Market Timing, Investment, and Risk Management*

PRELIMINARY AND INCOMPLETE

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March 8, 2010

Abstract

How should firms optimally time equity market opportunities? And how should they adapt their investment, payout and dynamic hedging decisions to changing market financing opportunities? We show that only firms with low cash-to-asset ratios should carry out new equity issues to take advantage of favorable equity market opportunities. Firms with high cash-to-asset ratios optimally respond to fleeting market opportunities by both delaying payouts to shareholders and by cutting back investment. We model market financing opportunities risk through switching probabilities between two states of nature with respectively low and high external costs of equity financing. The most striking result of our analysis is that market timing introduces convexity in the firm's value function, which gives rise to a non-monotonic investment policy in the firm's cash-to-asset ratio: investment at first declines with the cash-to-asset ratio, then rises and then again declines. This convexity also gives rise to a complex dynamic hedging policy, with the firm increasing its exposure to aggregate risk when its cash-to-asset ratio is both low and high, and decreasing its exposure for intermediate cash-to-asset ratios.

*We are grateful to Michael Adler, Charles Calomiris and Emi Nakamura for their comments.
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1 Introduction

What are the key factors that determine corporate equity issues? And, how do external equity financing costs affect corporate investment and cash holdings? There has been an ongoing and lively debate between the proponents of the *pecking order view* (Myers and Majluf, 1984) and the proponents of the *market timing view* (Loughran and Ritter, 1995, 1997, and Baker and Wurgler, 2002) on these questions. According to the pecking order view (in very broad terms) firms issue equity only as a last resort when they have exhausted all other forms of financing. According to the market timing view firms issue equity when the cost of equity capital is low. Underlying the differences between these two views are two fundamentally different conceptions of investor rationality.

Under the pecking order view, investors are assumed to be *fully rational* and to fully understand firms' motives for issuing equity. Thus, when equity issuers have better information than investors about the value of their business, investors tend to interpret equity issues as negative signals about the value of the business: They infer that issuers are more likely to issue equity when the stock is overvalued, and therefore lower their valuation of the firm in response to a new equity issue. The most direct evidence in support of this view are the studies by Asquith and Mullins (1986), Masulis and Korwar (1986), and Mikkelson and Partch (1986) among others, who have found that the average stock price announcement effect of a new common stock issue is a decline in stock price of the order of -3% to -4.5%.

Under the market timing view, investors are generally assumed to have *behavioral biases* such as overconfidence, which may lead them to sometimes overvalue stocks. Moreover, behaviorally biased investors are assumed to not fully undo a temporary overvaluation when they see firms issue more stock. As a result, firms may be able to take advantage of investors' behavioral biases and benefit from timing their equity issues in periods when investors are particularly favorably disposed towards their firm. [1]

Partly in reaction to the technology bubble and the dot-com IPO wave of the late 1990s, several studies have shown that firms do indeed try to time equity markets. Building on early findings of market timing by Loughran and Ritter (1995, 1997) in particular, Baker and Wurgler (2002) and Huang and Ritter (2009) have shown that market timing has a significant and persistent effect on firm leverage. Similarly, Graham and Harvey's (2001) survey of CFO corporate finance practices finds that most CFOs attempt to time equity markets. Fama and French (2005) also argue that the pecking order view is rejected by the data, to the extent that many firms are seen to tap equity markets even though they have not exhausted other financing opportunities. Yet the study by DeAngelo, DeAngelo and Stulz (2009) appears to contradict the market timing hypothesis, and suggests that firms' equity issuance policies are better explained by their concern for rebuilding cash reserves. They find that most firms with high market-to-book ratios do not issue stock and most firms who do issue stock look as if they are cash constrained.

In this paper we propose the first dynamic model of equity market timing, which may reconcile to a large extent the seemingly contradictory evidence of these studies. A dynamic model of market timing must be able to account for the facts that most firms only issue equity intermittently, and when they do go to the equity market, that they issue large amounts, as Fama and French (2005) have underscored. Moreover, the theory must account for the greater propensity to issue equity when the cost of equity capital is low. Accordingly, our dynamic theory of market timing builds on our model of corporate precautionary cash holdings in Bolton, Chen and Wang (2009) (BCW) by introducing stochastic financing opportunities. In this model we can analyze how equity market timing interacts with a firm's cash reserves, investment, and dynamic hedging policies.

The four main building blocks of the model are: 1) a long-run constant-returns-to-scale production function with *i.i.d.* earnings shocks, convex investment adjustment costs, and a constant

[1] It might also be possible to build an efficient market timing theory with fully rational investors who have time-changing risk-aversion, but such a theory has not yet been developed. One of the difficulties for such a theory is to translate changing risk-preferences into changes in the cost of equity capital *relative* to debt capital.

capital depreciation rate (as in Hayashi (1982)); 2) stochastic external equity financing costs; 3) constant cash carry costs; and 4) dynamic hedging opportunities through futures contracts. As the firm's external equity financing costs are sometimes low, we are able to study how the firm optimally times equity markets, and how it adjusts its investment, payout, and hedging policies to changing financing opportunities.

Our model can also be applied to study how firms' financing and investment decisions are affected by the impending risk of a financial crisis, which freezes up financial markets. Similarly, it can be applied to the situation of a firm in the midst of a financial crisis looking ahead to a return to normally functioning capital markets.

We show that, although the firm's equity issuance is driven by the need for cash, it is still optimal for firms to time equity markets. If the state of the world in which equity is cheap is (nearly) permanent then the firm only issues equity when it runs out of cash. If, however, the firm knows that the transition probability out of this favorable state is significantly different from zero then it will optimally time the market by issuing new equity before it runs out of cash. These results are consistent with the findings in both DeAngelo, DeAngelo and Stulz (2009) and in Fama and French (2005) and Huang and Ritter (2009). Moreover, we show that as the transition probability out of the favorable state rises the firm will issue equity in that state sooner. Similarly, the firm delays cash payouts to shareholders more, the more likely is the probability that equity market conditions will worsen. Finally, the firm also scales back its investment as the probability of exiting the good state rises. Overall, the firm's cash inventory rises in anticipation of a significant worsening of equity financing opportunities. These results also confirm the conjecture of Bates, Kahle and Stulz (2009), who have found that strikingly the average cash-to-asset ratio of US firms has nearly doubled in the past quarter century, and who attribute this rise to firms' perceived increase in risk. These results also help explain the investment and financing policies of many US non-financial firms in the years prior to the financial crisis of 2007-2008, to the extent that these firms had forebodings of a likely financial crisis .

We model a financial crisis by assuming that equity markets freeze up and asset sales are nearly impossible in a crisis state. We show that a firm entering the crisis state with a lot of cash will barely change its investment policy and will simply delay somewhat payouts to shareholders. The lower its cash reserves, however, the more the firm cuts back on investment and the more it engages in asset sales – to the extent that these are still feasible – in an effort to preserve cash and survive. These results are broadly in line with the findings of Campello, Graham and Harvey (2009) who surveyed a large sample of CFOs in the midst of the crisis and found that the more financially constrained their firms were the more they cut back on investment and the more they engaged in asset sales. If there is a redeeming aspect to crises it could be that looking ahead the future is expected to get brighter. The firm in the crisis state anticipates that with some probability it will exit the state and be able to return to more favorable equity market conditions. We show that, as one might expect, when the probability of exiting the crisis rises the firm invests more and is less conservative in its payout policy. One surprising result, when the firm is unable to engage in asset sales in the crisis state, is that the firm's payout boundary in that state may be non-monotonic in the probability of exiting the crisis. For very high and very low probabilities the firm pays out sooner than for intermediate probabilities. The reason is that when the firm is stuck in the crisis state for a long time the value of its investment opportunities is so low that it is best to payout cash to shareholders. When the probability of exiting the crisis is very high then the prospect of raising cheap equity in the future also encourages the firm to pay out more dividends in the crisis state. It is only for intermediate probabilities, when the value of the firm's investment opportunities is relatively high, but the risk of staying in a prolonged crisis is also high, that the firm is more conservative in its payout policy. This latter result, while intriguing is likely to be more of a curiosity than a quantitatively significant observation.

In contrast, the other surprising result of our analysis is important. We show that a major non-convexity in the firm's value – or the firm's average Q – as a function of its cash-to-asset ratio emerges as a result of equity market timing. When the firm times the market by choosing to issue equity before it runs out of cash, its value function is necessarily non-convex to the right of the (endogenous) equity issuance boundary. To see this, note first that when the firm issues equity it incurs a fixed issuance cost and raises a lumpy amount of cash. Second, note that a condition of optimality in equity issuance is that the marginal value of a dollar at the issuance boundary must be the same as the marginal value of the dollar at the return point for the firm's cash-to-asset ratio. This latter observation implies that the firm's value function is locally convex around the equity issuance boundary. This non-convexity in turn translates into a non-monotonicity in the firm's shadow value of cash, which in turn results in a non-monotonicity of investment as a function of the firm's cash-to-asset ratio.

The basic economics of this result can be explained as follows. When the firm's cash-to-asset ratio decreases and approaches the issuance boundary the firm increases its investment and thereby accelerates the depletion of its cash stock, as it is eager to time the favorable equity markets. By burning through its cash stock the firm brings forward the point at which it taps equity markets and replenishes its cash stock. However, when the firm's cash-to-asset ratio is already high it benefits less from timing the market and it is more concerned with preserving rather than renewing its cash hoard. This is why the firm responds locally to a small reduction in its cash-to-asset ratio by cutting back on investment and thereby replenish its cash stock. Our results are yet another warning of the complex dynamic interactions between firm savings and investment: when firms engage in market timing, not only is the cash-sensitivity of investment likely to be non-monotonic, but investment itself is non-monotonic.

This finding also has important consequences for the firm's dynamic hedging policy. Indeed, we show that the non-convexity in the firm's value function caused by market timing can induce the firm to load up on rather than hedge systematic risk. We are thus able to account for the possibility of risk-taking through derivatives positions, which far from being excessive, is efficient and maximizes the value of the firm. Note that these risk-taking benefits exist even when the firm has no leverage and are driven purely by market timing.

2 The Model

We build on the model for an all-equity firm in BCW by introducing stochastic external financing opportunities. The recent financial crisis among other large shocks to financial markets strongly suggest that firms' financing opportunities in reality are time-varying. In crisis periods, in particular, the cost of equity financing can be quite steep. Firms must adapt to these stochastic financing opportunities and change their financing and investment policy to be able to time favorable market conditions and hedge against unfavorable market conditions.

2.1 Two Regimes

We assume that the firm has exogenously given time-varying financing opportunities and production technology. We denote the *current regime* the firm is in by regime 1. We assume that this regime may change with a constant probability $\zeta \Delta t$ over each time period Δt. After a regime change has occurred at some random time T, the firm faces different financing costs and possibly also a different production technology. For our analysis in this paper we confine ourselves to only two possible regimes, a *favorable* and an *unfavorable* one. After the firm has entered a different regime, we then refer to this regime as regime 2. Under the first scenarios we consider, we also assume that once the firm has entered regime 2, it permanently remains in that regime. Regime 2 is then an *absorbing state* and the current regime (regime 1) is a *transitory state* with an average duration of $1/\zeta$. The assumption that regime 2 may be absorbing is made mainly for expositional reasons. Even in such a highly simplified setting our model can generate interesting financing and investment behavior that fundamentally differs from the firm's behavior in a stationary setting with constant investment/financing opportunities as in BCW. Thus, let s_t denote the regime at time t. We then have either $s_t = 1$ or $s_t = 2$. When regime 2 is absorbing we also have that when $s_t = 2$ then $s_u = 2$ for all $u \geq t$.

2.2 Production technology

The firm employs only physical capital as an input for production and, the price of physical capital is normalized to unity. We denote by K and I respectively the level of the capital stock and gross investment. As is standard in capital accumulation models, the firm's capital stock K evolves according to:

$$dK_t = (I_t - \delta K_t)\, dt, \quad t \geq 0, \tag{1}$$

where $\delta \geq 0$ is the rate of depreciation.

The firm's operating revenue at time t is proportional to its capital stock K_t, and is given by $K_t dA_t$, where dA_t is the firm's revenue (or productivity) shock over time increment dt. After risk adjustment (i.e. under the risk-neutral probability measure), the firm's revenue dA_t over time period dt are given by

$$dA_t = \mu\,(s_t)\, dt + \sigma\,(s_t)\, dZ_t,$$

where Z is a standard Brownian motion. Here, $\mu(s_t)$ denotes expected revenue per unit time in regime s_t and $\sigma(s_t)$ the volatility of revenue per unit of time in regime s_t.

The firm's incremental operating profit dY_t over time increment dt is then given by:

$$dY_t = K_t dA_t - I_t dt - G(I_t, K_t, s_t)dt, \quad t \geq 0, \tag{2}$$

where I is the cost of the investment and $G(I, K, s)$ is the additional adjustment cost that the firm incurs in the investment process if it is in regime s. Note that we allow the adjustment costs to be regime dependent. Intuitively, in expansion, the firm may be subject to lower transaction costs. Following the neoclassical investment literature (Hayashi (1982)), we assume that the firm's adjustment cost is homogeneous of degree one in I and K. In other words, the adjustment cost takes the homogeneous form $G(I, K, s) = g_s(i)K$, where i is the firm's investment capital ratio ($i = I/K$), and $g_s(i)$ is an regime-dependent increasing and convex function. Our analyses do not depend on the specific functional form of $g_s(i)$, and to simplify we assume that $g_s(i)$ is quadratic:

$$g_s\left(i\right) = \frac{\theta_s i^2}{2}, \tag{3}$$

The firm can liquidate its assets at any time. The liquidation value L_t is proportional to the firm's capital at time t in regime s_t, i.e. $L_t = l_s K_t$, where l_s is the recovery per unit of capital in regime s.

2.3 Stochastic Financing Opportunities

Neoclassical investment models (á la Hayashi (1982)) assume that the firm faces frictionless capital markets and that the Modigliani and Miller (1958) theorem holds. However, in reality, firms face important financing frictions for incentive, information asymmetry, and transaction cost reasons.[2] Our model incorporates a number of financing costs that firms face in practice and that empirical research has identified, while retaining an analytically tractable setting. The firm may choose to use external financing at any point in time. For the baseline model, we assume that the only source of external financing is equity. We leave the important extension of allowing the firm to also issue debt for future research.

We assume that the firm incurs a fixed and a variable cost of issuing external equity. The fixed cost is given by $\phi_s K$, where ϕ_s is the fixed cost parameter in regime $s = 1, 2$. For tractability, we take the fixed cost to be proportional to the firm's capital stock K. Mainly, this assumption ensures that the firm does not *grow out of its fixed issuing costs.* The firm also incurs a proportional issuance cost γ_s for each unit of external funds it raises, which may also be regime dependent. That is, after paying the fixed cost, the firm pays $\gamma_s > 0$ in regime s for each incremental dollar it raises. We denote by H the process for the firm's cumulative external financing, and hence by dH_t the incremental external financing over time dt, and by X the firm's cumulative issuance costs.

We also denote by W the process for the firm's cash stock. If the firm runs out of cash ($W_t = 0$) it needs to raise external funds to continue operating or its assets will be liquidated. If the firm

[2]See Jensen and Meckling (1976), Leland and Pyle (1977), and Myers and Majluf (1984), for example.

chooses to raise new external funds to continue operating, it must pay the financing costs specified above. The firm may prefer liquidation if the cost of financing is too high relative to the continuation value (e.g. when the firm is not productive; that is when μ is low). We denote by τ the firm's (stochastic) liquidation time, then $\tau = \infty$ means that the firm never chooses to liquidate.

Next, we denote by U the firm's cumulative non-decreasing payout process to shareholders, and hence by dU_t the incremental payout over time dt. Distributing cash to shareholders may take the form of a special dividend or a share repurchase. The benefit of a payout is that shareholders can invest the funds they obtain at the risk-free rate. For simplicity, we assume for now that the risk-free rate is constant r at all times. Finally, we denote by λ the *carry cost of cash inside the firm*.

Combining cash flows from operations dY_t given in (2), with the firm's financing policy given by the cumulative payout process U, the cumulative external financing process H, and the firm's interest earnings minus cash carry cost from its cash inventory, then W evolves according to:

$$dW_t = [K_t dA_t - I_t dt - G(I_t, K_t, s_t)]dt + (r - \lambda) W_t dt + dH_t - dU_t, \tag{4}$$

where, the second term is the interest income (net of the carry cost λ), the third term dH_t is the cash inflow from external financing, and the last term dU_t is the cash outflow to investors, so that $(dH_t - dU_t)$ is the net cash flow from financing. Note that this is a completely general financial accounting equation, where dH_t and dU_t are endogenously determined by the firm.

The homogeneity assumption embedded in the adjustment cost and the "AK" production technology allows us to deliver our key results in a parsimonious and analytically tractable way. Adjustment costs may not always be convex and the production technology may exhibit long-run decreasing returns to scale in practice, but these functional forms substantially complicate the formal analysis (see Hennessy and Whited, 2005, 2007, for an analysis of a similar, but non-homogenous, model). As will become clear below, the homogeneity of our model in K allows us to reduce the dynamics to a one-dimensional equation, which is relatively straightforward to solve.

2.4 Firm optimality

The firm chooses its investment I, its cumulative payout policy U, its cumulative external financing H, and its liquidation time τ to maximize firm value defined below:

$$\mathbb{E}\left[\int_0^\tau e^{-rt}\left(dU_t - dH_t - dX_t\right) + e^{-r\tau}\left(L_\tau + W_\tau\right)\right]. \tag{5}$$

The expectation is taken under the risk-adjusted probability. The first term is the discounted value of payouts to shareholders and the second term is the discounted value upon liquidation. Note that optimality may imply that the firm never liquidates. In that case, we simply have $\tau = \infty$. We impose the usual regularity conditions to ensure that the optimization problem is well posed. See the appendix for details.

Let $P(K, W, s)$ denote firm value, where $s = 1, 2$. We are interested in the impact of stochastic financing, investment opportunities on corporate decision making. In particular, we study how firms manage the risks of investment opportunities $\mu(s)$, volatility $\sigma(s)$, and costs of external financing (ϕ_s, γ_s).

3 First-best Benchmark

We first summarize the solution for the neoclassical q theory of investment, in which the Modigliani-Miller theorem holds. We solve the firm's investment and firm value using backward induction. As in BCW, for the absorbing state (i.e. regime 2), the firm's first-best investment policy is given by $I_2^{FB} = i_2^{FB} K$, where[3]

$$i_2^{FB} = (r+\delta) - \sqrt{(r+\delta)^2 - \frac{2}{\theta_2}\left(\mu_2 - (r+\delta)\right)} \,. \tag{6}$$

Next, we characterize firm investment and Tobin's q and in the *transitory* state (regime 1). Using backward induction, the firm's first-best investment policy in the transitory regime is given by

$$i_1^{FB} = r + \zeta + \delta - \sqrt{(r+\zeta+\delta)^2 - \frac{2}{\theta_1}\left(\mu_1 - (r+\zeta+\delta) + \zeta q_2^{FB}\right)} \,, \tag{7}$$

where q_2^{FB} is Tobin's q in regime 2 given below. [4] Finally, the first-order conditions (FOCs) for investment in both regimes ($s = 1, 2$) imply that firm value is $q_s^{FB} K$, where q_s^{FB} is Tobin's q in regime s and is given by:

$$q_s^{FB} = 1 + \theta_s i_s^{FB} \,, \; s = 1, 2. \tag{8}$$

Due to the homogeneity property in production and frictionless capital markets, marginal q is equal to average (Tobin's) q, as in Hayashi (1982). Next, we analyze the problem of a financially constrained firm.

[3]To ensure that the first-best investment policy is well defined, we impose the following parameter restriction: $(r+\delta)^2 - 2\left(\mu_2 - (r+\delta)\right)/\theta_2 > 0$.

[4]To ensure that the first-best investment policy is well defined in regime 1, we impose the following parameter restriction: $(r+\zeta+\delta)^2 - 2\left(\mu_1 + \zeta q_2^{FB} - (r+\zeta+\delta)\right)/\theta_1 > 0$.

4 Second-best Solution

We solve the firm's optimization problem using backward induction. First, consider the firm's problem in the *absorbing state* (regime 2).

4.1 The absorbing state $(s = 2)$

As in BCW, the following Hamilton-Jacobi-Bellman (HJB) equation for the firm's value defines $P(K, W, 2)$:

$$\begin{aligned} r_2 P(K,W,2) &= \max_I \left[(r - \lambda_2) W + \mu_2 K - I - G(I,K,2) \right] P_W(K,W,2) \\ &\quad + \frac{\sigma_2^2 K^2}{2} P_{WW}(K,W,2) + (I - \delta K) P_K(K,W,2). \end{aligned} \tag{9}$$

The endogenous payout boundary is given by

$$P_W(K, \overline{W}, 2) = 1. \tag{10}$$

As in BCW, we conjecture that firm value is homogeneous of degree one in W and K, so that

$$P(K, W, 2) = p_2(w)K,$$

where $p_2(w)$ solves the following ordinary differential equation (ODE):

$$r_2 p_2(w) = \left[(r - \lambda_2) w + \mu_2 - i_2 - g_2(i_2) \right] p_2'(w) + \frac{\sigma_2^2}{2} p_2''(w) + (i_2 - \delta) \left(p_2(w) - w p_2'(w) \right),$$

The first-order condition (FOC) for the investment-capital ratio $i(w)$ is then given by:

$$i_2(w) = \frac{1}{\theta_2} \left(\frac{p_2(w)}{p_2'(w)} - w - 1 \right). \tag{11}$$

At the optimally chosen endogenous payout boundary $\overline{w}_2$, we have the following value matching condition:

$$p_2'(\overline{w}_2) = 1, \tag{12}$$

which states that the marginal value of cash is one when the firm chooses to pay out cash. Moreover, the optimality of a payout implies the following smooth pasting condition holds:

$$p_2''(\overline{w}_2) = 0. \tag{13}$$

When the firm runs out of cash in regime 2, there are two scenarios. First, the firm may liquidate its capital. This implies the following boundary condition:

$$p_2(0) = l_2. \tag{14}$$

Alternatively, the firm can seek external financing. Since we focus on the case where external financing is in the form of equity, we then have the following value matching condition:

$$p_2(0) = p_2(m_2) - \phi_2 - (1 + \gamma_2)m_2\,. \tag{15}$$

If the firm uses external financing, it first pays the fixed equity issuance cost ϕ_2 per unit of capital and then incurs the marginal issuance cost γ_2 for each unit of equity it raises. The above equation gives the accounting relation for firm value immediately before and after issuance. Because the firm optimally chooses its external financing at the margin, we have the marginal benefit of issuance $p_2'(m_2)$ is equal to the marginal cost of doing so, i.e.

$$p_2'(m_2) = 1 + \gamma_2. \tag{16}$$

The firm chooses either liquidation or external equity financing to maximize its value. Technically, whichever strategy yields a higher value of $p_2(0)$ maps to the firm's optimal strategy.

Finally, if the firm has too much cash in regime 2 (i.e. $w > \overline{w}_2$), it will reduce its cash holding to $\overline{w}$. That is, we have

$$p_2\left(w\right) = p_2\left(\overline{w}_2\right) + \left(\overline{w}_2 - w\right). \tag{17}$$

4.2 The (current) transitory state $(s = 1)$

We now turn to the firm's optimization problem in the current regime. The firm chooses investment to solve the following HJB equation:

$$\begin{aligned} rP(K,W,1) &= \max_{I} \left[(r-\lambda_1) W + \mu_1 K - I - G(I,K,1)\right] P_W(K,W,1) + \frac{\sigma_1^2 K^2}{2} P_{WW}(K,W,1) \\ &\quad + (I - \delta K) P_K(K,W,1) + \zeta \left(P(K,W,2) - P(K,W,1)\right). \end{aligned} \tag{18}$$

Similarly, by exploiting homogeneity, we have the following ODE for $p_1(w)$:

$$\begin{aligned} (r+\zeta) p_1(w) &= \left[(r_1 - \lambda_1) w + \mu_1 - i_1 - g_1(i_1)\right] p_1'(w) \\ &\quad + \frac{\sigma_1^2}{2} p_1''(w) + (i_1 - \delta)\left(p_1(w) - w p_1'(w)\right) + \zeta p_2(w). \end{aligned} \tag{19}$$

As in regime 2, we have the following FOC for investment:

$$i_1(w) = \frac{1}{\theta_1}\left(\frac{p_1(w)}{p_1'(w)} - w - 1\right). \tag{20}$$

The endogenous upper payout boundary condition is determined in the same way as in regime 2. That is, $\overline{w}_1$ satisfies the following value matching and smooth pasting conditions:

$$p_1'(\overline{w}_1) = 1, \quad p_1''(\overline{w}_1) = 0. \tag{21}$$

For the lower endogenous financing boundary, the economic tradeoff is significantly different from the single-regime solution as in BCW. Let $\underline{w}_1$ denote the endogenous lower boundary for equity issuance in the current transitory regime (regime 1). Let m_1 denote the "return" target financing level in regime 1 per unit of capital. A key result in BCW is that the firm shall never raise external equity before it exhausts its cash because the firm always has the option to raise equity in the future and the financing term does not change over time (i.e. constant financing opportunity). That is, in the single-regime setting of BCW, the firm optimally chooses $\underline{w}_1 = 0$. However, this is often not optimal as we will show in our setting with stochastic financing opportunity.

Using essentially the same argument as for the absorbing regime, we have the following value matching and smooth pasting conditions:

$$p_1(\underline{w}_1) = p_1(m_1) - \phi_1 - (1+\gamma_1)(m_1 - \underline{w}_1), \tag{22}$$

$$p_1'(m_1) = 1+\gamma_1. \tag{23}$$

To determine the lower issuance boundary $\underline{w}_1$, we use the following argument. First, suppose that optimal $\underline{w}_1$ is interior, i.e. $\underline{w}_1 > 0$. Then, the standard optimality condition implies that the derivatives of the left and the right sides of (22) with respect to $\underline{w}_1$ are equal. This argument gives the following condition:

$$p_1'(\underline{w}_1) = 1+\gamma_1. \tag{24}$$

However, if there exists no $\underline{w}_1$ such that the above condition holds, we have the corner solution, i.e. $\underline{w}_1 = 0$.

Using the above procedure, we obtain the optimal lower boundary $\underline{w}_1 \geq 0$. Then, we need to compare whether external equity issuance (the above strategy) or liquidation is optimal. The firm has an option to liquidate. The firm's capital is productive and thus its going-concern value is higher than liquidation. Intuitively, the firm never chooses to exercise its liquidation option before it runs out cash. Therefore, under liquidation, we have $p_1(0) = l_1$. Therefore, the firm chooses equity issuance provided the equilibrium firm value $p_1(0)$ is greater than l_1.

5 Fighting for Survival in a Crisis

For the quantitative analysis of our model, we begin with the simplest possible scenario that involves a change in external financing opportunities. In this scenario, the firm finds itself in a financial crisis situation – state B – in which it has no access to external financing whatsoever. Should the firm run out of cash it would then be forced into liquidation. The liquidation value of the firm in the crisis state is given by $l_B K$. Given that there are not many potential acquirers of assets in a crisis, we take the liquidation value to be low in that state: $l_B = 0.7$.

The crisis, however, is a transitory state and the firm knows that it can exit the crisis state with transition density ζ. When the state of nature switches from the crisis state B to state G, in which financial markets operate normally, the firm is able to access external equity markets by incurring a fixed cost of financing ϕK and a variable cost $\gamma = 6\%$. The firm can also be liquidated for a higher value in that state: we set $l_G = 1$. For expositional clarity, we consider first the polar case where state G is *absorbing*. That is, once the firm finds itself in state G it never switches out of this state and remains in a good stationary environment. Thus, the only state in which the firm is affected by a possible change in financing opportunities is state B. In this state the firm's overriding concern is *survival* as it cannot get any external financing.

The other parameter values we take in this simple scenario are as follows: the riskfree rate is assumed to be constant and is set at $r = 6\%$. The rate of depreciation of capital is $\delta = 10\%$. The mean and volatility of the risk-adjusted productivity shock are also constant and set at $\mu = 18\%$ and $\sigma = 20\%$, respectively. The cash-carrying cost is $\lambda = 1\%$, and the adjustment cost parameter is set to $\theta = 1.5$. Although in reality these parameter values clearly change with the state of nature, we keep them fixed under this scenario so as to isolate the effects of changes in external financing costs ϕ and γ.

Note first that under this scenario the firm is not in a position to be able to time the equity market for new equity issues. Indeed, in state B it has no access to external equity markets and in state G it has *permanent* access to equity markets under constant terms. However, the firm still faces a timing problem with respect to payout and investment. To what extent should the firm time (i.e. delay) payout and investment when it is in state B? This is the question we address in this section.

To determine the firm's value in each state of the world, and the implications of market timing for the firm's investment and payout policies, we focus on numerical plots for the firm's average q, $q^a(w)$, the sensitivity of q to changes in cash holdings, $(q^a(w))'$, the firm's conditional investment policy $i(w)$, and the cash sensitivity of investment $i'(w)$. These variables are defined as follows:

1. The firm's average Q in any given state $s = B, G$ is given by the firm's *enterprise value* – $P(K, W, s) - W$ – divided by the firm's capital stock:

$$q_s^a(w) = \frac{P(K, W, s) - W}{K} = p_s(w) - w, \tag{25}$$

   Thus, the sensitivity of Q to changes in cash holdings is simply $(q_s^a(w))' = p_s'(w) - 1$.

2. The firm's conditional investment in each state $s = G, B$ is given by the first-order optimality conditions:

$$i_s(w) = \frac{1}{\theta}\left(\frac{p_s(w)}{p_s'(w)} - w - 1\right),$$

   and the cash sensitivity of investment in turn is:

$$i_s'(w) = -\frac{1}{\theta}\frac{p_s(w)p_s''(w)}{p_s'(w)^2}. \tag{26}$$

Figure 1 plots the solution for respectively $q_B^a(w)$, $(q_B^a(w))'$ and $q_B^a(w)$, $(q_G^a(w))'$ by varying the parameter values for the transition probability out of the crisis state ζ. It plots the solutions for respectively $\zeta = 0, 0.5,$ and 2.0.

Figure 1 uncovers the effects on the *timing* of payouts to shareholders of changes in the transition probability out of the crisis state. Panel A plots $q_B^a(w)$ for $\zeta = 0, 0.5,$ 2.0, and gives the optimal *payout boundary* $\overline{w}_B$ in the transitory state B. It reveals that when the probability of exiting a crisis

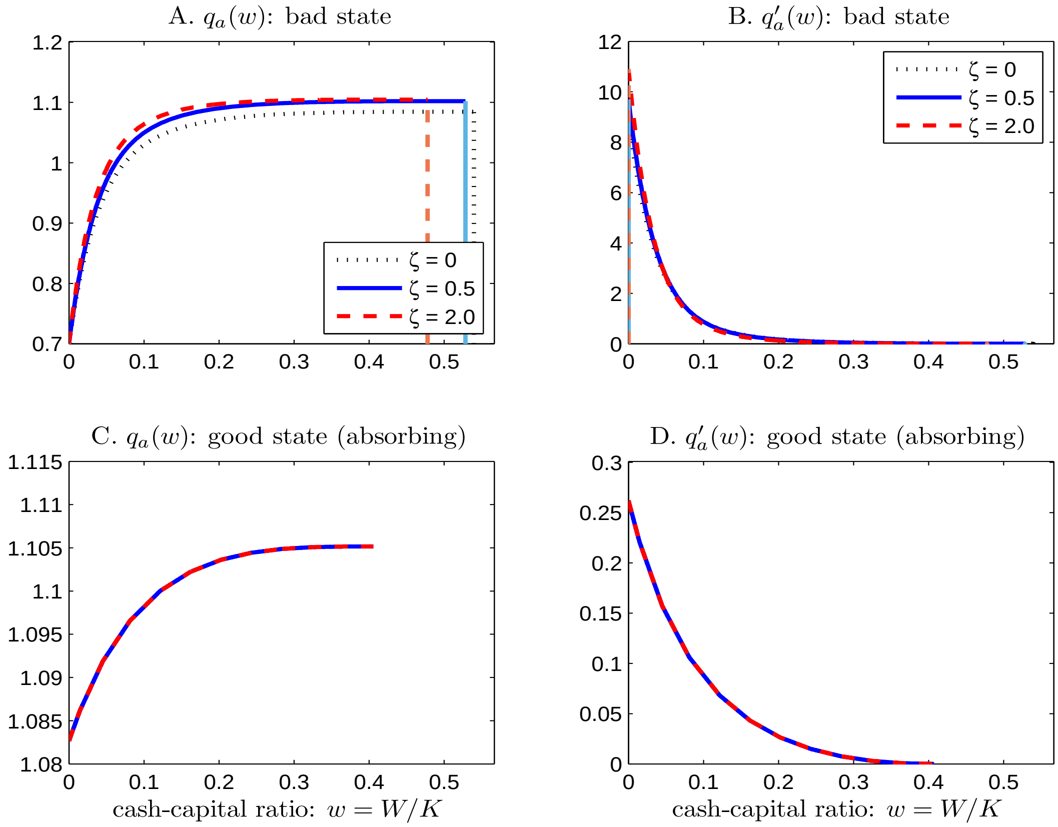

Figure 1: **From state B to G.** This figure plots the solution in the case when the firm has no access to external financing in state B, while state G is absorbing. The parameters are: riskfree rate $r_G = r_B = 6\%$, the mean and volatility of increment in productivity $\mu_G = \mu_B = 18\%$ and $\sigma_G = \sigma_B = 20\%$, adjustment cost parameter $\theta_G = \theta_B = 1.5$, capital depreciation rate $\delta_G = \delta_B = 10\%$, cash-carrying cost $\lambda_G = \lambda_B = 1.5\%$, and liquidation value-capital ratio $l_G = 1.0, l_B = 0.7$.

increases the firm responds by reducing its cash hoard and paying out more to its shareholders. Panel B plots $(q^a_B(w))'$, which immediately gives the shadow value of cash: $p'_B(w) = (q^a_B(w))' + 1$. It shows that as $w \longrightarrow 0$, the shadow value of the marginal dollar can rise as high as \$12 when $\zeta = 2.0$, \$11 when $\zeta = 0.5$, and \$9 when $\zeta = 0$. It is worth noting that the shadow value as $w \longrightarrow 0$ is higher when the probability of exiting the crisis state is also higher. This seems at first counter-intuitive, but it is simply a reflection of the fact that the *option to continue operations* is more valuable when the firm is more likely to transition out of the crisis state. Panels C and D respectively plot $q^a_G(w)$ and $(q^a_G(w))'$. As state G is *absorbing*, the firm is in a stationary environment once it ends

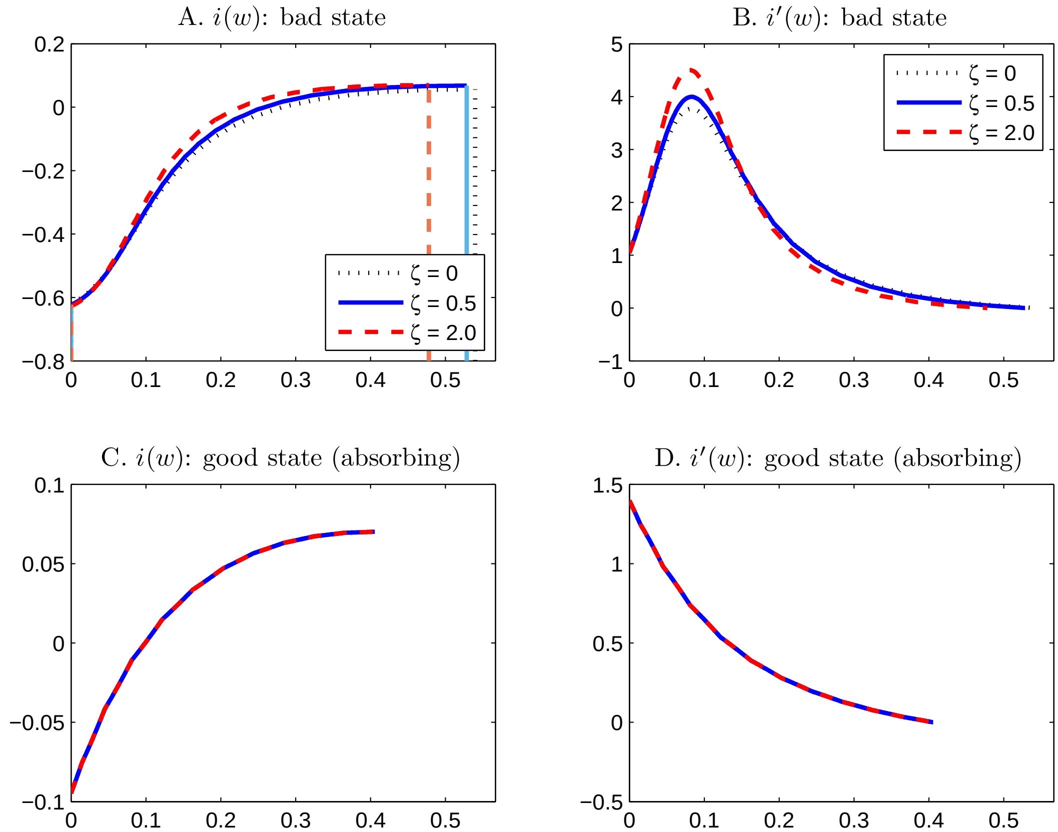

Figure 2: **Investment when transitioning from state B to G.** The parameters are the same as in Figure 1.

up in this state, so that the firm's dynamic investment, payout, and financing policy is as analyzed in BCW. It is interesting to compare the payout boundary $\overline{w}_G$ with the payout boundaries $\overline{w}_B$ in state B for different values of transition intensity ζ. Not surprisingly, the firm reduces its cash hoard once it enters state G. Also, the shadow value of cash is substantially smaller in state G, as it reaches at most \$1.25 as $w \longrightarrow 0$. Again, this is not surprising given that the firm has access to external equity markets in state G. Note finally that $q_G^a(w)$ is always larger than 1, while $q_B^a(w)$ can fall as low as 0.7. This difference simply reflects the fact that the firm has access to external financing in state G but not in state B.

Consider next Figure 2, which plots $i(w)$ and $i'(w)$ in states B and G for the same transition probabilities $\zeta = 0, 0.5$, and 2.0. Panel A shows that investment in state B is increasing in ζ, but the effect is not very large. The main driver of investment is actually the firm's cash stock. Panel B shows that the cash-sensitivity of investment $i'(w)$ is non-monotonic in w, and that this non-monotonicity becomes more pronounced as the firm is more likely to exit the crisis state. This non-monotonicity, however, does not translate into a non-monotonic investment policy. Panels C and D are not surprising and are consistent with the results obtained in BCW. Overall, this scenario reveals no major surprising finding. Market timing only affects payout and investment policy and it appears to affect these policies in an unsurprising way: when the firm is more likely to exit the crisis state it pays out more dividends and it (slightly) increases its capital expenditures. One small surprise is the tiny response in investment, but this may be due to our choice of parameter values.

One other surprising finding is captured in Figure 3. This figure plots the payout boundary $\overline{w}_B$ for the more realistic case where the firm cannot engage in asset sales ($i(w) \geq 0$) in state B, and highlights a non-monotonicity of $\overline{w}_B$ in ζ. The reason for this non-monotonicity seems to be the following. When $\zeta = 0$ the firm is stuck in a crisis state forever, in which investment returns net of financing costs are low. It then chooses to payout earnings to shareholders for lack of good investment opportunities. When ζ increases, however, the firm's investment opportunities are improved and it decides to retain more earnings. However, when ζ rises even more the firm knows that it is very likely to switch out of the crisis state, in which case it can again tap external equity markets. Therefore, it has less of a need to hold precautionary cash inventory.

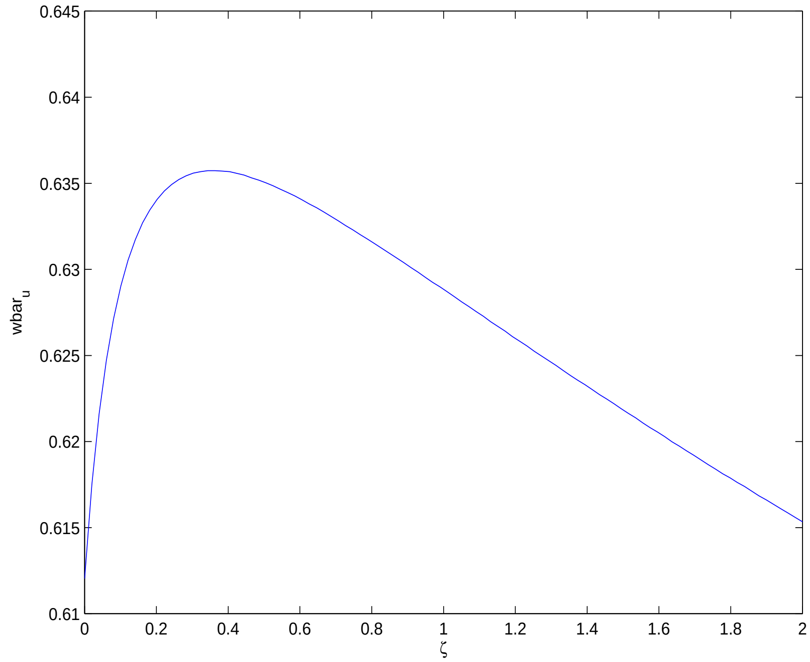

Figure 3: **Payout boundary in state B.** This figure plots the payout boundary in state B when the firm is waiting to move into state G for different values of the transition intensity ζ.

6 Market Timing: building a war-chest when financing is cheap

We now consider the scenario where the transitory state is state G, and the absorbing state is state B. This scenario describes in the simplest possible way the situation faced by a firm at the onset of a crisis. The firm understands that there is a risk that financial markets will *freeze up*. The firm therefore faces the decision in state G of whether to tap external equity markets while it still can, but at the cost of incurring issuing costs earlier than necessary, or take the chance that the crisis won't happen and save on these equity issuing costs. At the same time, the firm may choose to scale back its capital expenditures, and also to postpone payouts to shareholders, so as to preserve its *cash war-chest*.

As in the previous section, we take as our base line parameter values a constant riskfree rate of $r = 6\%$; a rate of depreciation of capital of $\delta = 10\%$; a constant mean and volatility of the risk-adjusted productivity shock of respectively $\mu = 18\%$ and $\sigma = 20\%$; a cash-carrying cost of

$\lambda = 1\%$; and, an adjustment cost parameter of $\theta = 1.5$. Although in reality these parameter values clearly change with the state of nature, we begin our analysis by keeping them fixed, so as to isolate the effects of changes in external financing costs from state G to state B.

As in the previous section, we focus on numerical plots for the firm's average Q, $q^a(w)$, the sensitivity of Q to changes in cash holdings, $(q^a(w))'$, the firm's conditional investment policy $i(w)$, and the cash sensitivity of investment $i'(w)$. Figures 4 and 5 plot the solution by varying in turn the parameter values for respectively the transition probability into the crisis state ζ and the fixed cost of external financing ϕ. Figure 4 plots the solution for respectively $q^a_G(w)$, $(q^a_G(w))'$ and $q^a_B(w)$, $(q^a_B(w))'$ for $\zeta = 0, 0.5,\ 2.0$, keeping ϕ fixed at $\phi = 0.01$. Next, Figure 5 plots the same functions $q^a_G(w)$, $(q^a_G(w))'$ and $q^a_G(w)$, $(q^a_B(w))'$ for $\phi = 0, 0.01, 0.05$, keeping ζ fixed at $\zeta = 0.05$.

Consider first Figure 4, which highlights the effects on *optimal market-timing* of changes in the transition probability into a crisis state. Panel A plots $q^a_G(w)$ for $\zeta = 0, 0.5,\ 2.0$, and gives: i) the optimal *market-timing boundary* $\underline{w}$, ii) the optimal *return point* for the cash stock m, and iii) the optimal *payout boundary* $\overline{w}$. Much insight into market timing can be gained from this panel. Note first that firm value is decreasing in ζ as one would expect. Second, there is no market to be *timed* when $\zeta = 0$. In that case, the dynamic pecking order of financing highlighted in BCW obtains, as the firm then finds itself in a stationary environment where it can always get access to external equity markets. Since it is costly to tap external equity it is optimal for the firm to postpone an equity issue until it runs out of cash. In other words, the firm's financing policy follows a dynamic pecking order by first relying on cash and only as a last resort relying on external equity financing. Market-timing arises however when $\zeta = 0.5$ or 2.0. In that case the firm decides to issue equity when its cash stock falls below a boundary $\underline{w} > 0$. This boundary is close to zero (around 0.02) when $\zeta = 0.5$, but is much higher (around 0.07) when $\zeta = 2.0$. The return point m also shifts to the right as ζ increases, but it appears that the firm does not respond to a greater crisis risk by raising more funds $(m - \underline{w})$; rather it responds by returning to the equity market sooner. The reason is

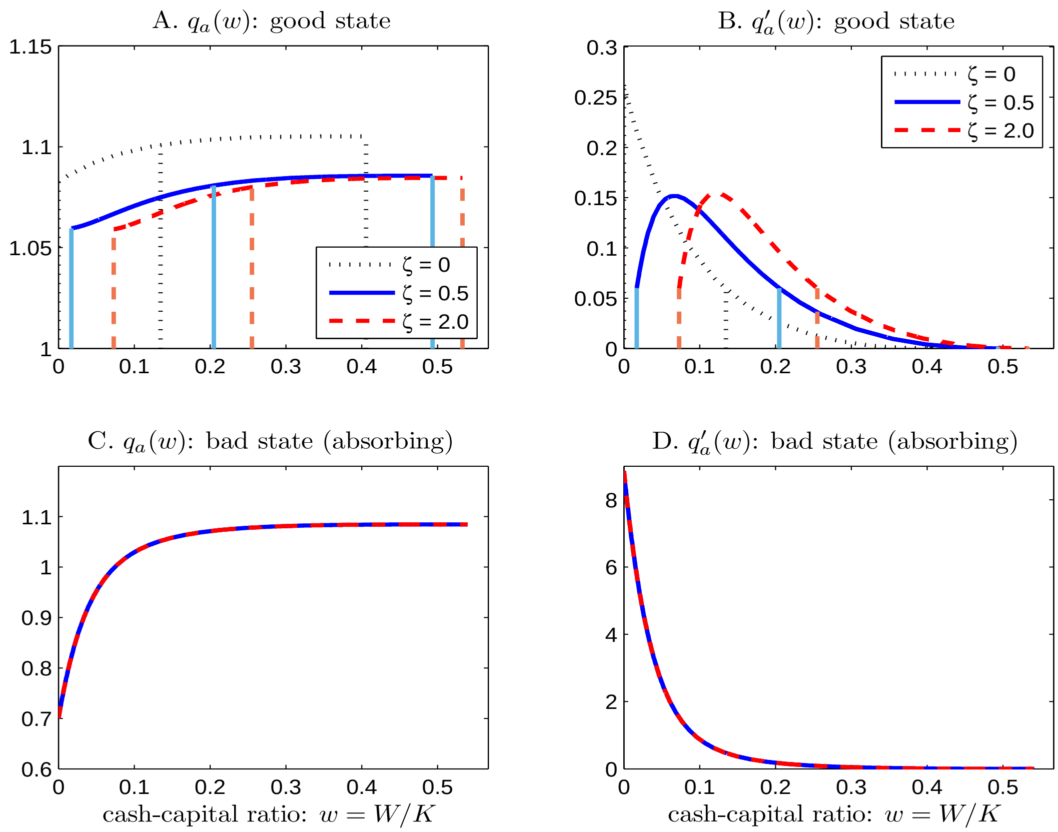

Figure 4: **From state G to B.** This figure plots the solution in the case when the firm has no access to external financing in state B, and state B is absorbing. The parameters are: riskfree rate $r_G = r_B = 6\%$, the mean and volatility of increment in productivity $\mu_G = \mu_B = 18\%$ and $\sigma_G = \sigma_B = 20\%$, adjustment cost parameter $\theta_G = \theta_B = 1.5$, capital depreciation rate $\delta_G = \delta_B = 10\%$, cash-carrying cost $\lambda_G = \lambda_B = 1.5\%$, and liquidation value-capital ratio $l_G = 1.0, l_B = 0.7$.

that there is an offsetting cash-carrying cost λ the firm is trying to avoid by holding too much cash. The firm also chooses to preserve its cash war-chest in response to a greater crisis risk by postponing payouts to shareholders. This can be seen by the sharp shift to the right in the payout boundary $\overline{w}$ as ζ rises. In sum, Panel A shows that through a combination of market-timing and reduced dividend payout it is optimal for a firm to respond to a greater crisis risk by holding more cash on average.

Panel B plots $(q_G^a(w))'$ the shadow value of cash as measured by the sensitivity of firm average Q to changes in cash holdings. When $\zeta = 0$ we obtain the expected result that the shadow value of cash is monotonically decreasing in the firm's cash holdings, and converges to one ($p'_G(w) \longrightarrow 1$ and therefore $(q^a_G(w))' \longrightarrow 0$) as the firm's cash stock approaches the payout boundary $\overline{w}$. Much more surprising is the non-monotonicity in the shadow-value of cash under market-timing (when $\zeta = 0.5$ or 2.0). The shadow-value of cash reaches a peak in the interior (around $w = 0.05$ when $\zeta = 0.5$ and $w = 0.13$ when $\zeta = 2.0$). This non-monotonicity in $p'(w)$ reveals a fundamental *non-convexity in firm value under market timing*. When $w \longrightarrow \underline{w}$ the firm's shadow value of cash decreases as the firm gets closer to the point where it optimally times the market. However, as already highlighted, timing the market involves deadweight equity issuance costs, which the firm would rather avoid in the absence of timing benefits. When the firm already has substantial cash holdings (say w is close to the payout boundary $\overline{w}$) there is no benefit in timing the market and the dominant concern for the firm is to avoid incurring deadweight equity issuance costs. This is why the shadow cost of cash is rising at that point when the firm's cash holding decline. In sum, the economics of market timing naturally give rise to a non-convexity in firm value.

Panels C and D respectively plot $q^a_B(w)$ and $(q^a_B(w))'$. Since state B is absorbing there is no market-timing in this state and firm value is unaffected by changes in ζ. The sharp concavity in $q^a_B(w)$ reflects the fact that the firm cannot tap external equity markets at all. If it runs out of cash it is simply liquidated. It is interesting to compare the payout boundary $\overline{w}_B$ with the payout boundary $\overline{w}_G$ in state G when $\zeta = 2.0$. The payout boundaries are almost the same at around 0.53. In other words, a firm in state G, which anticipates a high probability of a crisis is as conservative in its cash hoarding policy as a firm in the midst of a crisis, which has no access to external financing. The shadow cost of cash is, as expected, monotonically decreasing in the firm's cash holding in state B. It is worth noting, however, how much higher the shadow cost of cash is in state B than in state G.

Consider next Figure 5. In this figure the transition probability ζ is held constant at $\zeta = 0.5$ and the fixed cost of equity financing ϕ is varied from $\phi = 0$ to $\phi = 0.05$. This figure highlights how

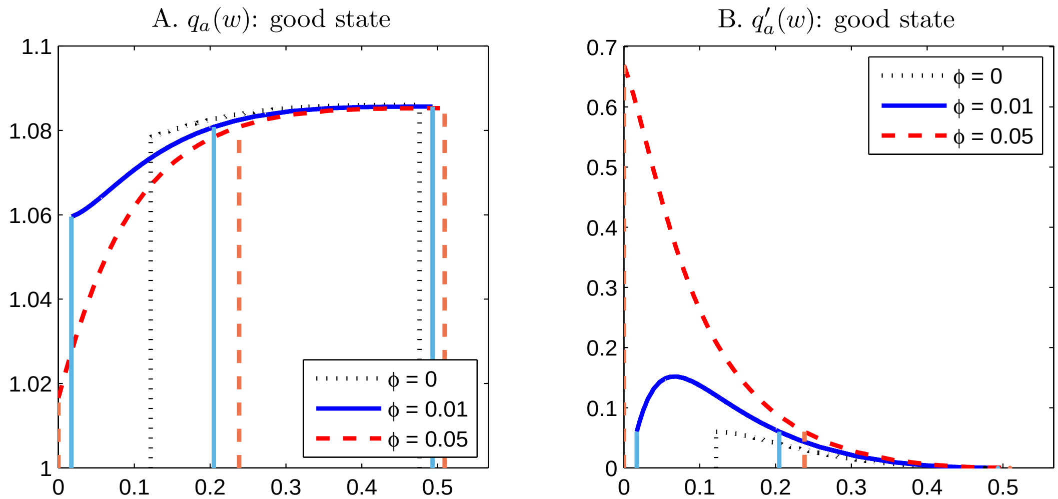

Figure 5: **From state G to B.** This figure plots the solution in the case when the firm has no access to external financing in state B, and state B is absorbing. We consider three different values of fixed cost of equity financing (ϕ_G) for state G. The transition intensity is $\zeta = 0.5$. The remaining parameters are the same as in Figure 4.

changes in equity issuance cost affect market timing. Not surprisingly when $\phi = 0$, the firm tends to time the market sooner (at a higher boundary $\underline{w}$) and much more frequently, as $(\underline{w} - m) = 0$ in that case. Most interesting is the implication for the shadow-value of cash (in Panel B) of the absence of a fixed cost of issuing equity. Indeed, in this case the shadow-value of cash is again monotonically decreasing in w, as the firm is not concerned about avoiding a fixed deadweight cost of issuing equity. When the fixed cost of issuing equity is very high, at $\phi = 0.05$, the shadow value of cash is again monotonically decreasing in w. Here the reason is simply that for such a high fixed cost it is not worth timing the market. The cost of equity has not fallen sufficiently to make it worthwhile for the firm to time the equity market. Thus, it is only for intermediate values of ϕ (here $\phi = 0.01$) that market timing results in a non-monotonic shadow value of cash and a non-convexity in the firm's value.

How is corporate investment affected by market timing? We answer this question in Figures 6 and 7, which plot $i(w)$ and $i'(w)$ in state G and B for different transition probabilities ζ and fixed cost of external financing ϕ. Consider first Panel A in Figure 6. This panel plots $i(w)$ for $\phi = 0.01$

and $\zeta = 0, 0.5$, and 2.0. Note first that when $\zeta = 0$, so that the firm finds itself in steady state in state G, then as expected investment is increasing (or disinvestment is decreasing) in the firm's cash reserves w. This result follows directly from the observation that the shadow cost of cash is decreasing in w, so that the firm effectively faces a lower cost of capital at higher w and therefore invests more. When there is a positive probability of switching into a crisis state, however, then *investment is non-monotonic in* w. This non-monotonicity is an inevitable consequence of market timing. Indeed, as we observed in Panel B of Figure 4, the shadow value of cash is non-monotonic when there is market timing. This translates into a non-monotonic cost of investment, which explains why investment is non-monotonic. This non-monotonicity is significant and is not a model artifact. It is to be expected when firms face changing financing opportunities and time equity markets. The reason is straightforward and is due to the non-monotonic behavior of the shadow value of cash under market timing. Comparing the behaviors of $i_G(w)$ and $i'_G(w)$ in respectively Panels A and B, with the behavior of $i_B(w)$ and $i'_B(w)$ in Panels C and D, we also note that although the cash-sensitivity of investment in state B may be non-monotonic, investment is always increasing monotonically with w in state B. This is because there is no market timing effect in state B.

Turning to Figure 7, we again see that since market timing is only possible for intermediate values of ϕ, the non-monotonicity of investment in w is only present for the intermediate value of ϕ ($\phi = 0.01$). For very low ($\phi = 0$) and very high ($\phi = 0.05$) costs of equity financing investment is monotonically increasing in w. The intermediate value of ϕ is, however, the most plausible one, so that it is to be expected that corporate investment does indeed exhibit this non-monotonic pattern in practice. This finding is yet another warning for empirical studies on the link between corporate investment and cash. A simple linear regression linking capital expenditure to average Q and cash will be misspecified and produce estimated coefficients on the cash variable that are impossible to interpret.

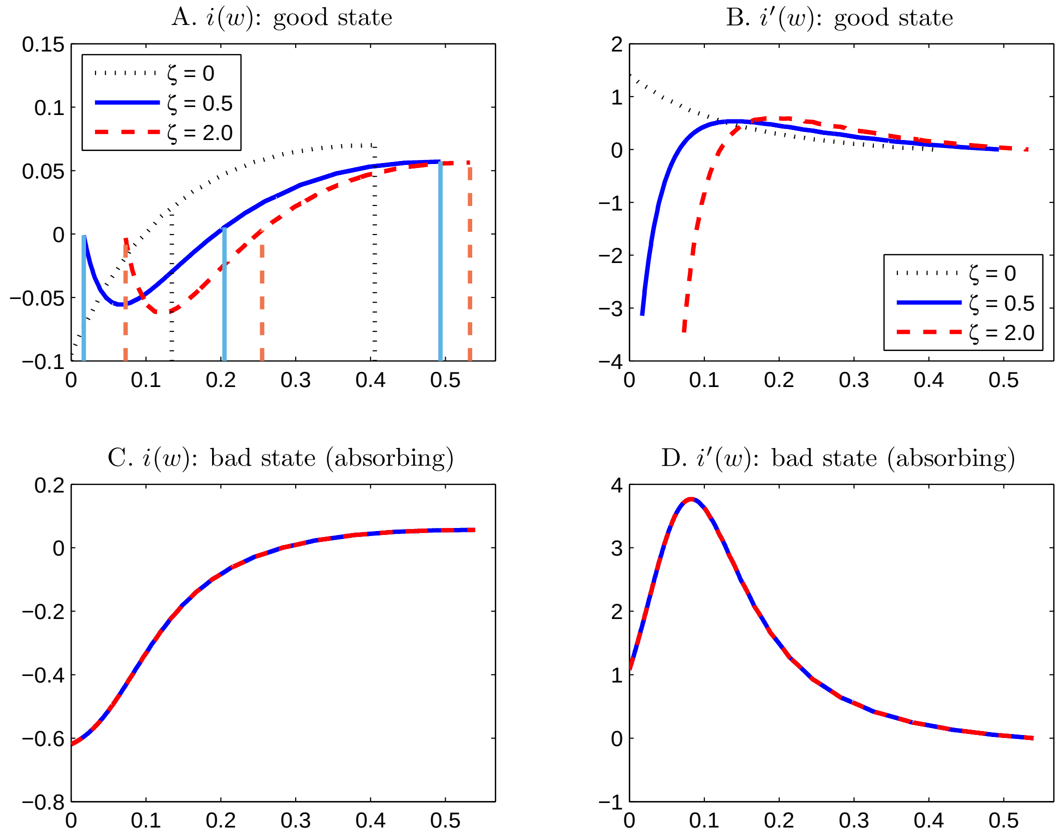

Figure 6: **Investment when transitioning from state G to B.** The parameters are the same as in Figure 4.

As suggestive as these results are, they nevertheless rely on rather implausible assumptions and firm behavior in the crisis state. In particular, as Panel C in in Figure 6 highlights, the firm in state B engages in substantial asset sales when its cash stock declines. However, in a crisis firms often find it impossible to sell assets at a reasonable price. To see the implications for market timing of constraints on asset sales in state B, we briefly explore a variant of our model in which asset sales are substantially more costly or even impossible in state B. Our results for this variant of our model are plotted in Figures 8 and 9.

Consider first Figure 8. This figure plots the solution for respectively $q_G^a(w)$, $(q_G^a(w))'$ and $q_B^a(w)$, $(q_B^a(w))'$ for fixed $\phi = 0.01$, fixed $\zeta = 0.5$, fixed $\theta_G = 1.5$, but variable investment adjustment costs θ_B in state B, ranging from $\theta_B = 1.5$ to $\theta_B = \infty$ (for asset sales, i.e. $i < 0$). The first notable

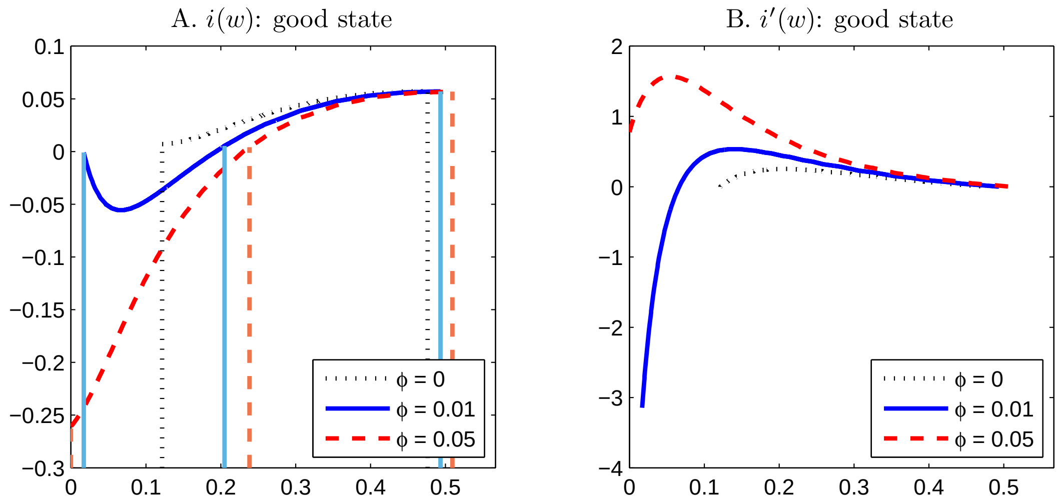

Figure 7: **Investment when transitioning from state G to B.** The parameters are the same as in Figure 5.

observation from Panel A is that the reduced flexibility in selling assets in state B substantially affects firm value in state G, and to a lesser extent also in state B (see Panel C). As expected, the firm times the market sooner in response to this reduced flexibility ($\underline{w}$ is higher other things equal) and also delays dividend payouts to shareholders. Turning to Figure 9, Panel A, the effect of the reduced flexibility in asset sales in state B does not significantly affect the non-monotonicity of investment in w in state G. The large, but somewhat hard-wired, effect is on asset sales in state B, which are virtually non-existent for $\theta \geq 10$. The other significant but expected effect of reduced asset sales in state B is on payout policy in state B, which becomes significantly more conservative.

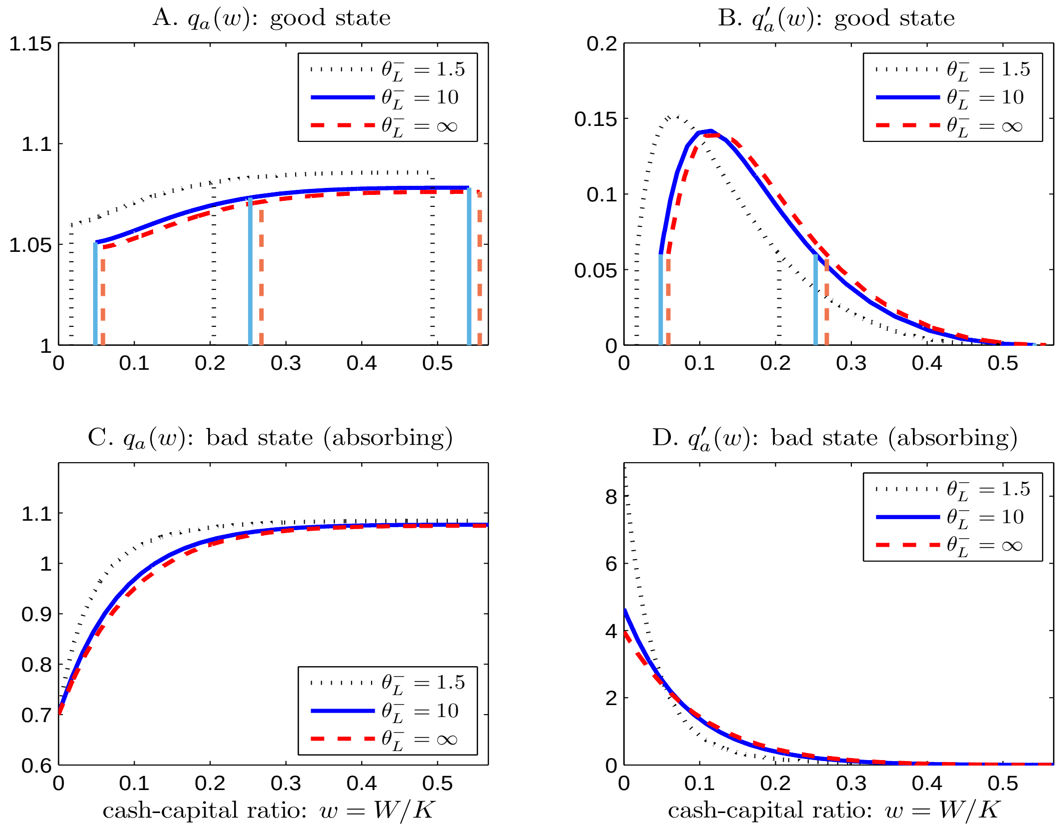

Figure 8: **From state G to B.** This figure plots the solution in the case when the firm has no access to external financing in state B, and state B is absorbing. We consider three different values of adjustment cost for asset sales in state B.

7 Market Timing and Lumpy Payout

In this section we introduce fixed costs in paying out cash to shareholders into the model. Specifically, we assume that the firm incurs a fixed cost $\overline{\phi}K$ when paying out cash to shareholders. This cost can, for example, be thought of as the cost of organizing and announcing a share-repurchase. [5] There are two reasons for introducing such a fixed cost. The first obvious reason is realism: payouts such as special dividends and share repurchases in reality are lumpy and infrequent. To be able to generate this type of payout behavior in our model, we need to introduce fixed costs in paying

[5] It can also reflect the "opportunity cost" of organizing a share repurchase. Under IRS rules a firm cannot put in place a "permanent" repurchase program and at the same time avoid dividend taxation of its payouts. This is one reason why in practice firms space out their share repurchases.

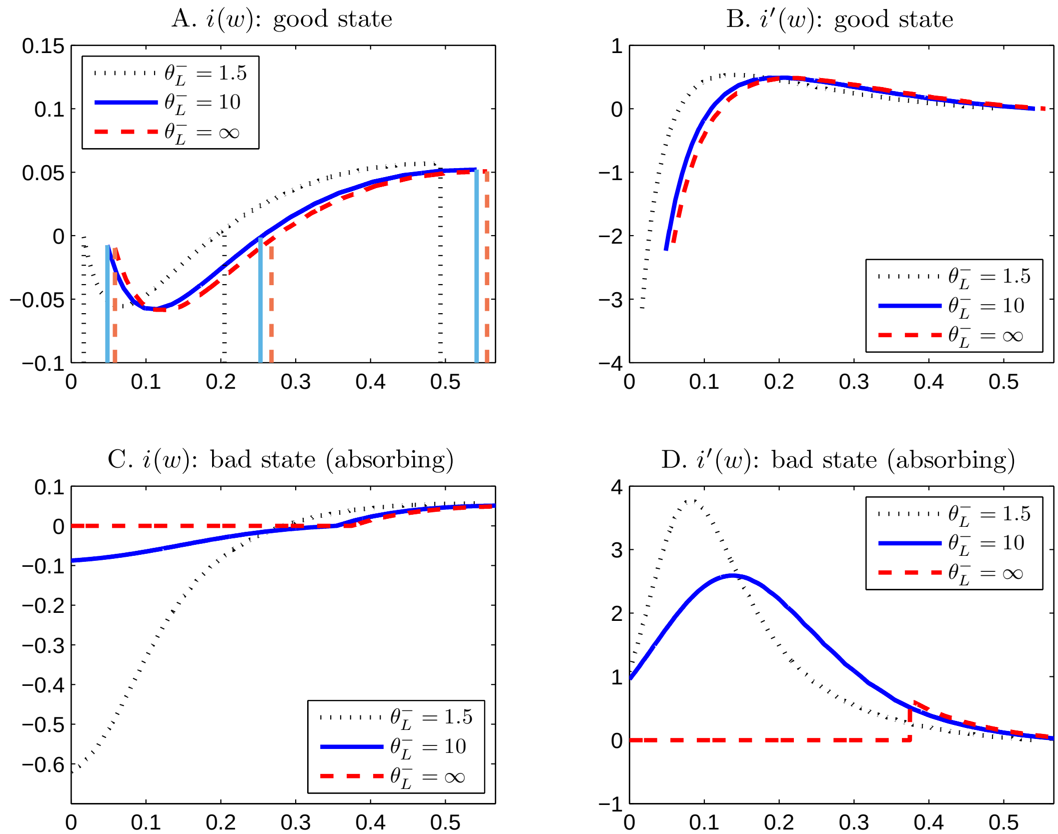

Figure 9: **Investment when transitioning from state G to B.** As in Figure 8, we consider three different values of adjustment cost for asset sales in state B.

out cash. The second reason is theoretical: the fixed payout cost generates another non-convexity in the firm's value function around the payout boundary. By highlighting this non-convexity, and its effects on the firm's investment policy, we gain further insight into the source of investment non-monotonicity associated with market timing.

Let $\overline{w}_s$ denote again the endogenous upper boundary for the cash-capital ratio at which the firm distributes cash to shareholders, and let $\overline{m}_s$ denote the *return* cash-capital ratio the firm targets when it distributes cash to shareholders. That is, if the firm decides to pay out, it chooses the optimal payout size $\overline{w}_s - \overline{m}_s$ per unit of capital. Since the firm value-capital ratio is continuous at the cash distribution point, we have:

$$p_s(\overline{w}_s) - \overline{\phi} - (\overline{w}_s - \overline{m}_s) = p_s(\overline{m}_s) .$$

Moreover, if the target cash-capital ratio following distribution $\overline{m}_s$ is chosen optimally, then the marginal value of the last dollar distributed must be equal to one, the marginal cost of distribution, so that

$$p_s'(\overline{m}_s) = 1.$$

Finally, the upper boundary $\overline{w}_s$ is also chosen optimally, so that

$$p_s'(\overline{w}_s) = 1.$$

The above three boundary conditions characterize the firm's optimal payout policy in state $s = B, G$ when the firm faces fixed payout costs $\overline{\phi}K$.

To avoid repetition, we only consider the effects of fixed payout costs in the scenario where state G is transitory and state B is absorbing. As before, our baseline parameter values are: $r = 6\%$ for the riskfree rate; $\delta = 10\%$ for the rate of capital depreciation; $\mu = 18\%$ and $\sigma = 20\%$ for respectively the mean and volatility of the productivity shock; $\lambda = 1\%$ for the cash-carrying cost; and, $\theta = 1.5$ for the adjustment cost parameter. As for the fixed cost parameters, we set $\underline{\phi} = 1\%$ as before, and $\overline{\phi} = 0.2\%$. We choose the latter cost to be somewhat higher than in reality in order to generate more visible effects in the figures below.

Figure 10 plots the solution for respectively $q_B^a(w)$, $(q_B^a(w))'$ and $q_G^a(w)$, $(q_G^a(w))'$ for a transition probability $\zeta = 0.05$.

Panel A plots $q_G^a(w)$ and reveals the non-convexity of the firm's value function near the external financing and the payout boundaries. The firm chooses to pay out cash when it hits the boundary $\overline{w}_G = 0.716$, and its return point is $\overline{m}_G = 0.383$, so that it makes a lumpy payment of $(\overline{w}_G - \overline{m}_G) = 0.333$ (or 46% percent of its liquid assets). Similarly, the firm chooses to raise equity when it hits the boundary $\underline{w}_G = 0.039$ and its lower return point is $\underline{m}_G = 0.22$, so that it raises a total amount $(\underline{m}_G - \underline{w}_G) = 0.18$. Panel B plots the (normalized) shadow value of cash $(q_G^a(w))'$. As a result of the introduction of fixed payout costs there are now two regions where $(q_G^a(w))'$ is non-monotonic in w. Most striking is the finding that *the shadow value of cash may now fall below one*: when

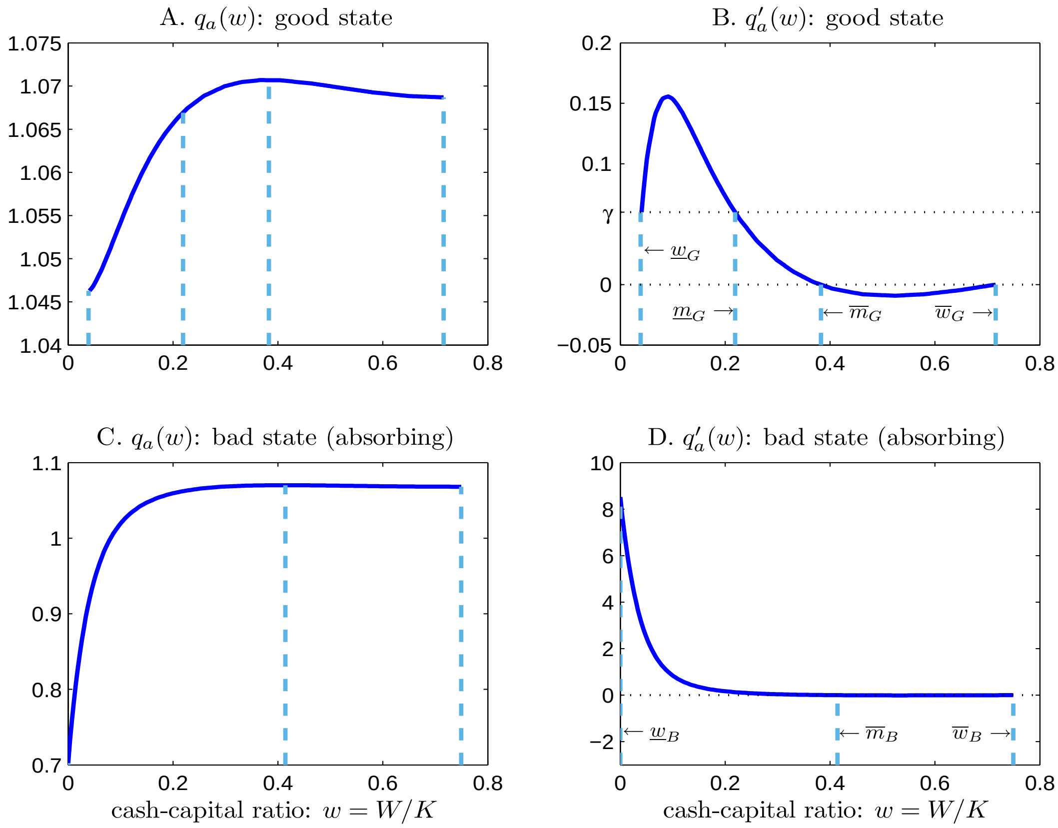

Figure 10: **From state G to B with fixed cost of payout.** The fixed cost of payout is $\psi_G = \psi_B = 0.2\%$.

$(q_G^a(w))' < 0$ this means that $p_G'(w) < 1$. When the shadow value of cash falls below one, the firm is effectively looking to do away with its cash.

Panels C and D respectively plot $q_B^a(w)$ and $(q_B^a(w))'$. Not surprisingly, in the crisis state (state B) the firm is both more conservative in when it chooses to pay out cash ($\overline{w}_B = 0.75$), but the amount it chooses to pay out is of similar size ($\overline{m}_B = 0.41$, so that $(\overline{w}_B - \overline{m}_B) = 0.34$). Although this is not clearly visible from the plots $q_B^a(w)$ and $(q_B^a(w))'$ it turns out that in this state the firm's value function is also non-convex around the payout boundary. We learn from this solution that the non-convexity at the payout boundary can be substantially attenuated (to the point of being virtually eliminated) by extreme external financing constraints.

Consider next Figure 11, which plots $i(w)$ and $i'(w)$ in states G and B.

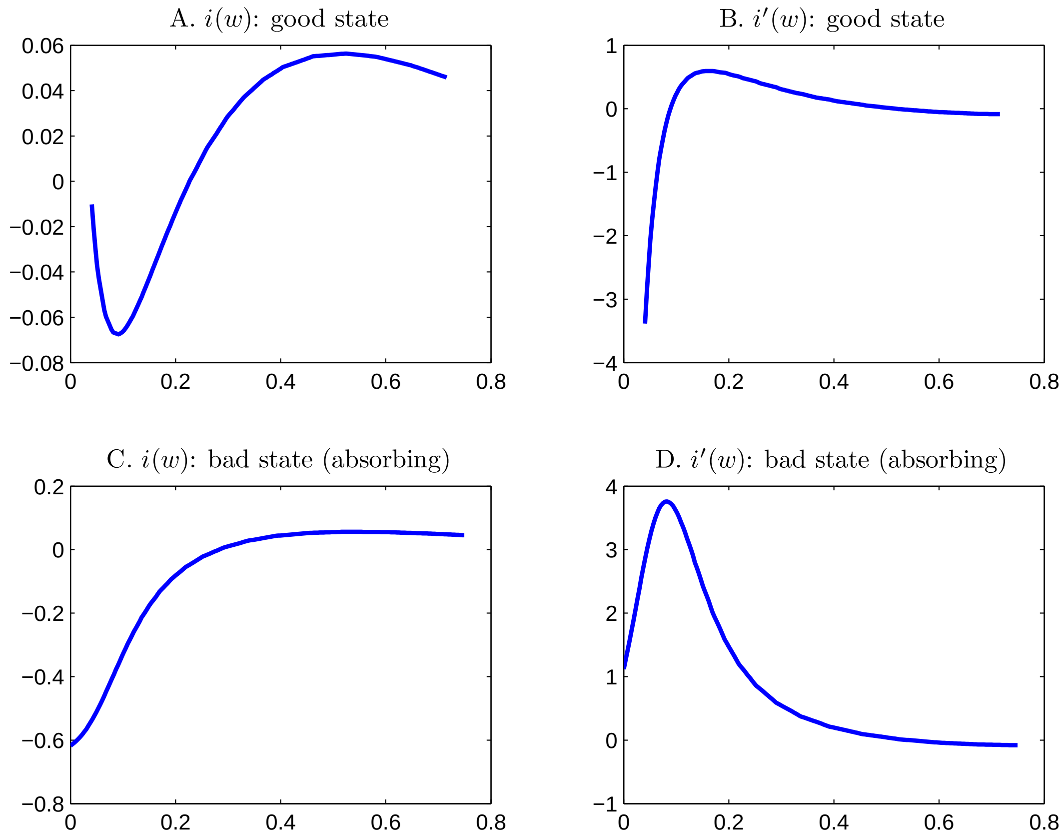

Figure 11: **Investment when transitioning from state G to B with fixed cost of payout.** The fixed cost of payout is $\psi_G = \psi_B = 0.2\%$.

Panel A displays the effects of the non-monotonicity of the shadow value of cash $(q^a_G(w))' = (p'_G(w) - 1)$ on investment in state G. Investment may now be decreasing in the firm's cash holdings w both near the external equity financing boundary and near the payout boundary. These effects follow directly from the first-order optimality conditions for investment:

$$i_G(w) = \frac{1}{\theta}\left(\frac{p_G(w)}{p'_G(w)} - w - 1\right).$$

Given that the non-monotonicity of $p'_B(w)$ is much more attenuated that that of $p'_G(w)$, we expect the decline in investment when w converges to $\overline{w}_s$ to be much smaller in state B than in state G. This is indeed what the comparison of panels A and C reveals. What is not apparent from this figure though is the surprising result that for some values of w close to $\overline{w}_s$ investment in state B

may actually be higher than in state G. Granted, this is a quantitatively tiny difference. Still, it highlights the potentially complex effects of changing financing opportunities and market timing on corporate investment.

Finally, note that changes in fixed payout costs can also have subtle effects on firm's market timing policy. A decrease in fixed payout costs from $\overline{\phi} = 0.2\%$ to $\overline{\phi} = 0.1\%$ lowers the payout boundary to $\overline{w}_G = 0.64$, and raises the return point to $\overline{m}_G = 0.39$ as one might expect. However, the equity financing boundary is also slightly higher $\underline{w}_G = 0.041$, meaning that the firm chooses to time the market earlier when payout is less costly. A simple prediction of our analysis is thus that as a result of the US dividend and capital gains tax cut of 2003, we should have observed not only more dividend payouts and stock repurchases (as has been documented by Auerbach and Hassett, 2007) but also (and somewhat counter-intuitively) more equity issues.

8 Market Timing and Dynamic Hedging

How does market timing affect the firm's other risk management decisions? And, vice-versa, how does the firm's dynamic hedging strategy affect its market timing behavior? We turn to these questions in this section by introducing the possibility for the firm to take hedging positions in an index futures contracts, and thus change its exposure to systematic risk.

We denote by F the index futures price. Under the risk-neutral probability measure F evolves according to:

$$dF_t = \sigma_m F_t dB_t \,, \tag{27}$$

where σ_m is the volatility of the market portfolio, and B_t is a standard Brownian motion that is correlated with Z_t with coefficient ρ. We denote by ψ_t the fraction of total cash W_t that the firm invests in the futures contract. Futures contracts require that investors hold cash in a *margin account*. Thus, let $\kappa_t \in [0,1]$ denote the fraction of the firm's total cash W_t held in the margin account; cash held in this margin account incurs a flow unit cost $\epsilon \geq 0$. Futures market regulations typically require that an investor's futures position (in absolute value) cannot exceed a regime-dependent multiple, $\pi(s_t)$, of the amount of cash $\kappa_t W_t$ put in the margin account. We therefore impose the constraint: $|\psi_t| \leq \pi(s_t)\kappa_t$. As the firm can costlessly reallocate cash between the margin account and its regular interest-bearing account it optimally holds the minimum amount of cash necessary in the margin account when $\epsilon > 0$. For simplicity, we shall ignore this small haircut on the margin account and assume that $\epsilon = 0$. Under that assumption we can set $\kappa_t = 1$, so that the evolution of the firm's cash stock when it takes a position $\psi_t W_t$ in the futures index is as follows:

$$dW_t = K_t \left(\mu(s_t) dA_t + \sigma dZ_t\right) - \left(I_t + G_t\right) dt + dH_t - dU_t + (r - \lambda) W_t dt + \psi_t W_t \sigma_m dB_t, \tag{28}$$

where $|\psi_t| \leq \pi(s_t)$.

Given that the firm has the possibility to engage in dynamic hedging of cash-flow risk by taking positions in a futures index, we now turn to the characterization of the firm's optimal hedging strategy in each state of nature and how dynamic hedging alters its' funding, investment and payout decisions. We begin with the easier case of hedging in the absorbing state.

Let the absorbing state be the "bad" state, i.e. the one with higher financing cost. This case is more interesting as we have shown because it potentially generates market timing in the current (good) state G.

8.1 Risk management in the absorbing state

In the absorbing state 2, the firm now chooses both its investment intensity I and its index futures position ψW to maximize its value $P(K, W, 2)$ given by the following HJB equation:

$$\begin{aligned} r_2 P(K,W,2) &= \max_{I,\psi} \left[(r_2 - \lambda_2) W + \mu_2 K - I - G(I,K,2) \right] P_W(K,W,2) + (I - \delta K) P_K(K,W,2) \\ &\quad + \frac{1}{2} \left(\sigma_2^2 K^2 + \psi^2 W^2 \sigma_m^2 + 2\rho \sigma_m \sigma_2 \psi W K \right) P_{WW}(K,W,2), \end{aligned}$$

subject to $|\psi| \leq \pi_2$.

Consider the case where $\rho > 0$. Note that the first two terms in the maximand of the firm only involve the control variable I, and the last term only involves the control variable ψ. The problem of optimizing I is therefore separable from the problem of optimizing ψ. The first-order condition for optimal investment I^* is therefore unchanged, and the first-order condition for ψ^* is given by differentiating the last term with respect to ψ:

$$\psi W \sigma_m^2 = -\rho \sigma_m \sigma_2 K.$$

Rearranging and dividing by W, we obtain the following optimal hedging rule:

$$\psi_2^*(w) = \max \left\{ \frac{-\rho \sigma_2}{w \sigma_m}, -\pi_2 \right\}. \tag{29}$$

This rule is the same as the one in the one-regime model (i.e. Bolton, Chen, and Wang (2009)) for the special case where $\epsilon = 0$ and $\kappa = 1$. Consider next, the solution in the transient state.

8.2 Risk management and risk taking in the transient state

In the transient state 1, the firm chooses its investment intensity I and its index futures position ψW to maximize the value $P(K,W,1)$ given by:

$$\begin{aligned} rP(K,W,1) &= \max_{I,\psi} \left[(r_1 - \lambda_1) W + \mu_1 K - I - G(I,K,1) \right] P_W(K,W,1) + (I - \delta K) P_K(K,W,1) \\ &\quad + \frac{1}{2} \left(\sigma_1^2 K^2 + \psi^2 \sigma_m^2 W^2 + 2\rho\sigma_m\sigma_1 \psi W K \right) P_{WW}(K,W,1) + \zeta \left[P(K,W,2) - P(K,W,1) \right] . \end{aligned}$$

subject to $|\psi| \leq \pi_1$.

Consider again the case where $\rho > 0$. When $P_{WW}(K,W,1) < 0$, the analysis is the same as in the absorbing state and we obtain:

$$\psi_1^*(w) = \max\left\{ \frac{-\rho\sigma_1}{w\sigma_m}, -\pi_1 \right\} . \tag{30}$$

Recall, however, that market timing combined with a fixed costs of equity issuance may result in a $P_{WW}(K,W,1) > 0$ for some range of $w = W/K$. With $P_{WW}(K,W,1) > 0$, the firm wants to *gamble* and the first-order condition no longer characterizes the optimal solution. The optimal hedging policy is then given by the corner solution $\psi_1(w) = \pi_1$, at which the firm maximizes its gambling opportunity.

Let $\widehat{w}_1$ denote the endogenously chosen point at which $P_{WW}(K,W,1) = 0$, or $p_1''(\widehat{w}_1) = 0$. The firm's optimal hedging policy in the transient state is then given by:

$$\psi_1^*(w) = \begin{cases} \max\left\{ \frac{-\rho\sigma_1}{w\sigma_m}, -\pi_1 \right\} & \text{for} \quad w \geq \widehat{w}_1 \\ \pi_1 & \text{for} \quad \underline{w}_1 \leq w \leq \widehat{w}_1 \end{cases}$$

8.3 Quantitative Analysis

As before our parameter values are: i) a constant risk-free rate of $r = 6\%$; ii) a rate of depreciation of capital of $\delta = 10\%$; iii) a constant mean and volatility of the risk-adjusted productivity shock of respectively $\mu = 18\%$ and $\sigma = 20\%$; iv) a cash-carrying cost of $\lambda = 1\%$; and, v) an adjustment cost parameter of $\theta = 1.5$ when asset sales are allowed.

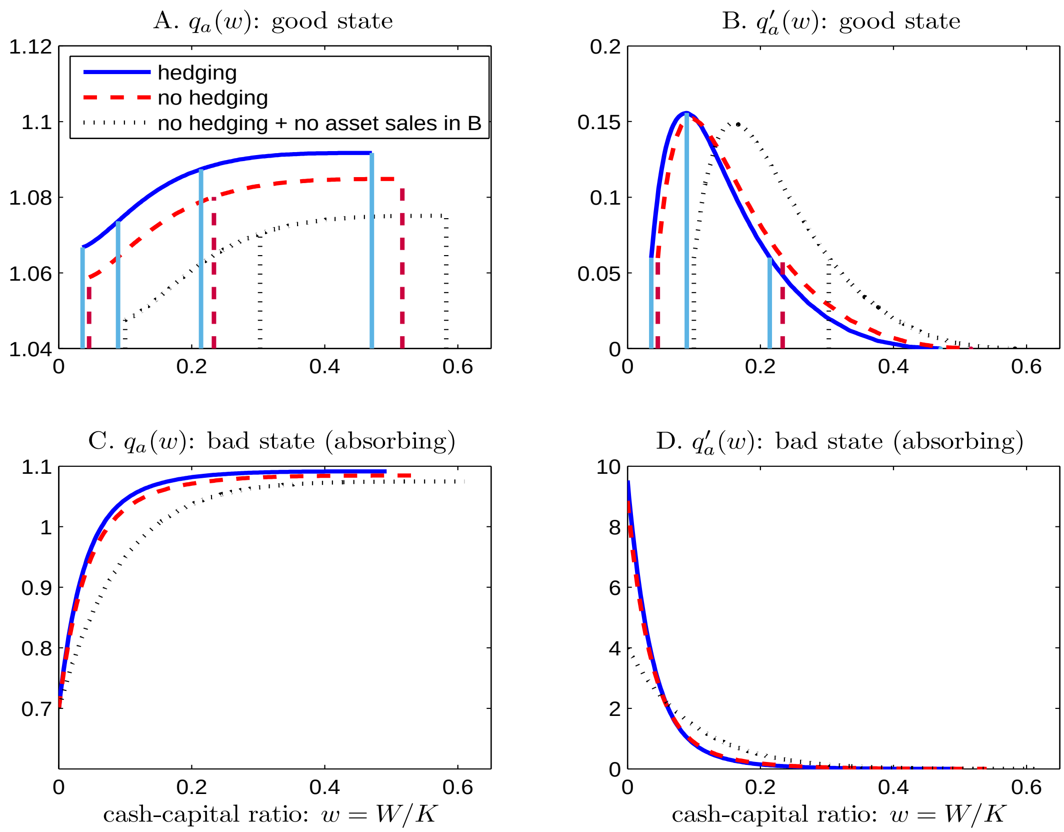

Figure 12: **Dynamic hedging.** This figure plots the solution in the case with dynamic hedging. The firm has no access to external financing in state B, and state B is absorbing. The solution with hedging is compared to the case without hedging and the case without either hedging or asset sales in state B. For the hedging case, we assume $\epsilon_G = \epsilon_B = 0$, $\pi_G = 5$, and $\pi_B = 2$.

We begin our discussion with the numerical plots for the firm's average Q, $q^a(w)$, and the sensitivity of Q to changes in cash holdings, $(q^a(w))'$. Figure 12 plots three solutions in the good and the bad state for a transition probability into the crisis state of $\zeta = 0.5$. The first solution is for a highly constrained firm that can neither engage in any dynamic hedging, nor in any asset sales in the bad state of the world. The second solution is for a less constrained firm that can engage in asset sales, but not in any dynamic hedging. The third solution is for a firm that takes full advantage of all its risk management tools and engages in both asset sales and in dynamic hedging. For this solution we take $\rho = 0.4$, $\sigma_m = 0.2$ and $\pi_G = 5$; $\pi_B = 2$.

It is striking to observe from Panel A how much the possibility of engaging in asset sales adds to

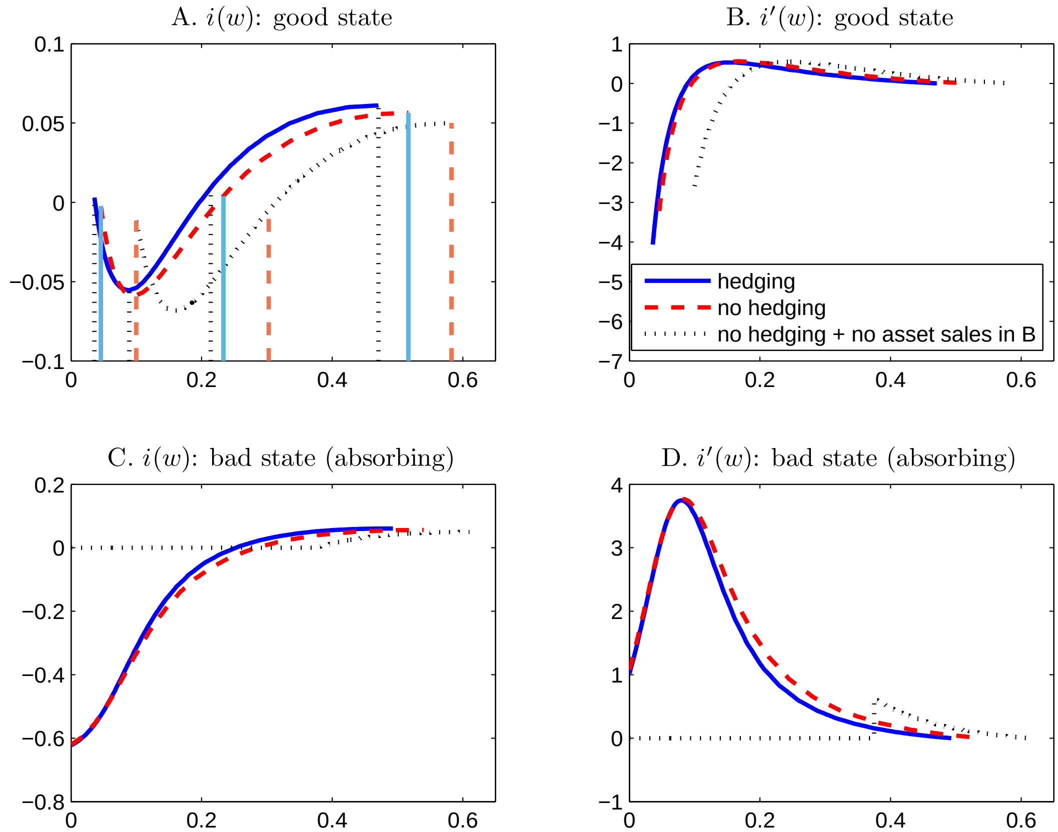

Figure 13: **Investment with dynamic hedging.** The parameters are the same as in Figure 12.

firm value and reduces the incentives for market timing. Dynamic hedging, in turn, barely affects market timing incentives when it comes to raising new equity. However, payout policy is more aggressive and firm value is significantly enhanced. It is also worth noting from Panel D that the marginal value of cash in the bad state is lower when the firm cannot engage in asset sales for w close to 0. This is due to the fact that firm value in the bad state is substantially lower when asset sales are not possible (as illustrated in Panel C), so that the *value of survival* (which is reflected in the marginal value of cash) is lower.

Consider next the firm's conditional investment policy $i(w)$ and the cash sensitivity of investment $i'(w)$. Figure 13 plots $i(w)$ and $i'(w)$ in state G and B for the three different scenarios. It is interesting to note from Panel A, which plots $i(w)$ in state G, that although *gambling* in the

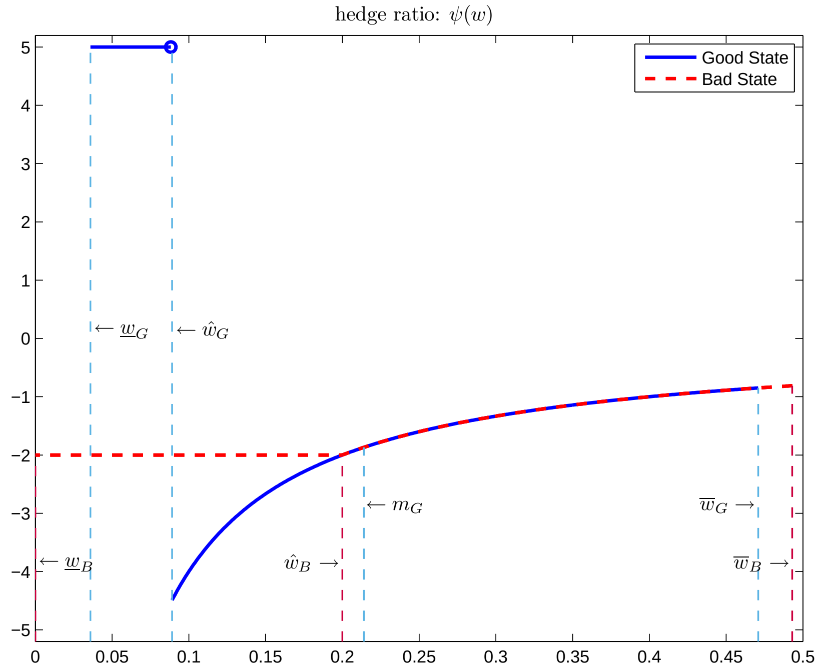

Figure 14: **Optimal hedge ratios in state G and B.**

region for $w \in [\underline{w}_G, \widehat{w}_G]$ tends to smooth out $p_G(w)$ this only has a marginal effect on investment close to the external financing boundary $\underline{w}_G$. In contrast, the effect of *hedging* on investment and payout for higher values of w is much more pronounced. Note also from Panel B that the effects of dynamic hedging on the cash sensitivity of investment is negligible. If anything the firm's gambling incentives close to the external financing boundary enhance the non-monotonicity of investment.

We close this section by illustrating the optimal hedging policy in states G and B. Figure 14 plots both solutions $\psi^*_G(w)$ and $\psi^*_B(w)$. It illustrates how the firm's dynamic hedging policy in the two states is radically different for w close to the lower boundary. While in state B the firm is desperate to reduce its exposure to systematic risk, and hedges that risk as much as its limited resources allow it to, in state G the firm engages in maximal gambling for $w \in [0.04, 0.09]$ and seeks to load up on aggregate risk as much as possible. For higher values of w, however, the hedging policies in the two states of the world are identical and the firm hedges aggregate risk less and less as it approaches the payout boundary, at which point it has no incentives to hedge systematic risk.

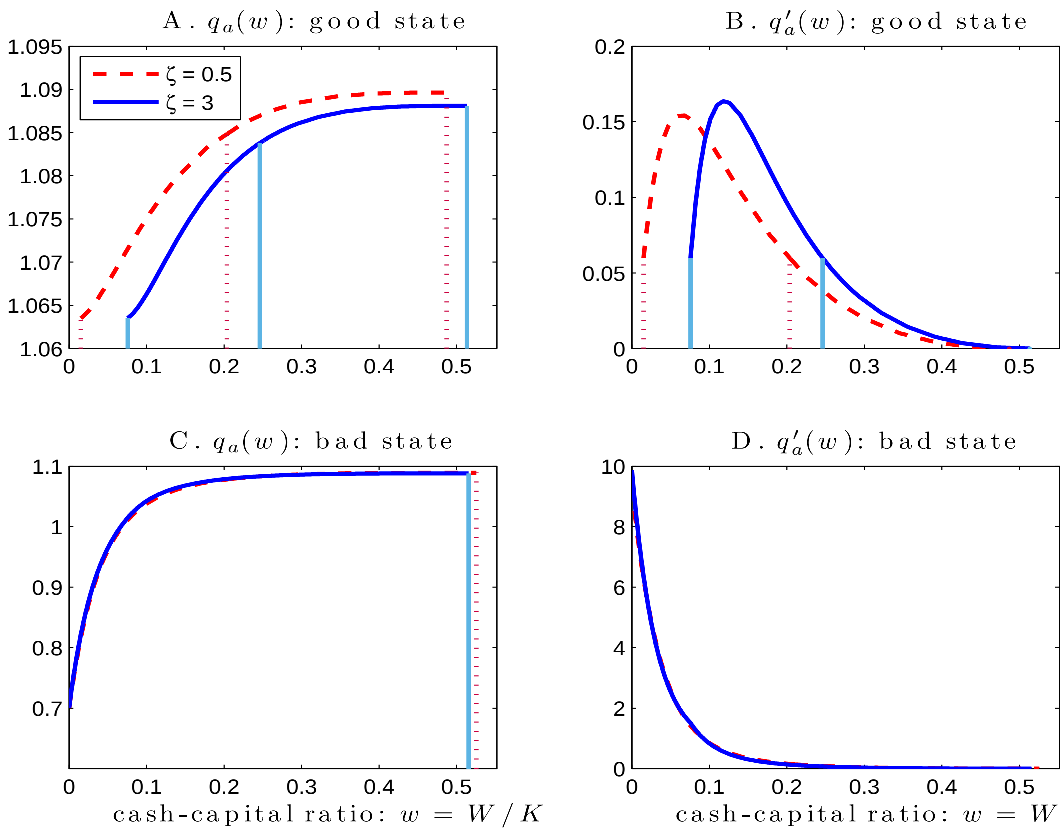

Figure 15: **Average q in Recurrent States.** This figure plots the solution in the case when neither state G nor B is absorbing. The risk-neutral transition intensities are assumed to be symmetric, $\zeta_G = \zeta_B$. The remaining parameters are the same as in Figure 4.

9 A recurrent two-state model

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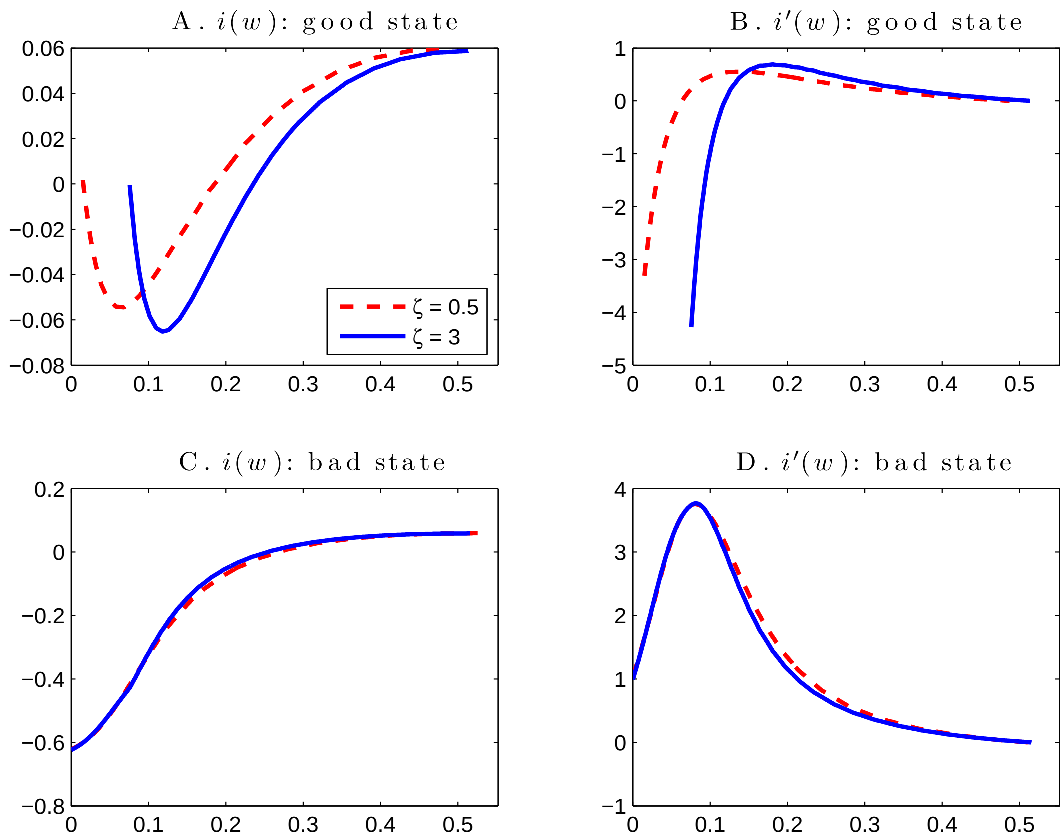

Figure 16: **Investment in Recurrent States.** This figure plots $i(w)$ and $i'(w)$ in the case when neither state G nor B is absorbing.

10 Conclusion

We provide a simple structural framework of dynamic market timing and risk management. Financing conditions and supply of capital change stochastically over time. Firms rationally respond to these stochastic financing opportunities. In particular, they optimally build war-chests by issuing equity and hoarding cash, when external financing is cheap. For firms anticipating an equity issuance, investment may be decreasing in the firm's cash-to-asset ratio: when firms get closer to equity issuance their investment policy is less constrained by the availability of internal funds, as the firm anticipates that more cash will be raised through an equity issue in the near future. We also show that market timing is consistent with risk-seeking behavior by the firm. The key driving mechanism for these surprising dynamic implications is the convexity of firm value in its cash holdings. One limitation of our analysis is that we take the stochastic financing opportunities that a firm faces to be exogenously given. It would clearly be desirable to consider a general equilibrium setting where the stochastic financing opportunities arise endogenously. We leave this for future research.

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