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Information acquisition and mutual funds

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Abstract

We study the formation of mutual funds by generalizing the standard competitive noisy rational expectations framework. In our model, informed agents set up mutual funds as a means of selling their private information to uninformed agents. We study the case of imperfect competition among fund managers, where uninformed agents invest simultaneously in multiple mutual funds. The size of the assets under management in the mutual fund industry is determined by endogenizing the agents' information acquisition decisions. Our model yields novel predictions on the informativeness of price, the optimal fees of mutual funds, and the equilibrium risk premium. In particular, we show that a sufficiently competitive mutual fund sector yields more informative prices and a lower equity risk premium.

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1 Introduction

One of the central ideas of modern financial economics is the concept of a noisy rational expectations equilibrium (Grossman and Stiglitz, 1980; Hellwig, 1980). Under the standard paradigm, agents trade only on their own accounts, which ignores the possibility that institutions such as mutual funds might influence equilibrium allocations and prices. This is highlighted in Ross (2005), who suggests that the standard paradigm is unstable since informed agents may choose to offer wealth management services to the uninformed agents. We explore this issue by modifying the standard noisy rational expectations model of Hellwig (1980) to allow informed agents to establish mutual funds. We then use our model to study the existence, the size of the assets under management, and the asset pricing implications of the mutual fund industry.

We show that informed agents are always better off establishing mutual funds rather than using their private information to trade on their own accounts. Thus mutual funds arise endogenously as the optimal strategy of informed utility maximizing agents. This establishes the existence of a mutual fund industry. Next, we endogenize the agents' information acquisition decisions, which determine the size of the assets under management. Lastly, we study how the existence of mutual funds impacts equilibrium asset prices. We show that a sufficiently competitive mutual fund industry produces more informative prices and a lower equity risk premium relative to an economy without mutual funds.

Since we endogenize the agents' information acquisition decisions, our paper can be viewed as extending Verrecchia (1982) to the case in which informed agents are allowed to sell their private information via mutual funds. Relative to the standard framework, such a possibility alters the incentives to acquire private information, which in turn impacts equilibrium prices. On the one hand, the existence of a vehicle for selling information increases the benefit of acquiring private information. Since informed agents trade more aggressively as mutual fund managers, this effect tends to increase the equilibrium price informativeness. On the other hand, mutual funds provide indirect exposure to the managers' private information. Thus the possibility of investing in mutual funds increases the attractiveness of remaining uninformed, which tends to reduce price informativeness. It is the interaction of these two effects that drives our equilibrium with endogenous mutual fund formation.

In our model, investing in a risky asset via a mutual fund is different than investing in a risky asset directly. Since households and fund managers have asymmetric information, the household sector's mutual fund payoff depends on the manager's risky asset demand. As in Dybvig and Ross (1985), from the household sector's perspective the distribution of the fund's payoff is different than that of the risky asset. Thus mutual funds increase the span of the market structure while allowing the uninformed agents to obtain exposure to the fund managers' private information. Even if the number of risky assets is small, multiple mutual funds might be established in equilibrium since households benefit from diversifying across

funds that possess different private signals.[1]

We derive our results for two types of investment management contracts. For the first type, fund managers are compensated via investment management fees that are proportional to their funds' final values. This mirrors the current fee structure of most no-load mutual funds. For the second type, we allow fund managers to use both proportional fees and fixed fees, where the latter quantity captures the effect of a sales load. Our focus on these types of contracts is motivated by the results in Admati and Pfleiderer (1997), who show that simple proportional contracts have more desirable properties than benchmark-based contracts. This distinguishes our paper from those of Brennan (1993), Das and Sundaram (2002), and Cuoco and Kaniel (2006), whose main emphasis is benchmark-adjusted and option-like compensation schemes.

Our mutual fund model involves selling private information indirectly, but our approach is different from the existing literature. While Admati and Pfleiderer (1986, 1988, 1990), Biais and Germain (2002), García and Urosević (2006) and Cespa (2007) are concerned with a monopolistic seller of information, we analyze the case of imperfect competition among mutual fund managers. Since we allow households to buy multiple mutual funds, a fund manager's fee setting decision is affected not only by the other managers' fees, which occurs naturally in an oligopolistic setting, but also by the household sector's equilibrium demand for mutual funds.[2] Thus, we go beyond the duopolistic cases in Fishman and Hagerty (1992) and Simonov (1999), analyzing a more general oligopolistic setting in which multiple fund managers compete for the investable funds of the household sector.

Although we study an imperfectly competitive mutual fund industry, our model has a perfectly competitive stock market. This allows us to connect our results to the existing literature (Hellwig, 1980; Verrecchia, 1982). It also adds parsimony to our model since we can analyze imperfect competition in the mutual fund industry without the additional complications that would arise if we allowed fund managers to behave strategically with respect to their risky asset trades. The general equilibrium nature of our model sets it apart from the partial equilibrium approaches in Chordia (1996), Nanda, Narayanan, and Warther (2000) and Christoffersen and Musto (2002).

Similar to our paper, Ross (2005) studies mutual funds within the standard competitive noisy rational expectations model. Ross (2005) takes the mutual fund industry as given and focuses on the signalling possibilities that arise when fund managers have different information precisions. In contrast, there is no signalling in our model since all fund managers are symmetric with respect to their risk tolerances and signal precisions. Instead, we study mutual fund

[1] Unlike Khorana and Servaes (1999) and Khorana, Servaes, and Tufano (2005), our explanation for the formation of mutual funds relies on private information.

[2] In the standard oligopoly game, it is assumed that agents value only one unit of the good in question. In the context of our mutual fund model, this is an undesirable assumption due to the benefits from investing in multiple funds. For related work on information acquisition and oligopolies, see Vives (1988) and Hwang (1993).

formation as the outcome of endogenous information acquisition. This allows us to make precise statements about the equilibrium size of the assets under management in the mutual fund industry, the industry's competitiveness, and the resulting impact on the stock market.[3] Our paper provides an explicit link between financial intermediation and equilibrium asset prices.

To facilitate our study of mutual funds, we make several simplifying assumptions that allow us to obtain most of our results in closed form. First, we fix the class of investment management contracts exogenously and we ignore moral hazard issues, which sets our paper apart from the optimal contracting literature.[4] Second, we assume managerial ability is common knowledge, which differentiates our paper from the screening models in Bhattacharya and Pfleiderer (1985), Huberman and Kandel (1993), Dow and Gorton (1997), and Das and Sundaram (2002). Lastly, we do not consider a host of issues that have been analyzed elsewhere in the literature. Specifically, we do not analyze mutual fund families (Massa, 2004; Gervais, Lynch, and Musto, 2005), fund manager turnover (Goldman and Slezak, 2003), dynamic issues (Berk and Green, 2004; Veldkamp, 2006), or trust-like funds (Mamaysky and Spiegel, 2002).

The remainder of our article is organized as follows. Section 2 outlines all of the model's primitives, including the overall structure of the model and our equilibrium definitions. Section 3.1 then assumes that mutual funds are available to the household sector and solves for a rational expectations equilibrium of the Hellwig (1980) type. Our equilibrium is summarized in Proposition 1. Section 3.2 endogenizes the fees of the mutual funds (Proposition 2) and shows that informed agents always establish mutual funds instead of trading on their own accounts using their private information. Section 3.3 endogenizes the fraction of agents that optimally becomes informed (Proposition 3), which determines the equilibrium size of the assets under management. Section 4 presents several extensions of our model, while section 5 concludes. All of our proofs are in the Appendix.

2 Assumptions and definitions

For ease of exposition, we describe our model using four dates. At date 0, we analyze the agents' information acquisition decisions to determine the equilibrium fraction of privately informed agents. At date 1, each informed agent chooses to either establish a mutual fund or trade on his own account. At date 2, we analyze the agents' optimal security demands and we

[3] Our modeling assumptions are also different than those in Ross (2005). For example, the household sector in Ross (2005) does not invest directly in the stock market, but instead obtains exposure to stocks only by purchasing mutual funds. We relax this constraint since allowing households to simultaneously have exposure to the stock market and mutual funds seems to be a better description of observed behavior.

[4] For optimal contracting models of delegated fund management, see Ross (1974), Dybvig and Spatt (1986), Stoughton (1993), Heinkel and Stoughton (1994), Rajan and Srivastava (2000), Arora and Ou-Yang (2001), Ou-Yang (2003), Palomino and Prat (2003), and Dybvig, Farnsworth, and Carpenter (2004).

determine equilibrium prices. Lastly, at date 3, the stock market pays a liquidating dividend, the mutual funds distribute their trading profits, and all agents consume their final realized wealth levels. We refer to dates 0, 1, and 2 as the information acquisition stage, the fund formation stage, and the trading stage, respectively.

2.1 Market structure

There is a continuum of groups in our model, where each group contains n agents. We index the groups by $k \in \left[0, \bar{k}\right]$ and we let $\bar{k} = 1/n$, which normalizes the economy so that the total mass of agents is equal to 1. There are h_k uninformed agents and m_k informed agents in group k, where $h_k + m_k = n$. We refer to the uninformed agents as "households" and the informed agents as "managers." The h_k households in group k are allowed to buy the mutual funds that are established in group k, but are precluded from buying any mutual fund in group $k' \neq k$. At the same time, the m_k informed agents in group k are allowed to establish mutual funds and serve the households in group k, but are precluded from serving any household in group $k' \neq k$. We also preclude the managers from holding positions in other funds. We index the managers by $i = 1, 2, \ldots, m_k$ and we index the households by $j = 1, 2, \ldots, h_k$. We use the double index ik to denote the ith manager in the kth group and we use jk to denote the jth household in the kth group. To describe the number of managers in each group across the entire economy, we define the set $\mathcal{M} = \{m_k : k \in [0, \bar{k}]\}$.

The main reason for our grouping structure is tractability.[5] We want to analyze how imperfect competition in the mutual fund industry affects asset prices in a competitive stock market. To preclude strategic trading in the stock market, it is necessary to have a continuum of informed agents, as in Hellwig (1980). However, to analyze imperfect competition in the mutual fund industry, it is necessary to limit the mutual fund competition to a finite number of managers. Our grouping procedure captures nicely both of these conditions. It allows for strategic behavior between the mutual fund managers in each group while maintaining the assumption of a perfectly competitive stock market. Although a similar set of competitive assumptions is used by Ross (2005), our model lays out an alternative structure that supports these assumptions.

Focusing on a model with a competitive stock market also allows us to compare our results to the existing literature. Although our grouping procedure restricts the contracting possibilities between managers and households, we allow all agents to invest directly in the risky asset.

[5] Alternatively, one could eliminate the grouping structure and work with a finite agent model in which every household is allowed to purchase every available mutual fund. In this case, the informed fund managers internalize their impact on the risky asset price when choosing their contingent fees. We analyzed this setting numerically and our results suggest that the intuition from our model with groups is robust. In particular, a sufficiently high level of competition in the mutual fund sector produces more informative prices and a lower equity risk premium. Thus we prefer our grouping procedure, which allows for closed-form expressions.

Thus by using a group size of $n = 1$, we can recover results from the existing literature, such as those in Hellwig (1980), Verrecchia (1982), and Diamond (1985). A group size of $n = 1$ precludes mutual funds since an informed agent has no opportunity to market his private information to an uninformed agent. The fact that we nest the standard model makes it easy to analyze how equilibrium properties such as price informativeness and the equity risk premium are altered in the presence of mutual funds.

Our grouping procedure also captures an important observed characteristic of the mutual fund industry. Although there are thousands of mutual funds available today, it is typical in practice for households to not hold positions in all of the available funds. One reason is that the cost of establishing and maintaining a large number of mutual fund accounts might outweigh the diversification benefit, which is decreasing in the number of funds purchased. Thus it seems unrealistic to allow the households to contract with a large number (e.g., a continuum) of funds. Another reason is that frictions may prevent households (resp. fund managers) from contracting with all of the existing mutual funds (resp. households). We do not model these frictions explicitly, but instead we rely on our grouping procedure as one way of capturing this feature of the mutual fund industry. While our grouping procedure is consistent with frictions between managers and households, it does not address why the same small set of mutual funds cannot be the same for every household. In addition, since we assume there is a continuum of groups, we are able to explain the relative proportion of mutual funds and households but not the absolute number of funds. We emphasize that explaining the structure of the mutual fund industry, which presumably can be done using a formal costly search model that might endogenize our grouping procedure, is beyond the scope of our paper.

2.2 Preferences and beliefs

The agents are expected utility maximizers with utility functions that exhibit constant absolute risk aversion (CARA). The ith manager in group k has the utility function $u(W_{ik}) = -\exp(-\tau W_{ik})$, where τ is the common risk aversion parameter and W_{ik} is the agent's date 3 wealth. Likewise, the jth household in group k has the utility function $u(W_{jk}) = -\exp(-\tau W_{jk})$. We relax the homogeneous risk aversion assumption in section 4.2, but for now it allows us to focus on the informational aspects of the model. We assume that all agents have zero initial wealth, which is without loss of generality due to the properties of CARA utility.

There are two primitive assets available for trading, a riskless asset and a risky asset. The riskless asset pays zero interest and has a perfectly elastic supply. The risky asset's price per share at date 2 is P_x and its payoff per share at date 3 is X, where X is normally distributed with mean μ_x and variance σ_x^2. We use the shorthand notation $X \sim \mathcal{N}(\mu_x, \sigma_x^2)$. The per capita supply of the risky asset is $U \sim \mathcal{N}(\mu_u, \sigma_u^2)$, which is interpreted as noise trading in the economy. In addition to the primitive assets, the investment opportunity set of the jth

household in group k includes the m_k mutual funds that are formed endogenously. The mutual funds are discussed in more detail shortly.

All agents have rational expectations in the sense of Hellwig (1980) and use the information revealed by price when forming their posterior beliefs. Following the literature, we solve for an equilibrium in which the risky asset price is an affine function of X and U, i.e.,

$$P_x = a + bX - dU\,, \tag{1}$$

where the coefficients a, b, and d are determined in equilibrium. Note that the private signals of the informed agents do not appear in (1), which is standard in models that use a continuum of agents with independent signal errors. We also conjecture that the prices of the mutual funds are constants, and we let P_{ik} denote the price per unit of the ith manager's fund in group k. These conjectures are verified to hold in the equilibrium that we present below.

At date 0, each agent can become privately informed by paying a fixed cost $c > 0$. Agents observe their private signals at date 2, which is the trading stage of the model. The ith informed agent in the kth group observes the private signal $Y_{ik} = X + \epsilon_{ik}$, where $\epsilon_{ik} \sim \mathcal{N}(0, \sigma_\epsilon^2)$. Thus the information possessed at date 2 by the ith informed agent in group k is the vector (P_x, Y_{ik}), while every uninformed agent observes only P_x. All agents have the same information at dates 0 and 1. We assume that X, U, and the signal errors ϵ_{ik} for all i and k are mutually independent random variables.

2.3 Mutual funds

At date 1, each informed agent faces a choice of whether to use his information to trade on his own account (as a proprietary trader) or to offer investment management services to the h_k households in his group. We model this problem by introducing the choice variable $\alpha_{ik} \in (0, 1]$, which is the *contingent management fee* of the ith informed agent in group k. The contingent management fee is the proportion of the fund's date 3 payoff that the informed agent retains as his compensation.[6] Choosing $\alpha_{ik} \in (0, 1)$ is equivalent to being a mutual fund manager since the fund's payoff is shared with the households. On the contrary, choosing $\alpha_{ik} = 1$ is equivalent to being a proprietary trader. To describe the contingent management fees within group k, we use the set $\{\alpha_{ik}\}_{i=1}^{m_k} = \{\alpha_{1k}, \alpha_{2k}, \ldots, \alpha_{m_k k}\}$. To describe the contingent management fees across the entire economy, we use the set $\mathcal{A} = \{\alpha_{ik} : i = 1, \ldots, m_k; k \in [0, \bar{k}]\}$.

The structure of our model involves two implicit assumptions that are worth mentioning. First, we assume there are no agency problems between the fund managers and the households in group k. Allowing a mutual fund's payoff to depend formally on managerial effort is feasible

[6] We consider affine contracts in section 4.1. Although our contingent fee α_{ik} is identical contractually to the indirect sales discussed in Admati and Pfleiderer (1990), our model is different since we analyze oligopolistic competition among mutual fund managers.

in our setting, but it makes our model very complicated. Second, we assume there are no opportunities for the managers to signal that they have superior private information. The households know the managers are privately informed with identical signal variance σ_ϵ^2.

If the ith informed agent in group k establishes a mutual fund, the fund's payoff at date 3 is

$$Z_{ik} = P_{ik} + \gamma_{ik}\left(X - P_x\right), \tag{2}$$

where γ_{ik} is the manager's risky asset demand and P_{ik} is the date 2 fund price. Without loss of generality, we normalize the fund's shares to be equal to 1 unit. Thus P_{ik} represents the ith fund's assets under management, i.e., it is the aggregate amount invested in the ith fund by the h_k households in group k. In equilibrium we show that $P_{ik} = 0$ when $\alpha_{ik} = 1$, which is consistent with the ith agent being a proprietary trader.[7]

For ease of exposition, we define the ith manager's *total fee* as $\alpha_{ik}P_{ik}$. We motivate this definition by examining the household sector's net payoff from investing in the ith mutual fund. Since the manager retains $\alpha_{ik}Z_{ik}$, the household sector's net payoff is $(1-\alpha_{ik})Z_{ik} - P_{ik}$. Using (2) to substitute for Z_{ik}, this payoff is equivalent to $(1-\alpha_{ik})\gamma_{ik}(X - P_x) - \alpha_{ik}P_{ik}$. This shows that the net fund payoff can be decomposed into variable and fixed parts. The variable part is $(1-\alpha_{ik})\gamma_{ik}(X - P_x)$, which represents the household sector's portion of the mutual fund's risky asset bet. The fixed part is $\alpha_{ik}P_{ik}$, which represents the total fee paid to the fund manager by the household sector. Thus the household sector in group k pays $\alpha_{ik}P_{ik}$ in exchange for the risky exposure $(1-\alpha_{ik})\gamma_{ik}(X - P_x)$.

2.4 Equilibrium definitions

We now define an equilibrium for our model. From our previous discussion, the optimization problem of the ith fund manager in group k at the trading stage is

$$\max_{\gamma_{ik}} \quad \mathbb{E}\left[-e^{-\tau W_{ik}} \,\middle|\, P_x, Y_{ik}\right], \tag{3}$$

where

$$W_{ik} = \alpha_{ik}Z_{ik} - c = \alpha_{ik}P_{ik} + \alpha_{ik}\gamma_{ik}\left(X - P_x\right) - c \tag{4}$$

is the fund manager's date 3 wealth. Using (4), note that (3) reduces to a standard mean-variance optimization problem. We use $\hat{\gamma}_{ik}$ to denote the manager's optimal risky asset demand and we use $\hat{Z}_{ik}$ to denote equation (2) evaluated at $\hat{\gamma}_{ik}$.

[7] From (2), note that P_{ik} remains in the mutual fund and is invested by the fund manager. This is a realistic assumption (see p. 907 of Admati and Pfleiderer, 1990) that corresponds to current practice in the mutual fund industry.

The date 3 wealth of the jth household in group k is

$$W_{jk} = \theta_{jk}\left(X - P_x\right) + \sum_{i=1}^{m_k} \phi_{ijk}\left[\hat{Z}_{ik}(1-\alpha_{ik}) - P_{ik}\right], \tag{5}$$

where θ_{jk} is the household's risky asset demand and ϕ_{ijk} is the household's demand for the ith mutual fund. The first term in (5) is the household's profit from trading the risky asset directly, while the second term is the profit from trading the m_k mutual funds. Using (5), the optimization problem of the jth household in group k at the trading stage is

$$\max_{\theta_{jk},\{\phi_{ijk}\}_{i=1}^{m_k}} \quad \mathbb{E}\left[-e^{-\tau W_{jk}} \middle| P_x\right]. \tag{6}$$

We use $\hat{\theta}_{jk}$ and $\{\hat{\phi}_{ijk}\}_{i=1}^{m_k}$ to denote the optimal risky asset demand and the optimal mutual fund demands, respectively.

The problem in (6) is non-standard in the sense that it does not reduce to the usual mean-variance optimization problem. This occurs because the mutual fund payoffs are not normally distributed conditional on the household's information set. The ith manager's optimal demand $\hat{\gamma}_{ik}$ depends on his private signal Y_{ik}, and thus from the household's perspective $\hat{Z}_{ik}$ involves a product of normally distributed random variables. This is precisely what makes the mutual fund payoffs valuable in our model – they depend on the managers' private signals, which are not part of the households' information set.

Our first definition generalizes the standard noisy rational expectations equilibrium to the case in which informed agents are allowed to establish mutual funds.

Definition 1. *For given sets $\mathcal{M}$ and $\mathcal{A}$, a rational expectations equilibrium at the trading stage is a collection of demands for the households $\{\hat{\theta}_{jk}, \{\hat{\phi}_{ijk}\}_{i=1}^{m_k} : j = 1, 2, \ldots, h_k; k \in [0, \bar{k}]\}$, a collection of demands for the managers $\{\hat{\gamma}_{ik} : i = 1, 2, \ldots, m_k; k \in [0, \bar{k}]\}$, a set of mutual fund prices $\{P_{ik} : i = 1, 2, \ldots, m_k; k \in [0, \bar{k}]\}$, and a risky asset price P_x such that:*

(a) $\hat{\theta}_{jk}$ and $\{\hat{\phi}_{ijk}\}_{i=1}^{m_k}$ solve (6) for the jth household in group k;

(b) $\hat{\gamma}_{ik}$ solves (3) for the ith manager in group k;

(c) the risky asset market and the mutual fund markets clear, that is

$$\int_0^{\bar{k}} \left[\sum\nolimits_{i=1}^{m_k} \hat{\gamma}_{ik} + \sum\nolimits_{j=1}^{n-m_k} \hat{\theta}_{jk}\right] dk = U, \tag{7}$$

$$\sum\nolimits_{j=1}^{n-m_k} \hat{\phi}_{ijk} = 1, \quad \text{for all } i \text{ and } k. \tag{8}$$

We remark that the optimal demands $\hat{\gamma}_{ik}$, $\hat{\theta}_{jk}$ and $\hat{\phi}_{ijk}$ in (7)-(8) depend on the risky asset price P_x and the mutual fund prices $\{P_{ik}\}_{i=1}^{m_k}$ in group k. Although our notation suppresses this dependence, the equilibrium prices are determined in the usual way by inverting the market clearing conditions (7)-(8).

To define an equilibrium at the other stages of our model, we need some additional notation. First, we define the set $\mathcal{M}_k = \{m_{k'} : k' \in [0, \bar{k}], k' \neq k\}$ which describes the number of managers in each group, excluding the kth group. Second, we define the set $\mathcal{A}_k = \{\alpha_{ik'} : i = 1, \ldots, m_{k'}; k' \in [0, \bar{k}], k' \neq k\}$ which gives the contingent fees of the managers, excluding those in the kth group. We use $\hat{\mathcal{A}}_k$ to denote the set of optimal fees. Next, we use $\hat{W}_{ik}$ and $\hat{W}_{jk}$ to denote the optimal wealth of the ith manager and the jth household, respectively, in group k. We construct $\hat{W}_{ik}$ by substituting $\hat{\gamma}_{ik}$ and the equilibrium prices into (4). Likewise, we construct $\hat{W}_{jk}$ by substituting $\hat{\theta}_{jk}$, $\{\hat{\phi}_{ijk}\}_{i=1}^{m_k}$, and the equilibrium prices into (5). Lastly, we define

$$\mathcal{U}_{ik}(m_k, \{\alpha_{lk}\}_{l=1}^{m_k}; \mathcal{M}_k, \mathcal{A}_k) \equiv -\frac{1}{\tau} \log\left(-\mathbb{E}\left[-e^{-\tau \hat{W}_{ik}}\right]\right)$$

and

$$\mathcal{V}_{jk}(m_k, \{\alpha_{lk}\}_{l=1}^{m_k}; \mathcal{M}_k, \mathcal{A}_k) \equiv -\frac{1}{\tau} \log\left(-\mathbb{E}\left[-e^{-\tau \hat{W}_{jk}}\right]\right),$$

where $\mathcal{U}_{ik}$ and $\mathcal{V}_{jk}$ are the unconditional certainty equivalent wealth levels of the ith manager and the jth household, respectively, in group k. Note that $\mathcal{U}_{ik}$ and $\mathcal{V}_{jk}$ depend directly on m_k and $\{\alpha_{lk}\}_{l=1}^{m_k}$, which characterize the mutual fund sector in group k. However, $\mathcal{U}_{ik}$ and $\mathcal{V}_{jk}$ also depend on $\mathcal{M}_k$ and $\mathcal{A}_k$, which characterize the mutual fund sectors for all of the other groups. The latter dependence shows up because $\mathcal{M}_k$ and $\mathcal{A}_k$ influence the price coefficients in (1). With this notation in place, we now define a fee setting equilibrium at the fund formation stage of our model.

Definition 2. *For a given set $\mathcal{M}$, an equilibrium at the fund formation stage is a collection of contingent fees $\mathcal{A}$ such that $\{\hat{\alpha}_{ik}\}_{i=1}^{m_k}$ constitutes a pure strategy symmetric Nash equilibrium for the m_k managers in group k. The equilibrium contingent fee of the ith manager in group k is given by*

$$\hat{\alpha}_{ik} = \arg\max_{\alpha_{ik}} \quad \mathcal{U}_{ik}(m_k, \{\alpha_{lk}\}_{l=1}^{m_k}; \mathcal{M}_k, \mathcal{A}_k)\big|_{\hat{\alpha}_{-i}^k, \hat{\mathcal{A}}_k}, \tag{9}$$

where $\hat{\alpha}_{-i}^k$ is the set $\{\hat{\alpha}_{lk}\}_{l=1}^{m_k}$ with the ith element omitted.

The equilibrium at the fund formation stage is the outcome of a non-cooperative game that is played by the m_k informed agents in group k. In the equilibrium we study, every informed agent in group k sets the same contingent fee, i.e., $\hat{\alpha}_{ik} = \hat{\alpha}_k$ for all $i = 1, \ldots, m_k$. However, we allow for different groups to have different contingent fees since the number of informed agents m_k may be different between groups. Thus our equilibrium involves symmetry within a group,

but allows for the possibility of asymmetry across groups.[8] To show that the equilibrium contingent fee depends on the group size, we write $\hat{\alpha}_k = \hat{\alpha}_k(m_k)$. Of course, $\hat{\alpha}_k(m_k)$ also depends on $\mathcal{M}_k$ and $\hat{\mathcal{A}}_k$, but we often suppress this dependence for notational simplicity.

Since the managers in group k set identical contingent fees, they have the same certainty equivalent of wealth. Thus we can write

$$\mathcal{U}_k(m_k, \hat{\alpha}_k(m_k); \mathcal{M}_k, \hat{\mathcal{A}}_k) = \mathcal{U}_{ik}(m_k, \{\hat{\alpha}_{lk}\}_{l=1}^{m_k}; \mathcal{M}_k, \hat{\mathcal{A}}_k),$$

for all $i = 1, 2, \ldots, m_k$. The function $\mathcal{U}_k$ is the unconditional certainty equivalent wealth for each manager in group k, evaluated at the optimal contingent fees $\hat{\alpha}_{ik}$ for all i and k. Likewise, since there is symmetry within a group, we can write

$$\mathcal{V}_k(m_k, \hat{\alpha}_k(m_k); \mathcal{M}_k, \hat{\mathcal{A}}_k) = \mathcal{V}_{jk}(m_k, \{\hat{\alpha}_{lk}\}_{l=1}^{m_k}; \mathcal{M}_k, \hat{\mathcal{A}}_k),$$

for all $j = 1, 2, \ldots, h_k$. The function $\mathcal{V}_k$ is the unconditional certainty equivalent wealth for each household in group k, evaluated at the optimal contingent fees $\hat{\alpha}_{ik}$ for all i and k.

At the information acquisition stage of our model, each agent decides whether to acquire private information, which costs c, or to remain uninformed, which costs nothing. When making this decision, each agent anticipates correctly the equilibrium at the trading stage and the fund formation stage. An equilibrium at the information acquisition stage is defined next.

Definition 3. *An equilibrium at the information acquisition stage is a set $\mathcal{M}$ such that: (i) none of the m_k informed agents in group k is better off being uninformed; and (ii) none of the $n - m_k$ uninformed agents in group k is better off being informed. When $1 \leq m_k \leq n-1$, conditions (i) and (ii) can be stated explicitly as*

$$\mathcal{U}_k(m_k, \hat{\alpha}_k(m_k); \mathcal{M}_k, \hat{\mathcal{A}}_k) \quad \geq \quad \mathcal{V}_k(m_k - 1, \hat{\alpha}_k(m_k - 1); \mathcal{M}_k, \hat{\mathcal{A}}_k) \tag{10}$$

and

$$\mathcal{V}_k(m_k, \hat{\alpha}_k(m_k); \mathcal{M}_k, \hat{\mathcal{A}}_k) \quad \geq \quad \mathcal{U}_k(m_k + 1, \hat{\alpha}_k(m_k + 1); \mathcal{M}_k, \hat{\mathcal{A}}_k) \tag{11}$$

for all groups $k \in [0, \bar{k}]$. If $m_k = n$, only the first inequality must hold; if $m_k = 0$, only the second inequality must hold.

Definition 3 spells out a simple Nash game at the information acquisition stage. We emphasize that conditions (10)-(11) are strategic since the agents anticipate that the contingent fees will be different at the fund formation stage if there is one less or one more informed agent. For example, (10) requires that a manager in a group with m_k informed agents is better off

[8] This is the natural case to study since different groups may have different m_k, but all agents in the economy have the same risk tolerance and all informed agents have the same signal precision.

as a manager rather than as a household facing $m_k - 1$ managers. As an alternative setup, we could instead allow the fund formation and information acquisition choices to be simultaneous. In this case, the equilibrium definition would still involve a condition like (9), but now conditions (10)-(11) would involve $\mathcal{U}_{ik}$ and $\mathcal{V}_{jk}$ instead of $\mathcal{U}_k$ and $\mathcal{V}_k$. Thus each agent would treat $\hat{\alpha}^k_{-i}$ as constant when choosing whether or not to acquire private information. Although this alternative setup would alter somewhat the strategic interaction among agents, it does not affect our main results.

3 Equilibrium

We now solve our model by working backwards through the three stages. First we fix the sets $\mathcal{M}$ and $\mathcal{A}$ and solve for a rational expectations equilibrium at the trading stage. Then we move back to the fund formation stage and we endogenize $\mathcal{A}$ while holding $\mathcal{M}$ fixed, which gives $\hat{\mathcal{A}}$. Lastly, we move back to the information acquisition stage and we endogenize the set of informed agents to get $\hat{\mathcal{M}}$.

3.1 Asset trading stage

For given sets $\mathcal{M}$ and $\mathcal{A}$, the following proposition characterizes the equilibrium at the asset trading stage of our model.

Proposition 1. *The equilibrium at the asset trading stage has the following properties:*

(i) the equilibrium risky asset price is given by (1) with price coefficients

$$d = \frac{1 + \frac{1}{\tau^2 \sigma_\epsilon^2 \sigma_u^2} \int_0^{\bar{k}} \bar{\alpha}_k^{-1} dk}{\left[\frac{1}{\tau \sigma_x^2} + \frac{1}{\tau \sigma_\epsilon^2} \int_0^{\bar{k}} \bar{\alpha}_k^{-1} dk + \frac{1}{\tau \sigma_u^2} \left(\frac{1}{\tau \sigma_\epsilon^2} \int_0^{\bar{k}} \bar{\alpha}_k^{-1} dk \right)^2 \right]}, \tag{12}$$

and

$$\frac{b}{d} = \frac{1}{\tau \sigma_\epsilon^2} \int_0^{\bar{k}} \bar{\alpha}_k^{-1} dk, \qquad \frac{a}{d} = \frac{\frac{\mu_x}{\sigma_x^2} + \left(\frac{b}{d} \right) \frac{\mu_u}{\sigma_u^2}}{\left[\tau + \left(\frac{b}{d} \right) \frac{1}{\sigma_u^2} \right]}, \tag{13}$$

where $\bar{\alpha}_k^{-1} = \sum_{i=1}^{m_k} \alpha_{ik}^{-1}$*;*

(ii) the equilibrium value of the ith mutual fund in group k is

$$P_{ik} = \frac{\left(\frac{1}{\tau\alpha_{ik}\sigma_\epsilon^2}\right)\left[\left(\frac{1-\alpha_{ik}}{\alpha_{ik}}\right) - \frac{1}{h_k}\left(\frac{1-\alpha_{ik}}{\alpha_{ik}}\right)^2\right]}{\left[\mathrm{var}(X|P_x)^{-1} + \frac{2}{h_k\,\sigma_\epsilon^2}\sum_{l=1}^{m_k}\left(\frac{1-\alpha_{lk}}{\alpha_{lk}}\right) - \frac{1}{h_k^2\,\sigma_\epsilon^2}\sum_{l=1}^{m_k}\left(\frac{1-\alpha_{lk}}{\alpha_{lk}}\right)^2\right]}; \tag{14}$$

(iii) the optimal risky asset demand of the ith informed agent in group k is

$$\hat{\gamma}_{ik} = \frac{\mathbb{E}\left[X|P_x, Y_{ik}\right] - P_x}{\alpha_{ik}\tau\,\mathrm{var}(X|P_x, Y_{ik})}; \tag{15}$$

(iv) the optimal risky asset demand of the jth household in group k is

$$\begin{aligned}\hat{\theta}_{jk} &= \left[\frac{\mathbb{E}\left[X|P_x\right] - P_x}{\tau\,\mathrm{var}(X|P_x)}\right]\left(1 + \frac{\mathrm{var}(X|P_x)}{h_k\,\sigma_\epsilon^2}\sum_{i=1}^{m_k}\left(\frac{1-\alpha_{ik}}{\alpha_{ik}}\right)\right) \\ &\quad -\frac{1}{h_k}\sum_{i=1}^{m_k}(1-\alpha_{ik})\,\mathbb{E}\left[\hat{\gamma}_{ik}|P_x\right];\end{aligned} \tag{16}$$

(v) the optimal demand for the ith mutual fund by household j in group k is $\hat{\phi}_{ijk} = \frac{1}{h_k}$.

Proposition 1 generalizes the standard noisy rational expectations equilibrium to the case in which informed agents are allowed to establish mutual funds and market their private information to uninformed households. Note that the proposition is consistent with our earlier price conjectures. First, the equilibrium risky asset price is an affine function of X and U, where the price coefficients are given by (12)-(13). Second, the mutual fund price in (14) is constant and thus is uninformative with respect to the asset's payoff X. To gain some intuition, note that from the household sector's perspective, the value of the ith fund depends on the covariance between the ith manager's demand and the risky asset's payoff. Due to the multivariate normal assumption, the covariances in our model are constants and thus the fund prices do not depend on X and U.

As in Hellwig (1980), the equilibrium risky asset price aggregates and partially reveals the informed agents' private information. The information revealed by price is measured by

$$\mathrm{var}(X|P_x)^{-1} = \frac{1}{\sigma_x^2} + \frac{b^2}{d^2\sigma_u^2}.$$

Thus the ratio of price coefficients b/d is a key determinant of the risky asset's price informativeness. We note that b/d depends on the sets $\mathcal{M}$ and $\mathcal{A}$ through the variable $\int_0^{\bar{k}} \bar{\alpha}_k^{-1} dk$, where $\bar{\alpha}_k^{-1} = \sum_{i=1}^{m_k} \alpha_{ik}^{-1}$. Since b/d measures the signal-to-noise ratio of the information that

is revealed by (1), the risky asset price is more informative when agents are fund managers instead of proprietary traders. This follows since the value of b/d when $\alpha_{ik} \in (0,1)$ for all i and k is greater than the value of b/d when $\alpha_{ik} = 1$ for all i and k. The intuition comes from the manager's optimal demand in (15), which is inversely proportional to α_{ik}. By establishing a mutual fund, the manager's effective risk aversion is reduced from τ, which prevails in the Hellwig (1980) setting, to $\alpha_{ik}\tau$. This reduction in the effective risk aversion increases the manager's trading aggressiveness, which in turn increases the informational content of the equilibrium price.

Since the ith manager's optimal demand in (15) depends on Y_{ik}, each household in group k views $\hat{\gamma}_{ik}$ as a random variable. Thus mutual funds increase the span of the financial market while allowing the household sector to get exposure to the manager's private information that is contained in $\hat{\gamma}_{ik}$. While each household believes that $(X - P_x)$ is normally distributed, the quantity $\hat{\gamma}_{ik}(X - P_x)$ is not normally distributed. Instead, $\hat{\gamma}_{ik}(X - P_x)$ is a noisy quadratic function of X.[9] Thus investing in a risky asset via a mutual fund is quite different than investing in the same risky asset directly.

To explore this more fully, note that the household sector's net payoff from investing in the ith mutual fund is

$$(1 - \alpha_{ik})Z_{ik} - P_{ik} = \frac{\rho_{ik} h_k}{\tau \sigma_\epsilon^2} Y_{ik}(X - P_x) + q(P_x)(X - P_x) - \alpha_{ik} P_{ik},$$

where $q(P_x)$ is an affine function of P_x and $\rho_{ik} = (1-\alpha_{ik})/(\alpha_{ik} h_k)$. The household's net payoff from a fund includes three terms, the last two of which can be replicated by a household that trades the risky and riskless assets. The first term, however, cannot be replicated since Y_{ik} is not part of the household's information set. Households therefore buy the ith mutual fund to get exposure to the first term, where ρ_{ik} controls the amount of the exposure. This first term involves X^2, which is always positive. However, it also involves the error term ϵ_{ik}. Thus there is a trade-off: investing in a mutual fund exposes the household to X^2, but it also exposes the household to the fund manager's signal error ϵ_{ik}. To diversify its exposure to ϵ_{ik}, each household invests in all of the funds in its group.

The exposure ρ_{ik} also affects the mutual fund price in (14). In fact, P_{ik} in (14) depends directly on the entire collection of exposures $\{\rho_{lk}\}_{l=1}^{m_k}$ for group k. This foreshadows the strategic fee setting equilibrium that we study at the fund formation stage in section 3.2. Finally, note that P_{ik} also depends on the exposures of the mutual funds in groups $k' \neq k$. This is because $\mathcal{A}_k$ and $\mathcal{M}_k$ affect $\text{var}(X|P_x)^{-1}$, which shows up in the denominator of (14).

[9] Although Brennan and Cao (1996) also analyze securities with a quadratic payoff, our model is different than theirs for three reasons. First, our quadratic function arises endogenously as the mutual fund's optimal payoff. Second, our quadratic function is noisy, i.e., it depends on the error ϵ_{ik}. Lastly, our quadratic function depends on α_{ik}, which implies that the fund manager controls the households' exposure to X^2.

While the ith manager's demand in (15) has the usual mean-variance form, the jth household's demand in (16) has some novel properties. The term in square brackets in (16) is the familiar mean-variance demand that shows up in Hellwig (1980). This mean-variance term is multiplied by a quantity in parentheses that is greater than 1. We refer to this as a risk aversion effect since we can interpret the household's effective risk aversion as being equal to τ divided by this quantity. Thus the risk aversion effect leads to aggressive trading. The second effect is a feedback effect, which is given by the final term in (16). The feedback effect captures the notion that the household decreases its long risky asset position (or increases its short position) if it believes the fund managers in its group are taking long positions in the risky asset. Thus the feedback effect offsets some of the trading aggressiveness of the risk aversion effect. As $\alpha_{ik} \to 1$ for all i and k, both effects vanish. In this case, all informed agents are proprietary traders and our model at the asset trading stage collapses to that of Hellwig (1980).

Lastly, the optimal mutual fund demands in part (v) of Proposition 1 are due to efficient risk sharing among the h_k households in group k. Since the h_k households are identical, each household buys $1/h_k$ of each of the available m_k funds. Later in the article (see section 4.2) we discuss how our asset trading stage is altered if the households are heterogeneous.

3.2 Fund formation stage

We turn now to the analysis of mutual fund formation. At this stage of our model, the ith manager in group k chooses his optimal contingent fee by solving the problem in (9). From our earlier discussion, recall that the ith manager's optimal wealth is constructed by substituting $\hat{\gamma}_{ik}$ and the optimal prices into (4). Since $\hat{\gamma}_{ik}$ is inversely proportional to α_{ik}, only the first term on the right-hand side of (4) depends on α_{ik}. Thus the ith informed agent's problem in (9) reduces to choosing the fee α_{ik} that maximizes $\alpha_{ik}P_{ik}$, where P_{ik} is given in (14). The ith manager takes $\hat{\alpha}^k_{-i}$ and $\hat{\mathcal{A}}_k$ as given when choosing α_{ik}.

Following Definition 2, we study an equilibrium in which every manager in group k chooses the same contingent fee, i.e., $\hat{\alpha}_{ik} = \hat{\alpha}_k$ for all i. The optimal fee depends on the number of fund managers in group k, but also on $\mathcal{M}_k$ and $\hat{\mathcal{A}}_k$. This is easily seen by noting that P_{ik} in (14) depends on $\text{var}(X|P_x)^{-1}$, which in turn is driven by $\mathcal{M}_k$ and $\hat{\mathcal{A}}_k$. Thus to be precise in our analysis, we let $\hat{\alpha}_k = \hat{\alpha}_k(m_k, \mathcal{R})$, where $\mathcal{R}$ is the equilibrium level of $\text{var}(X|P_x)^{-1}$. The next proposition characterizes the set of optimal contingent fees $\hat{\mathcal{A}}$.

Proposition 2.

(i) Fix the set $\mathcal{M}$ and define the function $\hat{\rho}(m, r)$ for $r > 0$ as the unique solution to the cubic equation

$$2(m-1)\hat{\rho}(m,r)^3 + (4-5m)\hat{\rho}(m,r)^2 + 2(m-1-\sigma_\epsilon^2 r)\hat{\rho}(m,r) + \sigma_\epsilon^2 r = 0 \qquad (17)$$

that satisfies $\hat{\rho}(m,r) \in (0,1/2)$. *Then for each group* k *with* $1 \leq m_k < n$, *the optimal contingent fee for every manager in the group is* $\hat{\alpha}_k(m_k,\mathcal{R}) = 1/(1+h_k\hat{\rho}(m_k,\mathcal{R}))$, *where* $\hat{\alpha}_k(m_k,\mathcal{R}) \in (1/(1+0.5h_k),1)$ *and* $\mathcal{R}$ *solves*

$$\mathcal{R} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_u^2}\left(\frac{1}{\tau\sigma_\epsilon^2}\right)^2 \left(\int_0^{\bar{k}} (1+h_k\hat{\rho}(m_k,\mathcal{R}))m_k dk\right)^2. \tag{18}$$

(ii) Suppose every group has $m_k = m$ *fund managers and* $h_k = h$ *households, where* $1 \leq m < n$. *Then* $\hat{\alpha}_{ik} = \hat{\alpha}(m)$ *for all* i *and* k, *where the optimal contingent fee* $\hat{\alpha}(m)$ *is given by the unique solution of the cubic equation*

$$k_3\hat{\alpha}(m)^3 + k_2\hat{\alpha}(m)^2 + k_1\hat{\alpha}(m) + k_0 = 0 \tag{19}$$

that satisfies $\hat{\alpha}(m) \in (1/(1+0.5h),1)$. *The coefficients of the equation are*

$$\begin{aligned}
k_0 &= \frac{2(m-1)}{\sigma_\epsilon^2 h^3} - \frac{2m^2}{\tau^2\sigma_u^2\sigma_\epsilon^4 hn^2},\\
k_1 &= \frac{m^2}{\tau^2\sigma_u^2\sigma_\epsilon^4 n^2} + \frac{2m^2}{\tau^2\sigma_u^2\sigma_\epsilon^4 hn^2} - \frac{(5m-4)}{\sigma_\epsilon^2 h^2} - \frac{6(m-1)}{\sigma_\epsilon^2 h^3},\\
k_2 &= \frac{2(m-1)}{\sigma_\epsilon^2 h} - \frac{2}{\sigma_x^2 h} + \frac{2(5m-4)}{\sigma_\epsilon^2 h^2} + \frac{6(m-1)}{\sigma_\epsilon^2 h^3},\\
k_3 &= \frac{1}{\sigma_x^2} - \frac{2(m-1)}{\sigma_\epsilon^2 h} + \frac{2}{\sigma_x^2 h} - \frac{(5m-4)}{\sigma_\epsilon^2 h^2} - \frac{2(m-1)}{\sigma_\epsilon^2 h^3}.
\end{aligned}$$

The characterization of the set of optimal contingent fees $\hat{\mathcal{A}}$ in part (i) of Proposition 2 is complex. There is one cubic equation for each group k, and the cubics are intertwined since each cubic depends on the other groups' choices through $\mathcal{R}$. Thus we have a system of cubics with a fixed point problem that must be solved to obtain $\hat{\mathcal{A}}$ in terms of the economic primitives. Our proof of the proposition shows that an equilibrium always exists.

If all groups are identical as in part (ii) of Proposition 2, the system reduces to a single cubic equation. In this case, the equilibrium level of $\text{var}(X|P_x)^{-1}$ that solves (18) is

$$\mathcal{R}(m) = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_u^2}\left(\frac{m}{n\tau\sigma_\epsilon^2\hat{\alpha}(m)}\right)^2. \tag{20}$$

After substituting $r = \mathcal{R}(m)$ into (17) and noting that $\hat{\rho}(m,\mathcal{R}(m)) = (1-\hat{\alpha}(m))/(h\hat{\alpha}(m))$, (17) can be rearranged to give (19).

The main implication of Proposition 2 is that $\hat{\alpha}_k < 1$, i.e., every informed agent establishes a mutual fund and markets his optimal investment strategy to the household sector. Rather than trade on their own accounts and keep their entire risky asset bets, which is the assumed course of action in the literature (Hellwig, 1980; Verrecchia, 1982), the informed agents in our

model find it optimal to share their risky asset bets with the household sector in exchange for the total fees. Thus Proposition 2 provides a foundation for the existence of mutual funds.

It is useful to contrast the optimal contingent fees in Proposition 2 with those that would prevail if risk sharing were efficient. Efficient sharing of the mutual fund's payoff would call for the ith manager to set $\rho_{ik} = 1$, which gives $\alpha_{ik} = 1/(1 + h_k)$. However, Proposition 2 calls for $\hat{\rho}_k \in (0, 1/2)$, which gives $\hat{\alpha}_k \in (1/(1 + 0.5h_k), 1)$. Thus the exposure $\hat{\rho}_k$ in Proposition 2 is more than 50% less than the efficient exposure. This result is driven by the fact that the ith manager maximizes the total fee $\alpha_{ik}P_{ik}$. While efficient risk sharing would produce a zero total fee since $P_{ik} = 0$ in (14) when $\rho_{ik} = 1$, Proposition 2 shows that the ith manager's optimal choice restricts the households' exposure to his private signal.

3.3 Information acquisition stage

We turn now to the analysis of information acquisition. As discussed in section 2.4, we solve for a set $\mathcal{M}$ such that none of the m_k informed agents in group k is better off being uninformed and none of the $n - m_k$ uninformed agents is better off being informed. Since our model involves a discrete number of agents in each group, multiple equilibria can arise at the information acquisition stage. Thus to make our analysis concise, we focus on the case in which there are at most two types of groups, where the two types differ only with respect to the number of informed agents, m_k. This simplifies the analysis while still allowing for comparisons to the existing literature.

Note that when $n = 1$, allowing for two types of groups is consistent with some of the groups having a single informed agent and some of the groups having a single uninformed agent. Since this is equivalent to the standard model without mutual funds (Diamond, 1985), Proposition 3 focuses on the case of $n \geq 2$.

Proposition 3. *Fix the cost $c > 0$ and let $n \geq 2$. Define the functions $\bar{c}(m)$ and $\underline{c}(m)$ as*

$$\begin{aligned}
\bar{c}(m) &= \frac{1}{2\tau}\log\left(\frac{\mathcal{R}(m) + 1/\sigma_\epsilon^2}{\mathcal{R}(m) + Q(m-1, \mathcal{R}(m))}\right) + f(m, \mathcal{R}(m)) + \frac{(m-1)}{(n-m+1)} f(m-1, \mathcal{R}(m)), \\
\underline{c}(m) &= \frac{1}{2\tau}\log\left(\frac{\mathcal{R}(m) + 1/\sigma_\epsilon^2}{\mathcal{R}(m) + Q(m, \mathcal{R}(m))}\right) + f(m+1, \mathcal{R}(m)) + \frac{m}{n-m} f(m, \mathcal{R}(m)),
\end{aligned}$$

where $\mathcal{R}(m)$ is given by (20), the functions $f, Q : \mathbb{R}^2 \to \mathbb{R}$ are defined as

$$\begin{aligned}
f(m, r) &= \frac{(n-m)\hat{\rho}(m, r)(1 - \hat{\rho}(m, r))}{\tau\sigma_\epsilon^2 \left(r + Q(m, r)\right)}, \\
Q(m, r) &= \frac{m\hat{\rho}(m, r)(2 - \hat{\rho}(m, r))}{\sigma_\epsilon^2},
\end{aligned}$$

and $\hat{\rho}(m, r)$ and $\hat{\alpha}(m)$ are given by (17) and (19), respectively. Then the following statements

hold:

(i) if $c \in [\underline{c}(\hat{m}), \bar{c}(\hat{m})]$ *for some* $\hat{m} \in \{1, 2, \ldots, n-1\}$*, there exists an equilibrium with identical groups where* $m_k = \hat{m}$ *for all* k*;*

(ii) if $c \in (\bar{c}(\hat{m}), \underline{c}(\hat{m}-1))$ *for some* $\hat{m} \in \{1, 2, \ldots, n\}$*, there exists an equilibrium with two types of groups. A fraction* λ *of the groups has* $\hat{m}$ *managers while a fraction* $1-\lambda$ *has* $\hat{m}-1$ *managers, where* $\lambda \in (0, 1)$*;*

(iii) if $c \leq \bar{c}(n)$*, there exists an equilibrium with identical groups where* $m_k = n$ *for all* k*;*

(iv) if $c \geq \underline{c}(0)$*, there exists an equilibrium with identical groups where* $m_k = 0$ *for all* k*.*

Collectively, the union of the intervals in parts (i)-(iv) of Proposition 3 contains $\mathbb{R}_+$. Thus for any cost $c > 0$, there always exists an equilibrium at the information acquisition stage of our model. Either all of the groups are identical, as described in parts (i), (iii), and (iv), or there are two types of groups, as described in part (ii).

To gain some intuition for the proposition, recall that Definition 3 involves $\mathcal{U}_k$ and $\mathcal{V}_k$, the ex-ante certainty equivalent wealth levels for a manager and household, respectively, in group k. These certainty equivalents can be written as

$$\mathcal{U}_k(m_k, \hat{\alpha}_k(m_k); \mathcal{M}_k, \hat{\mathcal{A}}_k) = \frac{1}{2\tau}\log(\mathcal{R} + 1/\sigma_\epsilon^2) + f(m_k, \mathcal{R}) - c + H, \tag{21}$$

$$\mathcal{V}_k(m_k, \hat{\alpha}_k(m_k); \mathcal{M}_k, \hat{\mathcal{A}}_k) = \frac{1}{2\tau}\log(\mathcal{R} + Q(m_k, \mathcal{R})) - \frac{m_k}{n - m_k} f(m_k, \mathcal{R}) + H, \tag{22}$$

where H is a quantity that is not affected by the decisions of any agent in group k. The function $f(m_k, \mathcal{R})$ is the fund manager's equilibrium total fee, $\hat{\alpha}_{ik} P_{ik}$, given that the manager's group contains m_k informed agents and the equilibrium level of $\text{var}(X|P_x)^{-1}$ is $\mathcal{R}$. The equilibrium total fee has a positive effect on the fund manager's certainty equivalent wealth in (21), while it has a negative effect on the household's certainty equivalent wealth in (22). The first term in (21) measures the ex-ante utility from the risky portfolio that the fund manager holds, which appears in the standard model. The first term in (22), which involves $Q(m_k, \mathcal{R})$, measures the ex-ante utility from the risky portfolio held by each household.

The conditions given in Proposition 3 are simply those from Definition 3, expressed in terms of the model primitives using (21) and (22). To see this, note that if all groups are identical, then $m_k = m$ for all k and the equilibrium level of $\text{var}(X|P_x)^{-1}$ is given by $\mathcal{R}(m)$ in (20). Upon substituting these quantities into (21)-(22), the equilibrium conditions in (10)-(11) are equivalent to a collection of intervals on the real line, where $[\underline{c}(m), \bar{c}(m)]$ is the mth interval. If the cost $c > 0$ of acquiring a private signal lies in at least one interval, we have an equilibrium in which all groups are identical, as in part (i) of Proposition 3. Likewise, an equilibrium with

identical groups may also occur if $c \leq \bar{c}(n)$ or $c \geq \underline{c}(0)$, in which case all agents are informed (part (iii)) or no agent is informed (part (iv)).

If the union of the intervals from parts (i), (iii), and (iv) does not contain $\mathbb{R}_+$, an equilibrium with identical groups may not exist. As shown in part (ii) of Proposition 3, this occurs if $\bar{c}(\hat{m}) < c < \underline{c}(\hat{m}-1)$ for some $\hat{m}$. In this case, we can construct an equilibrium with two types of groups in which a fraction λ of the groups has $\hat{m}$ managers and a fraction $1-\lambda$ of the groups has $\hat{m}-1$ managers. Since there are two types of groups, conditions (10)-(11) give two sets of inequalities, one set for each type of group. However, our proof of Proposition 3 in the appendix shows that these two sets of inequalities reduce to a single equation that identifies the value for λ.[10] The fact that we can construct an equilibrium with two types of groups whenever an equilibrium with identical groups fails to exist allows us to always have a well defined equilibrium at the information acquisition stage of our model.

From part (iv) of Proposition 3, we conclude that at least some agents will become informed if

$$c < \underline{c}(0) = \frac{1}{2\tau} \log\left(\frac{1/\sigma_x^2 + 1/\sigma_\epsilon^2}{1/\sigma_x^2}\right) + \frac{(n-1)\zeta(1-\zeta)}{\tau(\sigma_\epsilon^2/\sigma_x^2 + \zeta(2-\zeta))}, \tag{23}$$

where $\zeta = \hat{\rho}(1, 1/\sigma_x^2) = (\sigma_\epsilon^2/\sigma_x^2)(\sqrt{1+\sigma_x^2/\sigma_\epsilon^2} - 1)$ is obtained from Proposition 2.[11] Note that the right-hand side of (23) increases with n. Thus for arbitrary $c > 0$, (23) implies that information acquisition, and thus the establishment of mutual funds, is more likely to occur when the group size n is large. Given the high growth in the mutual fund industry over the past thirty years, a large value for n appears to be plausible empirically.

Two final remarks about the proposition are worth mentioning. First, although the union of the intervals in parts (i)-(iv) contains $\mathbb{R}_+$, this does not rule out the possibility that some of the intervals may lie on the negative half of the real line. For some parameter values there exists $\hat{m} < n$ such that $\underline{c}(\hat{m}) < 0 < \bar{c}(\hat{m})$. Thus as $c \to 0$, all agents may not become informed in our model. If the value of investing in mutual funds is sufficiently high, it is optimal for some agents to stay uninformed even if the cost of acquiring information is zero. The trade-off is whether to become informed and benefit from the acquired information (through trading profits and information sales) or to remain uninformed and benefit from investing in multiple mutual funds. This result is driven by our assumption that informed agents cannot invest in their competitors' mutual funds, an assumption that we revisit in section 4.2.

Second, although Proposition 3 shows that an equilibrium always exists at the information acquisition stage, it may not be unique. For example, suppose there exists $\hat{m}$ such that

[10] The reason follows from conditions (10)-(11). For an equilibrium to hold with two types of groups, an informed agent in a group with $\hat{m}$ managers must be indifferent between being informed or being uninformed. Likewise, an uninformed agent in a group with $\hat{m}-1$ managers must be indifferent between being uninformed or being informed.

[11] Substituting $n = 1$ into (23) shows that some agents will become informed as long as $c < \frac{1}{2\tau} \log\left(\frac{\sigma_\epsilon^2+\sigma_x^2}{\sigma_\epsilon^2}\right)$, which matches the expression in Diamond (1985, Lemma 3).

$\underline{c}(\hat{m}) < \underline{c}(\hat{m}-1) < c < \bar{c}(\hat{m}) < \bar{c}(\hat{m}-1)$. In this case c lies in the intersection of the intervals $[\underline{c}(\hat{m}), \bar{c}(\hat{m})]$ and $[\underline{c}(\hat{m}-1), \bar{c}(\hat{m}-1)]$. Thus part (i) of Proposition 3 applies to each interval and we have an instance of multiple equilibria. One equilibrium involves identical groups with $\hat{m}$ managers while the other equilibrium involves identical groups with $\hat{m}-1$ managers. We have verified numerically that multiple equilibria of this type are possible in our model for some parameter values.

3.4 Asset prices and mutual funds

We now analyze how mutual funds impact the price of the traded risky asset. We focus on the ex ante case in which the number of informed agents in each group is determined endogenously using Proposition 3. To facilitate our study, we compare our model with an arbitrary group size of $n \geq 2$, which allows for mutual funds, to our model with $n = 1$, which precludes agents from establishing mutual funds. When $n = 1$, our model is equivalent to that of Diamond (1985) and the symmetric version of Verrecchia (1982).

Previously we used $\mathcal{R} = \text{var}(X|P_x)^{-1}$ to denote the equilibrium level of price informativeness. Since now we are comparing models with different values of n, we modify our notation to show the group size. In particular, let $\mathcal{R}_n$ denote the equilibrium price informativeness when each group has size n. Thus $\mathcal{R}_1$ corresponds to the equilibrium price informativeness in the Diamond (1985) model. Diamond (1985) shows that when $\lambda_S \in (0,1)$, the endogenous fraction of informed agents is

$$\lambda_S = \tau\sigma_u\sigma_\epsilon\sqrt{\frac{1}{C(\tau)} - \frac{\sigma_\epsilon^2}{\sigma_x^2}},$$

where $C(\tau) \equiv e^{2\tau c} - 1$. Thus the equilibrium price informativeness in the standard model without mutual funds is $\mathcal{R}_1 = 1/(\sigma_\epsilon^2 C(\tau))$.

To characterize the price impact of mutual funds, we analyze the risky asset's ex ante equity risk premium. Following Cao (1999), O'Hara (2003), and Easley and O'Hara (2004), the ex ante risk premium is $\mu_x - \mathbb{E}[P_x]$, which we denote by ξ_n. Thus in our model we have

$$\xi_n \equiv \mu_x - \mathbb{E}[P_x] = \frac{\tau\mu_u}{\mathcal{R}_n + \tau b_n/d_n}, \tag{24}$$

where b_n and d_n are the equilibrium price coefficients when the group size is n. Note that ξ_n is increasing in both the risk aversion parameter τ and the expected asset supply μ_u, as one would expect. However, ξ_n also depends on the equilibrium price informativeness, $\mathcal{R}_n$, and the managers' fees, which impact b_n/d_n. In the standard model without mutual funds, one can verify that $\xi_1 = \tau\mu_u\sigma_\epsilon^2/(1/C(\tau) + \lambda_S)$.

Our first observation is that the presence of an endogenously determined mutual fund sector

may not be sufficient for the risky asset price to reveal more information.[12] To see this, consider an economy with the following primitives: $\tau = 3$, $\sigma_\epsilon^2 = 0.1$, $\sigma_u^2 = \sigma_x^2 = 1$, and $c = 0.15$. If $n = 1$, there are no mutual funds. In this case, a fraction (equal to 0.726) of the agents finds it optimal to become informed and $\mathcal{R}_1 = \text{var}(X|P_x)^{-1} = 6.8$. In contrast, if $n = 2$, there exists an equilibrium with identical groups where $\hat{m} = 1$ and $\mathcal{R}_2 = \text{var}(X|P_x)^{-1} = 6.3$. Thus an economy with mutual funds may not always produce a higher level of price informativeness. This is because the introduction of a mutual fund sector may discourage information acquisition. In our example, only 50% of the agents acquire a private signal when $n = 2$, while 72.6% of the agents acquire a signal when $n = 1$.

We can sharpen our results by characterizing our model for large values of n. This is a particularly interesting case in practice since fund managers and households should have low contracting and search costs when n is large. Furthermore, studying the case of large n seems important in light of the high growth of the mutual fund industry over the past thirty years. Our next proposition characterizes the equilibrium behavior of the optimal number of informed agents $\hat{m}$ for large n.

Proposition 4. *As n becomes large, either part (i) or part (ii) of Proposition 3 holds. For any $\epsilon > 0$, there exists $\bar{n}_\epsilon$ such that for all $n \geq \bar{n}_\epsilon$,*

$$\left| \frac{\hat{m}}{\sqrt{n}} - \sqrt{\frac{\tau \sigma_\epsilon^2 \sigma_u^2}{c}} \right| < \epsilon.$$

Furthermore, there exists $\bar{n}$ such that for all $n \geq \bar{n}$, an economy with mutual funds has a lower equity risk premium, $\xi_n < \xi_1$, and a higher price informativeness, $\mathcal{R}_n > \mathcal{R}_1$, relative to the standard economy without mutual funds.

The proposition shows that some, but not all, agents in each group become informed when n is large. Furthermore, the number of informed agents grows at the rate $\sqrt{n}$. Using Proposition 4 to evaluate the asymptotic behavior of (10)-(11) in Definition 3, one can verify that for sufficiently large n, the equilibrium reduces to the condition that the fee collected by the manager, $f(\hat{m}, \mathcal{R}_n)$, must equal the cost of information acquisition, c. Thus the total fee collected is just enough to offset the private information cost.

Proposition 4 establishes a sufficient condition (i.e., large n) for which our model with endogenous mutual fund formation produces a lower equity risk premium and higher level of price informativeness relative to the standard model. Although the number of managers grows rather slowly at rate $\sqrt{n}$ for large n, this slow growth rate is offset by the aggressive trading of

[12] This runs counter to our discussion following Proposition 1. While our earlier discussion assumed the existence of mutual funds, the current analysis endogenizes the mutual fund sector. This illustrates why the endogenous formation of mutual funds is an important consideration when analyzing the effect of financial intermediaries on asset prices.

the fund managers. The latter effect dominates in equilibrium, as informed agents are able to manage funds for more agents as n grows. This in turn produces a higher price informativeness and a lower equity risk premium relative to the standard economy without mutual funds.

Although a large value for n is sufficient for a higher price informativeness and a lower equity risk premium, it is not necessary. We have verified numerically that a similar statement can be made for some parameter values when n is small. For example, if we fix $n \geq 2$, both sufficiently large and sufficiently small values of σ_ϵ yield the conclusions of Proposition 4, i.e. $\xi_n < \xi_1$ and $\mathcal{R}_n > \mathcal{R}_1$. However, a general statement of the necessary and sufficient conditions for a lower equity risk premium and a higher level of price informativeness is cumbersome due to the nonlinear nature of the equilibrium in section 3.3.

4 Extensions

We now generalize our model of mutual fund formation by relaxing some of our earlier assumptions. First, in section 4.1, we relax the assumption that managers must use proportional compensation contracts. Although most mutual funds in practice have proportional investment management fees, some also have fixed account fees and front-end sales loads. Thus we analyze the case of affine compensation contracts.[13] The affine contracts allow the managers to choose their compensation levels separately from their funds' risky asset exposures. In our earlier model with only a proportional fee, these two items were intermingled. Second, in section 4.2, we relax the assumption that agents and group sizes are homogeneous. This allows us to study the cross-sectional variation of the managers' fees within each group.

4.1 Affine compensation contracts

We now allow the fund managers to use affine compensation contracts, but we maintain all of our other modeling assumptions. The ith fund manager in group k charges a fixed account fee δ_{ik} in addition to the proportional fee α_{ik}. Thus the date 3 wealth of the ith manager in group k is

$$W_{ik} = \alpha_{ik} Z_{ik} + \sum_{j=1}^{h_k} \delta_{ik} 1_{\{\phi_{ijk} \neq 0\}} - c,$$

where Z_{ik} is from (2). We use the indicator function $1_{\{\cdot\}}$ since a household pays the fixed account fee only if it takes a non-zero position in the ith fund. If the jth household in group k buys ϕ_{ijk} units of the ith fund, the household's net payoff is $\phi_{ijk}[Z_{ik}(1 - \alpha_{ik}) - P_{ik}] - \delta_{ik}$.

[13] Admati and Pfleiderer (1990) refer to this type of contract as a "general pricing scheme." We note that in general this may not be the optimal class of contracts, i.e., some contracts may include portfolio constraints (Dybvig, Farnsworth, and Carpenter, 2004). In contrast to Admati and Pfleiderer (1990), who analyze a monopolistic seller of information, we focus on oligopolistic competition with endogenous information acquisition.

Thus the date 3 wealth of the jth household in group k is

$$W_{jk} = \theta_{jk}\left(X - P_x\right) + \sum_{i=1}^{m_k} \phi_{ijk}\left[Z_{ik}(1-\alpha_{ik}) - P_{ik}\right] - \sum_{i=1}^{m_k} \delta_{ik} 1_{\left\{\phi_{ijk}\neq 0\right\}}.$$

As before, to describe the funds' management fees across the entire economy, we define a set $\mathcal{A} = \{\alpha_{ik}, \delta_{ik} : i = 1, \ldots, m_k; k \in [0, \bar{k}]\}$. At the fee setting stage, each manager must now choose his contingent fee α_{ik} and his fixed account fee δ_{ik}, taking the fees of all other managers as given. Thus Definition 2 must be modified slightly to accommodate the fixed account fees. However, the information acquisition stage and the trading stage continue to be defined as in section 2.4.[14]

To analyze the trading stage at date 2, suppose the kth group has m_k managers and let the set of fees $\mathcal{A}$ be given. As long as the fixed account fees are relatively small, which we verify to be true in equilibrium, each household holds a positive amount of each available mutual fund. In this case, due to the separability of the CARA utility function, the presence of the fixed account fees does not alter our results in Proposition 1. The risky asset demands of the ith manager and the jth household are given by (15) and (16), respectively, and the jth household's demand for the ith mutual fund is $\hat{\phi}_{ijk} = 1/(n - m_k)$. Furthermore, the mutual fund prices are given by (14) and the risky asset price is given by (1) with price coefficients (12)-(13).

Although Proposition 1 is unaltered by the presence of the fixed account fees, our results in Propositions 2 and 3, which describe the fund formation and information acquisition stages respectively, are altered. The new results are given in the following two propositions.

Proposition 5. *Suppose the informed agents are allowed to use affine compensation contracts if they establish mutual funds. Then the optimal contingent management fee of the ith manager in group k is*

$$\hat{\alpha}_{ik} = \frac{1}{1+h_k} = \frac{1}{1+n-m_k} \tag{25}$$

and the optimal fixed account fee of the ith manager in group k is

$$\hat{\delta}_{ik} = \frac{1}{2\tau} \log\left[\frac{\mathcal{R} + m_k/\sigma_\epsilon^2}{\mathcal{R} + (m_k - 1)/\sigma_\epsilon^2}\right], \tag{26}$$

[14] Instead of allowing the mutual funds to be traded simultaneously with the risky asset, we could allow the fund managers to contract directly with the households prior to the trading stage, as in Admati and Pfleiderer (1990). However, since the model with an interim contracting stage is isomorphic in terms of payoffs and prices to the one that we study, we continue to assume that mutual funds are traded alongside the risky asset. This assumption mirrors what is observed in practice.

where the price informativeness $\mathcal{R}$ is

$$\mathcal{R} = \frac{1}{\sigma_x^2} + \frac{1}{\tau^2 \sigma_u^2 \sigma_\epsilon^4} \left(\int_0^{\bar{k}} m_k (1 + n - m_k) dk \right)^2 .$$

Like Proposition 2, Proposition 5 shows that every informed agent finds it optimal to establish a mutual fund since $\hat{\alpha}_{ik} < 1$ for all i and k. However, due to the presence of the fixed account fee, the level of $\hat{\alpha}_{ik}$ in Proposition 5 is different than its counterpart in Proposition 2. In Proposition 2, surplus extraction and risk sharing are intermingled when each manager chooses α_{ik}. This is no longer the case in Proposition 5. Each fund manager uses the fixed account fee to extract as much consumer surplus as competition allows, without distorting risk sharing when choosing α_{ik}. Thus unlike Proposition 2, the optimal contingent fee in (25) is consistent with efficient risk sharing. The ith fund manager and the h_k households in group k each receive an equal share of the fund's payoff, Z_{ik}.

Using our previous notation, recall that $\rho_{ik} = (1 - \alpha_{ik})/(\alpha_{ik} h_k)$. Substituting (25) shows that the per capita exposure chosen by the ith fund manager is equal to its first best level, $\rho_{ik} = 1$. Thus the ith fund is simply a vehicle that implements the portfolio policy that each household would have chosen had it observed the information directly.[15]

Note that $\hat{\delta}_{ik}$ in (26) is consistent with each household being indifferent between investing and not investing in the ith fund. This is because the ith manager extracts the marginal surplus that each household gains from holding the ith mutual fund. However, since there are decreasing benefits to holding additional mutual funds, the total consumer surplus is greater than the sum of each manager's marginal surplus. Furthermore, as the number of informed agents m_k increases, the marginal surplus decreases.

To characterize the equilibrium at the information acquisition stage, we use the optimal fees from Proposition 5 to compute the ex ante certainty equivalent wealth levels for the informed and uninformed agents in each group. This generalizes the definitions of $\mathcal{U}_k$ and $\mathcal{V}_k$ in section 2.4. Assuming symmetrical fees within each group, let $\hat{\alpha}_{ik} = \hat{\alpha}_k$ and let $\hat{\delta}_{ik} = \hat{\delta}_k$ for all i and k. Furthermore let $\hat{\beta}_k(m_k) = \{\hat{\alpha}_k(m_k), \hat{\delta}_k(m_k)\}$, where m_k is used to show dependence on the number of informed agents. In this case, the ex ante certainty equivalent wealth levels are

$$\begin{aligned}
\mathcal{U}_k(m_k, \hat{\beta}_k(m_k); \mathcal{M}_k, \hat{\mathcal{A}}_k) &= \frac{\log\left(\mathcal{R} + 1/\sigma_\epsilon^2\right)}{2\tau} + \frac{(n - m_k)}{2\tau} \log\left(\frac{\mathcal{R} + m_k/\sigma_\epsilon^2}{\mathcal{R} + (m_k - 1)/\sigma_\epsilon^2} \right) - c + H, \\
\mathcal{V}_k(m_k, \hat{\beta}_k(m_k); \mathcal{M}_k, \hat{\mathcal{A}}_k) &= \frac{\log\left(\mathcal{R} + m_k/\sigma_\epsilon^2\right)}{2\tau} - \frac{m_k}{2\tau} \log\left(\frac{\mathcal{R} + m_k/\sigma_\epsilon^2}{\mathcal{R} + (m_k - 1)/\sigma_\epsilon^2} \right) + H,
\end{aligned}$$

[15] Using the terminology in Admati and Pfleiderer (1990), we now have "direct" information sales. After substituting (25) into (14), note that the equilibrium mutual fund price is zero. Thus the total fee $\alpha_{ik} P_{ik}$ is also zero. In a direct sale, managers earn their profits via the fixed account fees and households receive the payoff that they would choose if they observed the manager's information directly.

where $\mathcal{R} = \text{var}(X|P_x)^{-1}$ and H is a quantity that is not affected by any agent's information acquisition decision. The first term in the manager's certainty equivalent $\mathcal{U}_k$ is the ex ante value of the risky portfolio held by a manager, while the second term is the profit from selling information to the $n - m_k$ households in group k. Similarly, the first term in the household's utility $\mathcal{V}_k$ is the ex ante value of the risky portfolio held by a household, while the second term captures the payment to the mutual fund sector.

The next proposition provides an existence result at the information acquisition stage when informed agents are allowed to use affine compensation contracts. As in section 3.3, we focus on the case in which there are at most two types of groups.

Proposition 6. *Fix the cost $c > 0$ and let the group size satisfy $n \geq 2$. Define the functions $\bar{c}(m)$ and $\underline{c}(m)$ as*

$$\begin{aligned}
\bar{c}(m) &= \frac{1}{2\tau}\log\left(\frac{\mathcal{R}(m)+1/\sigma_\epsilon^2}{\mathcal{R}(m)+(m-1)/\sigma_\epsilon^2}\right)+\frac{(m-1)}{2\tau}\log\left(\frac{\mathcal{R}(m)+(m-1)/\sigma_\epsilon^2}{\mathcal{R}(m)+(m-2)/\sigma_\epsilon^2}\right)\\
&\quad+\frac{(n-m)}{2\tau}\log\left(\frac{\mathcal{R}(m)+m/\sigma_\epsilon^2}{\mathcal{R}(m)+(m-1)/\sigma_\epsilon^2}\right),\\
\underline{c}(m) &= \frac{1}{2\tau}\log\left(\frac{\mathcal{R}(m)+1/\sigma_\epsilon^2}{\mathcal{R}(m)+m/\sigma_\epsilon^2}\right)+\frac{m}{2\tau}\log\left(\frac{\mathcal{R}(m)+m/\sigma_\epsilon^2}{\mathcal{R}(m)+(m-1)/\sigma_\epsilon^2}\right)\\
&\quad+\frac{(n-m-1)}{2\tau}\log\left(\frac{\mathcal{R}(m)+(m+1)/\sigma_\epsilon^2}{\mathcal{R}(m)+m/\sigma_\epsilon^2}\right),
\end{aligned}$$

where

$$\mathcal{R}(m) = \frac{1}{\sigma_x^2}+\frac{m^2(1+n-m)^2}{\tau^2\sigma_u^2\sigma_\epsilon^4 n^2}.$$

Then the following statements hold:

(i) if $c \in [\underline{c}(\hat{m}), \bar{c}(\hat{m})]$ for some $\hat{m} \in \{1, 2, \ldots, n-1\}$, there exists an equilibrium with identical groups where $m_k = \hat{m}$ for all k;

(ii) if $c \in (\bar{c}(\hat{m}), \underline{c}(\hat{m}-1))$ for some $\hat{m} \in \{1, 2, \ldots, n\}$, there exists an equilibrium with two types of groups. A fraction λ of the groups has $\hat{m}$ managers while a fraction $1-\lambda$ has $\hat{m}-1$ managers, for some $\lambda \in (0,1)$;

(iii) if $c \leq \bar{c}(n)$, there exists an equilibrium with identical groups where $m_k = n$ for all k;

(iv) if $c \geq \underline{c}(0)$, there exists an equilibrium with identical groups where $m_k = 0$ for all k.

As in Proposition 3, either all groups have the same number of informed agents, as in parts (i), (iii), and (iv), or there exists an equilibrium with two types of groups, as in part (ii). Although Propositions 3 and 6 are similar qualitatively, there are obvious analytical differences

due to the surplus extraction that is afforded by the fixed account fees. For example, it is easy to verify that a sufficient condition for information to be acquired by at least some agents is

$$c < \underline{c}(0) = \frac{n}{2\tau} \log\left(\frac{1/\sigma_x^2 + 1/\sigma_\epsilon^2}{1/\sigma_x^2}\right).$$

Comparing this expression and (23) reveals that the interval of c values for which at least some agents become informed is larger in Proposition 6 relative to Proposition 3. In the model with only a proportional fee, the optimal exposure $\hat{\rho}_{ik}$ that is chosen by the ith fund manager is less than the efficient exposure. This impacts the household sector's willingness to pay and limits the c values for which at least some agents become informed. However, the exposure is efficient when affine contracts are allowed, which alters the household sector's willingness to pay relative to the model with only a proportional fee. Thus for some levels of c, all else equal, an equilibrium with mutual funds exists in the affine contract model while an equilibrium without mutual funds exists in the proportional fee model.

Next we analyze the asset pricing implications of the affine contract model. As in section 3.4, we focus on the case of a large group size n.

Proposition 7. *As n becomes large, either part (i) or part (ii) of Proposition 6 holds. For any $\epsilon > 0$, there exists $\bar{n}_\epsilon$ such that for all $n \geq \bar{n}_\epsilon$,*

$$\left|\frac{\hat{m}}{\sqrt{n}} - \sqrt{\frac{\tau\sigma_\epsilon^2\sigma_u^2}{2c}}\right| < \epsilon.$$

Furthermore, there exists $\bar{n}$ such that for all $n \geq \bar{n}$, an economy with mutual funds has a lower equity risk premium, $\xi_n < \xi_1$, and a higher price informativeness, $\mathcal{R}_n > \mathcal{R}_1$, relative to the standard economy without mutual funds.

As Proposition 7 shows, the asymptotic behavior of $\hat{m}$ for large n is similar to what is described in Proposition 4. In particular, the number of managers grows at the rate $\sqrt{n}$ and the proportion of managers in the economy decreases as n becomes large. However, the coefficient on $\sqrt{n}$ in Proposition 7 is smaller than its counterpart in Proposition 4. Thus for large n, the number of mutual fund managers in the model with affine contracts is smaller than in the model with only a proportional fee. Although the fixed account fees help the managers to extract surplus from the household sector, the managers trade more aggressively when affine contracts are allowed. Thus more information is revealed by price, which makes it more attractive to remain uninformed.

Proposition 7 also reveals that our prior statements concerning the impact of mutual funds on the stock market are robust. Although we have analyzed a different type of fund management contract, the equity risk premium is lower and the equilibrium level of price informative-

ness is higher when mutual funds are formed endogenously. This reinforces the arguments in Ross (2005), where it is claimed that the standard model without mutual funds is unstable with respect to informed agents offering to manage the wealth of uninformed agents. Not only does our model verify the validity of such an argument, we characterize explicitly how managed wealth impacts equilibrium prices.

4.2 Agent heterogeneity and other extensions

Our results in section 3 rely on several simplifying assumptions such as identical risk aversion, identical signal precisions, and identical group sizes. We now relax these assumptions to accommodate agent and group heterogeneity, which allows us to study the cross-sectional variation of the managers' fees within a given group. Although we focus on the model with only a proportional fee, our results can be extended to the affine case using section 4.1.

To generalize the model in section 2, we assume the kth group has m_k informed managers and h_k uninformed households, where $m_k + h_k = n_k$. We continue to index the groups by $k \in [0, \bar{k}]$, but now we normalize the total mass of agents to 1 by choosing $\bar{k}$ to satisfy $\int_0^{\bar{k}} n_k dk = 1$. The ith manager in group k has risk aversion parameter τ_{ik} and observes a private signal with noise variance σ_{ik}^2. Likewise, the jth household in group k has risk aversion parameter τ_{jk}. We let $\bar{\tau}_k = \left(\sum_{j=1}^{h_k} \frac{1}{\tau_{jk}}\right)^{-1}$, which implies that $1/\bar{\tau}_k$ is the total risk tolerance of the household sector in group k. All of our other assumptions remain unchanged from section 2. The next proposition extends our results from sections 3.1 and 3.2 to the case in which agents and groups are heterogeneous.

Proposition 8. *Suppose agents and groups are heterogeneous. Then our noisy rational expectations equilibrium with mutual funds has the following properties:*

(i) the risky asset's equilibrium price is given by (1) where the coefficients b and d satisfy

$$\frac{b}{d} = \int_0^{\bar{k}} \left(\sum_{i=1}^{m_k} \frac{1}{\alpha_{ik} \tau_{ik} \sigma_{ik}^2} \right) dk; \tag{27}$$

(ii) the jth household's optimal demand of the ith mutual fund is $\hat{\phi}_{ijk} = \bar{\tau}_k / \tau_{jk}$;

(iii) the optimal contingent fee of the ith manager in group k is $\hat{\alpha}_{ik} = 1/(1 + \tau_{ik}\sigma_{ik}^2 \hat{\omega}_{ik})$, which satisfies $\hat{\alpha}_{ik} \in \big((1 + 0.5(\tau_{ik}/\bar{\tau}_k))^{-1}, 1\big)$. The collection $\{\hat{\omega}_{ik}\}_{i=1}^{m_k}$ solves the system of nonlinear equations

$$1 - 2\bar{\tau}_k \sigma_{ik}^2 \hat{\omega}_{ik} = \frac{2\bar{\tau}_k \hat{\omega}_{ik} (1 - \bar{\tau}_k \sigma_{ik}^2 \hat{\omega}_{ik})^2}{\mathrm{var}(X|P_x)^{-1} + 2\bar{\tau}_k \sum_{l=1}^{m_k} \hat{\omega}_{lk} - \bar{\tau}_k^2 \sum_{l=1}^{m_k} \sigma_{lk}^2 \hat{\omega}_{lk}^2}; \quad i = 1, \ldots, m_k. \tag{28}$$

Parts (i) and (ii) of Proposition 8 have a similar flavor to the equilibrium described in section 3.1. In particular, the ratio b/d in part (i) is determined by the managers' total trading aggressiveness, which in turn depends on the managers' contingent fees, their risk aversion levels, and their signal precisions.[16] Furthermore, like Proposition 1, the jth household holds the same fraction of each available mutual fund, which is seen by noting that $\hat{\phi}_{ijk}$ in part (ii) of Proposition 8 does not vary with i. However, unlike Proposition 1, there is now cross-sectional variation in the households' mutual fund demands. This arises due to the households' heterogeneous risk aversion levels. The jth household's demand for the ith mutual fund is now equal to the household's risk tolerance divided by the total risk tolerance of all households in group k.

Part (iii) of Proposition 8 extends our results from Proposition 2. In particular, $\hat{\alpha}_{ik} < 1$ for all i and k, which shows that every informed agent finds it optimal to establish a mutual fund. However, unlike Proposition 2, the optimal fees cannot be determined analytically since now the fund managers have heterogeneous signal precisions. Instead the fees are characterized by the system of equations in (28), which appears to be intractable. However, if all agents have identical risk aversion, it is possible to show that the manager with the highest information precision charges the highest contingent fee, the manager with the next highest information precision charges the next highest contingent fee, and so forth. To see this, assume identical risk aversion levels for all agents. Then the system of equations in part (iii) of Proposition 8 implies that $\sigma_{lk}^2/\sigma_{ik}^2 = t_k(\alpha_{lk})/\,t_k(\alpha_{ik})$, where l and i are two arbitrary managers in group k and the function $t_k(z)$ is

$$t_k(z) = \frac{\left(\frac{1-z}{zh_k}\right)\left[1-\left(\frac{1-z}{zh_k}\right)\right]^2}{\left[1-2\left(\frac{1-z}{zh_k}\right)\right]}.$$

Since $\frac{dt(z)}{dz} < 0$, we conclude that $\sigma_{lk}^2 > \sigma_{ik}^2$ implies $\alpha_{lk} < \alpha_{ik}$. Thus ranking the agents according to their information precisions is equivalent to ranking the agents according to their contingent fees.

Since the equilibrium at the fee setting stage in part (iii) of Proposition 8 is characterized as a nonlinear system, we are unable to generalize the results in section 3.3 to study how heterogeneity impacts endogenous information acquisition. In the standard model (Verrecchia, 1982), the most risk tolerant agents are the ones who tend to acquire private information. However, in our model, it may be the case that the uninformed household sector is more risk tolerant than an informed fund manager. While an informed fund manager can invest only in his own fund, the household sector can invest in multiple funds. Thus a risk tolerant household may choose to remain uninformed if it places a high value on being able to invest in multiple

[16]The trading strategy of the ith manager in group k is of the form $\hat{\gamma}_{ik} = Y_{ik}/(\alpha_{ik}\tau_{ik}\sigma_{ik}^2) + q_{ik}(P_x)$, where $q_{ik}(P_x)$ is a linear function of P_x. The quantity $1/(\alpha_{ik}\tau_{ik}\sigma_{ik}^2)$, which is the coefficient on Y_{ik}, measures the manager's trading aggressiveness and thus determines b/d.

mutual funds.

Although Proposition 8 extends our model to accommodate agent and group heterogeneity, there are two other modeling assumptions that we do not relax. These assumptions deserve further discussion. First, we do not allow informed agents to establish mutual funds and simultaneously trade on their own accounts. While relaxing this assumption is possible, it results in a model in which only the agent's total risky asset exposure is identified in equilibrium. Furthermore, the agent is indifferent as to how this exposure is divided between the mutual fund and his own account.[17] If we assume that the informed agent acts in the best interest of the households when indifferent, which corresponds to the typical approach in the agency literature, then this relaxed model is equivalent to the one that we analyze in the article. Second, we do not allow mutual funds to hold positions in other mutual funds. This assumption is made solely for tractability, since otherwise the optimal fees cannot be obtained in closed form. However, our numerical results indicate that for large n, the equilibrium prices in the relaxed model are arbitrarily close to the prices that we characterize in our main model. Intuitively, the intra-fund holdings tend to zero as the group size increases. Thus the equilibrium prices have the same properties as those given in sections 3.4 and 4.1.

There are several other ways to extend our model. First, it would be interesting to endogenize the group formation, perhaps via a costly search model. Furthermore, if we combine costly search with the heterogeneity of Proposition 8, complex matching patterns may arise between informed and uninformed agents. Of course, these issues were not critical to our symmetric model in section 3. Second, in the case of heterogeneous agents, it would be interesting to relax the observability of the informed agents' signal precisions. This would open up possibilities for signalling by the informed agents, which is studied in Ross (2005). Third, even without a formal search model, it would be interesting to investigate an equilibrium in which the households have access to overlapping collections of mutual funds. In this case, the households' equilibrium holdings of mutual funds do not reduce to the optimal risk sharing holdings in Proposition 8. Instead, a system of nonlinear equations must be solved to obtain the households' equilibrium mutual fund holdings. A similar tractability problem arises if the household sector has heterogeneous information, e.g., if the households are endowed with heterogeneous priors. Lastly, it would be interesting to examine a multi-period model of mutual fund formation. In a multi-period environment, uninformed agents would presumably be able to estimate the risk tolerances and signal precisions of the mutual fund managers, which is discussed in Admati and Ross (1985). A formal analysis of all of these issues is beyond the scope of our current paper.

[17] This statement depends critically on the competitive aspects of the equilibria that we study. Strategic or reliability issues, which we assume away, may generate a rationale for having some trades inside the fund and others outside the fund.

5 Conclusion

We study the fund formation decision of rational informed agents to provide an explanation for why we observe a relatively high level of assets under management. Instead of trading on their own accounts, we show that informed agents are better off establishing mutual funds and marketing their investment strategies to the household sector. From the perspective of the household sector, a risky asset trade that is achieved by purchasing a mutual fund is very different than a direct trade on the risky asset. This follows due to the private information possessed by the fund managers. Our model provides a foundation for studying the link between financial intermediaries and asset prices.

Appendix

This Appendix sketches the main arguments for the proofs of the paper's Propositions. The Supplement to the paper, which is available on the journal's web site `http://www.nyu.edu/jet/supplementary.html`, contains further details on the proofs.

Proof of Proposition 1: Using the standard properties of Gaussian random variables, it is straightforward to evaluate the conditional expectation in (3). Solving the ith manager's problem in group k then produces the familiar mean-variance expression in (15). We note that the demand of the ith manager in group k can be written as $\hat{\gamma}_{ik} = v_{ik} + r_{ik}Y_{ik} + q_{ik}P_x$, where v_{ik}, r_{ik}, and q_{ik} depend on the price coefficients a, b, and d. One can verify that $r_{ik} = 1/(\tau\sigma_\epsilon^2\alpha_{ik})$.

To evaluate the jth household's problem in group k, we first note that the household's payoff is a quadratic function of the vector $(X, \epsilon_{1k}, \ldots, \epsilon_{m_k k})$. Let $\boldsymbol{V}_{jk}$ denote the variance-covariance matrix of $(X, \epsilon_{1k}, \ldots, \epsilon_{m_k k})$ conditional on price, which is a $(m_k+1)\times(m_k+1)$ diagonal matrix, and let $\boldsymbol{\phi}_{jk} \in \mathbb{R}^{m_k}$ denote the vector of mutual fund holdings for the jth household. By direct integration, the household's problem in (6) can be expressed as

$$\max_{\theta_{jk},\boldsymbol{\phi}_{jk}} \quad M(\theta_{jk},\boldsymbol{\phi}_{jk}) + \frac{1}{2\tau}\log(|\boldsymbol{B}(\boldsymbol{\phi}_{jk})|) - \frac{\tau}{2}\boldsymbol{g}(\theta_{jk},\boldsymbol{\phi}_{jk})^\top \boldsymbol{B}(\boldsymbol{\phi}_{jk})^{-1}\boldsymbol{V}_{jk}\boldsymbol{g}(\theta_{jk},\boldsymbol{\phi}_{jk}), \qquad (29)$$

where

$$\begin{aligned}
M(\theta_{jk},\boldsymbol{\phi}_{jk}) &= (\mathbb{E}[X|P_x] - P_x)\left(\theta_{jk} + \sum_{l=1}^{m_k}(1-\alpha_{lk})\phi_{ljk}\mathbb{E}[\hat{\gamma}_{lk}|P_x]\right) - \sum_{l=1}^{m_k}\alpha_{lk}\phi_{ljk}P_{lk};\\
g_1(\theta_{jk},\boldsymbol{\phi}_{jk}) &= \theta_{jk} + \sum_{l=1}^{m_k}(1-\alpha_{lk})\phi_{ljk}\left(\mathbb{E}[\hat{\gamma}_{lk}|P_x] + r_{lk}\left(\mathbb{E}[X|P_x] - P_x\right)\right);\\
g_{i+1}(\theta_{jk},\boldsymbol{\phi}_{jk}) &= (\mathbb{E}[X|P_x] - P_x)r_{ik}(1-\alpha_{ik})\phi_{ijk} \quad \text{for } i = 1,\ldots,m_k;\\
\boldsymbol{B}(\boldsymbol{\phi}_{jk}) &= \boldsymbol{I} + 2\tau\boldsymbol{V}_{jk}\boldsymbol{A}(\boldsymbol{\phi}_{jk});\\
A_{11}(\boldsymbol{\phi}_{jk}) &= \sum_{l=1}^{m_k}\phi_{ljk}r_{lk}(1-\alpha_{lk});\\
A_{1,i+1}(\boldsymbol{\phi}_{jk}) &= A_{i+1,1}(\boldsymbol{\phi}_{jk}) = r_{ik}(1-\alpha_{ik})\phi_{ijk}/2 \quad \text{for } i = 1,\ldots,m_k;\\
A_{ls}(\boldsymbol{\phi}_{jk}) &= 0 \quad \text{if } l > 1 \text{ and } s > 1;
\end{aligned}$$

with $\boldsymbol{I} \in \mathbb{R}^{(m_k+1)\times(m_k+1)}$ denoting the identity matrix. Subscripts on the vector $\boldsymbol{g}$ and the matrix $\boldsymbol{A}$ denote their elements. Note that $M(\cdot,\cdot) : \mathbb{R}^{m_k+1} \to \mathbb{R}$, $\boldsymbol{g}(\cdot,\cdot) : \mathbb{R}^{m_k+1} \to \mathbb{R}^{m_k+1}$, and $\boldsymbol{B}(\cdot), \boldsymbol{A}(\cdot) : \mathbb{R}^{m_k} \to \mathbb{R}^{(m_k+1)\times(m_k+1)}$. For notational simplicity, we have omitted the group subscript k on the functions M, $\boldsymbol{g}$, $\boldsymbol{A}$, and $\boldsymbol{B}$.

Given the form of (29), it is clear that the household's problem does not reduce to the familiar mean-variance expression. This is because the ith manager's signal shows up in the

ith fund's payoff. To proceed, we let $\boldsymbol{C}(\boldsymbol{\phi}_{jk}) \equiv \boldsymbol{B}(\boldsymbol{\phi}_{jk})^{-1}\boldsymbol{V}_{jk}$. Then the first order conditions for the problem in (29) are

$$\frac{\mathbb{E}\left[X|P_x\right] - P_x}{\tau} = \sum_{l=1}^{m_k+1} C_{1l}(\boldsymbol{\phi}_{jk}) g_l(\theta_{jk}, \boldsymbol{\phi}_{jk}); \tag{30}$$

$$\frac{\partial M}{\partial \phi_{ijk}} + \text{trace}\left(\boldsymbol{C}\frac{\partial \boldsymbol{A}}{\partial \phi_{ijk}}\right) = \tau\left(\frac{\partial \boldsymbol{g}}{\partial \phi_{ijk}}^{\top} - \tau \boldsymbol{g}^{\top}\boldsymbol{C}\frac{\partial \boldsymbol{A}}{\partial \phi_{ijk}}\right)\boldsymbol{C}\boldsymbol{g}, \qquad i = 1, \ldots, m_k. \tag{31}$$

Using (30) in (31), one can verify that the first order conditions (31) for the optimal mutual fund demands reduce to

$$\alpha_{ik}P_{ik} = \text{trace}\left(\boldsymbol{C}(\boldsymbol{\phi}_{jk})\frac{\partial \boldsymbol{A}}{\partial \phi_{ijk}}\right) = (1-\alpha_{ik})r_{ik}\left(C_{11}(\boldsymbol{\phi}_{jk}) + C_{1,i+1}(\boldsymbol{\phi}_{jk})\right) \tag{32}$$

for $i = 1, \ldots, m_k$. Given the symmetry of the household sector, the market clearing condition in (8) implies that $\hat{\phi}_{ijk} = 1/h_k$. Computing $\boldsymbol{C}$ explicitly and using the equilibrium fund holdings, one verifies that (32) reduces to (14). Substituting the optimal mutual fund demands into (30) gives, after some manipulation, the optimal risky asset demand in (16). Verification of the household's second order conditions for a maximum can be found in the online Supplement.

We extract the price coefficients a, b, and d from the risky asset's market clearing condition in (7). After substituting the optimal demands of the informed and uninformed agents into (7), we obtain three equilibrium conditions that can be solved uniquely to obtain the expressions for a, b, and d in (12)-(13). This verifies the functional form for P_x in (1) and completes the proof. □

Proof of Proposition 2: The ith manager's problem in (9) reduces to choosing α_{ik} to maximize $\alpha_{ik}P_{ik}$. Letting $\rho_{ik} = (1-\alpha_{ik})/(\alpha_{ik}h_k)$, the ith manager's problem can be written as

$$\max_{\rho_{ik}} \quad \frac{h_k\left(\rho_{ik} - \rho_{ik}^2\right)}{\tau\sigma_\epsilon^2\left[\mathcal{R} + \frac{2}{\sigma_\epsilon^2}\sum_{l=1}^{m_k}\rho_{lk} - \frac{1}{\sigma_\epsilon^2}\sum_{l=1}^{m_k}\rho_{lk}^2\right]},$$

where $\mathcal{R} = \text{var}(X|P_x)^{-1}$. The first order condition is

$$\sigma_\epsilon^2\left(1 - 2\hat{\rho}_{ik}\right)\left[\mathcal{R} + \frac{2}{\sigma_\epsilon^2}\sum_{l=1}^{m_k}\hat{\rho}_{lk} - \frac{1}{\sigma_\epsilon^2}\sum_{l=1}^{m_k}\hat{\rho}_{lk}^2\right] = 2\hat{\rho}_{ik}\left(1 - \hat{\rho}_{ik}\right)^2, \tag{33}$$

where we have used the fact that $\mathcal{R}$ does not depend on any individual manager's contingent fee. The second order condition is easily verified. After substituting $\hat{\rho}_{ik} = \hat{\rho}_k$ for all i, which imposes symmetry within group k, one can verify that (33) reduces to $\hat{\rho}_k = \hat{\rho}(m_k, \mathcal{R})$, where the function $\hat{\rho}(m, r)$ is given in (17). One can further show that the cubic equation that defines $\hat{\rho}(m, r)$ has a unique solution that lies in the interval $(0, 1/2)$. Details are in the online

Supplement. Since $\hat{\rho}_k = (1-\hat{\alpha}_k)/(\hat{\alpha}_k h_k)$ and $\hat{\rho}_k \in (0, 1/2)$, we have $\hat{\alpha}_k = 1/(1+h_k\hat{\rho}_k)$ and thus $\hat{\alpha}_k \in (1/(1+0.5h_k), 1)$.

Our characterization of the managers' optimal fees in group k holds for any $\mathcal{R} > 0$. We now identify the equilibrium $\mathcal{R}$ by showing that there is a fixed point to equation (18). Applying the implicit function theorem to (17), one can verify that $\partial\rho(m_k, \mathcal{R})/\partial\mathcal{R} > 0$ and $\lim_{\mathcal{R}\to\infty} \partial\rho(m_k, \mathcal{R})/\partial\mathcal{R} = 0$. Since $\rho(m_k, \mathcal{R}) \in (0, 1/2)$, the right-hand side of (18), as a function of $\mathcal{R}$, is bounded from above, whereas the left-hand side is not. Furthermore, the right-hand side of (18) is bounded away from zero as $\mathcal{R} \downarrow 0$, whereas the left-hand size tends to zero. Thus there always exists a fixed point to (18).

Lastly, to verify part (ii) of the proposition, we use (18) in (17) and express the resulting cubic equation in terms of $\hat{\alpha}$, where $\alpha_{ik} = \hat{\alpha}$ for all i and k. This completes the proof. □

Proof of Proposition 3: Fix $n \geq 2$. The ith manager's final wealth at date 3, as viewed from the information acquisition stage at date 0, is a quadratic function of the random vector (X, P_x, Y_{ik}). By direct integration, the manager's expected utility at date 0 is

$$\mathbb{E}\left[-e^{-\tau\hat{\alpha}_{ik}(P_{ik}+\hat{\gamma}_{ik}(X-P_x))+\tau c}\right] = -e^{-\tau(\hat{\alpha}_{ik}P_{ik}-c)-\frac{(\mu_x-\mathbb{E}[P_x])^2}{2\Lambda}}\sqrt{\frac{1}{F\Lambda}}, \tag{34}$$

where $F = \mathrm{var}(X|P_x, Y_{ik})^{-1}$ and $\Lambda = \mathrm{var}(X - P_x) = d^2\sigma_u^2 + (b-1)^2\sigma_x^2$. Since the managers in group k set the same proportional fees, the right-hand side of (34) implies that the ith manager's certainty equivalent wealth, $\mathcal{U}_k$, is identical to expression (21), where H is

$$H = \frac{(\mu_x - \mathbb{E}[P_x])^2}{2\tau\Lambda} + \frac{1}{2\tau}\log(\Lambda).$$

Likewise, the jth household's final wealth at date 3, as viewed from the information acquisition stage at date 0, is a quadratic function of the random vector $(X, P_x, \{Y_{ik}\}_{i=1}^{m_k})$. Again by direct integration, the household's expected utility at date 0 is

$$\mathbb{E}\left[-e^{-\tau\hat{W}_{jk}}\right] = -\,e^{-\frac{(\mu_x-\mathbb{E}[P_x])^2}{2\Lambda}+\frac{\tau}{h_k}\sum_{i=1}^{m_k}\hat{\alpha}_{ik}P_{ik}}\sqrt{\frac{1}{D\Lambda}}, \tag{35}$$

where $\mathcal{R} = \mathrm{var}(X|P_x)^{-1}$, $D = \mathcal{R} + Q(m_k, \mathcal{R})$, and $Q(m, r)$ is defined in Proposition 3. Since there is symmetry within a group, the right-hand side of (35) implies that the household's certainty equivalent wealth, $\mathcal{V}_k$, is given by (22).

Now suppose that $m_k = m$ for all k. In this case, $\mathcal{R}$ is given by (20). After substituting (20) into $\mathcal{U}_k$ and $\mathcal{V}_k$ in (21)-(22), the inequalities in (10)-(11) can be written as $c \leq \bar{c}(m)$ and $c \geq \underline{c}(m)$. Recall from Definition 3 that only (10) must hold if there are n informed agents in a group. Thus if $c \leq \bar{c}(n)$, there exists an equilibrium with identical groups where $m_k = n$ for all k. This proves part (iii) of the proposition. Likewise, recall from Definition 3 that

only (11) must hold if there are no informed agents in a group. Thus if $c \geq \underline{c}(0)$, there exists an equilibrium with identical groups where $m_k = 0$ for all k. This proves part (iv) of the proposition.

To prove part (i), let $1 \leq m \leq n-1$. It is tedious, but straightforward, to show that $\bar{c}(m) \geq \underline{c}(m)$, i.e., the interval $[\underline{c}(m), \bar{c}(m)]$ is non-empty. Details are in the online Supplement. By construction, if $c \in [\underline{c}(\hat{m}), \bar{c}(\hat{m})]$ for some $\hat{m} \in \{1, 2, \ldots, n-1\}$, the equilibrium conditions in (10)-(11) are satisfied, which shows there exists an equilibrium with identical groups where $m_k = \hat{m}$ for all k.

To prove part (ii), define $\bar{c}(m, \lambda)$ as

$$\begin{aligned} \bar{c}(m,\lambda) &= \frac{1}{2\tau}\log\left(\frac{\mathcal{R}(m,\lambda)+1/\sigma_\epsilon^2}{\mathcal{R}(m,\lambda)+Q(m-1,\mathcal{R}(m,\lambda))}\right) + f(m,\mathcal{R}(m,\lambda)) \\ &\quad + \frac{(m-1)}{(n-m+1)} f(m-1,\mathcal{R}(m,\lambda)), \end{aligned}$$

where $\mathcal{R}(m, \lambda)$ is given by

$$\mathcal{R}(m,\lambda) = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_u^2}\left(\frac{1}{n\tau\sigma_\epsilon^2}\right)^2 \left(\frac{\lambda m}{\hat{\alpha}_k(m)} + \frac{(1-\lambda)(m-1)}{\hat{\alpha}_k(m-1)}\right)^2. \tag{36}$$

In (36), $\hat{\alpha}_k(m) = 1/(1+(n-m)\hat{\rho}(m,\mathcal{R}))$ and $\hat{\alpha}_k(m-1) = 1/(1+(n-m+1)\hat{\rho}(m-1,\mathcal{R}))$, where $\hat{\rho}(m, \mathcal{R})$ and $\hat{\rho}(m-1, \mathcal{R})$ solve the pair of cubic equations that arise from part (i) of Proposition 2. The function $\bar{c}(m, \lambda)$ is continuous in λ with $\bar{c}(m, 1) = \bar{c}(m)$ and $\bar{c}(m, 0) = \underline{c}(m-1)$. Thus if $c \in (\bar{c}(\hat{m}), \underline{c}(\hat{m}-1))$ for some $\hat{m} \in \{1, 2, \ldots, n\}$, there exists $\lambda \in (0, 1)$ that solves $c = \bar{c}(\hat{m}, \lambda)$. One can verify that conditions (10) and (11) are satisfied for both the groups with $\hat{m}$ managers and the groups with $\hat{m} - 1$ managers. This proves part (ii) and completes the proof of the proposition. □

Proof of Proposition 4: Fix a finite cost $c > 0$. To see that either part (i) or part (ii) of Proposition 3 must hold for large n, we evaluate $\underline{c}(0)$ and $\bar{c}(n)$. Using (23), it is clear that $\underline{c}(0) \to \infty$ and $\bar{c}(n) \to -\infty$ as $n \to \infty$, which rules out parts (iii) and (iv) for sufficiently large n. Now suppose that $\lim_{n\uparrow\infty} \hat{m} n^{-\beta} = \nu$ for some $\nu > 0$ and $\beta \in (0, 1)$ which are constants to be determined. Using this in (17), we have $\lim_{n\uparrow\infty} \hat{\rho}_k = 1/2$. Furthermore, note that $\lim_{n\uparrow\infty} Q(\hat{m}, \mathcal{R}) n^{-\beta} = 3\nu/(4\sigma_\epsilon^2)$ and $\lim_{n\uparrow\infty} \hat{m}/\left[n^{1+\beta}\hat{\alpha}(\hat{m})\right] = 0.5\nu$. The latter fact implies that $\lim_{n\uparrow\infty} \mathcal{R} n^{-2\beta} = \nu^2/\left[4\tau^2\sigma_u^2\sigma_\epsilon^4\right]$, which applies to both (20) and (36). Given the asymptotic behavior of $Q(\hat{m}, \mathcal{R})$ and $\mathcal{R}$, it is easy to show that $\underline{c}(\hat{m})$ and $\bar{c}(\hat{m})$ depend only on the total fees f when n is large. One can also check that only $\beta = 1/2$ is consistent with the equilibrium conditions (10) and (11). In this case, $f(\hat{m}, \mathcal{R}(\hat{m})) \to \tau\sigma_u^2\sigma_\epsilon^2/\nu^2$ as $n \to \infty$, which implies $\underline{c}(\hat{m})$ and $\bar{c}(\hat{m})$ both converge to $\tau\sigma_u^2\sigma_\epsilon^2/\nu^2$ as $n \to \infty$. Since an equilibrium always exists in our model, $\underline{c}(\hat{m})$ and $\bar{c}(\hat{m})$ must be in the neighborhood of c for large n, so

$\nu = \sqrt{(\tau\sigma_u^2\sigma_\epsilon^2)/c}$. Lastly, the asymptotic behavior of $\mathcal{R}$ and b/d reveals that for sufficiently large n, the equity risk premium, given by (24), is lower in our model with mutual funds relative to the standard economy without mutual funds. This completes the proof. □

Proof of Proposition 5: It is clear that the ith informed agent in group k is always better off choosing α_{ik} and δ_{ik} such that $\phi_{ijk} > 0$ for all $j = 1, \ldots, h_k$. By the same arguments that we used in Propositions 1-3, the ex ante certainty equivalent wealth for the ith fund manager in group k is

$$\mathcal{U}_{ik}(m_k, \{\alpha_{lk}, \delta_{lk}\}_{l=1}^{m_k}; \mathcal{M}_k, \mathcal{A}_k) = \frac{1}{2\tau}\log\left(\mathcal{R} + 1/\sigma_\epsilon^2\right) + \alpha_{ik}P_{ik} + h_k\delta_{ik} - c, \tag{37}$$

where we have omitted terms that do not depend on the fees. Likewise, the certainty equivalent wealth for the jth household in group k is

$$\mathcal{V}_{jk}(m_k, \{\alpha_{lk}, \delta_{lk}\}_{l=1}^{m_k}; \mathcal{M}_k, \mathcal{A}_k) \;=\; \frac{1}{2\tau}\log\left(\mathcal{R} + \frac{1}{\sigma_\epsilon^2}\sum_{l=1}^{m_k}\left(2\rho_{lk} - \rho_{lk}^2\right)\right) - \sum_{l=1}^{m_k}\left(\frac{\alpha_{lk}P_{lk}}{h_k} + \delta_{lk}\right),$$

where $\rho_{ik} = (1 - \alpha_{ik})/(\alpha_{ik}h_k)$.

Taking the actions of the other $n-1$ agents as given, the ith manager chooses α_{ik} and δ_{ik} to maximize (37). The optimal δ_{ik} is given by the fee that makes each household indifferent between buying and not buying the ith fund, namely

$$\hat{\delta}_{ik} = \frac{1}{2\tau}\log\left(\frac{\mathcal{R} + \frac{1}{\sigma_\epsilon^2}\sum_{l=1}^{m_k}\left(2\rho_{lk} - \rho_{lk}^2\right)}{\mathcal{R} + \frac{1}{\sigma_\epsilon^2}\sum_{l=1,l\neq i}^{m_k}\left(2\rho_{lk} - \rho_{lk}^2\right)}\right) - \frac{\alpha_{ik}P_{ik}}{h_k}. \tag{38}$$

We substitute (38) back into (37) and maximize over α_{ik} to get (25), or $\hat{\rho}_{ik} = 1$. The second order condition for a maximum is verified easily. We then substitute (25) into (38) to get (26), where we note that when $\hat{\rho}_{ik} = 1$ we have $P_{ik} = 0$. Lastly, we construct the equilibrium price informativeness $\mathcal{R}$ using the equilibrium contingent fees in (25). This completes the proof. □

Proof of Proposition 6: Our proof is constructive and follows the arguments used in Proposition 3. First note that the expressions for $\mathcal{U}_k$ and $\mathcal{V}_k$ that precede the statement of Proposition 6 follow from the expressions for $\mathcal{U}_{ik}$ and $\mathcal{V}_{jk}$ in the proof of Proposition 5 once we substitute for the optimal $\hat{\alpha}_{ik}$ and $\hat{\delta}_{ik}$. Now suppose that $m_k = m$ for all $k \in [0, \bar{k}]$. In this case it is easy to verify that $\mathcal{R} = \text{var}(X|P_x)^{-1}$ is given by the expression in the Proposition. The inequalities (10)-(11) can be written as $c \leq \bar{c}(m)$ and $c \geq \underline{c}(m)$, where the functions $\bar{c}(m)$ and $\underline{c}(m)$ are given in the statement of Proposition 6. The two corner solutions $m_k = n$ and $m_k = 0$ provide parts (iii) and (iv) of the proposition.

To prove part (i), we note that $[\underline{c}(\hat{m}), \bar{c}(\hat{m})]$ is a non-empty interval for $\hat{m} \in \{1, \ldots, n-1\}$, and that the equilibrium conditions in (10)-(11) are equivalent to the condition $c \in [\underline{c}(\hat{m}), \bar{c}(\hat{m})]$

for some $\hat{m} \in \{1, \ldots, n-1\}$. Thus if $c \in [\underline{c}(\hat{m}), \bar{c}(\hat{m})]$ for some $\hat{m} \in \{1, \ldots, n-1\}$, there exists an equilibrium where all groups have $m_k = \hat{m}$ managers.

To prove part (ii), let

$$\begin{aligned}\bar{c}(m,\lambda) &= \frac{1}{2\tau}\log\left(\frac{\mathcal{R}(m,\lambda)+1/\sigma_\epsilon^2}{\mathcal{R}(m,\lambda)+(m-1)/\sigma_\epsilon^2}\right)+\frac{(n-m)}{2\tau}\log\left(\frac{\mathcal{R}(m,\lambda)+m/\sigma_\epsilon^2}{\mathcal{R}(m,\lambda)+(m-1)/\sigma_\epsilon^2}\right)\\ &\quad+\frac{(m-1)}{2\tau}\log\left(\frac{\mathcal{R}(m,\lambda)+(m-1)/\sigma_\epsilon^2}{\mathcal{R}(m,\lambda)+(m-2)/\sigma_\epsilon^2}\right),\end{aligned}$$

where

$$\mathcal{R}(m,\lambda)=\frac{1}{\sigma_x^2}+\frac{1}{\sigma_u^2}\left(\frac{1}{n\tau\sigma_\epsilon^2}\right)^2\left[\lambda m(1+n-m)+(1-\lambda)(m-1)(1+n-(m-1))\right]^2.$$

The function $\bar{c}(m,\lambda)$ is continuous in λ and satisfies $\bar{c}(m,0)=\underline{c}(m-1)$ and $\bar{c}(m,1)=\bar{c}(m)$. Thus if $c \in (\bar{c}(\hat{m}), \underline{c}(\hat{m}-1))$ for some $\hat{m} \in \{1,\ldots,n\}$, there exists $\lambda \in (0,1)$ such that $c=\bar{c}(\hat{m},\lambda)$. Furthermore, the equilibrium conditions (10)-(11) from Definition 3 are satisfied for both the groups with $\hat{m}$ managers and the groups with $\hat{m}-1$ managers. This completes the proof. □

Proof of Proposition 7: The proof is similar to that of Proposition 4, with the exception that $\underline{c}(m)$ and $\bar{c}(m)$ are now given in Proposition 6. Fix a finite cost $c>0$. As $n \to \infty$, $\underline{c}(0) \to \infty$ and $\bar{c}(n) \to -\infty$, which rules out parts (iii) and (iv) of Proposition 6. Thus it must be the case that part (i) or part (ii) holds for sufficiently large n. Now suppose that $\lim_{n\uparrow\infty}\hat{m}n^{-\beta}=\nu$ for some $\nu>0$ and $\beta \in (0,1)$ to be determined. One can verify that $\beta=1/2$ is the only equilibrium growth rate for $\hat{m}$. In this case, one can verify that $\lim_{n\uparrow\infty}\underline{c}(\hat{m})=\lim_{n\uparrow\infty}\bar{c}(\hat{m})=\tau\sigma_u^2\sigma_\epsilon^2/(2\nu^2)$. Since an equilibrium always exists in our model, $\underline{c}(\hat{m})$ and $\bar{c}(\hat{m})$ must be in the neighborhood of c for large n, so $\nu=\sqrt{(\tau\sigma_u^2\sigma_\epsilon^2)/2c}$. Lastly, note that $\mathcal{R}(\hat{m})$ satisfies $\lim_{n\uparrow\infty}\mathcal{R}(\hat{m})/n=1/(2\tau\sigma_\epsilon^2 c)$ when $\beta=1/2$. This asymptotic behavior for $\mathcal{R}$ implies that for sufficiently large n, the equity risk premium in (24) is lower and the price informativeness is higher in our economy with mutual funds relative to the standard economy without mutual funds. This completes the proof. □

Proof of Proposition 8: The demand functions for the informed traders are given by the usual mean-variance form in (15), but with τ replaced by τ_{ik}. The uninformed agents' optimization problem can be solved by the same approach as in Proposition 1. In particular, it is straightforward to check that the first order conditions of the jth household's optimization problem give

$$\alpha_{ik}P_{ik}=\frac{\omega_{ik}\left(1-\tau_{jk}\hat{\phi}_{ijk}\sigma_{ik}^2\omega_{ik}\right)}{\mathrm{var}(X|P_x)^{-1}+2\tau_{jk}\sum_{l=1}^{m_k}\hat{\phi}_{ljk}\omega_{lk}-\tau_{jk}^2\sum_{l=1}^{m_k}\hat{\phi}_{ljk}^2\sigma_{lk}^2\omega_{lk}^2}, \tag{39}$$

where $\omega_{ik} \equiv (1-\alpha_{ik})/(\alpha_{ik}\sigma_{ik}^2\tau_{ik})$. It is easy to verify that the demands $\hat{\phi}_{ijk}$ satisfy both the market clearing condition in (8) and the first order conditions in (39). We give details in the online Supplement. We also verify the household's second order conditions in the Supplement. Using the informed agents' optimal demands and risky asset's market clearing condition, one can obtain (27). Next, by the same arguments used in Proposition 2, the ith fund manager in group k solves $\max_{\alpha_{ik}} \alpha_{ik}P_{ik}$, which yields the system of equations in (28). Lastly, to see that $\alpha_{ik} = 1$ is never optimal, note that $P_{ik} = 0$ when $\alpha_{ik} = 1$. But given $\hat{\alpha}_{lk} \in (0,1)$ for $l \neq i$, there always exist $0 < \hat{\alpha}_{ik} < 1$ so that $P_{ik} > 0$. This completes the proof. $\square$

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