the same fraction of $\mathcal{R}$ investors, that is, when $s_j^{\mathcal{R}}$ is constant for all j. Equation (5) implies that this is the case if and only if a fraction $\alpha_j^{\mathcal{R}} = 1/N$ of $\mathcal{R}$ investors buys an amount $\max\{\frac{1}{\kappa}\left[\Pi^* - p^* - \lambda(1-\sigma^*)\right], 0\}$ of shares in each firm.

It follows that there can be only two types of *symmetric equilibria* where all firms have identical ownership structures and CSR policies: equilibria in which all firms are solely held by $\mathcal{N}$ investors and equilibria in which each firm has the same positive share of $\mathcal{R}$ investors.

No SRI. We first consider equilibria where responsible investors do not invest ($s_j^{\mathcal{R}} = 0$). In such equilibria, the CSR policy for each firm is given by $\sigma^* = \sigma_0$, which coincides with the equilibrium in which firms do not internalize shareholders' social preferences ($\eta = 0$). Let Π_0 denote the firms' expected profits and p_0 the equilibrium stock price. The symmetric equilibrium with no SRI exists if and only if $\mathcal{N}$ investors are willing to buy shares in any firm, whereas $\mathcal{R}$ investors are not. Therefore, the equilibrium share price must lie between the valuation of $\mathcal{R}$ and $\mathcal{N}$ investors:

$$p_0 \in [\Pi_0 - \lambda(1-\sigma_0), \Pi_0). \tag{9}$$

For the market clearing conditions to hold for all firms, it must be that $\alpha_j^{\mathcal{N}} = 1/N$. It then follows that $p_0 = \Pi_0 - \frac{N\kappa}{1-\chi}$, so the existence condition (9) boils down to $\chi \le \widehat{\chi}(\sigma_0, 0)$. Intuitively, a relatively small proportion of $\mathcal{R}$ investors implies that collectively $\mathcal{N}$ investors exert high price pressure on each firm. As a result, the dollar return, $\Pi_0 - p_0$, is small and $\mathcal{R}$ investors' disutility $\lambda(1-\sigma_0)$ prevents them from buying.

SRI without concentration. Next, we analyze the symmetric equilibria in which $\mathcal{R}$ investors buy shares. For each firm, the equilibrium stock price must, thus, be lower than their valuation:

$$p^* < \Pi^* - \lambda(1-\sigma^*). \tag{10}$$

As discussed above, the share of $\mathcal{R}$ investors is identical across firms in these equilibria. Therefore, the market clearing condition for each firm j is given by Equation (6) with $\alpha_j^{R} = 1/N$. Imposing symmetry, we find that this condition holds for all firms if and only if $\alpha_j^{\mathcal{N}} = 1/N$ for all $j \in \mathcal{J}$. The equilibrium share price is thus $p^* = \Pi^* - \chi\lambda(1-\sigma^*) - N\kappa$, where σ^* is the unique solution to:

$$\sigma^* = \frac{1}{c}\left[\eta\lambda s^{\mathcal{R}*} + \frac{\partial \pi_j}{\partial \sigma_j}(\sigma^*, \vec{\sigma}^*)\right]; \tag{11}$$

$$s^{\mathcal{R}*} = \frac{\chi}{N}\frac{1}{\kappa}\left[N\kappa - \lambda(1-\chi)(1-\sigma^*)\right]. \tag{12}$$

This system of equations is obtained by imposing symmetry in Equations (3) and (5), which describe firms' CSR policies and $\mathcal{R}$ investors' holdings, respectively. Note that firms' equilibrium ownership and CSR policies are characterized by a fixed-point problem. The number of shares each individual $\mathcal{R}$ investor demands depends on her expectation of the firm's CSR policy (σ^*), which in turn depends on the overall fraction of shares held by responsible investors ($s^{\mathcal{R}*}$).

Two opposing forces characterize the strategic interaction among $\mathcal{R}$ investors. On the one hand, the two-way feedback between σ^* and $s^{\mathcal{R}*}$ generates a source of strategic complementarity: when $\mathcal{R}$ investors demand more shares, $s^{\mathcal{R}*}$ and σ^* increase. The increase in σ^* reduces the expected disutility and so incentivizes each individual $\mathcal{R}$ investor to hold more shares at a fixed price p^*. On the other hand, higher demand also gives rise to the traditional strategic substitutability across investors: when $s^{\mathcal{R}*}$ increases, the price increases, and trading profits decrease. Holding σ^* fixed, the increase in p^* reduces the incentives of each individual investor to invest.

We show that under Assumption 1(i), the second effect (strategic substitutability) always dominates in equilibrium so that the system of Equations (11) and (12) admits a unique solution. Moreover, given the market clearing price and the firms' equilibrium investments, we show that the symmetric equilibrium with SRI exists if and only if $\chi > \widehat{\chi}(\sigma_0, 0)$.

Characterization. Collecting the results above, we have the following characterization of symmetric equilibria in Proposition 1.

Proposition 1 (Symmetric Equilibrium) *A symmetric equilibrium always exists and is unique:*

1. *If $\chi \leq \widehat{\chi}(\sigma_0, 0)$, the symmetric equilibrium features **No SRI**: $s_j^{\mathcal{R}} = 0$ and $\sigma_j = \sigma_0 \; \forall j \in \mathcal{J}$;*
2. *Otherwise, it features **SRI without concentration**: $s_j^{\mathcal{R}} = s^{\mathcal{R}} > 0$ and $\sigma_j = \sigma^* > \sigma_0 \; \forall j \in \mathcal{J}$.*

The threshold $\widehat{\chi}(\sigma_0, 0)$ for the mass of $\mathcal{R}$ investors is defined in Equation (8).

Proposition 1 shows that the game always admits a symmetric equilibrium and that this equilibrium is unique.[8] As discussed before, responsible investors are crowded out if their mass is too small. In this case, firms are still willing to implement CSR policies but only due to their effect on expected profits π_j. If the mass of responsible investors is sufficiently large, responsible and non-responsible investors acquire positions in all firms, which are then incentivized to implement greener CSR policies because managers internalize shareholders' social preferences.

Next, we describe how firms' CSR policies and SRI change with model parameters.

Lemma 2 *In the symmetric equilibrium of the game:*

[8]Since we have a continuum of agents, the symmetric equilibrium is unique up to permutations of investors' portfolios. Such permutations do not affect the aggregate equilibrium outcomes.

1. *Firms' CSR policies (σ^*) and the fraction of shares held by individual $\mathcal{R}$ investors ($s^{\mathcal{R}*}$) are both continuous and increasing in the fraction χ of $\mathcal{R}$ investors in the economy and the trading cost κ, strictly so for $\chi > \widehat{\chi}(\sigma_0, 0)$;*

2. *The threshold $\widehat{\chi}(\sigma_0, 0)$ is increasing in λ and decreasing in κ.*

In the symmetric equilibrium, CSR policies and SRI are increasing and continuous in χ and κ. As argued above, a higher fraction χ of responsible investors increases the willingness of an individual responsible investor to trade (through strategic complementarity) and hence encourages firms to increase their CSR investments. Similarly, an increase in the trading cost κ makes financial markets less efficient and, thus, leads to more SRI in equilibrium, which provides greater incentives for firms to implement CSR policies. Finally, when the externality λ becomes smaller, the wedge between $\mathcal{R}$ and $\mathcal{N}$ investors' valuations falls, and the equilibrium with SRI exists for a larger parameter space.

3.4 Asymmetric equilibria (SRI with concentration)

In addition to the symmetric equilibria characterized above, the game may also admit asymmetric equilibria. These equilibria feature *concentration* so that ex-ante identical firms differ in their share of responsible investors and, hence, their CSR policies ex-post.

To characterize all asymmetric equilibria of the game, consider any subset of firms in which $\mathcal{R}$ investors purchase a positive number of shares. As firms are ex-ante identical, it is without loss of generality to denote these firms by $j = 1, ..., \overline{n}$, with $\overline{n} \leq N$. To ensure market clearing, $\mathcal{N}$ investors must hold the shares of the other firms instead. That is, for all $\overline{n} < N$, firms $j = \overline{n}+1, ..., N$ are held exclusively by $\mathcal{N}$ investors. Finally, there may be firms where both $\mathcal{R}$ and $\mathcal{N}$ investors hold shares. Without loss of generality, let these firms be $j = \underline{n}+1, ..., \overline{n}$, with $\underline{n} \leq \overline{n}$, so that the particular cases $\underline{n} = 0$ and $\underline{n} = \overline{n}$ correspond to equilibria where $\mathcal{N}$ investors hold shares in all firms and only in firms without $\mathcal{R}$ investors, respectively. To sum up, equilibria are characterized by a pair $(\underline{n}, \overline{n})$, with $0 \leq \underline{n} \leq \overline{n} \leq N$,[9] such that (a) only $\mathcal{R}$ investors hold shares in firms $j \leq \underline{n}$; (b) both $\mathcal{R}$ and $\mathcal{N}$ investors hold shares in firms $j \in (\underline{n}, \overline{n}]$; (c) only $\mathcal{N}$ investors hold shares in firms $j \in (\overline{n}, N]$. The

[9]We rule out $(\underline{n}, \overline{n}) = \{(0,0), (0,N)\}$ as these correspond to the symmetric equilibria in Proposition 1. Note also that there cannot be equilibria with $\underline{n} = \overline{n} = N$ as $\mathcal{N}$ investors are never crowded out from the market. All other inequalities can hold with equality.

following proposition provides existence conditions for asymmetric equilibria for a given $(\underline{n}, \overline{n})$.

Proposition 2 (Asymmetric Equilibria) *All asymmetric equilibria feature* ***SRI with concentration****. In these equilibria, $\mathcal{R}$ investors hold shares in firms $j \leq \overline{n}$ and $\mathcal{N}$ investors hold shares in firms $j > \underline{n}$, where $\overline{n} \geq \underline{n}$, and firm j's CSR policy is:*

$$\sigma_j = \begin{cases} \overline{\sigma} & \text{for } j \leq \underline{n} \\ \widehat{\sigma} & \text{for } \underline{n} < j \leq \overline{n} \\ \underline{\sigma} & \text{for } j > \overline{n}, \end{cases} \tag{13}$$

where $\overline{\sigma} > \widehat{\sigma} > \underline{\sigma}$. For any pair $(\underline{n}, \overline{n})$, these equilibria exist if and only if:

1. *For $\overline{n} = \underline{n} = n$, $\widehat{\chi}(\overline{\sigma}, n) < \chi < \widehat{\chi}(\underline{\sigma}, n)$.*

2. *For $\overline{n} > \underline{n}$, $\widehat{\chi}(\widehat{\sigma}, \underline{n}) < \chi < \widehat{\chi}(\widehat{\sigma}, \overline{n})$.*

The threshold values for χ are defined in Equation (8), and the expressions for $\overline{\sigma}$, $\widehat{\sigma}$, and $\underline{\sigma}$ are described in Appendix B.4.

The equilibria described in Proposition 2 highlight a second source of strategic complementarity among $\mathcal{R}$ investors: complementarity in the choice of firms that are part of their portfolios.

To build intuition, consider two firms, j and $-j$, and start from an equilibrium in which $\mathcal{R}$ investors hold $\alpha\%$ of the shares in both firms. Now consider switching some of the $\mathcal{R}$ investors' shares in firm $-j$ with those of $\mathcal{N}$ investors in firm j, so that now $\mathcal{R}$ investors hold more than $\alpha\%$ of j's shares and less than $\alpha\%$ in $-j$. Due to the complementarity in their effects on firms' CSR policies, the swap makes $\mathcal{R}$ investors better off: the increase in $s_j^{\mathcal{R}}$ spurs more green investments by firm j, reducing the investors' expected disutility from holding its shares. Under substitutability in CSR investments, firm $-j$ now has lower incentives to invest in CSR: its green investments decrease after the swap so that $-j$ becomes less attractive for $\mathcal{R}$ investors. The interaction of these two effects generates a self-reinforcing mechanism, such that more and more $\mathcal{R}$ investors want to move from firm $-j$ to j if they expect others to do so.

It is worth noticing that the argument described above does not involve any increase in the total demand for shares for either firm, only a reallocation of shares across investor types. So, the strategic substitutability in investors' demands that we discussed in the context of symmetric equilibria does not play a role here. Put differently, while the complementarity in the number of shares $\mathcal{R}$ investors hold in a given firm is offset by the strategic substitutability in their demands, the complementarity in the choice of which firms to hold is not. Instead, this complementarity is *reinforced* by the substitutability in firms' CSR policies, which further strengthens the incentives of $\mathcal{R}$ investors to concentrate in a subset of firms and leads to dispersion in CSR policies across firms.

If CSR policies are strategic complements, when $\mathcal{R}$ investors move from firm $-j$ to j, σ_{-j} decreases less than under substitutability, since the increase in σ_j has a positive indirect effect on σ_{-j}. Since the spread $\sigma_j - \sigma_{-j}$ is smaller, moving to firm j is then relatively less attractive for $\mathcal{R}$ investors compared to the case of substitutability. Section 4.1 shows that, due to this effect, asymmetric equilibria are relatively less prevalent under strategic complementarity in firms' CSR policies.

4 Implications

In this section, we derive the central model implications. First, we describe how green capital tends to concentrate in a subset of firms in equilibrium. We then derive the consequences of this concentration for firms' CSR policies and welfare. Finally, we describe existing empirical evidence and novel testable implications surrounding our results.

4.1 SRI concentration

Section 3 characterizes two types of equilibria: symmetric and asymmetric. The following proposition compares the existence conditions for these two types of equilibria.

Proposition 3 (Concentration) *Suppose $\frac{N-1}{N} > \chi > \frac{c}{N\eta\lambda}$. Then, either the only equilibria with SRI are asymmetric, or the symmetric equilibrium with SRI coexists with asymmetric equilibria where $\mathcal{R}$ investors exclude at least one firm.*

For intermediate values of χ, $\mathcal{R}$ investors are more *likely* to participate in the financial market when the equilibrium is asymmetric. By concentrating on a subset of firms, these investors manage to have a larger impact on their CSR policies. The larger impact reduces the valuation gap between the two types of investors relative to the symmetric equilibrium, making it harder for $\mathcal{N}$ investors to crowd out $\mathcal{R}$ investors. Therefore, $\mathcal{R}$ investors hold shares of firms for a larger set of parameters in asymmetric equilibria featuring a concentration of green capital. Moreover, the number of possible asymmetric equilibria increases steeply with N, while the symmetric equilibrium is always unique. In this sense, the characterization of the equilibria suggests that concentration may be highest in relatively large or more competitive industries.

Although these results hold regardless of the kind of strategic interaction across firms, investors' tendency to concentrate is more pronounced under strategic substitutability. To see this, we consider the equilibria in which $\mathcal{R}$ investors hold n firms with CSR policies $\overline{\sigma}$, while $\mathcal{N}$ investors hold the other $N - n$ firms with CSR policies $\underline{\sigma}$.

Proposition 4 (Strategic interaction and concentration) *Consider the asymmetric equilibria in which $\mathcal{R}$ investors hold firms $j \leq n$ and $\mathcal{N}$ investors hold firms $j > n$, with $n \in \mathcal{J}\backslash\{N\}$. Fix the fraction of $\mathcal{R}$ investors χ and suppose $\frac{\partial^2 \Pi_j}{\partial \sigma_j^2}$ is a function of σ_j only. The following results hold in equilibrium:*

1. *The dispersion in CSR policies $\overline{\sigma} - \underline{\sigma}$ is higher under strategic substitutability;*
2. *These equilibria exist if $\widehat{\kappa}(\overline{\sigma}, n) < \kappa < \widehat{\kappa}(\underline{\sigma}, n)$; this range is larger under strategic substitutability.*

The first result in Proposition 4 relates the difference in CSR investments between targeted and excluded firms to the strategic interaction between firms.[10] Perhaps not surprisingly, CSR investments are more dispersed under substitutability. In this case, the investments of the targeted firms discourage other firms from investing in CSR, increasing the dispersion in CSR levels.

The second result speaks to the following question: for a given amount of responsible capital in the economy, which industries are most likely to have concentrated SRI? To tackle this question, we fix χ and describe the equilibrium existence conditions with respect to the trading cost κ. We then show that equilibria with concentrated SRI are most likely to arise (i.e., exist for a larger interval of values of κ) in industries where firms' CSR investments are strategic substitutes. We next show this is exactly the case when SRI concentration is the least desirable from a welfare perspective.

[10] The condition on $\partial^2 \pi_j / \partial \sigma_j^2$ in Proposition 4 allows us to compare the equilibrium CSR policies based only on the type of strategic interaction across firms (i.e., the sign of $\partial^2 \pi_j / \partial \sigma_j \partial \sigma_{-j}$), without having to specify a functional form for π_j. Notice that this condition is sufficient but not necessary for the results to hold, and is satisfied in the examples we describe in Section 4.3.

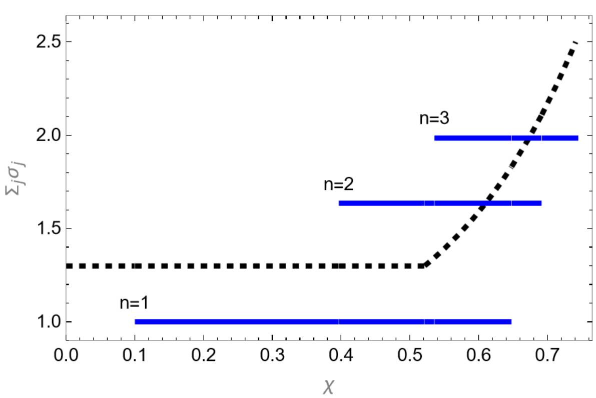
4.2 Aggregate CSR investments

We begin our analysis of the effects of SRI by examining its impact on aggregate CSR investments ($\sum_j \sigma_j$). Since the aggregate externality generated by firms is $\lambda(N - \sum_j \sigma_j)$, an increase in aggregate investments reduces firm externalities. The following proposition compares $\sum_j \sigma_j$ across the different types of possible equilibria.

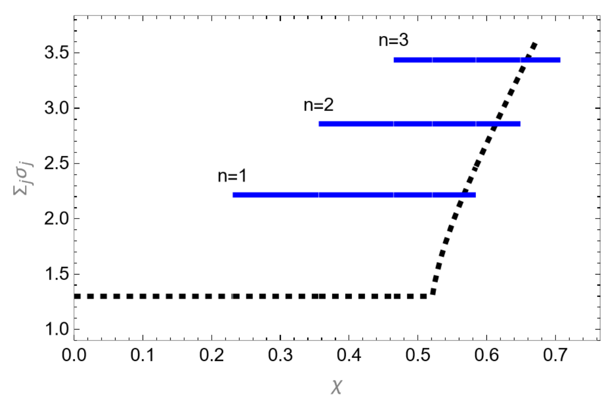
Proposition 5 (Aggregate CSR investments) *Compared to the equilibrium without SRI:*

1. *SRI without concentration always increases aggregate CSR investments $\sum_j \sigma_j$;*
2. *SRI with concentration may reduce $\sum_j \sigma_j$ only if firms' CSR policies are strategic substitutes.*

Responsible investors boost the greenness of the firms in their portfolios. However, in the case of strategic substitutability, they also crowd out the CSR investments of excluded firms. In equilibria where green capital is particularly concentrated, the crowding-out effect is strong, and so the overall effect of $\mathcal{R}$ investors on aggregate CSR investments is negative. In such cases, the economy's aggregate greenness is larger in the equilibrium in which $\mathcal{R}$ investors do not hold any firm.



(a) Strategic substitutability: $\frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_{-j}} < 0$.



(b) Strategic complementarity: $\frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_{-j}} > 0$.

Figure 1: This figure plots aggregate CSR investments $\sum_j \sigma_j$ as a function of the mass of responsible investors χ. The dotted black line corresponds to the symmetric equilibrium and the solid blue lines to the asymmetric equilibria described in Part 1 of Proposition 2, where n denotes the number of firms that are exclusively held by responsible investors. Parameters: $N = 10, \lambda = 12, \eta = \frac{1}{10}, c = \frac{33}{15}, \kappa = \frac{1}{2}$. Panel (a) features strategic substitutability with $\pi_j = \sigma_j \prod_{k \neq j}(1 - \sigma_k)$ (micro-founded in Example 1); Panel (b) features strategic complementarity with $\pi_j = \frac{2}{5}\sigma_j(1 - \prod_{k \neq j}(1 - \sigma_k))$ (micro-founded in Example 3).

Figure 1 illustrates how the equilibrium $\sum_j \sigma_j$ changes with the fraction χ of $\mathcal{R}$ investors for two numerical examples. Panel (a) describes a setting with strategic substitutability and Panel (b) one with complementarity. In both panels, when χ is small, the equilibrium is unique and features no SRI: $\mathcal{R}$ investors are too few to significantly impact CSR policies, so they prefer not to trade any share. For intermediate values of χ, the equilibrium without SRI coexists with asymmetric equilibria where $\mathcal{R}$ investors target only a subset of firms, where the number of targeted firms (n) increases with χ. When χ is sufficiently close to 1, the symmetric equilibrium also features SRI.

Now, focus on Panel (a). Here, the investments of the targeted firms *crowd out* those of the excluded firms. The crowding-out effect dominates in the equilibrium with $n = 1$, so $\sum_j \sigma_j$ is lower than in the equilibrium without SRI. On the contrary, SRI with concentration always increases $\sum_j \sigma_j$ in Panel (b): the CSR investments of the targeted firms *crowd in* those of the excluded firms under complementarity. Across the equilibria in both panels, the one where SRI reduces concentration (i.e., the equilibrium with $n = 1$ in Panel (a)) features the largest difference $\overline{\sigma} - \underline{\sigma}$ and, thus, it exists for the longest interval of values of χ. By the same intuition as for Proposition 4, equilibria with concentrated SRI are more likely to exist exactly when concentration is the least desirable.

4.3 Product markets and welfare

In this section, we explore the product market and welfare implications of our analysis. The model described in Section 2 captures firms' strategic interaction in a reduced-form way and is thus not suited to explore welfare implications. Here, we endogenize firms' profit functions by augmenting our main model with a unit mass of consumers, indexed by $h \in [0, 1]$. Introducing consumers also allows us to explore the equilibrium consequences of responsible consumption.

Product market setup. To link CSR policies with consumption, we assume that firms' CSR investments determine their production technologies, which in turn affect consumers' demand. We use the notation $a_j = 1$ ($a_j = 0$) to signify that firm j uses a green (brown) technology. The brown technology generates a negative externality λ, while the green one does not generate externalities. The random vector $\vec{a} \equiv (a_1, \ldots, a_N)$ describes the technology used by each firm. We denote the probability of $a_j = 1$ by σ_j and assume that a_j is independent of a_{-j} for any $j \in \mathcal{J}$. For a given realization of $\vec{a}$, firms compete à la Bertrand: firms simultaneously set their prices; having observed the price vector $\vec{\rho} \equiv (\rho_1, \ldots, \rho_N)'$ and technologies $\vec{a}$, consumers choose how much to buy from each firm.

Similar to investors, a fraction $\chi_c \in (0,1)$ of consumers are responsible, as they incur a disutility λ from consuming brown products, i.e., products produced using the brown technology. For a given $\vec{\rho}$ and $\vec{a}$, consumer h's demand vector for the different products is denoted by $\vec{x}_h$ and solves:

$$\max_{\vec{x}_h \geq \vec{0}} u\left(\vec{x}_h\right) - \vec{x}_h'\left[\vec{\rho} + \mathbb{1}_{h,\mathcal{R}}\lambda(\iota - \vec{a})\right], \tag{14}$$

where $\mathbb{1}_{h,\mathcal{R}} = 1$ for responsible consumers ($h \in \mathcal{R}$), 0 for non-responsible consumers ($h \in \mathcal{N}$), and $\iota \equiv (1, \ldots, 1)'$. We denote the utility of consumption by $u(\cdot)$ with $u : (\mathbb{R}_+)^N \to \mathbb{R}_+$.[11] Different specifications of $u(\cdot)$ can be used to micro-found both strategic substitutability and strategic complementarity in firms' CSR investments (see Example 1 and Example 2 below).

Firms' marginal cost of production is $\gamma \geq 0$ regardless of whether they produce brown or green products. For a given $\vec{a}$, fixing the prices charged by its competitors, firm j chooses the product price ρ_j to maximize its profit gross of the CSR investment costs:

$$\max_{\rho_j \geq 0}(\rho_j - \gamma)\int_0^1 x_{hj}(\vec{a}, \vec{\rho})\, dh. \tag{15}$$

Assuming that the price competition game admits a pure-strategy equilibrium, we can write firms' expected profits (gross of the CSR investments costs) as:

$$\pi_j \equiv \mathbb{E}\left[(\rho_j - \gamma)\int_0^1 x_{hj}(\vec{a}, \vec{\rho})\, dh\right], \tag{16}$$

where the expectation in Equation (16) is taken with respect to the random vector $\vec{a}$, and product prices and consumers' demands are evaluated at their equilibrium values for any realization of $\vec{a}$.

An equilibrium of the full game is a collection $\{\{s_{ij}\}, \vec{\sigma}, \vec{\rho}(\vec{a}), \{x_{hj}(\vec{a}, \vec{\rho})\}\}$, where $i, h \in [0,1]$, and $j \in \mathcal{J}$. The equilibrium collection jointly solves Programs (14) and (15) for any realization of $\vec{a}$, and Programs (1) and (2), which characterize investors' portfolio and firms' CSR policies, respectively.

It is worth emphasizing that, for a given realization of $\vec{a}$, the equilibrium strategies in the product

[11]The specification of the consumers' problem in Program (14) implicitly assumes that their budget constraints are not binding at the equilibrium consumption, so their choice is equivalent to an unconstrained problem. This assumption simplifies the exposition but does not affect our results.

market do not depend on $\{s_{ij}\}$ or $\vec{\sigma}$. Therefore, the analysis of the product market does not change the equilibrium characterization in Section 3. However, it yields an equilibrium specification for π_j that depends on the primitives of the model (i.e., consumers' preferences and firms' production costs), allowing us to explore the effects of SRI on consumer surplus and welfare.

To illustrate the characterization of the product market equilibrium, we describe two simple examples below, which we will use to perform numerical simulations of the model.

Example 1 (Strategic substitutes) *Suppose* $u(\vec{x}) \equiv \tilde{u}(\iota'\vec{x})$, *where* $\tilde{u}(x) : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ *is defined as* $\tilde{u}(x) \equiv -\frac{\alpha}{2}x^2 + \beta x$ *with* $\alpha > 0$ *and* $\beta \in (\gamma, \gamma + \lambda)$. *For* $N \geq 3$, *an equilibrium of the product market subgame always exists and is unique:*

1. *In equilibrium, each firm's expected gross profit is* $\pi_j = \sigma_j \prod_{k \neq j}(1 - \sigma_k)\chi_c \frac{(\beta-\gamma)^2}{4\alpha}$, *since:*
   (a) *If* $a_j = 0$ *for all firms or* $a_j = 1$ *or for at least two firms, all firms set* $\rho_j = \gamma$ *and make zero profits;*
   (b) *If* $a_j = 1$ *for firm* j *only, firm* j *sets* $\rho_j = \frac{\beta+\gamma}{2}$ *and makes profits equal to* $\chi_c \frac{(\beta-\gamma)^2}{4\alpha}$. *All other firms set* $\rho_{-j} = \gamma$ *and make zero profits.*
2. $\mathcal{R}$ *consumers only buy from green firms; if there is at least one green firm, they collectively buy* $\chi_c \frac{(\beta-\rho)}{\alpha}$, *with* $\rho = \min_{j \in \mathcal{J}:\, a_j = 1} \rho_j$. $\mathcal{N}$ *consumers always buy from the firm that charges the lowest price; collectively, they buy* $(1 - \chi_c)\frac{(\beta-\gamma)}{\alpha}$.

Example 1 describes the case where, aside from the disutility of consuming brown products for $\mathcal{R}$ types, firms' products are perfect substitutes in consumers' preferences. The parameter β is such that $\mathcal{R}$ consumers boycott brown firms because they are better off not consuming a brown product. Conditional on the technology, products are then homogeneous. Therefore, competition forces firms to set prices equal to marginal cost, except when only one firm is green. In this case, the green firm acts as a monopolist and serves only $\mathcal{R}$ consumers. The substitutability in consumers' preferences carries over to CSR policies ($\frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_{-j}} \propto -\prod_{k \neq j}(1 - \sigma_k) < 0$): j is more likely to be the only green firm (and capture the monopoly profit) when the other firms invest less in CSR.[12]

[12]Here we assume that a firm's marginal cost (γ) does not depend on its production technology. If green products are more expensive to produce, j also makes profits when it is the only firm to offer a brown product, as it can price slightly below the marginal cost of green firms and serve $\mathcal{N}$ consumers at a profit. This alternative assumption would then strengthen the strategic substitutability: when σ_{-j} increases, j is more likely to become a brown monopolist, further reducing its incentives to invest in CSR.

Example 2 (Strategic complements) *Suppose* $u(\vec{x}) \equiv \tilde{u}(\min_{j\in\mathcal{J}} x_j)$*, where* $\tilde{u}(x) = -\frac{\alpha}{2}x^2 + \beta x$ *with* $\alpha > 0$ *and* $\beta \in (N\gamma, N\gamma + \lambda)$*. An equilibrium of the product market subgame always exists and is unique:*

1. *Each firm sets* $\rho = \frac{\beta+\gamma}{N+1}$ *and makes expected gross profit* $\pi_j = \frac{(\beta-N\gamma)^2}{\alpha(N+1)^2}(1 - \chi_c + \chi_c \prod_{j\in\mathcal{J}} \sigma_j)$*.*

2. $\mathcal{R}$ *consumers only buy if all firms are green, in which case they collectively buy* $\chi_c \frac{\beta-N\gamma}{\alpha(N+1)}$ *from each firm.* $\mathcal{N}$ *consumers always collectively buy* $(1-\chi_c)\frac{\beta-N\gamma}{\alpha(N+1)}$ *from each firm.*

Example 2 describes the case where products are perfect complements (Leontief utility), which means that consumers only want to consume all products together. $\mathcal{R}$ consumers boycott brown firms, so they only consume if *all* firms are green. This means that firms can sell to a larger population of consumers (and make more profits) only if all firms are green at the same time, which translates into strategic complementarity in their CSR investments ($\frac{\partial^2 \pi_j}{\partial\sigma_j \partial\sigma_{-j}} \propto \prod_{k\neq j} \sigma_k > 0$).[13]

In both examples, the following assumption holds:

Assumption 2 *Consumer surplus* ($\int_0^1 \left[u(\vec{x}_h) - \vec{x}_h'\vec{\rho}\right] dh$) *and total market surplus* ($\int_0^1 \left[u(\vec{x}_h) - \vec{x}_h'\vec{\gamma}\right] dh$) *weakly increase with the number of green firms* ($n_g \equiv \sum_j a_j$) *in the economy.*

In what follows, we maintain this assumption for two reasons. First, we want to capture that firms' greenness aligns with consumers' interests. Second, this assumption also ensures that greenness is positively related to efficiency in the product market. Despite this assumption, we will show below that concentrated SRI may have negative welfare consequences in equilibrium.

Product market effects of SRI. To explore the product market implications of SRI, we need to evaluate its impact on consumer surplus and market power. The expected consumer surplus is

$$CS = \int_0^1 \mathbb{E}\left[u(\vec{x}_h) - \vec{x}_h'[\vec{\rho} + \mathbb{1}_{h,\mathcal{R}}\lambda(\iota - \vec{a})]\right] dh, \tag{17}$$

where the expectation in Equation (17) is taken with respect to the random vector $\vec{a}$, and firms' CSR policies, product prices, and consumers' demands are evaluated at their equilibrium values.

[13]The result that products' substitutability (resp. complementarity) translates into strategic substitutability (resp. complementarity) of firms' CSR policies carries over to more general environments, beyond the assumption of binary production technologies: an example with a utility function à la Singh and Vives (1984) is provided in Online Appendix E.3.

Firms' CSR investments affect expected consumer surplus only through the probability distribution of firms' technologies $\vec{a}$, which pins down the number of green firms (n_g) in the market. The following proposition compares CS across the possible types of equilibria:

Proposition 6 (Consumer surplus) *Compared with the equilibrium without SRI:*

1. *SRI without concentration always increases expected consumer surplus;*

2. *SRI with concentration may reduce CS only if firms' CSR policies are strategic substitutes.*

In equilibria featuring SRI with concentration, responsible capital creates dispersion in CSR policies, especially when these are strategic substitutes. Since excluded firms invest less in CSR compared to symmetric equilibria, high realizations of n_g can become less likely. If consumers benefit from having a larger number of green firms in the economy, the expected consumer surplus is lower than in equilibria without SRI or those featuring SRI without concentration.

To illustrate the results in Proposition 6, consider Example 1. Each firm makes positive profits only if it is the only green firm in the market so that it can charge a premium to $\mathcal{R}$ consumers. Consumer surplus is smallest without a green firm in the market ($n_g = 0$) since $\mathcal{R}$ types do not buy any product, and is largest when there are at least two green firms ($n_g \geq 2$), so all firms price at their marginal cost and all consumers buy. By reducing CSR investments of excluded firms, concentrated SRI can make $n_g \leq 1$ relatively more likely, reducing the expected consumer surplus.

It is worth emphasizing that even when concentrated SRI increases aggregate CSR investments, it may still hurt consumers. On the one hand, the increased CSR investment of the target firms (which are responsible for the larger aggregate investments) increases the likelihood that at least one firm is green, which benefits $\mathcal{R}$ consumers. On the other hand, the crowding out of the CSR investments of the excluded firms reduces the likelihood that a large number of firms are green so that those that become green face less competition. Reduced competition leads to higher markups for green products, which hurts $\mathcal{R}$ consumers.

Proposition 7 focuses on the competitive effects described above and shows that crowding out is crucial for the concentration of green capital to generate product market concentration.

Proposition 7 (Market power) *Let $\zeta \equiv \Pr[n_g = 1 | n_g \geq 1]$ denote the conditional probability that there is only one green firm in the economy (i.e., a monopoly provider of green products), given that at least one firm is green. Compared with the equilibrium without SRI:*

1. *SRI without concentration always reduces ζ;*
2. *SRI with concentration may increase ζ only if firms' CSR policies are strategic substitutes.*

Welfare. We conclude our welfare analysis by exploring the effects of SRI on the expected total surplus in the model. We can write the expected total surplus S as follows:

$$S = \int_0^1 \mathbb{E}\left[u(\vec{x}_h) - \iota' \vec{x}_h \gamma\right] dh - \frac{\kappa}{2} \int_0^1 \left(\iota' \vec{s}_i\right)^2 di - \sum_{j=1}^{N} \left[\lambda(1-\sigma_j) + \frac{c}{2}\sigma_j^2\right], \tag{18}$$

where investors' portfolio choices, firms' CSR investments, and consumers' consumption levels are at their equilibrium values.

Since share and product prices are transfers, they do not affect the total surplus. Following Dewatripont and Tirole (2022), our welfare measure includes the negative externality generated by firms but excludes the expected disutilities incurred by responsible investors and consumers to avoid double counting.[14] It follows that S comprises three components: (i) expected product market surplus, i.e., the difference between the sum of consumers' utilities and firms' production costs; (ii) investors' total trading cost; (iii) the sum of the expected negative externality and the green investment cost.

In equilibrium, firms' CSR investments may exceed or fall short of the socially optimal levels. On the one hand, firms do not fully internalize the negative externality, which pushes toward underinvestment. On the other hand, their private incentives are partly driven by the prospect of charging high markups to responsible consumers. This second feature might lead to overinvestment since these rents are not part of the social planner's objective.

SRI has a direct and an indirect effect on total surplus S. The direct effect represents the contribution of $\mathcal{R}$ investors to the total trading costs. In the Appendix, we formally show that this

[14]Economists hold differing views on whether warm-glow preferences of the type we use to model the responsible types should be counted as part of welfare. See Bergstrom (2006) for a discussion of the pros and cons of each approach.

direct effect is always negative.[15] The indirect effect operates through the impact of SRI on firms' CSR policies $\vec{\sigma}$, which in turn affect the other two components of S. When firms underinvest in CSR without SRI, then $\mathcal{R}$ investors can potentially improve welfare by moving firms' CSR policies closer to the social optimum. To highlight the most interesting results of our model, we focus on this case in the following proposition.

Proposition 8 (Welfare) *Suppose $\lambda > c$ so that firms' CSR investments in equilibrium fall short of the socially optimal level and that the equilibrium without SRI coexists with an asymmetric equilibrium. Then, if (and only if) CSR policies are strategic substitutes:*

1. *Total expected surplus (S) and aggregate CSR investments ($\sum_j \sigma_j$) may be higher in the equilibrium without SRI.*

2. *$\mathcal{R}$ investors may be better off in the equilibrium where S and $\sum_j \sigma_j$ are lower.*

Responsible investors boost the greenness of the firms in their portfolios, but they may also crowd out the CSR investments of excluded firms. The overall effect on aggregate CSR investments is negative in equilibria where green capital is particularly concentrated, and the crowding-out effect is strong. The economy's aggregate greenness is then larger in the equilibrium in which $\mathcal{R}$ investors do not hold any firm. The concentration of $\mathcal{R}$ investors creates dispersion in CSR policies, even when it does not reduce aggregate CSR investments. The dispersion generates market power and may lead to higher prices for green products, reducing the expected surplus in this market.

The two negative externalities described above hamper the positive effect of SRI and may lead to higher welfare in settings where $\mathcal{R}$ investors do not hold any firm. $\mathcal{R}$ investors do not consider the negative externalities of their investments to the broader economy but solely the expected externality of the firms in their portfolio. They are better off when participating in the financial market, even though aggregate CSR investments and welfare are lower in this equilibrium.

[15]This is because the individual trading costs are convex in the number of shares. Hence, as the overall amount of shares traded in equilibrium is fixed, the trading cost is unambiguously lower when more investors trade in the market.

4.4 Empirical implications

A central premise of our model is that a firm's CSR policy reflects both pressures from investors and its interaction with other firms. Regarding investors, a growing empirical literature finds that shareholders' social preferences shape their governance and firms' ESG policies (Flammer, 2015; Kim, Wan, Wang, and Yang, 2019; Bolton, Li, Ravina, and Rosenthal, 2020; Naaraayanan, Sachdeva, and Sharma, 2021). Consumption and employment decisions reflect CSR issues, and this has significant effects on firms' profits and stock prices (Mazar and Zhong, 2010; Albuquerque, Koskinen, Yang, and Zhang, 2020; Hacamo, 2022; Derrien, Krueger, Landier, and Yao, 2021). There is also evidence that pressure on raw materials influences investments in clean energy (Speirs, Contestabile, Houari, and Gross, 2014). Next, we discuss the evidence related to how the pricing of CSR in product markets and pressure on inputs can create interdependence in CSR policies.

Our results vary depending on whether firms' CSR policies are strategic substitutes or complements. The underlying rationale for substitutability is that increased competition for either socially motivated consumers or resources reduces an individual firm's return from CSR investments. If we interpret CSR as investments in "clean" innovation, the mechanism described above is consistent with the findings in Aghion, Bloom, Blundell, Griffith, and Howitt (2005), who show that competition reduces innovation in industries where it affects mostly post-innovation rents. Strategic complementarity in CSR policies may arise due to peer pressure or technological spillovers. Cao, Liang, and Zhan (2019) find evidence of peer-pressure effects in CSR proposals. Aghion, Dechezleprêtre, Hemous, Martin, and Van Reenen (2016) document evidence of positive technological spillovers from investments in low-emissions technologies in the auto industry.

The complementarity in responsible investors' portfolio choices, and the consequent concentration of green capital, are central to our results. Figure 2 shows that green funds are more concentrated than non-green funds in 20 out of 21 industry groups. Starks, Venkat, and Zhu (2017) find evidence that investors with longer horizons have preferences for, and tend to group in, firms with high ESG scores. Similarly, Dimson, Karakaş, and Li (2021) document the increasing prevalence of *coordinated* engagements: Groups of institutional investors who cooperate in promoting

CSR issues.[16] These findings are consistent with the complementarity in investors' choices.

Finally, our model links the concentration of green capital to dispersion in firms' CSR policies and product market position. We predict that increased concentration leads to a larger dispersion of CSR policies and, in cases where firms' CSR investments are strategic substitutes (e.g., when firms are close competitors or have price impact in the input market), to a reallocation of market shares in favor of the firms targeted by green investors. Figure 3 and Table 1 suggest a positive correlation between the concentration of green capital and firms' ESG scores. Importantly, causality runs in both directions in our model because increased dispersion in CSR policies also incentivizes $\mathcal{R}$ investors to tilt their portfolio towards firms with greener policies. Thus, a test of the causal effects of concentration would require an exogenous shock to green capital.

5 Extensions and robustness

This section briefly discusses the robustness of our results to alternative assumptions. We provide a complete analysis of each variation of the baseline model in the Online Appendix.

5.1 Financial market

Alternative trading cost. Our baseline model assumes that the trading cost investors incur for acquiring $\sum_j s_{ij}$ shares is $K_j = K\left(\sum_j s_{ij}\right)$. In Online Appendix D.1, we analyze a variation of the model in which the trading cost, rather than depending only on the sum of shares held by the investor across firms, depends on the individual holdings of each firm, i.e., $K_j = \sum_j K\left(s_{ij}\right)$. Assuming again a quadratic specification, the marginal cost of acquiring a small position in a given firm is zero, independently of the investor's holding in other firms. This implies that $\mathcal{N}$ investors always invest in *all* firms in equilibrium. $\mathcal{R}$ investors might instead still choose to exclude a subset of firms if the externalities they generate are too high. By concentrating in a subset of firms, $\mathcal{R}$ investors can have more impact on their CSR policies, reducing their externalities and the valuation gap with $\mathcal{N}$ investors. This result mirrors the central insight from our baseline model that responsible capital tends to concentrate in equilibrium. The equilibrium analysis, however, is

[16] Examples of such networks of investors include the Climate Action 100+ campaign, which is backed by 518 global investors with 47 trillion in assets, and the Coalition for Inclusive Capitalism, with 31 organizations representing over 30 trillion in assets

more cumbersome since we no longer obtain the tractable equilibria with full separation (where $\mathcal{R}$ and $\mathcal{N}$ investors hold different subsets of firms; see Part 1 of Proposition 2).

Reverse timing. In our model, firms choose their CSR policies after the investors trade. Online Appendix D.2 explores a variation of the model where this timing is reversed: firms choose their CSR policies to attract investors and maximize their stock prices. This version of the model describes contexts where firms can credibly commit to certain CSR policies.[17] We show that, although firms are ex-ante identical, they may select different CSR policies in equilibrium: some firms invest more in CSR to attract $\mathcal{R}$ investors, while others invest less and focus on $\mathcal{N}$ investors. This differentiation makes it harder for each firm to deviate and attract both types of investors since each equilibrium policy is tailored to the preferences of a specific type of investor. This equilibrium has properties similar to those with concentrated SRI of our baseline model. However, the equilibrium existence conditions are less tractable, so our baseline model is easier to analyze.

Broad mandate. In our main model, $\mathcal{R}$ investors only internalize the externalities generated by the firms in their portfolios.[18] Online Appendix D.3 considers a variation of the model where $\mathcal{R}$ investors internalize firms' externalities independently of their ownership. Following Oehmke and Opp (2020), we refer to these as investors with a *broad* investment mandate, as opposed to the *narrow* mandate of our main model. Since both types of investors have the same demand for shares, all the equilibria feature SRI in this version of the model. Yet, SRI concentration can still arise even when it is inefficient, as atomistic $\mathcal{R}$ investors may fail to coordinate on a more efficient symmetric equilibrium. However, unlike the main model, here $\mathcal{R}$ investors are worse off when they end up in equilibria with lower aggregate CSR. It is worth emphasizing that the equilibrium characterization in this version of the model is the same as in our main model if we add even a tiny extra disutility from *holding* firms that generate negative externalities. Except for $\mathcal{R}$ investors'

[17] See Albuquerque et al. (2019) for a discussion of the evidence surrounding firms' CSR commitments. This version of the model is similar to competitive screening games (e.g., Rothschild and Stiglitz, 1976), since investors have heterogeneous values for CSR policies and firms can use their CSR policies to target a specific group of investors.

[18] This modeling of socially responsible agents is consistent with traditional explanations of individuals' demand for CSR (e.g., warm-glow giving in Andreoni (1990), and image concerns in Bénabou and Tirole (2010)) and is similar to other papers on SRI (e.g., Heinkel et al., 2001; Moisson, 2020; Pástor et al., 2021; Goldstein et al., 2022).

preferences over equilibria, all our other results are thus robust to a setting where $\mathcal{R}$ investors internalize the externalities generated by *all* firms, but slightly more for the firms in their portfolio.

Cost of capital channel. In practice, investors can influence firms' externalities through three main channels. First, they can directly engage in governance, for example, by voting on ESG proposals or pressuring management to take certain actions. Like other papers (e.g., Oehmke and Opp 2020; Broccardo et al. 2022), our main model is most consistent with this channel. The pressure on management can also be *indirect*, since the investors' reaction to CSR issues may affect a firm's stock price, which feeds back to managerial incentives (e.g., through their pay packages). We study this second channel in the extension with reverse timing in Online Appendix D.2. Finally, investors can exclude dirty firms from their portfolios to reduce these firms' access to capital and reduce their production. Online Appendix D.4 considers a variation of our model that includes this strategy, showing that our results continue to hold when $\mathcal{R}$ investors' exclusions reduce firm production.

Social responsibility and markups. Firms reduce welfare in our model both by generating negative externalities and setting positive markups. Online Appendix D.5 explores a variation of the model where $\mathcal{R}$ investors care about firms' markups alongside their externalities. The equilibrium analysis reveals a tension between firms' CSR investments and markups. First, the prospect of charging higher prices to responsible consumers incentivizes firms to invest in CSR, so forcing firms to set lower prices crowds out their private incentives to become green. Second, the intention to control markups reinforces $\mathcal{R}$ investors' tendency to group together, which can reduce aggregate CSR.

5.2 CSR policies

Responsible consumption and CSR policies. Online Appendix E.1 uses the product market model from Section 4.3 to illustrate how socially responsible consumption (SRC) influences CSR policies, and compare these effects to those of SRI. Investing in CSR allows firms to charge higher prices (as in Example 1) or sell additional units (as in Example 2) to $\mathcal{R}$ consumers. Absent any distortion due to SRI concentration, SRC thus increases the CSR investments of *all* firms. Through this channel, an increase in the mass of $\mathcal{R}$ consumers also shrinks the valuation gap between $\mathcal{R}$ and $\mathcal{N}$ investors, so that the equilibrium that features SRI without concentration becomes more likely to exist.

Input market and CSR policies. Online Appendix E.2 micro-founds the properties of the profit function Π_j based on input markets, as opposed to the product market channels described in Section 4.3. We consider a version of the model where σ_j describes how much firm j chooses to rely on *green* inputs (e.g., clean energy), rather than *brown* inputs (e.g., energy generated from fossil fuels), for its production process. The model features technological spillovers, whereby firm j becomes more efficient in using a given input type when more firms use the same type.

Depending on the model parameters, we obtain either strategic substitutability (owing to price pressure on green inputs) or complementarity (due to technological spillovers) in CSR policies. Although their sales or product prices do not depend on σ_j in this model specification, firms continue to choose $\sigma_j > 0$ even in the equilibrium without SRI: when the aggregate demand for green inputs is small, their price is too low (relative to brown inputs) for firms not to use them for production. Our main results thus do not rely on the pricing of CSR issues in product markets or competition among firms, and so apply to firms at different layers of the production network.

Competition and strategic substitutability. The perverse effects of SRI arise when firms' CSR policies are strategic substitutes. Example 1 shows how product market competition leads to substitutability in a simple model of Bertrand competition. Online Appendix E.3 shows that this insight generalizes to a wide range of models where products are vertically differentiated along the CSR dimension.

Public good provision. In our model, CSR investments reduce the rate at which firms generate negative externalities (or *public bads*, like pollution or a negative corporate culture), and $\mathcal{R}$ investors suffer disutility from holding firms that generate negative externalities. Online Appendix E.4 considers a variation of the model where CSR investments entail positive externalities (or *public goods*, like corporate donations or a positive corporate culture), and $\mathcal{R}$ types attach a premium to holding firms that generate positive externalities. Contrary to our main model, here $\mathcal{R}$ investors have higher valuations than $\mathcal{N}$ investors, so all the equilibria feature SRI. Yet, SRI may still be concentrated in a subset of firms in equilibrium, even if this reduces aggregate CSR investments.

6 Conclusion

The practice of incorporating environmental and social factors into investment decisions and ownership policies is becoming increasingly popular. This paper develops a framework to explore how this socially responsible investing (SRI) affects firms' concerns with Corporate Social Responsibility (CSR) issues and analyzes the implications for product markets. We show that the typical SRI strategies (e.g., best-in-class investing and ESG exclusion) lead to distributional inefficiencies that can either prevent a transition to a greener economy from materializing or push one where firms have more market power, and consumers are ultimately worse off. However, we demonstrate that these inefficiencies are unlikely to persist if SRI becomes sufficiently popular among investors.

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A Figures and Tables

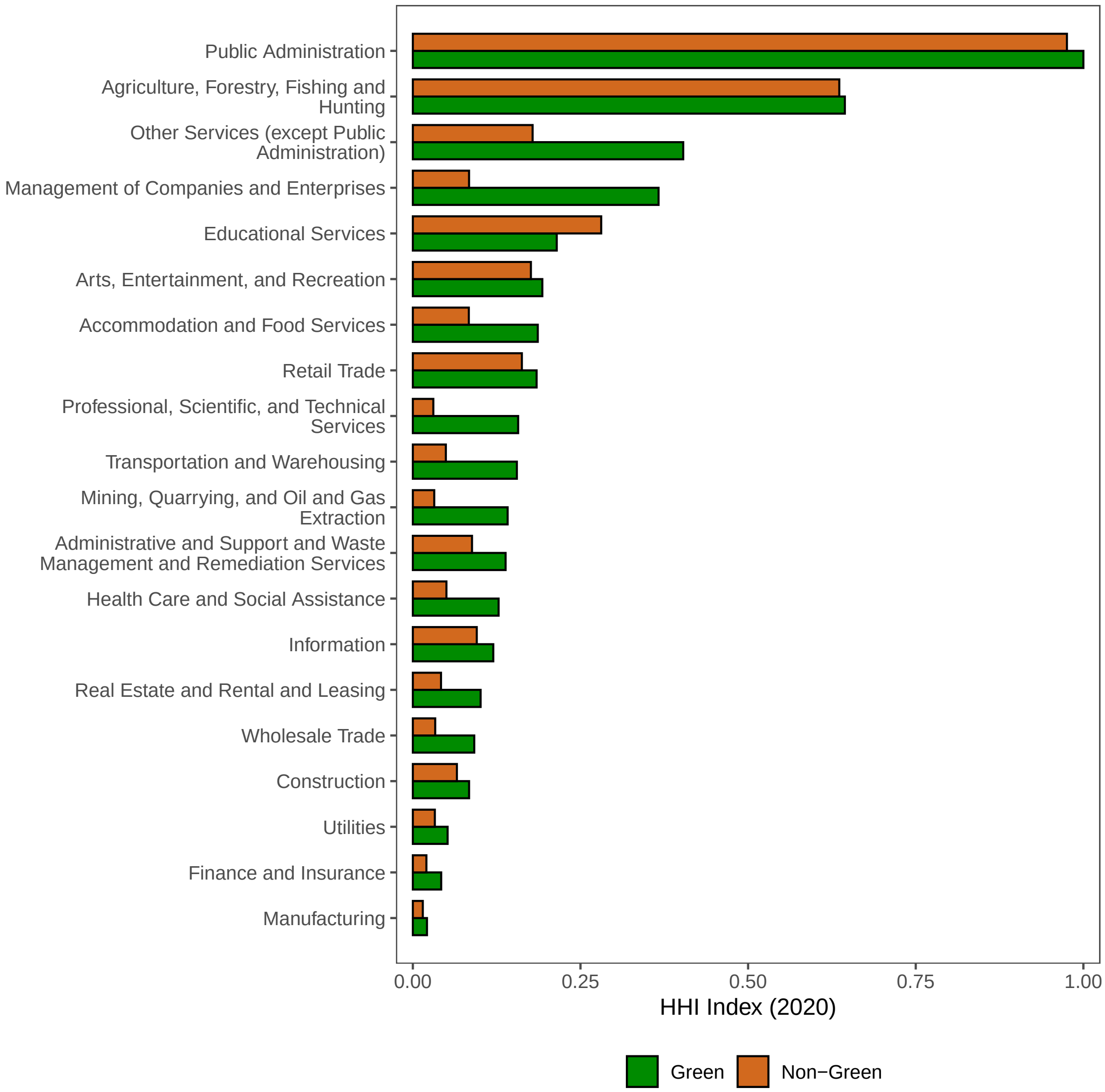
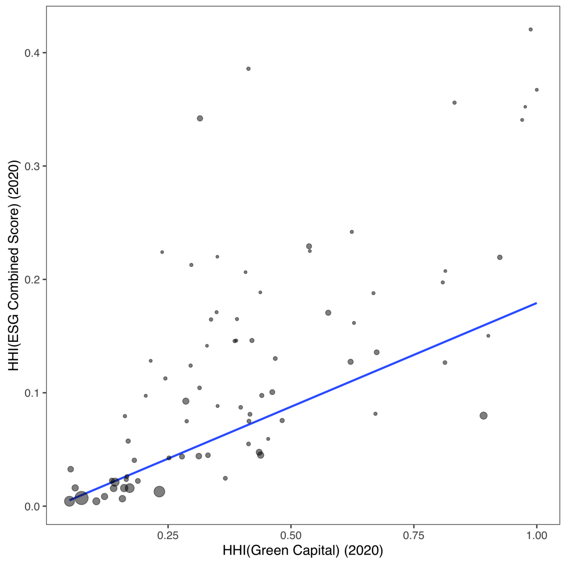
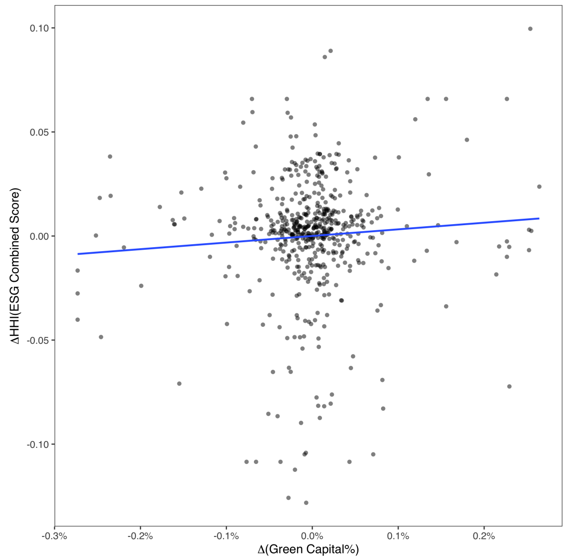


Figure 2: This figure plots HHI indexes of Green and Non-Green Capital at the industry level for NYSE/NASDAQ-listed stocks as of 2020. Industries are defined by NAICS 2-digit codes. NYSE/NASDAQ-listed stocks are from CRSP.



(a) Cross-sectional.



(b) Time series.

Figure 3: This figure summarizes the relationship between the concentration of Green Capital and ESG Combined Scores at the industry level. Panel (a) shows HHI indexes of Green Capital and ESG Combined Score at the industry level for NYSE/NASDAQ-listed stocks as of 2020. The size of the circles denotes the amount of green capital in the industry. Panel (b) shows changes in Green Capital (scaled by industry market capitalization) and changes in HHI indexes of ESG Combined Score at the industry level for NYSE/NASDAQ-listed stocks during the period 2012-2020. Both changes are demeaned by year. Industries are defined by NAICS 3-digit codes. All variables are winsorized at 1 percent. The blue lines denote a fitted line from a WLS regression of the y-axis on the x-axis.

	$\Delta HHI(ESG\ Combined\ Score)\ (t)$				$\Delta Green\ Capital\%\ (t)$	
	(1)	(2)	(3)	(4)	(5)	(6)
$\Delta Green\ Capital\%\ (t)$	6.495***	3.181**				
	(2.284)	(1.439)				
$\Delta Green\ Capital\%\ (t-1)$			1.950	2.938		
			(2.746)	(2.697)		
$\Delta HHI(ESG\ Combined\ Score)\ (t-1)$					0.001	0.000
					(0.001)	(0.001)
Constant	−0.012***		−0.013***		0.000	
	(0.001)		(0.001)		(0.000)	
Year Fixed Effects		Y		Y		Y
Observations	545	545	488	488	468	468
Adjusted R^2	0.014	0.282	−0.001	0.277	−0.001	0.027

Table 1: This table examines changes in green capital (scaled by industry market capitalization) and changes in HHI indexes of ESG Combined Score. Columns (1)-(4) use changes in HHI indexes of ESG Combined Score as the dependent variable. In columns (5)-(6), the dependent variable is a change in green capital. (t) denotes a variable is concurrent, while (t-1) denotes a variable is one year lagged. All variables are winsorized at 1 percent. Standard errors are clustered by industry and reported in parentheses. *, **, and *** indicate statistical significance at the 10%, 5%, and 1%, respectively.

B Proofs

B.1 Proof of Lemma 1.

We first define the lower bound for c used in Assumption 1(i):

$$\underline{c} \equiv \max\left\{\frac{\partial \pi_j}{\partial \sigma_j} + \eta\lambda,\ \frac{\partial^2 \pi_j}{\partial \sigma_j^2} - \frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_{-j}},\ \sum_{k=1}^{N} \frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_k} + \frac{\eta\lambda\chi(1-\chi)}{N\kappa}\right\}. \tag{B.1}$$

Next, we establish equilibrium existence. The following three assumptions guarantee that any equilibrium vector $\vec{\sigma}^*$ is obtained solving the system of first-order conditions (3): (i) $\pi_j \geq 0$ is increasing in σ_j, (ii) $C(0) = C'(0) = 0$, and (iii) $C'(1) = c \geq \frac{\partial \pi_j}{\partial \sigma_j} + \eta\lambda$ (see Assumption 1(i)). For all $j \in \mathcal{J}$, we can then define

$$f_j(\vec{\sigma}) \equiv \frac{1}{c}\left(\eta\lambda s_j^{\mathcal{R}} + \frac{\partial \pi_j}{\partial \sigma_j}\right), \tag{B.2}$$

with $s_j^{\mathcal{R}} \in [0,1]$ for all $j \in \mathcal{J}$. Note that $f_j : [0,1]^N \to [0,1]$ is a continuous function given the assumptions above. Hence, the vector-valued function

$$F(\vec{\sigma}) \equiv (f_j(\vec{\sigma}))_{j=1,\ldots,N} \tag{B.3}$$

is a continuous function mapping $[0,1]^N$, a non-empty, compact and convex subset of $\mathbb{R}^N$, into itself. By Brouwer's fixed point theorem, $F(\cdot)$ admits at least one fixed point — i.e., vectors $\vec{\sigma}^*$ such that $\vec{\sigma}^* = F(\vec{\sigma}^*)$. The definition of $F(\cdot)$ implies that fixed points correspond to the Nash equilibria of the CSR investment game.

Without loss of generality, consider $s_1^{\mathcal{R}} \geq s_2^{\mathcal{R}}$, for any $(s_k^{\mathcal{R}})_{k=3,\ldots,N}$. Then, FOC (3) for firms 1 and 2 imply

$$\frac{\partial \Pi_j}{\partial \sigma_j}(\sigma_2, \sigma_1, \vec{\sigma}_k) - \frac{\partial \Pi_j}{\partial \sigma_j}(\sigma_1, \sigma_2, \vec{\sigma}_k) = \eta\lambda(s_1^{\mathcal{R}} - s_2^{\mathcal{R}}) \geq 0, \tag{B.4}$$

where $\vec{\sigma}_k = (\sigma_k)_{k=3,\ldots,N}$ is the vector of other firms' equilibrium CSR policies.

Suppose, first, that $s_1^{\mathcal{R}} = s_2^{\mathcal{R}}$. To establish that $\sigma_1 = \sigma_2$, suppose, without loss of generality, that $\sigma_2 = \sigma + x$, with $\sigma \equiv \sigma_1$ and $x \geq 0$. Then, subtracting $\frac{\partial \Pi_j}{\partial \sigma_j}(\sigma, \sigma, \vec{\sigma}_k)$ from both sides, Equation (B.4) can be written as $H(x) = 0$, where

$$H(x) \equiv \int_0^x \left[\frac{\partial^2 \Pi_j}{\partial \sigma_j^2}(\sigma + \tilde{x}, \sigma, \vec{\sigma}_k) - \frac{\partial^2 \Pi_j}{\partial \sigma_j \partial \sigma_{-j}}(\sigma, \sigma + \tilde{x}, \vec{\sigma}_k) \right] \mathrm{d}\tilde{x}. \tag{B.5}$$

The function $H(x)$ is such that $H(0) = 0$ and $H'(x) < 0$ for all $x > 0$, given that: (i) $H'(0) = \frac{\partial^2 \Pi_j}{\partial \sigma_j^2} - \frac{\partial^2 \Pi_j}{\partial \sigma_j \partial \sigma_{-j}} = \frac{\partial^2 \pi_j}{\partial \sigma_j^2} - c - \frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_{-j}} \leq 0$ by Assumption 1(i); and (ii) $H''(x) = \frac{\partial^3 \Pi_j}{\partial \sigma_j^3} - \frac{\partial^3 \Pi_j}{\partial \sigma_j \partial \sigma_{-j}^2} < 0$ (thereby $H'(x)$ is decreasing) by Assumption 1(ii). Therefore, $H(x) = 0$ if and only if $x = 0$, i.e., $\sigma_1 = \sigma_2$.

These results imply that, if $s_j^{\mathcal{R}} \equiv s^{\mathcal{R}}$ for all $j \in \mathcal{J}$, then the game can only admit symmetric equilibria. To establish uniqueness, note that any symmetric equilibrium value σ^* is obtained from

$$\sigma^* = \frac{1}{c}\left(\eta \lambda s^{\mathcal{R}} + \frac{\partial \pi_j}{\partial \sigma_j}(\sigma^*, \vec{\sigma}^*) \right). \tag{B.6}$$

This equation can be rewritten as

$$c\sigma^* - \frac{\partial \pi_j}{\partial \sigma_j}(\sigma^*, \vec{\sigma}^*) = \eta \lambda s^{\mathcal{R}}. \tag{B.7}$$

The LHS is strictly increasing in σ^*, as $c > \sum_{k=1}^N \frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_k}$ under Assumption 1(i), while the RHS does not depend on σ^*. Therefore, these two functions intersect only once. Moreover, as $\eta s^{\mathcal{R}}$ grows larger, the RHS increases, and so the equilibrium CSR policy must also increase because the LHS is increasing in σ^*. It follows that $\sigma^*(s^{\mathcal{R}})$ is an increasing function of $s^{\mathcal{R}}$.

Next, suppose that $s_1^{\mathcal{R}} > s_2^{\mathcal{R}}$. To establish that $\sigma_1 > \sigma_2$ suppose, for the sake of contradiction, that $\sigma_2 = \sigma + x$, with $\sigma \equiv \sigma_1$ and $x \geq 0$. Then, by the same steps as above, Equation (B.4) can be written as $H(x) = \eta\lambda(s_1^{\mathcal{R}} - s_2^{\mathcal{R}}) > 0$, where $H(x) \leq 0$ for all $x \geq 0$. This contradiction implies that $\sigma_1 > \sigma_2$.

From the results above, it follows that if $s_1^{\mathcal{R}} \geq s_2^{\mathcal{R}} \geq \ldots \geq s_N^{\mathcal{R}}$, then $\sigma_1^* \geq \sigma_2^* \geq \ldots \geq \sigma_N^*$, and

$$\sigma_N^* = \frac{1}{c}\left(\eta \lambda s_N^{\mathcal{R}} + \frac{\partial \pi_j}{\partial \sigma_j}(\sigma_N^*, \vec{\sigma}_{-N}) \right) \geq \frac{1}{c}\frac{\partial \pi_j}{\partial \sigma_j}(\sigma_N^*, \vec{\sigma}_{-N}), \tag{B.8}$$

where $\vec{\sigma}_{-N} \equiv (\sigma_1^*, \ldots, \sigma_{N-1}^*)$, with $\sigma_j^* > \sigma_N^*$ for all j such that $s_j^{\mathcal{R}} > s_N^{\mathcal{R}}$.

Under strategic complementarity, $\sigma_N^* > \sigma_0$ because the last term in Equation (B.8) is equal (up to relabeling) to the RHS of Equation (B.6) for $s^{\mathcal{R}} = 0$ and, as $s_j^{\mathcal{R}} > s_N^{\mathcal{R}}$ for at least one rival,

$\frac{\partial \pi_j}{\partial \sigma_j}(\sigma, \vec{\sigma}_{-N}) > \frac{\partial \pi_j}{\partial \sigma_j}(\sigma, \vec{\sigma})$. Therefore, we conclude that $\sigma_1^* \geq \sigma_2^* \geq ... \geq \sigma_N^* > \sigma_0$, i.e., $\sigma_j^* > \sigma_0$ for all j.

On the contrary, under strategic substitutability, as the inequality in Equation (B.8) holds with equality if $s_N^{\mathcal{R}} = 0$, $\frac{\partial \pi_j}{\partial \sigma_j}(\sigma, \vec{\sigma}_{-N}) < \frac{\partial \pi_j}{\partial \sigma_j}(\sigma, \vec{\sigma})$ implies that it must be $\sigma_N^* < \sigma_0$. More generally, as $\sigma_j^* = \sigma_N^*$ for any j such that $s_j^{\mathcal{R}} = s_N^{\mathcal{R}} = 0$, we have $\sigma_j^* < \sigma_0$ for any such firm j. To show that at least one firm invests more than σ_0 in any equilibrium with SRI (i.e., $\sigma_1^* > 0$ provided that $s_1^{\mathcal{R}} > 0$), note that, even if $\sigma_N^* \leq ... \leq \sigma_2^* < \sigma_0$,

$$\sigma_1^* = \frac{1}{c}\left(\eta\lambda s_1^{\mathcal{R}} + \frac{\partial \pi_j}{\partial \sigma_j}(\sigma_1^*, \vec{\sigma}_{-1})\right) > \frac{1}{c}\frac{\partial \pi_j}{\partial \sigma_j}(\sigma_1^*, \vec{\sigma}_{-1}) > \frac{1}{c}\frac{\partial \pi_j}{\partial \sigma_j}(\sigma_1^*, \vec{\sigma}_0), \tag{B.9}$$

where $\vec{\sigma}_{-1} \equiv (\sigma_2^*, ..., \sigma_N^*)$, which implies that $\sigma_1^* > \sigma_0$ by the FOC (B.6) for $\vec{s}_j^{\mathcal{R}} = \vec{0}$.

B.2 Proof of Proposition 1.

The proof that the symmetric equilibrium with no SRI exists for all $\chi \leq \widehat{\chi}(\sigma_0, 0) = 1 - \frac{N\kappa}{\lambda(1-\sigma_0)}$ is given in the text. In what follows, we show the existence and uniqueness of the symmetric equilibrium with SRI. Substituting $s^{\mathcal{R}*}$ from Equation (12) into Equation (11) yields that

$$c\sigma^* - \frac{\partial \pi_j}{\partial \sigma_j}(\sigma^*, \vec{\sigma}^*) - \eta\lambda\chi\left(1 - \frac{\lambda(1-\chi)(1-\sigma^*)}{N\kappa}\right) = 0 \tag{B.10}$$

where we have used that $s^{\mathcal{R}*} \geq 0$, i.e.,

$$\chi \geq \widehat{\chi}(\sigma^*, 0). \tag{B.11}$$

Equilibrium uniqueness requires the LHS to be increasing in σ^*, i.e., $c > \sum_{k=1}^N \frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_k} + \frac{\eta\lambda^2\chi(1-\chi)}{N\kappa}$, as implied by Assumption 1(i). By the implicit function theorem, as the LHS of Equation (B.10) is decreasing in χ,[19] it follows that the symmetric equilibrium $\sigma^*(\chi)$ is an increasing function of χ.

From (B.11) we have that, at $\chi = \widehat{\chi}(\sigma^*, 0)$, $s^{\mathcal{R}*} = 0$ and so $\sigma^* = \sigma_0$ for $\chi = \widehat{\chi}(\sigma_0, 0)$. For larger values of χ, as $\sigma^*(\chi)$ is an increasing function of χ, $\sigma^*(\chi) > \sigma_0$ is well defined, because $\chi > \widehat{\chi}(\sigma^*, 0)$

[19]Differentiating the LHS of Equation (B.10) with respect to χ gives

$$-\frac{\eta\lambda(N\kappa - \lambda(1-\sigma^*)(1-2\chi))}{N\kappa}.$$

As this derivative is decreasing in χ, a sufficient condition for it to be negative for all $\chi \geq \widehat{\chi}(\sigma^*, 0)$ is

$$-\frac{\eta\lambda(N\kappa - \lambda(1-\sigma^*)(1-2\widehat{\chi}(\sigma^*, 0)))}{N\kappa} = -\frac{\eta\lambda(\lambda(1-\sigma^*) - N\kappa)}{N\kappa} < 0,$$

which holds for all $\kappa < \frac{\lambda(1-\sigma^*)}{N}$. This condition needs to hold as it is equivalent to $\widehat{\chi}(\sigma^*, 0) > 0$.

is implied by $\chi > \widehat{\chi}(\sigma_0, 0)$, given that $\sigma^* > \sigma_0$ and $\widehat{\chi}(\cdot)$ is decreasing in σ. By contrast, for lower values of χ, the symmetric equilibrium with SRI cannot exist, as it would feature $\sigma^* < \sigma_0$: this would exist if and only if $\chi > \widehat{\chi}(\sigma^*, 0)$, which is not satisfied for $\chi < \widehat{\chi}(\sigma_0, 0)$, as $\sigma^* < \sigma_0$ implies $\widehat{\chi}(\sigma_0, 0) < \widehat{\chi}(\sigma^*, 0)$.

We can thus conclude that, under Assumption 1(i), a symmetric equilibrium featuring SRI without concentration exists if and only if $\chi > \widehat{\chi}(\sigma_0, 0)$, and it is unique.

B.3 Proof of Lemma 2.

The arguments in the proof of Proposition 1 show that the symmetric equilibrium $\sigma^*(\chi)$ is a continuous function of χ such that $\sigma^*(\chi) = 0$ for $\chi \leq \widehat{\chi}(\sigma_0, 0)$ and $\frac{\partial \sigma^*}{\partial \chi} > 0$ for $\chi > \widehat{\chi}(\sigma_0, 0)$.

Note that the LHS of Equation (B.10) is decreasing in κ. By the implicit function theorem, we can therefore conclude that σ^* is increasing in κ as well.

To establish that also $s^{\mathcal{R}}$ is increasing in χ and κ, consider Equation (11). This expression implies that σ^* depends on χ (resp. κ) only indirectly through $s^{\mathcal{R}}$, and σ^* is increasing in $s^{\mathcal{R}}$. As a result, the sign of $\frac{\partial s^{\mathcal{R}}}{\partial \chi}$ (resp. $\frac{\partial s^{\mathcal{R}}}{\partial \kappa}$) must be the same as the sign of $\frac{\partial \sigma^*}{\partial \chi}$ (resp. $\frac{\partial \sigma^*}{\partial \kappa}$). These results hold not only for aggregate shares held by $\mathcal{R}$ investors in any firm but also for their individual holdings $s_i^{\mathcal{R}*} = \frac{1}{N\kappa}\left[N\kappa - \lambda(1-\chi)(1-\sigma^*)\right]$, which are increasing in κ and χ both directly and through σ^*.

Finally, since σ_0 does not depend on λ and κ, it follows that $\widehat{\chi}(\sigma_0, 0)$ is increasing in λ, and

$$\frac{\partial \widehat{\chi}(\sigma_0, 0)}{\partial \kappa} = -\frac{N - \frac{N^2\kappa - \lambda(N-2)(1-\sigma_0)}{\sqrt{(\lambda(1-\sigma) - N\kappa)^2 + 4\kappa\lambda(1-\sigma)}}}{2\lambda(1-\sigma_0)} < 0. \tag{B.12}$$

B.4 Proof of Proposition 2.

Let $\overline{\alpha}_j^{\mathcal{R}} > 0$ and $\underline{\alpha}_j^{\mathcal{R}} > 0$ be the fraction of $\mathcal{R}$ investors who buy shares in any firm $j \leq \underline{n}$ and $j \in (\underline{n}, \overline{n}]$, respectively, with $\sum_{j \leq \underline{n}} \overline{\alpha}_j^{\mathcal{R}} + \sum_{j \in (\underline{n}, \overline{n}]} \underline{\alpha}_j^{\mathcal{R}} = 1$. Similarly, let $\overline{\alpha}_j^{\mathcal{N}} > 0$ and $\underline{\alpha}_j^{\mathcal{N}} > 0$ be the fraction of $\mathcal{N}$ investors who buy shares in any firm $j > \overline{n}$ and $j \in (\underline{n}, \overline{n}]$, respectively, with $\sum_{j > \overline{n}} \overline{\alpha}_j^{\mathcal{N}} + \sum_{j \in (\underline{n}, \overline{n}]} \underline{\alpha}_j^{\mathcal{N}} = 1$.

Then, for any firm $j \leq \underline{n}$, the market clearing condition is

$$\chi \overline{\alpha}_j^{\mathcal{R}} \frac{1}{\kappa} \left[\Pi_j - p_j - \lambda(1 - \sigma_j) \right] = 1. \tag{B.13}$$

As $\mathcal{R}$ investors are indifferent between holding shares in any of these firms, $\Pi_j - p_j - \lambda(1 - \sigma_j)$ must be constant across these firms. This implies that the considered market clearing condition can hold for all $j \leq \underline{n}$ if and only if $\overline{\alpha}_j^{\mathcal{R}} \equiv \overline{\alpha}^{\mathcal{R}}$ for all such j.

Similarly, for any firm $j > \overline{n}$, the market clearing condition is

$$(1 - \chi) \overline{\alpha}_j^{\mathcal{N}} \frac{1}{\kappa} \left(\Pi_j - p_j \right) = 1. \tag{B.14}$$

As $\mathcal{N}$ investors are indifferent between holding shares in any of these firms, $\Pi_j - p_j$ must be constant across these firms. This implies that the considered market clearing condition can hold for all $j > \overline{n}$ if and only if $\overline{\alpha}_j^{\mathcal{N}} \equiv \overline{\alpha}^{\mathcal{N}}$ for all such j.

Finally, for any firm $j \in (\underline{n}, \overline{n}]$, the market clearing condition is

$$\chi \underline{\alpha}_j^{\mathcal{R}} \frac{1}{\kappa} \left[\Pi_j - p_j - \lambda(1 - \sigma_j) \right] + (1 - \chi) \underline{\alpha}_j^{\mathcal{N}} \frac{1}{\kappa} \left(\Pi_j - p_j \right) = 1. \tag{B.15}$$

In what follows, we first consider the case with $\underline{n} = \overline{n}$ and then move to $\underline{n} < \overline{n}$.

Proof of (i). For $\underline{n} = \overline{n} \equiv n$, from the above results it follows that $\overline{\alpha}^{\mathcal{R}} = \frac{1}{n}$ and $\overline{\alpha}^{\mathcal{N}} = \frac{1}{N-n}$. For this equilibrium to exist for any given n, $\mathcal{R}$ investors should be indifferent between buying shares in any of firms $j = 1, ..., n$, and strictly prefer doing so to buying shares in other firms or not buying any shares:

$$\Pi_1 - p_1 - \lambda(1 - \sigma_1) = ... = \Pi_n - p_n - \lambda(1 - \sigma_n) > \max \left\{ 0, \max_{j' > n} \Pi_{j'} - p_{j'} - \lambda(1 - \sigma_{j'}) \right\}. \tag{B.16}$$

By contrast, $\mathcal{N}$ investors should be indifferent between buying shares in any firm $j = n + 1, ..., N$, and strictly prefer doing so to buying shares in other firms or not buying any shares:

$$\Pi_{n+1} - p_{n+1} = ... = \Pi_N - p_N > \max \left\{ 0, \max_{j' \leq n} \Pi_{j'} - p_{j'} \right\}. \tag{B.17}$$

In this equilibrium, the results in Lemma 1 imply

$$\sigma_1^* = \ldots = \sigma_n^* \equiv \overline{\sigma} > \sigma_{n+1}^* = \ldots = \sigma_N^* \equiv \underline{\sigma}, \tag{B.18}$$

where $\overline{\sigma} > \underline{\sigma}$ solve the system obtained from the FOCs (3) for $s_j^{\mathcal{R}} = 1$ for $j \leq n$ and $s_j^{\mathcal{R}} = 0$ for $j > n$. Then, using the market clearing conditions (B.13)-(B.14), the existence conditions (B.16)-(B.17) yield

$$\frac{n\kappa}{\chi} > \frac{\kappa(N-n)}{1-\chi} - \lambda(1-\underline{\sigma}), \tag{B.19}$$

and

$$\frac{\kappa(N-n)}{1-\chi} > \frac{n\kappa}{\chi} + \lambda(1-\overline{\sigma}). \tag{B.20}$$

Both of these conditions hold if and only if $\widehat{\chi}(\overline{\sigma}, n) < \chi < \widehat{\chi}(\underline{\sigma}, n)$.

Proof of (ii). For any $\underline{n} < \overline{n}$, from Equation (B.13) and Equation (B.14) we find

$$\underline{n}\overline{\alpha}^{\mathcal{R}} = 1 - \sum_{j \in (\underline{n}, \overline{n}]} \alpha^{\mathcal{R}} = \frac{\underline{n}\kappa}{\chi\left[\Pi_j - p_j - \lambda(1-\sigma_j)\right]} \quad \forall j \leq \overline{n}, \tag{B.21}$$

and

$$(N - \overline{n})\overline{\alpha}^{\mathcal{N}} = 1 - \sum_{j \in (\underline{n}, \overline{n}]} \alpha^{\mathcal{N}} = \frac{(N-\overline{n})\kappa}{(1-\chi)\left(\Pi_j - p_j\right)} \quad \forall j > \underline{n}, \tag{B.22}$$

respectively. Summing the market clearing conditions (B.15) for $j \in (\underline{n}, \overline{n}]$ and using these results, we obtain after simple manipulations that

$$\chi\left[\Pi_j - p_j - \lambda(1-\sigma_j)\right] + (1-\chi)\left(\Pi_j - p_j\right) = N\kappa \quad \forall j \in (\underline{n}, \overline{n}]. \tag{B.23}$$

Since $s_j^{\mathcal{R}} = 1$ for all $j \leq \underline{n}$, and $s_j^{\mathcal{R}} = 0$ for all $j > \overline{n}$, from Lemma 1 it follows that in equilibrium $\sigma_1^* = \ldots = \sigma_{\underline{n}}^* \equiv \overline{\sigma}$, and $\sigma_{\overline{n}+1}^* = \ldots = \sigma_N^* \equiv \underline{\sigma} < \overline{\sigma}$. Moreover, $\Pi_j - p_j - \lambda(1-\sigma_j)$ must be constant for all $j \in (\underline{n}, \overline{n}]$ (by $\mathcal{R}$ investors' indifference conditions), and $\Pi_j - p_j$ must also be constant across these j (by $\mathcal{N}$ investors' indifference conditions). Taken together, these indifference conditions imply that also σ_j must be constant for $j \in (\underline{n}, \overline{n}]$: $\sigma_{\underline{n}+1}^* = \ldots = \sigma_{\overline{n}}^* \equiv \widehat{\sigma}$. From Lemma 1, this is the case if and only if, for all $j \in (\underline{n}, \overline{n}]$: $s_j^{\mathcal{R}} \equiv \widehat{s}^{\mathcal{R}}$, which in turn (by symmetry across $\mathcal{R}$ investors) holds if and only if $\underline{\alpha}_j^{\mathcal{R}} \equiv \underline{\alpha}^{\mathcal{R}}$ for all $j \in (\underline{n}, \overline{n}]$. Specifically, we have

$$\underline{\alpha}^{\mathcal{R}} = \frac{1 - \underline{n}\overline{\alpha}^{\mathcal{R}}}{\overline{n} - \underline{n}} = \frac{1}{\overline{n} - \underline{n}} \left[1 - \frac{\underline{n}\kappa}{\chi \left[N\kappa - \lambda(1-\chi)(1-\widehat{\sigma}) \right]} \right], \tag{B.24}$$

where the second equality uses the market clearing condition (B.13) and $\mathcal{R}$ investors' indifference condition. Then, for the market clearing condition (B.15) to hold for all firms $j \in (\underline{n}, \overline{n}]$, we must also have that $\underline{\alpha}_j^{\mathcal{N}} \equiv \underline{\alpha}^{\mathcal{N}}$ for all $j \in (\underline{n}, \overline{n}]$.

Next, the investment levels $(\overline{\sigma}, \widehat{\sigma}, \underline{\sigma})$ in equilibrium are obtained from the FOCs (3) with $s_j^{\mathcal{R}} = 1$ for $j \leq \underline{n}$, $s_j^{\mathcal{R}} = \chi \underline{\alpha}^{\mathcal{R}} \frac{1}{\kappa} \left[N\kappa - (1-\chi)\lambda(1-\widehat{\sigma}) \right] \in (0,1)$ for $\underline{n} < j \leq \overline{n}$, and $s_j^{\mathcal{R}} = 0$ for $j > \overline{n}$; hence, from Lemma 1, $\overline{\sigma} > \widehat{\sigma} > \underline{\sigma}$. It is immediate to check that, together with the indifference conditions of $\mathcal{R}$ and $\mathcal{N}$ investors, these inequalities imply that $\mathcal{R}$ investors strictly prefer firms $j \leq \overline{n}$ to firms $j > \overline{n}$, and $\mathcal{N}$ investors strictly prefer firms $j > \underline{n}$ to firms $j \leq \underline{n}$, as required for the existence of such equilibria.

Moreover, using (B.23) we find that $\mathcal{R}$ investors prefer buying shares in any firm $j \leq \overline{n}$ to not buying at all if and only if

$$\Pi_j - p_j - \lambda(1 - \sigma_j) > 0 \Leftrightarrow \chi > \widehat{\chi}(\widehat{\sigma}, 0). \tag{B.25}$$

Under this condition, it follows that $\mathcal{N}$ investors participate in the financial market.

Hence, to characterize existence conditions for these equilibria, we only need to impose conditions under which all shares $\underline{\alpha}^{\theta}$ and $\overline{\alpha}^{\theta}$, for $\theta \in \{\mathcal{R}, \mathcal{N}\}$, are positive. The market clearing conditions (B.13)-(B.14) immediately imply $\overline{\alpha}^{\theta} > 0$ whenever (B.25) holds. The condition $\underline{\alpha}^{\mathcal{R}} > 0$ is then equivalent to $\underline{n}\overline{\alpha}^{\mathcal{R}} < 1$. Using (B.21) and substituting the market clearing prices, after simple manipulations, we obtain that this condition holds if and only if $\chi > \widehat{\chi}(\widehat{\sigma}, \underline{n})$, where $\widehat{\chi}(\widehat{\sigma}, \underline{n}) \geq \widehat{\chi}(\widehat{\sigma}, 0)$, with equality at $\underline{n} = 0$ only.

Similarly, $\underline{\alpha}^{\mathcal{N}} > 0$ is equivalent to $(N - \overline{n})\overline{\alpha}^{\mathcal{N}} < 1$. Using (B.22) and substituting the market clearing prices, after simple manipulations, we obtain that this condition holds if and only if $\chi < \widehat{\chi}(\widehat{\sigma}, \overline{n})$.

Putting these conditions together yields that the considered equilibria exist for all $\widehat{\chi}(\widehat{\sigma}, \underline{n}) < \chi <$

$\widehat{\chi}(\widehat{\sigma}, \overline{n})$.

B.5 Proof of Proposition 3.

In order to derive sufficient conditions on χ, it is convenient to express the equilibrium characterization as a function of κ. Let

$$\widehat{\kappa}(\sigma, n) \equiv \frac{\chi(1-\chi)\lambda(1-\sigma)}{N\chi - n} > 0, \tag{B.26}$$

for $\sigma \in [0,1]$ and $n \in \{0, \ldots, N\}$. Then, rearranging the existence conditions of equilibria given in Proposition 1 and 2 yields:

- The symmetric equilibrium without (resp. with) SRI exists if and only if $\kappa \leq \widehat{\kappa}(\sigma_0, 0)$ (resp. $\kappa > \widehat{\kappa}(\sigma_0, 0)$);
- Asymmetric equilibria with $\overline{n} = \underline{n} = n$ exist if and only if $\chi > \frac{n}{N}$ and $\widehat{\kappa}(\overline{\sigma}, n) < \kappa < \widehat{\kappa}(\underline{\sigma}, n)$;
- Asymmetric equilibria with $\overline{n} > \underline{n}$ exist if either $\frac{\underline{n}}{N} < \chi \leq \frac{\overline{n}}{N}$ and $\kappa > \widehat{\kappa}(\widehat{\sigma}, \underline{n})$, or $\chi > \frac{\overline{n}}{N}$ and $\widehat{\kappa}(\widehat{\sigma}, \underline{n}) < \kappa < \widehat{\kappa}(\widehat{\sigma}, \overline{n})$.

Consider asymmetric equilibria with $\underline{n} = \overline{n} = 1$, which exist if $\chi > 1/N$ and $\widehat{\kappa}(\overline{\sigma}, 1) < \kappa < \widehat{\kappa}(\underline{\sigma}, 1)$. A sufficient condition for $\widehat{\kappa}(\overline{\sigma}, 1) < \widehat{\kappa}(\sigma_0, 0)$ is

$$\chi > \frac{1}{N(\overline{\sigma} - \sigma_0)}. \tag{B.27}$$

In turn, to obtain a sufficient condition for this inequality to hold, we note that

$$\overline{\sigma} = \frac{1}{c}\left[\eta\lambda + \frac{\partial \pi_j}{\partial \sigma_j}(\overline{\sigma}, \vec{\underline{\sigma}})\right] \geq \frac{1}{c}\left[\eta\lambda + \frac{\partial \pi_j}{\partial \sigma_j}(\sigma_0, \vec{\underline{\sigma}})\right] > \frac{1}{c}\left[\eta\lambda + \frac{\partial \pi_j}{\partial \sigma_j}(\sigma_0, \vec{\sigma}_0)\right] \Longrightarrow \overline{\sigma} - \underbrace{\frac{1}{c}\frac{\partial \pi_j}{\partial \sigma_j}(\sigma_0, \vec{\sigma}_0)}_{=\sigma_0} > \frac{\eta\lambda}{c}, \tag{B.28}$$

where the first inequality follows from $\overline{\sigma} > \sigma_0$ and $\frac{\partial^2 \pi_j}{\partial \sigma_j^2} \leq 0$, and the second one from $\underline{\sigma} < \sigma_0$ and $\frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_{-j}} < 0$ (resp. $\underline{\sigma} > \sigma_0$ and $\frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_{-j}} > 0$) under strategic substitutability (resp. complementarity).

Therefore, both under strategic substitutability and strategic complementarity,

$$\chi > \frac{c}{N\eta\lambda} \tag{B.29}$$

is a sufficient condition for $\widehat{\kappa}(\overline{\sigma}, 1) < \widehat{\kappa}(\sigma_0, 0)$.[20] Under this condition, there is a region of parameters, namely $\widehat{\kappa}(\sigma_0, 0) > \kappa > \widehat{\kappa}(\overline{\sigma}, 1)$, in which the only equilibria with SRI are asymmetric.

Next, consider asymmetric equilibria with $\underline{n} = 0 < \overline{n} = N - 1$, so that $\mathcal{R}$ investors exclude one firm. Suppose $\chi \leq \frac{N-1}{N}$. Then these equilibria exist for

$$\kappa > \widehat{\kappa}(\widehat{\sigma}, 0). \tag{B.30}$$

In what follows, we prove that the last condition is satisfied for all $\kappa > \widehat{\kappa}(\sigma_0, 0)$.

Substituting the market clearing price from Equation (B.23) into the individual demands $s_{ij}^{\mathcal{R}}$ of each $\mathcal{R}$ investor given in Equation (5) and rearranging, we have that any such equilibrium must satisfy

$$\Gamma(s_{ij}^{\mathcal{R}}) \equiv \kappa(N - s_{ij}^{\mathcal{R}}) - (1 - \chi)\lambda(1 - \widehat{\sigma}(s_{ij}^{\mathcal{R}})) = 0, \tag{B.31}$$

where $\widehat{\sigma}(\cdot)$ is obtained from the system of FOCs (3), given $s_j^{\mathcal{R}} = \chi s_{ij}^{\mathcal{R}}$ for $j = 1, \ldots, N-1$ and $s_{iN}^{\mathcal{R}} = 0$.

For the market clearing conditions $\frac{\chi}{N-1} s_{ij}^{\mathcal{R}} + (1-\chi)\underline{\alpha}^{\mathcal{N}} s_{ij}^{\mathcal{N}} = 1$ to hold, it must be $s_{ij}^{\mathcal{R}} \in \left(0, \frac{N-1}{\chi}\right)$. We have

$$\Gamma\left(\frac{N-1}{\chi}\right) < 0 \quad \forall \chi \leq \frac{N-1}{N}, \tag{B.32}$$

and

$$\Gamma(0) > 0 \Leftrightarrow \kappa > \widehat{\kappa}(\sigma_0, 0), \tag{B.33}$$

given that $\widehat{\sigma}(0) = \sigma_0$. This is because, as $s_{iN}^{\mathcal{R}} = 0$ in this equilibrium, if $s_{ij}^{\mathcal{R}} = 0$ for all $j = 1, \ldots, N-1$ as well, then $\mathcal{R}$ investors do not hold shares in any of the firms, so we obtain the equilibrium with no SRI.

Hence, for all $\kappa > \widehat{\kappa}(\sigma_0, 0)$ (i.e., in the region of parameters where there exists the symmetric equilibrium with SRI): $\Gamma(0) > 0 > \Gamma\left(\frac{N-1}{\chi}\right)$, which implies that asymmetric equilibria with $\underline{n} = 0 < \overline{n} = N - 1$ exist for all $\chi \leq \frac{N-1}{N}$. Therefore, under this condition, whenever the symmetric equilibrium features SRI, the game also admits an asymmetric equilibrium where $\mathcal{R}$ investors exclude one firm.

[20]Note that this sufficient condition imposes an upper bound on c, which is however in general compatible with the assumption $c \geq \frac{\eta\lambda^2\chi(1-\chi)}{N\kappa}$.

B.6 Proof of Proposition 4.

Considering again the equilibrium characterization as function of κ given in the previous proof, we have that the width of the region of parameters where the equilibria with $\underline{n} = \overline{n} = n$ exist is proportional to the dispersion $\overline{\sigma} - \underline{\sigma}$ in the equilibrium CSR policies:[21]

$$\widehat{\kappa}(\underline{\sigma}, n) - \widehat{\kappa}(\overline{\sigma}, n) = \frac{\chi(1-\chi)\lambda}{N\chi - n}(\overline{\sigma} - \underline{\sigma}).$$

From the FOC (3), we have that

$$\overline{\sigma} - \underline{\sigma} = \frac{1}{c}\left(\eta\lambda + \frac{\partial \pi_j}{\partial \sigma_j}(\overline{\sigma}, \underbrace{\overline{\sigma}, \ldots, \overline{\sigma}}_{n-1}, \underbrace{\underline{\sigma}, \ldots, \underline{\sigma}}_{N-n}) - \frac{\partial \pi_j}{\partial \sigma_j}(\underline{\sigma}, \underbrace{\overline{\sigma}, \ldots, \overline{\sigma}}_{n}, \underbrace{\underline{\sigma}, \ldots, \underline{\sigma}}_{N-n-1})\right). \tag{B.34}$$

The difference between the two partial derivatives can be rewritten as

$$\frac{\partial}{\partial \sigma_j}\left(\pi_j(\overline{\sigma}, \underbrace{\overline{\sigma}, \ldots, \overline{\sigma}}_{n-1}, \underbrace{\underline{\sigma}, \ldots, \underline{\sigma}}_{N-n}) - \pi_j(\underline{\sigma}, \underbrace{\overline{\sigma}, \ldots, \overline{\sigma}}_{n-1}, \underbrace{\underline{\sigma}, \ldots, \underline{\sigma}}_{N-n})\right) +$$
$$+\frac{\partial}{\partial \sigma_j}\left(\pi_j(\underline{\sigma}, \underbrace{\overline{\sigma}, \ldots, \overline{\sigma}}_{n-1}, \underbrace{\underline{\sigma}, \ldots, \underline{\sigma}}_{N-n}) - \pi_j(\underline{\sigma}, \underbrace{\overline{\sigma}, \ldots, \overline{\sigma}}_{n}, \underbrace{\underline{\sigma}, \ldots, \underline{\sigma}}_{N-n-1})\right) =$$
$$= \int_{\underline{\sigma}}^{\overline{\sigma}} \frac{\partial^2 \pi_j}{\partial \sigma_j^2}(\sigma, \underbrace{\overline{\sigma}, \ldots, \overline{\sigma}}_{n-1}, \underbrace{\underline{\sigma}, \ldots, \underline{\sigma}}_{N-n})\, \mathrm{d}\sigma - \int_{\underline{\sigma}}^{\overline{\sigma}} \frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_{-j}}(\underline{\sigma}, \sigma, \underbrace{\overline{\sigma}, \ldots, \overline{\sigma}}_{n-1}, \underbrace{\underline{\sigma}, \ldots, \underline{\sigma}}_{N-n-1})\, \mathrm{d}\sigma. \tag{B.35}$$

The first term is always weakly negative (as $\frac{\partial^2 \pi_j}{\partial \sigma_j^2} \leq 0$), whilst the second term is unambiguously positive under strategic complementarity ($\frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_{-j}} > 0$), and negative under strategic substitutability ($\frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_{-j}} < 0$). As this term appears with a negative sign, $\overline{\sigma} - \underline{\sigma}$ is higher under strategic substitutability whenever the first term is not too large (i.e., small in absolute value) under strategic complementarity relative to strategic substitutability.

This is always true if $\frac{\partial^2 \pi_j}{\partial \sigma_j^2}$ is a function of σ_j only. In this case, the value of the first term does not depend on the nature of strategic interactions among firms. To see this, suppose that the expected

[21] The argument n is omitted throughout the proof for notational simplicity.

profit can be written as

$$\pi_j \equiv F(\sigma_j) + \sigma_j G(\vec{\sigma}_{-j}) + H(\sigma_{-j}), \tag{B.36}$$

for three real-valued functions $F(\cdot)$, $G(\cdot)$ and $H(\cdot)$, where strategic substitutability (resp. complementarity) corresponds to $\frac{\partial^2 \pi_j}{\partial \sigma_j \sigma_{-j}} = G'(\cdot) < 0$ (resp. $G'(\cdot) > 0$), and so the nature of strategic interactions does not affect $\frac{\partial^2 \pi_j}{\partial \sigma_j^2} = F''(\sigma_j)$. In this case, Equation (B.34) can be written as

$$c(\overline{\sigma} - \underline{\sigma}) + F'(\underline{\sigma}) - F'(\overline{\sigma}) - \eta\lambda = -\int_{\underline{\sigma}}^{\overline{\sigma}} \frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_{-j}}(\underline{\sigma}, \sigma, \underbrace{\overline{\sigma}, \ldots, \overline{\sigma}}_{n-1}, \underbrace{\underline{\sigma}, \ldots, \underline{\sigma}}_{N-n-1})\, \mathrm{d}\sigma. \tag{B.37}$$

By the SOC of the firms' problem at the investment stage, the LHS is an increasing function of $(\overline{\sigma} - \underline{\sigma})$,[22] and, as argued above, the RHS is higher under strategic substitutability than under strategic complementarity. As a result, $\overline{\sigma} - \underline{\sigma}$ must be larger under strategic substitutability.

Finally, as strategic substitutability strengthens — i.e., $|\frac{\partial^2 \pi_j}{\partial \sigma_j \partial \sigma_{-j}}|$ increases — the RHS of Equation (B.37) increases, and so $\overline{\sigma} - \underline{\sigma}$ increases.

B.7 Proof of Proposition 5.

From Lemma 1 we know that, regardless of whether firms' CSR policies are strategic substitutes or complements, firms' investments are higher in the symmetric equilibrium with SRI than in the symmetric equilibrium without SRI: $\sigma^* > \sigma_0$, and so $\sum_j \sigma^* = N\sigma^* > \sum_j \sigma_0 = N\sigma_0$.

In any equilibrium featuring SRI with concentration, under strategic complementarity $\sigma_j^* > \sigma_0$ for all j (recall Lemma 1), and so again $\sum_j \sigma_j^* > N\sigma_0$. By contrast, under strategic substitutability, in an equilibrium with concentrated SRI, it may be that $\sum_j \sigma_j^* < N\sigma_0$. This possibility result is shown in Figure 1. In this example, the symmetric equilibrium with no SRI coexists with the equilibrium, featuring SRI concentration, with full separation and $n = 1$, and aggregate CSR investments are higher in the former.

[22] The derivative of the LHS with respect to $\overline{\sigma}$ (resp. $\underline{\sigma}$) is $c - F''(\overline{\sigma})$ (resp. $F''(\underline{\sigma}) - c$), which must be positive (resp. negative) for the SOC to hold. This condition is implied by Assumption 1(i).

B.8 Proof of Proposition 6.

The expected consumer surplus is

$$CS = \sum_{\tilde{n}_g=0}^{N} CS(\tilde{n}_g) \Pr[n_g = \tilde{n}], \tag{B.38}$$

where $CS(\tilde{n}_g)$ denotes consumer surplus when $\tilde{n}_g$ firms are green.[23] For any firm j, we have

$$\Pr[n_g = \tilde{n}_g] = \sigma_j \Pr[n_{-j,g} = \tilde{n}_g - 1] + (1 - \sigma_j) \Pr[n_{-j,g} = \tilde{n}_g], \quad \forall \tilde{n}_g = 1, \dots, N-1, \tag{B.39}$$

where $n_{-j,g} \equiv \sum_{-j \in \{\mathcal{J} \setminus \{j\}\}} a_{-j} \in \{0, \dots, N-1\}$ is the number of firm j's green rivals, and

$$\Pr[n_g = 0] = (1 - \sigma_j) \Pr[n_{-j,g} = 0], \quad \Pr[n_g = N] = \sigma_j \Pr[n_{-j,g} = N-1]. \tag{B.40}$$

Substituting these into CS and rearranging gives

$$CS = \sum_{\tilde{n}_g=0}^{N-1} \Pr[n_{-j,g} = \tilde{n}_g][(1-\sigma_j) CS(\tilde{n}_g) + \sigma_j CS(\tilde{n}_g + 1)], \tag{B.41}$$

whereby, under Assumption 2,

$$\frac{\partial CS}{\partial \sigma_j} = \sum_{\tilde{n}_g=0}^{N-1} \Pr[n_{-j,g} = \tilde{n}_g][CS(\tilde{n}_g + 1) - CS(\tilde{n}_g)] > 0 \quad \forall \sigma_j. \tag{B.42}$$

Therefore, as $\sigma^* > \sigma_0$ for all j, and making explicit the dependence of expected consumer surplus on firms' CSR investments,

$$CS(\sigma_0, \sigma_0, \dots, \sigma_0) < CS(\sigma^*, \sigma_0, \dots, \sigma_0) < CS(\sigma^*, \sigma^*, \dots, \sigma_0) < \dots < CS(\sigma^*, \sigma^*, \dots, \sigma^*), \tag{B.43}$$

where the first inequality follows from $\frac{\partial CS}{\partial \sigma_1} > 0$ holding fixed $\sigma_j = \sigma_0$ for all $j > 1$, the second inequality follows from $\frac{\partial CS}{\partial \sigma_2} > 0$ holding fixed $\sigma_1 = \sigma^*$ and $\sigma_j = \sigma_0$ for all $j > 2$, and so on.

Similarly, under strategic complementarity, the result that in any equilibrium with SRI $\sigma_j^* > \sigma_0$ for all j (see Lemma 1) implies that $CS(\vec{\sigma}_0) < CS(\vec{\sigma}_j^*)$.

Under strategic substitutability instead, considering, for instance, the equilibrium where $s_j^{\mathcal{R}} = 1$ one for firm j, and $\vec{s}_{-j}^{\mathcal{R}} = 0$, by Lemma 1 we have that $\sigma_j \equiv \overline{\sigma} > \sigma_0 > \sigma_{-j} \equiv \underline{\sigma}$ and, as a result, it

[23] That is, $CS(\tilde{n}_g) = \int_0^1 [u(\vec{x}_h) - \vec{x}_h' \vec{p}] dh$, where $\vec{p}$ and $\vec{x}_h$ are the equilibrium prices and consumers' demand when $\tilde{n}_g$ firms in the market are green.

might be that $CS(\sigma_0, \sigma_0, ..., \sigma_0) > CS(\overline{\sigma}, \underline{\sigma}, ..., \underline{\sigma})$. To see this, consider the product market game described in Example 1. We have that

$$\begin{aligned} CS(0) &= (1-\chi_c)\frac{(\beta-\gamma)^2}{2\alpha} < CS(1) = (1-\chi_c)\frac{(\beta-\gamma)^2}{2\alpha} + \chi_c\frac{(\beta-\gamma)^2}{8\alpha} < CS(2) = ... = CS(N) \\ &= \frac{(\beta-\gamma)^2}{2\alpha}. \end{aligned} \tag{B.44}$$

B.9 Proof of Proposition 7.

The conditional probability ζ is given by:

$$\zeta(\vec{\sigma}) \equiv \Pr[n_g = 1 | n_g \geq 1] = \frac{\Pr[n_g = 1]}{\Pr[n_g \geq 1]} = \frac{\Pr[n_g = 1]}{1 - \Pr[n_g = 0]} = \frac{\sum_{j=1}^{N} \sigma_j \cdot \prod_{y \neq j}(1-\sigma_y)}{1 - \prod_{j=1}^{N}(1-\sigma_j)}. \tag{B.45}$$

In a symmetric equilibrium where $\sigma_j \equiv \sigma$ for all j, we have that

$$\zeta = \frac{N\sigma(1-\sigma)^{N-1}}{1-(1-\sigma)^N} \tag{B.46}$$

is such that

$$\frac{\partial \zeta}{\partial \sigma} = -\frac{N(1-\sigma)^{N-2}((1-\sigma)^N + N\sigma - 1)}{(1-(1-\sigma)^N)^2} < 0 \Leftrightarrow (1-\sigma)^N > 1 - N\sigma, \tag{B.47}$$

which holds for all $\sigma \in [0,1]$ and $N \geq 2$ (Bernoulli inequality). Therefore, as (both under strategic substitutability and complementarity) the symmetric equilibrium with SRI features higher CSR investments than the symmetric equilibrium without SRI, we can conclude that SRI without concentration always reduces the probability ζ.

In any asymmetric equilibrium, differentiating ζ with respect to σ_j for an arbitrary j yields

$$\begin{aligned} \frac{\partial \zeta}{\partial \sigma_j} &= \left[\prod_{y \neq j}(1-\sigma_y) - \sum_{y \neq j} \sigma_y \prod_{k \neq j,y}(1-\sigma_k)\right]\left[1 - (1-\sigma_j)\prod_{y \neq j}(1-\sigma_y)\right] + \\ &- \left[\sigma_j \prod_{y \neq j}(1-\sigma_y) + (1-\sigma_j)\sum_{y \neq j}\sigma_y \prod_{k \neq j,y}(1-\sigma_k)\right]\prod_{y \neq j}(1-\sigma_y) = \\ &= \left[1 - \prod_{y \neq j}(1-\sigma_y)\right]\prod_{y \neq j}(1-\sigma_y) - \sum_{y \neq j}\sigma_y \prod_{k \neq j,y}(1-\sigma_k). \end{aligned} \tag{B.48}$$

As

$$\sum_{y\neq j}\sigma_y\prod_{k\neq j,y}(1-\sigma_k)=\prod_{y\neq j}(1-\sigma_y)\sum_{y\neq j}\frac{\sigma_y}{1-\sigma_y}, \tag{B.49}$$

we have

$$\frac{\partial\zeta}{\partial\sigma_j}<0\Leftrightarrow 1-\sum_{y\neq j}\frac{\sigma_y}{1-\sigma_y}-\prod_{y\neq j}(1-\sigma_y)<0. \tag{B.50}$$

Then, as $\sum_{y\neq j}\frac{\sigma_y}{1-\sigma_y}>\sum_{y\neq j}\sigma_y$, a sufficient condition for Inequality (B.50) to hold is

$$1-\sum_{y\neq j}\sigma_y<\prod_{y\neq j}(1-\sigma_y), \tag{B.51}$$

which holds for any $\sigma_y<1\,\forall y\neq j$ (generalized Bernoulli inequality).

Therefore, as, with strategic complementarity, in any equilibrium with SRI $\sigma_j^*>\sigma_0\,\forall j$, we have

$$\zeta(\sigma_0,\sigma_0,\dots,\sigma_0)>\zeta(\sigma_1^*,\sigma_0,\dots,\sigma_0)>\zeta(\sigma_1^*,\sigma_2^*,\dots,\sigma_0)>\dots>\zeta(\sigma_1^*,\sigma_2^*,\dots,\sigma_N^*), \tag{B.52}$$

where the first inequality follows from $\frac{\partial\zeta}{\partial\sigma_1}<0$ holding fixed $\vec{\sigma}_{-1}$, the second inequality from $\frac{\partial\zeta}{\partial\sigma_2}<0$ holding fixed $\vec{\sigma}_{-2}$, and so on.

By contrast, under strategic substitutability, SRI with concentration may increase ζ. To see this, consider the possibility result in Figure 4.

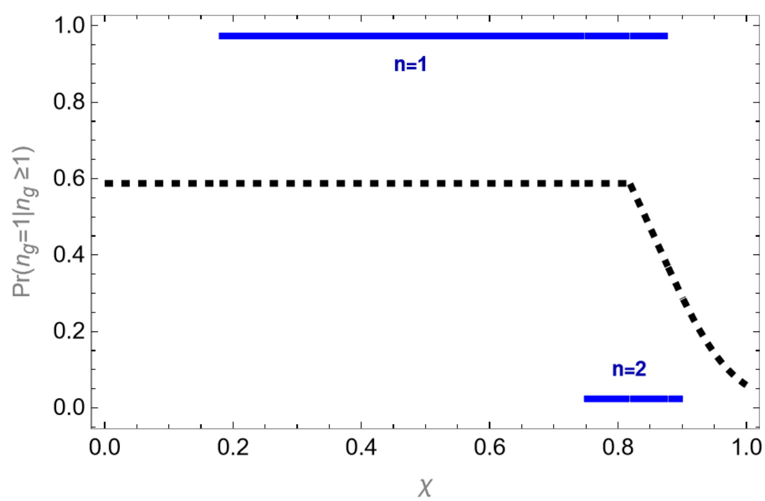


Figure 4: This figure plots the probability of having one green product, conditional on having at least one green product, as a function of the mass of responsible investors χ. The dotted black line corresponds to the symmetric equilibrium, and the solid blue lines to the asymmetric equilibria described in Part 1 of Proposition 2, where n denotes the number of firms that are exclusively held by responsible investors. Parameters: $N=6, c=\frac{33}{15}, \lambda=12, \eta=\kappa=\frac{1}{10}$. We set $\pi_j=\sigma_j\prod_{k\neq j}(1-\sigma_k)$; we micro-found this functional form in Example 1.

B.10 Proof of Proposition 8.

Suppose that a welfare-maximizing planner can choose shareholdings, CSR policies, and product prices and consider the model specification described in Example 1. Then, from the definition of social welfare in Equation (18), it follows that the maximization of expected surplus in the product market (first term) requires all firms being green and pricing at marginal cost. The trading costs (second term) do not (directly) depend on $\vec{\sigma}$. Finally, the third term is minimized when $\sigma_j = \min\{\lambda/c, 1\}\ \forall j \in \mathcal{J}$. From the remarks above, it follows that the social optimum prescribes $\sigma_j = 1\ \forall j \in \mathcal{J}$ for all $c < \lambda$.[24] Whenever this holds, under-investment unambiguously emerges in all equilibria of the game.

As for the aggregate trading cost, we can show the following result:

Lemma 3 *Suppose the symmetric equilibrium without SRI coexists with asymmetric equilibria. Then, the presence of concentrated SRI unambiguously lowers aggregate trading costs.*

Proof. In the symmetric equilibrium without SRI, $\mathcal{R}$ investors are crowded out from the market and so do not incur trading costs, whereas each $\mathcal{N}$ investor buys shares

$$s_i^{\mathcal{N}} = \frac{1}{\kappa}(\Pi_0 - p_0) = \frac{N}{1-\chi},$$

where the equality follows from the market clearing condition, and so aggregate trading costs are

$$(1-\chi)\frac{\kappa}{2}(s_i^{\mathcal{N}})^2 = \frac{\kappa N^2}{2(1-\chi)^2}.$$

In the asymmetric equilibria with $\underline{n} = \overline{n} = n$, denoting by $\overline{\Pi}$, $\overline{p}$ and $\overline{\sigma}$ (resp. $\underline{\Pi}$, $\underline{p}$ and $\underline{\sigma}$) the expected profits, share price and CSR policy of a firm held by $\mathcal{R}$ (resp. $\mathcal{N}$) investors, we have that each $\mathcal{R}$ investor buys

$$s_i^{\mathcal{R}} = \frac{1}{\kappa}\left[\overline{\Pi} - \overline{p} - \lambda(1-\overline{\sigma})\right] = \frac{n}{\chi},$$

and each $\mathcal{N}$ investor buys

$$s_i^{\mathcal{N}} = \frac{1}{\kappa}(\overline{\Pi} - \overline{p}) = \frac{N-n}{1-\chi},$$

[24] This condition is, in general, compatible with Assumption 1(i).

where in both cases, the equality follows from the market clearing conditions. The aggregate trading cost is then

$$\chi \frac{\kappa}{2}(s_i^{\mathcal{R}})^2 + (1-\chi)\frac{\kappa}{2}(s_i^{\mathcal{N}})^2 = \frac{\kappa}{2}\left[\frac{n^2}{\chi} + \frac{(N-n)^2}{1-\chi}\right].$$

This is lower than the aggregate trading costs in the equilibrium with no SRI for all $\chi > \frac{n}{2N}$, which is implied by the existence condition of these asymmetric equilibria: $\frac{n}{2N} < \frac{n}{N} < \widehat{\chi}(\overline{\sigma}, n)$.

Finally, consider any asymmetric equilibrium with $\underline{n} < \overline{n}$, and let $\widehat{\Pi}$, $\widehat{p}$ and $\widehat{\sigma}$ denote the expected profit, shares price, and CSR policy of any firm in which both types of investors hold positive shares. From the optimal shares demand, and using the indifference and market clearing conditions, we have that each $\mathcal{R}$ investor buys

$$s_i^{\mathcal{R}} = \frac{1}{\kappa}\left[\widehat{\Pi} - \widehat{p} - \lambda(1-\widehat{\sigma})\right] = \frac{1}{\kappa}\left[N\kappa - \lambda(1-\chi)(1-\widehat{\sigma})\right],$$

and each $\mathcal{N}$ investor buys

$$s_i^{\mathcal{N}} = \frac{1}{\kappa}\left[\widehat{\Pi} - \widehat{p}\right] = \frac{1}{\kappa}\left[N\kappa + \lambda\chi(1-\widehat{\sigma})\right],$$

so that the aggregate trading cost is

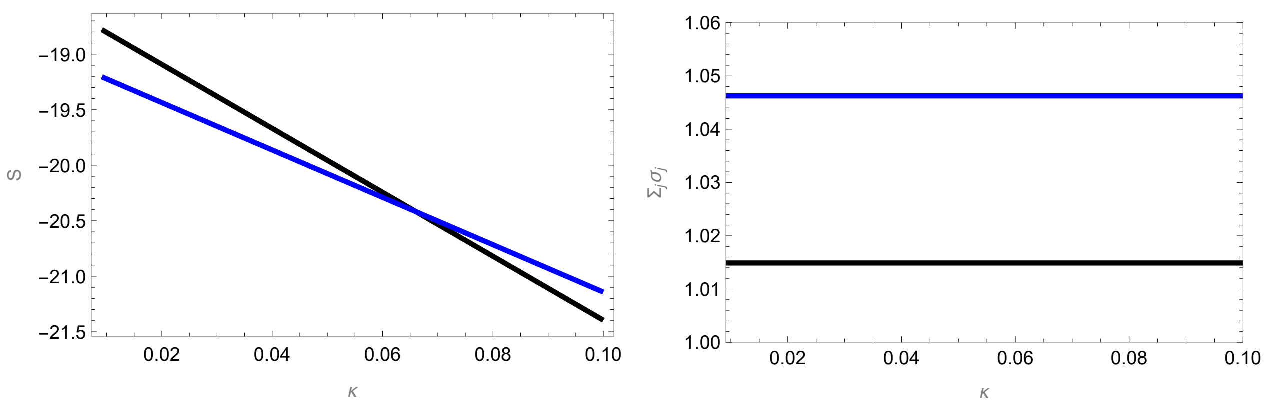
$$\chi \frac{\kappa}{2}(s_i^{\mathcal{R}})^2 + (1-\chi)\frac{\kappa}{2}(s_i^{\mathcal{N}})^2 = \frac{1}{2}\left[N^2\kappa + \frac{\lambda^2\chi(1-\chi)(1-\widehat{\sigma})^2}{\kappa}\right].$$

This is lower than the aggregate trading cost in the equilibrium without SRI for all $\chi > \widehat{\chi}(\widehat{\sigma}, 0)$, which is implied by the existence condition $\chi > \widehat{\chi}(\widehat{\sigma}, \underline{n})$ for these asymmetric equilibria, given that $\widehat{\chi}(\cdot)$ is increasing in n and so $\widehat{\chi}(\widehat{\sigma}, \underline{n}) > \widehat{\chi}(\widehat{\sigma}, 0)$.■

We continue with the proof of Proposition 8. Under strategic complementarity, all firms' CSR investments are higher with concentrated SRI than with no SRI. Therefore, even if concentrated, SRI reduces the underinvestment problem, reduces the aggregate trading costs, and also increases expected product market surplus. This is because in any equilibrium with SRI $\sigma_j^* > \sigma_0\, \forall j$ and, by Assumption 2, product market surplus is increasing in the number of green firms (the proof is analogous to the proof of Proposition 6). As a result, concentrated SRI unambiguously increases total surplus relative to no SRI.

By contrast, under strategic substitutability, concentrated SRI, by crowding out the investments of the excluded firms, may reduce aggregate CRS investments (Proposition 5) and market surplus (as we have seen in Proposition 6 for consumer surplus). These negative effects may outweigh the reduction in aggregate trading costs.

The two panels in Figure 5 display a numerical example based on the model specification given in Example 1 (and satisfying all the mentioned parametric conditions), in which the symmetric equilibrium without SRI coexists with an asymmetric equilibrium (with concentration). Total surplus and aggregate CSR investments are lower in the latter equilibrium, as shown in Panel (a) and Panel (b), respectively.



(a) Total Surplus

(b) Aggregate CSR investments

Figure 5: This figure plots total surplus S (Panel (a)) and aggregate CSR investments (Panel (b)) as a function of the trading cost κ. The blue line represents the symmetric equilibrium without SRI; the black line represents the asymmetric equilibrium with $n = 1$. The functional form for π_j is based on Example 1 with parameters: $N = 6$, $c = 8/15$, $\eta = 1/10$, $\alpha = 9/32$, $\beta = 1$, $\gamma = 0$, $\lambda = 4$, and $\chi_c = \chi_{in} = 3/8$.

Finally, in the equilibrium without SRI, $\mathcal{R}$ investors obtain zero utility because $s_{ij} = 0$ for $i \in \mathcal{R}$. It follows that $\mathcal{R}$ investors must obtain higher utility if they are willing to trade.

B.11 Analysis of Examples.

Example 1. Suppose firms' products are perfect substitutes, i.e., $u(\vec{x}) = \tilde{u}(\iota'\vec{x})$, where $u(x) \equiv -\frac{\alpha}{2}x^2 + \beta x$, with $\alpha > 0$ and $\beta \in (\gamma, \gamma + \lambda)$. The restriction $\beta > \gamma$ guarantees positive demand by $\mathcal{N}$ consumers, whereas $\beta < \gamma + \lambda$ implies that $\mathcal{R}$ consumers always *boycott* brown products — i.e., they are unwilling to purchase them even if these are priced at marginal cost.

Under these restrictions, letting $\rho = \min_{j \in \mathcal{J}} \rho_j$, $\mathcal{N}$ consumers demand

$$x^*(\rho) \equiv \max\left\{\frac{\beta - \rho}{\alpha}, 0\right\} \tag{B.53}$$

from the firm charging the lowest price (in case $n > 1$ firms charge the same minimum price ρ, we make the standard assumption that they purchase $x^*(\rho)/n$ from any of these firms), and zero from firms setting $\rho_j > \rho$. Similarly, $\mathcal{R}$ consumers buy the same quantity from the cheapest among green firms — formally, $x^*(\rho_g)$, with $\rho_g = \min_{j \in \mathcal{J}:\ a_j = 1} \rho_j$ (with a tie breaking condition analogous to that specified for $\mathcal{N}$ consumers).

As products are perfect substitutes, for all $N \geq 3$, we have:

- If all firms are brown, $\mathcal{R}$ consumers are de facto out of the market, and all firms compete à la Bertrand for $\mathcal{N}$ consumers: as a result, $\rho_j = \gamma$, and so $\pi_j = 0$, for all $j \in \mathcal{J}$.
- If exactly one firm, say j is green, this firm is a monopolist in the segment of $\mathcal{R}$ consumers, whereas all firms still compete à la Bertrand for $\mathcal{N}$ consumers. Hence, brown firms (which, as $N \geq 3$, are at least two) still set $\rho_{-j} = \gamma$, and make zero profits; the monopolist supplier of green products, as it cannot make profits in the segment of $\mathcal{N}$ consumers, optimally sets the monopoly price to $\mathcal{R}$ consumers, i.e., it solves

$$\max_{\rho_j \geq 0} \chi_c(\rho_j - \gamma)x^*(\rho_j), \tag{B.54}$$

  which gives

$$\rho_j = \rho^m \equiv \frac{\beta + \gamma}{2}, \tag{B.55}$$

  and a corresponding profit

$$\pi^m \equiv \chi_c \frac{(\beta - \gamma)^2}{4\alpha}. \tag{B.56}$$

- If there are at least two green firms, then there is perfect Bertrand competition in the segment of $\mathcal{N}$ consumers, and so $\rho_j = \gamma$ for all $j:\ a_j = 1$. As $\mathcal{R}$ consumers can buy from these firms at marginal cost, also brown firms must engage in marginal cost pricing to face positive demand: $\rho_j = \gamma$ also for $j:\ a_j = 0$. Hence, $\pi_j = 0$ for all $j \in \mathcal{J}$.

As a result, each firm j's expected profit at the investment stage (gross of CSR investments' cost) can be written as $\pi_j = \sigma_j \prod_{k \neq j}(1 - \sigma_k)\pi^m$.

Finally, let us derive consumer surplus and total market surplus (the expressions derived below have been used in the simulations to show the possibility results in Proposition 3 and 8). Substituting consumers' demand $x^*(\rho)$ back into the utility function, we find that each consumer that buys a positive quantity obtains surplus

$$CS(\rho) \equiv u(x^*(\rho)) - \rho x^*(\rho) = \frac{(\beta - \rho)^2}{2\alpha}, \tag{B.57}$$

with $CS(\rho)$ being decreasing in ρ. Note that Assumption 2 is satisfied, given that

$$CS(n_g = 0) = (1-\chi_c)CS(\gamma) < CS(n_g = 1) = (1-\chi_c)CS(\gamma)+\chi_c CS(\rho^m) < CS(n_g \geq 2) = CS(\gamma). \tag{B.58}$$

Expected consumer surplus can be written as

$$(1-\chi_c)CS(\gamma)+\sum_{j=1}^{N} \sigma_j \cdot \prod_{y \neq j}(1-\sigma_y)(\chi_c CS(\rho^m))+\left(1 - \prod_{j=1}^{N}(1 - \sigma_j) - \sum_{j=1}^{N} \sigma_j \cdot \prod_{y \neq j}(1 - \sigma_y)\right) \chi_c CS(\gamma). \tag{B.59}$$

Similarly, also product market surplus $T(n_g)$ is increasing in n_g (i.e., satisfies Assumption 2):

$$T(n_g = 0) = (1-\chi_c)CS(\gamma) < T(n_g = 1) = (1-\chi_c)CS(\gamma)+\chi_c CS(\rho^m)+\pi^m < T(n_g \geq 2) = CS(\gamma), \tag{B.60}$$

where the latter inequality follows from the standard deadweight loss from monopoly. Expected product market surplus T writes as

$$T = (1-\chi_c)CS(\gamma)+\sum_{j=1}^{N} \sigma_j \cdot \prod_{y \neq j}(1-\sigma_y)(\chi_c CS(\rho^m)+\pi^m)+\left(1 - \prod_{j=1}^{N}(1 - \sigma_j) - \sum_{j=1}^{N} \sigma_j \cdot \prod_{y \neq j}(1 - \sigma_y)\right) \chi_c CS(\gamma). \tag{B.61}$$

Example 2. Suppose firms' products are perfect complements, i.e., $u(\vec{x}) \equiv \tilde{u}(\min_{j \in \mathcal{J}} x_j)$, where $\tilde{u}(x)$ is as in Example 1. Under this utility specification, each consumer i optimally purchases the same quantity x_i from each firm (i.e., $x_{ij} = x_i \, \forall j$), with x_i determined solving

$$\max_{x_i \geq 0} \tilde{u}(x_i) - x_i \left[\sum_{j \in \mathcal{J}} \rho_j + \mathbb{1}_{i,\mathcal{R}} \lambda \sum_{j \in \mathcal{J}} (1 - a_j) \right]. \tag{B.62}$$

Solving this problem yields

$$x_i^*\left(\sum_{j\in\mathcal{J}}\rho_j\right)\equiv\max\left\{\frac{\beta-\sum_{j\in\mathcal{J}}\rho_j+\mathbb{1}_{i,\mathcal{R}}(1-a_j)}{\alpha},0\right\}. \tag{B.63}$$

We assume $\beta\in(N\gamma,N\gamma+\lambda)$, which ensures that $\mathcal{N}$ consumers have positive demand, whereas $\mathcal{R}$ consumers boycott all firms' products unless all of them are green.

Therefore, each firm j solves

$$\max_{\rho_j}(1-\chi_c+\chi_c\mathbb{1}_{\vec{a}=\iota})(\rho_j-\gamma)\frac{\beta-\sum_{j\in\mathcal{J}}\rho_j}{\alpha}, \tag{B.64}$$

which yields the unique Nash Equilibrium price

$$\rho^*=\frac{\beta+\gamma}{N+1}, \tag{B.65}$$

with corresponding demand

$$x^*=\frac{\beta-N\gamma}{\alpha(N+1)} \tag{B.66}$$

by any $\mathcal{N}$ consumer and by $\mathcal{R}$ consumers as well if and only if all firms are green.

Consumer surplus and product market surplus are given by

$$CS^*=(1-\chi_c+\chi_c\mathbb{1}_{\vec{a}=\iota})\frac{(\beta-N\gamma)^2}{2\alpha(N+1)^2} \tag{B.67}$$

and

$$T^*=(1-\chi_c+\chi_c\mathbb{1}_{\vec{a}=\iota})\frac{(\beta-N\gamma)^2(2N+1)}{2\alpha(N+1)^2}, \tag{B.68}$$

thereby Assumption 2 holds (i.e., $CS(0)=\ldots=CS(N-1)<CS(N)$, and the same for $T(n_g)$).

Example 3. In the remainder of this section, we micro-found the functional form we use for the numerical example in Panel (b) of Figure 1. Similar to Section 4.3, we use the notation $a_j=1$ ($a_j=0$) to signify that firm j uses a green (brown) technology, with $\sigma_j=\Pr[a_j=1]$, and let $\vec{a}$ denote the vector of firms' technologies. We assume that firm j generates an output $q_j=\underline{q}$ when $a_j=0$, and $q_j=\tilde{q}(\vec{a})$, with $\tilde{q}:(\mathbb{R}_+)^N\to\mathbb{R}_+$, when $a_j=1$, which captures technological spillovers across firms using the green technology. Example 3 below illustrates the specific choice of $\tilde{q}(\cdot)$ that yields the functional form we use in Figure 1. Since our focus is on technological spillovers, here we abstract from product market interactions and assume each firm sells its output at an exogenous price ρ.

Example 3 *Suppose* $\tilde{q}(\vec{a}) = \overline{q}$ *for* $\sum_{k \neq j} a_k \equiv n_{-j} \geq 1$*, and* $\tilde{q}(\vec{a}) = \underline{q}$ *otherwise. Each firm expected gross profit is* $\pi_j = \rho \left\{ \sigma_j \Pr[n_{-j} \geq 1]\overline{q} + (1 - \sigma_j \Pr[n_{-j} \geq 1])\underline{q} \right\}$*. Since* $\Pr[n_{-j} \geq 1] = 1 - \prod_{k \neq j}(1 - \sigma_k)$*, this expression simplifies to* $\pi_j = \rho \left\{ \underline{q} + \sigma_j (1 - \prod_{k \neq j}(1 - \sigma_k))(\overline{q} - \underline{q}) \right\}$.

Online Appendix

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C Data Appendix

We obtain mutual funds' stock holding information from Thomson/Refinitiv S12 (S12 hereafter). Since S12 does not have an indicator for ESG funds, following Dikolli et al. (2022), we rely on Morningstar's classification to identify ESG funds in S12 data (https://www.morningstar.com/esg-screener). We classify a fund as an ESG fund if Morningstar states that the fund's management identifies the fund as sustainability-focused in public filings ("Sustainable Investment by Prospectus"). Otherwise, we classify the fund as a non-ESG fund.

We match S12 data and Morningstar data using fund tickers. Since tickers can be reused, we manually compare fund names in S12 and Morningstar and verify the match if it can be inferred from the fund names that the same sponsor manages two funds. Throughout this process, we identify 82 ESG funds in S12 data. We define $GreenCapital_{it}$ as the aggregated value of stock holdings in firm i at year t by ESG funds. Similarly, $Non-GreenCapital_{it}$ is defined as the aggregated stock holdings in firm i at year t by non-ESG funds.

We obtain Refinitiv's ESG Combined Scores for US firms listed in NYES and NASDAQ during 2002-2020. The list of NYSE/NASDAQ-listed stocks and stock prices is obtained from CRSP.

To construct the HHI indexes for green (non-green) capital at the industry level, we first compute the share of green capital in a firm for a given year over the aggregated green (non-green) capital in the industry that the firm belongs to in the same year. The HHI indexes for green (non-green) capital are then obtained by summing the squared shares of green (non-green) capital in the industry. Similarly, we compute HHI indexes for ESG combined scores at the industry level are by summing the squared shares of ESG combined scores in a firm for a given year over the aggregated ESG combined scores in the industry that the firm belongs to in the same year. These HHI indexes allow us to measure the concentration of green and non-green capital and ESG combined scores within each industry.

D Financial market

Sections D.1 to D.5 develop the extensions discussed in Section 5.1. All proofs are in Section D.6.

D.1 Alternative trading costs

In this section, we consider an alternative specification of the trading cost, which depends on the individual holdings of each firm:

$$K(\vec{s}_{ij}) \equiv \kappa \sum_{j=1}^{N} s_{ij}^2. \tag{D.1}$$

Notice that, under the assumption that firms' terminal values are normally distributed with mean Π_j and exogenous variance $\frac{\kappa}{r}$, with $r > 0$, this specification is equivalent to a portfolio-choice problem where investors have CARA preferences with risk aversion coefficient r.

Equilibrium Analysis. For a given distribution of SRI, the optimal CSR policies are the same as in the main model, so Lemma 1 continues to hold. Moving backward to the ownership market stage, taking the FOCs of investors' problem, we now have

$$s_{ij} = \max\left\{\frac{1}{\kappa}\left[\Pi_j - p_j - \mathbb{1}_{i,\mathcal{R}}\lambda(1-\sigma_j)\right], 0\right\} \quad \forall j \in \mathcal{J}. \tag{D.2}$$

It follows that, in equilibrium, each $\mathcal{R}$ investor buys shares in any firm j for which $\Pi_j - p_j - \lambda(1-\sigma_j) > 0$, and each $\mathcal{N}$ investor buys shares in all firms. This is because (a) when $\mathcal{R}$ investors have positive demand, $\mathcal{N}$ investors must also have positive demand, and (b) in order for all markets to clear, $\mathcal{N}$ investors must have positive demand also for firms in which $\mathcal{R}$ investors have zero demand. Put differently, similar to our baseline model, in equilibrium $\Pi_j > p_j$ for all $j \in \mathcal{J}$. We then have:

Proposition 9 *All the equilibria of the game are such that $\mathcal{N}$ investors demand positive shares in all firms, whereas $\mathcal{R}$ investors demand positive shares in $n \in \{0, \dots, N\}$ firms. Any such equilibrium exists if and only if*

$$(1-\chi)\lambda(1-\overline{\sigma}(n)) < \kappa \leq (1-\chi)\lambda(1-\underline{\sigma}(n)), \tag{D.3}$$

where $\overline{\sigma}(n) > \underline{\sigma}(n)$ are the equilibrium CSR policies derived from the FOCs (3). Hence, all asymmetric equilibria of the game exhibit concentration of $\mathcal{R}$ investors in a subset of firms.

Thus, asymmetric equilibria (that feature SRI with concentration) arise also in this version of the model. Since the marginal cost of acquiring a small position in a given firm is zero, independently of the investor's holding in other firms, $\mathcal{N}$ investors invest in *all* firms in equilibrium. $\mathcal{R}$ investors, however, might still choose to exclude a subset of firms if the externalities they generate are too high. Similar to the main model, $\mathcal{R}$ investors' impact on CSR policies creates a strategic complementarity in their portfolio choices: by concentrating in a subset of firms, $\mathcal{R}$ investors have more impact on their CSR investments, which reduces the valuation gap with $\mathcal{N}$ investors and make these firms more attractive to $\mathcal{R}$ types.

Note that the equilibrium existence conditions are generally less tractable in this version of the model. The equilibrium values $(\overline{\sigma}(n), \underline{\sigma}(n))$ depend on κ (through $\mathcal{R}$ investors' aggregate demands in each firm), so the endpoints of the interval for κ defined by the existence conditions in Equation (D.3) are themselves a function of κ. This is not the case for the equilibria with full separation (where $\mathcal{R}$ and $\mathcal{N}$ investors hold different subsets of firms; see Part 1 of Proposition 2) we obtain in the main model. It follows that the analysis of the equilibria is more complicated in this setting.

Below, we illustrate the characterization of the equilibrium in a simple two-firm example using the functional form for π_j that we micro-found in Example 1 in the text.[1,2]

Example 4 *Suppose $N = 2$ and $\pi_j = \sigma_j(1-\sigma_{-j})\pi^m$, with $c > \pi^m + \eta\lambda\chi$. An equilibrium of the game always exists, and there may be more than one:*

1. *For $\kappa \leq \kappa_0 \equiv \frac{(1-\chi)\lambda c}{c+\pi^m}$, the equilibrium is unique and features no SRI. In this equilibrium, firms choose*

$$\sigma_0 = \frac{\pi^m}{c+\pi^m} \quad \forall j = 1, 2.$$

2. *For $\kappa > \kappa_0$, there exists an equilibrium featuring SRI without concentration. In this equilibrium, firms choose*

$$\sigma^* = \frac{\kappa(\pi^m + \eta\lambda\chi) - \eta\lambda^2\chi(1-\chi)}{\kappa(\pi^m + c) - \eta\lambda^2\chi(1-\chi)} \quad \forall j = 1, 2.$$

3. *For $\kappa_0 < \kappa < \frac{\eta\lambda^2\chi(1-\chi)}{c-\pi^m}$, there exists an equilibrium featuring SRI with concentration, where $\mathcal{R}$ investors buy shares only in firm j. In this equilibrium, firms choose*

$$\sigma_j^* = 1 - \frac{c\kappa(c - \pi^m - \eta\lambda\chi)}{\kappa(c^2 - (\pi^m)^2) - c\eta\lambda^2\chi(1-\chi)} > \sigma_{-j}^* = \frac{\kappa\pi^m(c - \pi^m - \eta\lambda\chi)}{\kappa(c^2 - (\pi^m)^2) - c\eta\lambda^2\chi(1-\chi)}.$$

The interval $\kappa_0 < \kappa < \frac{\eta\lambda^2\chi(1-\chi)}{c-\pi^m}$ is non-empty for all $\chi > \frac{c(c-\pi^m)}{\eta\lambda(c+\pi^m)}$.

D.2 Reverse timing

In the baseline model, the firms' objective is determined by their ownership so that CSR policies are chosen after the trading stage. In this section, we consider an alternative timing of the game and suppose that firms choose CSR policies to attract investors.

Each firm j's owner has one share to sell and commits to σ_j to maximize the share price. After each owner commits to σ_j, shares are traded. Finally, CSR policies are implemented, and firms' profits are realized and distributed to shareholders.

[1] Since $N = 2$, here we need a fringe of competitive brown firms to ensure that the product market equilibrium admits a pure-strategy equilibrium. Aside from that, the assumptions in Example 1 yield the same equilibrium expression for π_j also here.

[2] The detailed calculations for all the Examples in this Online Appendix are omitted for brevity and available upon request.

As we shall see, the equilibrium analysis becomes much more cumbersome under this alternative timing. Hence, in what follows, we again consider $N = 2$ strategic firms, and we content ourselves to show that an equilibrium where all $\mathcal{R}$ investors buy shares in firm j and all $\mathcal{N}$ investors buy shares in the other firm $-j$ can exist also under this alternative timing of the game. That is, owners of ex-ante identical firms may optimally commit to different CSR policies to target different investors.

Equilibrium Analysis. We shall derive investors' demand for any given vector $\vec{\sigma}$ of CSR policies to which firms' owners committed in the first stage. It turns out that investors' optimal choices are also as in the main model. This is because, in both models, they take $\vec{\sigma}$ as given:[3] the fact that here $\vec{\sigma}$ is observed, whereas in the main model it is correctly conjectured in equilibrium, plays no role on the equilibrium path (and off-path events are immaterial to the analysis, given the continuum of investors).

Given the two firms' CSR policies (σ_j, σ_{-j}), in an asymmetric equilibrium where all $\mathcal{R}$ investors buy shares in firm j, and all $\mathcal{N}$ investors buy shares in firm $-j$ (i.e., $\alpha_j^{\mathcal{R}} = \alpha_{-j}^{\mathcal{N}} = 1$ and $\alpha_{-j}^{\mathcal{R}} = \alpha_j^{\mathcal{N}} = 0$), market clearing prices are

$$p_j^*(\sigma_j, \sigma_{-j}) = \Pi_j(\sigma_j, \sigma_{-j}) - \frac{\kappa}{\chi} - \lambda(1 - \sigma_j), \tag{D.4}$$

and

$$p_{-j}^*(\sigma_j, \sigma_{-j}) = \Pi_{-j}(\sigma_j, \sigma_{-j}) - \frac{\kappa}{1 - \chi}. \tag{D.5}$$

In this candidate equilibrium, firm j's owner solves

$$\max_{\sigma_j} p_j^*(\sigma_j, \sigma_{-j}), \tag{D.6}$$

and similarly firm $-j$'s owner. We thus obtain the FOCs

$$\sigma_j = \frac{1}{c}\left[\lambda + \frac{\partial \pi_j}{\partial \sigma_j}(\sigma_j, \sigma_{-j})\right], \tag{D.7}$$

[3]Moreover, in both models, consumers take as given the shares' market clearing price and correctly anticipate firms' expected profits given $\vec{\sigma}$.

and

$$\sigma_{-j} = \frac{1}{c} \frac{\partial \pi_{-j}}{\partial \sigma_{-j}} (\sigma_{-j}, \sigma_j), \tag{D.8}$$

from which it follows that the candidate equilibrium CSR policies $(\sigma_j^*, \sigma_{-j}^*)$ correspond to the equilibrium CSR policies of the base model for $\eta = 1$. Hence, provided that this equilibrium exists, it has the same implications emphasized in the main model.

In the remainder of the analysis, we derive the existence conditions for this equilibrium. First, as seen in the paper, for $\mathcal{R}$ investors to optimally buy shares in firm j, and $\mathcal{N}$ investors to optimally buy shares in firm $-j$, it must be

$$\chi > \frac{1}{2} \quad \text{and} \quad \widehat{\kappa}(\sigma_j^*, 1) < \kappa < \widehat{\kappa}(\sigma_{-j}^*, 1). \tag{D.9}$$

Unlike in the base model, however, this is just a necessary existence condition for this equilibrium. The reason is that, while choosing σ_j^* is the best response to σ_{-j}^* *locally*, i.e., when taking as granted that all $\mathcal{R}$ (resp. $\mathcal{N}$) investors will buy shares in firm j (resp. $-j$), each firm's owner may have profitable *global deviations*, consisting in triggering different purchasing behaviors by investors. All possible global deviations are listed below:

1. Firm j's global deviations consist in attracting $\mathcal{N}$ investors, and
   (a) either keep attracting also $\mathcal{R}$ investors, or
   (b) attract only $\mathcal{N}$ investors: in this case, $\mathcal{R}$ investors may either buy shares in the other firm $-j$, or be crowded out.
2. Firm $-j$'s global deviations consist in attracting $\mathcal{R}$ investors, and
   (a) either keep attracting also $\mathcal{N}$ investors, or
   (b) attract only $\mathcal{R}$ investors: in this case, $\mathcal{N}$ investors buy shares in the other firm j.

In what follows, we assume that whenever (following a global deviation) there is no demand for a firm j, its shares are sold at zero ($p_j = 0$) — this firm is still present in the product market, though.[4]

[4]Note that deviations 1(a) and 2(a), which are the more problematic for equilibrium existence (see the proof of Proposition 10 in Section D.6), could be ignored by imposing market clearing conditions for both firms, which rule out the possibility for one firm to attract all investors (this entails removing the zero lower bound constraints on share prices).

Note that similar global deviations should also be considered starting from a candidate symmetric equilibrium. We can thus argue that, rather than undermining the prevalence of asymmetric equilibria in the game, this alternative timing reduces the scope for equilibrium multiplicity.

Considering each possible global deviation separately and providing conditions that rule out each of these deviations, we can show the following possibility result:

Proposition 10 *The game may admit an equilibrium where ex-ante identical firms commit to different CSR policies $\sigma_j^* > \sigma_{-j}^*$ to maximize the respective share prices, such that all $\mathcal{R}$ investors buy shares in firm j only, and all $\mathcal{N}$ investors buy shares in firm $-j$ only.*

This result suggests that, although the firms are ex-ante identical, they may select different CSR policies in equilibrium: some firms invest more in CSR to attract responsible investors, while others invest less and focus on non-responsible investors. This differentiation makes it harder for each firm to deviate and attract both types of investors since each equilibrium policy is tailored to the preferences of a specific type of investor.

D.3 Broad mandate

Suppose responsible investors have broad mandate, that is, internalize the negative externalities generated by all firms, irrespective of how many shares of these firms they hold in their portfolios. Formally, investors' problem is as follows

$$\max_{s_{ij} \geq 0} \sum_j s_{ij} \left(\Pi_j - p_j\right) - \sum_j \mathbb{1}_{i,\mathcal{R}} \lambda (1 - \sigma_j) - K_j. \tag{D.10}$$

Equilibrium analysis. As $\mathcal{R}$ investors' penalty is independent from their portfolio choices and, being atomistic, they take all firms' CSR policies as given, this penalty acts as a fixed cost and does not distort their demand compared to $\mathcal{N}$ investors. Therefore, each investor's demand is now

$$s_{ij} = \begin{cases} \max\left\{\frac{1}{\kappa}\left[\Pi_{j^*} - p_{j^*}\right], 0\right\} & \text{for } j^* \in \text{argmax}_j \{\Pi_j - p_j\} \\ 0 & \text{for } j \neq j^*. \end{cases} \tag{D.11}$$

Note that, as all investors have the same demand, equilibria without SRI do not exist in this framework. Moreover, the following holds:

Proposition 11 *When $\mathcal{R}$ investors have a broad mandate, the game admits a unique symmetric equilibrium and a continuum of asymmetric equilibria. In some of them, $\mathcal{R}$ investors invest only in a subset of firms, and aggregate CSR investments may be lower than in the symmetric equilibrium.*

Thus, also under broad mandate, the game features a prevalence of asymmetric equilibria. Since, for given ownership structure, firms' CSR policies are chosen as in the main model, these asymmetric equilibria entail dispersion in CSR investments, with firms more targeted by $\mathcal{R}$ investors investing more in CSR policies. As a result, under strategic substitutability SRI concentration can reduce aggregate greenness relative to a scenario without SRI and, a fortiori, relative to the symmetric equilibrium where SRI is uniformly present in all firms.

Finally, it is worth noticing that the equilibrium characterization is as in the main analysis whenever $\mathcal{R}$ investors' utility features a warm-glow component, even though this is arbitrarily small relative to the broad mandate component. Formally, suppose investors maximize $\sum_j s_{ij}\left(\Pi_j - p_j - \xi\lambda(1-\sigma_j)\right) - \sum_j \mathbb{1}_{i,\mathcal{R}}\lambda(1-\sigma_j) - K_j$. Then, for any $\xi > 0$, investors' demand is as in the main model, with $\xi\lambda$ in place of λ.[5]

D.4 Cost of capital channel

This section develops a variation of the main model where firms raise capital in a competitive capital market. The modeling of the capital market is similar to Huang and Kopytov (2023).

Firm j raises an amount of capital k_j, where $k_j = \int_i s_{ij} p_j$ and p_j is j's stock price in equilibrium. Let $\vec{k}$ denote the collection of firms' capital. Having raised k_j, j generates a total payoff $\Pi_j = k_j\left(1+\pi_j\right)$, where $\pi_j \equiv \pi\left(\sigma_j, \vec{\sigma}_{-j}, \vec{k}\right)$ is its expected return on capital. We assume that π_j is positive and concave in σ_j. Similar to the main model, firms' interaction in product and input markets is such that j's returns on capital depend on the other firms' CSR policies. Since firms may differ in their size, the interdependence in firms' returns here depends on their relative size (e.g., σ_{-j} has

[5] As in firms' problem we have instead $(1+\xi)\lambda$ in place of λ, the equilibrium CSR policies for given $\vec{s}_j^{\mathcal{R}}$ will be slightly different from the ones characterized in the main model, but the equilibrium characterization provided in Proposition 1 and 2 remains unchanged.

a larger effect on π_j as k_{-j} increases and firm $-j$ grows larger). So we assume that π_j depends on the collections of both CSR policies $\vec{\sigma}$ and capital $\vec{k}$. To model the link between firm size and externalities, here we assume the externalities firms generate depend on their capital. Given σ_j and k_j, firm j then generates externalities $\lambda(1-\sigma_j)k_j$.

At time $t = 1$, each investor chooses how to allocate its endowment, which is normalized to 1, across firms. Formally, for given vectors of CSR policies $\vec{\sigma}$ and capital $\vec{k}$, investor i solves:

$$\max_{s_{ij}\geq 0} \sum_j s_{ij} \left(\Pi_j - p_j - \mathbb{1}_{i,\mathcal{R}} \lambda(1-\sigma_j)k_j \right), \tag{D.12}$$

subject to $\sum_j s_{ij} p_j \leq 1$ and $s_{ij} \geq 0$ for all $j \in \mathcal{J}$, where s_{ij} is the number of shares investor i holds in firm j, and $\mathbb{1}_{i,\mathcal{R}} = 1$ if i is responsible ($i \in \mathcal{R}$) and 0 otherwise ($i \in \mathcal{N}$).

At time $t = 2$, given the vectors of capital $\vec{k}$ and distribution of SRI $\vec{s}^{\mathcal{R}}$, firms simultaneously choose their CSR policies to solve:

$$\max_{\sigma_j \in [0,1]} \Pi_j - s_j^{\mathcal{R}} \eta \lambda(1-\sigma_j)k_j. \tag{D.13}$$

As in the baseline model, the objective in Program (D.13) represents a weighted average of the expected payoff per share to investors, where the weights are proportional to the shares held by each shareholder.

Equilibrium analysis. To simplify the exposition, we focus on the analysis of symmetric equilibria and asymmetric equilibria with full separation (i.e., $\mathcal{R}$ investors hold firms $j \leq n$ and $\mathcal{N}$ investors hold firms $j > n$).

Proposition 12 *Depending the parameters of the model, the following two types of equilibria may exist:*

- *(Symmetric equilibria.) A symmetric equilibrium always exists. It features no SRI iff $\lambda(1-\sigma_0) \geq \pi_0$, where σ_0 is the equilibrium CSR policy and π_0 the return on capital when $k_j = \frac{1-\chi}{N}$ for all $j \in \mathcal{J}$.*

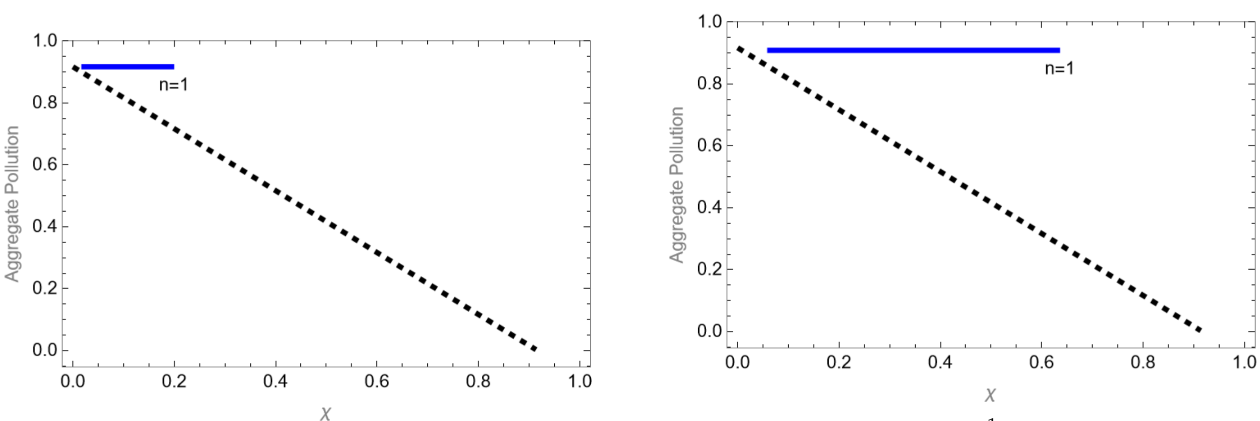
- *(Asymmetric equilibria.) Asymmetric equilibria with full separation where $\mathcal{R}$ investors target firms $j \leq n$, and CSR policies and capital are $\sigma_j = \overline{\sigma}$ and $k_j = \frac{\chi}{n}$ (resp. $\sigma_j = \underline{\sigma}$ and $k_j = \frac{1-\chi}{N-n}$) for $j \leq n$ (resp. $j > n$), exist if $\lambda(\overline{\sigma} - \underline{\sigma}) > \pi^*_{j>n} - \pi^*_{j\leq n} > 0$, where π^*_j is j's return on capital in equilibrium.*

The equilibrium CSR policies are characterized in the proof of Proposition 12 in Online Appendix D.6.

Example 5 *Suppose that, given capital k_j, firm j produces an output k_j which sells at a price $1 + \rho\sigma_j$, where ρ represents the pricing of CSR in product markets. Firm j generates externalities $\lambda(1 - \sigma_j)k_j$ if it uses an amount $\sigma_j k_j$ of an emission-reducing technology g, which is offered by a competitive upstream market at price p_g. Each upstream firm pays a cost $\frac{c}{2}x^2$ for producing a quantity x of g, which leads to an upward-sloping aggregate supply function $S(p_g) = \frac{1}{c}p_g$.[6] The aggregate demand for g is $\sum_j \sigma_j k_j$, so market clearing implies $p_g = c\sum_j \sigma_j k_j$. It follows that j's expected return on capital is $\pi_j = \sigma_j(\rho - c\sum_{s=1}^{N} \sigma_s k_s)$.*

- *In the symmetric equilibrium without SRI, we have $\sigma_0 = \frac{N\rho}{c}\left[(1-\chi)(1+N)\right]^{-1}$ and $\pi_0 = \sigma_0\left[\rho - c\sigma_0(1-\chi)\right]$; aggregate pollution is $\lambda(1-\sigma_0)(1-\chi)$.*
- *In the asymmetric equilibrium where $\mathcal{R}$ investors target firms $j \le n$, we have $\overline{\sigma} = \frac{n(\rho+\eta\lambda(N-n+1))}{c(N+1)\chi}$ and $\underline{\sigma} = \frac{(N-n)(\rho-\eta\lambda n)}{c(N+1)(1-\chi)}$; aggregate pollution is $\lambda\left[1 - \overline{\sigma}\chi - \underline{\sigma}(1-\chi)\right]$.*

Figure OA1 shows that aggregate pollution can be higher in the equilibrium with concentrated SRI compared to the one without SRI.



(a) $\eta = 0$ **(b)** $\eta = \frac{1}{10}$

Figure OA1: This figure plots aggregate pollution based on the specification in Example 5. Parameters: $N = 5, \rho = \frac{2}{10}, \lambda = 1, c = 2$.

D.5 Social responsibility and markups

In this section, we consider a variation of the product market model described in Section 4.3 and assume that responsible investors care about firms' markups alongside their negative externalities.

[6] The technology g may represent a technology to reduce the firm's toxic emissions or a less polluting (*green*) input in its production process. In Online Appendix E.2, we consider a more general model with markets for both green and brown inputs.

Consistent with our baseline model, we assume that $\mathcal{R}$ investors only care about the actions taken by firms in their portfolio. Therefore, they suffer disutility from holding firms that (a) generate negative externalities and (b) charge positive markups to consumers. In equilibrium, the disutility related to markups will affect firms' pricing strategies, similar to how the disutility related to the pollution externalities influences firms' CSR investments.

Let $\mu_j = \rho_j - \gamma$ denote firm j's markup. Consider, for simplicity, the model with binary production technologies $a_j \in \{0, 1\}$, with $\Pr[a_j = 1] = \sigma_j$. For a given vector of CSR policies $\vec{\sigma}$, investor i solves:

$$\max_{s_{ij} \geq 0} \sum_j s_{ij} \left\{ \Pi_j - p_j - \mathbb{1}_{i,\mathcal{R}} \lambda (1 - \sigma_j) - \mathbb{1}_{i,\mathcal{R}} \delta \mathbb{E} \left[\mu_j \right] \right\} - K_j, \tag{D.14}$$

where $\delta \geq 0$ captures the extent to which $\mathcal{R}$ investors suffer disutility from holding firms that charge markups. The expectation in Eqn. (D.14) is taken with respect to the random vector of firms' types $\vec{a}$ (like for the main model, prices and profits are evaluated at their equilibrium values for any realization of $\vec{a}$).

Note that the modeling of consumers does not change in this variation of the model. Thus, for given $\vec{a}$ and $\vec{\rho}$, their consumption decisions continue to solve Program (14). Since $\mathcal{R}$ investors care about markups, firms' pricing decisions depend on the distribution of SRI ($\vec{s}^{\mathcal{R}}$). For given $\vec{a}$ and $\vec{s}^{\mathcal{R}}$, fixing at equilibrium the prices charged by its competitors, firm j chooses the product price ρ_j to maximize its profit minus the markup disutility suffered by its shareholders:

$$\max_{\mu_j \geq 0} \mu_j \left(\int_0^1 x_{hj}(\vec{a}, \vec{\rho}) \, dh - \eta_2 s_j^{\mathcal{R}} \delta \right), \tag{D.15}$$

where $\eta_2 \in [0, 1]$ captures the extent to which firms internalize shareholders' markup preferences in their pricing strategies. Assuming that the price competition game admits a pure-strategy equilibrium for all $\vec{a}$, we can then write firms' expected profits (gross of the CSR investments costs) as $\pi_j \equiv \mathbb{E} \left[\mu_j \int_0^1 x_{hj}(\vec{a}, \vec{\rho}) \, dh \right]$, where the expectation is taken with respect to $\vec{a}$, and product prices and consumers' demands are evaluated at their equilibrium values for any realization of $\vec{a}$.

Moving one step backward, at the beginning of $t = 2$ and for a given $\vec{s}^{\mathcal{R}}$, firms simultaneously choose their CSR policies σ_j to solve:

$$\max_{\sigma_j \in [0,1]} \Pi_j - s_j^{\mathcal{R}} \left(\eta\lambda(1-\sigma_j) + \eta_2 \delta \mathbb{E}\left[\mu_j\right] \right). \tag{D.16}$$

As in the baseline model, the objectives in Programs (D.15) and (D.16) represent weighted averages of the expected payoff per share to investors, where the weights are proportional to the shares held by each shareholder.

Equilibrium analysis. To simplify the exposition, we focus on the analysis of symmetric equilibria and asymmetric equilibria with full separation (i.e., $\mathcal{R}$ investors hold firms $j \leq n$ and $\mathcal{N}$ investors hold firms $j > n$). Similar to the main model, it is helpful to introduce the function $\widetilde{\chi}$, which will be used as a threshold for the fraction χ of $\mathcal{R}$ investors:

$$\widetilde{\chi}(\sigma, \mathbb{E}[\mu], n) = \frac{1}{2} + \frac{\sqrt{4\kappa n(\delta\mathbb{E}[\mu] + \lambda(1-\sigma)) + (\delta\mathbb{E}[\mu] + \lambda(1-\sigma) - N\kappa)^2} - N\kappa}{2(\delta\mathbb{E}[\mu] + \lambda(1-\sigma))}, \tag{D.17}$$

for $\sigma \in [0,1]$, $n \in \{0, \dots, N\}$, and $\mathbb{E}[\mu] > 0$; $\widetilde{\chi}$ equals $\widehat{\chi}$ from the main model when $\delta = 0$.

Proposition 13 *The equilibria of the game are as follows:*

- *(Symmetric equilibrium.) The symmetric equilibrium always exists and is unique. It features no SRI if and only if $\chi \leq \widetilde{\chi}(\sigma_0, M_0, 0)$, where σ_0 and M_0 are the equilibrium CSR policies and expected markup, respectively. The threshold $\widetilde{\chi}(\sigma_0, M_0, 0)$ is increasing in δ.*
- *(Asymmetric equilibria.) Asymmetric equilibria with full separation where $\mathcal{R}$ investors target firms $j \leq n$, and CSR policies and expected markups are $\sigma_j = \overline{\sigma}$ and $\mu_j = \hat{\mu}$ (resp. $\sigma_j = \underline{\sigma}$ and $\mu_j = \tilde{\mu}$) for $j \leq n$ (resp. $j > n$), exist if and only if $\widetilde{\chi}(\overline{\sigma}, \hat{\mu}, n) < \chi < \widetilde{\chi}(\underline{\sigma}, \tilde{\mu}, n)$, where $\overline{\sigma}$ is decreasing in δ.*

The equilibrium CSR policies and expected markups are characterized in the proof of Proposition 13 in Online Appendix D.6.

The equilibrium characterization follows the same logic as in the main model, so we refer to Section 3 for a more detailed description of the steps and a discussion of the intuitions. Two novel results are worth discussing here. First, the additional objective to control markup increases $\mathcal{R}$

investors' penalty relative to $\mathcal{N}$ investors so that a symmetric equilibrium with SRI only exists for larger values of χ. Second, in the asymmetric equilibria with full separation, the higher the magnitude δ of $\mathcal{R}$ investors' markup disutility, the lower the expected markup and gross profit of the firms they hold in equilibrium. As a result, these firms invest less in CSR when δ increases. Put differently, $\mathcal{R}$ investors' efforts to lower firms' markups crowd out the firms' private incentives to increase their CSR investments.

Next, we use the specification of consumers' utility $u(\cdot)$ from Example 1 in Section 4.3 (where products are perfect substitutes) to derive closed-form expressions for the equilibrium markups. We then use these expressions to illustrate further properties of the equilibria.

Example 6 *Suppose $u(\vec{x}) \equiv \tilde{u}(\iota'\vec{x})$, where $\tilde{u}(x) : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ is defined as $\tilde{u}(x) \equiv -\frac{\alpha}{2}x^2 + \beta x$ with $\alpha > 0$ and $\beta \in (\gamma, \gamma + \lambda)$. Consider the asymmetric equilibria with full separation. For $N \geq 3$, $\eta_2 = 1$, and $\delta < \frac{\beta\chi_c - \gamma(1-\chi_c)}{\alpha}$, consumers' demand is as in Example 1, and the product market equilibrium is such that each firm's expected gross profit is $\pi_j = \sigma_j \prod_{k \neq j}(1-\sigma_k)\left[\frac{(\beta-\gamma)^2\chi_c}{4\alpha} - \frac{\alpha(s_j^{\mathcal{R}}\delta)^2}{4\chi_c}\right]$ and its expected markup is $\mathbb{E}[\mu_j] = \sigma_j \prod_{k \neq j}(1-\sigma_k)\frac{(\beta-\gamma)\chi_c - \alpha s_j^{\mathcal{R}}\delta}{2\chi_c}$, with $s_j^{\mathcal{R}} = 1$ for $j \leq n$ and $s_j^{\mathcal{R}} = 0$ for $j > n$, since:*

1. *If $a_j = 0$ for all firms or $a_j = 1$ for at least two firms, all firms set $\rho_j = \gamma$ (hence, $\mu_j = 0$) and make zero profits;*
2. *If $a_j = 1$ for firm j only, firm j sets $\rho_j = \frac{(\beta+\gamma)\chi_c - \alpha s_j^{\mathcal{R}}\delta}{2\chi_c}$ and makes profits equal to $\frac{(\beta-\gamma)^2\chi_c}{4\alpha} - \frac{\alpha(s_j^{\mathcal{R}}\delta)^2}{4\chi_c}$. All other firms set $\rho_{-j} = \gamma$ and make zero profits.*

Figure OA2 shows that the crowding out of the private CSR incentives of the firms targeted by $\mathcal{R}$ investors can be such that $\sum_j \sigma_j$ is highest when $\delta = 0$ (i.e., when $\mathcal{R}$ investors do not care about markups).

D.6 Proofs

Proof of Proposition 9. As we have argued that $\mathcal{N}$ investors always demand positive shares in all firms, and $\mathcal{R}$ investors are symmetric, it follows that any possible equilibrium is characterized by the subset of firms targeted by $\mathcal{R}$ investors. Without loss of generality, let these firms be $j \leq n$.

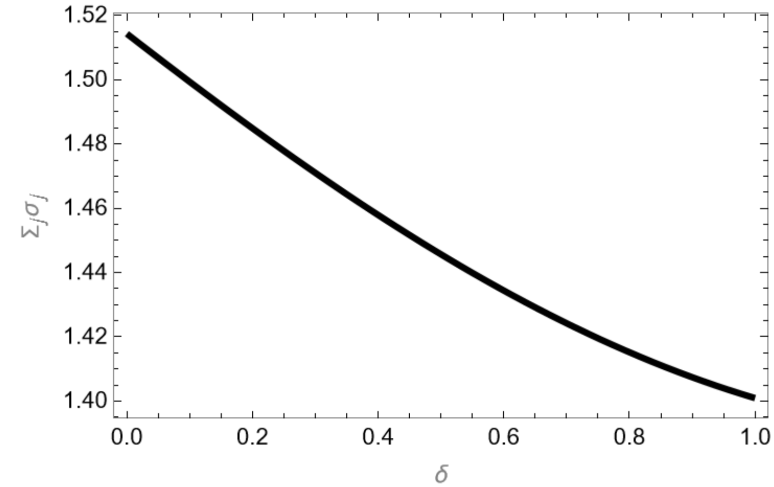


Figure OA2: Aggregate CSR investments $\sum_j \sigma_j$ as a function of responsible investors' concern for markups δ in the asymmetric equilibrium with concentration ($n = 2$); Parameters: $N = 6, c = \frac{33}{15}, \lambda = 12, \eta = \frac{1}{10}, \kappa = \chi = \chi_c = \frac{1}{2}, \eta_2 = 1$.

For any given n, the corresponding equilibrium exists when $\mathcal{R}$ investors have a positive net payoff from buying shares in firms $j \leq n$ only. That is, when

$$\Pi_j - p_j - \lambda(1-\sigma_j) = \kappa - (1-\chi)\lambda(1-\sigma_j) > 0 \Leftrightarrow \kappa > (1-\chi)\lambda(1-\sigma_j) \quad \forall j \leq n, \tag{D.18}$$

and

$$\Pi_j - p_j - \lambda(1-\sigma_j) = \frac{\kappa}{1-\chi} - \lambda(1-\sigma_j) \leq 0 \Leftrightarrow \kappa \leq (1-\chi)\lambda(1-\sigma_j) \quad \forall j > n, \tag{D.19}$$

where the equalities follow from the market clearing conditions

$$\frac{\chi}{\kappa}\left[\Pi_j - p_j - \lambda(1-\sigma_j)\right] + \frac{1-\chi}{\kappa}\left[\Pi_j - p_j\right] = 1 \quad \forall j \leq n; \quad \frac{1-\chi}{\kappa}\left[\Pi_j - p_j\right] = 1 \quad \forall j > n. \tag{D.20}$$

Lemma 1 implies that the equilibrium CSR policies satisfy

$$\sigma_1^* = \ldots = \sigma_n^* \equiv \overline{\sigma}(n) > \underline{\sigma}(n) \equiv \sigma_{n+1}^* = \ldots = \sigma_N^*, \tag{D.21}$$

where $(\overline{\sigma}, \underline{\sigma})$ solve the following system, obtained from the FOCs (3) given $\mathcal{R}$ investors' demand:

$$\begin{cases} \overline{\sigma} = \frac{1}{c}\left[\eta\lambda\chi\left(1 - \frac{(1-\chi)\lambda(1-\overline{\sigma})}{\kappa}\right) + \frac{\partial \pi_j}{\partial \sigma_j}(\overline{\sigma}, \underbrace{\overline{\sigma}, \ldots, \overline{\sigma}}_{n-1}, \underbrace{\underline{\sigma}, \ldots, \underline{\sigma}}_{N-n})\right]; \\ \underline{\sigma} = \frac{1}{c}\frac{\partial \pi_j}{\partial \sigma_j}(\underline{\sigma}, \underbrace{\overline{\sigma}, \ldots, \overline{\sigma}}_{n}, \underbrace{\underline{\sigma}, \ldots, \underline{\sigma}}_{N-n-1}). \end{cases} \tag{D.22}$$

The equilibrium existence conditions then can be written as in (D.3).

Proof of Proposition 10. In what follows, we provide conditions that rule out any of the possible global deviations identified in Section D.2:

Deviation 1(a). If firm j can attract all investors, its market clearing price is[7]

$$p_j^d(\sigma_j, \sigma_{-j}^*) = \Pi_j(\sigma_j, \sigma_{-j}^*) - \kappa - \lambda(1 - \sigma_j)\chi. \tag{D.23}$$

Given that $p_{-j}^d = 0$, all investors optimally buy shares in firm j if both

$$\Pi_j(\sigma_j, \sigma_{-j}^*) - p_j^d(\sigma_j, \sigma_{-j}^*) > \Pi_{-j}(\sigma_j, \sigma_{-j}^*), \tag{D.24}$$

and

$$\Pi_j(\sigma_j, \sigma_{-j}^*) - p_j^d(\sigma_j, \sigma_{-j}^*) - \lambda(1 - \sigma_j) > \Pi_{-j}(\sigma_j, \sigma_{-j}^*) - \lambda(1 - \sigma_{-j}^*), \tag{D.25}$$

are satisfied. These two conditions boil down to

$$\kappa > \kappa_{j,1}(\sigma_j) \equiv \Pi_{-j}(\sigma_j, \sigma_{-j}^*) - \lambda\chi(1 - \sigma_j), \tag{D.26}$$

and

$$\kappa > \kappa_{j,2}(\sigma_j) \equiv \Pi_{-j}(\sigma_j, \sigma_{-j}^*) - \lambda[1 - \sigma_{-j}^* - (1 - \chi)(1 - \sigma_j)]. \tag{D.27}$$

The condition for the deviation to be profitable, $p_j^d(\sigma_j, \sigma_{-j}^*) > p_j^*(\sigma_j^*, \sigma_{-j}^*)$, boils down to

$$\kappa > \kappa_{j,3}(\sigma_j) \equiv \frac{\chi}{1 - \chi}\left(\Pi_j(\sigma_j^*, \sigma_{-j}^*) - \Pi_j(\sigma_j, \sigma_{-j}^*) - \lambda(1 - \sigma_j^*) + \lambda\chi(1 - \sigma_j)\right). \tag{D.28}$$

Hence, for a given vector of parameters, this deviation destroys the considered equilibrium if there exists a value σ_j^d such that $\kappa > \max\{\kappa_{j,1}(\sigma_j^d), \kappa_{j,2}(\sigma_j^d), \kappa_{j,3}(\sigma_j^d)\}$.

Deviation 1(b). If firm j attracts $\mathcal{N}$ investors only, its market clearing price is

$$p_j^d(\sigma_j, \sigma_{-j}^*) = \Pi_j(\sigma_j, \sigma_{-j}^*) - \frac{\kappa}{1 - \chi}. \tag{D.29}$$

Supposing such deviation is implementable, it would be profitable if σ_j satisfies $p_j^*(\sigma_j^*, \sigma_{-j}^*) <$

[7]The superscript d, standing for *deviation*, is consistently used throughout.

$p_j^d(\sigma_j, \sigma_{-j}^*)$. This admits no solution for all

$$\kappa > \underline{\kappa} \equiv \frac{\chi(1-\chi)}{2\chi - 1}\left(\Pi_j(\sigma_j^d, \sigma_{-j}^*) - \Pi_j(\sigma_j^*, \sigma_{-j}^*) + \lambda(1-\sigma_j^*)\right), \tag{D.30}$$

where $\sigma_j^d \equiv \arg\max_{\sigma_j} p_j^d(\sigma_j, \sigma_{-j}^*)$. It follows that $\kappa > \underline{\kappa}$ is a sufficient condition to rule out the considered deviation.

Deviation 2(a). If firm $-j$ can attract all investors, its market clearing price is

$$p_{-j}^d(\sigma_j^*, \sigma_{-j}) = \Pi_{-j}(\sigma_j^*, \sigma_{-j}) - \kappa - \lambda(1-\sigma_{-j})\chi. \tag{D.31}$$

Given that $p_j^d = 0$, all investors optimally buy shares in firm $-j$ if both

$$\Pi_j(\sigma_j^*, \sigma_{-j}) < \Pi_{-j}(\sigma_j^*, \sigma_{-j}) - p_{-j}^d(\sigma_j^*, \sigma_{-j}), \tag{D.32}$$

and

$$\Pi_j(\sigma_j^*, \sigma_{-j}) - \lambda(1-\sigma_j^*) < \Pi_{-j}(\sigma_j^*, \sigma_{-j}) - p_{-j}^d(\sigma_j^*, \sigma_{-j}) - \lambda(1-\sigma_{-j}), \tag{D.33}$$

are satisfied. These two conditions boil down to

$$\kappa > \kappa_{-j,1}(\sigma_{-j}) \equiv \Pi_j(\sigma_j^*, \sigma_{-j}) - \lambda\chi(1-\sigma_{-j}), \tag{D.34}$$

and

$$\kappa > \kappa_{-j,2}(\sigma_{-j}) \equiv \Pi_j(\sigma_j^*, \sigma_{-j}) - \lambda[1-\sigma_j^* - (1-\chi)(1-\sigma_{-j})]. \tag{D.35}$$

Moreover, the deviation is profitable if and only if σ_{-j} is such that $p_{-j}^d(\sigma_j^*, \sigma_{-j}) > p_{-j}^*(\sigma_j^*, \sigma_{-j}^*)$, which gives

$$\kappa > \kappa_{-j,3}(\sigma_{-j}) \equiv \frac{1-\chi}{\chi}\left(\Pi_{-j}(\sigma_j^*, \sigma_{-j}^*) - \Pi_{-j}(\sigma_j^*, \sigma_{-j}) + \lambda\chi(1-\sigma_{-j})\right). \tag{D.36}$$

Hence, for a given vector of parameters, this deviation destroys the considered equilibrium if there exists a value σ_{-j}^d such that $\kappa > \max\{\kappa_{-j,1}(\sigma_{-j}^d), \kappa_{-j,2}(\sigma_{-j}^d), \kappa_{-j,3}(\sigma_{-j}^d)\}$.

Deviation 2(b). If firm $-j$ attracts $\mathcal{R}$ investors only, its market clearing price is

$$p_{-j}^d(\sigma_j^*, \sigma_{-j}) = \Pi_{-j}(\sigma_j^*, \sigma_{-j}) - \frac{\kappa}{\chi} - \lambda(1-\sigma_{-j}), \tag{D.37}$$

whereas the market clearing price of firm j, who ends up attracting $\mathcal{N}$ investors, is

$$p_j^d(\sigma_j^*, \sigma_{-j}) = \Pi_j(\sigma_j^*, \sigma_{-j}) - \frac{\kappa}{1-\chi}. \tag{D.38}$$

In order for firm $-j$ to attract $\mathcal{R}$ investors only, it must be

$$\Pi_j(\sigma_j^*, \sigma_{-j}) - p_j^d(\sigma_j^*, \sigma_{-j}) > \Pi_{-j}(\sigma_j^*, \sigma_{-j}) - p_{-j}^d(\sigma_j^*, \sigma_{-j}), \tag{D.39}$$

and

$$\Pi_j(\sigma_j^*, \sigma_{-j}) - p_j^d(\sigma_j^*, \sigma_{-j}) - \lambda(1-\sigma_j^*) < \Pi_{-j}(\sigma_j^*, \sigma_{-j}) - p_{-j}^d(\sigma_j^*, \sigma_{-j}) - \lambda(1-\sigma_{-j}). \tag{D.40}$$

These are simultaneously satisfied if and only if

$$\widehat{\kappa}(\sigma_{-j}^d, 1) < \kappa < \widehat{\kappa}(\sigma_j^*, 1), \tag{D.41}$$

which is not the case in the region of parameters defined in (D.9), where this equilibrium can exist.

Existence. The above results imply that deviation 2(b) can never be implemented in the relevant region of parameters defined in (D.9), and deviation 1(b) is ruled out for $\kappa > \underline{\kappa}$. The following example establishes the possibility result concerning the existence of asymmetric equilibria also under this alternative timing of the game.

Suppose $\pi_j = \sigma_j(1-\sigma_{-j})\pi^m$. In the candidate asymmetric equilibrium,

$$\sigma_j^* = \frac{c(\pi^m + \lambda) - (\pi^m)^2}{c^2 - (\pi^m)^2} > \sigma_{-j}^* = \frac{\pi^m(c - \pi^m - \lambda)}{c^2 - (\pi^m)^2}.$$

Accordingly, firms' share prices in this candidate equilibrium are

$$p_j^* = \frac{-2c^4\lambda + c^3(\lambda + \pi^m)^2 + 2(\pi^m)^2c^2(\lambda - \pi^m) + (\pi^m)^4c - 2(\pi^m)^4\lambda}{2\left(c^2 - (\pi^m)^2\right)^2} - \frac{\kappa}{\chi},$$

and

$$p_{-j}^* = \frac{(\pi^m)^2c(c - \lambda - (\pi^m))^2}{2\left(c^2 - (\pi^m)^2\right)^2} - \frac{\kappa}{1-\chi}.$$

We then have

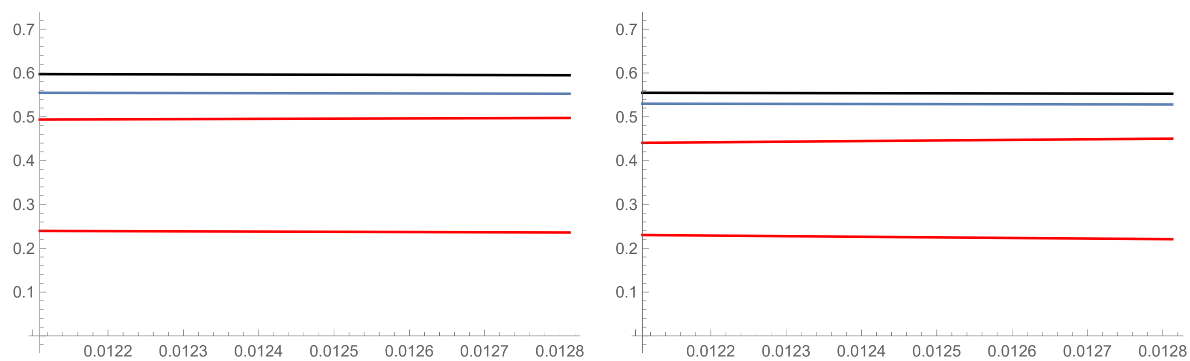
$$\underline{\kappa} = \frac{\lambda\chi(1-\chi)\left(2c^3 - c^2(\lambda + 2\pi^m) - \lambda(\pi^m)^2\right)}{2c(2\chi-1)\left(c^2 - (\pi^m)^2\right)} \in (\widehat{\kappa}(\sigma_j^*, 1), \widehat{\kappa}(\sigma_{-j}^*, 1)),$$

$$\kappa_{j,3}(\sigma_j) = \frac{\chi}{1-\chi}\Bigg(\frac{\left(c^2(\pi^m - \lambda) - (\pi^m)^2 c + 2(\pi^m)^2\lambda\right)\left(c(\lambda + \pi^m) - (\pi^m)^2\right)}{2\left(c^2 - (\pi^m)^2\right)^2} + \\ - \frac{\pi^m \sigma_j \left(c^2 - \pi^m c + \pi^m \lambda\right)}{c^2 - (\pi^m)^2} - \frac{c\sigma_j^2}{2} - \lambda\frac{c(c - \lambda - \pi^m)}{c^2 - (\pi^m)^2} + \lambda\chi(1 - \sigma_j)\Bigg),$$

and

$$\kappa_{-j,3}(\sigma_{-j}) \equiv \frac{(1-\chi)\left(c\left(c^2\sigma_{-j} - c\pi^m + \pi^m(\lambda - \pi^m\sigma_{-j} + \pi^m)\right)^2 + 2\lambda(1-\sigma_{-j})\chi\left(c^2 - (\pi^m)^2\right)^2\right)}{2\chi\left(c^2 - (\pi^m)^2\right)^2}.$$

The following figures show that also deviations 1(a) and 2(a) cannot be profitable and implementable at the same time for all $\kappa \in [\underline{\kappa}, \widehat{\kappa}(\sigma_{-j}^*, 1)]$, considering a numerical example with parameters' values $c = 2, \pi^m = 1, \lambda = 1/20, \chi = 3/4$. This establishes the existence of the asymmetric equilibrium for these values of the parameters.



(a) Firm j. The condition for the deviation to be profitable ($\kappa > \kappa_{j,3}$) requires σ_j to be within the two red lines. This is not compatible with $\kappa > \kappa_{j,1}$, which requires σ_j to be above the black line, nor with $\kappa > \kappa_{j,2}$, which requires σ_j to be above the blue line.

(b) Firm $-j$. The condition for the deviation to be profitable ($\kappa > \kappa_{-j,3}$) requires σ_{-j} to be within the two red lines. This is not compatible with $\kappa > \kappa_{-j,1}$, which requires σ_{-j} to be above the black line, nor with $\kappa > \kappa_{-j,2}$, which requires σ_{-j} to be above the blue line.

Figure OA3: X-axis: $\kappa \in [\underline{\kappa}, \widehat{\kappa}(\sigma_{-j}^*, 1)]$, Y-axis: σ_j (panel a), σ_{-j} (panel b).

Proof of Proposition 11. Given the shares' demand in Equation D.11, for the market clearing conditions to hold for all firms, it must be that $\Pi_j - p_j$ is positive and constant across firms — i.e., in any equilibrium, all investors must be indifferent between all firms' shares. The market clearing conditions then write as

$$\left[\chi\alpha_j^{\mathcal{R}} + (1-\chi)\alpha_j^{\mathcal{N}}\right]\frac{1}{\kappa}\left[\Pi_j - p_j\right] = 1 \quad \forall j \in \mathcal{J}. \tag{D.42}$$

Summing across firms, and using the fact that $\Pi_j - p_j$ does not depend on j, yields $\frac{1}{\kappa}\left[\Pi_j - p_j\right] = N$ and so

$$\chi\alpha_j^{\mathcal{R}} + (1-\chi)\alpha_j^{\mathcal{N}} = \frac{1}{N}. \tag{D.43}$$

Any pair $(\alpha_j^{\mathcal{R}}, \alpha_j^{\mathcal{N}}) \in [0,1]^2$ that satisfies Equation D.43, provided $\sum_j \alpha_j^{\mathcal{R}} = \sum_j \alpha_j^{\mathcal{N}} = 1$, yields an equilibrium of the game. Therefore, the game admits a continuum of equilibria.

In any equilibrium, firms' CSR policies are determined as in the base model. Indeed, under the proportional control assumption, firm j solves

$$\max_{\sigma_j\in[0,1]} \Pi_j - \eta s_j^{\mathcal{R}} \sum_k \lambda(1-\sigma_k), \tag{D.44}$$

which gives

$$\sigma_j = \frac{1}{c}\left(\eta\lambda s_j^{\mathcal{R}} + \frac{\partial\pi_j}{\partial\sigma_j}(\sigma_j, \vec{\sigma}_{-j})\right), \tag{D.45}$$

where, given the market clearing conditions, $s_j^{\mathcal{R}} = \chi N \alpha_j^{\mathcal{R}}$. Therefore, as in the main model, the game admits:

- A unique symmetric equilibrium, which here is obtained for $\alpha_j^{\mathcal{R}} = \alpha_j^{\mathcal{N}} = \frac{1}{N}$ for all j, where all firms have the same ownership and choose the same CSR policies (and accordingly make the same expected profits and have the same shares' prices);

- Multiple (in this case, infinitely many) asymmetric equilibria, in which firms more targeted by $\mathcal{R}$ investors invest more in CSR policies compared to firms that feature less prevalence of green capital (and, accordingly, firms make different profits and have different shares' prices).[8]

[8]Consider, for example, $N = 2$ and $\pi_j = \pi^m\sigma_j(1-\sigma_{-j})$. Then, for any $\alpha_1^{\mathcal{N}} \equiv \alpha^{\mathcal{N}} \in \left[0, \min\left\{\frac{1}{2(1-\chi)}, 1\right\}\right]$, the game admits an equilibrium with $\alpha_1^{\mathcal{R}} = \alpha^{\mathcal{N}} + \frac{1-2\alpha^{\mathcal{N}}}{2\chi}$ and CSR policies

$$\sigma_1 = \frac{\pi^m(\eta\lambda - \pi^m - 2\alpha^{\mathcal{N}}\eta\lambda(1-\chi) - 2\eta\lambda\chi) + c(\eta\lambda + \pi^m - 2\alpha^{\mathcal{N}}\eta\lambda(1-\chi))}{c^2 - (\pi^m)^2},$$

In particular, there may be equilibria where $\mathcal{R}$ investors concentrate in a subset of $n \leq N-1$ firms. As in the base model, consider $\alpha_j^{\mathcal{R}} = \frac{1}{n}$ for firms $j = 1, \dots, n$, and $\alpha_j^{\mathcal{R}} = 0$ for $j = n+1, \dots, N$. Then, the market clearing conditions imply that $\alpha_j^{\mathcal{N}} = \frac{n-N\chi}{nN(1-\chi)}$ for $j = 1, \dots, n$ and $\alpha_j^{\mathcal{N}} = \frac{1}{N(1-\chi)}$ for $j = n+1, \dots, N$. Note that for $\chi = \frac{n}{N}$, $\alpha_j^{\mathcal{N}} = 0$ for $j = 1, \dots, n$, so that we obtain the equilibrium with full separation characterized in the main model. The result in Proposition B.7 then implies that aggregate CSR investments may be lower in this equilibrium than in the scenario with no SRI ($\alpha_j^{\mathcal{R}} = s_j^{\mathcal{R}} = 0\,\forall j$), in which, in turn (by Lemma 1), aggregate investments are lower than in the symmetric equilibrium with SRI (where $\alpha_j^{\mathcal{R}} = \frac{1}{N}$ and so $s_j^{\mathcal{R}} > 0$ for all j), which here exists for all values of the parameters. Note that, by continuity of investors' and firms' equilibrium strategies, this possibility result extends to a non-zero measure set of the parameters.

Proof of Proposition 12. If firm j receives capital, it raises an amount $k_j = \int_i s_{ij} p_j di = p_j$, since $\int_i s_{ij} = 1$ by market clearing (each firm has a unit mass of shares). The Lagrangian function for investor i's optimization problem is:

$$\mathcal{L} = \sum_j s_{ij} \left[k_j \left(1 + \pi_j - \mathbb{1}_{i,\mathcal{R}} \lambda (1-\sigma_j) \right) - p_j \right] + \mu_i \left(1 - \sum_j s_{ij} p_j \right) \tag{D.46}$$

where $\mu_i \geq 0$ denotes the Lagrangian multiplier. Next, we use that $k_j = p_j$. Differentiating $\mathcal{L}$ with respect to s_{ij}, we then get the following first-order condition for an interior equilibrium:

$$\pi_j - \mathbb{1}_{i,\mathcal{R}} \lambda (1-\sigma_j) - \mu_i = 0. \tag{D.47}$$

Note that each investor buys shares only in the firm $j^* \in \arg\max_{j \in \mathcal{J}} \frac{\partial \mathcal{L}}{\partial s_{ij}}$, provided this maximum value (i.e., the LHS of the above FOC) is positive.[9]

In this case, the investor's budget constraint is binding, and so $\mu_i > 0$, which leads to the following existence condition:

$$\pi_j - \mathbb{1}_{i,\mathcal{R}} \lambda (1-\sigma_j) > 0. \tag{D.48}$$

and

$$\sigma_2 = \frac{c(\eta\lambda(2\alpha^{\mathcal{N}}(1-\chi) + 2\chi - 1) + \pi^m) - \pi^m(\eta\lambda + \pi^m - 2\alpha^{\mathcal{N}}\eta\lambda(1-\chi))}{c^2 - (\pi^m)^2},$$

where $\sigma_1 \geq \sigma_2$ if and only if $\alpha^{\mathcal{N}} \leq \frac{1}{2}$, with equality only at $\alpha^{\mathcal{N}} = \frac{1}{2}$ (symmetric equilibrium).

[9]As in the main model, without loss of generality we assume that, in case an investor is indifferent between shares of different firms, she only buys shares from one firm.

As we have assumed $\pi_j > 0$, it follows that $\mathcal{N}$ investors always participate in the financial market, whereas $\mathcal{R}$ investors are excluded whenever

$$\pi_0 \leq \lambda(1 - \sigma_0), \tag{D.49}$$

where by σ_0 and π_0 we denote the equilibrium CSR policy and return on capital for all firms when $\mathcal{R}$ investors do not participate to the financial market: formally, σ_0 solves Problem (D.13) for $s_j^{\mathcal{R}} = 0$ and $k_j = \frac{1-\chi}{N}$ for all $j \in \mathcal{J}$. Therefore, for all values of parameters such that Inequality (D.49) is satisfied, the game admits a symmetric equilibrium with no SRI. By the same steps of the main analysis it can be shown that whenever Inequality (D.49) does not hold the game admits a unique symmetric equilibrium that features SRI.

We next turn to consider the asymmetric equilibria with full separation, in which $\mathcal{R}$ investors hold firms $j \leq n$ and $\mathcal{N}$ investors hold firms $j > n$. we require that: (i) $\pi_j - \lambda(1 - \sigma_j) - \mu_i \geq 0$ for $j \leq n$ and $\pi_j - \lambda(1 - \sigma_j) - \mu_i < 0$ for $j > n$; (ii) $\pi_j - \mu_i < 0$ for $j \leq n$ and $\pi_j - \mu_i \geq 0$ for $j > n$. By the same logic as in the main model, all firms $j \leq n$ (resp. $j > n$) must choose the same CSR policy and have the same return on capital, denoted by $\overline{\sigma}$ and $\overline{\pi}$ (resp. $\underline{\sigma}$ and $\underline{\pi}$) in this equilibrium. It follows that we can write the existence conditions as:

$$\overline{\pi} < \underline{\pi} \quad \text{and} \quad \overline{\pi} + \lambda\overline{\sigma} > \underline{\pi} + \lambda\underline{\sigma}, \tag{D.50}$$

where firms' capital is determined as follows. In this equilibrium, we have that $s_{ij} = \frac{n}{\chi}$ for $i \in \mathcal{R}$ and $s_{ij} = \frac{N-n}{1-\chi}$ for $i \in \mathcal{N}$. As a result, firm $j \leq n$ raises $\frac{\chi}{n}$ and firm $j > n$ raises $\frac{1-\chi}{N-n}$, which follows from $s_{ij} = \frac{1}{p_j}$.

Proof of Proposition 13. The symmetric equilibrium without SRI is characterized as in the main model — i.e., it features the same expected markups $\mathbb{E}[\mu_j] = M_0$ and CSR policies $\sigma_j = \sigma_0$ of the main model. However, as $\mathcal{R}$ investors' utility is different in this version of the model, the condition under which they are not willing to buy shares in any firm now gives $\chi \leq \widetilde{\chi}(\sigma_0, M_0, 0)$. As σ_0 and

M_0 do not depend on δ, differentiating $\widetilde{\chi}(\sigma_0, M_0, 0)$ with respect to δ gives

$$\frac{\partial \widetilde{\chi}(\sigma_0, M_0, 0)}{\partial \delta} = \frac{N\kappa M_0}{(\delta M_0 + \lambda(1-\sigma_0))^2} > 0. \tag{D.51}$$

Proceeding as in the main model, it can be shown that, for all $\chi > \widetilde{\chi}(\sigma_0, M_0, 0)$, the game admits a unique symmetric equilibrium that features SRI without concentration. The equilibrium strategies jointly solve Programs (14) and (D.15), and the system of equations

$$c\sigma^* = \frac{\partial \pi_j}{\partial \sigma_j} + s^{\mathcal{R}*}\left[\eta\lambda - \eta_2\delta \frac{\partial \mathbb{E}[\mu_j]}{\partial \sigma_j}\right]; \tag{D.52}$$

$$s^{\mathcal{R}*} = \frac{\chi}{N}\frac{1}{\kappa}\left[N\kappa - \lambda(1-\chi)(1-\sigma^*) - \delta\mathbb{E}[\mu_j]\right]. \tag{D.53}$$

Moving to asymmetric equilibria with full separation,[10] we denote by $\overline{\sigma}$ and $\hat{\mu}$ the CSR policies and expected markup of the firms held by $\mathcal{R}$ investors ($j \leq n$), and by $\underline{\sigma}$ and $\tilde{\mu}$ the CSR policies and expected markup of the excluded firms ($j > n$). The equilibrium strategies jointly solve Programs (14) and (D.15), and the system of equations

$$\text{for } j \leq n: \qquad c\overline{\sigma} = \frac{\partial \pi_j}{\partial \sigma_j}(\overline{\sigma}, \underbrace{\overline{\sigma}, \ldots, \overline{\sigma}}_{n-1}, \underbrace{\underline{\sigma}, \ldots, \underline{\sigma}}_{N-n}) + \eta\lambda - \eta_2\delta \frac{\partial \mathbb{E}[\mu_j]}{\partial \sigma_j} \tag{D.54}$$

$$\text{for } j > n: \qquad c\underline{\sigma} = \frac{\partial \pi_j}{\partial \sigma_j}(\underline{\sigma}, \underbrace{\overline{\sigma}, \ldots, \overline{\sigma}}_{n}, \underbrace{\underline{\sigma}, \ldots, \underline{\sigma}}_{N-n-1}). \tag{D.55}$$

For a fixed n, imposing that $\mathcal{R}$ (resp. $\mathcal{N}$) investors prefer holding shares in firms $j \leq n$ (resp. $j > n$) gives that this equilibrium exists if and only if $\widetilde{\chi}(\overline{\sigma}, \hat{\mu}, n) < \chi < \widetilde{\chi}(\underline{\sigma}, \tilde{\mu}, n)$.

Finally, to prove that $\overline{\sigma}$ is decreasing in δ, note that δ enters the FOC (D.54) of firms $j \leq n$ both directly reducing its RHS, and indirectly, as μ_j and π_j depend on δ through its impact on firms' optimal prices. From Program (D.15) it is easy to see that, for any given $\vec{a}$, the larger δ, the lower $\mu_j(\vec{a})$ and firm j's profit $\mu_j(\vec{a})\int_0^1 x_{hj}(\vec{a}, \vec{p}(\vec{a}))\,dh$. As $\mathbb{E}[\mu_j] = \sum_{\tilde{a}\in\{0,1\}^N} \Pr[\vec{a} = \tilde{a}]\mu_j(\tilde{a})$ and $\pi_j = \sum_{\tilde{a}\in\{0,1\}^N} \Pr[\vec{a} = \tilde{a}]\mu_j(\tilde{a})\int_0^1 x_{hj}(\tilde{a}, \vec{p}(\tilde{a})\,dh$, it follows that both $\frac{\partial \mathbb{E}[\mu_j]}{\partial \sigma_j}$ and $\frac{\partial \pi_j}{\partial \sigma_j}$ are decreasing in δ. Hence, both the direct and indirect effect of an increase in δ entail a drop in the RHS of the FOC (D.54). By the corresponding SOC, this implies that (provided the strength of strategic interactions

[10] As in the main model, there may also be asymmetric equilibria where both $\mathcal{R}$ and $\mathcal{N}$ investors hold positive shares in some firms.

in CSR policies, i.e. $|\frac{\partial^2 \pi_j(\cdot)}{\partial \sigma_j \partial \sigma_{-j}}|$, is not too large, so that the effects through $\underline{\sigma}$ are second-order) $\overline{\sigma}$ is decreasing in δ.

E CSR policies

Sections E.1 to E.4 develop the extensions discussed in Section 5.2. All proofs are in Section E.5.

E.1 Responsible consumption and CSR

This section describes how socially responsible consumption (SRC) influences firms' CSR investments. $\mathcal{R}$ consumers incur a disutility from buying brown products, so they are willing to pay more for (and consume more of) a green product. The prospect of charging higher prices (as in Example 1) or selling additional units to $\mathcal{R}$ consumers (e.g., Example 2) incentivizes firms to invest more in CSR. The following proposition generalizes these insights and explores the implications of SRC for the distribution of SRI.

Proposition 14 *Suppose the marginal revenue from CSR investments increases with the fraction χ_c of $\mathcal{R}$ consumers in equilibrium (i.e., $\frac{\partial^2 \pi_j}{\partial \sigma_j \partial \chi_c} > 0$). Then, an increase in χ_c:*

1. *Increases aggregate CSR investments in all symmetric equilibria of the game;*
2. *Increases the set of parameters for which the symmetric equilibrium features SRI.*

Proposition 14 offers two main insights. First, absent any distortions induced by the concentration of green capital, SRC always increases all firms' CSR investments. Second, since firms invest more in CSR when χ_c increases, the valuation gap between $\mathcal{R}$ and $\mathcal{N}$ investors becomes smaller. As a result, it becomes easier for $\mathcal{R}$ investors to target a larger number of firms so that the symmetric equilibrium with SRI is more likely to exist.

E.2 Input market

In this section, we provide an alternative micro-foundation for the profit function in our baseline model. The micro-foundation here is based on the pricing of inputs and technological spillovers across firms, as opposed to the interactions in the product market described in Section 4.3.

Suppose that each firm j needs a fixed amount of total inputs, normalized to 1, to generate an output q_j. There are two types of inputs: brown ($l = b$) and green ($l = g$) inputs. Let σ_j denote the fraction of j's inputs that are green. Using the brown input generates negative externalities, so firm j overall generates a negative externality $(1 - \sigma_j)\lambda$. We discuss shortly how the choice of σ_j affects j's production. Since our focus is on the pricing of inputs, here we abstract from product market interactions and assume that each firm sells its output at an exogenous price ρ.

Inputs are produced by a unit mass of firms in a competitive market. To distinguish firms that operate in the input market from those operating in the output market, we refer to the former as *upstream* firms and the latter as *downstream* firms. Each upstream firm pays a cost $\frac{c_l}{2}x^2$ for producing a quantity x of input l, which leads to an upward-sloping aggregate supply function $S(p_l) = \frac{1}{c_l}p_l$ (see Chapter 5.3 of Tirole (1988)). The aggregate demand for the green input is $\sum_j \sigma_j$, so market clearing implies $p_g = c_g \sum_j \sigma_j$. By the same logic, we obtain $p_b = c_b \sum_j (1 - \sigma_j)$.

To micro-found the case of strategic complementarity in CSR investments, we assume that a downstream firm j's ability to turn input k into output may increase with the extent other firms also use the same input in their production process. This assumption captures technological spillovers among downstream firms using similar inputs. Formally, we assume $q_j = \tilde{q}(\vec{\sigma})$, with $\tilde{q} : (\mathbb{R}_+)^N \to \mathbb{R}_+$. Example 7 below illustrates the equilibrium for a specific choice of $\tilde{q}(\cdot)$. For the case of substitutability, it is sufficient to assume that inputs of different types are perfect substitutes in j's production technology, which is the case when q_j is a constant.

The downstream firms internalize the impact that their demands have on the equilibrium input prices. We can write firm j's expected terminal value as follows:

$$\begin{aligned}\Pi_j &= q_j\rho - \left(\sigma_j p_g + (1-\sigma_j)p_b\right) = q_j\rho - \sigma_j c_g \sum_j \sigma_j - (1-\sigma_j)c_b \sum_j (1-\sigma_j) \\ &= \underbrace{q_j\rho - \sigma_j c_g \Sigma_{g,-j} - (1-\sigma_j)c_b\Sigma_{b,-j} - c_b(1-2\sigma_j)}_{\equiv \pi_j(\sigma_j,\vec{\sigma}_{-j})} - \underbrace{(c_g + c_b)\sigma_j^2}_{\equiv C(\sigma_j)}\end{aligned} \tag{E.1}$$

where we have denoted $\Sigma_{g,-j} \equiv \sum_{k\neq j}\sigma_k$ and $\Sigma_{b,-j} \equiv \sum_{k\neq j}(1-\sigma_k)$.

Example 7 *Suppose* $\tilde{q}(\vec{\sigma}) = \sigma_j r_g \Sigma_{g,-j} + (1-\sigma_j) r_b \Sigma_{b,-j}$, *with* $r_l > 0$, $l = g, b$ *— i.e., the productivity of* *input l in firm j is proportional to its aggregate usage by rivals. Then,*

$$\frac{\partial^2 \Pi_j}{\partial \sigma_j \partial \sigma_{-j}} = (r_b + r_g)\rho - (c_b + c_g),$$

so that CSR policies are strategic complements (resp. substitutes) if and only if revenues are more (resp. less) responsive than costs to aggregate input demand (i.e., $(r_b + r_g)\rho >$ (resp. $<$)$c_b + c_g$).

Taking the FOC at the investment stage gives

$$\sigma_j = \frac{1}{2(c_b + c_g)} \left[\eta \lambda s_j^{\mathcal{R}} + [(r_b + r_g)\rho - (c_b + c_g)] \sum_{k \neq j} \sigma_k - (N-1) r_b \rho + (N+1) c_b \right].$$

Sufficient conditions for interior solutions are (i) $0 < (N+1)c_b - (N-1)r_b\rho < 2(c_g + c_b) - \eta\lambda$, and (ii) $-\frac{(N+1)c_b - (N-1)r_b\rho}{N-1} < (r_b + r_g)\rho - (c_b + c_g) < \frac{2(c_b + c_g) - \eta\lambda - ((N+1)c_b - (N-1)r_b\rho)}{N-1}$, which we assume throughout.

In the symmetric equilibrium with no SRI, each firm invests

$$\sigma_0 = \frac{(N+1)c_b - (N-1)r_b\rho}{(N+1)(c_b + c_g) - (N-1)(r_b + r_g)\rho}.$$

In asymmetric equilibria with full separation, denoting by $\overline{\sigma}$ (resp. $\underline{\sigma}$) the CSR policy chosen by any of the n (resp. $N - n$) firms owned by $\mathcal{R}$ (resp. $\mathcal{N}$) investors, we have

$$\underline{\sigma} = \frac{(c_b + c_g)((N+1)c_b - n\eta\lambda) + (c_b(2r_b + (N+1)r_g) + n(r_b + r_g)\eta\lambda - (N-1)r_b c_g)\rho - (N-1)r_b(r_b + r_g)\rho^2}{(N+1)(c_b + c_g)^2 + 2(c_b + c_g)(r_b + r_g)\rho - (N-1)(r_b + r_g)^2\rho^2},$$

and

$$\overline{\sigma} = \underline{\sigma} + \frac{\eta\lambda}{c_b + c_g + (r_b + r_g)\rho}.$$

Aggregate greenness in asymmetric equilibria,

$$n\overline{\sigma} + (N-n)\underline{\sigma} = \frac{n\eta\lambda + N(N+1)c_b - N(N-1)r_b\rho}{(N+1)(c_b + c_g) - (N-1)(r_b + r_g)\rho},$$

is increasing in n.

Moreover, as $(r_b + r_g)\rho$ grows larger, the strategic interactions in CSR policies move from strategic substitutability to strategic complementarity,[11] and:

[11] As $\frac{\partial^2 \Pi_j}{\partial \sigma_j^2} = c_b + c_g$, these cost parameters affect not only the nature and intensity of the strategic interactions among firms, but also the degree of convexity of their profit function. Therefore, it is natural to treat these parameters as fixed and consider $(r_b + r_g)\rho$, which

- *The region of parameters where the asymmetric equilibria exist, which (see the proof of Proposition 4 in Appendix B.6) is increasing in*

$$\overline{\sigma} - \underline{\sigma} = \frac{\eta\lambda}{c_b + c_g + (r_b + r_g)\rho},$$

  shrinks;

- *Concentrated SRI increases aggregate greenness relative to no SRI to a larger extent — i.e., the difference*

$$n\overline{\sigma} + (N-n)\underline{\sigma} - N\sigma_0 = \frac{n\eta\lambda}{(N+1)(c_b + c_g) - (N-1)(r_b + r_g)\rho}$$

  increases.

E.3 Competition and strategic substitutability

The unintended consequences of SRI in our model are linked to the case when firms' CSR policies are strategic substitutes. In Example 1, we capture this channel in a stark framework featuring perfect Bertrand competition. However, we can easily relax this assumption and consider more general settings:

Proposition 15 *Under binary production technologies $a_j \in \{0, 1\}$, a sufficient condition for CSR policies to be strategic substitutes is that the (average) extra profit that any firm obtains from being green decreases in the number of green rivals.*

The sufficient condition given in Proposition 15 is satisfied in standard models of imperfect competition: see the following example.

Example 8 (Hotelling) *Suppose consumers, both responsible and non-responsible ones, are uniformly located on a unit Hotelling line $h \in [0, 1]$. Two firms are located at the ends of the line: firm $j = 1$ is located at $h_1 = 0$; firm $j = 2$ is located at $h_2 = 1$. Each consumer located at h has unit demand and derives utility*

$$u_{hj} \equiv v - t|h - h_j| - \mathbb{1}_{h,\mathcal{R}}\lambda(1 - a_j) - \rho_j,$$

enter $\frac{\partial^2 \Pi_j}{\partial \sigma_j \partial \sigma_{-j}}$ but not $\frac{\partial^2 \Pi_j}{\partial \sigma_j^2}$, as a measure of strategic interactions among firms.

from buying firm j's product, with $v > 0$ and $t > 0$ being the gross utility from consuming any product and the unit transportation cost, respectively. In what follows, we normalize production costs to zero and assume

$$\lambda > v > \frac{2t}{3}\frac{3-\chi_c}{1-\chi_c}.$$

The first inequality implies that responsible consumers boycott brown firms' products — i.e., they are unwilling to purchase brown products, regardless of their price — while the second inequality ensures full market coverage.

The product market equilibrium is as follows:

- *If both products are green ($a_1 = a_2 = 1$), the two firms engage in standard Hotelling competition for all consumers. Hence, $\rho^*(1,1) = t$, and each firm makes a profit (gross of investment costs) $\pi^*(1,1) = t/2$.*

- *If both products are brown ($a_1 = a_2 = 0$), the two firms engage in standard Hotelling competition for non-responsible consumers, whereas responsible consumers boycott both firms. Hence, $\rho^*(0,0) = t$, and each firm makes profit $\pi^*(0,0) = (1-\chi_c)t/2$.*

- *If firm j sells a green product and the other firm $-j$ a brown product ($a_j = 1, a_{-j} = 0$), then, under the full market coverage assumption in the segment of non-responsible consumers, firm $-j$ solves*

$$\max_{\rho_{-j}}(1-\chi_c)\left[\frac{1}{2}+\frac{\rho_j-\rho_{-j}}{2t}\right]\rho_{-j} \quad \Longrightarrow \quad \rho_{-j} = \frac{t+\rho_j}{2}.$$

Assuming that it serves all responsible consumers, firm j solves

$$\max_{\rho_j}\left[(1-\chi_c)\left[\frac{1}{2}-\frac{\rho_j-\rho_{-j}}{2t}\right]+\chi_c\right]\rho_j \quad \Longrightarrow \quad \rho_j = \frac{\rho_{-j}}{2}+t\frac{1+\chi_c}{2(1-\chi_c)}.$$

From these best responses, we find the equilibrium prices $\rho_j^*(a_j, a_{-j})$:

$$\rho_j^*(1,0) = \frac{t}{3}\frac{3+\chi_c}{1-\chi_c} > \rho_{-j}^*(1,0) = \frac{t}{3}\frac{3-\chi_c}{1-\chi_c},$$

which, under the above parametric restrictions, satisfy the mentioned full market coverage conditions. The corresponding profits $\pi_j^*(a_j, a_{-j})$ are

$$\pi_j^*(1,0) = \frac{t}{18}\frac{(3+\chi_c)^2}{1-\chi_c} > \pi_{-j}^*(1,0) = \frac{t}{18}\frac{(3-\chi_c)^2}{1-\chi}.$$

The condition

$$\Delta\pi(0) = \pi_j^*(1,0) - \pi^*(0,0) > \pi^*(1,1) - \pi_j^*(0,1) = \Delta\pi(1) > 0,$$

can be immediately verified.

Finally, the result that product substitutability (resp. complementarity) implies that firms' CSR policies are strategic substitutes (resp. complements) carries over to more general environments, beyond the assumptions of binary production technologies, as shown in the following example:

Example 9 (Singh-Vives demand) *Consider a utility function à la Singh and Vives (1984):*

$$U_h(\vec{q}) \equiv \sum_j [1 - \mathbb{1}_{h,\mathcal{R}}\lambda(1-\sigma_j)]q_j - \frac{1}{2}\sum_j q_j^2 - \frac{b}{2}\prod_{j,j'} q_j q_{j'} - \sum_j \rho_j q_j,$$

with $\sigma_j \in [0,1]$ *and* $b \in (0,1)$ *(resp.* $b \in (-1,0)$*) if products are substitutes (resp. complements). Optimization yields the following (direct) demand functions for each consumer type* $\theta \in \{\mathcal{R}, \mathcal{N}\}$*:*

$$q_j^\theta(\vec{\sigma}_j, \vec{\rho}_j) = \frac{1 - b - \rho_j - (N-2)b\rho_j + b\sum_{k\neq j}\rho_k}{(1-b)(1+(N-1)b)} - \mathbb{1}_{h,\mathcal{R}}\lambda\frac{1 - b - \sigma_j - (N-2)b\sigma_j + b\sum_{k\neq j}\sigma_k}{(1-b)(1+(N-1)b)}.$$

Therefore, for any $(\vec{\sigma}, \vec{\rho})$*, firm* j*'s demand is* $Q_j(\vec{\sigma}, \vec{\rho}) \equiv \chi_c q_j^{\mathcal{R}}(\cdot) + (1-\chi_c)q_j^{\mathcal{N}}(\cdot)$*.*

Let us consider for simplicity the duopoly case ($N = 2$)*. Normalizing to zero firms' marginal costs, for given CSR policies, the Bertrand-Nash equilibrium prices are*

$$\rho_j = \frac{2 - b(1+b) - \lambda\chi_c(2(1-\sigma_j) - b(1+b+(1-b)\sigma_j - 2\sum_{k\neq j}\sigma_k))}{4-b^2},$$

and the corresponding profits,

$$\pi_j = \frac{(2 - b(1+b) - \lambda\chi_c(2(1-\sigma_j) - b(1+b+(1-b)\sigma_j - 2\sum_{k\neq j}\sigma_k)))^2}{(4-b^2)^2(1-b^2)},$$

are such that

$$\frac{\partial^2\pi_j}{\partial\sigma_j\partial\sigma_{-j}} = -\frac{4b\lambda^2\chi_c^2}{(2+b)^2(2-3b+b^2)} < 0 \Leftrightarrow b > 0.$$

That is, CSR policies are strategic substitutes (resp. complements) if and only if firms' products are substitutes (resp. complements).

E.4 Provision of public goods

In the main model, each firm j generates a negative externality λ at a rate $1 - \sigma_j$. In such a setting, responsible agents have lower valuations than non-responsible ones for brown firms' products and shares. This implies that in equilibrium $\mathcal{N}$ investors can crowd out $\mathcal{R}$ investors, but not the other way around.

Here we show that our qualitative results are robust under the opposite assumption. Namely, suppose the status-quo entails no externality, but CSR investments bring up a positive externality λ at a rate σ_j, which is internalized by responsible agents. Therefore, in this setting, $\mathcal{R}$ investors have higher valuations for firms' shares than $\mathcal{N}$ investors, and so responsible investors can crowd out non-responsible ones from the market.

Denoting again $\Pi_j = \Pi(\sigma_j, \vec{\sigma}_{-j})$ firm j's expected profit, which satisfies the same assumptions as in the base model, for a given vector of CSR policies $\vec{\sigma}$, investor i now solves:

$$\max_{s_{ij} \geq 0} \sum_j s_{ij}\Pi_j - p_j + \mathbb{1}_{i,\mathcal{R}}\lambda\sigma_j - K(\iota'\vec{s}_i). \tag{E.2}$$

Hence, the proportional control assumption implies that firm j chooses CSR policies solving:

$$\max_{\sigma_j \in [0,1]} \Pi_j + \lambda\sigma_j\eta s_j^{\mathcal{R}}. \tag{E.3}$$

Equilibrium Analysis. Taking the first-order condition at the investment stage, it is straightforward to see that firm j's profit is still maximized for σ_j solving Equation (3). Hence, all the results of Lemma 1 still hold.

Moving backward to the investment stage, we obtain the following shares' demand:

$$s_{ij} = \begin{cases} \max\{\frac{1}{\kappa}\left[\Pi_{j^*} - p_{j^*} + \mathbb{1}_{i,\mathcal{R}}\lambda\sigma_{j^*}\right], 0\} & \text{for } j^* = \text{argmax}_j\{\Pi_j - p_j + \mathbb{1}_{i,\mathcal{R}}\lambda\sigma_j\} \\ 0 & \text{for } j \neq j^*. \end{cases} \tag{E.4}$$

The market clearing conditions determine the share prices in equilibrium:

$$p_j = \Pi_j - \underbrace{\frac{\kappa}{\chi\alpha_j^{\mathcal{R}} + (1-\chi)\alpha_j^{\mathcal{N}}}}_{\text{liquidity discount}} + \underbrace{\frac{\lambda\sigma_j\chi\alpha_j^{\mathcal{R}}}{\chi\alpha_j^{\mathcal{R}} + (1-\chi)\alpha_j^{\mathcal{N}}}}_{\text{greenium}}, \tag{E.5}$$

where $\sum_j \alpha_j^{\mathcal{R}} = 1$ in all equilibria, whereas $\sum_j \alpha_j^{\mathcal{N}} \in \{0,1\}$, as $\alpha_j^{\mathcal{N}} = 0$ for all j in an equilibrium where $\mathcal{N}$ investors, having lower valuations than $\mathcal{R}$ investors, are crowded out from the market.

The equilibrium characterization mirrors the one shown in the main model:

Proposition 16 *The game always admits a unique symmetric equilibrium, and multiple asymmetric equilibria featuring SRI concentration in a subset of firms.*

Thus, the concentration of $\mathcal{R}$ investors in a subset of firms may still arise in equilibrium even when these investors have higher valuations compared to $\mathcal{N}$ investors. As the alternative assumption on externality considered here does not affect CSR investment strategies, it follows that the implications of concentration are as in the main model.

The complete equilibrium characterization in a setting that corresponds to the one in Example 1 is provided below:

Example 10 *Let $N = 2$ and $\pi_j = \sigma_j(1-\sigma_{-j})\pi^m$,*[12]

Defining

$$\kappa_R \equiv \frac{\lambda\chi(\pi^m + \eta\lambda)}{2(c+\pi^m)} < \widetilde{\kappa} \equiv \frac{\pi^m\lambda(1-\chi)\chi(c-\eta\lambda)}{c(c+\pi^m)(1-2\chi)},$$

and

$$\underline{\kappa} \equiv \frac{\pi^m\lambda(1-\chi)\chi(c-\pi^m-\eta\lambda)}{(c^2-(\pi^m)^2)(1-2\chi)} < \overline{\kappa} \equiv \frac{\lambda\chi(1-\chi)\left(c(\eta\lambda+\pi^m)-(\pi^m)^2\right)}{(c^2-(\pi^m)^2)(1-2\chi)},$$

for $\chi < 1/2$ and $c \in (\pi^m + \eta\lambda, \sqrt{2}\pi^m)$,[13] *the equilibrium characterization is as follows:*

[12] As in Example 1, this profit specification obtains supposing that products are perfect substitutes, up to the production technology $a_j \in \{0,1\}$, and that the positive externality entailed by green products ($a_j = 1$) is internalized by responsible consumers. Formally, for given prices $\vec{\rho}$ and types $\vec{a}$, consumer h's demand for firm j's product, denoted by x_{hj}, solves:

$$\max_{(x_{hj})_{j=1,\dots,N}\geq 0} u\left(\sum_{j=1}^{N} x_{hj}\right) + \sum_{j=1}^{N}[\mathbb{1}_{h,\mathcal{R}}\lambda a_j - \rho_j]x_{hj}.$$

Developing the new technology allows a firm to sell a product perceived as vertically differentiated by a fraction of consumers. Hence, a firm makes positive profits, denoted by π^m, if and only if it is the unique firm producing with the new technology (assuming again a competitive fringe of firms producing with the status-quo technology).

[13] The restriction $c < \sqrt{2}\pi^m$ is just a sufficient condition for $\widetilde{\kappa} \in (\kappa_R, \overline{\kappa})$.

1. *For* $\kappa < \min(\underline{\kappa}, \kappa_R)$, *with* $\kappa_R \equiv \frac{\lambda\chi(\pi^m+\eta\lambda)}{2(c+\pi^m)}$, *the unique equilibrium features exclusion of* $\mathcal{N}$ *investors and, accordingly,* $\sigma_j = \sigma_R = \frac{\eta\lambda+\pi^m}{c+\pi^m}$ *for* $j = 1, 2$.

2. *For* $\kappa \in (\min(\underline{\kappa}, \kappa_R), \max(\underline{\kappa}, \kappa_R))$:

   - *If* $\underline{\kappa} < \kappa_R$ *— i.e., for*

   $$\lambda > \frac{\pi^m(c-\pi^m)}{c(1-2\chi)+\pi^m} \quad \text{and} \quad \eta > \frac{\pi^m(c-\pi^m)}{\lambda(c(1-2\chi)+\pi^m)},$$

   the symmetric equilibrium with exclusion coexists with the asymmetric equilibria with full separation — i.e., $\alpha_j^{\mathcal{R}} = \alpha_{-j}^{\mathcal{N}} = 1$, *and* $(\sigma_j, \sigma_{-j}) = (\overline{\sigma}, \underline{\sigma})$, *with*

   $$\overline{\sigma} = \frac{c(\pi^m+\eta\lambda)-(\pi^m)^2}{c^2-(\pi^m)^2} > \underline{\sigma} = \frac{\pi^m(c-\pi^m-\eta\lambda)}{c^2-(\pi^m)^2};$$

   - *If instead* $\underline{\kappa} > \kappa_R$, *the symmetric equilibrium without exclusion of* $\mathcal{N}$ *investors, in which both firms choose*

   $$\sigma^* = \frac{2\kappa(\eta\lambda\chi+\pi^m)}{2(c+\pi^m)\kappa-\eta\lambda^2\chi(1-\chi)},$$

   coexists with asymmetric equilibria where $\mathcal{R}$ *investors buy shares in both firms, and all* $\mathcal{N}$ *investors buy shares of the same firm.*[14]

3. *For* $\kappa > \max(\underline{\kappa}, \kappa_R)$, *the symmetric equilibrium without exclusion of* $\mathcal{N}$ *investors coexists with the following equilibria:*

   - *for* $\kappa < \widetilde{\kappa}$, *the asymmetric equilibria with full separation, and the equilibria where* $\mathcal{R}$ *investors buy shares in both firms and* $\mathcal{N}$ *investors only buy shares in one firm;*

   - *for* $\kappa \in (\widetilde{\kappa}, \overline{\kappa})$, *the asymmetric equilibria with full separation only;*

   - *for* $\kappa > \overline{\kappa}$, *asymmetric equilibria where* $\mathcal{N}$ *investors buy shares in both firms, and all* $\mathcal{R}$ *investors*

[14] In these latter equilibria, $\alpha_{-j}^{\mathcal{R}} \equiv \alpha^{\mathcal{R}}$ and $\alpha_{-j}^{\mathcal{N}} = 0$, $\alpha_j^{\mathcal{R}} \equiv 1 - \alpha^{\mathcal{R}}$ and $\alpha_j^{\mathcal{N}} = 1$, with

$$\alpha^{\mathcal{R}} = \frac{(c+\pi^m)\left(c\kappa-\eta\lambda^2\chi(1-\chi)\right)}{\chi\left(2c^2\kappa+c\lambda(1-\chi)(\pi^m-\eta\lambda)+2\pi^m c\kappa-2\pi^m\eta\lambda^2(1-\chi)\right)},$$

and CSR policies

$$\sigma_j = \frac{\kappa(c(\eta\lambda(2\chi-1)+\pi^m)-2\pi^m\eta\lambda(1-\chi))}{(c+\pi^m)\left(c\kappa-\eta\lambda^2\chi(1-\chi)\right)}, \quad \sigma_{-j} = \frac{\eta\lambda+\pi^m}{c+\pi^m}.$$

buy shares of the same firm.[15]

Hence, provided the fraction of $\mathcal{R}$ investors is not too large,[16] *and trading costs are non-negligible, the game is likely to admit equilibria where $\mathcal{R}$ investors concentrate in a subset of firms.*

Moreover, this concentration is likely to be inefficient. To see this, consider $\kappa \in (\max(\underline{\kappa}, \kappa_R), \overline{\kappa})$, so that $\mathcal{N}$ investors trade shares in the market, and the symmetric equilibrium coexists with the asymmetric equilibrium with full separation. Then, aggregate greenness is lower under $\mathcal{R}$ investors' concentration provided that κ is not too large:

$$\kappa < \frac{\lambda(2\pi^m + \eta\lambda)\chi(1-\chi)}{2(c+\pi^m)(1-2\chi)} \in (\max(\underline{\kappa}, \kappa_R), \overline{\kappa}).$$

E.5 Proofs

Proof of Proposition 14. Recall that, with no SRI, the symmetric equilibrium investment level σ_0 is obtained from

$$c\sigma_0 - \frac{\partial \pi_j}{\partial \sigma_j}(\sigma_0, \vec{\sigma}_0) = 0, \tag{E.6}$$

where the LHS is increasing in σ_0 by Assumption 1(i). Then, by the implicit function theorem, we have that

$$\frac{\partial \sigma_0}{\partial \chi_c} \propto -\frac{\partial}{\partial \chi_c}\left[c\sigma_0 - \frac{\partial \pi_j}{\partial \sigma_j}(\sigma_0, \vec{\sigma}_0)\right] = \frac{\partial^2 \pi_j}{\partial \sigma_j \partial \chi_c}(\sigma_0, \vec{\sigma}_0). \tag{E.7}$$

Hence, $\frac{\partial^2 \pi_j}{\partial \sigma_j \partial \chi_c} > 0$ implies that σ_0 is increasing in χ_c.

Similarly, the symmetric equilibrium with SRI is pinned down by Equation (B.10), whose LHS is increasing in σ^* by Assumption 1(i) and decreasing in χ_c if and only if $\frac{\partial^2 \pi_j}{\partial \sigma_j \partial \chi_c} > 0$. Under this assumption, by the implicit function theorem, we have that $\frac{\partial \sigma^*}{\partial \chi_c} > 0$.

Finally, note that

$$\frac{\partial \widehat{\chi}(\sigma_0, 0)}{\partial \chi_c} = \frac{\partial \widehat{\chi}(\sigma, 0)}{\partial \sigma}\frac{\partial \sigma_0}{\partial \chi_c} < 0. \tag{E.8}$$

[15]In these latter equilibria, $\alpha^{\mathcal{R}}_{-j} = 0$ and $\alpha^{\mathcal{N}}_{-j} \equiv \alpha^{\mathcal{N}}$, $\alpha^{\mathcal{R}}_j = 1$ and $\alpha^{\mathcal{N}}_j = 1 - \alpha^{\mathcal{N}}$, with

$$\alpha^{\mathcal{N}} = \frac{c^2\kappa - c\eta\lambda^2(1-\chi)\chi - (\pi^m)^2\kappa}{(1-\chi)\left(2c^2\kappa - c\lambda\chi(2\eta\lambda + \pi^m) - (\pi^m)^2(2\kappa - \lambda\chi)\right)},$$

and CSR policies

$$\sigma_j = \frac{\kappa\left(c(2\eta\lambda\chi + \pi^m) - (\pi^m)^2\right)}{c^2\kappa - c\eta\lambda^2(1-\chi)\chi - (\pi^m)^2\kappa}, \quad \sigma_{-j} = \frac{\pi^m\left(c\kappa - \eta\lambda^2(1-\chi)\chi - \kappa(2\eta\lambda\chi + \pi^m)\right)}{c^2\kappa - c\eta\lambda^2(1-\chi)\chi - (\pi^m)^2\kappa}.$$

[16]For all $\chi > 1/2$, the only asymmetric equilibria are those in which $\mathcal{R}$ investors buy shares in both firms and all $\mathcal{N}$ investors buy shares of the same firm, which exist for all $\kappa > \kappa_R$. Hence, there is never concentration of $\mathcal{R}$ investors in equilibrium.

Therefore, $\widehat{\chi}(\cdot)$ being decreasing in σ, the result that σ_0 is increasing in χ_c implies that $\widehat{\chi}(\sigma_0, 0)$ is decreasing in χ_c, i.e., that the parameter space where the symmetric equilibrium features SRI expands as χ_c grows larger.

Proof of Proposition 15. To start with, note that we can express the expected profit for firm j as

$$\Pi_j = \sigma_j \mathbb{E}[\tilde{\pi}_j | a_j = 1] + (1 - \sigma_j)\mathbb{E}[\tilde{\pi}_j | a_j = 0] - C(\sigma_j), \tag{E.9}$$

where we define

$$\mathbb{E}[\tilde{\pi}_j | a_j = a] = \sum_{z=0}^{N-1} \Pr[n_{-j} = z]\mathbb{E}[\pi_j | a_j = a, n_{-j} = z], \tag{E.10}$$

for $a \in \{0, 1\}$,[17] with $n_{-j} \in \{0, \ldots, N-1\}$ denoting the number of green firm j's rivals. Maximizing Π_j with respect to σ_j then gives

$$\sigma_j = \frac{1}{c} \sum_{z=0}^{N-1} \Pr[n_{-j} = z]\Delta\pi(z), \tag{E.11}$$

where

$$\Delta\pi(z) \equiv \mathbb{E}[\tilde{\pi}_j | a_j = 1, n_{-j} = z] - \mathbb{E}[\tilde{\pi}_j | a_j = 0, n_{-j} = z] \tag{E.12}$$

is the (average) extra profit from being green when z rivals are green.

We obtain strategic substitutability whenever the expression in the RHS of Equation (E.11) is decreasing in any $\sigma_{j'}$. Denoting by $n_{-j,j'}$ the number of green firms excluding j and j', whose probability distribution does not depend on $(\sigma_j, \sigma_{j'})$, we have

$$\Pr[n_{-j} = z] = \sigma_{j'} \Pr[n_{-j,j'} = z - 1] + (1 - \sigma_{j'}) \Pr[n_{-j,j'} = z] \quad \forall z \geq 1, \tag{E.13}$$

$$\Pr[n_{-j} = 0] = (1 - \sigma_{j'}) \Pr[n_{-j,j'} = 0] \quad \text{for } z = 0. \tag{E.14}$$

Differentiating the RHS of Equation (E.11) with respect to $\sigma_{j'}$ and rearranging gives

$$\frac{\partial \sigma_j}{\partial \sigma_{j'}} = -\frac{1}{c} \sum_{z=0}^{N-2} \Pr[n_{-j,j'} = z] \left[\Delta\pi(z) - \Delta\pi(z+1)\right]. \tag{E.15}$$

A sufficient condition for this derivative to be negative is thus $\Delta\pi(z) \geq \Delta\pi(z+1)$, with strict

[17] We still take expectations in this expression because the product market competition model may also include additional sources of uncertainty, such as uncertain demand or production costs.

inequality for at least one z.

Proof of Proposition 16. Define

$$\kappa_R \equiv \frac{\lambda \chi \sigma_R}{N}, \tag{E.16}$$

where $\sigma_R \in (0,1)$ is the unique solution of

$$\sigma_R = \frac{1}{c}\left[\eta\lambda + \frac{\partial \pi_j}{\partial \sigma_j}(\vec{\sigma}_R)\right], \tag{E.17}$$

and corresponds to firms' CSR investments if $\mathcal{N}$ investors are crowded out (i.e., $s_j^{\mathcal{R}} = 1$ for all j). Proceeding as in the main model, it can be shown that the symmetric equilibrium always exists and is unique:

- For $\kappa \leq \kappa_R$, $\mathcal{N}$ investors are crowded out ($\alpha_j^{\mathcal{N}} = 0$ for all j), $\mathcal{R}$ investors are equally split across firms ($\alpha_j^{\mathcal{R}} = 1/N$ for all j), and all firms choose $\sigma_j = \sigma_R$.
- For $\kappa > \kappa_R$, both $\mathcal{N}$ and $\mathcal{R}$ investors are equally split across firms ($\alpha_j^{\mathcal{R}} = \alpha_j^{\mathcal{N}} = 1/N$ for all j), $\mathcal{R}$ investors hold shares $s_j^{\mathcal{R}} = s^{\mathcal{R}*}$ in all firms, and all firms choose $\sigma_j = \sigma^*$, where $(\sigma^*, s^{\mathcal{R}*})$ solve

$$\begin{cases} \sigma^* = \frac{1}{c}\left[\eta\lambda s^{\mathcal{R}*} + \frac{\partial \pi_j}{\partial \sigma_j}(\vec{\sigma}^*)\right]; \\ s^{\mathcal{R}*} = \frac{\chi}{N}\frac{1}{\kappa}\left[N\kappa + \lambda(1-\chi)\sigma^*\right]. \end{cases} \tag{E.18}$$

Asymmetric equilibria are also characterized as in the main model. Consider asymmetric equilibria with full separation, in which $\mathcal{R}$ investors entirely own firms $j \leq n$, whereas $\mathcal{N}$ investors entirely own the other firms $j > n$, for $n \in \{1, \ldots, N-1\}$. These equilibria feature the same CSR policies as the main model, i.e., $\sigma_j = \overline{\sigma}$ for $j \leq n$ and $\sigma_j = \underline{\sigma}$ for $j > n$, and now exist for

$$\chi < \frac{n}{N} \quad \text{and} \quad \frac{\chi(1-\chi)\lambda\underline{\sigma}}{n - N\chi} < \kappa < \frac{\chi(1-\chi)\lambda\overline{\sigma}}{n - N\chi}, \tag{E.19}$$

which can be shown following the same steps of the Proof of Proposition 2.