FTG Lecture on Dynamic Models of Sovereign Debt: Multiplicity and Maturity

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Framework: Basic environment

Main assumptions

- Incomplete markets but defaultable debt
  - Eaton-Gersovitz (Arellano, Aguiar-Gopinath)
- Small open economy (SOE) and international financial market
- A single, freely traded good
- Government of SOE cares about SOE's consumption
  - and controls it
- Stochastic evolution of exogenous state variables
- Infinite periods
- Markov equilibrium

EG Framework

- Discrete time for now (later switch to continuous time)
- Exogenous state vector in $s \in S$, with $\pi(s'|s)$ transition
- SOE receives an endowment, $y(s)$
- Government:
  - flow utility from expenditures, $u(c(s))$, strictly concave
  - discounts u flows with $\beta < 1$
  - default value: $V^D(s) \Leftarrow$ usually (semi) endogenous
- International financial market: risk neutral agents, $R^{-1} < 1$

EG Framework

Asset markets, default and budget constraints

- One period bonds: pay R next period (if no default)
- Amount issued: b
- Budget constraint (if no default):

$$c + Rb \leq y(s) + q \times b'$$

  where q is bond price
- If default: government gets $V^D(s)$, creditors get 0
- Pricing equation:

$$q = Prob(\text{no default tomorrow})$$

EG Framework

Equilibrium notion

- State variables (s, b)
- Value of repayment: $V(s, b)$
- Default is optimal: $V(s, b) < V^R(s, b)$
- Price *schedule*: $q(s, b, b')$

EG Framework

- Enter the period with b
- s realized
- Government decides to default or repay
- If repay: choose b' and

  auction bonds $\leftarrow$ not "allowed" to fail

Two equations

Pricing equation and government's optimality

$$q(s,b,b') = \sum_{s' \in S} \pi(s'|s)\mathbb{1}\{V(s,b') \geq V^D(s')\}$$

- Inherited debt b irrelevant: $q(s,b,b') \to q(s,b')$

$$V(s,b) = \max_{c,b'} \left\{ u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max\langle V(s',b'), V^D(s')\rangle \right\}$$

subject to:

$$c \leq y(s) - Rb + q(s,b')b'$$

Equilibrium: pair of functions V and q that solve the above.

EG: Properties of V

- Are there multiple equilibria?
- How do we find them?

With additional conditions (that rule out zero consumption):

- $V(s, b)$ is continuous in b and strictly decreasing in b
- Show a graph

Multiplicity

A strong (but incorrect!) intuition

- Suppose that $b' = b \longleftarrow$ exogenous "static" restriction
- $V^D(s)$ iid with F, $y(s) = y$
- From budget constraint: $c = y - Rb + qb'$
- Value conditional on repayment:

$$V = u(y - (R - q)b) + \beta \int \max\{V, v^D\} dF(v^D)$$

- But $q = F(V)$, so equilibrium is V such that

$$V = u(y - (R - F(V))b) + \beta \int \max\{V, v^D\} dF(v^D)$$

- LHS increasing in V, RHS increasing in V
- Easy to construct multiple eqm cases: multiplicity is pervasive!

EG: Uniqueness

- The above does not generalize to a dynamic environment

  Why? if facing "bad prices", reduce debt!

  Whole thing unravels

- There is a unique Markov equilibrium in EG (Auclert-Rognlie)
- Show different argument, based on duality
  (Aguiar-Amador-Hopenhayn-Werning, Aguiar-Amador)

EG: A fixed-point operator

- Define the operator $\hat{T}$:

$$[\hat{T}\ V](s,b) = \max_{c,b'} \left\{ u(c) + \beta \sum_{s'} \pi(s'|s) \max\left\{V(s,b), V^D(s)\right\} \right\}$$

subject to:

$$c \le y(s) + q'b' - Rb$$

$$q' = \begin{cases} \sum_{s'} \mathbb{1}\left\{V(b',s') \ge V^D(s')\right\} & b' > 0 \\ 1 & b' \le 0 \end{cases}$$

- Any equilibrium V is a fixed point of $\hat{T}$
- $\hat{T}$ is a monotone operator – but it is not a contraction
  - Monotonicity: iteration converges to an equilibrium

EG: A Dual representation

- let B: $B(s, V(s, b)) = Rb$

EG: A Dual representation

- let B: $B(s, V(s, b)) = Rb$
- B is fixed-point of dual operator T:

$$[TB](s, v) = \max_{c, \{v(s')\}, b'} \Big\{ y(s) - c + R^{-1} \min\langle 0, b' \rangle + R^{-1} \max\langle 0, b' \rangle \sum_{s' \in S} \pi(s'|s) \mathbb{1}\{v(s') \geq V^D(s')\} \Big\}$$

subject to:

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subject to:

$$v \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max\langle v(s'), V^D(s') \rangle$$

$$b' = B(s', v(s')) \text{ for } s' \in S \text{ such that } v(s') \geq V^D(s')$$

- Note this last constraint: incomplete markets

EG: A Dual representation

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$$
\begin{aligned}
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& + R^{-1}\max\langle 0, b'\rangle \sum_{s'\in S} \pi(s'|s)\mathbb{1}\{v(s') \geq V^D(s')\}\Big\} \\
& \text{subject to:} \\
& v \leq u(c) + \beta \sum_{s'\in S} \pi(s'|s) \max\langle v(s'), V^D(s')\rangle \\
& b' \leq B(s', v(s')) \text{ for } s' \in S \text{ such that } v(s') \geq V^D(s')
\end{aligned}
$$

- Note this last constraint: incomplete markets
- We can relax it with an inequality

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- Note this last constraint: incomplete markets
- We can relax it with an inequality
- Operator T is a contraction: Monotonicity and discounting

EG: Dual representation

- Contraction Mapping Theorem: existence and uniqueness
- Choosing $v(s)$ resembles complete markets
- Role of the constraint:

$$b' = B(s', v(s')) \text{ for all } s' \text{ such that } v(s') \geq V^D(s')$$

  Restricts freedom to allocate utility across states

- What about $q(s, b')$?

  q uniquely pinned down by V: only one eqm object

- Planning problem: Cannot find a better allocation that satisfies limited commitment and incomplete markets!

EG Planning Problem: Two frictions

$$B(s,v) = \max_{c, v(s'), b'} y(s) - c + R^{-1} b' \boxed{\sum_{s' \in S} \pi(s'|s) \mathbb{1}\{v(s') \geq V^D(s')\}}$$

subject to:

$$v \leq u(c) + \beta \sum_{s' \in S} \pi(s'|s) \max\langle v(s'), V^D(s') \rangle$$

$$\boxed{b' = B(s', v(s'))} \text{ for } s' \in S \text{ such that } v(s') \geq V^D(s')$$

- Incomplete markets: cannot insure fluctuations in $y(s)$
- Deadweight costs of default: increasing $v(s')$ has second order cost around $V^D(s')$ but first order gain $b'\pi(s'|s)$
- Both will provide an incentive to save (reduce debt)

Taking Stock and Next Steps

- One-period bond model is solution to planning problem
- Equilibrium unique and constrained efficient

- Efficiency has implications for maturity choice
- What about dynamics?
- What about long-term bonds

EG: Debt dynamics

- Focus on debt dynamics due to default risk
- $y(s) = y$ for all s
- Only risk left: $V^D(s)$, iid, cdf F
- State s drops from value function

EG: Debt dynamics

- Dual, ignoring $B < 0$

$$B(v) = \max_{c,v} \left\{ y - c + R^{-1} F(v') B(v') \right\}$$

subject to:

$$v \leq u(c) + \beta \int \max\{v', v^D\} dF(v^D)$$

- Suppose $\beta R = 1$. No debt dynamics without default risk.

- First order conditions (continuous F):

$$\underbrace{\frac{1}{u'(c')} = \frac{1}{u'(c)}}_{\text{standard Euler}} + \underbrace{\frac{f(v')}{F(v')} B(v')}_{\text{default wedge}}$$

- Backloading: $c' > c$ as long as $f(v') > 0$

- $v \to \overline{V}^D$: eventually no default risk

EG: Debt dynamics

- Rewriting the equation:

$$\underbrace{\frac{1}{u'(c')} = \frac{1}{u'(c)}}_{\text{standard Euler}} + \underbrace{\frac{f(v')}{F(v')}B(v')}_{\text{default wedge}}$$

- Intuition: save a bit, reduces default risk by bit.
- Second order loss in consumption smoothing $c' \approx c$
  First order gain in reduction of default deadweight losses
- What is this deadweight loss?
  - Default occurs at points where government is indifferent
  - But lenders are not – V^D has a *cost* in it
- Seems like a local intuition, but holds more generally

EG: Debt dynamics

Two shocks

- V^D takes on two values $V^D \in \{\underline{V}^D, \overline{V}^D\}$, $\underline{V}^D < \overline{V}^D$
- Probability of the high state, λ
- Split state space into three zones:
  - Safe zone: $v > \overline{V}^D$
  - Crisis zone: $v \in [\underline{V}^D, \overline{V}^D)$
  - Default zone: $v < \underline{V}^D$
- We can ignore default zone

EG: Debt dynamics

Two shocks

- Useful to take continuous time limit
- $\beta R = 1$, and ρ is instantaneous rate
- λ : Poisson probability of arrival of $\overline{V}^D$ value.

Safe Zone

- Safe Zone Bellman Equation

$$\rho B(v) = \max_{c} \left\{ y - c + B'(v) \underbrace{(\rho v - u(c))}_{\dot{v}} \right\}$$

- First-Order Condition:

$$-1 - B'(v)u'(c) = 0$$
$$\Rightarrow$$
$$-B'(v) = \frac{1}{u'(c)}$$

Safe Zone

Hamilton-Jacobi-Bellman Equation

- Safe Zone Bellman Equation

$$\rho B(v) = \max_{c} \left\{ y - c + B'(v) \left(\rho v - u(c) \right) \right\}$$

- First-Order Condition:

$$\begin{aligned} & -1 - B'(v)u'(c) = 0 \\ & \Rightarrow \\ & -B'(v) = \frac{1}{u'(c)} \end{aligned}$$

Safe Zone

Conjectured Solution

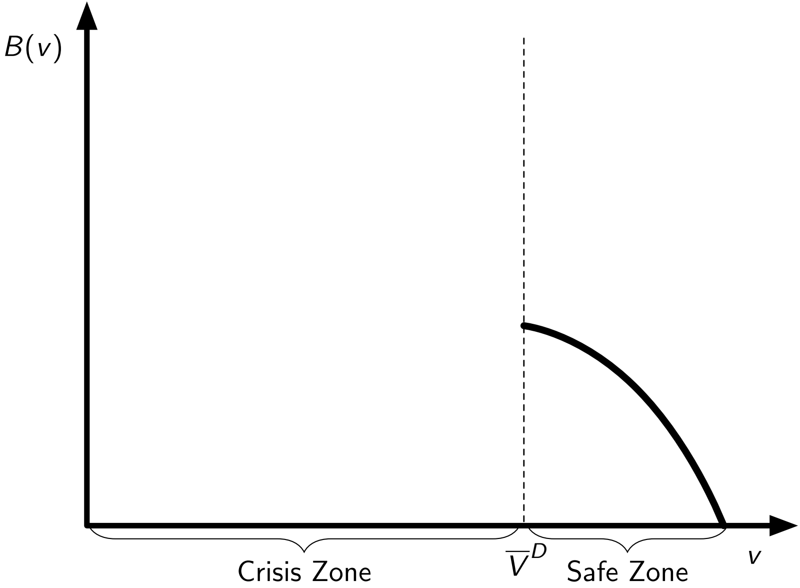
- In Safe Zone: no risk of default
- Lenders and government both discount at rate ρ
- Conjectured solution: $\dot{v} = 0$, $v = \frac{u(C(v))}{\rho}$

$$C(v) = u^{-1}(\rho v)$$

- Payoff to lenders:

$$B(v) = \frac{y - C(v)}{\rho}$$

- Note: $B'(v) = -\frac{C'(v)}{\rho} = \frac{-1}{u'(C(v))}$



$B(v)$
v
$\overline{V}^D$
Crisis Zone
Safe Zone

Crisis Zone

- Crisis Zone: Probability of default λ

$$(\rho + \lambda)B(v) = \max_{c} \left\{ y - c + B'(v)\dot{v} \right\}$$

with

$$\rho v = u(c) + \dot{v} + \lambda \left(\overline{V}^{D} - v \right)$$

Crisis Zone

- Crisis Zone: Probability of default λ

$$(\rho+\lambda)B(v)=\max_{c}\left\{y-c+B'(v)\dot{v}\right\}$$

with

$$\rho v=u(c)+\dot{v}+\lambda\left(\overline{V}^{D}-v\right)$$

or

$$(\rho+\lambda)v=u(c)+\dot{v}+\lambda\overline{V}^{D}$$

- Note: As if both discount at $(\rho+\lambda)$

Crisis Zone

Conjectured Solution

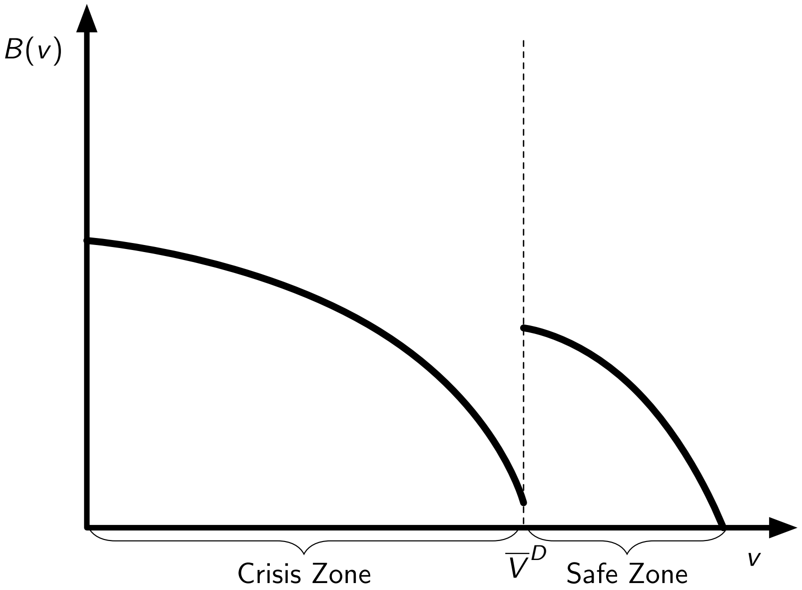
- Discount at actuarially fair prices
- Conjecture constant v and c:

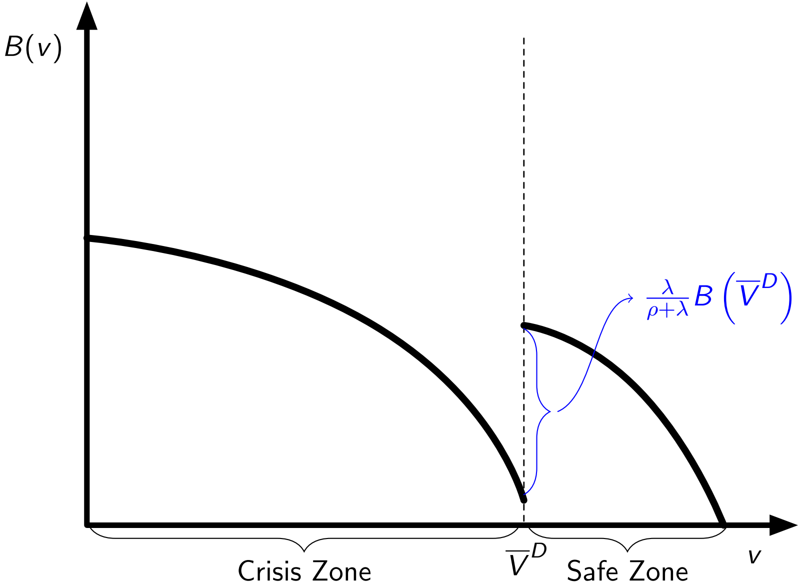
$$v = \frac{u\left(\tilde{C}(v)\right)}{\rho + \lambda} + \frac{\lambda}{\rho + \lambda}\overline{V}^D$$

- Payoff to lender:

$$B(v) = \frac{y - \tilde{C}(v)}{\rho + \lambda}$$

- Note: $B'(v) = \frac{-\tilde{C}'(v)}{\rho+\lambda} = \frac{-1}{u'\left(\tilde{C}(v)\right)}$

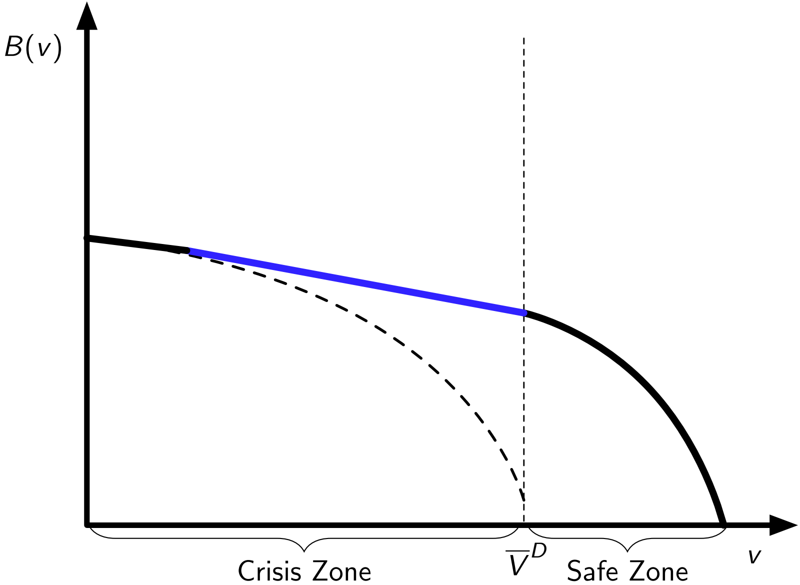

$B(v)$
$\overline{V}^D$
Crisis Zone
Safe Zone
v

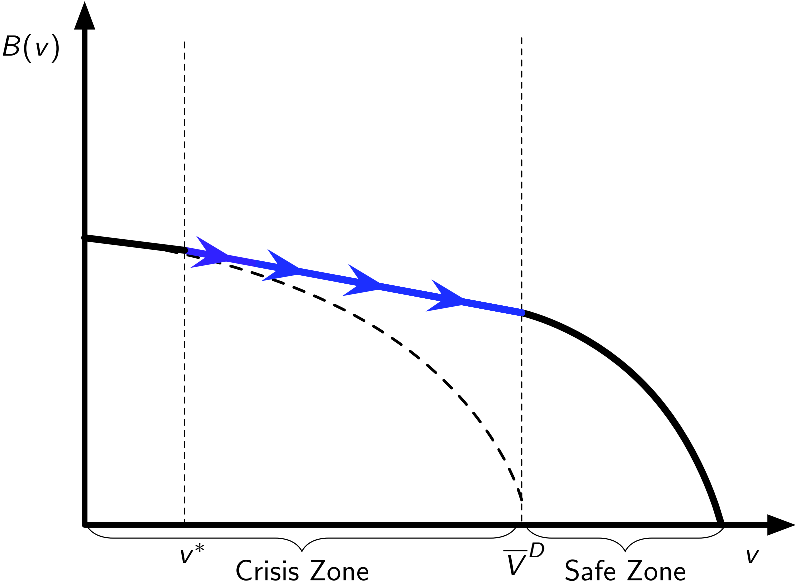

$B(v)$
$\frac{\lambda}{\rho+\lambda} B\left(\overline{V}^D\right)$
Crisis Zone
$\overline{V}^D$
Safe Zone
v

Inefficiency

Role for Backloading

- Discontinuity cannot be part of the solution
- Inefficiency:
  - To the left of $\overline{V}^D$, small decrease in c (increase in $\dot{v}$) generates discrete gain to lender with second order costs
  - Optimal to backload in neighborhood below $\overline{V}^D$


$B(v)$
Crisis Zone
$\overline{V}^D$
Safe Zone
v



$B(v)$
v^*
Crisis Zone
$\overline{V}^D$
Safe Zone
v

Two Shock Case

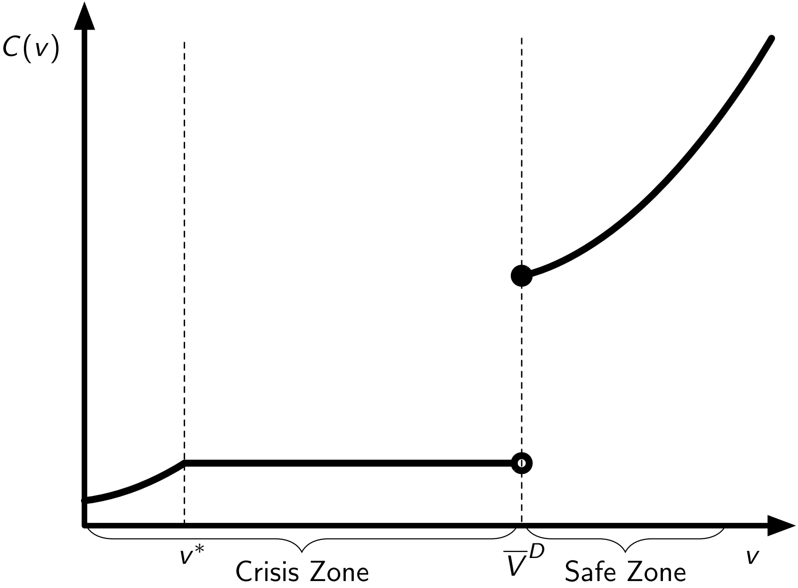
Role for Backloading

- In neighborhood of $v < \overline{V}^D$:
  - Set $c = \bar{c}$
  - $\bar{c}$ solves Bellman equation to the left of $\overline{V}^D$:

$$(\rho + \lambda)B(\overline{V}^D) = y - \bar{c} - \frac{1}{u'(\bar{c})}\left[(\rho + \lambda)v - u(\bar{c}) - \lambda\overline{V}^D\right]$$

- Threshold for saving:

$$v^* = \frac{u(\bar{c})}{\rho + \lambda} + \frac{\lambda}{\rho + \lambda}\overline{V}^D$$


$C(v)$
v^*
Crisis Zone
$\overline{V}^D$
Safe Zone
v

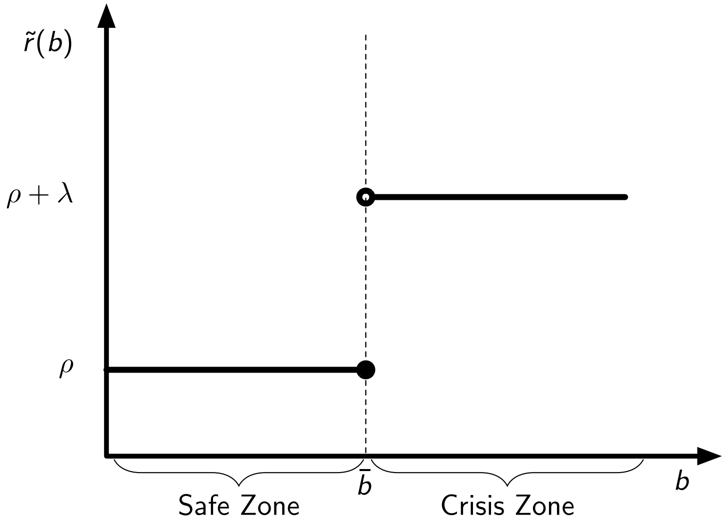
Dynamics

Two Shock Case

- Planner's solution:
  - If $v \geq \overline{V}^D$: keep consumption and v constant
  - If $v \in (v^*, \overline{V}^D)$ back load until $v = \overline{V}^D$
  - If $v \leq v^*$: keep consumption and v constant and default will eventually happen

- Key is that Planner delays consumption until reach Safe Zone
- Efficient from perspective of lender: Saves $\lambda B(\overline{V}^D)/(\rho+\lambda)$
- How is this decentralized in a competitive equilibrium?
  - Remember that default occurs when payoff V^D is high
  - In Crisis Zone, $\overline{V}^D$ is greater than value of repayment
  - Why not just rollover bonds until high payoff and then default?

Decentralization



Dynamics

Decentralization

- In Crisis Zone:
  - To left of $\bar{b} \equiv B(\overline{V}^D)$, pays $\rho + \lambda$ to roll over bonds
  - By saving to $\bar{b}$, pays only ρ
  - Saves $\frac{\lambda}{\rho+\lambda}\bar{b} = \frac{\lambda}{\rho+\lambda}B(\overline{V}^D)$
  - Completely internalizes efficiency cost via prices
- Important that government rolls over entire stock of debt each period
  - Otherwise, only internalizes fraction that is rolled over

Exogenous Default

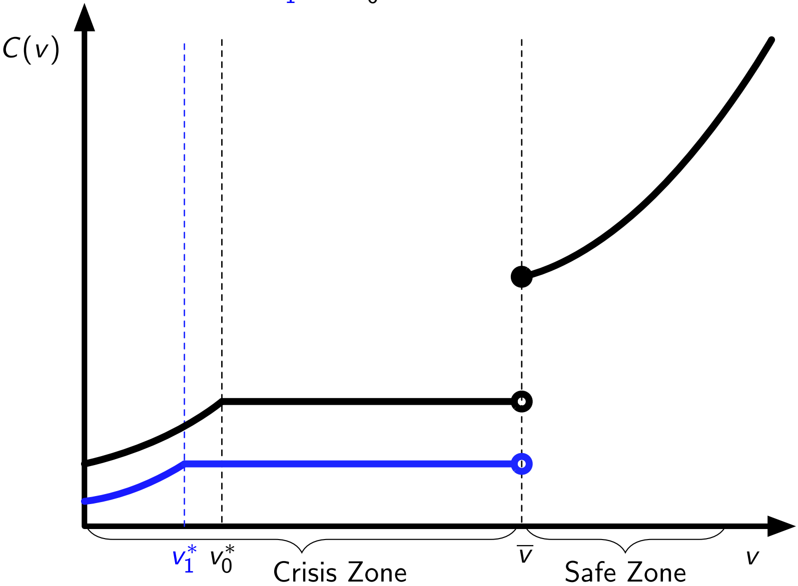
Role of V^D

- Consider arbitrary Crisis Zone: $v < \bar{v}$
- Consider arbitrary default payoff $V^D \leq \bar{v}$
- Safe Zone remains the same
- Consumption level in Crisis Zone solves:

$$(\rho + \lambda)B(\overline{v}) = y - \bar{c} - \frac{1}{u'(\bar{c})}\left[\rho v - u(\bar{c}) + \lambda(\bar{v} - V^D)\right]$$

- Note: $\frac{d\bar{c}}{dV^D} = \frac{u''(\bar{c})}{u'(\bar{c})^2}\dot{v} > 0$ as $\dot{v} < 0$.
- A decrease in V^D implies faster convergence in "saving" region

$V_1^D < V_0^D$



A Cole-Kehoe Interpretation

- Cole-Kehoe's run model:
  - Government defaults even when $V^D < v$
- Intuition: Failed auction forces a default
- Dynamics are the same – but faster

Taking stock

- General message in EG: reduce debt
- Decentralization through prices
  - Get out of regions where interest rates are high
- Quantitative models – challenge
- One way out: $\beta R < 1$ (political economy)
- (Show graph)
- Another way out: long-term bonds

Longer Maturity

Next Steps

- One-period bond model:
  - Constrained efficient
  - Equilibrium is unique

Longer Maturity
Next Steps

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  - Observed in practice
  - Improve quantitative fit of EG model

Longer Maturity

Next Steps

- One-period bond model:
  - Constrained efficient
  - Equilibrium is unique
- Longer maturities:
  - Observed in practice
  - Improve quantitative fit of EG model
- How does longer maturity change lessons from one-period bond environment?

Longer Maturity

Environment

- Continue with simplified environment
  - No output shocks: $y(s) = y$
  - Two default states: $V^D \in \{\underline{V}^D, \overline{V}^D\}$
  - *iid* transition: $\Pr(V^D = \overline{V}^D) = \lambda$
  - Safe Zone and Crisis Zone
  - Continuous time limit

Longer Maturity

Environment

- Random maturity (perpetual youth) bonds
  - Probability of maturity δ
  - *iid* across bonds and time
  - $\delta \to \infty$: Short-term debt
  - $\delta \to 0$: Perpetuitities
- Normalize coupon to r
- Assume $\rho > r$: Incentive to borrow

Longer Maturity

Environment

- Solve for equilibrium using "primal" approach:
  - Equilibrium is no longer solution to planning problem
- Let b denote face value of bonds
- Let $q(b)$ denote price per bond given face value b
- Let $V(b)$ denote value of repayment given b

Longer Maturity

Government's Problem

- Faced with price schedule q:

$$\rho V(b) = \max_{c} \left\{ u(c) + V'(b)\dot{b} + \lambda \left(\max\langle V(b), \overline{V}^D \rangle - V(b) \right) \right\}$$

- Subject to:

$$c = y - (r + \delta)b + q(b)\left(\dot{b} + \delta b\right)$$

Longer Maturity

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- Subject to:

$$c = y - (r+\delta)b + q(b)\left(\dot{b} + \delta b\right)$$

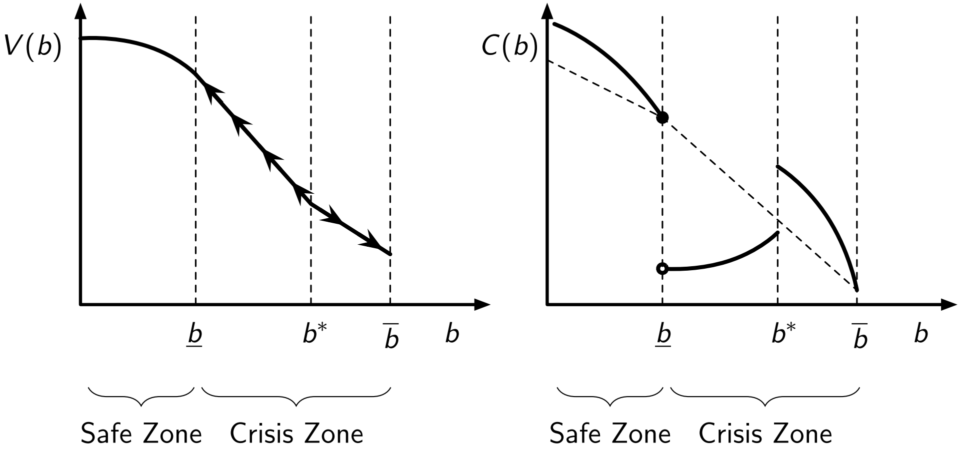
- Lenders' Break-Even Condition:

$$rq(b) = r + (1-q(b))\delta + q'(b)\dot{b} - \lambda q(b)\mathbb{1}_{\{V(b)<\overline{V}^D\}}$$

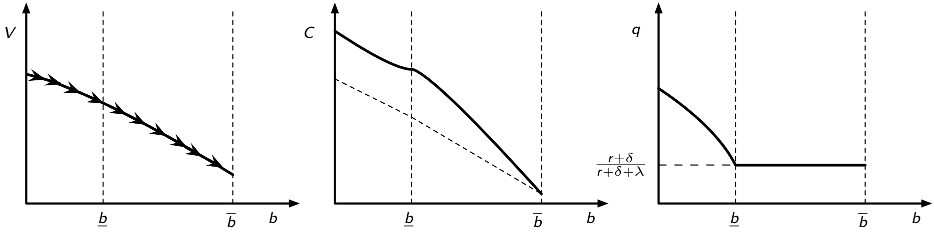
Constructing Equilibria

1. $\delta \to \infty$ (Uniqueness)
2. $\delta = 0$ (Uniqueness)
3. Intermediate case: $\delta \in (0, \infty)$ (Multiplicity)

Short-term Bonds: $\delta = \infty$



Perpetuities: $\delta = 0$



Short vs. Long

- Short-term bonds are "efficient" as government faces correct incentives to reduce default risk
  - At boundary of $\underline{b}$, government recognizes a small reduction in c lowers rollover costs
  - Prices correctly align incentives
  - Like a variable cost

Short vs. Long

- Short-term bonds are "efficient" as government faces correct incentives to reduce default risk
  - At boundary of $\underline{b}$, government recognizes a small reduction in c lowers rollover costs
  - Prices correctly align incentives
  - Like a variable cost
- Perpetuities provide no incentives to economize on default costs
  - When issued, price reflects future default probabilities
  - Never rolled over, so no incentive to reduce debt once issued
  - Like a sunk cost

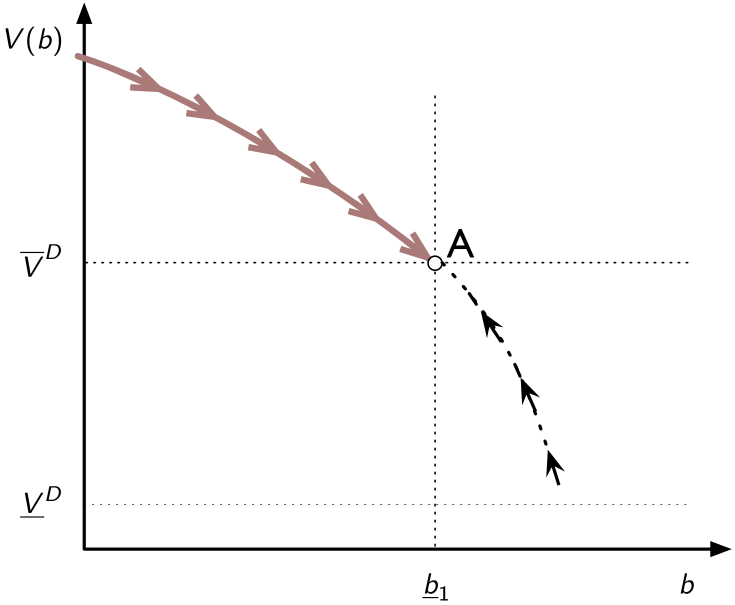
Intermediate Maturity

- Short-maturity type of equilibrium:
  - Need to roll over bonds in the future makes reducing debt worthwhile
  - $\underline{b}$ a stationary point

Intermediate Maturity

- Short-maturity type of equilibrium:
  - Need to roll over bonds in the future makes reducing debt worthwhile
  - $\underline{b}$ a stationary point
- "Perpetuity" type of equlibrium:
  - Borrow to the limit
  - $\overline{b}$ a stationary point

Constructing Equilibria with $\delta \in (0, \infty)$



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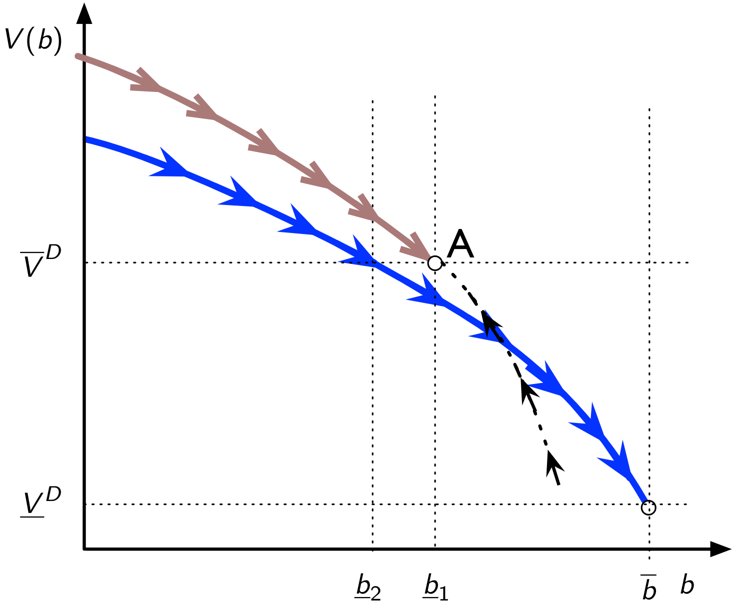

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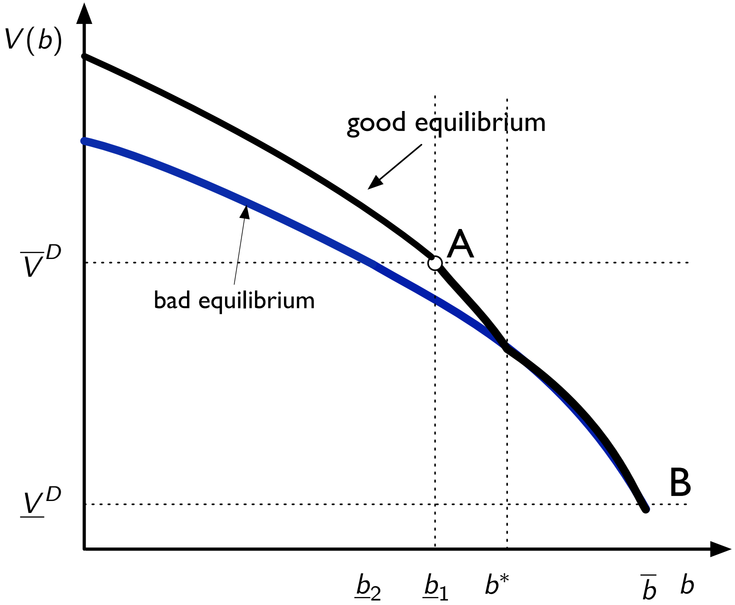

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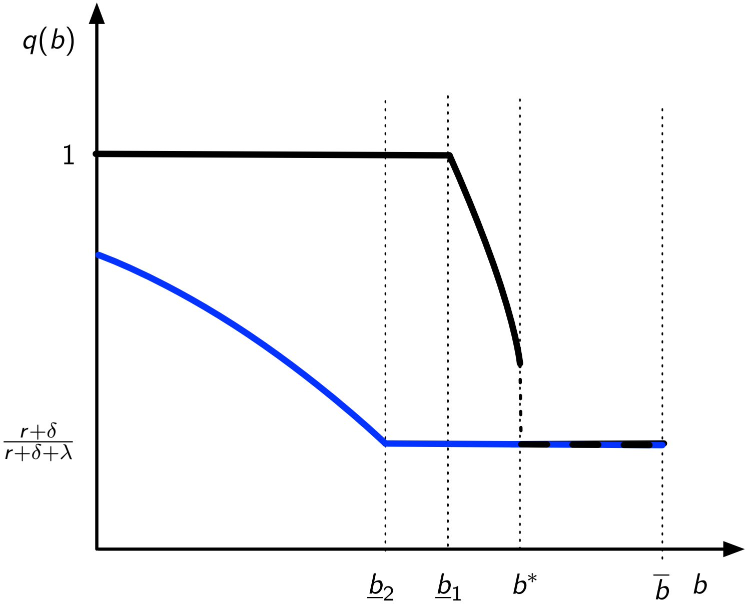
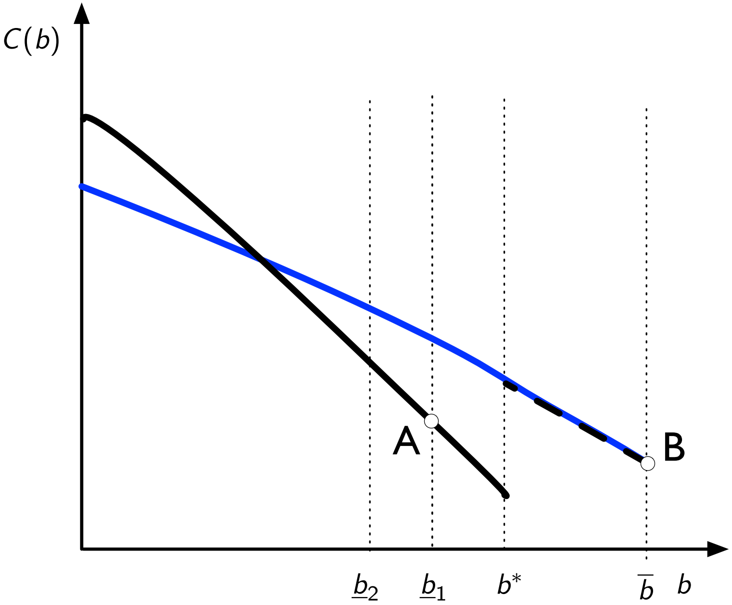


Constructing Equilibria with $\delta \in (0, \infty)$



Incentives behind Multiplicity

- Multiplicity due to creditor beliefs about future fiscal policy
  - Prices reflect creditor beliefs
  - Value functions reflect shape of price schedule
- Role of maturity:
  - With one-period debt, future fiscal policy irrelevant
  - With perpetuities, cannot support an interior stationary point (no need to roll over debt at stationary points)
- With endowment shocks same forces at work, but greater incentive to save due to precaution

Policy Implications

- How can an outside institution rule out bad equilibrium?
- Traditional policy: Price floor
  - Kills feedback from budget sets (Calvo)
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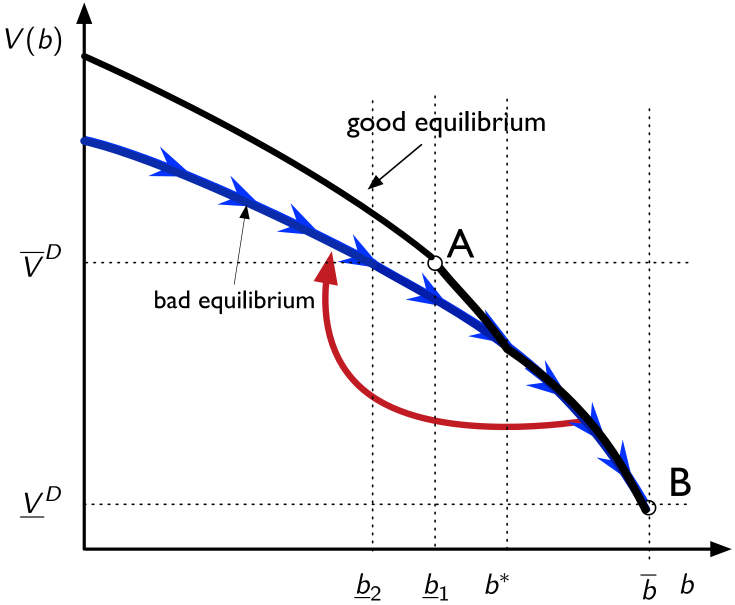
- In our version of EG model, price floor *selects* bad equilibrium
  - Kills incentive to save
  - "Flattens" price schedule
  - Sovereign borrows to limit
  - Requires third-party resources on equilibrium path

Other Policies

- Debt Forgiveness:
  - As long as sovereign relatively impatient, will resume borrowing
  - Does not rule out eventual default

Debt Forgiveness



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  - How to enforce non-market limits on debt?
- Costs to delay:
  - If b too high, unique equilibrium
  - Point emphasized by Lorenzoni-Werning in their framework

Some Remaining Questions

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- How to interpret episodes like Draghi's speech?
  - Were debt limits crucial to its success?

"Drying up" of Long Term Bond Markets

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- Yield curve flattens or inverts
  - Broner et al. argue risk premium of long-term bonds increases

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- Question: If issuing long-term debt is a bad idea, why not buy back?
- Builds on recent work with Hopenhayn and Werning

Extending the Framework

- Maintain from one-period bond analysis:
  - No output risk (kills spanning motive)
  - Outside option risk $v^D \sim F(v^D)$ and *iid*
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- Note: $b + \boldsymbol{l}_0$ is amount of debt due today

Competitive Equilibrium

Equilibrium Objects

- State variables at start of period $(b, \boldsymbol{I})$
- v^D realized
- If government does not default:
  - Value $V(b, \boldsymbol{I})$ if repays
  - Chooses $(b', \boldsymbol{I}')$
  - Faces prices $q(b', \boldsymbol{I}')$ and $Q(b', \boldsymbol{I}', \boldsymbol{I})$

Competitive Equilibrium

Equilibrium Objects

- One-period bond "break-even condition"

$$q(b', \boldsymbol{I}') = R^{-1} \Pr \left[v^{D\prime} < V(b', \boldsymbol{I}') \right]$$
$$= R^{-1} F \left(V(b', \boldsymbol{I}') \right)$$

Competitive Equilibrium

Equilibrium Objects

- Moving from $\boldsymbol{I}^t = \boldsymbol{I}$ to $\boldsymbol{I}^{t+1} = \boldsymbol{I}'$ raises net revenue:

$$Q(b', \boldsymbol{I}', \boldsymbol{I}) = \sum_{k=1}^{\infty} \rho_k(b', \boldsymbol{I}')(\boldsymbol{I}'_{k-1} - \boldsymbol{I}_k)$$

- $\boldsymbol{I}_k$ is k^{th} element of $\boldsymbol{I}$
- $\rho_k(b', \boldsymbol{I}')$ is the price of a unit promised in k periods (A "k-period zero coupon bond")

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- Revenue generated from *net* issuance of $t + k$ promises: $\rho_k \left(\boldsymbol{I}'_{k-1} - \boldsymbol{I}_k\right)$

Competitive Equilibrium

Equilibrium Objects

- Break-even condition for long-term bonds:

$$\rho_k(b', \boldsymbol{I}') = R^{-k}\Pi_{i=1}^{k} F(V_{t+i})$$

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- Expectations hypothesis holds:

$$\begin{aligned}\rho_k &= \rho_{k-1}q_k \\ &= \Pi_{i=1}^{k}q_i \\ &= R^{-k}\Pi_{i=1}^{k}F\left(V_{t+k}\right)\end{aligned}$$

where q_i is one-period price in $t+i-1$ for delivery in $t+i$

Competitive Equilibrium

Government's Problem

$$V(b, \boldsymbol{I}) = \sup_{c, b', \boldsymbol{I}'} \left\{ u(c) + \beta \int \max \left\langle V(b', \boldsymbol{I}'), v^D \right\rangle dF_t(v^D) \right\}$$

subject to:

$$c \leq y - b - \boldsymbol{I}_0 + q(b', \boldsymbol{I}')b' + Q(b', \boldsymbol{I}', \boldsymbol{I})$$

Competitive Equilibrium

Welfare Theorem

- Presence of long-term bonds makes the CE inefficient in the usual sense
- Nevertheless, CE solves a modified planning problem:
  - A government enters period t with legacy liabilities $(b, \boldsymbol{l})$
  - Contracts with a new set of lenders:
    - Receives $\{c_{t+k}\}_{k \geq 0}$
    - Pays $y_{t+k} - \boldsymbol{l}_k - c_{t+k}$ conditional on no default through $t+k$
    - Contract maximizes joint surplus between government and "new lenders"

Planning Problem

$$B^{\star}(v, \boldsymbol{I}) = \sup_{\{c_{t+k}, V_{t+k}\}} \sum_{k=0}^{\infty} \left(\prod_{i=1}^{k} R^{-1} F(V_{t+i}) \right) (y_{t+k} - \boldsymbol{I}_k - c_{t+k})$$

subject to:

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- Objective is payments net of consumption and payments to long-term bonds
- Respects legacy promises
- Does not maximize market value of long-term bonds

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$$b = B(v, \boldsymbol{I}) \equiv \sup_{c, b', \boldsymbol{I}', v'} \left\{ y_t - \boldsymbol{I}_0 - c + q(b', \boldsymbol{I}')b' + Q(b', \boldsymbol{I}', \boldsymbol{I}) \right\}$$

subject to:

$$v = u(c) + \beta F(v')v' + \beta \int_{v^D \geq v'} v^D dF(v^D)$$

$$v' = V(b', \boldsymbol{I}')$$

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- Need to show $B = B^\star$

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- Complication is the presence of long-term bond prices in Q
- Prices depend on path of $\{V_{t+i}\}$ not just next period's value
  - In CE: can commit to $(b', \boldsymbol{I}')$ today, but not future debt choices
  - Planning problem chooses entire sequence at start of contract
- The lack of commitment to fiscal trajectories featured in our previous discussion of long-term bonds

Toward's a welfare theorem...

- $B(v, \boldsymbol{I}) \leq B^{\star}$ is easy:
  - Given allocation $\{c_{t+i}, V_{t+i}\}$ define:

$$p_k \equiv R^{-k}\Pi_{i=1}^{k} F_{t+i}(V_{t+i})$$

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  - Now consider a CE allocation: $\{c_{t+i}, V_{t+i}\}$
  - Government's budget constraint:

$$\begin{aligned} B(v, \boldsymbol{I}) = b &\leq \sum_{k=0}^{\infty} \rho_k \left(y_{t+k} - \boldsymbol{I}_k - c_{t+k}\right) \\ &\leq \sup_{\{c_{t+k}, V_{t+k}\}} \sum_{k=0}^{\infty} p_k \left(y_{t+k} - \boldsymbol{I}_k - c_{t+k}\right) \\ &= B^{\star}(b, \boldsymbol{I}) \end{aligned}$$

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- Planning problem unconstrained by prices
- Always feasible to choose ocmpetitive allocation

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- To go the other way...$B(v, \boldsymbol{I}) \geq B^{\star}$

Toward's a welfare theorem...

Other way: $B(v, \boldsymbol{I}) \geq B^{\star}$:

- Always feasible for government to not trade long-term bonds: $Q(b', \boldsymbol{I}, \boldsymbol{I}) = 0$

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- Substitute in for $q(b', \boldsymbol{I}')b'$:

$$B(v, \boldsymbol{I}) \geq \sup_{c, b', v'} \left\{ y_t - \boldsymbol{I}_0 - c + F(v')B(v', \boldsymbol{I}_{\geq 1}) \right\}$$

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Other way: $B(v, \boldsymbol{I}) \geq B^{\star}$:

- Let $\{c_{t+k}, V_{t+k}\}$ be a choice for the planning problem
- Feasible in CE to choose $c = c_t$ and $v' = V_{t+1}$

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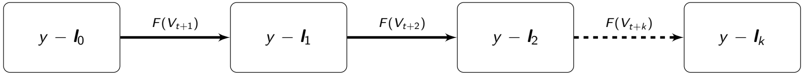
- Last term $\to 0$
- As $\{c_{t+k}, V_{t+k}\}$ was arbitrary, we have $B(v, \boldsymbol{I}) \geq B^{\star}(v, \boldsymbol{I})$

Modified Welfare Theorem

- CE implements a modified planning problem
- Trading only one-period bonds is enough to do this
- One-period bonds “solve” the time consistency problem regarding fiscal trajectories
- Legacy debt is a drag on efficiency, but must be respected absent default

Distortion due to Long-term Bonds



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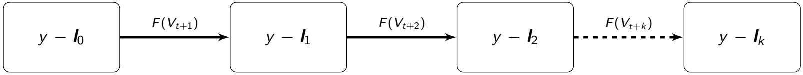

- Lower c_t and higher V_{t+k}:
  - Raises likelihood of "reaching" $t + k$ (and preceding path)
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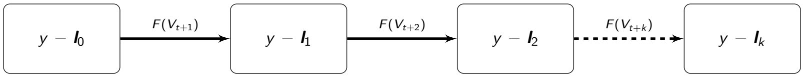

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  - Raises likelihood of "reaching" $t + k$ (and preceding path)
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- $\boldsymbol{I}_k \uparrow$:
  - Less surplus to split in $t + k$
  - Less incentive to reach that period
  - Default more likely

Convexity

- Let $\{c_{t+k}, V_{t+k}\}$ be the optimal allocation given $(v, \boldsymbol{I})$

$$B^{\star}(v, \boldsymbol{I}) = \sum_{k=0}^{\infty} p_k \left(y_{t+k} - \boldsymbol{I}_k - c_{t+k}\right)$$

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- Envelope condition wrt $\boldsymbol{I}$:

$$\nabla B^{\star} = \{-p_k\}_{k=0}^{\infty}$$

Convexity

- Compare $\boldsymbol{I}$ and $\boldsymbol{I}'$ given v

Convexity

- Compare $\boldsymbol{I}$ and $\boldsymbol{I}'$ given v
- Can always deliver v with same sequence $\{c_{t+k}, V_{t+k}\}$

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\begin{aligned}
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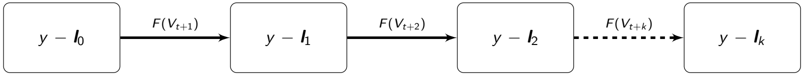
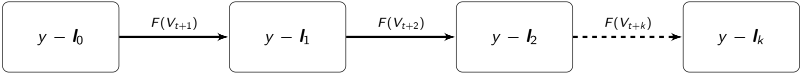
- $B^{\star}(v, \boldsymbol{I})$ is weakly convex in $\boldsymbol{I}$
- Strictly convex if default probability interior

Incentives and Prices

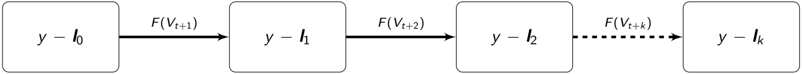


Incentives and Prices



- $\boldsymbol{I}_k \uparrow$:

Incentives and Prices



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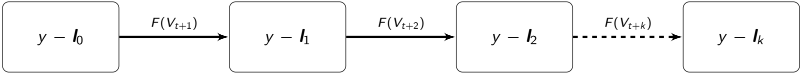
Incentives and Prices



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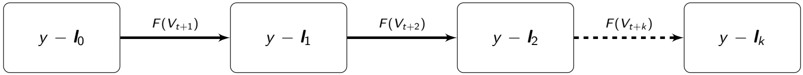

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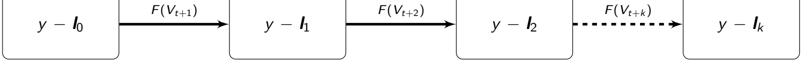
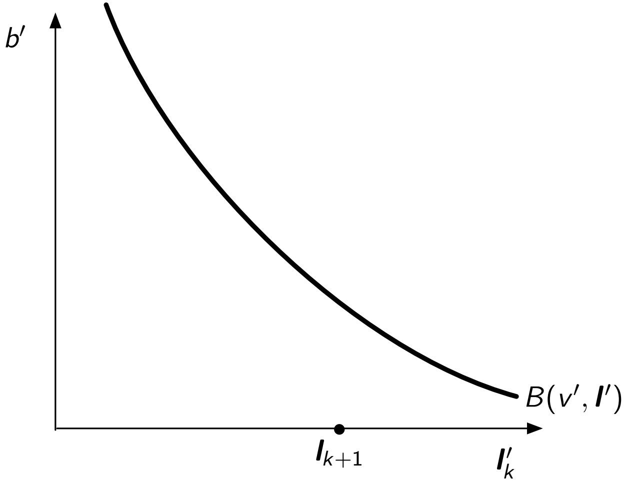
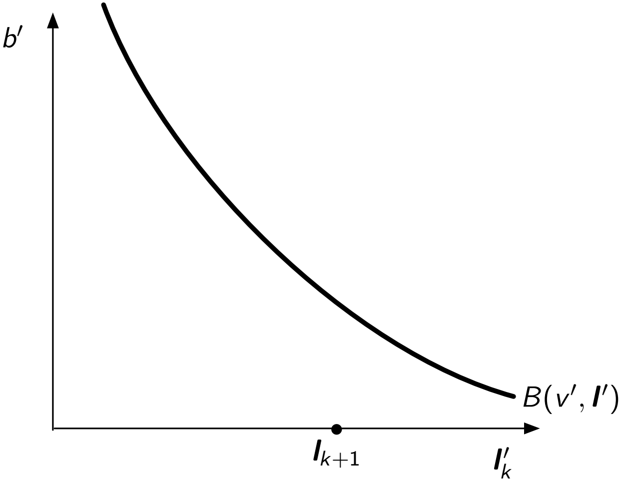


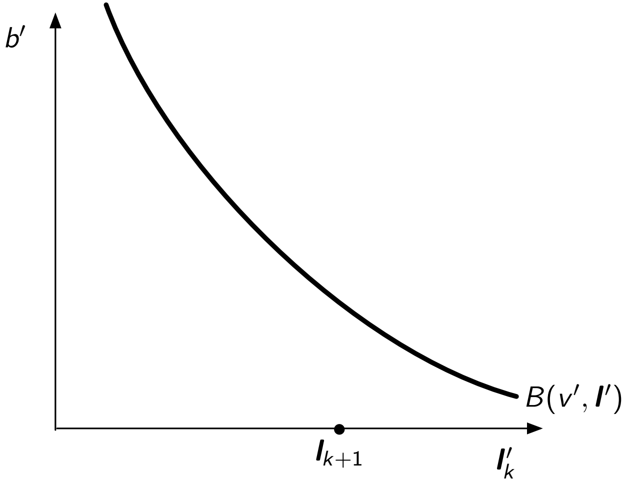
- $\boldsymbol{l}_k \uparrow$:
  - Less incentive to reach $t + k$
  - Lower V_{t+k} and move consumption ahead
  - Lowers $F(V_{t+k}) \Rightarrow p_k \downarrow$
- Issuing long-term debt lowers price
- Repurchasing long-term debt raises price



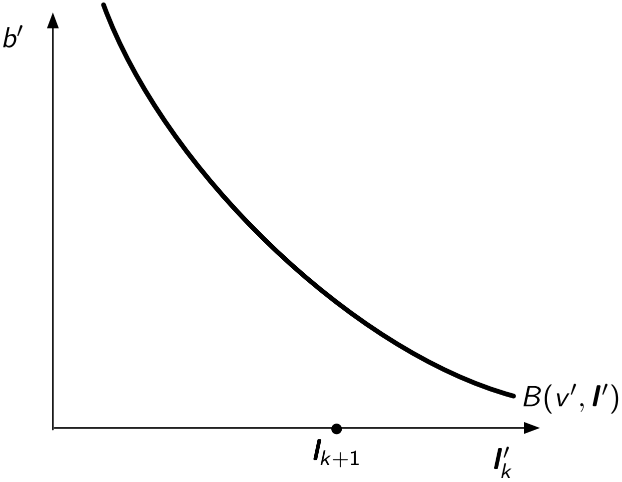
Consider starting out with $(b, \boldsymbol{I})$



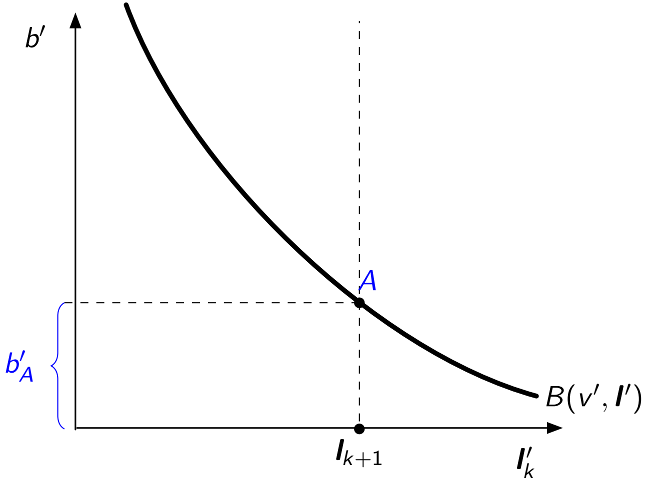
Choose to go to v' next period



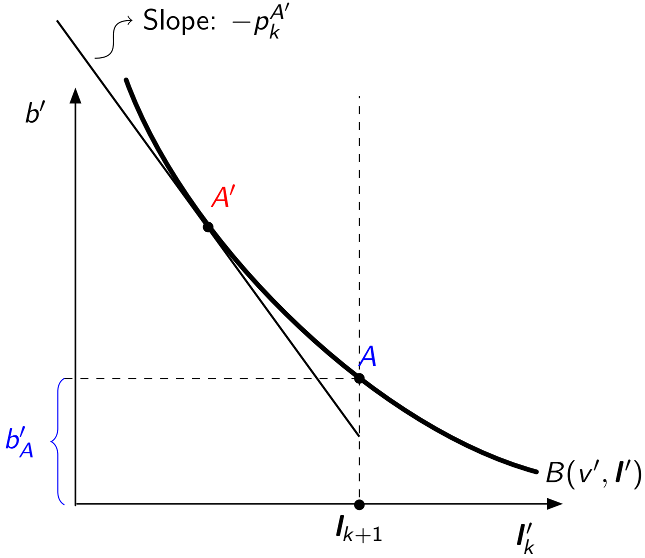
Which path $(b', \boldsymbol{I}'_k)$ maximizes today's consumption?



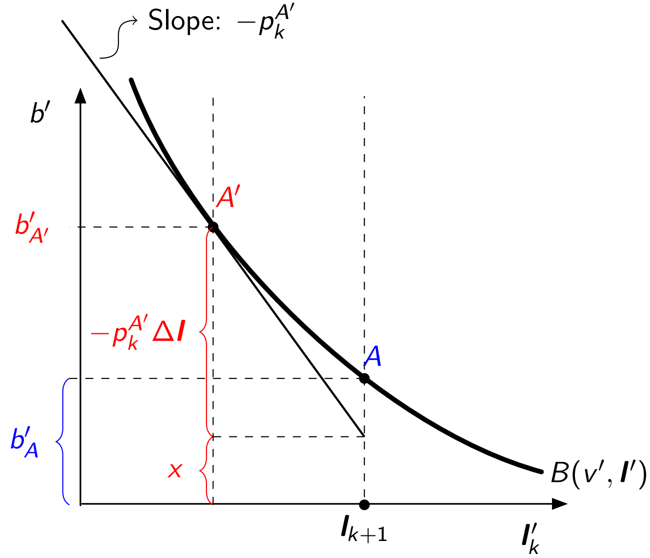
$$c = y - b + R^{-1}F(v')\,[b' + p_{k'}(\boldsymbol{I}'_k - \boldsymbol{I}_{k+1})]$$



No LT Debt Trade: $c = y - b + R^{-1}F(v')\,[b'_A]$



Alternative: $c = y - b + R^{-1}F(v')\left[\underbrace{b'_{A'} + p_k^{A'}(\boldsymbol{I}'_{A'} - \boldsymbol{I}_{k+1})}_{x}\right]$



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- Reminiscent but different mechanism than "Buy Back Boondoggle"

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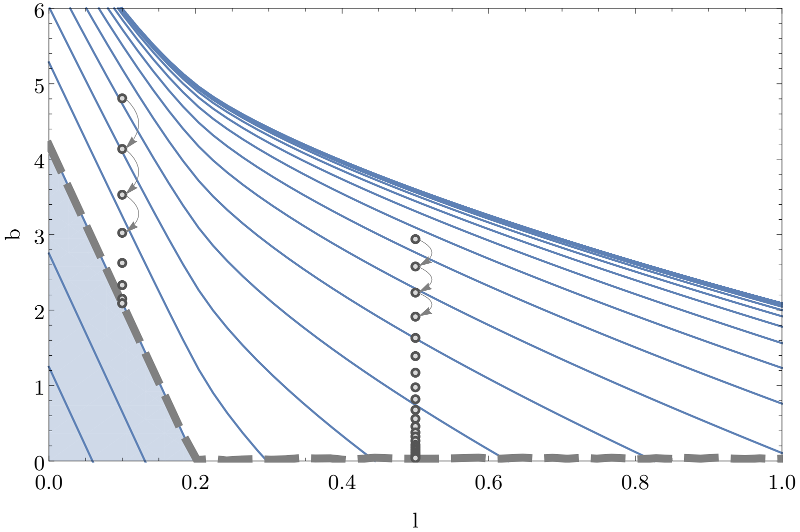
- If $\beta R = 1$, save to safety *or until* $b = 0$
- If $\beta R < 1$, may save or borrow depending on impatience
- Consider case of one-period bond and perpetuity:
  $\boldsymbol{l} = (l, l, l, l, \dots)$

Dynamics

$\beta R = 1$



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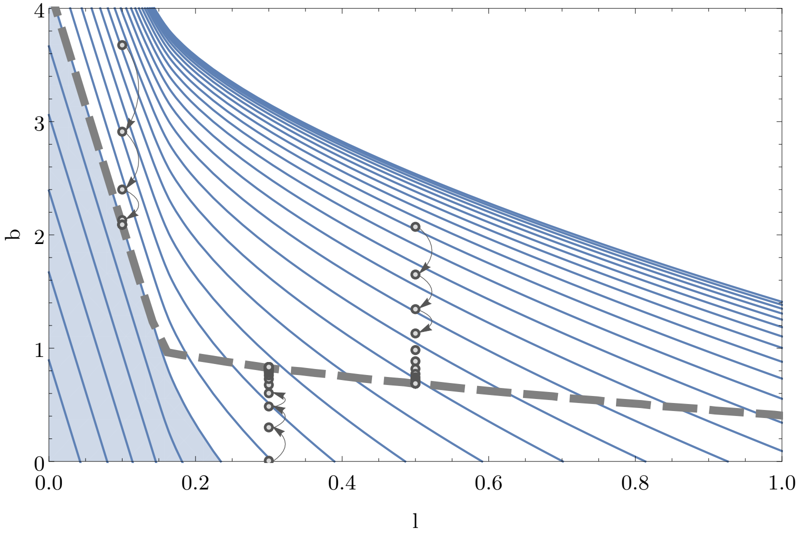

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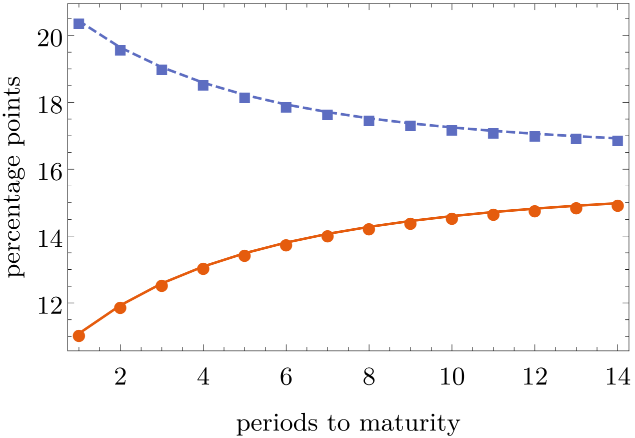
- Yield curve says nothing about *marginal* price

Dynamics

Yield Curve



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- Incentive to hold out
- Restructuring cannot be done via competitive trades

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- In model, market prices provide correct incentives
- No worries about enforcement

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- Default occurred when value of default exceeds value of repayment
- What about a coordination failure that induces inefficient default

Rollover Risk

No-Default Region

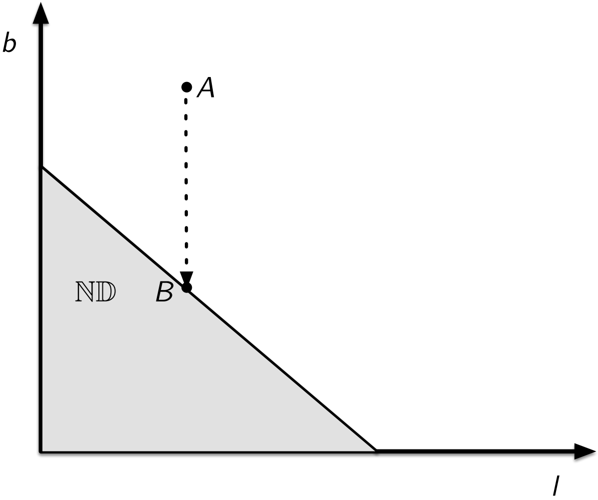
- When $\beta R = 1$, government saves to region of no default
- In benchmark model:

$$\frac{u(y - rb - l)}{1 - \beta} \geq \overline{V}^D$$

- Fundamental No-Default Region $\mathbb{ND}$: $rb + l \leq \overline{b}$

Fundamental Risk

Dynamics



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- Restrict to two-state case: $v^D \in \{\underline{V}^D, \overline{V}^D\}$
- Consider an equilibrium in which coordination failure is perfectly correlated with realization of $\overline{V}^D$

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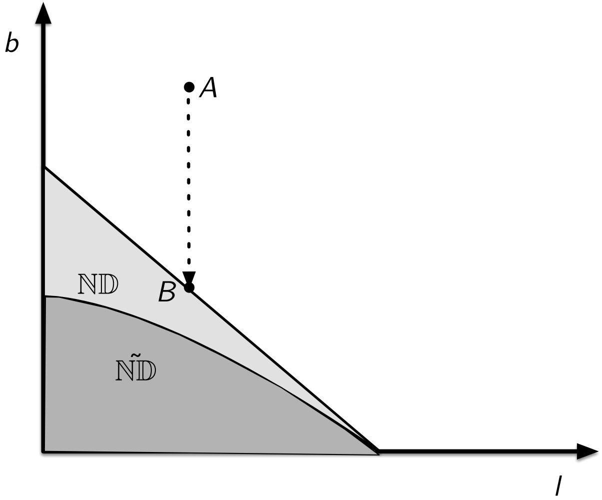
- Strictly inside if $b>0$

Fundamental Risk plus Rollover Risk

Dynamics



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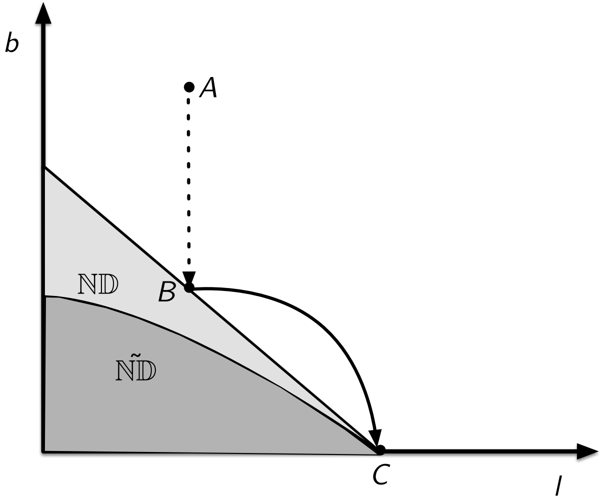

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- In our environment, clear sequencing:
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- More generally, solve a "stopping time" problem:
  - Reduce debt to mitigate fundamental risk
  - At some point, lengthen maturity to address rollover risk
  - Then continue to delever at slower pace to remove remaining fundamental risk

Hedging

- So far, downplayed insurance as a motive for longer maturities
- Well known that multiple maturities can help span business cycle risk
  - Not clear how important in practice
  - Quantitative models with commitment imply extremely large positions

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- Consider the following scenario:
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- Issuing perpetuities insures against the risk of a "bad" draw
  - Do not need to roll over debt
  - Consumption: $c = y - l$ until default regardless of F
- However, lenders internalize the bad incentives of long-term debt
  - Optimum maturity choice generally will include one-period debt
  - Provides commitment that if F is bad, government will mitigate default risk by saving

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- Complete markets
  - Back load consumption (i.e. reduce debt) to mitigate (off equilibrium) risk of repudiation

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- Long maturity
  - Generates an efficiency wedge between lenders and the government
  - Opens door to multiple equilibria
  - Issue is fiscal trajectory, not prices per se (as in a rollover crisis)

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- Additional questions:
  - Spillover from debt to other economic outcomes (output)?
  - Why don't governments save?

Debt Overhang

- Straightforward to incorporate a link between debt and output:

$$y \to f(k)$$
$$F(v^D) \to F(v^D|k)$$

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- Non-default requires: $W(b) \geq v^D(k)$

Planning Problem

$$B(v, w) = \sum_{t=0}^{\infty} R^{-t} \left(f(k_t) - (r + \delta)k - c_t \right)$$

subject to:

$$v \leq \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$w \leq \theta u(c_0) + \sum_{t=1}^{\infty} \beta^t u(c_t)$$

$$v^D(k_t) \leq \theta u(c_t) + \sum_{k=1}^{\infty} \beta^k u(c_{t+k}) \text{ for all } t$$

Planning Problem

Dynamics

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- First-order conditions:

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- As before, $\lambda_t \to 0$
- Implies $k_t \to k^*$ despite political economy frictions

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- Along path, debt falls and capital increases
- Consistent with data from developing economies

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- Explored models of sovereign debt and default
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Approach

- Explored models of sovereign debt and default
  - Mirrored complete-markets approach
  - Solved planning problems when appropriate
- Emphasized costs of debt
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  - Motivation for debt reduction

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- Debt overhang links debt dynamics with economic outcomes

Thank You