Costs and Benefits of Dynamic Trading in a Lemons Market
VERY PRELIMINARY

William Fuchs* Andrzej Skrzypacz

March 15, 2012

Abstract

We study a dynamic market with asymmetric information that induces the lemons problem. We compare efficiency of the market under different assumption about the timing of trade. We show that there generally exist conditions under which efficiency can be improved by temporarily closing the market as compared to continuous trading opportunities.

1 Introduction

Consider an owner of an asset who is facing liquidity needs and would like to sell the asset in a market where buyers compete. The seller is privately informed about the value of the asset. Although there is common knowledge of gains from trade, the buyers would not be willing to pay the average value if at that value the highest type would not be willing to trade. Hence, the competitive equilibrium price must be lower. As pointed out in the seminal paper by Akerlof (1970) this logic leads to an inefficiently low amount of trade.

The implicit assumption in Akerlof's model is that the seller has a unique opportunity to interact with the potential buyers. If the offered price is rejected there are no further opportunities to trade. It is natural to think that in many instances buyers will get additional

*Fuchs: Haas School of Business,University of California Berkeley (e-mail: wfuchs@haas.berkeley.edu). Skrzypacz: Graduate School of Business, Stanford University (e-mail: skrz@stanford.edu). We thank Ilan Kremer, Aniko Öry and Robert Wilson for comments and suggestions.

opportunities to trade with the seller. The possibility of being able to access the market again reduces the incentives of the seller to sell for two reasons: (1) by rejecting the first offer the buyers update upwards their posterior about the seller's type and are willing to make higher offers in the future (2) the fact that there is another opportunity to sell the good reduces the costs of not reaching an agreement. Both of these forces lead to less trade in the first period. Although this would decrease efficiency, allowing for an extra period might allow some higher types that would not have traded in the one shot model to trade in the second period. Depending on how many of these types are around and how the surplus from trade is distributed across types opening the market for a second round of trade might be either good or bad for efficiency.

In this paper we study the optimality of allowing for more opportunities to trade than the single initial offer. In addition, we also allow for the possibility that the adverse selection problem be only short lived. That is, there is some date T at which the seller's type is revealed and trade can take place efficiently. In the Akerlof model T=infinity but there might be situations were a finite T is a more natural assumption. <<In Section XXX we also allow for stochastic information revelation>>

We start our analysis with an example with linear valuations and uniform distribution of types and show that the market with restricted trading opportunities (allowing trades only at zero and at T) generates higher expected gains from trade than the continuous trading market. In general, there is a tradeoff: under dynamic (continuous) trading the lemons market problem gets worse and hence even fewer types trade early than in the classic static model. On the other hand, the buyers can use time to screen the seller types and eventually more types can trade in the continuous case. Indeed we show by construction that one can find combinations of distributions and valuations for which continuous trading dominates restricted trading if the informational asymmetry is long lived.

Intuitively, the restricted market design is useful because it gives a lot of incentives to trade at time zero (there is a big surplus loss if trade does not take place). On the other hand, because all the types that trade at zero must receive the same price this is a very blunt tool to separate the different types. With continuous trading we have the extreme opposite, there are little instantaneous incentives to trade but we can smoothly screen all types and hence eventually generate more trade. For continuous trading to dominate we need two things (1) that the equilibrium with restricted trading leave a lot of valuable trading opportunities unconsummated and (2) that frequent trading would lead to these trades taking place not too far away in the future.

If the private information is sufficiently short-lived then in general the market with continuous trading is strictly worse. Even the total amount of trade is larger when the market closes after the first offer than when it is continuously open. In the limit when T=0 both markets of course are equivalent with only the lowest type trading at exactly his value. As we start to increase T with continuous trading it is still only the lowest type that trades at zero with the same price and then over time we slowly screen to slightly higher types. With infrequent trading if the first price remained unchanged the buyers would be making profits since now higher seller types would be willing to trade at time zero. This leads to an even higher price which in turn generates even more trade. At the bottom of the distribution and for short T this feedback loop is twice as strong as the rate of increase we get from screening over time.

On the other extreme, if the private information is never revealed we use a mechanism design approach to provide a sufficient condition for the infrequent trading to dominate all other possible trading designs not just continuous trading. The condition coincides with the condition that is needed to guarantee that there exists a unique competitive equilibrium if trade is only allowed at zero and T.

Finally, we also show that short trading pauses are in general always useful either just after the initial period or just before the time the public information is released. The logic behind why having a pause at the beginning increases the surplus is very similar to the one when the private information is short lived. When there is a brake of trade at the end then although similar forces are at play the argument is not as straight forward. The mandated quiet period generates a mass of trade just before the trading ban takes place. Since buyers must break even, the price at which that mass of potential sellers trade has to be equal to the value of the expected type. Hence, it must be strictly higher than the value of the lowest type that trades in that last instant. Types below this type are being perfectly screened and hence trade at their own value. But then, there must be an additional endogenous period of no trade because otherwise some types would prefer not to trade when they are supposed to and wait for the jump in prices.

In practice there are several cases of restrictions to trading just before information is revealed. Our results show that short restrictions are welfare improving and also interestingly that there might be an additional period of low trading volume even before the restriction takes place.

1.1 Literature Review

To be included. Related papers include Akerlof (1970)

* Swinkels, “Education Signalling with Pre-emptive Offers,” 1999.

*Hörner and Vielle, “Public v. Private Offers in the Market for Lemons,” 2006

Skrzypacz and Kremer, “Dynamic Signaling and Market Breakdown,” 2007

* Daley and Green, “Waiting for News in the Market for Lemons,” 2011

Noldeke and van Damme, “Signalling in a Dynamic Labour Market,” 1990

Jansen and Roy, “Trading a Durable Good in a Walrasian Market with Asymmetric Information,” 1998

2 The Model

As in the classic market for lemons, a potential seller owns an asset. When the seller owns the asset it generates for him a revenue stream $c \in [0,1]$ that is a private information of the seller. c is drawn from a distribution $F(c)$, which is common knowledge, atomless and has a continuous, strictly positive density $f(c)$.

There is a competitive market of potential buyers. Each buyer values the asset at $v(c)$ which is strictly increasing, thrice differentiable, $v(c) > c$ for all $c < 1$ (i.e. common knowledge of gains from trade) and $v(1) = 1$ (i.e. no gap on the top). These assumptions imply that in the static Akerlof (1970) problem some types will trade, but that the lemons problem is present and not all the types trade in equilibrium.

Time is $t \in [0,T]$ and we consider different market designs in which the market is opened in different moments in that interval. We start the analysis with two extreme market designs: "infrequent trading" (or "restricted trading") in which the market is opened only twice at $t \in \{0,T\}$ and "continuous trading" in which the market is opened in all $t \in [0,T]$. Let $\Omega \subset [0,T]$ denote the set of times that the market is opened (we assume that at the very minimum $\{0,T\} \subset \Omega$).

Every time the market is opened buyers make public price offer to the seller and the seller either accepts one of them (which ends the game) or rejects and the game moves to the next time the market is opened. If no trade takes place by time T the type of the seller is revealed and the price in the market is $v(c)$, at which all seller types trade.

All players discount payoffs at a rate r and we let $\delta = e^{-rT}$. The values c and $v(c)$ are normalized to be in total discounted terms. Therefore, if trade happens at time t at a price p_t then the seller payoff is

$$\left(1 - e^{-rt}\right) c + e^{-rt} p_t$$

and the buyer payoff is

$$e^{-rt}\left(v\left(c\right)-p_t\right)$$

A competitive equilibrium of this market is a pair of functions $\{p_t, k_t\}$ for $t \in \Omega$ where p_t is the market price at time t and k_t is the highest type of the seller that trades at time t. These functions satisfy:

1) Zero profit condition: $p_t = E\left[v\left(c\right)|c \in [k_{t-}, k_t]\right]$ where k_{t-} is the cutoff type at the previous time the market is open before t (with $k_{t-} = 0$ for the first time the market is opened)[1]

2) Seller optimality: given the process of prices each seller type maximizes profits by trading according to the rule k_t.

3) No (Unrealized) Deals: in any period the market is open the price is at least $p_t \geq v\left(k_{t-}\right)$ since it is common knowledge that the value of the seller asset is at least that much (this condition removes some trivial multiplicity of equilibria, for example an equilibrium in which $p_t = k_t = 0$ for all periods).

These assumptions are analogous to the definitions in Daley and Green (2011) (see Definition 2.1 there).

We assume that the players publicly observe all the trades and hence after a buyer obtains the object, if he tries to put it back on the market the market can infer something about c based on the history. Since all buyers value the good at the same amount, there will not be any profitable trade between buyers after the first transaction with the seller and hence we ignore that possibility in our model. We abuse notation by specifying the prices and cutoffs for times in Ω other than T.

3 Motivating Examples

Before we present the general analysis of the problem, consider the following example. c is distributed uniformly over $[0, 1]$ and $v\left(c\right) = \frac{1+c}{2}$. Fix r and T and $\delta = e^{-rT}$.

We compare two possible market organizations. First, infrequent trading, in which case $\Omega = \{0, T\}$. Second, continuous trading, $\Omega = [0, T]$.

Infrequent Trading The "infrequent trading" market design corresponds to the classic market for lemons as in Akerlof (1970). The equilibrium in this case is described by a price

[1] In continuous time we use a convention $k_{t-} = \lim_{s \to t^-} k_s$, and $E\left[v\left(c\right)|c \in [k_{t-}, k_t]\right] = \lim_{s \to t^-} E\left[v\left(c\right)|c \in [k_s, k_t]\right]$ and $v\left(k_{t-}\right) = \lim_{s \to t^-} v\left(k_s\right)$.

p_0 and a cutoff k_0 that satisfy that the cutoff type is indifferent between trading at $t = 0$ and waiting till T:

$$p_0 = (1-\delta)\, k_0 + \delta \frac{1+k_0}{2}$$

and that the buyers break even in expectations:

$$p_0 = E\left[v\left(c\right)|c \leq k_0\right]$$

The solution is $k_0 = \frac{2-2\delta}{3-2\delta}$ and $p_0 = \frac{4-3\delta}{6-4\delta}$. The expected gains from trade are

$$S_0 = \int_0^{k_0} \left(v\left(c\right) - c\right) dc + \delta \int_{k_0}^{1} \left(v\left(c\right) - c\right) dc = \frac{4\delta^2 - 11\delta + 8}{4\left(2\delta - 3\right)^2}$$

Continuous Trading The above outcome cannot be sustained in equilibrium if there are multiple occasions to trade before T. If at $t = 0$ types below k_0 trade, the next time the market opens we require the price to be at least $v\left(k_0\right)$, but that means that types close to k_0 would be strictly better off delaying trade. As a result for any richer set Ω than in the infrequent case, there will be less trade in period 0.

If we look at the case of continuous trading, $\Omega = [0, T]$, then the equilibrium with continuous trade is a pair of two process $\{p_t, k_t\}$ that satisfy:

$$\begin{aligned} p_t &= v\left(k_t\right) \\ r\left(p_t - k_t\right) &= \dot{p}_t \end{aligned}$$

Since the process k_t is continuous, the zero profit condition is that the price is equal to the value of the current cutoff type. The second condition is the indifference of the current cutoff type between trading now and waiting for a dt and trading at a higher price. These conditions yield a differential equation for the cutoff type

$$r\left(v\left(k_t\right) - k_t\right) = v'\left(k_t\right)\dot{k}_t$$

with the boundary condition $k_0 = 0$. In our example this process has a simple solution:

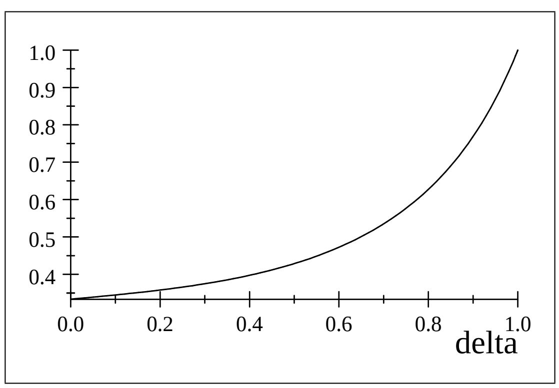
$$k_t = 1 - e^{-rt}.$$

The total surplus from continuous trading is

$$\begin{aligned} S_C &= \int_0^T e^{-rt}\left(v\left(k_t\right)-k_t\right)\dot{k}_t dt + e^{-rT}\int_{k_T}^1 \left(v\left(c\right)-c\right)dc \\ &= \int_0^T e^{-rt}\left(\frac{1}{2}e^{-rt}\right)\left(re^{-rt}\right)dt + e^{-rT}\int_{1-e^{-rT}}^1 \left(\frac{1-c}{2}\right)dc \\ &= \frac{1}{12}\left(2+\delta^3\right). \end{aligned}$$

Remark 1 *While we look at competitive equilibria, it is also possible to write a game-theoretic version of the model allowing two buyers to make public offers every time the market is opened. If we write the model having* $\Omega = \{0, \Delta, 2\Delta, ..., T\}$ *then we can show that there is a unique Perfect Bayesian Equilibrium in our example for every* T *and* $\Delta > 0$*. When* $\Delta = T$ *then the equilibrium coincides witht the equilibrium in the infrequent trading market we identified above. As we take the sequence of equilibria as* $\Delta \to 0$*, the equilibrium path converges to the competitive equilibrium we have identified for out "continunous trading" design. In other words, the equilibria we describe have a solid game-theoretic foundation.*

Comparing Infrequent and Continuous trading How do gains from trade compare in these two cases? The following graph shows the ratio $\frac{S_{FB}-S_0}{S_{FB}-S_C}$ where S_{FB} is the trading surplus if trade was efficient, so that the ratio represents the relative efficiency loss from adverse selection in the two markets:



When $\delta \to 0$ (i.e. when $T \to \infty$ which means the private information is long-lived) we get $\frac{S_{FB}-S_0}{S_{FB}-S_C} \to \frac{1}{3}$ so the efficiency loss of the continuous time trading is three times higher than infrequent trading! When $\delta \to 1$, which means that $T \to 0$, the private information is very short-lived and the organization of the market does not matter since even by waiting till T players can achieve close to full efficiency in either case.

Committing to only one opportunity to trade generates a big loss of surplus if players do not reach an agreement in the current period. This inefficiency upon disagreement is what helps overcome the adverse selection problem and increases the amount of trade in the current period. The problem is that closing the market for ever is a very coarse tool and does not allow for more than a simple partition of the seller types into two categories. Continuous trading on the other hand does not provide many incentives to trade in the current period since there a negligible loss of surplus from waiting an extra instant to trade. Leading to smooth trade. Although this leads to a slow screening of types and delay of trade the advantage is that eventually (in particular for large T) higher types will receive attractive offers (since they are no longer pooled with lower types) and there will be more trade. The tradeoff then that determines which trading environment is better has to weight the cost of delaying trade with low types that would trade immediately in the batch case and with delay in the continuous case with the advantage of eventually realizing additional surplus from trading with types that would not trade if there is only one opportunity to trade.

Since in our example types are uniformly distributed and there are higher gains from trade with the low types the advantage of getting more low types to trade without delay overcomes the benefit of getting to trade with higher types. In the next Sections we will formalize these ideas.

3.1 Can Continuous Trading be better?

Our example above demonstrates a case of $v(c)$ and $F(c)$ such that for every T the infrequent trading market is more efficient than the continuous trading market. Furthermore, the greater T, the greater the efficiency gains from using infrequent trading. Is it a general phenomenon? The answe is no:

Proposition 1 *There exist $v(c)$ and $F(c)$ such that for T large enough the continuous trading market generates more gains from trade than the infrequent trading market*

Proof. Consider a distribution that approximates the following: with probability ε c is drawn uniformly on $[0,1]$; with probability $\alpha(1-\varepsilon)$ it is uniform on $[0,\varepsilon]$; and with probability $(1-\alpha)(1-\varepsilon)$ it is uniform on $[c_1, c_1+\varepsilon]$ for some $c_1 > v(0)$. In other words, the mass is concentrated around 0 and c_1. Let $v(c) = \frac{1+c}{2}$ as in our example.

For small ε there exists $\alpha < 1$ such that

$$E[v(c)|c \leq c_1+\varepsilon] < c_1$$

so that in the infrequent trading market trade will happen only with the low types. In particular, if α is such that

$$\alpha v(0) + (1-\alpha) v(c_1) < c_1$$

then as $\varepsilon \to 0$ the infrequent trading equilibrium price converges to $v(0)$ and the surplus converges to

$$\lim_{\varepsilon \to 0} E[U_s(c)] = \alpha v(0) + (1-\alpha) c_1$$

The equilibrium path for the continuous trading market is independent of the distribution and hence

$$\begin{aligned} \lim_{\varepsilon \to 0} E[U_s(c)] &= \alpha v(0) + (1-\alpha) \left[e^{-r\tau(c)} v(c_1) + \left(1 - e^{-r\tau(c)}\right) c_1 \right] \\ &= \lim_{\varepsilon \to 0} E[U_s(c)] + (1-\alpha) \left(e^{-r\tau(c_1)} \left(v(c_1) - c_1 \right) \right) \end{aligned}$$

where $\tau(k)$ is the inverse of the function k_t. The last term is strictly positive for any $c_1 < v(c_1)$ .In particular, with $v(c) = \frac{1+c}{2}$, $e^{-r\tau(c)} = (1-c)$ and $v(c_1) - c_1 = \frac{1}{2}(1-c_1)$ so

$$\lim_{\varepsilon \to 0} E[U_s(c)] = \lim_{\varepsilon \to 0} E[U_s(c)] + \frac{1}{2}(1-\alpha)(1-c_1)^2$$

■

The example used in this proof illustrates well what is needed for the continuous trading market to dominate the infrequent one: we need a large mass in the bottom of the distribution, so that the infrequent trading market gets "stuck" with these types and the continuous market would reach them quickly too. But also we need some mass on higher types that would be reached in the continuous market after some time generating additional surplus (but that mass cannot be too large since then the static market would trade with those types too).

In the rest of the paper we offer general results that allow us to compare the continuous trading market design to several other designs, including the infrequent trading one.

4 Costs and Benefits of Continuous Trading

We now return to the general model. We first describe the equilibrium in the continuous time trading

Proposition 2 (Continuous trading) *For $\Omega = [0, T]$ the competitive equilibrium (unique up to measure zero of times) is the unique solution to:*

$$\begin{aligned} p_t &= v(k_t) \\ k_0 &= 0 \\ r(v(k_t) - k_t) &= v'(k_t)\dot{k}_t \end{aligned}$$

Proof. First note that our requirement $p_t \geq v(k_{t-})$ implies that there cannot be any atoms of trade, i.e. that k_t has to be continuous. Suppose not, that at time s types $[k_{s-}, k_s]$ trade with $k_{s-} < k_s$. Then at time $s + \varepsilon$ then price would be at least $v(k_s)$ while at s the price would be strictly smaller to satisfy the zero-profit condition. But then for small ε types close to k_s would be better off not trading at s, a contradiction. Therefore we are left with processes such that k_t is continuous and $p_t = v(k_t)$. For k_t to be strictly increasing over time we need that $r(p_t - k_t) = \dot{p}_t$ for almost all t: if price was rising faster, current cutoffs would like to wait, a contradiction. If prices were raising slower over any time interval starting at s, there would be an atom of types trading at s, another contradiction. So the only remaining possibility is that $\{p_t, k_t\}$ are constant over some interval $[s_1, s_2]$. Since the price at s_1 is $v\left(k_{s_{1-}}\right)$ and the price at s_2 is $v(k_{s_2})$, we would obtain a contradiction that there is no atom of trade in equilibrium. In particular, if $p_{s_1} = p_{s_2}$ (which holds if and only if $k_{s_{1-}} = k_{s_1} = k_{s_2}$) then there exist types $k > k_{s_1}$ such that

$$v(k_{s_1}) > \left(1 - e^{r(s_2 - s_1)}\right) k + e^{r(s_2 - s_1)} v(k_{s_1})$$

amd these types would strictly prefer to trade at $t = s_1$ than to wait till s_2, a contradiction again. ■

On the other extreme, with infrequent trading defined as $\Omega = \{0, T\}$ the equilibrium is:[2]

Proposition 3 (Infrequent/Restricted Trading) *For $\Omega = \{0, T\}$ there exists a competivie equilibrium $\{p_0, k_0\}$. Equilibria are a solution to:*

$$p_0 = E[v(c) | c \in [0, k_0]] \tag{1}$$

$$p_0 = \left(1 - e^{-rT}\right) k_0 + e^{-rT} v(k_0) \tag{2}$$

[2] The infrequent trading model is the same as the model in Akerlof (1970) if $T = \Delta = \infty$. Even with $T < \infty$ the proof of existence and inefficiency of the equilibrium is standard. The somewhat novel part of the proof is the sufficient condition for uniqueness in our environment.

If $\frac{f(c)}{F(c)}(v(c)-c)$ *is decreasing and* $v''(c) \geq 0$ *(or* $T=\infty$*) then the equilibrium is unique.*

Proof. 1) **Existence**. The equilibrium conditions follow from the definition of equilibrium. To see that there exists at least one solution to (1) and (2) note that if we write the condition for the cutoff as:

$$E[v(c)|c \leq k_0] = \left(1-e^{-rT}\right)k_0 + e^{-rT}v(k_0) \tag{3}$$

then both sides are continuous in k_0, the LHS is larger than the RHS at $k_0 = 0$ and the opposite ranking is true for $k_0 = 1$, so the two sides are equal for at least one k_0. [3]

2) **Uniqueness**. To see that there is a unique solution under the two assumptions, note that the derivative of the LHS of (3) at any k is

$$\frac{f(k)}{F(k)}(v(k) - E[v(c)|c \leq k])$$

When we evaluate it at points where (3) holds, the the derivative is $\frac{f(k)}{F(k)}\left(1-e^{-rT}\right)(v(k)-k)$ and that is by assumption decreasing in k.

Suppose that there are at least two solutions and select two: the lowest k_L and second-lowest k_H. Since k_L is the lowest solution, at that point the curve on the LHS cannot be steeper than the one on the RHS of (3). But then by our assumptions, at k_H the curve on the LHS is less steep than at k_L while the curve on the RHS is steeper at k_H than at k_L. That leads to a contradiction since by assumption between $[k_L, k_H]$ the LHS is less than the RHS, so with this ranking of derivatives they cannot be equal at k_H ■

4.1 Short-lived Private Information

Our first result is a generalization of our motivating example to arbitrary $v(c)$ and $F(c)$ when T is short:

Proposition 4 *Fix* $v(c)$, $F(c)$ *and* r. *There exists* $T^* > 0$ *such that for all* $T \leq T^*$ *the infrequent trading market generates higher expected gains from trade than the continuous trading market.*

Proof. Let k_0^* be the time-zero cutoff in the infrequent market. Let k_T^* be the cutoff at time T in the continuous trading market. The proof is by comparing k_0^* and k_T^* from the two models for T close to zero. If we can show that for small T, $k_0^* \geq k_T^*$ then we are done since

[3] If there are multiple solutions, a game theoretic-model would refine some of them, see section 13.B of Mas-Colell, Whinston and Green (1995) for a discussion.

due to discounting the surplus in the continuous trading market is smaller than the surplus in a infrequent market with a cutoff k_T^* (and the infrequent market is the more efficient the higher cutoff it supports in time 0).

In the limit, as $T \to 0$, both of these thresholds converge to 0. So it is sufficient to compare:

$$\lim_{T\to 0} \frac{\partial k_0^*}{\partial T} \text{ vs. } \lim_{T\to 0} \frac{\partial k_T^*}{\partial T}$$

The cutoff for the infrequent trading market, k_0^*, is defined implicitly by

$$v(k_0^*) - E[v(c)|c \le k_0^*] = \left(1 - e^{-rT}\right)(v(k_0^*) - k_0^*)$$

where the LHS is the benefit of rejecting the offer at time 0 and the RHS is the cost of waiting for the higher offer at T. For small T , $E[v(c)|c \le k_0^*] \approx \frac{v\left(k_0^*\right)}{2}$so the benefit of waiting is approximately$\frac{v\left(k_0^*\right)}{2}$ while the cost is approximately $rTv(0)$ so k_0^* for small T solves approximately

$$\frac{v(k_0^*)}{2} \approx rTv(0)$$

and more precisely, $\lim_{T\to 0} \frac{\partial k_0^*}{\partial T} = \frac{2rv(0)}{v'(0)}$

Now, turning to the continuous trading market, since k_t is defined by the differential equation

$$r(v(k_t) - k_t) = v'(k_t)\dot{k}_t$$

we have that for small T,

$$k_T^* \approx rT\frac{rv(0)}{v'(0)}$$

so

$$\lim_{T\to 0} \frac{\partial k_T^*}{\partial T} = \frac{rv(0)}{v'(0)}$$

Summing up, we have obtained

$$\lim_{T\to 0} \frac{\partial k_0^*}{\partial T} = 2\lim_{T\to 0} \frac{\partial k_T^*}{\partial T}$$

which implies the claim. ■

5 Other market designs

So far we have compared only the continuous trading market with the infrequent trading. But one can imagine many other ways to organize the market. For example, the market could clear every day, for some Δ which is smaller than T but larger than 0. Or the market could start with a positive Δ and then be opened continuously. Or, the market could start being opened continuously and close some Δ before T (i.e. at $t = T - \Delta$). In this section we consider some of these alternative timings.

We start with providing a sufficient condition for the infrequent trading to dominate all these other possible designs in case private information is long-lived:

Proposition 5 *If* $\frac{f(c)}{F(c)}(v(c) - c)$ *is weakly increasing then for* $T = \infty$ *infrequent trading* $\Omega = \{0\}$ *generates higher expected gains from trade than any other market design.*

Proof. We use mechanism design to establish the result. We expand the set of possible market designs to allow for any trading mechanism that is incentive compatible and does not require the buyers to lose money on average. For every market design, the equilibrium outcome can be replicated by such a mechanism. We then show that under the condition in the proposition infrequent trading replicates the outcome of the best mechanism and hence any other market design generates lower expected gains from trade.

A general (direct revelation) mechanism specifies for each type the distribution over the times of trade, $g_t(c)$. Let $x(c) = E_{g_t(c)}[e^{-rt}]$ be the expected discount factor at the time of trading. Note that $x(c) \in [0, 1]$, where $x = 1$ means trade for sure at $t = 0$ and $x = 0$ means no trade. The mechanism also specifies the expected payment to the seller, $P(c)$. In case there are payments only conditional on trade, with a payment p_t if there is trade at time t then $P(c) = E_{g_t(c)}[p_t e^{-rt}]$.

In the mechanism the seller gets an expected surplus

$$U(c) = (1 - x(c))\, c + P(c) = \max_{c'} (1 - x(c'))\, c + P(c') \tag{4}$$

The highest type $c = 1$ does not trade in any equilibrium hence $U(1) = 1$. Using the envelope condition,

$$\begin{aligned} U'(c) &= 1 - x(c) \\ U(c) &= U(1) - \int_c^1 (1 - x(c'))\, dc' \\ &= c + \int_c^1 x(c')\, dc' \end{aligned} \tag{5}$$

For the mechanism to be incentive compatible it has to satisfy the envelope condition (5) and $x(c)$ has to be non-increasing (otherwise it would not be optimal to report c truthfully).

Since in equilibrium the buyers make zero profit, the expected gains from trade are:

$$S = \int_0^1 (U(c) - c)\, f(c)\, dc = \int_0^1 f(c) \int_c^1 x(c')\, dc' dc$$

Integrating by parts

$$S = \int_0^1 F(c)\, x(c)\, dc \tag{6}$$

The constraint on the mechanism is that even if we allow for across-time cross-subsidization, the buyers do not lose money:

$$\int_0^1 (x(c)\, v(c) - P(c))\, f(c)\, dc \geq 0$$

Substituting $P(c) = U(c) - c + x(c)$ (from (4)) and the envelope formula (5) for $U(c)$ we can write the constraint as

$$\int_0^1 \left(x(c)\,(v(c) - c) - \int_c^1 x(c')\, dc' \right) f(c)\, dc \geq 0$$

again integrating by parts we get

$$\int_0^1 x(c)\,(v(c) - c)\, f(c) - F(c)\, x(c)\, dc \geq 0 \tag{7}$$

Consider the relaxed problem of maximizing (6) subject to (7) alone. What is the solution? The bang-for-the-buck for each $x(c)$ is

$$\frac{1}{(v(c) - c)\,\frac{f(c)}{F(c)} - 1}$$

In the solution, for any c such that $(v(c) - c)\frac{f(c)}{F(c)} - 1 > 0$ will have $x(c) = 1$ because that contributes positively to the objective function and relaxes the constraint. What about other $c's$? It is optimal to rank $(v(c) - c)\frac{f(c)}{F(c)}$ from the highest to the lowest and set $x(c) = 1$ for large values of that expression and $x(c) = 0$ for small ones, since that maximizes the ratio of the positive impact on the objective to the negative impact on the constraint (bang-for-the-buck). Now, recall the assumption in the proposition that $(v(c) - c)\frac{f(c)}{F(c)}$ is decreasing in c. That implies that the solution to this relaxed problem is to find a c^* such that all types $c \leq c^*$ trade immediately and all types $c > c^*$ never trade (and as a result this mechanism satisfies the monotonicity constraint). That mechanism is incentive compatible and has transfer $P(c) = E[v(c)|c \leq c^*]$ for all $c \leq c^*$ and $P(c) = 0$ for all higher types (to guarantee that the buyers make zero profit). Finally, since $U(c)$ is continuous, it must be that $\lim_{c\uparrow c^*} U(c) = \lim_{c\downarrow c^*} U(c)$ which implies that c^* solves $E[v(c)|c \leq c^*] = c^*$. This (plus the price) is exactly the equilibrium for the infrequent trading market, which completes the proof. ■

5.1 Closing the Market Briefly after Initial Trade.

Even if the condition in Proposition 5 does not hold, we can show that under very general conditions it is possible to improve upon the continuous trading market.

In particular, consider the following design: there is trade at $t = 0$, then the market is closed till $\Delta > 0$ and then it is opened continuously till T. We call this design "early closure". We claim that there always exists a small delay that improves upon continuous trading:

Proposition 6 *For every T there exists $\Delta > 0$ such that the expected gains from trade in the "early closure" market are higher than under continuous trading.*

The proof is analogous to the proof of Proposition 4 by observing that if in equilibrium types $\left[0, k_\Delta^B\right]$ trade at $t = 0$, then after the market re-opens at Δ, the price is $p_\Delta = v\left(k_\Delta^B\right)$. The rest of the proof compares the cutoff at Δ in the two designs (instead of at T in the previous proof) and follows exactly the same steps, so we omit it.

5.2 Closing the Market Briefly before Information Arrives

The final design we consider is the possibility of keeping the market opened continuously from $t = 0$ till $T - \Delta$ and then closing it till T. Such a design may be more realistic since in practice it may be easier to determine when some private information is likely to be revealed than when it is that the seller of the asset is hit by liquidity needs (i.e. when is $t = 0$).

The comparison of this "late closure" market with the continuous trading market is much more complicated than in the previous section for two related reasons. First, if the market is closed from $T-\Delta$ to T, there will be an atom of types trading at $T-\Delta$. As a result there will be "quiet period" before $T-\Delta$: there will be some time interval $[t^*, T-\Delta]$ such that despite the market being opened, there will be no types that trade on the equilibrium path in that time period. The equilibrium outcome until t^* is the same in the "late closure" and continuous trading designs, but diverges from that point on. That brings the second complication: starting at time t^*, the continuous trading market will benefit from some types trading earlier than in the "late closure" market. Therefore it is not sufficient to show that by T there are more types that trade in the late closure market, we actually have to compare directly the total surplus generated between t^* and T. These two complications are not present when we consider the "early closure" design since there is no t^* before $t=0$ for the early closure to affect trade before it.

The equilibrium in the "late closure" design is as follows. Let $p^*_{T-\Delta}, k^*_{T-\Delta}$ and t^* be a solution to the following system of equations:

$$E\left[v\left(c\right)|c\in\left[k_{t^*},k_{T-\Delta}\right]\right] = p_{T-\Delta} \tag{8}$$
$$\left(1-e^{-r\Delta}\right)k_{T-\Delta}+e^{-r\Delta}v\left(k_{T-\Delta}\right) = p_{T-\Delta} \tag{9}$$
$$\left(1-e^{-r(T-\Delta-t^*)}\right)k_{t^*}+e^{-r(T-\Delta-t^*)}p_{T-\Delta} = v\left(k_{t^*}\right) \tag{10}$$

where the first equation is the zero-profit condition at $t=T-\Delta$, the second equation is the indifference condition for the highest type trading at $T-\Delta$ and the last equation is the indifference condition of the lowest type that reaches $T-\Delta$, who chooses between trading at t^* and at $T-\Delta$. The equilibrium for the late closure market is then:

1) at times $t\in[0,t^*]$, (p_t,k_t) are the same as in the continuous trading market

2) at times $t\in(t^*,T-\Delta)$, $(p_t,k_t)=\left(v\left(k_{t^*}\right),k_{t^*}\right)$

3) at $t=T-\Delta$, $(p_t,k_t)=\left(p^*_{T-\Delta},k^*_{T-\Delta}\right)$

Condition (10) gurantees that given the constant price at times $t\in(t^*,T-\Delta)$ it is indeed optimal for the seller not to trade. There are other equilibria that differ from this equilibrium in terms of the prices in the "quiet period" time: any price process that satisfies in this time period

$$\left(1-e^{-r(T-\Delta-t)}\right)k_{t^*}+e^{-r(T-\Delta-t)}p_{T-\Delta}\geq p_t\geq v\left(k_{t^*}\right)$$

satisfies all our equilibrium conditions. Importantly, however, all these paths yield the same equilibrium outcome.

While we conjecture that the following result is more general, due to the two complications we mentioned, so far we have only established it for the leading example:

Proposition 7 *Suppose $v(c) = \frac{1+c}{2}$ and $F(c) = 1$. For every r, T there exists a $\Delta > 0$ such that the "late closure" market design generates higher expected gains from trade than the continuous trading market. Yet, the gains from late closure are of a smaller than the gains from early closure.*

Proof. In this case the equilibrium conditions simplify to

$$\begin{aligned} \frac{1}{2} + \frac{k_{t^*} + k_{T-\Delta}}{4} &= p_{T-\Delta} \\ \left(1 - e^{-r\Delta}\right) k_{T-\Delta} + \left(\frac{1}{2} + \frac{k_{T-\Delta}}{2}\right) e^{-r\Delta} &= p_{T-\Delta} \\ \left(1 - e^{-r(T-\Delta-t^*)}\right) k_{t^*} + e^{-r(T-\Delta-t^*)} p_{T-\Delta} &= v(k_{t^*}) \\ \left(1 - e^{-r\Delta_2}\right) k_{t^*} + e^{-r\Delta_2} p_{T-\Delta} &= \frac{1}{2} + \frac{k_{t^*}}{2} \end{aligned}$$

where $\Delta_2 = T - \Delta - t^*$.

Solution of the first two equations is:

$$\begin{aligned} k_{T-\Delta} &= \frac{k_{t^*} + 2 - 2e^{-r\Delta}}{3 - 2e^{-r\Delta}} \\ p_{T-\Delta} &= \frac{1}{2}\left(\frac{2 - e^{-r\Delta}}{3 - 2e^{-r\Delta}} k_{t^*} + \frac{4 - 3e^{-r\Delta}}{3 - 2e^{-r\Delta}}\right) \end{aligned}$$

Substituting the price to the last condition yields

$$\left(1 - e^{-r\Delta_2}\right) k_{t^*} + e^{-r\Delta_2}\left(\frac{1}{2}\left(\frac{2 - e^{-r\Delta}}{3 - 2e^{-r\Delta}} k_{t^*} + \frac{4 - 3e^{-r\Delta}}{3 - 2e^{-r\Delta}}\right)\right) = \frac{1}{2} + \frac{k_{t^*}}{2}$$

which can be solved for Δ_2 independently of k_{t^*} (given our assumptions about $v(c)$ and $F(c)$).

$$r\Delta_2 = -\ln \frac{3 - 2e^{-r\Delta}}{4 - 3e^{-r\Delta}}$$

Note that

$$\lim_{\Delta \to 0} \frac{\partial \Delta_2}{\partial \Delta} = r \lim_{\Delta \to 0} \frac{\partial}{\partial \Delta}\left(-\ln \frac{3 - 2e^{-r\Delta}}{4 - 3e^{-r\Delta}}\right) = r^2$$

so Δ_2 is on the same order as Δ.

In the continuous trading cutoffs follow $k_t = 1 - e^{-rt}$, $\dot{k}_t = re^{-rt}$. Normalize $T = 1$ (and rescale r appropriately). Then

$$\begin{aligned} k_{t^*} &= 1-e^{-r(1-\Delta-\Delta_2)} = 1-\frac{4-3e^{-r\Delta}}{3-2e^{-r\Delta}}e^{r\Delta}\delta \\ \text{where } \delta &= e^{-r} \text{ and} \\ t^* &= 1-\Delta-\Delta_2 = 1-\Delta+\frac{1}{r}\ln\frac{3-2e^{-r\Delta}}{4-3e^{-r\Delta}} \end{aligned}$$

We can now compare gains from trade in the two cases. The surplus starting at time t^* is (including discounting):

$$\begin{aligned} S_c(\Delta) &= \int_{k_{t^*}}^{1-e^{-r}} e^{-r\tau(c)}\left(v(c)-c\right)dc+\delta\int_{1-e^{-r}}^{1}\left(v(c)-c\right)dc \\ &= \int_{k_{t^*}}^{1-e^{-r}} (1-c)\left(\frac{1-c}{2}\right)dc+\delta\int_{1-e^{-r}}^{1}\left(\frac{1-c}{2}\right)dc \end{aligned}$$

where we used $e^{-r\tau(c)}=1-c$.

$$\frac{\partial S_c(\Delta)}{\partial\Delta} = -\frac{\partial k_{t^*}}{\partial\Delta}\frac{(1-k_{t^*})^2}{2}$$

and since $\lim_{\Delta\to 0}\frac{\partial k_{t^*}}{\partial\Delta}=-2r\delta$ we get that

$$\lim_{\Delta\to 0}\frac{\partial S_c(\Delta)}{\partial\Delta} = r\delta^3$$

For the "late closure" market the gains from trade are

$$S_{LC}(\Delta) = e^{-r(1-\Delta)}\int_{k_{t^*}}^{k_{T-\Delta}}\left(v(c)-c\right)dc+e^{-r}\int_{k_{T-\Delta}}^{1}\left(v(c)-c\right)dc$$

after substituting the computed values for k_{t^*} and $k_{T-\Delta}$ it can be verified that

$$\lim_{\Delta\to 0}\frac{\partial S_{LC}(\Delta)}{\partial\Delta} = r\delta^3$$

which is the same as in the case of continuous market, so to the first approximation even conditional on reaching t^* the gains from trade are approximately the same in the two market designs.

We can compare the second derivatives:

$$\lim_{\Delta\to 0}\frac{\partial S_{LC}^{2}\left(\Delta\right)}{\partial\Delta^{2}} = 3\delta^{3}r^{2}$$
$$\lim_{\Delta\to 0}\frac{\partial S_{c}^{2}\left(\Delta\right)}{\partial\Delta^{2}} = 3\delta^{3}r^{2}$$

and even these are the same. Finally, comparing third derivatives:

$$\lim_{\Delta\to 0}\frac{\partial S_{LC}^{3}\left(\Delta\right)}{\partial\Delta^{3}} = 13r^{3}\delta^{3}$$
$$\lim_{\Delta\to 0}\frac{\partial S_{c}^{3}\left(\Delta\right)}{\partial\Delta^{3}} = 9r^{3}\delta^{3}$$

so we get that for small Δ, the "late closure" market generates slightly higher expected surplus, but the effects are really small. ■

5.3 Stochastic Arrival of Information

So far we have assumed that it is known that the private information is revealed at T. However, in some markets even if the private information is short-lived, the market participants may be uncertain about the timing of its revelation. We now return to our motivating example to illustrate that trade-offs we have identified for the deterministic duration of private information apply also to the stochastic duration case.

Seller type c is distributed uniformly over $[0,1]$ and $v\left(c\right)=\frac{1+c}{2}$. Suppose that with a Poisson rate λ the type c gets publicly revealed and at that time the seller trades immediately at a price $v\left(c\right)$. Analogously to the previous definitions, let "infrequent trading" market be such that the seller can trade only either at $t=0$ or upon arrival of information. Also let the continuous trading market be such that the seller can trade at any time.

In the infrequent trading market, the equilibrium is determined by:

$$p_{0} = \frac{\lambda}{\lambda+r}v\left(k_{0}^{*}\right)+\frac{r}{\lambda+r}k_{0}$$
$$p_{0} = E\left[v\left(c\right)|c\leq k_{0}^{*}\right]$$

where the first equation is the indifference condition of the cutoff type and the second equation is the usual zero-profit condition. In our example we get

$$\begin{aligned} k_0 &= \frac{2r}{3r+\lambda} \\ p_0 &= \frac{4r+\lambda}{6r+2\lambda} \end{aligned}$$

In the continuous trading market the equilibrium is described by the same differential equation:

$$\begin{aligned} r\left(p_t - k_t\right) &= \dot{p}_t \\ p_t &= v\left(k_t\right) \\ k_0 &= 0 \end{aligned}$$

Since it is the same differential equation as in deterministic duration case, the solution is again

$$k_t = 1 - e^{-rt}$$

We now can compare the gains from trade. The total surplus in the infrequent trading market is

$$\begin{aligned} S_0 &= \int_0^{k_0} \left(v\left(c\right) - c\right) dc + \frac{\lambda}{\lambda + r} \int_{k_0}^{1} \left(v\left(c\right) - c\right) dc \\ &= \frac{1}{4} \frac{5z + 8 + z^2}{\left(3+z\right)^2} \end{aligned}$$

where $z \equiv \frac{\lambda}{r}$.

In the continuous trading market the surplus is

$$S_C = \int_0^{+\infty} \lambda e^{-\lambda t} \left(\int_0^{k_t} e^{-r\tau(c)} \left(v\left(c\right) - c\right) dc + e^{-rt} \int_{k_t}^{1} \left(v\left(c\right) - c\right) dc \right) dt$$

where $\tau\left(c\right)$ is the time type c trades if there is no arrival before his time of trade. In our example $\tau\left(c\right) = -\frac{\ln(1-c)}{r}$ and $e^{-r\tau(c)} = 1 - c$, so the expected surplus is:

$$S_C = \frac{1}{6} + \frac{z}{12\left(3+z\right)}$$

The difference is:

$$S_0\left(z\right) - S_B\left(z\right) = \frac{1}{2}\left(z+3\right)^{-2} > 0$$

So in our example, for every λ, the infrequent trading market is more efficient than the continuous trading market.

6 Appendix

References

[1] Akerlof, George. A. (1970). "The Market for "Lemons": Quality Uncertainty and the Market Mechanism." Quarterly Journal of Economics, 84 (3), pp. 488-500.

[2] Daley, Brendan and Brett Green (2011) "Waiting for News in the Market for Lemons." forthcoming in Econometrica.

[3] Mas-Colell, Andreu; Michael D. Whinston and Jerry R. Green (1995). Microeconomic Theory. New York: Oxford University Press.