Relative wealth concerns and complementarities in information acquisition

by

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Abstract

This paper studies how relative wealth concerns, in which a person's satisfaction with their own consumption depends on how much others are consuming, affect investors' incentives to acquire information. We find that such externalities can generate complementarities in information acquisition within the standard rational expectations paradigm. When agents are sensitive to the wealth of others, they herd on the same information, trying to mimic each other's trading strategies. We show that there can be multiple herding equilibria in which different communities pursue different information acquisition strategies. This multiplicity of equilibria generates price discontinuities: an infinitesimal shift in fundamentals can lead to a discrete price movement.

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Keywords: Keeping up with the Joneses; Consumption externalities; Complementarities in information acquisition.

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"Men do not desire merely to be rich, but to be richer than other men."

John Stuart Mill

1 Introduction

Economists have long believed that relative consumption effects, in which a person's satisfaction with their own consumption depends on how much others are consuming, are important (Veblen, 1899). Indeed, the growing literature on happiness in economics points to relative wealth concerns as one of the main explanations for why the growth in GDP over the last fifty years has not been accompanied by a similar increase in life satisfaction.[1] Starting with Abel (1990) and Galí (1994), such relative wealth concerns have been formally modeled in the asset pricing literature. To date, the theoretical literature has primarily focused on the price implications that these consumption externalities have in a symmetric information environment. In this paper, we identify an additional channel through which relative wealth concerns affect asset prices. By incorporating relative wealth concerns into a rational expectations equilibrium (REE) model, we examine how such consumption externalities influence the production of information and, as a consequence, asset prices.

The economic setting we use extends the model developed by Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982) to account for "keeping up with the Joneses" (KUJ) preferences. In particular, we adopt a preference specification similar to Galí (1994), in which an investor's marginal utility of consumption increases in the average consumption of the other investors in the economy. This allows for the idea that investors care not only about their own wealth, but also about how their wealth compares with that of others, in the spirit of John Stuart Mill's quote. In all other respects, our model is standard: agents decide whether or not to acquire costly information about asset payoffs before trading, and, based on that information, trade a risky and a riskless asset in a competitive market.

Our main result is to show that consumption externalities resulting from a KUJ preference specification can generate complementarities in information acquisition. In the standard model, an investor's expected benefit from collecting information is decreasing in the number of informed agents. The reason is that, as more agents acquire and act on their information, prices become more informative, and uninformed agents free-ride on the learning of others. When agents are sensitive to the wealth of others, this information revelation effect is counteracted by the investors' desire to keep up with their peers. A larger number of informed agents increases the expected trading profit of the average agent and, hence, reduces an un-

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[1]For two recent surveys of the topic, see Frey and Stutzer (2002) and Clark, Frijters, and Shields (2008).

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informed agent's relative position in the economy. The disutility associated with a higher average wealth level therefore raises the value of information. We find that relative wealth concerns can dominate the information revelation effect, making the marginal value of information increase in the number of agents who acquire it. This creates incentives for agents to coordinate their information production activities and introduces the possibility of multiple "herding" equilibria.

These complementarities in information acquisition have several important implications. First, they lead to an increase in informed trading, which improves the informativeness of asset prices. Second, they cause price discontinuities. In particular, we show that an infinitesimal shift in fundamentals can lead to a discrete jump in asset prices. The mechanism responsible for these price jumps is complementarities in the demand for information that, together with small changes in fundamentals, make investors switch between no-information and high-information equilibria. Finally, relative wealth concerns can also lead to a particular type of informational inefficiency. By giving agents a choice between perfectly correlated signals and signals that are independently distributed conditional on the asset's payoff, we demonstrate that, if relative wealth considerations are sufficiently strong, agents prefer the former signals. This inefficient allocation of research effort can arise because the gain from trading on new information may be more than offset by the disutility that an agent incurs when her consumption falls short of the average level.[2] This result is in stark contrast to the standard REE model (Grossman and Stiglitz, 1980; Hellwig, 1980), in which agents always prefer to acquire conditionally independent signals.

Most of our analysis is conducted under the assumption that relative wealth concerns are global, in the sense that agents care about their relative position with respect to the entire economy. However, the empirical literature on investors' portfolio choice (discussed below) has documented some anomalies that are more local in nature. In order to address these issues, we extend our model by grouping agents into different communities and by introducing relative wealth considerations within each community. Interestingly, we find that when the number of communities is large, symmetric equilibria in which different communities of agents pursue the same information acquisition strategy are unstable. There exist, however, stable equilibria with the property that agents in some communities collect information, while agents in other communities prefer to stay uninformed.

Our paper is related to several strands of the literature. First, it is related to the literature

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[2]We want to emphasize that the information inefficiencies that we discuss in this paper pertain to the agents' information acquisition decisions: relative wealth concerns induce agents to focus on the same source of information, rather than on a diverse set of signals. However, the market is efficient at the pricing stage. Once a set of information has been acquired, market prices incorporate this information in a Bayesian fashion.

that studies relative wealth concerns. This literature examines if and to what extent people's happiness depends on the consumption of others. In the finance literature, Abel (1990) was the first to introduce relative considerations with his "catching up with the Joneses" preference specification. Abel (1990) and Galí (1994) consider these consumption externalities as a potential resolution to the equity premium puzzle. Bakshi and Chen (1996) study their impact on stock price volatility. Gómez, Priestley, and Zapatero (2009b,a) study the cross-sectional implications of relative wealth concerns. DeMarzo, Kaniel, and Kremer (2004) present a model in which relative wealth considerations arise endogenously. They demonstrate that when investors care about their consumption relative to their local community, there may be a community effect whereby investors under-diversify and over-invest in local firms. We differ from this literature in that we examine the consequences that relative wealth concerns have on information acquisition. Rather than exogenously imposing an allocation of information, we endogenously derive the investors' incentives to engage in information collection activities. In contrast to symmetric information models, our model predicts that local investors will outperform non-local investors, which is broadly consistent with the empirical literature.[3]

A number of papers have presented models that generate complementarities in information production. Froot, Scharfstein, and Stein (1992) show how complementarities emerge when agents have short trading horizons. In Veldkamp (2006a,b), strategic complementarities result from fixed costs in information production. Goldstein, Ozdenoren, and Yuan (forthcoming, 2009) generate complementarities via a feedback mechanism from the financial market to the value of traded securities. In Mele and Sangiorgi (2008), ambiguity aversion is the source of complementarities. Hellwig and Veldkamp (2009) study information acquisition decisions in the context of a beauty contest game, showing that complementarities in actions drive complementarities in informational choices. This paper expands on this literature by showing that complementarities in information acquisition can arise rather naturally as a consequence of relative wealth considerations.[4]

Our paper also contributes to the literature on the home bias puzzle and on other local or community biases in portfolio choice.[5] Our model uses similar preferences to those endogenously derived in DeMarzo, Kaniel, and Kremer (2004) to explain the concentration of

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[3]See, for example, Coval and Moskowitz (2001) and Ivković and Weisbenner (2005).

[4]See also Barlevy and Veronesi (2000, 2008), Chamley (2008), and Ganguli and Yang (2009).

[5]For empirical evidence on this topic, see French and Poterba (1991), Coval and Moskowitz (2001), Grinblatt and Keloharju (2001), Duflo and Saez (2002), Feng and Seasholes (2004), Hong, Kubik, and Stein (2004, 2005), Bae, Stulz, and Tan (2008), Ivković and Weisbenner (2005, 2007), Brown, Ivković, Weisbenner, and Smith (2008), and Dorn, Huberman, and Sengmueller (2008). See Colla and Mele (2010) and Malinova and Smith (2009) for recent theoretical work in this area.

holdings. However, the actual mechanism is rather different and works via complementarities in the acquisition of information: agents have similar portfolios because they mimic each other's efforts to learn about securities. Van Nieuwerburgh and Veldkamp (2009) also present a model in which informational asymmetries are used to explain the home equity bias. In contrast to their paper, in which information is a strategic substitute, agents in our model buy information that others already have in order to make sure that their wealth is highly correlated with that of their peers.

Finally, our paper is related to the literature on bubbles and crashes in financial markets (see Brunnermeier, 2001, for an excellent survey of this literature). Whereas early papers by Gennotte and Leland (1990) and Jacklin, Kleidon, and Pfleiderer (1992) focus on the role of portfolio insurance, our paper is more closely related to the literature that links the existence of crashes to informational considerations (Barlevy and Veronesi, 2003; Bai, Chang, and Wang, 2006; Mele and Sangiorgi, 2008; Huang and Wang, 2009). In a dynamic setting with symmetric information, DeMarzo, Kaniel, and Kremer (2007, 2008) show that endogenous relative wealth concerns can create bubble-like deviations in asset prices. Our paper complements theirs by providing an alternative mechanism that generates crashes via an informational channel.

The remainder of this paper is organized as follows. Section 2 discusses the preferences we use to model consumption externalities and formally defines an equilibrium. Section 3 describes the equilibrium at the trading stage and the information acquisition stage. It also shows how complementarities in information acquisition can generate price discontinuities. Section 4 presents two extensions of our model: first, we introduce the notion of communities and study the implications of local relative wealth concerns; second, by allowing agents to choose between different signals, we demonstrate that consumption externalities resulting from a KUJ preference specification can lead to an inefficient allocation of research effort. Section 5 discusses empirical implications and relates our results to the literature on local biases in portfolio choice. Section 6 summarizes our contribution and concludes. All proofs are contained in the Appendix.

2   The model

This section introduces a model which extends the rational expectations model developed by Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982) to allow for consumption externalities. In particular, we assume that agents care not only about their own consumption, but also about that of their peers.

2.1 Preferences and assets

We study a three-period economy with consumption taking place only in the last period (i.e., at $t = 3$). At $t = 2$ there is a round of trade, whereas at $t = 1$ agents make a decision as to whether to become informed or not. These two stages are described in more detail below. Since consumption takes place only at the final date, we shall use the terms wealth and consumption interchangeably.

We assume that agent i has preferences of the form $\mathbb{E}\left[u(W_i, \bar{W})\right]$, where W_i denotes her terminal wealth, and $\bar{W}$ denotes the average wealth in the economy. Specifically, we assume that the agents' utility function is given by:

$$u(W_i, \bar{W}) = -\exp(-\tau(W_i - \gamma \bar{W})). \tag{1}$$

The particular functional form we have chosen captures the notion that agents care about the consumption of others in the most parsimonious way.[6] We note that the utility function satisfies the usual conditions with respect to an agent's own consumption W_i: it is increasing and concave in W_i, and the coefficient of absolute risk aversion is $-u_{11}/u_1 = \tau$. The parameter γ captures the extent of the consumption externality, i.e., how much agent i cares about other agents' wealth. The utility specification in (1) satisfies $u_{12}/u_1 = \gamma\tau$, which implies that an increase in the average wealth in the economy raises the marginal utility of consumption when γ is positive, as an agent tries to "keep up with the Joneses."

The preferences we introduce are essentially the CARA version of the standard KUJ preferences with CRRA utility.[7] A crucial feature of the specification in (1) is that agent i receives a negative utility shock when average wealth $\bar{W}$ is high and $\gamma > 0$. In order to mitigate such a shock, she will trade in the same direction as the average agent, in order to induce a high correlation between her wealth and that of others. This is the source of complementarities in the agents' decisions that will drive our results.

We want to emphasize that our contribution is to study the effect of consumption externalities on information acquisition activities in financial markets. The particular interpretation of the utility function introduced above is not critical. For example, one could interpret γ as

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[6]Of course, relative wealth concerns may enter an agent's utility function in some other way, rather than through average wealth. A natural alternative may be the rank of the agent's wealth in the economy or her wealth relative to some quantile of the wealth distribution. We chose the functional form in (1) mostly for tractability reasons.

[7]The properties of the utility function in (1) are identical to those discussed in Galí (1994), with the exception that there is no scaling by consumption. The additive structure we use in (1), as opposed to the multiplicative structure used in most of the asset pricing literature in conjunction with CRRA preferences, is more natural when coupled with CARA preferences, which are standard in the REE literature (Ljungqvist and Uhlig, 2000, use a similar formulation).

measuring jealousy (our preferences satisfy both the definition of jealousy and that of KUJ introduced in Dupor and Liu, 2003). Although we focus on the case where γ is positive, the model formally also allows for the case where agents view the consumption of others as a substitute for their own consumption. Finally, we should remark that the consumption externalities we consider are global rather than local, in the sense that agent i cares about the average consumption in the economy, not about the consumption of her neighbors (see, for example, Glaeser and Scheinkman, 2002, and Bisin, Horst, and Özgüra, 2006, for other utility specifications). We also want to point out that the type of preferences we study can be constructed from a purely axiomatic approach (Maccheroni, Marinacci, and Rustichini, 2010).

We study a competitive market that is populated by a continuum of agents, indexed by $i \in [0, 1]$. There are two assets available for trading in the market: a riskless asset in perfectly elastic supply with a price and payoff normalized to 1, and a risky asset with a random final payoff of $X \in \mathbb{R}$. We assume that all random variables belong to a linear space of jointly normally distributed random variables. In particular, we assume that the risky asset pays off $X \sim \mathcal{N}(\mu_x, \sigma_x^2)$. The aggregate supply of the risky asset is random and equals $Z \sim \mathcal{N}(\mu_z, \sigma_z^2)$. Such supply shocks are a typical ingredient of rational expectations models. The noise that they create prevents equilibrium prices from fully revealing the informed agents' private information. In the standard model without consumption externalities, the assumption of a stochastic stock supply is equivalent to assuming the presence of liquidity traders who have inelastic demands of $-Z$ shares of the stock, for reasons that are exogenous to the model. This is not the case, however, under KUJ preferences. While supply shocks affect the average wealth only indirectly through their effect on the equilibrium price, the interpretation of Z as the demand of liquidity traders has a direct effect on $\bar{W}$, since Z is then part of the agents' aggregate demand. We will examine the effects of this alternative interpretation of Z on the agents' information acquisition decision in Section 4.1.

A fraction λ of the agents receive a private signal prior to trading, whereas the rest of the agents are uninformed and must base their trading decision solely on their priors and on what they learn from prices. Specifically, we assume that, by incurring a cost of c, agent i can observe the signal $Y_i = X + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$. The error terms $\{\epsilon_i\}$ are assumed to be independent across agents (we relax this assumption in section 4.2). We use $\mathcal{F}_i$ to denote the information set of agent i at the time of trading.[8] Without loss of generality, we can label the informed agents with the subscripts $i \in [0, \lambda]$. We also let θ_i denote the number

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[8]Note that $\mathcal{F}_i = \sigma(Y_i, P)$ for informed agents $i \in [0, \lambda]$, and $\mathcal{F}_i = \sigma(P)$ for uninformed agents $i \in [\lambda, 1]$, where $\sigma(X)$ denotes the σ-algebra generated by a random variable X, and P is the price of the risky asset.

of shares of the risky asset bought by agent i, so that, assuming zero initial wealth, we have $W_i = \theta_i(X - P)$, where P denotes the price of the risky asset. Average wealth is then given by $\bar{W} = \int_0^1 \theta_i(X - P)di$. This expression assumes that the information acquisition cost c is a non-pecuniary cost that does not affect the average wealth $\bar{W}$. We want to point out, however, that including the agents' cost of information acquisition in our calculation of the average wealth would not affect any of our results, since it amounts to subtracting a constant from $\bar{W}$ that is the same for informed and uninformed agents and does not depend on an individual agent's actions.[9] Intuitively, all the action in our model comes from the covariation of the agent's wealth with that of her peers, and the cost of information production does not affect this covariation.

2.2 Definition of equilibrium

Fixing the fraction of informed agents, λ, a rational expectations equilibrium is characterized by a set of trading strategies $\{\theta_i\}$, $i \in [0, 1]$, and a price function P, such that:

(1) Each agent i chooses her trading strategy θ_i so as to maximize her expected utility conditional on her information set $\mathcal{F}_i$, i.e., θ_i solves:

$$\max_{\theta_i} \mathbb{E}\left[-\exp(-\tau(W_i - \gamma\bar{W}))|\mathcal{F}_i \right], \qquad i \in [0, 1]. \tag{2}$$

(2) Markets clear, i.e.:

$$\int_0^1 \theta_i \, di = Z. \tag{3}$$

As is customary in this literature, we restrict our attention to linear equilibria. Thus, we postulate that the equilibrium price is a linear function of the average signal and the aggregate stock supply, such that:

$$P = a + b \, X - d \, Z. \tag{4}$$

In the ensuing analysis, we derive a linear equilibrium in which this conjecture is confirmed to be correct.

At the ex-ante stage (i.e., at $t = 1$), agents must decide whether to spend c in order to get a private signal Y_i prior to trading. An equilibrium at the information acquisition stage is

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[9]Clearly, subtracting λc from $\bar{W}$ does not affect the agent's optimal trading strategy, since the cost term is a constant that does not depend on θ_i (see the proof of Proposition 1). Further, it does not affect the equilibrium at the information acquisition stage either, since the additional term increases the certainty equivalent of wealth for both informed and uninformed agents by the same fixed amount of $\gamma\lambda c$ (see the proof of Proposition 2).

defined by a fraction of agents $\lambda \in [0, 1]$ such that: (i) all informed agents $i \in [0, \lambda]$ are better off spending c in order to acquire information, taking all other agents' actions as given; and (ii) all uninformed agents $j \in (\lambda, 1]$ are better off staying uninformed, taking all other agents' actions as given. To be more precise, let $\hat{\theta}_i$ denote agent i's optimal trading strategy and define $\mathcal{U}_i$ to be the following monotone transformation of agent i's ex-ante expected utility:

$$\mathcal{U}_i = -\frac{1}{\tau} \, \log \left(\frac{\mathbb{E}\left[\exp\left(-\tau\left(\hat{\theta}_i(X-P) - \gamma \bar{W}\right)\right)\right]}{\mathbb{E}\left[\exp\left(\tau\gamma\bar{W}\right)\right]} \right). \tag{5}$$

In the absence of relative wealth concerns, $\mathcal{U}_i$ is the certainty equivalent of wealth gross of information acquisition costs. The above definition seems to be a natural extension of this concept to the case where an agent's utility depends on the average wealth in the economy.[10] However, independent of its interpretation, we can use the definition of $\mathcal{U}_i$ to formally define an equilibrium as follows: an interior equilibrium at the information acquisition stage is a fraction $\lambda \in (0, 1)$ such that $\mathcal{V}_I(\lambda) - c = \mathcal{V}_U(\lambda)$, where $\mathcal{V}_I(\lambda) \equiv \mathcal{U}_i$ for any informed agent $i \in [0, \lambda]$, and $\mathcal{V}_U(\lambda) \equiv \mathcal{U}_j$ for any uninformed agent $j \in (\lambda, 1]$. The non-interior equilibria are defined in the natural way: $\lambda = 0$ is an equilibrium if $\mathcal{V}_I(0) - c \le \mathcal{V}_U(0)$, and $\lambda = 1$ is an equilibrium if $\mathcal{V}_I(1) - c \ge \mathcal{V}_U(1)$.

The model outlined thus far reduces to a symmetric version of Verrecchia (1982) if $\gamma = 0$. Diamond (1985) solves such a model in closed form, showing, among other things, that the equilibrium at the information acquisition stage is unique, i.e., there is a unique $\lambda \in [0, 1]$ that satisfies the above definition. The focus of our analysis is to see how consumption externalities change the equilibrium at the trading stage, as well as the incentives to acquire information.

3 Characterization of equilibrium

In this section, we solve for the equilibrium defined above by backward induction. First, we conjecture that a fraction λ of agents become informed and solve for the equilibrium asset prices at the trading stage at $t = 2$. Then, we study the ex-ante information acquisition decision of agents at $t = 1$, given that they anticipate the equilibrium in the asset market at $t = 2$.

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[10]Our definition is essentially based on the same utility comparison as the standard definition: $\mathcal{U}_i$ is the payoff that an agent would have to receive in order to be indifferent between that payoff and her optimal portfolio $\hat{\theta}_i$.

3.1 Optimal trading strategies

When agents have relative wealth concerns, they must form beliefs about the trading strategies of all other traders, since their utility is directly affected by the average wealth of other investors. We start by assuming that a fraction λ of the agents are informed, i.e., they receive signals of the form $Y_i = X + \epsilon_i$. We conjecture that in equilibrium agents' trading strategies are linear in their signals and prices. This implies that the aggregate demand is given by:

$$\bar{\theta} = \int_0^1 \theta_i di = \xi + \lambda \beta X - \kappa P, \tag{6}$$

for some constants $\xi, \beta, \kappa \in \mathbb{R}$.[11]

We first note that average wealth $\bar{W} = \bar{\theta}(X - P)$ is a quadratic function in X, which makes the investment problem in (2) non-standard: the relevant payoff variable $W_i - \gamma \bar{W}$ is not normally distributed, conditional on the information set of either informed or uninformed agents. This is due to the fact that agents are asymmetrically informed. As they try to tilt their portfolios closer to those of their peers, they need to forecast the trades of other agents. The following proposition shows that the optimal investment problem is nonetheless tractable and that a rational expectations equilibrium exists under mild conditions.

Proposition 1. *Suppose that $\gamma < 1/(\tau \sigma_x \sigma_z)$. Then, for any fraction of informed agents λ, an equilibrium exists. This equilibrium is unique among the class of linear equilibria defined by (4). The optimal investment by agent i in the risky asset is given by:*

$$\theta_i = \frac{\mathbb{E}\left[X - P | \mathcal{F}_i\right]}{\operatorname{var}[X | \mathcal{F}_i]} \left(\frac{1}{\tau} - \gamma \operatorname{cov}[\bar{\theta}, X | \mathcal{F}_i] \right) + \gamma \mathbb{E}\left[\bar{\theta} | \mathcal{F}_i\right] \tag{7}$$

$$= \frac{\mathbb{E}\left[X - P | \mathcal{F}_i\right]}{\tau \operatorname{var}(X | \mathcal{F}_i)} + \gamma \big(\xi - P(\kappa - \lambda \beta)\big), \tag{8}$$

where $\mathcal{F}_i$ denotes the information possessed by agent i.

Equilibrium prices are as in (4); the equilibrium price coefficients are given in the Appendix.

Proposition 1 shows that an agent's optimal trading strategy contains (i) the standard mean-variance term, scaled down by the covariance of the average trade with the asset's payoff, and (ii) the expected average trade. The latter is rather intuitive. Agents care about their

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[11]This definition of the aggregate demand is consistent with the interpretation of Z as the stock supply. We will analyze the implications of the alternative definition of $\bar{\theta}$ that follows from the interpretation of Z as the aggregate demand of liquidity traders in Section 4.1.

relative position in the economy and, because of their risk aversion, suffer more from falling behind their peers than they gain from outperforming them. They hedge this relative wealth risk by mimicking the trading strategy of the average agent. Thus, Proposition 1 formalizes the intuition that investors exhibit herding behavior in their portfolio choice when they have relative wealth concerns. The reason for the reduction in information-based trading is that, in equilibrium, the average trade is (conditionally) positively correlated with the asset's payoff. The higher the payoff, the more shares the average agent wants to acquire. Since investors tilt their portfolios to imitate the average portfolio, they curtail the information-based portion of their desired holdings to account for this positive correlation.

Intuitively, the negative utility shock that an agent experiences when the average wealth is high can be interpreted as an endowment shock of $-\gamma\bar{\theta}$ shares of the risky asset. If $\bar{\theta}$ were deterministic, agents could simply hedge their endowment risk by acquiring an additional $\gamma\bar{\theta}$ shares. This is, however, not the case in our model. The quantity $\bar{\theta}$ is stochastic and positively correlated with the asset payoff X, conditional on the agents' information set. Thus, selling a position equal to the expected endowment shock would typically lead to a net short position when the asset payoff is high, because a lower-than-expected endowment shock (i.e., a higher-than-expected $\bar{\theta}$) is typically associated with a higher-than-expected payoff. The opposite is true when the asset payoff is lower than expected. In this case, the endowment shock is typically higher than expected, which leads to a net position that is long in the risky asset. The negative correlation between the endowment shock and the asset return therefore serves as a natural insurance against the investors' endowment risk and diminishes their hedging needs.

The rational expectations equilibrium presented in Proposition 1 shares many of the properties of the standard model. As in Hellwig (1980), price aggregates the disperse information possessed by agents, while the stochastic supply Z prevents prices from fully revealing the payoff X. The informational content of prices is given by:

$$\text{var}[X|P]^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_z^2}\left(\frac{\lambda}{\tau\sigma_\epsilon^2}\right)^2. \tag{9}$$

We note that price informativeness is exactly as in Hellwig (1980). The only difference in the information content of prices between the standard model and the one with KUJ preferences comes through a different fraction of informed agents, λ, which we endogenize below.

The existence condition in the above proposition, $\gamma < 1/(\tau\sigma_x\sigma_z)$, comes from the second-order condition of the agents' optimization problem. The agents' expected utility is not well-defined if they put too much weight on their peers' wealth. This technical problem also

arises in other models with this class of preferences (e.g., Galí, 1994, also needs to constrain the extent of consumption externalities in order to guarantee existence of an equilibrium). Although the above condition is not necessary, it is the minimal condition ensuring that the agents' objective function is strictly concave for all $\lambda \in [0,1]$. The necessary and sufficient condition for the existence of a linear rational expectations equilibrium, for a given λ, is provided in the proof of the proposition (see equation (46) in the Appendix).

3.2 Information acquisition

We next endogenize the fraction of traders that become informed. Intuitively, we expect the incentives to acquire information to differ from the standard model, since agents with KUJ preferences also care about the information that other agents possess. The following proposition solves for the equilibria at the information acquisition stage in closed form.

Proposition 2. Let $\hat{C} = 1/(\sigma_\epsilon^2(e^{2\tau c} - 1)) - 1/\sigma_x^2$ and assume that $\gamma < 1/(\tau \sigma_x \sigma_z)$.

1. If $\hat{C} > 0$, then there exists a unique equilibrium at the information acquisition stage. The fraction of agents that become informed is given by $\lambda = \min(\lambda^*, 1)$, where:

$$\lambda^* = \tau \sigma_\epsilon^2 \sigma_z^2 \left(\tau \gamma + \sqrt{\tau^2 \gamma^2 + \hat{C}/\sigma_z^2} \right). \tag{10}$$

2. If $\hat{C} \leq 0$, then two cases are relevant:

    (a) If $\tau^2 \gamma^2 + \hat{C}/\sigma_z^2 \geq 0$, then there are three equilibria at the information acquisition stage: $\lambda = \min(\lambda^*, 1)$ as given by (10), $\lambda = 0$, and $\lambda = \min(\lambda^{**}, 1)$, where:

$$\lambda^{**} = \tau \sigma_\epsilon^2 \sigma_z^2 \left(\tau \gamma - \sqrt{\tau^2 \gamma^2 + \hat{C}/\sigma_z^2} \right). \tag{11}$$

    (b) If $\tau^2 \gamma^2 + \hat{C}/\sigma_z^2 < 0$, the unique equilibrium is for all agents to stay uninformed, i.e., $\lambda = 0$.

Proposition 2 shows that there are three different types of equilibria, depending on the value of $\hat{C}$.[12] When information is very costly $\hat{C}$ is negative and large and, consequently, in the unique equilibrium no agent becomes informed. When the cost of gathering information is sufficiently low, $\hat{C}$ is positive, and the unique equilibrium involves a fraction of agents

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[12]Note that $\hat{C}$ is decreasing in the cost of gathering information, c.

$\lambda^* > 0$ gathering information. These are the only two regimes that arise in the standard model, i.e., when there are no consumption externalities ($\gamma = 0$).

The novel regime occurs when γ is sufficiently large relative to $\hat{C}$, and $\hat{C}$ is negative (i.e., for intermediate values of the cost parameter c). In this case, there are three equilibria at the information acquisition stage: one where no agent becomes informed, and two where a positive fraction of agents become informed. Intuitively, when agents expect other agents to purchase information, they have an incentive to purchase information as well, since they want to keep up with their peers, even if it is expensive to do so. At the same time, if no one expects others to purchase information, not acquiring information is an optimal strategy. This multiplicity of equilibria arises naturally from KUJ preferences and drives the core of the results discussed below.

Figure 1 plots the equilibrium values of λ as a function of the cost of gathering information. When there are no consumption externalities ($\gamma = 0$), the unique equilibrium is represented by the solid line. In this case, a positive fraction of agents acquire information if and only if the cost c is smaller than 0.55. Once investors care about their peers' wealth, multiplicity of equilibria arises. For example, when $\gamma = 0.4$, Figure 1 shows that when the cost parameter c takes on intermediate values, namely when $c \in (0.55, 0.62)$, there are three different equilibria, two of which have a positive fraction of informed agents. When c is lower than 0.55, there is a unique equilibrium that involves some agents gathering information. For costs higher than 0.62, the unique equilibrium coincides with the one in the absence of consumption externalities, since no agent becomes informed.

As the proof of Proposition 2 shows, agent i's ex-ante certainty equivalent of wealth, gross of information acquisition costs, is given by:

$$\mathcal{U}_i = \frac{1}{2\tau} \log \left(\text{var}[X|\mathcal{F}_i]^{-1} - \frac{2\lambda\gamma}{\sigma_\epsilon^2} \right) + H, \tag{12}$$

where H is independent of the information that the agent possesses. This expression shows that, in contrast to the standard model, the agents' utility may be decreasing in the fraction of informed investors when $\gamma > 0$.[13] The reason is that under KUJ preferences, informed agents impose a negative externality on their peers, because they earn, on average, higher profits. Furthermore, this externality generates complementarities in information acquisition decisions. In order to see this, let $\mathcal{R}(\lambda) = \mathcal{V}_I(\lambda) - \mathcal{V}_U(\lambda)$ denote the marginal value of information (gross of the cost of obtaining information). An interior equilibrium is given by the condition $\mathcal{R}(\lambda) = c$. One can easily verify that $d\mathcal{R}(\lambda)/d\gamma \geq 0$, which means that

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[13]Note, however, that the conditional precision $\text{var}[X|\mathcal{F}_i]^{-1}$ is increasing in λ and, hence, in γ.

the value of information is increasing in how much agents care about each other's wealth. Moreover, we have:

$$\text{sign}\left(\frac{d\mathcal{R}(\lambda)}{d\lambda}\right) = \text{sign}\left(\gamma - \frac{\lambda}{\tau^2\sigma_\epsilon^2\sigma_z^2}\right). \tag{13}$$

The value of information can therefore increase in the number of informed agents, as long as agents care about each other's wealth. In the standard case ($\gamma = 0$), increasing the number of informed agents lowers the value of information, because prices become more informative. When γ is positive, we find that as long as λ is low enough so that the information revelation effect does not dominate, there are complementarities in information acquisition in the sense that the marginal value of information is increasing in the number of agents who acquire information. Intuitively, if a trader's neighbors buy information, consumption externalities will increase the incentives of this trader to gather information herself.

From the above discussion, one might expect that, compared to the standard model, the emergence of strategic complementarities in information acquisition due to relative wealth considerations can lead to either more or less information production. This is not the case, however, as Proposition 2 shows. The fraction of informed investors under KUJ preferences (weakly) exceeds that in the standard model.[14] The reason for this result is that relative wealth concerns only affect an agent's decision to collect information when other agents are informed, because only informed agents impose a negative externality on their peers. If all agents are uninformed, there are no externalities and the value of information is identical to that in the case without relative wealth concerns. Thus, under KUJ preferences, an equilibrium with no information production only exists, if such an equilibrium also exists in the standard model. More generally, the above result that the marginal value of information increases in the KUJ parameter γ implies that an agent's incentive to acquire information under KUJ preferences is larger than in the standard model. It is therefore not surprising that more agents decide to become informed when they care about each other's wealth.

As Proposition 2 shows, multiple equilibria at the information acquisition stage exist when the cost of gathering information is in an intermediate range such that $\hat{C} \in [-\tau^2\gamma^2\sigma_z^2, 0]$. In order to assess the plausibility of the three different equilibria that arise in this case, we employ a refinement criterion based on dynamic stability. The definition of stability relies on an iterative process in which agents react to last period's outcome. An equilibrium is stable if it is the limiting outcome of such a process.[15] A simple method to determine whether an equilibrium is stable is to analyze the agents' optimal response to small deviations in the

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[14]In the standard model with $\gamma = 0$, the unique equilibrium is characterized by $\lambda = 0$ whenever there are multiple equilibria under KUJ preferences.

[15]For a formal definition, see, e.g., Mas-Colell, Whinston, and Green (1995), chapter 17.

equilibrium outcome.

Figure 2 plots an investor's optimal information acquisition decision as a function of the fraction of informed agents in the economy (the parameter values correspond to case 2(a) in Proposition 2). An equilibrium at the information acquisition stage is defined by the condition that the fraction of informed agents equals the probability that any agent acquires information. In our numerical example, the three points at which an investor's optimal response function crosses the 45° line are characterized by $\lambda = 0$, $\lambda^{**} = 0.38$, and $\lambda^* = 0.67$. Figure 2 clearly illustrates that information can be a complementary good in our model. Agents have no incentive to purchase the private signal Y_i if $\lambda < \lambda^{**}$; however, as λ increases to a level between λ^{**} and λ^*, the value of the signal goes up and investors find it optimal to acquire it. This observation establishes the following result.

Proposition 3. *Suppose that* $\hat{C} < 0 < \tau^2\gamma^2 + \hat{C}/\sigma_z^2$ *and* $\lambda^* < 1$. *Then, the only stable equilibria at the information acquisition stage are* $\lambda = 0$ *and* $\lambda = \lambda^*$.

Stability rules out the equilibrium in which a fraction $\lambda = \lambda^{**}$ of the agents become informed. As can be seen in Figure 2, any perturbation of this equilibrium will make agents switch to one of the other two equilibria. We will therefore ignore the unstable equilibrium in the ensuing analysis.

We conclude this section with a discussion of how complementarities in information acquisition can cause price discontinuities in our model. While the equilibrium price function exhibits discontinuities with respect to any of the model parameters, we illustrate the mechanism on the basis of the payoff variance σ_x^2. Figure 3 plots the stable equilibria from Proposition 2 as a function of σ_x^2. Over the range $\sigma_x^2 \in (0.87, 1.01)$, we have two stable equilibria: one where no agent becomes informed, and one with a positive fraction of informed agents. No matter what equilibrium one starts with, it is clear that changes in the payoff variance will induce discrete price jumps, as the measure of informed agents differs across these two equilibria. For example, suppose that we start with an equilibrium such that $\lambda > 0$ (i.e., σ_x^2 is sufficiently large). Then, as σ_x^2 falls below 0.87, the fraction of informed agents must jump from $\lambda = 0.42$ to $\lambda = 0$. An inspection of the price coefficients in Proposition 1 shows that this change in λ causes a discontinuity in prices: as agents stop acquiring information, asset prices experience a discrete jump down (i.e., a "crash"). The model also predicts jumps in the other direction. If the economy is in an equilibrium where $\lambda = 0$, then, as σ_x^2 increases above 1.01, the fraction of informed agents jumps from $\lambda = 0$ to $\lambda = 0.80$. In this case, we expect to see a jump up in prices, as agents move from the no-information equilibrium to the high-information equilibrium.

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4 Extensions

This section extends our model in several dimensions. In Section 4.1, we introduce the notion of communities and analyze the effects of relative wealth concerns within communities. Moreover, we modify our definition of aggregate wealth to include the trading profits of "liquidity traders." In Section 4.2, we study the possibility of herding at the information acquisition stage by allowing agents to purchase identical signals.

4.1 Community effects and liquidity traders

Our analysis so far has been conducted under the assumption that relative wealth concerns are global, in the sense that agents care about their relative position with respect to the entire economy. More realistic social interactions suggest a more local take on relative wealth considerations. The most natural interpretation is that of communities, where each agent has relative wealth concerns only with respect to other agents in her community. The empirical literature has indeed focused on geographical metrics and other social network variables that all hinge on the idea that agents care about their wealth relative to some reference group, rather than the entire investor population. We incorporate this idea into our model by generalizing our KUJ preference specification in the following way:

$$u(W_i, \bar{W}_k) = -\exp(-\tau(W_i - \gamma\bar{W}_k)), \tag{14}$$

where $\bar{W}_k$ denotes the average wealth of agents that belong to the same community k as agent i. To keep the model tractable, we assume that there are K identical communities, each consisting of a continuum of agents with measure $1/K$.

We further generalize our model with respect to the definition of the average wealth. Our analysis in the previous section is based on the assumption that agents only care about their wealth relative to other modeled individuals in the economy. This is consistent with the interpretation of Z as the aggregate stock supply. However, as discussed in Section 2, the quantity Z can also be interpreted as the aggregate demand of liquidity traders who trade for exogenous reasons. While this alternative interpretation does not affect the analysis in the standard model, this is no longer the case under KUJ preferences, since the wealth of liquidity traders needs then to be included in the calculation of the average wealth.[16] In order to examine the effects of this alternative interpretation of Z on the agents' incentive to

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[16]This assumes that strategic traders actually care about the wealth of *all* agents in their community, including liquidity traders. It is plausible that strategic traders only include other strategic individuals in their peer set, which corresponds to our definition of average wealth in Section 3.

acquire information, we modify our definition of the average wealth as follows:

$$\bar{W}_k = K \int_{\frac{k-1}{K}}^{\frac{k}{K}} \theta_i(X - P)di - \omega\, Z_k(X - P), \qquad k \in \{1, \ldots, K\}, \tag{15}$$

where $-Z_k$ denotes the aggregate demand of liquidity traders belonging to community k.[17] For simplicity, we assume that the liquidity shocks are perfectly correlated across communities, i.e., $Z_k = Z/K$ for all communities k.[18] In accordance with our analysis in the previous section, we further assume that informed and uninformed agents do not observe the realization of Z before submitting their orders. This is consistent with the interpretation that these liquidity trades are made by a different set of agents. Alternatively, they can also be interpreted as stemming from the same agents—for example, because of shocks to their endowment of the risky asset (see, e.g., Diamond and Verrecchia, 1981)—as long as each agent's liquidity demand does not reveal any information about the aggregate shock Z (as, e.g., in Grundy and McNichols, 1989). The parameter $\omega \in \{0, 1\}$ allows us to separate the community effect from the effect due to the inclusion of liquidity traders: the liquidity traders' profit is included in the calculation of the average wealth when $\omega = 1$, and excluded when $\omega = 0$. Note that this formulation comprises our analysis in Section 3 as a special case with $K = 1$ and $\omega = 0$.

All other aspects of the model are the same as in Section 2. As before, we conjecture that the average demand function in community k is linear in the asset payoff X and the equilibrium price P, and that the equilibrium price is a linear function of the average signal X and the aggregate liquidity shock Z.

The equilibrium at the trading stage is similar to the one characterized in Proposition 1. The only difference is that, rather than trying to mimic the average trade in the economy, investors now only care about trades executed by agents in their own community. Thus, while the total number of informed agents in the economy influences an investor's demand function through its effect on the conditional payoff variance $\text{var}[X|\mathcal{F}_i]$, the demand effect due to our KUJ preference specification only depends on the number of informed agents in community k, which we denote by λ_k. Moreover, the average trade now also includes the demand of liquidity traders when $\omega = 1$. The following proposition characterizes an agent's optimal trading strategy in the generalized model.

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[17]We subtract the demand of liquidity traders from the demand of informed and uninformed investors to be consistent with our previous analysis: a positive stock supply corresponds to a negative demand of liquidity traders.

[18]This assumption is only made for expositional clarity and does not affect our basic conclusions, as the proof of Proposition 5 shows.

Proposition 4. *There exists a $\bar{\gamma} > 0$ such that, for any $\gamma < \bar{\gamma}$, an equilibrium at the trading stage exists. This equilibrium is unique among the class of linear equilibria defined by (4). The optimal investment by agent i in the risky asset is given by:*

$$\theta_i = \frac{\mathbb{E}\left[X - P | \mathcal{F}_i\right]}{\text{var}[X|\mathcal{F}_i]} \left(\frac{1}{\tau} - \gamma \, \text{cov}[\bar{\theta}_k - \omega \, Z_k, X | \mathcal{F}_i] \right) + \gamma \, \mathbb{E}\left[\bar{\theta}_k - \omega \, Z_k | \mathcal{F}_i \right], \quad (16)$$

where:

$$\bar{\theta}_k = K \int_{\frac{k-1}{K}}^{\frac{k}{K}} \theta_i di, \qquad k \in \{1, \dots, K\}, \quad (17)$$

and $\mathcal{F}_i$ denotes the information possessed by agent i.

Equilibrium prices are as in (4); the relevant price coefficients are given in the Appendix.

Compared to the optimal trading strategy derived in Section 3, agents now try to mimic the average trade in their community, which in the above specification also includes the demand of liquidity traders. This is not surprising, given that the agents' relative wealth concerns are defined with respect to the average wealth in their community.

In order to derive the equilibrium number of informed agents, we have to compare the ex-ante expected utility of informed and uninformed investors in each community. Simple calculations (analogous to the ones in the proof of Proposition 2) show that an agent's certainty equivalent of wealth, gross of information acquisition costs, is given by:

$$\mathcal{U}_i = \frac{1}{2\tau} \log \left(\text{var}[X|\mathcal{F}_i]^{-1} - \frac{2\gamma}{\sigma_\epsilon^2} \left(\lambda_k - \omega \, \frac{\sum_{j=1}^{K} \lambda_j}{K^2} \right) \right) + H. \quad (18)$$

where H is independent of the agent's information set $\mathcal{F}_i$. The above expression is similar to the one derived for the single-community case in equation (12). Ignoring the demand of liquidity traders for the moment ($\omega = 0$), the only difference is that an agent's relative wealth concerns are now only affected by the number of informed investors in her own community, λ_k, rather than by the total number of informed investors. It is important to note, however, that the total amount of information acquired across all communities does affect the agent's expected utility in two ways: first, through its effect on the informativeness of the equilibrium price P, which is contained in the information set $\mathcal{F}_i$; and second, through its effect on the wealth of liquidity traders in community k, if these traders are included in the calculation of the average wealth (i.e., if $\omega = 1$). The latter effect is due to the fact that the wealth of liquidity traders is inversely related to that of informed agents: the more informed agents there are in the economy, the higher is the expected trading loss of liquidity traders. The following proposition follows immediately from the above expression.

Proposition 5. *If the wealth of liquidity traders is included in the calculation of the average wealth (i.e., if $\omega = 1$), relative wealth effects continue to influence an agent's information acquisition decision as long as there are $K \geq 2$ communities in the economy.*

Proposition 5 highlights an important aspect of our model. The mechanism through which KUJ preferences affect an agent's information acquisition decision requires that the average wealth is *endogenous*, that is, the average wealth has to depend on the agents' portfolio choice. If there is a single global community and the wealth of liquidity traders is included in the calculation of $\bar{W}$, then the agents' average wealth is a constant and relative wealth concerns play no role in the agents' investment decision. On the other hand, if there are two or more communities, relative wealth effects persist, because the aggregate wealth of liquidity traders in a community depends on informed trades across all communities. As more agents acquire information in community k, the trading losses of liquidity traders in all communities increase and, hence, the average wealth in community k goes up. In other words, when there are multiple communities, the average wealth in a community is again a function of the number of informed agents in that community, and all of our results derived in the previous section continue to hold.

We now proceed to examine the equilibria at the information acquisition stage when there are multiple communities. Unfortunately, a complete characterization of all possible equilibria is, for all practical purposes, unfeasible: the number of equilibria is of order K, the number of communities in the economy (even ignoring permutations).[19] To illustrate the role of communities in a tractable manner, we therefore focus our analysis on the following two types of equilibria: (i) symmetric equilibria in which the fraction of informed agents is identical across all communities; and (ii) asymmetric equilibria in which all agents that belong to a subset of m communities become informed, and all agents in the remaining $K-m$ communities stay uninformed. The following proposition provides sufficient conditions for the existence of these types of equilibria.

Proposition 6. *Let $\hat{C} = 1/(\sigma_\epsilon^2(e^{2\tau c}-1)) - 1/\sigma_x^2$ and assume that γ is sufficiently small such that a linear equilibrium at the trading stage exists.*

1. *If $\tau^2\gamma^2(1-\omega/K)^2 + \hat{C}/\sigma_z^2 \geq 0$, then there exists a symmetric equilibrium at the information acquisition stage in which a fraction $\lambda_k = \min(\lambda_c^*, 1)$ of agents in each community*

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[19]For any $m \in \{0, \ldots, K\}$, there exist parameter values such that it is an equilibrium for some agents in m different communities to become informed, whereas all agents in the remaining $K - m$ communities decide to stay uninformed. Furthermore, our analysis in the previous section suggests that there may be multiple equilibria with a different number of informed agents for each m.

$k \in \{1, \ldots, K\}$ *become informed, where:*

$$\lambda_c^* = \tau \sigma_\epsilon^2 \sigma_z^2 \left(\tau \gamma \left(1 - \frac{\omega}{K} \right) + \sqrt{\tau^2 \gamma^2 \left(1 - \frac{\omega}{K} \right)^2 + \frac{\hat{C}}{\sigma_z^2}} \right). \tag{19}$$

This equilibrium is stable if:

$$\hat{C} > \frac{K^4 - 2K^3 + \omega(2K - 1)}{K^2} \tau^2 \gamma^2 \sigma_z^2. \tag{20}$$

2. *Asymmetric equilibria at the information acquisition stage, in which all agents that belong to a subset of $m \in \{1, \ldots, K-1\}$ communities acquire information and all agents in the remaining $K - m$ communities stay uninformed, exist if:*

$$-\frac{2\gamma}{\sigma_\epsilon^2} < \hat{C} - \left(\frac{m}{K} \right)^2 \frac{1}{\tau^2 \sigma_\epsilon^4 \sigma_z^2} - \omega \left(\frac{m}{K^2} \right) \frac{2\gamma}{\sigma_\epsilon^2} < 0. \tag{21}$$

Such asymmetric equilibria are always stable.

The symmetric equilibrium characterized in the first part of Proposition 6 shares many of the same features of the stable equilibrium in the single-community case. In particular, λ^* defined in Proposition 2 is identical to λ_c^* if the wealth of liquidity traders is excluded from the calculation of the average wealth in a community (i.e., if $\omega = 0$).[20] The equilibria differ, however, with respect to their stability properties. While the equilibrium with $\lambda_k = \lambda_c^*$ is always stable in the single-community case, it may be unstable if the number of communities K is large. Specifically, the condition in (20) implies that, for any set of parameter values, there exists a $\bar{K}$, such that the symmetric equilibrium fails the stability criterion for any $K \geq \bar{K}$.

To obtain an intuitive understanding for this result, consider the effect that a small increase in λ_k has on an agent's expected utility. The reduction in the agent's certainty equivalent of wealth caused by the increase in the average wealth in her community is the same in both cases (see equations (12) and (18)). However, the positive effect due to the improved price informativeness (i.e., the increase in the precision $\text{var}[X|P]^{-1}$) is smaller in the case with multiple communities, since an increase in λ_k by Δ increases the total fraction of informed agents in the economy only by Δ/K. Thus, compared to the single-community case, relative wealth concerns play a more prominent role when there are multiple communities.

[20]Similar to the single-community case, there may be three different symmetric equilibria, depending on the value of $\hat{C}$. As in Section 3, the other symmetric equilibrium with a positive fraction of informed agents is always unstable.

This means that relative wealth concerns dominate the information revelation effect in the agents' information acquisition decisions, making information a complementary good even if there is a large fraction of informed agents.

Proposition 6 also identifies asymmetric equilibria in which agents in different communities follow different information acquisition strategies. In these equilibria, agents in "informed communities" are strictly better of acquiring information, whereas agents in "uninformed communities" strictly prefer to stay uninformed. Thus, a small change in the number of informed agents does not affect an agent's optimal information acquisition decision, which implies that these equilibria are always stable. Proposition 6 provides sufficient conditions for the existence of such asymmetric equilibria. Essentially, an asymmetric equilibrium of the form specified in Proposition 6 always exists in an economy with a large number of communities, as long as information is not too costly.

The above discussion indicates that, even though all agents and communities are ex-ante identical in our model, asymmetric equilibria are more likely to arise, because, in contrast to symmetric equilibria, they satisfy the stability requirements even for a large number of communities, which appears to be the empirically relevant case. We will discuss the implications of this observation in Section 5.

4.2   Herding and informational inefficiencies

In this section, we show that complementarities in the information market can lead to an inefficient allocation of the agents' research effort. In particular, we demonstrate that when relative wealth concerns are sufficiently strong, agents prefer to acquire perfectly correlated signals, even though their incremental value in predicting future asset payoffs is low. To this end, we extend our basic model by giving agents a choice between acquiring a conditionally independent signal $Y_i = X + \epsilon_i$ and a perfectly correlated signal $Y = X + \delta$. The collection of error terms $\{\epsilon_i\}$ and δ are assumed to be independently normally distributed with zero means and variances σ_ϵ^2 and σ_δ^2, respectively. At the information acquisition stage, agents can acquire the signal Y_i at a cost of c_ϵ, and the signal Y at a cost of c_δ. To distinguish the two types of informed agents, we refer to agents who choose the former (latter) signal as ϵ-informed (δ-informed). All other aspects of the model are the same as in Section 2.

As in the previous section, we restrict our attention to linear equilibria and conjecture that the equilibrium price function is of the following form:

$$P = a + b_x X + b_y Y - dZ. \tag{22}$$

We further postulate that the optimal trading strategy of ϵ-informed (δ-informed) agents is a linear function of the signal Y_i (Y) and the equilibrium price P. Letting β_ϵ (β_δ) denote the coefficient of the signal Y_i (Y) in the linear demand function of ϵ-informed (δ-informed) investors, we can therefore write the aggregate demand as follows:

$$\bar{\theta} = \int_0^1 \theta_i di = \xi + \lambda_\epsilon \beta_\epsilon X + \lambda_\delta \beta_\delta Y - \kappa P, \tag{23}$$

where λ_ϵ (λ_δ) denotes the fraction of ϵ-informed (δ-informed) agents, and ξ and κ are constants.[21]

The following proposition characterizes the equilibrium at the trading stage for a given number of ϵ-informed and δ-informed agents.

Proposition 7. *There exists a $\bar{\gamma} > 0$ such that for any $\gamma < \bar{\gamma}$, an equilibrium at the trading stage exists. This equilibrium is unique among the class of linear equilibria defined by (22). The optimal investment by agent i in the risky asset is given by:*

$$\theta_i = \frac{\mathbb{E}\left[X - P|\mathcal{F}_i\right]}{\tau \operatorname{var}[X|\mathcal{F}_i]} + \gamma\big(\xi - P(\kappa - \lambda_\epsilon \beta_\epsilon)\big) + \gamma \lambda_\delta \beta_\delta \left(\mathbb{E}\left[Y|\mathcal{F}_i\right] - \frac{\operatorname{cov}[X, Y|\mathcal{F}_i]}{\operatorname{var}[X|\mathcal{F}_i]} \mathbb{E}\left[X - P|\mathcal{F}_i\right] \right), \tag{24}$$

where $\mathcal{F}_i$ denotes the information possessed by agent i.

Equilibrium prices are as in (22); the relevant price coefficients are given in the Appendix.

Compared to the case analyzed in Section 3, the agents' optimal trading strategy contains an additional term that is proportional to λ_δ. As agents try to mimic the average demand $\bar{\theta}$ because of their relative wealth concerns, they now have to forecast the signal Y, since the error term δ has a non-negligible effect on $\bar{\theta}$ if $\lambda_\delta > 0$. The δ-informed agents directly observe Y and thus demand an additional $\gamma \lambda_\delta \beta_\delta Y$ shares to make sure that their wealth is close to the average level. The ϵ-informed agents, on the other hand, have to forecast Y based on their own signal Y_i and the equilibrium price P.

The proof of Proposition 7 reveals that the equilibrium trading intensities β_ϵ and β_δ of ϵ-informed and δ-informed agents are given by:

$$\beta_\epsilon = \frac{1}{\tau \sigma_\epsilon^2} \qquad \text{and} \qquad \beta_\delta = \frac{1}{\tau \sigma_\delta^2 \left(1 - \gamma \lambda_\delta + \dfrac{\lambda_\delta \lambda_\epsilon}{\tau^2 \sigma_\epsilon^2 \sigma_z^2} \right)}. \tag{25}$$

The trading intensity of ϵ-informed agents is not affected by the presence of δ-informed

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[21]As in Section 3, this definition of the aggregate demand is consistent with the interpretation of Z as the stock supply and, hence, does not include the demand of liquidity traders.

investors and does not depend on the KUJ parameter γ. It is identical to the expression derived in Section 3. The trading intensity of δ-informed agents, however, is influenced by their relative wealth considerations: as γ increases, they care more about each other's wealth and, hence, trade more aggressively on their common signal Y. It is also worth noting that β_δ is decreasing in the fraction of ϵ-informed agents. This is due to the fact that prices become more informative as λ_ϵ increases, making the signal Y less valuable to agents.

The above discussion has assumed that both types of informed agents coexist in the market. In order to demonstrate that this can indeed be the case, we have to calculate the expected utility of ϵ-informed, δ-informed, and uninformed agents at the information acquisition stage. The proof of Proposition 8 shows that agent i's certainty equivalent of wealth, gross of information acquisition costs, is given by:

$$\mathcal{U}_i = \frac{1}{2\tau} \log \left(\text{var}[X|\mathcal{F}_i]^{-1} - 2\tau\gamma \left(\lambda_\epsilon \beta_\epsilon + \lambda_\delta \beta_\delta \frac{\text{cov}[X,Y|\mathcal{F}_i]}{\text{var}[X|\mathcal{F}_i]} \right) - \tau^2 \gamma^2 \lambda_\delta^2 \beta_\delta^2 \, \text{var}[Y|X,\mathcal{F}_i] \right),$$
(26)

where we have omitted terms that are independent of the agent's information set $\mathcal{F}_i$.

Comparing the above expression to equation (12) in Section 3, we find that there are two additional terms that are negatively related to λ_δ. The first term, which is proportional to the linear regression coefficient of the signal Y on the asset payoff X (conditional on the equilibrium price P), captures the intuition that a more precise signal Y improves the trading profit of δ-informed agents, and thus hurts other investors. The second term is related to the fact that, because of the common error term in their signals, δ-informed agents increase the variance of the average wealth level. This imposes a negative externality on agents who do not observe this error term.

The definition of an equilibrium at the information acquisition stage in this extended model is analogous to the one given in Section 2.2. In order to calculate the equilibrium number of ϵ-informed, δ-informed, and uninformed agents, we have to compare the ex-ante expected utility of the different investor types to each other. We will say that an equilibrium exhibits "weak herding" if $\lambda_\delta > 0$. We will use the term "strong herding" to refer to equilibria for which $\lambda_\delta > 0$ and $\lambda_\epsilon = 0$. These equilibria are characterized by the fact that agents herd on the same information, even though private signals with orthogonal error terms are available to them. Of course, the existence of such herding equilibria depends on the signal-to-noise ratio of the two signals captured by the parameters σ_ϵ and σ_δ, as well as on the cost parameters c_ϵ and c_δ. The following proposition presents results for the case in which both types of signals have the same precision and are equally costly.

Proposition 8. *Suppose that the two signals $Y_i = X + \epsilon_i$ and $Y = X + \delta$ have the same*

precision (i.e., $\sigma_\epsilon = \sigma_\delta$) and are equally costly to investors (i.e., $c_\epsilon = c_\delta$). Then, in the absence of relative wealth concerns (i.e., when $\gamma = 0$), there are no weak herding equilibria. If, on the other hand, the agents' relative wealth concerns are sufficiently strong, there exist equilibria that exhibit strong herding (i.e., equilibria with $\lambda_\delta > 0$ and $\lambda_\epsilon = 0$).

The first result in Proposition 8 establishes the non-existence of herding equilibria in standard REE models without consumption externalities. In such models, the returns to acquiring information fall as the number of identically informed agents increases. These negative informational externalities encourage investors to acquire signals that are orthogonal to the information revealed by prices. Thus, when given a choice between the two equally informative (and equally costly) signals Y and Y_i, agents always prefer the conditionally independent signal Y_i when relative wealth considerations are not important to them.

The second result shows that strong herding equilibria, in which all informed agents acquire the same signal, can exist when agents care about their peer's consumption.[22] In fact, if γ is sufficiently large, relative wealth concerns dominate the information effect, and investors are better off gathering perfectly correlated information. This can be seen from equation (26). While the incremental value of the signal Y_i exceeds that of the signal Y (i.e., $\text{var}[X|Y_i, P] \leq \text{var}[X|Y, P]$ under our assumption that $\sigma_\epsilon = \sigma_\delta$), knowing Y allows agents to eliminate the uncertainty about the average wealth level caused by the common error term δ. By knowing what others know, agents can make sure not to fall behind their peers. In that sense, the signal's value to agents goes beyond its usefulness in predicting future asset payoffs.

Strong herding equilibria are clearly inefficient. Rather than acquiring signals that complement the information revealed by prices, agents exert costly effort to duplicate the information that is available to their peers. This inefficient allocation of research effort reduces the informational content of asset prices, which can affect social welfare through two distinct channels. First, it leads to less informed portfolio decisions and, hence, lowers the agents' expected utility from trading. Second, in a broader framework in which firms use asset prices to guide their production decisions, this informational inefficiency may also lead to a suboptimal allocation of investment resources.

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[22]We want to point out, however, that strong herding equilibria are typically not unique. There are other equilibria in which agents prefer to acquire conditionally independent signals or to stay uninformed. This is not surprising, given our results in Section 3.

5 Empirical implications

In this section, we discuss the empirical implications of our model and relate our results to the literature on community effects in portfolio choice. Of course, any such attempt requires a model that incorporates the notion of local assets and communities. We therefore first describe how our analysis extends to a multi-asset setting and then argue that our results on the asymmetry of equilibria at the information acquisition stage, which carry over from the single-asset case, imply that different communities of agents specialize in different assets.

Extending our model to a setting with multiple risky assets is rather straightforward.[23] Many of the insights gained from our analysis in Section 4.1 carry over to the multi-asset case. The only complication is that the number of potential equilibria grows exponentially with the number of assets and communities, which makes a general characterization of equilibria at the information acquisition stage intractable. However, one can show that symmetric equilibria in which all communities pursue the same information acquisition strategy are unstable if the number of communities is large (as in Proposition 6).

Our model thus predicts that agents in different communities will focus their information acquisition efforts on different assets.[24] Since better informed agents hold, on average, a larger position in an asset, our model best describes a situation in which agents' portfolios are concentrated in "local stocks" that these agents have more information about. This is consistent with the empirical findings of the growing literature on the role of geography in portfolio choice. For example, Hong, Kubik, and Stein (2004) show that investment decisions are related to social interactions, which are naturally linked to communities. A number of studies, including Grinblatt and Keloharju (2001), Feng and Seasholes (2004), and Ivković and Weisbenner (2005), document that investors are more likely to invest in firms that are geographically close to them.

Our model also provides an alternative explanation for the fact that investors located in the same city tend to buy similar stocks (see, e.g., Hong, Kubik, and Stein, 2005). The empirical literature has attributed this observation to "word-of-mouth" communication between fund managers. Although this is a plausible hypothesis, one has to question whether such "informal information exchanges" are actually incentive compatible. Why would physi-

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[23]The expression for the agents' certainty equivalent of wealth is similar to the one derived in the single-asset case, except that (i) the conditional payoff variance has to be replaced by the determinant of the conditional variance-covariance matrix and, (ii) the KUJ term now depends on the informed agents' demand for all assets. Details of the proof are available from the authors upon request.

[24]Of course, our results cannot explain why a community favors a specific set of assets. In order to establish a link between certain communities and assets, we could, for example, assume that agents have heterogenous information acquisition costs or endowments, so that acquiring information about certain assets is more valuable for some communities of agents than it is for others.

cal proximity facilitate the sharing of information? It seems that investorsr would optimally keep their private information for themselves, rather than lowering its value by sharing it with colleagues and neighbors. Our model suggests an alternative mechanism that operates through the investors' incentives to acquire correlated information. Agents herd on the same information because they care about their wealth relative to their peers.

The concentration of portfolio holdings is a common feature of asset pricing models with relative wealth concerns. For example, DeMarzo, Kaniel, and Kremer (2004) and Gómez, Priestley, and Zapatero (2009b) show in a symmetric information setting that agents with KUJ preferences optimally hold under-diversified portfolios. While our model generates similar predictions for the agents' optimal trading strategies, the actual mechanism that leads to concentrated portfolios is very different. In our economy, agents prefer to hold concentrated portfolios because of their incentives to coordinate their information production activities. This mechanism generates a new set of empirical implications that do not follow from the symmetric information models mentioned above. In particular, our model predicts that the agents' investments in local stocks should generate higher trading profits than their non-local investments, because of the superior information that agents have about these local stocks. By the same argument, local investors should outperform non-local investors. This is consistent with Coval and Moskowitz (2001) and Ivković and Weisbenner (2005), who show that professional and retail investors earn higher returns on local stocks than on their other investments.[25] While Van Nieuwerburgh and Veldkamp (2009) also offer an information-based explanation for this result, their model—in which information is a strategic substitute—relies on differences in the precision of the agents' prior information. In contrast, our model predicts that investors acquire more information about local stocks and, hence, outperform foreign investors in these stocks, even if all investors share the same prior information (and face identical information acquisition costs).

We conclude this section with a discussion of potential empirical tests that could distinguish our preference-based approach to generating strategic complementarities in information acquisition from other theories that have been suggested in the literature.[26] In our model, complementarities arise because agents have relative wealth concerns. Since these complementarities cause price discontinuities and, hence, increase volatility, our model predicts that these price patterns are more pronounced when agents care strongly about each other's wealth. Of course, relative wealth considerations are not directly observable. However, to the extent that they generate behavior that can be interpreted as "conspicuous consump-

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[25]See also Bae, Stulz, and Tan (2008), Massa and Simonov (2006), Ivković, Sialm, and Weisbenner (2008), and Seasholes and Zhu (forthcoming).

[26]See our discussion of alternative mechanisms in the introduction.

tion," proxies for their importance can be created from observed consumption expenditures. For example, Charles, Hurst, and Roussanov (2009) use data from the Consumer Expenditure Survey, collected by the United States Department of Labor, to construct a measure of conspicuous consumption based on expenditures in specific consumption categories. Our analysis suggests that these proxies should be positively related to the level of price volatility: assets that are predominantly held by individuals who exhibit a high level of conspicuous consumption should experience larger price jumps. Any such evidence would provide strong support for our theory relating complementarities in financial markets to a KUJ preference specification.

6 Conclusion

This paper extends the standard REE model with endogenous information acquisition developed by Grossman and Stiglitz (1980), Hellwig (1980), and Verrecchia (1982) to account for relative wealth considerations. In particular, we examine how consumption externalities resulting from a KUJ preference specification affect investors' incentives to acquire information. Our analysis shows that such consumption externalities can generate complementarities in information acquisition. A larger number of informed investors increases the average wealth, which imposes a negative externality on uninformed agents. We demonstrate that if the number of informed investors is not too high, relative wealth concerns dominate the information revelation effect, and the marginal value of information increases in the number of informed agents.

These complementarities in information acquisition can generate multiple herding equilibria. An agent's optimal decision as to whether she should gather information depends on her beliefs about the behavior of other agents. If she believes that most of her peers acquire information, she has an incentive to acquire information as well in order to keep up with them. On the other hand, if she expects others not to be informed, she may not find it worthwhile to spend resources on collecting information. In equilibrium, these beliefs are self-fulfilling. Some of these herding equilibria involve an inefficient allocation of the agents' research effort. In particular, we show that when relative wealth concerns are sufficiently strong, agents ignore signals about fundamentals in favor of signals that are informative about their peers' trades.

We also demonstrate that the multiplicity of equilibria at the information acquisition stage can cause price discontinuities: small changes in fundamentals can lead to large changes in asset prices. These price jumps are caused by changes in the risk premium as agents switch

from an equilibrium with many informed traders to an equilibrium with few informed traders (and vice versa). The model also generates empirical implications that link proxies for relative wealth concerns, e.g. conspicuous consumption, to asset prices.

Finally, we discuss how relative wealth concerns can help explain recent empirical findings regarding local biases in portfolio choice. By introducing relative wealth concerns at the community level, we show that in many cases, only equilibria in which different communities follow different information acquisition strategies exist. Consistent with the empirical evidence, our model predicts that local investors outperform non-local investors, since the concentration of asset holdings is driven by the superior information of local agents.

Appendix

The following lemma is a standard result on multivariate normal random variables (see, e.g., Marin and Rahi, 1999) and is used to calculate the agents' expected utility:

Lemma 1. *Let $X \in \mathbb{R}^n$ be a normally distributed random vector with mean (vector) μ and covariance matrix Σ. If $I - 2\Sigma A$ is positive definite, then $\mathbb{E}\left[\exp\left(X^\top A X + b^\top X\right)\right]$ is well-defined and given by:*

$$
\begin{aligned}
\mathbb{E}\left[\exp\left(X^\top A X + b^\top X\right)\right] &= |I - 2\Sigma A|^{-1/2} \exp\left(b^\top \mu + \mu^\top A \mu\right. \\
&\left. + \frac{1}{2}(b + 2A\mu)^\top (I - 2\Sigma A)^{-1}\Sigma(b + 2A\mu)\right),
\end{aligned} \tag{27}
$$

where $A \in \mathbb{R}^{n \times n}$ is a symmetric matrix and $b \in \mathbb{R}^n$ is a vector.

Proof of Proposition 1. Let V_i denote agent i's "relative payoff" $V_i = W_i - \gamma \bar{W}$. Simple calculations show that:

$$
V_i = (\theta_i + \gamma(P(\kappa - \lambda\beta) - \xi))(X - P) - \gamma\lambda\beta(X - P)^2. \tag{28}
$$

The agent's expected utility is therefore given by the exponential of a quadratic function of the normally distributed random variable $X - P$. Using Lemma 1, we have:

$$
\mathbb{E}\left[-e^{-\tau V_i}|\mathcal{F}_i\right] = -\Psi_i^{-1/2} \exp\left(-\tau\left(\Upsilon_i + \theta_i\eta_i - \frac{\tau}{2\Psi_i}\Gamma_i(\theta_i)^2\Sigma_i\right)\right), \tag{29}
$$

where:

$$
\Psi_i = 1 - 2\tau\gamma\lambda\beta\Sigma_i, \tag{30}
$$

$$
\Upsilon_i = \gamma\left(P(\kappa - \lambda\beta) - \xi - \lambda\beta\eta_i\right)\eta_i, \tag{31}
$$

$$
\Gamma_i(\theta_i) = \theta_i + \gamma\left(P(\kappa - \lambda\beta) - \xi - 2\lambda\beta\eta_i\right), \tag{32}
$$

with $\eta_i = \mathbb{E}\left[X - P|\mathcal{F}_i\right]$ and $\Sigma_i = \text{var}[X - P|\mathcal{F}_i]$.

Since Ψ_i and Υ_i are independent of θ_i, maximizing (29) with respect to θ_i is equivalent to maximizing $\theta_i\eta_i - \tau\Gamma_i(\theta_i)^2\Sigma_i/(2\Psi_i)$. Simple algebra shows that the optimal trading strategy is given by (8). The second-order condition reduces to $\Psi_i > 0$.

Furthermore, one can verify that:

$$\text{var}[X|Y_i, P]^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_z^2}\left(\frac{b}{d}\right)^2, \tag{33}$$

$$\text{var}[X|P]^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_z^2}\left(\frac{b}{d}\right)^2, \tag{34}$$

$$\mathbb{E}[X|Y_i, P] = \mu_x + \frac{\text{var}[X|Y_i, P]}{\sigma_\epsilon^2}(Y_i - \mu_x) + \frac{b\,\text{var}[X|Y_i, P]}{d^2\sigma_z^2}(P - \mathbb{E}[P]), \tag{35}$$

$$\mathbb{E}[X|P] = \mu_x + \frac{b\,\text{var}[X|P]}{d^2\sigma_z^2}(P - \mathbb{E}[P]). \tag{36}$$

We conjecture that an informed agent's trading strategy will be of the form $\theta_i = \alpha + \beta Y_i - \delta P$, whereas an uninformed agent's strategy is given by $\theta_i = \zeta - \nu P$, for some constants $\alpha, \beta, \delta, \zeta, \nu \in \mathbb{R}$. Then, $\kappa = \lambda\delta + (1-\lambda)\nu$ and $\xi = \lambda\alpha + (1-\lambda)\zeta$. Substituting the above conditional moments into the agent's demand function given by (8) yields the following expressions for the coefficients α, β, δ, ζ, and ν:

$$\alpha = \zeta = \frac{\mu_x}{\tau\,\text{var}[X|P]} - \frac{b}{\tau d^2\sigma_z^2}\mathbb{E}[P] + \gamma\xi, \tag{37}$$

$$\beta = \frac{1}{\tau\sigma_\epsilon^2}, \tag{38}$$

$$\delta = \gamma(\kappa - \lambda\beta) - \frac{b}{\tau d^2\sigma_z^2} + \frac{1}{\tau\,\text{var}[X|Y_i, P]}, \tag{39}$$

$$\nu = \gamma(\kappa - \lambda\beta) - \frac{b}{\tau d^2\sigma_z^2} + \frac{1}{\tau\,\text{var}[X|P]}. \tag{40}$$

The market clearing condition can therefore be written as:

$$\int_0^1 \theta_i di = \alpha + \lambda\beta X - \kappa P = Z. \tag{41}$$

This implies that the equilibrium price coefficients a, b, and d are characterized by the following three equations: $\kappa a = \alpha$, $\kappa b = \lambda\beta$, and $\kappa d = 1$. From these equations, it immediately follows that $b/d = \lambda/(\tau\sigma_\epsilon^2)$, which pins down the variances $\text{var}[X|P]$ and $\text{var}[X|Y_i, P]$.

From the definition of κ and the above expressions for δ and ν, we have:

$$\kappa = \gamma(\kappa - \lambda\beta) - \frac{\lambda\beta}{\tau\sigma_z^2}\left(\frac{1}{d}\right) + \frac{1}{\tau}\text{var}[X|P]^{-1} + \frac{\lambda}{\tau\sigma_\epsilon^2} \tag{42}$$

$$= \frac{\frac{1}{\tau}\text{var}[X|P]^{-1} + \frac{\lambda}{\tau\sigma_\epsilon^2} - \gamma\lambda\beta - \frac{\lambda\beta}{\tau\sigma_z^2}\left(\frac{1}{d}\right)}{1 - \gamma}. \tag{43}$$

This expression together with the equilibrium condition $\kappa d = 1$ can be used to solve for the price coefficient d. Further, the expression for a can be derived from the condition $\kappa a = \alpha$. One can readily verify that the price coefficients are uniquely defined in terms of the model's primitives as follows:

$$d = \frac{\tau(1-\gamma) + \dfrac{\lambda}{\tau\sigma_\epsilon^2\sigma_z^2}}{\dfrac{1}{\sigma_x^2} + \dfrac{1}{\sigma_z^2}\left(\dfrac{\lambda}{\tau\sigma_\epsilon^2}\right)^2 + \dfrac{\lambda(1-\gamma)}{\sigma_\epsilon^2}}, \tag{44}$$

$$\frac{a}{d} = \frac{\dfrac{\mu_x}{\sigma_x^2} + \dfrac{\mu_z\lambda}{\tau\sigma_\epsilon^2\sigma_z^2}}{\tau(1-\gamma) + \dfrac{\lambda}{\tau\sigma_\epsilon^2\sigma_z^2}}, \qquad \frac{b}{d} = \frac{\lambda}{\tau\sigma_\epsilon^2}. \tag{45}$$

Finally, we note that if the second-order condition $\Psi_i = 1 - 2\tau\gamma\lambda\beta\Sigma_i > 0$ is satisfied for an uninformed agent, then it is also satisfied for an informed agent. Therefore, a necessary and sufficient condition for a linear equilibrium with $\lambda > 0$ to exist is that $\Psi_i > 0$ holds for uninformed agents, which reduces to:

$$\frac{\sigma_\epsilon^2}{\sigma_x^2} + \frac{\lambda^2}{\tau^2\sigma_\epsilon^2\sigma_z^2} - 2\lambda\gamma > 0. \tag{46}$$

It is easy to check that $\gamma < 1/(\tau\sigma_x\sigma_z)$ is a sufficient condition for the above inequality to hold for all $\lambda \in [0,1]$. This completes the proof. ∎

Proof of Proposition 2. Substituting the agent's optimal demand θ_i given by equation (8) into the expression for the interim expected utility given by equation (29), we have:

$$\mathbb{E}\left[u(V_i)|\mathcal{F}_i\right] = -\Psi_i^{-1/2}\exp\left(-\frac{\eta_i^2}{2\Sigma_i}\right), \tag{47}$$

where, as before, $\eta_i = \mathbb{E}\left[X - P|\mathcal{F}_i\right]$ and $\Sigma_i = \text{var}[X - P|\mathcal{F}_i]$. The ex-ante expected utility (before P and Y_i are observed) is therefore given by the expectation of an exponential function of η_i^2. Since η_i is a normal random variable, it follows from Lemma 1 that:

$$\begin{aligned}
\mathbb{E}\left[\mathbb{E}\left[u(V_i)|\mathcal{F}_i\right]\right] &= -\Psi_i^{-1/2}\left(1 + \frac{\text{var}[\eta_i]}{\Sigma_i}\right)^{-1/2}\exp\left(-\frac{\mathbb{E}\left[\eta_i\right]^2}{2\,\Sigma_i}\left(1 - \frac{\text{var}[\eta_i]}{(\Sigma_i + \text{var}[\eta_i])}\right)\right) &\tag{48} \\
&= -\left(\Psi_i\,\frac{\text{var}[X-P]}{\Sigma_i}\right)^{-1/2}\exp\left(-\frac{(\mu_x - \mathbb{E}\left[P\right])^2}{2\,\text{var}[X-P]}\right), &\tag{49}
\end{aligned}$$

where we have used the fact that for normally distributed random variables, the variance

satisfies $\text{var}[X] = \text{var}[\mathbb{E}\,[X|\mathcal{F}_i]] + \text{var}[X|\mathcal{F}_i]$.

The certainty equivalent of wealth for informed agents defined in equation (5) is therefore given by:

$$\mathcal{V}_I(\lambda) = \frac{1}{2\tau}\log\left(\text{var}[X|Y_i, P]^{-1} - \frac{2\lambda\gamma}{\sigma_\epsilon^2}\right) + H, \tag{50}$$

where:

$$H = \frac{1}{2\tau}\log\left(\text{var}[X-P]\right) + \frac{(\mu_x - \mathbb{E}\,[P])^2}{2\tau\,\text{var}[X-P]} + \frac{1}{\tau}\log\left(\mathbb{E}\left[\exp\left(\tau\gamma\bar{W}\right)\right]\right). \tag{51}$$

For uninformed agents, we have:

$$\mathcal{V}_U(\lambda) = \frac{1}{2\tau}\log\left(\text{var}[X|P]^{-1} - \frac{2\lambda\gamma}{\sigma_\epsilon^2}\right) + H. \tag{52}$$

We first consider the case where $\hat{C} \equiv 1/(\sigma_\epsilon^2(e^{2\tau c}-1)) - 1/\sigma_x^2 > 0$. An interior equilibrium is defined by $\lambda \in (0,1)$ such that $\mathcal{V}_I(\lambda) - c = \mathcal{V}_U(\lambda)$. Using the expressions derived above, one can easily verify that such an interior equilibrium is given by the solution to the following quadratic equation:

$$\frac{\lambda^2}{\tau^2\sigma_\epsilon^4\sigma_z^2} - \frac{2\lambda\gamma}{\sigma_\epsilon^2} - \hat{C} = 0. \tag{53}$$

The discriminant of this quadratic equation is always positive when $\hat{C} > 0$. Furthermore, if $\hat{C} > 0$, if follows from Descartes' rule of sign that it has only one positive root. Thus, the unique interior equilibrium is given by $\lambda = \lambda^*$ as defined in equation (10). We are left to check whether there exist any corner equilibria. Clearly, $\lambda = 0$ cannot be an equilibrium when $\hat{C} > 0$. However, λ^* can exceed 1. In this case, the unique equilibrium is $\lambda = 1$.

Next, consider the case where $\hat{C} \leq 0$. In this case, the quadratic equation in (53) has real roots if and only if $\tau^2\gamma^2 + \hat{C}/\sigma_z^2 \geq 0$. If this condition is not met, the unique equilibrium is for all agents to stay uninformed (i.e, $\lambda = 0$). On the other hand, if this inequality holds, then there are two positive solutions. The corresponding equilibria are given by $\lambda = \min(\lambda^*, 1)$ and $\lambda = \min(\lambda^{**}, 1)$, where λ^* and λ^{**} are defined in Proposition 2. Note, however, that since $\hat{C} \leq 0$, we have $\mathcal{V}_I(0) - c \leq \mathcal{V}_U(0)$. Thus, $\lambda = 0$ is an equilibrium as well. ∎

Proof of Proposition 3. Let $\mathcal{R}(\lambda) = \mathcal{V}_I(\lambda) - \mathcal{V}_U(\lambda)$ denote the difference between the certainty equivalent of informed and uninformed agents. Then, a necessary (sufficient) condition for an interior equilibrium characterized by $\mathcal{R}(\hat{\lambda}) = c$ to be stable is that $d\mathcal{R}(\hat{\lambda})/d\lambda \leq 0$ $(d\mathcal{R}(\hat{\lambda})/d\lambda < 0)$. Substituting the expressions for $\mathcal{V}_I(\lambda)$ and $\mathcal{V}_U(\lambda)$ derived in the proof of Proposition 2 into the function $\mathcal{R}(\lambda)$ and using the fact that $\text{sign}(df(x)/dx) = \text{sign}(de^{f(x)}/dx)$,

we have:

$$\text{sign}\left(\frac{d\mathcal{R}(\lambda)}{d\lambda}\right) = \text{sign}\left(\frac{d}{d\lambda}\left(\frac{1}{\sigma_\epsilon^2\sigma_x^2} + \frac{\lambda^2}{\tau^2\sigma_\epsilon^2\sigma_z^2} - 2\lambda\gamma\right)^{-1}\right). \tag{54}$$

The above inequality can therefore be written as follows:

$$\gamma - \frac{\hat{\lambda}}{\tau^2\sigma_\epsilon^2\sigma_z^2} \leq 0. \tag{55}$$

If $\hat{C} < 0 < \tau^2\gamma^2 + \hat{C}/\sigma_z^2$ and $\lambda^* < 1$, the two interior equilibria are given by $\lambda = \lambda^*$ and $\lambda = \lambda^{**}$. From the expressions in equations (10) and (11), it follows immediately that $\lambda^* > \gamma\tau^2\sigma_\epsilon^2\sigma_z^2$ and that $\lambda^{**} < \gamma\tau^2\sigma_\epsilon^2\sigma_z^2$. This proves that only the equilibrium given by $\lambda = \lambda^*$ is stable. Clearly, the corner solution $\lambda = 0$ is a stable equilibrium as well. ∎

Proof of Proposition 4. The calculations involved in this proof are analogous to those in the proof of Propositions 1, but with the additional complication that we now also have to take into account the agents' beliefs about the demand of liquidity traders in their community, Z_k. The relevant payoff variable of agent i, $V_i = W_i - \gamma\bar{W}$, is given by:

$$V_i = (\theta_i + \gamma(P(\kappa_k - \lambda_k\beta_k) - \xi_k)(X - P) - \gamma\lambda_k\beta_k(X - P)^2 + \omega\gamma Z_k(X - P), \tag{56}$$

which is a quadratic function of the normal random vector $(X - P, Z_k)$. Using Lemma 1, we can therefore rewrite the agent's conditional expected utility as follows:

$$\mathbb{E}\left[-e^{-\tau V_i}|\mathcal{F}_i\right] = -\Psi_i^{-1/2}\exp\left(-\tau\left(\Upsilon_i + \theta_i\eta_{i,1} - \frac{\tau}{2\Psi_i}Q_i(\theta_i)\right)\right), \tag{57}$$

where:

$$\Psi_i = 1 - 2\tau\gamma(\lambda_k\beta_k\Sigma_{i,11} - \omega\Sigma_{i,12}) - \omega\tau^2\gamma^2|\Sigma_i|, \tag{58}$$

$$\Gamma_i(\theta_i) = \theta_i + \gamma\left(P(\kappa_k - \lambda_k\beta_k) - \xi_k - 2\lambda_k\beta_k\eta_{i,1} + \omega\eta_{i,2}\right), \tag{59}$$

$$Q_i(\theta_i) = \Gamma_i(\theta_i)^2\Sigma_{i,11} + 2\omega\gamma\Gamma_i(\theta_i)\eta_{i,1}(\Sigma_{i,12} - \tau\gamma|\Sigma_i|), \tag{60}$$

and Υ_i is independent of the agent's portfolio choice θ_i. As before, $\eta_i = \mathbb{E}[(X - P, Z_k)|\mathcal{F}_i]$ and $\Sigma_i = \text{var}[(X - P, Z_k)|\mathcal{F}_i]$. We use $\eta_{i,m}$ $(\Sigma_{i,mn})$ to denote the mth $(mn$th) element of the vector η_i (matrix Σ_i).

The agent's optimal trading strategy in (16) follows immediately from the first-order condition. The second-order condition is given by $\Psi_i > 0$. Equation (58) shows that this inequality holds for sufficiently small values of γ. The conditional moments η_i and Σ_i can

be calculated from the projection theorem. Substituting these conditional moments into the agent's demand function yields the demand coefficient $\beta_k = 1/(\tau\sigma_\epsilon^2)$. The equilibrium price coefficients are uniquely determined by the market clearing condition $\frac{1}{K}\sum_{k=1}^K \bar{\theta}_k = Z$. The ratio b/d is characterized by:[27]

$$\frac{b}{d} = \frac{\sum_{k=1}^K \lambda_k}{K\tau\sigma_\epsilon^2}. \tag{61}$$

∎

Proof of Proposition 5. Substituting the agent's optimal demand θ_i given by equation (16) into the expression for the interim expected utility given by equation (57), we have:

$$\mathbb{E}\left[u(V_i)|\mathcal{F}_i\right] = -\Psi_i^{-1/2}\exp\left(-\eta_{i,1}^2/(2\Sigma_{i,11})\right), \tag{62}$$

which yields the following expression for the agent's ex-ante expected utility:

$$\mathbb{E}\left[u(V_i)\right] = -\left(\Psi_i\frac{\operatorname{var}[X-P]}{\Sigma_{i,11}}\right)^{-1/2}\exp\left(-\frac{(\mu_x - \mathbb{E}\left[P\right])^2}{2\operatorname{var}[X-P]}\right). \tag{63}$$

Agent i's certainty equivalent of wealth, gross of information acquisition costs, can therefore be written as:

$$\mathcal{U}_i = \frac{1}{2\tau}\log\left(\operatorname{var}[X|\mathcal{F}_i]^{-1} - 2\tau\gamma\left(\lambda_k\beta_k - \omega\frac{\operatorname{cov}[X, Z_k|\mathcal{F}_i]}{\operatorname{var}[X|\mathcal{F}_i]}\right) - \omega\tau^2\gamma^2\operatorname{var}[Z_k|X, \mathcal{F}_i]\right) + H, \tag{64}$$

where H is independent of agent i's information set $\mathcal{F}_i$, and:

$$\frac{\operatorname{cov}[X, Z_k|\mathcal{F}_i]}{\operatorname{var}[X|\mathcal{F}_i]} = \frac{b}{d}\frac{\operatorname{cov}[Z_k, Z]}{\sigma_z^2}, \tag{65}$$

$$\operatorname{var}[Z_k|X, \mathcal{F}_i] = \operatorname{var}[Z_k] - \frac{\operatorname{cov}[Z_k, Z]^2}{\sigma_z^2}, \tag{66}$$

for all agents that belong to community k.

Note that up to this point, we have not made any assumptions about the correlation structure of the liquidity shocks $\{Z_k\}_{k=1}^K$. Assuming that these shocks are perfectly correlated across communities (i.e., $Z_k = Z/K$), we obtain $\operatorname{cov}[X, Z_k|\mathcal{F}_i]/\operatorname{var}[X|\mathcal{F}_i] = b/(dK)$ and $\operatorname{var}[Z_k|X, \mathcal{F}_i] = 0$. We can thus write the certainty equivalent of wealth, gross of information

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[27]Note that the normalized price $\tilde{P} = P/d$ is informationally equivalent to P. Thus, in order to characterize the equilibrium at the information acquisition stage, we only need to know the ratio b/d.

acquisition costs, of an agent in community k as follows:

$$\mathcal{U}_i = \frac{1}{2\tau} \log \left(\text{var}[X|\mathcal{F}_i]^{-1} - \frac{2\gamma}{\sigma_\epsilon^2} \left(\lambda_k - \omega \frac{\sum_{j=1}^K \lambda_j}{K^2} \right) \right) + H. \tag{67}$$

The agent's relative wealth concerns are captured by the second term inside the log in the above expression. Generically, this term is non-zero for all $K \geq 2$. Thus, for $K \geq 2$, relative wealth concerns affect an agent's information acquisition decision even if the wealth of liquidity traders is included in the calculation of the average wealth (i.e., even if $\omega = 1$). ∎

Proof of Proposition 6. The first part of the proof is analogous to the proof of Proposition 2. From the proof of Proposition 5, we know that the certainty equivalent of wealth of an informed agent in community k, gross of information acquisition costs, is given by:

$$\mathcal{V}_I(\lambda_k, \lambda_{-k}) = \frac{1}{2\tau} \log \left(\text{var}[X|Y_i, P]^{-1} - \frac{2\gamma}{\sigma_\epsilon^2} \left(\lambda_k - \omega \frac{\sum_{j=1}^K \lambda_j}{K^2} \right) \right) + H, \tag{68}$$

where $\lambda_{-k} \in \mathbb{R}^{K-1}$ denotes the fraction of informed agents in all communities but k and H is independent of the agent's information set. For uninformed agents, we have:

$$\mathcal{V}_U(\lambda_k, \lambda_{-k}) = \frac{1}{2\tau} \log \left(\text{var}[X|P]^{-1} - \frac{2\gamma}{\sigma_\epsilon^2} \left(\lambda_k - \omega \frac{\sum_{j=1}^K \lambda_j}{K^2} \right) \right) + H. \tag{69}$$

Further, it follows from the projection theorem and the equilibrium price coefficients specified in the proof of Proposition 4 that:

$$\text{var}[X|Y_i, P]^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_\epsilon^2} + \frac{1}{\sigma_z^2} \left(\frac{\sum_{k=1}^K \lambda_k}{K \tau \sigma_\epsilon^2} \right)^2, \tag{70}$$

$$\text{var}[X|P]^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_z^2} \left(\frac{\sum_{k=1}^K \lambda_k}{K \tau \sigma_\epsilon^2} \right)^2. \tag{71}$$

Let $\mathcal{R}(\lambda_k, \lambda_{-k}) = \mathcal{V}_I(\lambda_k, \lambda_{-k}) - \mathcal{V}_U(\lambda_k, \lambda_{-k})$ denote the difference between the certainty equivalent of informed and uninformed agents. Then, a symmetric interior equilibrium is defined by $\lambda_c \in (0, 1)$ such that $\mathcal{R}(\lambda_c, \lambda_c) = c$. Using the above expressions, one can easily verify that such an interior equilibrium is given by the solution to the following quadratic equation:

$$\frac{\lambda_c^2}{\tau^2 \sigma_\epsilon^4 \sigma_z^2} - \frac{2\lambda_c \gamma}{\sigma_\epsilon^2} \left(1 - \frac{\omega}{K} \right) - \hat{C} = 0. \tag{72}$$

This quadratic equation has real roots if and only if $\tau^2\gamma^2(1-\omega/K)^2+\hat{C}/\sigma_z^2 \geq 0$. The largest root is given by λ_c^* as defined by equation (19). Thus, $\lambda_k = \min(\lambda_c^*, 1)$, for all $k \in \{1, \ldots, K\}$, constitutes a symmetric equilibrium. A sufficient condition for this equilibrium to be stable is that $d\mathcal{R}(\lambda_c^*, \lambda_c^*)/d\lambda_k < 0$. Tedious but straightforward calculations show that this inequality reduces to (20).

In order to prove the second part of the proposition, we have to identify conditions that support an equilibrium in which all agents in m communities have an incentive to become informed and all agents in the remaining $K - m$ communities do not. For the former agents to be strictly better off acquiring information, we must have:

$$\frac{1}{2\tau} \log\left(\text{var}[X|Y_i, P]^{-1} - \frac{2\gamma}{\sigma_\epsilon^2}\left(1 - \omega\frac{m}{K^2}\right)\right) - c > \frac{1}{2\tau} \log\left(\text{var}[X|P]^{-1} - \frac{2\gamma}{\sigma_\epsilon^2}\left(1 - \omega\frac{m}{K^2}\right)\right),$$
(73)

whereas for the latter agents to optimally stay uninformed, we need:

$$\frac{1}{2\tau} \log\left(\text{var}[X|Y_i, P]^{-1} - \frac{2\gamma}{\sigma_\epsilon^2}\left(-\omega\frac{m}{K^2}\right)\right) - c < \frac{1}{2\tau} \log\left(\text{var}[X|P]^{-1} - \frac{2\gamma}{\sigma_\epsilon^2}\left(-\omega\frac{m}{K^2}\right)\right). \quad (74)$$

Simple manipulations using the expressions in (70) and (71) yield condition (21). These equilibria are always stable, as the strict inequalities ensure that the corner solutions $\lambda_k = 1$ and $\lambda_k = 0$ satisfy the stability criterion. This completes the proof. ∎

Proof of Proposition 7. The calculations involved in this proof are analogous to those in the proof of Proposition 1. The relevant payoff variable of agent i, $V_i = W_i - \gamma\bar{W}$, is given by:

$$V_i = (\theta_i + \gamma(P(\kappa - \lambda_\epsilon\beta_\epsilon) - \xi))(X - P) - \gamma\lambda_\delta\beta_\delta Y(X - P) - \gamma\lambda_\epsilon\beta_\epsilon(X - P)^2, \quad (75)$$

which is a quadratic function of the normal random vector $(X - P, Y)$. Using Lemma 1, we can therefore rewrite the agent's conditional expected utility as follows:

$$\mathbb{E}\left[-e^{-\tau V_i}|\mathcal{F}_i\right] = -\Psi_i^{-1/2} \exp\left(-\tau\left(\Upsilon_i + \theta_i\eta_{i,1} - \frac{\tau}{2\Psi_i}Q_i(\theta_i)\right)\right), \quad (76)$$

where:

$$\Psi_i = 1 - 2\tau\gamma\left(\lambda_\epsilon\beta_\epsilon\Sigma_{i,11} + \lambda_\delta\beta_\delta\Sigma_{i,12}\right) - \tau^2\gamma^2\lambda_\delta^2\beta_\delta^2|\Sigma_i|, \tag{77}$$

$$\Lambda_i = -\gamma\lambda_\delta\beta_\delta\eta_{i,1}, \tag{78}$$

$$\Upsilon_i = \gamma\left(P(\kappa - \lambda_\epsilon\beta_\epsilon) - \xi - \lambda_\epsilon\beta_\epsilon\eta_{i,1} - \lambda_\delta\beta_\delta\eta_{i,2}\right)\eta_{i,1}, \tag{79}$$

$$\Gamma_i(\theta_i) = \theta_i + \gamma\left(P(\kappa - \lambda_\epsilon\beta_\epsilon) - \xi - 2\lambda_\epsilon\beta_\epsilon\eta_{i,1} - \lambda_\delta\beta_\delta\eta_{i,2}\right), \tag{80}$$

$$Q_i(\theta_i) = \Gamma_i(\theta_i)^2\Sigma_{i,11} + 2\Gamma_i(\theta_i)\Lambda_i(\Sigma_{i,12} + \tau\gamma\lambda_\delta\beta_\delta|\Sigma_i|) + \Lambda_i^2(\Sigma_{i,22} - 2\tau\gamma\lambda_\epsilon\beta_\epsilon|\Sigma_i|), \tag{81}$$

with $\eta_i = \mathbb{E}\left[(X - P, Y)|\mathcal{F}_i\right]$ and $\Sigma_i = \text{var}[(X - P, Y)|\mathcal{F}_i]$. $\eta_{i,m}$ ($\Sigma_{i,mn}$) denotes the mth (mnth) element of the vector η_i (matrix Σ_i).

The agent's optimal trading strategy in (24) follows immediately from the first-order condition. The second-order condition is given by $\Psi_i > 0$.[28] From equation (77), this inequality holds for sufficiently small values of γ. The conditional moments η_i and Σ_i can be calculated from the projection theorem. Substituting these conditional moments into the agent's demand function and imposing the market clearing condition $\bar{\theta} = Z$, we obtain the following expressions for the demand coefficients β_ϵ and β_δ:

$$\beta_\epsilon = \frac{1}{\tau\sigma_\epsilon^2}, \tag{82}$$

$$\beta_\delta = \frac{1}{\tau\sigma_\delta^2} - \frac{\lambda_\delta\beta_\delta\lambda_\epsilon\beta_\epsilon}{\tau\sigma_z^2} + \gamma\lambda_\delta\beta_\delta. \tag{83}$$

The equilibrium price coefficients are uniquely determined by the market clearing condition. They satisfy the restrictions $b_x/d = \lambda_\epsilon\beta_\epsilon$ and $b_y/d = \lambda_\delta\beta_\delta$. ∎

Proof of Proposition 8. Analogous calculations to those in the proof of Proposition 2 show that the agents' ex-ante expected utility is given by:

$$\mathbb{E}\left[u(V_i)\right] = -\left(\Psi_i\frac{\text{var}[X - P]}{\text{var}[X|\mathcal{F}_i]}\right)^{-1/2}\exp\left(-\frac{(\mu_x - \mathbb{E}\left[P\right])^2}{2\,\text{var}[X - P]}\right), \tag{84}$$

where Ψ_i is defined in equation (77). The certainty equivalent of wealth, gross of information acquisition costs, can therefore be written as:

$$\mathcal{U}_i = \frac{1}{2\tau}\log\left(\text{var}[X|\mathcal{F}_i]^{-1} - 2\tau\gamma\left(\lambda_\epsilon\beta_\epsilon + \lambda_\delta\beta_\delta\frac{\text{cov}[X, Y|\mathcal{F}_i]}{\text{var}[X|\mathcal{F}_i]}\right) - \tau^2\gamma^2\lambda_\delta^2\beta_\delta^2\,\text{var}[Y|X, \mathcal{F}_i]\right) + H, \tag{85}$$

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[28]It can easily be verified that this condition is also sufficient for the agent's conditional expected utility in (76) to be well-defined.

where we have used the fact that $\text{var}[Y|X, \mathcal{F}_i] = \text{var}[Y|\mathcal{F}_i] - \text{cov}[X, Y|\mathcal{F}_i]^2/\text{var}[X|\mathcal{F}_i]$. As before, the H term is independent of agent i's information set $\mathcal{F}_i$.

For a weak herding equilibrium to exist, investors must be indifferent between acquiring the signal Y and the signal Y_i. Thus, under the assumption that $c_\epsilon = c_\delta$, we must have:

$$\text{var}[X|Y, P]^{-1} = \text{var}[X|Y_i, P]^{-1} - 2\tau\gamma\lambda_\delta\beta_\delta \frac{\text{cov}[X, Y|Y_i, P]}{\text{var}[X|Y_i, P]} - \tau^2\gamma^2\lambda_\delta^2\beta_\delta^2\, \text{var}[Y|X, Y_i, P]. \quad (86)$$

When $\gamma = 0$, this equation simplifies to:

$$\frac{1}{\sigma_\epsilon^2} + \frac{(\lambda_\epsilon\beta_\epsilon)^2}{\sigma_z^2} = \frac{1}{\sigma_\delta^2} + \frac{(\lambda_\epsilon\beta_\epsilon + \lambda_\delta\beta_\delta)^2}{\sigma_z^2 + (\lambda_\delta\beta_\delta)^2\sigma_\delta^2}, \quad (87)$$

where we have used the projection theorem to calculate the conditional moments:

$$\text{var}[X|Y, P]^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_\delta^2} + \frac{(\lambda_\epsilon\beta_\epsilon)^2}{\sigma_z^2}, \quad (88)$$

$$\text{var}[X|Y_i, P]^{-1} = \frac{1}{\sigma_x^2} + \frac{1}{\sigma_\epsilon^2} + \frac{(\lambda_\epsilon\beta_\epsilon + \lambda_\delta\beta_\delta)^2}{\sigma_z^2 + (\lambda_\delta\beta_\delta)^2\sigma_\delta^2}. \quad (89)$$

Substituting the equilibrium trading intensities β_ϵ and β_δ given by equation (25) into equation (87) and setting $\sigma_\epsilon = \sigma_\delta = \sigma_s$ yields:

$$\lambda_\delta^2\lambda_\epsilon^2 + \lambda_\delta(\lambda_\delta + 2\lambda_\epsilon)\tau^2\sigma_s^2\sigma_z^2 = 0. \quad (90)$$

Clearly, the above equation only holds for $\lambda_\delta = 0$. This proves that, in the absence of relative wealth concerns, there are no herding equilibria.

In order to prove the existence of strong herding equilibria, we have to show (i) that investors are indifferent between acquiring the signal Y and staying uninformed, and (ii) that informed investors strictly prefer to acquire the signal Y over the signal Y_i (i.e., $\lambda_\epsilon = 0$):

$$\text{var}[X|Y, P]^{-1}e^{-2\tau c} = \text{var}[X|P]^{-1} - 2\tau\gamma\lambda_\delta\beta_\delta\frac{\text{cov}[X, Y|P]}{\text{var}[X|P]} - \tau^2\gamma^2\lambda_\delta^2\beta_\delta^2\, \text{var}[Y|X, P], \quad (91)$$

$$\text{var}[X|Y, P]^{-1} > \text{var}[X|Y_i, P]^{-1} - 2\tau\gamma\lambda_\delta\beta_\delta\frac{\text{cov}[X, Y|Y_i, P]}{\text{var}[X|Y_i, P]} - \tau^2\gamma^2\lambda_\delta^2\beta_\delta^2\, \text{var}[Y|X, Y_i, P], \quad (92)$$

where we have again assumed that $c_\epsilon = c_\delta = c$. Using the projection theorem, one can show

that:

$$\frac{\text{cov}[X,Y|\mathcal{F}_i]}{\text{var}[X|\mathcal{F}_i]} = \frac{\sigma_z^2 - \lambda_\epsilon \beta_\epsilon \lambda_\delta \beta_\delta \sigma_\delta^2}{\sigma_z^2 + (\lambda_\delta \beta_\delta)^2 \sigma_\delta^2} \tag{93}$$

$$\text{var}[Y|X,\mathcal{F}_i] = \frac{\sigma_z^2 \sigma_\delta^2}{\sigma_z^2 + (\lambda_\delta \beta_\delta)^2 \sigma_\delta^2} \tag{94}$$

for both ϵ-informed and uninformed investors. Substituting these expressions and the conditional moments derived above into (92), and setting $\sigma_\epsilon = \sigma_\delta$, yields:

$$\lambda_\delta \beta_\delta - 2\tau\gamma\sigma_z^2 - \tau^2\gamma^2\lambda_\delta\beta_\delta\sigma_z^2\sigma_\delta^2 < 0. \tag{95}$$

Since β_δ is decreasing in γ, this inequality holds for sufficiently large values of γ. The equilibrium fraction of δ-informed investors can then be derived from equation (91), which is a quadratic equation in λ_δ, since:

$$\text{var}[X|P]^{-1} = \frac{1}{\sigma_x^2} + \frac{(\lambda_\epsilon \beta_\epsilon + \lambda_\delta \beta_\delta)^2}{\sigma_z^2 + (\lambda_\delta \beta_\delta)^2 \sigma_\delta^2}. \tag{96}$$

It can easily be verified that equation (91) has a positive root if the information acquisition cost c is not too large. This completes the proof. ∎

References

Abel, A. B., 1990, "Asset prices under habit formation and catching up with the Joneses," *American Economic Review*, 80(2), 38–42.

Bae, K.-H., R. M. Stulz, and H. Tan, 2008, "Do local analysts know more? A cross-country study of the performance of local analysts and foreign analysts," *Journal of Financial Economics*, 88(3), 581–606.

Bai, Y., E. C. Chang, and J. Wang, 2006, "Asset prices under short-sale constraints," working paper, MIT.

Bakshi, G. S., and Z. Chen, 1996, "The spirit of capitalism and stock market prices," *American Economic Review*, 86(1), 133–157.

Barlevy, G., and P. Veronesi, 2000, "Information acquisition in financial markets," *Review of Economic Studies*, 67(1), 79–90.

———, 2003, "Rational panics and stock market crashes," *Journal of Economic Theory*, 110(2), 234–263.

———, 2008, "Information acquisition in financial markets: a correction," working paper, University of Chicago.

Bisin, A., U. Horst, and O. Özgüra, 2006, "Rational expectations equilibria of economies with local interactions," *Journal of Economic Theory*, 127(1), 74–116.

Brown, J., Z. Ivković, S. Weisbenner, and P. Smith, 2008, "Neighbors matter: causal community effects and stock market participation," *Journal of Finance*, 63(3), 1509–1531.

Brunnermeier, M. K., 2001, *Asset pricing under asymmetric information*. Oxford University Press, Oxford.

Chamley, C., 2008, "On "Acquisition of information in financial markets"," *Review of Economic Studies*, 75(4), 1081–1084.

Charles, K. K., E. Hurst, and N. Roussanov, 2009, "Conspicuous consumption and race," *The Quarterly Journal of Economics*, 124(2), 425–467.

Clark, A. E., P. Frijters, and M. A. Shields, 2008, "Relative income, happiness, and utility: an explanation for the Easterlin paradox and other puzzles," *Journal of Economic Literature*, 46(1), 95–144.

Colla, P., and A. Mele, 2010, "Information linkages and correlated trading," *Review of Financial Studies*, 23(1), 203–246.

Coval, J. D., and T. J. Moskowitz, 2001, "The geography of investment: informed trading and asset prices," *Journal of Political Economy*, 109(4), 811–841.

DeMarzo, P., R. Kaniel, and I. Kremer, 2004, "Diversification as a public good: community effects in portfolio choice," *Journal of Finance*, 59(4), 1667–1715.

——— , 2007, "Technological innovation and real investment booms and busts," *Journal of Financial Economics*, 85(3), 735–754.

——— , 2008, "Relative wealth concerns and financial bubbles," *Review of Financial Studies*, 21(1), 19–50.

Diamond, D. W., 1985, "Optimal release of information by firms," *Journal of Finance*, 40(4), 1071–1094.

Diamond, D. W., and R. E. Verrecchia, 1981, "Information aggregation in a noisy rational expectations economy," *Journal of Financial Economics*, 9(3), 221–235.

Dorn, D., G. Huberman, and P. Sengmueller, 2008, "Correlated trading and returns," *Journal of Finance*, 63(2), 885–920.

Duflo, E., and E. Saez, 2002, "Participation and investment decisions in a retirement plan: the influence of colleagues' choices," *Journal of Public Economics*, 85(1), 121–148.

Dupor, B., and W.-F. Liu, 2003, "Jealousy and equilibrium overconsumption," *American Economic Review*, 93(1), 423–428.

Feng, L., and M. S. Seasholes, 2004, "Correlated trading and location," *Journal of Finance*, 59(5), 2117–2144.

French, K., and J. Poterba, 1991, "International diversification and international equity markets," *American Economic Review*, 81(2), 222–226.

Frey, B. S., and A. Stutzer, 2002, "What can economists learn from happiness research?," *Journal of Economic Literature*, 40(2), 402–435.

Froot, K. A., D. S. Scharfstein, and J. C. Stein, 1992, "Herd on the street: informational inefficiencies in a market with short-term speculation," *Journal of Finance*, 47(4), 1461–1484.

Galí, J., 1994, "Keeping up with the Joneses: consumption externalities, portfolio choice and asset prices," *Journal of Money, Credit, and Banking*, 26(1), 1–8.

Ganguli, J. V., and L. Yang, 2009, "Complementarities, multiplicity, and supply information," *Journal of the European Economic Association*, 7(1), 90–115.

Gennotte, G., and H. Leland, 1990, "Market liquidity, hedging and crashes," *American Economic Review*, 80(5), 999–1021.

Glaeser, E., and J. A. Scheinkman, 2002, "Non-market interactions," working paper, Harvard University.

Goldstein, I., E. Ozdenoren, and K. Yuan, 2009, "Trading frenzies and their impact on real investment," working paper, Wharton School, University of Pennsylvania.

———, forthcoming, "Learning and complementarities in speculative attacks," *Review of Economic Studies*.

Gómez, J.-P., R. Priestley, and F. Zapatero, 2009a, "The effect of relative wealth concerns on the cross-section of stock returns," working paper, Marshall School of Business, USC.

———, 2009b, "Implications of keeping-up-with-the-Joneses behavior for the equilibrium cross section of stock returns: international evidence," *Journal of Finance*, 64(6), 2703–2737.

Grinblatt, M., and M. Keloharju, 2001, "How distance, language, and culture influence stock-holdings and trades," *Journal of Finance*, 56(3), 1053–1073.

Grossman, S. J., and J. E. Stiglitz, 1980, "On the impossibility of informationally efficient markets," *American Economic Review*, 70(3), 393–408.

Grundy, B. D., and M. McNichols, 1989, "Trade and the revelation of information through prices and direct disclosure," *Review of Financial Studies*, 2(4), 495–526.

Hellwig, C., and L. Veldkamp, 2009, "Knowing what others know: coordination motives in information acquisition," *Review of Economic Studies*, 76(1), 223–251.

Hellwig, M. F., 1980, "On the aggregation of information in competitive markets," *Journal of Economic Theory*, 22(3), 477–498.

Hong, H., J. D. Kubik, and J. C. Stein, 2004, "Social interaction and stock-market participation," *Journal of Finance*, 59(1), 137–163.

———, 2005, "Thy neighbor's portfolio: word-of-mouth effects in the holdings and trades of money managers," *Journal of Finance*, 60(6), 2801–2824.

Huang, J., and J. Wang, 2009, "Liquidity and market crashes," *Review of Financial Studies*, 22(7), 2607–2643.

Ivković, Z., C. Sialm, and S. Weisbenner, 2008, "Portfolio concentration and the performance of individual investors," *Journal of Financial and Quantitative Analysis*, 43(3), 613–656.

Ivković, Z., and S. Weisbenner, 2005, "Local does as local is: information content of the geography of individual investors' common stock investments," *Journal of Finance*, 60(1), 267–306.

———, 2007, "Information diffusion effects in individual investors' common stock purchases: covet thy neighbors' investment choices," *Review of Financial Studies*, 20(4), 1327–1357.

Jacklin, C., A. Kleidon, and P. Pfleiderer, 1992, "Underestimation of portfolio insurance and the crash of October 1987," *Review of Financial Studies*, 5(1), 35–63.

Ljungqvist, L., and H. Uhlig, 2000, "Tax policy and aggregate demand management under catching up with the Joneses," *American Economic Review*, 90(3), 356–366.

Maccheroni, F., M. Marinacci, and A. Rustichini, 2010, "Social decision theory: choosing within and between groups," working paper, University of Minnesota.

Malinova, K., and L. Smith, 2009, "Informational diversity and proximity in rational expectations equilibrium," working paper, University of Toronto.

Marin, J. M., and R. Rahi, 1999, "Speculative securities," *Economic Theory*, 14(3), 653–668.

Mas-Colell, A., M. D. Whinston, and J. R. Green, 1995, *Microeconomic theory*. Oxford University Press, Oxford.

Massa, M., and A. Simonov, 2006, "Hedging, familiarity, and portfolio choice," *Review of Financial Studies*, 19(2), 633–685.

Mele, A., and F. Sangiorgi, 2008, "Ambiguity aversion and market efficiency," working paper, London School of Economics.

Seasholes, M. S., and N. Zhu, forthcoming, "Individual investors and local bias," *Journal of Finance*.

Van Nieuwerburgh, S., and L. Veldkamp, 2009, "Information immobility and the home bias puzzle," *Journal of Finance*, 64(3), 1187–1215.

Veblen, T. B., 1899, *The theory of the leisure class: an economic study of institutions*. Penguin, New York.

Veldkamp, L. L., 2006a, "Information markets and the comovement of asset prices," *Review of Economic Studies*, 73(3), 823–845.

——— , 2006b, "Media frenzies in markets for financial information," *American Economic Review*, 96(3), 577–601.

Verrecchia, R. E., 1982, "Information acquisition in a noisy rational expectations economy," *Econometrica*, 50(6), 1415–1430.
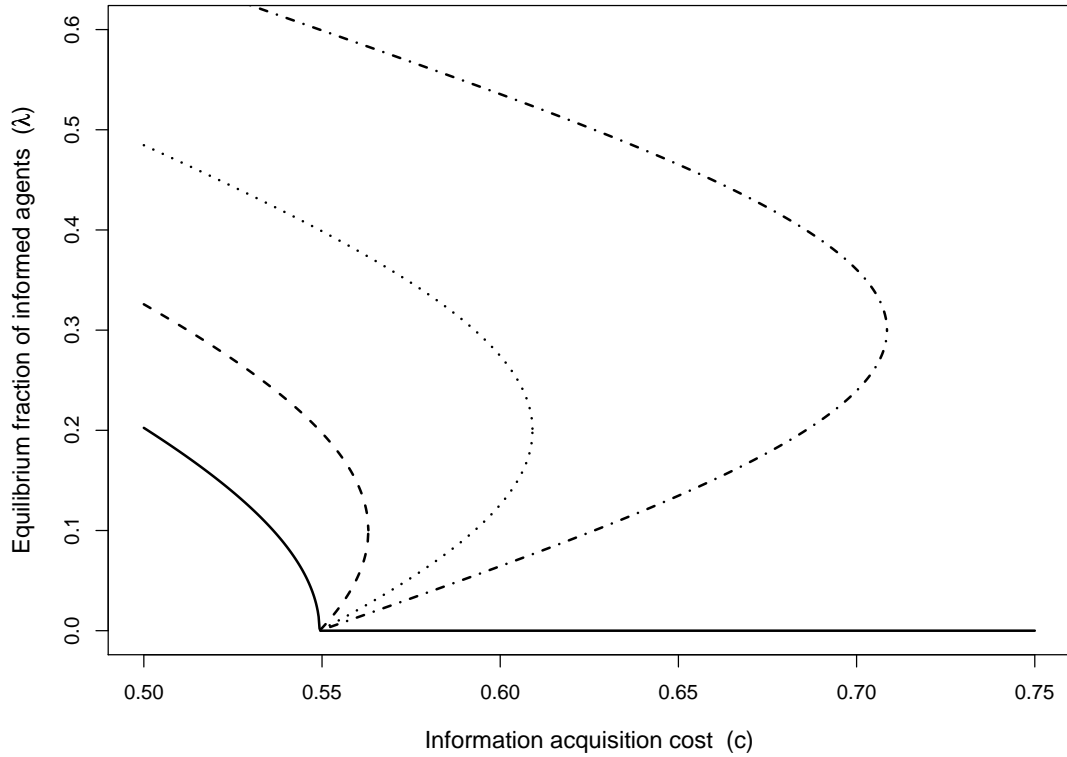
Figure 1: The graph presents the equilibrium fraction of informed agents, λ, as a function of the cost of gathering information, c. The solid line corresponds to the standard model with $\gamma = 0$. The dotted and dashed lines correspond to equilibria with $\gamma = 0.2$, 0.4, and 0.6, respectively. Other parameter values are $\sigma_\epsilon^2 = 0.5$, $\sigma_x^2 = \sigma_z^2 = \tau = 1$.
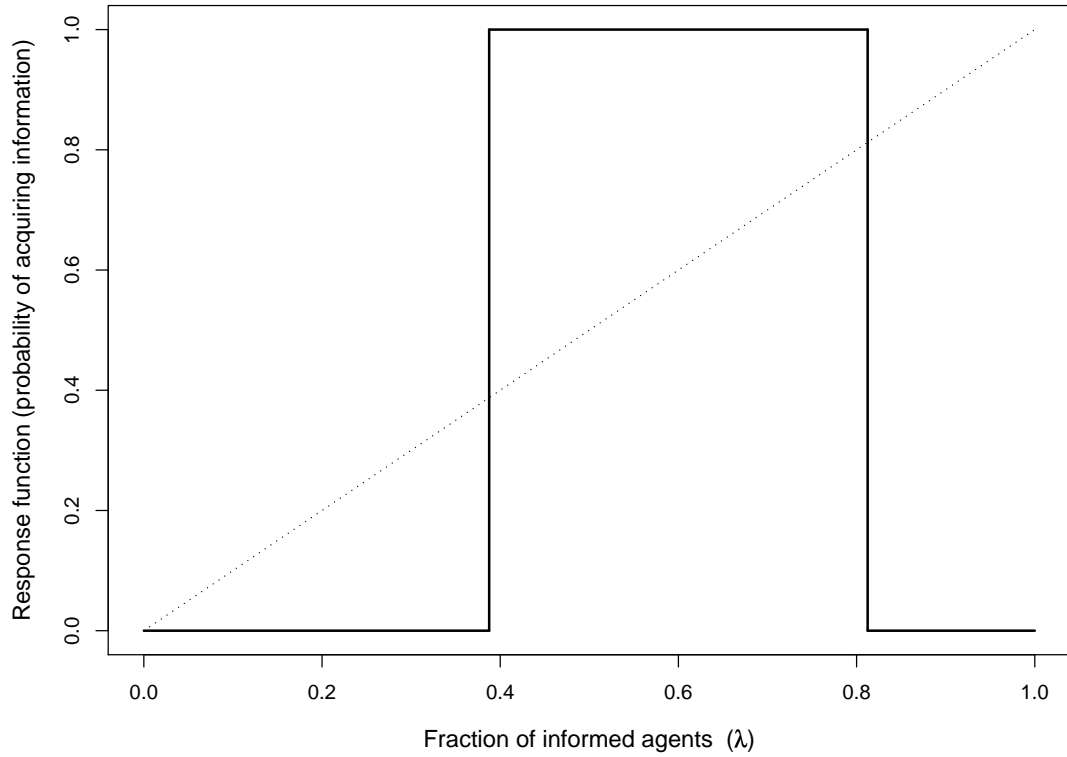
Figure 2: The graph presents the optimal information acquisition decision of an agent as a function of the fraction of informed agents. Parameters correspond to those satisfying $\hat{C} < 0 < \tau^2\gamma^2 + 4\hat{C}/\sigma_z^2$ in Proposition 2: $\sigma_x^2 = \sigma_\epsilon^2 = \sigma_z^2 = \tau = 1$, $c = 0.45$, and $\gamma = 0.6$.
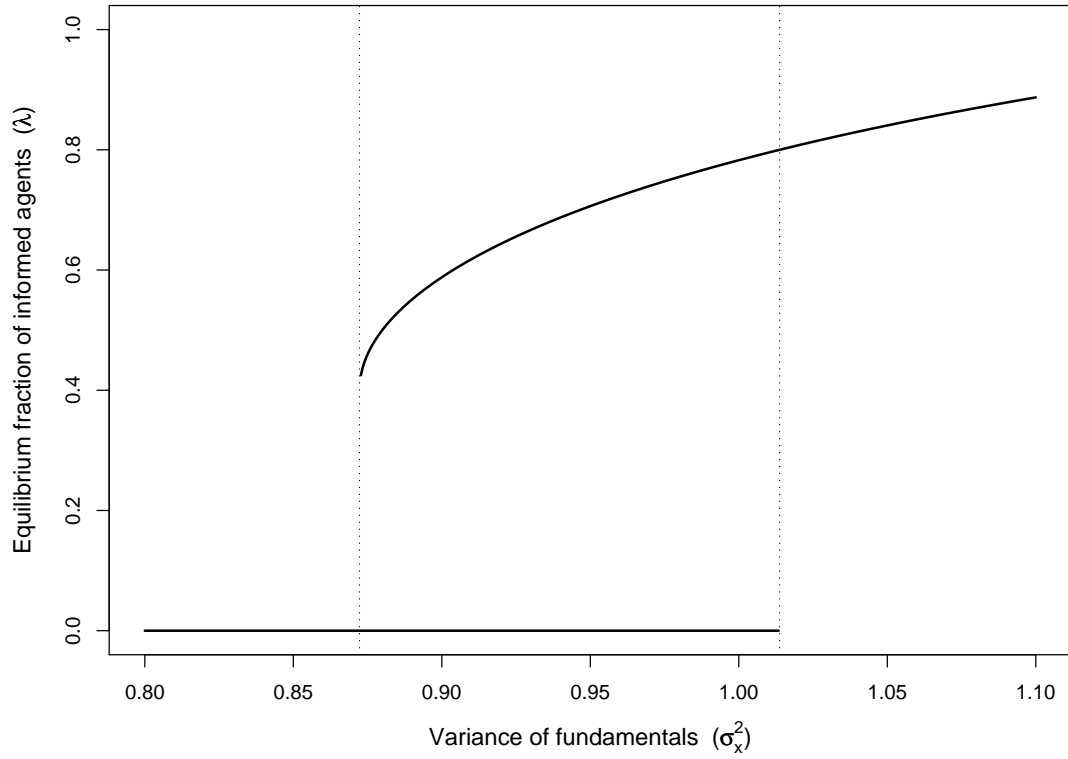
Figure 3: The graph presents the equilibrium fraction of informed agents, λ, at the two stable equilibria as a function of the payoff variance, σ_x^2. The dotted lines correspond to the points where the equilibrium switches from uniqueness to multiplicity. The parameter values used in the graph are $\sigma_\epsilon^2 = \sigma_z^2 = \tau = 1$, $c = 0.3$, and $\gamma = 0.2$.