Debt, Labor Markets and the Creation and Destruction of Firms

Andres Almazan
University of Texas

Adolfo de Motta
McGill University

Sheridan Titman*
University of Texas and the NBER

22 February 2012

ABSTRACT

We analyze the financing decisions of firms that face a labor market with search frictions and examine public policy choices that influence the firms' financing and liquidation choices. In our model, debt facilitates the process of creative destruction (i.e., the elimination of inefficient firms to facilitate the creation of new firms) but may induce excessive liquidation and unemployment; in particular, during economic downturns. Within this setting we examine the role of monetary policy, which can reduce debt burdens during economy-wide downturns, and tax policy, which can influence the incentives of firms to use debt financing.

*Almazan and Titman are from the McCombs School of Business, University of Texas at Austin, and de Motta is from McGill University. E-mail addresses: `andres.almazan@mccombs.utexas.edu`, `adolfo.demotta@mcgill.ca`, and `titman@mail.utexas.edu`. Preliminary version. Comments are welcome.

1. Introduction

Economic forecasters and policymakers have long recognized that financial structure at the corporate and household level can influence macro-economic conditions. The most recent economic crisis, which was triggered in part by the substantial leverage in the real estate and banking sectors, is perhaps the most visceral illustration of this point. While financial economists have responded to this crisis with a plethora of work that examines policy issues that relate to the leverage of financial institutions, the more general issue of the interaction between corporate financing choices and macro policy has received scant attention.[1]

To examine the interaction between corporate financing choices and macro policy we combine a corporate finance model in which debt plays a fundamental role with a model from the macro/search literature where production requires a match between workers and firms. In particular, we follow Hart and Moore (1995) and assume that debt choices are chosen by investors to indirectly control managers who enjoy private control benefits and thus never voluntarily liquidate their firms. We incorporate this into a macro labor search model along the lines of Pissarides (2000) to illustrate potential interactions between capital structure choices and macroeconomic outcomes. We are particularly interested in how capital structure, and its effect on liquidation, can affect the tightness of labor markets in both booms and recessions, and how these effects can in turn affect the emergence of new firms.

As we illustrate in our model, the nature of the economic shocks can be quite important. Some shocks can be firm specific while other shocks affect the entire economy.

[1] See for example Brunnermeier (2009).

This distinction can be important, since debt can lead to the exit of firms with negative firm specific shocks, and in doing so, facilitate the transfer of resources from less to more productive firms. However, debt contributes to unemployment during economy wide downturns, and can in this sense be harmful.

Within the context of this model we consider a number of policy issues. For example, U.S. tax policy tends to subsidize debt financing. Under what conditions is this good or bad? In addition, monetary policy, which affects future inflation, influences the value of a firm's fixed debt obligations. Hence, it is natural to ask how expectations about monetary policy influences the capital structure choices of firms, and through this channel, how monetary policy affects the economy.

We start with the simplest form of our model that includes a fixed number of existing firms and an unlimited number of ex-ante identical firms that can enter the market. In this setting there is no externality associated with debt financing, so the optimal subsidy or tax on debt is zero. However, even within the context of this simple model there can be an important role for policies that influence firms' liquidation choices. Specifically, by generating inflation, a loose monetary policy can reduce the real value of debt during economy wide downturns and improve expected values ex ante. Such a policy improves firm values by reducing bankruptcies in bad times, when liquidations tend to be costly. In addition, it leads to higher ex-ante debt ratios, and thus increases bankruptcies in good times, when liquidation is beneficial.

We next consider a setting where the number of potential entrants is fixed. When this is the case, there are externalities associated with firm liquidations as well as with their debt choices. There are negative externalities imposed on unemployed workers in the event of liquidation (i.e., liquidation causes the unemployed to have more workers to compete with for jobs), as well as positive externalities that benefit emerging new firms that need to hire labor. Depending on the magnitude of these two effects a social planner may want to use tax policy to tilt firms towards either more or less debt financing.[2]

In addition to Hart and Moore (1995) and Pissarides (2000), which provide the basis for our model, our analysis is related to a number of papers in the literature. In particular, most of the existing theories focus on potential negative spillovers created by debt financing. For instance, the fire-sale channel (Shleifer and Vishny 1992 and more recently Lorenzoni 2008), and effects related to collateral constraint (Kiyotaki and Moore 1997) suggest that firms' borrowing imposes negative externalities on other firms. Also, while the idea of positive externalities of liquidations is related to Schumpeter's (1939) ideas on creative destruction, and to more recent work by Kashyap et al.(2008), the role of debt in facilitating the creation and destruction of firms has not been considered. However, a contemporaneous paper by He and Matvos (2012) shows that debt can facilitate firm exit when companies compete for survival in a declining industry and finds that firms use less than the socially optimal amount of debt financing. To our knowledge, however, we are the first to consider a role for debt in an economy where externalities can be imposed on workers and well as emerging new firms and therefore the first to analyze potential effects on unemployment and the process of firm creation.

The rest of the paper is organized as follows. Section 2 presents the base model and

[2] The issue of the desirability of debt subsidies has been periodically raised. For instance during the Clinton and Bush administrations, the Congressional Budget Office (1997, 2005) considered proposals to eliminate the unequal treatment of debt and equity.

Section 3 analyzes it. Section 4 considers the policy implications that emanate from the base model and Section 5 presents a modification of the base model and revisits the policy implications. Section 6 discusses the main conclusions. Proofs and other technical derivations are related to the appendix.

2. The model

We consider a risk-neutral economy in which the discount rate is normalized to zero. The economy consists of two productive periods $t = 1, 2$ and an interim period in which existing firms can be liquidated and new firms can be created. Next, we describe the agents, technology, contracting environment and labor market.

2.1. Agents: workers and two generations of firms

Old generation firms

The economy starts in the first productive period, $t = 1$, with a continuum of size one of workers that are each employed by a firm i. Firms that initially employ workers are called *old generation firms*; these firms produce in period 1 and may retain their workers and produce in period 2. If the firm fails to retain its worker (i.e., is unable to pay the worker his outside option) the firm is liquidated and does not produce in period 2.

When active in period t, an old generation firm i produces a cash-flow of r_{it} that can be decomposed as follows:

$$r_{it} = s_t + \varepsilon_i. \tag{1}$$

The first component $s_t = \{s_1, s_2\}$ is an aggregate productivity shock that is common to all firms in the economy. Formally s_t is a sequence of two binomial variables positively correlated across time,

$$s_1 = s_2 = \begin{cases} s_h & \text{with prob. } p \\ s_l & \text{with prob. } 1-p \end{cases} \tag{2}$$

where

$$\Pr(s_2 = s_h | s_1 = s_h) = \Pr(s_2 = s_l | s_1 = s_l) = \rho \geq p. \tag{3}$$

The second cash-flow component ε_i is firm-specific, independent across firms, constant over time and drawn from a uniform distribution:

$$\varepsilon_i \sim U[-\overline{\varepsilon}, +\overline{\varepsilon}]. \tag{4}$$

New generation firms

New generation firms can enter the economy in the interim period between production periods 1 and 2. We assume a fixed entry cost $k > 0$ and free-entry, i.e., there is an unlimited number of potential entrants that are ex-ante identical.[3] This assumption implies that firms of the new generation firms enter the economy until their expected profits are zero.

After entering the economy, a new generation firm j needs to hire a worker to be productive. However, as discussed below, there are search frictions that may prevent these firms from finding a suitable worker. If a firm j succeeds in hiring a worker, it generates a cash-flow r_{j2} at the end of period 2, however, if it fails to hire a worker, firm j loses its investment k and liquidates. Similar to old generation firms, the cash flow of an active new generation firm j in period 2 is

$$r_{j2} = s_2 + \varepsilon_j$$

[3] In Section 5 we consider the alternative case when there is a limited number of ex-ante identical potential entrants.

where s_2 is the aggregated productivity shock in period 2 and ε_j is also a uniformly distributed firm-specific component independent across firms, i.e., $\varepsilon_j \sim U[-\overline{\varepsilon}, +\overline{\varepsilon}]$.

2.2. Contracting environment

Firms are initially controlled by *investors* who subsequently transfer control to *managers*. While this transfer of control is innocuous for firms of the new generation, as we show, it has important consequences for old generation firms. Specifically, we assume that managers of old generation firms enjoy private benefits of control and, following Hart and Moore (1995), we assume that because of the private benefits, an old generation firm continues to operate in period 2 *as long as the manager has access to the necessary funds to retain the firm's worker.* For simplicity, we assume that any funds available beyond those needed to retain the worker are paid out to the investors.

At the beginning of period 1, investors, before transferring control to the firm's manager, set their firm's capital structure. In particular, we assume that old generation firm i issues short-term debt with a face value $d_i \geq 0$ that matures at the end of period 1, just after the cash-flow r_{i1} is realized.[4] We assume that short-term debt is a "hard claim" which cannot be renegotiated with creditors and, therefore, the firm is forced to liquidate if it fails to meet this payment d_i. The firm can repay its short-term debt either from the period 1cash-flow, r_{i1}, or by borrowing funds against the period 2 cash-flow, r_{i2}. We exclude any other financial contract and in particular, we assume that debt cannot be made contingent on specific cash-flow components $\{\varepsilon_i, s_t\}$, which we assume are not verifiable.

[4] In Section 4, we discuss the possibility that the firm issues other securities such as senior long-term debt.

2.3. Labor market and workers' retention costs

To finish the description of the economy, we need to describe the labor market that allocates workers to firms during the interim period.

2.3.1. Labor market and search costs

We consider a labor market with search frictions, a framework that captures the fact that it is costly for firms and workers to find a suitable match. In particular, we consider a standard labor market, e.g., Pissarides (2000) that includes two main ingredients: (i) a matching technology, which describes the likelihood of a suitable match and (ii) a sharing rule, which indicates how the matched parties share the surplus created by the newly formed relationship. In particular, we characterize the labor market with the following constant returns-to-scale Cobb-Douglas matching function:[5]

$$m(a, v) = \lambda a^{\alpha} v^{(1-\alpha)} \tag{5}$$

where m, the number of matches, is determined by a, the number of workers looking for jobs, and v, the number of firms searching for workers. In this function, $0 < \alpha < 1$ is the elasticity of matches to workers seeking jobs and $\lambda > 0$ measures the efficiency of the matching technology.

Given this matching technology, if the ratio of firms to workers is $\theta \equiv \frac{v}{a}$, then each worker is hired with probability $q(\theta) \equiv m(1, \theta)$, and each firm hires a worker with probability $\frac{q(\theta)}{\theta}$.[6] For future reference, we follow the literature and refer to θ as the

[5] Due to its success in empirical studies, Cobb-Douglas with constant returns to scale is the most popular functional form for the matching function. (See Petrongolo and Pissarides, 2001.)

[6] Since $m(a, v)$ features constant returns-to-scale, it follows that $q(\theta) \equiv \frac{m(a,v)}{a} = m(1, \theta)$ and $\frac{q(\theta)}{\theta} \equiv \frac{m(a,v)}{v} = m\left(\frac{1}{\theta}, 1\right)$. Also, we assume an interior solution which require us to impose parametric

"labor market tightness". Notice that, in this setting, some firms and workers do not find a suitable match, that is, that some firms cannot hire while some workers remain unemployed. Thus, unemployment u consists of those unemployed workers that look for a job during the interim period but cannot find one, i.e., $u = a - m(a, v)$.

When there is a match between a firm and worker, the worker receives a wage

$$w_2 = \gamma + \beta E\left(r_2 | s_1\right) \tag{6}$$

where $\gamma \geq 0$ and $\beta \in [0, 1]$, and therefore, the firm keeps in expectation:

$$\pi_2 = (1 - \beta)E\left(r_2 | s_1\right) - \gamma. \tag{7}$$

Notice that this specification encompasses the case in which $\beta = 0$, where workers receive a constant wage, as well as the case in which $\gamma = 0$, that is where workers and firms bargain over the surplus generated by their relation with β being the workers' bargaining power.

To simplify the analysis we assume that

$$(1 - \beta)(s_l - \bar{\varepsilon}) \geqslant \gamma \tag{8}$$

which ensures that $w_2 \geq \gamma$ and $\pi_2 \geq 0$.

2.3.2. Workers' retention costs

We assume that old generation firms have already hired their workers and paid for their period 1 services prior to period 1.[7] However, to produce in period 2, an

constraints such that, in equilibrium, probabilities are well defined, *i.e.*, $m(a, v) < \min\{a, v\}$.

[7] This is done for simplicity. In a previous version of the model, we studied the case in which old generation firms also face a labor market with search costs.

old generation firm must pay the worker's outside option, U, namely the expected compensation if the worker quits his job and searches for an alternative job during the interim period,

$$U = q(\theta)w_2. \tag{9}$$

This assumption is without loss of generality and, as discussed below, simplifies the investor's design of the optimal capital structure of the old generation firm.[8]

2.4. Timing of events

There are two production periods and an interim period with the following relevant events in each.

Period 1 ($t = 1$): A measure of size 1 of old generation firms employ one worker each. At the beginning of the period, each firm i issues short-term debt d_i, and then transfers the control of its operations to a manager. At the end of the period, firm i produces a cash-flow r_{i1}, and its short-term debt d_i matures.

Interim period: Managers of old firms make their liquidation decisions and new generation firms invest k to enter the market.

Period 2 ($t = 2$): Each newly created firm j attempts to hire an unemployed worker. If a firm and a worker match, the firm becomes active. Old firms that are not liquidated and newly created active firms generate cash-flows $\{r_{i2}\}$ and $\{r_{j2}\}$ respectively.

The following time-line summarizes the relevant events:

[8] In particular this implies that the debt choices in period 1 cannot be used to extract rents from the workers in period 2.

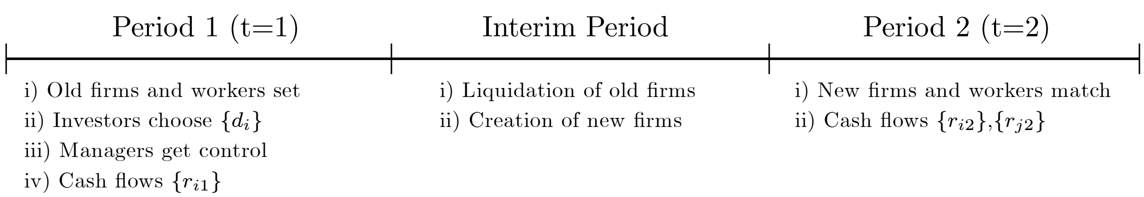

Figure 1: Timing of Events

3. Analysis of the model

The analysis of the model proceeds by backward induction. We start in Section 3.1 by characterizing the labor market during the interum period assuming an exogenous number of (unemployed) workers looking for jobs –labor supply– and endogenize the labor demand, i.e., the number of firms created. In Section 3.2, we consider the firms' liquidation decisions in the interim period as a function of the debt choices in $t = 1$ and the number of new firms created. Finally, in Section 3.3, we study the choice of debt by old generation firms in $t = 1$.

3.1. Labor market and the creation of new generation firms

The creation of new generation firms during the interim period determines the demand for labor. Specifically, for a given labor supply $a\left(s_1\right)$, the number of firms $v\left(s_1\right)$ created will be such that, due to the free-enty condition, the expected profit $V\left(s_1\right)$ of a firm created is zero. Expressed in terms of market tightness i.e., $\theta_1 \equiv \theta(s_1) = \frac{v(s_1)}{a(s_1)}$ the expected profit of firms is given by:

$$V\left(s_1\right) = -k + \frac{q(\theta_1)}{\theta_1}\left[(1-\beta)E(r_2|s_1) - \gamma\right], \tag{10}$$

where the probability of finding a worker is given by $\frac{q(\theta_1)}{\theta_1}$ and the expected cash-flow (i.e., profit) retained by the firm is given by the term in brackets, i.e., $(1-\beta)E(s_2|s_1)-\gamma$. Equalizing (10) to zero and under the specific Cobb-Douglas matching function (5) that we are considering, the following proposition:

Lemma 1 *Workers face a labor market tightness* $\theta_1^* = \left(\frac{\lambda[(1-\beta)E(s_2|s_1)-\gamma]}{k}\right)^{1/\alpha}$ *and have a reservation utility*

$$U^*(s_1) = \lambda\theta_1^{*(1-\alpha)}(\gamma + \beta E(s_2|s_1)). \tag{11}$$

From the previous lemma we can make a number of observations. First, the market tightness θ_1^* (and consequently $v(s_1)$, the number of firms created) increases with the efficiency of the matching technology (i.e., λ) and the expected surplus generated in the match (i.e., $E(s_2|s_1)$) and decreases with the worker's compensation (both on β and γ), the fixed cost of creating the firm (i.e., k) and the elasticity of the matching technology to the labor supply (i.e., α). Second, workers' utility increases with matching efficiency i.e., λ, and the expected surplus $E(s_2|s_1)$ and decreases with the cost of entry k and the match elasticity to labor supply α. The effect of the worker compensation, however, is ambiguous and will play an important role when we consider the policy implications below.

Third, it is worth noting that since there is positive serial correlation among aggregate shocks e.g., $E(r_2|s_h) \geq E(r_2|s_l)$ there are intertemporal effects on w_2^*, $U^*(s_1)$, and θ_1^*. In particular, a positive aggregate shock in the first period (i.e., $s_1 = s_h$) is followed by an increase in wages, w_2^*, and workers' reservation utility, $U^*(s_h)$, and by an increase in market tightness, θ_h^*.

Finally it is worth noting that Lemma 1 also establishes that θ_1^* and U_1^* are independent of the labor supply $a^*(s_1)$. Specifically, due to the adjustments in the number of firms that enter the market, i.e., $v^*(s_1)$, any labor supply effects $a(s_1)$ are offset in equilibrium by adjustments in firm entry $v(s_1)$ until the market tightness θ_1^* reaches its equilibrium level. This effect simplifies the analysis by isolating any supply of labor effects on the labor market conditions. In other words, since all newly created firms are ex-ante identical, shocks to the labor supply are fully accommodated by a perfectly elastic labor demand. This, in turn, implies that there is no effect in market tightness or workers' compensation.

3.2. Liquidation decisions of old generation firms

While we have characterized the labor market conditions in terms of θ_1^* and U_1^*, to fully characterize the labor market equilibrium we need to solve for the labor supply, $a^*(s_1)$. In this setting, the labor supply corresponds to the workers employed in $t = 1$ by those firms that are liquidated, $l^*(s_1)$:

$$a^*(s_1) = l^*(s_1). \tag{12}$$

Hence determining $a^*(s_1)$, requires us to characterize the liquidation decision of the old generation firms. In our setting, managers of the old generation firms enjoy private benefits of control and choose to liquidate their firms only when they are unable to retain their workers. Such worker retention requires firms to pay the workers' outside option, U_1^* with either internally generated or borrowed funds. Formally, since an old generation firm i generates a period 1 cash-flow $r_{1i} = \varepsilon_i + s_1$ and expects to generate a period 2 cash-flow $E(r_{2i}|r_{1i}) = \varepsilon_i + E(s_2|s_1)$, the manager liquidates the firm's operations when:[9]

$$G(\varepsilon_i, s_1, d_i) \equiv 2\varepsilon_i + s_1 + E(s_2|s_1) - d_i - U^*(s_1) < 0. \tag{13}$$

[9]The firm pays the initial wage to the worker at the begining of period 1.

Hence, for a certain amount of debt, d_i, and an aggregate shock, s_1, the firm is liquidated when its idiosyncratic shock is smaller than $\varepsilon_{d_i}^{*1}$, where $\varepsilon_{d_i}^{*1}$ is such that $G(\varepsilon_{d_i}^{*1}, s_1, d_i) = 0$, and can be rewritten as

$$\varepsilon_{d_i}^{*1} = \frac{1}{2}[d_i - s_1 - E(s_2|s_1) + U^*(s_1)]. \tag{14}$$

Given $\varepsilon_{d_i}^{*1}$ and the distributional assumptions on ε_i, the probability that a firm i with debt d_i is liquidated after period 1 is

$$\Pr(\varepsilon_i < \varepsilon_{d_i}^{*1}) = \frac{d_i - s_1 - E(s_2|s_1) + U^*(s_1) + 2\bar{\varepsilon}}{4\bar{\varepsilon}}, \tag{15}$$

and has the following properties:

Proposition 1 *The probability of liquidation increases (i) if the firm employs more debt, d_i, in its capital structure, (ii) if there is a low realization of the aggregate shock $s_1 = s_l$ and (iii) with increases in the workers' reservation utility in period 2, $U^*(s_1)$.*

Intuitively, managers liquidate their firms when they cannot raise the necessary funds to retain their workers, which is more likely to occur when firms have more debt, when they suffer a negative productivity shock, and when the workers' reservation utility is high.[10]

3.3. Debt choice of old generation firms

To close the model we need to characterize the optimal capital structure, *i.e.*, the choice of debt, d_i, made by investors to maximize firm value. To determine the

[10] Notice that while a negative productivity shock, $s_1 = s_l$, decreases the workers outside option, that is, $U_2^*(s_l) \leq U_2^*(s_h)$, it also decreases the amount of funds that a firm can raise against its future cash-flow, r_{i2}.

optimal amount of debt, firm i's investors solve the following problem:

$$\max_{d_i} \; p \int_{\varepsilon^{*h}_{d_i}}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_h) - U^*(s_h)}{2\overline{\varepsilon}} d\varepsilon_i + (1-p) \int_{\varepsilon^{*l}_{d_i}}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_l) - U^*(s_l)}{2\overline{\varepsilon}} d\varepsilon_i \qquad (16)$$

where $\varepsilon^{*h}_{d_i}$ and $\varepsilon^{*l}_{d_i}$ correspond to $\varepsilon^{*1}_{d_i}$ when $s_1 = s_h$ and $s_1 = s_l$, respectively.
In the objective function (16), the first and second terms are the expected profits in period 2 when $s_1 = s_h$ and $s_1 = s_l$, respectively. These profits are affected by the debt choice because such a choice determines when the firm is liquidated, i.e., it changes the liquidation cut-offs $\varepsilon^{*h}_{d_i}$ and $\varepsilon^{*l}_{d_i}$.

Problem (16) yields the following f.o.c.,

$$\frac{1}{4\overline{\varepsilon}}[p(\varepsilon^{*h}_{d_i} + E(s_2|s_h) - U^*(s_h)) + (1-p)(\varepsilon^{*l}_{d_i} + E(s_2|s_l) - U^*(s_l))] = 0,$$

which, using the definition of $\varepsilon^{*1}_{d_i}$ in (14) and the fact since old generation firms (which are ex-ante identical) choose the same amount of debt (i.e., $d^* \equiv d^*_i$ for all i), can be rewritten as[11]

$$d^* = E(s_1) - E\left[E(s_2|s_1) - U^*(s_1)\right]. \qquad (17)$$

As the above equation illustrates, the optimal debt level d^* increases with the expected cash-flow in period 1, (a higher expected cash-flow increases the severity of the managerial free cash-flow problem), and decreases with the expected value of the firm in period 2, (which is related to the severity of the debt overhang problem created by the issuance of debt).[12]

[11] *To guarantee $d > 0$ we need to impose parametric constraints on the distribution of shocks.*

[12] Since, for any given amount of debt d_i and shock s_1, an increase in $E(s_2|s_1) - U^*_2(s_1)$ decreases $\varepsilon^{*1}_{d_i}$ by only $\frac{1}{2}\left[E(s_2|s_1) - U^*_2(s_1)\right]$, the value of the marginal firm liquidated (*i.e.*, $\varepsilon^{*1}_{d_i} + E(s_2|s_1) - U^*_2(s_1)$) increases by $\frac{1}{2}[E(s_2|s_1) - U^*_2(s_1)]$.

Intuitively, the optimal amount of debt d^* is chosen so that the marginal firm liquidated has an expected value of zero, that is, d^* solves

$$E[\varepsilon_d^{*1} + E(s_2|s_1) - U^*(s_1)] = 0. \tag{18}$$

In other words, since s_l and s_h are equally likely, the optimal debt choice d^* equates the value lost from liquidating the (profitable) marginal firm when $s_1 = s_h$ to the value saved from liquidating the (unprofitable) marginal firm when $s_1 = s_h$.

Finally, notice that d^* determines ε_d^{*1}, which, in turn, determines the quantity of workers whose employers are liquidated at the end of period:

$$l^*(s_1) = \Pr(\varepsilon_i < \varepsilon_d^{*1}) \tag{19}$$

From Proposition 1, it follows that the number of firms liquidated after period 1 increases when firms employ more debt and in downturns, *i.e.*, when $s_1 = s_l$.

4. Policy implications

Our previous analysis shows a number of connections between firm financial choices and the creation of firms in the economy. Next, we examine whether the use of debt creates inefficiencies in the allocation of resources that can be alleviated by some public intervention. More specifically, we consider two sets of questions. First, we examine the role of public policy to address situations of excessive or insufficient liquidation of firms once aggregate shocks are realized. Second, we examine whether public policy should foster the use of debt by firms. In our analysis, the main difference between these policy interventions has to do with the timing of its potential applicability. We refer to ex-post public interventions as those that take action after the aggregate shock s_1 is realized, and ex-ante public interventions, as taking action before s_1 is realized.

In what follows, we identify social welfare as the sum of firm value added, that is, the sum of firm profits and wages, and exclude the private benefits of managerial control from the social welfare function.[13] To the extent that managerial control benefits are of a smaller order of magnitude than the value of the firm, ignoring them simplifies the analysis without affecting results in a substantial manner.[14]

It is worth noting that our social welfare function does not imply that the optimal policy maximizes employment. Since what matters is the sum of firm profits and wages, from the social point of view it may be more desirable a situation in which fewer but more efficient firms employ fewer workers that another situation in which more but less efficient firms employ more workers.

4.1. Ex-post interventions

We start by considering the role of public policy after the aggregate shock s_1 is realized. As a benchmark, Proposition 2 below describes the social inefficiencies of the market equilibrium after s_1 is realized assuming that no public intervention takes place.

[13] Ignoring managerial private benefits would be consistent with a political system in which managers have a negligible influence in the outcome of political elections. Alternatively, it would also be consistent with a situation in which the marginal benefit of the last dollar of managerial compensation are negligible in comparison with the benefits of the marginal dollar of worker and investor compensation.

[14] See Hart (1995) p. 126-130 for a more through discussion of the conditions in which ignoring managerial private benefits for the analysis of capital structure can be justified.

Proposition 2 *In equilibrium, from the social point of view, there is excessive liquidation in recessions (when $s_1 = s_l$) and too little liquidation during booms (when $s_1 = s_h$).*

Intuitively, when the economy suffers a negative productivity shock, the amount of debt chosen by firms is ex-post too high, which leads to a *debt overhang* problem in the economy. Alternatively, when the economy suffers a positive productivity shock, too few firms are liquidated. Notice that this result relies on the assumption that the overall state of the economy is not verifiable, so that debt cannot be made contingent on specific cash-flow components $\{\varepsilon_i, s_t\}$.

We now consider the possibility of public intervention. In particular, we will consider "monetary" interventions that alter the real value of debt obligations. More specifically, we abstract from institutional details of implementation and consider government interventions that change the face value of the firms' debt after s_1 is realized but before the liquidation decisions are made. We examine two alternative cases, one in which monetary interventions are unanticipated by firms and another in which monetary interventions are fully anticipated.

We model monetary policy as a technology in the hands of the government such that, by incurring a social cost, the government can change the general price level of the economy. In our framework, this implies that at a social cost $c(\tau)$ the face value of any debt contract due at $t = 1$ can be modified from d to $d - \tau$ in real terms, where $c(0) = c'(0) = 0$ and $c'' > 0$. We refer to an expansionary or inflationary monetary policy when $\tau > 0$ and to restrictive or deflationary policy when $\tau < 0$.

4.1.1. Unantipated monetary policy

Consider first the case of unanticipated monetary policy. Specifically, we examine the price distortion that is chosen to affect the process of firm creation and destruction, which is determined as a function of the realization of the aggregate shock s_1, *i.e.*, $\tau(s_1) \equiv \tau_1$. Since we are considering the case of unanticipated monetary policy, we can solve this problem by simply taking as given the equilibrium choices of job creation $m(1, v^*(s_1))$, posted wages $w^*(s_1)$ and debt choices d^* (which are oblivious to monetary interventions) and consider the ex-post price distortion that would be performed by the government. Formally the government solves the following problem at the end of period 1:

$$\max_{\tau_1} \int_{\varepsilon(\tau_1)}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_1) - U^*(s_1)}{2\overline{\varepsilon}} d\varepsilon_i - c(\tau_1) \tag{20}$$

$$\text{s.t.} \quad \varepsilon(\tau_1) = \frac{1}{2}[d^* - \tau_1 - s_1 - E(s_2|s_1) + U^*(s_1)]. \tag{21}$$

The following proposition characterizes the optimal unanticipated monetary policy.

Proposition 3 *It is optimal to follow an inflationary monetary policy during recessions,* $\tau^*(s_l) > 0$, *and a restrictive one during booms,* $\tau^*(s_h) < 0$.

Intuitively, since the ex-ante socially optimal amount of debt leads to excessive liquidation in recessions and to too little liquidation during booms, it is optimal to increase the real value of debt during booms and to decrease it during recessions. The implicit assumption behind this result is that debt contracts are set in nominal terms, and hence, through inflation or deflation the government can change the real value of the liabilities that firms face. Notice that setting debt contracts in nominal terms is indeed optimal for firms as it leads to better liquidation decisions during the interim period.

4.1.2. Anticipated monetary policy

Consider now the case of anticipated monetary policy. In this case, firms foresee that if the economy suffers from debt-overhang (*free cash-flow*), the government will follow an inflationary (*restrictive*) monetary policy and adjust their debt choice accordingly. Whether firms react by increasing or decreasing their debt obligations depends on the relative cost of inflation and deflation. For instance, all else equal, if inflation is less costly than deflation, e.g., $c'(-\tau) > c'(\tau)$ then firms have incentive to increase their face value of debt, d, ex-ante anticipating an inflationary policy ex-post by the government. The following proposition summarizes this discussion.

Proposition 4 *When the monetary policy is anticipated, firms increase their choice of debt beyond d^* if and only if firms expect, on average, an inflationary policy, that is, if $E(\tau_1) > 0$. This choice of debt is socially optimal.*

Note that the monetary policy $\{\tau_l,\tau_h\}$ is determined by the overhang or free-cash flow problem faced by the whole economy rather than by the problem faced by any one firm. As a consequence, firms have incentives to coordinate their capital structures. For instance, if many firms use debt and hence an active inflationary policy is expected, then firms have more incentives to use debt. Alternatively, if few firms use debt, there is less incentives for other firms to use it. While each firm takes the monetary policy as given when choosing its debt, and the amount of debt is a given when monetary authority chooses its policy $\{\tau_l,\tau_h\}$, the firms' choice of debt is socially optimal. Intuitively, while firms do not internalize the cost of monetary policy, since firms are infinitesimally small, they also do not have incentives to change its debt in order to influence the monetary policy, which results in debt being socially optimal.

4.1.3. Other issues

So far we have assumed that firms issue short-term debt to control the managers' reluctance to liquidate. However, long-term debt, since it limits the ability to borrow against future cash flows, can also be used as an instrument to control the managerial unwillingness to liquidate. In the case of monetary policy, the distinction between long-term and short-term debt can be of particular importance since long-term debt is likely to be more sensitive than short-term debt to inflation.

The previous observation suggests some implications on optimal debt maturity for firms in different industries. For instance, industries with different sensitivity to the aggregate shock require different active monetary policies. Since monetary policy is not tailored to one specific industry but to the whole economy, firms in industries with higher sensitivity will have an incentive to use debt structures that have longer maturity, that is, a higher sensitivity to monetary policy.[15]

4.2. Ex-ante policy interventions: Corporate tax policy

We now analyze ex-ante policy interventions, i.e., those interventions that may take place before the aggregate shock s_1 is realized. In particular, we consider whether the government should provide incentives for the use of debt, for instance, through tax

[15] Notice that according to the same logic, firms in industries with a higher sensitivity to the aggregate shock will have an incentive to use less inflation-protected bonds.

policy. We start by stating the following result.

Proposition 5 *Without public intervention, investors choose the socially optimal amount of debt to fund their firms at* $t = 1$*.*

The previous proposition indicates that there is no need for public intervention ex-ante. Hence, tax incentives that distort the use of debt financing by firms reduce social welfare. This result is somewhat surprising since, as it is well-known in the search literature, in general the entry and liquidation decisions by firms need not be socially optimal. Intuitively, a firm's entry into (*exit from*) the market creates a positive (*negative*) externality for unemployed workers and a negative (*positive*) externality for firms with vacancies. Stated differently, the labor market tightness θ_1 can be too high or too low from a social point of view.

In our setting, however, the assumption that there is an unlimited number of ex-ante identical potential entrants implies that the market tightness is independent of how many firms are liquidated, which in turn, implies that market tightness is also independent of the leverage choices made by firms at $t = 1$. That is, θ_1 does not depend on $l(s_1)$ or d.[16] Thus since the choice of debt at $t = 1$ does not create any externalities in the labor market, the (ex-ante) social and private choices of leverage coincide.

While the the assumption of unlimited entry is analytically convenient (and may be not unreasonable in the long-run) it also represents a polar case of perfect flexibility in entry. This case, however, obscures the effects that arise when firm entry reacts

[16] To illustrate, an exogenous increase in d would force firms of the old generation to liquidate more often but would also encourage more firms of the new generation to enter until the initial market tightness, $\theta(s_1)$, would be restored.

imperfectly (or with some delay) to firms' liquidation decisions. In the next section, we relax this assumption and consider the alternative polar case in which there is a limited number of firms of the new generation that will enter in period 2. Within this setting of limited rather than free entry, we will reexamine the effects of debt tax policy on welfare and discuss the role of government intervention.

5. Limited firm entry

5.1. Debt choices under limited firm entry

In this section, we consider a modified setting with the same features and timing as before except that now there is a limited number of potential entrants $v(s_1)$ among firms of the new generation. This specification allows for the number of entrants to depend on the aggregate shock s_1. In particular, we assume that there can be more entry in good than in bad economic times, *i.e.*, $v(s_h) \geq v(s_l)$. For simplicity, we also assume that the entry cost is nil, *i.e.*, $k = 0$, so that indeed all the potential entrants $v(s_1)$ enter the market during the interim period.[17]

To solve the model we proceed as follows:[18]

1. For each realization of the aggregate shock s_1 (and given the amount of limited entry $v(s_1)$) we derive the workers' outside option at $t = 2$ as a function of the

[17] While we are taking the polar case of a fixed number of entrants, qualitatively similar results can be derived with free-entry as long as entrants are not all identical (*i.e.*, when potential entrants have different entry costs). In our polar case, one can think of a number of firms $v(s_1)$ that can enter without cost ($k = 0$) and then other potential entrants with a high enough cost of entry.

[18] Please refer to the Appendix for details.

amount of firms liquidated after period 1, i.e., $l(s_1)$:

$$U(s_1) = \lambda\theta_1^{(1-\alpha)}\left(\gamma + \beta E\left(s_2|s_1\right)\right) \tag{22}$$

where

$$\theta_1 = \frac{v(s_1)}{l(s_1)}. \tag{23}$$

2. We derive the number of firms liquidated $l(s_1)$ as a function of the workers' reservation utility $U(s_1)$ and the firms' debt choice d at $t = 1$:

$$l(s_1) = \frac{d + \bar{\varepsilon} - s_1 - E(s_2|s_1) + U(s_1)}{4\bar{\varepsilon}} \tag{24}$$

3. We solved for the optimal amount of debt taking as given the workers' reservation utility:

$$d = E(s_1) - E\left[E(s_2|s_1) - U(s_1)\right] \tag{25}$$

Notice that while the expressions for $U(s_1)$, $l(s_1)$ and d show a high resemblance to the corresponding expressions (11), (19) and (17) obtained for the free-entry case, there are also important differences. First, technically, the model cannot be solved sequentially since now $U(s_1)$, $l(s_1)$, and d depend on each other which requires to solve the three equations, (22), (24) and (25), simultaneously. Second, from an economic point of view, these expressions show that now the labor market tightness depends on the liquidation decision and hence on the choice of debt.

5.2. Corporate tax policy under limited firm entry

We are now ready to examine the optimal corporate tax policy. We consider two cases, one in which the economy has no aggregate uncertainty in period $t = 1$ and then the more general case in which there is aggregate uncertainty in period $t = 1$.[19] Let, $\tilde{\beta}(s_1) \equiv \frac{w_2(s_1)}{E(s_2|s_1)} = \frac{\gamma}{E(s_2|s_1)} + \beta$, which corresponds to the worker share of the surplus, then the following proposition considers the case of no aggregate uncertainty:

Proposition 6 *Assume that there is no aggregate uncertainty at $t = 1$, (i.e., s_1 equals s_l or s_h with probability 1) then if $\tilde{\beta}(s_1) < \alpha$ firms have less debt than is socially optimal. Alternatively, if $\tilde{\beta}(s_1) > \alpha$, firms have more debt than is socially optimal.*

A firm's liquidation decision imposes a negative externality on unemployed workers and a positive one on firms with vacancies. These two externalities exactly offset each other only when $\tilde{\beta}(s_1) = \alpha$, that is, when the worker's share of surplus $\tilde{\beta}(s_1)$ is equal to the elasticity of the matching function with respect to unemployment α. In such case, the equilibrium and socially optimal market tightness coincide.[20] However, if the worker's share of surplus is lower (*higher*) than the elasticity of the matching function with respect to unemployment, the equilibrium market tightness is higher (*lower*) than the optimal social tightness. Intuitively, a social planner would like to increase the number of workers looking for jobs –the number of liquidations– to the point where the marginal benefit in terms of additional matches with new firms is equal to the cost. Since old generation firms do not internalize the benefit that a new match has on new generation firms, (i.e., $(1-\beta)E(s_2|s_1) - \gamma$) if this benefit is too large, *i.e.*, if $\tilde{\beta}(s_1)$ is small enough, there is not enough liquidation and the labor market becomes too tight, that is, there are too few unemployed workers in

[19] As it will be clear below the case of no aggregate uncertainty in period $t = 1$ (when either $p = 1$ or $p = 0$) allows firms to perfectly solve their managerial agency problem, that is, at the end of period 1 there will be no privately inefficient liquidation or continuation.

[20] See Hosios (1990) for a detailed analysis of externalities in search models.

relation to the number of vacancies. Hence if $\tilde{\beta}(s_1) < \alpha$, an increase in debt, which in turn, increases liquidation, has a positive net externality as it loosens a labor market that it is too tight from the social point of view. It is also worth noticing that that the relation between $\tilde{\beta}(s_1)$ and α does not depend on the number of entrants, $v(s_1)$. That is $v(s_1)$ affects both the market tightness in equilibrium and the socially optimal market tightness but not whether one is smaller or larger than the other.

Notice that an increase in expected $E(s_2|s_1)$ decreases $\tilde{\beta}(s_1)$, and makes it more likely that the labor market is too tight from the social point of view. Intuitively, this occurs because an increase in $E(s_2|s_1)$ does not translate into a proportional increase in the wage $w_2(s_1)$, *i.e.*, there are real wage rigidities. The main implication that arises from this observation is that the labor market tends to be too tight during economic booms (that is, too many firms looking for workers relative to the number of unemployed workers) and too loose during recessions (that is, too many unemployed workers looking for jobs relative to the number of job vacancies). Hence, the previous proposition highlights a reason to promote debt at $t = 1$ when good economic times are expected at $t = 2$, (*i.e.*, when the expected productivity in the economy $E(s_2|s_1)$ is high). When this is the case, the positive externalities that liquidation creates on new firms looking for workers are greater than the negative externalities that liquidation has on other unemployed workers. When bad economic times are expected (i.e., when $E(s_2|s_1)$ is low), however, firms can easily find workers, and hence, additional liquidations do not help these firms much while it hurts the unemployed workers who already a small probability of finding a job.

These search externality effects are on top of free cash-flow and debt overhang problems that may arise when there is aggregate uncertainty. In fact, in the case absence of aggregate uncertainty in period 1, firms would be able to perfectly tune their debt choices to resolve the managerial agency problem, i.e., his reluctance to liquidate the firm. As the previous proposition show, even when this is the case, since when entry is not totally flexible, firm choices create externalities in the labor market, an tax policy as described above would increase social welfare.

Finally for the general case with aggregate uncertainty and if $\tilde{\beta}(s_h) < \alpha < \tilde{\beta}(s_l)$, the social optimum amount of debt will trade-off the possibility of ending up with too much debt in recessions against too little debt during booms. Notice that these two effects, unlike the free cash-flow and debt overhang problem, will not be internalized by firms, and hence, in general firms' debt choice will not be ex-ante socially optimal. In addition, when there is aggregate uncertainty there is also a general equilibrium feedback effect that firms do not internalize: firms' debt choices affect the workers' outside option in the interim period, U, which in turn, affects the firms' liquidation in period 1.[21] From an ex-post point of view, there are now two reasons to increase (*reduce*) the value of debt during good economic times, that is to resolve the free-cash low (*debt overhang*) problem and to increase (*decrease*) the tightness of the labor market.

5.3. Monetary policy with limited entry

Finally we consider the issue of monetary policy under limited firm entry. For brevity, we focus the discussion on two issues, namely the effect of monetary policy when there is no aggregate uncertainty in the economy and the complementarity between monetary and tax policy when there is aggregate uncertainty.

[21] We refer to the appendix for the general derivations in the case of aggregate uncertainty.

Regarding the first issue, in the absence of aggregate uncertainty, authorities face the typical problem of time inconsistency in ex-post monetary policy interventions (e.g., Kydland and Prescott, 1977). For instance, if firms anticipate that the government is going to inflate, firms will take on more debt ex-ante and the economy will trapped into a costly high inflation equilibrium. Hence, the monetary authority would like to commit not to have an active monetary policy.

Finally for the general case with aggregate uncertainty and if $\tilde{\beta}(s_h) < \alpha < \tilde{\beta}(s_l)$, the optimal combination of monetary and fiscal policy may need to do with their implementation costs. In general, the optimal policy combination will take into account the ex-post reasons to affect the value of debt during good and bad economic times. Specifically, the analysis suggests the following complementarity in the use of monetary and fiscal policy: the government may rely on incentives to affect debt choices at $t = 1$ and use monetary policy to accommodate the realization of the aggregate shock in the interim period.

6. Concluding remarks

Since the seminal work of Modigliani and Miller (1958), economists have examined the costs and benefits of financial leverage from the perspective of firms seeking financing. In this paper, we extend this analysis and examine how corporate financing choices influence the aggregate economy. In particular, we consider a setting where financial leverage can increase the probability of a firm liquidating following economic shocks, and within this setting we consider potential externalities. For example, corporate liquidations can have negative externalities during economic recessions, if they contribute to excess slack in the labor markets. In contrast, liquidations may have positive externalities during economic booms, if they facilitate the emergence of more productive startup companies.

The framework we develop provides intuition about economic effects of policies that influence corporate capital structures. In particular, we consider inflation policy, which affects the real value of debt obligations, and show that in some situations an active policy that decreases debt obligations during economy-wide downturns can improve ex ante firm values. In addition, we identify conditions under which welfare can be improved with subsidies or taxes that alter the firms' use of debt financing.

While we do not explore this in our paper, there are a number of other policy choices that may be evaluated within the framework of our model. For example, the U.S. government provides subsidized debt for emerging industries that might otherwise fail and are likely to create positive externalities, like renewable energy, as well as for failing industries, like automobiles, that might otherwise create negative spillovers. Since a primary motivation for these initiatives is to create and save jobs, a model, such as ours, that explicitly considers the effect of financing on the labor market might be relevant. In addition to calculating the relevant spillovers, an evaluation of these policies should also consider the alternative of subsidized equity and the effect of that choice on future decisions by the firm that may also influence the job market and the creation of new firms.

There are a number of important aspects of our analysis that merit further attention. In addition to considering the study of a richer set of policy tools, it would be interesting to calibrate the magnitude of debt induced effects in the labor market tightness. Furthermore, future research should also extend the scope of the model. For instance, we consider very limited dynamics (firms interact in a single period) and therefore ignore how these policy choices influence business cycles and the growth rate of the economy. An analysis of the optimal debt policy and public interventions in a more dynamic (and more complicated) setting is a challenge that is left to future work.

References

[1] Acemoglu, D. (2001). Good jobs versus bad jobs. *Journal of Labor Economics*, 19(1): 1-21.

[2] Acemoglu, D. and R. Shimer (1999a). Holdups and efficiency with search frictions. *International Economic Review*, 40: 827-49.

[3] Acemoglu, D. and R. Shimer (1999b). Efficient unemployment insurance. *Journal of Political Economy*, 107(5): 893-928.

[4] Brunnermeier, M. K. (2009). Deciphering the Liquidity and Credit

[5] Crunch 2007–2008. *Journal of Economic Perspectives*, 23(1): 77-100.

[6] Congressional Budget Office, (1997). "The economic effects of comprehensive tax reform."

[7] Congressional Budget Office, (2005). "Corporate income tax rates: International comparisons."

[8] Hart, O. (1995). Firms, Contracts, and Financial Structure, Clarendon Lectures in Economics, Oxford University Press.

[9] Hart, O. and J. More (1995). Debt and seniority: An analysis of the role of hard claims in constraining management. *American Economic Review*, 85: 567-85.

[10] He, Z. and G, Matvos (2012). Debt and creative destruction: Why is subsidizing corporate debt optimal? Working paper University of Chicago.

[11] Caballero, R., T. Hoshi and A. Kashyap (2008) "Zombie Lending and Depressed Restructuring in Japan." *American Economic Review*, 98(5): 1943-77.

[12] Kiyotaki N. and Moore J., (1997). "Credit cycles," *Journal of Political Economy*, 105 (2): 211-248.

[13] Kydland, F. E.; Prescott, E. C. (1977). "Rules rather than discretion: The inconsistency of optimal plans". *Journal of Political Economy,* 85 (3): 473–492.

[14] Lorenzoni G., (2008), "Efficient credit booms," *Review of Economics Studies*, 75, 809–833.

[15] Moen, E. R. (1997). Competitive search equilibrium. *Journal of Political Economy* 105: 385-411.

[16] Montgomery, J. (1991). Equilibrium wage dispersion and the interindustry wage differentials. *Quarterly Journal of Economics* 106: 163-79.

[17] Peters, M. (1991). Ex ante price offers in matching games non-steady states. *Econometrica* 59: 1425-54.

[18] Pissarides, C.A. (2000). Equilibrium Unemployment Theory, MIT press

[19] Schumpeter, J. (1939). "Business Cycles: A Theoretical, Historical and Statistical Analysis of the Capitalist Process", McGraw Hill.

[20] Shleifer, A. and R. Vishny (1992) Liquidation values and debt capacity: A market equilibrium approach. *Journal of Finance*, 47: 1343-66.

APPENDIX

Proof of Lemma 1

Note that $E(r_2|s_1) = E(s_2|s_1)$ and hence, the free-entry $V = -k + \frac{q(\theta_1)}{\theta_1}[(1-\beta)E(r_2|s_1) - \gamma] = 0$ implies: $\theta_1^* = (\frac{\lambda[(1-\beta)E(s_2|s_1)-\gamma]}{k})^{1/\alpha}$ and

$$U^* = q(\theta_1^*)(\gamma + \beta E(s_2|s_1)) = \lambda^{1/\alpha}(\frac{(1-\beta)E(s_2|s_1) - \gamma}{k})^{\frac{1-\alpha}{\alpha}}(\gamma + \beta E(s_2|s_1))\blacksquare \quad \text{(A.1)}$$

Proof of Proposition 2

From the social point of view firm i should be liquidated when its expected cash-flow in period 2, $E(r_{i2}|r_{i1})$, is lower than the workers' outside option, $U^*(s_1)$. (Note that $U^*(s_1)$ is pinned down by the free-entry condition and hence it is not affected by the liquidation decision or the amount of debt.)

$$H(\varepsilon_i, s_1) \equiv E(r_{i2}|r_{i1}) - U^*(s_1) = \varepsilon_i + E(s_2|s_1) - U^*(s_1) < 0. \quad \text{(A.2)}$$

In equilibrium the marginal firm liquidated is (see equation (14)):

$$\varepsilon_{d_i}^{*\,1} = \frac{1}{2}[d^* - s_1 - E(s_2|s_1) + U^*(s_1)] \quad \text{(A.3)}$$

which implies that:

$$H(\varepsilon_{d_i}^{*1}, s_1) = \frac{1}{2}[d^* - s_1 + E(s_2|s_1) - U^*(s_1)]. \quad \text{(A.4)}$$

There is too much liquidation in recessions (a debt overhang) if the marginal firm liquidated in recession has a positive social value (that is, if $H(\varepsilon_d^{*l}, s_1) > 0$). Symmetrically, there is too little liquidation in recessions (a free cash-flow problem) if the marginal firm liquidated during booms has a negative social value (that is, $H(\varepsilon_d^{*h}, s_1) < 0$). Since

$$d^* = E(s_1) - E\left[E(s_2|s_1) - U^*(s_1)\right], \quad \text{(A.5)}$$

then $H(\varepsilon_d^{*l}, s_1) > 0$ and $H(\varepsilon_d^{*h}, s_1) < 0$ if and only if:

$$s_h - E(s_2|s_h) + U^*(s_h) > s_l - E(s_2|s_l) + U^*(s_l). \quad \text{(A.6)}$$

The above expression can be rewritten as: $2(s_h - s_l)(1-\rho) + U^*(s_h) > U^*(s_l)$. Since $U^*(s_h) > U^*(s_l)$ it follows that $H(\varepsilon_d^{*l}, s_1) > 0$ and $H(\varepsilon_d^{*h}, s_1) < 0\blacksquare$

Proof of Proposition 3. The government solves:

$$\max_{\tau_1} \int_{\varepsilon(\tau_1, s_1)}^{\overline{\varepsilon}} \frac{H(\varepsilon_i, s_1)}{2\overline{\varepsilon}} d\varepsilon_i - c(\tau_1) \quad \text{(A.7)}$$

$$\text{s.t.:} \quad \varepsilon(\tau_1, s_1) = \frac{1}{2}[d^* - \tau_1 - s_1 - E(s_2|s_1) + U^*(s_1)] \quad \text{(A.8)}$$

where

$$H(\varepsilon_i, s_1) \equiv E(r_{i2}|r_{i1}) - U^*(s_1) = \varepsilon_i + E(s_2|s_1) - U^*(s_1). \tag{A.9}$$

The problem yields the following f.o.c.:

$$-\frac{d^*}{4\overline{\varepsilon}} H(\varepsilon(\tau_1), s_1) - c'(\tau_1) = 0. \tag{A.10}$$

Consider first the case in which $s_1 = s_h$. If $\tau = 1$, then $\varepsilon(1, s_h) = \varepsilon_d^{*h}$, and from the proof of Proposition 2 above we know that $H(\varepsilon_d^{*h}, s_1) < 0$. In that case, the f.o.c. evaluated when $\tau_1 = 1$ has a positive sign and hence there is incentives to increase τ_1 above 1. (Note that $H(\varepsilon(\tau_1), s_1)$ is linear on τ_1 and hence the problem is well defined.)

Consider now the case in which $s_1 = s_l$. If $\tau_1 = 1$, then $\varepsilon(1, s_l) = \varepsilon_d^{*l}$, and from the proof of Proposition 2 we know that $H(\varepsilon_d^{*l}, s_1) > 0$. In that case, the f.o.c. evaluated when $\tau_1 = 1$ has a negative sign and hence there is incentives to decrease τ_1 below 1.■

Proof of Proposition 4

Firms solve the following optimization problem taken as given $\{\tau_l, \tau_h\}$:

$$\max_d E[\int_{\varepsilon(\tau_1, s_1)}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_1) - U(s_1)}{2\overline{\varepsilon}} d\varepsilon_i] \tag{A.11}$$

$$\text{s.t.} \quad : \quad \varepsilon(\tau_1, s_1) = \frac{1}{2}[d - \tau_1 - s_1 - E(s_2|s_1) + U(s_1)]. \tag{A.12}$$

which yields the following f.o.c.:

$$d = E[\tau_1 + s_1 - E(s_2|s_1) + U(s_1)] \tag{A.13}$$

Since

$$d = E[s_1 - E(s_2|s_1) + U(s_1)] \tag{A.14}$$

then

$$d > d^* \Leftrightarrow E(\tau_1) > 0. \tag{A.15}$$

Given d the monetary authority solves the following problem given s_1:

$$\max_{\tau_1} \int_{\varepsilon(\tau_1, s_1)}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_1) - U^*(s_1)}{2\overline{\varepsilon}} d\varepsilon_i - c(\tau_1) \tag{A.16}$$

$$\text{s.t.:} \quad \varepsilon(\tau_1, s_1) = \frac{1}{2}[d - \tau_1 - s_1 - E(s_2|s_1) + U(s_1)] \tag{A.17}$$

which yields

$$\frac{d - \tau_1 - s_1 + E(s_2|s_1) - U(s_1)}{4\overline{\varepsilon}} - c'(\tau_1) = 0. \tag{A.18}$$

Next we prove that the equilibrium choices are socially optimal:

$$\max_{d, \tau_h, \tau_l} E(\int_{\varepsilon(\tau_1, s_1)}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_1) - U^*(s_1)}{2\overline{\varepsilon}} d\varepsilon_i - c(\tau_1)) \tag{A.19}$$

$$\text{s.t.:} \quad \varepsilon(\tau_1, s_1) = \frac{1}{2}[d - \tau_1 - s_1 - E(s_2|s_1) + U(s_1)]. \tag{A.20}$$

which yields the following three order conditions:

$$d = E\left[\tau_1 + s_1 - E(s_2|s_1) + U(s_1)\right] \tag{A.21}$$

$$\frac{d - \tau_h - s_h + E(s_2|s_h) - U(s_h)}{4\overline{\varepsilon}} - c'(\tau_h) = 0 \tag{A.22}$$

$$\frac{d - \tau_l - s_l + E(s_2|s_l) - U(s_l)}{4\overline{\varepsilon}} - c'(\tau_l) = 0. \tag{A.23}$$

Notice that the three first order conditions that solve the social optimum are identical to the ones that solve the private optimum■

Proof of Proposition 5

The social planner solves the following optimization problem:

$$\max_d E\left[\int_{\varepsilon_{d_i}^{*1}}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_1) - U(s_1)}{2\overline{\varepsilon}} d\varepsilon_i\right] + E\left[U(s_1)\right] \tag{A.24}$$

s.t.

$$\varepsilon_{d_i}^{*1} = \frac{1}{2}[d_i - s_1 - E(s_2|s_1) + U^*(s_1)]. \tag{A.25}$$

Since the reservation utility $U^* = q_2(\theta_2^*)\left(\gamma + \beta E\left(s_2|s_1\right)\right)$ and $\theta^* = (\frac{\lambda[(1-\beta)E(s_2|s_1)-\gamma]}{k})^{1/\alpha}$, then $E(U(s_1))$ does not depend on d and hence the solution the above problem coincides with the private optimum, that is: $d^* = E[s\ _1 - E(s_2|s_1) + U(s_1)]$■

Proof of Proposition 6

Assume that there is no aggregate uncertainty at $t = 1$ that is s_1 equals either s_h or s_l with probability one. (Note that still $\Pr(s_2 = s_1|s_1) = \rho$.) Then we have the following equilibrium:

$$d^* = s_1 - E(s_2|s_1) + U^*(s_1) = s_1 - E(s_2|s_1) + \lambda(\frac{v(s_1)}{l^*(s_1)})^{1-\alpha}\left[\gamma + \beta E(s_2|s_1)\right] \tag{A.26}$$

The social planner solves the following problem

$$\max_d \int_{\varepsilon_d}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_1) - U(s_1)}{2\overline{\varepsilon}} d\varepsilon_i + U(s_1) + v(s_1)\frac{q(\theta_1)}{\theta_1}\left[(1-\beta)E(s_2|s_1) - \gamma\right] \tag{A.27}$$

where

$$U(s_1) = \lambda(\theta_1)^{1-\alpha}\left[\gamma + \beta E(s_2|s_1)\right] \;\; ; \;\; \varepsilon_d = \frac{1}{2}[d - s_1 - E(s_2|s_1) + U^*(s_1)] \tag{A.28}$$

$$\frac{q(\theta_1)}{\theta_1} = \lambda(\theta_1)^{-\alpha} \;\; ; \;\; \theta_1 = \frac{v(s_1)}{l(s_1)} \tag{A.29}$$

$$l(s_1) = \Pr(\varepsilon_i < \varepsilon_d^1) = \frac{d + 2\bar{\varepsilon} - s_1 - E(s_2|s_1) + U^*(s_1)}{4\bar{\varepsilon}} \tag{A.30}$$

The derivative of the social planner's objective function (SPOF) w.r.t. debt is:

$$\begin{aligned}
\frac{\partial \text{SPOF}}{\partial d} &= \frac{-1}{4\bar{\varepsilon}}(\varepsilon_d^1 + E(s_2|s_1) - U(s_1)) - \frac{1}{4\bar{\varepsilon}}\frac{\partial U(s_1)}{\partial d}(\varepsilon_d^1 + E(s_2|s_1) - U(s_1)) \\
&\quad + \underbrace{\left(1 - \Pr\left(\varepsilon_i \geq \varepsilon_d^1\right)\right)}_{l(s_{11})}\frac{\partial U(s_1)}{\partial d} \\
&\quad + \lambda \frac{\partial (\theta_1)^{-\alpha}}{\partial d} v(s_1)\left[(1-\beta)E(s_2|s_1) - \gamma\right]
\end{aligned} \tag{A.31}$$

Since

$$\frac{\partial U(s_1)}{\partial d} = (1-\alpha)\lambda\,(\theta_1)^{-\alpha}\,\frac{\partial \theta_1}{\partial d}\left[\gamma + \beta E\left(s_2|s_1\right)\right] \tag{A.32}$$

then we can rewrite $\frac{\partial \text{SPOF}}{\partial d}$ as:

$$\begin{aligned}
\frac{\partial \text{SPOF}}{\partial d} &= \frac{-1}{4\bar{\varepsilon}}\left[1 + \frac{\partial U(s_1)}{\partial d}\right](\varepsilon_d^1 + E\left(s_2|s_1\right) - U(s_1)) \\
&\quad + l(s_1)\lambda\,(\theta_1)^{-\alpha}\,\frac{\partial \theta_1}{\partial d}\left[(1-\alpha)\left[\gamma + \beta E\left(s_2|s_1\right)\right] - \alpha\left[(1-\beta)E\left(s_2|s_1\right) - \gamma\right]\right]
\end{aligned} \tag{A.33}$$

At the private optimum

$$d^* = U^*(s_1) = \lambda\left(\frac{v(s_1)}{l^*(s_1)}\right)^{1-\alpha}\left[\gamma + \beta E\left(s_2|s_1\right)\right] \tag{A.34}$$

and

$$\varepsilon_d^{1*} = \frac{1}{2}[d^* - s_1 - E\left(s_2|s_1\right) + U^*(s_1)] = U^*(s_1) - s_1. \tag{A.35}$$

Hence evaluating $\frac{\partial \text{SPOF}}{\partial d}$ at the private optimum (PO):

$$\left|\frac{\partial \text{SPOF}}{\partial d}\right|_{\text{PO}} = l^*(s_1)\lambda\,(\theta^*)^{-\alpha}\left|\frac{\partial \theta}{\partial d}\right|_{\text{PO}}\left[(1-\alpha)\left[\gamma + \beta E\left(s_2|s_1\right)\right] - \alpha\left[(1-\beta)E\left(s_2|s_1\right) - \gamma\right]\right] \tag{A.36}$$

Next we show that $\frac{\partial \theta}{\partial d} < 0$:

$$\begin{aligned}
\frac{\partial \theta}{\partial d} &= -\frac{v(s_1)}{(l(s_1))^2}\frac{\partial l(s_1)}{\partial d} = \frac{-1}{2\bar{\varepsilon}v(s_1)}\left(1 + \frac{\partial U(s_1)}{\partial d}\right) = \\
&= \frac{-1}{2\bar{\varepsilon}v(s_1)}\left[1 + (1-\alpha)\lambda\theta^{-\alpha}\frac{\partial \theta}{\partial d}\left[\gamma + \beta E\left(s_2|s_1\right)\right]\right]
\end{aligned} \tag{A.37}$$

and solving for $\frac{\partial \theta}{\partial d}$:

$$\frac{\partial \theta}{\partial d} = \frac{-1}{2\bar{\varepsilon}v(s_1) + (1-\alpha)\lambda\theta^{-\alpha}\left[\gamma + \beta E\left(s_2|s_1\right)\right]} < 0 \tag{A.38}$$

Hence

$$\left|\frac{\partial \text{SPOF}}{\partial d}\right|_{\text{PO}} > 0 \Leftrightarrow (1-\alpha)\left[\gamma+\beta E\left(s_2|s_1\right)\right]-\alpha\left[(1-\beta) E\left(s_2|s_1\right)-\gamma\right]<0 \quad (A.39)$$

$$\Leftrightarrow \quad (1-\alpha)\widetilde{\beta} E\left(s_2|s_1\right)-\alpha(1-\widetilde{\beta}) E\left(s_2|s_1\right)<0 \Leftrightarrow \widetilde{\beta}<\alpha \blacksquare$$

AGGREGATE UNCERTAINTY & LIMITED ENTRY

Equilibrium Debt

A worker's outside option at $t=2$ if he quits and looks for a job at $t=2$ is:

$$U\left(s_1\right)=\lambda(\theta_1)^{1-\alpha}\left[\gamma+\beta E\left(s_2|s_1\right)\right] \lambda(\frac{v_1}{l_1})^{1-\alpha}\left[\gamma+\beta E\left(s_2|s_1\right)\right] \quad (A.40)$$

where

$$l_1 \equiv l(s_1) \quad ; v_1 \equiv v(s_1) \quad (A.41)$$

The firm liquidates if the manager cannot retain the worker, that is, if

$$G(\varepsilon_i, s_1, d_i) \equiv 2\varepsilon_i + s_1 + E(s_2|s_1) - d_i - U(s_1) < 0 \quad (A.42)$$

which implies that the marginal firms liquidated, $\varepsilon_{d_i}^{*1}$, is determined by the following equation,

$$G(\varepsilon_{d_i}^{*1}, s_1, d_i) \equiv 0 \quad (A.43)$$

which boils down to

$$\varepsilon_{d_i}^{*1} = \frac{1}{2}[d_i - s_1 - E(s_2|s_1) + U(s_1)]. \quad (A.44)$$

and since there is a continuum [0,1] of firms at $t=1$, then:

$$l_1 = \Pr(\varepsilon_i < \varepsilon_{d_i}^{*1}) = \frac{d_i + 2\bar{\varepsilon} - s_1 - E(s_2|s_1) + U(s_1)}{4\bar{\varepsilon}} \quad (A.45)$$

The debt choice at $t=0$ solves:

$$\max_{d_i} E \int_{\varepsilon_{d_i}^{*1}}^{\bar{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_1) - U(s_1)}{2\bar{\varepsilon}} d\varepsilon_i \quad (A.46)$$

which yields the following f.o.c.:

$$\frac{-1}{4\bar{\varepsilon}} E[\varepsilon_{d_i}^{*1} + E(s_2|s_1) - U^*(s_1)] = 0, \quad (A.47)$$

which yields the following amount of debt

$$d^* = E\left[(s_1) - E(s_2|s_1) + E\left(U^*(s_1)\right)\right]. \quad (A.48)$$

Social Optimum Debt

The social planner solves the following problem:

$$\max_d E[\int_{\varepsilon_{d_i}^{*1}}^{\overline{\varepsilon}} \frac{\varepsilon_i + E(s_2|s_1) - U(s_1)}{2\overline{\varepsilon}} d\varepsilon_i] + E(U(s_1)) + \tag{A.49}$$

$$+ E\left[v_1 \frac{q(\theta_1)}{\theta_1}\left[(1-\beta)E(s_2|s_1) - \gamma\right]\right]$$

s.t.

$$\varepsilon_d^{*1} = \frac{1}{2}[d - s_1 - E(s_2|s_1) + U_2^*(s_1)]. \tag{A.50}$$

$$U(s_1) = \lambda(\theta_1)^{1-\alpha}\left[\gamma + \beta E\left(s_2|s_1\right)\right] \tag{A.51}$$

$$\frac{q(\theta_1)}{\theta_1} = \lambda\theta_1^{-\alpha} \quad ; \quad \theta_1 = \frac{v_1}{l_1} \tag{A.52}$$

$$l_1 = \Pr(\varepsilon_i < \varepsilon_d^{*1}) \tag{A.53}$$

Note that unlike individual firms the social planner internalizes the effect that the choice of debt has on θ_1, and hence on ε_d^{*1} and $U(s_1)$. Deriving the social planner's objective function (SPOF) w.r.t. debt:

$$\frac{\partial \text{SPOF}}{\partial d} = -E[[1 + \frac{\partial U(s_1)}{\partial d}]\frac{\varepsilon_d^{*1} + E(s_2|s_1) - U^*(s_1)}{4\overline{\varepsilon}}] + E[\frac{\partial U(s_1)}{\partial d} l_1] \tag{A.54}$$

$$+E[\frac{\partial(\frac{q(\theta_1)}{\theta_1})}{\partial d} v_1((1-\beta)E(s_2|s_1) - \gamma)]$$

Notice that

$$\frac{\partial U(s_1)}{\partial d} = -(1-\alpha)\left[\gamma + \beta E(s_2|s_1)\right]\lambda(\theta_1)^{-\alpha}\frac{\partial \theta_1}{\partial d} \tag{A.55}$$

and

$$\frac{\partial\left(\frac{q(\theta_1)}{\theta_1}\right)}{\partial d} = -\alpha\lambda\theta_1^{-\alpha-1}\frac{\partial\theta_1}{\partial d} \tag{A.56}$$

so we can rewrite $\frac{\partial \text{SPOF}}{\partial d}$ as:

$$\frac{\partial \text{SPOF}}{\partial d} = -E\left[\left[1 + \frac{\partial U(s_1)}{\partial d}\right]\frac{\varepsilon_d^{*1} + E(s_2|s_1) - U^*(s_1)}{4\overline{\varepsilon}}\right] + \tag{A.57}$$

$$+E\left[\left[l_1\lambda(\theta_1)^{-\alpha}E(s_2|s_1)\frac{\partial\theta_1}{\partial d}\right]\left[(1-\alpha)\widetilde{\beta}(s_1) - \alpha(1-\widetilde{\beta}(s_1))\right]\right]$$

which evaluated at the private optimal (PO), that is,

$$\frac{-1}{4\overline{\varepsilon}}E[\varepsilon_{d_i}^{*1} + E(s_2|s_1) - U^*(s_1)] = 0, \tag{A.58}$$

gives:

$$\left|\frac{\partial \text{SPOF}}{\partial d}\right|_{\text{PO}} = -E\left[\left|\frac{\partial \text{SPOF}}{\partial d}\right|_{\text{PO}}\left(\frac{\varepsilon_d^{*1}+E(s_2|s_1)-U^*(s_1)}{4\overline{\varepsilon}}\right)\right]+ \tag{A.59}$$
$$+E\left[\left[l_1^*\lambda(\theta_1^*)^{-\alpha}E(s_2|s_1)\left|\frac{\partial\theta_1}{\partial d}\right|_{\text{PO}}\right]\left[(1-\alpha)\widetilde{\beta}(s_1)-\alpha(1-\widetilde{\beta}(s_1))\right]\right]$$

Hence there are two effects determining if at the equilibrium the social planner has incentives to increase or decrease debt, that is, whether $\left|\frac{\partial \text{SPOF}}{\partial d}\right|_{\text{PO}}$: the "feedback effect" (in the first line) and the "search externality effect" (in the second line).

(1) The "*feedback effect*":

$$-E\left[\left|\frac{\partial \text{SPOF}}{\partial d}\right|_{\text{PO}}\left(\frac{\varepsilon_d^{*1}+E(s_2|s_1)-U^*(s_1)}{4\overline{\varepsilon}}\right)\right] \tag{A.60}$$

When firms choose the amount of debt, d^*, they do not take into account that d^* affects $U^*(s_1)$ and hence ε_d^{*1}. The marginal firm destroyed in good times has a value:

$$\varepsilon_d^{*h}+E(s_2|s_h)-U_2^*(s_h) \tag{A.61}$$

and the marginal firm destroyed in bad times has a value:

$$\varepsilon_d^{*l}+E(s_2|s_l)-U_2^*(s_l) \tag{A.62}$$

Since d^* affects $U^*(s_1)$ and hence ε_d^{*1} the net effect depends on whether d moves the outside option more in good or bad times multiplied by the value of the marginal firm destroyed, $\varepsilon_d^{*1}+E(s_2|s_1)-U_2^*(s_1)$, which is positive in bad times and negative in good times..

(2) The "*search externality effect*":

$$E\left[\left[l_1^*\lambda(\theta_1^*)^{-\alpha}E(s_2|s_1)\left|\frac{\partial\theta_1}{\partial d}\right|_{\text{PO}}\right]\left[(1-\alpha)\widetilde{\beta}(s_1)-\alpha(1-\widetilde{\beta}(s_1))\right]\right] \tag{A.63}$$

First notice that $\frac{\partial\theta_1}{\partial d}<0$:

$$\frac{\partial\theta_1}{\partial d} = -\frac{v_1}{(l_1)^2}\frac{\partial l_1}{\partial d}=\frac{-1}{2\overline{\varepsilon}v_1}\left(1+\frac{\partial U(s_1)}{\partial d}\right)= \tag{A.64}$$
$$= \frac{-1}{2\overline{\varepsilon}v_1}\left[1+(1-\alpha)\lambda\theta_1^{-\alpha}\left[\gamma+\beta E(s_2|s_1)\right]\frac{\partial\theta_1}{\partial d}\right]$$

and solving for $\frac{\partial\theta_1}{\partial d}$,

$$\frac{\partial\theta_1}{\partial d}=\frac{-1}{2\overline{\varepsilon}v_1+(1-\alpha)\lambda\theta^{-\alpha}\left[\gamma+\beta E(s_2|s_1)\right]}<0. \tag{A.65}$$

Hence the sign depends on $\left[(1-\alpha)\widetilde{\beta}(s_1)-\alpha(1-\widetilde{\beta}(s_1))\right]$. For instance consider the case in which $\widetilde{\beta}(s_h)<\widetilde{\beta}(s_l)<\alpha$. In that case the search externality would tend push things towards increasing debt since the labor market would tend to be too tight. Alternatively, if $\alpha<\widetilde{\beta}(s_h)<\widetilde{\beta}(s_l)$, the search externality would tend push things towards decreasing debt. Finally, if $\widetilde{\beta}(s_h)<\alpha<\widetilde{\beta}(s_l)$ the search externality induces increasing debt in good time and decreasing debt in bad time, hence , ex-ante, whether search externality induces an increase or decrease in debt depends on which one of the two effects dominates.